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WEB BASED SPRING ELEMENT LEARNING MODULE FOR THE FINITE ELEMENT METHOD

An Interactive Qualifying Project Report

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1. Computers
2. The Internet
3. Learning

Abstract

A web based finite element learning module for the one-dimensional spring element has been developed. This paper will provide an introduction to the finite element method and review the format of the module to demonstrate the development of the one-dimensional spring element. This module has been designed to be integrated into the existing Finite Element Method Universal Resource (FEMur) web site and can be accessed at http://femur.wpi.edu/learning-modules/.

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1 Introduction

1.1 Overview of Finite Element Method

The finite element method (FEM) is a numerical method for solving a wide variety of complex engineering problems. The method was originally developed in 1956 to study stresses in complex aircraft frame structures. It has since been applied to solve problems in vibration analysis, heat transfer, fluid flow, mass transport, electromagnetic potential, etc.

Often times engineers are called upon to solve complicated problems which could include determining; the deflections of machine elements with complicated geometry, vibrations in machine bases, the stress in an aircraft wing, the flow of fluid through an arbitrary shape, etc. These types of problems require the solution of ordinary or partial differential equations and most often do not have analytical solutions. The finite element method is a numerical approach in which a complicated problem can be broken down into smaller, more manageable parts that have simpler solutions. The problem is then reconstructed with the parts to determine the overall effects to the system. The advantage to this approach is the formulation of the problem results in a system of linear algebraic equations which can be readily solved with the computer. Although the finite element method is a numerical approximation technique, the accuracy can be improved by breaking the system down into more parts. The fact that there are more pieces to solve is of little consequence as the computer is doing the work. Modern FEM software is becoming easy to use and can produce solutions to complex problems very quickly.

The ability to solve complicated problems by breaking it into smaller parts makes the finite element method diverse and flexible. Although originally developed for structural analysis it has been recognized that this problem solving technique can be applied across a wide variety of engineering disciplines and applications. For that reason both the finite element method and commercial FEM software continues to be developed so it can be applied to solve a wide variety of complex engineering problems. FEM is a powerful tool and is available to the engineering community.

1.2 Review of FEM Resources on the Web

There are many resources and types of information related to FEM available on the web. The information comes in different formats such as html based, java applets, pdf files, and PowerPoint presentations. This section will review some web sites that provide instructional information related to the development of the spring element.

1.2.1 Southern Illinois University

Professor James N. Craddock teaches a graduate Finite Element Analysis course using the internet to distribute PowerPoint presentations[1] of his class lectures at Southern Illinois University. The lectures can be downloaded from http://civil.engr.siu.edu/CE551 /download.htm. Several chapters of PowerPoint presentations cover topics such as beam stiffness, plane stress/strain equations, linear strain triangular elements among others. Chapter 2 presentation deals with the stiffness method and uses the one-dimensional spring element as an example. The presentation is useful at reinforcing the understanding of spring elements as he demonstrates the stiffness method using two and three spring assemblages.

1.2.2 University of Buffalo

Professor Abani K. Patra has developed a web site that delivers an html based introduction to Finite Element Method[2] in conjunction with an undergrad CAD Applications course. There are several sections which cover various topics concerning the finite element method. In the basic ideas section is a sub-section titled "springs and assemblages of springs" which is used to expand upon the basic ideas of FEM. Additional sections cover application of loads and constraints, trusses and transformations, the energy approach, structure types and choice of elements among others. The page can be found at http://www.eng.buffalo.edu/~abani/ fem/fem.html.

1.2.3 University of Dayton

Professor David H. Myszka teaches an undergrad Applied Finite Element Analysis course[3]. The web page for this class includes a section which covers various topics in pdf format related to the course. One of these topics titled "Notes on Spring Element Theory" gives a review of the one-dimensional spring element with an example of an assemblage of springs. The page can be found at http://www.engr.udayton.edu/faculty/dmyszka/ WebPages/mct446/mct446.htm

1.2.4 University of Cincinnati

Dr. Yijun Liu teaches Introduction to Finite Element Method – I and II. The class web page http://urbana.mie.uc.edu/CAELab.htm provides information related to the conduct of the course[4]. Lecture notes are available for download in pdf file format. In addition is a showcase section which has examples of "finite element analysis in action". Various projects displaying actual finite element analysis using FEA software are presented.

1.2.5 F.E.A. Resources on the Internet

This web site is a list of links to other FEA/FEM resources on the internet[5]. The resources are grouped onto the following four different sections:

- FEA/FEM at Some Government and Non-Profit Sites
- FEA/FEM at Some University and Academic Sites
- FEA/FEM at Some Commercial Sites
- MetaSites with Multiple Links to FEA

There are literally hundreds of links to various web sites that provide a wide range of information related to FEA/FEM.

1.2.6 FEA Information

The title of this site is a good description of what you will find there. It has a large amount of general information related to finite element analysis. The mission of this site is "to explain the fundamental concepts with as little formulae as possible"[6]. A good understanding of FEA and some important aspects of the method can be gained by reading the information at this site. It is located at http://www.dermotmonaghan.com.

1.3 Finite Element Method Universal Resource

The Finite Element Universal Resource (FEMur) web site was established and is maintained by faculty and students of Worcester Polytechnic Institute[7] since 1996. It is a resource which can be used by anyone interested in the finite element method. The site has sections which provide learning modules, computer assisted learning, and finite element resources among others. The FEMur-LAM section of the site provides learning modules for industrial applications, review of linear algebra, the basic finite element equation, and stress analysis. The learning modules available in the stress analysis section cover the development of the truss, torsional, and beam elements. The learning modules provide element definitions, assumptions made for each element, sign convention used, element stiffness matrix formulation, solution characteristics, element rigid body modes, and an example problem. The spring element module developed in this project will be integrated into the stress analysis section. A more detailed review of the content and format of the FEMur site is available in the article titled "Learning Modules for Finite Element Method on World Wide Web" found in the ASEE Computers in Education Journal[8]. The web site is located at http://femur.wpi.edu.

1.4 Goal and Objectives

The goal of this project is to develop a web based learning module for the one-dimensional spring element that can be used by the world-wide engineering community, practitioners, and students. The objectives of this project are the following:

1. Provide an introduction to finite elements.
2. Know the physical behavior and limitations of the spring element.
3. Be able to solve simple problems using spring elements.
4. Be able to determine when and where a spring element can be applied.

2 Spring Element Module

An overview of the spring element module will be given in this chapter. The spring element module was developed using Microsoft Front Page 2002[9]. Each component will be discussed and some features illustrated to provide a general understanding of the layout of the module. The spring element module can be found on the World Wide Web at http://femur.wpi.edu/learning-modules/.

2.1 Front Page

The front page of the spring element learning module is shown in Figure 2-1. The spring element is defined and an illustration is used to show what the element looks like. Below is a table of hyperlinks in which sub-modules can be accessed. Each sub-module shown in Figure 2-1 will be discussed in that order.

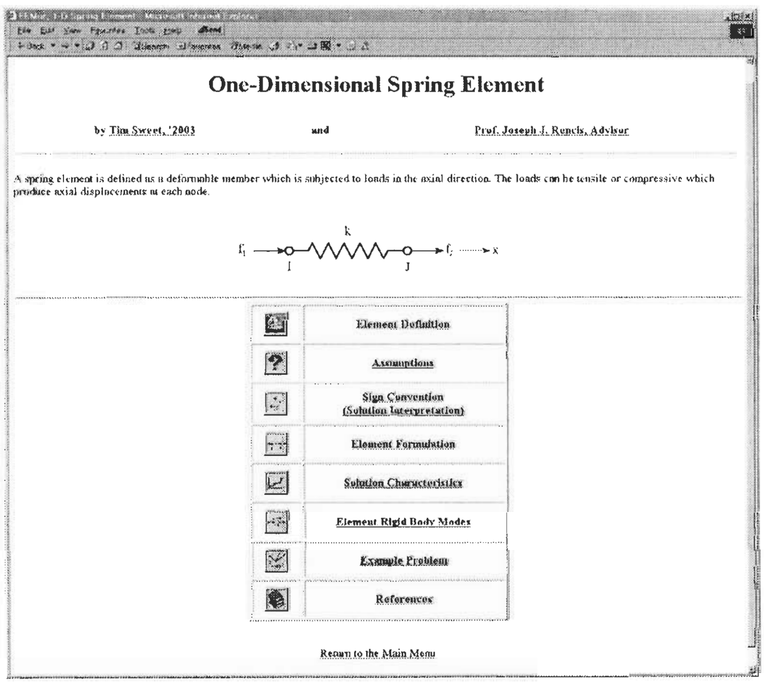


Figure 2-1: Spring Element Front Page

2.2 Element Definition Sub-module

The element definition is provided for reference and the important characteristics of the spring element are organized into a table of hyperlinks. Figure 2-2 shows a portion of the element characteristics. The table provides a column of links to access a detailed definition, a graphic illustration, and a description of each characteristic. The following characteristics are described: physical discipline, element dimensionality, geometric shape, spring law, types of degrees of freedom per node, and method of element formulation.

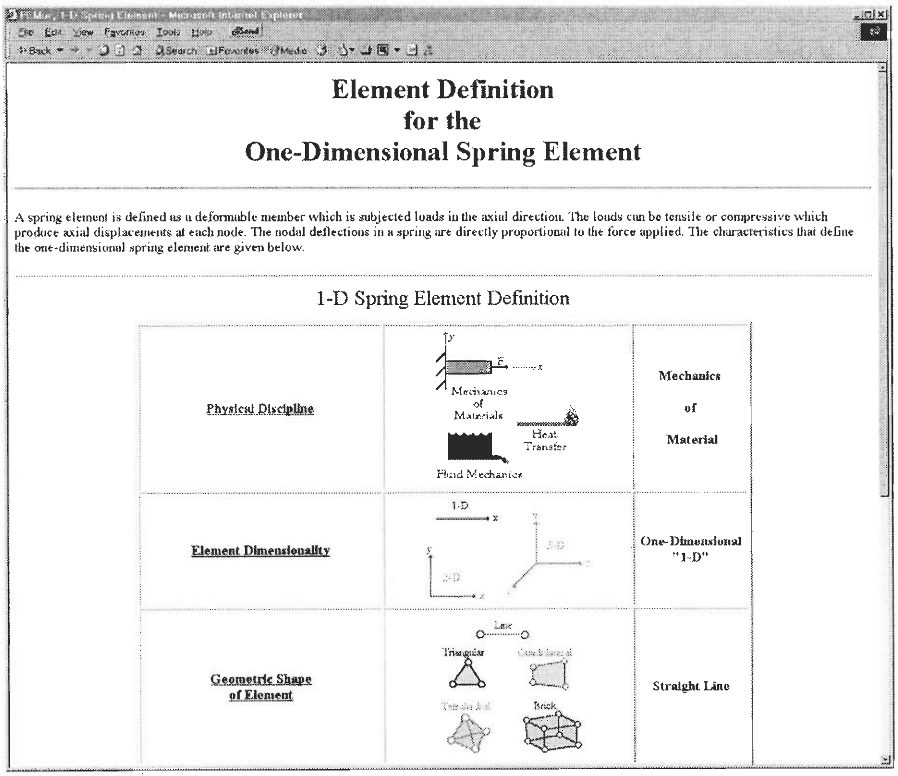


Figure 2-2: Element Definition Page

2.3 Assumptions Sub-Module

The assumptions associated with the spring element formulation are listed in table form as shown in Figure 2-3. The left column has links that describe the assumptions. As a link is selected the description will open up in the bottom frame of the page. The following assumptions are described: spring deformation, spring stiffness, force direction, and weightless member.

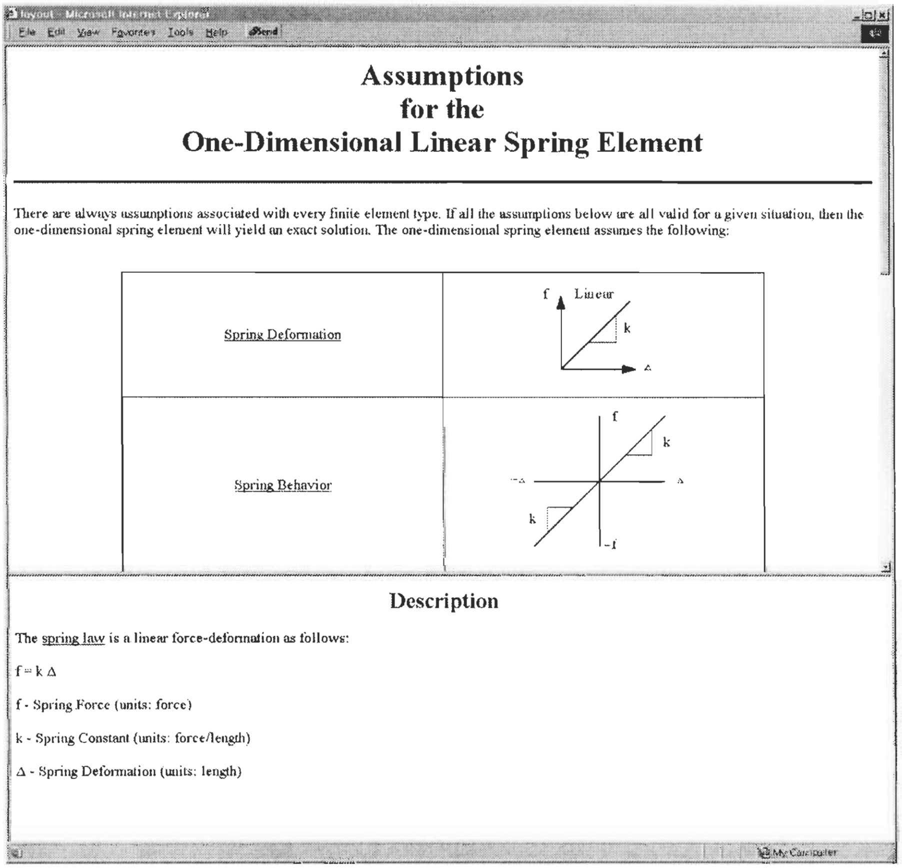
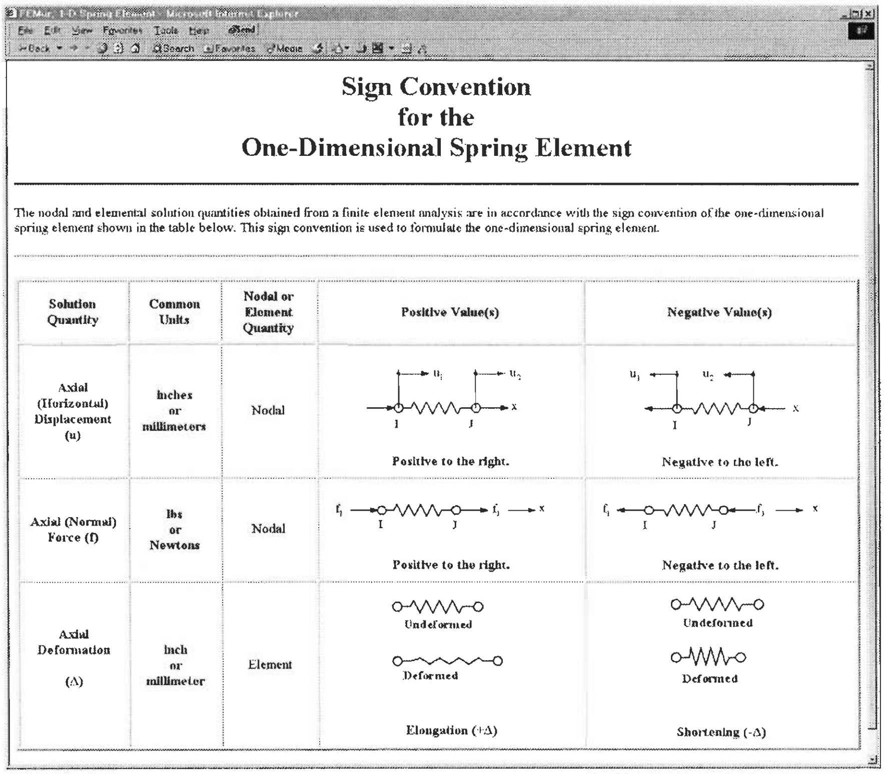


Figure 2-3: Assumptions Sub-module Page

2.4 Sign Convention Sub-Module

This page has a table that lists the sign conventions for each solution quantity, its common units, and type of quantity. The table is shown in Figure 2-4. Conventions for axial displacement, axial force, and axial deformation are addressed.



Sign Convention
for the
One-Dimensional Spring Element

The nodal and elemental solution quantities obtained from a finite element analysis are in accordance with the sign convention of the one-dimensional spring element shown in the table below. This sign convention is used to formulate the one-dimensional spring element.

Solution Quantity	Common Units	Nodal or Element Quantity	Positive Value(s)	Negative Value(s)
Axial (Horizontal) Displacement (u)	inches or millimeters	Nodal	u_1 u_2 I J x Positive to the right.	u_1 u_2 I J x Negative to the left.
Axial (Normal) Force (f)	lbs or Newtons	Nodal	f_i I J f_j x Positive to the right.	f_i I J f_j x Negative to the left.
Axial Deformation (Δ)	inch or millimeter	Element	Undeformed Deformed Elongation (+Δ)	Undeformed Deformed Shortening (-Δ)

Figure 2-4: Sign Convention Table

2.5 Element Formulation Sub-Module

This module is used to formulate the stiffness matrix for the one dimensional spring element. It begins by listing the conditions required to determine the stiffness matrix as shown in Figure 2-5. Some preliminary definitions follow and reference links are available to access previous descriptions of element definition, assumptions, sign convention, and degrees of freedom. Next is a step by step process using descriptions and illustrations to formulate the element stiffness matrix using the direct method. Figure 2-6 shows some of the steps. A list of comments which will help understand the process is provided after the stiffness matrix has been formulated.

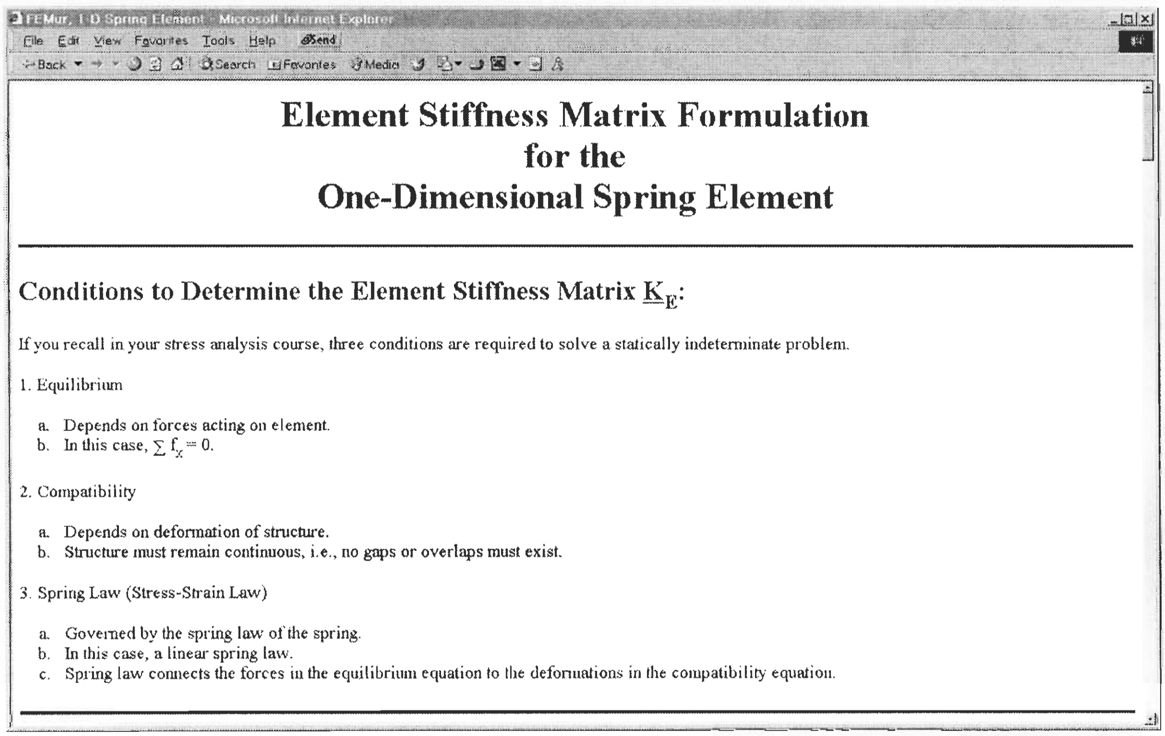


Figure 2-5: Element Formulation Sub-Module

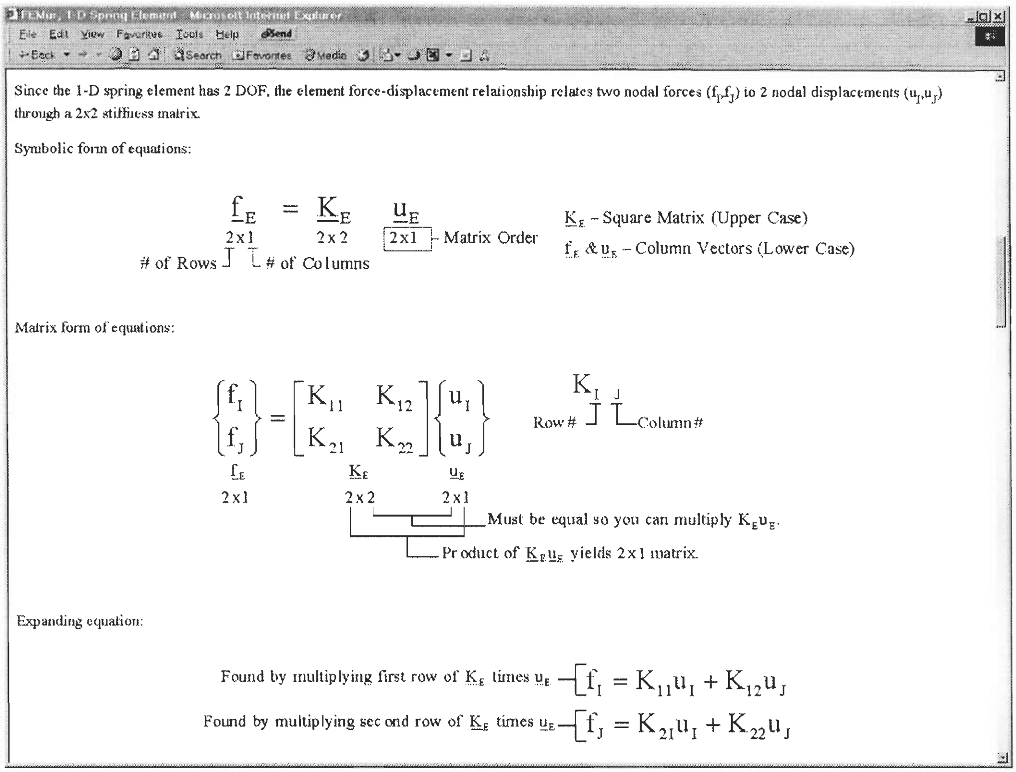


Figure 2-6: Stiffness Matrix Formulation Steps

2.6 Solution Characteristics Sub-Module

The solution characteristics associated with the spring element are used to interpret the results of the FEM model. There are two tables in this module. One table lists the characteristics at the element level and the other table lists the characteristics at the assemblage level. A portion of each table is shown in Figures 2-7 and 2-8.

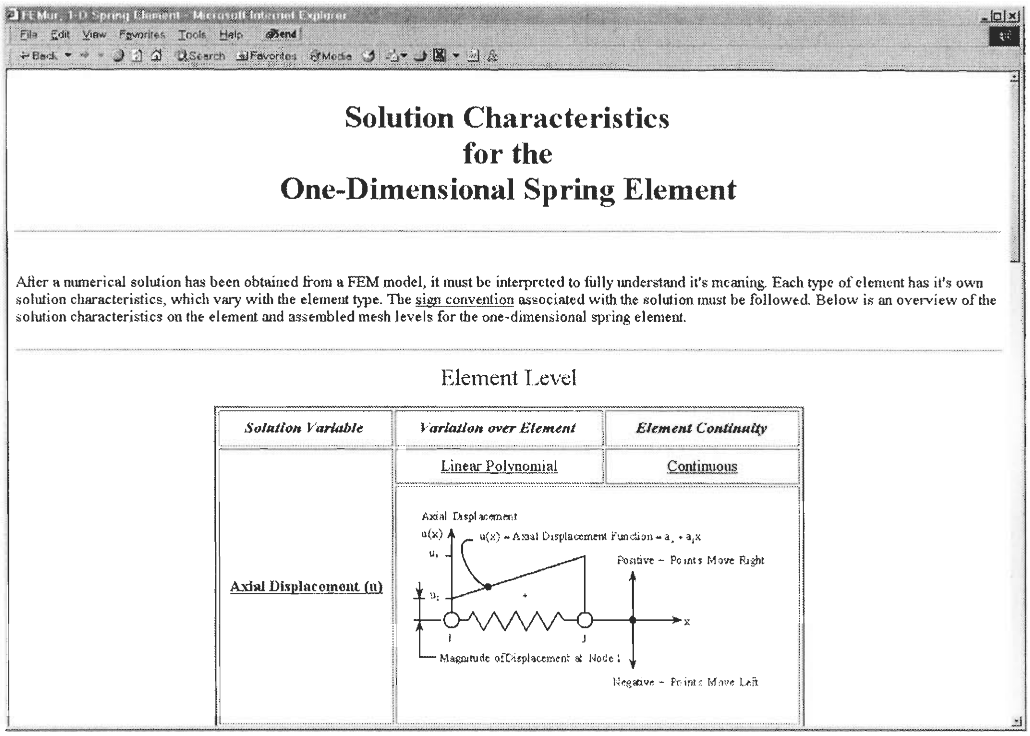


Figure 2-7: Element Level Solution Characteristics

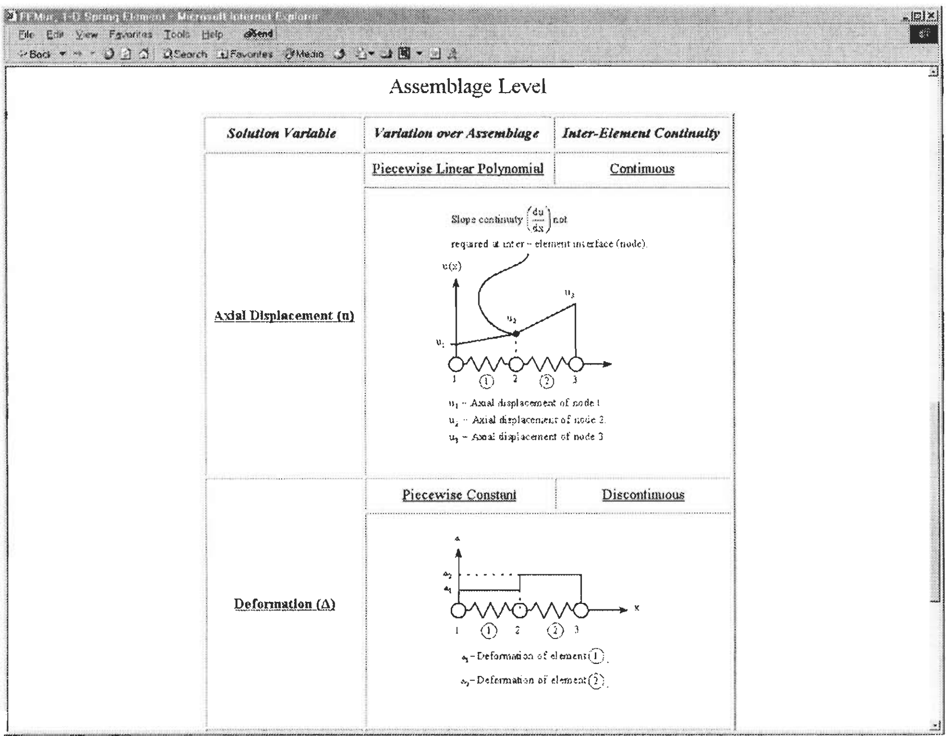
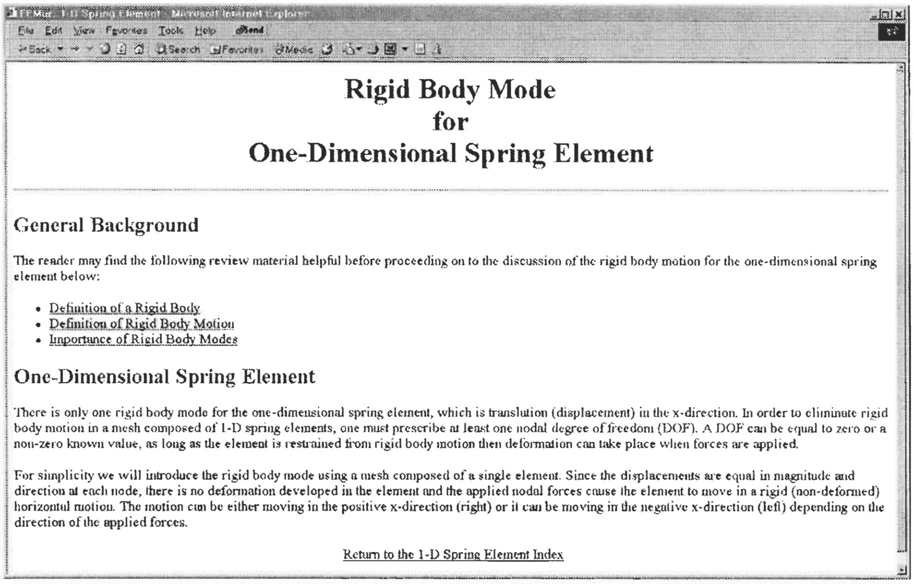


Figure 2-8: Assemblage Level Solution Characteristics

2.7 Element Rigid Body Mode Sub-Module

Rigid body modes need to be considered to obtain a unique FEM solution. This page provides links to access a general background for a rigid body, rigid body motion, and the importance of rigid body motion. The rigid body modes of the spring element are described after the background. Figure 9 shows the rigid body mode page.



Rigid Body Mode for One-Dimensional Spring Element

General Background

The reader may find the following review material helpful before proceeding on to the discussion of the rigid body motion for the one-dimensional spring element below:

- Definition of a Rigid Body
- Definition of Rigid Body Motion
- Importance of Rigid Body Modes

One-Dimensional Spring Element

There is only one rigid body mode for the one-dimensional spring element, which is translation (displacement) in the x-direction. In order to eliminate rigid body motion in a mesh composed of 1-D spring elements, one must prescribe at least one nodal degree of freedom (DOF). A DOF can be equal to zero or a non-zero known value, as long as the element is restrained from rigid body motion then deformation can take place when forces are applied.

For simplicity we will introduce the rigid body mode using a mesh composed of a single element. Since the displacements are equal in magnitude and direction at each node, there is no deformation developed in the element and the applied nodal forces cause the element to move in a rigid (non-deformed) horizontal motion. The motion can be either moving in the positive x-direction (right) or it can be moving in the negative x-direction (left) depending on the direction of the applied forces.

Return to the 1-D Spring Element Index

Figure 2-9: Rigid Body Mode Page

2.8 Example Problem Sub-Module

A simple example problem is modeled and solved to demonstrate the finite element method. The problem is solved in the following four steps: nodal displacements, reactions at nodes, element forces, and element elongation. There is a detailed explanation with illustrations to assist in understanding for each step. Figure 2-10 shows a statically indeterminate example problem of three springs in series. Spring rates and initial force conditions are given. Determining nodal displacements is step one and is shown in Figure 2-11. The left hand column of each step has three frames. The left-hand top frame has links that take the user through sub-steps of the problem solution. The other two frames have links for reference while working through the solution. The middle frame is linked to the full problem statement. The left-hand bottom frame is linked to the problem mesh.

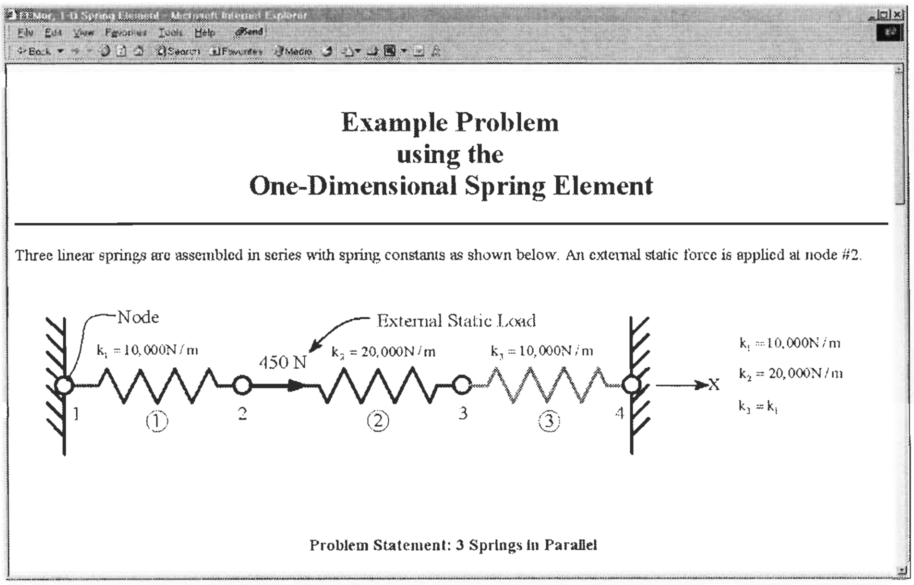


Figure 2-10: Example Problem

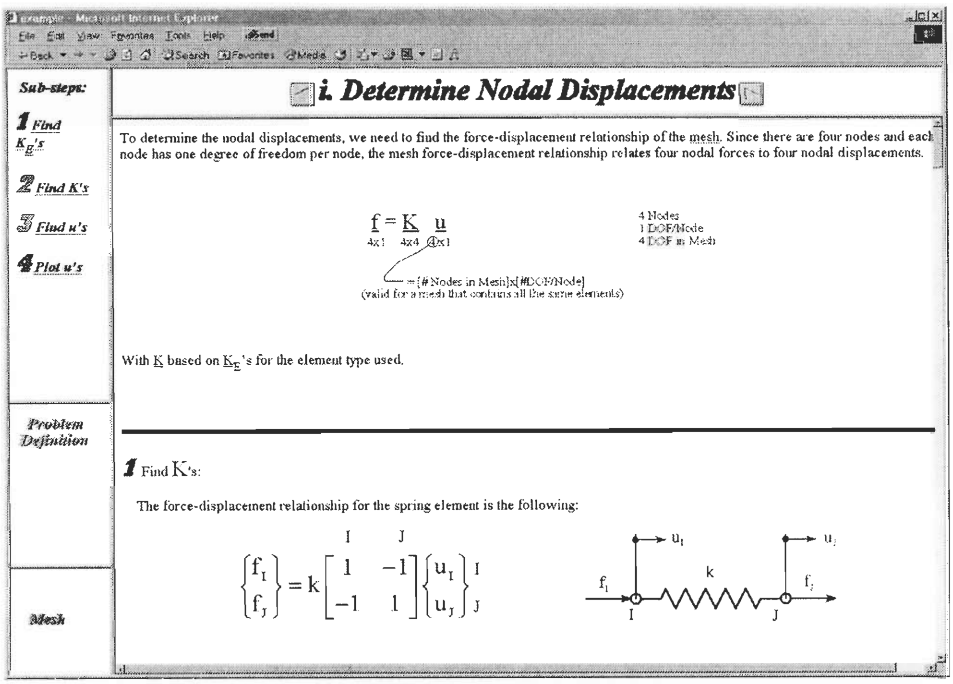
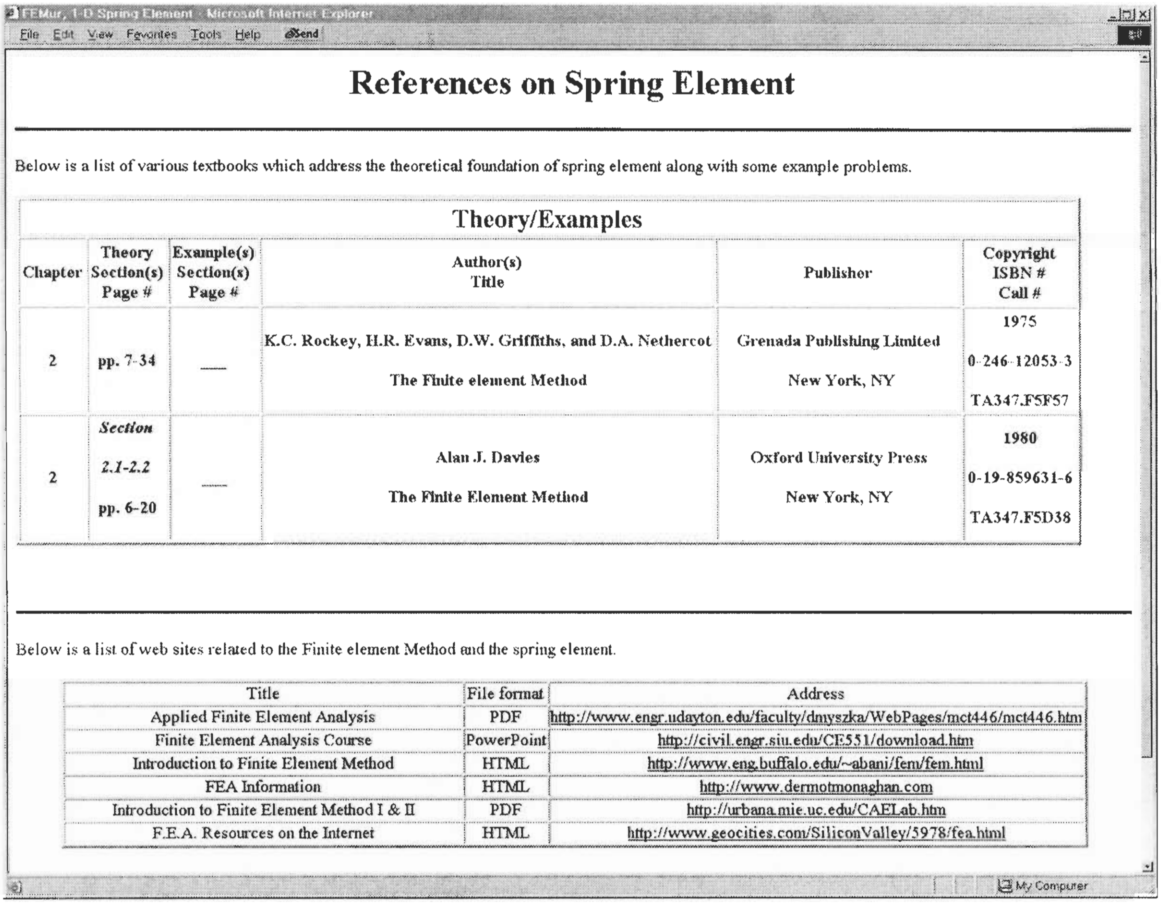


Figure 2-11: Determine Nodal Displacements

2.9 Reference Sub-Module

Figure 12 shows the reference table page which consists of two reference tables. One table provides a list of books that cover the theory of the spring element and has some examples. The other table provides a list of links which provide additional information related to the spring element and FEM in general.



FEMur, 1-D Spring Element - Microsoft Internet Explorer

File Edit View Favorites Tools Help Send

References on Spring Element

Below is a list of various textbooks which address the theoretical foundation of spring element along with some example problems.

Theory/Examples					
Chapter	Theory Section(s) Page #	Example(s) Section(s) Page #	Author(s) Title	Publishor	Copyright ISBN # Call #
2	pp. 7-34	—	K.C. Rockey, H.R. Evans, D.W. Griffiths, and D.A. Nethercot The Finite element Method	Grenada Publishing Limited New York, NY	1975 0-246-12053-3 TA347.F5F57
2	*Section* *2.1-2.2* pp. 6-20	—	Alan J. Davies The Finite Element Method	Oxford University Press New York, NY	1980 0-19-859631-6 TA347.F5D38

Below is a list of web sites related to the Finite element Method and the spring element.

Title	File format	Address
Applied Finite Element Analysis	PDF	http://www.engr.udayton.edu/faculty/dmyszka/WebPages/mct446/mct446.htm
Finite Element Analysis Course	PowerPoint	http://civil.engr.siu.edu/CE551/download.htm
Introduction to Finite Element Method	HTML	http://www.eng.buffalo.edu/~abani/fem/fem.html
FEA Information	HTML	http://www.dermotmonaghan.com
Introduction to Finite Element Method I & II	PDF	http://urbana.mie.uc.edu/CAELab.htm
F.E.A. Resources on the Internet	HTML	http://www.geocities.com/SiliconValley/5978/fea.html

My Computer

Figure 2-12: Table of references

3 Conclusion

The one-dimensional spring learning module has accomplished the goal of this project. Working through the module from start to finish will allow an individual to become familiar with finite elements and know the physical behavior of the spring element. The example problem in the module will make the user familiar with the solution procedure. After completing the module the individual should understand the characteristics of the element and be able to determine when and where it can be used.

4 References

1. Craddock, J. N., "Download Lecture Slides," CE551, Finite Element Analysis, Department of Civil Engineering, Southern Illinois University, Carbondale, Illinois, 2003, http://civil.engr.siu.edu/CE551 /download.htm.

2. Patra, A.K., "Introduction to the Finite Element Method for Computer Aided Engineering," MAE 477/577, Cad Applications, Department of Mechanical and Aerospace Engineering, University of Buffalo, Buffalo New York, 2003, http://www.eng.buffalo.edu/~abani/ fem/fem.html.

3. Myszka, D.H., "Applied Finite Element Analysis," MCT446, Applied Finite Element Analysis, Department of Mechanical Engineering Technology, University of Dayton, Dayton, Ohio, 2003, http://www.engr.udayton.edu/faculty/dmyszka/ WebPages/mct446/mct446.htm.

4. Liu, Y., "CEA Research Laboratory at UC," 20-MECH-525 & 526, Introduction to Finite element Method I & II, Mechanical Engineering Department, University of Cincinnati, Cincinnati, Ohio, 2003, http://urbana.mie.uc.edu/CAELab.htm.

5. Gorni, A. A., "F.E.A. Resources on the Internet", 2003, http://www.geocities.com /SiliconValley/5978/fea.html.

6. Monaghan, D., "FEA Information," Belfast, Ireland, 2003, http://www. dermotmonaghan.com.

7. Rencis, J.J., "Finite Element Method Universal Resource," Department of Mechanical Engineering, Worcester Polytechnic Institute, Worcester, Massachusetts, 2003, http://femur.wpi.edu.

8. Rencis, J.J., Flory, E., Kwok, P. and Alam, J., "Learning Modules for Finite Element Method on World Wide Web," ASEE Computers in Education Journal, Vol. IX, No. 4, October – December, 1999.

9. Microsoft ™ Front Page ™, Software, http://www.microsoft.com/frontpage.