Lagrangian Mechanics in a High School Environment

A Survey on Practical Applications for the Progression of Physics Education

A Major Qualifying Project
Submitted to the Faculty of
Worcester Polytechnic Institute
In partial fulfillment of the requirements for the
Degree of Bachelor of Physics

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Date: April 27, 2011

Table of Contents

Preface: Our Universe, The Jester

One of the greatest frustrations and realities that we face as physicists is that there is most certainly an answer to every problem; in many cases the problem is that we simply don't know how to approach finding the solution. This great truth, infuriating as it may be at times, is a revelation that many find to be their greatest inspiration. To know that our universe has been so kind as to have provided a solutions manual of sorts, written in the complex language of mathematics and the abstract forms it takes, is comforting. The responsibility falls to us, the readers of this tome of knowledge we call reality, to sift through and diligently piece together the cryptic temperament of existence. As time has passed, the puzzle has only grown larger; the more we uncover of our cosmos, the less knowledge we realize we truly possess.

Of course, we could never be handed such a terrifically large puzzle without more than one way of putting it together; there are undoubtedly multiple ways to solve every problem, branching from multiple facets of our field. Some prefer to solve puzzles by branching together the corner pieces, while others look for pieces from the same image. Likewise, a problem in thermodynamics can be solved using statistics, a problem in electrodynamics can be solved using solid state physics or quantum mechanics, and a problem in kinematics can be solved using just about anything. This "bigger picture" is something that, in passing down our knowledge from generation to generation, is often lost in the tedious nature of learning the foundations needed to progress further into the field. To further the issue, so *much* knowledge is now required to be able to progress the field that information must undoubtedly be passed down through the lower ranks of academia to ensure that when they arrive at the moment in which they must contribute something new, they know all that we know and more, and the novelty will lie solely in their imagination and their approach to the solution.

The premise of this experiment follows this pretense, that with each passing generation, more is expected of those that arrive on the steps of our institutions with the hopes of naming the next major theory, law, or particle after themselves. This reality that surrounds us, in all its grandeur and mystery, will go forever unsolved and unrealized if we do not continue to pass down everything that we know as fast as we find it. It is my sincerest hope that the result of this experiment, for better or worse, be taken into consideration as exactly what it preaches: an alternative solution to a truly intricate problem For although we can comfortably acknowledge that all we know of existence may be determined, predicted, and evaluated on paper through the complexities of mathematics and physics, the transmission of knowledge, of intelligence, and the awareness of we, the humble human element, is still one of most difficult problems mankind faces. Surely this is something we should address, as I intend to in this paper.

Thank you,

James Hopkins
WPI Physics 2011

Chapter 1 : A Brief History of Lagrangian Mechanics [With Derivation]

Lagrangian Mechanics is a reformulation of the Newtonian Mechanics that flourished as the dominant form of Classical Mechanics in the 18^{th} century. Derived by Joseph Louis Lagrange in 1788, the principles of conservation of momentum and energy were combined in order to simplify formulas and to ease calculations, which could become extensively difficult beyond the simplest of problems in Newtonian Formalism. Lagrange's work in analytical geometry, number theory and his re-formulation of Classical Mechanics shaped mathematical physics for the greater part of the 19^{th} century.

Lagrangian Mechanics allows the derivation of an object's equation of motion, determining the object's trajectory by solving the Lagrange Equations in one or two forms, called *Lagrange Equations of the First Kind*, which treat constraints explicitly as separate equations to be solved individually, or *Lagrange Equations of the Second Kind*, which considers the constraints a part of the equation by carefully choosing a generalized coordinate with which to consider the entire system. As we will go into further in a moment, the fundamental theorem of the calculus of variations allows us to solve Lagrange equations to show that the action variable is stationary, an essential proof in the derivation of a geodesic or any situation where we seek to determine our deviance from the path of least distance.

While Lagrange Equations of the First Kind are useful, the Second Kind are more prominent in physics, as a simple choice of coordinate allows us to simplify problems that would be incredibly difficult to solve using Newtonian Mechanics down to something much less taxing. Of course, without knowing the equations, this is not plain to see. Let us begin by reproducing Lagrange's derivation of his famous equations.

Deriving the Formula

The derivation of Lagrange Equations of the Second Kind begins from a pre-existing theorem, D'Alembert's Principle; also known as the Lagrange-D'Alembert Principle, shared between Lagrange and the French physicist Jean le Rong d'Alembert, which is a statement on the classical laws of motion. It states that the sum of the differences between the forces acting on a system and the time derivatives of the momenta of the system itself along any virtual displacement consistent with the constraints of the system is zero. This is the basis of the derivation,

$$\sum_i \left(\mathbf{F}_i - m_i\mathbf{a}_i\right)\cdot \delta\mathbf{r}_i = 0 \qquad \textbf{(1.1)}$$

where $\mathbf{F}_i$ are the applied forces, m_i are the masses of the system, $\mathbf{a}_i$ are the accelerations of the particles in the system, $\delta\mathbf{r}_i$ is the virtual displacement of the system, which keeps consistent with the constraints placed on the system, and the summation is over *i* particles in the system.

This principle is an incredibly flexible and dynamic formulae, and historically has been called more general than Hamilton's Principle, which formulates the principle of stationary action, avoiding the use of holonomic constraints entirely. The advantage of this is that holonomic constraints are dependent on coordinates and time, but not velocity, which we need to consider.

Thus to start, consider Newton's Second Law for a system of *i* particles:

$$\sum_i \mathbf{F}_i = m_i \mathbf{a}_i \qquad \textbf{(1.2)}$$

Or, for manipulation purposes, we can adjust the summation to a superscript to denote a total force:

$$\mathbf{F}_i^{(T)} = m_i \mathbf{a}_i \qquad \textbf{(1.3)}$$

Still one more small adjustment allows us to visually imagine the system in a quasi-static equilibrium:

$$\mathbf{F}_i^{(T)} - m_i \mathbf{a}_i = 0 \qquad \textbf{(1.4)}$$

Now, if we recognize that each of these elements, when paired with a virtual displacement, produces a virtual work, we still produce a zero identity, but we can move into the more dynamic topic of work, which correlated closely with energy.

$$\delta W = \sum_i^n \mathbf{F}_i^{(T)} \cdot \delta \mathbf{r}_i - \sum_i^n m_i \mathbf{a}_i \cdot \delta \mathbf{r}_i = 0 \qquad \textbf{(1.5)}$$

Fortunately this is as far as we need to travel into D'Alembert's Principle before we can turn towards where Lagrange began to pick up the derivation, in which he now assumes that we have a system of equations that hold, representing transformation equations of *m* independent generalized coordinates, q_j:

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{r}_1(q_1, q_2, ..., q_m, t) \\ \mathbf{r}_2 &= \mathbf{r}_2(q_1, q_2, ..., q_m, t) \\ &\;\;. \\ &\;\;. \\ &\;\;. \\ \mathbf{r}_n &= \mathbf{r}_n(q_1, q_2, ..., q_m, t) \end{aligned} \qquad \textbf{(1.6)}$$

If we wish to compact this into a more useable form, we can write this expression of virtual displacement, $\delta \mathbf{r}_i$ of the system for the exclusively time-independent constraints as

$$\delta \mathbf{r}_i = \sum_{j=1}^m \frac{\partial \mathbf{r}_i}{\partial q_j} \cdot \delta q_j \qquad \textbf{(1.7)}$$

where *j* denotes some variable corresponding to a generalized coordinate.

We can then express our applied forces in terms of generalized coordinates, as generalized forces, Q_j

$$\mathbf{Q}_j = \sum_{i=1}^{n} \mathbf{F}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} \qquad \textbf{(1.8)}$$

Now, if we combine the equations for δW, $\delta \mathbf{r}_i$, and $\mathbf{Q}_j$, and pull out the sum inside the dot product of the second term, we get the beginnings of an interesting result:

$$\delta W = \sum_{j=1}^{m} \mathbf{Q}_j \cdot \delta q_j - \sum_{j=1}^{m} \sum_{i=1}^{n} m_i \mathbf{a}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} = 0 \qquad \textbf{(1.9)}$$

From here we must look elsewhere in order to obtain the next step of the derivation, towards kinetic energy relations, where we can find a way to substitute a more useable term for the second element of that equation.

<u>A Brief Interlude into Kinetic Theory</u>

We begin with the simplest relation of a kinetic energy term:

$$T = \frac{1}{2} \sum_{i=1}^{n} m_i \mathbf{v}_i \cdot \mathbf{v}_\text{i} \qquad \textbf{(1.10)}$$

From here, we need to consider the partial derivative of this energy term with respect to the generalized coordinates we devised, and thus must employ the product rule,

$$\frac{d}{dx}\left(\mathbf{f}(x) \cdot \mathbf{g}(x)\right) = \mathbf{f}(x) \cdot \frac{d}{dx}\mathbf{g}(x) + \mathbf{g}(x) \cdot \frac{d}{dx}\mathbf{f}(x) \qquad \textbf{(1.11)}$$

By acknowledging that the dot product (in our case) is a summation of terms, and that both **f** and **g** are **v** for this function, the one half term disappears, and our result is

$$\frac{\partial T}{\partial \dot{q}_j} = \sum_{i=1}^{n} m_i \mathbf{v}_i \cdot \frac{\partial \mathbf{v}_i}{\partial \dot{q}_j} \qquad \textbf{(1.12)}$$

Now, if we consider that $\mathbf{v}_i$ is nothing more than the time derivative of the generalized coordinates $\mathbf{r}_i$ whose transformation equations we listed above, then by applying the chain rule to those functions we obtain an expression for $\mathbf{v}_i$:

$$\mathbf{v}_i = \sum_{k=1}^{m} \frac{\partial \mathbf{r}_i}{\partial q_k} \cdot \dot{q}_k + \frac{\partial \mathbf{r}_i}{\partial t} \qquad \textbf{(1.13)}$$

Thus, if we consider our defined values for $\mathbf{v}_i$, and the complete differential $d\mathbf{r}_i$, we see that

$$\frac{\partial \mathbf{v}_i}{\partial \dot{q}_j} = \frac{\partial \mathbf{r}_i}{\partial q_j} \qquad \textbf{(1.14)}$$

because for a function *f*

$$\frac{\partial}{\partial \dot{q}_k} f \cdot \dot{q}_k = f \qquad \textbf{(1.15)}$$

and also because there is only one $\dot{q}_j$ in the sum. In this form we can now substitute our original equations for kinetic energy into our result, which yields:

$$\frac{\partial T}{\partial \dot{q}_j} = \sum_{i=1}^{n} m_i \mathbf{v}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} \qquad \textbf{(1.16)}$$

Taking the time derivative of this function gives us

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \sum_{i=1}^{n}\left[m_i \mathbf{a}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} + m_i \mathbf{v}_i \cdot \frac{d}{dt}\left(\frac{\partial \mathbf{r}_i}{\partial q_j}\right)\right] \qquad \textbf{(1.17)}$$

which requires that we use the chain rule on the last term. This gives the expression,

$$\frac{d}{dt}\left(\frac{\partial \mathbf{r}_i}{\partial q_j}\right) = \sum_{k=1}^{m} \frac{\partial^2 \mathbf{r}_i}{\partial q_j \partial q_k} \dot{q}_k + \frac{\partial^2 \mathbf{r}_i}{\partial q_j \partial t} \qquad \textbf{(1.18)}$$

a rather messy expansion, but upon seeing that

$$\frac{d}{dt}\left(\frac{\partial \mathbf{r}_i}{\partial q_j}\right) = \frac{\partial \mathbf{v}_i}{\partial q_j} \qquad \textbf{(1.19)}$$

we can make the simplification in the last term such that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \sum_{i=1}^{n}\left[m_i \mathbf{a}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} + m_i \mathbf{v}_i \cdot \left(\frac{\partial \mathbf{v}_i}{\partial q_j}\right)\right] \quad . \qquad \textbf{(1.20)}$$

From here, things get messy for just a moment, and then simplify once more to our final result. If we now take the partial derivative of the kinetic energy term with respect to our generalized coordinates, we get our messy penultimate step:

$$\frac{\partial T}{\partial q_j} = \sum_{i=1}^{n} \frac{\partial\left[\frac{1}{2} m_i \mathbf{v}_i^{\,2}\right]}{\partial q_j} = \sum_{i=1}^{n} \frac{\partial\left[\frac{1}{2} m_i \left(\mathbf{v}_i \cdot \mathbf{v}_i\right)\right]}{\partial q_j} = \frac{1}{2}\sum_{i=1}^{n}\left[m_i \mathbf{v}_i \cdot \frac{\partial \mathbf{v}_i}{\partial q_j} + m_i \mathbf{v}_i \cdot \frac{\partial \mathbf{v}_i}{\partial q_j}\right] = \sum_{i=1}^{n} m_i \mathbf{v}_i \cdot \frac{\partial \mathbf{v}_i}{\partial q_j}$$

(1.21)

The final result comes from the symmetry of the two functions, which will ultimately obtain the same result, canceling out the one half term. We can now finally combine our last two equations to provide us with an equation for the inertial forces in terms of the kinetic energies of the system:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = \sum_{i=1}^{n} m_i \mathbf{a}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} \qquad \textbf{(1.22)}$$

This final result is what we will take back to our derivation of Lagrange Equations of the Second Kind

<u>Lagrange Equations Revisited</u>

Now that we finally have a familiar element in terms of quantities we can use, we can return to our original stopping point, which was an expression for virtual work:

$$\delta W = \sum_{j=1}^{m} \mathbf{Q}_j \cdot \delta q_j - \sum_{j=1}^{m}\sum_{i=1}^{n} m_i \mathbf{a}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} = 0 \qquad \textbf{(1.23)}$$

By replacing the second term (and sum) with the expression we derived, we arrive at

$$\delta W = \sum_{j=1}^{m} \mathbf{Q}_j \cdot \delta q_j - \sum_{j=1}^{m} \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} \right] \cdot \delta q_j \qquad \textbf{(1.24)}$$

We know that $\mathbf{Q}_j$ must hold for each term since δq_j is completely arbitrary (as we designed it). So our expression for $\mathbf{Q}_j$ can be reformulated as

$$\mathbf{Q}_j = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} \qquad \textbf{(1.25)}$$

Returning to our notation for the total inertial forces $\mathbf{F}_i$, we must consider, since our constraints are built into the equations, that the forces are conservative, and by extension can be written as a scalar potential field *V*.

$$\mathbf{F}_i = -\nabla V \Rightarrow Q_j = -\sum_{i=1}^{n} \nabla V \cdot \frac{\partial \mathbf{r}_i}{\partial q_j} = \frac{\partial V}{\partial q_j} \qquad \textbf{(1.26)}$$

Now, if we recall that the operator in question is the Lagrangian Operator

$$\mathbf{L} = T - V \qquad \textbf{(1.27)}$$

And that the potential is dependent only on position and not velocity, then we obtain the Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0 \qquad \textbf{(1.28)}$$

Or, if we leave it as the full operator:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = 0 \qquad \textbf{(1.29)}$$

This completes our proof.

Chapter 2: The Massachusetts Curriculum Frameworks

The Massachusetts State Frameworks for Physics students in the grades 8-12 are an outline of all the material and fundamental physics principles deemed "basic and essential" knowledge by the Board of Education. In its current state, it is a large, intimidating list of topics, covering very nearly the entire spectrum of the subject as we know it; classical mechanics, electrostatics, electromagnetism, quantum mechanics, nuclear physics, optics, waves and oscillations, heat, and fluid flow all make the list, with a number of subtopics within each point of interest to specify which principles to cover in each section. The Frameworks in their entirety may be found in the appendix, but the summation of the field it covers is a daunting 31 standards to teach to, categorized under 6 essential topics of physics: Motion and Forces, Conservation of Momentum and Energy, Heat and Heat Transfer, Waves, Electromagnetism, and Electromagnetic Radiation.

For a school year of just under 40 weeks, barring snow days or other delays, this is not a simple task to accomplish. Just recently, in 2009, changes were made in the sections of Heat and Magnetism, eliminating all of phase changes and moving magnetic systems, based primarily on various complaints from faculty across the state that there was simply too much to teach in one year. Teachers not charged with "teaching to a test" such as is expected for AP classes have taken to carefully choosing what they feel is the most useful material for students to know out of the list of standards, and teaching those, ignoring or brushing past less significant components. Even AP teachers, who are expected to teach according to material that will be on a standardized test, are breaking down the exam into how many questions are expected from each section, and strategically eliminating small segments from their teaching roster in order to cover the more important material. With knowledge of the difficulty that teachers are having with the vast amount of material that needs to be covered, this begs the question: why adjust the standards to include something *else*?

A Necessary Adjustment

In the process of acquiring schools for the project, teachers were naturally curious as to why this experiment was necessary to begin with, and the answer caught their attention. Any of the teachers on board with the experiment will agree that thirty years ago, high school students were most certainly not learning electrostatics, nuclear physics, quantum physics, or fluid flow. Most classes were limited to classical mechanics, and any chance of a calculus-based curriculum, or even a small subtopic there within, was out of the question because calculus wasn't *taught* in high school at that time. The takeaway from this anecdote is that the educational expectations have progressed with time. Information deemed "essential" has been handed down from the highest level of academia down through the ranks of graduates, undergraduates, and finally high schools, until we reach today's standards which expect much more of students than would ever have been made available to the public school systems thirty years ago.

As a result of this natural progression, further research in every field will eventually cause this cascade of information to pass through the educational system, which means that educators will need to find inventive ways to present not only the original material, but the new material as well. The next wave of material to be added to the Frameworks has not been announced, nor is there discussion of note on the topic in the Board of Education, but when the time comes, the topics voted on will have to be those that are the most easily absorbed by students, relatable to topics already in the Frameworks, and those most practical in use. While the Framework standards in other fields can be dissected in terms of practicality, such as the Mathematics standards which still insist on teaching synthetic division and y-intercept form, both obsolete by modern standards, physicists must be aware of all corners of their field. Thus, it is a difficult discussion of how to combine basic topics into useable forms.

Current methods include individually teaching all the subtopics of classical mechanics, such as kinematics, energy, momentum, and forces, and then introducing methods of solving problems from one subtopic using principles from another. The method proposed and tested in this paper is somewhat novel, suggesting that we simply combine multiple topics together and teach them as one; in short, introduce the necessary elements as they arise. Although the nature of Lagrangian Mechanics is such that it requires a brief background in calculus (i.e. the ability to integrate and take derivatives of simple functions with respect to specific variables), these methods can be taught as a precursor to the actual material. I feel that while there are many subtleties and intricate tricks that can be performed with derivatives and integrals, not all are necessary. While the purpose of the experiment and its suggested course of action are to progress the skill level of high school students, there are still methods and materials best left to higher academia.

Simply having the ability to take derivatives and integrals in their most basic form would be enough to give students access to an enormous quantity of topics previously out of their reach. Imagine, the relationship between position, velocity, and acceleration, instantly mathematically correlated. Charge and current, force and energy and momentum, all connected by this simple mathematical abstraction. Students would be able to be introduced to more advanced problems by simple association with this tool.

The premise, of course, is to use this precursor to bring about the introduction to Lagrangian Mechanics. While Newtonian mechanics is useful for solving component-heavy problems such as inclines or collisions, Lagrangian Mechanics can be used to derive an objects equation of motion with nothing more than a convenient choice of coordinate. Each has its merits, and I am by no means proposing that Newtonian Mechanics be phased out, but rather that, like other methods in other fields that have a less than broad range of uses, we simply pick and choose when to use it; a convenient choice of *approach*, if you will. With this mentality, students could use Newtonian mechanics to solve problems that are more convenient to solve in that realm, and Lagrangian problems to solve more advanced systems that would be difficult to solve using Newtonian mechanics. With the Lagrangian, the entirety of forces, momentum, energy, and even kinematics (when applied to a ball in free space and seen through to the equation of motion) can be introduced simultaneously, or one by one as a part of teaching the operator.

It is certainly expected that such a proposal would come under major criticism for suggesting that one of the fundamental methods of solving classical mechanics problems be picked apart for its practical applications, but the reality is that this is what the high school environment is forced to do; take what is most applicable, most easily absorbed, and most related to other curriculum standards, and integrate it into the Frameworks to further progress the expectations of education. The Lagrangian is the next logical step in the advancement of edification in the field of physics; the method by which we can take problems that range from slightly more advanced to considerably more advanced, and unite them under the same basic formula, while engaging students towards a more imaginative solution. This approach to solving classical mechanics problems requires not only the use of the equations, which is something any physics course will do, but it requires that you truly *understand* what is happening in the problem and consider the most convenient approach (generalized coordinate) with which to solve it.

A Novel Approach

Boasting that the Lagrangian can solve the issue of the overcrowding of education standards for physics is certainly a tall order, one that must be substantiated with some sort of explanation. Admittedly, the Lagrangian in and of itself is not sufficient to solve the problem, but rather the approach to education that can be introduced as a result of it. The Lagrangian is an amalgam of several topics of interest to the State Frameworks, although some of them are hidden in the equations. Momentum, Forces, Motion, and Energy all make appearances in the Lagrangian Formalism, and as a consequence of this, they can be introduced in any order the educator deems most efficient for their students, while uniting them under the same approach to a problem. The following example illustrates this.

A Particle in Free Space

The Lagrangian Operator is denoted by the difference of kinetic and potential energies of a system,

$$\mathbf{L} = KE - PE \qquad \textbf{(2.1)}$$

Where the energies are dependent on some generalized coordinate q and its time derivative (equivalent to velocity), denoted $\dot{q}$, or “q-dot”. In the Lagrangian Formalism, which allows us to derive an objects equation of motion based solely on a convenient choice of generalized coordinate where

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \qquad \textbf{(2.2)}$$

If we were to look carefully at what the formula becomes for an object in free space, we quickly see that our Operator is simply

$$\mathbf{L} = \frac{1}{2}mv^2 - mgh \quad \textbf{(2.3)}$$

Which makes our generalized coordinate *h*, since it represents our position coordinate in this problem. We can certainly solve this if we know how to take partial derivatives. Remembering that for a simple derivative of a function of the form

$$f(x) = x^n \quad \textbf{(2.4)}$$

its derivative takes the form

$$\frac{df}{dx} = nx^{n-1}, \qquad \textbf{(2.5)}$$

And also recalling that for a partial derivative, the same algorithm applies, but only for the variable specified by the derivative operator, and that all other variables are treated as constants, then the partial derivative is very simple:

$$\frac{\partial L}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}mv^2 - mgh\right) = \frac{\partial}{\partial h}\left(\frac{1}{2}mv^2 - mgh\right) = -mg \qquad \textbf{(2.6)}$$

This is simply the object's weight, and therefore this particular derivative represents a *generalized force*. From here we recognize fairly plainly that *force is the derivative of energy with respect to position*.

The next derivative is the inner derivative of the right side:

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial}{\partial \dot{q}}\left(\frac{1}{2}mv^2 - mgh\right) = \frac{\partial}{\partial v}\left(\frac{1}{2}mv^2 - mgh\right) = mv \qquad \textbf{(2.7)}$$

Which is quickly recognized as a simple expression for momentum, and we therefore call this derivative the *generalized momentum.* If we take the time derivative once more, for a system in which mass is not changing, then velocity is the only variable affected by a time derivative, and thus the equation becomes

$$\frac{d}{dt}(mv) = m\frac{d}{dt}(\mathrm{v}) = ma \quad \textbf{(2.8)}$$

Or more simply put, Newton's Second Law. The full Lagrangian Formalism now reads

$$-mg = ma \qquad \textbf{(2.9)}$$

Which outright states that for a particle in free space, its motion is governed solely by the only force acting on it, its weight. In just this portion of the derivation, we arrived within the realm of Forces, Energy and Momentum, all because we were able to take a simple derivative. The next relation is slightly trickier, but involves no more than the remembrance of the simple algorithm of an integral.

For a function of the form

$$f(x) = x^n \qquad \textbf{(2.10)}$$

The integral of the function is simply

$$\int f(x)dx = \frac{x^{n+1}}{n+1} + \mathrm{C(x)} \qquad \textbf{(2.11)}$$

Where C(x) is some constant that depends on *x*, of whatever variable the function is being integrated with respect to. If we then choose to integrate our result from the Lagrangian Formalism with respect to time, so as to see how the system's position will change with time, we begin to recover very familiar formulas:

$$\int -mg \cdot dt = \int ma \cdot dt \qquad \textbf{(2.12)}$$

or

$$\int -g \cdot dt = \int a \cdot dt \qquad \textbf{2.13)}$$

Since we are integrating with respect to time, we include the bounds of the integral as well:

$$\int_0^t -g \cdot dt = \int_0^t a \cdot dt \qquad \textbf{(2.14)}$$

By recognizing that $a = \frac{dv}{dt}$, and making this substitution:

$$\int_0^t -g \cdot dt = \int_0^t \frac{dv}{dt} \cdot dt = \int_0^t dv \qquad \textbf{(2.15)}$$

Which leads to the result:

$$-gt + v_0 = v(t) \qquad \textbf{(2.16)}$$

This is simply the formula to find the velocity of an object in free fall, from basic kinematics. The v_0 term is the constant that results from the integration, which must depend on velocity. Already we have recovered one of the familiar equations of motion, but we can recover still one more if we integrate again:

$$\int_0^t -gt \cdot dt + \int_0^t v_0 \cdot dt = \int_0^t v(t) \cdot dt = \int_0^t \frac{dx}{dt} \cdot dt = \int_0^t dx \qquad \textbf{(2.17)}$$

We have made the connection that velocity is the time rate of change of position, and made a substitution similar to that made in the integration of acceleration. As a result of this, we acquire the formula for position with respect to time:

$$x(t) = x_0 + v_0 t - \frac{1}{2} g t^2 \qquad \textbf{(2.18)}$$

The constant this time is a result of the integration with respect to position, and the result is one of the major equations of kinematics, which describes an object's position in free fall. The generalized version of this is more recognizable:

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad \textbf{(2.19)}$$

Thus we see that from the simple equation of the Lagrangian Formalism, four topics of interest to the State Frameworks can be introduced with little issue. Teachers can choose whether to teach the derivative first (if necessary), or whether to introduce the generalized coordinates, velocities, forces, and momentum in some order that best suits them. In any case, although unorthodox, the Lagrangian Formalism consolidates a number of topics into one applicable, dynamic and imaginative function, while teaching the principles that govern each. Upon completion of this introduction, teachers might find it a suitable time to cover the missing principles such as conservation of energy and momentum, or the component method of resolving vectors, which are not explicitly introduced by the function. From this we see that the Lagrangian offers a unique and practical introduction to four subjects in an environment that displays its worth and flexibility, a quality that students desperately need in a time where, if something isn't useable in the world they live in, it must therefore be useless.

Chapter 3: A Look at Pre-existing Programs, their Merits, and Drawbacks

As any good experimentalist will attest to, a theory cannot be proven unless several methods of analysis are used to determine its worth. First and foremost, an experiment must be devised to provide tangible, quantifiable data, which must then be analyzed and conclusions drawn there from; clearly that is what the purpose of this paper and the experiment are for. Second, it must be compared to all other experiments that have been performed prior, and effectively picked apart for loopholes. Analogous to this experiment, we must look at other programs that have attempted to either introduce new, more difficult material into the high school environment, or to have taken a novel approach in general to the current curriculum. There are a number of educational programs, which have gained strength in the past years, which are certainly worth looking at.

Physics First

The first (no pun intended) program, and the most notable for its movement throughout the country as an alternative approach to physical science education is Physics First, a program which quite literally suggests turning the current physical science curriculum around entirely, and teaching the more mathematically-based classes to freshmen, and the generally less mathematically-based classes to the seniors. The organization seeks to improve student understanding of biology and chemistry based on the notion that by exposing students to physics in their freshmen year, the experimental, more tangible nature of most introductory physics courses will lead students to grasp physical concepts more naturally, allowing for a better conceptual understanding of the less intuitive subjects of biology and chemistry, which are not so easily studied experimentally as physics is in high school.

The program's tentative schedule places a freshman in an introductory physics course, learning about kinematics, forces, energy, momentum, waves, optics, heat, electromagnetism, and the basics of circuitry, all with very minimal mathematical application, expecting students to have their pre-algebra and algebra I skills intact from middle-school. Following this, sophomores are brought into chemistry, where they are introduced to more physics-based concepts than would a typical high school chemistry class. The junior then follows into either biology or a more advanced physics course. The biology course draws from the freshman and sophomore years and dedicates time to more advanced, interrelated concepts with the pretense that students could now make more complex conceptual connections between topics, having seen all of it before. The physics course is simply a more advanced mathematical version of the introductory course, expanding the previous material into its mathematical equivalents and using the students' now more advanced math skills to elaborate topics that were previously left unexplained.

While the premise of the project is an ambitious and certainly novel approach to the current educational system, it fails to address a number of issues. The first, and most important, is that it fails to address the ever-increasing flow of "basic information" which is trickling slowly downstream from college graduates all the way down to high schools, which will inevitably make its way into the system. As such, the adjustment of the curriculum has no bearing with respect to that consideration. In fact, the adjustment of the curriculum in this direction is almost a hindrance in consideration of the second item it fails to address. While it is encouraging and moderately useful for students to understand the conceptual aspect of physics, it is also a whimsical daydream to expect that this is enough. Physics is based on two principles: conceptual understanding and mathematical application. Without the first, students can never hope to grasp the subtleties of the second, and without the second, physics becomes nothing more than a philosophical discussion of what would happen if X interacted with Y in Z ways", which is useful in theory, but nothing more.

Our generation has reached a fundamental understanding of the topics at hand, and acknowledging nothing more than a conceptual understanding in the beginning, without at least insisting that it be followed up with a mandatory mathematical expansion, is to doom students who take the path of the physics major to a painful, if not impossible, freshman year in college. The aim of secondary education is to provide students with a solid, working knowledge of a broad range of materials that can carry them smoothly into higher education, and while Physics First places an admirable emphasis on chemistry and biology by teaching the physical principles that govern them first, it neglects the science which initially *requires* a more rigorous mathematical background.

I concede that every physical science, upon reading the higher levels of education, is strongly rooted in complex mathematical theory; such is the nature of nature. However, the theories and principles of chemistry and biology are subjects that require, without question, that students have a firm visual and conceptual understanding of what is taking place, most of which can easily be taught in the high school level without much math at all. Physics, however, makes the unique connection with mathematics and conceptual understanding, that frequently to understand one is to understand the other, and vice versa. This distinct quality of the subject makes it difficult to consider downgrading the mathematical quality of a physics class in high school, for fear of detrimental effects on the students' understanding of the material.

Take, for example, the topic of DNA to mRNA transcription. Even on the college level (freshman and sophomore classes), the highest level of mathematical skill necessary is the ability to multiply, divide and add by pairs of 3 (amino acid and nucleotide chains). Most introductory classes on biology, environmental biology, and biotechnology, don't travel much farther into the mathematical basis of the topic, but rather seek to lay conceptual foundations of what *exactly* goes on inside a cell. An introductory physics course, on the other hand will, at the very *least* require that students be able to solve algebraic equations, simultaneous equations, have a working knowledge of trigonometric functions, and be able to work in multi-step processes to acquire a solution. Calculus based physics courses will also expect students to be able to quickly pick up laws and principles of mathematics such as integrals, derivatives, dot products, cross products, and gradients. Although students may not necessarily be expect to have these skills at the start, the ability to quickly pick up and apply mathematical principles is one that is (as well it should be) taught in high schools, and is best applied in physics classes, under similar environments.

Any program which seeks to better the educational experience and subsequent transitions into higher education of students, is admirable and worth considering. The issue taken with Physics First is simply that is leaves several necessary considerations in the dark. Were the program to consider mandating the second year of physics, so that students are never left with the choice of abandoning the important mathematical skills required to *use* physics, then it would certainly warrant further reflection. Without this key element, however, it leaves the necessary tools of functional physics in the dust.

Four Years of Science

Alternative arrangements of the Physics First program have arisen which warrant far more merit, as they do exactly what is suggested above. A number of educators in administrative positions are calling for a law in Massachusetts that mandates not only 4 years of English in high schools, but 4 years of science as well. This would open schools to the potential of two full physics classes, one that could certainly teach the concepts, and another that could expand in to the mathematical background. As still another approach in light of the 4-year requirement, but still maintaining the prospect of two physics courses, the possibility arises of having *both* physics course be mathematically based, with still more options available as to what to do with them. One consideration may be to expand the current physics courses outward time-wise, spending more time on the current Framework Standards and potentially laying a calculus based introduction on each topic as a way of solidifying more advanced material. A second notion might simply use the first class for the current curriculum standards, and the entire second class for more advanced material, such as Lagrangian and Hamiltonian Mechanics, Maxwell's Equations for Electrodynamics, further discussion on magnetic fields (which have been significantly displaced from the Curriculum Standards, despite being so very important), or potentially even wave functions from Quantum Mechanics (without using differential equations).

The prospect of having two years to teach physics to students is a beautiful idea, but it also places constrictions on other classes. While physics is arguably the more mathematically rigorous (especially in high school), I am quite certain that the biology and chemistry teachers might feel a little upset to have been pushed out of their opportunity for a second class; in short, the idea may be misconstrued as favoritism. While it is not my interest to consider the political and bureaucratic mess that usually entails the public educational system (I once encountered a teacher who had been asking the administration to fix the phone jack in his classroom for ten years. Ten *years.*), in order to accurately consider the merits and drawbacks of this program, we must be realistic, and unfortunately that is part of the reality.

There is still one more unique risk that becomes apparent in mandating the four years of science, specifically with mandating the four years with two years of physics. Historically, physics in the high school environment has acquired certain notoriety for being a particularly difficult class, both conceptually and mathematically. As such, without the class being mandated, statistically students, who are, at the very least, slightly interested or slightly motivated in the subject, take the class voluntarily. The subsequent results of this voluntary class are twofold: students who take the class are generally more likely to participate, to listen, and to manage themselves appropriately due to their self-incentive in the class, and secondarily, the absence of students who are disruptive due to their disinterest in the class provides an environment far more conducive to learning than in their presence. A removal of these perks through the mandating of four years of science (and potentially two of physics) could pose a difficulty for educators in terms of class management, and could hinder the learning experience of students who are making a genuine effort.

Ultimately the call from educators for a mandated four years of science does not immediately call for the two years of physics that I am suggesting; this is merely a speculation and an alternative application to the idea. The fourth year has not been considered as to its subject matter, but I feel that, were the law to come to fruition, that a subsequent argument would begin within the science department deciding which class would receive the fourth year. Surely I am not the only one to ask for more time with their students (if I am, then the educational system has truly lost its way), and the idea for Four Years of Science, while overall worth the consideration, still have developmental flaws to fix before it can be fully executed.

<u>Modeling Instruction in High School Physics [Jane C. Jackson, Arizona State University]</u> [1]

Jane Jackson's tutorial and explanation of the Modeling Instruction method is one of the more compelling alternatives to current physics education. The program emphasizes two major additions and modifications to a typical curriculum: increased teacher-student interaction in order to facilitate a more comfortable environment for the students, and project-based learning for the purpose of debunking "impetus", or naïve thinking [2] . The first intention of the program is actually a consequence of the second, but both are equally important in achieving a higher efficiency of learning on the part of the student.

The typical high school environment makes a very clear line of distinction between the authority of the teacher and the authority of the student, namely that the teacher has the authority and the student has none. Doing this, at a time in most high school students' lives where they are beginning to become aware of themselves and their role in society, can cause a resentment towards the teacher, as the student is at a moment of questioning what constitutes the grounds for one's authority in society, and may thus challenge the teacher for this role. A safer alternative that is far more conducive to cooperation is something similar to a vocational school, in that the teachers and students work side-by-side on a project, with expert guidance from the educator, but without direct instruction. The integration of the projects as part of the curriculum allows for this cooperative environment and opportunity for the instructor to act as a "coach" rather than an authority figure.

The nature of the projects is nature itself, aiming to explain the inner workings of the basic branches of physics by means of demonstration and self-developed methods of testing. Emphasizing the computer as "an essential educational tool", the program suggests using modern software and equipment as a means of helping students perform labs and projects that allow them to translate and quantify the physical aspects of nature into the language of mathematics, providing them with the ability to derive their own equations. In contrast with other programs, or even vocational schools, the instructors do not deny students the ability of their experiments to fail; in fact it becomes an integral part of the explanation if they do. Several methods of thought are inevitably developed when students are allowed to design their own experiment to test a theory, and a number of them are selected for presentations to defend that theory and the subsequent results.

The benefit of this exhibition for the students in groups is myriad. For those that fully understand the material, it is a chance to gain experience with explaining their findings in a more articulate manner than they would normally expect, with the teacher questioning their methods to probe their understanding. For those in a group with some understanding and some confusion, the probing seeks to identify unique difficulties, and using Socratic Dialogue, guide them to the solution of their error. Finally, for those in a group in which no one understands, the method allows the instructor to guide the entire group through a process by which to understand the flaw in their method.

The success of the program is cause enough to praise this program for its innovative thought process, as several other programs attempted this method, but with limited success due to the diluted attempts that were made. In the context of teaching the current Frameworks, this is a method that provides actual experience as a method of phasing out misunderstandings of the physical world with proper models and experiments to develop the ability to see the world as it really is. While the program does not address in the incoming need to expand the Frameworks, most programs rarely do, and this one, at the very least, addresses the efficiency with which we teach what we currently have.

<u>Physics By Inquiry, Tutorials in Physics [Lillian McDermott, Paula Heron] </u>[3]

Lillian McDermott and Paula Heron's paper (and subsequent PowerPoint) on Tutorials in Physics is an informational guideline for teachers addressing the ever-growing issue of large introductory classes in college. The paper itself isn't a novel approach to teaching, but it offers an approach to managing large classes that is oddly similar to WPI's current methods. In the face of campuses much larger than our own, introductory, mandatory classes for a variety of technology and science related majors can rise to well over 1000 students, resulting in packed lecture halls in several sections, several times a day. The suggestion made by McDermott and Heron is that, by addressing some of the strains and constraints placed on students in these situations, and varying the teaching methods over a wide spectrum, a larger population of the students can be reached at a greater efficiency.

Upon first reading the proposed curriculum in this paper, I was confused, as it was proposing something that so closely resembled WPI's method of presenting introductory courses, I could hardly call it novel, or even non-intuitive. The realization eventually dawned on me that not every college incorporates this approach, although from experience I believe they could surely benefit from it. The method suggests teaching several sections of the same lecture in order to break up the enormous student body and prevent general management issues. In addition, several times a week the students should meet in even smaller conferences, to work in still smaller groups, with a team of TA's and professors, to address one-on-one difficulties, as would be expected of something more like a high school environment.

The transfer of the comparatively small high school classroom into universities and institutions that accept students from all over the world, instead of being mandated by district laws, is a difficult one; one which must be addressed with care towards the students, rather than the professors. The paper makes mention that the faculty on hand "are conscientious teachers, but many are heavily engaged in research", which typically makes them less accessible than incoming freshman are wanting of. One of the issues that the paper doesn't make particular note of that freshman (and, admittedly, sophomores and even juniors) are quick to complain about is that TA's are often several years removed from the material that the student is engaged in, and is thus not able to relate the topics in the same context that the professor has been teaching it in. As a result, students often find themselves more confused than at the start, having taken in two explanations of the same topic, and understanding neither.

The authors, of course, can't be blamed for the drastically varying competency of the TA's from college to college, or even the realism of the availability of professors. The model presented in the paper is an ideal one, one which we can certainly all hope for, and strive towards, but ultimately we must concede that the model is not, at this time, reachable within the context of human variance. The suggestions in their own right, however, are certainly effective, and if we move past the premise of difficulty within the human element, the overall effectiveness on students trying to grasp new material is very impressive, as is clear, at the very least, here at WPI.

Socratic Dialogue-Inducing Labs [Richard Hake] [4]

One of the fundamental methods of the transition from “teach by telling” to “teach by understanding” is to lead the student to the answer by subtle questions and analogies; in other words, Socratic Dialogue. The premise of Richard Hake’s paper on Socratic Dialogue Inducing Labs is to tap into this principle, using thought-provoking lab experiments to bring students to a full understanding of the material by means of interactive, engaging research. Richard Hake had worked for nearly 30 years primarily as a researcher in a number of branches of physics, and came to realize that his ability to communicate with those many years behind him in terms of experience was lacking; he found that his work had taken his vocabulary and understanding to the point of obscurity and esotericism. Thus, he gradually began developing a program to help him, as well as future educators, convey the rigorous nature of introductory college and high school physics by means of lab experiments, which he was most familiar with.

While the idea is very simple, the title of the paper is deceiving; at no point does true Socratic dialogue take place. This method in its purest form of class discussion is an extremely effective method helping students become more confident in their understanding by reaching the solution by their own word of mouth. In combination with Pat and Ken Heller’s *Context Rich Problem Solving* methods, the means of bringing a student’s understanding to fruition through questioning and real-life analogies are further compounded. By contrast, Richard Hake’s labs appear to be nothing more than normal lab experiments that convey much more rigorous examples, using acronyms to establish the understanding of specific terms, while imposing a number of very specific constraints on lab requirements, such as color-coding free body diagrams based on types of forces.

Even if we were to ignore the nit picking about the use of the term “Socratic Dialogue”, the method itself almost seems forced, no pun intended. The inclusion of complicated lab instructions, extensive descriptions of what could likely be explained simply in no more than two or three sentences, and a number of graphs that upon first glance, are not by any means intuitive as to what they are asking or portraying, make everything much more difficult to incoming prospective physics majors, let alone students who would simply take the class as a requirement within some other field.

One of the major merits of the labs, however, is the list of goals that precedes each experiment. A clear table of things that students should expect to understand is listed just before each lab, and in this particular case, the level of detail that permeates the entire paper is useful here. One of the difficulties that plague freshmen in college or even seniors in high school, is the confusion as to what exactly a student is meant to take away from a lab. Typically the labs available to students present principles that students could either have guessed, or are simply not enlightened by performing themselves, and thus the message is lost. Having the list prior to the lab itself shows the students what specifically to look for in order to gain and understanding of the significance of the experiment, which is certainly an improvement to the typical lab format.

Categorization and Representations of Physics Problems by Experts and Novices [Paul Feltovich, Michelene Chi, Robert Glaser, University of Pittsburgh, 1987] [6]

It is certainly more than speculation to note that a physics professor is much more quick on the draw to solve any problem within the range of easy to advanced difficulty when compared with a freshman in college, and surely there is a reason for that. Feltovich et.al. perform an experiment in which they test the manner in which those experienced in solving physics problems go about approaching them, as opposed to the manner in which novices do so. In a series of trials which sought to help identify the cause of the expert's alleged superiority at solving problems, they began to uncover the solution

Their first trial was a simple sorting game, in which 24 problems, 3 problems each from 8 chapters of a physics text, were selected, and the test subjects were asked to categorize them by the similarity of their solution. They were not allowed to ever actually solve the problems, so they must have been able to recognize certain cues in the problems in order to understand the nature of the solution. As it turns out, the number of sorting piles, and the consistency with which they sorted the same piles, varied very little between eight Ph.D track graduate students, and eight freshmen who had just completed a course on mechanics. However, upon analysis of their responses to *why* they categorized certain solutions together, the answers were more revealing. The undergraduates used more shallow approaches to their knowledge of the content of the solution, relying on "surface structure" of the problem, such as the nature or shape of the object, which in their limited experience cued them to a certain kind of problem, like a block on an incline.

The experts, however, categorized the problems on what was described as a "deep structure" level, relying on the principles of physics that would be used to solve the actual problem in order to categorize them, such as Newtonian Mechanics, or conservation of energy. In a second study, which sought to confirm this theory, the two groups were asked to identify problems once more by category of their solution, but this time based on the wording of the problem, rather than with images and words. This confirmed the original theory, as novices tended to lump problems together based solely on the manner in which they were worded, regardless of the nature of the solution. In cases where the problem gave quantities based on energies, the novices categorized these based on their "cover stories", and thus called them energy problems, although the underlying principle of the problem was conservation of momentum.

The general result of the study was that experts tend to rely on certain schemas that activate specific cues and triggers of prior knowledge, while novices rely on the surface structure of the problem, gleaning minor details about the superficial nature of the problem to attempt a hypothesis. Experts were usually able to identify the nature of the solution after reading 20% of less of the problem, having noticed certain cues that guide the rest of the problems towards a certain approach. The takeaway from this study is a useful guideline of the cues and triggers that educators should hope and try to teach as they are introducing the material to students. Students who can regurgitate equations and random bits of knowledge are not useful or productive in pushing the field, and they do not gain an appreciation for the material itself. Only when they learn to understand, learn to grasp, and learn to *think* beneath the surface of the problem, can they truly begin to maneuver through problems in this way [2] .

Concentration Analysis: A Quantitative Assessment of Student States [Lei Bao] [7]

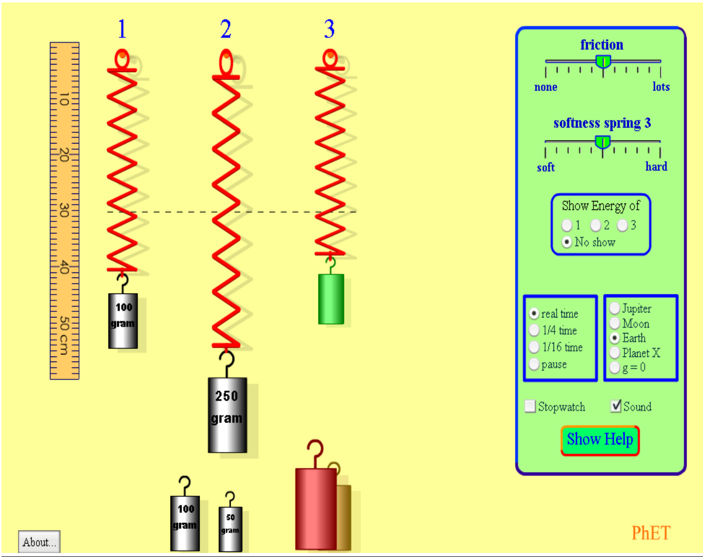
Quantum mechanics is a topic that many hear and shy away from, due to its dense mathematical nature. Its applications, however, are surprisingly vast, as it describes the behavior of particles on a subatomic level, so can it be applied to a large system of statistical data to quantify it likewise. Lei Bao take an impressive and innovative approach to quantum mechanics, by applying it as the primary method of determining the naïve schemas and physical models that students exhibit on multiple-choice exams. The premise of the experiment is to use the *incorrectly* chosen answers by students, rather than the correct ones, to identify the problem areas, a technique which is commonly used by high school teachers today. His method, however, represents the distribution of potential "impetus" schemas in a mathematical function based on statistical analysis.

By recognizing that there are a small, finite number of choices on each multiple choice question, and obviously a finite number of students, he concludes that there must be a limited number of significant distributions of answers on each question, and organizes them into three categories of statistical significance. The first category assumes a completely random distribution in which the same numbers of student answers are given to each possible choice in the question. The second category assumes the other extreme end in which all of the student responses are concentrated on one possible choice. These two categories represent the extreme ends of the statistical spectrum: complete randomness and complete predictability. The third category is simply any distribution between those two categories, which he then quantifies as some number as a measure of concentration **C**, on the interval [0,1], with 1 being the perfectly correlated answers (100% distribution to one answer), and 0 being complete randomness (even distribution between the answers).

From here he goes on to apply this "Concentration Factor" to an average statistical distribution, quantifying the factor into a function that assumes m possible choices on a single multiple-choice question, and N students taking part in the exam. Here he uses basis statistical analysis to define the minimum values and maximum values of the function that would satisfy these constraints of maximum and minimum "concentration", and in an ironic twist of fate, uses the Lagrange multipliers as a method of obtaining them.

In effect, what Bao has done is to *quantify human thought* in a very specific situation, which is impressive in its own right. His subsequent experiments and computer simulations with said formula were also successful, with the constraints holding in every conceivable combination of choices. He then goes on to plot the concentration factor against the function itself to get an idea for the optimal choices for each student mistake. This sounds a bit strange, but the premise is that, with very specific (but nonetheless numerous) combinations of choices, the responses will help to identify which naïve schema the student used to select their answer. Although the paper is certainly an intimidating and rigorous approach to educational research, the novelty and practicality in multiple-choice sections of exams is invaluable, and is indeed one of the more impressive approaches to identifying methods of improving student efficiency.

Technology in Physics: Simulated Methods of Physical Models [Noah Finkelstein, Kathy Perkins; University of Colorado] [9]



http://phet.colorado.edu/

Although there are formal papers written by the duo on the matter of applying technology as a primary or secondary method of approaching physical models, the website that couples with this method is the more impressive, and forthcoming explanation of the topic. Working with the web designers of the University of Colorado, Finkelstein and Perkins raised a website that features a plethora of simulated physical models, ingeniously disguised as games for students to play. As a matter of fact, these games were used as a teaching method during my student teaching, and were spectacularly effective.

The site features just about every topic in physics, covering the spectrum of the MA Frameworks requirements and then some; everything from Motion, Forces, Momentum, Energy, to Heat, Fluid Flow, Nuclear, Quantum Mechanics, and Relativity is represented in mini-games that challenge the students to adjust values until they achieve certain goals. Not all of them are challenges, however, as some are simply open ended simulations that let the students play at will and see what happens when they raise the temperature, adjust the gravity, increase the velocity, and so on.

The premise of this approach is very close to *Modeling Instruction* by Jane Jackson, as it suggests using hands-on methods to show students that physics is not only interesting, but practical and approachable as well. Although the simulations take away a bit of the approachability, the advent of the playful nature of the programs makes the physical principles more available, more simplistic, and overall more visual; something that students desperately need with the more non-intuitive topics like electrostatics and quantum mechanics. By making topics visual, which is one of the most statistically effective methods of making a model more accessible to a student, the comfort of at least understanding the physical model typically allows students to advance into the mathematical translation more easily.

Ultimately, the approach is nothing novel, but the thoroughness with which they complete each simulation is what truly makes this a useful tool for any aspiring teacher. Even on the topic of integrating more difficult material into the current curriculum, these simulations are not simplistic models. They can be *reduced* to the simple models for the current curriculum, but the equations governing the simulations are the fullest form of each branch of physics, and can simulate more advanced phenomena, making this an extremely broad tool educational instrument.

Qualitative Methods of Physics Education Research [Valerie Otero] [8]

Another unique approach to the field of physics education comes in the form of Valerie Otero's "coding tactics", which uses transcript analysis of interviews or responses from students in order to decipher clues and categories as to how the students are approaching the problems, and with what level of assuredness. By analyzing the verbal processes that students use to interact with each other, or even to work their way through a problem on their own, certain patterns of speech and vocabulary may emerge to identify the categories in which they are placing certain topics.

To begin, "coding" is dividing the speech, written transcript, or written response of a student (or any other person) into small, manageable chunks in order to find meaning in each section that could be categorized into schemas or simple though patterns which would guide the student's response. In a beginning example, she does this with three teachers who are discussing why they believe the student gives certain responses. In the transcript of this discussion, she breaks each line into a text segment that represents a "turn of speech", or a significant, distinct thought. She then (having either recorded the discussion and transcribed it later, or having transcribed it on the spot) goes on to annotate the discussion with the speech patterns of the teachers; overlapping speech, continuous utterances, elongated pauses, stretched words, body language, and speech patterns are all given their own functional description in a special code (no pun intended) that makes the shorthand of these tendencies much easier.

The use of all of this is not just to identify the written and visual aspects of the categories place certain problem solving strategies into, but also the auditory tendencies that may give away some of their uncertainty (or certainty, as it were). In the transcripts which she then begins to pick apart, it is noticeable that some of the students stretch their words in thought or uncertainty before they give a potential solution. She uses this to categorize the topics in which they seem uncertain, and also classifies their thought processes into educational standards, including "sense making", "meta-cognition", and "the nature of science".

Thus from these programs we gain insight as to what sorts of programs already exist, and what research has been performed in the field of physics and science education to better structure and deliver the foundations of physical sciences that students need to know in order to become efficient purveyors and implementers of scientific inquiry [10] . With this in mind I begin discussion on this particular experiment, beginning with the schools that took part, the curriculum that the students will take part in, and moving into the analysis and project findings upon completion.

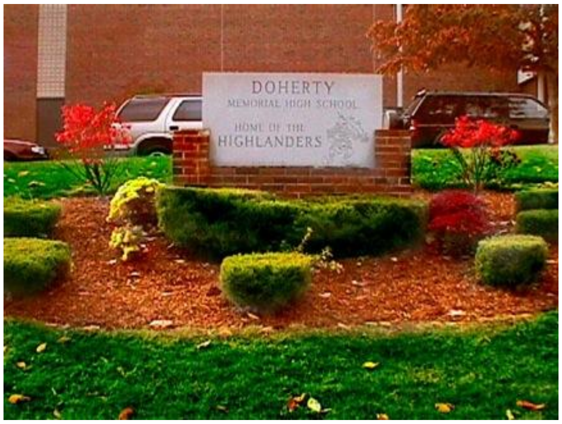
Chapter 4: The Schools

As a foreword to the analysis of the schools, it would be pertinent to consider the audience of students we will be examining, as the human element of analysis is so vast and complicated that we must know everything we can about the environment we step into before we enter. Indeed, with all the human variables making their way into our considerations, our analysis almost borders on the psychological and sociological rather than the academic, but certainly there are ways to consider these variants in the study. The schools under consideration are all within the Worcester county area, spread from the close neighborhood of Highland Street in Worcester (Doherty) to the boondocks of Barre (Quabbin Regional), each with its own reflection of the neighborhood's demographics. Each school will be considered for their MCAS scores (Massachusetts Comprehensive Assessment System) [5] and their progressive improvement over the years; this will provide a baseline by which we can gauge how each class (by level) should perform in light of their pre-existing skills.

As an experimenter, I hoped to acquire as much data as I possibly could, which meant taking absolutely any school that would be so considerate as to provide me with five days' worth of their precious time. This also meant that every level available to me needed to be considered in the experiment in order to it to be a fair analysis, from the college preparation (lowest) to the AP (highest). As such, I asked for as many classes as the teacher's would provide me, which in most cases was not an issue. The primary concern for most teachers was the AP classes, who were already weeks behind due to the inordinate number of snow days this year. Nonetheless, I was given access to AP classes in several schools, and the lower levels in all of them, which provided me with a reasonable data pool.

Ultimately, what must be considered as we look at each school is a very simple fact that is easily ignored: they are all different. They each have a different set of demographics, a different historical background, a different faculty, and a different progression of MCAS scores. Additionally, each student that steps into each school comes from a different place, a different family, and a different neighborhood, with different opinions not only on education as a whole, but also of specific teachers, classes, and social dynamics (discussed earlier), and teaching methods, all of which contribute to their reception of the material. With all of this in mind we must concede that there is a large margin within which we must consider these variables, all of which we will try to consider when we eventually reach the analysis. With that in mind, I welcome you to figuratively step into each school, as the stage is set, as I did literally.

Doherty Memorial High School



http://dmhs1986.com/reunion.html

Location: 299 Highland St. Worcester MA, 01602
Student Population: 1,327
Male: 661
Female:666
Faculty Population: 102
Student:Teacher Ratio: Approximately 13:1
4-year Graduation Rate: 77%
Dropout Rate: 4.2%
Truancy Rate: 36%
Low-Income: 46%

Doherty High is an urban high school set on the quiet end of Highland Street in Worcester, MA. Constructed in the fall of 1966, it replaced to of the existing high schools, Worcester Classical High and Worcester Commerce High, becoming somewhat of a super-school in terms of class size at the time. As it stands today, Doherty regularly houses between 1200 and 1600 students per year, depending on the incoming class size, churn rates, and placement appeals. On the average, Doherty tends to take in a larger number of transfers each year than other schools due to the programs available that are exclusive to its walls, including (but not limited to) a four-year Latin program, AP classes in all the major subjects, and a wide range of extracurricular programs after school hours.

Being dead-center between several high-income neighborhoods and several low-income neighborhoods, as well as sitting on a rift between a number of religious communities, all of various ethnic backgrounds, Doherty also boasts the greatest demographic diversity of the city. Of the 1300 students this year, 47.2% are white, 25.9% Hispanic, 13.6% African American, 10.9% Asian, 1.7% Multi-Race, non-Hispanic, and 0.7% Native America. In addition, 48.3% of the school is considered to be Low-Income, which qualifies Doherty High as a “high needs” school, placing it under consideration for various grants for equipment and scholarships for students based on financial need. The school also boasts a high stability enroll rate for selected populations that are typically churned or transferred to other schools, with every population above the 80% mark for stability. Overall, the school holds a great retention and graduation record for its qualifications, as an urban high-needs school.

On the topic of its advanced courses, of which Doherty boasts AP classes for Biology, Chemistry, Calculus, Statistics, Physics and History, the one of most interest to this experiment is obviously Calculus. The age breakdown for the classes I will be teaching place most of the students as seniors, which means that most of them will have been through at least Algebra, if not Pre-Calculus. It is expected as a trend of ambitious students, that most students in the AP Physics class will have taken, or be taking, AP Calculus, and almost all of them will have taken pre-calculus. With these expectations in mind, it is a reasonable presumption that students will have experience with memorizing mathematical algorithms, steps to approaching a solution in similarly patterned problems, and thus simple derivatives should not pose a major problem. Obviously it is naïve to expect that everyone will understand it completely; there are always exceptions to which teachers must adjust their teaching methods in order to accommodate them, but there are definitely standards to which students may be held at this point in their high school career.

In any case, the fact that I will be teaching not just upperclassmen but a majority of seniors places high hopes on Doherty for their performance in this experiment. Logic would stand to dictate that students in the higher levels will likely perform better, either for their increased mathematical abilities or for their better sense of motivation and interest; without data to argue otherwise at this point, logic is what we shall expect to prevail.

<u>Worcester Technical High School</u>

Worcester Technical Vocational High School.
http://www.wbjournal.com/news42289.html

Location: 1 Skyline Drive Worcester MA 01605
Student Population: 1400
Male: 668
Female: 732
Faculty Population: 130
Student:Teacher Ratio: Approximately 11:1
4-Year Graduation Rate: 87.3%
Dropout Rate: 1.0%
Truancy Rate: 25.5%
Low Income: 61.2%

Worcester Technical High School, formerly known as "Worcester Voke" is a newly rebuilt trade school for students who typically want to enter to the workforce immediately upon graduation. As such, the school has a very unique schedule which they follow with students, having one full week of classes, followed by one full week of projects and training, which alternate. This raised an interesting issue in acquiring the school for this experiment, because it meant that if I was given more than one class to teach, I would likely be teaching for two weeks instead of one. As it turned out I was given two classes to teach the first week, and two classes the next; four classes in all, with two coming in after projects, leaving, followed by two more classes coming in from projects. This also meant that I would have to have an unusually fast turnaround rate for grading papers and providing data back to the teachers.

The school itself now sits on a private road just off of Route 9, and rightfully so, as the building is *very* large. The newly rebuilt facility stands on 165 acres of land, covering 139,077 square feet in all, a vast structure that houses a number of workshops, pre-engineering programs, and biomedical engineering programs, as well as all the STEM standards and MA State Curriculum requirements. All of this is provided to a class of approximately 1400 students, an average class size for what amounts to another super-school of sorts; Worcester Tech provides for a district housing 3 high schools and one middle school, allowing for transfers from typical high schools into the trade school as well.

The school's demographic breakdown is not so diverse as other schools, as it is situated firmly in a distinctly urban neighborhood. Of its 1400 students, 50.2% are White, 34.5% Hispanic, 10.0% African American, 3.9% Asian, 1.1% Multi-Race Non-Hispanic, 0.2% Native American, and 0.1% Native Hawaiian. Additionally, 61.2% of the students are considered Low-

Income, which qualifies them for free/reduced lunch, and qualifies the school as another high-needs facility. Historically, the high-need rating on Worcester Tech is what began the considerations of rebuilding the trade school, since many of the shops, machinery, and materials were run-down and desperately in need of replacement. One other statistic of note for this school is the 82% of students who intend to either work immediately upon graduation, or complete a 2-year public college, with the split being 34% and 48%, respectively. With this school being designed to allow students to learn only what is practical and necessary for their trade, this statistic is intimidating in the face of teaching them something that would require several more years of education in order to put to good use. I immediately recognize that this could pose a problem in terms of class interest, which I predict will show on the initial and final surveys.

The school's environment is immediately deceptive, as the entrance is guarded by a police officer that checks everyone in under the strictest of questioning. Beyond this first impression however, is the enormous expansion of the building, which houses brand new machinery, state of the art equipment, and new classrooms as well. The condition of a school is said to show the facility's level of respect for its students, and this building clearly indicates an appreciation for its pupils. The staff and students are still mutually enjoying the new environment, as students seem focused and hard at work in the shops and in the classrooms. The rules for obedience are somewhat stricter in this school, as the workshops contain machinery and materials that are most certainly dangerous if one is unfocused or careless in their tasks. This seems to be an effective method for these students, as the 4-year graduation rate is an impressive 87.5% for an unadjusted rate, and 90.5% for adjusted cohort.

I have been told in advance that this class will not have any prior knowledge of derivatives, which is ultimately the expectation that brought me to build it into the curriculum. This knowledge, however, implies (through the tone of voice it was conveyed in) that I should not have heavy hopes for their basic math skills either, and thus my initial prediction is to see initial difficulty with simple derivatives. If this obstacle can be overcome, hopefully partial derivatives don't prove to be too much more of a challenge, and we can move on to the more difficult material without resistance.

Quabbin Regional High School

http://www.qrsd.org/our-schools/high-school/

Location: 800 South St. Barre MA 01005
Student Population: 895
Male: 441
Female: 454
Faculty Population: 64
Student:Teacher Ratio: Approximately 15:1
4-Year Graduation Rate: 81.7%
Dropout Rate: 3.0%
Truancy Rate: 0.0%
Low Income: 17.4%

Quabbin Regional High school holds a special interest in this experiment because, in the interest of acquiring any school that I could for the purposes of having a larger pool of data, I managed to come in contact with a school outside of the Worcester area, which places the study in a slightly broader neighborhood. Ideally, it would have been nice to acquire other schools from outside of Worcester County; there was discussion with schools in Wareham, but ultimately there wasn't enough time to pull the schedule together. As a result, we have a comparative study case that can be contrasted against the Worcester County schools, which allows a brief look into how schools outside of the County might be expected to perform based off of their comparative MCAS [5] and experimental performances. Even so, for initial analysis, it can be included in the data pool as all the others, and considered later for its unique qualities.

One of the most distinct features about Quabbin is its demographic, which is virtually mono-racial in nature. An *astounding* 91.7% of the school's population, numbered at just under 900 students, is White, while 3.4% is Multi-Racial Non Hispanic, 2.6% is Hispanic, 1.1% African American, 0.9% Asian, and 0.3% Native American. This means that roughly 810 students are White, 31 Multi-Racial Non Hispanic, 23 Hispanic, 10 African American, and 3 Native American. What this means in terms of expected results is completely unclear as of the moment, but with such an obvious trend of racial breakdown, some sort of pattern will emerge that we will analyze later. The school also has an unusually low Low-Income population, with only 17.4% of students qualifying for free/reduced lunch, which means that this is also the only *non*-high-need school in the experiment as well. In a way, this school will provide a unique baseline of high-need vs. non-high-need, as well as a curious potential demographic analysis.

In terms of expectations, the school has an interesting class system that I feel is very well implemented to introduce a student to the appropriate level of mathematical expectations corresponding to each science. Students are given Calculus 1 and Calculus 2 courses, which at the very least introduce them to the concept of a derivative in Calculus 1 and an integral in Calculus 2. Thus, the rumors handed down from the teacher and a number of his students suggest that I can expect that most students will have seen a derivative and understand its value both in math and in physics. This would be a tremendous help in offering a physical explanation of a simple and partial derivative, as well as helping to explain exactly what the Lagrangian Formalism means as a formula rather than a concept.

Quabbin Regional High School has offered me the chance to teach a number of its classes, but there is no formal AP class to be spoken of; only an Honors level with students that have taken at least Calculus 1, and of course the lower levels. I have no reason to put merit into the title of the class students are taught in, especially with the educational system putting such interesting constraints on students in terms of what classes they can join; students frequently sacrifice AP for Honors or even college level classes based solely on scheduling conflicts, instead of their own level of ambition or interest in the subject. As such, the knowledge that they might already know derivatives is encouraging and suggests promising results.

North High School

http://wn.com/North_High_School_(Worcester,_Massachusetts)

Location: 450 Harrington Way Worcester MA
Student Population: 1149
Male: 619
Female: 540
Faculty Population: 100
Student:Teacher Ratio: Approximately 11:1
4-Year Graduation Rate: 62.5%
Dropout Rate: 6.1%
Truancy Rate: 42.1%
Low Income: 77.1%

North High School is the last school to take interest in the project, and certainly an important part of Worcester County's public school system. This was the most difficult school to acquire for a number of reasons. The teacher, Joseph Marzilli, was fairly convinced that I wouldn't want to teach his AP class, which consists of only 11 students, for the notion that the project would fall primarily on deaf ears. In the interest of keeping true to the scientific method, however, no data that *can* be acquired can be ignored, so I assure him that I was willing to take on the project. The second reason was more related to time constraints. As expected, the number of snow days that overtook the city this year had a severe hindrance on the pace teachers were keeping with their classes, and by extension Mr. Marzilli was reluctant to give me the gracious period of 5 days. When he realized that he would be absent from March 9th-11th, however, he resolved to give me the time I needed, under the consideration that I would hand out his class work upon completion of my daily lecture.

The school itself is deeply rooted in the stereotypes that surround the city of Worcester, as it is one of the two schools (North and South) which are regarded as the absolute highest need schools in the city. The schools demographic breaks down to a 43.2% Hispanic population, 27.8% White, 19.0% African American, 8.7% Asian, 1.2% Multi-Racial, Non-Hispanic, and 0.1% Pacific Islander and Native American, respectively. Additionally, the school has an unusually large Low-Income population at 77.1%. Statistically, this demographic closely reflects the county as a whole, being within 6% of every demographic and statistic.

The expectations of this school do not necessarily have to reflect the assumptions that come from being in a very high need school, but discussion with the AP teacher confirms certain suspicions. Although the school does have an AP Calculus class, I have been warned that students are not proficient in derivatives, although a few of them have seen them in practice. As for any material beyond that, Mr. Marzilli had no major speculations. My own expectations are likely higher than they should be, as the results up to this point have been encouraging despite initial thoughts prior to each school. I believe that, as with most students, the initial material will provide obstacles for some and pose no challenge for others, but that everyone will have to hang on tightly when approaching the Lagrangian and its uses.

School data acquired from the Worcester County district website and the Massachusetts Department of Elementary and Secondary Education.

Chapter 5: A Word On Class Dynamics

The human element, the consideration of the incredibly diverse, dynamic, and fickle nature of the high school classroom, is a variable that is difficult to analyze. A room full of teenagers, at a milestone transition in their lives, socially and physically, as well as hormonally and educationally, is not to be trifled with. Creating an atmosphere that promotes an environment of acceptance, while maintaining a comfortable authority that is conducive to learning is one of the educator's greatest tasks. A teacher who cannot keep control of their students, cannot keep their attention, or cannot keep their interest, is not going to gain the full potential of their students, and is consequently not going to achieve the desired results in terms of progress. A look at the dynamics that accompanied each experience within the schools is certainly warranted as a means of analyzing the effort that students put into their work as a consequence of my teaching, as well as a consequence of their own interests and considerations.

Within each school lie deeply rooted mores, social scripts, social dynamics, and etiquette for each interaction. While the demographics might be expected to speak for what a prospecting educator would find upon stepping inside, the experience has been altogether humbling. The environment, or more precisely, the community, that a school forms creates a unique dynamic between educators and students that varies greatly based on the nature of the school, but nonetheless creates something altogether new. For a moment, in ideal conditions, when students have come to trust their mentors and the mentors have learned to respect their students, the social dynamics can be temporarily transcended for the purpose of education, even if the students don't realize it (and certainly they don't).

Ideal conditions are difficult to come across, however, and even more difficult to synthesize. As such, I did my best in each new environment to create the atmosphere of excitement, and to keep students interested in what effectively accounted for a very widely used mathematical skill, and a very useful physics tool as well. The prospects of college appeal to students knowing calculus, as well as any sort of competitive nature, was left as a minor (but mentioned) point, as it was not my intention to pit students against one another. It was mentioned to every class that colleges certainly find calculus an appealing student skill when considering applicants, but the purpose was to interest students in the material itself, not its academic weight.

Class management as a whole was the greatest challenge faced throughout the experiment, and probably the greatest unknown in terms of calculation; not just in terms of behavioral disorders, but rather in terms of maintaining class interest, enthusiasm, and participation. In a social environment where it can be detrimental to one's status to show interest in school or proficiency in the material, pulling responses from students in order to facilitate Socratic dialogue and ease the lesson flow into their hands is a taxing task. Nonetheless, the pretense expected and held throughout is that every student, regardless of social status, race, ethnic background, age, or skill level, can learn; certainly this much is true.

Doherty Memorial High

The school's atmosphere is very stereotypical, in particular because the school is very old. The floors are a randomly colored checkerboard due to tiles being pulled or kicked up, warranting replacement without having the right colored tiles to replace them with; teachers and staff make do with what they have here. Lockers permeate the walls, down every hallway and branch of the school, with some whole sections of lockers being taped off or removed from maluse or potential replacement. Between classes, students saturate the halls with noise and shouts above one another, with MP3 players draping from pockets, phones being whipped out to text, and an overall chaotic four minutes passes before the bell rings and they trickle into their respective classrooms. Up the stairs, down the long hallway to the right, past the bathrooms, down another long hallway passing the vice-principal's office, tucked away in a hallway hidden to the left of a staircase, is the physics wing. The room opens up to the right into eight large black science tables, separated into fours against each wall, with a long walkway to the front of the classroom in between, where the teacher's desk (which is just another table) sits level with the others.

The room itself creates an instant dynamic, keeping four students to a table (32 maximum, which has never been reached), which minimizes the social chaos that typically accompanies 20-something students being lumped together in desk-chairs. The walkway allows ample room for the instructors to walk between tables and offer advice, to step back from the board and pose questions, or to keep the class managed by the simple close proximity of authority. Additionally, as the teacher's "desk" is simply another one of the tables, it poses no additional sense of authority on the students, and reduces the discomfort of approaching the front desk for questions or requests. While the room is certainly a luxury for any science teacher in terms of class dynamic and management, the teacher is obviously responsible for initiating the environment.

Honors Level (Period 1)

Drew Merrill's first period class was my first period of the day. While I had taught two classes at Doherty already, this was a group of students I had not encountered until now. Mr. Merrill directed his students to give me their full attention, which I did my best to keep in any case, but his directive set helped to set the authority of the situation, which had not yet been established. The class layout was a typical classroom, with the desk-chairs arranged in block form in front of a large marble slab table for the teacher's desk, and the whiteboard behind this. Maneuvering around the classroom was particularly difficult due to this layout, and thus students in the back of the room were difficult to manage at times, as I could not use my presence to maintain an influence over their behavior by proximity.

One of the first challenges encountered in this class was the most obvious, that being that it was the first period of the day, and thus students were either absent or simply not awake just yet. Class response was minimal in most instances for this period, although engaging the class in methods of their choice brought a few students to the surface of interaction. The initial difficulty of assuring them that the material was well within their reach was overcome as soon as it became apparent that the pattern of taking a derivative was extraordinarily simple, which brought the "light" back to their eyes, in a sense. Students are much more prone to maintain their focus when they feel that the material is understandable. The intention, to avoid the pitfall of students that get lost and then subsequently fear asking questions in front of their peers, was to explain things in the absolute simplest method possible, using Patrick and Ken Heller's *Context Rich Problem Solving* methods as a guide. By introducing topics such as derivatives (simple and partial) in a physical sense, rather than the non-intuitive mathematical abstraction, students can understand that the correlation between the math and the reality is full circle, and feel that what they are doing is significant and applicable. Without this in mind, students may readily find the curriculum uninteresting, as it presents just another mathematical rule to memorize, and another principle of physics to consider.

Student interest was admittedly much greater in the first portion of the curriculum than the second, as students on this level were not accustomed to problems that contained more than, at most, two steps. When we passed partial derivatives and moved into the Lagrangian Operator and Formalism, which require not just the mathematical skills, but also, as I told my students, "a bit of imagination as well". The prospect of completing a problem with more than two steps (the operator, two derivatives, and the final time derivative) was intimidating, and thus the students began to wane in their responsiveness towards the end. Drew Merrill, the attending teacher, was helpful in spurring his students on by reminding them of previous lessons when we arrived at the physics portion of the curriculum, which assisted in my Socratic dialogue with the students to guide them towards the answer.

Homework response in this class was extremely low, not by scores, but by sheer number of submissions. Of the 114 assignments that should have been cumulatively submitted, a shocking 47 of them were un-submitted, which left a large hole in the data and also a large population of the students who either weren't asking questions or weren't taking the time to work on the material. From experience, I can respect that students at that age have other obligations, but the academic obligation, for them majority of the demographic within the class and the school as a whole, is the largest. Drew Merrill assured me that this level of submission was unusual, which suggested an unfortunate lack of student interest that I was forced to concede.

As a whole, the period one honors class was quick to admit to themselves that the time of day was not conducive to an introduction to such advanced material, but they nonetheless showed promise in some of their resultant scores. An overall lack of interest in the later half of the lessons made it a very rough process to continue through each day, so a brief synopsis of the previous day's work usually generated questions that students had had a moment to consider over the night. This brought several students back from their haze, as I touched upon several points that I had considered overnight to need either more reinforcement or more clarification.

College Level (Period 2)

The college level is also referred to as a "college preparation" period, which is the lowest level offered at Doherty. Consisting entirely of seniors by the second week, during which the only two juniors transferred classes, there was an immediate air of superiority and familiarity with the students; they had done their time here already, and were very clearly ready to be finished. They were also very familiar with both John Staley and myself, as we had just spent 15 weeks working together on my IQP, which had been teaching the students myself for 150 hours through the MA Frameworks Curriculum. As such, they were already comfortable with me as a teacher, which had the potential to either help or hinder the process of engaging them. In my experience during my 15 weeks at Doherty for my required student teaching hours, it became apparent that my age was a distinct variable in the students' level of respect and response for me.

Without the air of authority that students correlate with the age of a teacher, and being that I am well within their generation and can thus still relate to their interests, distinguishing myself as a teacher rather than a peer was part of the challenge. Over an extended period of time, as my students realized that I am still in college and share (to some degree) the sense of humor associated with this generation, they slowly began to toe the line between educator and peer, which I had to set them back from several times. This was my concern upon re-entering the classes that I had previously taught, especially under conditions that required me to acquire as much data as possible while gaining the full potential of my students.

Initially my students were happy to see me again, which quickly translated them immediately becoming a bit too comfortable with their environment, and talking amongst themselves. Being that I had dealt with this class in the past, I was able to quickly quiet them down in order to explain the schedule for the next five days. Students were immediately turned off at the idea of doing calculus when their mathematical skills were already so low, and I assured them that the patterns they would follow would be very simple and easily applied. The starting survey revealed students who had previously seen derivatives and remembered the equations of energy, but only a few. To this degree, the scores on the first homework were extremely impressive considering the low scores most students had attained on the survey.

Each day was effectively a struggle to prepare the students mentally for the material that was to come. I am of the distinct belief that these particular students were far more capable than they gave themselves credit for, and were thus far more likely to give up at the first sign of trouble; they were not accustomed to fighting their way through a problem using logic. Having become familiar with this class, I was aware that they learned best when given a problem to work with together in steps, which forced them to ask questions in order to achieve as a group. Unfortunately, for time constraints and statistical analysis purposes, I could not adjust the curriculum to tailor to their needs, as any teacher would do if the curriculum were officially in place. Thus, it was expected to some degree that the lecture style lessons would not be as efficient as they could be if, say, class work were incorporated into the curriculum.

No behavioral disorders were present in this level, although keeping students more interested in Lagrangian Mechanics than on their texts (which they were frequently warned about) was an admittedly difficult task. Overall the lack of submissions towards the end suggests a falling out with regards to understanding, which is more than understandable with regards to the Pendulum problem, but less understandable for simple problems such as the Operators worksheet. One particular student, in a complete overturn of the expectations, scored 20/21 on the final survey, with impressive scores throughout, with no previous understanding of derivatives. This diamond in the rough, while not representative of the majority of the scores in this class, was an encouraging reminder that some students can be pushed to their full potential when engaged correctly. In his case, he preferred the language of mathematics than the English language.

<u>*AP Physics (Period 3)*</u>

The AP Physics class, which takes place in periods three and four of the day, is also one of the classes I had taught during my student teaching at Doherty. As such, this class was also familiar with my teaching methods, and also presented the immediate potential problem of judging me as a peer. Unfortunately, this particular class *did* present that problem frequently while I did my student teaching, and also during this project. Their close relation to my age gave them the false impression that I was less authoritative than other teachers, a point which I had to clear up more often than I would have liked. John Staley's presence in the room was extremely helpful in maintaining class cohesion, although in most cases I was able to focus the class' thoughts, although at the expense of valuable time.

Academically, the AP class was expected to set the standard for the experiment by the sheer nature of the educational level of the class; fast-paced, hard working, structured to meet stringent standards, the class was basically *made* for a curriculum like this. Indeed, the students in the AP class were not only prepared to accept one form of learning for a short time, they were used to it. John Staley structures his class in a staggered lesson-style form, varying the methods he uses to present information to his students, in order to see what works best, and to keep their interest. They had recently come off of a lesson-style, which was very PowerPoint heavy, so returning to the board wasn't much of a chore for them. Many of the students later noted (see Chapter ___ on Feedback) that with more time the curriculum could have been much more effective, an opinion which was widespread throughout the project. With more time there could have been in-class worksheets, discussions on the equations, all the things the students are used to using in order to fully understand the material.

One of the interesting points to make about this particular class is that their focus actually increased as the curriculum went on. Students at this level tend to fight very hard to overcome a challenge, and the first two lectures of the series were hardly a challenge; most students had already seen derivatives, and didn't find partial derivatives too much of a stretch either, which is reflected in their initial scores. Once the material became new, a rift was effectively created in the classroom, which separated students who were only interested in AP Physics for the grade, and the students who were interested in the material. The students interested in the material were inquisitive, prompting Socratic dialogue to work their way through the logical idea of the Lagrangian Operator and Formalism in general, and contributing to the class as a whole. The students who were only there for the prestige and the grade began to tune out at the mention of difficult material, and it was my interest to keep them on board throughout.

The Lagrangian Operator was a non-issue for them, as their initial surveys showed that they were proficient in their prior knowledge of energy. Putting two and two together to take the difference of energies was never the problem, but the idea of the "dummy variables", or the generalized coordinates and velocities, was a new prospect for them. "Dummy variables" are a common term for a variable that effectively acts as a placeholder until the real variable arrives. In this case, I explained that the purpose of the generalized variables was "to ensure that the Lagrangian could work for any problem, regardless of what kinds of energy the problem contains, which places the responsibility on us, the scientists, to use our imagination to determine exactly what is happening in the problem, and translate this to the type of energies we're dealing with". The dot notation for time derivatives was also a brand new topic for them, and the relationship was established between position, velocity, and acceleration, thus making the connection between the generalized coordinate and the generalized velocity, both of which bore the new dummy variable "q", but one using the dot notation to denote a velocity.

During the first three days of the lectures I believe the students were still feeling well within their comfort zone and this was reflected in their behavior. The general demeanor of the class was "above" the material and thus unchallenged, which led to side discussions and students doing other work or simply not paying attention. The side discussions were dealt with when it became apparent that they were a hindrance to both myself and those students, and that it was the students' intention to continue in that discussion rather than make a side comment and continue listening. Although it is never my wish to be confrontational about this issue, when necessary I address it as calmly as possible; my tendency is to stop speaking, which brings the class' attention to me as to why I have stopped speaking, which inevitably brings their attention to the student who *is* speaking, bringing this to a stop with nothing more than a slight nod in their direction once they stop. All of this is normally accomplished in a few seconds, so one incident doesn't lose very much time; when the incident becomes frequent, a verbal warning is usually warranted to avoid losing more time.

In this class, where the students made the greatest relation to myself, even going to far as to try and find my Facebook page (which is completely locked) and add me as a friend, a verbal warning can be a difficult means by which to control a student. The false impression of a peer-to-peer relationship means that any outward exertion of authority stands to challenge that (false) relationship, and on two or three occasions, caused the student to question my authority. These challenges were quietly dealt with in one of two ways: either by addressing the issue after class, and warning the student that challenging my requests to pay attention weren't going to accomplish anything, or by addressing it directly in class by pointing out logical flaws in the student's argument.

Typically the former is the better choice, since more often than not the student's argument *isn't* logical, and to continue the argument in class would be to waste time. The AP classes are the highest level that a high school can offer, and thus students of this caliber often feel they are closer to their teachers than they really are, which is even more difficult in my case, as it is by no means apparent to them as to just *how* far apart academically these students are, seeing as they are only 4-5 years younger than me.

Overall, despite a few of these lost moments, the AP class certainly lived up to its expectations. Of the four classes at Doherty, it had the largest number of papers turned in as a whole, with just under half the students attempting the Pendulum problem, and many of them getting points on it. The final survey was also impressive, with two perfect scores several near-perfect scores, as well as a large range of scores that showed improvement. The Doherty AP class could be called the ideal environment for a curriculum like this, as students are well prepared mathematically, and can spend more time on the physics, rather than the math. Additionally, students are prepared to face problems with multiple steps, as it is expected of them on the AP Exam in May.

Honors Level (Period 5)

Drew Merrill's fifth period physics class was also new to me, but welcomed me almost immediately after my introduction. A few of the students I had met in the previous year while doing my student observation, sitting in on various science and mathematics classes and thus encountering a number of them in their pre-requisite classes. As such, they were aware of who I was, but had never formerly had me as a teacher, which resulted in my reputation by word of mouth of my AP and College level students preceding me. The students in this class had something called "third lunch", which meant that, during the fifth period, students would be sent to lunch in 20-minute blocks, and this class went last. As such, it would be left to logic that they would be eager to get to lunch and thus not give their full attention, but these expectations were flawed.

Despite their empty bellies, students in this class were very eager to learn and also very energetic, without being disruptive. Although many had never seen derivatives before, their inquisitive nature and fast tendency to pick up on the subtleties and difficulties of taking a derivative allowed them to predict and question all of their perceived problems in understanding, all before the end of the first class. Their level of awareness of their own logical flaws was truly impressive, which is reflected in the first night's work (See Chapter _____ on *Analysis*). Despite this, time constraints would eventually give way to pitfalls that couldn't be patched in time.

The class itself had no behavioral issues, but as with the first period class, maintaining full cohesion by simple proximity was difficult due to the block arrangement of the chairs, so small side discussions did persist, if only for moments. Without these issues to deal with, the day's lectures often went smoothly, with students questioning some of the tools and methods being used, and me responding with an alternative way of presenting the solution. In all reality, I only had two or three alternative ways of explaining each technique, and I was fortunate to not have had to cycle through all of them. Context-rich problem solving only takes one so far when the analogies are buried deep in the abstractions of mathematical concepts, and thus when we reached the more obscure form of the Lagrangian Formalism, describing what was happening couldn't be taken much further than proving to students that each piece of the equation was a relevant item that they had seen before; i.e. a generalized force, momentum, etc.

The fifth period class proceeded without hindrance for the entirety of the project, and students continued to be inquisitive throughout, although this was also an encouragingly accurate reflection of their homework submission as well; almost all students in this class turned in at least 80% of their work, with several outliers to skew the average. Their effort was exactly the level of dedication I requested from them, and despite their lacking mathematical background, a number of them achieved levels on the final survey that qualified as an overall improvement. With more time, this class could have easily mastered the material, but with them just having reached the end of energy concepts prior to my arrival, the pace of the class indicates that they were unprepared for such a fast-paced curriculum, which ultimately was the greatest difficulty of all.

Worcester Technical High

Worcester Technical High School, formerly known as "Worcester Vocational High School", colloquially known as "Worcester Voke", was recently rebuilt in light of the necessity of new equipment and a safer environment for students to work in. Along with the classrooms, workshops, library, computer labs, and other things you might expect to find in a high school, the building houses what effectively amounts to a small mall or shopping center, with a pharmacy, deli, pastry shop, coffee shop, bank, restaurant, craft store, and generic provisions store. One would think upon entering the east wing of the high school that you had stepped into Union Station, or perhaps wandered into the Greendale Mall, but in reality every store and shop has its purpose in the school.

One of the unique elements that quite literally defines Worcester Technical High is that it is a vocational school, where students who typically wish to learn a trade for immediate use in the ever-competitive job market can come to spend their four years becoming young specialists in their field. Here, students are taught their trade right alongside with their classes, by alternating one week of trade classes in the workshops, with one week of academic classes to broaden their horizons as any normal high school would. The benefit of this is that students can graduate in the same four years as a typical high school student, but with four years of experience and expertise in their trade as well. But what of the strange placement of the stores?

A student a Worcester Vocational High doesn't just learn their trade in the classroom; they are given real-world experience as well. The stores that line the east wing of the school are, in fact, real businesses that are open to the public, which explains its close proximity to the entrance of the school, rather than placing it knee-deep in classrooms. Here, students *are* the pastry chefs, the waiters and waitresses, the bank tellers, the nurses at the front desk, and the cashiers at the pharmacy. They are all gaining experience in their trade by means of actual businesses; truly ingenious. Of course, the rest of the school deserves a description as well. Venturing out of the east wing, the foyer offers entrance to the west wing, or stairs down into the sparkling new cafeteria, arranged café style with circular tables spread across the floor. Our destination, however, is in the west wing. One long walk down the half completed hallways, with the left side being solid cement and the right side being newly installed tile, a long trek up the stairs to the fourth floor, and to the right, down the pristine, well-lit hallways, is our classroom, D423, where all four classes took place.

If you've noticed the glowing reviews and descriptions of the building, it's with good reason: the environment can have a great impact on the way students behave and feel towards their school. The building that students and teachers alike come to in the morning reflects upon the administration's level of dedication towards their employees and their pupils, and if this is so then much can be said about the administration's dedication to WTHS. A student or teacher that steps into a building of this caliber can confidently say that someone genuinely cares about their well-being; for a student this means a certain level of appreciation for those who care about their education, and for a teacher this means appreciation for their teaching skills. Either consideration ultimately leads to a better mood and better results overall. One potential statistic would be interesting to note, if someone were to compare my results at North High, which is currently a level 3 school, in a very run-down building, to a new survey taken several months or years into the new building, which is currently under construction. Regardless, the environment at WTHS is pristine enough to warrant a note about its effect on student behavior. At the first bell in the morning, contemporary pop music begins to play over the intercom until the second bell, reminding students to enjoy their day and work hard.

The classroom, D423, very closely resembles the environment at Doherty, in that it has a very wide walking space between the tables, which were lined up parallel to each other in long lines down the sides of the class. This leaves a terrific spot walk between the tables, working with students one-on-one without having to push between desks and chairs; again, very similar to Doherty. Ultimately I believe that this arrangement is commonly used for classes that are typically highly engaging and require efficient instruction on class-work in order to gain the full potential of the students. With all this in place, all that was left was to meet the students.

Week 1: Period 1 (Honors)

My introduction to my first group of students was more than welcoming. Although students piled in fairly late with regards to the second bell (a habitual tendency of most students during my stay here), they were open to listening from the moment they sat down, and greeted the stranger (i.e., me) in the room as though he had been there the entire time. This initial interest in a student teacher was unheard of up to this point, so it was immediately apparent, although not fully understood, that the unique dynamic of the school affected the student-teacher interactions, which I paid close attention to for the next two weeks. After my introduction to the class, it was obvious that the students were very uncomfortable with the idea of doing calculus, as there were no calculus courses offered at that school. When speaking with Tom Gusek, the resident teacher of the four classes I oversaw during the project, he had this to say on the matter:

"When the math department refuses to show the kids calculus, that sends them a message. It tells them that 'Well, if the school doesn't even want to offer it, doesn't even want to show it, it must be way too hard for me'. So they get this idea that it must be this impossible forbidden art that's way, way over their head, and that's not what we want to be telling them."

-Tom Gusek, WTHS
Science Teacher

Tom has been trying to convince the math department for nearly a decade that the students at WTHS are fully capable of doing calculus, and that most of the more engineering or mechanical based trades learned here would benefit from seeing it; if not for the practical applications, then at least because higher academia truly appreciates seeing that level of skill in a student. Ultimately, his decision to take on the project was partially in the hopes of acquiring some evidence that he was right. To do this, however, the students needed to be assured that what they were doing was not rocket science, and the recommended course of action was to present the material as a series of rules. While this seems like a small and arbitrary point, the environment of this school dictates that rules are extremely important. Students graduating directly into their trade need to know all the rules and regulations or they will inevitably face legal issues. As such, here, in the classroom, indicating that something is a rule makes it a top priority of the student in terms of knowledge.

Thus, the initial approach to derivatives, partial derivatives, and Lagrangians were all laced with the idea that what we were doing was just a series of rules. Admittedly, the algorithms for taking a derivative (simple or partial) are not obvious or forthcoming in any way, and thus for a student on this level, a certain amount of blatant memorization is necessary in order to continue on to the physical relevance of the topic. When presented with the statement that these were just simple rules, the students took very well to them, and for quite some time I believe that they simply forgot that they were doing calculus, as they approached and mastered the material easily. Beyond this, however, was a still more fascinating experience with the students.

After the bell had rung and I was sitting in the library grading the initial surveys, one of the students approached my table and asked if he could ask a question. This, although I wish it hadn't, struck me as very strange, as there is a general air of superior authority typically seen in the teacher-student relationships at most schools, and as a result students usually don't approach teachers outside of class. I, of course, offered him a seat, and answered his question, at which point several more students approached the table with questions. As I sat with a number of them, clarifying some of the points made in class, I realized that the students are less timid towards their teachers at this school because in alternating weeks, they work side-by-side with them. Working in a business with your teacher is no more than having a friendly boss or co-worker, and thus the relationship develops into a more approachable entity, which subsequently allows students to approach their classroom teachers as well.

The students in this class took an interest in the material up to the point of the Lagrangian Operator. At this point the material seemed to become a bit too obscure, and the message slowly lost its potency as we drew closer towards the Formalism. Using the material as analogous to real-life situations, and showing students that we could, in fact, predict the solution of several problems before moving into more difficult topics, was something that brought their interest back somewhat. Of course, there were the few students who resigned themselves to failure and slumber on the first day, and this naturally reflects in their scores and survey responses. The majority of them, however, were still riding on their own self-satisfaction at having the chance to not only learn calculus, but to *use* it as well, which overall led to a reasonably impressive array of final scores, one of which would have been perfect, had he been present on the day trig functions were covered.

Week 1: Period 5

The second class was significantly smaller, numbering only 14 students, which at any given time were never all present. The intimidation factor of teaching at a new school had long worn off since my encounter with my first period class, and these students were equally inviting, albeit slightly less enthusiastic. They faced the same concern with calculus as the first class, but ultimately found their footing on the material. One thing of interest in this class was that the students were engaged enough in the material to ask questions about what was going on, about the origin of the methods, and to request repeat explanations. Additionally, this interaction also resulted in the students tending to blurt out answers more often, which was encouraging to say the least.

While there were no behavioral disorders in this class (indeed, there were no behavioral disorders in *any* of the classes), there were certainly students who had begun to tune out a little early. To re-engage the students, I encouraged those who were not participating to answer questions regarding the material on the board, and when they invariably could not answer, I questioned their concerns or difficulties on the topic. On several occasions this led to confessions on not understanding certain aspects, which had brought them to just disengage rather than ask me to clarify. This is a common pitfall for students and teachers alike, as the teacher can ask students as a whole if they have any questions, but the social repercussions of singling yourself out for clarification of a subject are allegedly damaging, and often results in a quiet smile and nod from the class. The instructor, of course, has no choice but to continue, or else lose time lecturing the class on the merits of speaking up when you truly have a question. If no students had questions, theory would suggest that all students would acquire perfect scores, and as this is not the case, we are left to assume that there *are* questions, but they are simply not being asked.

Socratic Dialogue was the method of choice when guiding students through an explanation that they did not understand. When at last a difficulty had been identified, the process was a matter of leading them to realize the logical flaw in their argument and correct it themselves, which builds their confidence in their own ability to work through an issue while also correcting the specific problem that they were having. This clarification by means of guiding questions, as several students confessed later, also cleared up a number of their own questions that they had been afraid to ask.

One of the issues that is practically unavoidable in a public school system with a student-dependent social dynamic is the group-think mentality, which tends to facilitate one answer speaking for the group, even if the group doesn't agree. The social environment of *any* high school, vocational, academic, private, public, or otherwise, is determined entirely by the students, and invariable we encounter the same issue: asking questions singles a student out. In what manner it singles them out depends on the social dynamic of the school. It may indicate to their peers that they are ignorant in some sense, which would in turn give them a feeling of insignificance. It could also indicate an interest in the subject, which in some social environments is also unacceptable, and they are labeled as a "geek" or "nerd". The socially damaging effects of students isolating themselves typically outweigh the benefits of clarifying a question, which leads to a dilemma on the part of the educator.

This class was particularly engaged in the material, and a quick glance at the Scholarship Recipients wall at the entrance to the school would show practically my entire class roster. As such, it can be assumed that the students felt that they were in good, safe company, and were thus more inclined to ask questions when they had them. There were, again, a wide variety of final scores, but encouraging overall; it is difficult, in the face of the lower scores, to remember that all of the students initially scored 0/10 on the initial survey, and that *any* score higher than that constitutes improvement. Nonetheless, their level of interaction and tendency to ask questions in and out of class is likely what benefited them most.

Week 2: Period 1

The second week began just as the first had, with an introduction to the purpose of the project, and a solid assurance that the material, while advanced, was by no means over their heads. The initial response in this class was not nearly as encouraging as it had been in the previous classes; where the first two classes had been concerned but willing, this class seemed snide and sarcastic about the thought of approaching this kind of material. As I later found out by means of the surveys, as well as discussion with the teacher, a number of the students in this class were not only habitually disinterested in the material, they were also culinary students. Although this seems like an arbitrary trade to cause concern for an educator, when examining all the other trades, it becomes apparent that no skill taught in Worcester Technical High requires *less* math or physics than culinary. Thus, it was a noticeable trend for the culinary students (as I reflected on the past classes as well) to be generally disinterested in the topic, primarily due to its inapplicability in their lives.

Nonetheless, the response to the first day's lecture was relatively encouraging, with students averaging at least 50% on the homework. Students in this class were unusually quiet when compared to the others, and thus a more forced interaction was required to engage them. Questions on simple solutions to derivatives, or perhaps their physical significance, sparked discussion in the class, eventually sparking questions on the methods and physical significance of the topics I was describing. Socratic dialogue was difficult to use, as students were unable to answer most of the guiding questions being asked of them, even with analogous situations from their everyday lives. The difficulty with this class was not the mathematics; introducing the simple rules to follow for simple and partial derivatives was just as effective as with any other class at this school, and their abilities to latch onto and apply rules was engrained in them. The physical significance, however, seemed to fade and waver for them, as it became more and more difficult to visualize that, somehow, the functions *themselves* had a physical activity about them, the fact that their derivatives were a motion of sorts.

As the week progressed, the class effectively got quieter and quieter, leaving most of the interactive questions to me, and even in those situations, they frequently did not respond. It can safely be said that the material intimidated the students, and if not that, then they had simply become disinterested with it entirely. A number of students held on until the end, holding out to see if the message would finally become clear, and for a number of them seeing the original Newtonian solution come to light with a new method lifted the fog for a moment, but when asked to apply it, the skills were still lacking. I highly regard the low level of progression in this class to their respective trades, which in most cases did not involve any semblance of use for physics. Of course, in their defense, the entire class (along with every other class taught at WTHS) scored zeroes across the board on the initial survey, which allows for vast amounts of improvement overall. Their ability to pick up calculus was just as admirable as any other class, but the variable of interest was sadly a recessive trait in this particular room, and one that is reflected in their final scores and survey responses as well.

Week 2: Period 5

As the closing chapter of Worcester Technical High for this project, the period 5 class held the torch high and surpassed every expectation that I, or Tom Gusek, had while beginning the experiment. That is certainly a very lofty statement, and one that I intend to back up. This particular class epitomized the level of interest and effort that every teacher wakes up for in the morning, if only to see what they are capable of that day. Their introduction to the material was met with the same concern for difficulty as the other classes, but their willingness to approach it with questions and practice was unique. During the first day's lecture, students took well to the algorithm of taking a derivative, and also seemed receptive of its physical significance. A number of a students asked questions regarding special situations such as the derivative of constants or of a variable to the first power, often being one step ahead of me in terms of material. At the close of the first day, Mr. Gusek said that he had been investigating some side discussions that had been going on, and to his amazement found that the discussions were actually about the material, when he had thought it was just inconsequential chatter.

The second day with this class went equally well, as their introduction to partial derivatives brought in only one new rule, and their homework scores were enough to prove that they could certainly do simple derivatives. When the nature of the partial derivative operator was explained, and the fact that it only specifies that one variable is changing brought forward, a number of students began doing examples at their desks without my prompting, which ultimately led them to ask questions about more specific functions. I gave an example that had a function of two variables that were mutually exclusive, and another one that had them multiplied together, and described how to solve both using the logic of the operator, and the product rule for the second function. Luckily enough time was available for the students to ask why the product rule works the way it does, and I could explain what exactly it was doing: checking the rate of change of each variable separately while the others remained constant.

During the time before and after class, students would approach me in the library, much like the first class had, to ask questions on the homework or the lecture. As the entire class was in the library but performing separate tasks, I felt it fair that I was present for any of their questions, but that those who wished to improve their ability should have the chance, so I sat and worked with them when they needed it. The questions were, more often than not, surprisingly not related to the mathematics, but rather were aimed towards an understanding of what all of this was for, what it did, and where it could be used. These answers I provided in detail, as the understanding and appreciation for the topic is, in my opinion, far superior to mastery of the mathematics. Surely, students must be able to complete the problems, but these are simple rules that can be memorized; going home and telling their parents what they learned and why it is useful, and having that admiration for mathematics and physics is what inspires the next generation of scientists.

As with every class, the introduction beginning of the Lagrangian Operators portion of the lectures caused great difficulty in some, but their perseverance and persistent questioning shed light on their trouble. The Operator was a simple item to introduce, but the Formalism required some explanation. Students in this class were loathe to simply accepting something, and expected some sort of explanation to justify the use of any formula. For this, I spent some time showing them that the derivatives involved in the Lagrangian Formalism were not just random happenstance, but rather they represented the generalized force, and the time derivative of the generalized momentum, and that the Formalism could unify all of mechanics: motion, forces, energy, and momentum, all in one place.

Although the class kept asking questions, the homework submissions made it clear that the material was beginning to go over their heads, but their effort never waned. Most of the submissions on the later homework that received little to no credit were not for lack of effort, but rather a lack of understanding or uncertainty of how to approach the problem. Some, however, hung on until the very end, with outstanding scores on all the homework leading up to the final survey. This class is all the proof that this project needs to show that, with enough time, students of *at least* a higher-level class could master this kind of material.

North High School

North High was the first school in which I would only be teaching one class, an AP Physics course with eleven students, all of whom Mr. Joseph Marzilli, their residing instructor, assured me would be enticed by the idea of a greater challenge than their current curriculum. It was also the first school that I would be operating alone in; Mr. Marzilli had to leave after the first day to attend a teacher's conference, and thus assigned another instructor to watch my lessons, as student teachers could not be left alone by state law. Neither of these facts particularly bothered me, as I had begun feeling comfortable and at-home in the classroom environment long ago.

The first day began, I felt, much like all of the other first days had started, since the students all tend to "feel out" the new instructor and gauge whether they like them or not. In my case, I felt that the students warmed up very quickly, in part because I had begun implementing several colloquial jokes into my introduction of simple derivatives that showed the students that I didn't just know the material, but I could apply it to things that they felt directly connected to as well. Even in such a small class as this, where one student laughing about a teacher's joke can't go unnoticed, and is at risk for judgment by peers, the students seemed at ease with directing their undivided attention to the task at hand, laughing at jokes, and making them right back. This environment felt the most like a traditional high school that I had come in contact with yet, as Doherty had felt very stoic in their desire to absorb knowledge, and WTHS had been an entirely new experience altogether; here the students were content to pick up the knowledge, and enjoy it along the way.

As we progressed into Partial Derivatives, the student became more inquisitive as to why it was we were learning this new material, which prompted a discussion on how one would be very hard-pressed to find an equation that simultaneously accurately describes a piece of our physics world, and also depends on only one variable. I then began to allude to the fact that checking partial derivatives in "certain directions" (i.e.- with respect to certain variables of a function) may yield *other* physically significant quantities, which then brought us to the night's homework, where a number of problems had them unknowingly taking important derivatives of energy equations in order to prime them for the Lagrangian Operator.

When at last we reached the Operator and Formalism itself, the students were well-versed in taking simple and partial derivatives, with many of them having seen derivatives before, but not having understood the physics significance associated with them. Many of them asked of the physical significance of the Operator and the Formalism, to which I could only tell them that the derivations themselves were outside the scope of the curriculum, but that if they wanted I could show them to them after class or some other time. The applicability, however, I could easily display by giving them the prepared examples of a ball in free-space and a spring-block system. Having a slightly longer period than usual to work in, I held a class discussion on the dummy variables, since it seemed to be the biggest sticking point, to clarify what they were, and their function. Most of the other classes had picked up the topic quickly, but failed to apply it later, whereas these students had identified their immediate issue and addressed it with me.

One of the more interesting questions that a student asked me was an inquiry as to why we were doing "such easy problems with the Lagrangian". My response was simply that the curriculum had to be designed for all levels across all schools, and thus had to take into account some simplistic problems. The nature of these problems, however, was not to show us the level of difficulty that could be reduced by the Lagrangian; that was what the pendulum problem was included for. The simplistic problems were only meant to show us that the method *works*, and if we can't show that it works, then it is useless to us as scientists. The pendulum problem was everyone's stumbling point, but there were certainly a disproportionate number of correct submissions when compared with previous schools; these students were extremely well adjusted to performing in short periods of time, and turning in assignments consistently.

This level of dedication (or at least time management) was impressive, and something rare even in students of this caliber. This was very clearly reflected in their final and total scores, which boasted an impressive ratio of perfect scores, as well as scores in the advanced and proficient range. While this class is certainly a reflection of the "best and brightest" that North has to offer, and I received no other glimpse into the lower levels to broaden my perspective, the work ethic of these students was a truly unique thing to see, and something that by all accounts one would not generally expect to find tucked away in a corner classroom of a Level 3 school.

Quabbin Regional High School

The only school taking part in the project that was not considered "high needs", Quabbin boasts a large and beautiful building several years older than WTHS, but still very well kept and adorned with banners promoting the benefits of education. This was the second school that I would be teaching a single class in, but this one was considerably larger, an AP Physics course of 27 students; roughly the same size as Doherty's, but twice and more of North's class. This would also be the last class of the project, which I was fairly disappointed about, but I kept my enthusiasm nonetheless. The room was laid out in a same, methodical pattern as WTHS, with lab tables organized in a row and column shape around the room so as to allow the instructor, Robert Kolesnik, to walk between them with plenty of room without disturbing the surrounding students. Although I never found it necessary to do this in this room, the layout is welcoming to both the instructor and the students as a sort of "free forum" that encourages questions simply by its structure.

The class itself, upon first meeting, had clearly absorbed some of Robert Kolesnik's enthusiasm, as they were especially excited to be starting a new, advanced topic. Quabbin High offers a multi-tiered Calculus program, so it came as no surprise that most of the students knew how to do derivatives; happily, not all of them did, or else the first day might have felt like a waste of their time. Those students who did not know how to perform a derivative were extremely adept at picking up the new material and applying it elsewhere. In particular, those students who did not originally have that knowledge were the first to ask what happens if you apply it to a constant, or a variable raised to the first power, topics which were two steps ahead of my place in the day's lecture.

Partial Derivatives were something that many in the class had originally thought they knew, but found that the notation, and the rules associated there with, were not what they were expecting; in fact, many students in all the schools confused partial derivatives with implicit differentiation, but the new notation separated the two ideas entirely. The new material, however, didn't phase the students in the slightest, with many scores being near-perfect, since most of them understood the algorithm and merely had to apply it in a new context. By this same extension, the Lagrangian Operator offered no trouble for them either, as they had already become accustomed to working with dynamic equations, and this was nothing more than an exercise in imagination, something they definitively did not lack.

The Formalism, as with most of the other classes, was the point where the students felt that the physical significance, something very important to grasping their individual physical models of the subject, was fading fast. The introduction of the two simplistic problems, however, reaffirmed their confidence in the material, and there was little issue taken with it until we arrived at the pendulum. As with the other schools, the pendulum was not a matter of imagination, but the difficulty of transitioning from using familiar variables such as x, v, and a, to using polar coordinates, and taking corresponding derivatives. This was very clearly not a pitfall for them, however, as their final scores reflect a series understanding of the overall material, and their enthusiasm conveyed in the open response surveys shows that it was not a simple matter of regurgitating knowledge, but that they actually felt a deeper understanding of the conceptual aspects, having encountered this new material.

The class has requested, through Robert Kolesnik, that I return with a friend and perform the classic Liquid Nitrogen ice cream feat, and perhaps show some other interesting topics, like the "ghost fingers" effect of vibrating non-Newtonian Fluids on a Faraday wave. Their interest in physics as a whole shows that there was some distinct correlation between their level of interest and their level of perception, or perhaps even between their final scores; this remains to be seen in the analysis.

Chapter 6: The Curriculum

The following chapter will present the worksheets and handouts given to students in order to provide supplementary material for their nightly assignments and for their own edification. The curriculum presents an initial survey to test their prior knowledge, followed by four nightly assignments which reflect the day's work. On the final day, a final survey is presented which tests all of the original material as well as two problems which require students to acquire the generalized forces [which lead to acquiring the equations of motion] using Lagrangian Formalism. There are survey questions on the beginning and final surveys which acquire data regarding the student's background mathematically, with regards to other physics courses, and with regards to their expectations for grades and effort in the class they are currently in. There are also questions which acquire data regarding their opinion on the material presented, as well as my own teaching ability, which is certainly a factor in the experiment.

Each survey or worksheet will have a synopsis of its intended purpose at the end which describes in detail what it was meant to accomplish. Every piece of the curriculum is meant to further progress the student towards understanding, not only mathematically, but also conceptually, what is happening in each problem, and why the Formalism works as it does. Joe Redish's paper, *Introducing Students to the Culture of Physics: Explicating elements of the hidden curriculum*, believes that, “Typically, when we prepare our syllabi, we select the content that we intend to ‘cover’. But what we really want our students to learn is more than just a set of facts: it’s a way of thinking – the manner and ‘adaptive expertise’ of the professional.”

The implication of this is that our students cannot forever regurgitate material back to the teacher like an oscillator that reaches a strong damping coefficient. We must not only present the material, we must present it in the way that is best absorbed, understood, and fundamentally rooted into them. Thus, the curriculum designed here, which informal in writing, is designed to put students at ease and assure them that the material they are poring over is well within their reach, and that physics and mathematics, particularly calculus, are not the terrifying monsters under their beds that we have led them to believe.

With this in mind, the worksheets, which sum up the days’ lectures, are designed to be a recap and a flashback to the statements made in class, often laced with familiar analogies, which they can relate to in order to shed light on the black box that mathematics is typically kept in. Each problem introduced is meant to demonstrate a specific quality of either the derivative operator or the Lagrangian itself, and the problems likewise reflect this. In this way students are introduced to a new mathematical item and instantly convinced that it works by means of familiar examples.

In-class Day 1: Initial Survey

Name:

Grade: (9, 10, 11, 12?) :

Level: (College Prep, Honors, AP, etc.):

School Name:

Initial MQP Survey

Lagrangian Formalism in High Schools

1) Have you ever encountered derivatives before? Please circle one: Yes No (if no, please bare with me)

2) Please take the following derivatives:

i) $\left(\frac{d}{dx}\right)\left(x^2\right)$

ii) $\left(\frac{d}{dx}\right)(3x+4x^3)$

iii) $\left(\frac{d}{dy}\right)(5y-2y^2)$

3) Have you ever encountered derivatives of trigonometric functions? Please circle one: Yes No
(once again, if the answer is no, please try your best nonetheless)

4) Please take the following derivatives:

i) $\left(\frac{d}{d\theta}\right)\sin\theta$

ii) $\left(\frac{d}{d\theta}\right)\cos\theta$

iii) $\left(\frac{d}{d\theta}\right)(\sin\theta - \cos\theta)$

5) In your math or physics courses, have you ever encountered partial derivatives?
Please circle one: Yes No

6) Please take the following partial derivatives:

i) $\left(\frac{d}{dx}\right)(x+y)$

ii) $\left(\frac{d}{dx}\right)(x^2 + y^2)$

iii) $\left(\frac{d}{dy}\right)(y^2 + 2xy)$

7) Have you ever worked with energy concepts (i.e., kinetic, potential)? Yes No

8) Please provide the formulae for both kinetic and potential energy (assume regular variables, mass *m*, height *h*, velocity *v*, etc.)

____

Thanks very much ☺, all the physics and math is done now, I just need a few other things.

9) Have you ever taken a physics course before this one? Yes No

If so, which ones? ____

10) Do plan to pursue a career in the physical sciences? Yes No Not sure

11) How many hours a week do you spend working on the homework for this class? 0-2 2-5 5-8 8-12 12+

12) How many hours a week do you spend studying the material for this class? 0-2 2-5 5-8 8-12 12+

13) Have you ever performed experiments in your class? Yes No

14) As of this survey, what is your current level of interest in Physics?

1 2 3 4 5 6 7 8 9 10

(not interested) (very interested)

15) What grade do you expect to receive in this course?

A+ (95-100) A (92-95) A- (90-92) B+ (85-89) B (82-85) B- (80-82)

C+ (75-79) C (72-75) C- (70-72) D (65-70) F (< 65)

Thank you for participating in my survey. You'll be given a similar one near the end of the lectures to see if anything's changed. In the meantime, I ask you to have an open mind while I introduce the new material. There are a number of ways to solve every problem; it's just a matter of whether you have the proper tools to solve them. The Lagrangian method of solving basic kinematics problems is simply another tool for situations where you are given less than you believe necessary to find the solution.

Synopsis:

The initial survey serves as a determinant of the knowledge of calculus that students already possess with regards to the experimental curriculum, as prior familiarity with the material would certainly change the level of expectation to hold that particular student to. At Doherty Memorial High, for instance, many of the students entering into AP Physics have likely taken Pre-Calculus, and are normally engaged in AP Calculus while also taking AP Physics. As such, they are extremely likely to have dealt with derivatives before, making the explanation of partial derivatives a much smoother task, and the over curriculum an easier transition. The students at Worcester Technical High, however, are in an environment that has no calculus classes, and thus are unlikely to have ever seen the derivative operator in any form up to this point.

The surveys are also used to determine an image of the student's level of effort in the class prior to my arrival, their interest in the subject, and a number of other data points that are useful for comparative purposes, such as race, gender, class level, etc. With these data points, along with the statistical data available for the schools online, which include MCAS scores, selected populations analyzed against their scores, racial demographics, and annual improvement rates, the experiment can be evaluated in a number of ways, from a number of angles, in order to better illustrate the overall result.

Night 1 : Simple Derivatives 101

A derivative has two "methods of thought", or two ways you can think about it. It has a physical meaning, something that you could look around and see. It also has a more abstract mathematical meaning, which is how we manage to take that physical meaning and give it something mathematicians and scientists can use it for. We'll go a little into both; this worksheet is meant to supplement the lecture for when you are at home.

What *is* a derivative? (The physical meaning)

If you recall from our studies in kinematics, the idea of a **position**, a **velocity**, and an **acceleration** are all related to each other by some mathematical concept that was just out of our reach at the time. That concept is a derivative; a **derivative** is a measure of how fast something is changing with respect to something else. When we talked about velocity, we wrote it like this:

$$\mathbf{v} = \frac{\Delta x}{\Delta t}$$

Remember? Velocity is the *change* in position as *time* is also changing. Velocity is the **first derivative** of position. What that means is that if we look at the way position changes as time changes, we find the velocity. There's another way to write this that means exactly the same thing, but takes a form we need to become more familiar with; we can write it as a derivative.

$$\mathbf{v} = \frac{dx}{dt}$$

The "*d"* that replaced the triangle *delta* means exactly the same thing, a change, but it's written in the form that mathematicians use. By that same idea, we can write acceleration (a change in velocity as time changes), which was originally in the form:

$$a = \frac{\Delta v}{\Delta t} \qquad \textbf{as} \qquad a = \frac{dv}{dt}$$

This means that acceleration is the **first derivative** of velocity. Now, if we look very closely at the new way of writing acceleration, we see that since we've already related velocity to position, and acceleration to velocity, there must be a way to relate position and acceleration, and indeed there is. If we replace *v* in the formula

$\mathbf{a} = \frac{dv}{dt}$ with the formula that we found for velocity,

$\mathbf{v} = \frac{dx}{dt}$, then we get

$$\mathbf{a} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

This strange-seeming result doesn't seem to tell us anything immediately. What it tells us is that acceleration is the derivative *of the derivative of* position, or the **second derivative** of position. It means that if we take a look at the change in position while time changes, and then take a look at how *that* is changing, we get acceleration. In other words, acceleration is the rate of change of velocity.

With all that said and done, I only need you to take away two things:

1) Derivatives, physically, are the rate or speed at which something changes with respect to something else (like position and time, velocity and time, etc.)
2) The new notation of a derivative is only for convenience, as you will see when we get to the mathematical portion of this. If we left the triangle deltas in our notation, things would become confusing very quickly. Understand that they mean the same thing as before, but we're going to attach a new meaning to the "d" notation.

How do we *use* a derivative? (The mathematical meaning)

Now that we understand what a derivative *is*, we have to dive a little deeper into how we actually use it. The reason I introduced the physical meaning first is because the mathematical explanation is by no means something you would look at and say "Aha! That makes perfect sense!" The derivative's historical background is extremely mathematically intense, and although the purpose of this lesson is to show you some more advanced material, how the derivative came to be is far too complex. What I'd like to show you is simply the pattern we will *always* follow when taking a derivative.

First, we start with something called a **function**. A function is basically just a formula that has a few variables, and the answer depends entirely on what numbers you put into it. The notation we use for a function is *f( )*, which, read out loud, means "some function that depends on the variables inside the parentheses."

Let's say that our function is the formula x^2. So we would write

$$f(x) = x^2$$

because our formula is a function, and it depends on the variable *x*. Now that we have a function, let's take the derivative of it.

<u>Taking a derivative</u>

Taking a derivative, no matter what function or formula we're working with, will always follow the same pattern:

1) Take the exponent (the number above the variable, so in this case the 2) and drop it down to the left of the variable (it becomes the coefficient, the number being multiplied by the variable).

2) Subtract 1 from the number you just dropped down, and place *that* number above the variable, where the first number was. In other words, it becomes the new exponent.

There are obviously special cases and more interesting functions that we could work with, but this is the basic idea. Let me write that out so you see how that works:

The derivative of our function, the derivative of *f(x)* with respect to the variable *x*, is written

$$\frac{df(x)}{dx} = \frac{d}{dx}\left(x^2\right)$$

With step 1, I take my function x^2, and I drop the number above the variable down:

So x^2 becomes 2*x*. Then I subtract 1 from that number and place it above the variable:

2x becomes $2x^1$

Which in this particular case, are exactly the same thing.

So $\frac{d}{dx}\left(x^2\right)$ = 2x.

What if our function has more than one piece to it? (Which it will)

If our function has more than one piece to it, we go to each piece of the function and do the same thing to each of them, paying attention *only* to the piece we're on until we've done each part, and then putting them all back together in the end.

Ex. $f(x) = 2x + 3x^3$

Focus on one part at a time:

2x is also written $2x^1$, so we start by dropping down the number above the variable and multiplying it by what's already there, then reducing that number by 1 and placing it above the variable.

So $2x^1$ becomes $2*1*x^0$ = $2x^0$, and anything to the 0^{th} power is just 1, so 2x becomes just 2.

Now $3x^3$:

Drop down the 3 and multiply it by what's already there, which makes 9. Then we subtract 1 from 3 and place it back above the variable.

So $3x^3$ becomes $9x^2$.

Now we have all the pieces, so we combine them to get our full derivative of the function:

$$\frac{df(x)}{dx} = 2 + 9x^2.$$

This pattern is literally how we're going to do it every single time. We may not always have *x* for a variable, but the pattern will be the same. If we were taking a derivative of a function that only had the variable *y* in it, then the derivative would look like this:

$$\frac{df(y)}{dy}$$

Or if it depended on the variable "fish"

$$\frac{df(fish)}{d(fish)}$$

And so on.

What happens if my function has a constant? (Like a number)

A number, all by itself, can't change as some other variable is changing. It isn't attached or connected to anything, so why would it? If a derivative is a measure of how something changes, then the derivative of a constant is simply 0. So if you see a number sitting all by itself, and you need to take the derivative of it, it's just zero.

What this means is that we're taking the derivative *with respect to* some variable. We're paying attention only to that variable. In simple derivatives, there is only one variable to worry about, so this hardly matters. Later we'll look into some trickier problems where that idea will matter a lot. For now, let's do some practice.

Problem Set: Please take the following derivatives (no tricks here, they're all done using the above method):

1) $f(x) = 4x + 2x^2$

2) $f(x) = x^1$

3) $f(x) = 8x^{10}$

4) $f(x) = 5$

5) $f(x) = x^2$

6) $f(y) = 5y - 2y^2$ (take the derivative with respect to y)

Synopsis:

The tutorial on derivatives is meant to act as a supplement to the day's lecture, which offers an introduction to the mathematical abstraction known as the "derivative operator", and its corresponding notation. In situations where a student has already seen derivatives, it is meant to illuminate the physical description of a derivative, which is hardly touched upon in mathematics courses. Students are prone to respond well to analogous experiences, and the comparison of a derivative to the speed at which current flows through the wires over our heads, the speed of water flowing out of a faucet, or the rate of change of position of a ball kicked across a field, help them see that realistically, there is a physical and a mathematical explanation for everything; the reality is all around us, but the responsibility lies with us to quantify it in some way.

With this explained, the mathematical aspect is nothing more than a simple rule to memorize, which students are accustomed to doing in a high school environment; if something is not fully explained, students are generally expected to memorize, and later regurgitate, equations, facts, laws, principles. The idea is to help them make the connection between the physical and the mathematics, and draw this connection into a level of significance great enough for students to appreciate the relationship between the visual and the abstract. Secondarily, becoming familiar with the idea that variables are just symbols, and that absolutely anything could be considered a variable, is a subtle segue into the idea of partial derivatives, which is covered the next day.

Night 2: Partial Derivatives 101

Now that we've seen what we can do with derivatives, and how useful they can be, we want to take a look at something slightly more realistic. In reality, things change in every direction, not just one. An object's position can change in the x, y, z directions; it can rotate, it can spin, and it can translate. Reality is terribly, terribly complicated, but don't panic, because there are always partial derivatives.

Where a simple derivative could find the rate of change of something that had a function of only one variable, a **partial derivative** can find the rate of change of something with respect to only one variable in a function that has *more* than one variable. In effect, if we had a function that represented an object's motion in three-dimensions (the variables x, y, and z), and we took a partial derivative with respect to, say, *x*, what we've found is it's velocity in the *x*-direction, while leaving the other two directions untouched. Interesting!

So what's so special about a partial derivative?

A partial derivative, as I explained, means that we're looking at a function that has multiple variables to consider, but we only want to take a derivative with respect to one variable. So there are a few things that we need to know:

1) This is a big one. When we take the partial derivative of a function, we only take the derivative of the function with respect to the variable in the denominator of the $\left(\frac{d}{dx}\right)$ symbol (so in that example, it's *x*). What this means is that we've decided that that is the *only* variable that is changing, so everything else is staying the same. So if the variable you're looking at is *not* the one marked in the derivative symbol, then we treat it in one of two ways, either as:
   a) Constants, if they're all by themselves and not attached to anything (and remember, the derivative of a constant is zero)

   Or

   b) A coefficient if it is being multiplied by a variable you *do* care about. (in other words, when you do the pattern to take the derivative, leave that variable right where it is).

2) A partial derivative has a slightly different notation:

Where before we wrote $\left(\frac{d}{dx}\right)$, we'll now be considering a function where there are multiple variables, so to distinguish that fact we write it as $\left(\frac{\partial}{\partial x}\right)$. I wish I had a good name for that slightly squiggly "d", but there actually isn't

a name for it. So "squiggly 'd' " it is.

3) Once we've established the other rules, understand that the *method* of taking the derivative is still the same, the pattern we followed is exactly as before.

Fortunately, there aren't that many extra rules in taking a partial derivative, so let's take a look at some examples before I hand off some problems for you to do:

Examples:

Let's take the function

$$f(x,y) = x^2 + y^2$$

You'll notice that the f(x) has suddenly become f(x,y). This is because the function now depends on two variables, and our notation must reflect that.

So, what can we do with this? Let's say we want to take the derivative of this function with respect to the variable *y*.

We write this as $\left(\frac{\partial}{\partial y}\right) f(x, y)$

So now let's see how we can do this according to our rules. Ask yourself these questions:

1) What variable am I paying attention to?
   Answer: *y*

2) Which variable(s) am I *not* paying attention to, or treating as coefficients/constants?
   Answer: *x*

Easy enough. Alright, so if we look at the function:

$$f(x, y) = x^2 + y^2$$

We see that the variable we're paying attention to is all by itself, and the variable we aren't paying attention to is also all by itself. That's convenient. This means that we can do the usual method of taking a derivative to the variable *y*, and treat x like a constant. Since we know the derivative of a constant is zero, we know that *y* is going to disappear from the answer entirely. Looking back at our examples from simple derivatives, we see that the derivative of y^2 with respect to the variable y is just 2y.

So let's put it all together. We realized that x is going to be treated as a constant here. Why? Because we're only concerned with y in this problem. Since the derivative of a constant is zero, x disappears. We take the derivative of the remaining part of the problem the way we normally would.

Thus we end up at the result:

$$\left(\frac{\partial}{\partial y}\right) f(x, y) = \left(\frac{\partial}{\partial y}\right) x^2 + y^2 = \left(\frac{\partial}{\partial y}\right) x^2 + \left(\frac{\partial}{\partial y}\right) y^2 = 0 + \left(\frac{\partial}{\partial y}\right) y^2 = 2y .$$

Voila! Can you guess what the answer would be if we did the same thing taking the derivative of that function with respect to x?

$$\left(\frac{\partial}{\partial x}\right) f(x, y) = \left(\frac{\partial}{\partial x}\right) x^2 + y^2 = \left(\frac{\partial}{\partial x}\right) x^2 + \left(\frac{\partial}{\partial x}\right) y^2 = 2x + 0 = 2x.$$

Let's see an example that treats the variable we don't care about as a coefficient. We'll be needing this very soon, so pay close attention to this one.

Given the function

$f(x, y) = 2xy^2$

Find the partial derivative with respect to y.

So we want

$\left(\frac{\partial}{\partial y}\right) 2xy^2$

In this problem, the variable we pay attention you *is* attached to a variable we don't want to pay attention to. In this case, all we have to do is treat *x* as though it were just another number attached to the variable. We can pretty much consider 2*x* to be one big coefficient constant. So imagine, in this case, that 2x is just a number, and take the derivative of y^2 as you normally would. So y^2 becomes 2y, and then we bring in the part that we left alone:

(2x)*2y = 4xy   And there's our answer.

What would happen if we took the derivative with respect to x? Well then the 2 and the y^2 are both things we don't care about, and we take the derivative of x as we normally would. Since the derivative of x with respect to x is just 1, then our answer is just

$\left(\frac{\partial}{\partial x}\right) 2xy^2 = 2y^2 \left(\frac{\partial}{\partial x}\right) \mathrm{x} = 2y^2 \left(\frac{\partial x}{\partial x}\right) = 2y^2.$

You'll notice I did something interesting here. Because I'm taking the derivative of a function that has parts I don't need to pay attention to and parts I do , I can basically just push the derivative over to the part I care about, and pull the other variables/numbers out until I'm done.

So what's the take away from all of this?

Key Points:

-Taking a partial derivative just means that we only use the "derivative method" on the variable in question. That is, the one marked in the denominator symbol.
-All other variables we treat just like we would a number, whether it be a constant, a coefficient, etc. Just pretend it's a number, but leave it in variable form.
-Once we've established what variable we're paying attention to, we use the same method of taking a derivative that we used for simple derivatives. No exceptions.

Now you try!

Problems: Please take the following partial derivatives with respect to the variables requested.

1) $\left(\frac{\partial}{\partial x}\right)$ for the function $f(x,y) = 2x^2 + 2y^2$

2) $\left(\frac{\partial}{\partial y}\right)$ for the function $f(x,y) = 2x^2 + 2y^2$

3) $\left(\frac{\partial}{\partial x}\right)$ for the function $f(x,y) = 2x^3 + y^3$

4) $\left(\frac{\partial}{\partial y}\right)$ for the function $f(x,y) = 2x^3 + y^3$

5) $\left(\frac{\partial}{\partial z}\right)$ for the function $f(x,y,z) = 4xyz$

6) $\left(\frac{\partial}{\partial m}\right)$ for the function $f(m,h) = mgh + \frac{1}{2}mv^2$

7) $\left(\frac{\partial}{\partial m}\right)$ for the function $f(m,h) = mgh$

8) $\left(\frac{\partial}{\partial h}\right)$ for the function $f(m,h) = mgh + \frac{1}{2}mv^2$

9) $\left(\frac{\partial}{\partial v}\right)$ for the function $f(m,h) = mgh + \frac{1}{2}mv^2$

10) $\left(\frac{\partial}{\partial x}\right)$ for the function $f(m,x,v) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

Synopsis:

The topic of partial derivatives is clearly an important one when considering, in foresight, the idea of the Lagrangian Formalism. Before students learn the additional rules to apply to functions of more than one variable, the physical explanation is necessary in order to make the mathematics significant. Understanding that the rate of change now takes place in only one dimension while nothing else changes in a multidimensional space is key to grasping the meaning of the equations given. In reality, outside the comfort of the idealizations and simple situations that are typically handed to students as a basis for accepting certain principles, everything works in three dimensions, and almost everything is a function of something else, which means that simple derivatives are a rarity in explaining the true nature of nature.

As such, the tutorial on partial derivatives is explained to be not only widely used in physics, but widely used *everywhere*. A partial derivative could describe heat flow in an oven for those engaged in culinary arts, or fluid flow from a faucet for a plumber. It could describe charge traveling across a wire for an electrician, or the velocity of a ball kicked across the field in one particular direction for a soccer player. Of course, it’s mathematical uses are more practically used in physics and mathematics, but nonetheless, the point is made that a partial derivative has a physical meaning, as does any abstraction made in physics.

The new rule associate with the partial derivative is simple to remember, but difficult to apply without first understanding what is happening to the function. Students were given a function with four parts: one with an order 3 polynomial, one with an order 2, order 1, and finally a piece of the function that was simply a number. They were instructed to imagine what the result would be if one of the variables had the value unity. They worked their way through the function and acquired separate answers for each piece, ultimately concluding that the number was simply itself. When asked what would happen if that variable were now zero instead, they realized that each part of the function containing that variable would now be zero, but that the number still remained as itself. The connection made, then, was that if we considered those two values of the variable to be points on a graph, we would be taking the slope of the graph, which we had previously realized was, by nature, a derivative.

As such, we were looking at a rate of change, or a "final minus an initial" value. We notice, then, that the number has remained the same throughout, implying that any *constant* is never going to change, and thus its derivative must be zero. Applying this to the idea of a partial derivative, they were led to understand that variables, despite their name, do not all have to vary at once, they can vary one at a time. Since this is true, then anything that isn't changing (which is what the derivative operator specifies) must then be a constant, which makes its derivative zero. The mathematical idea is related to a physical equivalent, and the student can begin understanding what is physically happening to the function piecewise.

Night 3: Lagrangian Operators 101

Now it's finally time to put those derivatives to use, and I can think of no better use to put them to than Lagrangian Operators. An **operator**, in mathematics, is an abstract mathematical object that *does* something. For instance, there is an operator called "del" which, if used on a function, is understood to take the partial derivative of the three directions x, y, and z, and add them all together. Our operator is actually a function all by itself, and what we *do* to the function is what matters. When used correctly, the Lagrangian Operator will tell us the object's equation of motion (how it will move), regardless of the difficulty of the problem.

The Lagrangian Operator

The operator itself is denoted by a cursive capital L, or L.

The function that the operator contains is something oddly familiar, bringing back a topic we've certainly seen before; energy! The Lagrangian Operator is simply the difference of the Kinetic Energy and the Potential Energy of the system you're working with.

$$L = KE - PE$$

Easy enough, right? However, there is something that we must become used to when dealing with these problems, which is that **the energies listed in that equation are the kinds of energies it will have at the moment it is moving.** What this means is that, even if the problem states that the object in question isn't moving, we have to imagine the kinds of energies that it will have when it does. We need a complete picture of how the system is going to move, so we need to imagine the moment that all the energies the object can have are in play.

In reality, there are only three kinds of energies that we'll ever have to worry about:

Kinetic Energy: $\frac{1}{2}mv^2$

Gravitational Potential Energy: mgh

Spring Potential Energy: $\frac{1}{2}kx^2$

With that in mind, making the Operator itself shouldn't be a problem. Once we've constructed the Operator, we need to take the derivative of it.

<u>Taking Derivatives of the Operator</u>

Remember awhile back when I mentioned that absolutely any mathematical formula could be called a "function", and that we could take it's derivative? Well the Lagrangian Operator is most certainly a function, and we're about to take its derivative. A lot.

There are a number of derivatives that we'll take of the Lagrangian Operator, but first we have to establish the variables we're taking the derivatives with respect to.

Generalized Position Coordinate: When we construct our Operator, KE – PE, we're always going to have at least one position (displacement, distance) coordinate, which will always be in the Potential Energy term. Whether that Potential Energy is Gravitational or Spring, both of those energies have some position in it [Gravitational has *h*, and Spring has *x*, both of which are "positions" in space]. Luckily the problems we will be dealing with will only have one of those at a time, so you won't be seeing problems with both spring and gravitational potential energy.

Because we're going to take the derivative of the Operator with respect to some position coordinate, but *which* position variable we use may change from problem to problem, we have something called a Generalized Position Coordinate, denoted by "q".

One of the derivatives we'll be taking is the derivative of the operator with respect to "q", and it's your job to replace "q" with whatever position variable you're using in this problem, whether it be *x* or *h*.

So when you see $\left(\frac{\partial L}{\partial q}\right)$ that means "take the derivative of the Lagrangian Operator with respect to the variable "q", where "q" is whatever position coordinate you're using in this problem.

Ex. 1 : A ball rolling down a hill will have kinetic energy and gravitational potential energy

$$L = KE - PE = \frac{1}{2}mv^2 - mgh$$

$$\text{So } \left(\frac{\partial L}{\partial q}\right) \text{ is really } \left(\frac{\partial L}{\partial h}\right)$$

Then $\left(\frac{\partial L}{\partial h}\right)$= 0 – mg (since *h* is nowhere in the first term, that whole term is a constant, and its derivative is 0, and in the second term, I use the normal way of taking a derivative, keeping *m* and *g* as constants, and get –mg as my answer).

Finally,

$$\left(\frac{\partial L}{\partial h}\right) = mg$$

So we got a somewhat familiar answer from this term, the weight of the object. This will be important later, but this is how we go about taking the derivative of the Operator with respect to a " generalized position coordinate"

Generalized Velocity: In the same way that there is a generalized position, there is also a generalized velocity, but for our purposes it's much less complicated. We'll only be encountering problems with one kind of velocity (with the small exception of a particularly difficult example I'll show you), so this will be quick.

The Generalized Velocity is denoted by $\dot{q}$, or "q dot". In physics, a dot above a variable indicates the rate of change of that variable; in other words, its velocity. Since we know that "q" is some sort of position, we know that the rate of change of position is its velocity, so $\dot{q}$ is going to mean a velocity. When we take the derivative of the Operator with respect to $\dot{q}$, we're saying that the variable to pay attention to while taking the derivative is the velocity.

Time Derivatives: This part is simple, we just need to set a couple ground rules:

1) Mass is never changing, so we'll never be taking a derivative of mass with respect to time.
2) Remember that position, velocity, and acceleration are related by rates of change over time.

$$a = \frac{dv}{dt} \quad \text{and} \quad v = \frac{dx}{dt}$$

So with all of that being said, let's get into how we put this all together.

Lagrangian Formalism

The Lagrangian Formalism is how we put all the derivatives together in a special way, such that given any problem, we can figure out how the object will move, otherwise known as its "equation of motion". At the very least we'll be able to recover some familiar laws. This equation is what we're doing all of this for, so here it is:

Deriving the Equation of Motion

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

It doesn't look like much, but there it is.

The left side of the equation is just the partial derivative of the Operator with respect to the position variable, no questions there. The right side is more interesting.

On the right side we have a partial derivative of the Operator with respect to the velocity variable, which I introduced above. Following that on the outside of the parentheses is a lone time derivative. Here's the order we follow this in (on the right side)

1) Take the derivative of the operator with respect to the velocity coordinate
2) Take the derivative of the RESULT of that derivative with respect to time.*

*Note: You'll notice that none of the energies we've dealt with *have* time in them, so what are we to do? The answer is simple: The energies we've dealt with have variables that *depend* on time, and their time derivative is something special. In most cases you'll take the time derivative of velocity, and what's that? Acceleration. Just be aware that you want to wedge that time derivative into your answer until it's working on something that actually *is* time dependent.

Then of course, on the left side, we'll take the derivative with respect to the position variable, and hope to see something interesting (which we will ☺).

Now that we understand the formula, and we've seen some piece-by-piece examples, let's try some problems. Before we begin, let me make these clarifications:

- The Lagrangian **Operator** is the function that we create based on the problem we're working with: **L= KE – PE.**

- The Lagrangian **Formalism** is the series of derivatives that we take *on* the Operator with respect to certain variables. The Formalism has a set pattern and we follow it every time.

- In short, we have to build the Operator before we can use the Formalism. Building the Operator is just a matter of imagining what kinds of energy a system will have while it's moving, and plugging their formulae into the Operator.

Problems:

1) $\left(\frac{\partial L}{\partial q}\right)$ for the function **L =** $\frac{1}{2}mv^2 - mgh$

2) $\left(\frac{\partial L}{\partial \dot{q}}\right)$ for the function **L =** $\frac{1}{2}mv^2 - \frac{1}{2}kx^2$

3) $\left(\frac{\partial L}{\partial q}\right)$ for the function **L =** $\frac{1}{2}mv^2$ - $\frac{1}{2}kx^2$

4) Create the Lagrangian Operator for a ball falling (mid-air) back to earth.

5) Use your answer for 1) to solve the Lagrangian Formalism for that problem. You should see a familiar answer if you've solved it correctly. Remember that a time derivative acts on any variable that we know changes with time, like position or velocity.

Synopsis:

With simple and partial derivatives out of the way, the students now have all the mathematical background that they need in order to begin working with the Lagrangian Formalism. Unfortunately, explaining the physical significance is difficult here, because the formula is so non-intuitive, and so deeply based in the Calculus of Variations, that explaining its origin in order to enlighten students as to its innermost machinations is impossible. As such, explaining the formula to its full level of detail, as had been done previously with derivatives, is also impossible, which leaves the students at a loss for why we're using this obscure formula.

All that is left to do to regain their confidence in this strange equation, is to prove that it works. The students are first introduced to the Lagrangian Operator, which is a simple, general equation that takes the difference of two energies. The origin of this equation is also rooted in Calculus of Variations, but previous examples from the homework are used to explain that there is a definite physical significance to the derivatives of this function with respect to certain variables. Furthermore, calculus can be used to relate all of the topics of classical mechanics: Forces, Motion, Energy and Momentum, all using this one formula.

The Formalism is brought in as the series of derivatives that we take of the Operator in order to connect the pieces of significant data together. We notice right away that taking the derivative of the difference of energies with respect to position and velocity yields interesting results: the generalized force and generalize momentum. In a simple example using a ball in free fall, which has kinetic and gravitational potential energy, the derivative with respect to velocity produces the obvious term for momentum, and the derivative with respect to position yields the force, which in this case is just weight. This connection to physical concepts already covers establishes a trust in the equation being used, provided that students are aware of what is happening mathematically.

Once students are comfortable with taking the two partial derivatives in the Formalism, the last step is more complicated. Most students immediately realized that the time derivative leftover was acting on a function that *had* no time term, which brought up the idea that a variable could be intrinsically time dependent. Position, velocity, and acceleration had all been correlated earlier in the lessons by calculus, describing the derivative forms of each of them. With this in mind, time derivatives were said to act "only on things that are, in fact, time dependent" such as those three variables. Connecting a time derivative to one of those variables produces the next step down in the derivative chain, so in this case, taking the derivative of velocity with respect to time produced acceleration, as it often did in most problems.

The concept of the time derivative is the most difficult idea to convey, simply because it is very hard to think of a variable as varying based on some *other* variable; a variable within a variable, so to speak. Conveying the concept that some variables change naturally with time, while others change naturally with respect to variables other than time, is a strange abstraction to make, but ultimately one that is necessary to understand the physical consequence of the Formalism.

Night 4 Lagrangian Problems

Now that all the math is solid, all the derivations complete, and all the formulas available to us, we can finally begin digging into the actual problems we might encounter with this new method of solving problems. Surely after all of this we'd like to prove to ourselves that this works, right?

Ex.1: A ball in free space (i.e.- Monday Night's Homework)

For a ball in free space, free to move and pretty much do as it pleases, we can (and will) safely assume that it will, at some point, have both a kinetic and a gravitational potential energy. We haven't been told anything about a spring, so it's rather difficult to imagine that it could have that kind of energy. We won't assume that it has that kind of energy.

Our Lagrangian Operator, then, will look like this:

$$\mathbf{L} = \frac{1}{2}mv^2 + mgh$$

From here, we apply our Lagrangian Formalism, and go through the steps to take each derivative. Our Formalism is written as

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

Recall that "q" is our "Generalized Position Coordinate", and it is our job to look at our Operator and determine which one we'll be using. Since we're using gravitational potential energy, our position coordinate is "h", and "h" will replace the "q" in the first term, so our Formalism looks like this:

$$\frac{\partial L}{\partial h} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

Finally, we look at that pesky $\dot{q}$ and decide what is going to replace that. Fortunately it's as simple as I promised in class. For the problems you'll be expected to do, $\dot{q}$ is going to be replaced by "v" absolutely every time, since $\dot{q}$ is a time derivative of position, which is (as we know) a velocity. Now our

Formalism looks like this:

$$\frac{\partial L}{\partial h} = \frac{d}{dt}\left(\frac{\partial L}{\partial v}\right)$$

So now lets go through and replace the Lagrangian Operator with the actual function in each of the pieces it appears, and see what we get from it:

$$\frac{\partial}{\partial h}\left(\frac{1}{2}mv^2 + mgh\right) = \frac{d}{dt}\left(\frac{\partial}{\partial v}\left[\frac{1}{2}mv^2 + mgh\right]\right)$$

Now, we can distribute the Operator into every term so we can take a look and see if we're interested in it:

$$\frac{\partial}{\partial h}\left(\frac{1}{2}mv^2\right) + \frac{\partial}{\partial h}(mgh) = \frac{d}{dt}\left(\frac{\partial}{\partial v}\left(\frac{1}{2}mv^2\right) + \frac{\partial}{\partial v}(mgh)\right)$$

Ignore the time derivative for now, and go through each piece of the function to see if it contains the variable specified by the operator. A close look at each piece shows that the first term on the left side has no "h" and is thus going to be 0. The second term on the right side has no "v", and will also cancel out, leaving us with:

$$\frac{\partial}{\partial h}(mgh) = \frac{d}{dt}\left(\frac{\partial}{\partial v}\left(\frac{1}{2}mv^2\right)\right)$$

Well that's dandy! Now we only have to take two derivatives instead of four. Also, pardon my use of the term "dandy". Moving on.

So let's take these derivatives shall we? We know that everything we have leftover has variables of interest to us, so we *know* we're going to be using the pattern of taking a derivative. Start with the first term:

$$\frac{\partial}{\partial h}(mgh)$$

The variable of interest is "h" so we pull out all the constants (**don't drop them, just pull them out**), and the operator pushes its way into the function until it reaches the variables it's interested in:

$$\left(mg\frac{\partial}{\partial h}h\right)$$

We know that this derivative will just be 1, so our final result on this side is just *mg*.

Now our formula looks like this:

$$mg = \frac{d}{dt}\left(\frac{\partial}{\partial v}\left(\frac{1}{2}mv^2\right)\right)$$

Let's take the derivative of the term on the right, ignoring the time derivative until we need to pay attention to it.

$$\left(\frac{\partial}{\partial v}\left(\frac{1}{2}mv^2\right)\right)$$

Our variable of interest is "v", so we pull all the other constants to the outside of the derivative so it can do its job:

$$\left(\frac{1}{2}m\frac{\partial}{\partial v}v^2\right)$$

The $\frac{1}{2}m$ can hang on the outside until the derivative is finished. Taking the derivative of v^2 is exactly as we remember it, following the pattern and ending up with 2*v*.

So if we bring that term together, we get

$$\left(\frac{1}{2}m[2v]\right) = mv$$

Finally, let's pull the time derivative back in and take a look and what's going to happen:

$$\frac{d}{dt}(mv)$$

Since we know mass is never going to change (because we let that be true from the very start), then the only thing that is time dependent is velocity, and we push the time derivative over to the velocity, and produce something we've seen before:

$$\left(m\frac{dv}{dt}\right)$$

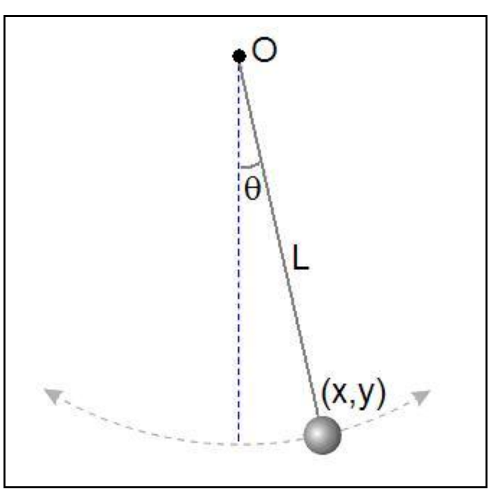
This final term is *definitely* something we've seen before, since the time derivative of velocity is acceleration. Thus the right side of our equation is just *ma*.

Now, finally, let's put it *all* together, the left side and the right side, to show how the ball is going to behave:

$$mg = ma$$

This should look terribly familiar. All this statement says is that the ball's movement is determined solely by its weight, *mg*!

Phew! That seems like a heck of a lot of work to produce something we could have *guessed* , right? Yeah, you win on that one. Unfortunately I needed to show you it really did work before I could show you a harder problem. So without anymore ado, here is



Ex. 2: A Pendulum That Barely Touches The Floor at the Bottom

Here we have a pendulum of length *l* holding a mass *m* at the end, and the pendulum can swing back and forth in two-dimensions (x,y). This problem is going to be tricky, and it's going to show exactly how simple a problem like this can become if we only choose our position coordinate correctly.

In this problem, the energies are not immediately obvious. We have to think long are hard about what they'll look like, because now we technically have circular motion, so our usual idea of using just "v" doesn't quite work here.

To begin, we have to stop and consider what "position" variable we want to choose to use throughout the problem to make this easiest. The pendulum is moving in the x and y direction, true, but those movements are both controlled by *one* variable, namely theta, θ. This will be our generalized variable. So before we construct our operator, we know that our Formalism will look like this:

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}} \right)$$

What the Operator looks like is for us to figure out.

The Kinetic Energy term normally looks like :

$$\frac{1}{2}mv^2$$

The mass term can obviously stay, that isn't going to change, but we need some sort of velocity that depends on theta, and the length of the string. As it turns out, we can get a velocity in units of m/s by multiplying the length of the string (meters), by the time derivative of theta (the speed at which the angle changes). So our expression for *v* will be denoted

$$v = l\dot{\theta}$$

Notice that the speed of the angle change is denoted by "theta-dot". Now, if we substitute that expression into our Kinetic Energy, we have something we can work with!

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(l\dot{\theta} \right)^2 = \frac{1}{2}ml^2\dot{\theta}^2$$

So KE = $\frac{1}{2}ml^2\dot{\theta}^2$

The gravitational energy term is a little more difficult, but close your eyes and imagine this if you will: Well, first open your eyes and read what I have to tell you, *then* imagine it. Up to this point, our choice of what to call the "zero point" for height has been obvious. Now we must be careful.

We'll choose the ground as our starting point, and call it zero. The height that the pendulum reaches is equal to the full length of the string minus the y-component of the length of the string when it is off the ground. It seems strange to use the length of the string to measure our height from the ground, but the formula says it all:

$$\text{PE} = mgl(1-\cos\theta)$$

What this says is that, if the pendulum were at the bottom of its swing, facing straight down, the angle would be 0, and since $\cos(0) = 1$, then the formula would read

$$\text{PE} = mgl(1-1) = 0$$

Since we would have no height. Make sense? Now we have KE and PE, and our Operator looks like this:

$$\mathbf{L} = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$$

And our Formalism is

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)$$

So now what? Well folks, that's your homework.

Homework: On a separate sheet of paper, take the Operator and Formalism and work out the solution to the Formalism.

You may not recognize the solution you get. The truth is that scientists very rarely *expect* a certain solution, they simply go where the theory takes them. That's what you'll do tonight. Make sure that everything you're doing is mathematically legal, and you'll arrive at the answer no problem.

Synopsis:

The fourth day's lecture has a twofold purpose. The first problem, the ball in freefall (and, if enough time is available, the spring block problem), is given as a way of exposing students further to the methods by which we solve problems using Lagrangian Mechanics. The solutions follows through the steps of acquiring the Operator by determining the types of energies that exist in the system, then using that operator to crank through the derivatives and acquire the equation of motion. For the ball in freefall, the solution obviously predicts that the only force on the ball will be its weight, neglecting air resistance of course.

The following problem, however, is something somewhat more difficult. The pendulum problem takes students of every level beyond their comfort zone, as no part of the MA Frameworks dictates the study of rotational motion. Using the pendulum problem, students are introduced to the idea that physics does not only require the ability to regurgitate equations and to memorize facts, but it also at times requires the ability to imagine the most convenient solution. In the Lagrangian approach, we must choose a convenient variable, and only one variable, on which to base our reference frame and subsequent solution. This problem suggests that we must find a way to determine the position of the pendulum using only one position variable, and the most convenient of these must be the variable that determines both the *x and y* positions of the bob, which happens to be theta. Doing this, however, places us in the polar space of the pendulum's reference frame, a space we have never been to before in these classrooms.

Students are then walked through the process of creating the Lagrangian Formalism based on the use of theta as a position, and theta-dot as a velocity. The operator is then constructed, using principles of geometry to determine the rate of change of the arc length, which is our "v" term for kinetic energy, and the potential energy term which must depend solely on the length of the string, and the angle at which it swings. Once this is complete, the students effectively have all they need to work through the Formalism, which is the night's homework. Although it seems difficult and complicated, they are assured that it is just as doable as any other problem, so long as they recognize the derivatives that they must take with respect to the proper variables.

The point of the problem in its entirety is to prove that physicists, or even scientists in general, may not always be confident as to the physical meaning of the solution they acquire. We, having trained in the field long enough, recognize the final term acquired from applying the Lagrangian Formalism to a pendulum as the angular frequency of oscillation, but to students who have never seen this term, the solution, even when fully correct, may not be obvious. Having proven to them that the Formalism works, and produces something physically significant, the idea of this exercise was to assure them that, with confidence in your mathematical skills, and provided that no silly mistakes are made along the way, the solution *will* be correct, even without a full grasp on its meaning. As physicists, a new theory may make perfect sense conceptually, but the mathematical translation of a theory is not always forthcoming in its nature.

In-Class: Day 5, Final Survey

Name:

Grade: (9, 10, 11, 12?) :

Level: (College Prep, Honors, AP, etc.):

School Name:

Final MQP Survey (Quiz)
Lagrangian Formalism in High Schools

1) Please take the following derivatives of these functions:

iv) $\left(\frac{d}{dx}\right)\left(x^2\right) =$

v) $\left(\frac{d}{dx}\right)(3x + 4x^3) =$

vi) $\left(\frac{d}{dy}\right)(5y - 2y^2) \quad =$

2) Please take the following derivatives of trig functions:

iv) $\left(\frac{d}{d\theta}\right)\sin\theta \quad =$

v) $\left(\frac{d}{d\theta}\right)\cos\theta \quad =$

iii) $\left(\frac{d}{d\theta}\right)(\sin\theta \; - \; \cos\theta) \quad =$

3) Please take the following ***partial*** derivatives:

iv) $\left(\frac{\partial}{\partial x}\right)(x + y) \quad =$

v) $\left(\frac{\partial}{\partial x}\right)(x^2 \; + \; y^2) \quad =$

vi) $\left(\frac{\partial}{\partial y}\right)(y^2 + 2xy) \quad =$

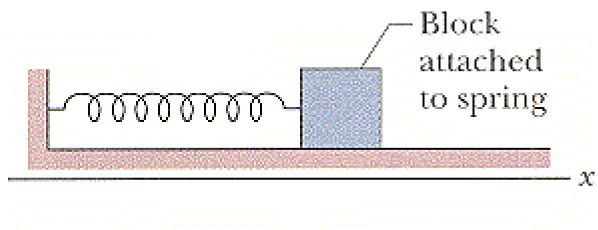
4) Please write the symbol for the Lagrangian Operator, and what it is equal to:

5) Please write the formula used to find an object’s equation of motion (The Lagrangian Formalism):

Alright, that’s the “Did you remember this?” portion of the quiz. The next section is the Open Response, which tests your ability to *apply* what we learned. There will be one question that you’ve seen before (with an answer you could certainly expect), and one question that you’ve seen before that is asking for you to solve for something we haven’t done yet, but I assure you the answer is within your reach. Work hard!

6) A ball of mass *m* is free to move around space in any direction as it pleases. As the ball is moving around with velocity *v* at height *h*, please use the Lagrangian Formalism to find its equation of motion. Show all steps! Remembering the answer isn’t enough, I need to see your work ☺

7) A block of mass *m* is attached to a spring of spring-constant *k*, and the spring is compressed a distance *x*. When the spring is let go, use the Lagrangian Formalism to find its equation of motion. [Hint: a part of the Lagrangian Operator has Potential Energy in it. What *kind* of PE are we dealing with in this problem? Remember that when you write your formulas].



That's it! Now for the usual survey material:

How well do you feel I communicated the material?

1 2 3 4 5 6 7 8 9 10
(poorly) (perfectly)

How understandable were my explanations, or responses to questions?

1 2 3 4 5 6 7 8 9 10
(not understandable) (crystal clear)

How interesting did you find the material?

1 2 3 4 5 6 7 8 9 10
(uninterested) (very interested)

How do you think you did on this survey (quiz)?

1 2 3 4 5 6 7 8 9 10
(very poorly) (very well)

Do you have any additional comments, questions, or concerns? Difficulty in certain sections? Was the curriculum too fast? Too slow? Do you feel you could have mastered the material better with more time? (I know the survey isn't anonymous, but feel free to speak your mind) :

Thank you again for being a part of my project. Physics is a subject that continues to grow as we learn more and more about our universe and how it works. Remember that while we teach subjects with ideal situations, simplifications or generalized explanations, absolutely *nothing* in our universe is simple; a fact that is both terrifying and inspiring. Even with the vast amount of information that we know now, the reality is that we know next to nothing compared to how much we *could* know. Every problem in physics has more than one way to approach it; in fact there are usually several ways to approach any problem. This curriculum was designed to show you that we could take two entirely different branches of physics and accomplish this. I hope that from this, you take away the knowledge that our reality is filled with problems for us to solve in any way we please, and that the solutions are out there as well. All of them.

Synopsis:

The final survey is, naturally, a measure of how much students have learned as a whole. Admittedly, there is a certain amount of memorization required for the final, which students were made aware of beforehand as any good teacher would do to prepare students for an examination. The extent of the memorization includes knowing the algorithm of a derivative, knowing the constant rule associated with partial derivatives, knowing the Lagrangian Operator, and knowing the Lagrangian Formalism. Beyond this, the rest of the examination is meant to test their applied knowledge of these subjects, and to gauge how well they picked it up. The first section tests their knowledge of three derivatives, three partial derivatives, and three trigonometry functions. The second section tests their knowledge of the Operator and the Formalism, and the third section asks them to apply these concepts to two problems.

Following the examination portion of the survey, the actual survey questions ask students how they feel about a number of important factors in the curriculum. They are asked how well they feel I communicated the material, and how understandable the explanations were, to get a sense of my own ability to teach as a variable in the experiment. Their expectations for the survey score reflect exactly how much they truly believed they knew, against what they actually knew. Lastly, an open response survey asks them if there is anything else they would like to add. The purpose of this is to identify any unique issues students may have been having with the material that I could not have foreseen as a variable, and to encourage them to make mention of it. One of the assurances made to the students is that, while the survey is not anonymous, I am open to criticism if it serves to improve the program and my own abilities, and that all data is statistically relevant; if they felt it was too fast, too slow, too complicated, or if they thought I was a jerk, that's a variable. Ultimately any data that I could glean from the students was valuable towards piecing together the human element of the analysis.

Chapter 7: Analysis and Discussion

7.1 Preface

At last we arrive to the moment of truth; all the data has been collected from the schools, all the classes have been taught, and the papers handed in, and all the grading finished and correlated. Before we begin in earnest picking apart the correlated data there are a few things that I wish to clarify, in order to better illuminate the purpose of the project. First and foremost is that, although it was my sincerest hope for the project to prove that students *can* do advanced mathematics and physics on a lower level of education than previously anticipated, it was not the true motive behind the project; the project was simply meant to decide *whether* students could do it, and to make an informed conclusion about its potential in the MA Frameworks there from. One of the original intentions was to examine the Lagrangian in context with its possible application as part of the MA Frameworks, in order that it be integrated into the curriculum to facilitate the advent of new, more advanced material being introduced to students in the high school environment. I ask that you keep this in mind as your peruse through the findings of this project.

Secondarily, I must readily admit that, even without statistically analyzing the contents of my own work, I am well aware of the novelty, and therefore likely erred nature of this experiment. In doing research before and during the project, for purposes of understanding the more current topics of education, and also in comparing my experiment to those that may already have been performed, I found precious little in the way of similar experiments. John Norbury's tutorial on "Lagrangians and Hamiltonians for High School Students" outlined a method for teaching the difficult material to students, but the experiment itself was never carried out. As such, statistically I am only n=1 in the way of quantity of experiments, although my one experiment contained upwards for 180 students. Quantum mechanics dictates that in observing a wave form, it is possible to perform an experiment in one of two ways: the observer can set up a million of the same wave form and observe them all simultaneously, or he can set one wave form up at a time and observe it one after another, one million times. In my case, I chose the former, having only so much time to perform my quantum study, and not enough schools to perform one million observations.

Ideally, had I enough time, and more willing students (which, encouragingly enough, all of them seemed to be), I would continue the experiment until I achieved smooth curves on all my graphs and error bars within fractions of a person. As this is not the case, I bring one last point to the table: statistical analysis can only bring us so far in the way of small groups, but human observation, which intrinsically speaks the language of understanding and analysis, can be an equally powerful tool. Investigation of the correlations between certain values in this experiment may provide statistically significant or insignificant data, depending on the sample size, the pool size, mean, standard deviation, and limits to the function, but on a person-to-person basis, the functions could never describe whether one single student could achieve on this level. In particular this is true because statistics dictates that n=1 is the most statistically insignificant pool size you could have, aside from n=0, and thus no paper, publication or magazine would ever house it. In the field of education, however, n=1 is the *most* statistically significant value: the individual, the student, on a 1:1 correlation, is the most important part of the job, and we must never forget this. With that, let us examine the findings, with brief discussion on the quality and diversity of the data accumulated.

7.2 Procedure

7.2.1 Acquiring Schools

Obviously for the project to succeed, I needed students to teach to; it's a little difficult to gauge how good one's teaching is, or how well the students are absorbing material, when your only company are empty chairs. The original intention of the project was to obtain schools from multiple districts, possibly even multiple cities, to get a solid representation of the state statistics. This was formed under two regrettably naïve principles one year before the start of the project. The first misconception was that the curriculum would be taught in two to three days, which ultimately proved to be a very unfair timeline to place student up against. The second problem arose when it came to my attention that, in light of all the snow days that the state was handing out, teachers were understandably unwilling to provide five days of their precious time to a stranger promising advanced, unrelated topics to their students.

As is mentioned in the preface and discussion of the MA State Frameworks, teachers are already very crunched for time with the sheer number of standards they are required to teach, and students are under similar pressure to learn it. My sincerest thanks are extended to the schools and teachers who generously provided me with their time and resources to complete my project. Ultimately, due to these constraints, it was most effective to choose a city, which closely (or closest) mirrored the state statistics, as it could be expected to provide a rough outline of how the state would perform, were we to acquire all the schools within that city. As luck would have it, Worcester is a very close approximation to the state statistics as a mean value, and thus it was decided to contact the surrounding schools. One school remains the outlier in the experiment, which is Quabbin Regional High School. This school became a part of the project when one of my peers informed me that his former high school teacher would be interested in taking part, and provided me with his email address. As the project was to invite and accept all schools without discrimination, I contacted Robert Kolesnik as Quabbin High, and the school became a part of the data pool.

In order to obtain schools for the project, an email was sent to all the physics teachers, and all the science department heads, in the city. It outlined the purpose of the project, as well as the benefits towards the students, and requested either a meeting time or a phone call for discussion and negotiation. Of the major schools in the area, those being Doherty, Burncoat, South, North, and Worcester Technical High, Burncoat and South rejected the project; Burncoat through a denial email, and South through silence. South was pursued at length, but no response was ever given. Thus, Doherty, North, WTHS, and Quabbin became the four schools to take part in the project. In analyzing these schools simply by their diversity, the qualities that each school brings to the experiment are surprisingly distinct from one another.

Doherty is, by all accounts, the average of the city, having MCAS scores, graduation and dropout rates, and male/female ratios within single digit values of the city and state averages [5] . North High's racial distribution is almost *exactly* that of the city and state distributions, and thus accurately reflects the diversity expectations that may interest some analysts. Worcester Technical High is a vocational school, and thus offers an inherently unique look at education from a trade-based perspective. Lastly, Quabbin's racial distribution is 93% White, and also happens to be the only school that does not qualify as "high needs", as under 50% of their student population receives free or reduced lunch.

Thus, each school brings something unique and diverse to the table, and we should expect a diverse spread of results from each of them. If any of the school statistics (race, gender, etc.) are proper indicators of score predictions, than some of the analysis should reflect these correlations. If, however, they are not, then other factors may be taken into consideration, such as the class dynamics, the school environment, and the social aspects of the surrounding neighborhoods.

7.2.2 The Curriculum "Script"

Statistical significance demands that each experiment be performed as closely to the last as possible. The issue with this in the field of education, and also the reason it is difficult to tack on error bars to any experiment *done* in the field of education, is the human condition. We, as sentient beings, capable of spatial, visual, verbal, tactile, oral, and olfactory analysis, are simply too complex to consider the variance of each action within this experiment, and from this point on in the paper, that much will be assumed. A simple hand gesture in one class may unlock the key to understanding my point of discussion, but have no effect whatsoever in another; more to the point, I may not even *do* the same hand gesture, or any at all, in each class. This, leaving alone the differences in my wording of explanations, is simply too large a variant to consider.

Every educator, like an artist, seeks to improve their craft, and thus from class to class, and school to school, my explanations morphed in small but significant ways depending on class feedback. My explanation of simple derivatives was unconsciously adapted in WTHS to include more practical applications, simply because students in a trade school would naturally look for immediate uses for a concept they learn. Increasing student interest became a focus at Doherty, as students seemed turned off by the new and challenging material, so Batman suddenly became a variable in one of my functions. This simple change got a laugh out of the class, and a few more students perked up and participated. I carried this change with me to the other classes, and it was the highlight of the partial derivatives lecture. Statistically, this means that small changes to the variable of the curriculum were made, but certain standards were held in terms of the worksheets and the project rules.

Students were greatly discouraged at the beginning of the project from consulting other students for solutions, or from blatantly copying answers before turning in. They were assured that, while the teacher's decision of what to do with the actual grades was not up to me, my only grading system to turn in would simply be to check off whether the student attempted the homework. With this in mind, they were assured that copying or consulting other students was not useful in any way, and that it detracted from my ability to know whether they were actually learning the material, which was the point of the project in its entirety. As a whole, there were very, very few cases of what appeared to be blatant copying, and students were observed each morning upon homework collection for sudden fervent writing, which never occurred in any school. It is, then, my opinion that the data itself, which includes all six worksheets from Initial Survey to Final Survey, is legitimate and significant, containing very few instances of academic dishonesty.

The question of "self-selection" became an issue when it became apparent that for some students, the net worth of my project was not enough to warrant their participation. Even in high level honors or AP classes, the prospect of extra homework grades, extra credit points, or an additional quiz score could not persuade them to care for the material. This must be taken into account when considering the score distribution, as there are several students of note who considerably skewed the averages (in both directions) due to a very high or low interest in the material, or its intrinsic worth to them. In the face of these self-selected students, and their uninterested counterparts, there were only so many incentives that could be offered to keep them interested. Overall I believe that there was only a handful, perhaps 6 or 7 students in the pool of 185, who were genuinely tuned out. This, of course, is a rough estimate based on my own observation, under the pretense that "genuinely tuned out" implies a student who never, under any circumstance or method of persuasion, displayed any sense of interest or initiative in the material; this also does not include students who tried lazily, or students who tried, but were too frequently absent to achieve much of anything. In one case, a student was going into the culinary arts, and thus had absolutely no use for this kind of work. In another case, the student had resigned himself in identity foreclosure to believe that he absolutely could not achieve on this level, and thus, with only five days to convince him otherwise, the time constraints won out in the end.

One of last points about the project curriculum is with regards to the nature of the teaching style. While within less rigid constraints and a more dynamic timeline, I would have liked to vary my teaching methods using the more common assessments such as group work, in-class assignments, and PowerPoint presentations, due to time constraints I modeled the curriculum after the experience expected of a college student, which is the lecture style. In larger introductory college courses, a student is effectively 100% guaranteed to encounter the lecture style of teaching, with labs and conferences at the discretion of the professor. As such, in fulfilling the dual role of the project to provide data for myself while providing students with a genuine college experience, lecturing was the primary means of edifying, and homework was the primary means of assessment, along with the frequently used, and affectionately named "dipsticking" or simply asking questions during class to check up on student progress.

Overall, the message to take away from this discussion is that there were an enormous number of variables to consider outside the realm of what educational scientists generally regard as quantifiable data. Student backgrounds, interest levels, peer pressures, social aspects, teaching quality of communication, and time constraints all factor into equations that cannot yet accommodate them.While physics has had large working databases to compare experiments to for nearly 40 years, educational science has only begun compiling significant data within the last 20, and thus pioneering experiments such as this are difficult to consider on a verifiable statistical level. Thus, as we finally begin to pick apart the data, we will discuss the significance (verifiable or potential) of each correlation we find.

7.3 Analysis

7.3.1 Cohen's d [11]

Statistical significance of all the data in this project was determined using Cohen's *d* value, which provides a quantitative look at the correlations between a sample size of data against the pooled values. Cohen's D is calculated using the formula

$$d = \frac{\overline{x_1} - \overline{x}_2}{s} \qquad \textbf{(7.1)}$$

where $\overline{x_1}$ is the mean value of the school data, $\overline{x_2}$ is the mean value of the pool data, and s is the pooled standard deviation, which is calculated by the equation

$$s = \sqrt{\frac{(n_1 - 1)s_1^{\ 2} + (n_2 - 1)s_2^{\ 2}}{n_1 + n_2}} \qquad \textbf{(7.2)}$$

and the individual standard deviations are calculated by the familiar equation

$$s_1^{\ 2} = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_{1,i} - \overline{x}_1)^2 \qquad \textbf{(7.3)}$$

The upper and lower limits to the Cohen's *d* equation are trivial calculations that I allowed a calculator [11] to define, and they set the confidence levels of my data, providing a quantifiable error term in each comparison. It is worth noting at this point that, due to the variance in the data, and the fact that this experiment was only performed once, with no other comparable data available, the error bars are very large on all of the graphs. The point marked as the true value of Cohen's *d* is the indicator of how statistically "worthwhile" the data is, with values between $-0.24 < d < 0.24$ being considered statistically insignificant for any correlation. Values of $-0.49 < d < -0.24$ or

0.24 < *d* < 0.49 are considered to be educationally significant (i.e., significant for publication purposes, and values *d* > 0.5 or *d* < -.50 are considered "practically" or "clinically" significant, which means something very large must have changed.

The resolution for this, or at least for the purpose of coming to some state of conclusion until further data is available, is to come to some reasonable compromise between the data and the error. For instance, a *d* value that has error bars *only* within the positive range must represent some sort of positive correlation, as it is not possible for the value to lie anywhere in the negative region. Likewise must a *d* value in the negative region with error values exclusively in the negative region represent a negative correlation, as the value may not enter the positive region. These values will be trivial to understand and decide upon their significance. It should also be noted that the 95% Confidence Interval is based on two standard deviations, and thus they are literally twice as large as a standard error bar.

In the graphs displayed for the Cohen's *d* values, some elucidation may be necessary for some of the values. I would like to preface these graphs with an explanatory chart.

Value	Meaning						
Mean 1	Mean value for the sample group in question						
Std.Dev 1	Standard Deviation for the sample group in question						
N1	Sample size of the group in question						
Mean 2	Mean value of the entire category pool						
Std.Dev 2	Standard Deviation of the entire category pool						
N2	Pool size for the category in question						
Confidence Level		Two standard deviations of the pool variance					
Mean Difference		Mean 1 - Mean 2					
d lower limit		Lowest possible value of *d* under given conditions					
d upper limit		Highest possible value of *d* under given conditions					
Pooled Variance		Variance of the category pool					
$z_{\alpha/2}$		Measure of the confidence intervals of noncentral parameters					

Chart 7.1: Definition chart for terms in the Cohen's *d* graphs.

7.3.2 Analysis of Individual Major Data Correlations

While analyses of gender and race were performed primarily for the sake of answering questions many would have on those topics, the data considered significant for this project was composed of two major values: Final Score and Total Score. The names sound synonymous, but the term Final Score is short for Final Survey Score, the last worksheet which tested student knowledge on simple and partial derivatives, Lagrangian Operators and Lagrangian Mechanics. The Total Score is the cumulative point value obtained from all six worksheets, the Initial Survey included. The maximum score obtainable on the Final was 21, and the maximum score obtainable for the Total was 57. We begin by looking at how each school performed individually, and then work our way up to the pooled data. For reference purposes, the notations of P1, P2, etc. are the Periods, or classes, in which the study took place, and the "Stud. #" are the number of students who achieved a certain score.

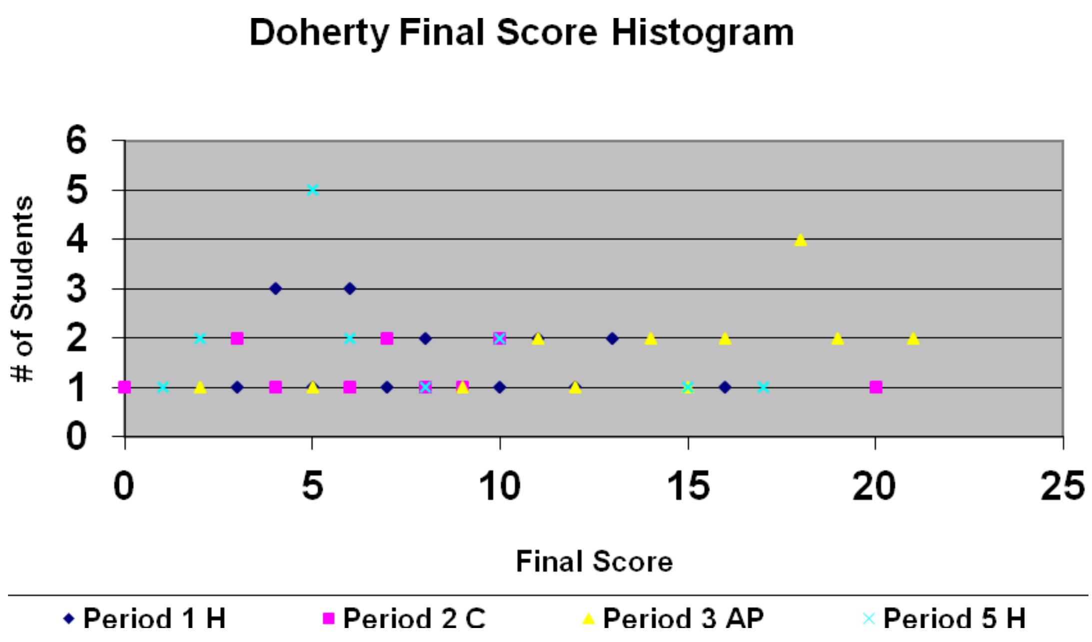


Figure 7.1: A Histogram of Doherty's Final Scores by class reveals the performance of each level [Honors (H), College (C) , and (AP)].

P1 Score	Stud. #	P2 Score	Stud. #	P3 Score	Stud #	P5 Score	Stud. #
3	1	0	1	2	1	1	1
4	3	3	2	5	1	2	2
5	1	4	1	9	1	5	5
6	3	6	1	11	2	6	2
7	1	7	2	12	1	8	1
8	2	8	1	14	2	10	2
9	1	9	1	15	1	15	1
10	1	10	2	16	2	17	1
11	2	20	1	18	4		
12	1			19	2		
13	2			21	2		
16	1						

Chart 7.2: The score distributions of each period, organized by the number of students who scored certain values.

The graph of Doherty's Final scores has a few interesting points, along with the pattern that it discloses. The class levels produced an expected trend amongst themselves, with AP performing better than Honors, and Honors performing better than College Prep. The data points between the score values of 3 and 8 show that, while in some cases the College Prep students did, in fact, outperform a number of Honors students, they also perform on the lowest end of the data spectrum as well, while several Honors data points escape into the higher, 10-20 score range. For a short time in the lower range, the Honors and College Prep points co-mingle, with

11 Honors students and 10 College Prep students in the lower range. The single outlier from the College Prep class offers an interesting look into the dynamics of the project.

One of the highest scores in the entire school-wide project, and this student was located in the lowest level. The reason for this was due to his inability to speak fluent English, although he understood the language of mathematics without error. In a brief discussion with this student, I asked him if he would please ask me for help if he found himself unable to understand the questions of the worksheets due to language barriers. When he did approach me, I clarified what certain words meant, or simply what the question was asking for. As a result, he ended with by far the highest average in the class and one of the highest in the school.

Overall the discussion on this graph is limited, as it effectively speaks for itself, but the interest of it is primarily in the co-mingling of all the honors classes within the same range of the College Prep class. Clearly there is some level of ability within all of these students to do these tasks; in most cases the lower scores on the final were caused by difficulties with the execution of multi-step problems, which are second nature to Lagrangian problems.

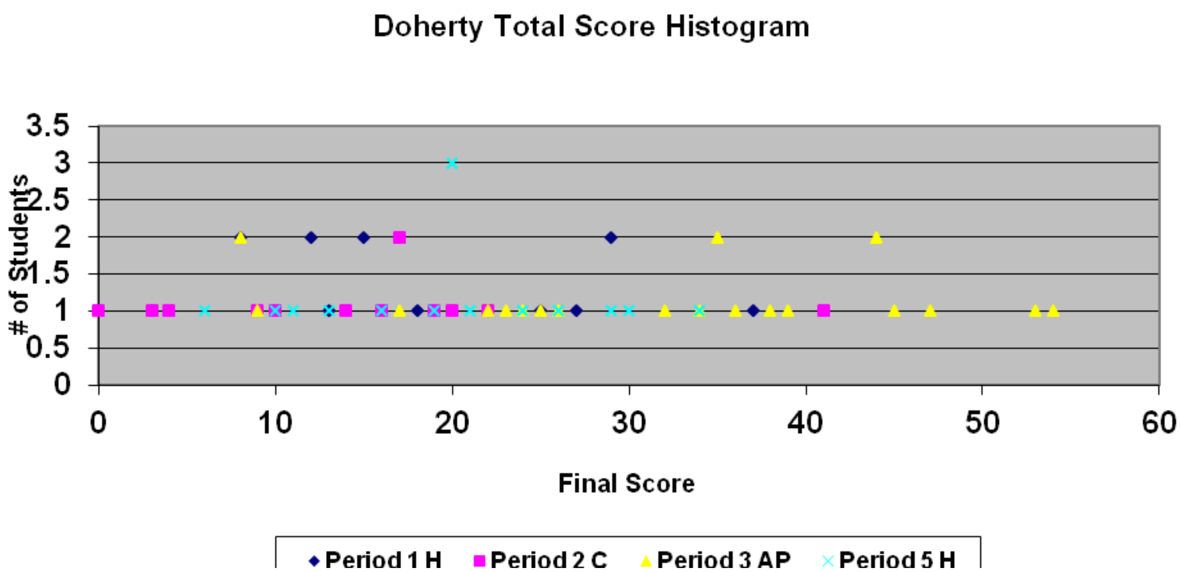


Figure 7.2: The Total Score distribution of Doherty by class shows the same co-mingling patterns of certain levels, with similar outliers.

P1 Score	Stud. #	P2 Score	Stud. #	P3 Score	Stud. #	P5 Score	Stud. #
4	1	1	1	9	2	7	1
9	2	4	1	10	1	11	1
10	1	5	1	18	1	12	1
13	2	10	1	23	1	14	1
14	1	11	1	24	1	17	1
15	1	15	1	25	1	20	1
16	2	18	2	26	1	21	3
19	1	20	1	27	1	22	1
20	1	21	1	33	1	25	1
23	1	23	1	36	2	27	1
26	1	42	1	37	1	30	1
30	2			39	1	31	1
38	1			40	1	35	1
				45	2		
				46	1		
				48	1		
				54	1		
				55	1		

Chart 7.3: The score distributions of each period, organized by the number of students who scored certain values.

In the graph of Doherty's Total scores, we see similar patterns appear that confirm the initial suspicions raised in the Final Survey Graph. The outlier from the College Prep group is, of course, the same student, who nearly doubled the next highest score, and the same mixing pattern between both Honors classes and the College Prep occurs at a similar place in the graph, this time between the 10-30 point range. Although, as before the Period 5 Honors group has a number of score values in the upper range, between the Period 1 and Period 5 Honors group, there are nearly the same number of students in the range of 10-30 in Honors as there are in College Prep. We also notice the same spike between 10-20 from the Period 1 Honors group, surrounded by both lower and higher scores from College Prep. Were it not for the point values, this would almost be a mirror image of the Final Survey Histogram.

Cohen's *d* for the mean difference (v1.3)

Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
9.569231	5.587288	65	0.95	-0.30	**-1.41**	1.96
Mean 2	**Std. Dev.2**	***N* 2**	**Pooled Variance**	**Cohen's *d***	***d* upper limit**	
9.867925	5.93287	157	5.81	**-0.05**	**0.88**	

Cohen (1977) this is a:	Wolf (1986) this is a:
0 or near zero effect	**0 or near 0 effect**
.20 = small	
.50 = moderate	.25 = educationally significant e.g., something was learnt
.80 = large	significant e.g., something really changed

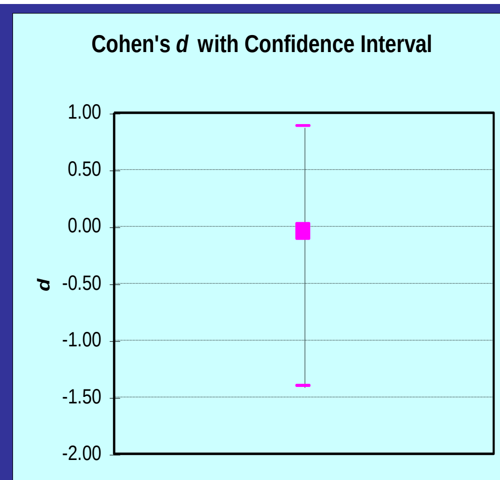


Figure 7.3a

Cohen's *d* for the mean difference (v1.3)							
Enter sample sizes (N), means, standard deviations (SDs) and confidence level							
Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$	
22.94203	12.65246	69	0.95	0.35	**-2.96**	1.96	
Mean 2	Std. Dev.2	*N* 2	Pooled Variance	**Cohen's *d***	*d* upper limit		
22.58857	13.54184	175	13.24	**0.03**	**2.03**		
Interpreting the effect size. Handy guide - according to:							
Cohen (1977) this is a:			Wolf (1986) this is a:				
0 or near zero effect			**0 or near 0 effect**				
.20 = small							
.50 = moderate			.25 = educationally significant		e.g., something was learnt		
.80 = large			significant		e.g., something really changed		

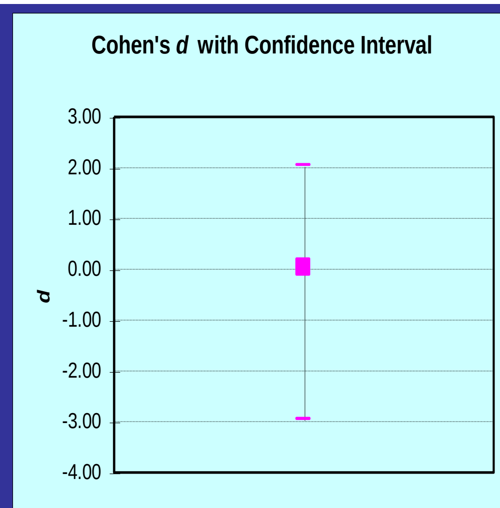


Figure 7.3b

Figure 7.3a: Cohen's *d* values for Doherty Total Scores
Figure 7.3b: Cohen's d values for Doherty Final Scores against the pool data. While both graphs have enough error in both quadrants to make it statistically insignificant, the actual *d* value states that it would not be significant in any case.

The calculated values for the Doherty data points, set against the pool data of approximately 160-175 students, respectively, not only vary surprisingly little from the pool mean and standard deviation (shockingly so), but as a result, as hardly considered significance in the effect the project had as a whole on the students. In terms of scores, Doherty Memorial High School is the closest school of any to the city and state averages, so it comes as no surprise that there was so very little variance as a whole. The huge standard deviation values stem from the difficulty of teaching the same curriculum, using the same methods, to three different levels of classes. In other schools, the standard deviations were considerably lower for the primary reason that there were no College Prep classes to teach, and thus the scores were expected to be a bit closer together in terms of scoring, since the level gap was no larger than one.

In the case of the both graphs, the means were literally no more than half of a point from the mean, but one full standard deviation was valued at approximately 25% of the maximum score in each case. This obviously creates a very large margin within which the *d* value could exist, but because the values themselves speak so plainly for almost null variance from the mean value, it is fairly safe to conclude that there was little gain or loss in either case without the aid of the proposed error bars or confidence level.

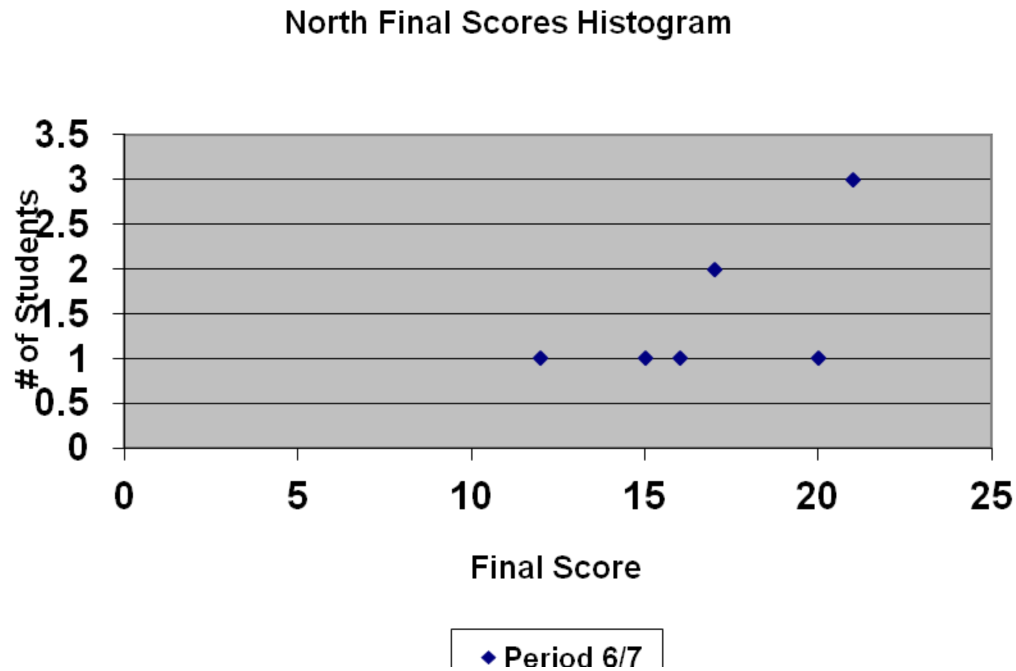


Figure 7.4: North Final Survey Histogram shows students in the North High AP Physics class perform at a very high level.

Period 6/7					
Score	12	15	16	17	20
Stud. #	1	1	1	2	1

Chart 7.4: Score Distributions for North Final Survey Scores. Albeit a small class, score distributions stayed fairly close together in the proficient-advanced range.

North High School's AP Physics class is a sparse one indeed, housing only 11 students this year. Unfortunately due to absence, these numbers dwindled even lower, as is the case of the final scores, which amount to only 6 students. On any given day there were realistically only 8-10 students, but the final scores dropped lower than this due to the statistical insignificance of some of the testers. Two students were present for only 40% of the lectures, one of which was the lecture on partial derivatives, and thus they would not have been able to take the final with any level of confidence whatsoever. The big note to make of this graph is simply the level to which the students performed. The highest scored value of the entire class was, in fact, a perfect score of 21, with scores only dropping as low as 12, which is still well into the Proficient range.

One of the benefits that the AP Physics class at North had was that most of them had already seen calculus at one point or another; there was only one student who had not, and as an aside, they ended up out-performing almost everyone on each individual homework assignment. As a result, the first two days in which I introduced derivatives were mostly review for them. On the other hand, none of them had ever seen the visual aspect of a derivative, which is to show the correlation between position, velocity, and acceleration, using simple variable vs. time graphs and take the slope. Many of them commented that this made the correlation between the calculus and the physics much simpler in the later days of the lecture series.

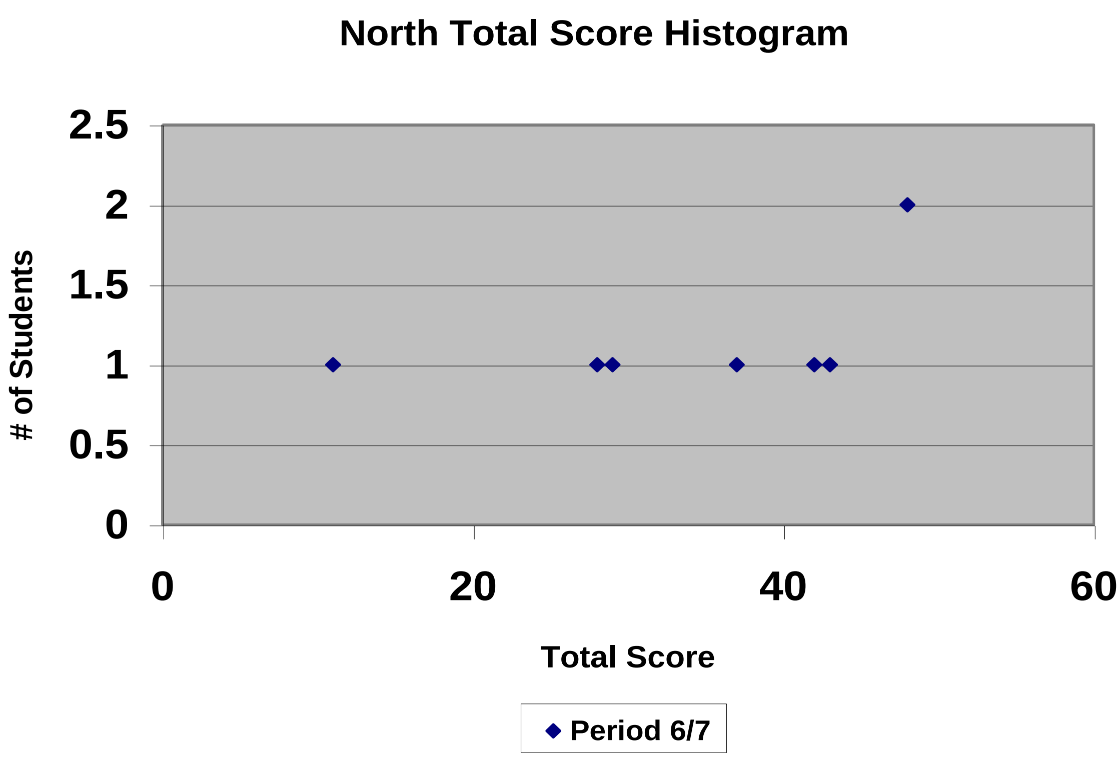


Figure 7.5: The Histogram of North High's Total scores show similar trends in high levels of performance.

Score	11	28	29	37	42	43	48
Stud. #	1	1	1	1	1	1	2

Chart 7.5: Score Distribution for North Total Scores .The same general trend appears in the score distributions as did in the Final chart, with scores well into the Proficient-Advanced values.

In examining the Total Score graph for North, a similarly encouraging pattern appears. The peak number of students scored in the highest bracket, with most of the others taking scores in the advanced range of values. The outlier was a student who had been present for 60% of the lectures and knew derivatives, so in all fairness qualified to take the final survey. They, of course, had not practiced the step-by-step methods of solving the Lagrangian-type problems, and thus was able to successfully navigate the calculus portions of the lectures, and had trouble with the Lagrangian homework and final as well. Overall, however, the class proved itself to be very successful in picking up the new material and adapting it. One student even went so far as to say it was "too easy", and that he would have preferred that it be more rigorous, as he did not understand how such simplistic examples could be applied to more difficult problems that affected the real world.

To resolve his question, I was brought anecdotally back to Professor Zozulya's Classical Mechanics class, in which he prescribed one of Goldstein's *Classical Mechanics (3rd Edition)* problems involving a cylinder with a bead constrained on a helical wire wrapped around the cylinder. All the system's components were free to translate and rotate, and we were left to our own devices to solve for the forces on the object, and the object's equation of motion.

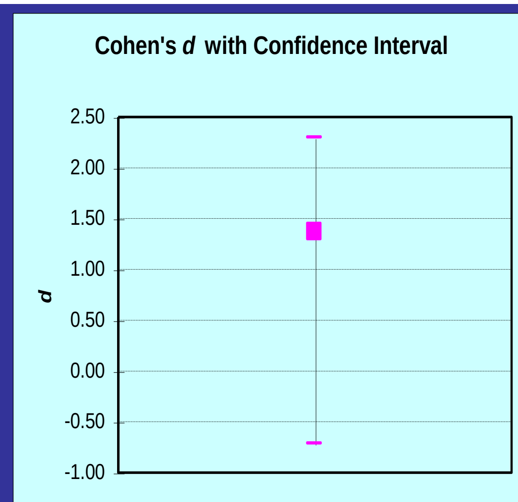
"Imagination," as I told him, "is the most difficult part of the problem. Proving to ourselves that this works, as we have done here this week, is the most important."

In this case, the students were very well prepared for this kind of material; they were used to dealing with multi-step problems, a fast pace with multiple concepts of both a physical and abstract nature being introduced, and high expectations as to their study habits. As with every class, the introduction of the Lagrangian was the stumbling point, but that will be left for later discussion.

Cohen's *d* for the mean difference (v1.3)

Enter sample sizes (N), means, standard deviations (SDs) and confidence level

Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
17.77778	3.192874	9	0.95	7.91	**-0.72**	1.96
Mean 2	Std. Dev.2	*N* 2	Pooled Variance	**Cohen's *d***	*d* upper limit	
9.867925	5.93287	157	5.79	**1.37**	**2.29**	
Interpreting the effect size. Handy guide - according to:						
Cohen (1977) this is a:			Wolf (1986) this is a:			
LARGE +ve effect			**PRACTICAL/CLINICAL +ve effe**			
.20 = small						
.50 = moderate			.25 = educationally significant		e.g., something was learnt	
.80 = large			significant		e.g., something really changed	



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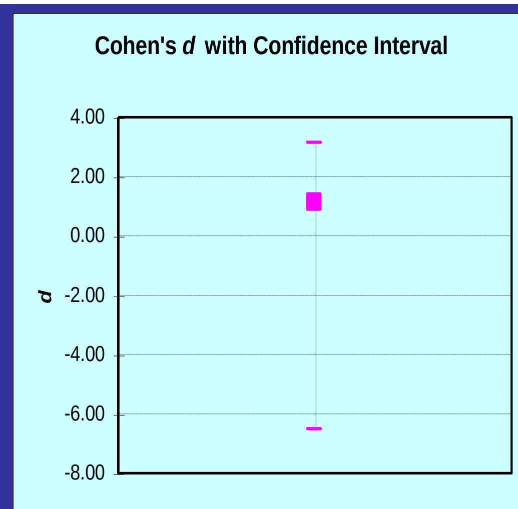
Cohen's *d* for the mean difference (v1.3)						
Enter sample sizes (N), means, standard deviations (SDs) and confidence level						
Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
37.5	11.71182	9	0.95	14.91	**-6.54**	1.96
Mean 2	Std. Dev.2	*N* 2	Pooled Variance	**Cohen's *d***	*d* upper limit	
22.58857	13.54184	175	13.39	**1.11**	**3.12**	
Interpreting the effect size. Handy guide - according to:						
Cohen (1977) this is a:			Wolf (1986) this is a:			
LARGE +ve effect			**PRACTICAL/CLINICAL +ve effe**			
.20 = small						
.50 = moderate			.25 = educationally significant		e.g., something was learnt	
.80 = large			significant		e.g., something really changed	



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e 7.6b

Figure 7.6a: Cohen's *d* values for North's Final Scores and
Figure 7.6b: Cohen's *d* values for North's Total Scores are clearly indicative of significant positive correlations, but their confidence levels are questionable in one case.

The calculations of Cohen's *d* values show positive correlations well into the clinically significant spectrum, which is plainly visible from the large mean value of the student body for the school. There are certainly a number of things to take into account as we examine the data for these graphs, however, that are both for against their use as significant indicators. Both graphs have error bars that extend well into the positive and negative values, each one far beyond that of clinical significance. The Final graph, however, has only 23% of its confidence level within the error range, which yields 77% confidence that the correlation was a positive one, 69% of which is considered statistically significant. In any case, a solid 84% of the confidence levels are considered significant. The Total graph, however, has much of its error range in the negative region, despite having score values that suggest great improvement. Ultimately the confidence levels on both graphs must be as wide as they are solely because the class was so very small, and without a larger sample size, it is difficult to determine exactly how well they did against the average. The main consideration however, as mentioned earlier, is the individual, and the individual scores in this class show tremendous potential to work with this level of material. The impressive thing to remember is that this entire curriculum, at each school, is taught in only five days; these students are given a very short period of time in which to absorb and adapt to this new material, which a great number of them managed to do.

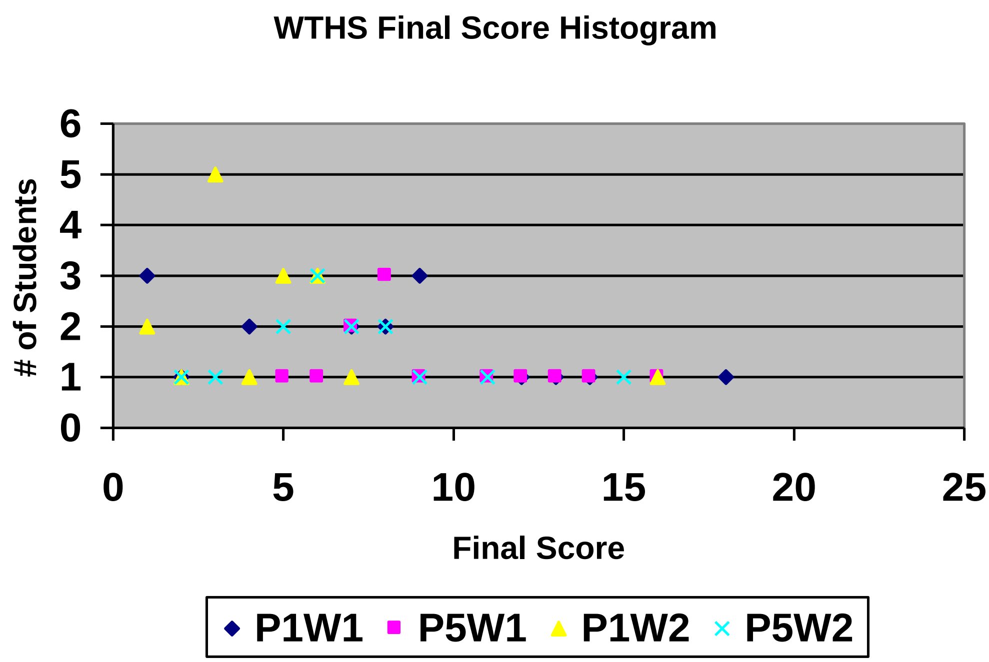

Figure 7.7: WTHS Final Score Histogram shows an interesting "clumping" effect of several classes, with an unusual spread of points despite their level playing field.

P1W1Scr	Stud. #	P5W1Scr	Stud. #	P1W2Scr	Stud.#	P5W2Scr	Stud.#
1	3	5	1	1	2	2	1
2	1	6	1	2	1	3	1
4	2	7	2	3	5	5	2
7	3	8	3	4	1	6	3
8	2	9	1	5	3	7	2
9	3	11	1	6	3	8	2
12	1	12	1	7	1	9	1
13	1	13	1			11	1
14	1	14	1			15	1
18	1	16	1				

Chart 7.6: Score distributions of the four classes at WTHS show a particularly low range of scores, the reason for which is up for discussion.

A small alteration is made in the WTHS distribution charts, which is that, because the same periods of classes were taught for two weeks, both sets of classes are technically Period 1 and Period 5. For notation purposes, W1 and W2 is added to distinguish between Week 1 and Week 2. Although discussed earlier, I feel it is a worthwhile topic to bring up the class dynamic I experienced while at Worcester Technical High, if for nothing else than simply to point out once more how very different the environment of a trade-based school has proven itself to be. Students at WTHS were, unquestionably, the most interactive of any school I visited, which of course leads us to ask “why?”. The conclusion I reached was twofold, with one resolve pointing towards the condition of the school, and the other pointing towards the relationship between the teachers and students. While WTHS is certainly an uplifting building to be a part of, I still hold that the student-mentor/colleague relationship that students become a part of when learning their trade in their no-class weeks encourages them to ask questions and work *with* authority, rather than against it.

Tom Gusek, the teacher who allowed me to teach his classes for two weeks (two classes each week), told me that

“When they asked me to sub in one of the auto-body shops, the students thought ‘Oh this guy’s just a sub, I don’t need to listen to him.’ But I built my house, I plumbed it, I wired it, and I built my car. When they saw that I knew my stuff they said ‘Well I guess I can listen to this guy.’ It’s not enough to teach the stuff, you have to use it around here.”

This was naturally the approach I took to these classes, by including applications of the material in real-world situations as often as I could, in the hopes of stimulating their engrained habits of searching for implication of their work. While this was a useful method in engaging students who might have some use for this work, there were still a number of students who simply tuned out due to their intended professions, either in the culinary arts, manicurists, or other distinctly non-mathematically based fields.

As a result of this, the scores from WTHS are admittedly in the lower range, with a distinct clumping of scores in the 0-10 range, and a small distribution of scores spread out through the 10-20 values. A similar result comes from looking at the Total Score Graph, and both correlations warrant the same discussion.

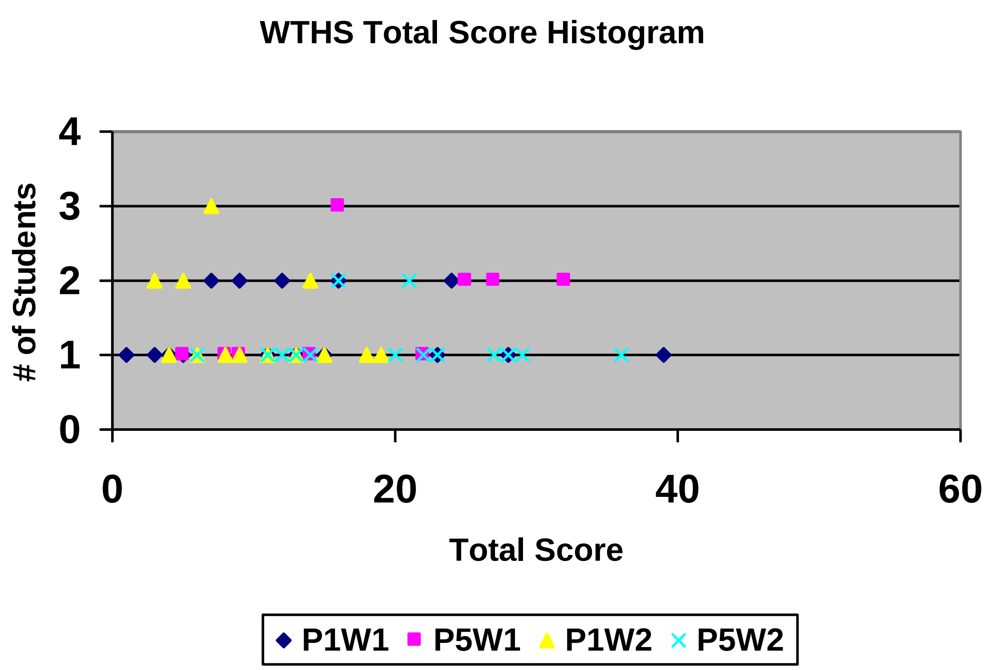

Figure 7.8: WTHS Total Score Histogram shows a doubled, but almost identical pattern to the Final chart, with large clumping in the lower regions of the score spectrum.

P1W1Scr	Stud. #	P5W1Scr	Stud. #	P1W2Scr	Stud.#	P5W2Scr	Stud.#
1	1	5	1	3	2	6	1
4	1	8	1	4	1	11	1
5	1	9	1	5	2	12	1
6	1	14	1	6	1	13	1
8	2	16	3	7	3	14	1
10	2	22	1	8	1	16	2
12	1	25	2	9	1	20	1
13	2	27	2	11	1	21	2
14	1	32	2	13	1	22	1
16	1			14	2	23	1
17	2			15	1	27	1
24	1			18	1	28	1
25	2			19	1	29	1
29	1					36	1
40	1						

Chart 7.7: Score distributions of the four classes (all Honors level) at WTHS range widely from failing values into proficient-advanced.

As with the Final Survey Graph, the Total Score Distribution looks eerily similar, with the same clumping in the lower regions. At this point, at the very least, we can start noticing a correlation between the scores on the Final against the Total Scores, in terms of expected range based on performance. The range of scores in the Total distribution, however, is at the very least slightly more encouraging, in particular the number of students scoring in the 20-40 ranges, which would be considering significant on all accounts. We cannot forget as we sift through the data that there are two major factors that have been hidden in the data.

The first pertains directly to WTHS, which is that the school itself *has* no Calculus program. Up until this point, all the schools discussed have had students who, to some extent, had been exposed to calculus; here the class isn't even offered to begin with. Thus, there was only one (read: *one*) student of the 79 taking part in the project who didn't score a 0/10 on the initial survey (and they scored a 1/10). This brings me to my second point, which pertains to every school in the project. The true value of their potential could be considered by considering their scores out of 47 instead of 57; in general the Initial Survey is included in the total because it is, by all technicalities, one of the means of assessment, and is thus important to the overall value.

By considering this, and taking another look at the scores, they seem much more impressive, all things considered. This was probably the most disadvantaged school of the four, as it was the only one without a calculus program, and yet in my opinion, they were the most interactive, the most interested, and the most dynamic of the groups I taught. As an aside, vocational schools are often thought to be the underdog of the school system (and not in the good way), which I admittedly expected upon entering into the school. The student performance, however, when combined with the fact that WTHS is the only Level 1 school in the district, and their impressive MCAS scores [5] , clearly tell a different story.

Cohen's *d* for the mean difference (v1.3)						
Enter sample sizes (N), means, standard deviations (SDs) and confidence level						
Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
6.868852	3.985287	75	0.95	-3.00	**-1.46**	1.96
Mean 2	Std. Dev.2	*N* 2	Pooled Variance	**Cohen's *d***	*d* upper limit	
9.867925	5.93287	157	5.36	**-0.56**	**0.37**	
Interpreting the effect size. Handy guide - according to:						
Cohen (1977) this is a:			Wolf (1986) this is a:			
MODERATE -ve effect			**EDUCATIONAL -ve effect**			
.20 = small						
.50 = moderate			.25 = educationally significant		e.g., something was learnt	
.80 = large			significant		e.g., something really changed	

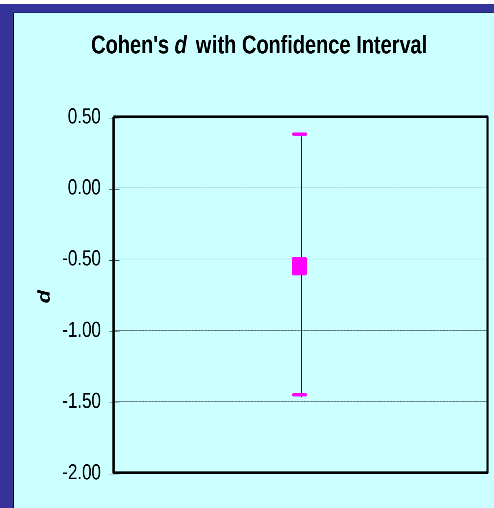

Figure 7.9a

Cohen's *d* for the mean difference (v1.3)						
Enter sample sizes (N), means, standard deviations (SDs) and confidence level						
Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
15.28986	9.039296	69	0.95	-7.30	**-2.72**	1.96
Mean 2	Std. Dev.2	*N* 2	Pooled Variance	**Cohen's *d***	*d* upper limit	
22.58857	13.54184	175	12.39	**-0.59**	**1.42**	
Interpreting the effect size. Handy guide - according to:						
Cohen (1977) this is a:			Wolf (1986) this is a:			
MODERATE -ve effect			**EDUCATIONAL -ve effect**			
.20 = small						
.50 = moderate			.25 = educationally significant		e.g., something was learnt	
.80 = large			significant		e.g., something really changed	

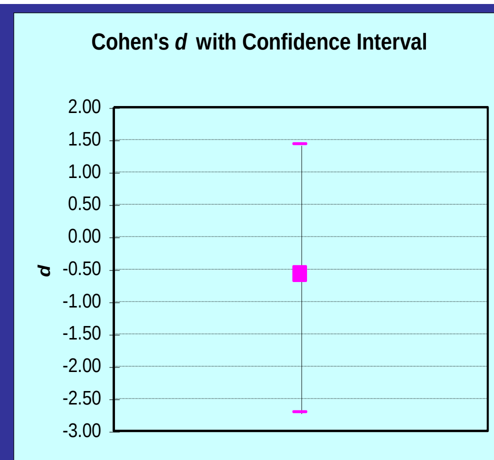

Figure 7.9b

Figure 7.9a: Cohen's *d* values for WTHS Final and
Figure 7.9b: Cohen's *d* values for WTHS Total scores, respectively, show a moderate negative effect on the student body.

The values obtained through the calculations of Cohen's *d* are, in this particular case, not what we were looking for, showing a moderate, and considerable negative correlation. This is not to say that the project caused the students to do *worse*, but rather just to say that WTHS was outperformed on several levels. It is definitively worth
reiterating that WTHS was the only school to participate in the project without any mathematical background in calculus whatsoever; every other school had a severe advantage going in, which is why this school's performance is so admirable nonetheless. Additionally, with 34.4% of the graph's error region in the positive, and the rest in the negative, we can just barely conclude that this data could be considered significant to show a small negative correlation.

Again, the negative value is not necessarily an indicator that the students were incited to do poorly because of the project, it only indicates a trend, and does not touch upon the individual interests of the students, which, as Tom Gusek noted, had gone considerably up. He and I had discussed earlier in the project that the project could hold a third intention, although I can't claim credit for it. Tom noted a particular student who, despite never having taken calculus, was certain that he was very good at mathematics, but sadly flunked the course when he reached college. Had he been made aware of the material that was coming his way, or introduced to it in some fashion, he might have made more informed decisions about his career choice and what he felt comfortable taking on. The project at WTHS, then, while not a positive correlation statistically, served a tertian purpose, which was to inform the students who had no previous knowledge of the mathematical foundations expected of science majors of the road the lay ahead.

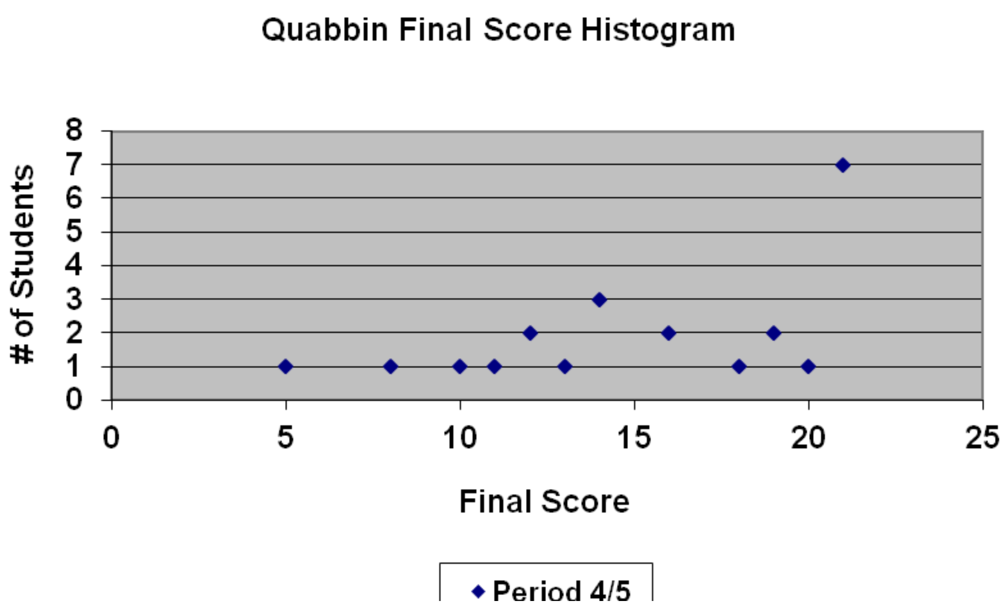

Figure 7.10: Quabbin Final Score Histogram shows a remarkable 7 students scoring perfect scores, with an average distribution otherwise.

Score	5	8	10	11	12	13	14	16	18	19	20	21
Stud.#	1	1	1	1	2	1	3	2	1	2	1	7

Chart 7.8: The score distributions of Quabbin's Final Scores show a strange pull towards the high end.

Although the initial reaction to the Quabbin Final Histogram might be to admire the nice, reasonably Gaussian distribution we obtained, a second cursory glance reveals a somewhat surprising outlier, namely the seven students who obtained perfect scores, four students greater than the next highest point distribution. In this case, it was the outlier that was *most* worth considering in relation to all the other variables, as it is not only the highest score, but also the most frequent one. Quabbin outperformed every school on *almost* every level, coming second only to North in terms of lowest score values, and WTHS on the grounds of sheer interest in the material.

While the data is certainly remarkable, the implications towards certain aspects of educational psychology are also fairly profound. Any of the school's advantages could be the cause of this, from its location in a rural suburban area, the fact that it is the only school in the project that is not considered "high-needs", or any of a number of other potential candidates. But with most of the other schools having similar programs in physics, with calculus available to them, what makes Quabbin so special? Let us examine the Total Histogram and see what we can come up with.

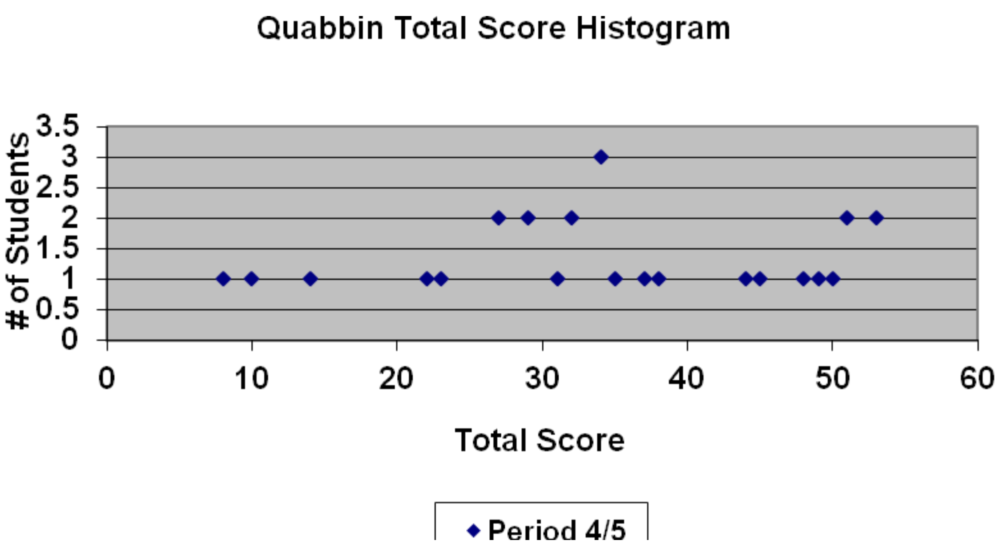

Figure 7.11: Quabbin Total Score Histogram shows a distinct average in the 30s range, with a number of others in the advanced region as well.

Score	8	10	14	22	23	27	29	31	32	34
Stud #	1	1	1	1	1	2	2	1	2	3
Score	35	37	38	44	45	48	49	50	51	53
Stud #	1	1	1	1	1	1	1	1	2	2

Chart 7.9: Score distributions for Quabbin's Total Scores show a wider score spectrum than the Final Histogram might have suggested.

A closer look at the Total scores Histogram and the corresponding data chart shows a much wider range of total scores than originally anticipated. Why this occurred is apparent through the feedback received from students in their individual worksheets. While many did very well on the final due to prescribed studying and hard work, everyone, at one point or another, stumbled on pieces of the material. The lower score distribution obviously represents the students who had trouble with some of the earlier material and were never able to pick it up afterwards.

The average distributions that make up the majority of scores centered around the 20-40 ranges are representative of the students who had trouble with partial derivatives, understanding the concept of "dummy variables" within the Lagrangian Formalism, or the Pendulum problem (the most common difficulty among students, rightfully only valued at 5 points due to its novelty). Thus, the scores are as spread out, if not more so, than could be seen at Doherty or North. This does not detract in the slightest from the highest scores, however, some of which are between only 3-5 points from a perfect 57. The implication of *that*, however, is naturally that they have seen calculus and derivatives before, or else they could never have scored higher than a 48 (the additional point comes from their ability to know the equations for Kinetic and Potential energy). Now, what of the significance of this data? Are we far enough into the positive correlation to achieve something undeniably verifiable?

Cohen's *d* for the mean difference (v1.3)						
Enter sample sizes (N), means, standard deviations (SDs) and confidence level						
Mean 1	Std. Dev.1	*N* 1	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
16	4.824182	23	0.95	6.13	**-0.91**	1.96
Mean 2	Std. Dev.2	*N* 2	Pooled Variance	**Cohen's *d***	*d* upper limit	
9.867925	5.93287	157	5.77	**1.06**	**1.99**	
Interpreting the effect size. Handy guide - according to:						
Cohen (1977) this is a:			Wolf (1986) this is a:			
LARGE +ve effect			**PRACTICAL/CLINICAL +ve effe**			
.20 = small						
.50 = moderate			.25 = educationally significant		e.g., something was learnt	
.80 = large			significant		e.g., something really changed	

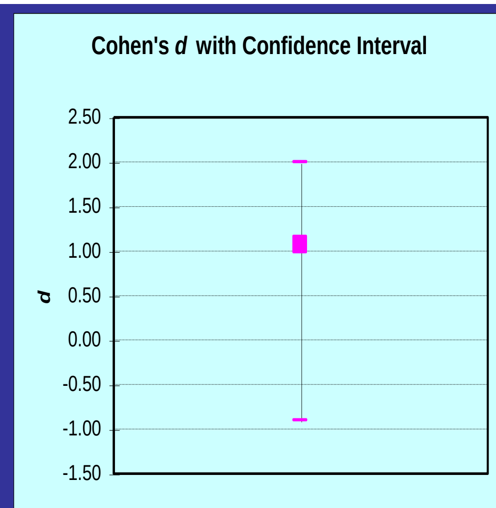

Figure 7.12a

Cohen's *d* for the mean difference (v1.3)						
Enter sample sizes (N), means, standard deviations (SDs) and confidence level						
Mean 1	Std. Dev.1	*N 1*	Confid. Level	Mean Difference	*d* lower limit	$z_{\alpha/2}$
34.81481	12.84235	27	0.95	12.23	**-3.93**	1.96
Mean 2	Std. Dev.2	*N 2*	Pooled Variance	**Cohen's *d***	*d* upper limit	
22.58857	13.54184	175	13.39	**0.91**	**2.92**	
Interpreting the effect size. Handy guide - according to:						
Cohen (1977) this is a:			Wolf (1986) this is a:			
LARGE +ve effect			**PRACTICAL/CLINICAL +ve effec**			
.20 = small						
.50 = moderate			.25 = educationally significant		e.g., something was learnt	
.80 = large			significant		e.g., something really changed	

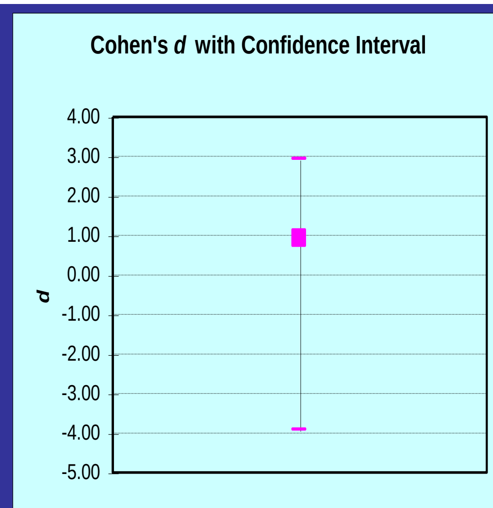

Figure 7.12b

Figure 7.12a: Cohen's *d* values for Quabbin High's Final and
Figure 7.12b: Cohen's *d* values for Quabbin High's Total scores, respectively, show definitively positive point values, but uncertainty within the confidence levels.

Certainly the data points support our conjecture that this school had a very powerful positive correlation, but the error bars are still as large as always, primarily due to the comparatively small size of the class against the pool. Clearly the school mean is well above pool mean, almost by one full standard deviation in both cases, which places it well into the positive range. The issues, once again, are within the confidence levels, which in the case of the Final graph are 65.7% positive, and therefore, by our criteria, relatively significant. The Total graph, however, shows 54% of the error bar within the negative region, which makes it statistically insignificant for our use.

We won't simply ignore the data, but it cannot be used any further either. We will, then, leave the evaluation of this data with the consideration the Quabbin is, as a part of this project, most definitely an outlier, but an encouraging one at that, and while the data may not be significant statistically, we as educators cannot simply brush aside the suggestion that if I were to take the best scorers from all the classes that were taught from each school, I could easily fill *several* advanced physics classes, and prepare the next generation of scientists for a more rigorous path than previously charted.

7.3.3 Analysis of Pooled Major Data Correlations

For consistency purposes, I would like to preface this section with a rating system on which to base the pooled scores. Bear in mind that the Final scores are graded out of 21 points, and Total scores out of 57, with Levels of Achievement determined by a system similar to the MCAS (Massachusetts Comprehensive Assessment System) grading methods.

Score Rating	Qualifying Score(Final)	Qualifying Score (Total)
Advanced	16 to 21	38 to 57
Proficient	11 to 15	24 to 37
Needs Improvement	7 to 10	11 to 23
Failing	0 to 6	0 to 10

Chart 7.10: Grade ratings for corresponding Final and Total score values. The large area that qualifies in the Total Advanced category is due to the consideration that the "true" score does not include the Initial Survey, worth 10 points.

Examining the individual schools has the advantage of being able to explain some of the more subtle points of each experience, and identify the causes of the outliers that will inevitably appear again when we pool the data together. On the other hand, looking at all four schools as one large data sample not only provides us with a larger, more significant sample size, but also reveals the smoother patterns that get lost without enough data points to fill in the curve. Disregarding race, gender, religion, and previous experience, we can look at the system as one big melting pot, as is often the case in human social experiments.

In the case of pooling data together, certain considerations had to be taken with regard to the scaling; frankly it is unfair to hold a College Prep student, or an Honors student with no calculus experience, against an AP Physics student taking AP Calculus, and call it fair. As such, the grades themselves were discarded for a scaled ratio score that reflects their score against the pool mean value, which in the Final Histogram, ranged from 0 to approximately 2.3, and 0 to 2.5 on the Total Histogram. In this way, we can compare the performance of a student against the pool, rather than pitting them against the significantly higher averages of the upper levels, and everyone is on fair, level ground. We then break down the Total pool scores by individual high schools in order to confirm or deny whether the trends predicted by each school's yearly performance hold true within the context of the experiment as well.

The Final score breakdown is omitted because the score distributions are so close (21 points distributed between 185 students) that the breakdown is too cluttered to glean anything useful from it. Lastly, we will take a look at the average score distributions for each individual worksheet, which will allow us to see where students performed well, and where they began to stumble. These four graphs are, in my opinion, the most significant indicators (or at the very least, the smoothest curves to make speculation) of the outcome of the project, and thus we will carefully consider their implications. As we are examining the pooled values, a few new abbreviations appear which bear clarification. Within the charts that display the score distributions, the quantities "*ScaledScr*" and "*Stud. #*" represent the Scaled Score achieved by a student, as determined by averaging all the pool data together, and the Number of Students who achieved this value.

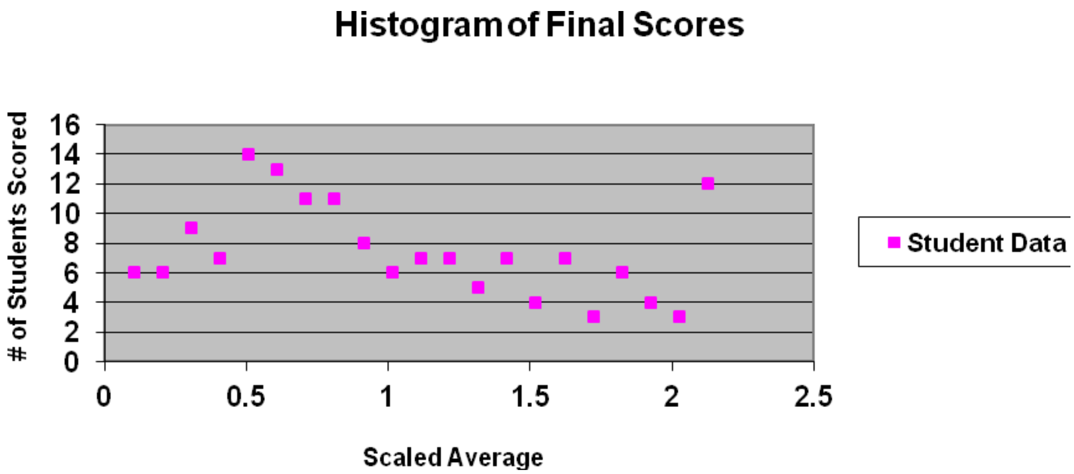

Figure 7.13: The Final Score Histogram of Pooled Data shows the average around 0.5 as a ratio of individual student scores against the pool mean.

ScaledScr	0.101338	0.202677	0.304015	0.405354	0.506692	0.608031	0.709369	0.810707	0.912046	1.013384	1.114723
Stud. #	6	6	9	7	14	13	11	11	8	6	7
ScaledScr	1.216061	1.3174	1.418738	1.520076	1.621415	1.722753	1.824092	1.92543	2.026769	2.128107	
Stud. #	7	5	7	4	7	3	6	4	3	12	

Chart 7.11: The score distribution of the Pooled Final scores show two major spikes, one near the proficient range, and another, more encouraging spike, at the perfect score mark.

A cursory glance of the Histogram reveals two major points of interest. The first is simply the average described by the Gaussian curve, which is situated roughly in the "Needs Improvement" to "Proficient" range of approximately 0.45-0.55, and the second is a spike in the upper limit of the scores. As we are looking at the Final score distribution, the upper limit must correspond to those students who came close to, or achieved, a perfect score. The data chart notes that 12 students achieved this average, with 23 more within the same range of advanced scores. Thus 35 students, or approximately 23% of the student pool managed to score in the advanced range, with about 19% in the proficient range, 23% in the "Needs Improvement" range, and 35% in the "Failing" range. The second spike at the end of the graph is clearly the result of the large number of students who received perfect scores on the final, with seven from Quabbin, two from Doherty, and three from North.

The interesting nature of this graph is that, despite the large number of students who scored in the advanced range, a human experiment has no potential to "pull" the curve right or left because of the individual participation that determines the score distribution. Thus, while in mathematical experiments, a large spike at one end should pull the Gaussian curve one way or the other, the average was determined simply by the achievement capacity and willingness of the students, rather than the relying on the data points to move the average. Additionally, the data point split around 1.3-1.4 mark can be averaged by the values it dips low to and the peaks it reaches to achieve a semi-smooth flat line leveling near the 5 or 6 person range.

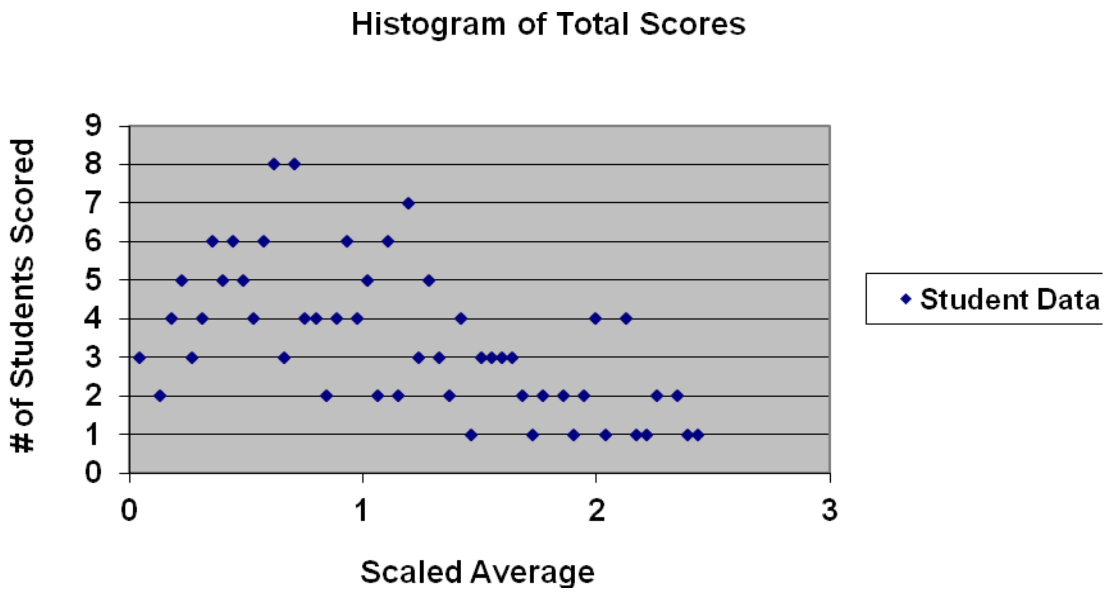

Figure 7.14: Total score Histogram of Pooled Data averages the scores around 0.7 -0.8 range, with fewer outliers in the upper range than previously.

ScaledScr	0.04427	0.132811	0.177081	0.221351	0.265621	0.309891	0.354161	0.398432	0.442702	0.486972	0.531242
Stud. #	3	2	4	5	3	4	6	5	6	5	4
ScaledScr	0.575512	0.619782	0.664053	0.708323	0.752593	0.796863	0.841133	0.885404	0.929674	0.973944	1.018214
Stud. #	6	8	3	8	4	4	2	4	6	4	5
ScaledScr	1.062484	1.106754	1.151025	1.195295	1.239565	1.283835	1.328105	1.372375	1.416646	1.460916	1.505186
Stud. #	2	6	2	7	3	5	3	2	4	1	3
ScaledScr	1.549456	1.593726	1.637997	1.682267	1.726537	1.770807	1.859347	1.903618	1.947888	1.992158	2.036428
Stud. #	3	3	3	2	1	2	2	1	2	4	1
ScaledScr	2.124969	2.169239	2.213509	2.257779	2.346319	2.39059	2.43486				
Stud. #	4	1	1	2	2	1	1				

Chart 7.12: Total score of Pooled Data point distribution, where we see a distinct peaking of scores between 0.5 and 0.9, with the upper range having fewer guests than had been seen in the Final score distribution.

The Total score distribution is by far one of the most clarifying graphs of the pool analyses, as the point values vary so greatly that we can see many of the more subtle qualities of the graph. In particular, while the peak is very clear, located in the 0.7-0.8 range, the second, smaller peak that appears in the Advanced range is also worth examining. The appearance of a large number of students scoring just above 50 points warrants this peak, and implicates, by virtue of that fact that one could not obtain a Total score higher than 48 without knowing some level of calculus already, that the higher scores went towards students who had the advantage of prior exposure to the initial material. What might be still *more* enlightening would be for us to examine this graph in pieces, based on each high school's performance.

Before we do this, take a moment to consider the merits and disadvantages of each school. North, despite being the smallest class, had exposure to Calculus beforehand, as did most of Doherty's classes, and all of Quabbin. WTHS, despite having no Calculus, is the only Level 1 (scores meet expectations) school in the city, but vocational schools frequently receive the title of "underdog" due to their faulty association that trade-based teaching must mean some sort of lack of education. Doherty is frequently heralded by its reputation and location in a rural, semi-suburban neighborhood, but frankly all the schools in Worcester, for anyone that hasn't lived here, are assumed to be "dangerous". The city itself receives many unwarranted stereotypes for its size and urban nature, and likewise do the schools receive their own labels, dependent upon their location and the socioeconomic status of the surrounding neighborhoods. Having mentioned this, reflect for a moment and glance down at the breakdown. Try covering up the legend, and deciding which points correspond to which school before you peek.

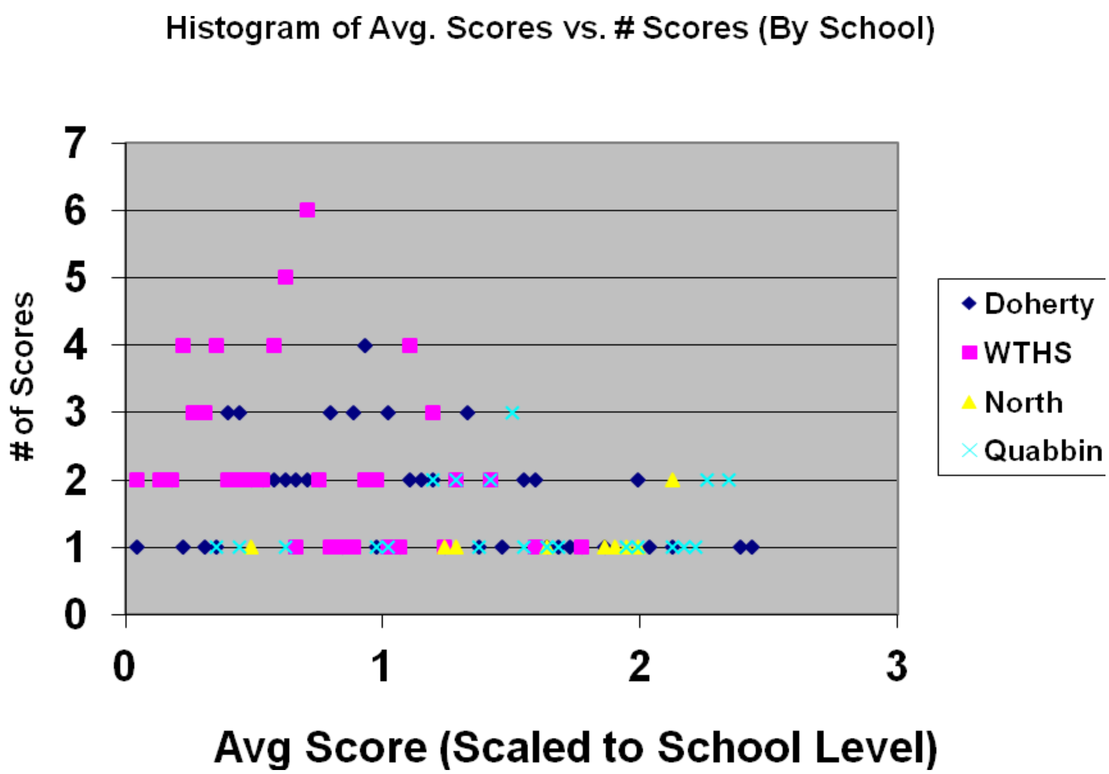

Figure 7.15: The individual high school breakdown of the Total score distribution reveals the overall expected trends, with a number of co-mingling data values.

Certainly there are some noticeable trends that conform to the expectations many would place on the schools, while there are a number of interesting points that defy the norm. WTHS, being the most disadvantaged mathematically, has the lowest peak average, having had no calculus to back up their initial learning experience. The specific values can be re-examined in

Chart 7.10. Doherty represents the *average* average, if you can imagine such a thing. In the Cohen's *d* calculations, we saw that, compared to the data pool, the mean and standard deviation were within fractions of one point value of the pool value. Here, Doherty also represents the space and scoring direction between all the others, with its peak somewhere in proximity with 1.0 and spreading out all the way from 0 to 2.4 (one of the highest). Next comes North, who had too few students to produce a genuine "peak", but rather we see a clumping of points around the 1.8-2.0 range. Lastly is Quabbin, who strangely enough seems to have two distinguishable peaks. The more obvious, vertical peak is right around 1.5, whereas there is also a large horizontal clumping of scores in the 2.0-2.3 range. The two "peaks" are both legitimate, amounting to approximately 8-10 students each, and thus could conclude that either there are two averages for this class, or that there must be an average of the average, at approximately 1.8-2.0, which would roughly tie them with North.

Thus, while some of our initial suspicions are confirmed by the average placements of the schools, the general co-mingling of data points, so many so that they can hardly be called outliers, suggests that there is a certain level of performance that the schools have in common with one another, despite the very black and white images painted by the city labels. Of course, up until this point our own analysis has been fairly black and white, as we have not yet considered the "in-between" work that comes before the final and the total, that being the homework. Our final graph brings forward several interesting conclusions.

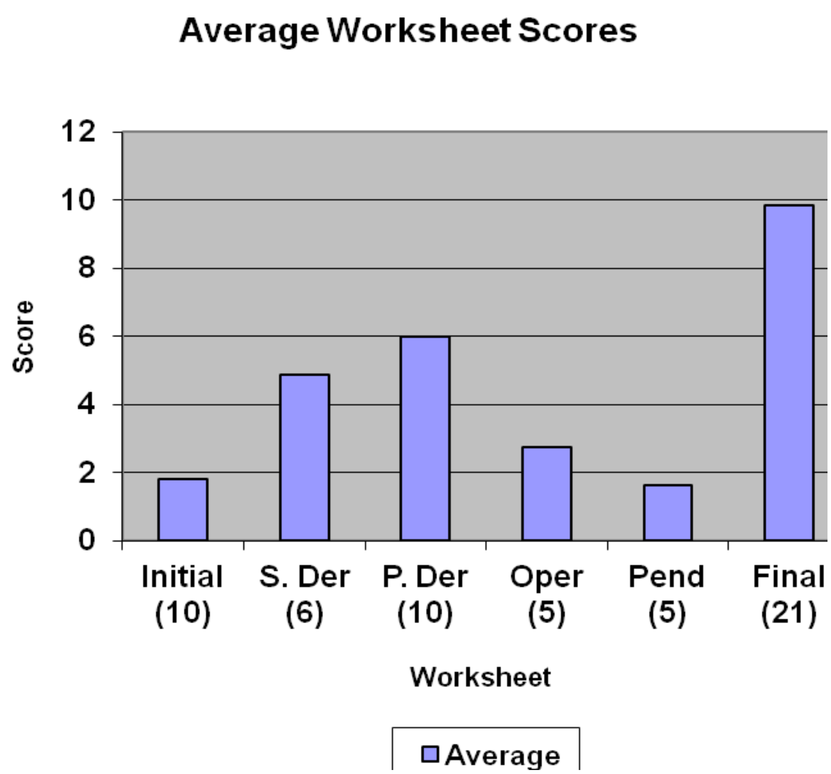

Figure 7.16: The Average Individual Worksheet scores show an impressive performance in the calculus section, and moderate values in the physics portion. The total possible score for each worksheet is listed under each worksheet.

Wrksheet	Initial (10)	S. Der (6)	P.Der (10)	Oper (5)	Pend (5)	Final (21)
Average	1.796296	4.87037	5.978417	2.760684	1.630435	9.859873

Chart 7.13: Average values for the individual worksheets throughout the curriculum.

The scale of this graph is deceptive, as it appears that students did spectacularly on the final despite comparatively lower scores on the other worksheets. Each worksheet, however, has a different maximum score potential, and thus we must consider the implications of each value. The initial survey comes as no surprise, as even those students who had seen calculus before tended to stumble on the partial derivatives and trigonometric derivatives, usually averages 3 or 4 points. With such a large population of students at WTHS, and the average from that school being approximately 0, and also considering the general anxiety and uncertainty with regards to the notation of derivatives, the low score average on this worksheet is more than warranted. It came as somewhat of a surprise, however, that most students didn't remember the terms for kinetic and gravitational potential energy, which is why the calculus students averaged between 3 and 4 points, that last potential point coming from the energy equations.

The first night's homework was following the first lecture on simple derivatives, which was composed of 6 problems designed to test their ability to use all the qualities of the derivative operator. These problems included:

- Ability to recognize that the derivative of a constant is zero
- Ability to recognize that the derivative of a variable raised to the first power is simply unity multiplied by whatever coefficient existed beforehand
- Ability to take a derivative raised to a high power in order to practice the algorithm
- Ability to use the distributive property to take the derivative of a function with more that one piece

The results of this homework are the most surprising of any of the worksheets (which is rather anticlimactic since there are four more to discuss) as the average is so very high. The average value of 4.87 out of a potential 6 is an extremely high score considering that the students from WTHS, numbering approximately 75 out of the pool of 185, had never seen calculus before. To have the average be this high implies very concisely that the students across all of the schools did very well in the initial calculus section, whether they had seen it or not.

The derivative algorithm was presented as a simple rule to follow, provided that we could use the properties of the derivative operator (the distributive property and the product rule, primarily) to separate the function into pieces. At WTHS, students in trade-based learning are taught the importance of rules and regulations, and thus the concept of absorbing a rule was the easiest portion of the project for them. This placed them on level ground, even with students who had seen the material beforehand, and the average reflects this ability.

The partial derivative worksheet, with an average of approximately 6 out of a potential 10 points, also shows an impressive improvement over the initial scores. The partial derivative homework was designed to test specific properties of both the simple derivative (which carry into the partial derivative) and also the new qualities introduced that day. These properties included all of the initial qualities of the simple derivative (listed above), along with the concept of breaking a multivariable function down, either by the product rule, or by virtue of the fact that all variables not specified by the derivative operator are considered constants, whose derivatives are therefore zero.

The first five problems were normal functions, using the familiar *x*, *y*, *z* variables, whereas the second half consisted of taking derivatives of physics equations; specifically equations of energy. This allowed them some practice in partial derivatives, while also becoming comfortable taking derivatives of functions they'd seen before, but hadn't thought to manipulate in this way. The result is a surprisingly mixed bag of scores. Individual analysis of the worksheets showed that about 40% of the students managed to solve at least one of the five physics equation derivatives, with the majority managing to solve only the first five. This segued into the next day's lecture, which taught them how to consider the derivative operator when looking at physics equations.

The operator worksheet primarily tested the students' ability to substitute the "dummy variables" q and $\dot{q}$ ("q-dot") with the proper variables by closely examining a given equation. Three problems were dedicated to this task, and the last two consisted of constructing the Lagrangian Operator (the difference of two energies), and solving that Operator for the final solution using the Lagrangian Formalism. This was done with equations from kinetic, gravitational, and spring potential energy, with only one kind of potential energy at a time. The average of 5 points, which about 2.75, primarily reflects the confusion that accompanied the last two problems, which were admittedly difficult for having just seen the material. Nonetheless, their fluency is fairly well solidified in the 2.75 problems they managed to solve on average, with the first and second being the most easily solved, as they related energies that absolutely everyone had seen prior to the curriculum (kinetic and gravitational potential).

The pendulum worksheet followed a particularly difficult lecture which was meant to demonstrate the level of difficulty that the Lagrangian can quickly escalate to when one has to "imagine" the proper position coordinate, and design the Operator around that presumption. The movement from Cartesian space into polar space to accommodate the proper position variable of theta meant that the linear velocity and position terms used in the Operator would no longer suffice, and thus had to be re-designed in terms of the available quantities. This problem was meant to prove a point, above everything else; that as scientists, we must be confident in our skills and follow through with legal mathematical operators, provided that we have thought the problem through. Only then, in the face of a new theory with unpredictable results, can we be confident that our solutions are correct. Thus, this problem was meant to be more difficult than the last, as students would not be able to anticipate the solution, but rather would have to build confidence in taking derivatives of new quantities in order to produce a satisfactory solution. The five points were distributed on a basis of whether students managed to:

- Identity the correct Operator
- Appropriately substitute the "dummy" variables with the proper ones
- Correctly take the partial derivatives of the Operator
- Correctly recognize the time derivative's effect on a new "velocity" variable
- Produce the correct final solution

While the average suggests that most could only, at best, perform the first two steps, the average is skewed by a large number of students who attempted the problem, but received no points for mistakes, either in the operator's energy terms, replacement of the dummy variables, or taking the derivatives. In five days, most students had properly learned the steps to take to solve a more advanced problem, but the curriculum was simply too fast to fully develop the subtler skills.

7.3.4 Discussion of Minor Data Correlations

While the Final and Total scores are certainly the most significant data produced by means of this project, there are other comparisons of interest that were performed, primarily for the purpose of being able to satisfy the curiosity that comes with such experiments. Race and gender were considered comparative topics outside the scope of this project, and thus were documented but not considered significant in producing a result of whether or not this material could be integrated into the high school environment. For the sake of clarity, the pool data will be our focus in this section; individual school data was taken and sorted by race and gender, and this information is available in the appendix.

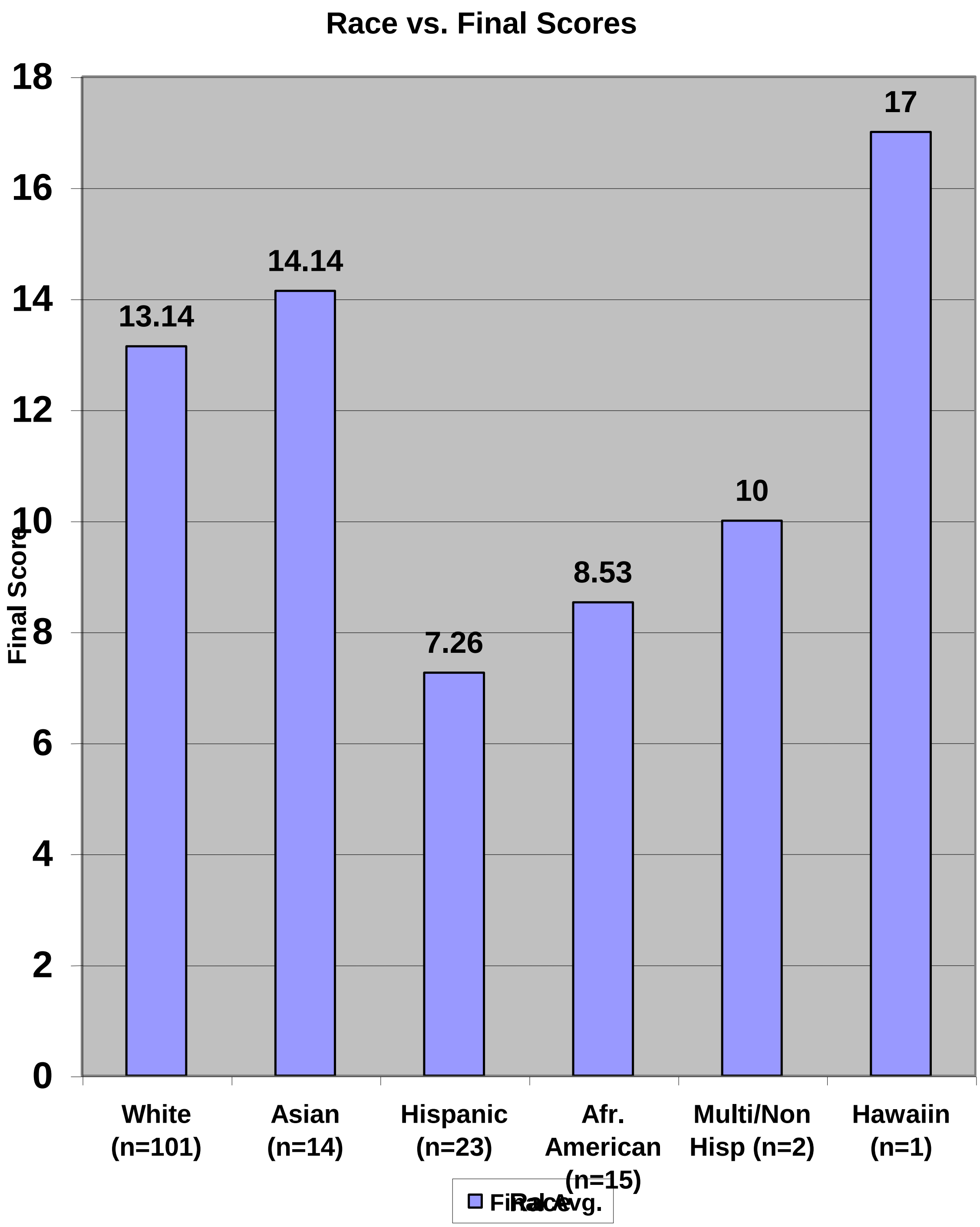

Figure 7.17: Race vs. Final Average

Race comparative studies are often thought to be the “third rail”, or touchy subject, of education studies, as it is every educator’s goal to practice equality and fairness regardless of creed. This, however, is an incomplete outlook, as we must also be sensitive to diversity and cross-cultural relations, which ultimately means we must examine if there are any kinks in the curriculum, or the educator’s method, that could be clashing with a cultural more or expectation. Regrettably, discussion on these graph is limited, as it is clear in Figure 7.16 that the majority of the study took place, albeit unintentionally, with the white demographic. Thus, we can at least make the conjecture that the white Final Score is an accurate reflection of their ability, which is within the Proficient Range. The major outlier is the one Hawaiian student, which again is statistically insignificant, as it is only relevant to this one student, who cannot represent the entirety of that demographic.

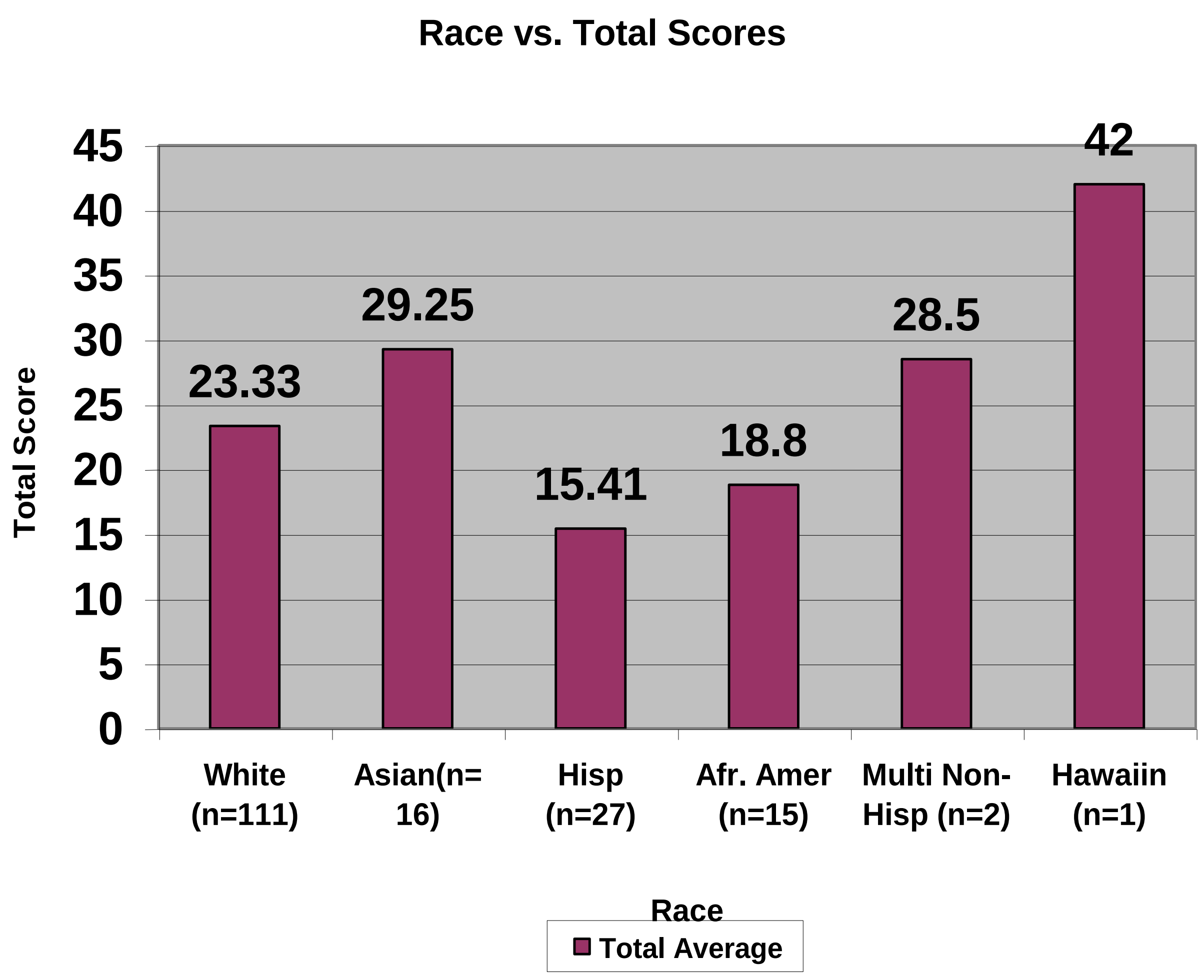

Figure 7.18: Race vs. Total Average

A similar, if not identical, pattern emerges in Figure 7.17, which shows each race graphed against their average scores. For similar reasons, the pool size of the white demographic is the only statistically significant one, as the others have considerably smaller sample sizes. One thing that is noticeable in both graphs is the approximately similar values that the Asian, Hispanic, and African American demographics have, despite their small numbers. One of the potential reasons for the comparatively lower scores is what clearly amounts to a disproportional white demographic when compared to the others, which suggests that perhaps their learning styles weren't taken into careful enough consideration. The repeated trend in Figure 7.16 and 7.17 only strengthens the notion that the level of effort placed into the final was reflective of the level of effort exerted throughout the whole project, and thus we expect exactly the result we obtained.

Unfortunately, beyond a cursory analysis of the graph, the results are up for speculation as to the social, socioeconomic, cultural, or lingual differences that may have affected the study. Splitting the statistics apart in this way was never truly the intention of the project, but rather a necessary consideration, if only for the purpose of gaining as much insight into the logistics and mechanics of the experiment as possible. As such, the results are forthcoming but not necessarily apparent as to their implications, and with such a wide variety of schools taking part in the project, the difference, as most of the minority demographics were centralized in WTHS, may have been based on the environment, or any other of a number of factors. There are, however, a considerably less diverse set of deliberations to make for the next set of graphs.

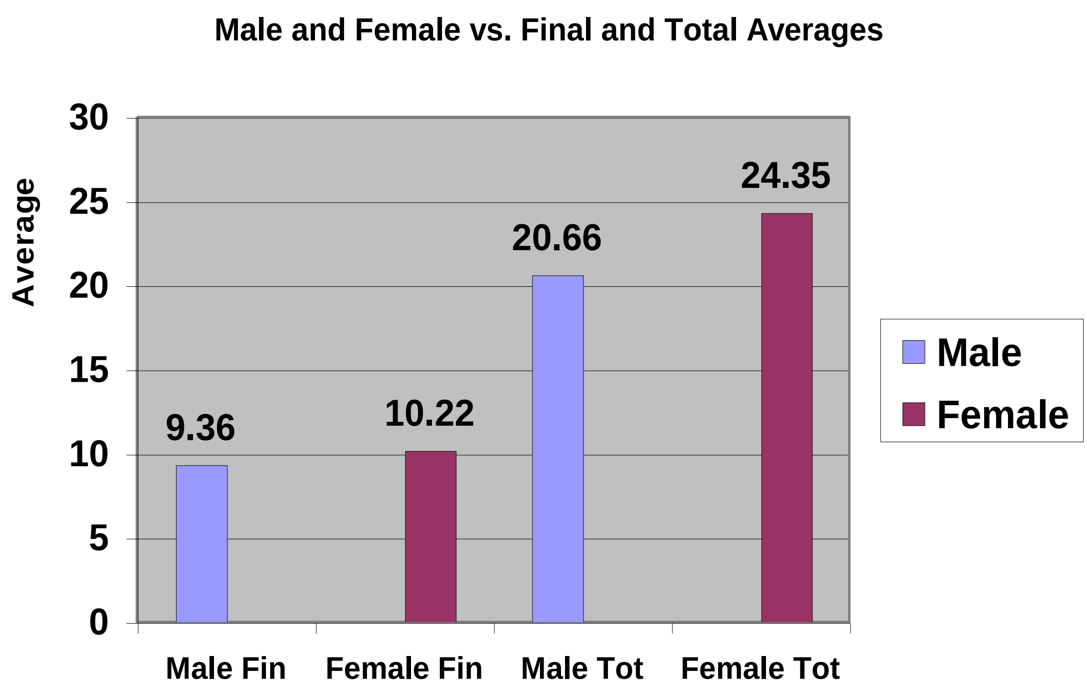

Figure 7.19: Male and Female vs. Final and Total Averages

The condensed chart that Figure 7.19 shows is a representation of the averaged achieved by each gender under both of the major categories of Final and Total Scores. The immediate pattern to note is that, as expected, a pattern developed in the Final Scores is reflected once more in the Total Scores as well; this result happens to be that the females have outperformed the males. Unlike race, which mixes, changes, and spans the globe in different forms and cultural implications, gender is universal; it is only the societal role that defines it. Here, an interested result has manifested unintentionally as a possible result of the general environment the students are placed into in today's educational system. Several psychological phenomena have shown that, when reminded of their gender, females tend to do poorer in mathematics and science, while male's tend to do better. Additionally, when threatened in their dominance, the same trend appears for males and females, which males having a "threat response" and performing better as a result, and females performing worse.

What appears here, as a result of making *no* mention of gender throughout the curriculum, at least as some sort of intentional factor, is a fairly clean result. The individual school results for gender below show that in almost every case, the females out performed the males, *except* for WTHS, where 3 of 4 classes had the males outperforming the females. In traditionally taught high schools, there is a subtle action taking place to urge women into science and mathematics, a movement that began in the 70s when the ratios were absurdly disproportional and favorable to men. This trending societal pressure has been subtle and encouraging enough to almost completely flip the ratios in the last thirty years, and women are now, on average, outperforming men in these fields, as is demonstrated by this result. Why, then, did WTHS diverge so greatly from this condition?

As stated earlier, vocational schools are often mislabeled for their trade-based nature as a school for those that cannot do mathematics, science, or other abstract topics, and are forced to “submit” to a life in trade. Today’s model of vocational schools is quite different, and in fact WTHS is one of the only Level 1 schools in the area, having met its expected improvement rate on the 2010 MCAS [5] , along with several years prior. This being said, the condition does not change that there is still a disparity of female confidence in vocational schools, as well as trades that are typically labeled as “girl jobs”, such as manicurists, nurses, and culinary arts. As such, we can speculate to some degree that, due to the large number of girls still choosing trades in these areas, that their interest in mathematics and science may have suffered decline, bringing their scores lower than one would expect. This decline in interest is also one of the considerations we can make towards correlating scores with certain variables.

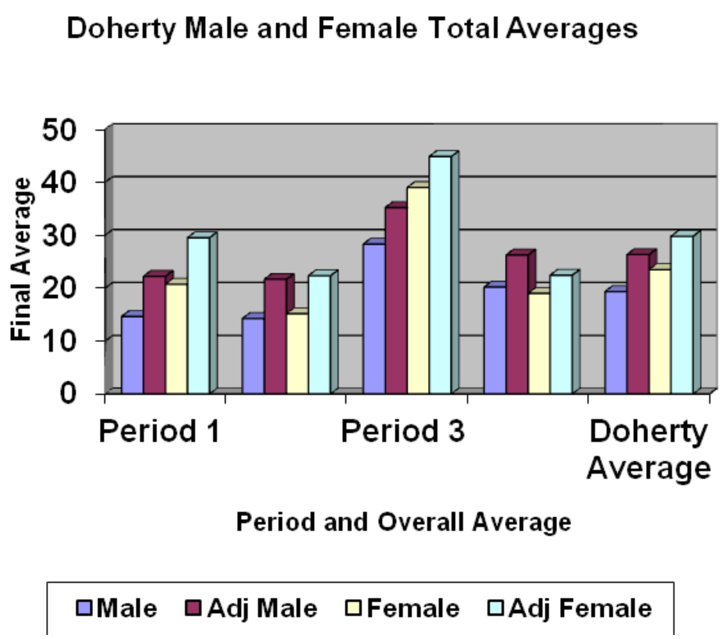

Figure 7.20: Doherty Male and Female Total Averages by Period, also including the school average. Adjusted values account for the maximum potential score, had everything been turned in.

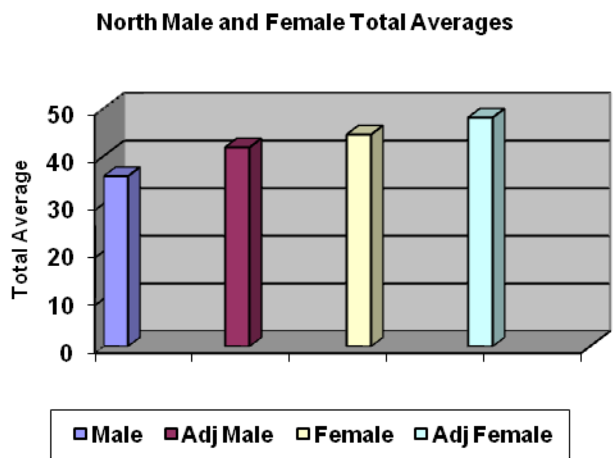

Figure 7.21: North Male and Female Total Averages suggest a common trend of females o outperforming males.

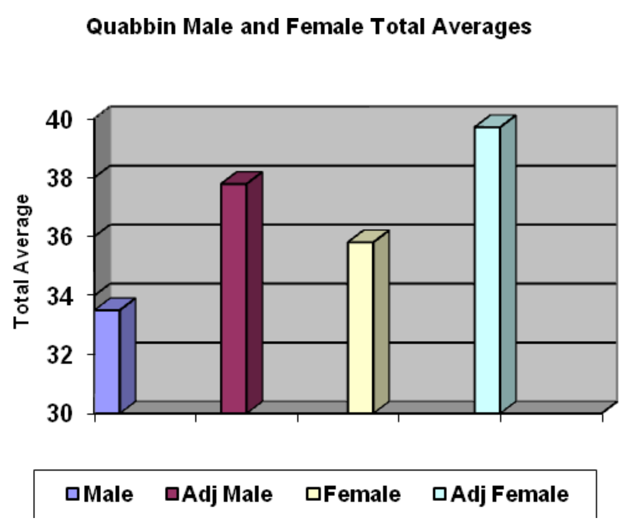

Figure 7.22: Quabbin Male and Female Total Averages show females performing approximately two points better than males on average.

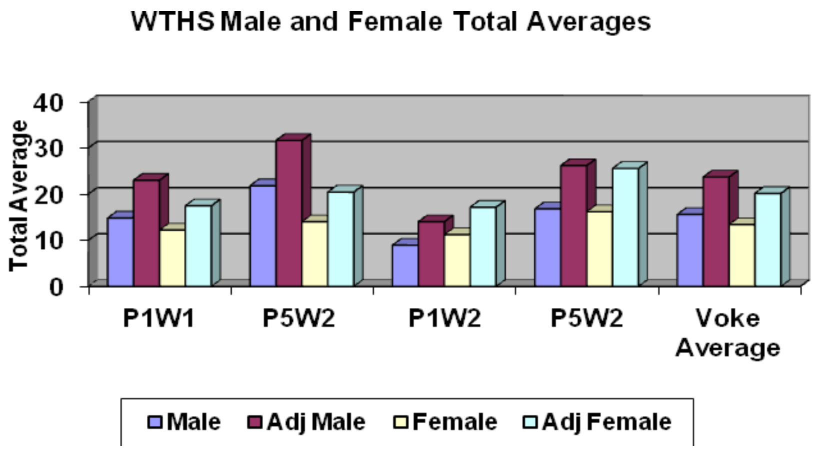

Figure 7.23: WTHS Male and Female Total Averages show males breaking the trend set by other schools, outperforming females by the same margin of two points.

One of the last correlations made during the course of the experiment was between three particular rated variables, and the final scores. The three variables were rated from a scale of 1 to 10 on the topics of the understandability of the material, how well I communicated the material, and their interest in the topics discussed. The hypothesis initiating this correlation was that higher rated scores should correspond to higher final scores, as it should reflect, in some sense, that an improved sense of understanding will generate better scores.

The final exam was based on several of the earlier designs of individual worksheets combined. The different kinds of problems that demonstrate knowledge of the properties of simple derivatives, partial derivatives, energy equations, and Lagrangian Mechanics are used, and the open response problems were designed and scored with the same idea as the Operator homework sheet. Thus, at the very least the scoring system was designed to reflect a necessity of understanding, propagated by communication, and generated by interest.

The three graphs pool together these values as a method of identifying potential correlations between those that knew, or at least felt, that they grasped the material well, and those that scored well. Due to the similarities in the graphs, they are worth discussing in tandem.

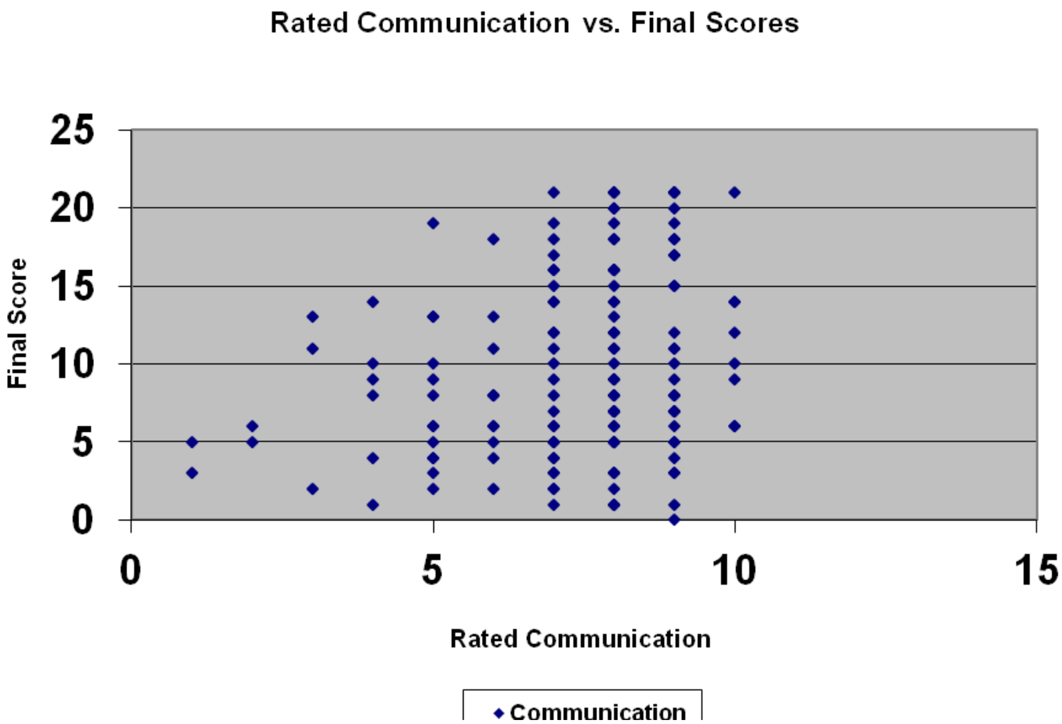

Figure 7.24: Rated Communication vs. Final Scores shows an interesting horizontal threshold around the 3-5 range.

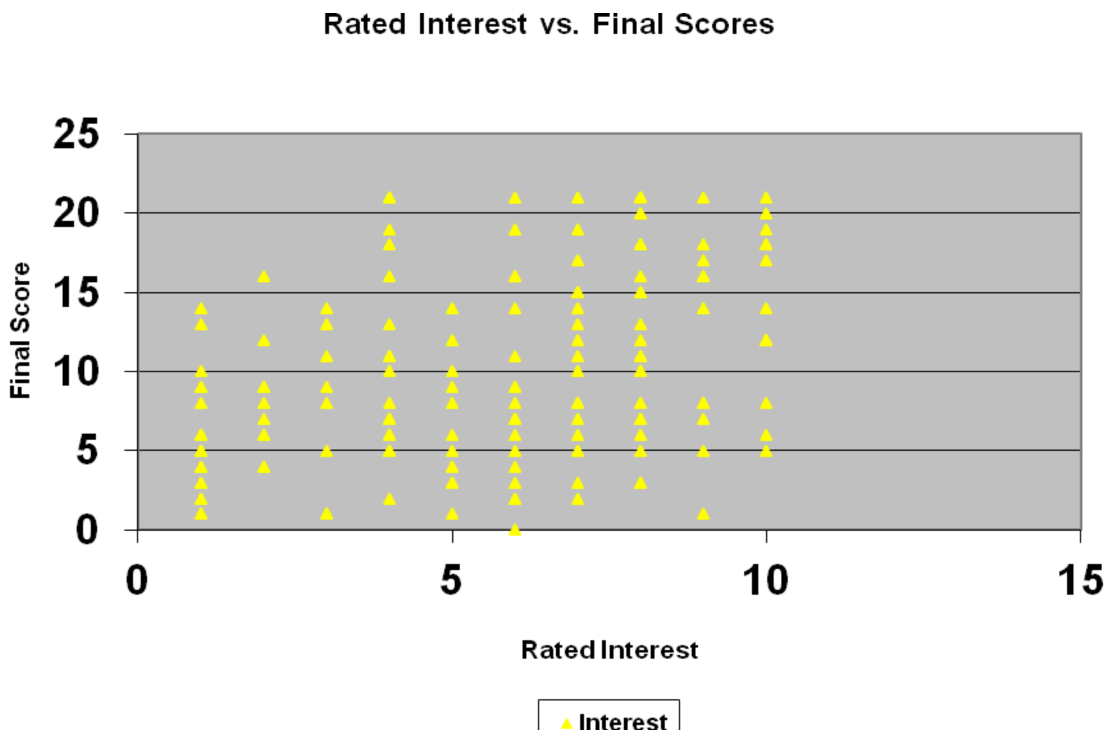

Figure 7.25: Rated Interest vs. Final Scores repeats the potential threshold in a similar range.

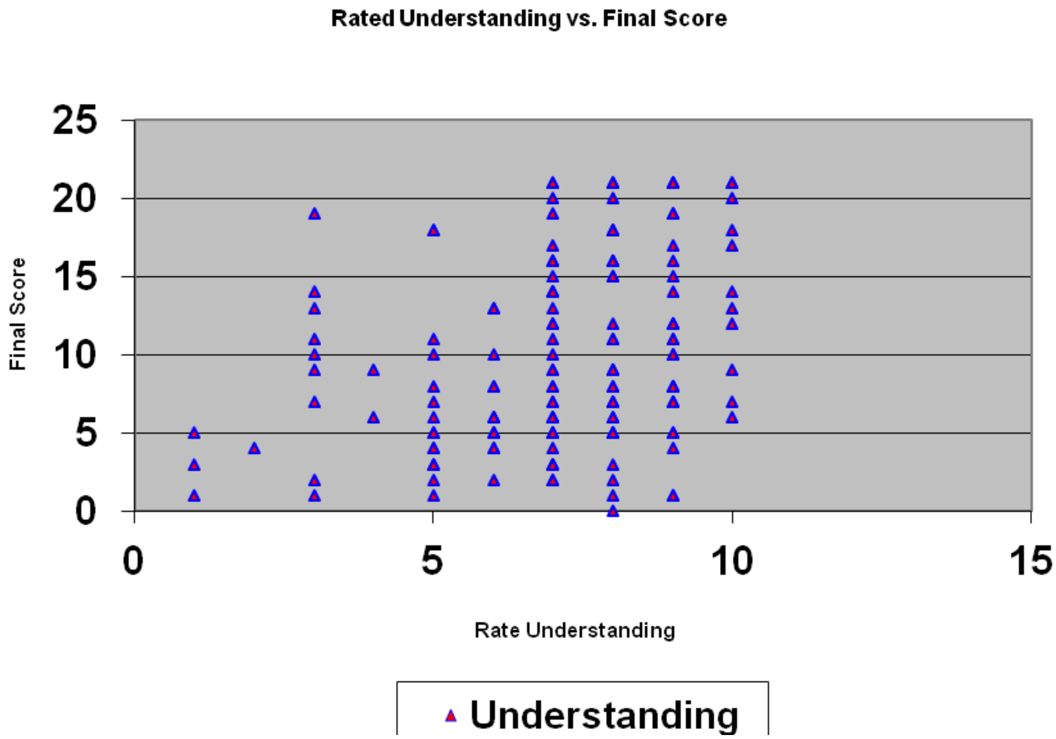

Figure 7.26: Rated Understanding vs. Final Score has a threshold at a rating of three that fluctuates at certain values.

The three graphs, upon first glance, appear to be nothing more than scattered points with no distinguishable pattern. Figure 7.26 offers the best look at what appears more vaguely on the other graphs, which is a sudden increase in the Final Scores upon reaching a certain rated value. This threshold, which appears at 4, 3, and 6, on the Communication, Interest, and Understanding graphs, respectively, are horizontal lines drawn across the maximum score achieved at that value, at which point a noticeable increase in the maximum score appears beyond that value. This indicates a value that genuinely distinguishes some correlation between their rated value and their score. Although this value varies from each graph, there is some approximation that establishes a minor relationship between a student's level of interest and attentiveness, and their final score.

Even under the consideration that much of the graph is very rough and scattered, all the graph tells us in this case is that, quite frankly, anyone can learn. Some of the most interested students scored the lowest, and some of the least interested students scored the highest. There were high and low scorers for both high and low communication and understandability as well, which conveys that to some extent, these students were prepared long before I arrived to teach them this material.

Chapter 8: Conclusions

The intention of this project, in its infancy, was simply to provide some evidence as to whether or not more rigorous, advanced material could successfully be brought into the high school environment. In reflecting upon this, several other more philosophical reasons began to persist. In particular, the point made by Tom Gusek from Worcester Technical High School, that students are consistently surprised by the level of difficulty that they encounter if and when they reach college, and that this is potentially a reflection of our need as educators to raise the bar even higher. Recognizing that students were not learning electrostatics, fluid mechanics, or nuclear physics thirty years ago is an important realization to make, one that implicates our responsibility to continue pushing the bar. With this in mind, the project became something more important, as I found that with each high school I passed through, the teachers were ecstatic to see their students trying harder material, and often felt bound to the less rigorous tasks set forward by the MA State Frameworks. Most of the teachers kept copies of the material, either to try teaching it as a side project the next year, or to show to their math department as proof that students could, in fact, do calculus, and deserved the shot at higher math classes.

The importance of preparing the next generation for what is to come stems from the fact that higher academia is, and has been, consistently producing new theories, new equations, and better understanding to explain our physical world on a biological, chemical, and atomic level, to the point where eventually the material that is being taught to college sophomores will have to be taught to freshman in order to make room for the new wave of information placed in the senior classes. This will iterate once more as the now-freshman material will have to be passed down into high schools, and herein we find another purpose of the experiment. The Lagrangian is general, adaptable, dynamic, and useful in myriad ways for solving Newtonian Mechanics problems using the simple concept of the scalar energy. The results of this experiment, albeit on a relatively small scale, were indicative of two important considerations. When examining the Total Score Distribution [pp 107] , pooled together and broken down by high schools, we saw that, while there was a distinct mixing of scores between high schools around a certain score range, each high school performed to the level predicted of them by their available statistics, if they didn't simply perform better than them.

Worcester Technical High School, being the most disadvantaged school for not having a calculus program, had the lowest average, with a large number of data points spreading into the Proficient and Advanced Range. Doherty, with an average within tenths of a point of the pool mean, was considered the baseline, as their MCAS scores [5] reflect scores very close to that of the district and state averages. They did not disappoint, as their pooled scores, with two Honors classes, one College Prep, and one AP, produced a score that set the average for the experiment. North High, despite having a very small class, performed extremely well, with 27% of the students achieving perfect scores, actually outperforming their expected scores based on their available MCAS data. Lastly, Quabbin, who was considered the outlier due to its status as the only non-high needs school in the experiment, averaged an amazing 26% perfect score rate in a class of 27, embodying the potential that students can reach when exposed to advanced material earlier on.

Figures 7.17 [pp 112] and 7.18 [pp 113] show the race correlations on Total and Final scores, but frankly have no context in the experiment, nor enough relevant data to be considered pertinent in our speculations. Regrettably, and unintentionally, there simply weren't enough students within some of the demographics to accurately represent the potential they reached during the course of the project. The gender graphs, however, shown in Figures 7.19, 7.20, 7.21, 7.22, and 7.23, display an interesting trend that often goes unspoken of in modern American society, which is that the females consistently outperformed the boys on Final and Total scores, save for one exception. The great push that began in the 1970s to get females involves in science and mathematics has actually flipped the stereotype, as males are now being outperformed in almost every category of mathematics and science courses. At WTHS, however, the males actually ended up on top, and we speculated that this could strongly be correlated with the fact that many females in the sample group were pursuing distinctly non-mathematically rigorous careers such as culinary arts and manicurists, whereas the boys were in trades such as car repair and engineering, that still required skills such as arithmetic and equation manipulation for measurements.

While some of the graphs were not particularly forthcoming with their applicable significance, the pool graphs were certainly the most elucidating, showing a clean pattern of aptitude in certain areas, and areas that require improvement as well. Figures 7.13 [pp 105] and 7.15 [pp 107] display the pooled Final and Total score values for all the students involved in the survey, with 7.15 [pp 107] breaking down Total scores by individual high schools. Both graphs reveal distinct, Gaussian curves that show averages in the range of scores qualifying as "Needs Improvement" up to "Proficient", and thus set a precedent for potential future endeavors in this field. Figure 7.15 [pp 107] , in particular, shows that the Total scores, when analyzed by the high schools that achieved them, obey a rough correlation of each school's standings in the yearly MCAS against one another. It can further be speculated from this data, that due to this correlation, should it be found to be consistent from experiment to experiment, that the worksheets used to assess the students are consistent with state standards, as it achieves similar results to the state's.

Furthermore, with the notion that the project allows a relatively accurate projection of the school's scores against the district and school averages, it may be possible to establish an expected performance record for each school within this specialized curriculum, thus allowing a standard of performance to be set on an even more advanced level than is currently expected of the highest level students. This, of course, would require further tests to be performed, and in order to be truly accurate, would have to be performed in other states to ensure the accuracy of the material in producing district and state comparative averages. While the scope of this project does not seek to implement this material in other states, it may be a worthwhile study to consider the same experimental curriculum being implemented in the same five-day time span in other states, if only to begin a cross-reference of data from other areas.

With regards to whether this material should be implemented in the high school MA State Frameworks for Physics as a consequence of this experiment, I believe that there are still a number of factors to consider. The actual process of making the material a part of the curriculum is, admittedly, a bureaucratic mess that is most certainly beyond the reach of this paper, but it is worth mentioning. As there are so very many theories regarding the best method of teaching, the proper material to introduce, when to introduce it, and other discussions, it is difficult to imagine that all teachers across the state could be convinced that that is what should be taught. This project brings quantifiable data, along with video evidence, in order to best show that students, on the average, are prepared to attempt material that would be considered as “advanced” by most standards.

The realization that must be made is that the term “advanced” is subjective, and there has been significant evidence showing that students who are never assured that the material is “difficult”, but rather is lead through being assured that it is a simple process, or series of steps, perform better. This sense of identity foreclosure, that students believe they are innately capable of certain “levels” of difficulty, indicated that educators should not be placing labels with respect to the difficulty of the subject, but instead do their best to make it seem appealing, simple, and fun, without sacrificing the rigor of the material itself.

There are a number of considerations that we must acknowledge in order for difficult material of *any* kind to reach the student, and in keeping with the central dogma of education that “any student can learn”, it is likely that many of the limitations are psychological, which has lead to the boom in quantifiable research in the field of education in the last 20 years. This being said, the findings of this paper suggest that students of virtually every level are capable of learning both the qualitative and quantitative aspects of advanced mathematical and physical material in a relatively fast period of time. One of the greatest complaints of the student surveys was simply that “there was not enough time to absorb the material”, with the most frequent follow-up of “I feel I could have mastered the material, given more time”, which I am inclined to believe, considering the impressive array of scores achieved within a mere five days.

Although this experiment has concluded, the project is only in its infancy, as there are still many more factors that may be considered, were someone to take up the mantel and find a new array of schools. Some of the difficulties in acquiring schools for the project made the choices much more random, which is certainly statistically desirable, but for the purpose of making concise conclusions about specific schools in the area, or perhaps districts, it would have been more beneficial to have chosen an array of schools that more closely resembled one another’s statistics and teaching styles. The project, having a very wide spectrum of schools, encompasses a rough outline of the average of potential of an amalgam, void of attention towards race, gender, trade, age, interest, or experience, and while this is a good starting point, the focus must be narrowed to examine a larger sample size in more specific groups; perhaps only vocational schools, or perhaps grouping schools by average socioeconomic status, as a method of testing individual potential. One interest that I would like to see in future projects would be to perform the same experiment the same year, and several years after, North High transfers to its new school building, to examine the effects of a newer environment on student participation and effort.

While there were several inconsequential mistakes made over the course of the curriculum, such as the absence of a ninth question on the initial survey for quite some time, there were no major blockades that affected the experiment detrimentally. As an aside, there was only one student who actually noticed the “missing” ninth question, which had no effect on the experiment, but rather was curiously noted. I do feel, in part, that the project would do better in Doherty with an instructor who the students were unfamiliar with, but equally excited about the material as I was; my continued presence there, I feel, had much to do with the noticeable issue of missing homework. Having taught them for sixteen weeks already, they had blurred the line between peer and instructor some time ago, and thus it was a bit difficult to illicit the same response as those students in other schools who felt I was something new and a worthy distraction from their normal curriculum.

Re-implementing the project, for prospective IQP or MQP students, would be as simple as using the curriculum outlined in this paper, or perhaps redesigning it to suit one's needs, and contacting the schools in the same manner I did. It is very easy to forget that one term at WPI is only seven weeks, and thus at best, one hopes to accomplish an absolute maximum of twenty-three schools in the three-term timeline (the extra two schools would be performed during our normal breaks). Such a large scope would be beneficial to providing the larger sample size I sought out, and a more narrowed focus can be gleaned from watching and adjusting the video lecture series available with the paper.

Thus, as we conclude our discussion on the ability of students in the high school environment to take on bigger and better things, the outlook is encouraging. It is my distinct belief, as I told one teacher over the course of the curriculum, that if I were to take the highest scorers, the most interested students from each school (with those two not necessarily being one and the same, but not necessarily being mutually exclusive either) I could fill several classrooms with students who were capable of achieving on very high levels. If I was given more time for the project, I wager that I could fill several more, and this very likely has little to do with the teacher, so much as it does the subject itself. The capacity of the average high school student is clearly higher than the bar we have set; the challenge is making such abstractions, such seemingly esoteric knowledge, and such difficult material interesting and practical for the student, because only then do we see the effort we so closely associate with achievement. The gauntlet has been thrown down, and the students answered with vigor.

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Acknowledgements

I would like to extend my sincere thanks to all those that participated in this project, both subjects and advisors, for allowing the project to grow into something far larger than itself and its original intentions. Without your help, none of this would have been possible, and as became clear over the course of the experiment, none of the abundant evidence in favor of more advanced material in the high school environment would have been discovered. Many thanks to Professor Carolann Koleci (advisor), John Staley (Department Head of Science, Doherty Memorial High), Tom Gusek (Physics Teacher, WTHS), Azad Chaparian (Department Head of Science, WTHS), Joseph Marzilli (Physics Teacher, North High), Robert Kolesnik (Physics Teacher, Quabbin Regional High), Channel 11 for providing me with their time and recording equipment, and of course, all the students who took part in the project, who were spectacularly cooperative and, for the most part, enthused by the material.

Chapter 9: Appendix

Introductory Physics, High School

Learning Standards for a Full First-Year Course

I. Content Standards

1. Motion and Forces

Central Concept: Newton's laws of motion and gravitation describe and predict the motion of most objects.

1.1 Compare and contrast vector quantities (e.g., displacement, velocity, acceleration force, linear momentum) and scalar quantities (e.g., distance, speed, energy, mass, work).
1.2 Distinguish between displacement, distance, velocity, speed, and acceleration. Solve problems involving displacement, distance, velocity, speed, and constant acceleration.
1.3 Create and interpret graphs of 1-dimensional motion, such as position vs. time, distance vs. time, speed vs. time, velocity vs. time, and acceleration vs. time where acceleration is constant.
1.4 Interpret and apply Newton's three laws of motion.
1.5 Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.
1.6 Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.
1.7 Describe Newton's law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.
1.8 Describe conceptually the forces involved in circular motion.

2. Conservation of Energy and Momentum

Central Concept: The laws of conservation of energy and momentum provide alternate approaches to predict and describe the movement of objects.

2.1 Interpret and provide examples that illustrate the law of conservation of energy.
2.2 Interpret and provide examples of how energy can be converted from gravitational potential energy to kinetic energy and vice versa.
2.3 Describe both qualitatively and quantitatively how work can be expressed as a change in mechanical energy.
2.4 Describe both qualitatively and quantitatively the concept of power as work done per unit time.
2.5 Provide and interpret examples showing that linear momentum is the product of mass and

velocity, and is always conserved (law of conservation of momentum). Calculate the momentum of an object.

3. Heat and Heat Transfer

Central Concept: Heat is energy that is transferred by the processes of convection, conduction, and radiation between objects or regions that are at different temperatures.

3.1 Explain how heat energy is transferred by convection, conduction, and radiation.
3.2 Explain how heat energy will move from a higher temperature to a lower temperature until equilibrium is reached.
3.3 Describe the relationship between average molecular kinetic energy and temperature. Recognize that energy is absorbed when a substance changes from a solid to a liquid to a gas, and that energy is released when a substance changes from a gas to a liquid to a solid. Explain the relationships among evaporation, condensation, cooling, and warming.

3. Heat and Heat Transfer (cont.)

3.4 Explain the relationships among temperature changes in a substance, the amount of heat transferred, the amount (mass) of the substance, and the specific heat of the substance.

4. Waves

Central Concept: Waves carry energy from place to place without the transfer of matter.

4.1 Describe the measurable properties of waves (velocity, frequency, wavelength, amplitude, period) and explain the relationships among them. Recognize examples of simple harmonic motion.
4.2 Distinguish between mechanical and electromagnetic waves.
4.3 Distinguish between the two types of mechanical waves, transverse and longitudinal.
4.4 Describe qualitatively the basic principles of reflection and refraction of waves.
4.5 Recognize that mechanical waves generally move faster through a solid than through a liquid and faster through a liquid than through a gas.
4.6 Describe the apparent change in frequency of waves due to the motion of a source or a receiver (the Doppler effect).

5. Electromagnetism

Central Concept: Stationary and moving charged particles result in the phenomena known as electricity and magnetism.

5.1 Recognize that an electric charge tends to be static on insulators and can move on and in conductors. Explain that energy can produce a separation of charges.
5.2 Develop qualitative and quantitative understandings of current, voltage, resistance, and the connections among them (Ohm’s law).
5.3 Analyze simple arrangements of electrical components in both series and parallel circuits. Recognize symbols and understand the functions of common circuit elements (battery, connecting wire, switch, fuse, resistance) in a schematic diagram.
5.4 Describe conceptually the attractive or repulsive forces between objects relative to their

charges and the distance between them (Coulomb's law).

5.5 Explain how electric current is a flow of charge caused by a potential difference (voltage), and how power is equal to current multiplied by voltage.

5.6 Recognize that moving electric charges produce magnetic forces and moving magnets produce electric forces. Recognize that the interplay of electric and magnetic forces is the basis for electric motors, generators, and other technologies.

6. Electromagnetic Radiation

Central Concept: Oscillating electric or magnetic fields can generate electromagnetic waves over a wide spectrum.

6.1 Recognize that electromagnetic waves are transverse waves and travel at the speed of light through a vacuum.

6.2 Describe the electromagnetic spectrum in terms of frequency and wavelength, and identify the locations of radio waves, microwaves, infrared radiation, visible light (red, orange, yellow, green, blue, indigo, and violet), ultraviolet rays, x-rays, and gamma rays on the spectrum.

II. Scientific Inquiry Skills Standards

Scientific literacy can be achieved as students inquire about the physical world. The curriculum should include substantial hands-on laboratory and field experiences, as appropriate, for students to develop and use scientific skills in introductory physics, along with the inquiry skills listed below.

SIS1. Make observations, raise questions, and formulate hypotheses.

- **Observe the world from a scientific perspective.**
- **Pose questions and form hypotheses based on personal observations, scientific articles, experiments, and knowledge.**
- **Read, interpret, and examine the credibility and validity of scientific claims in different sources of information, such as scientific articles, advertisements, or media stories.**

SIS2. Design and conduct scientific investigations.

- **Articulate and explain the major concepts being investigated and the purpose of an investigation.**
- **Select required materials, equipment, and conditions for conducting an experiment.**
- **Identify independent and dependent variables.**
- **Write procedures that are clear and replicable.**
- **Employ appropriate methods for accurately and consistently**
    - **making observations**
    - **making and recording measurements at appropriate levels of precision**
    - **collecting data or evidence in an organized way**

- **Properly use instruments, equipment, and materials (e.g., scales, probeware, meter sticks, microscopes, computers) including set-up, calibration (if required), technique, maintenance, and storage.**
- **Follow safety guidelines.**

SIS3. Analyze and interpret results of scientific investigations.

- **Present relationships between and among variables in appropriate forms.**
  - **Represent data and relationships between and among variables in charts and graphs.**
  - **Use appropriate technology (e.g., graphing software) and other tools.**
- **Use mathematical operations to analyze and interpret data results.**
- **Assess the reliability of data and identify reasons for inconsistent results, such as sources of error or uncontrolled conditions.**
- **Use results of an experiment to develop a conclusion to an investigation that addresses the initial questions and supports or refutes the stated hypothesis.**
- **State questions raised by an experiment that may require further investigation.**

SIS4. Communicate and apply the results of scientific investigations.

- **Develop descriptions of and explanations for scientific concepts that were a focus of one or more investigations.**
- **Review information, explain statistical analysis, and summarize data collected and analyzed as the result of an investigation.**
- **Explain diagrams and charts that represent relationships of variables.**
- **Construct a reasoned argument and respond appropriately to critical comments and questions.**
- **Use language and vocabulary appropriately, speak clearly and logically, and use appropriate technology (e.g., presentation software) and other tools to present findings.**
- **Use and refine scientific models that simulate physical processes or phenomena.**

Students are expected to know the content of the *Massachusetts Mathematics Curriculum Framework,* through grade 8. Below are some specific skills from the *Mathematics Framework* that students in this course should have the opportunity to apply:

- ✓ Construct and use tables and graphs to interpret data sets.
- ✓ Solve simple algebraic expressions.
- ✓ Perform basic statistical procedures to analyze the center and spread of data.
- ✓ Measure with accuracy and precision (e.g., length, volume, mass, temperature, time)
- ✓ Convert within a unit (e.g., centimeters to meters).

- ✓ Use common prefixes such as *milli*-, *centi*-, and *kilo*-.
- ✓ Use scientific notation, where appropriate.
- ✓ Use ratio and proportion to solve problems.

The following skills are not detailed in the *Mathematics Framework*, but are necessary for a solid understanding in this course:

- ✓ Determine the correct number of significant figures.
- ✓ Determine percent error from experimental and accepted values.
- ✓ Use appropriate metric/standard international (SI) units of measurement for mass (kg); length (m); time (s); force (N); speed (m/s); acceleration (m/s^2); frequency (Hz); work and energy (J); power (W); momentum (kg•m/s); electric current (A); electric potential difference/voltage (V); and electric resistance (Ω).
- ✓ Use the Celsius and Kelvin scales.

Accelerating Cars

Introductory Physics, High School

Acceleration is a concept Ms. Luke chooses to teach students early in her introductory physics class. Many students are aware that acceleration means that an object moves faster, but Ms. Luke has found that students often have difficulty articulating how to measure acceleration and graphically relating acceleration to changes in speed. She decides to teach these concepts by using something with which all her students are familiar; cars.

In an opening dialog, Ms. Luke and her students together define speed and velocity, and how they are calculated. They then move on to the more challenging concept of acceleration, including deceleration, no acceleration, and constant acceleration. Ms. Luke asks, “How can you tell something is accelerating?” One student quickly mentions using a speedometer. Another student mentions “that thing that measures how fast you walk,” which Ms. Luke identifies as a pedometer. “How can you use a speedometer, for example, to measure acceleration?” she asks. “Or, if you didn’t have an speedometer or pedometer, how would you know that the object is accelerating?”

After listening to student responses, without accepting or dismissing any of them, Ms. Luke proposes that the class go outside to observe whether cars that drive by the front of the school build up speed, slow down, or maintain a constant speed over a given distance. With the data students collect, they will relate what they see and hear to a graph of each car’s speed and an analysis of its acceleration.

The students are organized into small groups. Each group stands on the sidewalk along a stretch of road identified by Ms. Luke, separated from the next group by twenty meters. Ms. Luke has already marked off 20-meter increments. She has chosen to use a strip of road that begins at the stop sign in front of the school and includes the downward sloping hill beyond. Here she knows her students will have a good opportunity to observe different rates of speed and acceleration. The students are equipped with stopwatches and their lab notebooks. Each group knows to measure and record the time it takes a car to travel from the stop sign to their position. They are also instructed to record observations of each car while it is in their assigned zone, including the sound of its engine and whether the brake lights are on. The groups record data for five cars identified by Ms. Luke before going back into class to work through their calculations, graph their data, and answer the key questions of the activity.

Upon reentering the classroom, the students record their data on the board. Ms. Luke asks one student to demonstrate how to calculate the speed of one car, within that student’s assigned zone, using the data from the student’s group plus the data of the group positioned just uphill of them. Each group then records the speed of each car in their zone on a class chart for everyone to see. Ms. Luke also asks students to relate these calculations to their observations of the cars. Ms. Luke then asks her students to consider, “What does the graph of the speed of each car over the entire stretch of road look like?” She has each student make a position vs. time graph and a velocity vs. time graph for each car. Ms. Luke has the students annotate each graph with their observations of that car. From these graphs the class compares change in speed for the cars relative to each other.

Ms. Luke then asks the class to focus on the speed vs. time graph of the first car, which she projects for everyone to see. They notice that the points on the graph do not form a continuous straight line across the grid, but instead go up, straight across, and then down slightly in the last segment. “What does this mean?” she asks. “It means that the car sped up and slowed down,” offers one student. “It means that the car accelerated from here to here," another student points out on the graph, "but then it stopped speeding up from here to here." She asks the students to confirm this against their observations of the car.

Ms. Luke then says to the class, "Determine if each car accelerated, decelerated, or showed no acceleration over any period of time. If a car did accelerate or decelerate at some time, did it keep doing so at the same rate?"

Finally, Ms. Luke instructs the students to circle and notate the places on each graph where that car possibly accelerated, decelerated, or showed no acceleration. To quantify the areas circled, she has the students calculate the acceleration from one zone to the next, pointing out that a negative result means that the car slowed down or decelerated, and a zero result means that the car maintained its speed. Ms. Luke also instructs her students to look for instances where the acceleration is the same for two or more adjacent places on the graph, and to label those instances as constant acceleration.

Assessment Strategies

- Students should pay particular attention to the construction and labeling of graphs. They should use units appropriately throughout their work.
- Students can write out a scenario that aligns with the changes in speeds on the graphs they have created themselves. Students should properly use the terms "speed," "velocity," "acceleration," "deceleration," "no acceleration," and "constant acceleration" in their scenarios.
- As a follow-up assignment, the students can create a data chart that includes distance, time, and speed of a fictitious vehicle. With this data, they create a speed vs. time graph. Their data must show acceleration, deceleration, no acceleration, and constant acceleration on their graph. They should also calculate acceleration.

Introductory Physics Learning Standards

High School

1.1 Compare and contrast vector quantities (e.g., displacement, velocity, acceleration, force, linear momentum) and scalar quantities (e.g., distance, speed, energy, mass, work).

1.2 Distinguish between displacement, distance, velocity, speed, and acceleration. Solve problems involving displacement, distance, velocity, speed and constant acceleration.

1.3 Create and interpret graphs of 1-dimensional motion, such as position vs. time, distance vs. time, speed vs. time, velocity vs. time, and acceleration vs. time where acceleration is constant.

Scientific Inquiry Skills Standards that apply

High School

SIS2. Design and conduct scientific investigations.
- Employ appropriate methods for accurately and consistently
  - making observations
  - making and recording measurements at appropriate levels of precision
  - collecting data or evidence in an organized way

SIS3. Analyze and interpret results of scientific investigations.
- Use mathematical operations to analyze and interpret data results.

SIS4. Communicate and apply the results of scientific investigations.
- Explain diagrams and charts that represent relationships of variables.