

Paired Comparison Models for Ranking National Soccer

Teams

by

Shawn E. Hallinan

A Project Report

Submitted to the Faculty

of

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Degree of Master of Science

in

Applied Statistics

by

May 2005

APPROVED:

Dr. Andrew W. Swift, Advisor

Dr. Bogdan M. Vernescu, Department Head

Abstract

National soccer teams are currently ranked by soccer's governing body, the Federation Internationale de Football Association (FIFA). Although the system used by FIFA is thorough, taking into account many different factors, many of the weights used in the system's calculations are somewhat arbitrary. It is investigated here how the use of a statistical model might better compare the teams for ranking purposes. By treating each game played as a pairwise comparison experiment and by using the Bradley-Terry model as a starting point some suitable models are presented. A key component of the final model introduced here its ability to differentiate between friendly matches and competitive matches when determining the impact of a match on a teams ranking. Posterior distributions of the rating parameters are obtained, and the rankings and results obtained from each model are compared to FIFA's rankings and each other.

Acknowledgements

First and foremost, I would like to thank my advisor, Dr. Andrew Swift, without whose guidance I would have been lost on many occasions in this project. I would also like to thank the Mathematical Sciences Department for the financial assistance provided, without which I would not have been able to afford graduate school. In particular, I would like to thank the professors that have helped me in my two years as a graduate student: Dr. Carlos Morales, Dr. Joe Petrucci, and Dr. Jason Wilbur. Thank you to Greg Matthews who helped make my experience as a graduate student enjoyable with his similar interests in statistics and sports. In addition, thank you to all of the teaching assistants who added to my experience at WPI, Raj, Alina, Yan, Rajesh, Eric, Gang, Owen, Ren, Didem, Flora, Scott, Liz, Bijaya, and Ashley. I would like to thank my family; my mother Lynne, my father Jeff, and my sisters, Caitlin and Megan for putting up with me when I was cranky after long days of work. Finally, I would also like to thank Becca Nacewicz for listening to me vent when I was frustrated.

Table of Contents

Introduction.....	3
Chapter 1: The Model	6
1.1: The General Bradley – Terry Model.....	6
1.2: Ties.....	9
1.3: Inclusion of Other Parameters in the Model.....	10
1.4: A Dynamic Model	14
Chapter 2: The Data	16
Chapter 3: The Model for Soccer Data	18
Chapter 4: Results and Convergence	20
5.1: General Bradley – Terry Model.....	21
5.2: Extended Bradley – Terry Model with Ties and Home – Field Advantage	24
5.3: Extended Bradley – Terry Model with Ties, Home – Field Advantage, and Neutral Site Distinction.....	29
5.4: Extended Bradley – Terry Model with Ties and Home – Field Advantage, Neutral Site Distinction, and Game Type	33
5.5: Dynamic Bradley – Terry Model.....	39
Chapter 5: Conclusions and Future Work.....	47
References.....	49
Appendices.....	50
Appendix A - FIFA Rankings as of December, 2004.....	50
Appendix B – WinBugs Code for Each Model	53
B.1 – General Bradley Terry Model	53
B.2 – Extended Bradley Terry Model with Ties and Home Field Advantage.....	55
B.3 – Extended Bradley Terry Model with Ties and Neutral Site Distinction.....	57
B.4 – Extended Bradley Terry Model with Ties, Neutral Site Distinction, and Game Type.....	59
B.5 – Dynamic Bradley Terry Model with Ties and Neutral Site Distinction	62
Appendix C – Full Rankings.....	65
C.1 – Full Rankings for Extended Bradley Terry Model with Ties, Neutral Site Distinction, and Game Type.....	65
C.2 – Full Rankings for Dynamic Bradley Terry Model with Ties and Neutral Site Distinction.....	69
Appendix D – Non - Dynamic Model with Game Type Parameter	73
D.1 – Posterior Trace – Top Twenty Teams.....	73
D.2 – Smoothed Posterior Density ($s = .15$) – Top Twenty Teams.....	76
Appendix E – Dynamic Model	78
E.1 – Posterior Trace – Top Twenty Teams	78
E.2 – Smoothed Posterior Density ($s = .15$) – Top Twenty Teams	81
E.3 – Team Parameter Strength vs. Time	83

List of Tables

Table 1: Burn – In Convergence of Parameters	20
Table 2: Results From General BT Model	22
Table 3: General BT Model Rankings Compared with FIFA	22
Table 4: Results for BT Model with Ties and HFA	26
Table 5: BT Model with Ties and HFA Rankings Compared with FIFA	27
Table 6: Results for Extended BT Model with Ties, HFA, and Neutral Site	31
Table 7: BT Model with Ties, HFA, and Neutral Site Rankings Compared with FIFA.....	31
Table 8: Results for Extended BT Model with Ties, HFA, Neutral Site, and Game Type ..	35
Table 9: BT Model with Ties, HFA, Neutral Site, and Game Type Rankings Compared with FIFA.....	35
Table 10: MSE for Each Model Considered.....	37
Table 11: Results for Dynamic BT Model with Ties, HFA, and Neutral Site	42
Table 12: Dynamic BT Model with Ties, HFA, and Neutral Site Rankings Compared with FIFA	43

List of Figures

Figure 1: Trace of Parameters for Bradley - Terry Model.....	23
Figure 2: Smoothed Kernel Densities for Bradley - Terry Model ($s = .15$)	24
Figure 3: Trace of Parameters for BT with Ties and HFA.....	28
Figure 4: Smoothed Kernel Densities for BT Model with Ties and HFA ($s = .15$)	29
Figure 5: Winning Percentages for Home Teams Under Different Model Assumptions....	32
Figure 6: Trace of Parameters for BT with Ties, HFA, and Neutral Site	33
Figure 7: Smoothed Kernel Densities for BT Model with Ties, HFA, and Neutral Site ($s = .15$)	33
Figure 8: Trace of Parameters for BT with Ties, HFA, Neutral Site, and Game Type	38
Figure 9: Smoothed Kernel Densities for BT with Ties, HFA, Neutral Site, and Game Type ($s = .15$).....	39
Figure 10: Convergence of Sigma with a $U(0,100)$ Prior Distribution.....	40
Figure 11: Convergence of Sigma with a $U(0,10)$ Prior Distribution for 4,000 and 85,000 Burn – Ins	41
Figure 12: Trace of Parameters for Dynamic BT with Ties, HFA, and Neutral Site.....	44
Figure 13: Smoothed Kernel Densities for Dynamic BT Model with Ties, HFA, and Neutral Site ($s = .15$).....	44
Figure 14: Zoom of Trace for Home - Field and Tie Parameters from Figure 12.....	45
Figure 15: Team Strength vs. Time.....	46

Introduction

In the world of professional soccer, there is one governing body that determines all rules, and rankings, called the Federation Internationale de Football Association (FIFA). The FIFA rankings system, developed by FIFA and Coca Cola allows for published rankings of all senior national teams. The rankings are based on a number of criteria including match outcome, date of match, type of match, home field advantage, the number of goals scored, and points for regional strength of the competing teams. For each match, the two competing teams each earn a score based on the aforementioned criteria. A teams ranking is based on its cumulative score from all games during the ranking time period, which dates back to the previous eight years.

The scoring is done as follows:

1. The winning team is awarded points, based on the strength of the team that they beat.
2. Each team is awarded points based on the number of goals scored, with the more weight attached to the first goal scored rather than the subsequent ones, in order to avoid overweighting of goals.
3. To account for home field advantage, the away team is awarded a small bonus of three points.
4. To account for the importance of the game, an arbitrary weighting is assigned to each type of game as follows:
 - Friendly match x 1.00
 - Continental championship preliminary x 1.50
 - World Cup preliminary match x 1.50
 - Continental championship finals match x 1.75
 - FIFA Confederations Cup match x 1.75
 - World Cup finals match x 2.00
5. If two strong teams from the same continent play each other, than a weighting based on which confederation they are a part of is used. For two strong teams from different continents, the average weighting for each teams respective confederation is used. The weights are as follows:
 - UEFA x 1.00
 - CONMEBOL x 0.99
 - CAF x 0.94
 - CONCACAF x 0.94

- AFC x 0.93
- OFC x 0.93

6. Last, FIFA considers all games played for the previous eight years, and arbitrarily weights the games based on when it was played with the following system(from 2003):

- + previous year (2002) : 7/8 value
- + previous year (2001) : 6/8 value
- + previous year (2000) : 5/8 value
- + previous year (1999) : 4/8 value
- + previous year (1998) : 3/8 value
- + previous year (1997) : 2/8 value
- + previous year (1996) : 1/8 value

In addition to the above, FIFA only uses each teams 7 best results each year, so that teams cannot improve in the rankings by simply playing an excessive number of games. To see how this works, consider an example. Suppose a team plays twelve games in a year, their score for that year would be calculated as follows:

- The best seven of the 12 results are identified
- The total score for these seven matches is calculated (X)
- The total score for all 12 matches is calculated
- This total is divided by 12 and multiplied by seven (Y)
- The total for the seven best results is added to the seven "average" results (X+Y)
- This total (X+Y) is divided by two for the final score

The FIFA rankings of all teams considered in this project can be found in Appendix A. While this system is very thorough; all of the weights used in the ranking process are arbitrary chosen. Thus a statistical model that could take certain criteria and use it to compare teams in a non – arbitrary manner would be useful.

When teams are being compared with one another based on certain characteristics, this is called a paired comparison design (Wilkinson, 1957). In practice, the Bradley-Terry model has often been used for paired comparisons designs used to rank sports teams in the past (Knorr – Held, 1999 and Glickman, 1993). The goal of this project was to develop a less arbitrary system

to rank national soccer teams, and the Bradley-Terry model is a good starting point for achieving this goal.

Chapter 1: The Model

In order to compare soccer teams, certain criteria are necessary. The results from each game clearly would be the most important criteria, but there are other criteria, such as home – field advantage, what type of game is being played, and a team’s ability to get better or worse over time that also can not be ignored. In building a model to incorporate all these criteria, there has to be a starting point, and the Bradley – Terry model, which compares teams based on wins and losses only, will be used to serve this function. This chapter describes the methodology behind the Bradley – Terry model, and how it can be extended to incorporate the desired criteria listed above, which will eventually lead to the final model. Some of the extensions to the Bradley – Terry model discussed here have been previously implemented (see Glickman, 1993). However, the extension of the Bradley – Terry model to rank teams using different types of games is an original idea.

1.1: The General Bradley – Terry Model

The Bradley – Terry model was first introduced by Ralph Bradley and Milton Terry in their 1952 paper, *Rank Analysis of Incomplete Block Designs I: The Method of Paired Comparisons*. In its most basic form, the Bradley-Terry model assumes that:

$$P_{ij} = \frac{\pi_i}{\pi_i + \pi_j}, \quad (1)$$

where p_{ij} denotes the probability that team i defeats team j , and π_i and π_j are parameters representing player strengths or abilities. It is trivial to see that $p_{ij} + p_{ji} = 1$, and thus the model only accepts one of two possible outcomes: team i wins, or team j wins. One derivation of this model can be found in Glickman(1993). The model assumes that when team i competes, it produces a “score”, S_i . This score is unobserved, and considered independent of the opposing team’s “score”, S_j . The observed variable is the result of the game, which is determined by the larger of the two scores. S_i follows an extreme value distribution with location parameter (the parameter that describes the location of the distribution) $\log \pi_i$ (Gumbel, 1961). Thus, the cumulative distribution function of S_i , $F_i(s)$ is of the form

$$F_i(s) = \exp(-e^{-(s-\log \pi_i)}),$$

and it follows that the difference, $S_i - S_j$ follows a logistic distribution with mean $\log \pi_i - \log \pi_j$:

$$S_i - S_j \sim F_i(s) = \frac{1}{1 + e^{-(s-(\log \pi_i - \log \pi_j))}}.$$

Therefore, using the properties of the cdf it is implied that

$$\Pr(S_i > S_j) = P(S_i - S_j > 0) = 1 - \frac{1}{1 + e^{-(0-(\log \pi_i - \log \pi_j))}} = \frac{e^{(\log \pi_i - \log \pi_j)}}{1 + e^{(\log \pi_i - \log \pi_j)}} = \frac{\left(\frac{\pi_i}{\pi_j}\right)}{1 + \left(\frac{\pi_i}{\pi_j}\right)} = \frac{\pi_i}{\pi_i + \pi_j},$$

which is now in the form originally stated.

Now consider that teams i and j compete a total of n_{ij} times with team i winning y_{ij} times and team j winning $n_{ij} - y_{ij} = y_{ji}$ times. Then, if $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_p)$, the distribution of $\mathbf{y} = (y_{ij}, i, j = 1, 2, \dots, p)$ is multinomial with with probability p_{ij} and density

$$f(\mathbf{y} | \boldsymbol{\pi}) = \prod_{i < j} \binom{n_{ij}}{y_{ij}} \left(\frac{\pi_i}{\pi_i + \pi_j} \right)^{y_{ij}} \left(\frac{\pi_j}{\pi_i + \pi_j} \right)^{y_{ji}}.$$

Now, it follows that the likelihood for $\boldsymbol{\pi}$, given the data can now be written

$$L(\boldsymbol{\pi} | \mathbf{y}) \propto \frac{\prod_{i=1}^p \pi_i^{y_i}}{\prod_{i < j} (\pi_i + \pi_j)^{n_{ij}}},$$

where $y_i = \sum_{j=1}^p y_{ij}$. The likelihood of $\boldsymbol{\pi}$ is a function which describes the amount of support given to particular values of $\boldsymbol{\pi}$ by the data. In other words, if $L(\boldsymbol{\pi}_1 | \mathbf{y}) > L(\boldsymbol{\pi}_2 | \mathbf{y})$ then the data gives more support to $\boldsymbol{\pi}_1$ being the true value of $\boldsymbol{\pi}$ than it does to $\boldsymbol{\pi}_2$. Thus, maximizing this likelihood with respect to $\boldsymbol{\pi}$ will give a “best” estimate in the sense that the data supports it more than all other values of $\boldsymbol{\pi}$. The estimates resulting from maximizing the likelihood are called Maximum Likelihood Estimates (MLE’s).

If the likelihood for a parameter depends on the data only through the value of a summary statistic, that statistic is called a sufficient statistic. Thus, it clearly follows that given n_{ij} , y_{ij} are sufficient statistics for π_i , and so only the number of times that each team beat all the other teams in the model are needed to estimate $\boldsymbol{\pi}$.

This model can be viewed in the Bayesian context as well. Bayesian data analysis assumes that the parameters come from a distribution instead of the assumption that they are simply values. The parameters are given a prior distribution and that distribution is combined with the data to produce a posterior distribution, from which inference on the parameters can be made. Thus by using Bayesian data analysis instead of conventional analysis, we will gain more information about each of the parameters of interest. The Bayesian model used here is from

Leonard (1977). First, consider the model established in (1), and let $\alpha_i = \log \pi_i$ and place a multivariate normal prior distribution on α with mean vector μ and non singular covariance matrix C . Reparametrizing the model in terms of α_i (the team's "rating") gives

$$p_{ij} = \frac{e^{\alpha_i}}{e^{\alpha_i} + e^{\alpha_j}}. \quad (2)$$

The implementation of this model is relatively simple. Before the games are played, the α_i 's are assumed to have a normal prior distribution. When games are played, the likelihood is used to obtain parameter estimates, and these updated estimates are used as the prior parameters for the next games played. This process continues until the parameter estimates converge to certain values. If a team has yet to compete, a large prior variance can be used to show the uncertainty of the team's rating.

1.2: Ties

Any model used to rank soccer teams will obviously need to be able to handle tied game results. Davidson (1970) proposed the following extension to the Bradley-Terry model to handle tied games:

$$p_{ij1} = \Pr(i \text{ defeats } j | \theta) = \frac{e^{\alpha_i}}{e^{\alpha_i} + e^{\alpha_j} + e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j)}}$$

$$p_{ij2} = \Pr(j \text{ defeats } i | \theta) = \frac{e^{\alpha_j}}{e^{\alpha_i} + e^{\alpha_j} + e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j)}} \quad (3)$$

$$p_{ij3} = \Pr(i \text{ ties with } j | \theta) = \frac{e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j)}}{e^{\alpha_i} + e^{\alpha_j} + e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j)}},$$

where the parameter δ measures the effect of ties. Now the model considers three possibilities: a win, a loss, or a tie. Consider how δ affects the model. Large positive values of δ will imply that there is a high probability of ties, since p_{ij3} will be larger for large values of δ . Large negative values of δ will imply a small probability of ties, since p_{ij3} will be small for large negative values of δ . In the case that δ is close to zero, this will imply that there the probabilities of a win, loss, or tie are about equal subject to the team strengths. In the Bayesian context, δ has to be given a prior distribution. Since there is no prior information on how ties affect the model, a vague prior is given. The prior distribution on δ is a normal distribution with mean zero and a large variance, to reflect the lack of knowledge about it. Now, with the inclusion of ties, the model is beginning to look more suitable for modeling the situation of interest to us.

1.3: Inclusion of Other Parameters in the Model

The model established in the previous section is an improvement over the general Bradley - Terry model, but still does not account for a few important factors in ranking soccer teams. For one, home field advantage is very important in soccer and can not be overlooked. A team playing in its own country should have a big advantage over the team they are playing. Hence an extension of the model in (3), also proposed by Davidson(1970) is given by

$$\begin{aligned}
p_{ij1} = \Pr(i \text{ defeats } j|\theta) &= \frac{e^{\alpha_i}}{e^{\alpha_i} + e^{\alpha_j + \eta} + e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j + \eta)}} \\
p_{ij2} = \Pr(j \text{ defeats } i|\theta) &= \frac{e^{\alpha_j + \eta}}{e^{\alpha_i} + e^{\alpha_j + \eta} + e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j + \eta)}} \\
p_{ij3} = \Pr(i \text{ ties with } j|\theta) &= \frac{e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j + \eta)}}{e^{\alpha_i} + e^{\alpha_j + \eta} + e^{\delta + \frac{1}{2}(\alpha_i + \alpha_j + \eta)}} ,
\end{aligned} \tag{4}$$

where η determines the relative home field advantage (note that it should be negative if it gives the home team an advantage, because it will decrease the probability of team j , the road team winning).

Let y_{ijk} , $k = 1, 2, 3$ be the data representing the number of wins losses and ties for team i in games with team j for which team i was the home team and let n_{ij} be the total number of games played between teams i and j . The y_{ijk} 's are just counts of wins, losses, and ties. Now consider the parameter $\mu_{ijk} = n_{ij} \cdot p_{ijk}$, which is the total number of games played between teams i and j multiplied by the probability that team i defeats team j . This parameter should be a good representation of the number of times that team i defeats team j . Thus, y_{ijk} can be modeled as a Poisson random variable with mean μ_{ijk} . Also consider μ_{ijk} . The logarithm of μ_{ijk} can be equated to a linear predictor:

$$\log(\mu_{ij1}) = \log(n_{ij}) + \alpha_i - A_{ij}$$

$$\log(\mu_{ij2}) = \log(n_{ij}) + \alpha_j + \gamma - A_{ij}$$

$$\log(\mu_{ij3}) = \log(n_{ij}) + \delta + \frac{1}{2}\alpha_i + \frac{1}{2}\alpha_j + \frac{1}{2}\gamma - A_{ij},$$

where A_{ij} is the logarithm of the denominator in the p_{ijk} 's. Thus, now the model described in this section (and the model in (3)) can be parameterized as a Poisson generalized linear model with a logarithmic link. The A_{ij} term serves as a nuisance parameter, in that it is in the model to ensure that $y_{ij1} + y_{ij2} + y_{ij3} = n_{ij}$, however it will not be sampled from the posterior distribution since it is not of interest. It should also be noted that this model is over parameterized, so one of the α_i should be set to zero.

Two other important factors that go into ranking teams are whether or not the game was played at a neutral site and what type of game was played. For example, a win in a friendly match should not count towards a teams' "rating" as much as a win in a world cup match. While the models in the previous sections have all been used before, this idea of including game type as a parameter is a new concept that could potentially have a significant contribution to the model. Similarly, if a team is playing in its own country instead of a neutral site, it should have a higher probability of winning the game. Thus, the model has to be extended to include game type and games at neutral sites. For this project, two game types were considered: friendly matches and competitive matches. The model to include these ideas can be seen by

$$p_{ijhm1} = \Pr(i \text{ defeats } j | \theta) = \frac{e^{\alpha'_{im}}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}$$

$$p_{ijhm2} = \Pr(j \text{ defeats } i | \theta) = \frac{e^{\alpha'_{jm} + (h-1)\eta}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}} \quad (5)$$

$$p_{ijm3} = \Pr(i \text{ ties with } j|\theta) = \frac{e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}},$$

where $\alpha'_{im} = \lambda_m \alpha_i$ so that only a fraction of the team's true "rating" parameter actually contributes to the probabilities. For competitive matches, $\lambda_m = 1$ so that $\alpha'_{im} = \alpha_i$ and the team's full parameter will only be included in the model. For friendly matches the data will determine λ_m , which should estimate the weight that friendly matches take on relative to competitive matches.

Multiplying $(h - 1)$ by η allows the model to account for games played at a neutral site. The $(h - 1)$ term works like an indicator variable in that when $h = 1$ (neutral games), the home field advantage parameter, η will not be in the model, whereas when $h = 2$ (team i is the home team), the home field advantage parameter will be in the model. Thus, this "indicator" method allows the model to consider games in which there is no home team.

In the Bayesian context, prior distributions need to be assigned to the home – field advantage and game type parameters. Since there is no information about the home – field advantage parameter, and it can be positive or negative, the same prior distribution as the tie parameter is assigned, a normal distribution with mean zero and large variance. For the game type parameter however, there is some prior information. It must be greater than zero because it represents the weight that friendly matches contribute relative to competitive matches. Thus, a beta prior distribution was assigned to ensure that it was greater than zero. Since there is no other information about the parameter, a vague beta distribution was assigned to the game type parameter, $\lambda_m \sim \text{beta}(1,1)$, for $m = 1$ (friendly matches).

1.4: A Dynamic Model

The model developed in this section can also be used with dynamic parameters (see Glickman, 1993). Similar to the reparameterized Bayesian Bradley – Terry model, let p_{ijt} be the probability that team i defeats team j during time period t . So, like the model in (2),

$$p_{ijt} = \frac{e^{\alpha_{it}}}{e^{\alpha_{it}} + e^{\alpha_{jt}}}, \quad (6)$$

where α_{it} is team i 's rating in time period t . Now team ratings progress over time following an AR(1) model with $\phi = 1$:

$$\alpha_{it} = \alpha_{i(t-1)} + w_t,$$

where w_t is the amount that team strengths change from time period $t - 1$ to time period t . w_t is assumed to be stochastically independent of $\alpha_{i(t-1)}$, and

$$w_t | \sigma^2 \sim N(\gamma_t, \sigma^2 \mathbf{I}_p),$$

where γ_t is the mean amount by which teams change between time periods $t - 1$ and t , and σ^2 is the variance of the change that occurs between time periods. An initial prior distribution on α_1 must be specified,

$$\alpha_1 \sim N(\boldsymbol{\mu}_1, \mathbf{C}_1),$$

where $\boldsymbol{\mu}_1$ and \mathbf{C}_1 are p – dimensional (for the number of teams). A gamma prior distribution was

placed on the precision, $\omega = \frac{1}{\sigma^2}$, as it is the conjugate prior distribution for precision:

$$\omega \propto \omega^{\alpha_0 - 1} e^{-b_0 \omega},$$

where a_0 and b_0 are specified in advance. This system will now show how the Bradley – Terry ratings change over time. The model used for this paper defines $\gamma_t = 0$ for all t , thus making w_t follow a random walk model.

The dynamic Bayesian analysis of the extended Bradley – Terry model to include ties, home – field advantage, and game type follows the same methodology as that of the previous Bradley – Terry model, where now the model describes probabilities at a given time period,

$$\begin{aligned}
 p_{ijhm1t} &= \Pr(i \text{ defeats } j \text{ at time } t | \theta) = \frac{e^{\alpha'_{im}}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}} \\
 p_{ijhm2t} &= \Pr(j \text{ defeats } i \text{ at time } t | \theta) = \frac{e^{\alpha'_{jm} + (h-1)\eta}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}} \quad (7) \\
 p_{ijhm3t} &= \Pr(i \text{ ties with } j \text{ at time } t | \theta) = \frac{e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}} .
 \end{aligned}$$

The prior distribution of all the parameters is the same as described previously,

$$(\gamma_t, \delta, \eta) \sim N(\boldsymbol{\mu}_1, \mathbf{C}_1)$$

$$\lambda \sim \text{beta}(1,1),$$

where $\boldsymbol{\mu}_1$ and \mathbf{C}_1 are now $(p + 2)$ dimensional, and the precision between time periods follows the gamma prior distributions specified previously. Now the model is finalized, and the analysis can be conducted.

Chapter 2: The Data

For the general Bradley-Terry model in (1), the data had to be entered in the form:

$$y_{ij} = 1, \text{ if team } i \text{ beats team } j.$$

$$y_{ij} = -1, \text{ if team } j \text{ beats team } i.$$

Notice that the data for this model can not take ties into account, as there are only two possible values for the data, so ties were thrown out of the data set for this model. The data was obtained from FIFA's website, www.fifa.com.

For the model with ties, and all models including ties, the data had to be entered in the form:

$$y_{ij1} = \text{the number of wins team } i \text{ had against team } j.$$

$$y_{ij2} = \text{the number of wins team } j \text{ had against team } i.$$

$$y_{ij3} = \text{the number of ties between team } i \text{ and team } j.$$

Data from over 2,300 games from the past ten years was collected and separated based on time period, home field advantage, and game type. For the final model, the following data had been collected:

- Wins, Losses, and Ties between all teams in competitive games for which there was a home team, in T different time periods.

- Wins, Losses, and Ties between all teams in competitive games for which there was not a home team, in T different time periods.

- Wins, Losses, and Ties between all teams in friendly matches for which there was a home team, in T different time periods.

- Wins, Losses, and Ties between all teams in friendly matches for which there was not a home team, in T different time periods.

T represents the number of different time periods considered in the model, and thus all games in all time periods under all possible circumstances in the model are accounted for.

Chapter 3: The Model for Soccer Data

The final model, established in section 2.4 is used for the soccer data described in Chapter 3. It has been tailored to describe the data as it will be seen in the following section. The model for soccer data can be specified as

$$P_{ijhm1t} = \Pr(i \text{ defeats } j \text{ at time } t | \theta) = \frac{e^{\alpha'_{im}}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}$$

$$P_{ijhm2t} = \Pr(j \text{ defeats } i \text{ at time } t | \theta) = \frac{e^{\alpha'_{jm} + (h-1)\eta}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}$$

$$P_{ijhm3t} = \Pr(i \text{ ties with } j \text{ at time } t | \theta) = \frac{e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}{e^{\alpha'_{im}} + e^{\alpha'_{jm} + (h-1)\eta} + e^{\delta + \frac{1}{2}(\alpha'_{im} + \alpha'_{jm} + (h-1)\eta)}}$$

where,

$i = 1, 2, \dots, 158$ for the 158 teams considered in the model.

$j = 1, 2, \dots, 158$.

$h = 1, 2$ (1 for when i is home team, 2 for when there is no home team).

$m = 1, 2$ (1 for friendly matches, 2 for competitive matches).

$t = 1, 2, \dots, T + 1$ (corresponding to each of the T time periods considered),

and where $\alpha'_{im} = \lambda_m \alpha_{it}$ and $\alpha_{it} = \alpha_{i(t-1)} + w_t$,

where $w_t | \sigma^2 \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_p)$. The prior distributions for γ_t , δ , η , and λ are as follows:

$$\gamma_I \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_p),$$

$$(\delta, \eta) \sim N(\mathbf{0}, \mathbf{1000} * \mathbf{I}_2),$$

$$\lambda \sim \text{beta}(1,1), \text{ and}$$

$$\omega = \frac{1}{\sigma^2} \propto \omega^{a_0-1} e^{b_0 \omega}.$$

Notice that only T time periods are considered, but there are T + 1 values of t. This is due to the random walk assumption on α_t , because α_1 must be simulated before applying the random walk to α_2 . Thus, t = 2 corresponds to the time period. Now, the model can be applied to the soccer data, and the ranking system can be tested. The program used to run the model was Winbugs, a sophisticated statistics program which uses Markov Chain Monte Carlo (MCMC) methods and is particularly helpful with Bayesian data analysis. The code for each of the models in the results section can be found in Appendix B.

Chapter 4: Results and Convergence

We wanted to compare the results we obtained from the model as we made it more complex, so the rankings and statistics produced by each model will be presented, along with the FIFA rankings for the top twenty teams in each ranking system. In addition, a trace of the posterior samples for certain parameters will be presented to show that the parameters are converging, which is important because the inference made on the posterior distributions is not valid unless convergence is met.

For all models, the number of samples to be burned (were thrown out before calculations on the samples began) had to be determined. The method used to determine this involved burning 4000, 5000, and 6000 samples and comparing parameter estimates and a fit statistic called the Deviance Information Criterion (DIC). The reason that the smallest number of burned samples is 4000 is that WinBugs would not allow any less. If the parameter estimates and the DIC do not change by more burning more samples, then there is strong evidence that convergence is being reached. For example, Table 1 shows the some of the parameter estimates of 5000 iterations after burning 4000, 5000, and 6000 samples respectively, along with the DIC for the model including only ties and home – field advantage.

node	mean - 4000	mean - 5000	mean - 6000
Brazil	0.7535	0.7576	0.7591
England	0.5456	0.5452	0.5475
USA	0.526	0.5198	0.5265
home	-0.4599	-0.4599	-0.4594
tie	-0.08565	-0.08676	-0.08739
DIC	10021.1	10022.2	10021.4

Table 1: Burn – In Convergence of Parameters

It can be seen that neither the parameters nor the DIC changes as a result of burning more samples. Thus, it can be concluded that the parameters are converging by 4000 burns, and it is only necessary to burn 4000 samples. This method of finding convergence was done for all models, and 4000 samples proved to be enough for all models used in this project. In each section, figures depicting selected parameter's history and density will be given as further proof of convergence.

5.1: General Bradley – Terry Model

The general Bradley – Terry model which is a simple pairwise comparison model assuming that all games played, at all times carry equal weight. In other words, a Friendly match played eight years ago contributes exactly as much as a World Cup match played within the past few years. It also assumes no ties, so one team must win and one team must lose. Thus, all the games for which ties occurred were not counted. The results for the this model were not expected to be that close to FIFA's rankings, because of the complexity of FIFA's rankings and the simplicity of this model. The results (of the top 20 teams) are displayed in Table 2, and the rankings compared to those of FIFA are displayed in Table 3.

Team	mean	sd	MC_error	Ratio	val2.5pc	median	val97.5pc
Brazil	0.962	0.2026	0.0022	0.010859	0.5698	0.9599	1.363
France	0.6843	0.2384	0.00271	0.011367	0.2187	0.6846	1.153
Mexico	0.6128	0.2054	0.002428	0.011821	0.2146	0.6133	1.014
Japan	0.4144	0.2265	0.002431	0.010733	-0.02844	0.4128	0.8635
Nigeria	0.4132	0.2382	0.002304	0.009673	-0.0628	0.4136	0.8743
Argentina	0.3962	0.2338	0.002345	0.01003	-0.06315	0.396	0.8616
Netherlands	0.3936	0.2595	0.002635	0.010154	-0.1041	0.3951	0.8978
Australia	0.3859	0.2448	0.002425	0.009906	-0.08624	0.3859	0.8772
Germany	0.372	0.2336	0.002273	0.00973	-0.08214	0.3716	0.8362
Spain	0.3698	0.2616	0.00273	0.010436	-0.1512	0.3693	0.8856
USA	0.3645	0.2106	0.002148	0.010199	-0.04502	0.3611	0.7808
Iran	0.3338	0.2616	0.002752	0.01052	-0.1746	0.3338	0.8438
Italy	0.33	0.2579	0.002442	0.009469	-0.175	0.3335	0.8311
Norway	0.279	0.2826	0.002763	0.009777	-0.2745	0.2787	0.8245
Egypt	0.2768	0.2481	0.002861	0.011532	-0.2012	0.2764	0.7673
Colombia	0.2562	0.224	0.002274	0.010152	-0.1831	0.2569	0.7007
England	0.2313	0.2612	0.00259	0.009916	-0.2813	0.2328	0.746
Croatia	0.2201	0.2544	0.002626	0.010322	-0.2809	0.22	0.7164
Cameroon	0.2137	0.2435	0.002744	0.011269	-0.2692	0.2131	0.6926
Portugal	0.2097	0.2588	0.002388	0.009227	-0.3004	0.2072	0.7261

Table 2: Results From General BT Model

Rank	Team - Model Ranking	Team - FIFA Ranking
1	Brazil	Brazil
2	France	France
3	Mexico	Argentina
4	Japan	Czech Republic
5	Nigeria	Spain
6	Argentina	Netherlands
7	Netherlands	Mexico
8	Australia	England
9	Germany	Portugal
10	Spain	Italy
11	USA	USA
12	Iran	Ireland
13	Italy	Sweden
14	Norway	Denmark
15	Egypt	Turkey
16	Colombia	Uruguay
17	England	Japan
18	Croatia	Greece
19	Cameroon	Germany
20	Portugal	Iran

Table 3: General BT Model Rankings Compared with FIFA

The top two teams are the same for both, however there are some teams in the model's top twenty teams that FIFA, and general knowledge say probably are not correct. Three teams in particular stand out: Australia, Norway, and Egypt. Australia, is ranked 57th by FIFA, a large differential from the 8th that they are ranked by the model. Norway and Egypt, ranked 14th and 15th by the model are ranked 35th and 34th respectively by FIFA. Thus, as would be expected from this model which has very simple assumptions, the rankings produced are not good.

These results are based on the assumption that the parameters are converging. Evidence of this was produced in the previous section in Table 1. In addition, a method for determining if convergence is failing is to compare the MCMC error to the standard error of the parameter. If the ratio of the MCMC error to the posterior standard error of the parameter exceeds 5%, failure of convergence is indicated (Conroy, et al., 2005). Note that this value is included in the Table 2 under the column labeled ratio (and will be included in the following sections as well). A trace of the parameter history and their posterior densities would only further evidence of convergence. A trace of the top two teams and their posterior densities are presented in Figures 1 and 2. This will be done in each section to further the evidence that each model is converging.

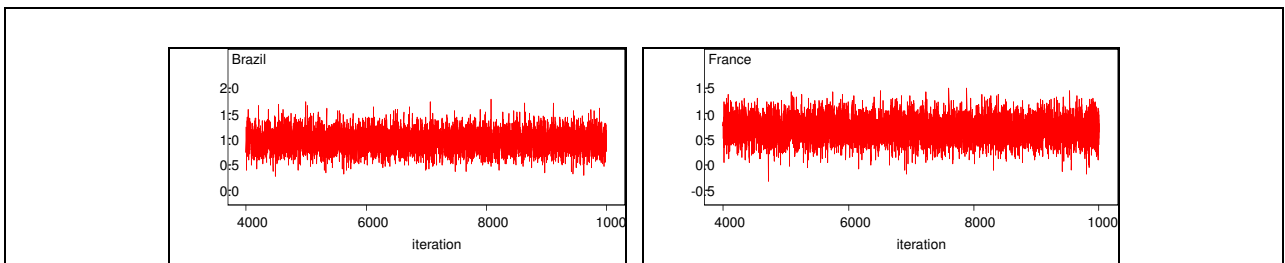


Figure 1: Trace of Parameters for Bradley - Terry Model

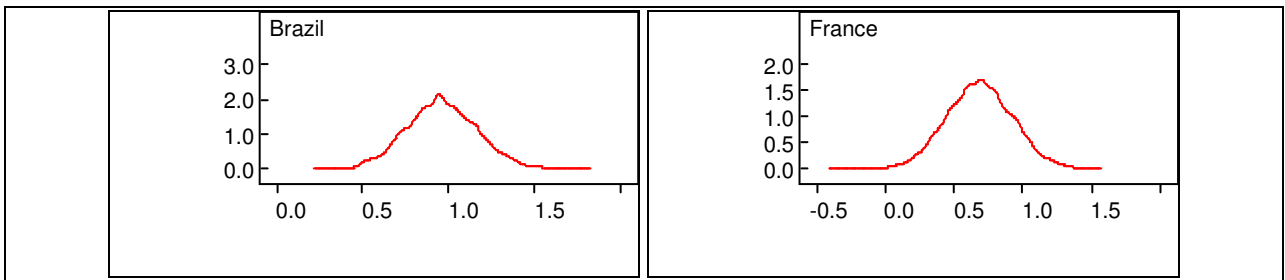


Figure 2: Smoothed Kernel Densities for Bradley - Terry Model ($s = .15$)

The trace of each parameter definitely appears to be stationary and the densities both clearly appear to follow a normal density centered at their respective mean (.9620 for Brazil and .6843 for France). Thus, there is enough evidence that the parameters in the model are converging, and the results can thus be accepted. However, as stated previously the results are not great because of the very simple model assumptions.

5.2: Extended Bradley – Terry Model with Ties and Home – Field Advantage

The extended Bradley – Terry model to include ties and home – field advantage should produce better results than the previous model because it now includes some of the aspects that are very important in soccer. Allowing for the possibility of ties is something that any soccer model must have if it is to be considered credible and home – field advantage seems as if it would be more important in soccer than it would be in other sports, simply because of the importance attached to soccer in most countries around the world.

The model is still not complete, as it does not address many important issues important to soccer. It still considers all games played at any time to be equally weighted, and it does not account for the fact that not all types of games are equally weighted as well. In addition, the manner in which the model addresses home – field advantage is very simple, in that it assumes

there is a home team in every game played, which is clearly not the case, with a majority of international games being played at neutral sites. With that said, the results should still show improvement over those from section 5.1.

The results for the top twenty teams with the tie and Home – Field Advantage Parameter from the model (4), with 4000 burn – in iterations and a sample size of 6000 are displayed in Table 4, and the rankings compared with those from FIFA are displayed in Table 5. The Monte Carlo errors are very small again and the standard errors are also fairly constant. Notice that the home parameter is negative, as it should be if the home team has an advantage (this was explained in section 1.3). It is also very large in magnitude, as it is larger than most of the team’s parameter estimates. This implies that home – field advantage is very important, and in most cases, more important than the teams playing. To see this, consider a game played by England and the USA. If the game is played in England,

$$\text{Pr(England Wins)} = .4272$$

$$\text{Pr(USA Wins)} = .2644$$

$$\text{Pr(Tie)} = .3083,$$

whereas if the game is played in the United States,

$$\text{Pr(England Wins)} = .2706$$

$$\text{Pr(USA Wins)} = .4202$$

$$\text{Pr(Tie)} = .3093.$$

So between these two teams, the probability of the road team winning is less than the probability of tie, and each team's probability of winning largely depends on where the game is played. Thus, under this model, home – field advantage is very influential in predicting the probabilities of a win, a loss, and a tie. The tie parameter is also negative, which implies that a tie is less likely to occur than a win or a loss. Its magnitude is relatively small however, so it is not that much less likely. Also, the standard errors for the tie and home – field advantage parameters are very small, meaning that narrow credible intervals are obtained for these parameters and more confidence can be had in these estimates than the team parameter estimates.

Team	mean	sd	MC error	Ratio	2.50%	median	97.50%
Brazil	0.7535	0.179	0.005115	0.028575	0.3995	0.7521	1.095
Mexico	0.6974	0.1793	0.005432	0.030296	0.3556	0.7002	1.05
France	0.6874	0.2239	0.006077	0.027142	0.2556	0.6746	1.117
Spain	0.6808	0.2359	0.007182	0.030445	0.217	0.682	1.138
Italy	0.6513	0.2269	0.007164	0.031573	0.2185	0.65	1.095
Germany	0.6134	0.2161	0.006022	0.027867	0.2211	0.6036	1.051
Argentina	0.581	0.2031	0.006347	0.031251	0.1678	0.5811	0.9721
Netherlands	0.5621	0.2383	0.006506	0.027302	0.08817	0.5739	1.024
Denmark	0.5485	0.2173	0.006488	0.029857	0.1287	0.5468	0.9672
Iran	0.5485	0.2257	0.006915	0.030638	0.1058	0.5504	0.9852
Nigeria	0.5467	0.2289	0.006699	0.029266	0.08978	0.5466	0.9944
England	0.5447	0.2306	0.006209	0.026925	0.07717	0.5574	0.9932
Japan	0.5337	0.2085	0.005411	0.025952	0.1333	0.5342	0.9729
Sweden	0.5288	0.2382	0.006197	0.026016	0.07951	0.5264	0.983
USA	0.5249	0.2062	0.006134	0.029748	0.1248	0.5245	0.9396
Croatia	0.4887	0.2334	0.007019	0.030073	0.04289	0.4823	0.9619
Czech Republic	0.4589	0.2344	0.005861	0.025004	0.01398	0.4662	0.9088
Costa Rica	0.4222	0.207	0.006388	0.03086	0.01121	0.4235	0.8268
Ireland	0.4095	0.2416	0.007304	0.030232	-0.07726	0.4106	0.8978
Australia	0.3958	0.2335	0.006442	0.027589	-0.1064	0.3985	0.846
home	-0.4599	0.04332	0.001242	0.02867	-0.5435	-0.46	-0.3766
tie	-0.08635	0.03939	0.001015	0.025768	-0.165	-0.08508	-0.01021

Table 4: Results for BT Model with Ties and HFA

Rank	Team - Model Ranking	Team - FIFA Ranking
1	Brazil	Brazil
2	Mexico	France
3	France	Argentina
4	Spain	Czech Republic
5	Italy	Spain
6	Germany	Netherlands
7	Argentina	Mexico
8	Netherlands	England
9	Denmark	Portugal
10	Iran	Italy
11	Nigeria	USA
12	England	Ireland
13	Japan	Sweden
14	Sweden	Denmark
14	USA	Turkey
16	Croatia	Uruguay
17	Czech Republic	Japan
18	Costa Rica	Greece
19	Ireland	Germany
20	Australia	Iran

Table 5: BT Model with Ties and HFA Rankings Compared with FIFA

The rankings are closer to FIFA's rankings than the rankings from section 5.1. Like the rankings from section 5.1, most of the teams at the top of the rankings are there for both ranking systems. However, the teams that differ do not differ as much under this model. Of the teams that are in the top twenty in the for the model that do not appear in FIFA's top twenty, Nigeria, Croatia, and Costa Rica are all in FIFA's top thirty teams, which is not a glaring difference. However there are still some large differences between the rankings. Australia is again in the top twenty, however they move down 13 spots under the new model. Since the home – field advantage parameter was added to the model, this probably implies that Australia won a lot of their games at home. Under the old model, these wins would only have added to their parameter strength, but under the new model each home win adds to the home – field advantage parameter as well. Uruguay and Greece, who are in FIFA's top twenty are ranked 41st and 46th respectively by the model. The model still contains some simple assumptions, so the rankings are not

expected to be great yet, however clear improvements can be seen by the addition of the ties and home – field advantage to the model.

As stated in the previous section, these results are under the assumption that the parameters converge. Again, evidence of convergence was found by burning 4000, 5000, and 6000 samples and noting that the parameter estimates and DIC did not change based on the number of burns. Also, the ratio of MCMC error to posterior standard error of parameters does not exceed 5% as seen in Table 4. For more evidence of convergence Figures 3 and 4 contain the posterior traces and density of Brazil and France, along with the posterior traces and densities of the tie and home – field advantage parameters.

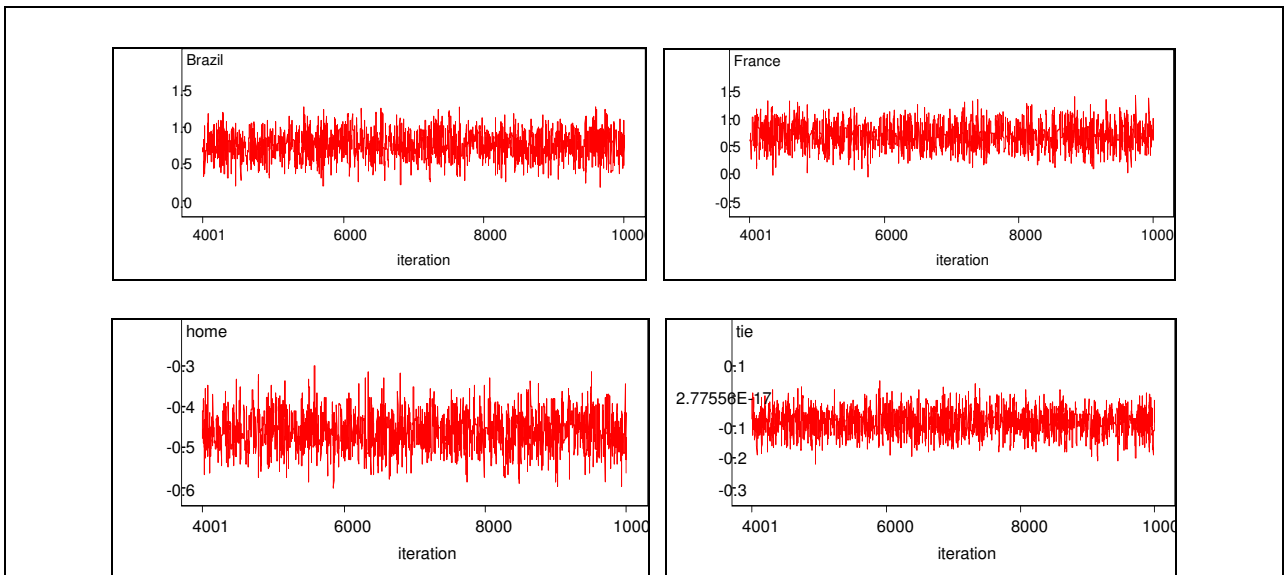


Figure 3: Trace of Parameters for BT with Ties and HFA

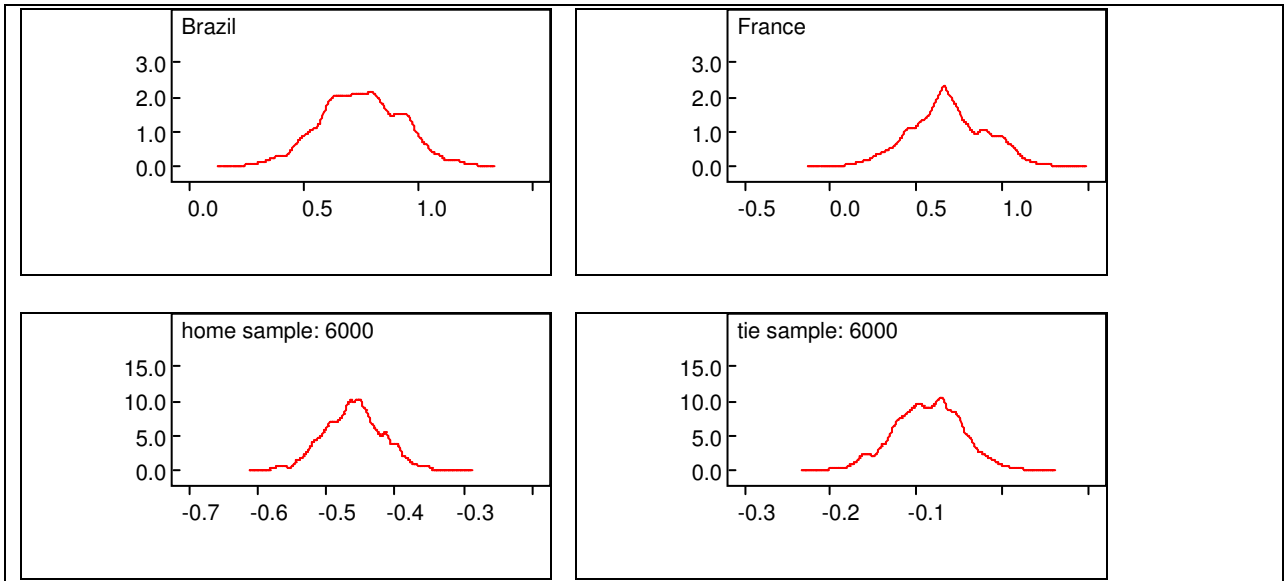


Figure 4: Smoothed Kernel Densities for BT Model with Ties and HFA ($s = .15$)

Again, like in section 5.1, the trace of each team's parameter definitely appears to be stationary (once they reach their posterior means the densities both clearly appear to follow a normal density centered at their respective mean (.7535 for Brazil and .6874 for France). In addition, the convergence the tie and home – field advantage parameters of needs to be investigated. Notice that the traces appear stationary for both parameters, and their densities come from their Normal posterior distributions, centered at their posterior means. Thus, it can be concluded that convergence is reached for all parameters in the model, and the results from this section are credible.

5.3: Extended Bradley – Terry Model with Ties, Home – Field Advantage, and Neutral Site Distinction

The extended Bradley – Terry model to include ties, home – field advantage, and the ability to distinguish between games played at a neutral site and games in which there is a home

team should produce better results than the previous model because of the large number of international games played at neutral sites. Many of the biggest games are played at a neutral site (World Cup games for example), and any model for soccer data should take this into account.

The model is still not finalized, as again it considers team parameter strength to be constant over time and all games of equal weight. However, the results should improve over the results from section 5.2, since the model is now able to distinguish between games at a home site and games at a neutral site. The results for the top twenty teams with the tie and Home – Field Advantage Parameter from the model (4), with 4000 burn – in iterations and a sample size of 6000 are displayed in Table 6, and the rankings compared with those from FIFA are displayed in Table 7.

Team	mean	sd	MC error	Ratio	2.50%	median	97.50%
Brazil	0.8778	0.1737	0.005193	0.029896	0.5308	0.8787	1.219
France	0.7085	0.2239	0.006581	0.029393	0.254	0.7033	1.144
Mexico	0.7057	0.1774	0.005499	0.030998	0.3524	0.714	1.064
Spain	0.6985	0.2395	0.007175	0.029958	0.2408	0.6977	1.185
Italy	0.6905	0.2245	0.006762	0.03012	0.2588	0.6915	1.131
Argentina	0.6644	0.2021	0.00646	0.031964	0.2649	0.6684	1.043
Germany	0.6551	0.2214	0.007095	0.032046	0.233	0.6528	1.101
Netherlands	0.6176	0.2372	0.006476	0.027302	0.1834	0.621	1.076
Denmark	0.5657	0.2288	0.006831	0.029856	0.1179	0.5685	1.006
Nigeria	0.5619	0.227	0.006511	0.028683	0.1289	0.5615	1.001
England	0.5435	0.2337	0.006387	0.02733	0.08671	0.5402	1.001
Japan	0.5427	0.2164	0.006283	0.029034	0.1276	0.5418	0.9689
Sweden	0.5321	0.2381	0.005891	0.024742	0.058	0.5408	1.01
USA	0.5287	0.2101	0.006333	0.030143	0.1318	0.5347	0.9246
Iran	0.5231	0.227	0.006707	0.029546	0.07009	0.5215	0.9648
Croatia	0.4689	0.2322	0.005553	0.023915	0.03458	0.4627	0.9364
Czech Republic	0.4512	0.2416	0.00686	0.028394	-0.01174	0.4453	0.9095
Costa Rica	0.4446	0.2066	0.006786	0.032846	0.04244	0.4446	0.8407
Australia	0.4243	0.2351	0.006021	0.02561	-0.05332	0.4358	0.8642
Ireland	0.4052	0.2405	0.007745	0.032204	-0.06381	0.4042	0.8846
home	-0.5865	0.05519	0.001272	0.023048	-0.6977	-0.5875	-0.4764
tie	-0.08614	0.03924	0.001078	0.027472	-0.1674	-0.08405	-0.01327

Table 6: Results for Extended BT Model with Ties, HFA, and Neutral Site

Rank	Team - Model Ranking	Team - FIFA Ranking
1	Brazil	Brazil
2	France	France
3	Mexico	Argentina
4	Spain	Czech Republic
5	Italy	Spain
6	Argentina	Netherlands
7	Germany	Mexico
8	Netherlands	England
9	Denmark	Portugal
10	Nigeria	Italy
11	England	USA
12	Japan	Ireland
13	Sweden	Sweden
14	USA	Denmark
14	Iran	Turkey
16	Croatia	Uruguay
17	Czech Republic	Japan
18	Costa Rica	Greece
19	Australia	Germany
20	Ireland	Iran

Table 7: BT Model with Ties, HFA, and Neutral Site Rankings Compared with FIFA

Notice that the rankings and results barely changed from the previous section. This makes sense because the only thing that was added to the model was the model's ability to differentiate between games played at one team's home site and games played at a neutral site. The only team that made a jump of more than a couple spots in the rankings was Iran, who moved from 10th in the previous model to 14th in the current model. This makes sense because in the 15 neutral site games that they played, they were considered the "road" team 10 times, and went 7 – 3 in those games. They were considered the "home" team 5 times, and went 3 -2 in those games. Thus, because they did so well in games in which they were considered the "road" team, their parameter strength would be increased. However, under the new model, they are not considered the "away" team in these games, and their parameter strength will reflect that.

The tie parameter remains the same from the previous model, however the one parameter that would be expected to change, the home – field parameter does change. It moves from a mean of -0.4599 to a mean of -0.5865 (again, larger negative magnitudes imply more home – field advantage). So, by adding the neutral site distinction to the model, the home – field advantage becomes larger. As Figure 5 shows, this is also expected. In general, a team will win more games at home than they will at neutral sites, so if all games are considered home (even if they are really neutral site games), then the winning percentage for home teams will not be as large than it would be if only true home games were considered. Thus, the results from this section follow as expected from the previous section.

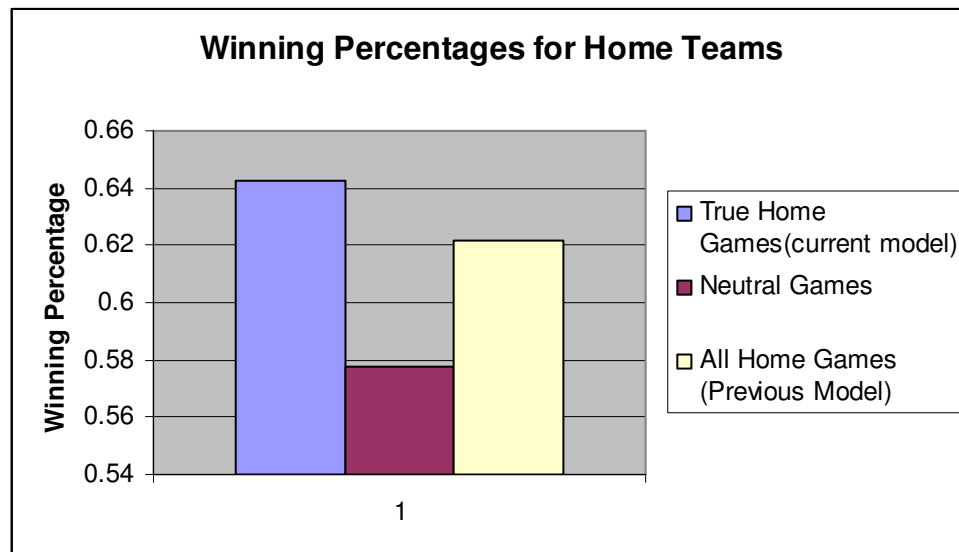


Figure 5: Winning Percentages for Home Teams Under Different Model Assumptions

Again, all the results from this section are based on the assumption that the parameters converge. As seen in Table 6, the ratio of MCMC error to posterior standard error is less than 5% for all the parameters. Further evidence of their convergence can be seen in Figures 6 and 7, which contain the history and densities of two team parameters and the tie and home – field advantage parameters after the 4000 burned iterations. As in the previous sections, the histories

of each parameter appear stationary and the densities all appear to come from a normal distribution, as they are supposed to. Thus, the results from this section are credible.

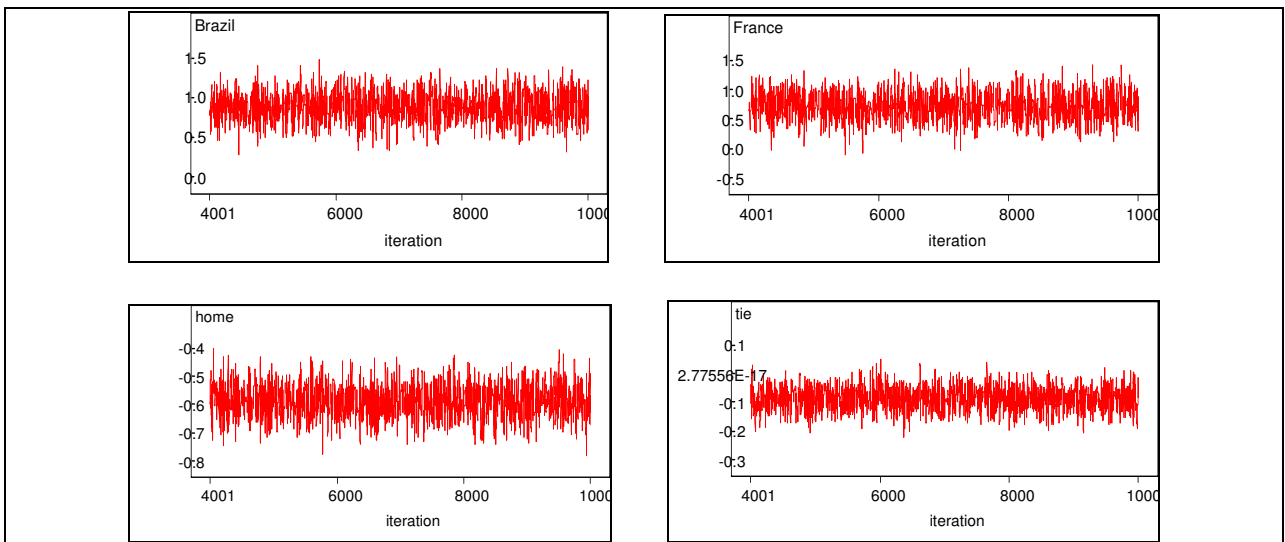


Figure 6: Trace of Parameters for BT with Ties, HFA, and Neutral Site

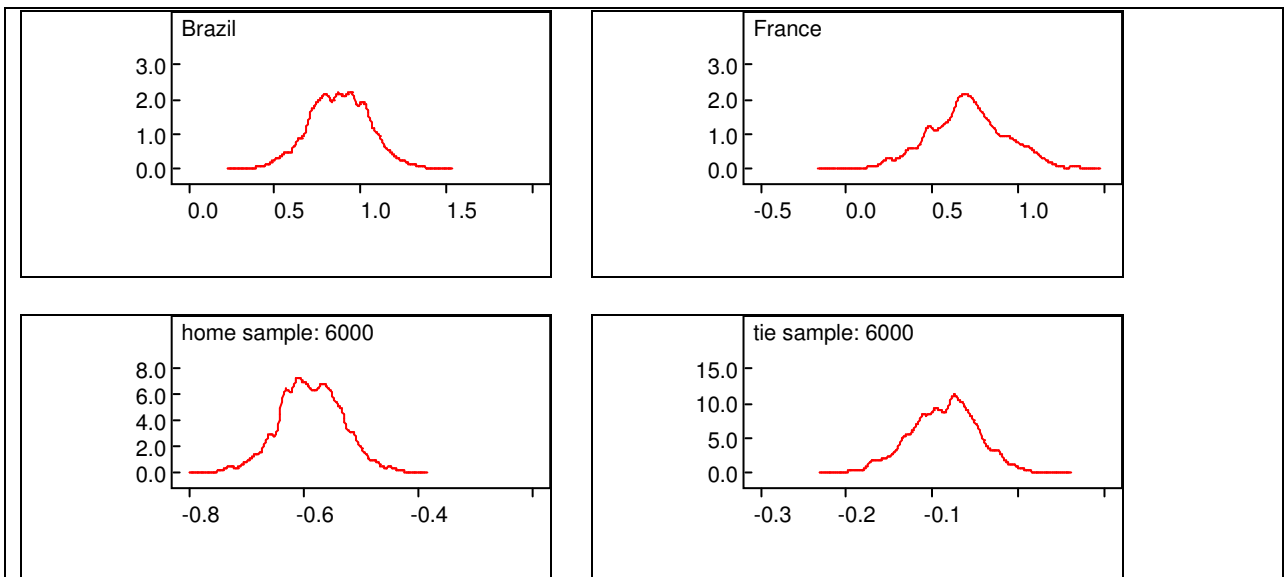


Figure 7: Smoothed Kernel Densities for BT Model with Ties, HFA, and Neutral Site ($s = .15$)

5.4: Extended Bradley – Terry Model with Ties and Home – Field Advantage, Neutral Site Distinction, and Game Type

The results from the previous section showed improvement over the other models, but still lacked one key component that is vital to soccer. The model in the previous section assumed that all games were of equal importance, which is clearly not true in the world of professional soccer. Often times, when a team plays a friendly match it does not send its best team, instead it sends younger or less experienced players to enable them to gain international experience. By introducing a parameter into the model to distinguish between friendly matches and competitive matches, the model should produce better results. Recall from section 1.3 that the model uses a team's full parameter for competitive games, and some multiple of the parameter determined by the data for friendly games. This will allow the model to give the appropriate weight to friendly matches and produce a more credible weighting system.

As stated in the previous sections, the model is still not complete because it still assumes team parameter strength remains constant over time, however the results should still show improvement over previous models. The results for the top twenty teams with the tie, Home – Field Advantage Parameter, and Friendly match parameter from the model (5), with 4000 burn – in iterations and a sample size of 6000 are displayed in Table 8, and the rankings compared with those from FIFA are displayed in Table 9.

Team	mean	sd	MC error	Ratio	2.50%	median	97.50%
Brazil	1.089	0.1999	0.006202	0.031026	0.6789	1.09	1.483
France	0.8148	0.2441	0.007059	0.028918	0.3465	0.8089	1.296
Mexico	0.8144	0.194	0.006292	0.032433	0.4426	0.8118	1.198
Argentina	0.7352	0.2178	0.005623	0.025817	0.3227	0.7414	1.171
Italy	0.6861	0.24	0.006852	0.02855	0.2124	0.6909	1.164
Spain	0.649	0.2426	0.00734	0.030256	0.164	0.651	1.126
Netherlands	0.6322	0.2536	0.00673	0.026538	0.1239	0.6309	1.137
Germany	0.6086	0.2328	0.005635	0.024205	0.1596	0.6053	1.065
Japan	0.6061	0.2346	0.007058	0.030085	0.1285	0.6108	1.062
Australia	0.6012	0.2497	0.00708	0.028354	0.1099	0.6062	1.113
England	0.5496	0.2498	0.006883	0.027554	0.07462	0.554	1.029
Nigeria	0.5333	0.2225	0.006333	0.028463	0.09634	0.5345	0.9587
USA	0.5282	0.2004	0.006123	0.030554	0.1322	0.5228	0.9041
Iran	0.511	0.233	0.005962	0.025588	0.06543	0.5117	0.9735
Denmark	0.5109	0.2421	0.007154	0.02955	0.03662	0.5134	0.9882
Sweden	0.5058	0.245	0.007748	0.031624	0.01239	0.4966	0.9954
Croatia	0.474	0.2455	0.007788	0.031723	-0.00196	0.4697	0.942
Costa Rica	0.4654	0.2137	0.005693	0.02664	0.05305	0.4684	0.8862
Czech Republic	0.4652	0.2482	0.008242	0.033207	-0.02944	0.4576	0.9682
Portugal	0.4651	0.2531	0.006133	0.024232	-0.04178	0.4652	0.9451
friendly	0.2143	0.1534	0.003928	0.025606	0.01069	0.187	0.5737
home	-0.6073	0.05392	0.001444	0.02678	-0.7149	-0.6058	-0.5024
tie	-0.08822	0.03899	0.001096	0.02811	-0.1636	-0.08768	-0.0137

Table 8: Results for Extended BT Model with Ties, HFA, Neutral Site, and Game Type

Rank	Team - FIFA Ranking	Team - Model Ranking
1	Brazil	Brazil
2	France	France
3	Argentina	Mexico
4	Czech Republic	Argentina
5	Spain	Italy
6	Netherlands	Spain
7	Mexico	Netherlands
8	England	Germany
9	Portugal	Japan
10	Italy	Australia
11	USA	England
12	Ireland	Nigeria
13	Sweden	USA
14	Denmark	Iran
14	Turkey	Denmark
16	Uruguay	Sweden
17	Japan	Croatia
18	Greece	Costa Rica
19	Germany	Czech Republic
20	Iran	Portugal

Table 9: BT Model with Ties, HFA, Neutral Site, and Game Type Rankings Compared with FIFA

Both the rankings and results have barely changed from the previous section's results, as the top nineteen teams are the same for both sections and Portugal, ranked 21st in the previous section's model replaced Ireland, ranked 24th in this model, as the 20th ranked team. The home – field advantage parameter and the tie parameter estimates also remain very similar to their estimates in the previous section. It is interesting that the results remain very similar because the model does make a large distinction between competitive matches and friendly matches. The parameter for friendly matches has a mean of .2143, and its 95% credible interval of (0.003928, 0.5737) is still showing that friendly matches are given far less weight than competitive ones. Consider the mean of the parameter estimate. It implies that friendly matches are worth only about one – fifth of what competitive matches are worth for estimating team parameter strength. This is a large difference that would be expected to change the results more significantly. This leads to the question of whether the model is actually worth using if it is more complicated but does not improve the results.

One possible way to determine this would be to look at the squared error. The only problem is how to define a squared error for these type of results. One measure of squared error

that could be useful would be $MSE = \frac{\sum_{i=1}^{158} (ModelRanking_i - FifaRanking_i)^2}{N}$, where N is the

number of teams included in the model. This measure will show which model gives results that match up closet to FIFA's results. While the goal of the project was not to get ranking that were similar to those of FIFA (in fact it was the opposite; to obtain a better ranking system than FIFA's), FIFA's rankings are considered very credible by most people. Thus, although an improved ranking system is trying to be created, one that differed greatly from FIFA's rankings

may be considered good statistically but would not be considered credible in the real world.

Thus, the MSE defined above is an accurate measure of how well the model is performing.

Table 10 gives the MSE for each of the models considered so far.

Model (Section Found In)	MSE
General BT Model (5.1)	2,227
BT Model with Ties and HFA (5.2)	783
BT Model with Ties, HFA, and Neutral Distinction (5.3)	760
BT Model with Ties, HFA, Neutral Distinction, and Game Type (5.4)	810

Table 10: MSE for Each Model Considered

The three models considered with ties all perform similarly with the model from section 5.3 performing slightly better than the other two. However, because of the extra information that the model considered in this section reveals about game type, it is worth using. Also, since the goal of the project was not to find a model that would compare favorably with FIFA's rankings, then one model would have to outperform another one significantly with regards to the MSE to be considered a superior model. For example, all three models that consider ties greatly outperform the general Bradley – Terry model and can be considered superior. As stated before, these are not the final models, they are only building blocks in the construction of the final model.

The results from this section are based on the assumption that the parameters converge. Evidence of their convergence can be seen in Figures 8 and 9, which contain the history and densities of two team parameters and the tie, home – field advantage, and friendly parameters after the 4000 burned iterations. As in the previous sections, the histories of each parameter appear stationary (for the friendly parameter, remember that it comes from a beta distribution so it must be greater than zero). The densities all appear to come from a normal distribution for the team parameters and the tie and home – field advantage parameters as they should. The Friendly parameter appears to come from a beta distribution, also as it should. Thus, there is enough

evidence that the parameters converge and the results presented in this section can be viewed as credible.

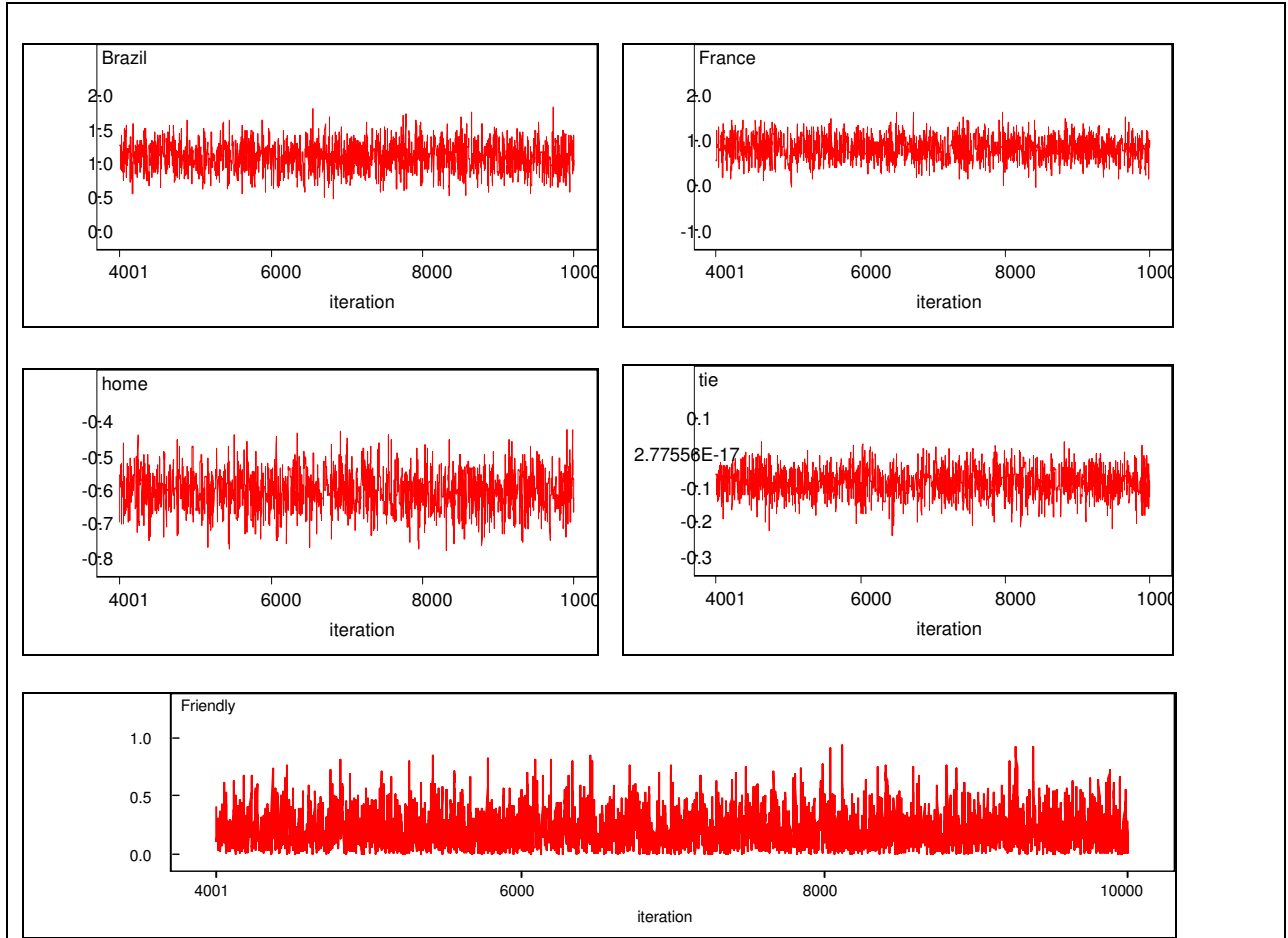


Figure 8: Trace of Parameters for BT with Ties, HFA, Neutral Site, and Game Type

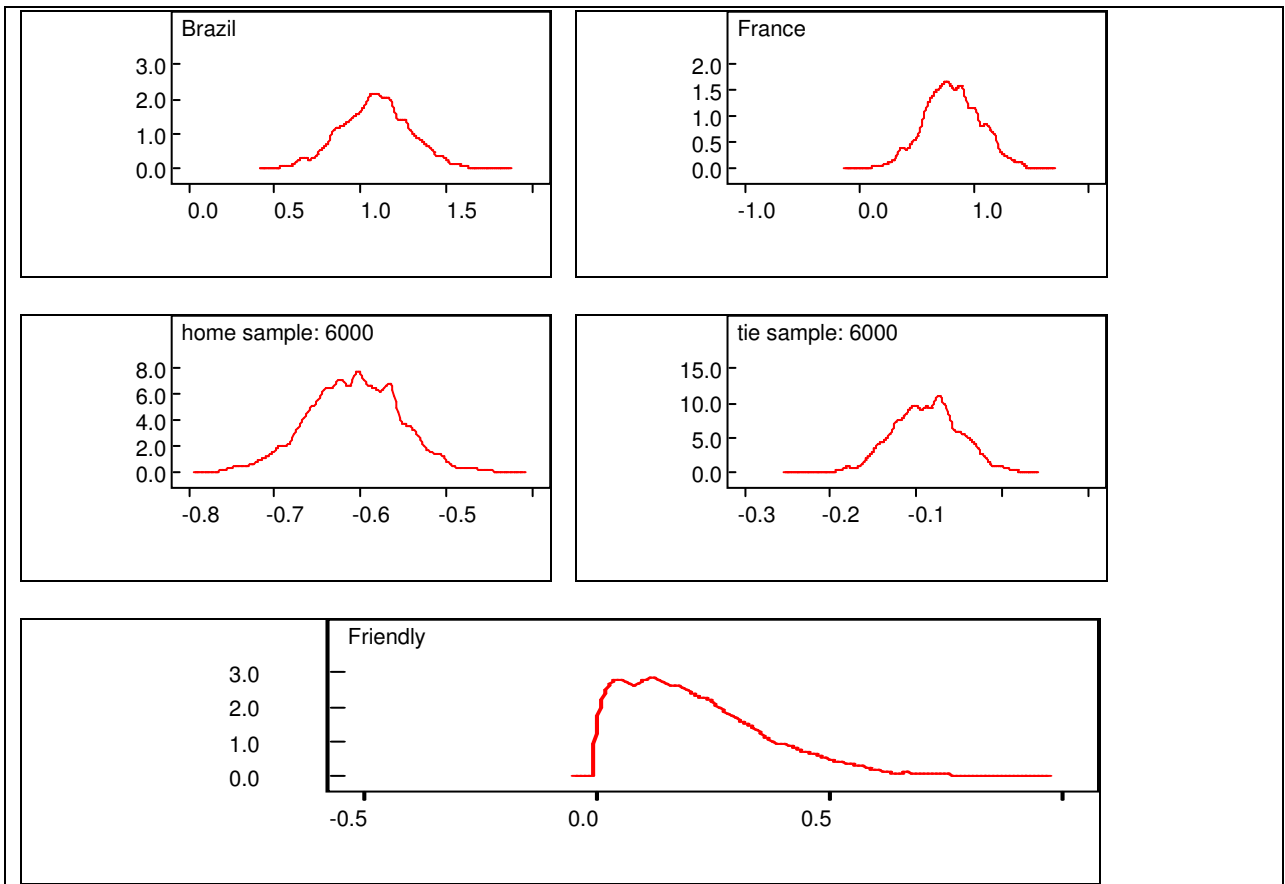


Figure 9: Smoothed Kernel Densities for BT with Ties, HFA, Neutral Site, and Game Type (s = .15)

5.5: Dynamic Bradley – Terry Model

Up until this point, each section has incorporated more information about the games and the teams involved than the previous section. However, the final step in the model does not add any new information about the games themselves. The last step drops the assumption that each team's parameter strength is constant. The final model, found in Chapter 3 drops this assumption and assumes that each team's strength parameter changes over time. The model in Chapter 3 does not specify the number of time periods. The limitations of Winbugs played a major role in determining the number of time periods to use. When the dimensions k (wins, loss,

or tie), h (home or neutral), t (time period 1, 2, ..., T), and g (competitive game or friendly) in

y_{ijkhtg} exceeded 36 Winbugs would crash while attempting to compile. Thus, there were

limitations on the number of time periods to be used. Since k , h , and g all were fixed ($k = 3$, $h = 2$, and $g = 2$), then the only dimension that could vary was t . Considering t should be as large as possible, and

$$k \cdot h \cdot t \cdot g = 3 \cdot 2 \cdot t \cdot 2 = 12 \cdot t \leq 36,$$

thus $t \leq 3$, so $T = 3$ was chosen. Therefore $t = 1$ corresponds to all games played up to and including 2002, $t = 2$ includes all games played in 2003, and $t = 3$ includes all games played in 2004. When the model was ran using the gamma prior (regardless of the choices for a_0 and b_0 , Winbugs ran crashes while the burn – in iterations were taking place. Thus, instead of placing a prior on ω , a noninformative Uniform distribution was placed on σ . Using a $U(0,100)$ prior distribution, σ did not converge. Figure 10 shows the history of a sample of size 6000 after 4000 burn – in iterations.

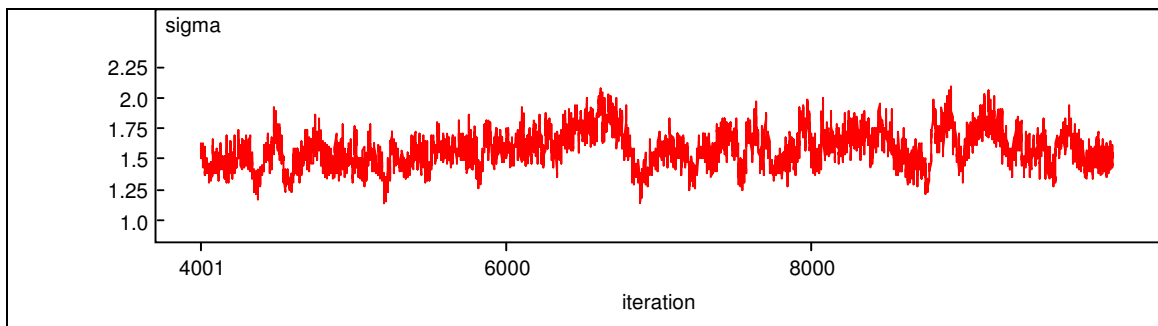


Figure 10: Convergence of Sigma with a $U(0,100)$ Prior Distribution

It is clear that convergence is not being reached in Figure 10, as the plot is clearly not stationary. Next, an attempt was made to make the prior distribution slightly informative, and a $U(0,10)$ prior distribution was placed on σ . However, σ did not converge again after 4000 burn – in iterations. In order to be certain that σ would not converge if more samples were burned, 85,000 samples were burned. Figure 11 shows the history of σ after both 4,000 and 85,000 burn – in iterations. In each case, it is easy to see that σ is not converging. In addition, the ratio of MCMC error to posterior standard error exceeded 5% for many of the parameters, thus indicating failure of convergence.

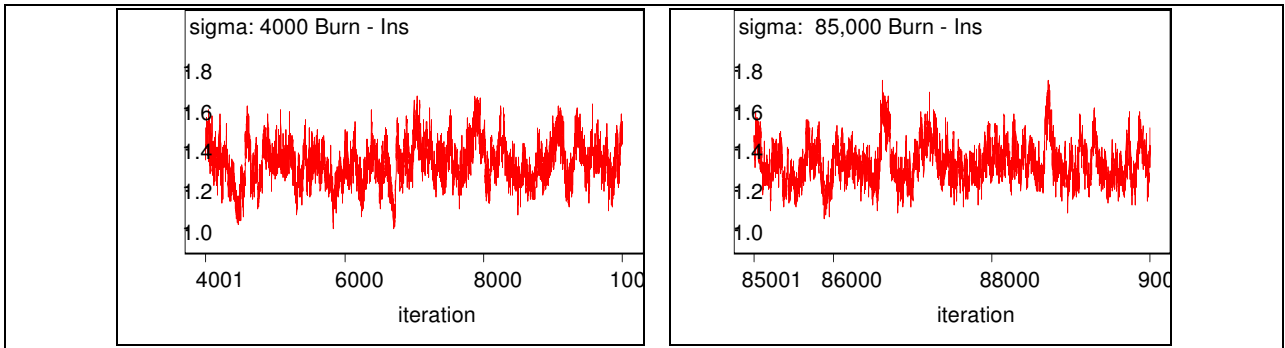


Figure 11: Convergence of Sigma with a U(0,10) Prior Distribution for 4,000 and 85,000 Burn – Ins

Since the parameters for this model did not converge, an attempt was made to run the model with σ^2 being held constant. However, convergence failed for all values assigned to σ^2 . Due to the manner in which the data was set up, there were a lot more games in the first time period (all games up to and in 2002) than in either of the last two time periods. This may have been the cause of failed convergence. By removing the friendly match parameter from the model, the number of time periods could be doubled. Thus, the game type parameter was removed from the model which allowed 6 time periods to be considered (1999 and before, 2000, 2001, 2002, 2003, and 2004).

A small variance, $\sigma^2 = .1$ was assigned to the noise in the AR(1) model described in section 1.4. The results of 6000 samples after 4000 burn – in iterations can be seen in Tables 11 and 12.

Team	mean	sd	MC error	Ratio	2.50%	median	97.50%
Mexico	2.472	0.5546	0.01333	0.024035	1.412	2.467	3.561
Argentina	2.312	0.5566	0.01374	0.024686	1.206	2.309	3.405
Brazil	2.276	0.498	0.01209	0.024277	1.304	2.276	3.251
England	2.132	0.5846	0.01356	0.023195	1.006	2.112	3.32
France	2.08	0.5577	0.01153	0.020674	0.9793	2.074	3.159
Spain	2.072	0.5839	0.01223	0.020945	0.9292	2.06	3.229
Japan	2.068	0.5247	0.01128	0.021498	1.032	2.069	3.103
Iran	1.99	0.5755	0.01275	0.022155	0.8678	1.991	3.135
Czech Republic	1.983	0.5718	0.0134	0.023435	0.862	1.982	3.117
Italy	1.872	0.5787	0.01489	0.02573	0.7452	1.873	3.048
Nigeria	1.838	0.5627	0.01106	0.019655	0.7805	1.823	2.955
USA	1.814	0.5215	0.01182	0.022665	0.8271	1.807	2.869
Sweden	1.734	0.5628	0.01301	0.023117	0.6592	1.736	2.85
Ireland	1.716	0.5872	0.01469	0.025017	0.5282	1.724	2.866
Netherlands	1.713	0.609	0.01246	0.02046	0.5349	1.708	2.935
Greece	1.681	0.5619	0.01251	0.022264	0.5978	1.683	2.778
Germany	1.675	0.5981	0.01338	0.022371	0.5006	1.686	2.851
Denmark	1.585	0.5487	0.01146	0.020886	0.5503	1.575	2.68
Croatia	1.531	0.5785	0.01244	0.021504	0.4188	1.525	2.695
Turkey	1.413	0.5513	0.01159	0.021023	0.3223	1.405	2.501
home	-0.676	0.06253	0.001638	0.026195	-0.7986	-0.6781	-0.555
tie	0.04315	0.04105	0.001253	0.030524	-0.03286	0.04081	0.1263

Table 11: Results for Dynamic BT Model with Ties, HFA, and Neutral Site

Rank	Team - FIFA Ranking	Team - Model Ranking
1	Brazil	Mexico
2	France	Argentina
3	Argentina	Brazil
4	Czech Republic	England
5	Spain	France
6	Netherlands	Spain
7	Mexico	Japan
8	England	Iran
9	Portugal	Czech Republic
10	Italy	Italy
11	USA	Nigeria
12	Ireland	USA
13	Sweden	Sweden
14	Denmark	Ireland
14	Turkey	Netherlands
16	Uruguay	Greece
17	Japan	Germany
18	Greece	Denmark
19	Germany	Croatia
20	Iran	Turkey

Table 12: Dynamic BT Model with Ties, HFA, and Neutral Site Rankings Compared with FIFA

The results from this model appear to be much more consistent with what would be expected than the results from previous sections. The two teams that appear in the top twenty for the model that do not appear in FIFA's top twenty, Nigeria and Croatia, rank 21st and 23rd respectively in FIFA's rankings. Australia, the team that had inexplicably been in the model's top twenty for the previous models is now ranked 31st. In addition, the MSE, as defined in the previous section, is 541, a large improvement over all of the previous models. Thus, the model performs well despite missing the game type parameter. It is also interesting to note that this model is the first in which Brazil did not rank first, as Mexico held that honor.

The ratio of MCMC error to standard error is much less than 5% as seen in Table 11, and Figures 12 and 13 show the history and density of Brazil and France in the last time period, the home – field advantage parameter, and the tie parameter. In this instance, the histories are displayed starting with the first iteration, to show the parameters path of convergence.

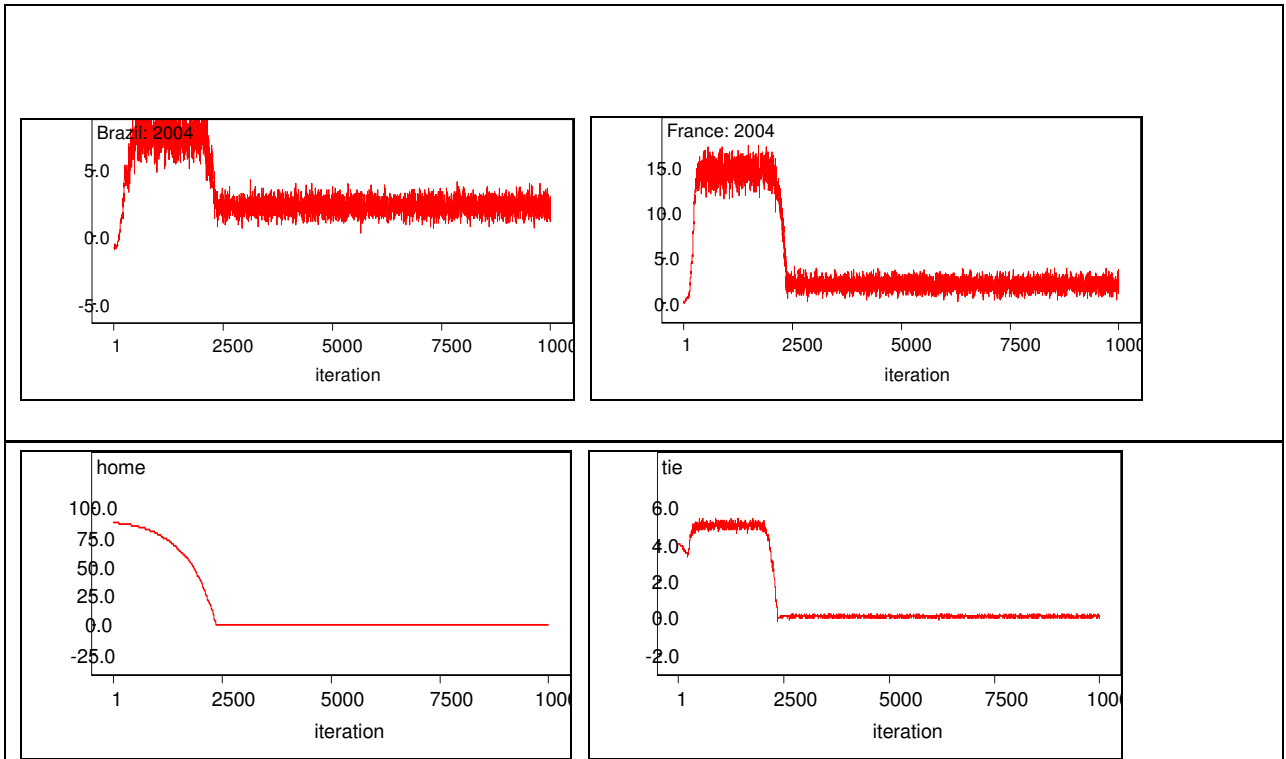


Figure 12: Trace of Parameters for Dynamic BT with Ties, HFA, and Neutral Site

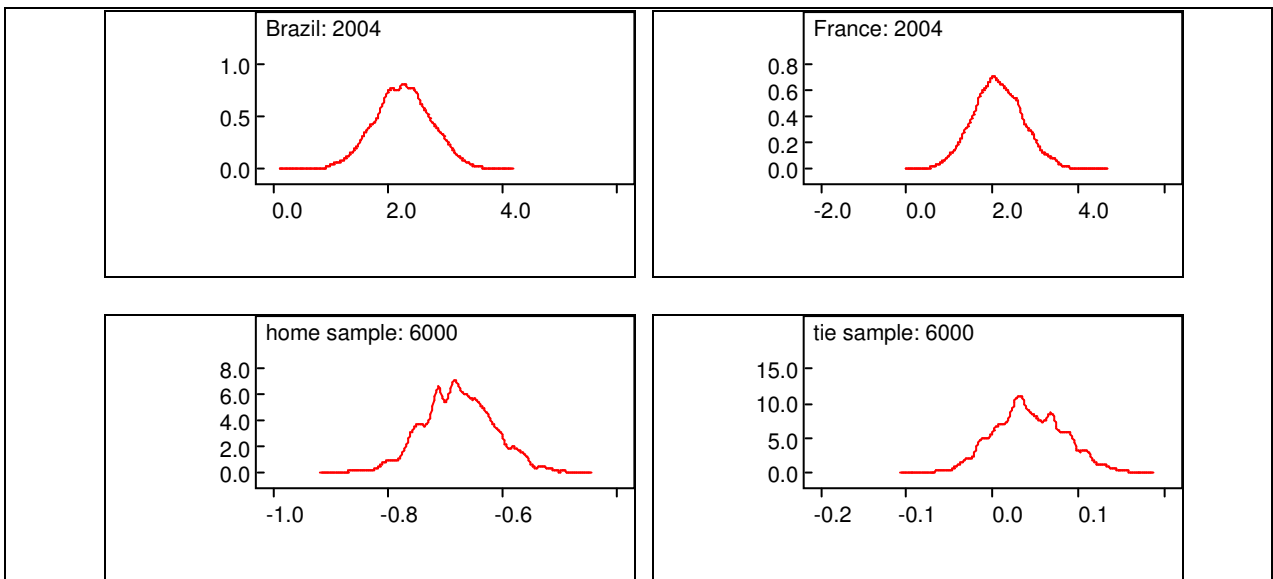


Figure 13: Smoothed Kernel Densities for Dynamic BT Model with Ties, HFA, and Neutral Site ($s = .15$)

Convergence is clearly met in the case of Brazil and France, however is difficult to see in the home – field and tie parameters. Figure 14 shows home – field and tie parameters zoomed in to show that they are clearly converging.

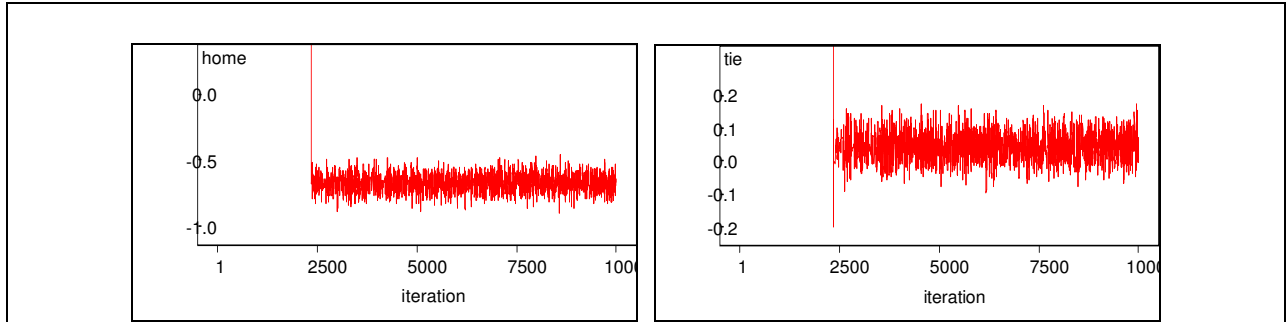


Figure 14: Zoom of Trace for Home - Field and Tie Parameters from Figure 12

Since the model is now dynamic, the team strengths change over time. This can be seen in Figure 12, which displays the team's mean strength parameter as it changes over time. Figure 12 displays the team strength of Brazil and France (who have been discussed throughout the paper), Mexico (ranked first), Australia (also discussed throughout the paper), and Luxembourg (ranked last by every model considered). It is interesting to see how the time dynamic works. Brazil appears to be very good throughout time, without much improvement however. Mexico on the other hand, starts off good, but gets dramatically better, particularly between the years 2001 and 2004 eventually surpassing Brazil in 2003. France's dynamic is also interesting, as they start out strong, then get worse in 2000, before vast improvement between 2000 and 2002, where they level off.

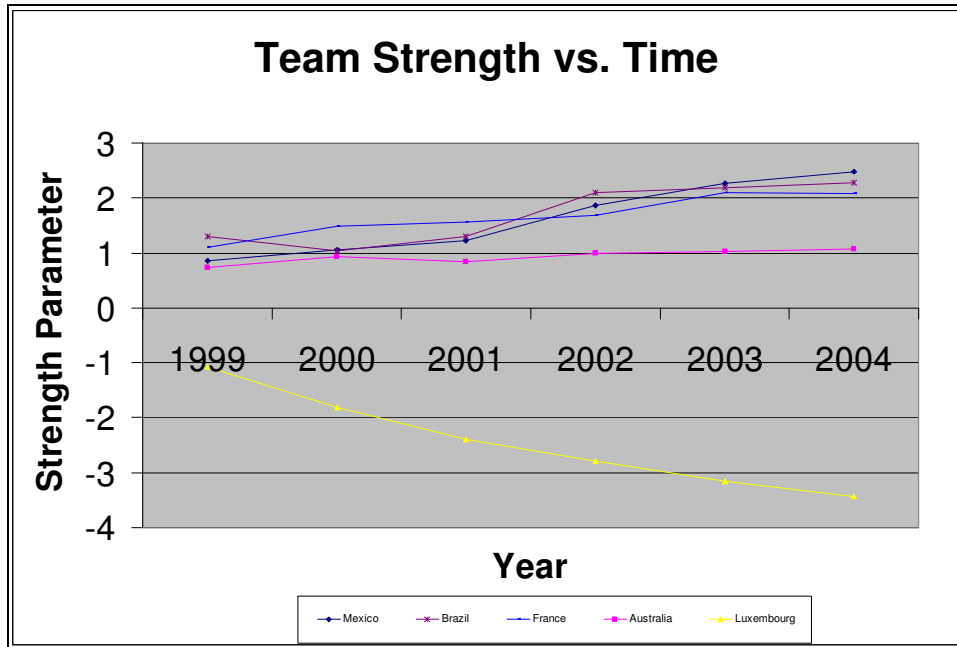


Figure 15: Team Strength vs. Time

Chapter 5: Conclusions and Future Work

Although the final model was not able to be achieved due to convergence failure and limitations of Winbugs, the paper produces two models that give substantial information about the world of soccer. Each of the models from the final two sections produces valuable information. The full results for both models can be found in Appendix C, and the parameter histories and densities can be found in Appendix D. Since the home field advantage and tie parameters do not differ greatly in each model, the same inference can be made on them from both models. Home field plays a major role in the outcome of games. According to both models, for two teams very close in strength, home – field can swing the probability of one team beating another by about 15%. Also, in both models, the tie parameter is negative and small in magnitude, implying that a tie is about slightly less likely to occur than a win or a loss for all things considered equal (similar team strength, neutral site). In addition to the inferences provided by both models, each model provides valuable information on its own.

The model from section 5.4, which includes game type as a parameter but is not dynamic, provides information on how much a friendly match contributes to a team's strength compared with a competitive match. According to the model, a friendly is worth about 21% of what a competitive match is worth when estimating team strength. This is a significant difference from the 50% weight that it carries in FIFA's ranking system when compared with World Cup matches. For other matches, it is worth more than 50%.

The model from section 5.5, which is dynamic but does not include game type as a parameter shows how team strength can change over time. The method allows the parameter strength to change over time in a non – arbitrary manner, different from FIFA's method of weighting games from different years arbitrarily. Because of the time variability in team

strength Mexico and France both surpass Brazil by 2004, Brazil is still currently ranked first in the world by FIFA.

Due to limitations, a final model that incorporated both the game type parameter and the dynamic nature of the team strength parameters could not be created. If more dimensions could be added, then a model that includes more game types (include all 6 game types as parameters, not just friendly and competitive matches) and more time periods (which could lead to convergence) could be tried. If this could be achieved, then rankings superior to FIFA's, both statistically and in the real – world, could be created. If a statistical model would not be accepted in the real world, the work done in this paper can still contribute to the FIFA rankings. Clearly, according to the results produced by all the models, FIFA's small bonus of 3 points awarded to the away team is not enough, as home – field advantage plays a much more prominent role in the outcome of games. In addition, the weight assigned to friendly matches appears to be too high based on the results from the model in section 5.4. Thus, extensions of this work could produce rankings superior to FIFA's. If this type of ranking procedure is not accepted by the real world, this work could still have an impact. FIFA could use this statistical model, or a similar one to determine the arbitrary weights that they assign throughout their ranking system.

References

- Bradley, Ralph Allen and Terry, Milton E. (1952), “Rank Analysis of Incomplete Block Designs I: The Method of Paired Comparisons”, *Biometrika*, Vol. 39, No. 3/4 324-345.
- Conroy, Michael J., Fonnesebeck, Christopher, and Zimpfer, Nathan. (2005), “Modeling Regional Waterfowl Harvest Rates Using Markov Chain Monte Carlo”, *Journal of Wildlife Management* 69(1): 77-90.
- Davidson, Roger R. (1970), “On Extending the Bradley – Terry Model to Accommodate Ties in Paired Comparison Experiments”, *Journal of the American Statistical Association*, Vol. 65, No. 329 317-328.
- Fahrmeir, Ludwig, Tutz, Gerhard. (1994), “Dynamic Stochastic Models for Time – Dependent Ordered Paired Comparison Systems”, *Journal of the American Statistical Association*, Vol. 89, No. 428 1438-1449.
- Gelman, Andrew, Carlin, John B., Stern, Hal S., and Rubin, Donald B. (2004), *Bayesian Data Analysis*, Second Edition, Chapman and Hall/CRC.
- Glickman, Mark E. (1993), “Paired Comparison Models with Time Varying Parameters”, *Doctoral Thesis, Harvard University Department of Statistics*.
- Glickman, Mark E. (1999), “Parameter Estimation in Large Dynamic Paired Comparison Experiments”, *Applied Statistics*, Vol 48, No. 3 377-394.
- Gumbel, E.J. (1961), “Sommes et differences de valeurs extremes independantes”, *C.R. Acad. Sci. Paris*, 253, 2838-2839.
- Knorr-Held, Leonhard. (1999), “Dynamic Rating of Sports Teams”, *Institute fur Statistik, Ludwig – Maximilians – Universitat Munchen*.
- Kobayashi, Kei, Kawasaki, Hideki, and Takemura, Akimichi. (2003), “Parallel Matching for Ranking all Teams in a Tournament”, *Department of Mathematical Informatics, Graduate School of Information Science and Technology, University of Tokyo*.
- Leonard, T. (1977), “An Alternative Bayesian Approach to the Bradley – Terry Model for Paired Comparisons”, *Biometrics*, 33, 121-132.
- Wilkinson, John W. (1957), “An Analysis of Paired Comparison Designs with Incomplete Repetitions”, *Biometrika*, Vol. 44, No. 1/2 97-113.
- Shumway, Robert H. and Soffer, David S. (200), *Time Series Analysis and Its Applications*, Springer – Verlag, New York, Inc.

Appendices

Appendix A - FIFA Rankings as of December, 2004

Rank	Team	Points
1	Brazil	843
2	France	792
3	Argentina	785
4	Czech Republic	777
5	Spain	765
6	Netherlands	758
7	Mexico	753
8	England	752
9	Portugal	747
10	Italy	738
11	USA	726
12	Ireland	716
13	Sweden	715
14	Denmark	711
14	Turkey	711
16	Uruguay	708
17	Japan	707
18	Greece	706
19	Germany	705
20	Iran	697
21	Nigeria	690
22	Korea Rep.	688
23	Cameroon	677
23	Croatia	677
25	Poland	672
26	Colombia	669
27	Costa Rica	668
28	Saudi Arabia	665
29	Romania	664
30	Paraguay	661
31	Senegal	657
32	Russia	652
33	Morocco	646
34	Egypt	644
35	Norway	633
35	Tunisia	633
37	Bulgaria	623
38	South Africa	619
39	Ecuador	616
40	Côte d'Ivoire	613
41	Jordan	611
42	Slovenia	608
43	Finland	607
44	Iraq	603
45	Belgium	600
46	Serbia and Montenegro	599
47	Uzbekistan	598
48	Israel	595

49	Bahrain	594
49	Jamaica	594
51	Mali	589
51	Switzerland	589
53	Slovakia	587
54	China PR	585
54	Kuwait	585
56	Oman	582
57	Australia	577
58	Honduras	575
59	Zimbabwe	572
60	Libya	566
61	Venezuela	565
62	Trinidad and Tobago	564
63	Hungary	562
64	Latvia	560
65	Peru	558
65	Qatar	558
67	Wales	555
68	Belarus	554
69	Zambia	552
70	Guatemala	551
71	Angola	545
72	Algeria	536
73	Chile	533
73	Kenya	533
75	Cuba	532
76	Ghana	528
77	Congo DR	527
78	Bosnia-Herzegovina	526
78	Thailand	526
80	Estonia	525
81	United Arab Emirates	523
82	Austria	522
83	Burkina Faso	520
84	Syria	516
85	Albania	515
85	Guinea	515
85	Scotland	515
88	Togo	513
89	Canada	509
90	Indonesia	504
91	Iceland	495
92	Bolivia	485
93	Haiti	481
93	Korea DPR	481
93	New Zealand	481
96	Turkmenistan	478
97	Rwanda	475
98	Lithuania	472
98	Panama	472
100	Botswana	462
101	Vietnam	457
102	Georgia	455
103	Lebanon	448
104	El Salvador	446
105	Northern Ireland	443
106	Cyprus	442
107	Gabon	438
107	Malawi	438

109	Singapore	435
110	Azerbaijan	432
111	Moldova	429
111	St. Lucia	429
111	Sudan	429
114	Congo	420
115	St. Kitts and Nevis	418
116	Armenia	402
117	Malaysia	401
118	Barbados	393
119	Liberia	384
120	Tahiti	381
120	Yemen	381
122	Mozambique	379
122	Palestine	379
122	Swaziland	379
125	Cape Verde Islands	377
126	Solomon Islands	375
127	India	365
128	Malta	355
129	Fiji	354
130	St. Vincent and the Grenadines	337
131	Andorra	336
132	Maldives	335
133	Liechtenstein	330
134	Vanuatu	325
135	Grenada	322
135	Lesotho	322
135	Myanmar	322
138	Kazakhstan	313
138	Madagascar	313
140	Surinam	299
141	Kyrgyzstan	293
142	Gambia	260
143	Luxembourg	259
144	Namibia	239
144	Nicaragua	239
146	Sierra Leone	232
147	Papua New Guinea	231
148	Netherlands Antilles	222
149	San Marino	221
150	British Virgin Islands	218
150	Dominica	218
152	Dominican Republic	195
153	Samoa	167
154	New Caledonia	119
155	Cook Islands	91
156	Bahamas	90
157	US Virgin Islands	70
158	Guam	15

Appendix B – WinBugs Code for Each Model

B.1 – General Bradley Terry Model

```

model{
for (i in 1:1357)
  {
Y[i]<-1;
N[i]<-1;
Y[i] ~ dbin(p[i], N[i]);
A[i]<-
X[i,1]*Alb+X[i,2]*Alg+X[i,3]*And+X[i,4]*Ang+X[i,5]*Arg+X[i,6]*Arm+X[i,7]*Austral+X[i,
8]*Austria+X[i,9]*Azer+X[i,10]*Bahamas;
B[i]<-
X[i,11]*Bahrain+X[i,12]*Barb+X[i,13]*Belar+X[i,14]*Belg+X[i,15]*Bol+X[i,16]*BosHer+X[i
,17]*Bot+X[i,18]*Bra+X[i,19]*BriVirIsl;
C[i]<-
X[i,20]*Bul+X[i,21]*BurFas+X[i,22]*Cam+X[i,23]*Can+X[i,24]*CapVerIsl+X[i,25]*Chil+X[i
,26]*ChiPR+X[i,27]*Col+X[i,28]*Con;
D[i]<-
X[i,29]*ConDR+X[i,30]*CookIsl+X[i,31]*CosRic+X[i,32]*CoteIvo+X[i,33]*Cro+X[i,34]*Cub
a+X[i,35]*Cyp+X[i,36]*CzechRep;
E[i]<-
X[i,37]*Den+X[i,38]*Dom+X[i,39]*DomRep+X[i,40]*Ecuad+X[i,41]*Egypt+X[i,42]*ElSal+X
[i,43]*Eng+X[i,44]*Est+X[i,45]*Fiji;
F[i]<-
X[i,46]*Fin+X[i,47]*Fra+X[i,48]*Gab+X[i,49]*Gam+X[i,50]*Geo+X[i,51]*Ger+X[i,52]*Gha+
X[i,53]*Gre+X[i,54]*Gren;
G[i]<-
X[i,55]*Guam+X[i,56]*Gua+X[i,57]*Guin+X[i,58]*Hai+X[i,59]*Hon+X[i,60]*Hun+X[i,61]*I
ce+X[i,62]*India+X[i,63]*Indon;
H[i]<-
X[i,64]*Iran+X[i,65]*Iraq+X[i,66]*Ire+X[i,67]*Isr+X[i,68]*Ita+X[i,69]*Jam+X[i,70]*Jap+X[i,
71]*Jor+X[i,72]*Kaz+X[i,73]*Ken;
I[i]<-
X[i,74]*KorDPR+X[i,75]*KorRep+X[i,76]*Kuw+X[i,77]*Kyr+X[i,78]*Lat+X[i,79]*Leb+X[i,
80]*Les+X[i,81]*Liber+X[i,82]*Libya;
J[i]<-
X[i,83]*Lie+X[i,84]*Lit+X[i,85]*Lux+X[i,86]*Mad+X[i,87]*Malaw+X[i,88]*Malay+X[i,89]*
Maldi+X[i,90]*Mali+X[i,91]*Malt+X[i,92]*Mar;
K[i]<-
X[i,93]*Mex+X[i,94]*Mol+X[i,95]*Mor+X[i,96]*Moz+X[i,97]*Mya+X[i,98]*Nam+X[i,99]*N
etAnt+X[i,100]*Net+X[i,101]*NewCal;

```



```

L[i]<-
X[i,102]*NewZea+X[i,103]*Nic+X[i,104]*Nig+X[i,105]*NorIre+X[i,106]*Nor+X[i,107]*Oma
n+X[i,108]*Pal+X[i,109]*Pan;
M[i]<-
X[i,110]*PapNewGui+X[i,111]*Par+X[i,112]*Peru+X[i,113]*Pol+X[i,114]*Por+X[i,115]*Qat
+X[i,116]*Rom+X[i,117]*Rus+X[i,118]*Rwa;
N2[i]<-
X[i,119]*SauAra+X[i,120]*Sam+X[i,121]*SanMar+X[i,122]*Sco+X[i,123]*Sen+X[i,124]*Ser
Mon+X[i,125]*SieLeo+X[i,126]*Sin;
O[i]<-
X[i,127]*Slovak+X[i,128]*Sloven+X[i,129]*SolIsl+X[i,130]*SouAfr+X[i,131]*Spa+X[i,132]*
StKitNev+X[i,133]*StLuc;
P[i]<-
X[i,134]*StVinGre+X[i,135]*Sud+X[i,136]*Sur+X[i,137]*Swaz+X[i,138]*Swe+X[i,139]*Swit
+X[i,140]*Syr+X[i,141]*Tah;
Q[i]<-
X[i,142]*Thail+X[i,143]*Togo+X[i,144]*TriTob+X[i,145]*Tun+X[i,146]*Tur+X[i,147]*Turk
men+X[i,148]*UniAraEmi+X[i,149]*Uru;
R[i]<-
X[i,150]*USVirIsl+X[i,151]*USA+X[i,152]*Uzb+X[i,153]*Van+X[i,154]*Ven+X[i,155]*Vie+
X[i,156]*Wal+X[i,157]*Yem+X[i,158]*Yug;
S[i]<-X[i,159]*Zai+X[i,160]*Zam+X[i,161]*Zim;
logit(p[i])<-
A[i]+B[i]+C[i]+D[i]+E[i]+F[i]+G[i]+H[i]+I[i]+J[i]+K[i]+L[i]+M[i]+N2[i]+O[i]+P[i]+Q[i]+R[i]
+S[i];
}
}

```

B.2 – Extended Bradley Terry Model with Ties and Home Field Advantage

```

model{

for (i in 1:161)
{
for (j in 1:161)
{
p[i,j,1] <- (exp(alpha[i]))/(exp(alpha[i]) + exp(alpha[j] + home) + exp(tie + (alpha[i] +
alpha[j] + home)/2));
p[i,j,2] <- (exp(alpha[j] + home))/(exp(alpha[i]) + exp(alpha[j] + home) + exp(tie + (alpha[i]
+ alpha[j] + home)/2));
p[i,j,3] <- (exp(tie + (alpha[i] + alpha[j] + home)/2))/(exp(alpha[i]) + exp(alpha[j] + home) +
exp(tie + (alpha[i] + alpha[j] + home)/2));
}
}

for (n in 1:161)
{
for (q in 1:161)
{
Y[n,q,1] <- WNO1[n,q] + WNF1[n,q] + WNO2[n,q] + WNF2[n,q] + WNO3[n,q]
+ WNF3[n,q] + WNO4[n,q] + WNF4[n,q] + WNO5[n,q]+ WNF5[n,q] + WNO6[n,q] +
WNF6[n,q] + WHO1[n,q] + WHF1[n,q] + WHO2[n,q] + WHF2[n,q] + WHO3[n,q] +
WHF3[n,q] + WHO4[n,q] + WHF4[n,q] + WHO5[n,q] + WHF5[n,q] + WHO6[n,q] +
WHF6[n,q];
Y[n,q,2] <- LNO1[n,q] + LNF1[n,q] + LNO2[n,q] + LNF2[n,q] + LNO3[n,q] +
LNF3[n,q] + LNO4[n,q] + LNF4[n,q] + LNO5[n,q]+ LNF5[n,q] + LNO6[n,q] + LNF6[n,q] +
LHO1[n,q] + LHF1[n,q] + LHO2[n,q] + LHF2[n,q] + LHO3[n,q] + LHF3[n,q] + LHO4[n,q] +
LHF4[n,q] + LHO5[n,q] + LHF5[n,q] + LHO6[n,q] + LHF6[n,q];
Y[n,q,3] <- TNO1[n,q] + TNF1[n,q] + TNO2[n,q] + TNF2[n,q] + TNO3[n,q] +
TNF3[n,q] + TNO4[n,q] + TNF4[n,q] + TNO5[n,q]+ TNF5[n,q] + TNO6[n,q] + TNF6[n,q] +
THO1[n,q] + THF1[n,q] + THO2[n,q] + THF2[n,q] + THO3[n,q] + THF3[n,q] + THO4[n,q] +
THF4[n,q] + THO5[n,q] + THF5[n,q] + THO6[n,q] + THF6[n,q];

}
}

for (v in 1:161)
{
for(w in 1:161)
{
N[v,w] <- Y[v,w,1] + Y[v,w,2] + Y[v,w,3];
}
}

```

```
}  
  
for (d in 1:161)  
{  
  for (f in 1:161)  
  {  
    for (g in 1:3)  
    {  
      mu[d,f,g] <- N[d,f]*p[d,f,g];  
      Y[d,f,g] ~ dpois(mu[d,f,g]);  
    }  
  }  
}  
  
for (o in 1:160)  
{  
  alpha[o] ~ dnorm(0.0, 10);  
}  
  
alpha[161] <- 0;  
home ~ dnorm(0, .0001);  
tie ~ dnorm(0,.0001);  
  
}
```

B.3 – Extended Bradley Terry Model with Ties and Neutral Site Distinction

```

model{

for (i in 1:161)
{
for (j in 1:161)
{
for(m in 1:2)
{
p[i,j,1,m] <- (exp(alpha[i]))/(exp(alpha[i]) + exp(alpha[j] + (m-1)*home) + exp(tie +
(alpha[i] + alpha[j] + (m-1)*home)/2));
p[i,j,2,m] <- (exp(alpha[j] + (m-1)*home))/(exp(alpha[i]) + exp(alpha[j] + (m-1)*home) +
exp(tie + (alpha[i] + alpha[j] + (m-1)*home)/2));
p[i,j,3,m] <- (exp(tie + (alpha[i] + alpha[j] + (m-1)*home)/2))/(exp(alpha[i]) + exp(alpha[j]
+ (m-1)*home) + exp(tie + (alpha[i] + alpha[j] + (m-1)*home)/2));
}
}
}

for (n in 1:161)
{
for (q in 1:161)
{
Y[n,q,1,1] <-
WNO1[n,q]+WNF1[n,q]+WNO2[n,q]+WNF2[n,q]+WNO3[n,q]+WNF3[n,q]+WNO4[n,q]+WN
F4[n,q]+WNO5[n,q]+WNF5[n,q]+WNO6[n,q]+WNF6[n,q];
Y[n,q,1,2] <-
WHO1[n,q]+WHF1[n,q]+WHO2[n,q]+WHF2[n,q]+WHO3[n,q]+WHF3[n,q]+WHO4[n,q]+WH
F4[n,q]+WHO5[n,q]+WHF5[n,q]+WHO6[n,q]+WHF6[n,q];
Y[n,q,2,1] <-
LNO1[n,q]+LNF1[n,q]+LNO2[n,q]+LNF2[n,q]+LNO3[n,q]+LNF3[n,q]+LNO4[n,q]+LNF4[n,q
]+LNO5[n,q]+LNF5[n,q]+LNO6[n,q]+LNF6[n,q];
Y[n,q,2,2] <-
LHO1[n,q]+LHF1[n,q]+LHO2[n,q]+LHF2[n,q]+LHO3[n,q]+LHF3[n,q]+LHO4[n,q]+LHF4[n,q
]+LHO5[n,q]+LHF5[n,q]+LHO6[n,q]+LHF6[n,q];
Y[n,q,3,1] <-
TNO1[n,q]+TNF1[n,q]+TNO2[n,q]+TNF2[n,q]+TNO3[n,q]+TNF3[n,q]+TNO4[n,q]+TNF4[n,q
]+TNO5[n,q]+TNF5[n,q]+TNO6[n,q]+TNF6[n,q];
Y[n,q,3,2] <-
THO1[n,q]+THF1[n,q]+THO2[n,q]+THF2[n,q]+THO3[n,q]+THF3[n,q]+THO4[n,q]+THF4[n,q
]+THO5[n,q]+THF5[n,q]+THO6[n,q]+THF6[n,q];
}
}
}

```

```
for (v in 1:161)
{
for(w in 1:161)
{
N[v,w] <- Y[v,w,1,1] + Y[v,w,1,2] + Y[v,w,2,1] + Y[v,w,2,2] + Y[v,w,3,1] + Y[v,w,3,2];
}
}

for (d in 1:161)
{
for (f in 1:161)
{
for (g in 1:3)
{
for(h in 1:2)
{

mu[d,f,g,h] <- N[d,f]*p[d,f,g,h];
Y[d,f,g,h] ~ dpois(mu[d,f,g,h]);
}
}
}
}

for (o in 1:160)
{
alpha[o] ~ dnorm(0.0, 10);
}

alpha[161] <- 0;
home ~ dnorm(0, .0001);
tie ~ dnorm(0,.0001);

}
```

B.4 – Extended Bradley Terry Model with Ties, Neutral Site Distinction, and Game Type

```

model{

for (i in 1:161)
{
for (j in 1:161)
{
for(m in 1:2)
{
for(l in 1:2)
{
prob[i,j,1,m,l] <- (exp(gtype[l]*alpha[i]))/(exp(gtype[l]*alpha[i]) + exp(gtype[l]*alpha[j] +
(m-1)*home) + exp(tie + (gtype[l]*alpha[i] + gtype[l]*alpha[j] + (m-1)*home)/2));
prob[i,j,2,m,l] <- (exp(gtype[l]*alpha[j] + (m-1)*home))/(exp(gtype[l]*alpha[i] +
exp(gtype[l]*alpha[j] + (m-1)*home) + exp(tie + (gtype[l]*alpha[i] + gtype[l]*alpha[j] + (m-
1)*home)/2));
prob[i,j,3,m,l] <- (exp(tie + (gtype[l]*alpha[i] + gtype[l]*alpha[j] + (m-
1)*home)/2))/(exp(gtype[l]*alpha[i] + exp(gtype[l]*alpha[j] + (m-1)*home) + exp(tie +
(gtype[l]*alpha[i] + gtype[l]*alpha[j] + (m-1)*home)/2));
}
}
}
}

for (n in 1:161)
{
for (q in 1:161)
{
Y[n,q,1,1,1] <- WNO1[n,q] + WNO2[n,q] + WNO3[n,q] + WNO4[n,q] + WNO5[n,q] +
WNO6[n,q];
Y[n,q,1,2,1] <- WHO1[n,q] + WHO2[n,q] + WHO3[n,q] + WHO4[n,q] + WHO5[n,q] +
WHO6[n,q];
Y[n,q,2,1,1] <- LNO1[n,q] + LNO2[n,q] + LNO3[n,q] + LNO4[n,q] + LNO5[n,q] +
LNO6[n,q];
Y[n,q,2,2,1] <- LHO1[n,q] + LHO2[n,q] + LHO3[n,q] + LHO4[n,q] + LHO5[n,q] +
LHO6[n,q];
Y[n,q,3,1,1] <- TNO1[n,q] + TNO2[n,q] + TNO3[n,q] + TNO4[n,q] + TNO5[n,q] +
TNO6[n,q];
Y[n,q,3,2,1] <- THO1[n,q] + THO2[n,q] + THO3[n,q] + THO4[n,q] + THO5[n,q] +
THO6[n,q];

Y[n,q,1,1,2] <- WNF1[n,q] + WNF2[n,q] + WNF3[n,q] + WNF4[n,q] + WNF5[n,q] +
WNF6[n,q];
Y[n,q,1,2,2] <- WHF1[n,q] + WHF2[n,q] + WHF3[n,q] + WHF4[n,q] + WHF5[n,q] +

```

```

        WHF6[n,q];
Y[n,q,2,1,2] <- LNF1[n,q] + LNF2[n,q] + LNF3[n,q] + LNF4[n,q] + LNF5[n,q] +
        LNF6[n,q];
Y[n,q,2,2,2] <- LHF1[n,q] + LHF2[n,q] + LHF3[n,q] + LHF4[n,q] + LHF5[n,q] +
        LHF6[n,q];
Y[n,q,3,1,2] <- TNF1[n,q] + TNF2[n,q] + TNF3[n,q] + TNF4[n,q] + TNF5[n,q] +
        TNF6[n,q];
Y[n,q,3,2,2] <- THF1[n,q] + THF2[n,q] + THF3[n,q] + THF4[n,q] + THF5[n,q] +
        THF6[n,q];

}
}

for (v in 1:161)
{
for(w in 1:161)
{
for (h in 1:2)
{
for (m in 1:2)
{
N[v,w,h,m] <- Y[v,w,1,h,m] + Y[v,w,2,h,m] + Y[v,w,3,h,m];
}
}
}
}
for (d in 1:161)
{
for (f in 1:161)
{
for (g in 1:3)
{
for(h in 1:2)
{
for (m in 1:2)
{

mu[d,f,g,h,m] <- N[d,f,h,m]*prob[d,f,g,h,m];
Y[d,f,g,h,m] ~ dpois(mu[d,f,g,h,m]);
}
}

}
}

}
}

for (o in 1:160)
{

```

```
alpha[o] ~ dnorm(0.0, 10);  
}
```

```
alpha[161] <- 0;
```

```
gtype[1] <- 1;  
gtype[2] ~ dbeta(1,1);  
home ~ dnorm(0, .0001);  
tie ~ dnorm(0,.0001);  
}
```


B.5 – Dynamic Bradley Terry Model with Ties and Neutral Site Distinction

```

model{

for (j in 1:161)
{
alpha[j,1]~dnorm(0,10);
}

for (i in 1:161)
{
for (j in 1:161)
{
for(m in 1:2)
{
for(t in 2:7)
{
prob[i,j,1,m,t] <- (exp(alpha[i,t]))/(exp(alpha[i,t]) + exp(alpha[j,t] + (m-1)*home) + exp(tie
+ (alpha[i,t] + alpha[j,t] + (m-1)*home)/2));
prob[i,j,2,m,t] <- (exp(alpha[j,t] + (m-1)*home))/(exp(alpha[i,t]) + exp(alpha[j,t] + (m-
1)*home) + exp(tie + (alpha[i,t] + alpha[j,t] + (m-1)*home)/2));
prob[i,j,3,m,t] <- (exp(tie + (alpha[i,t] + alpha[j,t] + (m-1)*home)/2))/(exp(alpha[i,t]) +
exp(alpha[j,t] + (m-1)*home) + exp(tie + (alpha[i,t] + alpha[j,t] + (m-1)*home)/2));
}
}
}
}

for (n in 1:161)
{
for (q in 1:161)
{
Y[n,q,1,1,1] <- WNO1[n,q]+WNF1[n,q];
Y[n,q,1,1,2] <- WNO2[n,q]+WNF2[n,q];
Y[n,q,1,1,3] <- WNO3[n,q]+WNF3[n,q];
Y[n,q,1,1,4] <- WNO4[n,q]+WNF4[n,q];
Y[n,q,1,1,5] <- WNO5[n,q]+WNF5[n,q];
Y[n,q,1,1,6] <- WNO6[n,q]+WNF6[n,q];
Y[n,q,1,2,1] <- WHO1[n,q]+WHF1[n,q];
Y[n,q,1,2,2] <- WHO2[n,q]+WHF2[n,q];
Y[n,q,1,2,3] <- WHO3[n,q]+WHF3[n,q];
Y[n,q,1,2,4] <- WHO4[n,q]+WHF4[n,q];
Y[n,q,1,2,5] <- WHO5[n,q]+WHF5[n,q];
Y[n,q,1,2,6] <- WHO6[n,q]+WHF6[n,q];
Y[n,q,2,1,1] <- LNO1[n,q]+LNF1[n,q];
Y[n,q,2,1,2] <- LNO2[n,q]+LNF2[n,q];
Y[n,q,2,1,3] <- LNO3[n,q]+LNF3[n,q];
}
}
}
}

```

```

Y[n,q,2,1,4] <- LNO4[n,q]+LNF4[n,q];
Y[n,q,2,1,5] <- LNO5[n,q]+LNF5[n,q];
Y[n,q,2,1,6] <- LNO6[n,q]+LNF6[n,q];
Y[n,q,2,2,1] <- LHO1[n,q]+LHF1[n,q];
Y[n,q,2,2,2] <- LHO2[n,q]+LHF2[n,q];
Y[n,q,2,2,3] <- LHO3[n,q]+LHF3[n,q];
Y[n,q,2,2,4] <- LHO4[n,q]+LHF4[n,q];
Y[n,q,2,2,5] <- LHO5[n,q]+LHF5[n,q];
Y[n,q,2,2,6] <- LHO6[n,q]+LHF6[n,q];
Y[n,q,3,1,1] <- TNO1[n,q]+TNF1[n,q];
Y[n,q,3,1,2] <- TNO2[n,q]+TNF2[n,q];
Y[n,q,3,1,3] <- TNO3[n,q]+TNF3[n,q];
Y[n,q,3,1,4] <- TNO4[n,q]+TNF4[n,q];
Y[n,q,3,1,5] <- TNO5[n,q]+TNF5[n,q];
Y[n,q,3,1,6] <- TNO6[n,q]+TNF6[n,q];
Y[n,q,3,2,1] <- THO1[n,q]+THF1[n,q];
Y[n,q,3,2,2] <- THO2[n,q]+THF2[n,q];
Y[n,q,3,2,3] <- THO3[n,q]+THF3[n,q];
Y[n,q,3,2,4] <- THO4[n,q]+THF4[n,q];
Y[n,q,3,2,5] <- THO5[n,q]+THF5[n,q];
Y[n,q,3,2,6] <- THO6[n,q]+THF6[n,q];
}
}

for (v in 1:161)
{
for(w in 1:161)
{
for (n in 1:2)
{
for (t in 2:7)
{
N[v,w,n,t] <- Y[v,w,1,n,t-1] + Y[v,w,2,n,t-1] + Y[v,w,3,n,t-1];
}
}
}
}

for (d in 1:161)
{
for (f in 1:161)
{
for (g in 1:3)
{
for(h in 1:2)
{
for(t in 2:7)
{

```

```
mu[d,f,g,h,t] <- N[d,f,h,t]*prob[d,f,g,h,t];
Y[d,f,g,h,t-1] ~ dpois(mu[d,f,g,h,t]);
}
}

}
}
}

for (m in 1:160)
{
for (n in 2:7)
{
alpha[m,n]~dnorm(alpha[m,n-1], 10)
}
}

for (p in 2:7)
{
alpha[161,p]<-0;
}

home ~ dnorm(0, .0001);
tie ~ dnorm(0,.0001);
}
```

Appendix C – Full Rankings

C.1 – Full Rankings for Extended Bradley Terry Model with Ties, Neutral Site Distinction, and Game Type

Rank	Team	mean	sd	MC error	2.50%	median	97.50%
1	Brazil	1.089	0.1999	0.006202	0.6789	1.09	1.483
2	France	0.8148	0.2441	0.007059	0.3465	0.8089	1.296
3	Mexico	0.8144	0.194	0.006292	0.4426	0.8118	1.198
4	Argentina	0.7352	0.2178	0.005623	0.3227	0.7414	1.171
5	Italy	0.6861	0.24	0.006852	0.2124	0.6909	1.164
6	Spain	0.649	0.2426	0.00734	0.164	0.651	1.126
7	Netherlands	0.6322	0.2536	0.00673	0.1239	0.6309	1.137
8	Germany	0.6086	0.2328	0.005635	0.1596	0.6053	1.065
9	Japan	0.6061	0.2346	0.007058	0.1285	0.6108	1.062
10	Australia	0.6012	0.2497	0.00708	0.1099	0.6062	1.113
11	England	0.5496	0.2498	0.006883	0.07462	0.554	1.029
12	Nigeria	0.5333	0.2225	0.006333	0.09634	0.5345	0.9587
13	USA	0.5282	0.2004	0.006123	0.1322	0.5228	0.9041
14	Iran	0.511	0.233	0.005962	0.06543	0.5117	0.9735
15	Denmark	0.5109	0.2421	0.007154	0.03662	0.5134	0.9882
16	Sweden	0.5058	0.245	0.007748	0.01239	0.4966	0.9954
17	Croatia	0.474	0.2455	0.007788	-0.00196	0.4697	0.942
18	Costa Rica	0.4654	0.2137	0.005693	0.05305	0.4684	0.8862
19	Czech Republic	0.4652	0.2482	0.008242	-0.02944	0.4576	0.9682
20	Portugal	0.4651	0.2531	0.006133	-0.04178	0.4652	0.9451
21	Cameroon	0.4008	0.2373	0.007182	-0.05814	0.4048	0.8856
22	South Africa	0.3977	0.2301	0.006502	-0.05095	0.3995	0.8432
23	Morocco	0.3701	0.2383	0.005775	-0.1049	0.3714	0.854
24	Ireland	0.3409	0.2597	0.008056	-0.1769	0.3362	0.87
25	Senegal	0.2989	0.2417	0.007711	-0.1711	0.3029	0.7559
26	Egypt	0.2761	0.2398	0.006993	-0.1901	0.2897	0.7238
27	Colombia	0.2727	0.2134	0.005958	-0.149	0.271	0.697
28	Turkey	0.2713	0.242	0.006474	-0.2059	0.2673	0.7578
29	China PR	0.2681	0.2437	0.006782	-0.2074	0.2687	0.7637
30	Bahrain	0.2628	0.2521	0.008505	-0.2223	0.2694	0.7528
31	Honduras	0.2621	0.2321	0.006303	-0.1915	0.2517	0.726
32	Saudi Arabia	0.2517	0.2146	0.006574	-0.1795	0.2525	0.6625
33	Uzbekistan	0.2495	0.2613	0.006515	-0.2771	0.2595	0.7521
34	Belgium	0.2393	0.2488	0.007753	-0.2426	0.2394	0.7376
35	Jordan	0.2362	0.2775	0.007832	-0.2943	0.22	0.747
36	Cote d'Ivoire	0.2294	0.2455	0.007065	-0.2522	0.2223	0.7161
37	New Zealand	0.2276	0.2483	0.006325	-0.2537	0.227	0.7053
38	Romania	0.2264	0.2466	0.00628	-0.2558	0.2264	0.6896
39	Uruguay	0.2134	0.2178	0.005792	-0.2224	0.2104	0.6493

40	Zambia	0.2117	0.2383	0.006765	-0.2519	0.2025	0.6703
41	Guinea	0.1996	0.2778	0.008926	-0.3368	0.2005	0.7328
42	Jamaica	0.183	0.228	0.006867	-0.2349	0.1792	0.6464
43	Cuba	0.1806	0.2534	0.007062	-0.3474	0.1777	0.674
44	Bulgaria	0.1774	0.254	0.007796	-0.3135	0.1742	0.6774
45	Paraguay	0.1722	0.2299	0.007791	-0.2646	0.1692	0.6292
46	Poland	0.1702	0.2527	0.007542	-0.3328	0.1808	0.6374
47	Kuwait	0.1662	0.2527	0.007238	-0.3221	0.1721	0.6307
48	Serb. and Mont.	0.1662	0.2884	0.007919	-0.3963	0.1745	0.7293
49	Greece	0.1631	0.2672	0.00682	-0.3788	0.1618	0.6931
50	Trin. and Tob.	0.1601	0.2218	0.007279	-0.2741	0.1661	0.6092
51	Ghana	0.1463	0.2504	0.007033	-0.3549	0.1487	0.6126
52	Angola	0.1426	0.2665	0.008572	-0.3961	0.1398	0.6475
53	Korea Rep.	0.1411	0.2296	0.006075	-0.3104	0.1357	0.5847
54	Scotland	0.1236	0.2713	0.008528	-0.4141	0.117	0.6252
55	Russia	0.1215	0.2512	0.007545	-0.3835	0.1265	0.624
56	Tunisia	0.1054	0.2287	0.006695	-0.349	0.1061	0.5471
57	Oman	0.08357	0.2722	0.008554	-0.4511	0.07664	0.6255
58	Guatemala	0.07478	0.2438	0.007891	-0.4169	0.08134	0.5638
59	Qatar	0.064	0.2523	0.008754	-0.4562	0.07614	0.5365
60	Haiti	0.06088	0.2592	0.008472	-0.4416	0.06031	0.5859
61	Fiji	0.05986	0.2665	0.007763	-0.4533	0.06697	0.6026
62	Israel	0.05212	0.2702	0.008198	-0.4616	0.05226	0.5929
63	Norway	0.05175	0.2524	0.00685	-0.4142	0.03948	0.5587
64	Finland	0.047	0.2813	0.007666	-0.4903	0.04181	0.5858
65	Slovenia	0.04691	0.2476	0.00745	-0.4386	0.05729	0.517
66	Solomon Islands	0.04114	0.2697	0.008438	-0.464	0.03651	0.5632
67	Togo	0.03713	0.2625	0.007424	-0.4952	0.03557	0.5263
68	Tahiti	0.02419	0.2688	0.007261	-0.489	0.0163	0.5656
69	Ecuador	0.01729	0.2323	0.006878	-0.4213	0.02168	0.4929
70	Canada	0.01435	0.2494	0.006966	-0.4848	0.01799	0.4986
71	Iraq	0.003045	0.2656	0.006919	-0.5358	0.003737	0.5294
72	Zimbabwe	0	0	0	0	0	0
73	Slovakia	-0.00884	0.281	0.008589	-0.5536	0.005442	0.5504
74	Bosnia-Herz	-0.00942	0.2765	0.007925	-0.5568	-0.01377	0.5487
75	Congo DR	-0.01405	0.2398	0.007242	-0.4871	-0.00581	0.4468
76	St. Lucia	-0.01434	0.2811	0.007588	-0.5749	-0.0057	0.5587
77	Unit. Arab Emir.	-0.02007	0.2431	0.006117	-0.4835	-0.01917	0.4262
78	Liberia	-0.0493	0.2683	0.008422	-0.579	-0.04702	0.4639
79	Austria	-0.05068	0.2764	0.008361	-0.5661	-0.04882	0.4882
80	Turkmenistan	-0.0509	0.2796	0.007843	-0.5857	-0.04826	0.4851
81	Kenya	-0.0536	0.2778	0.007864	-0.6137	-0.05717	0.4938
82	Kyrgyzstan	-0.05893	0.3112	0.009324	-0.6674	-0.0621	0.5446
83	Switzerland	-0.06175	0.2743	0.007967	-0.5879	-0.06033	0.486
84	St. Vin. & Gren.	-0.07126	0.2699	0.00704	-0.6294	-0.06236	0.4481
85	Namibia	-0.07904	0.3003	0.008584	-0.6697	-0.08365	0.4963
86	Guam	-0.07909	0.3192	0.01055	-0.7184	-0.07896	0.5536
87	Belarus	-0.08249	0.2762	0.00713	-0.638	-0.08105	0.4642
88	Latvia	-0.1038	0.2751	0.007835	-0.6459	-0.09822	0.4224
89	Syria	-0.1075	0.2835	0.007293	-0.6803	-0.1155	0.4276
90	Dominica	-0.11	0.2955	0.008629	-0.7294	-0.09961	0.4574
91	Wales	-0.1132	0.2755	0.007954	-0.6633	-0.1082	0.4215

92	Hungary	-0.1143	0.2634	0.006818	-0.6205	-0.1099	0.409
93	Rwanda	-0.1203	0.2872	0.009016	-0.6827	-0.1276	0.4458
94	New Caledonia	-0.1208	0.2919	0.008581	-0.6798	-0.1108	0.4768
95	Mali	-0.1209	0.2677	0.008004	-0.6706	-0.1195	0.3864
96	El Salvador	-0.123	0.2589	0.007099	-0.6371	-0.112	0.3561
97	Peru	-0.123	0.2277	0.006672	-0.5798	-0.1196	0.3137
98	Gambia	-0.1238	0.2929	0.008051	-0.7052	-0.1131	0.4589
99	Cape Verde Isl.	-0.125	0.2894	0.007528	-0.6886	-0.1239	0.4397
100	Pap. New Guin.	-0.1307	0.2885	0.008553	-0.6902	-0.1369	0.4429
101	Netherlands Ant.	-0.1389	0.2991	0.00829	-0.7053	-0.1457	0.4507
102	Swaziland	-0.1441	0.2708	0.007409	-0.6784	-0.1367	0.3952
103	Georgia	-0.1465	0.2844	0.00784	-0.7075	-0.1577	0.3973
104	Lebanon	-0.1657	0.2727	0.00733	-0.694	-0.1636	0.3778
105	Libya	-0.1664	0.2685	0.008809	-0.6629	-0.1637	0.3641
106	Korea DPR	-0.175	0.2798	0.009436	-0.728	-0.179	0.3844
107	Gabon	-0.1835	0.2778	0.007689	-0.7427	-0.183	0.338
108	St. Kitts & Nev.	-0.1842	0.2819	0.007793	-0.7452	-0.1954	0.3581
109	Bahamas	-0.1868	0.3056	0.00852	-0.8121	-0.1811	0.4041
110	Myanmar	-0.1877	0.2906	0.008272	-0.7456	-0.1958	0.4153
111	Surinam	-0.1877	0.2927	0.009564	-0.7494	-0.1891	0.3775
112	Madagascar	-0.1887	0.2743	0.008058	-0.733	-0.1806	0.332
113	Iceland	-0.1897	0.268	0.007452	-0.7191	-0.211	0.3529
114	Armenia	-0.2019	0.2689	0.007617	-0.7353	-0.1966	0.3282
115	Estonia	-0.2044	0.2786	0.008039	-0.782	-0.1983	0.3505
116	Vanuatu	-0.2131	0.2587	0.008096	-0.7206	-0.2191	0.2916
117	Lesotho	-0.2151	0.2954	0.00824	-0.8124	-0.2228	0.3487
118	Kazakhstan	-0.2244	0.2916	0.00867	-0.8258	-0.2264	0.348
119	Indonesia	-0.2278	0.2721	0.006797	-0.7567	-0.2099	0.2976
120	Panama	-0.2311	0.2545	0.008173	-0.7454	-0.2268	0.2647
121	India	-0.235	0.283	0.008153	-0.7867	-0.2371	0.3217
122	Dominican Rep.	-0.2462	0.2995	0.008943	-0.8186	-0.2475	0.344
123	Vietnam	-0.248	0.2808	0.008504	-0.8014	-0.2454	0.3029
124	Congo	-0.249	0.2623	0.0097	-0.7613	-0.2435	0.2505
125	Botswana	-0.2516	0.2851	0.007976	-0.792	-0.2577	0.3003
126	Sierra Leone	-0.2563	0.2787	0.008899	-0.7994	-0.2599	0.2843
127	Northern Ireland	-0.2649	0.2677	0.008292	-0.7935	-0.2654	0.28
128	Albania	-0.2661	0.2761	0.007923	-0.8053	-0.2649	0.2765
129	Algeria	-0.2714	0.2589	0.007273	-0.7676	-0.2676	0.2486
130	Barbados	-0.2725	0.2736	0.007745	-0.8071	-0.2752	0.2777
131	Bolivia	-0.2786	0.2354	0.007018	-0.7337	-0.2786	0.1879
132	Lithuania	-0.2839	0.2766	0.007393	-0.8262	-0.2814	0.2861
133	Thailand	-0.308	0.2521	0.008775	-0.7959	-0.2994	0.181
134	Malawi	-0.3088	0.2897	0.008249	-0.8954	-0.3134	0.2752
135	Mozambique	-0.3204	0.2854	0.008001	-0.8661	-0.3316	0.2488
136	British Virgin Isl.	-0.3246	0.3019	0.007946	-0.9321	-0.3219	0.2613
137	Yemen	-0.3264	0.2808	0.008934	-0.8965	-0.3204	0.2073
138	Malaysia	-0.3294	0.2844	0.007334	-0.8727	-0.3207	0.235
139	Cyprus	-0.3352	0.2777	0.008002	-0.8876	-0.3409	0.2035
140	Moldova	-0.3353	0.2723	0.007991	-0.8545	-0.3421	0.1938
141	Sudan	-0.3386	0.2748	0.008081	-0.8799	-0.3332	0.2366
142	Chile	-0.3586	0.234	0.007271	-0.8496	-0.3598	0.09612
143	Cook Islands	-0.3598	0.286	0.007469	-0.9066	-0.3618	0.2243

144	Burkina Faso	-0.3758	0.2458	0.00724	-0.8414	-0.3717	0.08878
145	US Virgin Isl.	-0.396	0.2923	0.009389	-0.9742	-0.3997	0.2016
146	Samoa	-0.4139	0.2952	0.009102	-0.9793	-0.4063	0.1852
147	Palestine	-0.4153	0.2807	0.007981	-0.9678	-0.4219	0.1517
148	Azerbaijan	-0.4311	0.2821	0.008614	-0.9654	-0.4435	0.1535
149	Grenada	-0.4362	0.29	0.00732	-1.002	-0.4353	0.1431
150	Singapore	-0.4438	0.288	0.006808	-1.014	-0.4401	0.1233
151	Nicaragua	-0.4867	0.2773	0.007109	-1.028	-0.4924	0.07125
152	Venezuela	-0.548	0.2532	0.007148	-1.029	-0.5632	-0.05385
153	Maldives	-0.5594	0.2811	0.008458	-1.101	-0.5654	-0.01052
154	Liechtenstein	-0.6114	0.2728	0.007044	-1.126	-0.6189	-0.06118
155	San Marino	-0.7049	0.2716	0.008084	-1.242	-0.7066	-0.1942
156	Malta	-0.7097	0.2824	0.007813	-1.284	-0.7108	-0.1521
157	Andorra	-0.7312	0.2734	0.006823	-1.28	-0.74	-0.2219
158	Luxembourg	-0.8362	0.2735	0.008511	-1.354	-0.8386	-0.2855

C.2 – Full Rankings for Dynamic Bradley Terry Model with Ties and Neutral Site Distinction

Rank	Team	MC					
		mean	sd	error	2.50%	median	97.50%
1	Mexico	2.472	0.5546	0.01333	1.412	2.467	3.561
2	Argentina	2.312	0.5566	0.01374	1.206	2.309	3.405
3	Brazil	2.276	0.498	0.01209	1.304	2.276	3.251
4	England	2.132	0.5846	0.01356	1.006	2.112	3.32
5	France	2.08	0.5577	0.01153	0.9793	2.074	3.159
6	Spain	2.072	0.5839	0.01223	0.9292	2.06	3.229
7	Japan	2.068	0.5247	0.01128	1.032	2.069	3.103
8	Iran	1.99	0.5755	0.01275	0.8678	1.991	3.135
9	Czech Republic	1.983	0.5718	0.0134	0.862	1.982	3.117
10	Italy	1.872	0.5787	0.01489	0.7452	1.873	3.048
11	Nigeria	1.838	0.5627	0.01106	0.7805	1.823	2.955
12	USA	1.814	0.5215	0.01182	0.8271	1.807	2.869
13	Sweden	1.734	0.5628	0.01301	0.6592	1.736	2.85
14	Ireland	1.716	0.5872	0.01469	0.5282	1.724	2.866
15	Netherlands	1.713	0.609	0.01246	0.5349	1.708	2.935
16	Greece	1.681	0.5619	0.01251	0.5978	1.683	2.778
17	Germany	1.675	0.5981	0.01338	0.5006	1.686	2.851
18	Denmark	1.585	0.5487	0.01146	0.5503	1.575	2.68
19	Croatia	1.531	0.5785	0.01244	0.4188	1.525	2.695
20	Turkey	1.413	0.5513	0.01159	0.3223	1.405	2.501
21	Cote d'Ivoire	1.409	0.6751	0.01571	0.1045	1.409	2.739
22	Norway	1.272	0.5966	0.0126	0.1035	1.267	2.469
23	Senegal	1.255	0.5601	0.01388	0.1546	1.261	2.369
24	Morocco	1.231	0.5597	0.009774	0.1566	1.218	2.349
25	Romania	1.205	0.6504	0.01484	-0.05292	1.196	2.504
26	Costa Rica	1.192	0.5012	0.01031	0.1965	1.191	2.187
27	Colombia	1.174	0.5149	0.009978	0.1928	1.168	2.179
28	Portugal	1.154	0.5904	0.01176	0.02909	1.161	2.324
29	Uzbekistan	1.114	0.6371	0.01474	-0.11	1.108	2.407
30	Angola	1.072	0.6284	0.01377	-0.1435	1.065	2.282
31	Australia	1.066	0.6374	0.01385	-0.1734	1.054	2.349
32	Cameroon	1.04	0.5932	0.01577	-0.1274	1.045	2.188
33	Saudi Arabia	1.031	0.5767	0.01165	-0.09746	1.032	2.135
34	China PR	1.018	0.5438	0.01205	7.42E-04	1.01	2.106
35	Jamaica	0.9862	0.5236	0.01261	-0.02169	0.978	2.027
36	Solomon Islands	0.9643	0.6767	0.01404	-0.3572	0.955	2.321
37	Poland	0.9257	0.5725	0.01296	-0.2001	0.936	2.046
38	Paraguay	0.9123	0.5807	0.01246	-0.2027	0.9062	2.05
39	Guatemala	0.8763	0.5112	0.01209	-0.1512	0.8781	1.879
40	Uruguay	0.8676	0.5575	0.01348	-0.2287	0.8697	1.93
41	Jordan	0.8379	0.5114	0.009052	-0.1576	0.8391	1.873
42	Korea Rep.	0.8138	0.5281	0.01017	-0.2209	0.8163	1.836
43	Cuba	0.8005	0.6301	0.01424	-0.4231	0.7981	2.026

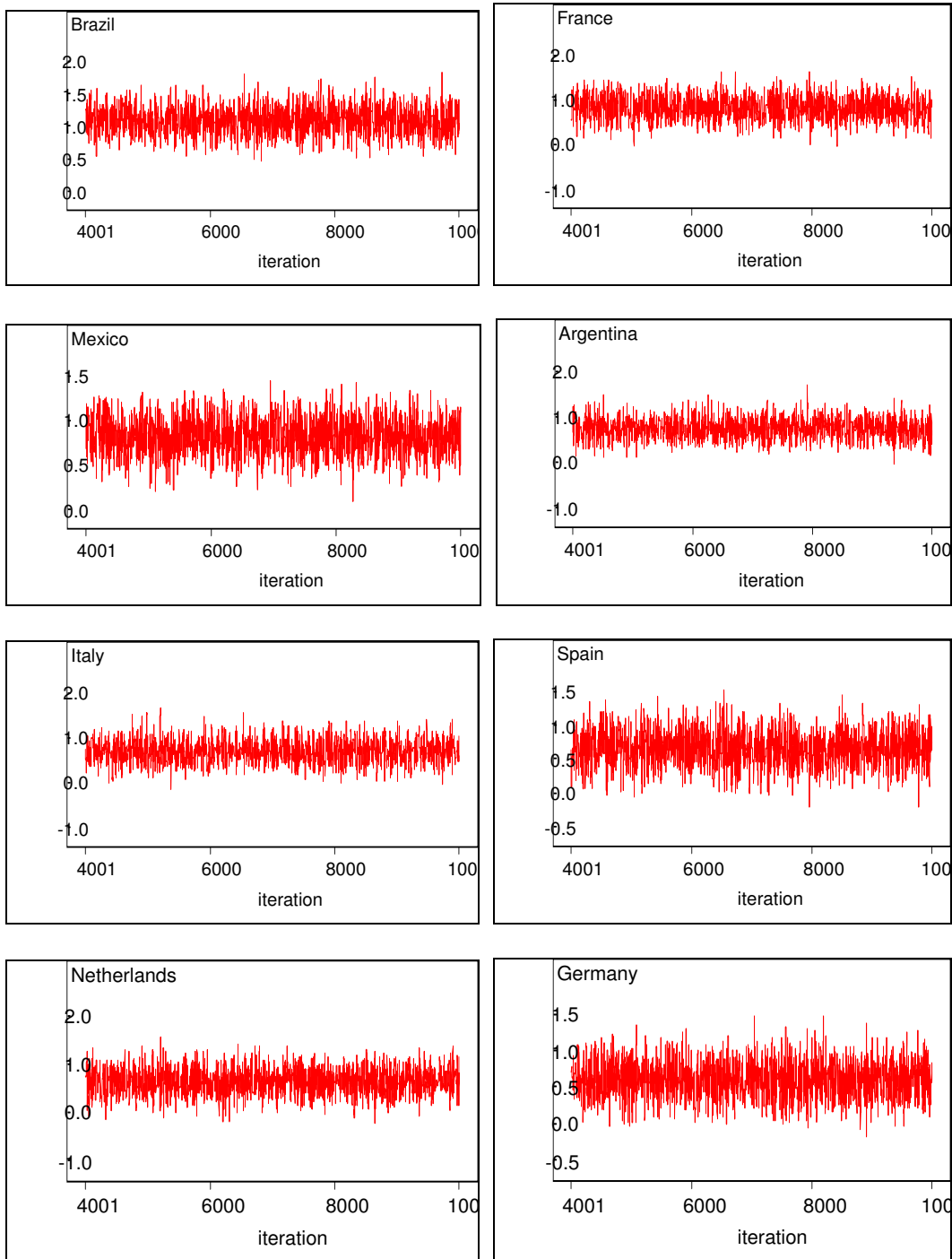
44	Bosnia-Herz.	0.7259	0.7147	0.01509	-0.7063	0.7304	2.12
45	Bulgaria	0.7255	0.606	0.01321	-0.4561	0.7342	1.925
46	Bahrain	0.7217	0.4841	0.01143	-0.1913	0.7169	1.667
47	Honduras	0.6977	0.5009	0.01208	-0.2839	0.698	1.677
48	Serbia and Mont.	0.6873	0.6536	0.01272	-0.5888	0.6905	2.017
49	Guinea	0.6857	0.611	0.0121	-0.4789	0.6806	1.883
50	Egypt	0.6447	0.5699	0.0125	-0.4363	0.621	1.814
51	New Zealand	0.6395	0.7031	0.01813	-0.7062	0.6355	2.054
52	Russia	0.6253	0.5908	0.01308	-0.534	0.6263	1.81
53	Togo	0.5557	0.6652	0.01801	-0.7288	0.5375	1.874
54	Zambia	0.5142	0.6295	0.01208	-0.6969	0.5094	1.794
55	Hungary	0.511	0.601	0.0116	-0.6728	0.5125	1.683
56	Israel	0.4902	0.6424	0.01331	-0.7212	0.4813	1.728
57	Wales	0.49	0.6175	0.01345	-0.715	0.4983	1.703
58	Oman	0.4884	0.5605	0.01286	-0.5858	0.4843	1.58
59	Belgium	0.4883	0.6248	0.01571	-0.7494	0.491	1.686
60	Tunisia	0.4647	0.5608	0.01278	-0.6216	0.4591	1.54
61	Belarus	0.4646	0.6375	0.01334	-0.759	0.4547	1.713
62	South Africa	0.4572	0.6064	0.0135	-0.7474	0.4568	1.629
63	Libya	0.4241	0.6272	0.0138	-0.7791	0.4182	1.637
64	Ecuador	0.3819	0.5901	0.01386	-0.7674	0.3772	1.553
65	Ghana	0.3768	0.6506	0.01344	-0.9155	0.3768	1.644
66	Tahiti	0.2155	0.6989	0.01566	-1.143	0.212	1.603
67	Canada	0.1615	0.6406	0.0137	-1.091	0.1726	1.396
68	Panama	0.1135	0.5301	0.01086	-0.9496	0.1086	1.164
69	Peru	0.09667	0.5847	0.01575	-1.087	0.1089	1.179
70	Qatar	0.09286	0.5346	0.01199	-0.9456	0.09453	1.162
71	Mali	0.09178	0.5547	0.01116	-1.003	0.09281	1.172
72	Switzerland	0.08194	0.6196	0.01355	-1.144	0.08916	1.281
73	Chile	0.0664	0.6108	0.01497	-1.155	0.07265	1.244
74	Finland	0.0589	0.6342	0.01322	-1.171	0.06074	1.332
75	Scotland	0.04652	0.6061	0.01306	-1.145	0.04593	1.234
76	Zimbabwe	0	0	0	0	0	0
77	Iraq	-0.00557	0.5542	0.0115	-1.106	-8.45E-04	1.068
78	Haiti	-0.03306	0.5734	0.01394	-1.144	-0.03289	1.108
79	Trin. and Tobago	-0.04713	0.4835	0.01123	-0.9787	-0.04194	0.9029
80	Kuwait	-0.06396	0.5376	0.01091	-1.123	-0.06513	0.9575
81	Austria	-0.08772	0.6416	0.01373	-1.387	-0.07908	1.132
82	Slovenia	-0.09495	0.5938	0.01316	-1.275	-0.08144	1.065
83	Syria	-0.1211	0.572	0.01168	-1.237	-0.1099	0.9726
84	Fiji	-0.1378	0.7279	0.01544	-1.575	-0.1259	1.271
85	Venezuela	-0.1447	0.581	0.0131	-1.293	-0.1351	0.9987
86	Congo DR	-0.157	0.5797	0.01177	-1.323	-0.1526	0.9754
87	Kyrgyzstan	-0.1754	0.8293	0.02104	-1.805	-0.1761	1.446
88	Albania	-0.2019	0.6467	0.01537	-1.44	-0.1978	1.077
89	Northern Ireland	-0.2136	0.5969	0.01354	-1.403	-0.2006	0.9532
90	Gabon	-0.2251	0.6935	0.01571	-1.605	-0.2099	1.144
91	Slovakia	-0.2689	0.7163	0.01797	-1.685	-0.2585	1.125
92	Gambia	-0.3026	0.9302	0.02123	-2.141	-0.2921	1.547
93	Kenya	-0.3034	0.6279	0.01389	-1.547	-0.3003	0.912
94	Congo	-0.3158	0.6962	0.01589	-1.686	-0.316	1.026
95	Guam	-0.3491	1.134	0.02402	-2.598	-0.3615	1.891

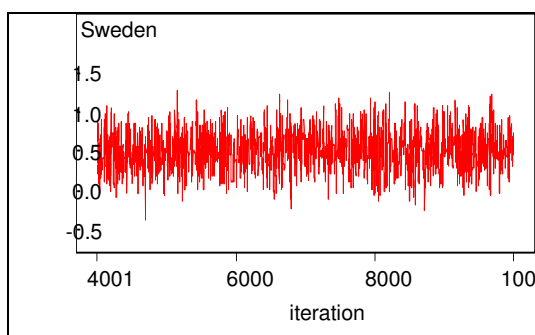
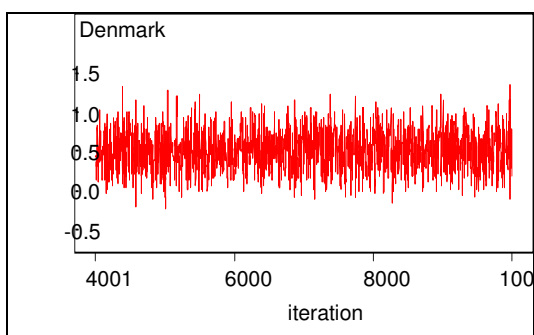
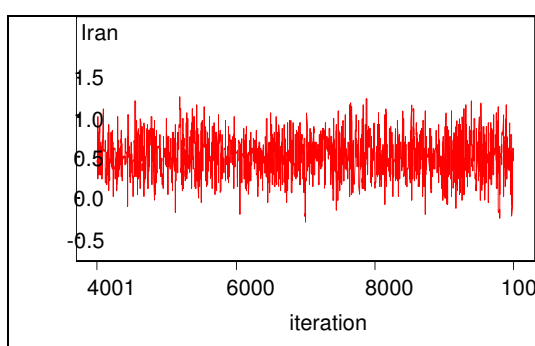
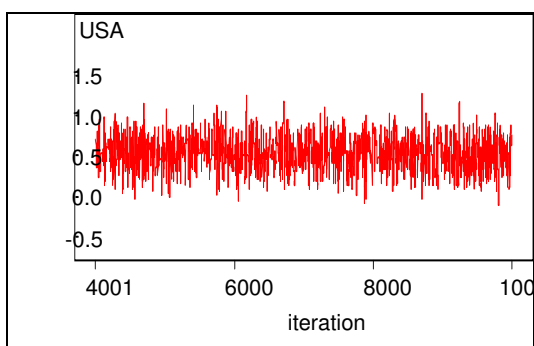
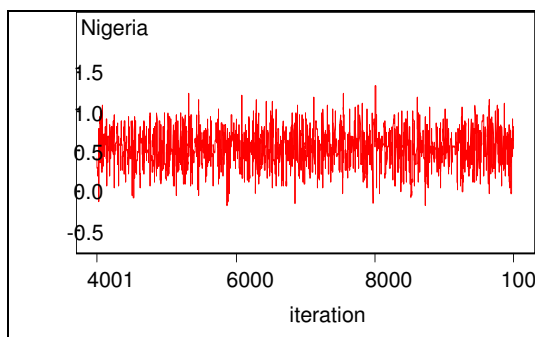
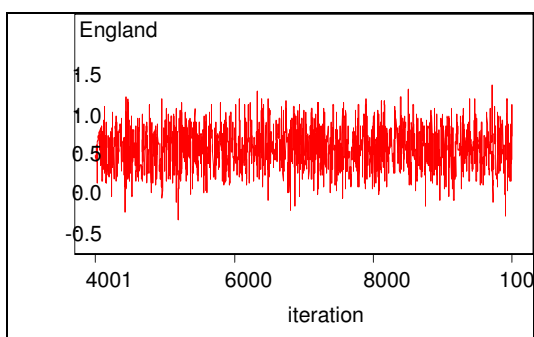
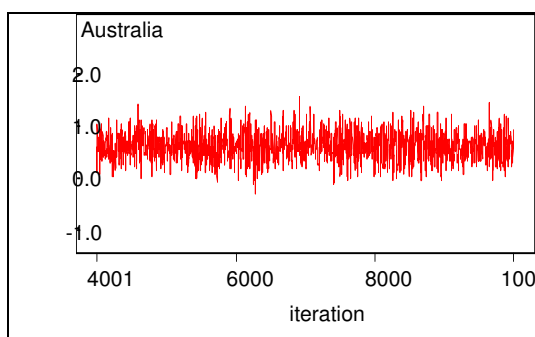
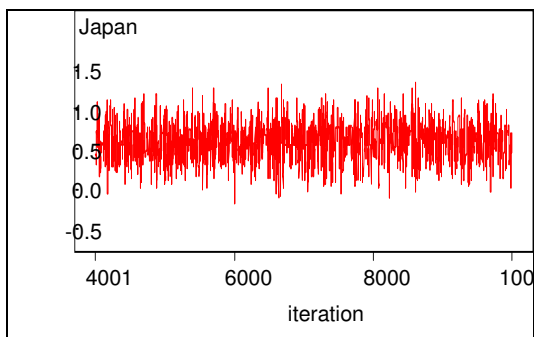
96	Bolivia	-0.3532	0.6376	0.01338	-1.612	-0.3419	0.8924
97	Latvia	-0.4079	0.5469	0.01005	-1.481	-0.4126	0.6641
98	Vanuatu	-0.4127	0.6566	0.01315	-1.674	-0.4206	0.9053
99	Liberia	-0.4363	0.7021	0.01672	-1.852	-0.4312	0.8744
100	St. Vint. & Gren.	-0.4401	0.63	0.01408	-1.694	-0.4333	0.805
101	United Arab Emir.	-0.4444	0.5565	0.01232	-1.541	-0.4458	0.623
102	Turkmenistan	-0.4707	0.6452	0.0154	-1.737	-0.4681	0.7972
103	Armenia	-0.5083	0.6598	0.0147	-1.794	-0.4958	0.7399
104	Netherlands Ant.	-0.5269	0.8463	0.0186	-2.13	-0.5442	1.164
105	New Caledonia	-0.5316	0.8743	0.01991	-2.211	-0.5362	1.186
106	Pap. New Guinea	-0.5386	0.8595	0.02249	-2.237	-0.5518	1.153
107	Namibia	-0.5561	1.013	0.02743	-2.489	-0.5726	1.412
108	Lebanon	-0.557	0.6114	0.01376	-1.759	-0.5605	0.6503
109	Korea DPR	-0.5573	0.6733	0.01262	-1.885	-0.5556	0.755
110	Rwanda	-0.5735	0.6903	0.01651	-1.935	-0.5721	0.7806
111	Burkina Faso	-0.5971	0.5959	0.01303	-1.778	-0.5991	0.5744
112	Iceland	-0.6676	0.6536	0.01465	-1.952	-0.6645	0.605
113	Cape Verde Isl.	-0.6926	0.7329	0.01782	-2.165	-0.6782	0.7256
114	Indonesia	-0.7033	0.6096	0.01351	-1.891	-0.7202	0.5055
115	El Salvador	-0.7399	0.5712	0.01373	-1.857	-0.7408	0.3508
116	Algeria	-0.7459	0.6041	0.01241	-1.964	-0.7405	0.4504
117	Sierra Leone	-0.7513	0.8561	0.01932	-2.429	-0.7516	0.9328
118	Botswana	-0.7649	0.6513	0.01335	-2.061	-0.7523	0.4791
119	Swaziland	-0.7861	0.6987	0.01594	-2.177	-0.7762	0.5754
120	Mozambique	-0.7868	0.6916	0.01361	-2.185	-0.7802	0.5616
121	Azerbaijan	-0.8089	0.6349	0.01659	-2.057	-0.7918	0.4108
122	St. Lucia	-0.8199	0.6628	0.01399	-2.145	-0.8194	0.4859
123	Dominica	-0.8247	0.7741	0.01738	-2.319	-0.8257	0.7103
124	Lithuania	-0.8363	0.7157	0.01444	-2.244	-0.8287	0.51
125	Estonia	-0.8536	0.584	0.01188	-1.984	-0.8621	0.285
126	Cyprus	-0.8937	0.6526	0.01369	-2.193	-0.8935	0.381
127	Moldova	-0.92	0.619	0.01417	-2.132	-0.9152	0.2766
128	Thailand	-0.9247	0.5917	0.01234	-2.08	-0.9254	0.2167
129	Madagascar	-0.9431	0.8829	0.02176	-2.665	-0.9421	0.8149
130	St. Kitts and Nev.	-0.9681	0.605	0.01504	-2.145	-0.9686	0.2249
131	Georgia	-1.014	0.6823	0.01469	-2.393	-1.001	0.2804
132	Lesotho	-1.017	0.8828	0.02452	-2.738	-1.031	0.7233
133	Vietnam	-1.032	0.7058	0.01633	-2.451	-1.023	0.3464
134	India	-1.098	0.7166	0.01571	-2.503	-1.088	0.2788
135	Malawi	-1.104	0.6598	0.01482	-2.401	-1.106	0.1573
136	Cook Islands	-1.161	0.8359	0.01906	-2.846	-1.164	0.4812
137	Barbados	-1.182	0.6661	0.01869	-2.521	-1.181	0.1066
138	Surinam	-1.188	0.7954	0.01734	-2.793	-1.183	0.3521
139	Malaysia	-1.208	0.6727	0.01451	-2.528	-1.198	0.1285
140	Bahamas	-1.377	0.9199	0.0228	-3.203	-1.366	0.4382
141	Sudan	-1.508	0.7293	0.01676	-2.937	-1.494	-0.08135
142	Palestine	-1.576	0.704	0.01615	-2.989	-1.568	-0.1992
143	Nicaragua	-1.614	0.7537	0.01547	-3.108	-1.592	-0.144
144	Dominican Rep.	-1.643	0.8766	0.02155	-3.411	-1.634	0.05599
145	Myanmar	-1.647	0.7901	0.01925	-3.232	-1.629	-0.09014
146	Singapore	-1.799	0.6807	0.01684	-3.177	-1.791	-0.5031
147	Yemen	-1.8	0.6145	0.01427	-3.022	-1.799	-0.613

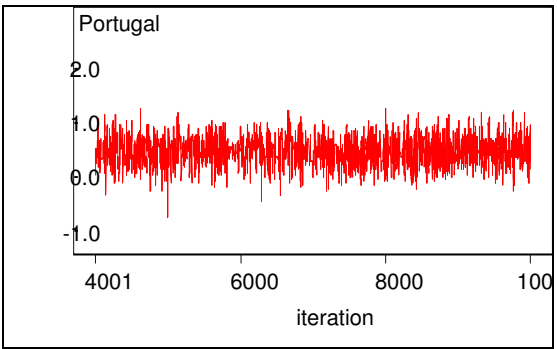
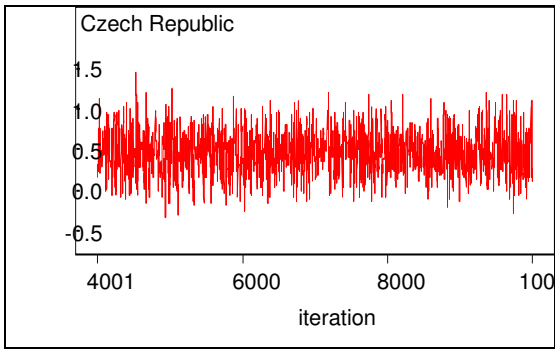
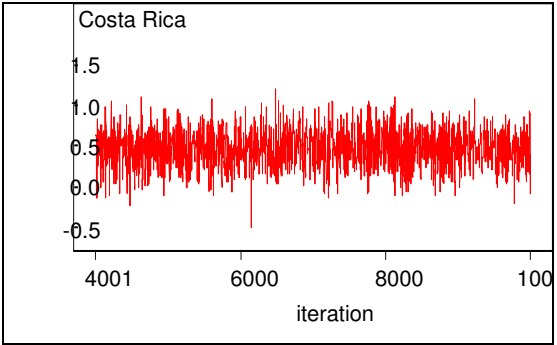
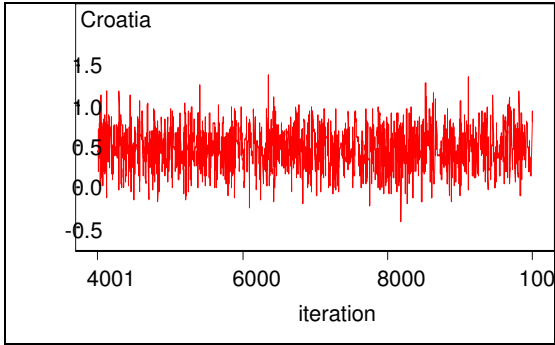
148	Kazakhstan	-1.816	0.8064	0.01818	-3.451	-1.819	-0.3091
149	Grenada	-1.874	0.7121	0.01623	-3.331	-1.87	-0.5047
150	US Virgin Islands	-2.153	0.794	0.01823	-3.732	-2.143	-0.6203
151	Samoa	-2.291	0.8337	0.01739	-3.951	-2.277	-0.6438
152	British Virgin Isl.	-2.336	0.7472	0.01667	-3.811	-2.336	-0.8712
153	Malta	-2.35	0.6978	0.01564	-3.766	-2.342	-1.038
154	Maldives	-2.423	0.7447	0.01669	-3.897	-2.422	-1.008
155	Andorra	-2.528	0.8126	0.019	-4.126	-2.517	-0.9451
156	San Marino	-2.568	0.7633	0.01839	-4.077	-2.554	-1.101
157	Liechtenstein	-2.653	0.7335	0.01616	-4.126	-2.639	-1.278
158	Luxembourg	-3.428	0.798	0.0188	-5.038	-3.406	-1.925

Appendix D – Non - Dynamic Model with Game Type Parameter

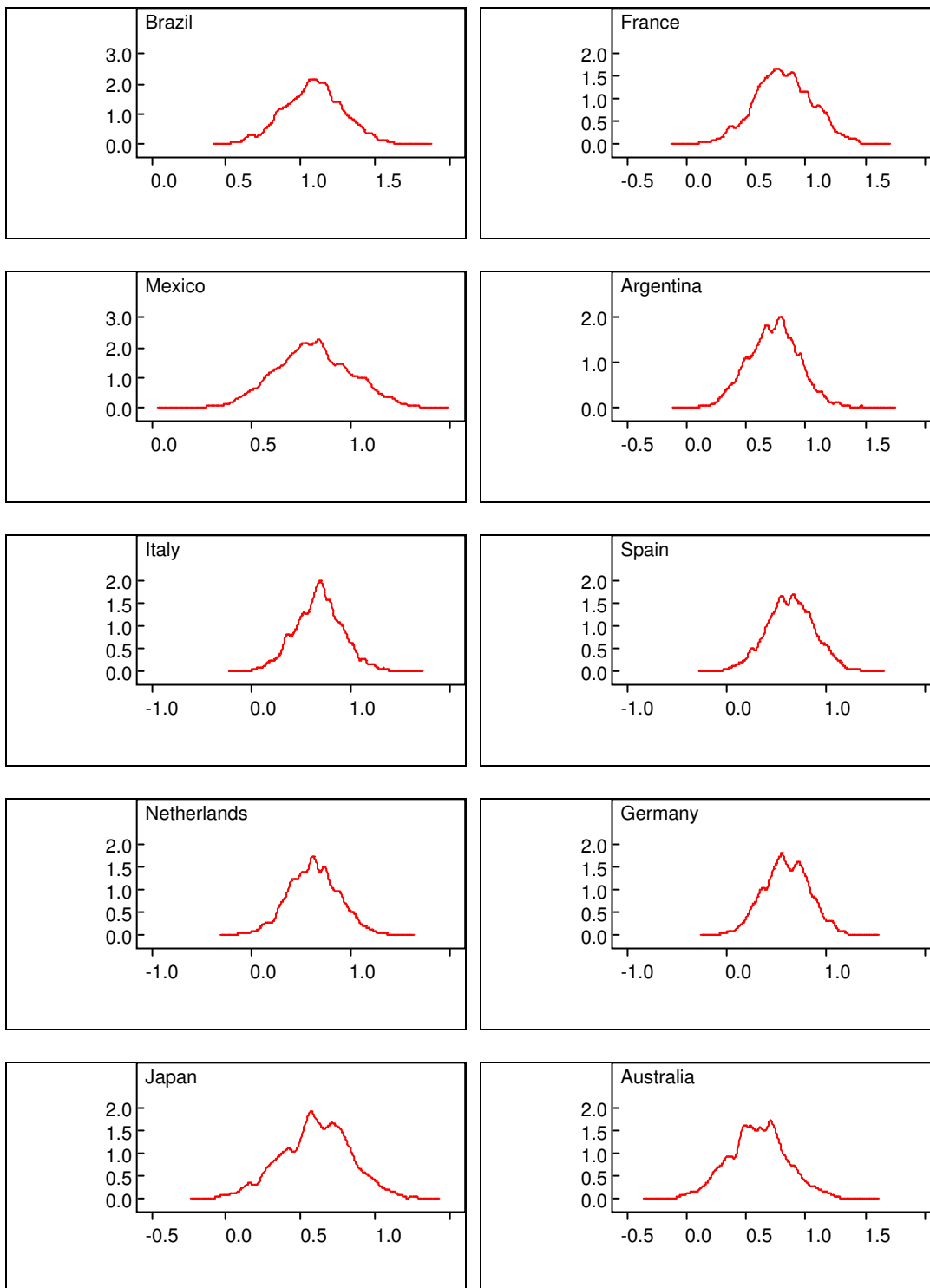
D.1 – Posterior Trace – Top Twenty Teams

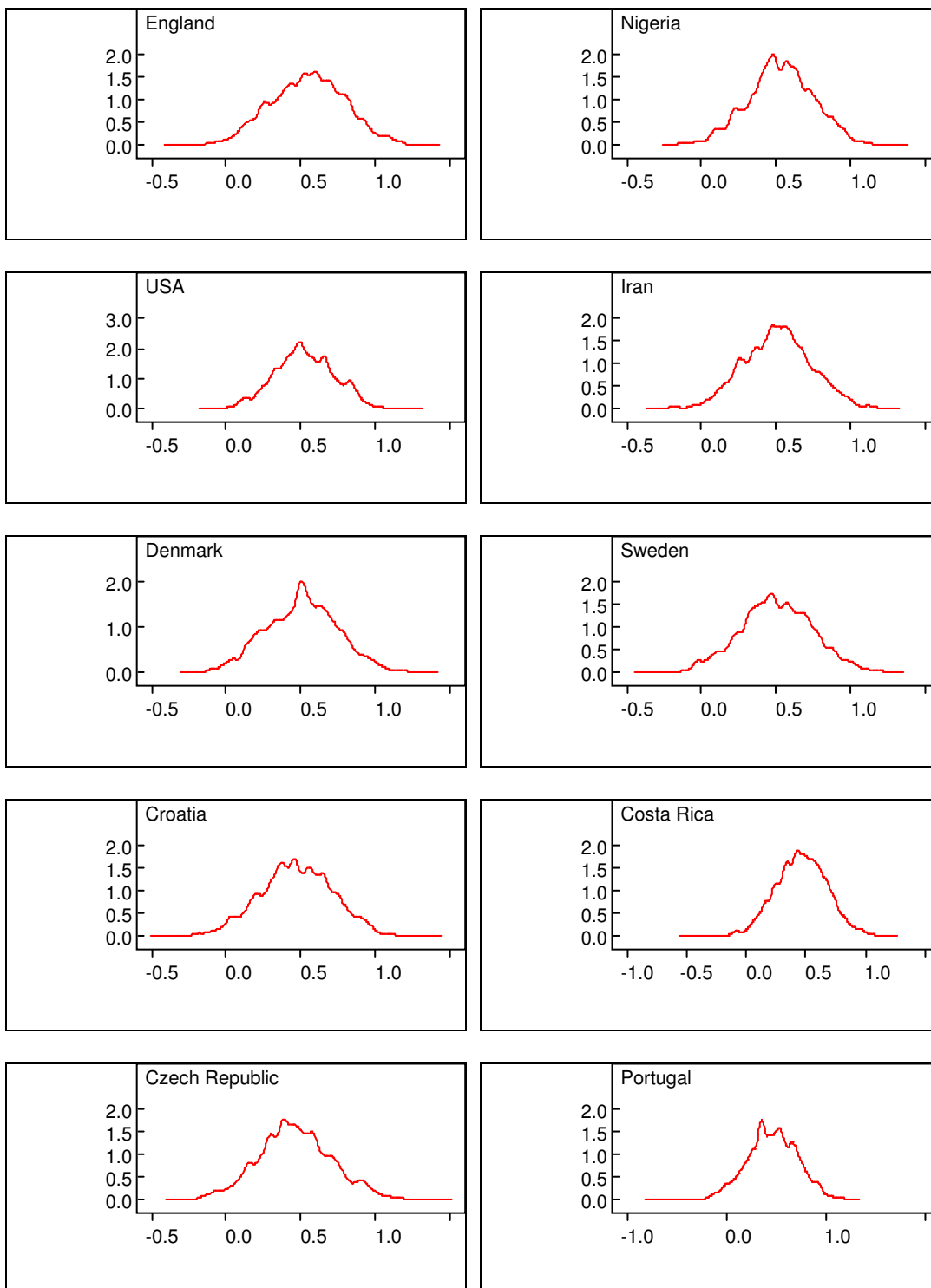






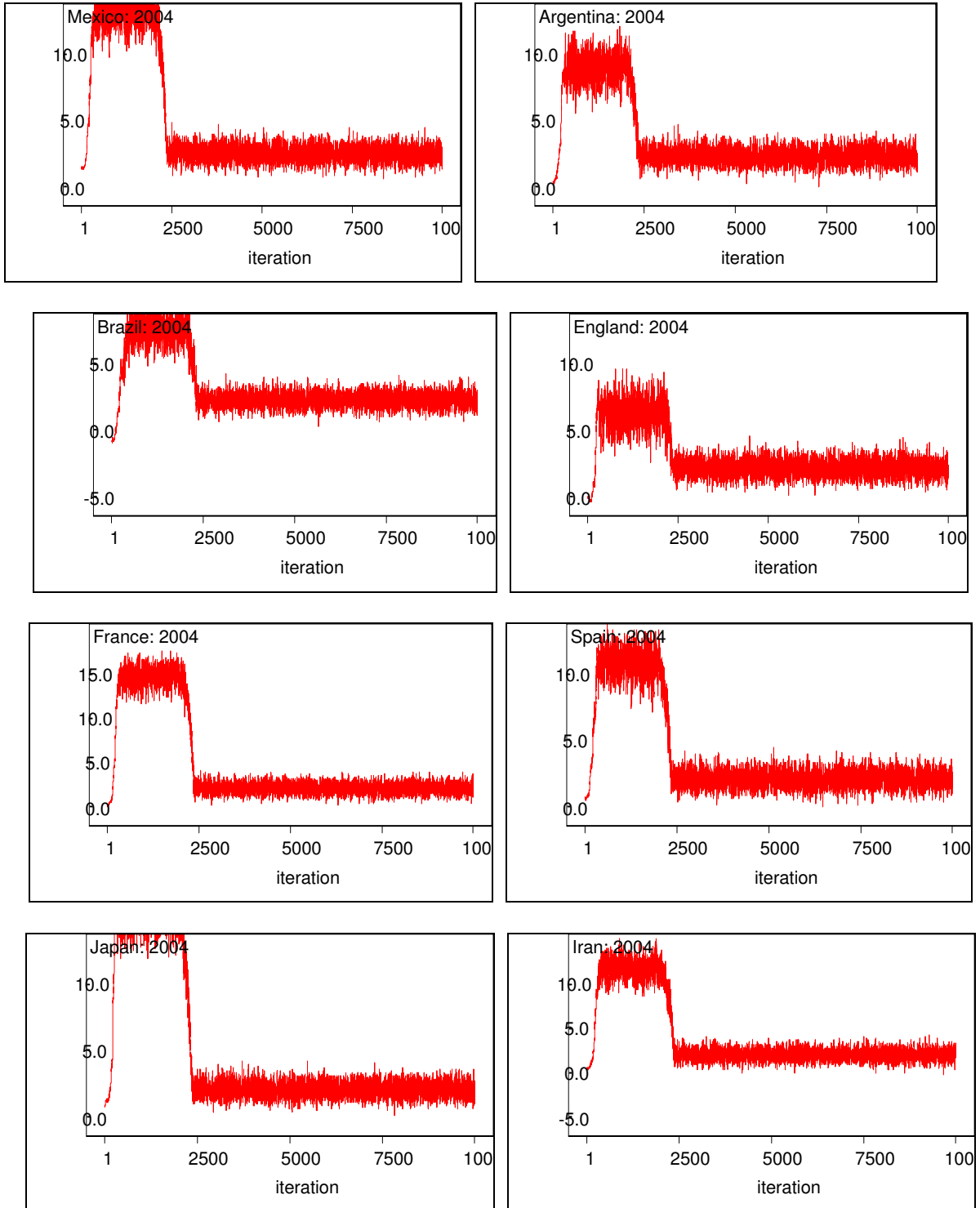
D.2 – Smoothed Posterior Density ($s = .15$) – Top Twenty Teams

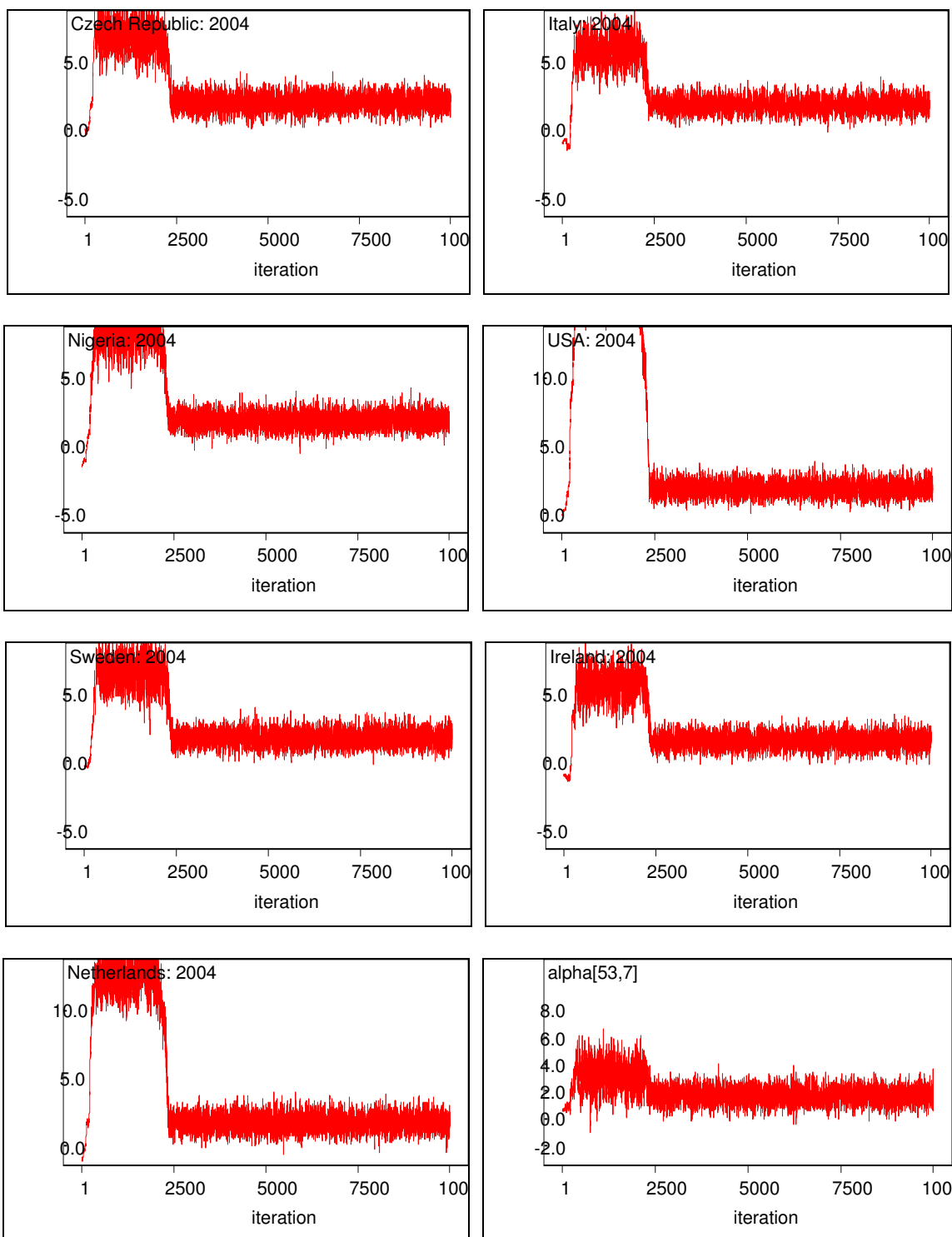


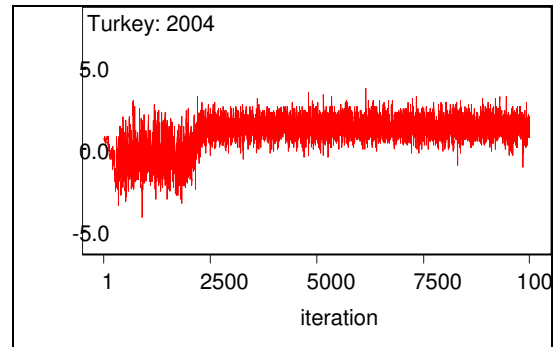
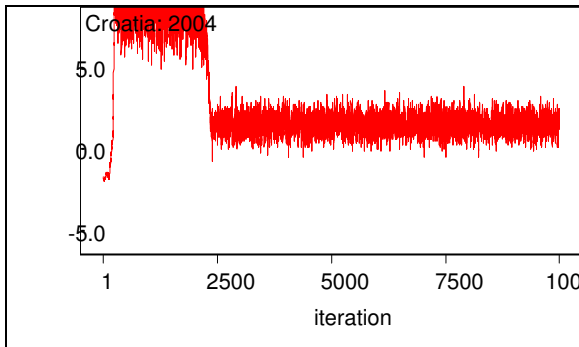
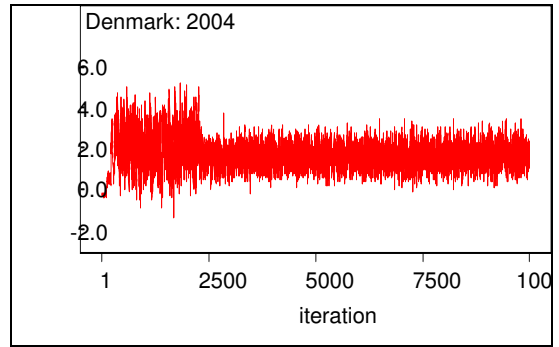
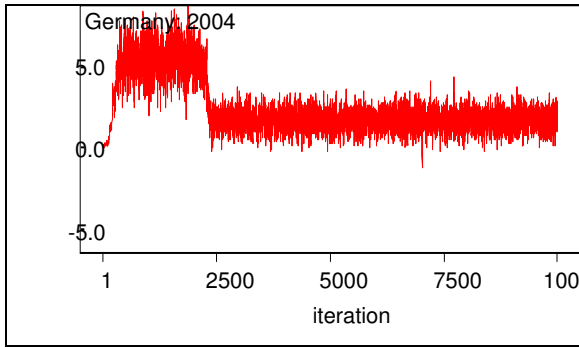


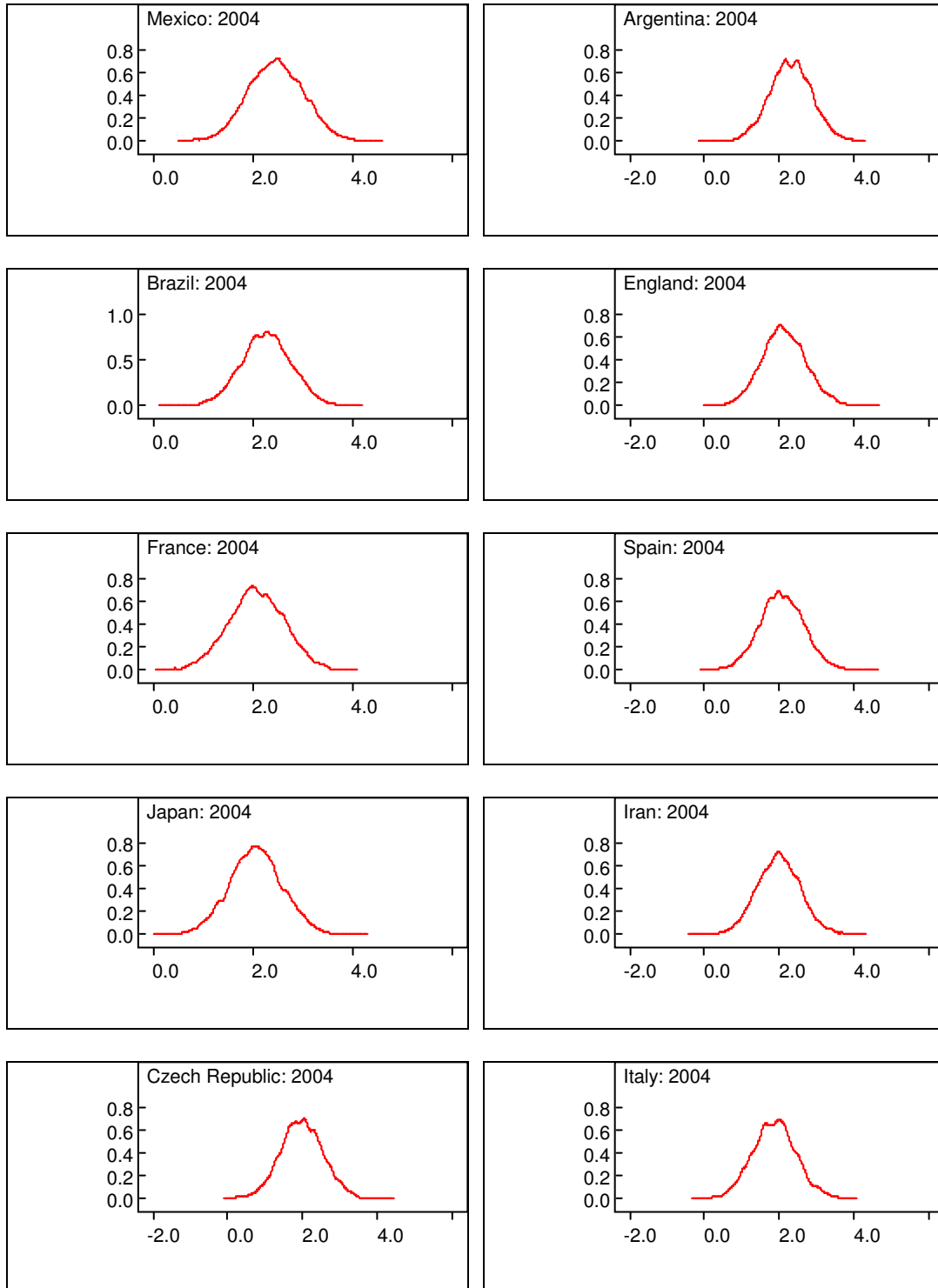
Appendix E – Dynamic Model

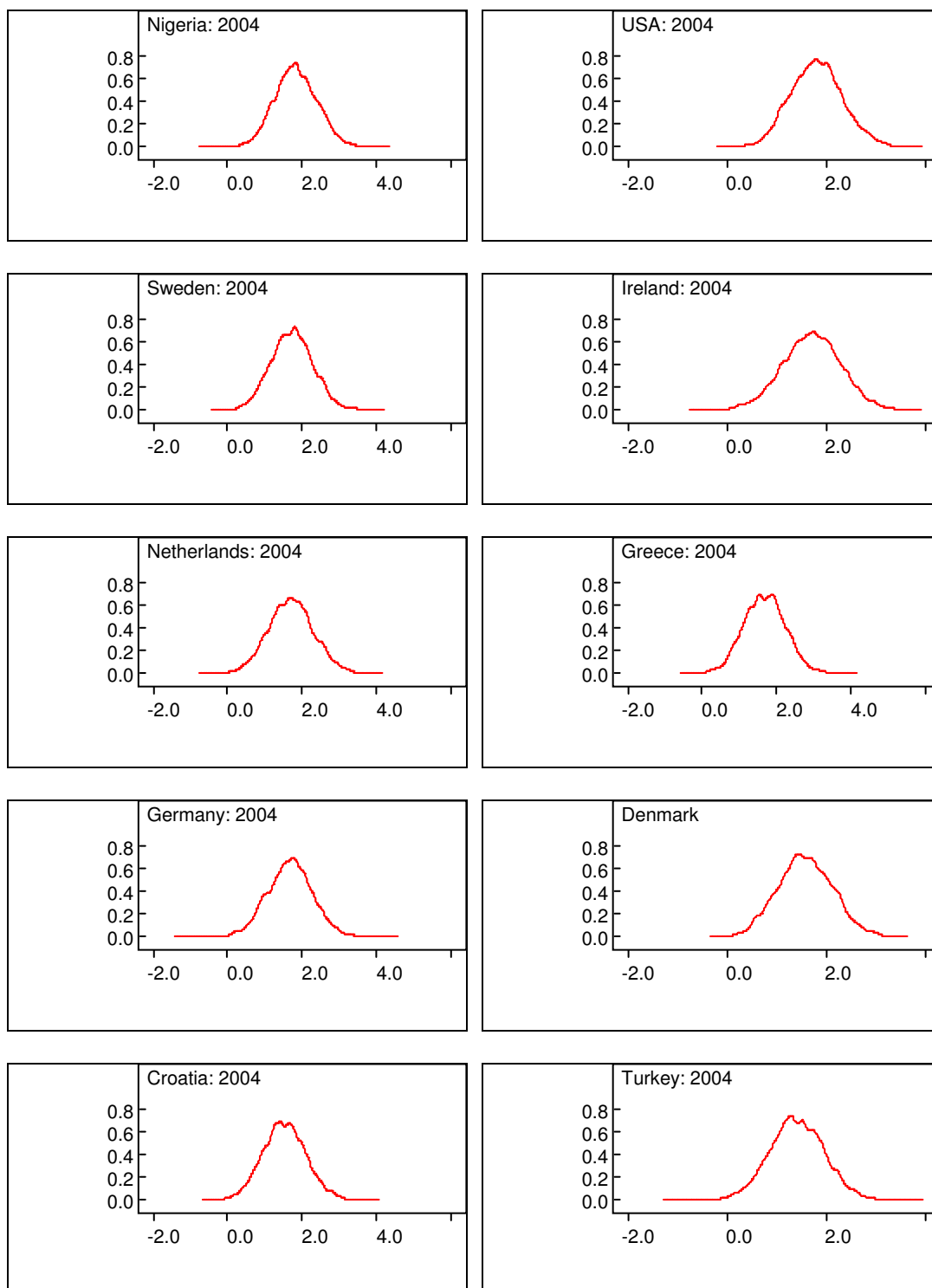
E.1 – Posterior Trace – Top Twenty Teams







E.2 – Smoothed Posterior Density ($s = .15$) – Top Twenty Teams



E.3 – Team Parameter Strength vs. Time

