

Valuation of Mortgage Backed Securities with Prepayment using BDT Model and Monte Carlo Methods

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Abstract

After the subprime crisis in 2007, mortgage backed securities are concerned more, because people fear to loss the control of the mortgage backed security's risk but still need such kind of financial instruments to help promote the asset liquidity. Thus the valuation of mortgage backed securities are increasingly popular for scholars to research.

In this article, mortgage is introduced comprehensively with its main attributes, structure and the relative risk initially. This is the basis for understanding the mortgage backed securities and its valuation. Then the mortgage backed security's mechanism, market and basic cash flow is summarized to prepare for its valuation. In the valuation part, the no- arbitrage Black-Derman-Toy (BDT) Model and PSA prepayment model are applied to simulate the interest rate binominal path and the prepayment cash flow respectively. Monte Carlo simulation method has been chosen to calculate the mortgage price for its simple implementation and high accuracy.

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Introduction

Residential mortgage, as one of the largest asset pool in the world is a permanent topic for people who need to finance houses and the rest whole market. Also, the young but well developed financial instrument Mortgage backed security (MBS) which helps in increasing the liquidity of whole asset market has always been noticed. Moreover, ever since the subprime mortgage crisis in 2007, the mortgage market had gone through from steep drop of house financing to generally recovery of selling rate. The mortgage backed security (MBS) market has undergone a comprehensive shift simultaneously. MBS is still a useful instrument for liquidity but it needs more sophisticated valuation methods to make itself not fail again. Thus this paper is going to get acknowledge of the mortgage and MBS market, mechanism and find a explainable method to price fixed rate fully amortizing mortgage MBS product.

In this paper, because of the time limitation, only the fixed rate fully amortizing mortgage will be constructed and priced. As mentioned before, such kind of mortgage's price might be influenced by interest rate and prepayment rate mainly, thus I use BDT model and PSA model to simulate the interest rate and prepayment rate respectively to help pricing the mortgage.

Mortgage and Mortgage Market

Mortgage is a loan that has some underlying assets as collateral which can be repossessed when default occurs. Over the past few decades, the residential mortgage in US has developed promptly into one of the world's largest asset classes. Simultaneously, the mortgage has experienced deep going shift during its young life.

As in the pretext, mortgage is a loan to the owner of one- to four- family who needs to buy dwelling.

The following several attributes define the mortgage comprehensively:

Lien status, original loan term:

Lien is a common concept in the mortgage loan, which means the creditor have the ownership of obligor's property during the loan period and once the obligor default, creditors have the right to deal with the mortgage property according to the law. Lien statues indicates that when an obligor default the loan's seniority to force the liquidation of the property to creditor. People will allocate loans mostly to 30 years,

while the most popular short loan term is 15 years for people who wants to amortize loan faster. 10 and 20 years loans are also available in the market.

Credit classification:

There are usually 3 credit classes for the mortgage: primes loans, subprime loans and the alternative-A loans. Prime loans are defined for obligors who have strong employment, nice credit record, and sufficient income to pay the loan. Subprime loans are for obligors who has unstable employment, blemished credit record and low level of income to pay the loan. Such obligors have higher default rate. Alternative-A loans are for those loans who are nebulous to classify.

Interest rate type:

Interest rate for loans are often divide into 2 basic categories. First one is fixed rate mortgage. In this situation, the even monthly cash flow of payment is available once the loan balance, term and fixed rate are settled. Second one is adjustable rate mortgage (ARM). As we can tell from the name, the note rate alters over the loan's life time. Whenever comes to a reset date, the cash flow will be reallocated by the rest term of the loan life, the current using rate and the to-be paid principle. The rate can be set refer to many criteria like London Interbank Offering rate (LIBOR) ,one year Constant Maturity Treasury(CMT), or the 12-month Moving Treasury Average(MTA)¹. The most common ARMs are fixed period ARM and hybrid ARM, and their peak time was from 2001 to 2007.

Amortization type:

Conventional amortization type for mortgage is fully amortization in both fixed and adjusted rate loans. Fully amortization means that obligors will pay the interest and principal simultaneously in equally increment payment term. However, some innovative amortization like IO and PO are more popular between 2001 and 2007. IO is Interest Only, such loans only needs interest to be paid in some predetermined terms of the loan's life time and delay the beginning date of principle payment; PO indicates principal only which is analogous to IO.

Except for the previous attributes that may influence mortgage loan's price, other things like credit guarantees, loan balance for the agencies and the prepayment penalty are also important in maintaining the stability of mortgage market.

¹ Frank J. Fabozzi, Anand K. Bhattacharya, and William S. Berliner (2011) "Mortgage-Backed Securities: Products, Structuring, and Analytical Techniques".

Risk factor of mortgage

Once the mortgage loan is defined with those previous attributes, the future scheduled cash flows can be calculated. However, the loan is still facing with many risk factors that may influence its payment.

Interest rate risk mostly relate to the fixed rate mortgage holders when the market rate changes. When the interest rate increases, the obligor will benefit from fixed lower rate of loan, as a zero game the creditors have to suffer the same amount of interest lose conversely. When the interest rate decreases, the situation of obligors and creditors just switch.

Since the obligors have the right to prepay the loans before its maturity, once they have the ability to do so, this will disorder the predetermined cash flow of the mortgage and influence its performance. When the market rate decreases, obligors tend to prepay and refinance the loans more and to the other hand, when market rate increases, obligors have low inclination to prepay the loan.

Credit risk is the risk that creditors may loss the principal or interest payment stemming from the obligors' fail to pay the loan. Credit risk contains default risk which is the point where obligors loss their title of the properties.

Mortgage Backed Security

As the scale of residential mortgage is so large that eliminates the creditors' asset liquidity significantly, government supported companies like Fannie Mae (FNMA) play a crucial role in promoting the loans liquidity. After lending the loans to obligors, companies put the loans in a large "pool" and pack their future cash flows together into securities backed by obligors' properties and sell those securities to investors, offering stable monthly payments with some special interest rates. This process of packing the illiquid assets to generate new securities is what we call the securitization. Through this process, we can create more financial instruments to increase the liquidity in the marketplace. Asset backed securities is the most popular product of securitization.

Asset backed securities (ABS) is a security whose income payment is backed on a pool of collateralized underlying asset. In most cases, the asset pool is composed by some small assets with low liquid.

Mortgage backed security (MBS) is a kind of ABS whose cash flow is secured by mortgage. It was one of the earliest ABS that appeared in the market. MBS's basic unit is a pool, during which there are a lot of

mortgages similar in characteristics like rate, term structure, credit quality and loan balance. After combining the previous mentioned mortgage loans, the new sizeable and homogenous component MBS will be liquid in the market.

During the whole process of MBS's securitization, we need more participators besides the investors, obligors and intermediate company such as FNMA. For the whole process including the collection of loan, the designation of the MBS pool, credit evaluation or enhancement and other supplementary links to ensure the whole process to be finished successfully.

Originator: An institution or individual who initiates the securitization. Originators first will use the fundamental asset that are proper to securitize to compose the asset pool and pass the pool of asset to the special purpose vehicle. They are lender of the mortgager and part of the primary mortgage market.

Special Purpose Vehicle (SPV): SPV is a subsidiary corporation that set up by originator of the mortgage. The SPV will buy the fundamental asset from originator by the rule of "true sale". "True sale" is the core of legal issue in securitization which ensures investors to get legal right of the receivables.

Servicer: They are the intermediary companies that help originator to collect the mortgage refund.

Trustee: Another kind of intermediate company that manage the asset pool and give the payments to investors.

Credit enhancement Provider: The entity that use methods to enhance the credit profile of the pool asset.

Credit rating agency: Agent institutions associate originators to settle the credit enhancement method and scale to offer investors accurate and affordable credit criteria.

Distributor: Corporations that sell the MBS to investors.

Investor: Individuals of company that invest on the MBS.

Main products of MBS

The process of public MBS can be similar but there are many different types of MBS created to satisfy investors with diverse necessities.

Pass through MBS is a pool of securities backed by the mortgage assets. The SPV collects the payments every month from originator and passes them pro rata to the investors after subtracting the service fee and the insurance fee. The most significant trait of pass through MBS is that mortgages of the MBS pool are similar in structure. Moreover, originators only need to transfer the original cash flow of the mortgages to investors after eliminate some fee, that is why people call it pass through MBS.

Collateralized Mortgage Obligation (CMO) is a kind of MBS constructed by relocating cash flow of mortgages in the pool according to differentiate rewards, maturity and risk. The technology used in separating cash flows is called “Tranche”. CMO offers investors more choices related to their distinct preferences like risk tolerance and investing objectives. In the market, there are many kinds of CMO designed for different requests. Investors who need to separate the prepayment risk can invest in the Planned Amortization Classes (PAC) or the Targeted Amortization Classed (TAC), while for institutions who want to release the press of supervise organizations the Very Accurately Defined Maturity (VADM) should be a good choice. One thing to notice is that CMO products do not eliminate the prepayment risk but relocate the risk into different classes to cater investors with distinct necessities.

Originators separate the principle and interest in some particular way of mortgages in a pool into two or more part to get the Stripped Mortgage-Backed Securities (SMBSs).The most common way is to strip the original mortgage in to interest only (IO) part and principle only (PO) part. Owners of IO SMBS only have right to claim interest payment and similarly owners of PO SMBS may only have the principle payment. The earnings of IO and PO are asymmetric and very sensitive to the fluctuation of interest rate. When the prepayment rate increase, IO owners’ payment will be undermined but PO investors will benefit more, and vice versa.

Risk of MBS

Same as mortgage, MBS also has to confront many risk factors and because the MBS is instruments based on mortgage loans, it may facing more than the mortgage do.

Prices of securities are exposed to fluctuation of interest rate and so does MBS, particularly for the fixed rate MBS which is included in the following valuation part. When interest rate increases, the investors of MBS will receive lower return for the unchanged interest rate, inversely, when the interest rate drops, investors will benefit from the fixed rate of loans. In order to avoid such kind of risk, we need to simulate the path of market interest rate and to give a reasonable discount rate of the loans.

Prepayment refers to the activity that borrower of mortgage loans pay more than the scheduled monthly payment amount, where the excess part will be used to pay the remaining outstanding balance principle which leads principle be paid faster than original scheduled amortization. Prepayment is one of the risk factor that will interrupt the scheduled cash flow of MBS influencing the structure and profitability.

There are three factors mainly incent obligors to prepay the loans. One is turnover of the property, if house owners need to move or sale the property or the property has been destroyed for some reasons, prepayment will occur. The second is refinancing, borrowers incline to refinance their loan if the current loan rate is low enough to reduce their monthly payment. Third factor which leads prepayment is borrower's financial situation, like people who gain credit level will give them opportunities to get lower loans to refinance with a lower cost. However, in addition to interest rate, the interest rate path is also a major impact of prepayment. In real world, when borrowers make decisions about prepayment there is always a phenomenon called Burnout effect that they do not incline to prepay even the interest rate is low than the loan's rate after a period of time.

Considering the crucial influences of prepayment activity, many scholars did a lot of research to model the prepayment cash flow. Multi-factor models are prevailing nowadays to elaborate the prepayment cash flows. However, one factor seemed to be enough for this paper as time is limited.

For the delinquent borrowers who eventually fail to resume to give the payments and result in losing the title to the property, we call this activity default. Default is severe for the loan structure since once it occurs, originators may need long time to go through all the procedures to get the obligors' properties sold which may hurt the cash flows. Also, in most cases the payments get from selling of properties may not cover the default loans otherwise property owners will sell them by themselves.

Measurement of risk in prepayment, delinquency and default:

There are already many methods to quantize the influence of risk factors to cash flow fluctuations which helps to price the MBS.

Prepayment

Many agencies and associations build their own models to simulate the speed of prepayment for MBS pricing. And here are some of them:

Federal Housing Administration (FHA)

The earliest method to calculate prepayment rate which assumed that the prepayment rate would be identical to the FHA experience rate. It is no longer used but meaningful in the prepayment history.

The conditional prepayment rate (CPR)

CPR approach is an annual rate refer to the percentage of monthly prepaid principle of the outstanding balance, it can be used as input of other methods. The advantage of this method is that it used the history actual to predict the future. Since in the real calculation people will use monthly data to get the prepayment rate, Single Monthly Rate (SMM) is the monthly rate we get and we can annualize it to get CPR by:

$$CPR = 1 - (1 - SMM)^{12} \quad (1)$$

Public Securities Association(PSA)

The PSA is known as Securities Industry and Financial Market Association (SIFMA) because of name change, but the content is unchanged.

For the PSA, it assumes that:

The first month CPR is 0.2%, then CPR increases 0.2% per month during the following 29 months to 6%.

After 30 months, CPR remains to be 6%

The benchmark denotes a “100% PSA”, or “100PSA”, if there is need to alter the prepayment speed, people can use like 150PSA to fast the prepayment rate, in which 150 refers to 1.5 times of CPR in the PSA. If the total payment month is less than 360 months, the first rate start from the (361-t whole payment months) month CPR rate.

Prospectus prepayment curve(PPC)

PPC will be used when originators need to specified the prepayment ramp based on the prospectus of products. For instance, a PPC may be designed as “6%-12%” 100CPR over 12 months, in which the

prepayment CPR evenly increased 0.5% per month from 6% to 12% during the first 12 months and stay at 12% in the rest payment life. This approach is rather flexible but not commonly used for it is so diversified to manage.

Home Equity Prepayment(HEP)

The HEP is created for prepayment rate of home equity and manufactured housing loans. However, such fashion of securities is out of favor since 2000, we only need to know about it. HEP is designed similar to PPC with a 10 months ramp.

Manufactured housing prepayment(MHP)

MHP is the curve that helps to measure the prepayment of manufactures housing securities and it is in the similar fashion of PSA. 100MHP equates the 3.6% CPR at month zero and in the next 24 months the prepayment rate increases 0.1% CPR per month evenly.

Delinquency

In the situations when an obligor do not make scheduled payment in time, the loan is said to be delinquent. Delinquency will influence the cash flow of loan payment, thus in order to record and manage it companies usually use two methods to gauge the seriousness, and they are Office of Thrift Supervision (OTS) and the Mortgage Bankers Association (MBA) respectively.

In OTS, delinquency are classified as:

Payment due day to 30 days late: Current

30-60 days late: 30 days delinquent

60-90 days late: 60 days delinquent

More than 90 days late: 90+ days delinquent

MBS method is similar to OTS but stricter than OTS and they will give very different report of delinquency.

Default

As mentioned before, once the borrower lose the title to their property, the loss caused by default should be calculated to help value the MBS. And people commonly use two methods to quantify the default. One is cumulative default rate (CDX) and other is conditional default rate (CDR).

$$CDX = \frac{\text{total default face value in he loan}}{\text{total face value of the loan}}$$

AS for CDR, it is annualized value thus we use monthly default rate (MDR) to convert:

$$MDR = \frac{\text{Default loan balacne in month } t}{\text{Begining balance for month } t - \text{Scheduled principle payment for month } t}$$

Also,

$$CDR_i = 1 - (1 - MDR)^{12} \quad (2)$$

Basic Cash flow allocation of MBS:

When we talk about pricing the security, the essential idea is to use a proper discount rate to get the present value of the security's all future cash flow. And take a Level payment fixed rate mortgage which requires same payment each period as an example. Let R be the obligor's mortgage rate in annual, MB_n be the remaining balance at n of mortgage backed security, and N be the total month length of mortgage ($0 \leq n \leq N$), then the monthly payment is calculated as:

$$MP = MB_0 * \frac{R/12(1 + R)/12^N}{(1 + R/12)^N - 1} \quad (3)$$

And the remaining balance is:

$$MB_n = MB_0 * \frac{(1 + R)/12^N - (1 + R)/12^n}{(1 + R/12)^N - 1} \quad (4)$$

From which we can get the scheduled interest and principle to be paid of each month respectively, let I_n and P_n to be monthly interest and principle payment:

$$P_n = MB_{n-1} - MB_n \quad (5)$$

$$= MB_0 * \frac{R/12(1 + R)/12^{n-1}}{(1 + R/12)^N - 1} \quad (6)$$

And,

$$I_n = MB_{n-1} * R/12 \quad (7)$$

$$= MB_0 * \frac{R/12[(1 + R)/12^N - [(1 + R)/12^{n-1}]]}{(1 + R/12)^N - 1} \quad (8)$$

When we take servicing fee into account, the relationship between interest rate and servicing fee should be:

$$\text{servicing fee} = I_i * \left(1 - \frac{S}{12}\right)$$

Where I_i =the monthly interest payment

S = annual servicing fee rate

Pricing the Mortgage Backed Security

As mentioned before, the valuation of mortgage backed security is crucial in stabilizing the market. Interest rate and prepayment are considered to be the two main factors that will impact MBS's cash flow, thus we need to find some models that simulate the cash flow's fluctuation under these two factors.

Interest rate Model

In chapter two, we find out that the price of MBS price is exposed to interest rate risk and prepayment risk depend not only on the interest rate but also on its path. Thus we need to find a model to simulate the future interest rate path. One thing before building the interest rate path, we need to get knowledge to the term structure of interest rate. It is the relationship between the

theoretical spot rate and maturity. With the help of term structure model, we can simulate the interest rate path to price the MBS.

Black-Derman-Toy Model (BDT)

BDT model is a one factor short rate no arbitrage model initially developed by Fischer Black, Emanuel Derman, and Bill Toy in 1990. The model proved to be prevailing in valuing interest rate derivative for its proper assumptions and simple computing process. Contrast to the equilibrium model like Vasicek Model, no arbitrage model like BDT model is used to generate the term structure with a given series of long term spot rate with different maturities.

The following are Assumptions for BDT

1. Short rate is lognormal and the volatility only depends on time.
2. Expected returns on all securities over one period are equal.
3. Changes in all bond yields are perfectly correlated.
4. No tax or transaction cost.

Although the model is originally developed algorithmically, and described by a discrete time binomial lattice, when we take the time limit to zero, the implied stochastic differential equation for BDT should be like:

$$d\ln r(t) = \left[\theta(t) - \frac{\partial \sigma(t)/\partial t}{\sigma(t)} \ln(r) \right] dt + \sigma(t) dW \quad (9)$$

Firstly, the lognormal distribution assumption of interest rate indicates that r is non negative. And from the Equation (9) we can figure that $\frac{\partial \sigma(t)/\partial t}{\sigma(t)}$ part means the speed of interest rate's reversion, and $\theta(t)$ divided by the speed of the mean reversion is a time-dependent mean-reversion level.

Modeling the interest rate process with binomial tree

In order to describe the interest rate process, we use the term structure of Treasury bond as an example to derive the interest rate binomial tree, and the volatilities are settles randomly.

Table.1 Black-Derman-Toy Model		
maturity	Zero-coupon rates (%)	Zero-coupon Volatilities (%)
3 months	0.020	0.04
6 months	0.07	0.035
12 months	0.14	0.05
2 years	0.53	0.03
3 years	0.96	0.04
5 years	1.63	0.045

For any node, suppose the bond will mature in time T with face value 1 and the spot rate is R , also assume r to be the spot rate with any maturity time t between time 0 and T , Δt be the time between time t and T . Under the assumption of BTD, the forward interest rate between time t and T will go up or down:

$$R_u = R * e^{mt+\sigma\sqrt{t}} \quad (10)$$

$$R_d = R * e^{mt-\sigma\sqrt{t}} \quad (11)$$

In which:

m refers to the drift during each time period t

σ refers to the volatility of each time period T

m, σ both might alter in different period of time

Here is a sample of the BDT model:

When we use the R to calculate the bond price, the price should be:

$$P_1 = \frac{1}{(1 + RT)^T} \quad (12)$$

Similarly, when we utilize the spot rate r and the forward rate R_u and R_d , the bond price will be:

$$P_2 = \frac{1}{(1 + rt)^t} * \frac{1}{2} \left(\frac{1}{(1 + R_u \Delta t)^{\Delta t}} + \frac{1}{(1 + R_d \Delta t)^{\Delta t}} \right) \quad (13)$$

Under the no-arbitrage pricing theory, P_1 and P_2 should be same, i.e.:

$$\frac{1}{(1+RT)^T} = \frac{1}{(1+rt)^t} * \frac{1}{2} \left(\frac{1}{(1+R_u \Delta t)^{\Delta t}} + \frac{1}{(1+R_d \Delta t)^{\Delta t}} \right) \quad (14)$$

In equation (14), the only unknown parameter is m which is easy to calculate, take the Treasury bond's 3 months rate and 6 months rate as an example, the T is 0.5, t is 0.25 and R is 0.07%, r is 0.02% respectively. Thus,

$$P_1 = \frac{1}{(1 + 0.07\%/2)^{0.5}} \quad (15)$$

$$P_2 = \frac{\left(\frac{1}{(1 + 0.02 * e^{0.25m + 0.035\% * \sqrt{0.25}})^{0.25}} + \frac{1}{(1 + 0.02 * e^{0.25m - 0.035\% * \sqrt{0.25}})^{0.25}} \right)}{2 * (1 + 0.02\%/4)^{0.25}} \quad (16)$$

Also,

$$P_1 = P_2 \quad (17)$$

With this equation, we can get:

$$R_u = 0.020008 \quad (18)$$

$$R_d = 0.019992 \quad (19)$$

When we apply this to all of the binominal tree, the forward rate between each node is shown in following table:

3months	6months	12months	2years	3years	5years
0.02	0.020008	0.020018	0.020038	0.02005	0.020073
0	0.019992	0.020002	0.020022	0.020034	0.020057
0	0	0.019982	0.020002	0.020014	0.020037
0	0	0	0.019962	0.019974	0.019997
0	0	0	0	0.01995	0.019973
0	0	0	0	0	0.019928

The original aiming of this model is mean reverting and the volatility is variable under the assumption, however once the term structure of volatility is set to be satisfied, the model will become mean fleeing conversely. This paradox lead practitioners of the model use constant volatility and that will be used in the pricing part as well.

Prepayment model

PSA model

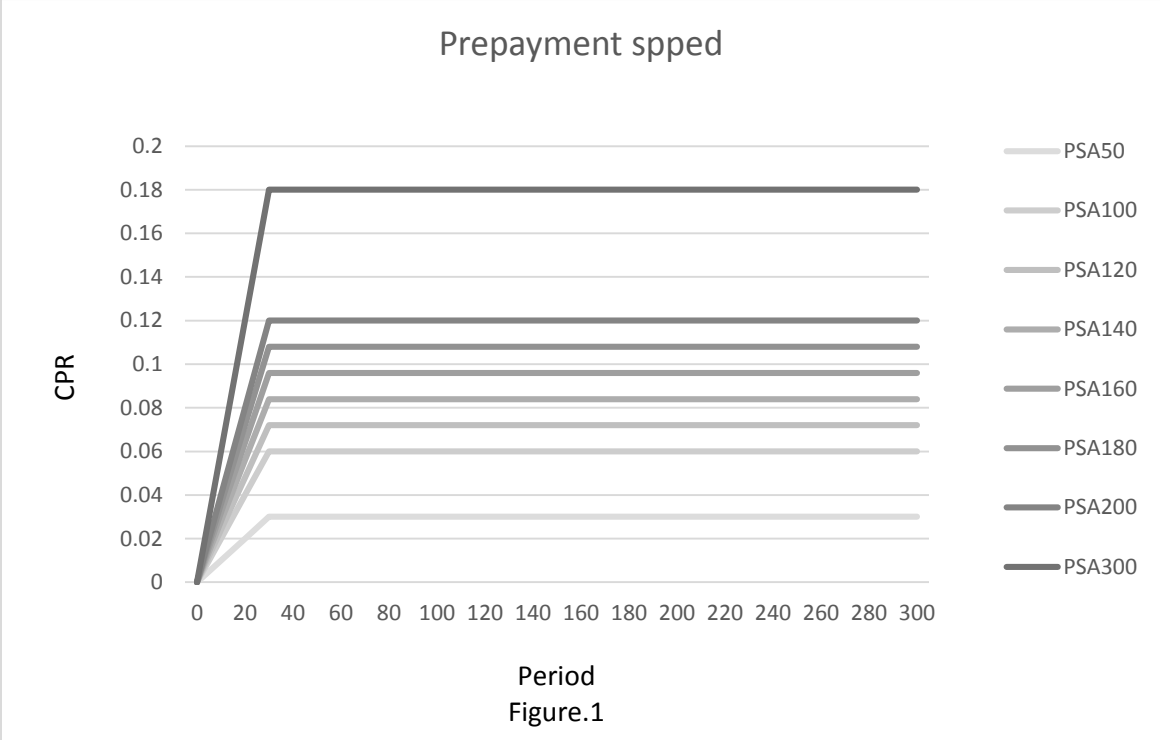
As mentioned before, prepayment will influence the cash flow of MBS significantly as it is a crucial part in pricing. PSA will be the model used to calculate prepayment cash flow in the paper because the model fits the reality better.

In PSA100 model, we assume that the prepayment rate starts from 0.2%, increase evenly during the next 29 months to 6% and remain constant at 6% in the rest of loan life, which can be summarized as:

$$CPR = t * 6\% / 30, \quad t \leq 30$$

$$CPR = 6\%, \quad t > 30$$

The 100 in PSA100 indicates a multiplier of the loan prepayment speed. If it is a PSA200, the CPR need to time 2 to fit the prepayment speed. Figure.1 indicate the relationship between different multiplier of PSA and the relative prepayment rate.



Utilize Monte Carlo Simulation to calculate the MBS price

After the cash flow can be simulated by the previous two models, we need to find an effective way to price the MBS and Monte Carlo is a good choice for its simplicity and accuracy. The following steps are a guideline for pricing MBS with Monte Carlo method.

Step one: Use BDT model to get the interest rate binomial tree. Then transfer interest rate tree to the discount factors tree for simpler calculation.

When structuring the interest rate binomial tree, people commonly use Treasury bond rate curve or LIBOR rate curve. The choice depend on many factors like investors' objective or the prepayment model. However, LIBOR performs better when portfolio managers concentrate more on the spread they can earn comparing to the funding cost. Thus, LIBOR rate is the one used in the model.

02/2014	03/2014	04/2014	05/2014	06/2014	07/2014	08/2014	09/2014	10/2014	11/2014
0.4359	0.437428	0.438962	0.440501	0.442046	0.443595	0.445151	0.446711	0.448278	0.449849
0	0.434377	0.4359	0.437428	0.438962	0.440501	0.442046	0.443595	0.445151	0.446711
0	0	0.432859	0.434377	0.4359	0.437428	0.438962	0.440501	0.442046	0.443595
0	0	0	0.431347	0.432859	0.434377	0.4359	0.437428	0.438962	0.440501
0	0	0	0	0.42984	0.431347	0.432859	0.434377	0.4359	0.437428
0	0	0	0	0	0.428338	0.42984	0.431347	0.432859	0.434377
0	0	0	0	0	0	0.426842	0.428338	0.42984	0.431347
0	0	0	0	0	0	0	0.42535	0.426842	0.428338
0	0	0	0	0	0	0	0	0.423864	0.42535
0	0	0	0	0	0	0	0	0	0.422383

Table.3 is the lattice for the simulated binomial tree generated by the LIBOR term structure, all the data are annual forward rate between each node in the tree and the time between each node is month in the model. In fact, the matrix dimension is 360 by 360 for we used the longest period a loan might have and it is easy to pick any period of binomial tree according to different MBS life time.

After the rate lattice is generated, it is more convenient in the discount step if we change the rate into discount factors lattice. And the way to get discount factor for each period is as:

$$d_{i,j} = \frac{1}{(1 + r_{i,j}/12)} \quad (20)$$

Then the discount factors are presented in table.4 with also part of the whole lattice.

1	0.999637	0.999636	0.999634	0.999633	0.999632
2	0	0.999638	0.999637	0.999636	0.999634
3	0	0	0.999639	0.999638	0.999637
4	0	0	0	0.999641	0.999639
5	0	0	0	0	0.999642

Step two: Project theoretical monthly payment including prepayment based on PSA model

As we know, obligors' monthly payment is composed by scheduled principle, interest rate and prepayment part. The scheduled principle and interest rate has been settled at the beginning, thus the unknown prepayment is significant in the valuing process. In this paper, PSA100 model is used to simulate the prepayment cash flow.

The assumptions of constructing the MBS are presented in Table 5. The original amount of mortgage is settles as \$400,000, the loan life is 60 months and weighted average cost is 4.023% with 0.523% of servicing fee. Also, the initial SMM is same as the 100PSA threshold rate.

Original Amount (MM)	\$400,000	WAC	4.023%
WAL (yrs.)	2.4	Servicing Fee	0.523%
WAM	60	Initial SMM	0.200%
Monthly Coupon	0.34%		

In order to get the simulated MBS's cash flow, a table contains each component of the monthly cash flow can be utilized (table.6).

Table.6 Simulated cash flow of MBS

Month	Begin Principal	Scheduled Monthly Payment	Interest Payment	Scheduled Principal Payment	Scheduled End Principal	Prepayment
1	\$400,000	\$1,915	\$1,341	\$574	\$399,426	\$67
2	\$399,359	\$1,915	\$1,339	\$576	\$398,783	\$133
3	\$398,650	\$1,915	\$1,336	\$578	\$398,072	\$199
4	\$397,872	\$1,915	\$1,334	\$581	\$397,291	\$265
5	\$397,026	\$1,915	\$1,331	\$584	\$396,442	\$331
6	\$396,111	\$1,915	\$1,328	\$587	\$395,524	\$396
7	\$395,128	\$1,915	\$1,325	\$590	\$394,538	\$461
8	\$394,077	\$1,915	\$1,321	\$594	\$393,483	\$525
9	\$392,957	\$1,915	\$1,317	\$598	\$392,360	\$589

10	\$391,770	\$1,915	\$1,313	\$602	\$391,169	\$653
Month	Actual	Actual	Cumulative	Cumulative	Cash Flow	WAL
	Principal	End	Interest	Principal	after	
	Payment	Principal	Paid	Paid	Servicing	
					Fees	
1	\$641	\$399,359	\$1,341	\$641	\$1,981	0.000133
2	\$709	\$398,650	\$2,680	\$1,350	\$2,048	0.000296
3	\$778	\$397,872	\$4,016	\$2,128	\$2,114	0.000486
4	\$846	\$397,026	\$5,350	\$2,974	\$2,180	0.000705
5	\$915	\$396,111	\$6,681	\$3,889	\$2,245	0.000953
6	\$983	\$395,128	\$8,009	\$4,872	\$2,311	0.001229
7	\$1,051	\$394,077	\$9,334	\$5,923	\$2,375	0.001533
8	\$1,119	\$392,957	\$10,655	\$7,043	\$2,440	0.001865
9	\$1,187	\$391,770	\$11,972	\$8,230	\$2,504	0.002226
10	\$1,255	\$390,516	\$13,286	\$9,484	\$2,567	0.002614

With the help of table. 6, we can depict whole simulated cash flow process of the MBS by multiplying every month's theoretical principle remained with the assumed prepayment rate of PSA model.

Step three: Get the present value of the cash flows of loan under each interest rate path.

As we already get the discount factor paths in step one, the present value of each path will be easily calculated using following equation:

$$PV_t(n) = [d_1(n) * d_2(n) * d_3(n) \dots * d_t(n)] * CF_t(n)$$

Where

$PV_t(n)$ = the present value of t month cash flow from n path.

$CF_t(n)$ = the cash flow of t month in path n

$d_t(n)$ =the discount factor of month t in path n

In this step, the $d_t(n)$ s are the monthly discounted factors we get in step one which contains the simulation of interest path based on BDT model.

Step four: Compute the value of MBS.

Once we have the present value of each path, we can get the theoretical value of Mortgage Backed Security by:

$$\textit{Theoretical value} = \frac{PV(\textit{path1}) + PV(\textit{path2}) + PV(\textit{path3}) \dots + PV(\textit{pathN})}{N}$$

Conclusion

AS we can get the fixed rate fully amortizing mortgage backed security price as 423147.7. This price equate 105.78 when transfer the MBS scale to a 100 face value one. Compare to the real price online, although not 5 years MBS price are available, it is not hard to compare with the longer loan period with same coupon rate that the price is a little higher than is should be in real case. For any kind of security, the pricing problem will finally generalized to the simulation of its cash flow and so does for fixed rate fully amortized MBS. In this paper, interest rate process and prepayment can explain most of the MBS's cash flow oscillation, however, the default rate it also an important factor which remain to be evaluate in further discussion. Also, recent papers are concerning more about modeling the prepayment process with more complicated methods. It is also a good research direction to develop more sophisticated prepayment models.

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