Trace Gases and Their Impact on Humanity

An Interactive Qualifying Project Report

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Abstract

In this project we investigated the impact of trace gases such as carbon dioxide on the Earth's climate using a seven reservoir model for its ecosystem. To obtain quantitative insights, we performed numerical simulations for the concentration of CO_2 in the different reservoirs. Based on these results we modeled and computed the mean temperature of the Earth. Various strategies were explored to mitigate the production of anthropogenic carbon dioxide in the atmosphere and its impacts on human society.

Executive Summary

Trace gases in the Earth's atmosphere trap enough radiation from the Sun to keep the planet warm. When the concentration of gases is too high, the planet's temperature rises resulting in a very hot climate. As the concentration of these greenhouse gases decreases, the temperature of the planet drops, resulting in a very cold climate. Over the past centuries, the concentration of these gases has been perfectly suited for the existence of human civilization. However, since the industrial revolution, mankind has slowly been increasing the levels of greenhouse gases in the atmosphere which are beginning to have an effect on the Earth's climate. Studies have shown that the atmospheric level of CO_2 has increased from 270 parts per million (ppm) to 380 ppm since the industrial revolution.

Global temperatures fluctuate naturally in many complex cycles. The changes in temperature have begun to steadily increase, perhaps unnaturally, which may be correlated to the increase of $CO_{2.}$ It has been proven that over the past 100 years, the mean global temperature of the Earth has risen by approximately 1.3°F. Projections indicate that this rate in heating will continue to increase in correlation with increasing greenhouse gas emissions. One major problem is that temperature increase has occurred with greatest intensity at the poles, thus putting the ice caps at risk. The melting of the polar region can result in extreme climate change including the possibility of an ice age, as well as the obvious result of coastal flooding. The increase of temperatures has already created irregularities in our climate which are manifested as more frequent and stronger hurricanes, tornadoes and other natural disasters. This will affect humans bringing more migration from hot equatorial areas to cooler areas of higher latitude (often across national borders), as well as the increase of tropical diseases and pests.

Humans have the ability to hinder the increase of Greenhouse gas emissions either by decreasing fossil fuel use, or by the use of mitigation techniques. Advancing technology allows for both options to be employed through the use of alternative fuel vehicles, carbon sequestration, and many other options. Predictions indicate that the Earth can return to its natural balance if humans are able to reduce the amount of carbon being emitted into the atmosphere each year. If policies for change are not enacted very soon, computer model predictions show that the climate will change drastically as temperatures oscillate between very low and very high before increasing to extremely high temperatures with enough CO_2.

Introduction

Studies showed that the Earth's climate behavior has changed radically over the past five decades. Scientist concluded that the recent climate changes are due to an increase in the mean temperature of the Earth. The result of the rise in temperature is due to the increasing amount of different gases in the atmosphere known as trace gases. Trace gases include carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and chlorofluorocarbons (CFCs). Studies also showed that some of these trace gases are increasing in larger amounts every year. The role of these gases is to trap and reflect back the energy that Earth emits, which is known as the Greenhouse effect. Therefore their increase will result in a rise in the temperature of the Earth. Although different types of trace gases exist, our main focus is going to be on carbon dioxide (CO_2).

The temperature of the Earth is controlled by the amount of electromagnetic waves of sunlight that reach the Earth's surface. Part of this energy is absorbed by our planet and part is emitted back out as infrared waves. Without the presence of the trace gases, the temperature of the Earth would plummet and remain far below freezing. Differences in sunlight cause cold polar and warm tropic regions on the Earth which provide for uneven heating of the Earth's surface and in turn create differences in temperature. These differences in temperature power the Earth's wind system and storm movements. An increase in carbon dioxide in the atmosphere absorbs more infrared radiation from the Earth, heating the lower atmosphere and resulting in warmer ocean temperatures, which fuel hurricanes.

Different models have been created that display the grave outlook that the increase of CO_2 in the atmosphere has for our planet. Predictions indicate that if no changes are made, the Earth's climate could change so drastically that human life could be at risk in the near future. The following is an in depth study of the effects of trace gases found in the atmosphere. Using atmospheric data from the past, including human influences on CO_2 levels, we used computer models of trace gas concentrations in the atmosphere to predict future changes. These results are then used to predict the temperature change of the Earth in the future. Many scientists agree that global warming is occurring. The debate is whether this is caused by natural fluctuations or by human actions.

Background

1. Greenhouse Gasses

Carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and chlorofluorocarbons (CFCs) are four main greenhouse gasses that are produced naturally; however, they have been increased drastically in recent years due to anthropogenic effects. The CFCs is a main greenhouse gas which is not produced naturally. The largest source of anthropogenic greenhouse gases is the use of fossil fuels, which has also provided the energy to power our modern lifestyle (mostly in the form of electricity production and automobiles).

New technologies over the past century have resulted in large environmental problems. Coal power plants, for instance, produce severe regional air pollution crises in industrial areas, such as London or Donora, PA, where particulates from the burning of coal resulted in killer smog. The solution for many of these problems usually has been a temporary patch, such as the building of smokestacks or catalytic converters. These environmental "band-aids" still generate the emission of greenhouse gases and particulates into the troposphere.

Humans have changed the composition of the Earth's atmosphere over a very short amount of time. Over the past 100 years, CO_2 levels have increased by 25%, and are predicted to rise another 50% over the next few decades. The global production of N_2O exceeds consumption by 40 %. This has a staying power in the atmosphere that results in one ton of nitrous oxide being equivalent to 296 tons of carbon dioxide. CFCs are also extremely harmful in that they both destroy stratospheric ozone (the ozone layer that blocks damaging UV light) as well as trap as much heat per ton as 5,000-8,100 tons of CO_2. [i]

Fossil fuels cause three main problems which occur locally as well as regionally. Particulates released into the atmosphere through mining and combustion, cause a wide array of respiratory ailments in humans. This problem has been remedied to some extent, by restricting coal use to major power plants and by requiring these power plants to install devices to remove particulates from their emissions. The second problem is elevated levels of O_3 near ground level, which can cause damage to animals' lungs and plants. Oxygen, nitrogen oxides (NO_x), hydrocarbons, and sunlight are the main reactants responsible for producing O_3 in the lower troposphere. Of these reactants, humans can only stop the production of NO_x, which is released during combustion. This harmful compound produces both ground-level ozone as well as acid rain. When SO_2 and NO combine with atmospheric water they result in sulfuric and nitric acid which mixes with rain in the atmosphere. Acid rain results in the acidification of soil and fresh water, which then releases toxic elements such as aluminum.

In recent years, steps have been taken to reduce harmful emissions such as CFCs. The Montreal Protocol in 1987 was the first international agreement formed in order to protect the atmosphere. This agreement came after a hole in the ozone layer above Antarctica was discovered, leading to a greater public awareness of the fragile environment. Governments around the world since then, have enforced strict regulations on industrial emissions; there has been, however, criticism of these regulations (as well as those that have not yet been enacted), due of the difficulty scientists still have in predicting what will happen if greenhouse gas production continues. There is a general consensus among scientists that CO_2 levels must be reduced, but some countries (most notably the United States and China) continue to resist serious policies, because of the lack of accurate predictions about the future of the environment. [i]

2. Atmospheric Carbon Dioxide

Carbon dioxide (CO_2) is accounted for 355 parts per million (ppm) of our current atmosphere. This concentration has steadily been rising by 2ppm every year, due largely to the domestic consumption of fossil fuels such as coal, oil, and natural gases. A famous geologist named C. D. Keeling, has provided us with measurements which show evidence of a long-term increase in the concentration of atmospheric CO_2 in both the Northern and Southern hemispheres. This evidence also shows a saw-tooth pattern in the trend of the rise of CO_2 in the atmosphere, which tells us that the abundance of CO_2 decreases during spring and summer due to photosynthesis and rises during fall and winter due to respiration and decay.

Photosynthesis is the process by which plants use solar energy to transform H_2O and CO_2 to carbohydrates, releasing O_2

$$He + CO_2 + H_2O \rightarrow CH_2O + O_2$$

Respiration is the process by which we convert carbon in organic form to CO_2; we inhale O_2 and exhale the oxidation product CO_2, recovering a portion of solar energy.

$$CH_2O + O_2 \rightarrow CO_2 + H_2O + energy$$

Carbon that is fixed by photosynthesis is converted into a variety of chemical formulas by plants and animals. The chemical formula CH_2O in the reactions above provides an umbrella description for the diversity of organic species in nature; its utility lies in the fact that it correctly accounts, on the average, for the oxidation state of the composite of these species. [i]

3. The Carbon Cycle

Carbon is the fourth most abundant element in the universe (not including dark matter) after hydrogen (~75%), helium, and oxygen. Carbon in the atmosphere usually forms CO_2 molecules which are 3.67 times heavier than pure carbon. This means when one ton of carbon is released into the atmosphere, 3.67 tons of CO_2 is being produced. About 7 billion tons of carbon (26.7 billion tons of CO_2) are produced each year amounting to about 6 parts per million (ppm) of the total atmosphere (4.41 trillion tons) through the burning of fossil fuels. Currently, the atmosphere consists of approximately 380 ppm of CO_2, meaning that the burning of fossil fuels adds 1.8% of the current CO_2 concentration to the air every year. Due to its abundance and tetra valence, CO_2 is the most important element in sustaining life on Earth, given that it represents 50% of the dry weight of plants and animals. The amount of carbon on Earth has remained relatively unchanged since the formation of the planet. The combination of atmospheric CO_2 and water forms carbonic acid (H_2CO_3), which is a weak acid. Carbonic acid slowly combines with calcium and magnesium to form insoluble carbonates in a process called weathering. These carbonates eventually erode to the bottom of the sea floor, where they are slowly absorbed into the center of the planet by subduction between continental plates. The insoluble carbonates melt and form magma, which is eventually released in volcanic eruptions and becomes atmospheric CO_2 once again. [i]

There are two forms of the carbon cycle on Earth. The geological carbon cycle occurs over a period of millions of years, while the biological carbon cycle occurs over a period ranging from days to thousands of years. Geological carbon is released into the atmosphere primarily during volcanic eruptions and has been shown to drastically affect global temperatures; however, it cannot be controlled by humans. Biological carbon, on the other hand, is stored in plants, animals, and fossil fuels and can be controlled by humans by limiting the amount of fossil fuels burned each year and by increasing the amount of plants on the Earth.

The biological and physical carbon cycle occurs on much smaller time scales than the geological carbon cycle, and can be used much more easily to facilitate a reduction in atmospheric carbon. The biological carbon cycle absorbs and releases 1000 times more carbon in a typical year than the geological carbon cycle. Plants absorb atmospheric carbon when they use sunlight, CO_2, and water, to form sugar in a process known as photosynthesis. Many other organisms metabolize sugars with O_2 and release CO_2 back into the atmosphere through respiration. Decomposition is usually respiration by fungi and bacteria, which takes place on dead plants and animals. During periods in history when photosynthesis has outweighed respiration, dead organic matter surged resulting in the coal and oil deposits tapped today.

The geological carbon cycle has likely been one of the most important factors in global temperatures. Ice cores (taken primarily from Greenland and Antarctica) allow scientists to determine CO_2 levels and temperatures (along with much more) over the past 740,000 years by determining the isotope concentrations of hydrogen and oxygen, as well as by testing trapped air bubbles of trace gases and aerosols. These have provided a strong correlation between an increase in global atmospheric CO_2 levels and an increase in global temperature.

Some organisms, including many microorganisms in the oceans, take part in both biological and geological carbon cycles, because they use carbon to make shells of calcium carbonate ($CaCO_3$), which settle to the ocean floor, thus, becoming a part of the geological carbon cycle.[i]

The carbon cycle can be broken up into groups of sinks and sources. Sinks are carbon deposit locations which remove CO_2 from the atmosphere, whereas sources add CO_2 to the atmosphere. The Kyoto Treaty recommends creating carbon sinks to offset anthropogenic carbon, therefore resulting in a carbon-neutral process (e.g. plant a tree for every ton of coal burned). There are three main natural sinks on Earth; they include forests, oceans, and soil.

Growing forests absorb CO_2 to build trees and plants, forming a natural terrestrial sink. Tropical rainforests absorb large amounts of CO_2 all year round, however, they tend to be comprised of thin soil due to high temperatures and moist conditions; the outcome is an increased speed in the return rate of CO_2 to the atmosphere. New growth taigas (sub arctic evergreen forests) have a large amount of growth and soil decays very slowly. This allows for a large ingestion of CO_2 by the soil, which keeps it out of the atmosphere. Old forests do not usually have enough growth to absorb large quantities of CO_2. Fires, however, often burn these forests down resulting in a release of CO_2. Char from forest fires becomes part of the geological carbon cycle, and is never released back into the atmosphere in the biological carbon cycle. New growth can then begin, which is a net loss of CO_2 from the atmosphere in the long run.

Another terrestrial sink is soil, which contains more carbon than is currently in the atmosphere and all vegetation combined. Humus is an organic carbon stored in soil and can be formed in large quantities where temperatures are below 25°C. Above that temperature, soil is very quickly oxidized (the reason that tropical forests have very thin soil), which does not allow the soil to act as a carbon sink. Grasslands work well as sinks due to their extensive root structures, which do not decay easily especially in their dry conditions.

Figure 1 below, shows the terrestrial carbon cycle. In this model, carbon is distributed among six reservoirs. The model was devised to study the response of the atmospheric/terrestrial carbon system to changes in land use and the combustion of fossil fuels. It serves as a useful function in focusing our thoughts on the manner and rate at which carbon is exchanged between the atmosphere and biosphere, biosphere and soil, and soil and atmosphere. The model shows the amount of carbon each reservoir contains as well as the rate of exchange between it and other reservoirs. The cycle is shown in steady state prior to the large scale of disturbance by land use and combustion of fossil fuels.

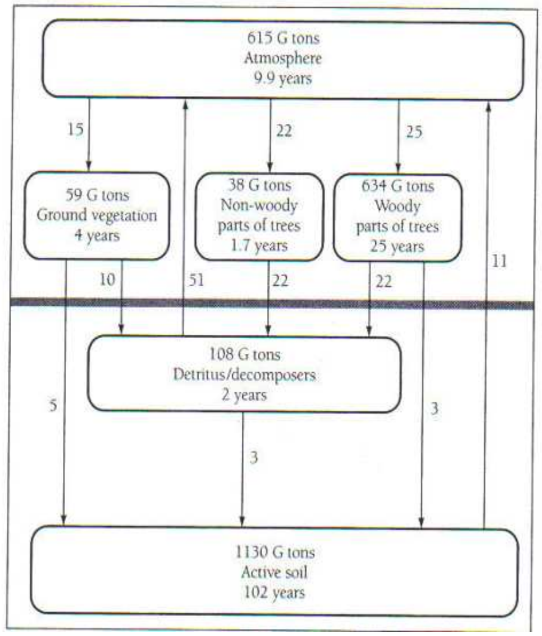

Figure 1

The resident time for carbon in a particular reservoir is given by dividing the content of the reservoir by the rate at which carbon either enters or leaves the reservoir. The model also shows paths carbon takes when returning to the atmosphere and the lifetime of it while in different reservoirs. [i]

The ocean represents the largest sink on the planet. It removes carbon by dissolving CO_2 into carbonic acid, and by photosynthetic organisms absorbing CO_2 before burying their carbon at the bottom of the sea, which eventually forms oil and natural gas reservoirs. The dissolution of CO_2 into the ocean is the most important factor, by far, and results in the ocean being able to absorb about 1/3 of all anthropogenic carbon. The ocean is buffered enough that its pH has not yet dropped due to levels of carbonic acid; however, if the ocean continues to absorb as much CO_2 as does today, its pH will drop enough to disrupt its biological CO_2 absorption and turn the ocean into a carbon source rather than a sink. This occurs because a vast variety of sea life use carbonates to create protective shells. Larger amounts of CO_2 absorption by the ocean result in a progressively lower pH, therefore decreasing the concentration of basic HCO_3^- ions. Shells dissolve when bicarbonate ion concentrations drop, and acid concentrations increase, which is what occurs when CO_2 is added to the water.

When the shells dissolve, they release additional carbon into the ocean, further propagating the overall effect until the ocean water begins releasing CO_2 back into the atmosphere as a gas in order to once again achieve equilibrium. When this happens, the ocean will have turned from a carbon sink into a carbon source. [xxvii]

Average Ocean Surface pH

Time	pH	pH change
Pre-industrial <1700s	8.179	0
1994	8.104	-0.075
2050 w/560 ppm CO_2	7.949	-0.230
2100	7.824	-0.355

Figure 2 below, shows carbon cycles within the ocean. As you can see, the ocean is split into 4 reservoirs at different depths. The abundance of carbon in the ocean, as it is shown, is 60 times larger than that of the atmosphere. We can see that there is a large amount of carbon transferring from the ocean's surface to the underlying ocean, which is mediated by biological activity near the surface, a process referred to as a "biological pump". This process draws carbon from the atmosphere and sequesters it into the deep sea.

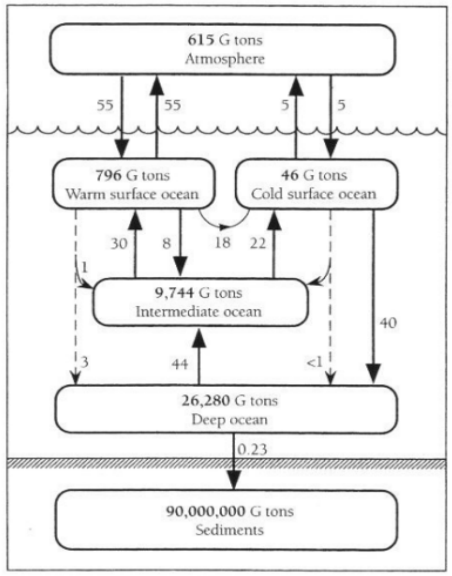

Figure 2

Figure 3 shows the overall carbon cycle, a composite of the data in figure 1 and 2. This figure explicitly represents the sedimentary reservoir. It's clear that the carbon atom spends most of its life in the ocean (160,930 years). The residence time of the atom in the sediment is estimated to be 390 million years which makes him circulate 10 times through the sedimentary compartment. This is due to the motion of the tectonic plates carbon is either uplifted or sub ducted over time.

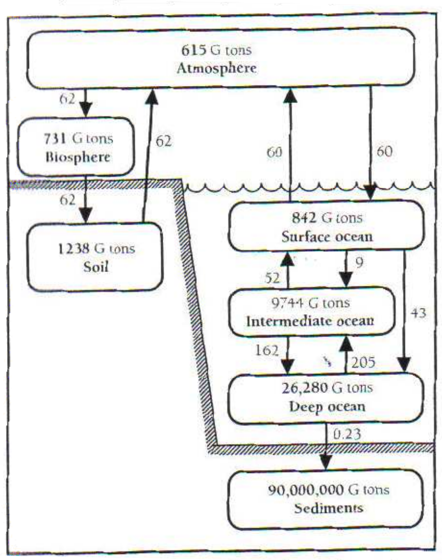

Figure 3

The carbon cycles presented in the three figures above are in steady state. Our interest is now in the amount of CO_2 released into the atmosphere that is associated with the combustion of fossil fuels and factors influencing the capacity of the ocean to absorb industrial CO_2.

As shown in figure 3, the ocean is filled with positive (cations) and negative charges (anions). Carbon dioxide behaves as a weak acid when in solution.

$$CO_2 + H_2O \rightarrow HCO_3^- + H^+$$

In the ocean this reaction becomes:

$$CO_2 + CO_3^{2-} + H_20 \rightarrow 2\ HCO_3^-$$

The ability of the ocean to absorb CO_2 is limited by the availability of CO_3^{2-}. This conversion is accompanied with an increased level of acidity.

Exchange of CO_2 between the ocean and atmosphere depends upon the partial pressure of CO_2 in air. If the partial pressure of carbon dioxide (pCO_2) in the ocean is larger than the amount of CO_2 that is transferred from ocean to the atmosphere, then CO_2 is transferred from the atmosphere to the ocean. This process continues until the pressure is at equilibrium.

A rise in the temperature of water results in an increase of the concentration of H^+ and a rise of pCO_2, therefore transferring CO_2 from the ocean to the atmosphere. When the temperature of the ocean decreases, the opposite result occurs. An increase in pCO_2 also results in an increase in the acidity of the ocean, since CO_2 is a weak acid. The fractional change in pCO_2 is called the Revelle factor and is a measure of the ocean's capability to take up additional CO_2 from the atmosphere.

Now we are able to calculate the fraction of CO_2 released by the combustion of fossil fuels since the industrial revolution that persists in the atmosphere. As previously stated, the ability of the ocean to absorb CO_2 is limited by the availability of CO_3^{2-}. The concentration of CO_3^{2-} decreases with the increase of pCO_2 which means the more CO_2 we have in the ocean, the less CO_3^{2-} there is present.

Due to the fact that the ocean turns over slowly (only 10% of the ocean total volume has had contact with the atmosphere since the industrial revolution) the actual fraction of CO_2 retained by the atmosphere from fossil fuel carbon is much larger than the one implied. The ability of the ocean to take up extra CO_2 is limited both by the low concentration of CO_3^{2-} and by the sluggish nature of the vertical circulation of the ocean. The capacity of the ocean as a sink for fossil carbon would be enhanced if the abundance of CO_3^{2-} were higher. This could occur as a result of the dissolution of calcium carbonate in sediments.

$$CaCO_3(s) \leftrightarrow Ca^{2+} + CO_3^{2-}$$

The abundance of CO_3 is relatively large in the surface and comparatively low in the depths and determines the stability of the calcite. This variation in abundance is controlled by the production in surface water and decay in deeper waters of organic matter and its associated carbonate. [i]

"The increase in the abundance of carbon contained in a unit volume of sea water is given by

$$(\Delta C)_{ocean} = ([CO_3^{2-}]_0 - [CO_3^{2-}])$$

Assuming that the volume of the ocean water exposed to the atmosphere since the industrial revolution (i.e., the quantity of water that has had the opportunity to interact directly with the atmosphere) is given by V, the net increase in the carbon abundance of the ocean is given by

$$(\Delta C)_{ocean} = V([CO_3^{2-}]_0 - [CO_3^{2-}])$$

The reduction in the concentration of CO_3^{2-} is offset by a proportional rise in the value of pCO_2. If the initial partial pressure of the CO_2 is given by $(pCO_2)_0$ with the present value equal to pCO_2, it follows that we may write the product pCO_2 $[CO_3^{2-}]$:

$$(PCO_2)_0\,[CO_3^{2-}]_0 = PCO_2[CO_3^{2-}]$$

Using this previous equation to substitute then for $[CO_3^{2-}]$ in $(\Delta C)_{ocean} = V([CO_3^{2-}]_0 - [CO_3^{2-}])$ we find

$$(\Delta C)_{ocean} = \frac{V[CO_3^{2-}]_0}{PCO_2}(PCO_2 - (PCO_2)_0)$$

Suppose that the mass of carbon initially present in the atmosphere is given be M_0. The mass of carbon in the atmosphere at any given time is, of course, proportional to the value of the partial *pressure* PCO_2*. It follows that the change in the mass of carbon in the atmosphere since the industrial revolution (beginning about 1800 is given by*

$$(\Delta C)_{atmos} = \frac{PCO_2 - (PCO_2)_0}{(PCO_2)_0} M_0$$

The fraction of the total carbon added to the atmosphere-ocean system persisting in the atmosphere is given by

$$f_{atmos} = \frac{\Delta C)_{atmos}}{(\Delta C)_{atmos} + (\Delta C)_{ocean}}$$

Using $(\Delta C)_{ocean} = \frac{V[CO_3^{2-}]_0}{PCO_2}(PCO_2 - (PCO_2)_0)$ *and* $(\Delta C)_{atmos} = \frac{PCO_2 - (PCO_2)_0}{(PCO_2)_0} M_0$ *to substitute for* $(\Delta C)_{ocean}$ *and* $(\Delta C)_{atmos}$*, we find, after some algebraic manipulation,*

$$f_{atmos} = [\,1 + \frac{(PCO_2)_0}{PCO_2}[CO_3^{2-}]_0 \frac{V}{M_0}\,]^{-1}$$ ”[i]

4. Carbon Sources and Sinks

The inputs and outputs contributing to the abundance of CO_2 in the contemporary environment are another topic of debate. According to steady state hypothesis, the inputs and outputs from many different parts of the biosphere were in balance, however this hypothesis should not be considered for the present-day environment. Studies show that the concentration of CO_2 in the atmosphere has increased from about 315 ppm in 1958 to about 380 ppm to the present day. Polar ice core studies suggest that the concentration of CO_2 has increased rapidly since the time of industrial modernization, beginning in the 18th century. [vii]

It is clear that the industrial modernization, among many other factors, has played a large role in the increase of CO_2 in the atmosphere. One area that accounts for this large increase of CO_2 in the atmosphere is the burning of fossil fuels, such as coal, which has come as a direct result of modernization. An additional reason for the increase of CO_2 is that when wood was the primary fuel source in the United States during the 18th century, forests were depleted and the soil was used for agriculture. Another anthropogenic influence on the atmospheric carbon level is the depletion of sinks for CO_2 around the globe. Recent deforestation in the tropics has been reducing a large CO_2 sink, and burning of the forest has become a carbon source.

Ralph Keeling (the son of C.D. Keeling) invented one way to measure carbon in the atmosphere by using an instrument to measure the abundance of O_2. This approach carefully measures the abundance of CO_2 and O_2 coupled in the atmosphere and can provide information about the ocean-biosphere-soil system as a sink for CO_2. The theory is that with a given measurement of the change in the quantity of atmospheric CO_2, and knowing the contribution of this change from the factors contributing to the CO_2 abundance, we can estimate the quantity of carbon transferred from the atmosphere to the ocean and the net exchange of the atmospheric carbon in the biosphere and the soil.

Over the last 40 years the biosphere has played a minor role in studies of contribution of CO_2 to the atmosphere, but with new information this view has recently changed. The increase of the temperature may drastically change the biosphere, which may result in the ocean becoming a less efficient carbon sink. Studies show that the United States has been responsible for an increase of the CO_2 abundance by about 20 ppm over the last 100 years. China is also greatly responsible for contributing to the increase of CO_2 and is expected to emit about 100 ppm by the 2100. These two countries are the biggest contributors of CO_2 in the atmosphere due to their industrialization and increased use of fossil fuel. Carbon emissions need to significantly be reduced in order to balance its abundance in our atmosphere. [i]

5. Solar Radiation Absorption and Transmission

Solar energy is delivered to Earth in the form of light from the sun. Earth, in return, releases most of the solar energy it receives, back into space. The balance between the energy the Earth absorbs and what it emits is crucial because the atmospheric temperature could rapidly change due to it.

Radiation propagates through space as an electromagnetic wave, and the intensity of this light is determined by its frequency and wavelength. Radiation emitted by the sun as a function of wavelength is known as its spectrum. The following graph exhibits the electromagnetic (EM) spectrum of a ray of sunlight.

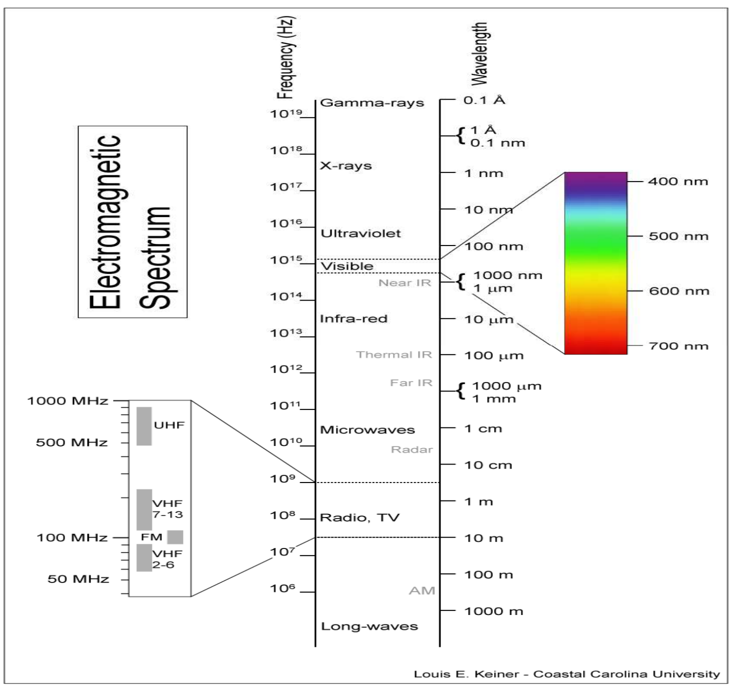

Figure 4

These electromagnetic waves have wavelength characteristics of the color of light as seen in the diagram above. Wavelength (λ) defines the distance between successive crests of the wave and the frequency (f) of the wave is given by the number of wave cycles that pass a particular reference point in unit time.

Wave speed (c) = frequency x wavelength

The energy of a light particle E is given in terms of the frequency of the radiation field by a relation first derived by Einstein.

$$\lambda = \frac{c}{f}$$

And

$$E = hf$$

Or

$$E = \frac{hc}{\lambda}$$

Where c is the speed of light 299,792,458 m/s and h is Planck's constant, 6.62×10^{-27} erg sec.

"*Radiation includes an infinite array of radio, microwave, infrared, visible-light, ultraviolet, X-ray, and gamma-ray emissions. The un-attenuated path of radiation to the Earth's surface is called transmission. The Earth's atmosphere has a dramatic effect on the transmission of solar radiation, as one can easily tell when looking outside on a dark rainy day. Whether it is a completely clear or dark and gloomy day, the Earth's atmosphere will affect the solar radiation at various wavelengths. As solar radiation travels through our atmosphere, it can be absorbed, scattered, or reflected*."[xxxix] When viewed from space, the Earth's spectrum looks like the back of a double-humped camel; the short wavelength hump is contributed by solar radiation reflected by clouds, the atmosphere, and the surface on the day side of the planet. The long wavelength structure is due to thermal emissions from the atmosphere and the surface. The figure below shows solar radiation in the atmosphere.

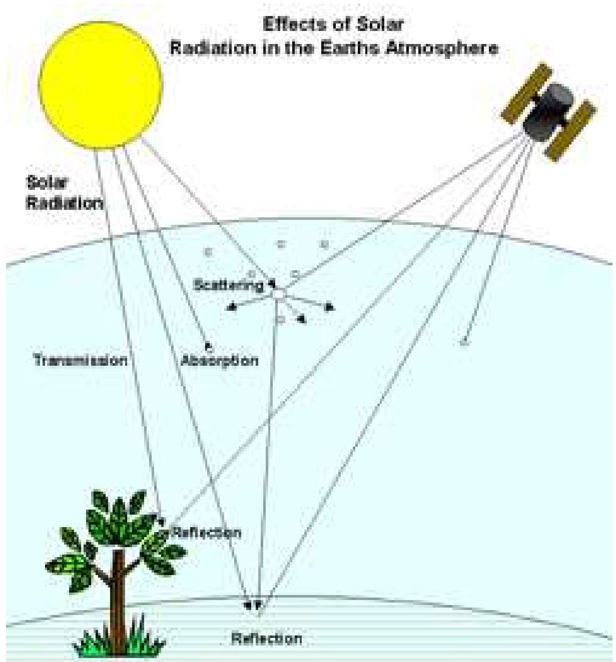

Figure 5

Transmissions of radiation extending to about 100 µm (10^{-2} cm) are referred to as infrared and are limited by clouds and trace constituents of the atmosphere such as H_20, CO_2, O_3, CH_4, CO, N_2O, SF_6 as well as industrial chlorofluorocarbons. Energy radiated by the surface in the infrared region is partially trapped by the atmosphere and is reradiated back to the surface. Atmospheric trapping of infrared radiation is primarily responsible for what is known as the Greenhouse Effect. The importance of the role played by H_20, CO_2, O_3, CH_4, CO, N_2O in atmosphere absorption and emission of infrared radiation is shown in the figure below.

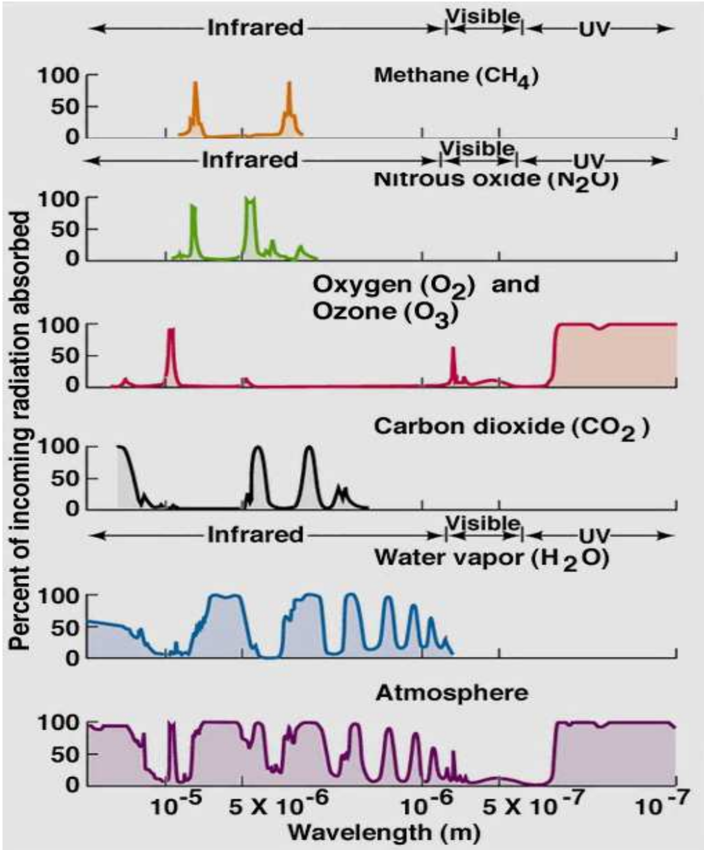

Figure 6

Of the 70 units of solar energy absorbed by the atmosphere-surface system, 64 units are returned into space in the form of infrared radiation from the atmosphere. High altitude clouds and infrared absorbing gases such as H_2O and CO_2 are primarily responsible for the emission of infrared radiation from the atmosphere. The energy that is reflected determines the albedo of Earth. The fraction of the total incoming solar energy reflected into space [(20 + 10) / 100 = 0.30] defines the albedo of Earth.

Research

1. Methods of Storing Anthropogenic CO_2

Natural resources such as coal, oil, and natural gas are harvested from beneath the ground and burned to produce electricity, heat transportation, and industrial power. They are released to the atmosphere in the form of carbon dioxide. Since the industrial revolution in the beginning of the 20th century, CO_2 levels in the atmosphere have increased by approximately 95 parts per million (ppm). This anthropogenic rise in carbon dioxide levels is one of the major reasons Earth has suffered an increase in its temperature.

The figure (from The Carbon cycle Group of the NOAA Climate Monitoring and Diagnostics Laboratory) shows the rise in CO_2 levels in the atmosphere from the late 1950's to present day in Mauna Loa, Hawaii. [xlv] Predictions of global energy use in the next century suggest a continued increase in carbon emissions and concentration of CO_2 in the atmosphere unless major changes are made in the way we produce and use energy. Since it is clear that the problem of atmospheric CO_2 accumulation will not simply go away, we must seek to discover new methods of reducing CO_2 emissions and harness existing CO_2 in the atmosphere. Carbon sequestration is one of the processes that are being investigated. This involves the removing of CO_2 from the atmosphere and storing it in the terrestrial biosphere, underground, or in the oceans. [xi]

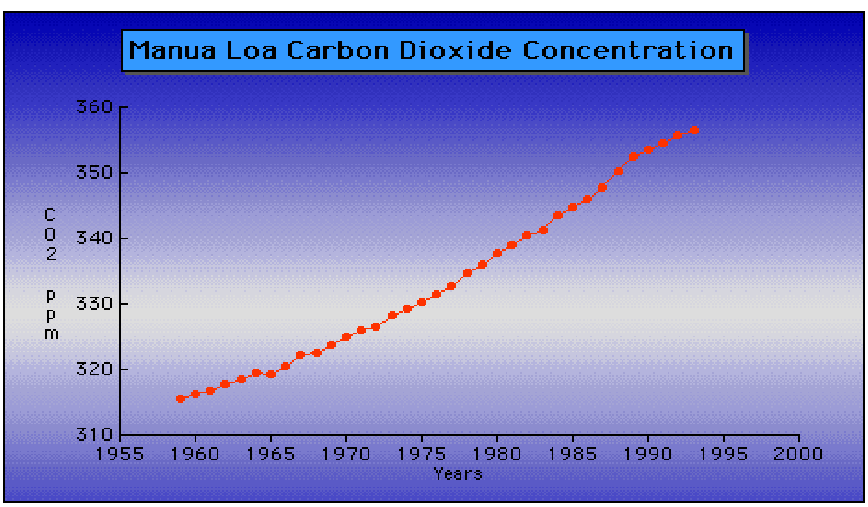

Figure 7

Carbon sequestration usually occurs naturally through absorption of carbon dioxide by plants and oceans. According to Rowland Benjamin, president of Information for Action, most methods of enhancing natural carbon sequestration are already being implemented; one is the planting of trees and forests. On land, carbon dioxide is stored in wood products, such as tree trunks, and soil. Re-growth of plant-life in previously devastated areas, such as farmland and areas where forest fires have occurred helps in reducing anthropogenic carbon dioxide. Studies have shown that the terrestrial biosphere is a larger contributor in the global carbon cycle than the ocean. However, the rate at which forests can absorb carbon given the availability of land is far exceeded by the rate at which it is released by the burning of fossil fuels. Reducing U.S. carbon emissions by 7%, as set by the Kyoto Protocol, would require the planting of a forest the size of Texas every thirty years.

In the ocean, carbon dioxide molecules react with carbonate ions to produce bicarbonate ions. Surface water carrying the anthropogenic carbon dioxide descends and spreads throughout the ocean as part of the thermohaline circulation. The time frame for this process to be completed is more than 1000 years.

Today there are a variety of technologies being researched to artificially capture and store CO_2. Two new ways of carbon sequestration are being explored; they include enhancing the existing natural carbon sequestration processes, and capturing and storing carbon artificially.

One way of enhancing natural sequestration is to add iron sulfate particles into the water. This stimulates the growth of plankton which results in a greater amount of photosynthesis in oceans. The impact of increasing the amount of plankton on the biosphere is currently unknown. Because of this, this method is only implemented on a smaller scale.

Since the natural enhancement of carbon sequestration does not seem to be particularly effective, many scientists have been researching artificial ways to capture and store carbon. Carbon can be captured from the atmosphere using solvents and hydroxides. After the carbon is captured there are a variety of methods that can be used for storage. One of the most common methods is geo-sequestration; the process of capturing CO_2 in a non-gaseous state and storing it underground. To prevent CO_2 from entering the atmosphere coal is burned and turned into a mixture of gases. When coal is burned in oxygen, higher concentrations of CO_2 are produced which are easier to separate. Carbon dioxide is removed from the gas mixture and compressed into liquid form. The liquid CO_2 is then pumped deep underground into crevices that exist in solid rock formations where it is secured and stored for thousands of years. Other potential storage sites include old oil and gas fields, coal beds or under the deep seafloor. The gas that remains after processing is CO_2 free and can be used by power stations.

Although geo-sequestration seeks to reduce CO_2 admissions, thereby reducing greenhouse gas concentrations, there are many potential problems with this method. Almost all the existing power stations will require major modifications or will have to be replaced so that they are able to capture the CO_2. This will require major financial investments. The safety and security of storage also needs to be considered due to potential CO_2 leakage out of the rock it is stored in. Public acceptance is potentially the most challenging obstacle, as the community needs to be convinced of the benefits of geo-sequestration and support such an activity despite the cost and associated risk factors.

2. Methods of Measuring Anthropogenic CO_2

It is known that due to climate changes and precipitation glaciers around the world change dimensions. Scientist's records show that glaciers have been shortening at an almost constant rate since early 18th century. Contrary to popular global warming beliefs, about 70 percent of this process occurred before 1940 when human usage of hydrocarbons and production of CO_2 began to spike. Humans have been adding vast amounts of CO_2 to the atmosphere for nearly 70 years (8Gt C/yr), during which, the glacier shortening rate has remained nearly unchanged. The same trends occurred before the Medieval Climate Optimum and if they continue, we can expect sea levels to increase by about 14 inches over the next two centuries.

One method of gathering climate data from glaciers involves taking ice core samples. Snow falls to the Earth carrying with it compounds that exist in the air. These compounds include different ions like sulfate and nitrate, dust, radioactive fallout, as well as trace metals. The snow accumulates into area where the temperature is constantly below freezing. Over years the weight of the new snow turns the older snow into ice while trapping small air bubbles inside of it. Both compounds as well as the air bubbles trapped in the snow when analyzed give the scientists information of the atmospheric composition at different times.

Water in the oceans contains primarily oxygen with an atomic weight of 16 (^{16}O) but also small amounts of oxygen with an atomic weight of 18 (^{18}O). The later one is 12% heavier than "typical" oxygen. Heavier isotopes have a lower vapor pressure, which is why when the temperature falls, the heavier water molecules (^{18}O) condense faster than the normal water molecules (^{16}O). The comparative concentrations of the heavier isotopes in the condensate indicate its temperature at the time. The ratio of ^{18}O to ^{16}O will vary depending on the temperature of evaporation and how far the water has had to travel before it fell as snow. Over short time scales a very clear oscillation in the $^{18}O/^{16}O$ isotopic ratio, known as $d^{18}0$, is produced by the change in temperature from summer to winter. This oscillation is used to determine the age of the core, simply by counting how many oscillations there are within a certain depth. The $^{18}O/^{16}O$ isotopic ratio can be measured very accurately using a mass spectrometer over longer time periods. This ratio is able to indicate the average temperature of the region between the evaporation site and the coring site. Annual and even seasonal estimates are possible with ice cores, but because they are in high resolution, they are only able to produce direct data up to several hundred thousand years ago. Figure-8 shows a “19 cm long section of GISP 2 ice core from 1855 m showing annual layer structure illuminated from below by a fiber optic source. Section contains 11 annual layers with summer layers (arrowed) sandwiched between darker winter layers.” [xvii, xviii]

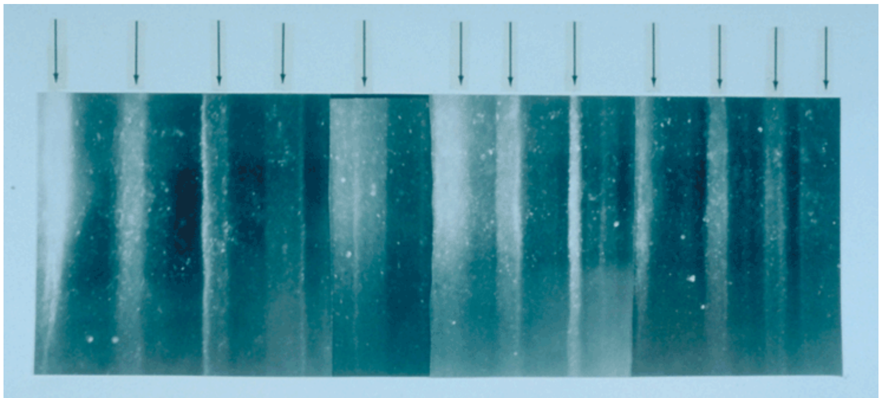

Figure 8

Some scientists believe glacier shortening is regulated by solar activity (not CO_2 emissions), which is directly correlated with the surface temperature. Geographical characteristics, also greatly affect the temperature ranges of each region. Most locations do show evidence of experiencing both the Medieval Climate Optimum and the Little Ice Age. However when this temperature information is placed alongside hydrocarbon usage graphs, trends in each of the two do not correlate with each other, thus demonstrating evidence that manmade CO_2 emissions have had little effect on the Earth's temperature. A recently experienced warming trend between 1978 and 1979 has also been established as naturally occurring due to an El Nino that year. Temperature graphs for each region, as well as global, indicate declining temperatures just before El Nino and directly after, which would conclude that the increase in temperature was unrelated to CO_2 emissions.

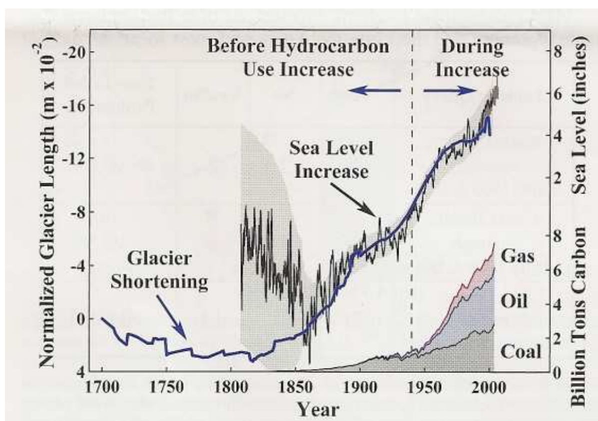

Figure 12: Glacier shortening (4) and sea level rise (24,25). Gray area designates estimated range of error in the sea level record. These measurements lag air temperature increases by about 20 years. So, the trends began more than a century before increases in hydrocarbon use.

Figure 9

Another method, similar to ice cores, involves deep sea cores taken from accumulated sediments on the ocean floor. This can also provide information about climate, although by indirect means. Sediments on the other hand, accumulate very slowly which results in much longer records from deep see cores than ice cores, but have a much reduced ability to determine short term changes (low resolution). One example of this indirect evidence is the method for determining temperature. When sediment cores are analyzed, researchers meticulously sort out plankton shells which become twisted in different directions depending on the temperature of the water they grew in. Through counting the number of shells that twist each way they can determine the temperature of the surface water at a particular time. Sediment cores can provide records which are as long as several million years, but further understanding of the behavior of this plankton is necessary to produce a historical record of temperature for the ocean. Because of the differences between sediment cores and ice cores, data taken from each are able to provide complimentary climate information.

3. Global Dimming

The term "global dimming" was coined by Gerry Stanhill after studying the apparent decrease in sunlight hitting the Earth in locations worldwide. Several studies conducted in the 1980s discovered that the sunlight hitting the terrestrial surface of the Earth had been dimming by approximately 2-3% per decade. The amount of light hitting the surface of oceans has not been studied, and the amount of light hitting the highest levels of the atmosphere has been constant indicating a change in the composition of the Earth's atmosphere. [xxvi]

Global dimming reduces direct solar energy reaching the surface of the Earth for two reasons. Particulate matters in the air (aerosols) are one of the reasons. Pollution particulates can also be seeds for clouds, giving water something to stick to. The clouds that are formed either would not exist without these particles, or would be much less reflective. These two factors block different wavelengths of the Suns light excluding the UV range. The particles and clouds mostly block light during the day and trap heat at night, leading to cooler days and warmer nights.

Proof of solar dimming, outside of actual light readings on the planet's surface, has come from factors like the evaporation of water at a global scale (pan-evaporation). Despite an increase in global temperature, water evaporation has actually decreased. These readings are easy to verify through agricultural companies readings for over 50 years. While it turns out ambient temperature is not a serious factor in evaporation (vapor pressure deficit and wind speed are much more important), some scientists consider pan-evaporation the greatest proof of global dimming. [xxvii]

According to NASA the global dimming effect has become a brightening effect especially over the oceans since 1990. Countries all over the world began reducing aerosol emissions around that time. These were mostly developing nations who usually did not mitigate their production of CO_2. This has led to a stronger warming trend.

In 1974, Mikhail Budyko suggested that the global dimming effect be used to mitigate global warming, if it ever became a problem. His solution would be difficult to implement because the particles we use to pollute the air with, contain elements which cause acid rain, change in rainfall patterns and lead to droughts in some areas. Eventually, the particles in the atmosphere should be reduced leading to a cleaner and healthier environment. If this is done before we solve the problem of global warming, it could lead to a greater disaster in that area. [xxviii]

4. Effect of Climate Change on Human Health

A rise in the mean temperature of the Earth has many consequences that affect future generations. Scientists have predicted that an increasing global temperature will result in disastrous effects such as the rise of sea levels due to melting glaciers, severity of storms, and an increase in precipitation. It is also clear that the effects of global warming form a chain reaction resulting in global chaos which may ultimately end with the extinction of life on planet Earth.

In the US, according to the EPA, it has been recorded that instances of extreme heat have lead to twice as many deaths than extreme cold annually. Extended periods of extreme heat affect individuals differently. Particularly the young, sick, and elderly, may have respiratory problems and eventually respiratory failure, due to weaker cardiovascular functioning which the body utilizes in order to produce sweat to cool itself.

Increase in temperature can also lead to an increase in air pollution. Higher temperatures cause ozone to become trapped in the lower atmosphere which is extremely harmful to breathe. According to the US EPA, an increase in temperature of about 4 degrees Fahrenheit could increase ozone concentrations by as much as 5 percent. Exposure to ozone destroys lung tissue which could lead to respiratory problems, nausea, pulmonary congestion, and even death.

Another harmful effect of global warming would be the spread of disease. As temperatures increase in cooler areas, disease carrying insects and vermin have a greater territory in which to multiply. Diseases such as the West Nile Virus, Lyme disease, and Cholera become an even greater threat as mice, ticks, and mosquitoes thrive in warm weather infecting humans as well as other animals.

State	Encephalitis/ Meningitis	Fever	Other Clinical/ Unspecified	Total	Fatalities
Alabama	8	0	0	8	0
Arizona	65	77	8	150	11
Arkansas	24	5	0	29	4
California	81	186	11	278	7
Colorado	66	279	0	345	7
Connecticut	7	2	0	9	1
Florida	3	0	0	3	0
Georgia	2	5	1	8	1
Idaho	115	827	54	996	21
Illinois	122	69	24	215	10
Indiana	27	8	45	80	5
Iowa	21	13	3	37	0
Kansas	17	13	0	30	4
Kentucky	5	1	0	6	1
Louisiana	91	89	0	180	9
Maryland	9	1	1	11	1
Massachusetts	2	1	0	3	0
Michigan	43	10	2	55	7
Minnesota	31	34	0	65	3
Mississippi	89	94	0	183	14
Missouri	50	11	1	62	5
Montana	12	21	1	34	0
Nebraska	45	219	0	264	1
Nevada	34	76	14	124	1
New Jersey	2	2	1	5	0
New Mexico	3	5	0	8	1
New York	16	8	0	24	4
North	1	0	0	1	0
North Dakota	20	117	0	137	1
Ohio	36	12	0	48	4
Oklahoma	26	21	1	48	6

Oregon	7	50	12	69	2
Pennsylvania	8	1	0	9	2
South	1	0	0	1	0
South Dakota	38	75	0	113	3
Tennessee	16	6	0	22	1
Texas	233	121	0	354	32
Utah	56	102	0	158	5
Virginia	0	0	5	5	0
Wisconsin	11	10	0	21	1
Wyoming	15	40	10	65	2
Totals	1459	2616	194	4269	177

Figure 10

The rise in sea level alone would start a chain of catastrophic effects. Increasing water levels would cause global flooding especially in coastal areas and islands. This flooding would affect humans by potentially contaminating fresh water supplies with sea water, reducing the amount of habitable land, and killing animals in those habitats. These disasters may lead to a spread in disease by the contamination of drinking water via flooded sewers. People would flea in search of alternative locations in which to settle, resulting in a massive migration to certain areas of the Earth. Governments would then try to prevent migration and chaos would erupt.

5. Effect of Climate Change on Human Migration

Humans have probably existed for about five hundred thousand years, depending on the definition used for the species. During the majority of this time human migration was due to finding a better source of food. Within the past 100 thousand years, farming has allowed humans to live in a wider variety of climates than before. The graph below shows the temperature change over the past 450,000 years calculated using EPICA and Vostok ice cores, as well as the associated ice volume. [ix]

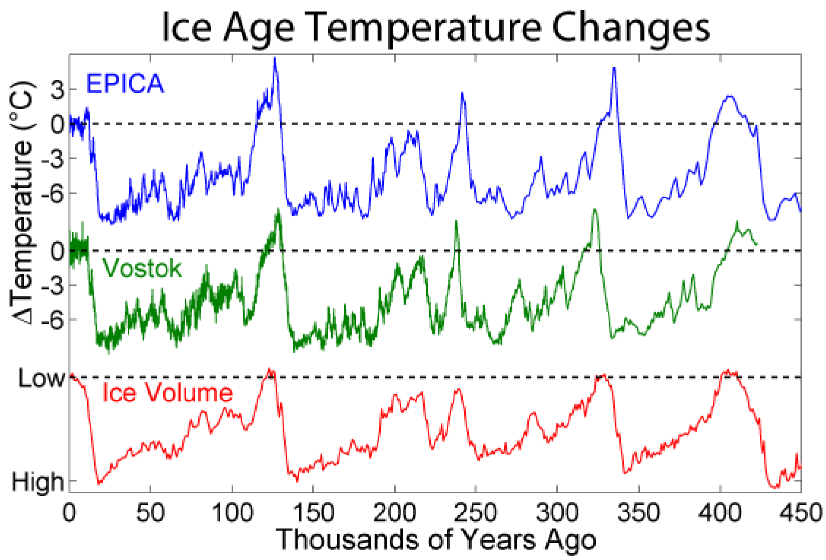

Figure 11

This gives a rough idea of the glacial and interglacial periods that have had the largest affect on human migration. For example humans began to populate North America during a glacial period. They moved from northeastern Asia into North America hunting mammoths across the frozen Bering Strait.

In most areas of the world, farming has completely replaced hunting as the source of food during the current interglacial period. Modern farming techniques allow the increase of population because the production of food is easier. Temperature and rainfall have an important effect in the growth of crops in an area, but we must consider the human labor required to plant and harvest them. Countries closer to the equator tend to be on a lower socioeconomic bracket with the current global conditions because of the difficulty of labor in hot climates. Mild climates, such as the American Midwest, are excellent for grain farming by humans because of moderate rainfalls, mild temperatures, and perfect amount of sunlight.

The migration of humans over the past 100 thousand years is an important factor to be considered as temperatures continue to change dramatically. Rising greenhouse gas levels are expected to push the global temperature even higher, which may have severe consequences for people in tropical and subtropical climates. As these climates worsen, living conditions may become unsuitable for humans and they will need to migrate toward the poles (most of the world's landmass is in the northern hemisphere, so humans will likely migrate north). This massive amount of migration becomes a problem for different countries due to their laws. These days illegal immigration is one of the major issues in the U.S. Migration is seen even within the United States where numerous people or corporations have moved from the South to the North.

Another large problem with land use is the prediction of rising sea levels. Thermal expansion means that as the oceans temperature increase, the current mass of water will occupy more volume, causing sea levels to rise. Monitoring to this point has not shown a calculable rise in sea levels due to thermal expansion, but a much larger problem lies under the surface. Global temperature affects the movement of ice at the North and South poles. Ice on land does not affect sea levels, but when the ice moves into the ocean, a rise occurs. Over the past 100 years, sea levels have risen by an estimated 6 to 12 inches due to glacial melting. Greenland glaciers are melting at an extraordinary pace, and may result in 20 feet of sea level rise. Antarctic ice shelves, which were predicted to melt very slowly, have recently shown signs that they may melt within the next 50 to 100 years. This will result in a sea level rise of approximately 200 feet. [x]

A large portion of the human population lives within 200 feet of altitude of the current sea level, and they will need to migrate inland if sea levels inundate their cities. The consequences could be catastrophic. Figure-12 below shows the increases amount of Greenland ice sheet melted in summer from 1992 on the left side to 2002 on the right side.

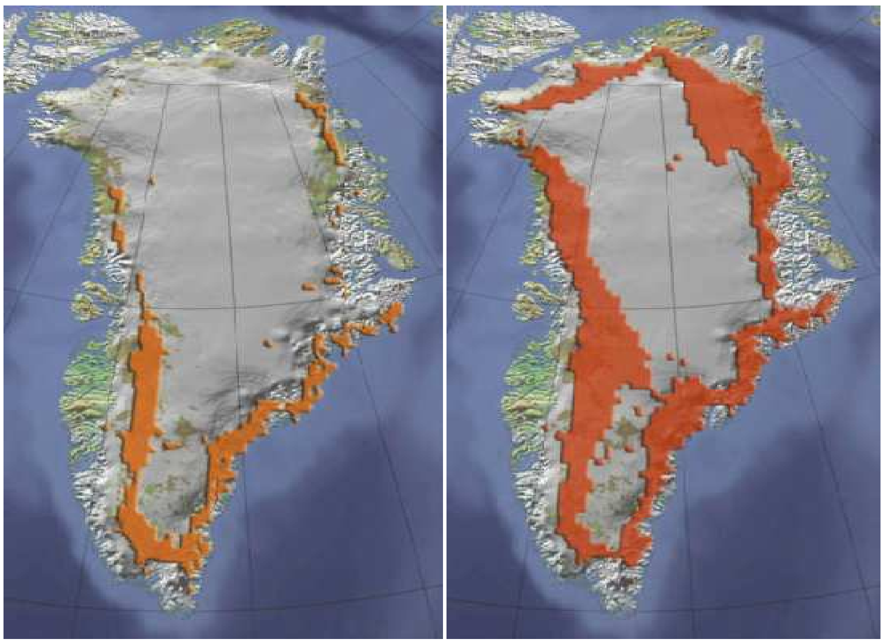

Figure 12

Russia and Canada, among other nations, are already disputing ownership of the North Pole. As northern areas become more habitable than southern areas there will likely be many more disputes, and possibly wars, over land that was once unwanted. Sea level rises may destroy sources of fresh water, and as glaciers disappear, mountain runoff will slow resulting in even less fresh water. These factors may lead to a water and land crises in the north in the near future.

6. Climate Simulation: Global CO_2 Model (GCM)

The increase in the CO_2 accumulation in the atmosphere creates large uncertainties on the Earth's environment with unknown long-term consequences. This accumulation causes the famous "greenhouse effect" which may increase the mean temperature of the Earth significantly. The change in temperature may produce major changes in the Earth's environment.

Studies of the CO_2 accumulation are important to understand the factors that are contributing to this increase. In the following discussion, we will develop an introductory mathematical model of these factors which can be used for example to study the effect of increasing fossil fuel burning and destruction of the forest areas. This model addresses only the problem of CO_2 buildup, but its output can be used in other models for climate prediction.

The structure of the model is illustrated in Figure 13 below.

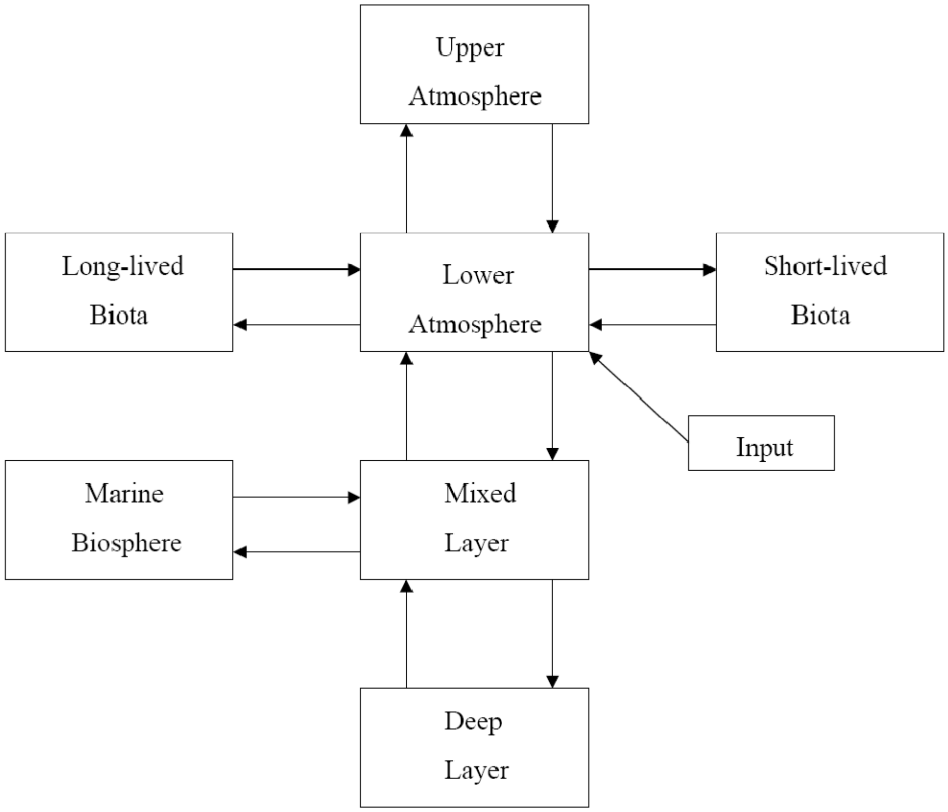

Figure 13

As you can see it consists of seven interconnected reservoirs for:

1. upper atmosphere
2. lower atmosphere
3. long-lived biota
4. short lived biota
5. mixed ocean layer
6. deep sea
7. marine biosphere

Figure 13 shows that the lower atmosphere is the only one which has an input of CO_2 from human activities. These seven reservoirs represent different parts of Earth's environment and as shown in the figure above they all interacted with each other in a different way.

The model is derived from a carbon balance on each of these seven reservoirs. Each of these reservoirs is represented by one ordinary differential equation (ODE) that uses time as an independent variable. Therefore there exist seven ODEs in the model with associate initial conditions. This system is in the end integrated over the interval $1700 \leq t \leq 2000$. The time period and the initial conditions will be subject to changes as they are being adapted to measured values over recent years.

Below are shown the seven ODEs for each reservoir:

Upper atmosphere	$\frac{dC_{ua}}{dt} = D_{lu}$	(1)
Long-lived biota	$\frac{dC_{lb}}{dt} = D_{lb}$	(2)
Short-lived biota	$\frac{dC_{sb}}{dt} = D_{sb}$	(3)
Lower atmosphere	$\frac{dC_{la}}{dt} = -D_{lu} - D_{sb} - D_{lb} + D_{ml} + \gamma(t)$	(4)
Ocean mixed layer	$\frac{dC_{ml}}{dt} = -D_{ml} - D_{md} - D_{mb}$	(5)
Marine biosphere:	$\frac{dC_{mb}}{dt} = D_{mb}$	(6)
Deep sea	$\frac{dC_{ds}}{dt} = D_{md}$	(7)

In each equation the carbon concentration in the derivative is given by the formula below:

Equation 8

$$C_{ir}(t) = \frac{C_i(t) - C_i(1700)}{C_{ta}(1700)}$$

This means that the carbon concentration in any reservoirs is equal to the amount present at the time measure minus the initial amount at year 1700, all divided by the total amount of the carbon dioxide in the seven reservoirs at 1700. In our case, we are making the carbon concentration in all reservoirs at t= 1700 zero. This will be changed in the future when we use measured values for the different reservoirs.

Important to the model are the fluxes in the right hand side of equations 1-8. These six fluxes can be written in two ways. Below we have shown the simplified version which we use in our code. Also as you will notice there are six fluxes instead of seven due to the fact that the lower atmosphere does not have an individual flux, but is dependent on 4 fluxes and the fossil fuel consumption rate.

Equation 9

$$D_{lu} = (1/T_{ul})[\frac{P_1}{1 - P_1} C_{la} - C_{ua}]$$

Equation 10

$$D_{lb} = \beta(F_{lb0}/N_{a0})C_{la}$$

Equation 11

$$D_{sb} = (1/N_{a0})F_{sb0}(\beta C_{la} + C_{lb}/P_2 - C_{sb}/P_3)$$

Equation 12

$$D_{ml} = (1/T_{am})\lfloor\frac{1 + C_{org}/C_{mt}}{P_4}\epsilon C_{ml} - \frac{1}{1 - P_1}C_{la}\rfloor$$

Equation 13

$$D_{mb} = F_{mb0}/N_{a0}(1 + C_{mb}/p_5)\beta C_{ml}/P_4$$

Equation 14

$$D_{md} = (1/T_{dm})[(W_s/W_m - 1)C_{ml} - C_{ds}]$$

The parameters used in the equations above are explained in detailed in the appendix with the appropriate values. The fossil fuel consumption rate$\gamma(t)$, (1/year) is given by

Equation 15

$$\gamma(t) = \gamma o e^{rt}$$

Where t is the calendar year starting in our case at 1700 and r is a constant. The carbon added to the lower atmosphere in tons/years, $T_{py}(t)$ is given by

Equation 16

$$T_{py}(t) = \frac{\gamma(t)N_{a0}}{(453.6)(2000.0)}$$

To summarize, our model consists of ODEs (1) to (7), the initial condition (8), the RHS fluxes of equations (9) to (14) and the yearly addition of carbon to the lower atmosphere, equations (15) and (16). [xxxii, xxxiii]

The code is written using the program MATLAB. We created one M-file (co2sys.m) for the differential equations and the variables for the CO_2 concentration of the seven reservoirs, and a second M-file (co2sim.m) to contain the parameters that we used, the time domain and executive plots of these concentrations versus time. As you can see in our code shown in the appendix, we set the carbon concentration through time for the seven reservoirs to a variable x as shown below:

Code 1

```
Cua=x(1);   %upper atmosphere
Clb=x(2);   %longed-lived biota
Csb=x(3);   %short-lived biota
Cla=x(4);   %lower atmosphere
Cml=x(5);   %mixed ocean layer
Cmb=x(6);   %marine biosphere
Cds=x(7);   %deep sea
```

To plot this concentration through time we first have to set the error tolerance and the method for the numerical integrator of the differential equations. This part of the code is shown below in code 2 where 'co2sys' is the M-file that contains the differential equations.

Code 2

```
% set error tolerance and method for the numerical
%integrator of the differential equations.
odeset('maxstep',1.,'reltol',1.e-3,'bdf','on')

%perform the integration 'lorsys' is a file that defines the system
%[0,300] time interval for the integration
%[5,5,5] define the initial conditions
[t,x]=ode15s('co2sys',[0,300],[0.;0.;0.;0.;0.;0.;0.]);
```

We used the command `plot (t, x(:,1),'r')` for each of the reservoirs to plot their CO_2 concentration with respect to time starting at 1700. Also to show all the figures at the same time we had to put the command figure before each one of the plots. The figures below show these concentration with initial conditions set to zero.

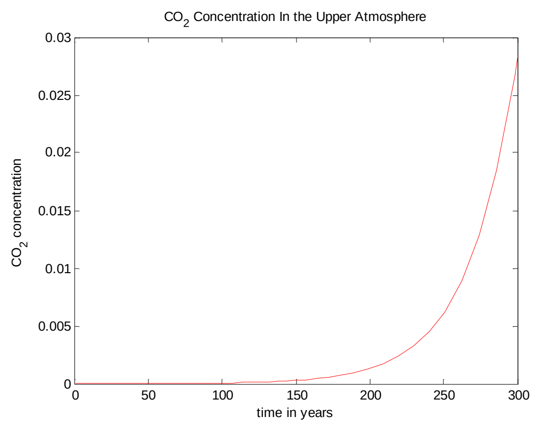

Figure 14

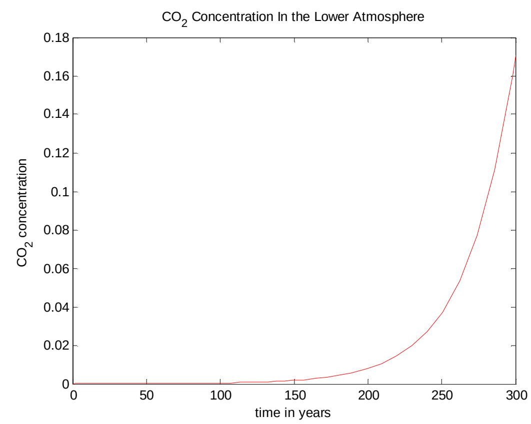

Figure 15

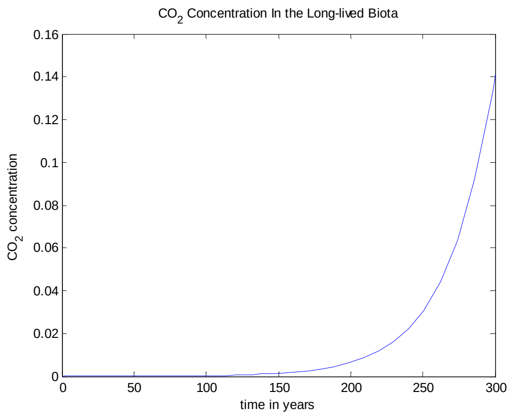

Figure 16

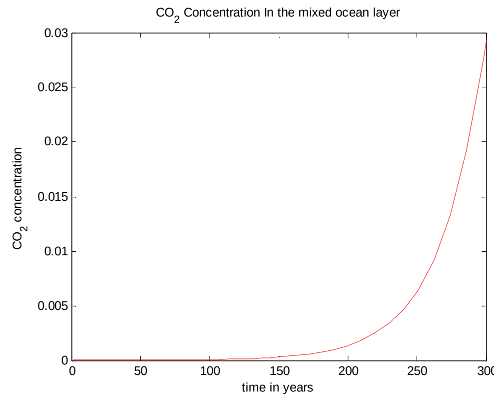

Figure 18

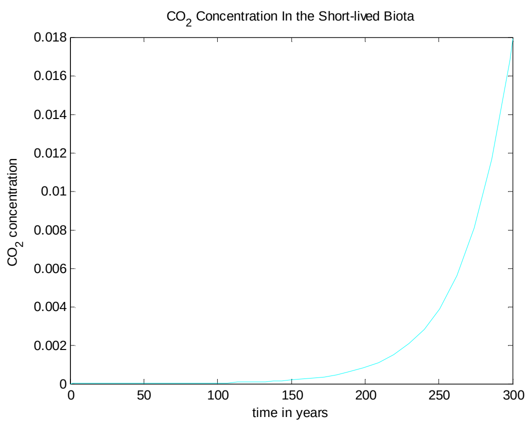

Figure 17

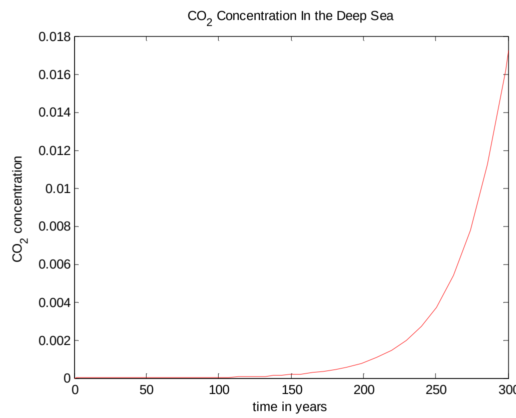

Figure 19

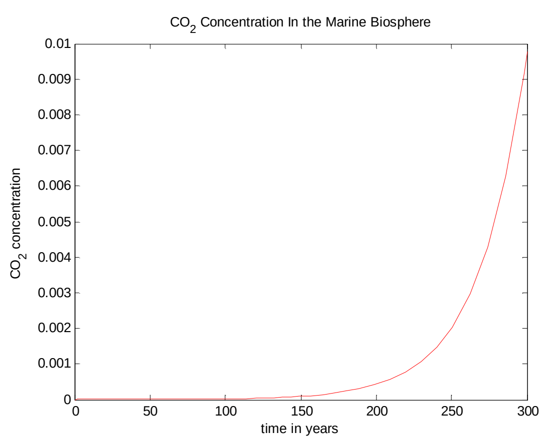

Figure 20

As you can see from the figures above the level of the CO_2 concentration in the seven reservoirs is increasing at an exponential scale in the 300 years time frame. However, these graphs are inaccurate due to the fact that the initial conditions could not be zero at the year 1700. Also the fossil fuel consumption rate (gamma) is set at predefined value that doesn't take into consideration the measured data of the CO_2 concentration in the lower atmosphere. This brought us to different modifications of the program. To change the gamma we researched the internet and found recorded measurements of the CO_2 concentration in lower atmosphere from the year 1958. These data were collected from the Mauna Loa research center in Hawaii. The data is shown in the appendix and, as you can see, is recorded for each month of the year starting from the year 1958.[xliv] These data were saved in a .dat file so that it would be easier to use by MATLAB. We imported the file using the function import and selecting the appropriate file. This function created a variable of our data in the workspace which we can use for the following manipulations. To create a new function for the gamma we open a new M-file. At the beginning of the file we write the code `load co2.dat` which makes the data available for use. The data that is shown is calculated in parts per million (ppm) which we changed into grams per litter (g/l) to match the predefined parameters. The code that we used to convert each data into g/l and the creation of the yr vector is shown in code three.

Code 3

```
%converts the relevant data into a vector
for i=1:47
for j=2:13
xco2((i-1)*12+j-1)=(co2(i,j)*10^(-6)*44)/22.4;
end
end
for i=1:47*12
yr(i)=(i-1)/12;
end
```

As you can see all the variables in the co2 data file are converted and stored in the xco2 vector. After the data is imported into the program and converted we used the least square function to represent them for our gamma parameter.

Least square is a mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offsets (residuals) of the points from the curve. Actual data almost never lie exactly on a straight line, but we sometimes want to approximate them with a straight line that fits as well as possible, this is where least square becomes beneficial. The least-squares line uses a straight line

$$y = a + bx$$

to approximate the given set of data, (x_1,y_1), (x_2,y_2) ....(x_n,y_n) , where n>2. The best fitting curve F(x) has the least square error, i.e.,

Equation 17

$$\Pi = \sum_{i=1}^{n}[y_i - f(x_i)]^2 = \sum_{i=1}^{n}[y_i - (a + bx_i)]^2 = \min.$$

This method allows us to determine the equation of the line and from its derivation we can also get the "error function". The data is fairly compact meaning that each value is close to the previous one, so a gradual increase or decrease can be represented by a line or a curve through the data points. There are many different models of least square, but our main discussion is surrounding quadratic and linear least square. The figure below illustrates the concept of least square, the first for a straight line and the second for a parabola.

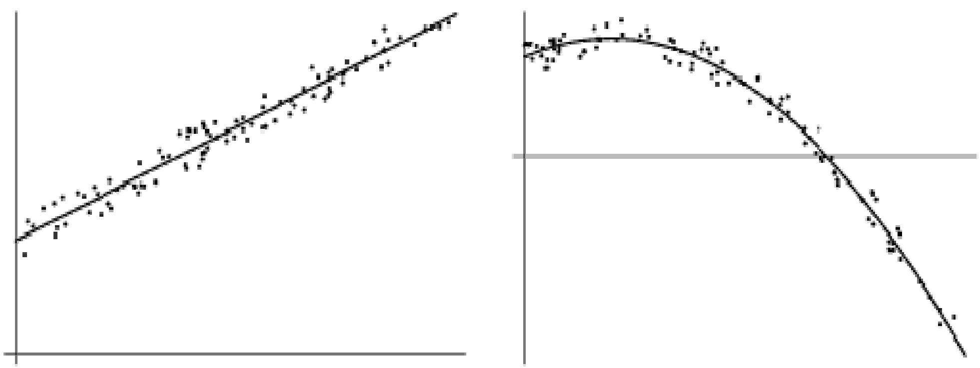

Figure 21

The figure is from a paper titled; "The Least Square fitting" written by Eric Weisstein, creator of the award winning Math World site the world's most widely accessed online mathematics resource. According to Mr. Weisstein, *"The linear least squares fitting technique is the simplest and most commonly applied form of linear regression and provides a solution to the problem of finding the best fitting straight line through a set of points. In fact, if the functional relationship between the two quantities being graphed is known to within additive or* *multiplicative constants, it is common practice to transform the data in such a way that the resulting line is a straight line, say by plotting T vs.*$\sqrt{L}$ *instead of T vs. L in the case of analyzing the period (T) of a pendulum as a function of its length (L). For this reason, standard forms for exponential, logarithmic and power laws are often explicitly computed."*

The offsets (residuals) form the basis of the error function and must be taken into account when deriving least square equations. Residuals are the horizontal or vertical distance from the offset to the least square representation.

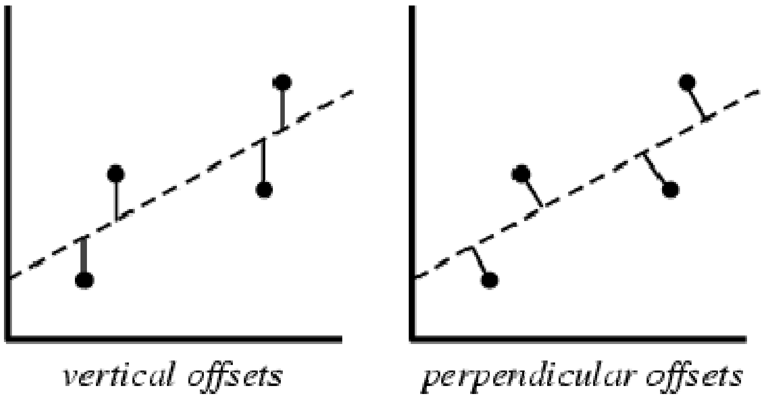

Figure 22

The sum of the squares of the offsets is used instead of the offset absolute values because this allows the residuals to be treated as a continuous differentiable quantity. In practice, the vertical offsets from a line (polynomial, surface, hyper plane, etc.) are almost always minimized instead of the perpendicular offset. [xxxiv]

The MATLAB code that will give us the least square of our data is shown below:

Code 4

```
p1=polyfit(yr,xco2,1);
t1 = 0:1:50;         % Define a uniformly spaced time vector
y1=polyval(p1,t1);   % Evaluate the polynomial at t1
```

The polyfit(x,y,n) functions find the coefficients of a polynomial p(x) of degree n that fits the data, p(x(i)) to y(i), in a least squares sense. The result p is a row vector of length n+1 containing the polynomial coefficients in descending powers. Therefore if we wanted different degrees polynomial for the best least square fit, we only have to change the degree (n) of the polyfit functions. We computed the least square up to the third polynomial and we found the exponential least square of our data. The code for the exponential least square is shown below in code 5:

Code 5

```
p=polyfit(yr,log(xco2),1);
% for exponential fit C*exp(\alpha*x) we have
C=exp(p(2));
alpha=p(1);
y4=C*exp(t1*alpha);
```

The calculations that we computed to find the exponential fit are shown below in equation 18

Equation 18

$$y = C * e^{\alpha x} \rightarrow \ln(y) = \ln(C * e^{\alpha x}) \rightarrow \ln(y) = \alpha x + \ln(C)$$

After we found all the possible least square fits for our data we plot the different graphs to compare which one is the best fit. The figures for each of our least squares fit are shown below:

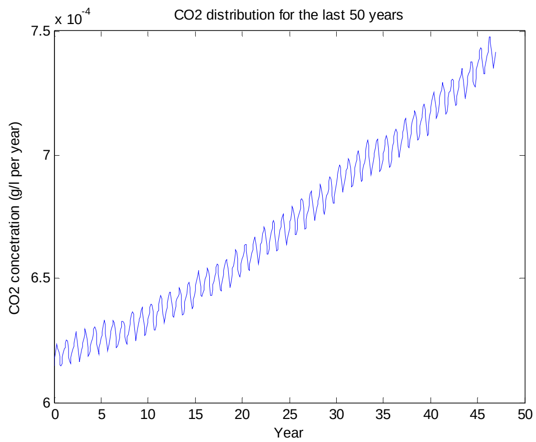

Figure 23 Data vs. Year

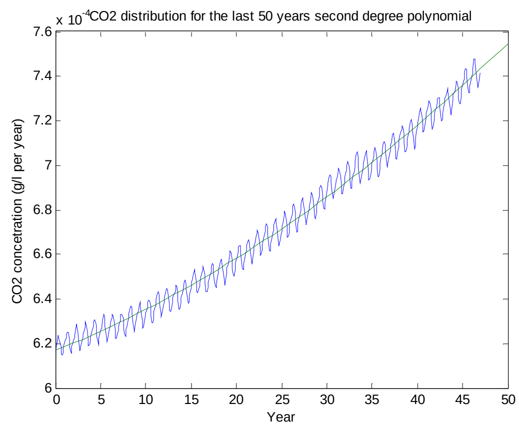

Figure 25 Second degree

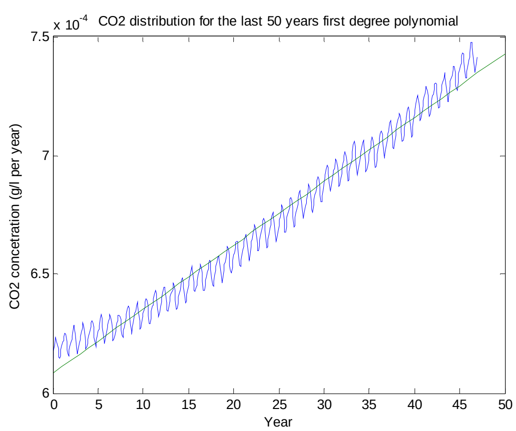

Figure 24 First degree

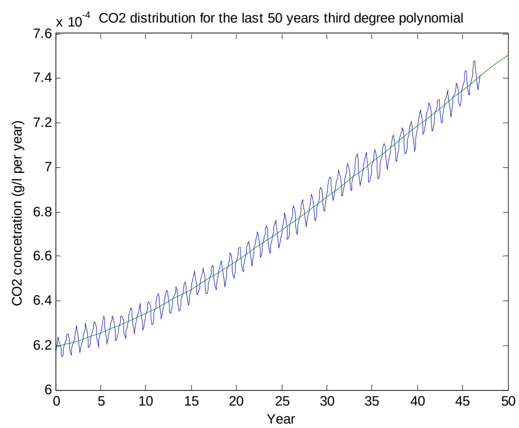

Figure 26 Third degree

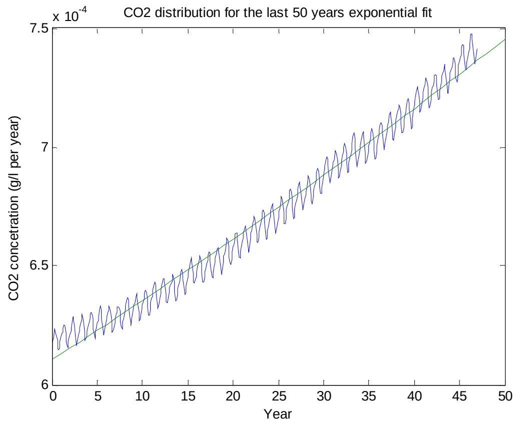

Figure 27

As you can see from the figures above the best fit for our data is the second degree polynomial. The code of the least square polynomial is shown in the appendix. The second degree polynomial that we found was used in our main program defining the gamma parameter. Below I have shown the part of the program where we changed the gamma.

Code 6

```
%REMEMBER: we have to add only the forcing which are not included in
%the initial conditions
%linear fit
%gam= p1*t;
%p1=0.00268605036723e-3
%p2=0.60848369726802e-3 this is included already in the initial conditions
%quadratic fit
p1=0.00002297543249e-3;
p2=0.00160811965954e-3;
%p3=0.61689754529194e-3 this is included already in the initial conditions.
gam= p1*t^2+p2*t;
%exponential fit: gam= C*(exp(alpha*t)-1) so that the forcing is 0 at t=0;
%gam=6.106170969309385e-04*(exp(0.00398734415689*t)-1);
```

The p constants above are our coefficients for the different types of polynomials that we created. As mentioned before we selected the second degree polynomial least square for our gamma. The p3 coefficient that we get from this polynomial is the amount of the CO_2 concentration that exists in the lower atmosphere in the year 1958.

Next, since the data that we used for the gamma begins at 1958, we have to change the initial conditions for the other regions to start from this year. After intense research we were unable to find actual data for these regions. Therefore we used the previous program and ran it for only 258 year to calculate the value of the CO_2 for the year 1958 for all the regions except the lower atmosphere. These values are now our new initial conditions for the different regions. Below we have shown the part of the code where the initial conditions are changed.

Code 7

```
%perform the integration 'lorsys' is a file that defines the system
%[0,300] time interval for the integration
%[0.00758014512126 ...] define the initial conditions.
%These initial conditions are %obtained from the program co2sim
%(at yr 258 ~~ 1958) and from the actual Mauna Loa data
% the simulation will run for 300yrs starting from 1958. If you
%want a shorter period change the "300" to some other number.
for i=1:300
tt(i)=i-1;
end
[t,x]=ode15s('co2sys2',tt, ...
[0.00758014512126,0.03758430343092,0.00477927269048,6.148214285714287e-04,
...
0.00783797139839,0.00251675633673,0.00459320292789]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The graphs below show the CO_2 distribution for 300 years starting at 1958 for all seven regions with new initial conditions and gamma calculated in measured values.

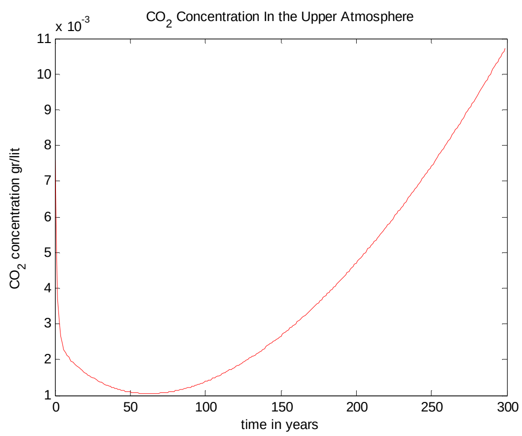

Figure 28

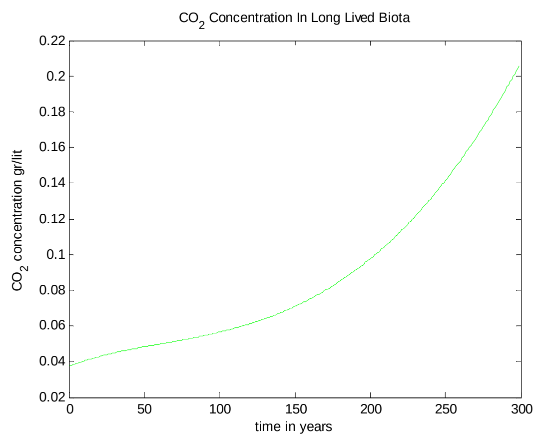

Figure 29

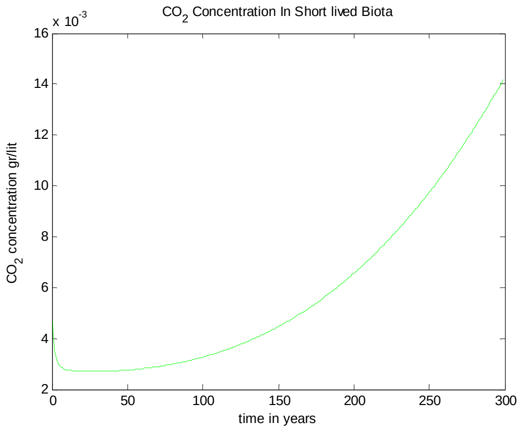

Figure 30

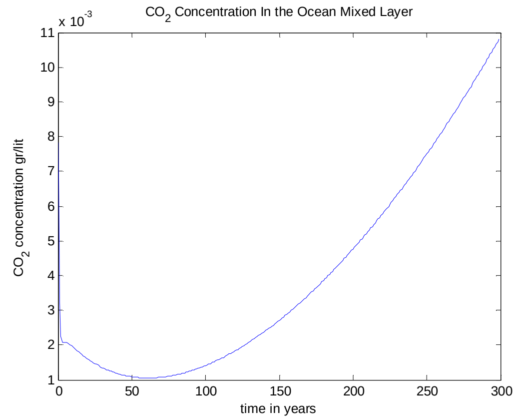

Figure 32

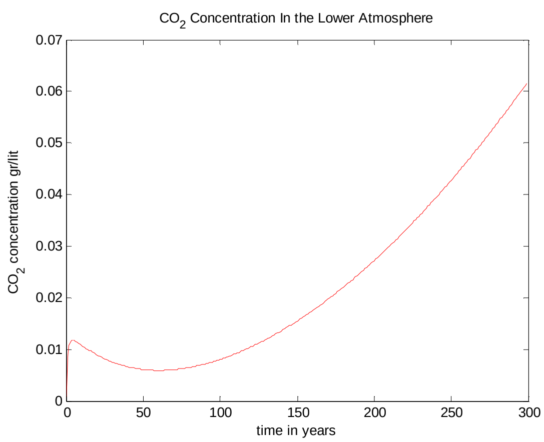

Figure 31

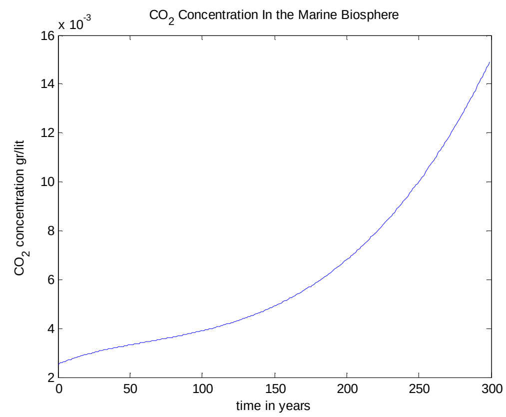

Figure 33

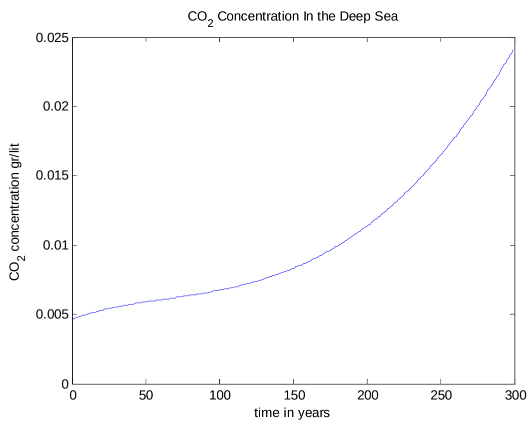

Figure 34

As you can see the figures above are not all smooth exponential increases. This is due to the fact that the values for the initial conditions of the regions are not exactly accurate.

The solution to get better predictions for the values that we don't know, can be achieved by changing the initial conditions to 0,0,0,4e-4,0,0,0 (with the 4^{th} value representing the lower atmosphere) starting at 1700. This is because the approximate level of CO_2 in the lower atmosphere, according to glacial ice cores, was 4e-4 g/L at 1700. Finding the values after 200 years gives decent starting conditions at 1900 for everything except the lower atmosphere, which can now be adjusted to 6e-4, which is an approximation of the CO_2 levels in the lower atmosphere at that point. Finding the values after 58 more years with these new initial conditions gives us a good approximation of the CO_2 levels in the world in 1958, with the lower atmospheric CO_2 level set to the measured value.

The new graphs produced by the program show a much more accurate picture over the first 100 years, because there is not a large adjustment phase, while the program corrects the incorrect values. These graphs are shown in the figures below:

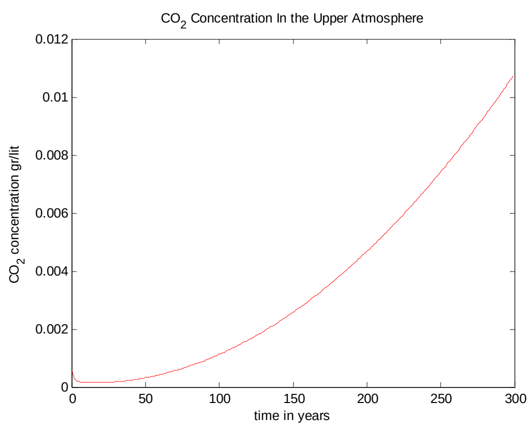

Figure 35

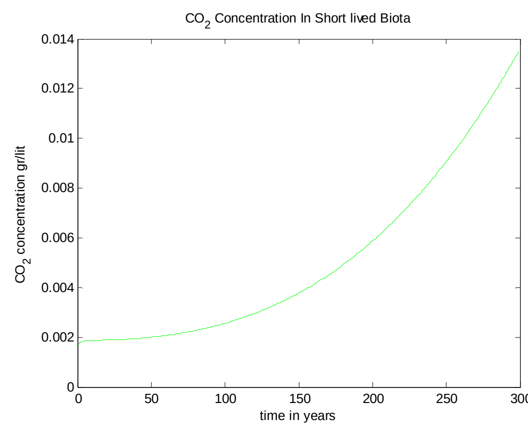

Figure 37

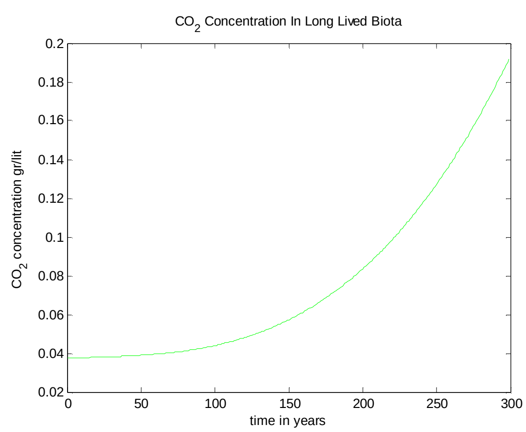

Figure 36

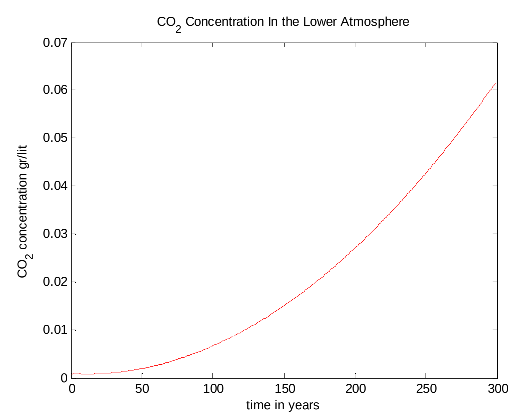

Figure 38

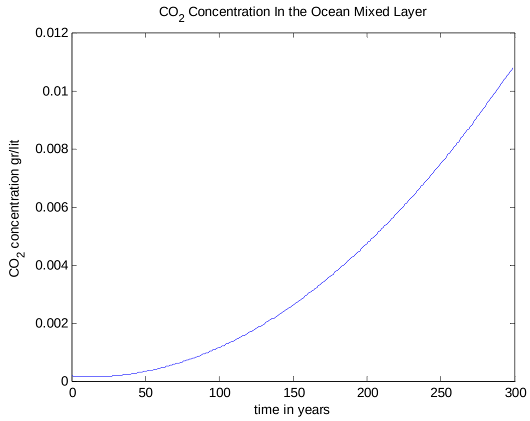

Figure 39

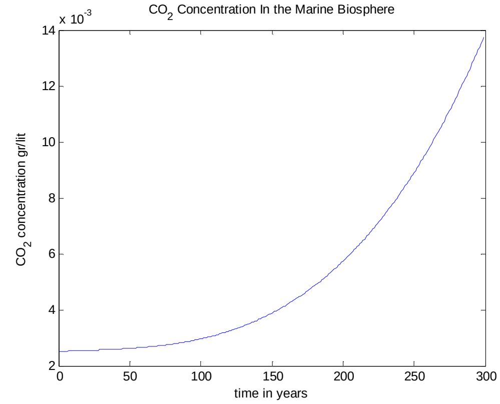

Figure 40

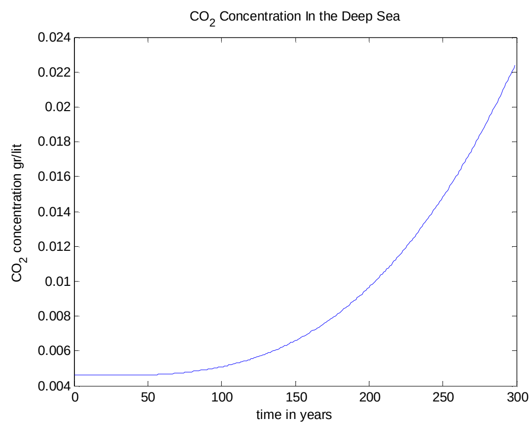

Figure 41

The curves are increasing exponentially as expected. The only effect that we have is in the upper atmosphere where we notice a decrease in the CO2 concentration during the stating years. The lower atmosphere curve now has no bumps or extreme changes as before. Now our model is complete and up to date. As you can see from the graphs above the concentration of CO_2 in the atmosphere is increasing exponentially. This event may produce significant increase in the Earth's mean temperature. A second model describing the change in the temperature due to the increase in the CO_2 will be explained in the section ahead.

This program is useful for the fact that we can change the fossil fuel consumption (gamma) in respect to any policies that are applied and observe their effect to the accumulations of CO_2 in the atmosphere. Due to the importance of this issue different types of policies have emerged to prevent the impact of this increase to our planet Earth. After searching the World Wide Web we came across different types of policies that could be applied to our program. In the next chapter of this report we go in depth of the different types of solutions to the global warming.

The U.S. transportation sector alone represents approximately 10% of all energy-related greenhouse gas emissions worldwide and over a third of all transportation emissions worldwide. Technology available within the next 10 years will allow for reduction of CO_2 emissions in this area to 35% of their current value. Therefore, applying plug-in hybrid electric vehicle (PHEV) technology ubiquitously in the United States will reduce total CO_2 emissions by 6.5%. If the rest of the world began to use PHEV's, the reduction would be tripled to 19.5%. Changing over the entire world power grid from fossil fuel burning to He^3 would represent a reduction in CO_2 emissions of 99% for power production. Power production represents about 40% of the world's CO_2 emissions, so the reduction in CO_2 emissions would be approximately 39.5%.[xxviii-xxxi]

Applying these policies to our fossil fuel constant in our model will change the CO_2 concentration in the seven reservoirs in different ways. The changes that we made to the code to represent each of the different policies are shown below in code

Code 8

```
%quadratic fit
p1=0.00002297543249e-3
p2=0.00160811965954e-3
%p3=0.61689754529194e-3 this is included already in the initial conditions.
gam1= p1*t^2+p2*t;
%exponential fit: gam= C*(exp(alpha*t)-1) so that the forcing is 0 at t=0;
%gam1=6.106170969309385e-04*(exp(0.00398734415689*t)-1);
%gam=gam1-0.065*gam1   %U.S. using all PHEV technology
%gam=gam1-0.195*gam1   %World using all PHEV technology (including U.S.)
%gam=gam1-0.395*gam1   %World using Helium-3 instead of fossil fuels
gam=gam1-0.590*gam1;   %World using both PHEV and Helium-3
dx=[Dlu;Dlb;Dsb;-Dlu-Dsb-Dlb+Dml+gam;-Dml-Dmd-Dmb;Dmb;Dmd];
```

Different policies will give different results. In the figures below we have shown what would happen to the CO_2 concentration in the reservoirs if the world started to use both PHEV and Helium-3.

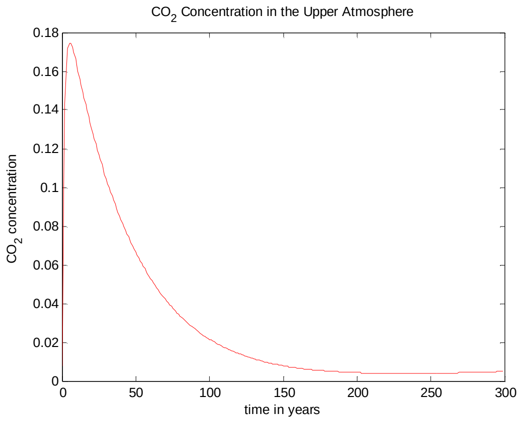

Figure 42

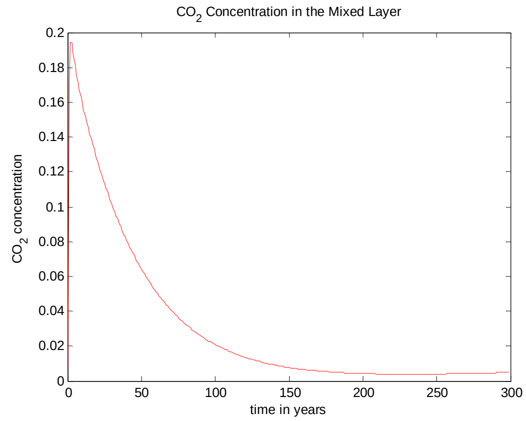

Figure 45

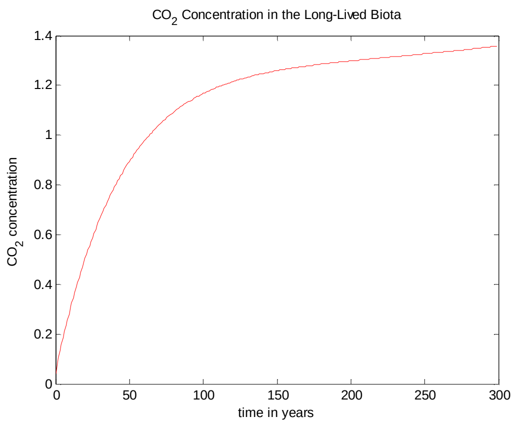

Figure 43

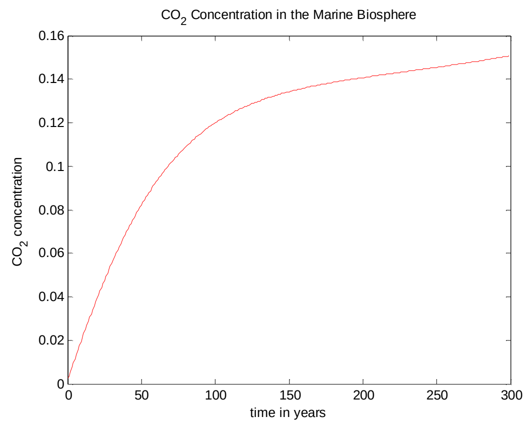

Figure 46

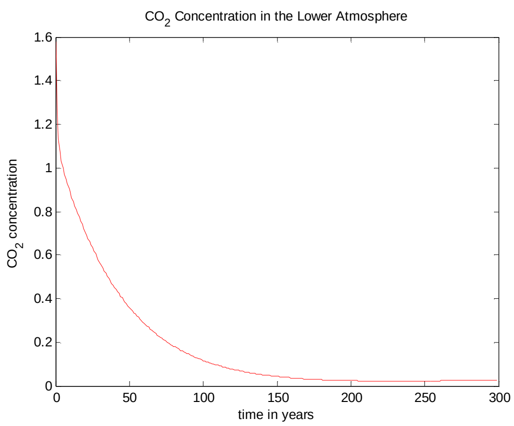

Figure 44

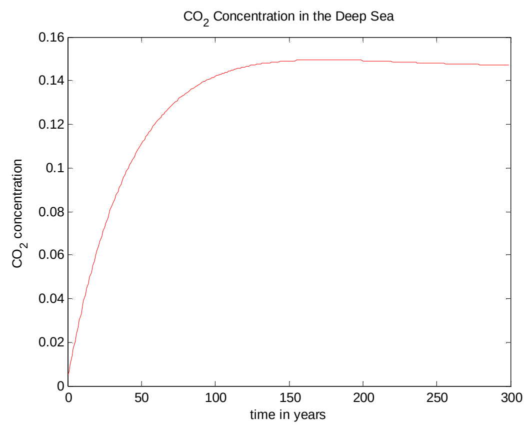

Figure 47

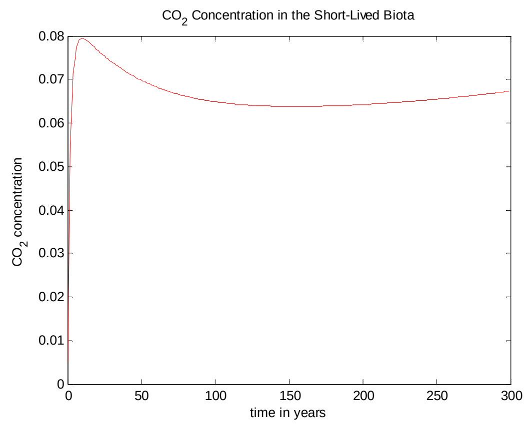

Figure 48

The figures above show that with the right policies we can decrease the accumulation of the CO_2 in our atmosphere and reduce the risk of increasing the mean temperature of the Earth. In the following pages we have used the information that we gathered from this model and constructed a zero dimensional model to represent the changes in the mean temperature of the Earth. The modified program that contains the fossil fuel consumption (gamma) parameter is shown in the appendix.

7. Climate Simulation: Modeling Geophysical Phenomena

The increase or decrease of CO_2 in the atmosphere results in changes to the greenhouse effect towards the Earth. This is noticed at large in the unusual changes of the Earth mean temperature due to this effect. To observe the effect of the greenhouse gases on the temperature of the Earth we will build a zero dimensional model using MATLAB that will incorporate all the different parameters that are relevant to this change. The following is an examination of some of these parameters and also the importance that they have in the construction of our model.

The energy that comes from the sun to Earth is "usually" reflected back to space. Different trace gases that exist in the atmosphere, like CO_2 or methane, can block this energy and reflect it back to Earth increasing its temperature. This is known as the greenhouse effect.

Many models have been developed in an effort to understand the impacts of the Greenhouse effect. They differ from each other in the number of spatial dimensions, their sophistication and resolution. We are going to be focused on the zero dimensional models where all the variables are dependent on time.

There are three types of bodies that explain the mechanism of the greenhouse effect. The Perfect Mirror body reflects all the light that is directed at it with no change in temperature. The Black Body will absorb all the energy and emit it back at a rate proportional to the forth power of its temperature, which is represented by the Boltzmann equation $P = \sigma T4$. The Grey Body will reflect part of the electromagnetic radiation, absorb the rest and release part of the absorbed energy as thermal energy. The ratio of the reflected radiation to the amount incident upon it is called the "albedo" of the body which is expressed in percentage. The formula for the grey body therefore becomes: $(1 - A)\, P = \sigma T4$ where A represents the "albedo". The equation of this parameter will be shown in the later paragraphs.

If the Earth didn't have an atmosphere it would be a good example of a grey body with a mean temperature of approximately -25^{o}C. The effect of the atmosphere on the temperature of the Earth is related to the absorption and emission of energy, carried by the light particles photons, from atoms. The particles carry energy of $\boldsymbol{hv}$ where $\boldsymbol{h}$ is the Planck constant and $\boldsymbol{v}$ is the frequency. An atom absorbs such a photon only if it has two energy levels, E1, E2 such that

$$E2 - E1 = \boldsymbol{hv}$$

If an atom is at energy level E1 and absorbs a photon of $\boldsymbol{hv}$ then it will get "excited" until it reaches an energy level or E2. Then this atom will get back to its original energy E1 by releasing the photon with energy $\boldsymbol{hv}$ in the opposite direction. This photon will be trapped in the Earth's atmosphere and maybe absorbed by Earth which will increase its temperature. This transfer of energy is made possible only by the trace gases that have the right energy band needed for trapping this energy and thereby leading to the greenhouse effect.

To create a zero dimensional model for the amount of CO_2 in the atmosphere we divide the Earth-atmosphere into seven reservoirs and the contribution of CO_2 due to man-made effects and natural catastrophes. The rate of CO_2 transfer between the seven reservoirs is proportional to the concentration of CO_2 in the reservoir. If we denote CO_2 by Ci where i is the number of reservoirs than the rate can be written by the equation below:

Equation 19

$$\frac{dCi}{d\,t} = \sum_{i=1}^{7} KijCj + Fi,\ i = 1,....., 7$$

Kij are constant and *F* is the forcing factor. To achieve and accurate value for the amount of CO_2 in our atmosphere we need to obtain a proper estimation of these coefficients. Once we determine the concentration of CO_2 in the lower and upper atmosphere another model is needed to determine the impact of this concentration on the mean temperature of Earth.

The Earth receives energy from the sun in the form of short waves and emits it back to space in the form of long waves. The average energy flux from the sun to Earth is Fs=1372/4 (Fs = 348 Watts/(m^2sec)) while the flux emitted by Earth is Fs = 390 Watts/(m^2sec). Some of this energy is stored in clouds and aerosols and some is released by conduction and evaporation but both of these processes are balanced with each other to 106 watts/(m^2 sec). Long wave and short wave distribution is used to calculate the temperature of the Earth using the Black-Body Stefan-Boltzmann law which for Earth becomes:

Equation 20

$$(1 - A)F_s \pi R^2 = 4\,\pi R^2 \sigma T_e^4$$

In this case R is the radius of Earth and σ is the Boltzmann constant which in our model is known as sigma with a value of $5.67 * 10^{-8}$. By balancing the two radiations we discover that the temperature of the Earth is 255 K which is 30° lower than the mean temperature of the Earth. The difference is due to the Greenhouse effect and the structure of the atmosphere and the oceans.

Our atmosphere is divided into three regions: homosphere 0-100 km heterosphere 100-500 km, exosphere above 500 km. Due to the fact that in the homosphere the molecular mean free path is small, there is a homogeneous composition of 78% N2 and 21% O2. The homosphere is divided into troposphere, stratosphere and mesosphere. The stratosphere is the only layer where the temperature increases due to ozone heating.

The Earth is composed of convective currents which circulate in the atmosphere. Air heats and cools differently over land versus above water which produces cells of circulating warm and cool air. Winds are turned east and west by the Coriolis force created by the rotation of the Earth. In the Northern hemisphere winds travel in a westerly direction while winds in the Southern hemisphere travel in an easterly direction. These westerly winds create the jet streams and conversely, easterly winds traveling toward the equator create what is known as the trade winds.

Solar radiation warms the Earth's surface greatest over the equator. Heat then radiates into the atmosphere via infrared waves, resulting in a vast amount of warm moist air. The difference in heat distribution between land and oceans produces a longitudinal cell called the Walker Circulation, named after Sir Gilbert Walker. In this loop, air near the equator (located over Indonesia) rises at warmer longitudes, creating low pressure at the surface. This air is pushed to the east, and descends at cooler longitudes (west of South America) as high pressure where it is pushed along the surface back to the west by the trade winds. A reverse in this circulation leads to warmer than usual ocean surface temperatures which leads to unseasonable temperatures and precipitation patterns known as "El Nino" in the Pacific Ocean.

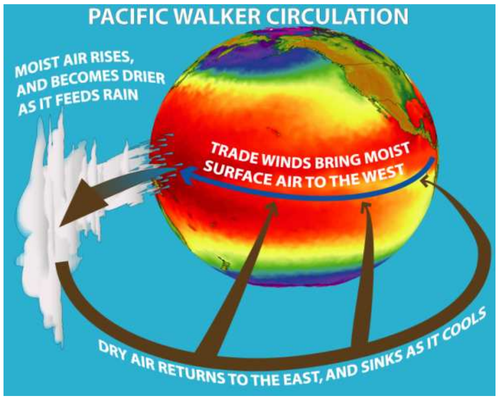

Figure 49: Water circulation in the pacific ocean.

Rising warm air enters the tropopause and travels in an easterly pole ward direction (latitudinal) until it reaches about 30 degrees north and 30 degrees south latitude, or the Horse latitudes. Here the air cools and becomes more dense. This high pressure zone is known as the subtropics and contains desert areas due to the extremely dry descending air. At these latitudes the dry air sinks to the Earth's surface and travels in a westward flow back towards the equator where it is warmed again. As a result, the return of air to the equator completes the tropical overturning cycle known as the Hadley Cell, formulated by George Hadley in 1735.

A secondary circulation known as the Ferrel cell, named after William Ferrel, is located next to the Hadley cell. It is an open thermal loop located between 30 and 60 degrees north and south latitudes, or between the Hadley cell in the midlatitudes and the Polar cells located nearest the poles. The Ferrel cell produces westerly winds which flow pole ward near the Earth's surface from the Hadley cell until it meets the Polar cell. This area is known as the polar front and creates the jet streams. Air in the Ferrel cell that is located closest to the Polar cell is still relatively warm compared to the air inside the Polar cell. This causes it to rise and travel easterly towards each pole which returns it towards the equator in higher altitudes until it reaches the Hadley cell and descends again.

The Polar cell is located in the arctic region beginning at 60 degrees latitude in both the northern and southern hemispheres. Warmer air from the Ferrel cell travels in higher altitudes until it reaches the poles, cools and descends. When the dry, high pressure air descends, it travels along the Earth's surface as easterly winds towards the Ferrel cell where it meets the polar front; it warms, and rises once again.

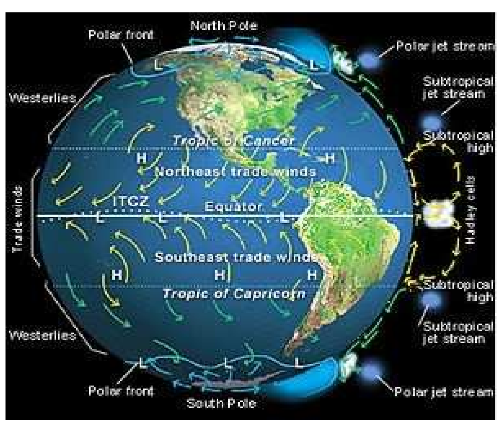

Figure 50

Models for climate prediction and the computation of the mean temperature of the Earth are classified by degrees of sophistication. In the following we will consider only the 0-dimensional and the 1-dimensonal models.

For the zero dimensional models, we only take into consideration the balance between the (total) incoming and outgoing radiations which are denoted by Ri and Ro respectively.

Equation 21

$$C\frac{dTm}{dt} = Ri - Ro$$

Where Tm is the global mean temperature, t is the time and C is the heat capacity of the Earth system. To model the incoming and outgoing radiations we let:

Equation 22

$$Ri = Q\{1 - A(T_m)\} \quad Ro = \sigma g(T_m)T_m^4$$

Here Q is the flux of the solar radiation which in our model is represented by Fs=1367/4, C the heat capacity of the Earth system which in our case is 1, $A(T_m)$ is the mean albedo and $g(T_m)$ is a "grayness factor" which measures the deviation of the Earth emissions from black body radiation due to the greenhouse effect. If we neglect the spatial heat distribution on Earth, the prototype model for the global mean temperature can be derived by the equations above in the form.

Equation 23

$$\frac{CdT_m}{dt} = Q\{1 - A(T_m)\} - \sigma g(T_m)T_m^4$$

In MATLAB we write this differential equation as `dx=[Fs*(1-A)-sigma*g*x^4];`where x represents our changing temperature T_m and where $g(T_m)$ was modeled by Seller as:

Equation 24

$$g(T_m) = 1 - k\tanh\left(\frac{T_m}{To}\right)^6, To^{-6} = 1.9 * 10^{-15}$$

where k is the portion of the Earth covered by clouds (k ~ 0.5). In our MATLAB code we write the equation above as: `g=1-kappa*tanh(x/T0) + alpha;` where k is kappa and x is our T_m. We choose T_O as 275 K which is the ambient temperature for the greenhouse effect. As you already noticed our code includes a variable alpha which is not mentioned before. This parameter is a "lump parameter" which represents the effect of greenhouse gases and clouds. The alpha is the only one that will be controlled by us. We will show that by lightly changing this parameter the mean temperature of the Earth is going to change dramatically.

The albedo is introduced by Sellers in the following linear interpolation function.

Equation 25

$$A(T) = \begin{cases} \alpha_M, & T < T_1 \\ \alpha_M - \frac{T-T_1}{T_2-T_1}(\alpha_M - \alpha_m), & T_1 < T < T_2 \\ \alpha_m & T_2 < T \end{cases}$$

Where in our model α_M= 0.85 (alphaM), α_m=0.25 (alpham) are the albedo values assigned to ice-covered and ice-free surface respectively and T_1=240°K, T_2= 275°K. The equation above is represented as conditional statements in our code which is shown below:

Code 9

```
%compute the albedo
if (x < T1)
A=alphaM;
elseif (x > T2)
A=alpham;
else
A=alphaM - (x-T1)/(T2-T1)*(alphaM-alpham);
end
```

This model has three equilibrium points two of which are stable while the third (middle) is unstable. This is the last equation that we needed to create a zero dimensional model of the mean temperature of the Earth. All the differential equations that we used were put in an M-file called temper and were executed in a second M-file which contained all our fixed constants and the temperature range of T_m. Below are shown the range of this temperature and the different initial conditions for the different alphas (as the comments suggest). Also is shown the code for the execution of the file temper which contains the differential equations and the "lump parameter".

Code 10

```
%[0,300] time interval for the integration
%[TEMP0] define the initial conditions
TEMP0=275;
%TEMP0=235;
for i=1:30000
tt(i)=i-20;
end
[t,eqtemp]=ode15s('tmpr',tt,[TEMP0]);
```

First we found the alpha for which the temperature goes up to 400 K with initial conditions as at TEMPO = 235. After many tries we found the value of alpha that made the equilibrium temperature go up to 400K is -0.37504. This value was picked due to the fact that the oscillations of the temperature at the moment of increase were the lowest.

The figure below shows the curve that the temperature makes at 400 degrees for this alpha:

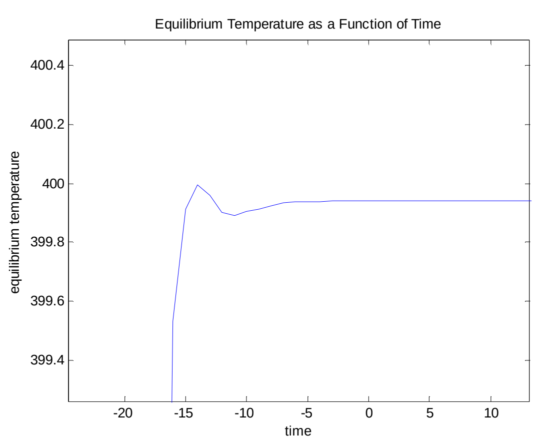

Figure 51: Temperature increase to 400 K

Next we found the alpha for which the temperature drops 200 degrees. First we changed the value of our initial condition TEMPO to 275 and we know that the value alpha is between 0.17 and 0.18 so by trial and error we found that alpha=0.17121359

This gave us the change that is shown in Figure 52.

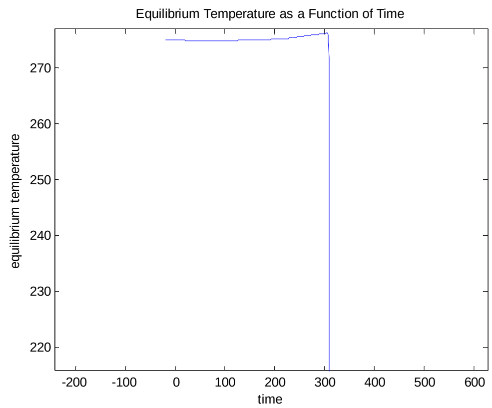

Figure 52: Decrease of the Temperature by 200 K

As you can see using all the parameters and the greenhouse effect we can build a zero dimensional model that represents the mean temperature of the Earth dependent only on time. Also our model taught us that by changing the amount of greenhouse gases in the atmosphere we run the risk of having devastating changes in the temperature of our planet. The code for this model is shown in the appendix.

To create a one or more dimensional model we can make different changes to the equations above. We can refine equations above when we let Q depend on the possible variations in the sun radiations.

Equation 26

$$Q = \lambda(t)Qo$$

We elaborate in the albedo model by taking in account the different albedos of ocean, land and ice and the meridional extent of the ice cover of the Earth. Thus:

Equation 27

$$A = \gamma(\alpha_a + \alpha L) + (1 - \gamma)\alpha_{oc}$$

Where γ is the land and ice percentage of the Earth surface and $\alpha_{0c}=\alpha_{0c}(T)$ is the albedo of the ocean. In equation the albedo of land and ice is a linear function of the meridional extent of the ice sheet L. As a result L is defined by the following differential equation.

Equation 28

$$\dot{L} = \lambda L^{-1/2}[(1+\epsilon(T))L_T - L]$$

L_T is the meridional extent of the ice accumulation zone and ϵ(T) is the ramp function.

In this formulation the climate of the Earth can be represented in two differential equations which depend on several parameters that control the various bifurcations of the climate.

To introduce the spatial dependence in these models we change the dT/dt into

Equation 28

$$DT/Dt = \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T$$

where **u** is the wind speed. However since the climate time-scale is long we eliminate this velocity **u** by applying the "eddy diffusivity approximation"

Equation 29

$$-(u \cdot \nabla)T \cong \nabla \cdot (\nu_e \nabla T)$$

where v_e is the "eddy diffusivity coefficient". This changes equations 23 into the form of:

Equation 30

$$C(\mathbf{x})\frac{\partial T}{\partial t} = QS(\mathbf{x})\{1 - A(\mathbf{x},T)\} - \sigma g(x,T)T^4 + \nabla(\nu_e \nabla T)$$

Here S(x) is the distribution of the solar flux on Earth. To simplify the equation we can assume that the variables depend only on time and latitude. This will transform our equation into:

Equation 31

$$C(\phi)\frac{\partial T}{\partial t} = QS(\phi)\{1 - A(\phi, T)\} - \sigma g(\phi, T)T^4 + \frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left\{\nu_e(\phi)\cos\phi\frac{\partial T}{\partial\phi}\right\}$$

As you can see it is difficult to predict the future of the climate of the Earth due to its complexity. A major issue is whether these models can predict reliably the impact of man-made inputs to this system.

Next we focused on the large scale motions in the atmosphere and the oceans. These motions are characterized by shallowness, stratification and the influence of the Earth rotation on the dynamical system under consideration. Shallowness means that the lateral scales is large compared to its depth. This means that the ratio between lateral scale (L) and depth (D) is:

Equation 32

$$\delta = \frac{D}{L} \ll 1$$

Stratification is due to differential heating of the Earth by the sun. This brings stability between the atmosphere and the ocean on the large scale. As a result of this stratification motions parallel to the local vertical are inhibited and therefore large scale motions are nearly horizontal. [xxxviii]

Problems and Solutions

Avoiding the catastrophes of global warming is the reason many individuals, organizations, and even nations, are taking this issue seriously.   Based on current data of atmospheric levels of CO_2 and credible model predictions for the future, scientists and environmentalists around the world are growing increasingly concerned about the effects these pollutants are having on our environment. Concerns about the dangers (greenhouse effect) of the emission CO_2 prompted law and policy makers to establish strategies to reduce carbon emissions. The figure below, taken from an article written by Robert Rohde, of the University of California Berkley depicts the rise in CO_2 concentration in developed countries from 1800 to the year 2000.

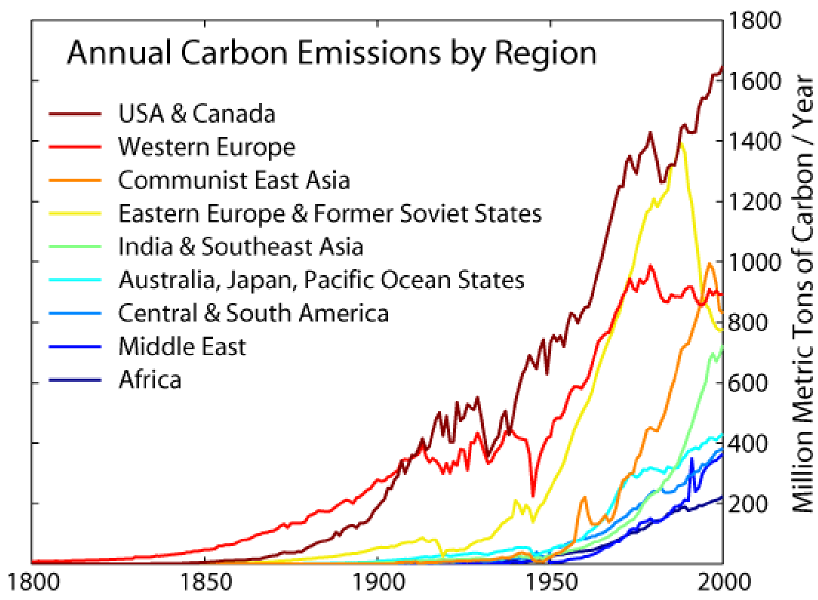

Figure 53

The graph shows a dangerous increase in CO_2 emission in developed countries over the past two hundred years. The United States have had the greatest increase since the industrial revolution. In addition, from the graph it is evident that all developed nations CO_2 emission increased and this trend will definitely continue unless corrective measures are implemented. The most well known approach, the Kyoto Protocol, developed in 1997 and came into force in 2005 requires developed countries to reduce their CO_2 emissions below levels specified for each of them in the treaty. These targets must be met within a five year time frame between 2008 and

2012 and add up to a total cut in CO_2 emissions of at least 5% against the baseline of 1990. The Kyoto Protocol is seen as an important first step towards a truly global emission reduction regime that will stabilize greenhouse gas (GHG) concentration at a level that will avoid dangerous climate change. Of the one hundred and seventy-two countries and government entities that have signed the treaty, United States was not one of them. American experts, scientists and critics questioned the usefulness of the protocol. According to Sandra Brown author of, Philosophical Transactions: Mathematical, Physical and Engineering Sciences,

"the controversy is based primarily on two arguments: sinks may allow developed nations to delay or avoid actions to reduce fossil fuel emissions, and the technical and operational difficulties are too threatening to the successful implementation of land use and forestry projects for providing carbon offsets. Here we discuss the importance of including carbon sinks in efforts to address global warming and the consequent additional social, environmental and economic benefits to host countries." [xxxv]

Policy makers and organizations believed emission trade would be the most effective approach to reduce the quantity of pollutants that are being emitted. They argued that emission trading would be beneficial for everyone by lowering their emission rate companies could sell credits to other companies who needed credits. Under the policy emission permits would be given to companies and the amount they would emit categorized by credits. In effect , companies who lowered their rate of emission would have more credits at their disposal. The Bayer Climate Program glossary agrees, according to them "*it is an administrative approach used to control pollution by providing economic incentives for achieving reductions in the emissions of pollutants*." [xxxvii] It would force companies to reduce the volume of emission they produce each year or decrease their financial profits.

Policy makers suggests the replacement of coal with natural gas in some power plants through the re-powering of existing plants, retirement of older coal plants, construction of new gas turbines and combined cycle plants and increased dispatch of gas-fired plants. According to Joseph Romm, *"to be most effective, the emission trading system needs to be combined with a technology strategy to developed and deploy energy-efficient and low-carbon technology that varies by sector. In the energy supply sector the permit-trading system is the important policy* *because it will promote the use of low-carbon fuels*." [xxxvi] Many Coal-fired power plants exist today which are responsible for 25% of smog-forming pollution therefore playing a major role in the issue that the environment is undertaking.

Figure 54

Carbon tax, is also another policy being investigated as a means to reduce emission. It is a direct tax to energy sources that emit carbon dioxide into the atmosphere. The purpose of the tax is to reduce CO_2 that is emitted in the environment and thereby slow global warming. It can be implemented by taxing the burning of fossil fuels, coal, petroleum products such as gasoline and aviation fuel, and natural gas in proportion to their carbon content. It has the benefit of being easily understood and can be popular with the public if the tax is used to fund environmental projects.

The Kyoto Protocol is a convening of international Governing bodies to discuss steps and measures that could be undertaken to reduce pollution world wide. The debate over carbon credits or carbon taxes is ongoing, but the signatories of the Kyoto Protocol favor carbon credits. Joseph Romm say's that, *"criticism of the tax-raising schemes is that taxation raised by a government may be applied inefficiently or not used to benefit the environment. If emission was treated as a market commodity it will benefit both business and economists".* [xxxvi]

Despite whatever policy is adopted the bottom-line remains the same, we must act quickly and collectively to reduce emission of pollutants in our atmosphere because global warming is not going to disappear. The responsibility belongs to each and everyone, we must play a positive role in reducing the pollutants we release in the atmosphere. The international governing bodies that signed the Kyoto Protocol must endeavor to reach their assigned targets because the Protocol ensure that all investment goes into genuine sustainable carbon reduction schemes, through its internationally agreed validation process.

Today technology provides many alternative energy sources that do not produce CO_2 or other greenhouse gases. One alternative includes looking to other renewable energy sources such as wind power which can supply vast amounts of energy for homes and even cities. European countries have begun using this alternative energy source in many parts of their countries. A study by the Stanford University found that in some locations around the world there is more than enough wind power to satisfy the global demands for energy. *"The locations with sustainable Class 3 winds could produce approximately 72 terawatts and that capturing even a fraction of that energy could provide the 1.6-1.8 terawatts that made up the world's electricity usage in the year 2000."* At a height of 33 feet, class 3 winds reach about 12.5mph and at a height of 164 feet class 3 winds reaches about 15.7mph3. The research concluded that the strongest winds are found in Northern Europe followed by the southern tip of South America and in Tasmania, Australia. Strong winds were found even in the United States from the ocean breezes along all of the coasts. As with many alternative energy sources, this method also has some complications. Wind turbines would cause noise, be hazardous for birds, and we would require large fields of wind turbines to keep up with the energy demanded when winds are low. This problem can easily be solved by finding methods to reduce noise as well as methods to protect the birds from wind turbines. They should be installed only in places with strong enough winds to keep up with the demand of energy. Countries around the globe should work together to settle upon the usage of this source of energy for the benefit of people and the planet on a large scale. [iii]

Figure 55

Another alternative energy source is solar energy. The solar energy method uses photovoltaic cells that absorb the photons from solar radiation to produce electricity. However, photovoltaic cells can only provide a short-term energy supply since climate conditions such as clouds or fog affect the amount of solar radiation that these cells are able to use. This method should be used in countries where the solar radiation is strong enough the weather is usually clear.

Since transportation is also another major source of CO_2, policies should be created that will constrain the automobile industry to use alternative fuel resources like hydrogen. This should be suitable for urban areas where no heavy-duty trucks are necessary. Hybrid cars are already on the market and even though their price is higher compared to a fuel vehicle, it is an improvement that will help preventing global catastrophic events.

There are also methods an individual can take part in helping reduce the amount of CO_2 in the atmosphere. Such methods include opting for alternative energy sources such as the ones mentioned earlier which actually result in a low cost alternative energy source. Individuals can restrict their driving by exploring public transportation on urban areas, walk or ride a bike as methods of traveling. Methods in which humans can help reduce CO_2 emission include buying reusable products to help decrease waste, recycling paper, plastic, glass, insulating the walls of houses, shutting off the heat and lights while no one is at home, replacing current light bulbs with compact fluorescent light bulbs and many other events. Planting trees also helps, because plants produce oxygen and absorb CO_2. [iv]

Global warming is a major issue which is happening now and needs to be addressed with great importance. As mentioned in this segment of our report there exist methods which can prevent it, but the main issue would be to encourage each other in the realization of these methods.

Recommendations for Future Projects

During our research, we discovered different project topics that can be based on or complement this project. These projects could significantly contribute to the study and prevention of global warming.

Clean energy sources today include wind and solar power among many others. A good project topic would be a study of the potential applications of these types of power, as well as the effect the implementation of these energy sources would have on CO_2 levels. As it was mentioned above, both are natural resource that can be used to supply energy to many cities around the world. Exploring the comprehension of these renewable energy sources in a large scale could be an interesting project.

Following our report as an example, individuals can explore the harm that the other trace gases, such as CFCs, have in our climate. Through the use of our models, or other models, they can predict the change in the mean temperature of the Earth due to the increase of methane (CH_4), nitrous oxide (N_2O), or chlorofluorocarbons (CFCs). This project would explore the possible consequences that the increase of these gases can have on society, on the planet's ecosystem and many other aspects of Earth's climate.

One of the major topics of our project was the study of different types of policies that would bring the greenhouse gas emission to a normalized level. Researchers should explore these different types of regulations and demonstrate their effect on global warming. This type of project could analysis methods of mitigating the concentrations of the trace gases on the atmosphere, if it were implemented in countries with serious pollution problems, such as China, Mexico, and the United States.

These are different recommendations that individuals might want to pursue if they are interested in this topic. It is clear that their objective is to reduce the concentration of trace gases in the atmosphere therefore, preventing global warming.

Conclusion

There is undeniable evidence that CO_2 has been increasing steadily since the dawn of the industrial revolution. There is a correlation between atmospheric CO_2 levels and mean global temperature. Ice core data shows that the climate of the Earth fluctuates with respect to the changes in CO_2 concentration. This has had a monumental effect on the course of human history.

Human migration towards the northern hemisphere is due to the steady increase of temperature. A likely cause has been the tropical diseases and the drastic effect of heat on human labor over time. Diseases and pests, such as the West Nile Virus spread by mosquitoes, once found only at low latitudes are now present in wider regions. The obvious anomalies of the weather, such as higher occurrences of hurricanes, tornadoes and thunderstorms, are also due to the climate change. Even though these changes create larger landmass that can successfully sustain common crops, they also cause coastal flooding. Approximately 70% of the world's population lives in an area that could experience flooding if the polar ice caps completely melt into the oceans. Swift melting of fresh water could also stop or alter the flow of currents, such as the Gulf Stream (which brings warm water to higher latitudes on the Atlantic Ocean), which would lead to unknown consequences. The last severe glacial period may have been a result of fresh water melting into the north Atlantic, halting the oceanic conveyor. The planet could be changed for millennia, and the human race might not survive the extreme changes.

Techniques to prevent these numerous disasters are being tested and practiced right now. After CFCs were banned in many countries the ozone hole caused by these chemicals quickly began to restore itself. The computer models used in this paper show that the same result would occur if CO_2 emissions ceased. Since CO_2 emission can't be stopped in the near future, the best alternatives include mitigation and energy conservation. Mitigation can be accomplished in many ways, such as increasing photosynthesis through planting or genetically engineering new trees, or seeding the ocean with iron to increase phytoplankton. Other mitigation techniques focus on sequestering the carbon in underground porous rocks or absorbing it from the atmosphere by other means. Energy conservation can occur through reduction of power use and/or increasing the percentage of renewable electricity production, such as wind or fusion power. The reduction of fossil fuels can occur easily in automobiles, which can be engineered to run on electricity or other energy sources instead of gasoline. Mitigation and conservation are being studied and will be part of a future solution to the increase of greenhouse gasses.

As the computer models showed, global temperatures may very soon begin to fluctuate unless the anthropogenic carbon surplus is significantly cut. Stopping the annual increase of CO_2 emissions is no longer enough, since our planet can't remove CO_2 excess quickly. The world's carbon emissions need to be reduced each year, until the atmospheric CO_2 level is stabilized. Keeping the concentration below 450 ppm in coming years is an attainable goal that may just be enough to prevent a global climate catastrophe. Important tools in accomplishing this goal include carbon credits, which keep carbon neutral for biggest polluters, and upcoming technologies for efficient energy use. Hybrid electric engines can drastically cut carbon emission, reduce oil use, and retain same performances. New sources of energy, including geothermal, new solar plants, and possibly ^3H fusion power could replace much of the coal power in use today. Carbon sequestration and photosynthetic conversion can play roles in removing CO_2 that is already in the atmosphere. The execution of these methods will result in the annual decline of carbon emission within 10 years.

Governments around the globe should take immediate action in using alternative energy and transportation sources that exist today. Individuals should be educated about global warming so each can contribute and make a difference in the future health of our planet. Countries should work together to solve the issue of global warming and begin the process of implementing policies that will alleviate the amount of anthropogenic CO_2 in the atmosphere.

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xlv. Dr. Pieter Tans, NOAA/ESRL <http://www.esrl.noaa.gov/gmd/cgg/trends>

Appendix

Model Parameters

The numerical values of the model parameters are given below:

Na06 156_1017 grams
Tam5:8 years
Tul2:0 years
Tdm 1500 years
β 0:6
Flb0 2.6 x 10^{16} grams/year
Fsb0 3.0 x 10^{16} grams/year
Fmb0 2.0 x 10^{16} grams/year
Cmt (12.011)(0.002057) grams/liter
Corg 0.001 grams/liter
Є 12.5
P1 0.15
P2 2.534
P3 0.122
P4 2.0
P5 0.1
Ws 1.370 x 10^{21} liters
Wm P4Na0/(Cmt + Corg) liters
γo 2.3937 x 10^{-29} 1/year
r 0.03077(1/year)

Nomenclature

β biota growth rate factor
Cds carbon in the deep sea
Cla carbon in the lower atmosphere
Clb carbon in the long-lived biota
Cmb carbon in the marine biosphere
Cml carbon in the ocean mixed layer
Cmt inorganic carbon concentration in the mixed ocean layer
Corg organic carbon in the ocean mixed layer
Csb carbon in the short-lived biota
Cta carbon in the total atmosphere in 1700 (grams)
Cua carbon in the upper atmosphere
dCds/dt time derivative of Cds (1/year)
dCla/dt time derivative of Cla (1/year)
dClb/dt time derivative of Clb (1/year)
dCmb/dt time derivative of Cmb (1/year)
dCml/dt time derivative of Cml (1/year)
dCsb/dt time derivative of Csb (1/year)

dCua/dt time derivative of Cua (1/year)
Dlb;Dlu net fluxes of carbon between reservoirs
Dmb;Dmd as defined in the model ODEs (1/year)
Dml;Dsb
Є ocean evasion factor
Flb0 initial flux from lower atmosphere to long-lived biota (grams/year)
Fmb0 initial flux from mixed layer to marine biosphere (grams/year)
Fsb0 initial flux from lower atmosphere to short-lived biota (grams/year)
γ(t) fossil fuel combustion rate (1/year)
γo constant in eq. (21) (1/year)
Kam transfer coefficient between lower atmosphere and mixed layer (1/year)
Kdm transfer coefficient between deep sea and mixed layer (1/year)
Kul transfer coefficient between upper and atmosphere (1/year)
Na0 total carbon in the total atmosphere in base year (1700) = Nua + Nla (grams)
Nla total carbon in lower atmosphere in base year (grams)
Nlb total carbon in long-lived biota in base year (grams)
Nmb total carbon in marine biosphere in base year (grams)
Nml total carbon in mixed layer in base year (grams)
Nsb total carbon in short-lived biota in base year (grams)
Nua total carbon in upper atmosphere in base year (grams)
P1 Nua/Na0
P2 Nlb/Na0
P3 Nsb/Na0
P4 Nml/Na0
P5 Nmb/Na0
R constant in eq. (21) (1/year)
T calendar year, starting with t = 1700
Tam lower atmosphere to mixed layer exchange time (years)
Tdm mixed layer to deep sea exchange time (years)
Tul upper to lower atmosphere exchange time (years)
Wm volume of ocean mixed layer = Nml=(Cmt + Corg) (liters)
Ws total ocean water volume (liters)

Original Program for CO_2 Concentration (written by Mayer Humi)

```
% help command prints help on the command
% define some global constants that will be valid
% when you call a function or a subroutine.
% cof is the cofficient matrix for the interaction between the reservoirs
global Na0 Tam Tul Tdm betta Flb0 Fsb0 Fmb0 Cmt Corg ...
eps P1 P2 P3 P4 P5 Ws Wm gam0 r
%request high accuracy
format long
% give values to the global constants
Na0=6.156d17;
Tam=5.8;
Tul=2.;
Tdm=1500.;
betta=0.6;
Flb0=2.6d16;
Fsb0=3.d16;
Fmb0=2.d16;
Cmt=(12.011)*0.002057;
Corg=0.001;
eps=12.5;
P1=0.15;
P2=2.534;
P3=0.122;
P4=2.;
P5=0.1;
Ws=1.370d21;
Wm=(P4*Na0)/(Cmt+Corg);
gam0=2.3937d-29;
r=0.03077;
% set error tolerance and method for the numerical
%integrator of the differential equations.
odeset('maxstep',1.,'reltol',1.e-3,'bdf','on')
%perform the integration 'lorsys' is a file that defines the system
%[0,300] time interval for the integration
%[5,5,5] define the initial conditions
[t,x]=ode15s('co2sys',[0,300],[0.;0.;0.;0.;0.;0.;0.]);
%plot x=x(:,1) vs. time(t0=1700) in red
plot(t,x(:,1),'r')
%title for the plot
title('CO_2 Concentration In the Upper Atmosphere')
xlabel('time in years')
ylabel('CO_2 concentration')
%save the plot to a postscript file
print -dpsc2 -r600 CO2sim.ps
figure
plot(t,x(:,2),'r')
title('CO_2 Concentration In the Long-Lived Biota')
xlabel('time in years')
ylabel('CO_2 concentration')
print -dpsc2 -r600 -append CO2sim.ps
figure
plot(t,x(:,3),'r')
title('CO_2 Concentration In the Short-Lived Biota')
```

```
xlabel('time in years')
ylabel('CO_2 concentration')
print -dpsc2 -r600 -append CO2sim.ps
figure
plot(t,x(:,4),'r')
title('CO_2 Concentration In the Lower Atmosphere')
xlabel('time in years')
ylabel('CO_2 concentration')
print -dpsc2 -r600 -append CO2sim.ps
figure
plot(t,x(:,5),'r')
title('CO_2 Concentration In the Mixed Layer')
xlabel('time in years')
ylabel('CO_2 concentration')
print -dpsc2 -r600 -append CO2sim.ps
figure
plot(t,x(:,6),'r')
title('CO_2 Concentration In the Marine Biosphere')
xlabel('time in years')
ylabel('CO_2 concentration')
print -dpsc2 -r600 -append CO2sim.ps
figure
plot(t,x(:,7),'r')
title('CO_2 Concentration In the Deep Layer')
xlabel('time in years')
ylabel('CO_2 concentration')
print -dpsc2 -r600 -append CO2sim.ps

%file co2sys.m
function dx =co2sys(t,x)
global Na0 Tam Tul Tdm betta Flb0 Fsb0 Fmb0 Cmt Corg ...
eps P1 P2 P3 P4 P5 Ws Wm gam0 r
Cua=x(1);
Clb=x(2);
Csb=x(3);
Cla=x(4);
Cml=x(5);
Cmb=x(6);
Cds=x(7);
%Dlu=-Kul*Cua+Kul*(Nua/Nla)*Cla;
Dlu=1/Tul*(P1/(1-P1)*Cla-Cua);
%Dlb=betta*(Flb0/Nla)*Cla;
Dlb=betta*(Flb0/Na0)*Cla;
%Dsb=-(Fsb0/Nsb)*Csb+(Fsb0/Nlb)*Clb+betta*(Fsb0/Nla)*CLa;
Dsb=1/Na0*Fsb0*(betta*Cla+Clb/P2-Csb/P3);
%Dml=-Kam*(Na0/Nla)*Cla +Kam*Na0/(Cmt*Wm)*eps*Cml;
Dml=1/Tam*((1+Corg/Cmt)/P4*eps*Cml-1/(1-P1)*Cla);
%Dmb=betta*(Fmb0/Na0)*(1+Cmb*Na0/Nmb)*Cml*Na0/Nml;
Dmb=Fmb0/Na0*(1+Cmb/P5)*betta*Cml/P4;
%Dmd= -Kdm*Cds+((Ws-Wm)/Wm)*kdm*Cml
Dmd=1/Tdm*((Ws/Wm-1)*Cml-Cds);
gam= gam0*exp(r*(t+1700));
%gam= gam0+r*t;
Tpy=gam*Na0/(2000*453.6);
dx=[Dlu;Dlb;Dsb;-Dlu-Dsb-Dlb+Dml+gam;-Dml-Dmd-Dmb;Dmb;Dmd];
```

Measured data at Mauna Loa for the CO_2 concentration in Lower Atmosphere

1958	313.00	314.50	315.71	317.45	317.50	316.50	315.86	314.93	313.19	313.00	313.34	314.67	315.00	315.00
1959	315.58	316.47	316.65	317.71	318.29	318.16	316.55	314.80	313.84	313.34	314.81	315.59	315.98	316.00
1960	316.43	316.97	317.58	319.03	320.03	319.59	318.18	315.91	314.16	313.83	315.00	316.19	316.91	316.91
1961	316.89	317.70	318.54	319.48	320.58	319.78	318.58	316.79	314.99	315.31	316.10	317.01	317.65	317.63
1962	317.94	318.56	319.69	320.58	321.01	320.61	319.61	317.40	316.26	315.42	316.69	317.69	318.45	318.46
1963	318.74	319.08	319.86	321.39	322.24	321.47	319.74	317.77	316.21	315.99	317.07	318.36	318.99	319.02
1964	319.57	320.00	321.00	321.50	322.23	321.89	320.44	318.70	316.70	316.87	317.68	318.71	319.00	319.52
1965	319.44	320.44	320.89	322.13	322.16	321.87	321.21	318.87	317.81	317.30	318.87	319.42	320.03	320.09
1966	320.62	321.59	322.39	323.70	324.07	323.75	322.40	320.37	318.64	318.10	319.79	321.03	321.37	321.34
1967	322.33	322.50	323.04	324.42	325.00	324.09	322.55	320.92	319.26	319.39	320.72	321.96	322.18	322.13
1968	322.57	323.15	323.89	325.02	325.57	325.36	324.14	322.11	320.33	320.25	321.32	322.90	323.05	323.11
1969	324.00	324.42	325.64	326.66	327.38	326.70	325.89	323.67	322.38	321.78	322.85	324.12	324.62	324.60
1970	325.06	325.98	326.93	328.13	328.07	327.66	326.35	324.69	323.10	323.07	324.01	325.13	325.68	325.65
1971	326.17	326.68	327.18	327.78	328.92	328.57	327.37	325.43	323.36	323.56	324.80	326.01	326.32	326.32
1972	326.77	327.63	327.75	329.72	330.07	329.09	328.05	326.32	324.84	325.20	326.50	327.55	327.46	327.52
1973	328.54	329.56	330.30	331.50	332.48	332.07	330.87	329.31	327.51	327.18	328.16	328.64	329.68	329.61
1974	329.35	330.71	331.48	332.65	333.09	332.25	331.18	329.40	327.44	327.37	328.46	329.58	330.25	330.29
1975	330.40	331.41	332.04	333.31	333.96	333.59	331.91	330.06	328.56	328.34	329.49	330.76	331.15	331.16
1976	331.74	332.56	333.50	334.58	334.87	334.34	333.05	330.94	329.30	328.94	330.31	331.68	332.15	332.18
1977	332.92	333.42	334.70	336.07	336.74	336.27	334.93	332.75	331.58	331.16	332.40	333.85	333.90	333.88
1978	334.97	335.39	336.64	337.76	338.01	337.89	336.54	334.68	332.76	332.54	333.92	334.95	335.50	335.52
1979	336.23	336.76	337.96	338.89	339.47	339.29	337.73	336.09	333.91	333.86	335.29	336.73	336.85	336.89
1980	338.01	338.36	340.08	340.77	341.46	341.17	339.56	337.60	335.88	336.01	337.10	338.21	338.69	338.67
1981	339.23	340.47	341.38	342.51	342.91	342.25	340.49	338.43	336.69	336.85	338.36	339.61	339.93	339.95
1982	340.75	341.61	342.70	343.56	344.13	343.35	342.06	339.82	337.97	337.86	339.26	340.49	341.13	341.09
1983	341.37	342.52	343.10	344.94	345.75	345.32	343.99	342.39	339.86	339.99	341.16	342.99	342.78	342.75
1984	343.70	344.51	345.28	347.08	347.43	346.79	345.40	343.28	341.07	341.35	342.98	344.22	344.42	344.44
1985	344.97	346.00	347.43	348.35	348.93	348.25	346.56	344.69	343.09	342.80	344.24	345.56	345.90	345.86
1986	346.29	346.96	347.86	349.55	350.21	349.54	347.94	345.91	344.86	344.17	345.66	346.90	347.15	347.14
1987	348.02	348.47	349.42	350.99	351.84	351.25	349.52	348.10	346.44	346.36	347.81	348.96	348.93	348.99
1988	350.43	351.72	352.22	353.59	354.22	353.79	352.39	350.44	348.72	348.88	350.07	351.34	351.48	351.44
1989	352.76	353.07	353.68	355.42	355.67	355.13	353.90	351.67	349.80	349.99	351.30	352.53	352.91	352.94
1990	353.66	354.70	355.39	356.20	357.16	356.22	354.82	352.91	350.96	351.18	352.83	354.21	354.19	354.19
1991	354.72	355.75	357.16	358.60	359.34	358.24	356.17	354.03	352.16	352.21	353.75	354.99	355.59	355.62
1992	355.98	356.72	357.81	359.15	359.66	359.25	357.03	355.00	353.01	353.31	354.16	355.40	356.37	356.36
1993	356.70	357.16	358.38	359.46	360.28	359.60	357.57	355.52	353.70	353.98	355.33	356.80	357.04	357.10
1994	358.36	358.91	359.97	361.26	361.68	360.95	359.55	357.49	355.84	355.99	357.58	359.04	358.88	358.86
1995	359.96	361.00	361.64	363.45	363.79	363.26	361.90	359.46	358.06	357.75	359.56	360.70	360.88	360.90
1996	362.05	363.25	364.03	364.72	365.41	364.97	363.65	361.49	359.46	359.60	360.76	362.33	362.64	362.58
1997	363.18	364.00	364.57	366.35	366.79	365.62	364.47	362.51	360.19	360.77	362.43	364.28	363.76	363.84
1998	365.32	366.15	367.31	368.61	369.29	368.87	367.64	365.77	363.90	364.23	365.46	366.97	366.63	366.58
1999	368.15	368.87	369.59	371.14	371.00	370.35	369.27	366.94	364.63	365.12	366.67	368.01	368.31	368.30
2000	369.14	369.46	370.52	371.66	371.82	371.70	370.12	368.12	366.62	366.73	368.29	369.53	369.48	369.47
2001	370.28	371.50	372.12	372.87	374.02	373.30	371.62	369.55	367.96	368.09	369.68	371.24	371.02	371.04
2002	372.43	373.09	373.52	374.86	375.55	375.40	374.02	371.49	370.71	370.24	372.08	373.78	373.10	373.08
2003	374.68	375.63	376.11	377.65	378.35	378.13	376.62	374.50	372.99	373.00	374.35	375.70	375.64	375.61
2004	376.79	377.37	378.41	380.52	380.63	379.57	377.79	375.86	374.06	374.24	375.86	377.48	377.38	377.43

Modified gamma in program.

```
%file co2sys2.m
function dx =co2sys(t,x)
global Na0 Tam Tul Tdm betta Flb0 Fsb0 Fmb0 Cmt Corg ...
eps P1 P2 P3 P4 P5 Ws Wm gam0 r
Cua=x(1);
Clb=x(2);
Csb=x(3);
Cla=x(4);
Cml=x(5);
Cmb=x(6);
Cds=x(7);
%Dlu=-Kul*Cua+Kul*(Nua/Nla)*Cla;
Dlu=1/Tul*(P1/(1-P1)*Cla-Cua);
%Dlb=betta*(Flb0/Nla)*Cla;
Dlb=betta*(Flb0/Na0)*Cla;
%Dsb=-(Fsb0/Nsb)*Csb+(Fsb0/Nlb)*Clb+betta*(Fsb0/Nla)*CLa;
Dsb=1/Na0*Fsb0*(betta*Cla+Clb/P2-Csb/P3);
%Dml=-Kam*(Na0/Nla)*Cla +Kam*Na0/(Cmt*Wm)*eps*Cml;
Dml=1/Tam*((1+Corg/Cmt)/P4*eps*Cml-1/(1-P1)*Cla);
%Dmb=betta*(Fmb0/Na0)*(1+Cmb*Na0/Nmb)*Cml*Na0/Nml;
Dmb=Fmb0/Na0*(1+Cmb/P5)*betta*Cml/P4;
%Dmd= -Kdm*Cds+((Ws-Wm)/Wm)*kdm*Cml
Dmd=1/Tdm*((Ws/Wm-1)*Cml-Cds);
%REMEMBER: we have to add only the forcing which are not included in
%the initial conditions
%linear fit
%gam= p1*t;
%p1=0.00268605036723e-3
%p2=0.60848369726802e-3 this is included already in the initial conditions
%quadratic fit
p1=0.00002297543249e-3
p2=0.00160811965954e-3
%p3=0.61689754529194e-3 this is included already in the initial conditions.
gam1= p1*t^2+p2*t;
%exponential fit: gam= C*(exp(alpha*t)-1) so that the forcing is 0 at t=0;
%gam1=6.106170969309385e-04*(exp(0.00398734415689*t)-1);
%gam=gam1-0.065*gam1   %U.S. using all PHEV technology
%gam=gam1-0.195*gam1   %World using all PHEV technology (including U.S.)
%gam=gam1-0.395*gam1   %World using Helium-3 instead of fossil fuels
gam=gam1-0.590*gam1;   %World using both PHEV and Helium-3
dx=[Dlu;Dlb;Dsb;-Dlu-Dsb-Dlb+Dml+gam;-Dml-Dmd-Dmb;Dmb;Dmd];
%-----------------------------------------------------
```

Program 2 Mean Temperature of the Earth (written by Mayer Humi)

```
% file temper
function dx =temper(t,x)
%the differential eq for the mean temperature of the Earth
global C   %heat capacity of Earth
global sigma %Stephan Boltzman constant 5.67 x 10-8 Watts/(m^2 K^4)
global T0  %ambient temp for green house effect
global T1  %freeze temp for albedo
global T2  % nonfreezing temp for albedo
global kappa % cloud cover for green house effect
global Fs    %solar forcing radiation /4
global alphaM % albedo for freezing temp
global alpham % albeado for non freezing Earth
%compute the albedo
if (x < T1)
A=alphaM;
elseif (x > T2)
A=alpham;
else
A=alphaM - (x-T1)/(T2-T1)*(alphaM-alpham);
end
%compute the greenhouse factor
% for the first setting of TEMP0 try to find the breaking point between
%alpha=0.17 and 0.18
alpha=0.18;
g=1-kappa*tanh(x/T0) + alpha;
% for the 2nd setting of TEMP0 try to find the breaking point between
%alpha=-0.37 and -0.38
%g=1-kappa*tanh(x/T0) + alpha
dx=[Fs*(1-A)-sigma*g*x^4];


%zero dim model for the mean temperature of the Earth
%Author Mayer Humi
global C   %heat capacity of Earth
global sigma %Stephan Boltzman constant 5.67 x 10-8 Watts/(m^2 K^4)
global T0  %ambient temp for green house effect
global T1  %freeze temp for albedo
global T2  % nonfreezing temp for albedo
global kappa % cloud cover for green house effect
global Fs    %solar forcing radiation /4
global alphaM % albedo for freezing temp
global alpham % albeado for non freezing Earth
C=1;
sigma= 5.67e-8;
T0=275;
T1=240;
T2=275;
kappa=0.5;
Fs=1367/4;
alphaM=0.85;
alpham=0.25;
% set error tolerance and method for the numerical
%integrator of the differential equations.
```

```
odeset('maxstep',1.,'reltol',1.e-3,'bdf','on')

%perform the integration 'temper' is a file that defines
%the differential equation

%[0,300] time interval for the integration
%[TEMP0] define the initial conditions
TEMP0=275;
%TEMP0=235;
for i=1:30000
tt(i)=i-20;
end
[t,eqtemp]=ode15s('tmpr',tt,[TEMP0]);
plot(t,eqtemp)
xlabel('time')
ylabel('equilibrium temperature')
title('Equilibrium Temperature as a Function of Time')
```