

# System Identification of Structures subjected to Human-Induced Vibrations using State-Space Modelling

A Major Qualifying Project  
Submitted to the Faculty  
of  
WORCESTER POLYTECHNIC INSTITUTE  
In partial fulfillment of the requirements for  
the Degree of Bachelor of Science  
by:

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April 27, 2016

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This report represents the work of one WPI undergraduate student submitted to the faculty as evidence of completion of a degree requirement. WPI routinely publishes these reports on its website without editorial or peer review. For more information about the projects program at WPI, please see <http://www.wpi.edu/Academics/Project>.

## Abstract

This report combines concepts of structural engineering with the field of biomechanics, through human-induced, structural loading. The study analyzes the response of a steel I-beam under the activity of jumping. A state-space system identification approach was utilized to produce a model which could adhere to various impact tests and code simulations. This model was then used to generate simulated output responses in order to monitor the overall health of the structure. The report includes the analysis and design of the provided structure, along with the system identification and continuous-time representation of the state-space model. A number of PZT devices and computational software were used for this process. Results of this study were evaluated to determine effects on human comfort and structural performance. The paper concludes with finishing thoughts and future recommendations.

*Keywords:* human-body force (HBF), state-space, system identification (SID), structural dynamics, natural frequency, fast Fourier transform (FFT)

## Acknowledgements

I would like to thank everyone who provided insight and support throughout the duration of this Major Qualifying Project. Furthermore, I would like to give a special thanks to the following:

- Advisor/Assistant Professor, Yeesock Kim, of the Civil and Environmental Engineering Department for aiding me in my studies and connections
- Dr. Yeon Sik Noh of the University of Connecticut for helping me to apply wireless accelerometers to human bodies for data testing and acquisition
- Laboratory technician, Russ Lang, for installing the steel I-beam used in this project
- Laboratory manager, Don Pellegrino, for allowing and aiding me with the use the laboratory wired accelerometers and impact hammer

## Capstone Design Statement

To achieve the necessary criteria developed by the Accreditation Board for Engineering and Technology (ABET), this Major Qualifying Project (MQP) incorporated a design experience which considered appropriate engineering standards and realistic constraints. This section provides a description of the design problem, the taken approach, and how the constraints were addressed.

### Design Problem

This design problem encompasses the structural response analyzation of human-induced loading with concerns towards human comfort and structural performance. Human-induced activity can account for a substantial portion of the live loads acting on any given structure. Such structures could be office floors, stadiums, pedestrian footbridges, or residential buildings. As a dynamic impact, these induced loads may generate excessive vibrations, affecting comfort levels, and safety and serviceability performance. This issue has been a concern in the civil industry for many years, however the study of human-induced loading on a structure poses a vast amount of complexity. Unlike natural impacts such as wind, snow, and seismicity, human-induced activity depends solely on the individuals'. For instance, situations where system fatigue or human discomfort may be present are often a result of various human motion responses under diverse loading situations. Due to this intricacy, computational software such as MATLAB and Simulink were needed.

### Approach

To solve this problem, training was instituted for various piezoelectric (PZT) devices, structural concepts, and engineering software. Once trained, a system had to be established for testing purposes. With the aid of the Worcester Polytechnic Institute (WPI) Civil & Environmental

Engineering Department a standard steel I-beam was bolted into the floor, producing a nearly-fixed connection. Numerous data tests were then generated and assessed accordingly. From this data, output responses were developed using system identification and state-space representation methods. These responses were analyzed to perceive the potential effects on human comfort and structural performance, and recommendations were made accordingly.

## Realistic Constraints

According to ABET General Criterion, realistic design constraints must be assessed within the capstone experience. To achieve this standard, the following constraints were considered within the aspects of the MQP: economic, health and safety, ethical, sustainability, social, and constructability.

### Economic

When matched with a structural design, perceived human comfort levels may be utilized to design structures which can withstand excessive floor vibrations. Furthermore, future research will entail fatigue limits, which may allow for a general understanding of the amplitude of cyclic stress that can be applied to the steel I-beam prior to systematic failure. This data could prove useful for establishments attempting to design future space, while constructing a more relaxed environment.

### Health and Safety

Since this project deals with human-induced loading and a standard steel beam, codes such as the International Building Code (IBC) and ASCE-7 are not too beneficial. However, by

generating and scaling the human comfort levels perceived through various activities, the health and overall safety of future participants can be ensured.

## Ethical

This major qualifying project upholds the standards of the American Society of Civil Engineers' (ASCE) Code of Ethics. The Code of Ethics consists of seven canons, and although all canons were considered during the process, Canon 1 appeared to be the most relevant and important. Canon 1 states that, "Engineers shall hold paramount the safety, health and welfare of the public and shall strive to comply with the principles of sustainable development in the performance of their professional duties" [14]. Recommendations in this report abide by this law through the promotion public safety and structural enhancement.

## Sustainability

Sustainability is not a direct factor in this project. However, though the utilization of structural response data and the correlation amongst human comfort levels, establishments may be able to develop structures with less internal vibrations. Through the reduction of floor vibrations, human productivity may increase due to sustainable comfort levels, and therefore generate a greater output in work.

## Social

Social constraints also play an important role as the goal behind this project is to allow establishments such as office buildings, sport complexes, or other high-activity environments to operate in vibration-free zones. This could result in a more approachable surrounding for people to socialize and attain productivity in.

## Constructability

Although a complete system was not analyzed during this project, the steel I-beam underwent numerous impact tests in distinct locations, and utilized various piezoelectric (PZT) devices to verify optimal selections. Furthermore, a number of state-space models were implemented using MATLAB's system identification application. As a result, an *SSEST* model estimation method was selected to support the tested load responses.

## Professional Licensure Statement

A licensure is the granting or regulation of a license(s) which allows one to perform specified acts legally. In this case, a professional licensure plays a significant role in one's career as it symbolizes that one has become a professional engineer (PE) and meets specific qualifications in their education and work experience to uphold and protect the public. In the United States, engineering licensure is governed by state through each state's licensure board. However, a general four-step process is required regardless:

1. Earn a four-year degree in engineering from an accredited engineering program
2. Pass the Fundamentals of Engineering (FE) exam
3. Complete four years of progressive engineering experience under a PE
4. Pass the Principles and Practice of Engineering (PE) exam

To further detail, the process of achieving a professional licensure first begins with becoming an engineer intern. By graduating from an accredited four-year engineering program approved by one's state licensure board and successfully completing the FE exam, it signifies that one has mastered the fundamental requirements necessary to continue forwards. It is then necessary to gain professional experience in the field of engineering. Every state requires that candidates complete at least four years of qualifying engineering experience, which is typically held under the supervision of a PE. It is helpful to note that most of these qualifying experiences occur after graduation rather than during. After years of experience, the next step is to learn one's state licensure requirements. Every state has its own licensure board which administers distinct exams and required qualifications. Once this is understood, an engineer can prepare for and take the PE exam in hopes to attain a professional licensure and become a PE. In order to maintain a

PE license, most states require that the PE continues their professional competency. In all cases, however, a renewal fee must be instituted either annually or bi-annually. [15]

Obtaining a licensure proves significant importance to the profession of engineering, the public, and the individuals themselves. A licensure is important to the profession as it proves that one has met the standards, education, and experience to serve as a professional engineer and uphold the standards of those with the same mastery. To the public, a licensure exhibits that one has committed their art in protecting and servicing the people. It signifies that one has the credentials to earn trust, respect, and take on higher levels of responsibility. For the individual, a licensure symbolizes a sense of pride and achievement. It demonstrates an adoration for the field and future work of the engineer. [15]

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## 1.0 Introduction

The study of human-induced vibrations plays a vital role in the field of civil engineering. Each day, human body forces (HBF) generate a substantial portion of the live loads expelled on structures such as floors, stadiums, assembly halls, and footbridges. These HBF can produce excessive vibrations within a system, which can potentially effect human comfort levels and structural performance. This has become a major concern in areas such as the United Kingdom and South America, where “predicting the structural response due to crowd-induced vibrations is now recognized as an important aspect in the design and/or performance assessment” [3].

The effects of human-induced loads date back all the way to the early 1800’s when soldiers used to march in unity across footbridges. One of the first appearances of these effects occurred in April 1831 where “a brigade of soldiers marched in step across England’s Broughton Suspension Bridge...the bridge broke apart beneath the soldiers, throwing dozens of men into the water” [2]. Due to the synchronization of the soldiers’ movement, mechanical resonance was generated within the structure which amplified the internal vibrations and resulted in systematic failure. In order to avoid further issues armies commanded their men to break stride when crossing bridges; however further analyzation of the occurrence was not prominent.

Human-induced loads have affected the design of more recent structures, too. Although the United States has not received many concerns involving human-induced issues, regions such as Europe, South America, and the United Kingdom have experienced a considerable amount of detriment and human discomfort. This may be a result of the increased excitation of dynamic crowd loads during sporting and musical events. Some examples of these occurrences have been provided below:

- In the summer of 1985, the Nya Ullevi Stadium, located in Gothenburg, Sweden, experienced effects of human-induced vibrations during a number of concerts. During these concerts, the audience jumped in time to the music which excited the clay layer from the surface with the same frequency as the beat of the music. It was found that the frequency was close to 2.4 Hz. This vibrated the whole deposit underneath the stadium, causing structural damage and people to seek safer locations. [5]
- In 1993, the Maracanã Stadium, located in Rio de Janeiro, Brazil, encountered heavy human-induced vibrations during a number of ongoing football games. With a stadium capacity of roughly 150,000 people, high levels acceleration and large displacements could be felt during the games. The lowest natural frequency of the empty cantilever stands was reported to be 4.6 Hz. Due to these vibrations, cracks were developed in the beams. These cracks may have led to systematic failure if not taken care of sooner. [6]
- On June 10, 2000, the Millennium Bridge, located in London, England, opened for a crowd of over 1,000 people. As the crowd began to walk across the bridge, there was an unexpectedly pronounced lateral movement of the deck. The frequency of excitation was close to the mean pacing rate of walking of about 2 Hz. This movement was enough to cause people to stop walking and retain their balance. The bridge was immediately closed and limited to the number of people it could carry. [7]

Today, human-induced loads pose an increased concern within the civil industry as “the aesthetic demand of human beings and recent advances in material and fabrication technologies have enabled the design and construction of stylish, light and slender long span structures” [1]. Through the use of this design criteria, structures have become more flexible, “resulting in relatively low natural frequencies that are more easily excitable by human occupants” [4]. However, the effects of these HBF are a topic not typically discussed within the civil industry. Perhaps this is due to the intricacy and lack of technology necessary to produce accurate results,

or that excitations such as wind, fire, and seismicity are more influential in society. Nevertheless, the impact of human-induced loads has become a crucial topic for future engineers.

This report focuses on the response analyzation of a steel I-beam under human-induced jumping, using concepts of structural dynamics. The study includes the analysis and design of the provided structure, along with the model estimation and continuous-time representation of the state-space function. Results of the study were evaluated to determine the potential effects on human comfort and structural performance. The report concludes with finishing thoughts and future recommendations.

## 2.0 Background

Chapter 2.0 provides insight on the key concepts and background information utilized within this study. This chapter was written to pertain specifically towards jumping for correlation purposes. Section 2.1 discusses the context of human body forces and explores the potential effects of human loading from both a structural and personal perspective. Section 2.2 evaluates a number of past studies who have conducted dynamic analyzations of jump-like movements. Various methods of analyzation were observed, along with their results and conclusions.

### 2.1 Human Body Forces

Human body forces (HBF), such as walking, running and jumping, generate a significant amount of live loads deployed on structures each day. Increased attention has been brought to this topic due to its potential effects on system performance and human comfort. However, HBFs prove a difficult topic for engineers due to their complex and time-consuming nature. During this study, the human activity of jumping was analyzed and evaluated based on the potential effects it had on a provided structural member. In order to assess the situation properly, background information was researched and implemented as seen necessary.

Like all human body forces, jumping or jump-like movements, act as a type of dynamic loading. These loads obey under Newton's Second Law, which states that:

$$F = ma \quad (1)$$

*Where,*

*F = Force*

*m = Mass*

*a = Acceleration*

Likewise, a corresponding exerted force, called a ground reaction force (GRF), can be determined through the equation [31]:

$$F_{GR} = \sum_{i=1}^s m_i(a_i - g) \quad (2)$$

Where,

$F_{GR}$  = Ground Reaction Force

$g$  = Static Acceleration due to Gravity ( $9.81 \frac{m}{s^2}$ )

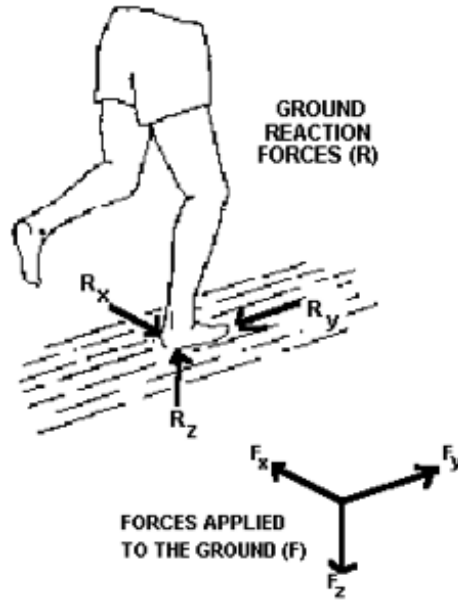


Figure 1: Ground Reaction Force (GRF)

The nature of jumping is an appropriate topic when introducing one to the field of structural dynamics. This is due to the somewhat ‘simplicity’ of the movement. Unlike walking and running, which contain various impacting factors due to the gait cycle, jumping acts as a rhythmic activity. This can be noted for both continuous and single-jump patterns. Jumping is also typically measured as either countermovement or squat [32].

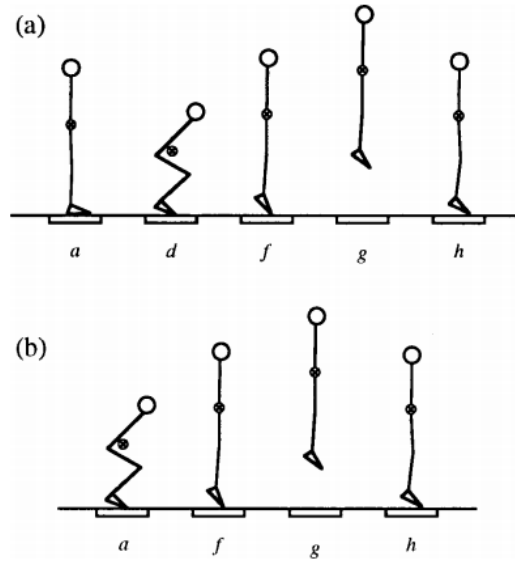


Figure 2: (a) Countermovement Jump; (b) Squat Jump

In a countermovement jump, “the jumper starts from an upright standing position, makes a preliminary downward movement by flexing at the knees and hips, then immediately and vigorously extends the knees and hips again to jump vertically up off the ground” [32]. A squat jump embodies most of the same actions as a countermovement, however the jumper starts from a stationary semi-squatted position and does not employ a preliminary downward phase. In terms of this report, a countermovement jump was analyzed as it is a “much more natural jumping movement and most people can jump several centimeters higher in a countermovement jump than in a squat jump” [32].

### 2.1.1 Effects on Structural Performance

The nature of jumping, or jump-like movements, has a high probability of impacting structural performance compared to other movements. This is potentially due to the isolation of system loading, which can be observed to a greater extent in single jumps and heel-drop impacts as they act as impulsive loads [24]. Furthermore, the effects of crowd synchronization on jump-like movements have been notorious for producing structural fatigue; as noted in Chapter 1.0.

According to recent studies, the typical frequency range for movements such as jumping hovers roughly around 1.5 Hz to 3.5 Hz [22]. This can be detrimental for systems that are not designed to handle such low frequencies, as mechanical resonance, or an increase in amplitude range due to frequency correlation, could occur. However, a number of building codes have been developed for structural guidance on addressing this issue. For instance, vibrations due to rhythmic activities were recognized in a Commentary to the 1970 National Building Code of Canada (NBC), where it was stated that “resonance due to human activities can be a problem if the floor frequency is less than 5 Hz” [24]. It was also researched that structures that move significantly make it impossible to jump at (or very near) the natural frequency of a structure; thus removing any possible chance of resonance [28]. This is due to the fact that “as the human-induced vibrations increased, feet-structures contact time simultaneously increased, so the body motion became more bouncing-like than jumping” [28]. However, if structures are built stiffer, they are more likely to generate higher human-induced forces. These are key concepts to keep in mind when designing for human body forces.

### 2.1.2 Effects on Human Comfort Levels

Similar to structural performance, human comfort levels can also be significantly impacted by jump-like movements. This is most likely a result from excessive vibrations, or an increase in the response amplitude range within a system. However, when attempting to design for comfort there are a few points to consider.

The first, is that the reaction of people who feel vibration depends very strongly on what they are doing. For instance, according to *Floor Vibrations due to Human Activity* by Murray et al., when dealing with a frequency range of 4-8 Hz, “people in offices or residences do not like ‘distinctly perceptible’ vibration (peak acceleration of about 0.5% of g), whereas people taking

part in an activity will accept vibrations approximately 10 times greater (5% of g)” [24]. Likewise, people dining beside a dance floor, lifting weights in a gym, or standing in a shopping mall may tolerate vibration up to 1.5% of g [24].

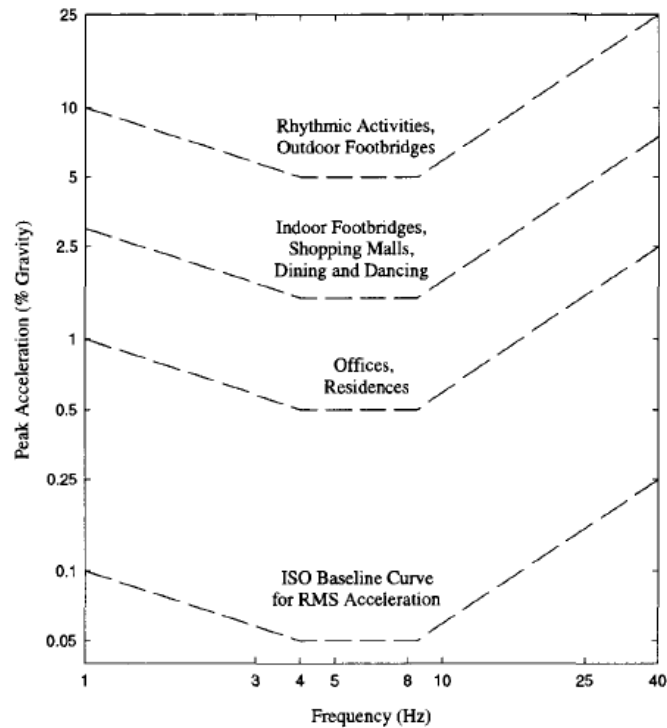


Figure 3: Recommended Peak Acceleration for Human Comfort

The second consideration, is that “there are no generally accepted international standards for comfort criteria” [27]. Research from the *Vibrations and Shock Handbook* by Clarence W. de Silva, indicated that levels of human comfort perception were generated based on a number of physiological and physiological parameters. These parameters included the “occupant’s expectations and experience, activity, body posture, and orientation; visual and acoustic cues; and the amplitude, frequency, and acceleration of both the translational and rotational motion to which the occupant was subjected” [27]. In this study, affected human perceptions of motion and vibration in the frequency range of 0-1 Hz were produced. The following figure can provide one with a suitable baseline for design criteria of tall structures such as offices and hotels.

| Level | Acceleration<br>(m/sec <sup>2</sup> ) | Effect                                                                                                                                |
|-------|---------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| 1     | <0.05                                 | Humans cannot perceive motion                                                                                                         |
| 2     | 0.05 to 0.1                           | Sensitive people can perceive motion; hanging objects may move slightly                                                               |
| 3     | 0.1 to 0.25                           | The majority of people will perceive motion; the level of motion may affect desk work; long-term exposure may produce motion sickness |
| 4     | 0.25 to 0.4                           | Desk work becomes difficult or almost impossible; ambulation still possible                                                           |
| 5     | 0.4 to 0.5                            | People strongly perceive motion; it is difficult to walk naturally; standing people may lose their balance                            |
| 6     | 0.5 to 0.6                            | Most people cannot tolerate the motion and are unable to walk naturally                                                               |
| 7     | 0.6 to 0.7                            | People cannot walk or tolerate the motion                                                                                             |
| 8     | >0.85                                 | Objects begin to fall and people may be injured                                                                                       |

Figure 4: Human Perception Levels

## 2.2 Prior Studies

Recent studies that have analyzed human induced loads have been investigated to assess potential methods for research. All three studies pertain to the action of jump-like movements, strictly for correlation purposes. Analyzing this data allowed for a stronger understanding of both the issues and the possible solutions utilized.

### 2.2.1 Study 1

The first study, *Human Induced Dynamic Loads Estimation Based on Body Motion*, conducted by Paolo Mazzoleni and Emanuele Zappa, discussed the effects of human jumping and bobbing on a dynamometric platform [22]. 42 tests (21 jumping, 21 bobbing) were performed and evaluated for “the vertical force induced by the movement of the volunteer on the ground, the vertical acceleration of 8 parts of the body, and a video of the motion of the volunteer” [22]. The investigated frequency range was 1.5 to 3.5 Hz, with a step frequency of 0.1 Hz.

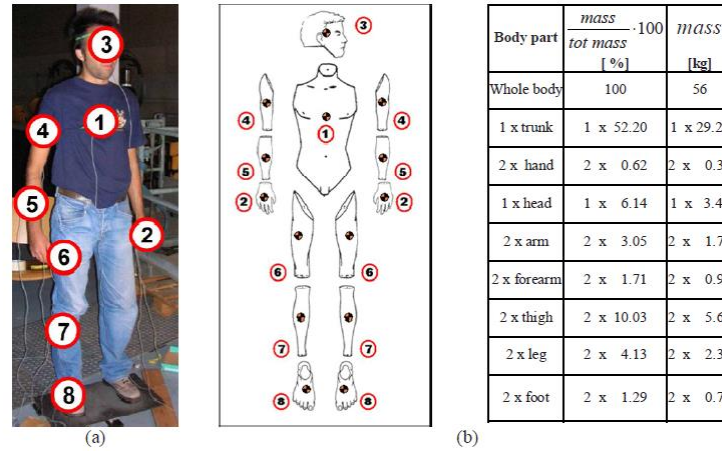


Figure 5: (a) Picture of accelerometers during tests; (b) Reference Chandler's Model

The first evaluation, vertical forces induced by the volunteers' movement on the ground, was the main focus when analyzing this report. Results compared the data between measured and estimated forces at a frequency of 2.5 Hz. A formula for percentage error was the used to determine the accuracy of the recorded data. Although the approximation was acceptable, Mazzoleni and Zappa concluded that "high frequency errors may be due to an imperfect fastening of the accelerometers" and "low frequency discrepancies...may be attributed to no negligible rotations in such kind of movement" [22].

Evaluations for the vertical acceleration of body parts and volunteer motion were also analyzed in this report. These tests used digital image correlation (DIC) code to develop reliable acceleration estimations of individual body parts. The DIC results were then arranged in a 3D matrix and video processing of human motion was applied using a greyscale 1024x521 global shutter CCD camera. The processing system combined previous data with that of the 3D matrix, using velocity as a third dimension. The results proved to be more accurate, when implanting the percentage error formula, than those previously in the study. The authors concluded that this may have occurred due to the fact that "a non-contact measuring technique is not subjected to problems

related to sensor fastening”, but rather, “the neglecting rotation of some parts of the body in these kinds of movements” [22].

### 2.2.2 Study 2

Study 2, *Structural Health Monitoring of a Stadium for Evaluating Human Comfort and Structural Performance*, administered by Hasan Ozerk Sazak, analyzed the human induced vibrations occurring during football games with respect to human comfort levels defined by international codes [8]. The structure under monitoring was a steel-framed stadium encompassing 25 acres and an approximate 45,000 seating capacity. The report focused on two student sections of the stadium, as “students mostly get more excited during the games and also the university band was located close” [8]. High sensitivity accelerometers, PCB 393C, were utilized to collect crowd-induced, structural responses over a span of eight games. These responses were recorded before, during and after each game in order to distinguish results. A single set of the experimental data was then extracted and used as a forcing function for a developed finite element (FE) model.

To develop an FE model, Sazak initially compared the measured data with a symmetric half-sine function, as it was assumed that “a series of identical half-sine wave pulses may be represented by measured jumping force pulses” [8]. However, none of the jumping harmonics aligned with previous data. Therefore, to achieve a reliable model, a small scale experiment was conducted on a fixed, 4-foot, C-type beam. Accelerometers, PCB 603C01, were placed under the girder of the beam at identical spans, lengthwise. Crowd impacts for jumping were then simulated using a total of three participants and a number of different loading combinations. Data sets for acceleration were recorded and compared with the acceleration results from the FE beam model in order to “verify the validity of the time-history application” [8]. These results were based on the

first mode frequencies. A single matching forcing frequency, depicting a random-jumping scenario, was extracted for further use.

Once the validated loading function was achieved, it was applied to a retrofitted FE model representing the structure of the selected stadium sections at full capacity. The purpose of the model was to “identify the changes in vibration level and dynamic characteristics with increasing the stiffness of the section by implementing a retrofit to structural members” [8]. After retrofitting a number of beams, and comparing their time domain and frequency domain results, “It was identified that retrofit application was successful in both decreasing the vibration level and increasing the stiffness [modal frequencies]” [8]. The observed results showed that “vibration levels dropped nearly to half of its value before retrofitting and modal frequencies increased by nearly thirty percent for the first two main frequencies”, thus removing the first mode frequency out of damaging range [8].

The author concludes the study by stating that “synchronized motion events mostly created uncomfortable or very uncomfortable situation for the spectators as per the ISO 2631 code” and that “even when the maximum value of the vibration (raw signal) of an event was not very high, the situation it created for the spectators (RMS signal for ISO 2631) may induce more uncomfortable conditions depending on the duration of the event” [8].

### 2.2.3 Study 3

Study 3, *Floor Vibration Induced by Dance-Type Loads: Verification*, by B. R. Ellis and T. Ji, discussed the testing procedures performed for verifying an analytical method of calculating floor vibrations induced by dance-type loads [29]. Tests were recorded on a precast reinforced concrete beam with dimensions 3.0 x 0.4 x 0.083 m and simply supported ends 0.1 m in. Vibrational responses from movements such as jumping and stamping were monitored using

“three transducers mounted under the center of the beam, to measure acceleration, velocity and displacement” [29]. Ellis and Ji recorded a series of experiments depicting human involvement for a number of possible hypotheses:

1. Human involvement as an additional mass-spring-damper system on the beam
2. Human involvement as a load on the beam
3. Human involvement as a load and an additional mass-spring-damper system on the beam

Scenarios 1 and 3 were evaluated based on the assumption that “crowd[s] act as a rigid mass, which implies that the added mass will effectively reduce the fundamental frequency of the floor” [29]. The fundamental frequency of the beam was recorded both bare and damped situations. When there was a person standing or sitting on the beam, the “frequency increased and the damping value increased significantly whereas, when a mass equivalent to the mass of the person was placed on the beam, the frequency decreased and the damping value remained unchanged” [29]. Although this data suggested that a person acts as an additional mass-spring-damper system, it was “identified experimentally that people jumping only act as a load and not as an additional mass on the floor” [29].

Scenario 2 discussed human involvement strictly as a load on the beam. An analytical model was developed using a set of equations which calculated the gravitational acceleration for vibration due to rhythmic, dance-type activities. These equations were derived using “beam theory and the first Fourier component of load” [29].

$$f_{11} \geq f_p \sqrt{1.0 + 1.3 \frac{r_1 g m_l}{a_0 m}} \quad \dots(5)$$

This criterion was derived from the following formula by assuming  $\zeta = 0$

$$\frac{a}{g} = 1.3 \frac{r_1 m_l / m}{\sqrt{\left[ \left( \frac{f_{11}}{f_p} \right)^2 - 1 \right]^2 + \left( 2\zeta \frac{f_{11}}{f_p} \right)^2}} \quad \dots(6)$$

where

$a/g$  is the acceleration as a fraction of the acceleration due to gravity  
 $r_1$  is the first dynamic load factor  
 $a_0$  is the limiting value of acceleration

Figure 6: Design Criteria for Analytical Model

The model was then compared with numerical measurements obtained from responses of human jumping at roughly 2.0 Hz. Measurements were accomplished using a direct integration method within the finite element program, LUSAS, along with the standard dynamic equation for damped forced vibration [29].

*Correlation between the analytical and numerical solutions*  
The basic dynamic equilibrium equation for a finite element system is

$$M\ddot{U} + C\dot{U} + KU = F(t) \quad \dots(2)$$

where

$M$ ,  $C$  and  $K$  are the mass, damping and stiffness matrices  
 $F(t)$  is the external load vector in which the variation with time is defined by eqn. (1) in ref. 1  
 $\ddot{U}$ ,  $\dot{U}$  and  $U$  are the displacement, velocity and acceleration vectors of the finite element assemblage

Figure 7: Dynamic Equilibrium Equation for Finite Element System

Results of this study noted that the analytical and numerical methods coincided once they reached a steady-state; roughly 1.5 seconds in. Furthermore, the peak acceleration calculated through the numerical method was nearly identical to that of the analytical method, illustrating that the results were quite accurate. Ellis and Ji concluded the report stating that “it is not safe to apply design criterion based on simply supported beam for all types of floors” [29].

### 3.0 Methodology

This chapter discusses the structural properties, technology and process taken towards achieving a baseline model for the analysis of human-induced jumping loads on a steel I-beam. The process employed during this project was broken down into two phases. The first phase was considered to be the “trial and error” phase. This phase focused on the development of a state-space model using matrix and beam property alteration. The second phase was an “alternative method” phase. During this phase, a number of estimation algorithms were performed using system identification (SID).

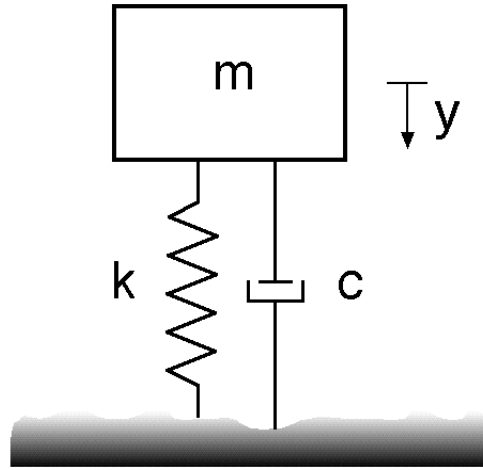
#### 3.1 Structural Properties

This project focused on the impact analysis of human-induced jumping on a steel I-beam provided by the Worcester Polytechnic Institute (WPI) Civil & Environmental Engineering Department. Due to the rugged layout of the floor, the structure was bolted down to produce a nearly-fixed support. Calculations were based on the length of the beam from support to support and disregarded the one foot overhang on each side as shown below.



*Figure 8: Semi-Fixed Steel I-Beam*

During this study, the beam was treated as a damped single-degree-of-freedom (SDOF) system, consisting of a mass ( $m$ ), a stiffness ( $k$ ), and a damping coefficient ( $c$ ). These factors enabled the use of functions such as state-space modelling and Fast-Fourier Transform (FFT), which have been discussed later in the report. The standard process used to achieve these values has been presented below. Further detail of these calculations can be found in Appendix A.



*Figure 9: SDOF System*

The mass, stiffness and damping parameters of the steel I-beam were generated using a number of formulas, and deduced and calculated values. A fixed support system was assumed to develop a foundation for testing.

$$m = \rho V \quad (3)$$

*Where,*

$\rho = \text{Density}$

$V = \text{Volume}$

$$k = \frac{12EI}{L^3} \quad (4)$$

*Where,*

$E = \text{Modulus of Elasticity}$

$I$  = Moment of Inertia around the Centroid

$L$  = Length

$$c = 2 * m * \omega_n * \zeta \quad (5)$$

Where,

$\zeta$  = Damping Ratio (2%)

$\omega_n$  = Natural Frequency

In order to achieve the natural frequency of the structure, the following equation was used:

$$\omega_n = \sqrt{\frac{k}{m}} \quad (6)$$

Using the presented material, values for mass, stiffness and damping were calculated out to be roughly 1215 lb, 1.2E5 lb/in and 488 lb\*s/in, respectively. The natural frequency was also computed as approximately 10 Hz. These values were utilized throughout the study for comparison and alteration purposes.

### 3.2 Technology

During this study two accelerometers, one wireless and one wired, and an impact hammer were tested. All three mechanisms functioned under the piezoelectric (PZT) effect by using “microscopic crystal structures that get stressed by accelerative forces, causing a voltage to be generated” [16]. This section aims to discuss the following PZT devices along with their corresponding specifications.

An accelerometer is a device that measures the acceleration of a system and illustrates the data through a frequency response. Accelerometers evaluate using a specified sensitivity, or scale factor, which can be noted as “the ratio of the sensor’s electrical output to mechanical input” [26].

This sensitivity may also have a stated tolerance, such as  $\pm 5\%$  or  $\pm 10\%$  [26]. Furthermore, each accelerometer operates based on its own amplitude range, which defines the “maximum [or minimum] amplitude vibration that can be measured before distortion occurs” [30]. This energy may be specified in units of vibration, “g”, or gravitational acceleration; indicating the intensity of a gravitational field expressed in meters per second squared ( $\text{m/s}^2$ ).

The wireless accelerometer, MMA8652FC, was implemented using the Bluetooth program, BlueSoleil, and a portable dongle. The MMA8652FC is a three-axis sensor with 12 bits of resolution. It contains a dynamic range of  $\pm 2\text{g}$ , which produces a digital output sensitivity of 1024 counts/g [9]. During testing, the accelerometer was set to record data at a rate of 360 Hz, or 360 samples/sec. Furthermore, velcro padding was applied to the back to allow for a more straightforward and secure application.



*Figure 10: Wireless Accelerometer*

A wired accelerometer was also needed to gather accurate testing data. This device, 780C, was connected to the steel beam using a screw-in connection. The accelerometer contains an acceleration range of  $\pm 80\text{g}$  and produces an analog output sensitivity of  $100 \text{ mV/g}$  [10]. During testing, the accelerometer recorded data at a rate of 1652 Hz.



*Figure 11: Wired Accelerometer*

To produce the most accurate model possible, an impact hammer was synchronized to the wired accelerometer system. This device, Model 2304, had an output range of 5000 lbf and could produce a maximum force of 8000 lbf. The weight of the head was 3.0 pounds and the device's total length was 15.2 inches [11].



*Figure 12: Impact Hammer*

The engineering programs applied in this project were LabVIEW, MATLAB and Simulink. LabVIEW, or Laboratory Virtual Instrument Engineering Workbench, is a system design software used for visual programming. This software aided in the gathering of impact responses which were converted into text and excel files for further implementation. For this report, two virtual

instrument (vi) layouts were utilized to produce the given results. MATLAB, or Matrix Laboratory, is a numerical computing program which uses fourth-generation programming language to solve complex problems. MATLAB was employed to develop and analyze various response files through a number of generated codes. Concepts such as state-space modelling and Fast-Fourier Transform (FFT) were also evaluated through the use of this program. Simulink, a program within MATLAB, was applied to this study as well. Simulink is a graphical program used for modelling, simulating, and analyzing dynamic systems. This software was applied in order to construct the final model of the project, along with the ability to compare multiple frequencies using set model configuration parameters.

### 3.3 Phase I: Matrix & Property Altercation

Phase I focused on the development of a state-space model using matrix and structural property alteration. State-space models “use state variables to describe a system by a set of first-order differential or difference equations, rather than by one or more nth-order differential or difference equations” [19]. This modelling method was utilized because it can handle systems with multiple inputs and outputs, provides quick estimations, and produces time-domain solutions [12]. The steps taken during this phase have been broken down into three segments:

1. Gather structural responses using the impact hammer
2. Run the impact force through a state-space system
3. Match the impact hammer structural response with the model output response

These steps have been discussed in further detail within the following sections.

### 3.3.1 Step 1: Gather Structural Responses

In order to develop a baseline model for human-induced loads, responses were generated using the provided impact hammer and wired accelerometer. Originally the goal was to simply use the wireless accelerometer for all tests, however, the device seemed to not produce any structural response signals. This may have been a result of too low of sensitivity, or possible digital connectivity issues through Bluetooth.

With the aid of Russ Lang and Don Pellegrino, laboratory technicians for the Civil & Environmental Engineering Department at WPI, the steel I-beam and PZT devices were setup for testing through National Instruments (NI). In order to record data, the wired accelerometer was screwed in at the top-center of the beam's flange. Impacts were struck at the direct center and vibrational responses were recorded in a corresponding LabVIEW VI file.



*Figure 13: Test in Progress*

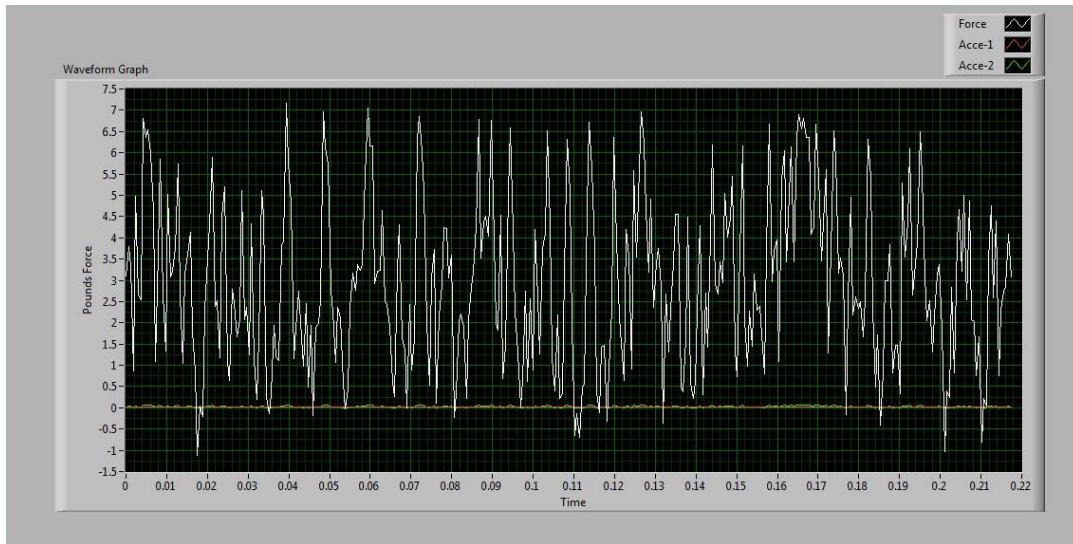


Figure 14: VI File Layout for Wired Accelerometer & Impact Hammer

Impacts during the testing process attempted to hover around 200 pounds, as this was the general weight of my body at the time. However, since the sensitivity of the impact hammer was so high, numerous data sets needed to be generated to provide accurate results. For the purposes of the developed model in Phase I, the following response selections were used.

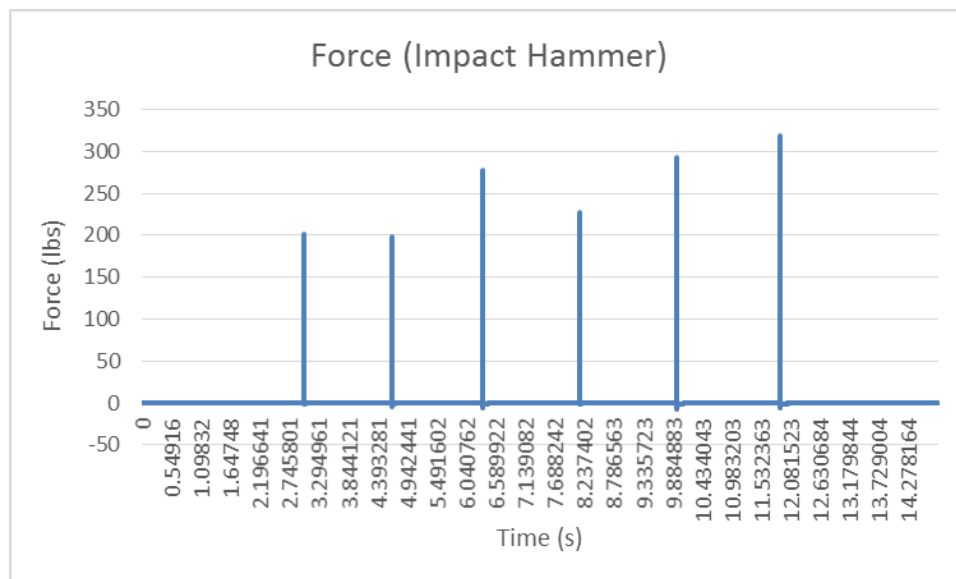


Figure 15: Input Response (Impact Hammer)

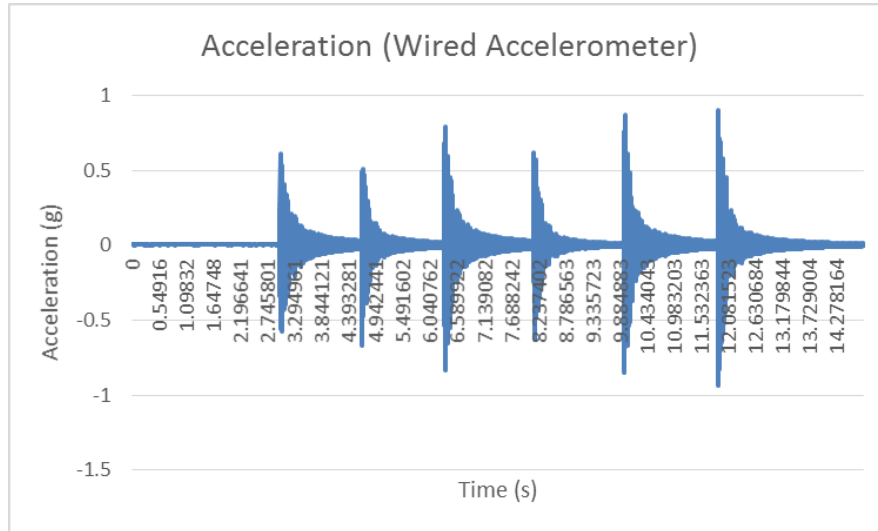


Figure 16: Output Response (Wired Accelerometer)

### 3.3.2 Step 2: Model Responses

Step two was to model the gathered responses through state-space system design. To accomplish this, a code was developed using the engineering software MATLAB and Simulink. Input data focused on a single frequency response for simplification and modelling purposes. The selected impact force was registered at 201.5 pounds.

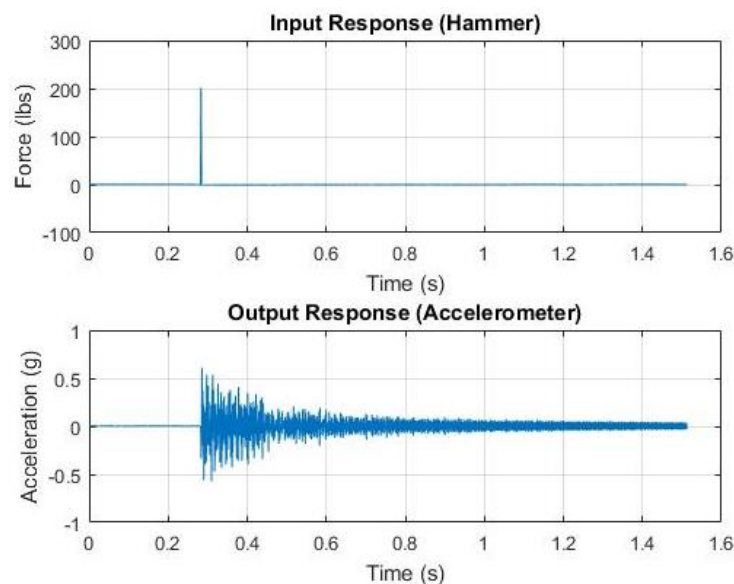


Figure 17: Response Data for Model

The general state-space representation of a linear system for a given number of inputs ( $u$ ), outputs ( $y$ ), and state variables ( $x$ ) has been illustrated below in Figure 18. This representation is applicable to both single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) systems.

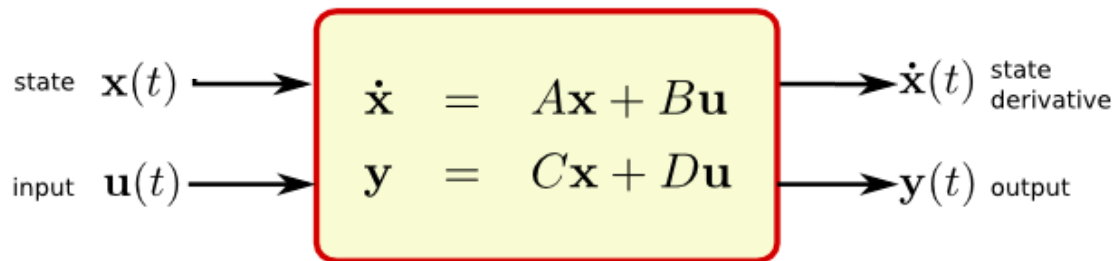


Figure 18: General State-Space Representation for a Linear System

The final development of Phase I's model required the implementation of the force and acceleration data responses, along with the parameters set for the state-space system equation mentioned above.

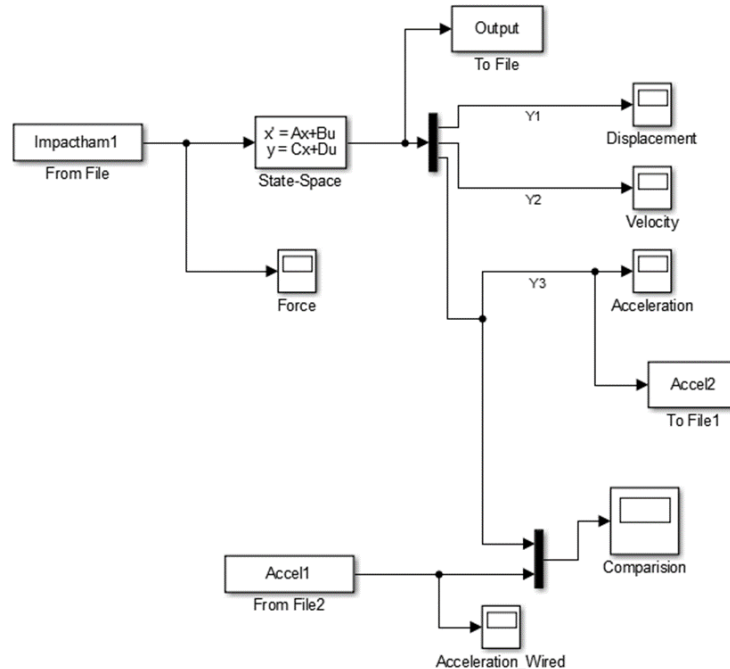
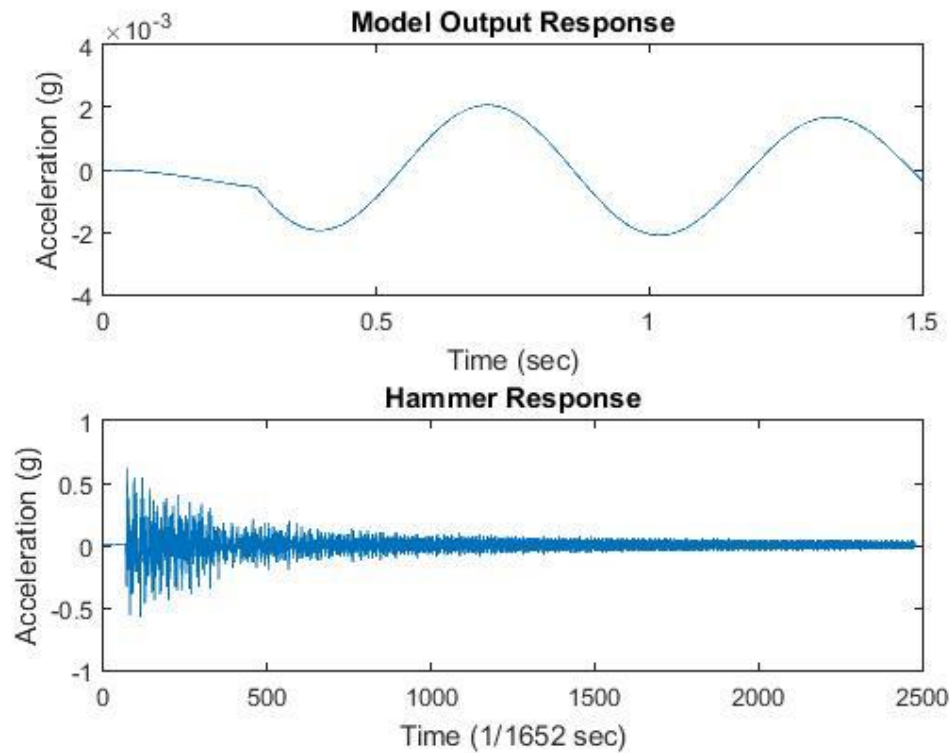


Figure 19: State-Space Model (Phase I)

A model output response was generated from the force of the impact hammer, labelled as “Accel2”. The response was then compared to with that of the wired accelerometer for further data analysis and comparison.



*Figure 20: Model Response Comparison*

When comparing the two responses, the model output response was coded as yellow while the wired accelerometer response was coded as blue. One can tell that the initial two responses do not align; the model output response seems to follow a standard sinusoidal curve at a lower amplitude range than that of the wired accelerometer. In order to match these frequencies, the matrices and structural properties used within the equation needed to be adjusted.

### 3.3.3 Step 3: Match Responses

Matching the frequency responses was a key aspect towards achieving an accurate state-space model. By matching the model output response to the collected raw data response, future

implementations and advancements could be assessed. For example, if a model was developed which illustrated the effects of a 200 pound jumping force on a given structure, tests could then be run for other weight classes and implemented into the model to improve range and accuracy. Likewise, if one was to integrate additional input variables such as walking and running, a matched-response model would act as a suitable base.

Matching the state-space model output response with the response generated by the wired accelerometer required the adjustment of state-space matrices and structural properties of the steel I-beam. However, this was quickly discovered to be a long and tedious process. Since time was a factor, an alternative method was essential for forwards progression.

### 3.4 Phase II: Model Estimation & System Identification

Phase II focused on the development of a baseline, state-space model using model estimation and system identification. System identification (SID) is the process of using statistical methods to construct mathematical models of dynamic systems. This process was chosen as it allowed for quick estimation, provided the necessary state-space matrices, and was directly applicable to multivariable systems with vector inputs and outputs [20]. The steps taken during this phase have been broken down into three segments:

1. Determine the best state-space model estimation method
2. Gather human body responses
3. Integrate the responses into the estimation model

These steps have been discussed in further detail within the following sections.

### 3.4.1 Step 1: Determine Best Estimation Method

Incorporating the ideals of system identification into a state-space model required the selection of an estimation method. To determine the most accurate model possible, black-box estimation was implemented into the program MATLAB. Black-box estimation, or functional estimation, treats software under tests without the knowledge of its internals [17]. The approach works by evaluating what the available inputs for an application are and what the corresponding expected outputs should be [18]. To allow for this, one must “specify the model order, and, optionally, additional model structure attributes that configure the overall structure of the state-space matrices” [19]. Black-box estimation was chosen because of its ease of use, simplicity, and quick test case development [18].

Black-box estimation encompasses three estimation commands; `n4sid`, `ssest`, and `ssregest`. `N4SID` is a non-iterative, subspace method that works on both time-domain and frequency-domain data. This method can be used to generate an initial model and then refined later using the `SSEST` command. `SSEST` is an iterative method that uses prediction error minimization algorithm. This method also works on both time-domain and frequency-domain data, however is slower than `N4SID` to process data. `SSREGEST` is a non-iterative method that works on discrete time-domain and frequency-domain data. This command utilizes balance reduction and typically works best on short, noisy data sets [19].

To determine the best estimate state-space model, all three estimation commands were tested. Each command was programmed to focus on prediction and select the optimal model order, 1-10. The model order for commands `N4SID` and `SSEST` was 8, while the model order for `SSREGEST` was 10. An input-output data object, denoted as ‘`iddata`’, was also required. This was represented by the same force and frequency response set used in Phase I. Noise was added to each model before simulation.

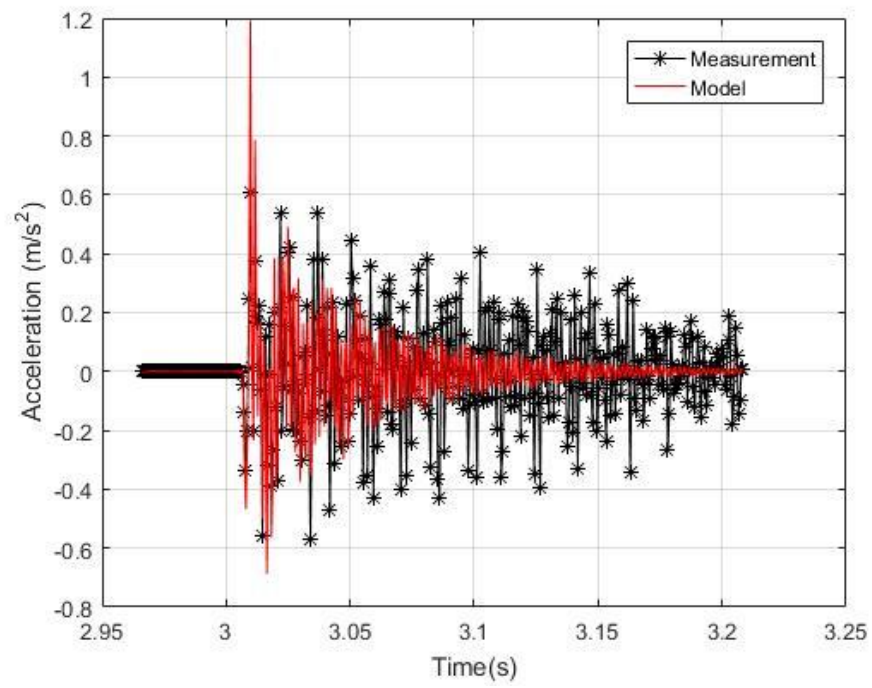


Figure 21: Method I (n4sid)

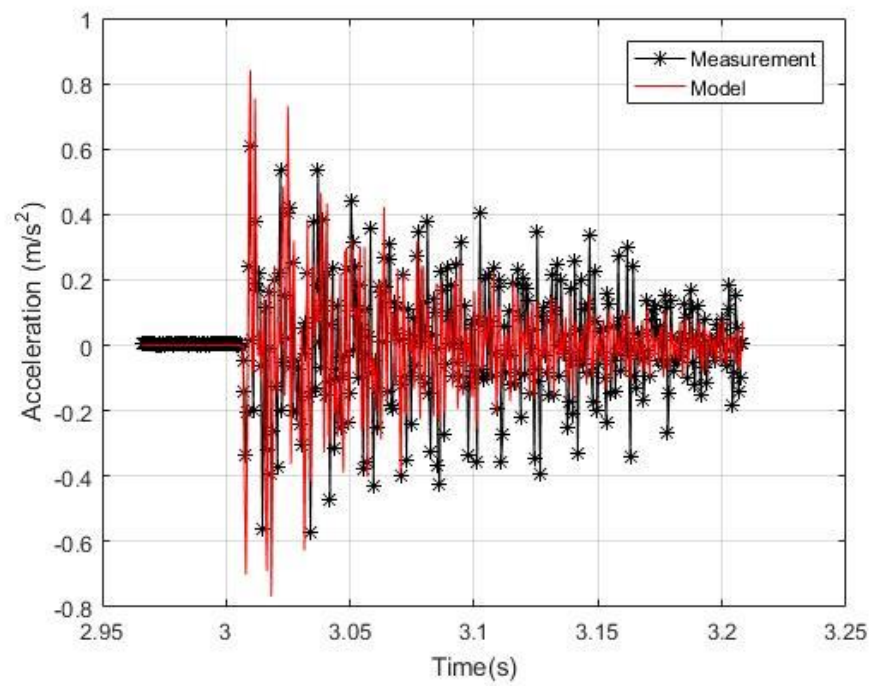


Figure 22: Method II (ssest)

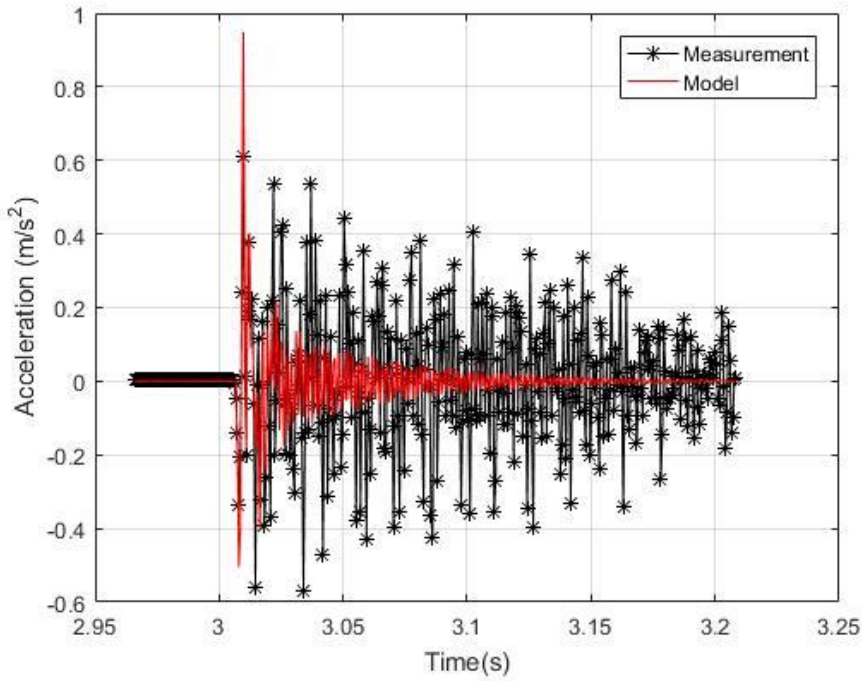


Figure 23: Method III (ssregest)

Based on the data above, estimation command SSEST was selected to represent the state-space model. This command generated a 60.11% match with the wired accelerometer data, far greater than N4SID, at 31.33%, and SSREGEST, at 46.46%. This can be verified as the command SSEST utilizes that of N4SID in order to initialize the state-space matrices. Therefore, although the command takes longer to estimate, it typically provides a better fit to data [19]. Furthermore, the SSEST command was the only command that automatically estimated continuous-time models. Although the other two functions were also able to estimate continuous-time models, accomplished through setting the sampling time to zero, there was no visible change when doing so.

### 3.4.2 Step 2: Gather Human-Body Responses

To attain human-induced jumping responses, a single human load was examined. For the sake of convenience, my body was setup as the test subject. During the time of testing my weight

and height were roughly 200 pounds and 5 feet 9 inches. During the process the wireless accelerometer was tightly attached around my right ankle. This location was chosen as it was the closest to ground level and generated symmetric responses within the body. However, a degree of error was almost guaranteed for this step since “the effect of accelerometer (and other sensors) placement on different parts of the body... has received little consideration” [21]. A stiff-legged jump height of approximately 4 inches was measured based on the rise of my heels as “the rise of the jumper’s heels during the jump is very nearly the same as the rise of the jumper’s center of mass” [32]. This was accomplished through the use of a meter ruler and a keen eye. Furthermore, all tests were recorded barefoot in hopes of excluding any additional cushion, or damping factor, which may have affected the data accuracy.



*Figure 24: Wireless Accelerometer Setup*

Tests were performed in the center of the beam and recorded for both continuous and delayed patterns (spaced roughly 1.5 seconds apart) to illustrate two common activity scenarios. A total of 15 tests were recorded throughout the study for each scenario, as previous studies state that “at least 12 trials are needed to establish stable data in jumping” [28]. Six simulations for each

jumping pattern have been noted within Appendix A for further illustration. Through the use of LabVIEW, frequency vibrations were established based on the x, y, and z-axes of my body. Once the data was collected, the LabVIEW files were then converted into excel files which illustrated time in Column A, x-axis responses in Column B, y-axis responses in Column C, and z-axis responses in Column D.

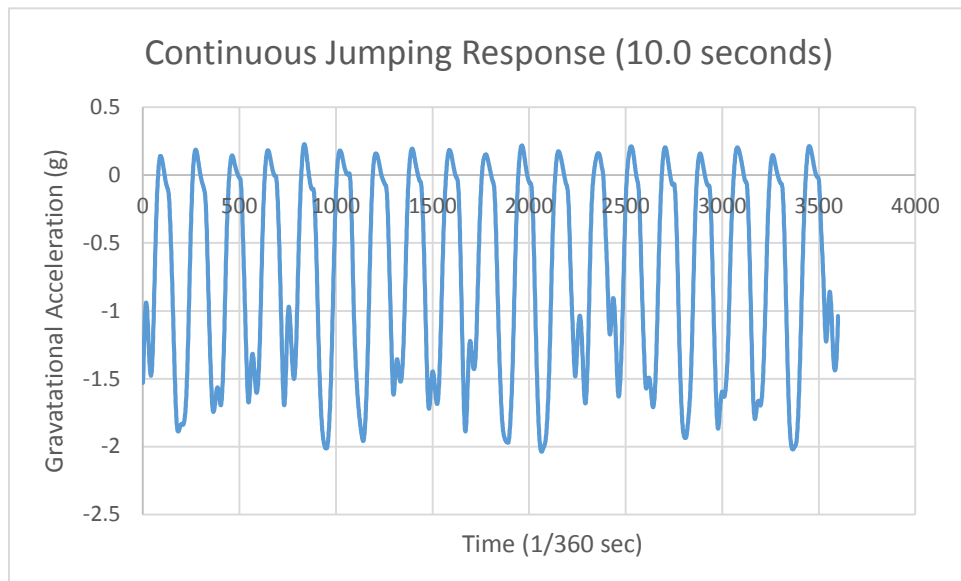


Figure 25: VI File Layout for Wireless Accelerometer

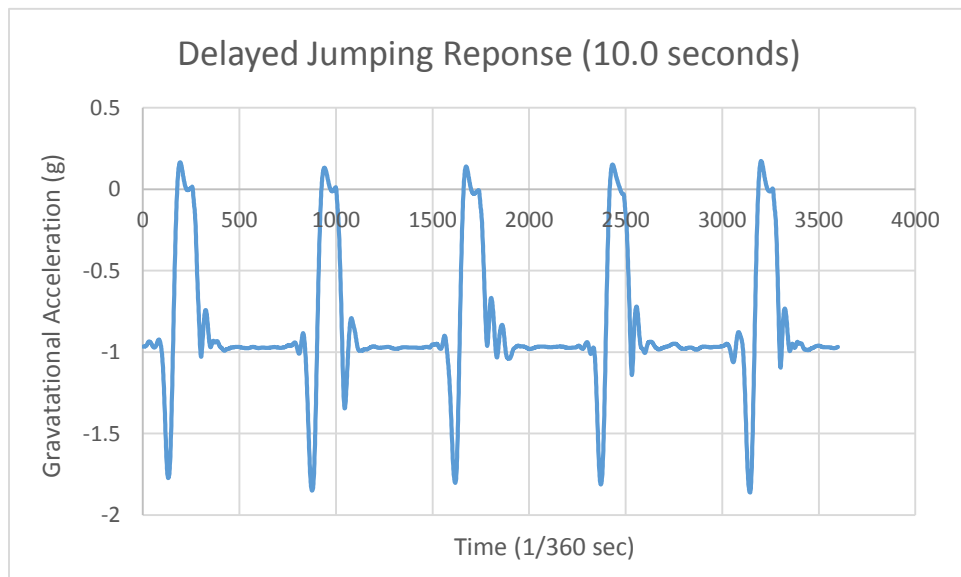
|    | A  | B        | C        | D        |
|----|----|----------|----------|----------|
| 1  | 0  | -0.88659 | 0.348808 | 0.221398 |
| 2  | 1  | -0.88656 | 0.348871 | 0.221282 |
| 3  | 2  | -0.88652 | 0.348933 | 0.221172 |
| 4  | 3  | -0.88648 | 0.348991 | 0.221061 |
| 5  | 4  | -0.88643 | 0.349042 | 0.220944 |
| 6  | 5  | -0.88638 | 0.349082 | 0.220825 |
| 7  | 6  | -0.88632 | 0.349112 | 0.2207   |
| 8  | 7  | -0.88626 | 0.34913  | 0.22056  |
| 9  | 8  | -0.88619 | 0.349139 | 0.2204   |
| 10 | 9  | -0.88613 | 0.349141 | 0.220225 |
| 11 | 10 | -0.88607 | 0.349137 | 0.220038 |
| 12 | 11 | -0.88602 | 0.349126 | 0.21984  |
| 13 | 12 | -0.88597 | 0.349107 | 0.219633 |

Figure 26: Excel Data Conversion (sample)

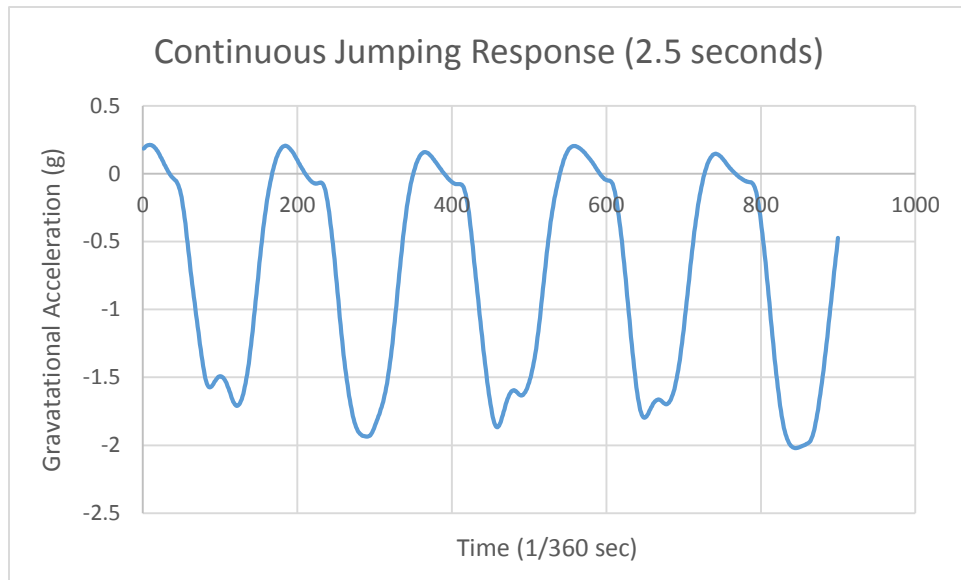
By positioning the accelerometer with respect to the x-axis, Column B was analyzed to provide the forcing frequency vibration. A selected 10 second (3600 cycles) response was examined for each scenario. Additionally, a 2.5 second (900 cycles) component for both tests was extracted and analyzed further.



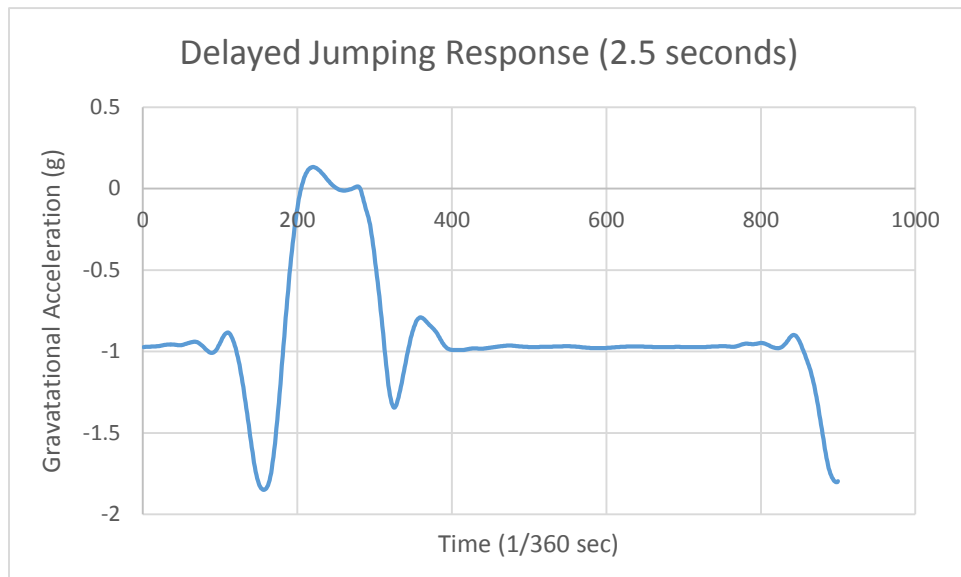
*Figure 27: Continuous Jumping Response (10 sec)*



*Figure 28: Delayed Jumping Response (10 sec)*



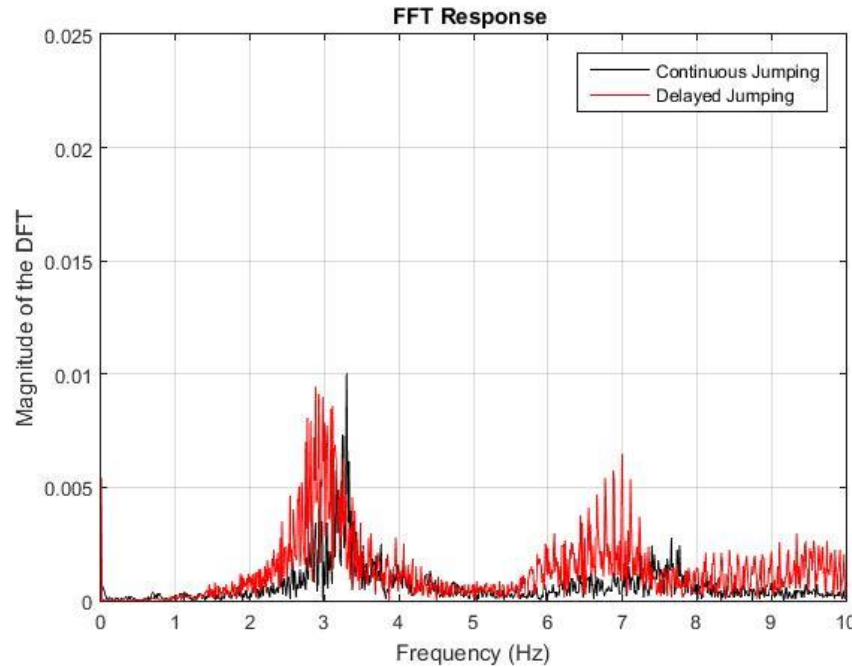
*Figure 29: Continuous Jumping Response (2.5 sec)*



*Figure 30: Delayed Jumping Response (2.5 sec)*

The selected vibrations were run through a fast Fourier transform (FFT) function to determine if the data sets were both accurate and reliable. Fast Fourier transform was implemented as it is a useful tool for converting time or space-domain representations into a frequency-domain

representation, or vice-versa [23]. For this project, the FFT function was able to generate the dominant frequency ranges of the recorded responses.



*Figure 31: FFT of Continuous and Delayed Jumping Responses*

Referring back to recent studies, the typical frequency range for movements such as jumping and bobbing hovers roughly around 1.5 Hz to 3.5 Hz. [22]. When analyzing the recorded FFT responses, one can note that the dominant frequencies between the two scenarios were fairly similar and met standard criteria. The continuous jumping response illustrated a distinct dominant frequency of roughly 3.3 Hz, whereas the delayed response displayed a dominant frequency of about 2.9 Hz.

### 3.4.3 Step 3: Model Responses

Once validated, a single jumping response of two seconds was extracted from each scenario and inserted into a state-space model using the selected estimation method developed in Step 1.

These responses were multiplied by a mass of 200 pounds in order to generate the appropriate forcing frequency.

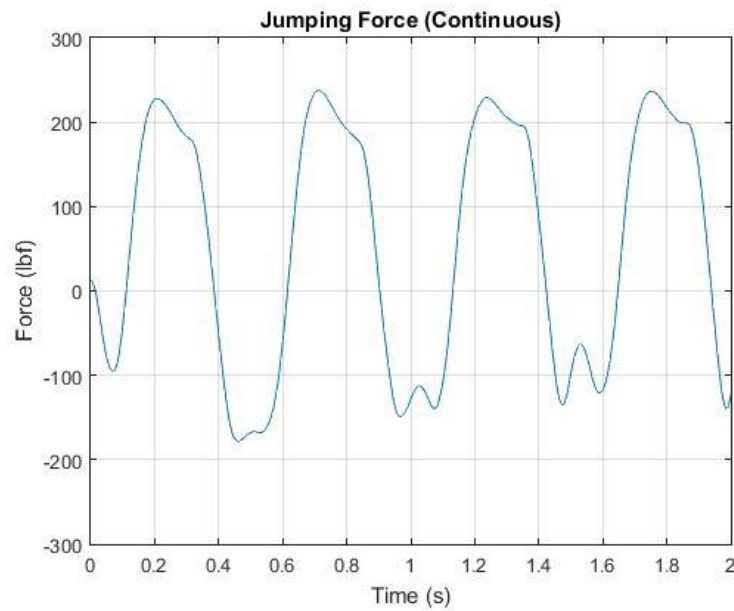


Figure 32: HBF (Continuous Jumping)

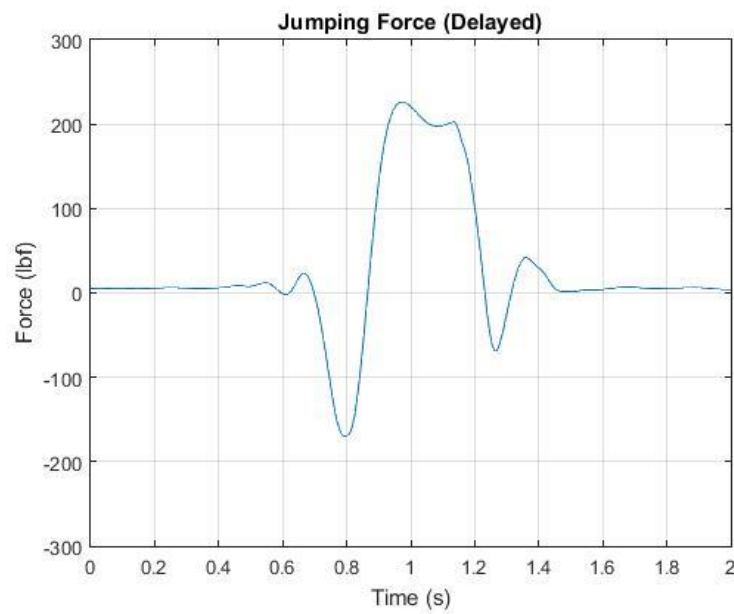
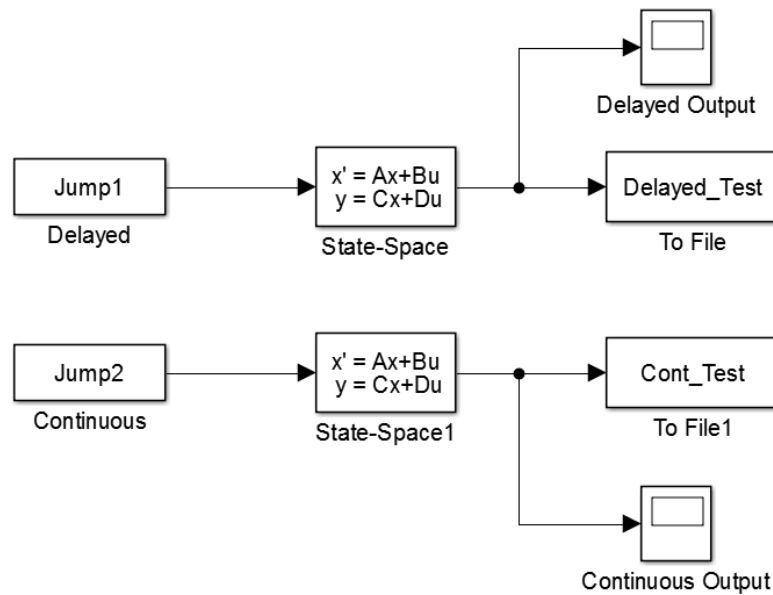


Figure 33: HBF (Delayed Jumping)

Phase II's model applied the provided, pre-formulated matrices for A, B, C and D of the state-space equation. The output responses for each scenario were also recorded over a period of two seconds.



*Figure 34: State-Space Model (Phase II)*

Once the output frequencies were assessed, structural responses were then generated with the use of human-loading and the wired accelerometer; one should note that these responses were not tested in synchronization with those from the human body. Similar to Phase I, a selected 10 second response and 2.5 second extraction for each jumping scenario was analyzed and compared with that of the model.

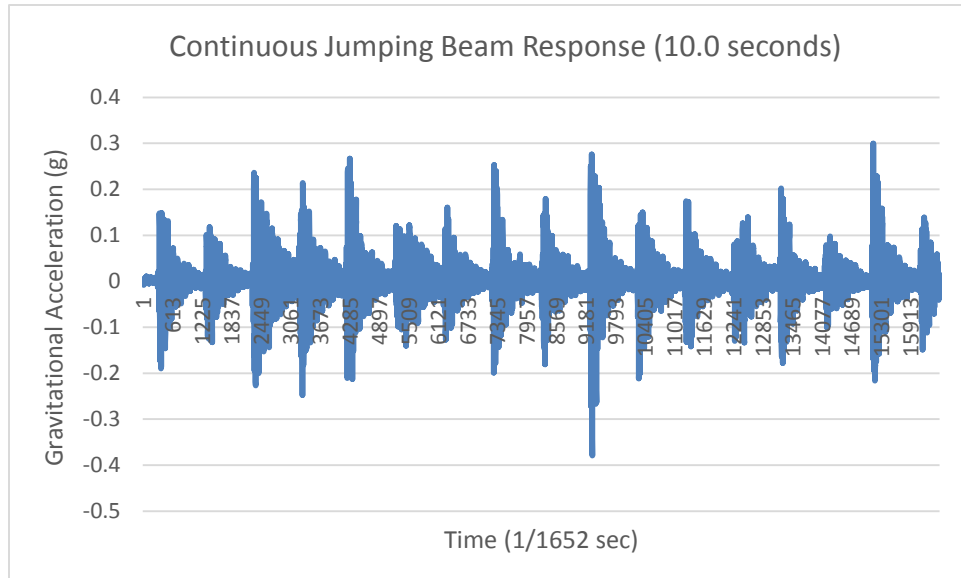


Figure 35: Continuous Jumping Beam Response (10 sec)

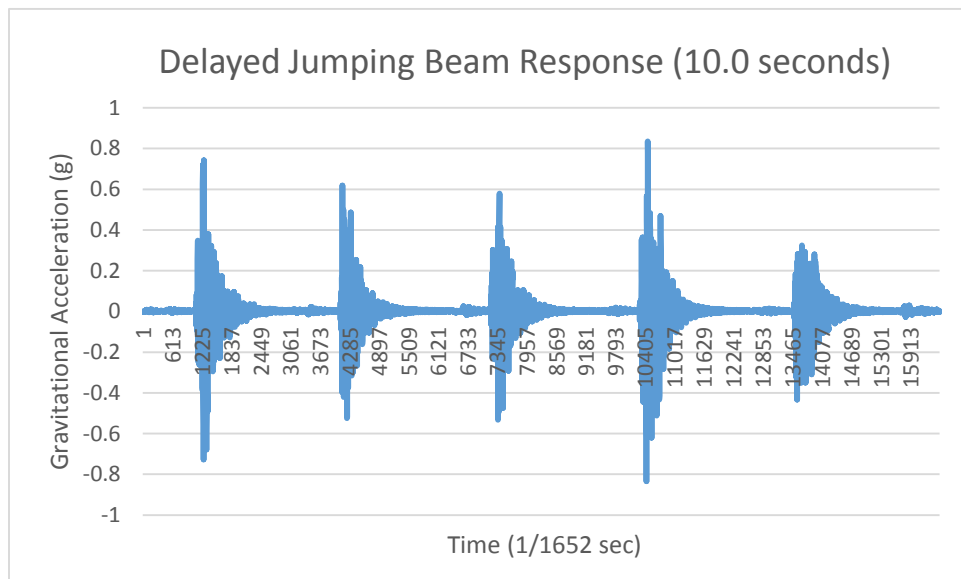


Figure 36: Delayed Jumping Beam Response (10 sec)

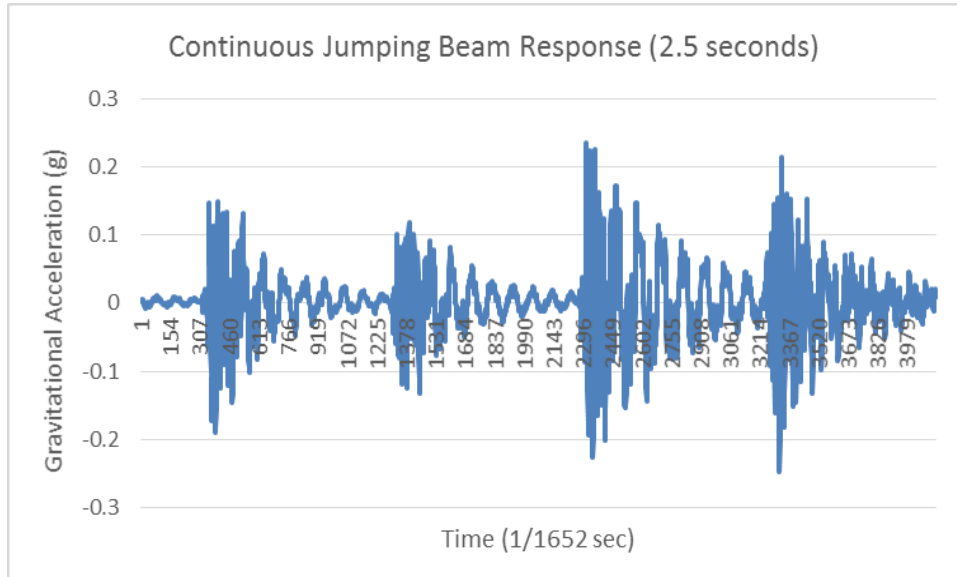


Figure 37: Continuous Jumping Beam Response (2.5 sec)

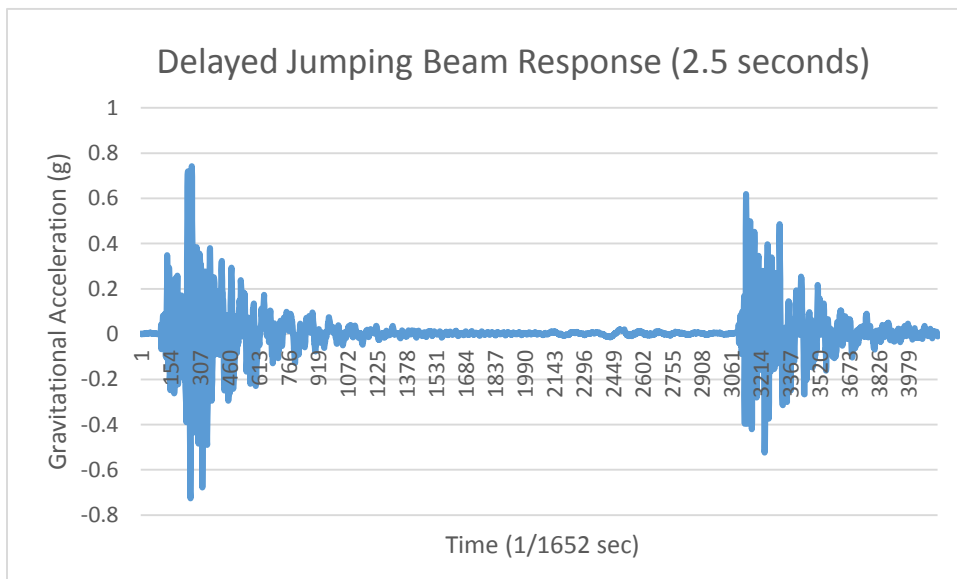


Figure 38: Delayed Jumping Beam Response (2.5 sec)

It was determined that the predictions of this model for real jumping loads did not match perfectly with that of the measured responses. This may have been a result of a number of issues, such as time synchronization between the wireless and wired sensors, and excessive noise generation. Recommendations to counteract these matters have been discussed within the following sections.

## 4.0 Results

This project combined concepts of structural engineering with the field of biomechanics, through human-induced, system loading. The report captured and analyzed the responses of a steel I-beam under the activity of jumping, along with a number of human body vibrations, using state-space system identification design. Originally, the established model would be used to generate simulated output responses in order to monitor the overall health of the structure. However, with limited time, the study focused solely on the development of a baseline model for future implementation.

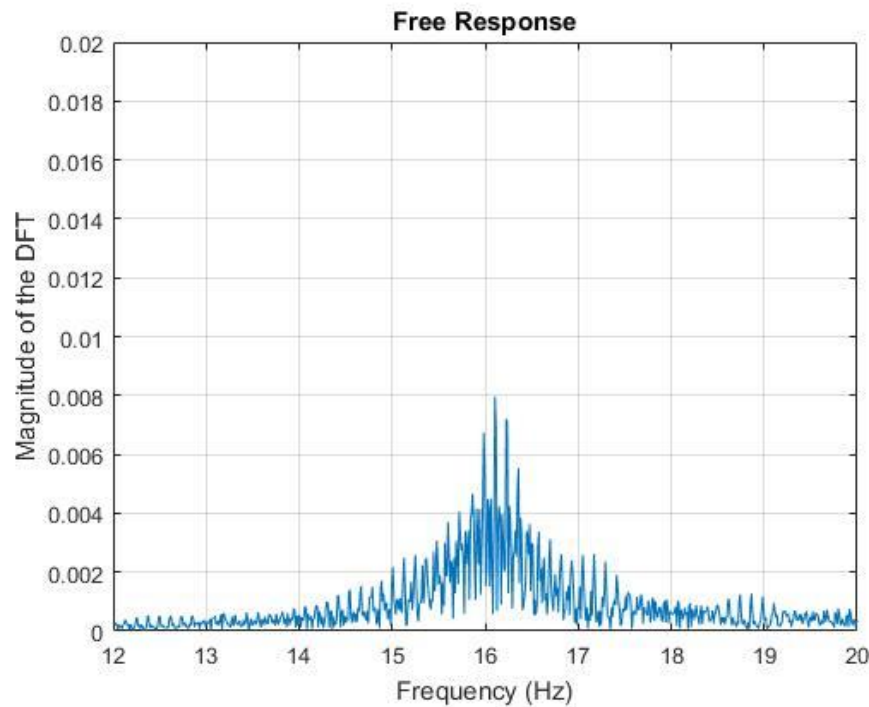
The final baseline model for human-induced jumping analyzation was generated using methods of state-space modelling and system identification (SID). State-space modelling was able to handle systems with multiple inputs and outputs, provide a quick estimations, and produce time-domain solutions. Likewise, SID allowed for quick estimation, was directly applicable to multivariable systems with vector inputs and outputs, and provided the necessary state-space matrices for baseline creation. The SID estimation method, SSEST, was selected to produce these necessary state-space matrices. This iterative method utilized prediction error minimization algorithm, which generated a 60.11% match with that of the impact hammer and wired accelerometer; far greater than N4SID, at 31.33%, and SSREGEST, at 46.46%. Once the matrices were established, a number of human-induced vibrations for continuous and delayed jumping patterns were recorded for suitable model integration.

The action of continuous jumping, opposed to delayed, is one that is much more frequently observed in the world. This was no surprise as the continuous-rhythmic patterns appeared in every study involving the analyzation of jump-like movements. Therefore, continuous data sets from this report should align with those from previous studies; a value between 1.5 to 3.5 Hz. Results

indicated that this was true as continuous jumping produced a dominant frequency of roughly 3.3 Hz. On the other hand, delayed jumping was not evaluated in any of the researched studies, and therefore, there was no reference for data precision. However, delayed jumps proved to be quite similar, generating a dominant frequency of approximately 2.9 Hz. It was presumed that this frequency would have been higher, however, the lower value may have been due to the fact that my body acted as a damper when recording. Furthermore, in the *article Floor Vibrations due to Human Activity*, it's stated that "with the advent of limit states design and the more common use of lightweight concrete, floor systems have become lighter, resulting in higher natural frequencies for the same structural steel layout" [24]. Therefore, since steel is heavier than concrete per unit volume, these tests may have generated a low frequency to that of an actual establishment.

The final baseline model consisted of the developed state-space SID matrices, as well as the continuous and delayed jumping responses. The generated output responses from this model were compared with the human-induced, structural responses. Frequencies were not perfectly correlated with these measurements, possibly due to time synchronization issues between wireless and wired sensors and/or excessive noise. Potential solutions for these complications include the use of the same accelerometers for both structure and human movement and the application of digital filters on the measured responses. These have been detailed further in the following section. Nevertheless, these responses were still evaluated for their potential effects on structural performance and human comfort.

Structural performance was determined using the approximated natural frequency of the beam calculated in Section 3.1; roughly 10 Hz. To check for this value's accuracy, an impact hammer test was run through the fast Fourier transform function. This output also symbolized the general dominant natural frequency of the system as the modal hammer generated a free response.



*Figure 39: Steel Beam Free Response (Dominant Frequency)*

It was noted that the two values did not coincide, which was more possibly due to an error in structural properties. Therefore, the free response value of roughly 16 Hz was used to portray the system's natural frequency. The FFT of the actual structural, jump responses were then determined and evaluated. This has been portrayed below in Figure 40; where green, black and red indicate free vibration, continuous jumping and delayed jumping, respectively.

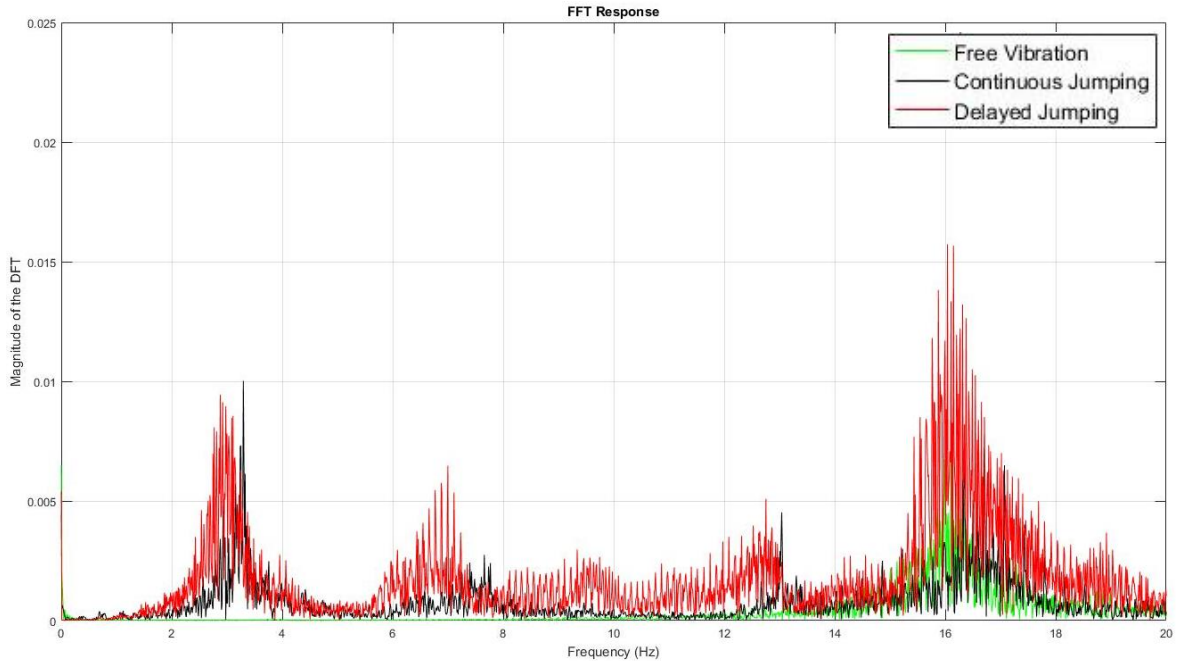


Figure 40: FFT Response Comparison

Results revealed that there could be a potential indication of resonance within the beam around 16 Hz. Continuous jumping patterns seemed to generate frequencies closer to the free response of the system. This implied that continuous, harmonic movements are more likely to impact steel over a period of time than delayed.

Human comfort was determined using criteria from Chapter 2.0. The “actual” responses, Figures 35 and 36, were first tested based on the comfort perception chart (Figure 4) from the *Vibrations and Shock Handbook* by Clarence W. de Silva. The continuous response illustrated an average peak acceleration of roughly 0.25g, which verified a comfort perception level of 3. This stated that “the majority of people will perceive motion; the level of motion may affect desk work; long-term exposure may produce motion sickness” [27]. On the other hand, the delayed response produced an average peak acceleration of approximately 0.60g, which indicated a comfort perception level of 6; “most people cannot tolerate the motion and are unable to walk naturally” [27]. This made relative sense as the delayed vibration was an impulsive load and therefore caused

more disruption. Criteria (Figure 3) from *Floor Vibrations due to Human Activity* by Murray et al. was evaluated next. Values were determined based on the percent gravity of  $9.81 \text{ m/s}^2$ . Using the average peak accelerations above, continuous jumping produced a percent gravity of roughly 2.5%, whereas delayed jumping established a percent gravity of approximately 6.1%. Both of these values stated that single-load jumping can be felt within indoor footbridges, shopping malls, and dining and dancing venues [24]. Overall, the assessed values above seemed extremely high. This was probably due to the fact that the perception levels were based on impacts within tall building structures rather than a steel beam. Furthermore, this data revolved around the peak acceleration point; illustrating the most significant effects.

## 5.0 Discussion & Conclusions

Chapter 5.0 discusses the overall results of the report, as well as concluding remarks and recommendations for subsequent studies. It should be noted that the vision of this project was to design a model which could analyze various human-induced loads for a given structure. However, a steel beam was used to account for time, the number of group members and gain insight on the subject of structural dynamics. Therefore, future work should be geared towards monitoring the health of high-activity structures, such as the WPI Rooftop Field.

Key observations have been developed based on final results of this study. These points have been listed below:

- (1) Continuous jumping is the most typical pattern studied, whereas, delayed jumping patterns are rarely evaluated
- (2) Continuous jumping generates a greater frequency and is more likely to resonate on the steel I-beam than delayed jumping
- (3) State-space system modelling can be an excellent tool for handling systems with multiple inputs and outputs, providing a quick estimations, and producing time-domain solutions
- (4) System identification modelling is beneficial for its quick estimation, direct applicability to multivariable systems with vector inputs and outputs, and provision of pre-formulated state-space matrices
- (5) Tweaking matrices and structural properties for model correlation is a timely and tedious process that is slightly aided by SID estimation methods

Through these observations, future studies should have a less difficult time in achieving specified model parameters for proper data acquisition.

A number of propositions have also been provided for the future development of a more precise and enhanced model. These propositions include details on accelerometer usage, the application of filters, fabricated weight ranges and activity inputs, and simulated crowd excitations.

The first proposition for future work is to alter the usage of accelerometers. For instance, model results might have indicated time synchronization issues between the wireless and wired sensors. Therefore, it is recommended that future teams should use the same accelerometers for both structure and human body, which would improve the performance of system identification. Furthermore, there should be a change in the location of the accelerometer from the ankle to the shin. In the study, *Accelerometer Based on Body Sensor Localization for Health and Medical Monitoring Applications* by Navid Amini, Majid Sarrafzadeh, Alireza Vahdatpour, and Wen Yao Xu, they were able to “classify the shin area flawlessly, as the impact of each step on the shin [was] significantly higher than other locations” [21]. This increase in impact would be valuable as it would state the max amplitude of the system which could be used to set safety boundaries.

The application of digital filters is also recommended in order to diminish excessive noises within the collected signals. Through the use of a digital filter, one can focus on a specific range of frequencies to depict a much clearer response. For example, using a low-pass filter could allow one to clearly identify the frequency range of human movements; 1.5-3.5 Hz. This occurs as an ideal low-pass filter “leaves unchanged all frequency components of a signal below a designated cutoff frequency,  $w_c$ , and rejects all components above  $w_c$ ” [34]. Furthermore, the initial use of a Finite Impulse Response (FIR) filter design is recommended over an Infinite Impulse Response (IIR) filter design. This is due to the fact that FIR filters are subsequently easier to handle as they “are widely used due to the powerful design algorithms that exist for them, inherent stability when implemented in non-recursive form (the ease with which one can attain linear phase), have simple extensibility to multi-rate cases and ample hardware support that exists for them among other reasons” [35].

The third proposition is to fabricate a number of weight ranges (i.e. 140-160lbs, 160-180lbs), along with the introduction of multiple activities (i.e. running, walking, jogging). One could accomplish this by generating and evaluating the activity responses of 30 individuals for each weight range. For example, 10 people around 160-166 lbs, 10 people around 167-173 lbs, and 10 people around 174-180 lbs. For each individual and activity, at least 12 tests should be recorded as reference to Racic et al., *Experimental Identification and Analytical Modelling of Human Walking Forces: Literature Review*.

Simulated crowd excitation is the final proposition, as most structural issues revolving around human impact are due to the effects of multiple people. Through the use of a human induced-based equation, such as the one below in Figure 41, one can determine the synchronized dynamic loading of an individual and/or group. This equation originates from the book *Structural Detailing in Concrete* by M. Y. H. Bangash, and can be used to calculate the synchronized dynamic load caused by “activities, such as jumping and dancing, which are periodic” [25] and mainly dependent upon the static weight of human, period of loads, and contact ratio [6].

|                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|--------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>VIII.3.3.2. Synchronized dynamic loading</b></p> | <p>It is caused by activities, such as jumping and dancing, which are periodic and mainly dependent upon:</p> <ul style="list-style-type: none"> <li>(a) the static weight of the dancer(s) (<math>G</math>)</li> <li>(b) the period of the dancing load(s) (<math>T_p</math>)</li> <li>(c) the contact ratio (<math>\alpha</math>), i.e. the ratio of the duration within each cycle when the load is in contact with the floor and the period of the dancing.</li> </ul> <p>The load at any time (<math>t</math>) may be expressed as</p> $F(t) = G \left[ 1 + \sum_{n=1}^{\infty} r_n \sin \left( \frac{2n\pi}{T_p} t + \varphi_n \right) \right]$ <p>where <math>n</math> is the harmonic being considered 1, 2, 3, . . . , <math>\gamma_n</math> is the dynamic load factor for the <math>n</math>th harmonic, and <math>\varphi_n</math> is the phase angle of <math>n</math>th harmonic.</p> <p>The values of <math>\gamma_n</math> and <math>\varphi_n</math> are functions of the value of the contact ratio <math>\alpha</math>, which is given in Table VIII.3.1.</p> |
|--------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Figure 41: Synchronized Dynamic Load Equation

| <b>Table VIII.3.1. Typical values of contact ratio for various activities</b> |                                          |
|-------------------------------------------------------------------------------|------------------------------------------|
| <b>Activity</b>                                                               | <b>Contact ratio <math>\alpha</math></b> |
| Pedestrian movement                                                           | 0.667                                    |
| Low-impact aerobics                                                           |                                          |
| Rhythmic exercises                                                            | 0.500                                    |
| High-impact aerobics                                                          |                                          |
| Normal jumping                                                                | 0.333                                    |
| High jumping                                                                  | 0.250                                    |

*Figure 42: Typical Values of Contact Ratio for Various Activities*

Further developments should also involve a combination of direct measurement and Monte Carlo simulation. Monte Carlo simulation is “an analytical technique in which a large number of simulations are run using random quantities for uncertain variables and the distribution of results infer which values are most likely best” [33]. Therefore, the general approach would be to “collect data for a small group of people, introduce certain parameters to quantify the main characteristics and use the Monte Carlo method of sampling from some fitted probability distributions to generate the loads for a large crowd” [6]. In order to take into account the lack of synchronization between each individual, a phase lag,  $\psi_n$ , should be introduced to the Fourier series to represent the jumping load such that:

$$F(t) = G \left[ 1.0 + \sum_{n=1}^{\infty} r_n \sin \left( \frac{2n\pi}{T_p} t + \phi_n + \psi_n \right) \right] \quad (7)$$

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## Appendix A: Structural Properties

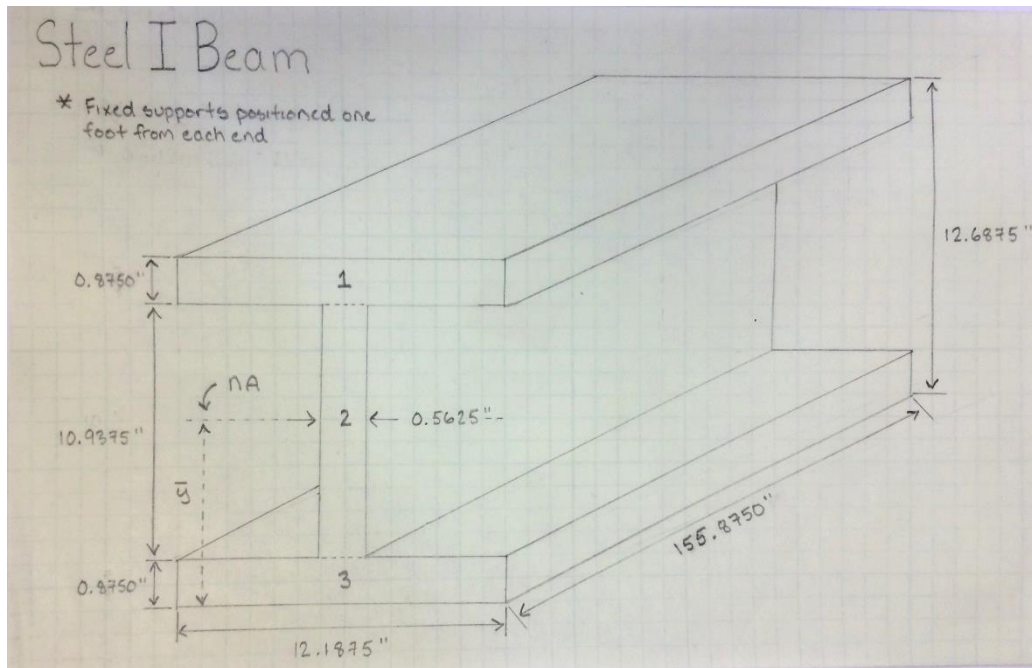


Figure 43: Steel I-Beam Dimensions

$$\text{Mass (m)} = \text{Density } (\rho) \times \text{Volume (V)}$$

$$\text{Flanges} = 2 [0.8750 \text{ in} \times 12.1875 \text{ in} \times 155.8750 \text{ in}]$$

$$\hookrightarrow 3324.5215 \text{ in}^3$$

$$\text{Web} = 0.5625 \text{ in} \times 10.9375 \text{ in} \times 155.8750 \text{ in}$$

$$\hookrightarrow 958.9966 \text{ in}^3$$

$$V_{\text{Total}} = 3324.5215 \text{ in}^3 + 958.9966 \text{ in}^3$$

$$\hookrightarrow 4283.5181 \text{ in}^3$$

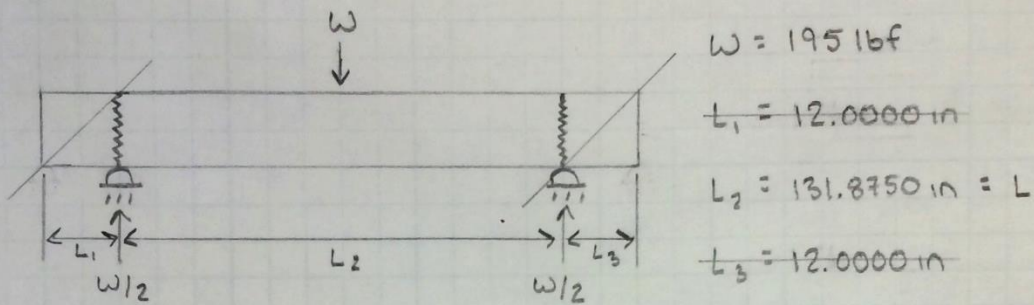
$$\text{Material Type} = \text{Common Steel}$$

$$\hookrightarrow \rho = 0.2836 \text{ lb/in}^3$$

$$\text{Mass} = (0.2836 \frac{\text{lb}}{\text{in}^3}) (4283.5181 \text{ in}^3) = 1214.8057 \text{ lb}$$

Figure 44: Beam Mass Calculations

Stiffness (using my weight + neglecting overhang)



Moment of Inertia (I)

↳ Centroid (symmetrical) = 6.3438 in from bottom ( $\bar{y}$ )

$$I_{\text{tot}} = \sum (\bar{I}_i + A_i d_i^2), \text{ where:}$$

$$\bar{I}_i = \text{Moment of inertia about centroid} = \frac{1}{12} b h^3$$

↳  $b$  = base ;  $h$  = height

$A_i$  = Area of the individual segment

$d_i$  = Vertical distance from the centroid to neutral axis

Segment 1:

$$I_1 = \frac{1}{12} (12.1875 \text{ in}) (0.8750 \text{ in})^3 = 0.6804 \text{ in}^4$$

$$A_1 = (12.1875 \text{ in}) (0.8750 \text{ in}) = 10.6641 \text{ in}^2$$

$$d_1 = |y_1 - \bar{y}| = |0.8750 \text{ in} + 10.9375 \text{ in} + \frac{0.8750}{2} \text{ in} - 6.3438 \text{ in}|$$

$$\hookrightarrow 5.9062 \text{ in}$$

Figure 45: Beam Stiffness Calculations (Pt. 1)

Segment 2:

$$I_2 = \frac{1}{12} (0.5625 \text{ in}) (10.9375 \text{ in})^3 = 61.3332 \text{ in}^4$$

$$A_2 = (0.5625 \text{ in}) (10.9375 \text{ in}) = 6.1523 \text{ in}^2$$

$$d_2 = |y_2 - \bar{y}| = |0.8750 \text{ in} + \frac{10.9375}{2} \text{ in} - 6.3438 \text{ in}|$$

↪ 0

Segment 3:

$$I_3 = \frac{1}{12} (12.1875 \text{ in}) (0.8750 \text{ in})^3 = 0.6804 \text{ in}^4$$

$$A_3 = (12.1875 \text{ in}) (0.8750 \text{ in}) = 10.6641 \text{ in}^2$$

$$d_3 = |y_3 - \bar{y}| = | \frac{0.8750}{2} \text{ in} - 6.3438 \text{ in} |$$

↪ 5.9062 in

$$\begin{aligned} I_{tot} &= [0.6804 \text{ in}^4 + 10.6641 \text{ in}^2 \times (5.9062 \text{ in})^2] \\ &\quad + [61.3332 \text{ in}^4 + 6.1523 \text{ in}^2 \times (0 \text{ in})^2] \\ &\quad + [0.6804 \text{ in}^4 + 10.6641 \text{ in}^2 \times (5.9062 \text{ in})^2] \end{aligned}$$

↪ 306.6898 in<sup>4</sup>

Figure 46: Beam Stiffness Calculations (Pt. 2)

Stiffness (K): Based on a fixed-fixed connection

$$\begin{aligned} \hookrightarrow K &= \frac{12EI}{L^3} = \frac{(12)(2.9 \times 10^7 \text{ psi})(806.6898 \text{ in}^4)}{(131.875 \text{ in})^3} \\ &= 122404.78 \text{ lb/in} \end{aligned}$$

Natural Frequency ( $\omega_n$ )

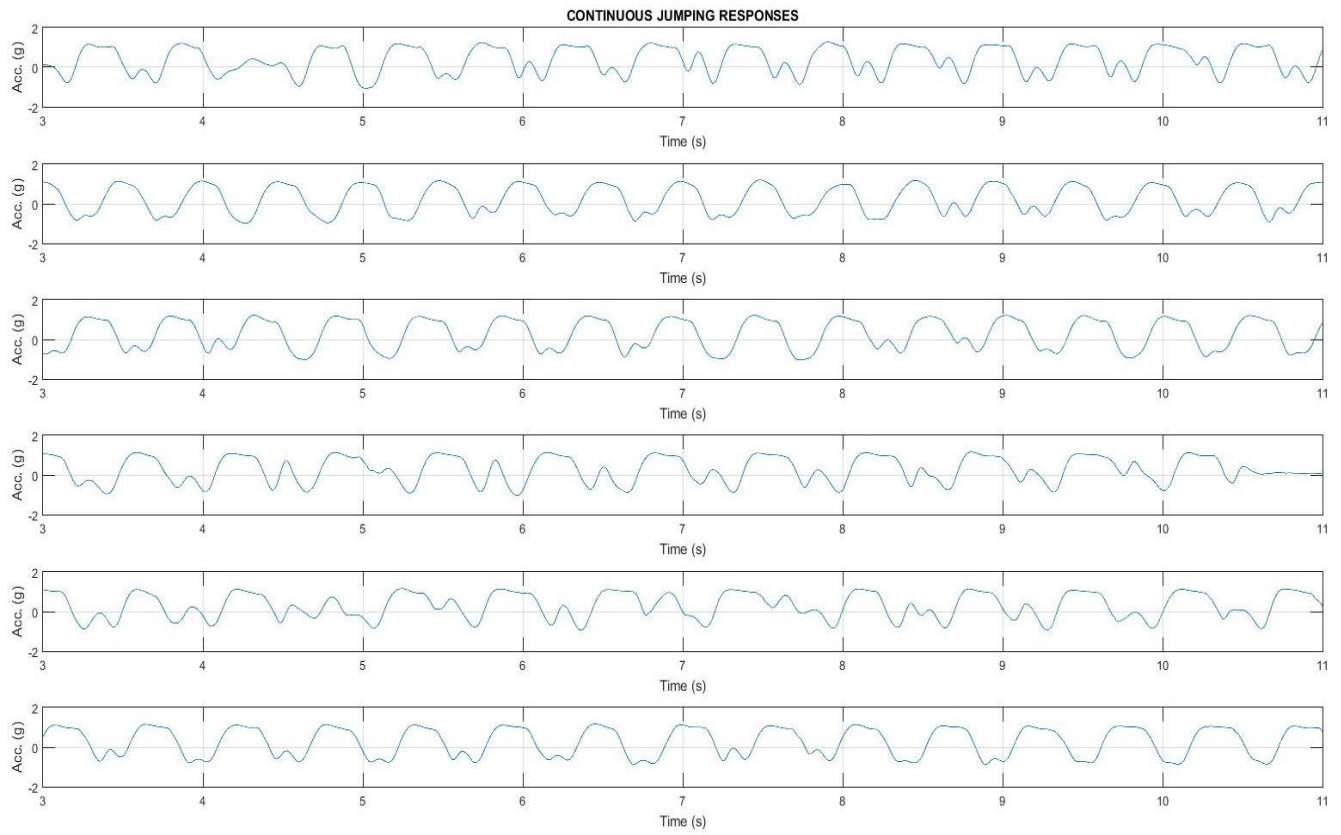
$$\begin{aligned} \hookrightarrow \omega_n &= \sqrt{\frac{K}{m}} = \sqrt{\frac{122404.78 \text{ lb/in}}{1214.805716}} \\ &= 10.04 \text{ Hz} \end{aligned}$$

Damping Coefficient (C)

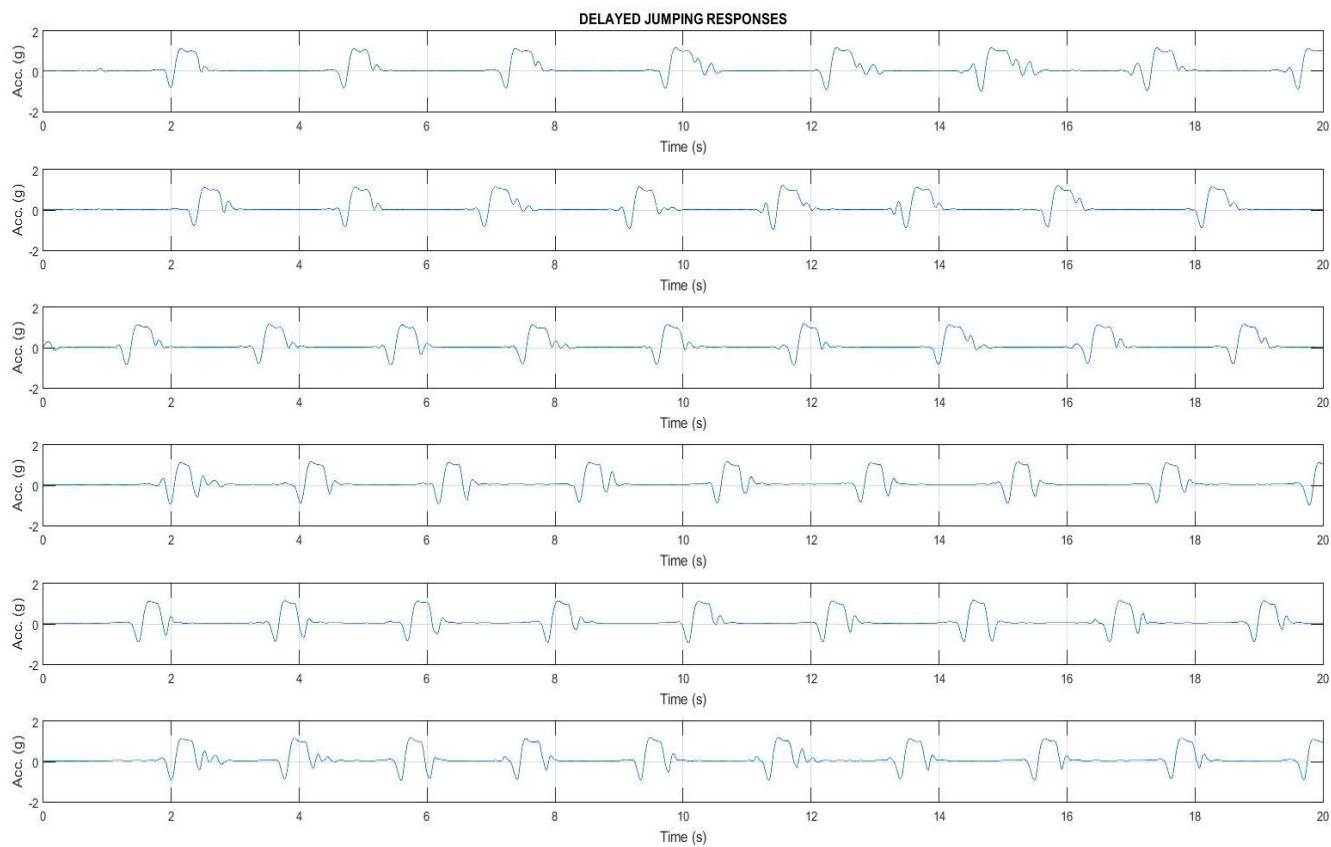
$$\begin{aligned} \hookrightarrow C &= 2 \times m \times \omega_n \times \zeta = (2)(1214.805716)(10.04 \text{ Hz})(0.02) \\ &= 487.87 \text{ lb·s/in} \end{aligned}$$

Figure 47: Beam Stiffness (Pt. 3), Natural Frequency & Damping Coefficient Calculations

## Appendix B: Jumping Responses



*Figure 48: Continuous Jumping Responses*



*Figure 49: Delayed Jumping Responses*