Data Mining for Car Insurance Claims Prediction

By

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1.0 Introduction

A key challenge for the insurance industry is to charge each customer an appropriate price for the risk they represent. Risk varies widely from customer to customer, and a deep understanding of different risk factors helps predict the likelihood and cost of insurance claims. The goal of this project is to see how well various statistical methods perform in predicting bodily injury liability Insurance claim payments based on the characteristics of the insured customer's vehicles for this particular dataset from Allstate Insurance Company.

The data was found at the Kaggle website(www.kaggle.com), which is a website that specializes in running statistical analysis and predictive modeling competitions. The data consist of automobile insurance claims from the Allstate Insurance Company, and were posted for the Kaggle competition called the "Claim Prediction Challenge", which was run from July 13 to October 12 2011. The contest's goal was to use data—three years of information on drivers' vehicles and their injury claims from 2005 to 2007 to predict insurance claims in 2008.

A number of factors will determine bodily injury rates, among them a driver's age, past accident history, and domicile, etc. However, this contest focused on the relationship between bodily injury claims and vehicle characteristics well as other characteristics associated with the insurance policies.

In this project, we implemented different statistical models to test their performances using the contest data. The original training data consists of observations from 2005 to 2007. Observations from 2008 make up the test data used to score the public leaderboard. Since the response variable (Claim_Amount) is not provided in the test set, we created our own training set and test set to evaluate model performance. The metric for the leaderboard used to score entries was the "normalized Gini coefficient" (named for the similar Gini coefficient/index used in Economics), and we used it to evaluate model performance. We also compared our results to those of the 290 entries from 202 contestants competing in 107 teams using the same evaluation method but a different test set.

The 2005-2007 data consist of 34 variables and 13184290 cases. The meanings of most of the predictor variables are unknown to us because the information was not provided due to company privacy.

Challenges of this project included: (1) A weak relation between claims and predictors. Vehicle characteristics are not the main factor in car accidents and the severity of the accident. (2) High dimensionality. The data has 33 the covariates including a number of categorical variables with many levels. (3) Missing values. The data naturally contains numerous missing observations. (4) Big data. The whole training data set contains 13184290 observations, and algorithmic efficiency is a problem that needs to be considered.

We tried several statistical methods, including logistic regression, Tweedie's compound gamma-Poisson model, principal component analysis (PCA), response averaging, and regression and decision trees. From all the models we tried, PCA combined with a with a Regression Tree produced the best results. This is somewhat surprising given the widespread use of the Tweedie model for insurance claim prediction problems.

2.0 Data exploration and Basic Summary Statistics

Each of the 13184290 cases in the 2005-2007 data sets contains one year's information for an insured vehicle. The response variable Claim_Amount (dollar amount of claims experienced for that vehicle in that year) has been adjusted to disguise its actual value. Among the other variables given definitions on the Kaggle website, Calendar_Year is the year that the vehicle was insured, Household_ID is a household identification number that allows year-to-year tracking of each household, and Model_Year is the year when the specific vehicle's model came into market, ranging from 1999-2009. Since a customer may insure multiple vehicles in one household, there may be multiple vehicles associated with each household identification number. The variable Vehicle identifies these vehicles (but the same Vehicle number may not apply to the same vehicle from year to year). The data set also has coded variables denoting make (manufacturer), model, and submodel. From these, it is impossible to determine the actual make, model or submodel of a vehicle.The remaining variables contain miscellaneous vehicle characteristics, as well as other characteristics associated with the insurance policy. There are not any details about what these variables are on the Kaggle website.

The dataset contains a substantial number of missing values for the categorical variables. Table 2.1 shows the number and percentage of missing values. In this project, 'missing' was counted as a new level of the category for a categorical variable.

Table 2.1

Variable Name	Blind_ Model	Blind Make	Blind_ Submodel	Cat1	Cat2	Cat3	Cat4
Number of Missing Values	8431	8431	8431	25981	4874164	3999	5631649
Percentage of Missing	0.064%	0.064%	0.064%	0.197%	3.697%	0.030%	42.7%
Variable Name	Cat5	Cat6	Cat7	Cat8	Cat10	Cat11	Cat12
Number of Missing Values	5637321	25981	7167634	3364	3917	31469	28882
Percentage of Missing	42.7%	0.197%	54.4%	0.026%	0.029%	0.239%	0.219%

The Kaggle contest provided two datasets: training data and test data. Contestants could use the training set to train their methods and the test set to compute their predictions for contest submission. However, the insurance claim responses for the test set have never been published. As a result, we split the training set into two sets. Observations from 2005 and 2006 were used as a training set, and observations from 2007 was used as a test set for this project.

A basic statistics summary for the continuous variables in the original training set with 13184290 observations is shown in Appendix A. A statistics summary for categorical variables can be found in Appendix B.

The response variable “Claim_Amount” is highly skewed with only 95,605 non-zero values (about 0.73%) among the 13,184,290 records. A frequency histogram of the non-zero claim amounts is shown in Figure 2.1, and of all claim amounts is shown in Figure 2.2.

Figure 2.1

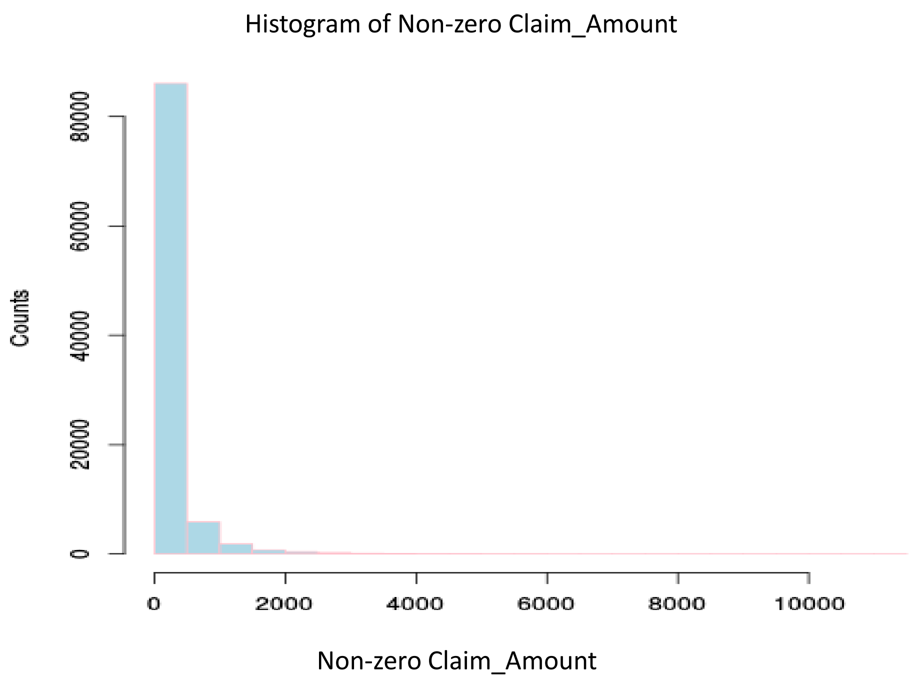
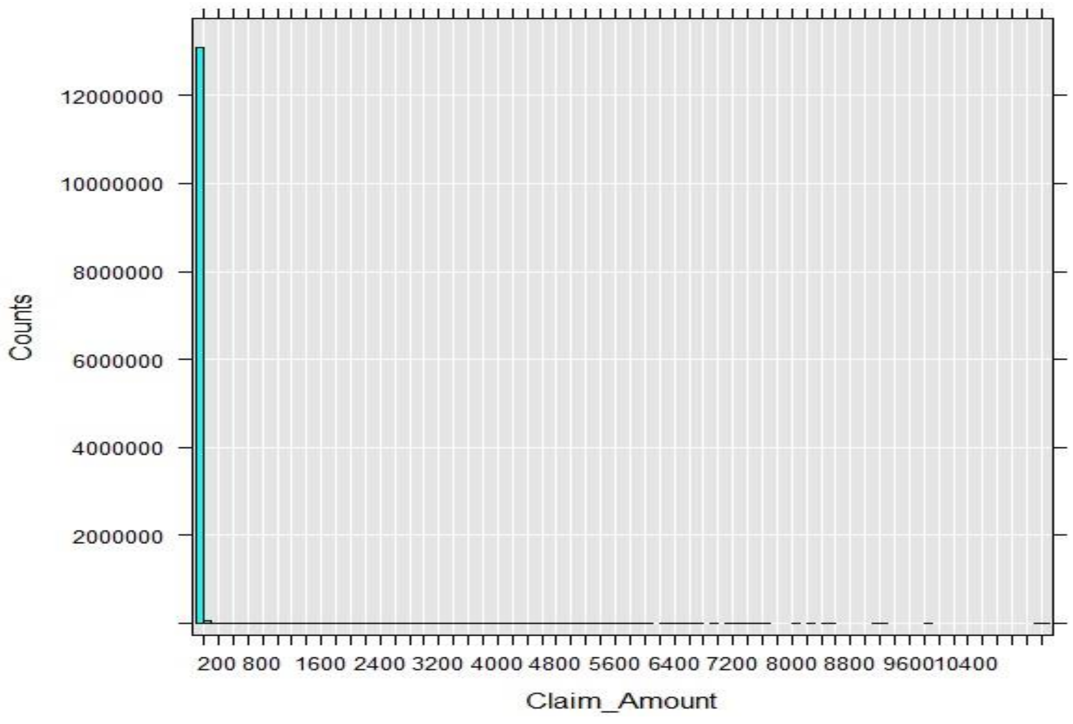


Figure 2.2



Some of the other categorical covariates have a large number of levels (see Appendix B), and this became a complication when we trying to get the statistical algorithms to work.

The correlation matrix between the continuous variables is shown in Table 2.2. There are 12 numerical predictors, named Var1 - Var8 and NVVar1 - NVVar4, and the response, insurance claim amount. We can see some strong correlations between some of the covariates,e.g.:Var2 and Var4. The correlation coefficients bigger than 0.7 are highlighted in Table 2.2. However, none of the covariates is even modestly correlated with the response.

Table 2.2

	Var1	Var2	Var3	Var4	Var5	Var6	Var7
Var1	1.0000						
Var2	0.5585	1.0000					
Var3	0.7464	0.6457	1.0000				
Var4	0.5759	0.9826	0.6570	1.0000			
Var5	0.9062	0.5718	0.7861	0.5924	1.0000		
Var6	0.7787	0.7722	0.8272	0.7159	0.7904	1.0000	
Var7	0.6768	0.5079	0.6682	0.5201	0.4975	0.8024	1.0000
Var8	0.2689	0.7017	0.3586	0.6909	0.3020	0.5767	0.2491
NVVar1	-0.0216	-0.0198	-0.0377	-0.0196	-0.0242	-0.0386	-0.0331
NVVar2	-0.0436	-0.0524	-0.0482	-0.0533	-0.0437	-0.0566	-0.0516
NVVar3	-0.0094	-0.253	-0.0176	-0.0243	-0.0115	-0.0234	-0.0176
NVVar4	-0.0654	-0.0575	-0.0736	-0.0599	-0.0663	-0.0796	-0.0696
Claim_Amount	-0.0009	-0.0013	-0.0011	-0.0012	-0.0002	-0.0013	-0.0014

	Var8	NVVar1	NVVar2	NVVar3	NVVar4	Claim_Amount
Var1						
Var2						
Var3						
Var4						
Var5						
Var6						
Var7						
Var8	1.0000					
NVVar1	-0.0356	1.0000				
NVVar2	-0.0427	-0.0089	1.0000			
NVVar3	-0.0350	-0.0394	0.0206	1.0000		
NVVar4	-0.0500	0.0762	-0.0483	-0.0433	1.0000	
Claim_Amount	-0.0015	0.0004	0.0014	0.0019	0.0001	1.0000

3.0 Methodology

Predicting the claim amount value can be regarded as a regression problem, because the claim amount is a continuous variable. The insurance claim prediction problem could also be considered as a classification problem by transforming the claim amount variable into a binary variable, taking response variable as one (Y = 1) if its value is greater than zero and zero otherwise (Y = 0). In this case, the desired outcome would be correctly predicting whether or not there would be a claim. We considered both approaches.

Since this problem can be considered as both classification and regression, there are a number of choices for analytical models and methods. To predict insurance claim amounts, we tried Tweedie's compound gamma-Poisson model, a gamma general linear model with gamma distribution and natural log link function, and regression trees. To classify observations as having or not having a claim, we tried logistic regression, and classification trees. However, given the computer resources available to us, we found that due to high dimensionality and the size of the data set, none of the algorithms used to apply those models and methods would converge without some form of dimension reduction.

a. Methods used to deal with high dimensionality

i. Principal Component Analysis (PCA)

Given a large set of correlated variables, principal component analysis (PCA) can be used to summarize the pattern of variation in those variables with a smaller set of variables called principal components (PCs). In our data set, we investigated whether the 12 continuous variables Var1-Var8 and NVVar1-NVVar4 could perhaps be adequately represented by a smaller number of principal components.

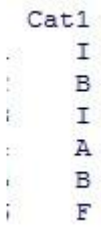
The variation in a set of variables can be summarized by their variances and covariances. Their PCs are a new set of variables formed by taking linear combinations of the original variables that are uncorrelated and that explain as much variation as possible in the fewest number of PCs. There are the same number of PCs as there are original variables. The first PC is the normalized linear combination that has maximum variance. The second PC is the normalized linear combination that has maximum variance among all normalized linear combinations uncorrelated with the first PC. The third PC is the normalized linear combination that has maximum variance among all normalized linear combinations uncorrelated with the first two PCs, and so on. When used for dimension reduction, the hope is that a large proportion of the total variation can be represented by a relatively small number of PCs, which then substitute for the original variables. Principal component analysis can also be used to identify meaningful underlying variables. Another benefit of PCA is that many statistical procedures work better conceptually and algorithmically with uncorrelated variables.

To find the principal components, we compute the covariance matrix of the data, and calculate the eigenvalues and corresponding eigenvectors of this covariance matrix. Then we need to normalize each eigenvector to make the set orthonormal. The proportion of the variance that each eigenvector represents can be calculated by dividing the eigenvalue corresponding to that eigenvector by the sum of all eigenvalues.

There are several methods to apply PCA to categorical variables, among these an optimal scoring method due to Fisher. However, due to computational constraints, the most feasible method for our purposes is to apply PCA to categorical variables by establishing a design matrix that assigns dummy variables to the categorical variables. For example, if variable Cat1 has 10 levels A to J from 1-6 individuals, the transformed matrix will be of 6 * 10 dimension, as shown in Figure 3.1:

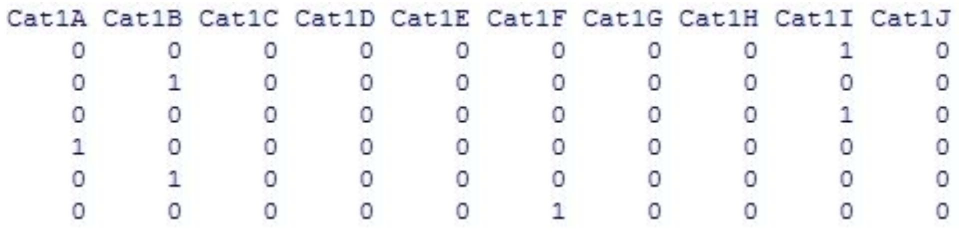
Figure 3.1

Original Cat1:



```
 Cat1
    I
    B
    I
    A
    B
    F
```

Transformed Cat1:



```
Cat1A Cat1B Cat1C Cat1D Cat1E Cat1F Cat1G Cat1H Cat1I Cat1J
    0     0     0     0     0     0     0     0     1     0
    0     1     0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0     1     0
    1     0     0     0     0     0     0     0     0     0
    0     1     0     0     0     0     0     0     0     0
    0     0     0     0     0     1     0     0     0     0
```

PCA is then applied to the combined set of quantitative and dummy variables.

Another method called Fisher's optimal scoring [1] can be used to transform nominal variables by scoring the categories. PCA could then be applied for dimension reduction [2]. However, this method proved impractical due to its intensive use of computational resources.

ii. Response Averaging

Three categorical variables with large numbers of values made fitting models and obtaining predictions particularly challenging. These variables were Blind Make (75 values), Blind Model (1303 values), and Blind Submodel (2740 values). The approach we used was to change the categorical values into numerical values by replacing each of those categorical values with the average of the insurance claims in the training set corresponding to that categorical value.

b. Classification

i. Logistic Regression

Binomial or binary logistic regression models the probability the response belongs to one of two possible categories as a function of one or more predictors. In our case the categories are that an insurance claim occurred (Y=1), or that an insurance claim didn't occur (Y=0). The logistic regression fits the model

$$\Pr(\mathrm{Y}=1|\mathrm{X}) = \frac{e^{\beta^T \mathrm{X}}}{1+e^{\beta^T \mathrm{X}}} \quad ,$$

where X is the vector of predictors.

c. Regression

i. The Tweedie Model

This dataset has a large number of zero responses, and non-zero positive responses. We assumed that T is the number of claims one car have and X_j is the amount of the jth claim, so the total claim for one car over the course of the year follows a Tweedie distribution [3]. Due to its ability to simultaneously model the zeros and the continuous positive outcomes, the Tweedie GLM is a widely used method in predicting insurance claims.

We assume the observed response vector is Y = $(Y_1; \ldots\ldots; Y_n)'$, where Y_i is distributed as a compound gamma-Poisson distribution (Tweedie distribution) with parameters μ_i, ϕ, p, if the Y_i are independently distributed as

$$\sum_{j=1}^{T} X_j, T \sim Pois(\lambda), X_j \sim Ga(\alpha,\gamma), T \perp X_j$$

where *Pois(λ)* denotes a Poisson random variable with mean λ , and Ga(α,γ) denotes a Gamma random variable with mean and variance equal to αγ and $\alpha\gamma^2$, respectively. As a result, the compound Poisson distribution has a probability mass at zero accompanied by a skewed continuous distribution on the positive real line.

The Tweedie distribution belongs to the exponential dispersion family. A two-parameter representation of the exponential dispersion model is

$$p(y|\theta,\phi) = a(y,\phi)exp\left(\frac{y\theta - \kappa(\theta)}{\phi}\right)$$

Where α and *κ* are known functions, *θ* is the natural parameter and *Φ* > 0 is the dispersion parameter. For the exponential family of distributions, we have the well-known relationships E(y) = μ=κ′(θ) and Var(y) =Φκ′′(θ). Since the mapping from θ to μ is one-to one, κ′′(θ) can also be represented as a function of μ, denoted by V (μ) = μ^p in which the value of the index parameter p lies in the interval (1, 2). By means of deriving and equating the cumulant generating functions for the above equation, we can work out the relationship between the two sets of parameters in the two representations as:

$$\mu = \lambda\alpha\gamma$$

$$\alpha = \frac{2-p}{p-1}$$

$$\lambda = \frac{\mu^{2-p}}{\phi(2-p)}$$

$$\phi = \frac{\lambda^{1-p} \cdot (\alpha\gamma)^{2-p}}{2-p}$$

$$\lambda = \frac{\mu^{2-p}}{\phi(2-p)}$$

$$p = \frac{\alpha+2}{\alpha+1}$$

In the Tweedie model, the mean μ= E(Y) is stipulated as a function of some linear predictors through a link function as η(μ) = Xβ, where *X* is the design matrix and *β* is the vector of fixed effects. For unknown *p*, parameter estimation can be done using the profile likelihood approach.

There is also a zero-inflated Tweedie model, which adds additional mass at 0. The zero-inflated Tweedie model assumes that, for the ith observation of the count data, Y_i is generated as below:

$$Y_i \sim \begin{cases} 0 & \text{with probability } q_i \\ Tweedie(\mu_i, \phi, p) & \text{with probability } 1 - q_i \end{cases}$$

d. Decision Tree Method

A decision tree is a tree-based method that partitions the predictor space X into a set of rectangles and fits a simple model in each one. The majority of trees use a simple two-stage algorithm: First, partition the observations by univariate splits in a recursive way and second, fit a model in each cell of the resulting partition. The algorithm decides on the predictor giving the best split by doing an exhaustive search over all possible splits and choosing the predictor and split giving the maximum of an information measure. A tree is built recursively through this process; a process known as recursive partitioning. Decision tree methods can be both used for classification and regression, yielding classification and regression trees, respectively. The partitions are chosen to maximize a measure of classification or prediction accuracy, respectively.

i. Regression Tree

A regression tree stratifies the predictor space into rectangular regions, and fits a simple model in each of the regions to predict a continuous response.

We chose an ANOVA method similar to linear regression as the splitting criterion. Specifically, the splitting criterion minimizes SST − (SSL + SSR), where SST $= \sum (y_i - \bar{y})^2$ is the sum of squares for the node, and SSR, SSL are the sums of squares for the right and left split node, respectively.

This splitting or partitioning is then applied to each of the new branches. The process continues until each node reaches a user-specified minimum node size and becomes a terminal node. We then used cross-validation to prune the tree. At each pair of leaf nodes with a common parent, we evaluated the error on the validation/testing data, and saw whether the testing sum of squares would shrink if we removed those two nodes and made their parent a leaf. If so, we pruned; otherwise, not. This was repeated until pruning no longer improved the error on the testing data.

ii. The Conditional Inference Decision Tree [5][6]

However, the standard algorithm does not perform well on highly imbalanced data such as these, due to overfitting and towards predictors with many possible splits. The conditional inference tree algorithm nullifies this bias through the use of a significance test procedure to screen the predictors.

Specifically, a permutation test is conducted to test the hypothesis of independence between any of the predictors and the response. If the test fails to reject the null hypothesis, the recursive partitioning stops. Otherwise, the predictor with strongest association with the response is chosen and the optimal split determined. These steps are recursively repeated until the tree is completed.

The conditional inference decision tree can both be used for classification and regression.

e. Other models

Other models like GLM Gamma Regression, SVM, and clustering methods were also tried to solve the problem. For GLM Gamma Regression, the original thought was to use logistic regression first to predict whether an insurance claim would occur or not, and then use the gamma regression to fit the non-zero responses. However, the logistic regression's prediction performance kept this method from being competitive, as shown in the next section. Other methods including SVM and k-means clustering were also implemented; however, due to the highly unbalanced nature of the data and its high dimensionality, these methods proved impractical. Consequently, in what follows we only provide information on models that gave a relatively good result.

4.0 Evaluation Methods - Normalized Gini Coefficient

a. Normalized Gini Coefficient

For the Kaggle contest, the specified evaluation measure is the normalized Gini coefficient. Traditionally, statistical response models have been evaluated based on some form of goodness of fit. Assumptions are made regarding underlying data distributions and models are evaluated based on how well predicted values fit the observed data values from a sample data set. Various statistical measures (Likelihood, R^2, the F statistic, the Chi Square statistic, classification indices and so on) are used to evaluate or produce the goodness of fit. In the insurance claim prediction, the Kaggle contest required the contestants to use the normalized Gini coefficient as the evaluation measure.

To define the normalized Gini coefficient, we first need to define Lorenz curve: “Let $p \in [0,100]$, and define the function $L(p)$to be the proportion of all claims associated with the largest p percent of predicted values. The graph of $L(p)$ versus p is the Lorenz curve. The line at 45 degrees represents the results expected from a “null model”, that is, from predicting by randomly choosing a claim amount from the data set, so the area between L(p) and the 45 degree line represents the improvement of the prediction method over chance prediction.. Let $L_1(p)$ denote the Lorenz curve for the prediction method being evaluated and $L_2(p)$ the Lorenz curve for perfect prediction, and let A_1 and A_2 be the respective areas between the Lorenz curves and the 45 degree line.. The normalized Gini coefficient is the ratio A_1/A_2.

As a result of this definition, the actual predicted values do not matter for evaluating the normalized Gini coefficient, only the relative ordering of these predictions does.

b. **Confusion Matrix**

The confusion matrix, also known as the contingency table, is a specific table layout that displays the performance of a classification test. For a binary classification it contains two rows and two columns that report the number of false positives (FP), false negatives (FN), true positives (TP), and true negatives (TN) (Table 4.1).

Table 4.1

	Model Prediction Positive	Model Prediction Negative
Truth: Positive	TP	FN
Truth: Negative	FP	TN

5.0 Experimental Results

The original training set has information from years 2005 to 2007, with 13184290 observations. In the Kaggle contest, participants developed their predictors using the training data, and used them to predict claims in the test set. They submitted these predictions and the sponsors evaluated the results using the normalized Gini coefficient. The claims data in the test set were never published, so we cannot evaluate our predictors on that data. However, the leader board scoring can be found on the Kaggle website and shows how the best prediction methods (which were also not disclosed) performed on the test data.

While we don't have the test set claims data, we can assess the similarity of the covariates in my test set (Table 5.1) and the Kaggle test sets (Table 5.2).

Table 5.1

Name	Mean	SD	Min	Max
Var1	0.01759	0.9849655	−2.57822	5.14339
Var2	0.01301	0.9957419	−2.49339	7.82942
Var3	0.01553	1.009001	−2.79033	5.56322
Var4	0.02049	0.995602	−2.50822	7.58926
Var5	0.02844	0.9944316	−3.35034	4.01817
Var6	0.008497	0.9910882	−2.376657	4.584289
Var7	0.008933	1.001785	−2.778491	4.127148
Var8	0.00117	1.025582	−2.16304	47.35072
NVVar1	−0.009784	1.02622	−0.23153	6.627110
NVVar2	0.01242	1.035767	−0.26612	8.88308
NVVar3	−0.01515	1.042242	−0.27234	8.69114
NVVar4	0.005654	1.015079	−0.25142	6.388802

Table 5.2

Name	Mean	SD	Min	Max
Var1	−0.29122	0.44803	−2.57822	3.08644
Var2	−0.05289	0.898978	−2.14757	7.82942
Var3	−0.2088	0.692303	−2.4664	2.0691
Var4	−0.08893	0.881487	−2.16994	7.94445
Var5	−0.1727	0.615788	−5.0572	2.8763
Var6	−0.37134	0.55005	−2.02925	2.85897
Var7	−0.5528	0.502969	−2.2133	1.6819
Var8	0.08771	1.095868	−1.4848	46.72172
NVVar1	−0.02708	0.930115	−0.23153	6.62711
NVVar2	−0.01003	0.984403	−0.26612	8.88308
NVVar3	−0.04531	0.895811	−0.27234	8.69114
NVVar4	0.009567	1.042618	−0.25142	6.3888

We can see that given the large samples, the continuous covariates Var1, Var3, Var6 and Var7 in the training set and test set have some significant differences (highlighted in yellow). For the other continuous variables, the mean and standard deviations are close, and the maximum and minimum values for all variables are very close. While these summary statistics do not rule out the possibility that similar results would be obtained by using our methods on the Kaggle test data, they do indicate the possibility of substantial differences due to differences in the distributions of the covariates in the two sets, .

We further checked the correlation matrix of our test set (Table 5.3) and the Kaggle test set (Table 5.4)

Table 5.3

	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8	NVVar1	NVVar2	NVVar3	NVVar4
Var1	1.0000											
Var2	0.5582	1.0000										
Var3	0.7488	0.6491	1.0000									
Var4	0.5795	0.9837	0.6623	1.0000								
Var5	0.9112	0.5691	0.7876	0.5925	1.0000							
Var6	0.7800	0.7700	0.8332	0.7867	0.7530	1.0000						
Var7	0.6770	0.5064	0.6762	0.5215	0.5100	0.8066	1.0000					
Var8	0.2682	0.6999	0.3641	0.6898	0.2976	0.5714	0.2462	1.0000				
NVVar1	-0.0197	-0.0191	-0.0356	-0.0188	-0.0222	-0.0367	-0.0331	-0.0337	1.0000			
NVVar2	-0.0444	-0.0529	-0.0494	-0.0538	-0.0443	-0.0569	-0.0520	-0.0418	-0.0106	1.0000		
NVvar3	-0.0107	-0.0274	-0.0201	-0.0264	-0.0124	-0.0252	-0.0200	-0.0348	-0.0381	0.0192	1.0000	
NVVar4	-0.0631	-0.0561	-0.0704	-0.0583	-0.0637	-0.0761	-0.0680	-0.4830	0.0675	-0.0454	-0.0414	1.0000

Table 5.4

	Var1	Var2	Var3	Var4	Var5	Var6
Var1	1.0000					
Var2	0.4568	1.0000				
Var3	0.8403	0.4851	1.0000			
Var4	0.5485	0.9964	0.4915	1.0000		
Var5	0.8236	0.4744	0.7947	0.4809	1.0000	
Var6	0.8504	0.6873	0.8503	0.6912	0.7309	1.0000
Var7	0.5480	0.0601	0.5585	0.0643	0.2422	0.5168
Var8	0.3623	0.7328	0.3128	0.7321	0.2805	0.5798
NVVar1	-0.0439	-0.0201	-0.0503	-0.0205	-0.0358	-0.0510
NVVar2	-0.0478	-0.0556	-0.0527	-0.0556	-0.0463	-0.0596
NVvar3	-0.0269	-0.0183	-0.0181	-0.0183	-0.0144	-0.0207
NVVar4	--0.0726	-0.0705	-0.0827	-0.0709	-0.0646	-0.0936

	Var7	Var8	NVVar1	NVVar2	NVVar3	NVVar4
Var1						
Var2						
Var3						
Var4						
Var5						
Var6						
Var7	1.0000					
Var8	-0.0677	1.0000				
NVVar1	-0.0471	-0.0275	1.0000			
NVVar2	-0.0243	-0.0429	-0.0092	1.0000		
NVvar3	-0.0133	-0.0153	0.0284	0.0284	1.0000	
NVVar4	-0.0532	-0.0652	-0.0396	-0.0396	-0.0334	1.0000

To better compare the above tables, we subtracted the Table 5.4 entries from the corresponding entries in Table 5.3. The results are shown in Table 5.5.

Table 5.5

	Var1	Var2	Var3	Var4	Var5	Var6
Var1	0					
Var2	0.1014	0				
Var3	-0.0915	0.164	0			
Var4	0.031	-0.0127	0.1708	0		
Var5	0.0876	0.0947	-0.0071	0.1116	0	
Var6	-0.0704	0.0827	-0.0171	0.0955	0.0221	0
Var7	0.129	0.4463	0.1177	0.4572	0.2678	0.2898
Var8	-0.0941	-0.0329	0.0513	-0.0423	0.0171	-0.0084
NVVar1	0.0242	0.001	0.0147	0.0017	0.0136	0.0143
NVVar2	0.0034	0.0027	0.0033	0.0018	0.002	0.0027
NVvar3	0.0162	-0.0091	-0.002	-0.0081	0.002	-0.0045
NVVar4	-0.1357	0.0144	0.0123	0.0126	0.0009	0.0175

	Var7	Var8	NVVar1	NVVar2	NVVar3	NVVar4
Var1						
Var2						
Var3						
Var4						
Var5						
Var6						
Var7	0					
Var8	0.3139	0				
NVVar1	0.014	-0.0062	0			
NVVar2	-0.0277	0.0011	-0.0014	0		
NVvar3	-0.0067	-0.0195	-0.0665	-0.0092	0	
NVVar4	-0.0148	-0.4178	0.1071	-0.0058	-0.008	0

We can see that for most variables, the correlation coefficients are very close, except for variable Var7. The differences in the correlation coefficients of Var7 with Var2, Var4, Var5, Var6 and Var8 are bigger than 0.2.

Since we don't have the response on the Kaggle test set, in order to evaluate the performance of the various methods considered, we divided the original training set into a training and a test set. The training set contains information from 2005 to 2006 with 8473402 (64%) observations, and the test set contains information from 2007 with 4710888 (36%) observations. We trained our predictors on this training set and tested them on this test set, in the hope is that the results would be similar to what we would have obtained if we had applied them to the true test set. In what follows, "training set" and "test set" will refer to these two sets.

One question is the similarity of the training set and test set. To check this, we created summary statistics for some of the variables in the training and test sets. For the response variable Claim_Amount, the summary statistics of non-zero and zero values are shown in Table 5.6. The histogram of non-zero values in the training set is shown in Figure 5.1 and some summary statistics are shown in Table 5.7. The corresponding displays for the test set are shown in Figure 5.2 and Table 5.8.

Table 5.6 Comparison of Zero and Non-Zero Values

	Counts of non-zero values	Counts of -zero values	Percentage of none-zero values
Training Set	61838	8411564	0.735%
Test Set	33767	4677121	0.722%

Figure 5.1

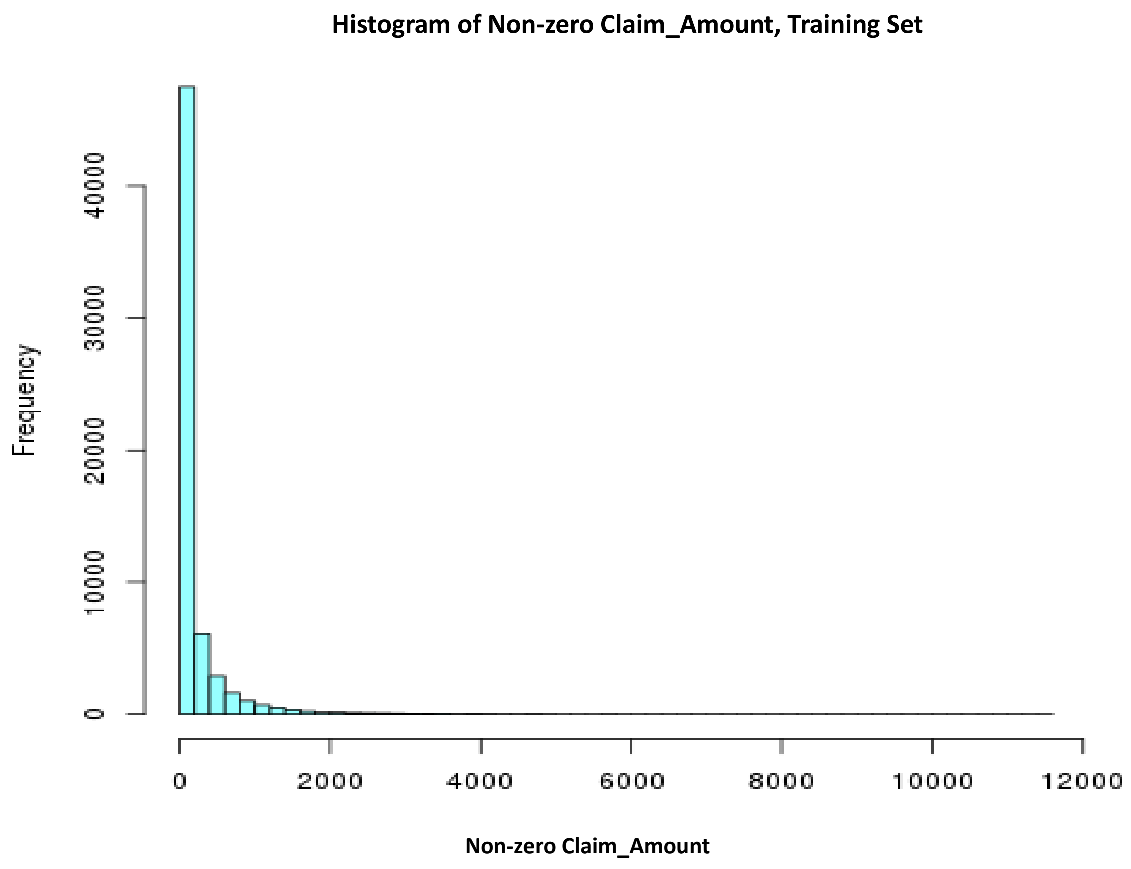


Table 5.7 Summary Statistics, Training Set

Non-zero Claim Amount	mean	Standard deviation	min	max
	201.00	465.2236	0.00	11440.00

Figure 5.2

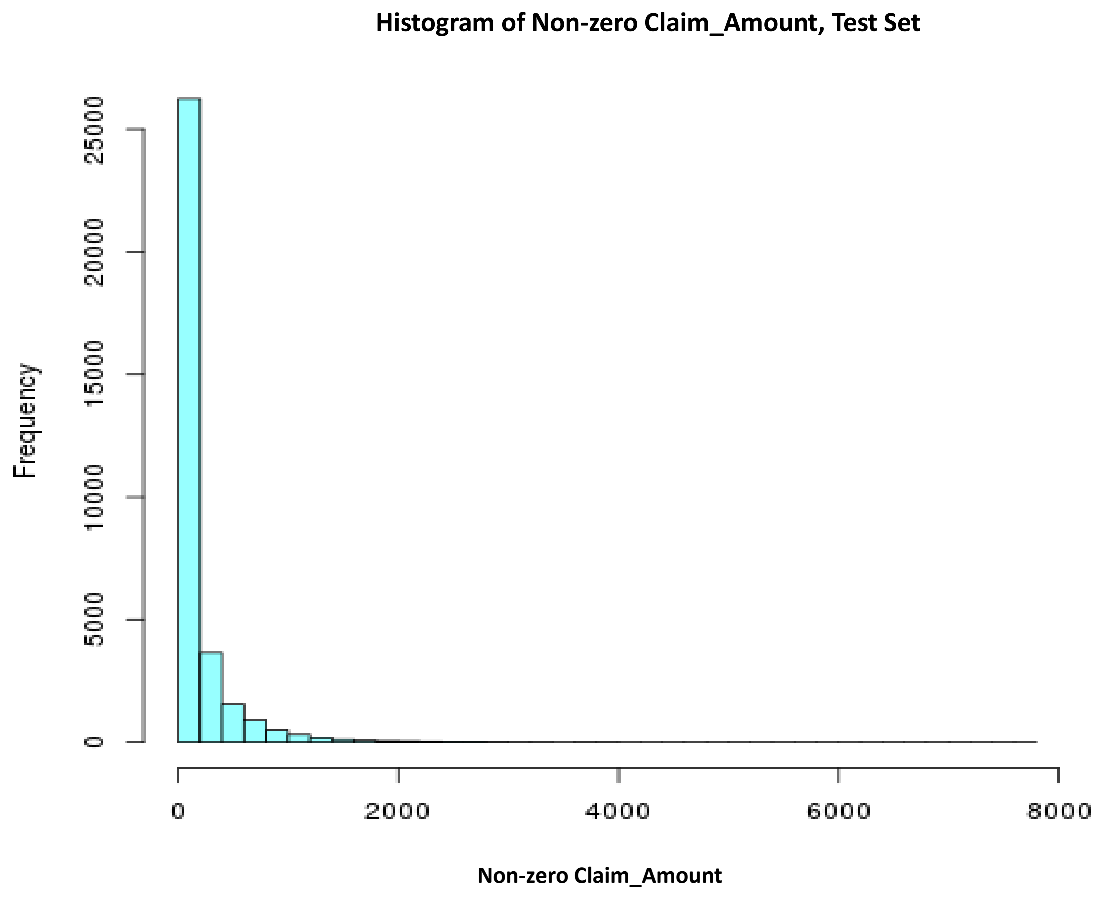


Table 5.8 Summary Statistics, Test Set

Non-zero Claim Amount	mean	Standard deviation	min	max
	163.20	312.5247	0.00	7667.00

We can see is that the percentage of zero and non-zero values are similar in the training set and test set, but the summary statistics of non-zero values are not as similar. Essentially, there are substantial differences between our test set and Kaggle's and between our training and test set.

a. Logistic Regression

Logistic regression can be used to classify according to whether or not an insurance claim occurred. However, 162G of memory are needed to fit a logistic regression with all available predictor variables, which is not available on our computing cluster. So we first tried to delete the two variables that have over 1000 levels each: Blind_Model and Blind_Submodel.

In the logistic regression, we set the response equal to 1 for any positive claim amount. The result of fitting the logistic regression on the test set is shown in Table 5.9.

Table 5.9 Confusion Matrix

Real value \ Prediction	Positive	Negative
Truth: positive	4643325	33183
Truth: negative	33796	584

We can see the prediction is not good. From the above, we can see the model fails to predict non-zeros.

We then tried to use response averaging to transform the three categorical variables Blind_Make, Blind_Model and Blind_Submodel into numerical values, and then fitted the logistic regression. The result is shown in Table 5.10

Table 5.10 Confusion Matrix with Response Averaging

Real value \ Prediction	Positive	Negative
Truth: positive	4643293	33215
Truth: negative	33828	552

We can see that by using response averaging the result is worse than the previous logistic regression model. We further tried using PCA and response averaging as predictors, the result is shown in Table 5.11

Table 5.11 Confusion Matrix with Response Averaging and PCA

Real value \ Prediction	Positive	Negative
Truth: positive	4643272	33236
Truth: negative	33849	531

From Table 5.11 we can see the result is worse than the other two. In this case, response averaging and PCA doesn’t help improving the prediction accuracy, on the contrary, they make it worse.

b. Tweedie Model with Sampling Subset

We tried to fit our data with the Tweedie model. Because of the large size of data as well as the many levels of the categorical variables, the algorithms wouldn't converge. To get some idea of the performance of this model, we used random sampling without replacement to draw four samples, each with sampling fraction around 1/80 giving approximately 100,000 observations as training sets. The Tweedie model was fit to each of these training sets. Figures 5.3-5.6 show the resulting residual plots versus fitted values.

It can be seen in all the four plots that the residuals have a relatively larger range, indicating that the fits are poor for responses which have values around 0, and that the models failed to predict high insurance claims.

Figure 5.3

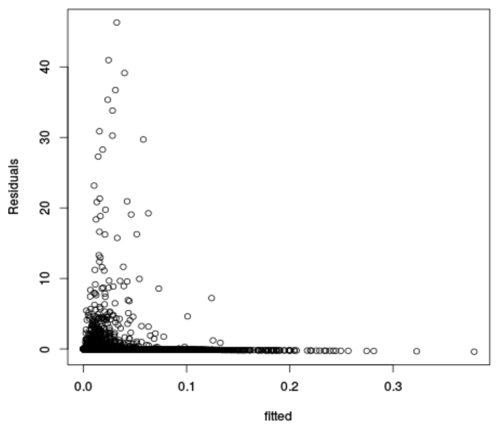


Figure 5.4

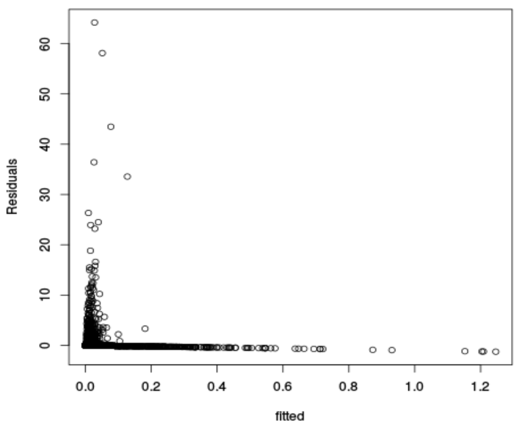


Figure 5.5

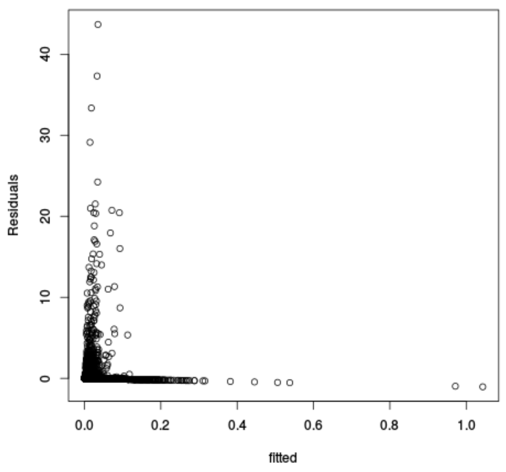
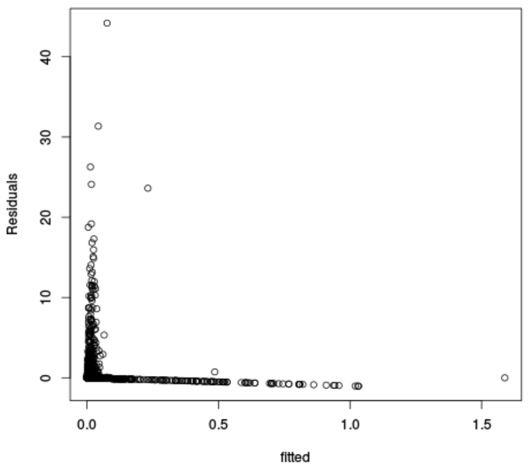


Figure 5.6



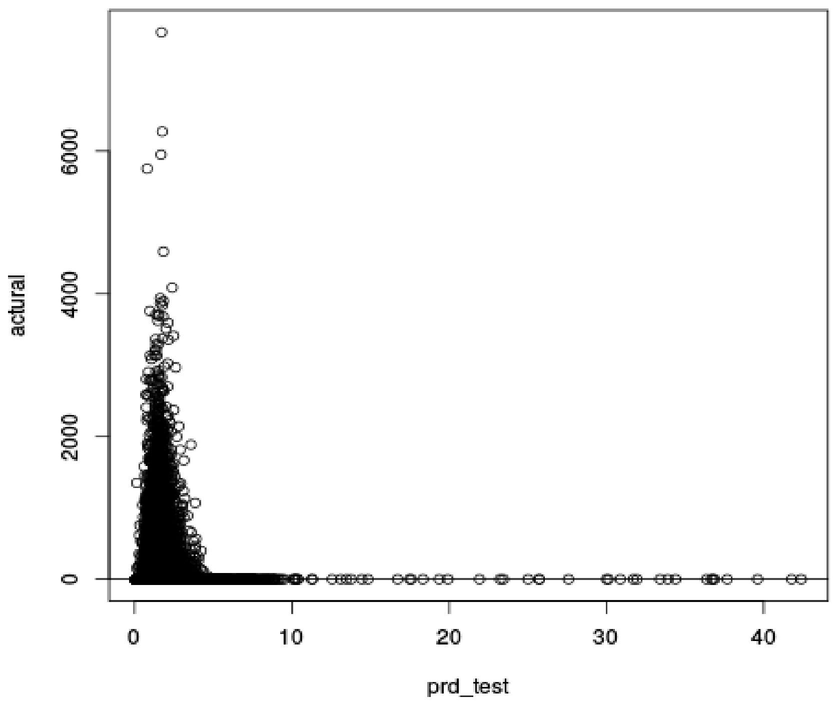
We took two samples (sample_trainB, sample_trainC) among the four samples above to calculate the normalized Gini coefficient and get 0.38 for sample_trainB and 0.39 for sample_trainC. However, applying the model trained on training sample trainB to predict the responses for sample trainC, resulted in a normalized Gini coefficient of only 0.004039204.

Because the algorithm failed to converge for the full data set, we needed to find a way to deal with the high dimensionality problem. Our first attempted solution was to use PCA to reduce the dimension of the predictor space.

c. PCA

To apply PCA to the categorical variables, we first transformed them to dummy variables as described in the methodology section. Taking all the resulting the variables, continuous and categorical, we find that 38 principal components account for 98% of the variation. Fitting the Tweedie model to the training data with these as predictors, we obtain a normalized Gini coefficient on the test set, 0.02249483. The resulting residual plot is shown in Figure 5.7:

Figure 5.7



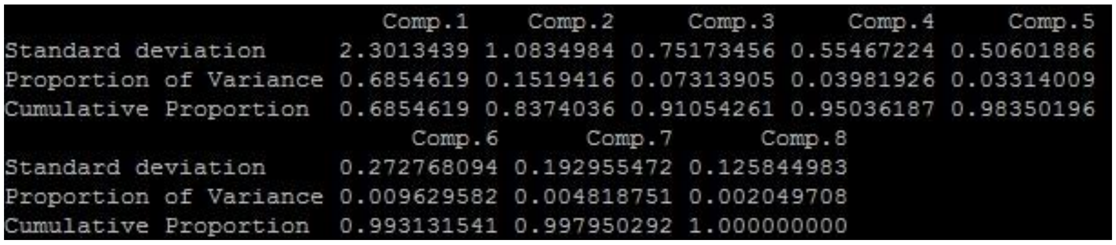
We can see data possibly contains a lot of noise and this model probably suffers from over-fitting. However, we do not have an effective remedy for over-fitting in the Tweedie model. As a result, we attempted another approach. We first obtained principal components for the dummy categorical variables, obtaining the results shown in Table 5.12 (shown only for the first 12 PCs).

Table 5.12

PC	Comp 1	Comp 2	Comp 3	Comp 4	Comp 5	Comp 6
Standard deviation	9.16E-01	7.15E-01	6.44E-01	5.93E-01	5.85E-01	5.55E-01
Cumulative proportion	0.157674	0.096012	0.078068	0.066011	0.064264	0.057933
Proportion of variance	0.157674	0.253687	0.331755	0.397766	0.462031	0.519964
PC	Comp 7	Comp 8	Comp 9	Comp 10	Comp 11	Comp 12
Standard deviation	5.42E-01	5.20E-01	4.76E-01	4.68E-01	4.51E-01	3.78E-01
Cumulative proportion	0.0552	0.050903	0.042554	0.041085	0.038228	0.026909
Proportion of variance	0.575164	0.626067	0.668621	0.709707	0.747935	0.774844

Next, we applied PCA to the continuous variables. In our data, the continuous variables are Var1-Var8 and NVVar1-NVVar4. Since all of the correlations of variables NVVar1-NVVar4 are smaller then 0.3, we chose to obtain the principal components only for Var1-Var8 (Table 5.13)

Table 5.13

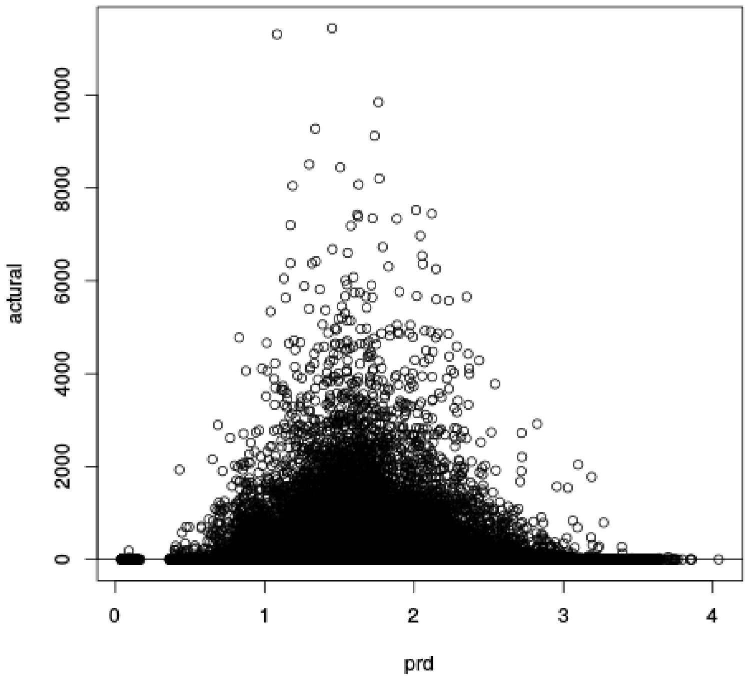


```
                         Comp.1    Comp.2     Comp.3     Comp.4     Comp.5
Standard deviation     2.3013439 1.0834984 0.75173456 0.55467224 0.50601886
Proportion of Variance 0.6854619 0.1519416 0.07313905 0.03981926 0.03314009
Cumulative Proportion  0.6854619 0.8374036 0.91054261 0.95036187 0.98350196
                           Comp.6      Comp.7      Comp.8
Standard deviation     0.272768094 0.192955472 0.125844983
Proportion of Variance 0.009629582 0.004818751 0.002049708
Cumulative Proportion  0.993131541 0.997950292 1.000000000
```

We chose to use three PCs, explaining 91% of the variation.

Using these PC's in the Tweedie model. The algorithm ran successfully. The fitted against actual value plot is shown in Figure 5.8:

Figure 5.8

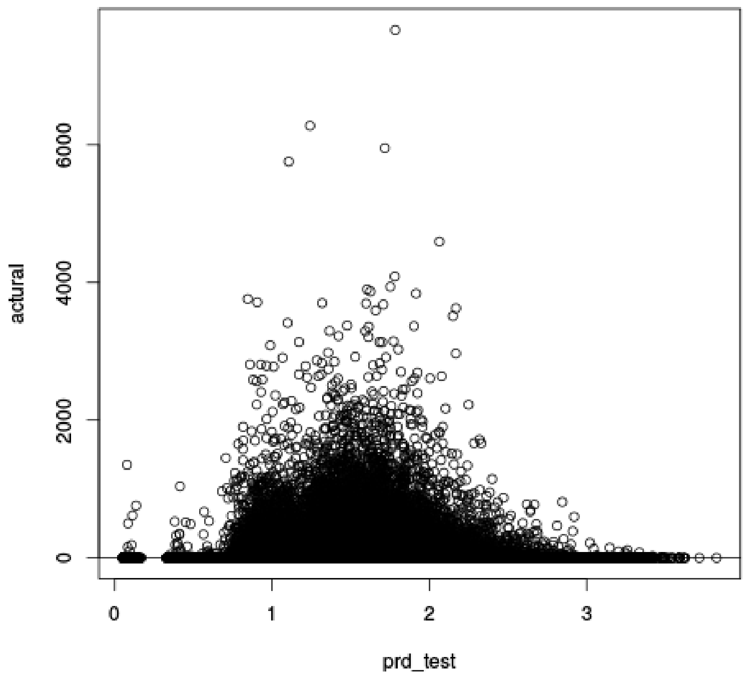


The resulting normalized Gini coefficient is 0.3751337 in the training set, very similar to the previous result in the sampling data set.

To run the Tweedie model with separate PCs (take 5 PC from 12 categorical variables, and 3 PCs from the 8 continuous variables) on the test set, the true against fitted values plot is shown in Figure 5.9.

Applying the model to the test set, we get a normalized Gini coefficient of 0.05972099, considerably better than the previous results, and very similar to the internal benchmark 0.05933 from the website, ranking 52 among the 102 participating teams.

Figure 5.9



d. Regression Tree with PCA

Regression trees are very prone over-fitting. To reduce the over-fitting as much as possible, we further divided the test set into two equal parts, a validation set and test set.

The validation set was used to tune the parameters of a classifier and determine the termination of the algorithm. The test set was used only to assess the performance of the fully-trained classifier. We used the test set to estimate the error rate after we had chosen the final model to compute the normalized Gini coefficient.

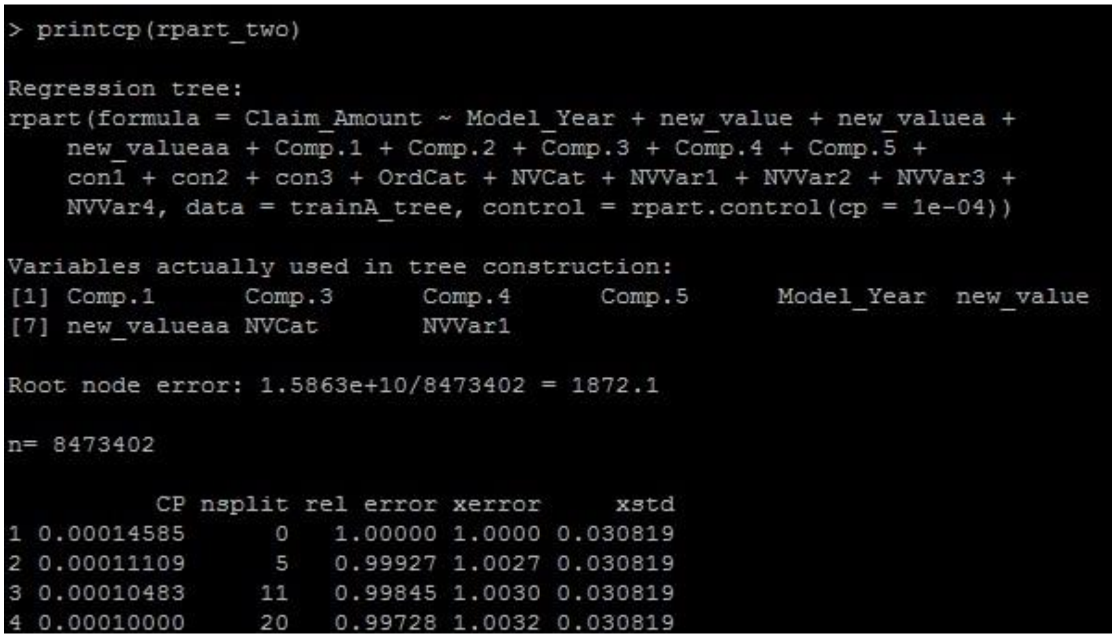
First we transformed the three variables Blind Make, Blind Model, and Blind Submodel using the response averaging technique mentioned previously: in the training set, we replaced the categorical levels by the average of the corresponding insurance claims, while in the validation set and test sets, we replaced those categorical levels with the numerical values obtained from the training set.

Implementing a decision tree model on the transformed three variables in the training set, we obtained a normalized Gini coefficient 0.48, the highest one so far. However, this model is seriously over-fit with the Gini coefficient of 0.040 on the test set.

As we did for the Tweedie model, we used PCA to transform the 12 categorical variables Cat1- -Cat12 (using the design matrix set-up), and as before took 5 PCs with 46% of their total variance and among the 8 continuous variables Var1-Var8, we took 3 PC's with 91% of their total variance.

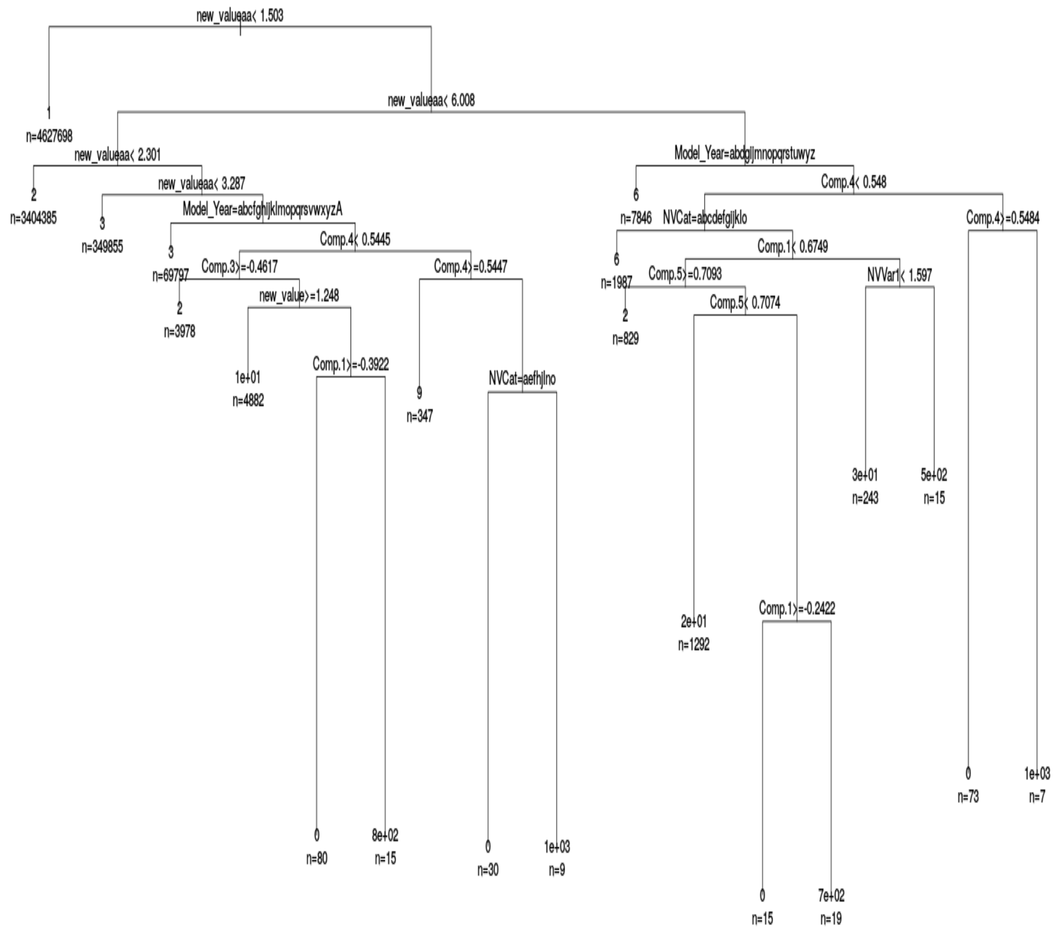
Finally, we implemented the decision tree model with the PCs from PCA as the predictors, and pruned the tree using ten-fold cross-validation. The final model is shown in Figure 5.10 and 5.11

Figure 5.10



```
> printcp(rpart_two)

Regression tree:
rpart(formula = Claim_Amount ~ Model_Year + new_value + new_valuea +
    new_valueaa + Comp.1 + Comp.2 + Comp.3 + Comp.4 + Comp.5 +
    con1 + con2 + con3 + OrdCat + NVCat + NVVar1 + NVVar2 + NVVar3 +
    NVVar4, data = trainA_tree, control = rpart.control(cp = 1e-04))

Variables actually used in tree construction:
[1] Comp.1      Comp.3      Comp.4      Comp.5      Model_Year  new_value
[7] new_valueaa NVCat       NVVar1

Root node error: 1.5863e+10/8473402 = 1872.1

n= 8473402

          CP nsplit rel error xerror     xstd
1 0.00014585      0   1.00000 1.0000 0.030819
2 0.00011109      5   0.99927 1.0027 0.030819
3 0.00010483     11   0.99845 1.0030 0.030819
4 0.00010000     20   0.99728 1.0032 0.030819
```

Figure 5.11



The variable new_valueaa is the variable transformed from the original variable Blind_Submodel which has over 2000 levels and is the least granular of the car model descriptors. We can see this is the most important variable in the decision tree. By using the model above, the normalized Gini coefficient on the training set is 0.152567, and on the test set is 0.080135, which, if it applied to the Kaggle test data, would rank 27th among contest results.

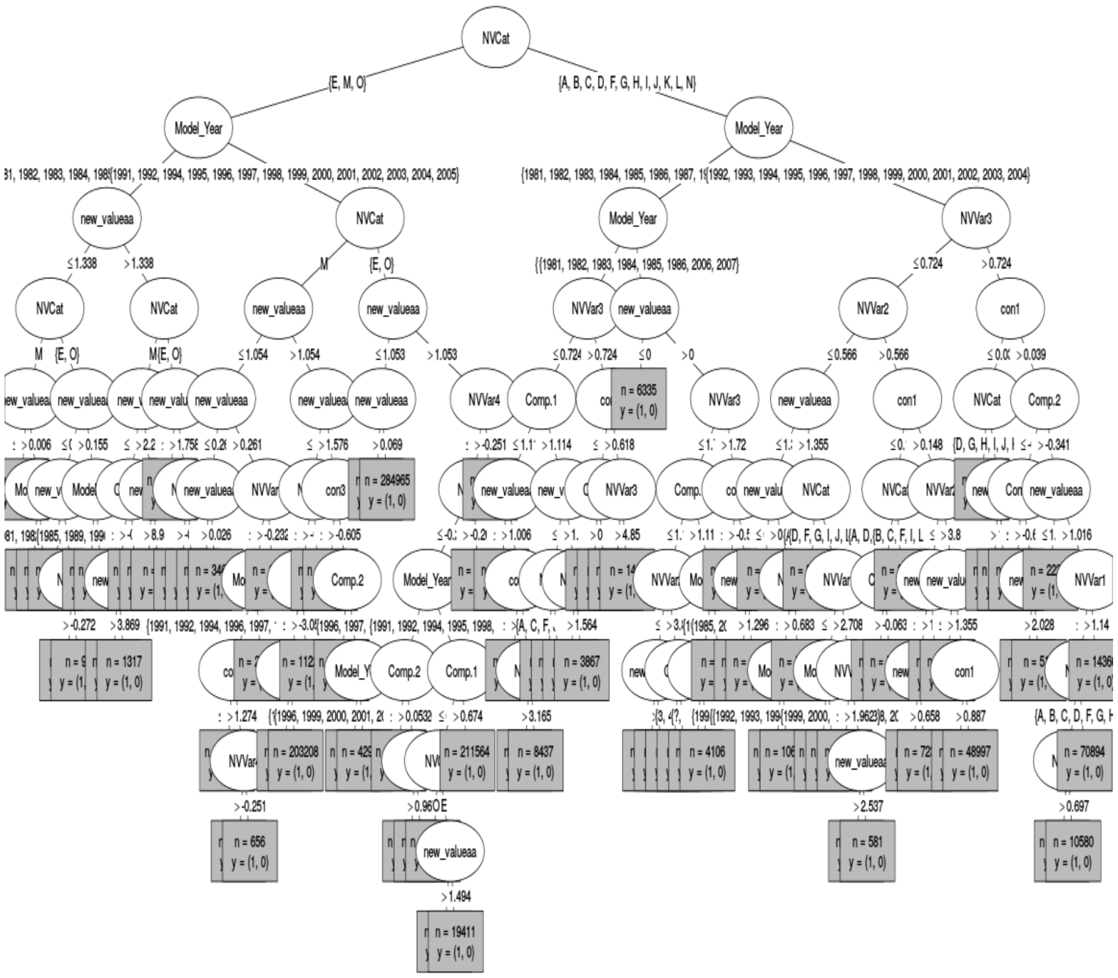
e. Conditional Inference Tree with PCA

As mentioned in the methodology section, a conditional inference method for tree fitting was developed to deal with highly imbalanced data, such as ours.

The covariates used were the same as it is for the Regression Tree. Since this method can both be used for regression and decision trees, we first tried the decision tree for classification.

We can see from the resulting decision tree in Figure 5.12, the top three important predictor variables are: NVCat, Model_Year, and new-valueaa (the sub_model transformed by the response averaging method).

Figure 5.12



And the confusion matrix for the prediction evaluation is shown below in Table 5.14

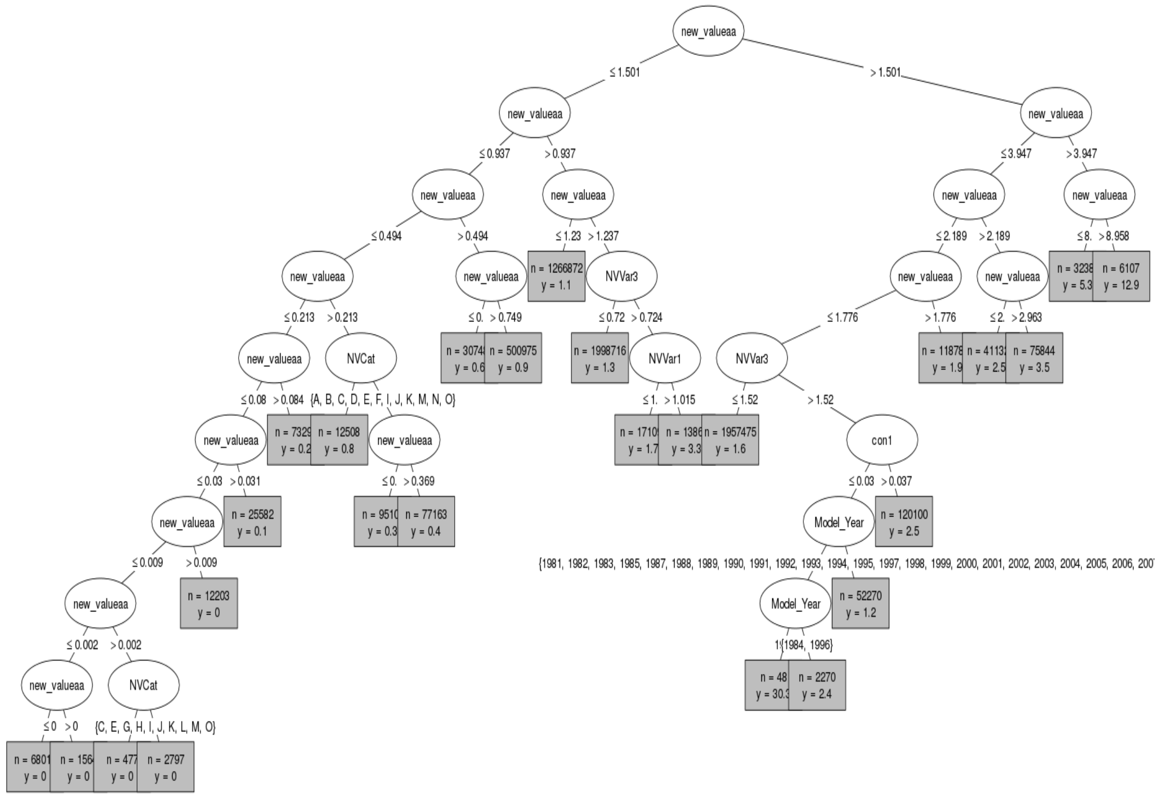
Table 5.14

Real value \ Prediction	Positive	Negative
Truth: positive	4643310	33198
Truth: negative	33852	528

This result is worse than all the results from logistic regression; the conditional inference classification tree notably fails to identify nonzero claims.

We then tried the conditional inference regression tree, and the covariates used are the same as for the classification tree. The tree model is shown as below in Figure 5.13:

Figure 5.13



The final normalized Gini coefficient of the conditional inference regression tree on the test set is 0.06190335, which, if it applied to the Kaggle test data, would rank 47th among contest results.

Comparing the result of Gini coefficient obtained by regression tree using different fitting method, the traditional regression tree performs better for our data set. The decision tree's result is worse than logistic regression model.

6.0 Computer Resources

All the models were run using R on a linux cluster, which contains two Intel E5-2670 2.60Ghz 20M Cache 8-Core 115W Processor, with 378 GB of shared memory used by other people, and we were not able to access all the 378 GB memory.

The cluster we used doesn't support parallel computing, so it takes long time to run most of the models.

7.0 Conclusion

The aim of this project was to compare the performances of various statistical models and methods on predicting the bodily injury liability insurance claim payments based on the characteristics of the insured's vehicles in a particular data set, which was used in a Kaggle data competition. We tried a number of methods, including principal component analysis, response averaging, the Tweedie model, and decision tree methods. The most successful methods, based on the normalized Gini coefficient, were regression tree with PCA, which on our test set gave a value of 0.080135. If our test set were comparable to the test set in the competition (a questionable assumption), our results would have earned 27th in the ranking. We also tried viewing the problem as classification, using logistic regression and classification trees to see how well these methods could predict whether a car will have insurance claim or not. The evaluation method for classification is the confusion matrix. Classification trees performed worse than logistic regression for this data.

One difficulty we faced was not knowing the exact values of many of the predictor variables, as these had been anonymized. This prevented us from making some informed choices we might have made in the presence of full information.

Another difficulty in predicting insurance claims using only the predictors given, is that many other factors contribute to the frequency and severity of car accidents including how, where and under what conditions people drive, as well as what cars they are driving.

But the most important influential predictors are actually related to the drivers, including their driving history, driving behavior, etc. Therefore, prediction only based on the car characteristics is almost an impossible mission, and the best predictions we could make were not particularly accurate, if the criterion were the difference between the prediction and the actual claim. Indeed, in this case, a not unreasonable strategy would be to predict a claim amount of 0 for all cases, since more than 99% of all claims are 0. In fact, in terms of classification, this strategy proved better than both the classification tree and logistic regression methods.

However, using the normalized Gini coefficient, which uses only the rankings of the predictions, as the measure of prediction quality, we were able to obtain respectable results as measured by the best contest entries.

Another challenging part of the project was to find a way to process the data and create a data set that could be used by the algorithms. Not all the algorithms are able to handle the presence of categorical variables, so those variables have to be transformed into numerical variables. Furthermore, transforming categorical variables that have over 2000 levels is even more difficult. From all the models we tried, PCA combined with Regression Tree has the best result. Although the Tweedie model is famous for the insurance claim prediction problem, the regression tree gave the most accurate result in this insurance claim prediction project.

Among the several methods we have tried, the regression tree algorithm has been proven the most efficient model for prediction given existing and limited resources. However, grid search has been identified as a method to find out the best settings of parameters for a decision tree analysis. As part of the future work, it might be beneficial to implement this method using parallel computing to better tune the parameters of the regression tree.

Reference

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Appendix A

Statistics Summary for Numerical Variables:

Name	Mean	Standard Deviation	Min	Max
Var1	-0.01011925	9.800609e-01	-2.5782218	5.143392e+00
Var2	-0.06508703	9.684165e-01	-2.4933927	7.829420e+00
Var3	-0.02543391	1.018902e+00	-2.7903352	5.563325e+00
Var4	-0.05456793	9.680170e-01	-2.5082161	7.589262e+00
Var5	0.003838594	9.910490e-01	-3.3503442	4.018167e+00
Var6	-0.04012272	9.792078e-01	-2.3766568	4.584289e+00
Var7	-0.02421288	1.006433e+00	-2.7784905	4.127148e+00
Var8	-0.05856059	1.003954e+00	-2.1630421	4.735074e+01
NVVar1	0.01468409	1.031040e+00	-0.2315299	6.627110e+00
NVVar2	0.01751169	1.038212e+00	-0.2661168	8.883081e+00
NVVar3	0.01354226	1.027748e+00	-0.2723372	8.691144e+00
NVVar4	0.01851377	1.034274e+00	-0.2514189	6.388803e+00
Claim_Amount	1.360658	3.900103e+01	0	1.144075e+04

Appendix B

Variable Name	Category Counts	**Examples and Counts**
Blind_Make	75	K: 1657185, Q 233255 AR 202083 D 174362 ……
Blind_Model	1303	A.1, A.2,…,A.15,…,B.1,…
Blind_Submodel	2740	A.1.1,…,B.2.0,…,D.5.2,…
Cat1	11	D 2487951 B 4017739 J 233968 G 782602 ……
Cat2	4	C 5895027 ? 4874164 A 2191054 B 224045
Cat 3	7	F 872031 A 7488029 B 2256802 ……
Cat 4	4	? 5631649 A 5723163 C 1454425 B 375053 ……
Cat 5	4	? 5637321 A 6683980 C 779280 B 83709 ……
Cat 6	6	C 3677694 E 1173316 ? 25981 ……

Cat 7	5	? 7167634 C 4618653 A 1050621 ……
Cat 8	4	C 880481 A 8626513 B 3673932 ? 3364
Cat 9	2	A 2333508 B 10850782
Cat 10	4	B 3969170 A 8573092 C 638111 ? 3917
Cat 11	7	F 787998 B 3174528 E 816595 ……
Cat 12	6	D 3525723 B 4348276 C 3619974 ……
OrdCat	8	4 5935475 5 2964704 2 4146321 ……
NVCat	15	M 5767944 O 3416948 F 325556 ……