Portfolio Construction and Risk Measurement:

Practical Issues and Examples

A Thesis

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Abstract

This thesis describes some of the practical issues faced by a portfolio manager in analyzing the risk associated with a portfolio of assets. The main tools used are the mean-variance optimization algorithm introduced by Markowitz and multi-factor models for risk decomposition. A sample portfolio designed to track the Russell 1000G stock index is constructed that minimizes tracking error while satisfying constraints on the exposure of the portfolio to particular factors (growth and market capitalization).

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Table of Contents

1. Introduction and Background

Modern portfolio theory began with the fundamental work of Harry Markowitz. (References [4, 5, 6].)   His was the first work that gave a clear mathematical definition to "risk" in portfolio analysis. No work prior to Markowitz was able to give a mathematical explanation for the fact that diversification reduced the risk in a portfolio of stocks. Markowitz did not actually use the word "risk" in his original paper; he spoke only of variance in return as the quantity that an investor should wish to minimize (or control) while maximizing return.

Markowitz's original work still defines the main analytical tool for choosing "optimal" portfolios. In practice, however, most of the work of the portfolio manager is done in preparing the *inputs* for the Markowitz model (the forecasts for portfolio return and portfolio variance), and in interpreting the *outputs* of the model. The most recent advances in portfolio management have focused on ways to analyze in more detail the different sources contributing to the total risk in a portfolio. (See [8].)

1.1 Mean-variance Analysis

In the simplest example, the investor chooses a fraction of total wealth x_i to invest in an asset with (random) return R_i for each stock $i = 1,2,...N$.   The expected return on the portfolio is the weighted average of the individual expected returns:

$$E_P = E[R_P] = \sum_{i=1}^{N} x_i E[R_i] = \sum_{i=1}^{N} x_i \mu_i = x^T \cdot \mu$$

In Markowitz's work, the risk associated with the portfolio is defined to be variance (or standard deviation) in the return on the portfolio:

1

$$V_P = Var(R_P) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j = x^T V x$$

where V is the $N \times N$ covariance matrix with entries

$$\sigma_{ij} = E\left[(R_i - \mu_i) \cdot (R_j - \mu_j)\right].$$

The intuitive definition of risk is the probability of suffering harm or loss. Much of the history of risk is tied to attempts to quantify the "probability" mentioned in the intuitive definition. (See [1] for an engaging discussion of the history of risk, from its development in parallel with the mathematical theory of probability up to modern finance including portfolio theory and derivatives.) Any mathematical definition for risk must capture and quantify the intuitive idea that return is a random variable and risk is the probability of loss.

To say that the return on a portfolio is a *random variable* means that the (future) return is not known in advance but the analyst has some way of modeling the distribution of possible returns and their associated probabilities. For example, if the return on a particular stock is assumed to have a normal distribution, with mean $\mu_i = 0.12$ and variance $\sigma_i^2 = (0.05)^2 = 0.025$, then about 60% of all returns will be in the interval $[0.12 - 0.05 \, , \, 0.12 + 0.05] = [0.07 \, , \, 0.17]$ and about 95% of all returns will fall in the interval $[0.12 - 0.1 \, , \, 0.12 + 0.1] = [0.02 \, , \, 0.22]$. The investor should be about 97.5% confident that the return on the stock will exceed 0.02 or 2%. The normal distribution has (at least) two more crucial properties:

- The distribution is symmetric about the mean;

- Two parameters, the mean μ_i and the variance σ_i^2, capture all of the information needed to answer any question about the distribution.

The second fact implies that the variance is the only information you need to quantify the risk associated with this investment.

1.2 Efficient Portfolios

Markowitz defined as *efficient* portfolios that minimized risk for a given level of return and maximized return for a given level of risk. The set of all efficient (feasible) portfolios was called the *efficient frontier*. (See [4,6].) In addition to giving the fundamental definition, Markowitz developed computer algorithms that could efficiently find the efficient frontier.

The search for good portfolios is reduced to a standard mathematical optimization problem. It can be formulated in several equivalent forms. For example:

Problem 1:

Minimize $V_P = Var(R_P) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j = x^T C x$

subject to the constraint

$E_P = E[R_P] = \sum_{i=1}^{N} x_i \mu_i = x^T \cdot \mu$ is equal to a specified constant,

and

$\sum_{i=1}^{N} x_i = 1$

Problem 2:

Maximize $E_P = E[R_P] = \sum_{i=1}^{N} x_i \mu_i = x^T \cdot \mu$

subject to the constraint

$V_P = Var(R_P) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j = x^T C x$ is equal to a specified constant,

and

$\sum_{i=1}^{N} x_i = 1$

Problem 3:

Minimize $U = V_P - \lambda_E \cdot E_P$

subject to the constraint

$\sum_{i=1}^{N} x_i = 1$

The first two problems simply require that the "optimal" portfolio is efficient. The third problem introduces (quietly) the notion of a utility function where the parameter λ_E is a measure

of the risk tolerance for the investor (actually the reciprocal of the risk tolerance). There may

also be additional constraints of the form $A \cdot x = b$ or $A \cdot x \geq b$ or $x_i \geq 0$ for $i = 1, 2, ..., N$. (In the

last case, short-selling is forbidden.)

1.3 Alternate Risk Measures

In practice, at least in some applications, stock returns are not normal and so variance

may not be the best measure of risk for a stock or portfolio. (See Chapter 2 in [8] for a

discussion of the implications of non-normality.) In addition, there are other measure for risk

that are easier to interpret and easier to explain to a customer or client. Other risk measures have

been developed and applied (see, for example, Chapter 3 in [3]). These measures include:

- Semivariance (also called downside risk or downside variance)

- Target semivariance

- Shortfall probability

- Value at Risk

Semivariance simply assumes that the investor only cares about large shifts in the price of

a stock if the large shifts are down. If the distribution is symmetric, then semivariance is simply

a multiple of variance and so no new information is recorded. If the distribution is not

symmetric, then semivariance does capture useful information.

Target semivariance goes one step further and records only drops in price larger than a

certain (target) threshold.

Shortfall probability records the part of the distribution in returns that is below a critical

threshold. It answers the question "What is the probability that returns will be below X?" for a

specified X.

4

Value at risk is perhaps the most used of the alternate risk measures. It records the actual loss that would occur if the returns were in the worst 5% of the distribution. (Other thresholds can be set.) Note that, as with the semivariance, when the distribution of returns is normal, then the value at risk is a multiple of the variance. Even in this situation where the two measures really provide the same information, some clients will demand a report of value at risk for a portfolio.

1.4 Practical Issues in Portfolio Management

The theory developed by Markowitz remains the core of portfolio theory, but the portfolio manager must deal with many practical issues that change the focus of the analysis. These difficulties include:

- The universe of available investments can change.

- Estimating the input parameters for the model is expensive.

- There is always error in the parameter estimates.

- Parameters change over time (non-stationary processes).

- It is expensive to change the holdings in a portfolio (transaction costs).

Most of the work of the portfolio managers must be done before the Markowitz machinery can be applied. In particular, sophisticated statistical/econometric models have been developed to estimate the necessary parameters. (See [2,3,8] and the references cited therein.)

Even if there is enough data to estimate all of the inputs to the model, these inputs are still forecasts of stock behavior and so subject to error. In fact, this estimation error is a problem for another reason; mean-variance optimizers tend to be "error optimizers" in the sense that the optimization algorithm will put too much weight on assets with unusually high returns. It is

highly likely that the unusually high return is due more to estimation error than to actual performance.

The mean variance approach assumes that two numbers can record all of the information that an investor needs or should use to make an investment decision. The most restrictive assumption, perhaps, is that all of the "risk" inherent in an investment can be captured in a single number. Many economists and financial analysts have taken the view that there is much more information to be gained and used by studying the sources of risk within the portfolio. Which assets contribute the most to the total risk of the portfolio? If risk is reduced by diversification, is there another way to compare the level of diversification in two portfolios with roughly the same predicted variance (or total risk).

It is not always possible or practical to maintain a truly optimal portfolio. If the estimates for the input parameters change, then there may be a significant change in the allocations in the optimal portfolio. Transaction costs could quickly wipe out any profits made by frequent re-optimization of a portfolio.

Finally, in many situations, the manager's job may be to construct a portfolio that tracks a specific index or set of stocks, the *benchmark*. In this situation, the appropriate measure for return is *active return*, defined by

$$\text{Active Return} = R_P - R_B$$

and the appropriate measure for risk is the tracking error:

$$TE = std(R_P - R_B)$$

where R_B is the return on the benchmark and R_P is the return on the manager's portfolio. Roughly, the idea is that the manager is rewarded for beating the benchmark in returns while

carrying the same risk load as that of the benchmark. This will be the measure of risk used in the example portfolio constructed in Section 4.

1.5 Overview of the Thesis

This thesis focuses on the practical side of portfolio analysis and risk analysis in particular. The next section gives an overview of the different approaches to estimating risk. We then turn to the details of the interpretation for the factors in the risk model. The final section provides a complete analysis of a portfolio designed to track the Russell 1000G.

2. Estimating Risk

2.1 Total Risk and Tracking Error

A portfolio's total risk and tracking error can be computed from the time series of the portfolio's returns. The total risk is just the standard deviation in the portfolio returns:

$$\sigma_P = Std\left(R_P\right)$$

where R_P is portfolio's total return.

As mentioned in the first section, there is another way to look at risk for a portfolio of investments. For many portfolio managers, the goal is to use a smaller set of stocks to mimic working the performance and risk characteristics of a certain benchmark portfolio. *Active return* is defined to be the difference between the portfolio return and the return of the benchmark:

$$\text{Active Return} = R_P - R_B,$$

and the *tracking error* is the standard deviation of active return:

$$TE = \sigma_p \equiv Std\left(R_P - R_B\right).$$

The terms of *total active risk* and *tracking error* are used interchangeably. Tracking error is used more specifically for active risk. Active risk is the standard deviation of *active returns*. Notice that, even for a portfolio that has no benchmark, we can think of active return for a portfolio that is benchmarked against cash.

In estimating realized tracking errors, different time frames are used. For example, the standard deviation of 36 monthly returns is widely used by agencies that rate mutual funds, but for portfolio management purposes, many believe that a 36 month period is too long for

detecting any patterns and less informative than statistics based on shorter periods and more frequent time intervals, such as 120 day, 60 day, 20 day standard deviation of daily returns.

Tracking errors are expressed as percents and, just as returns and tracking error, is annualize for comparability. The standard deviation based on monthly return or daily return is then annualized by multiplying $\sqrt{12}$ or $\sqrt{252}$, respectively.

2.2 Estimating Total Risk from Asset Covariance

There are "standard" statistical tools designed for estimating the return vector and the covariance matrix for a set of stocks. A portfolio's risk can also be calculated from asset level:

$$\sigma_P = \sqrt{w \cdot V \cdot w^T} = \sqrt{\sum w_i^2 \sigma_i^2 + \sum_i \sum_j w_i w_j \sigma_{ij}} = \sqrt{\sum w_i^2 \sigma_i^2 + \sum_i \sum_j w_i w_j \rho_{ij} \sigma_i \sigma_j}$$

where,

$w = 1 \times N$ vector of asset weights,

$w_i = \dfrac{(price_i) \cdot (shares_i)}{\sum\limits_{i=1}^{n} (price_i) \cdot (shares_i)}$ is the weight of ith asset,

$V = N \times N$ covariance matrix for the asset returns, and

$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$ introduces the correlation coefficient.

For any portfolio, we are able to calculate its risk if we know the weights and covariance matrix of assets.

2.3 The Single-index Model

One barrier to the application of the Markowitz optimization approach lies in number of parameters that must be estimated for a large universe of stocks. If the portfolio manager wishes

9

to choose from among 500 stocks, then she must estimate 500 returns and $\frac{500 \cdot 501}{2} = 125{,}250$

covariance entries. To obtain independent estimates of all parameters requires an incredibly long

history of returns for all stocks in the universe.

One of the simplest methods used to simplify the inputs for the portfolio optimization

problem starts with the assumption that the variability in each stock's return is a (linear) function

of the return on some larger market:

$$R_i = \alpha_i + \beta_i \cdot R_M + e_i$$

Here, R_M is the return on the market, α_i is the intercept and β_i is a the slope of the regression

line. The key (technical) assumption made here is that the residuals e_i are uncorrelated:

$$\text{cov}(e_i, e_j) = 0, \text{for all } i \text{ and } j.$$

This reduces the number of parameters significantly; it is now necessary to estimate the return

for the markets, the N individual returns, the N betas, and the N specific variances, $\sigma_i^2 = \text{var}(e_i)$

($3N + 1$ parameters instead of $(N \cdot (N+1))/2$). For a complete overview of the single-index

model, as well as other index models, see Chapters 7 and 8 in [2].

2.4 Multi-factor risk models

Each particular stock has some associated risk due to fluctuations in return. Some of

these fluctuations can be explained by the fact that the company is part of a particular industry

and that industry, as a whole, is doing well (or not). Some of the fluctuations are explained by

more general economic factors such as inflation and some are explained by fundamental

characteristics of the portfolio such as growth, value, and size. (A table listing the risk factors

used in this thesis is included in Appendix A.)

The need and desire to analyze, estimate, predict and decompose a portfolio's risk gives rise to multifactor risk models and risk-model-enabled risk analysis. The following is the general form of a multi-factor risk model:

$$R_P = \sum_{j=i}^{K} X_j \cdot f_j + u$$

where

$x_j = j^{th}$ factor's factor exposure,

$f_j =$ returns attributed to the jth factor, and

$u =$ specific returns.

That is, a multi-factor risk model is used to model factor returns and then to estimate the covariance matrix of factor returns. Specific returns are returns that are unaccounted for by factors and therefore unique to the security. Specific risks are the variance (or standard deviation) of specific returns.

The process of building and maintaining a risk model is rather complex. It involves the following steps:

1. Test, and select a pool of factors. For example *price to book* and *price to cash flow*, among many others, are selected as significant predictors. (See Appendix A.)

2. For the selected factors, formulate the factors and obtain factor exposures. For example, *price to book* and *price to cash flow* are similar and should be grouped as one factor to describe security's valuation aspect; call this the factor *value*. The factor *value* could be formulated as *value*= weight · (price to book) + weight· (price to sales). Price to book and price to sales each has its own scoring scheme.

3. After obtaining factor and factor exposures (scores) for each asset, estimate factor returns and then estimate the covariance matrix for factor returns.

4. Estimate specific returns and estimate specific risks. Specific returns are uncorrelated with factor returns, and therefore the covariance of specific return with factors is zero. Specific returns, on the other hand, are related with the level of market volatility.

This process is diagramed on the following page.

```
                    ┌──────────────────┐
                    │      Model       │
                    │  -Fundamental?   │
                    │ -Macroeconomic?  │
                    │ -Microeconomic?  │
                    └──────────────────┘
                             │
                             ▼
                    ┌──────────────────┐
                    │ Universe Selection│
                    │ -Focus on sector? │
                    │ -Focus on country?│
                    └──────────────────┘
                             │
                             ▼
                    ┌──────────────────┐
                    │ Factor Selection &│
                    │ Factor formulation│
                    │   (K factors)     │
                    └──────────────────┘
                        │           │
              ┌─────────┘           └─────────┐
              ▼                                ▼
      ┌──────────────┐                 ┌──────────────┐
      │Estimate Factor│                 │Estimate Specific│
      └──────────────┘                 └──────────────┘
          │        │                           │
      ┌───┘        └───┐                        ▼
      ▼                ▼                 ┌──────────────┐
┌──────────────┐ ┌──────────────┐       │ Specific Risk │
│Covariance Matrix│ │Factor Exposures│    │   (N×1)       │
│   (K×K)       │ │   (N×K)       │       └──────────────┘
└──────────────┘ └──────────────┘
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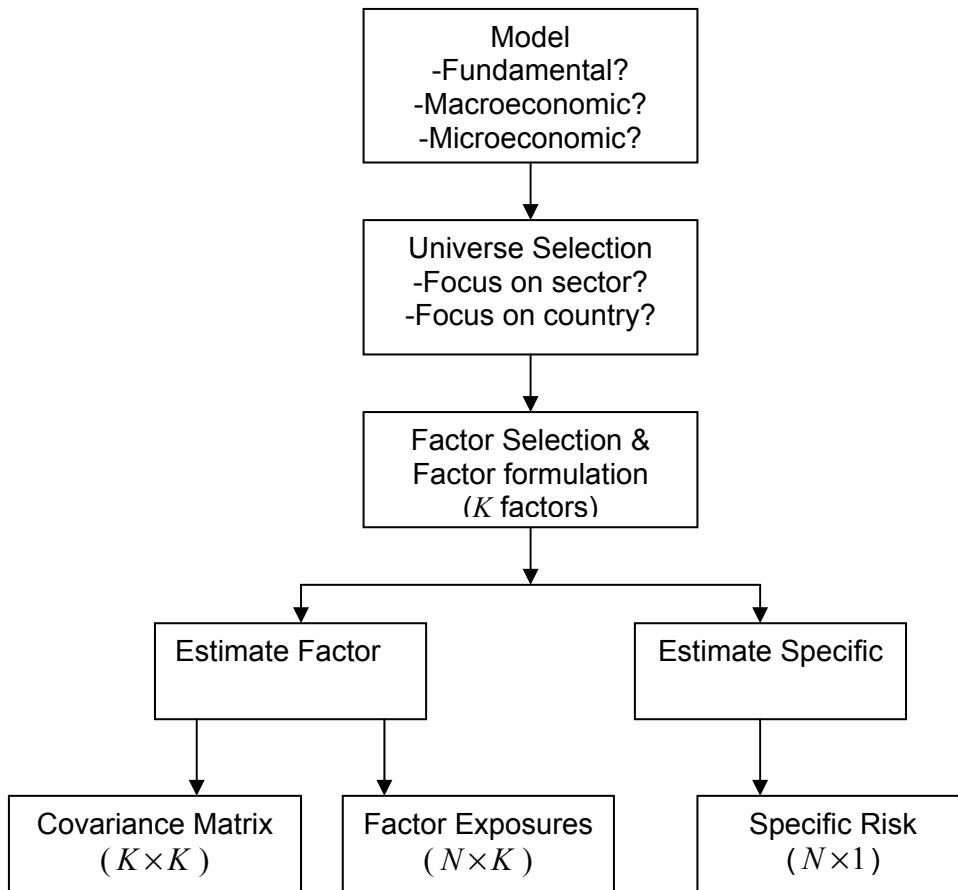
Figure 2.1: Building a structured risk model

The techniques used for finding and formulating factors vary and can be very involved. These techniques are beyond the scope of this project. We will focus on risk analysis from the point of view of a portfolio manager or portfolio construction professional. They are the end-users of the risk model. From the above diagram, three key sets of inputs are the output of risk models: the *factor covariance matrix*, the *factor exposures,* and the *specific risk*. Section 3 discusses risk analysis based on the outputs of a multifactor risk model.

2.4 Tracking Error Target Range

Except for the case of index funds, the lowest tracking errors are not always desirable. Low tracking errors imply a conservative approach towards risk and limit the portfolio's return potential. High tracking errors on the other hand expose the portfolio to greater risk. A tracking error range should be determined before any further risk analysis takes place. After the tracking error range is set, it needs to be periodically reexamined for appropriateness under the current market conditions.

Define first the *market cross sectional volatility* as the standard deviation of all market security's returns at one point in time.

$$Std[R_M] = \sqrt{VarR_M}$$

$$Var[R_M] = E\left[\left(r_i - \bar{r}\right)^2\right]$$

where r_i = return of the i^{th} asset in the market. It is important to remember that the market portfolio is unobservable. Usually a broad index, such as the SP500, is used as proxy and is referred to as the market.

Knowledge of market cross-sectional volatility is helpful in assessing an appropriate tracking target range. Let's use the single factor risk model. In a single factor risk model, each stock's return is given by:

$$r_i = \beta_i \bullet r_M + \varpi_i$$

where $r_i = \beta_i \bullet r_M + \varpi_i$,

$r_i = $ stock i's return,

$\beta_i = $ stock i's beta,

$r_M = $ market return, and

$\varpi_i = $ stock i's residual return.

Residual returns are assumed to be uncorrelated, and hence

$$Cov(r_i, r_j) = \beta_i \cdot \beta_j \cdot \sigma_M^2$$

and so

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \varpi_i^2$$

To see the relation between asset correlation and market volatility, notice that

$$\rho_{ij} = \frac{\beta_i \beta_j \sigma_M^2}{\sqrt{(\beta_i^2 \sigma_M^2 + \varpi_i^2) \cdot (\beta_j^2 \sigma_M^2 + \varpi_j^2)}}$$

Values for stock β's are relative stable. Let's assume $\beta = 1$, then the above reduces to

$$\rho_{ij} = \frac{\sigma_M^2}{\sigma_M^2 + \varpi^2}$$

Thus, there is a positive correlation between market risk and correlation between assets. Now let's take a look at how asset correlation would affect tracking error. In a multifactor model, a portfolio's tracking error is given by:

$$TE^2 = x \cdot F \cdot x^T + w \cdot \Delta \cdot w^T$$

The first term of the righthand side is the portion of risk explained by the factor model. Keeping factor exposure constant, with correlation among assets increasing(decrease), factor risk will increase(decrease).

By the model design, specific risks are the risks unaccounted for by any factors, but the level of specific risks are affected by the market overall volatility. The specific risks are estimated by two steps - the first step is a scaling and standardizing of overall market risk and the second step is a regression analysis of any residual volatility.

Both factor risks and specific risks are affected by market overall volatility, and therefore the tracking error is not constant. It is important for risk analysis to start with an assessment of overall market volatility. If there is no significant change in market volatility, then the preset tracking error range should be upheld, and more focus should be given to risk decomposition.

However, we have examined residual risks of BARRA model and found residual risks *are* correlated, which makes the specific risk portion susceptible to asset correlation change also.

3. Risk Decomposition

Decomposing risk allows portfolio managers to identify and isolate the sources of risk. It is important primarily for two reasons:

1. It allows the portfolio manager to align the portfolio's risk with a particular strategy.

2. It gives additional information regarding the level of real diversification in the portfolio.

Two portfolios with the same estimated tracking error may have very different risk characteristics. For example, a portfolio loaded with high beta stocks may have high tracking errors due to volatility, while another portfolio may tilt towards momentum, which favors recent winners. Risk characteristics should be in sync with portfolio's intended management strategy.

A portfolio that is heavily tilted towards one type of factor is very different from a portfolio with relatively diversified factor risks. Generally, diversification among factors is more desirable than overly concentrated factor risk. But some factors are by design more volatile (explaining more variance) than others. For example, in Barra's USE3 risk model, volatility and momentum are the volatile factors among common factors, and Gold, Internet, Semi conductors account for more risk among industry factors.

3.1 Risk Decompositions along Factors

Tracking Error can be decomposed into *factor risk* and *specific risk*:

$$TE^2 = \sigma_F^2 + \sigma_S^2$$

Here, the *specific risk* is the weighted sum of the individual asset-specific risks.

$$\sigma_S^2 = \sum_{i=1}^{N} w_i^2 \sigma_{si}^2$$

where $w_i = i^{th}$ asset's active weight, and

$\sigma_{si} = i^{th}$ asset's specific risk.

The *factor risk* (industry factor and common factor) is given by

$$\sigma_F^2 = xFx^T$$

where $x =$ Factor Active Exposure ($1 \times K$, and K is the number of factors), and

$F =$ Factor Covariance ($K \times K$ matrix).

A portfolio's exposure to each factor is then computed as

$$x_j = \sum_{i=1}^{N} Coeff_{ij} \cdot w_i \text{ for } j = 1, 2, ..., K$$

where $Coeff_{ij}$ is the *exposure coefficient* (*score*) for the i^{th} security on the j^{th} factor.

Factor exposure measures how a portfolio is exposed to particular factors in the risk model. This is just the weighted sum of factor scores. Factor exposures are easy to understand and easy to manage, i.e. to include one or many factor exposure constraints are just adding one or more linear constraints, while to target factor risk to a range will be harder as they are quadratic constraints.

Factor Risk (A factor's contribution to TE)

$$\sigma_{Fj}^2 = xF_j x_j$$

where $\sigma_{Fj}^2 =$ factor risk attributed to j^{th} factor

$x =$ factor exposure vector ($1 \times K$)

$F_j = j^{th}$ factor's covariance with other factors ($K \times 1$)

$x_j = j^{th}$ factor's exposure

The marginal contribution to active risk by each factor is then

$$\text{MCAR}_j = \frac{\sigma_{Fj}^2}{TE^2}$$

Each factor's contribution to risk is expressed as a percentage of the total risk. It is similar to common size accounting analysis; using percentages we are able to compare different portfolios even though they have different levels of tracking error.

3.2 Sources of Factor Exposure

After a factor is identified as a major risk contributor, i.e. a factor accounting for a high percentage of total risk, further examinations are required:

1. Is the high percentage of risk caused by high exposure or by the variance and covariance matrix?
2. Is the factor exposure concentrated on a few stocks or it is rather diversified?

Individual security contribution to each risk factor's exposure can be computed as

$$x_{ij} = Coeff_{ij} \cdot w_i$$

This analysis is simple and useful in isolating the sources of risk exposures. We will see that in some cases a handful of securities accounts for so much of a factor's exposure such that if those are removed, the factor exposure will change dramatically. In other cases, such analysis exposes data error; sometimes it provides more reasonable calculations for factor scores.

As an example consider the Barra factor *momentum*. Barra defines *momentum* as a combination of *relative strength* and *historical alpha*:

a) Relative Strength is defined as cumulative *excess return* over the past 12 months, using continuously compounded monthly returns. The excess return is defined as excess return over the compounded monthly risk-free rate.

b) Historical alpha is defined as the excess return from a 60 month regression of the stocks

excess returns on the SP500 excess returns. Again, excess returns are defined as the

excess return over the risk-free rate.

Let us use MRVL as example: MRVL's IPO price was at $15 but the first trade was $50.

In essence, MRVL has a positive return over the past three years but a meaningful loss (-70%) if

calculated from when investors could actually buy the stock. Thus, momentum scores based on

IPO price for cases like MRVL can be misleading. There are might be plenty of cases similar to

MRVL that IPOed in 1999/2000. And such a problem will be easy to spot when we analyze

factor exposure by security contribution.

Taking action to adjust factor coefficients for certain securities, (which can be costly), or

not taking action but being able to readjusting tolerance to certain risk factors, either way, both

are direct benefits from understanding the source of factor risk.

3.3 A Security's contribution to One Risk Factor's Factor Risk

It is useful to focus more closely on how an individual security contributes to a specific

risk factor. We have

$$\sigma_{ij}^2 = x \cdot F_j \cdot Coeff_{ij} \cdot w_i \qquad (i^{th} \text{ security's risk contribution to } j^{th} \text{ factor's risk}),$$

as well as

$$\sigma_i^2 = \sum_{j=1}^{K} \sigma_{ij}^2 \qquad (i^{th} \text{ security's risk contribution to factor risk}).$$

This shows that one asset's contribution to one risk factor depends on

1. the security's active weight,

2. the security's exposure coefficient for that factor, and

3. the factor's relative "significance" (active exposure times covariance with other

    factors)

However, a security also contributes to the factor exposure, making it difficult to assign and interpret. .

3.4 Risk Decompositions across Securities

Each security's contribution to tracking error can be expressed as the sum of security risks:

$$TE^2 = \sum_{i=1}^{N} \sigma_i^2$$

where σ_i^2 = the i^{th} security's contribution to TE

   N = the number of assets in the portfolio.

It is important to note that σ_i^2 is a security's contribution to the portfolio's tracking error and has to be put into the context of the portfolio being analyzed. This is not the intrinsic risk or variance of a security that is measured based the securities historical movements alone, rather it is calculated given a security's active weight in the portfolio, its factor risk and specific risk in the portfolio.

Each individual security's contribution to tracking error can be expressed as

$$\sigma_i^2 = \sum_{j=1}^{K} x \cdot F_j \cdot Coeff_{ij} \cdot w_i + w_i^2 \cdot \sigma_{si}^2$$

where

   $Coeff_{ij}$ = the exposure coefficient for the i^{th} security on the j^{th} factor

One asset's contribution to TE is the sum of the asset's

1) factor risk contribution, and

2) specific risk contribution.

Thus, the marginal contribution to active risk (MCAR) can be calculated as:

$$\text{MCAR}_i = \frac{\sigma_i^2}{TE^2}.$$

4. Portfolio Construction and Risk Analysis Based on a Paper Portfolio

This chapter discusses an example based on the Barra USE3 Risk model used to construct an index fund that tracks the R1000G.   The Russell 1000G is a large-cap index developed and maintained by Russell, an investment services firm. The index measures the performance of the largest 1,000 U.S. incorporated companies.

The Russell 1000G measures the performance of those Russell 1000 companies with higher price-to-book ratios and higher forecasted growth values.

4.1 Tracking the Russell 1000G

Indexed funds are growing popular due to several factors:

- Low cost. An indexed mutual fund charges around 0.3% of asset, whereas an actively managed mutual fund normally costs around 1.3% of asset to investors.

- The assumption of market efficiency.

- A proliferation of quantitative risk models and portfolio construction tools.

The primary goal of indexing is to track the target portfolio as closely as possible. For this exercise, our goal is to choose a portfolio with the lowest tracking error. There are also other constraints set for this index fund:

- no more than 100 assets

- no more than 5% cash holding

- single asset will exceed 2.5% of weight of the portfolio

- sector neutral

Initial construction as of date 2001/12/06.

Step 1: Generate 100 names

On the portfolio formation date, R1000G had around 550 assets. To randomly choose 100

from 550 assets, we need to generate a set of random number with probability of

0.1818(=100/550), that is, the selection of an asset can be represented as:

$$P(X = 1) = 0.1818$$

where X is a 0, 1 variable indicating in or out of sample.

We also wish to mimic the distribution of *growth* and *market capitalization* in the

R1000G. Growth and capitalization are basic characteristics of a portfolio, and are widely used

to determine the style of a portfolio. Assuming growth and market cap are independent, the joint

probability can be represented as:

$$P(X = 1) = P(X_g = 1) \ P(X_m = 1) = 0.1818$$

and

$$P(X_g = 1) = P(X_m = 1) = 0.4264$$

where the subscripts g, and m indicate whether the indicator is for growth or market cap.

In the algorithm, first rank each of R1000 constituents by growth and market cap, and

then each security takes a uniform random number with probability of 0.4264.

Step 2: Assign weight of holdings

To mimic sector exposure of R1000G,

$$\sum_{i=1}^{n_j} w_i I_j = W_j \quad i = 1...n_j, j = 1..m$$

where w_i = the weight of a security in the portfolio

n_j = the number of securities in the portfolio that belong to sector j

W_j = the sector weight in the benchmark

In this step, W_j is determined by the benchmark; n_j is determined after the names are

chosen. If we set $w_i I_j = \bar{w}_j$, that is, a security's weight equals the average weight of assets in

the sector, we have

$$w_i I_j = \bar{w}_j = W_j / n_j$$

The final adjustment is if any single security holding exceeds 2.5%, set the holding to be

2.5%. The choice of 2.5% is arbitrary and in fact 2.5% is rather large in a diversified portfolio,

some investors may have higher or lower tolerance.

Step 3: Calculate the estimated tracking error for the synthesized portfolios

The tracking errors and other information, such as portfolio's average market cap,

average long term growth rate, are calculated as a basis for choosing the initial portfolio.

4.1 A Complete Step-By-Step Risk Analysis

Analysis of the 100 portfolios begins by reviewing the composition of factor risk and specific

risk:

	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TE	TE Var	Factor Risk		% Specific Risk		%
Average	15404	22.3	98.8	6.6	9.6	8.8	77.5	*49.2*	63.2	*28.3*	36.8
Min	8311	20.7	73.0	2.2	8.1	7.7	59.8	34.9	56.0	20.9	26.0
Max	24054	24.4	125.0	12.0	11.5	10.4	108.6	76.5	74.0	39.9	44.0
Std	3475	0.8	11.5	1.7	0.7	0.6	10.2	8.0	3.4	3.5	3.4

					Lowest 15 TE Portfolios							
	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TE	TE Var	Factor Risk		% Specific Risk		%
Average	17563	21.7	100.9	5.7	8.5	8.0	63.7	*38.2*	60.0	*25.5*	40.0
Min	12346	20.8	85.0	2.2	8.1	7.7	59.8	34.9	57.0	20.9	34.0
Max	21308	22.5	117.0	9.4	8.9	8.2	66.9	41.5	66.0	28.3	43.0
Std	2697	0.6	9.0	2.2	0.2	0.1	2.2	2.1	2.5	1.8	2.5

Highest 15 TE Portfolios

	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TE	TE Var	Factor Risk	%	Specific Risk	%
Average	12694	22.4	92.9	6.5	10.6	9.7	94.8	*61.8*	65.1	*32.9*	34.9
Min	8621	20.7	76.0	4.4	10.1	9.4	87.7	54.7	59.0	28.2	30.0
Max	16033	23.5	115.0	7.8	11.5	10.4	108.6	76.5	70.0	39.9	41.0
Std	2149	0.7	8.7	1.1	0.4	0.3	6.6	5.7	2.9	3.5	2.9

The table shows that with the number of securities fixed, the level of factor risk has more impact on tracking error than specific risk, and factor risk is in a much wider range (twice as wide as specific risk) and is more volatile.

The following charts illustrate the same pattern:

Specific Risk

Specific Risk – ∎ – % Poly. (Specific Risk)
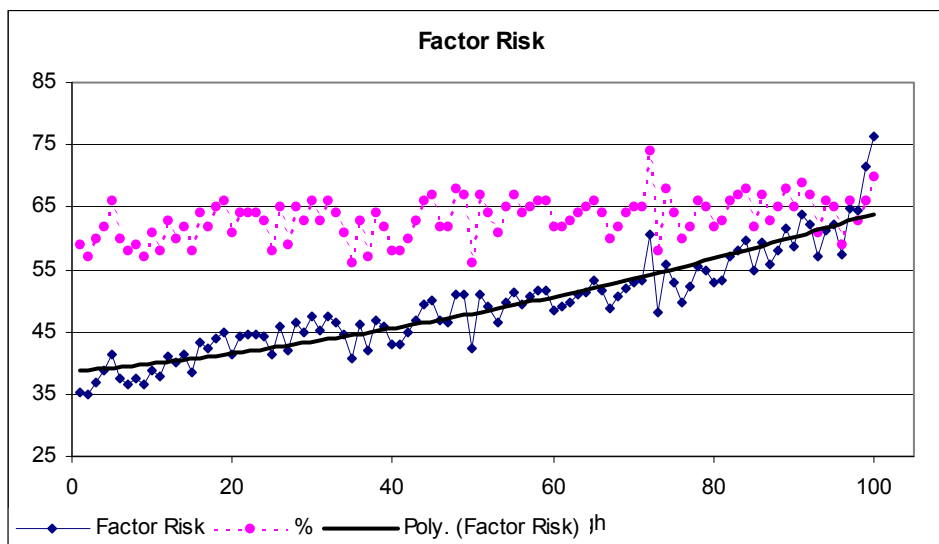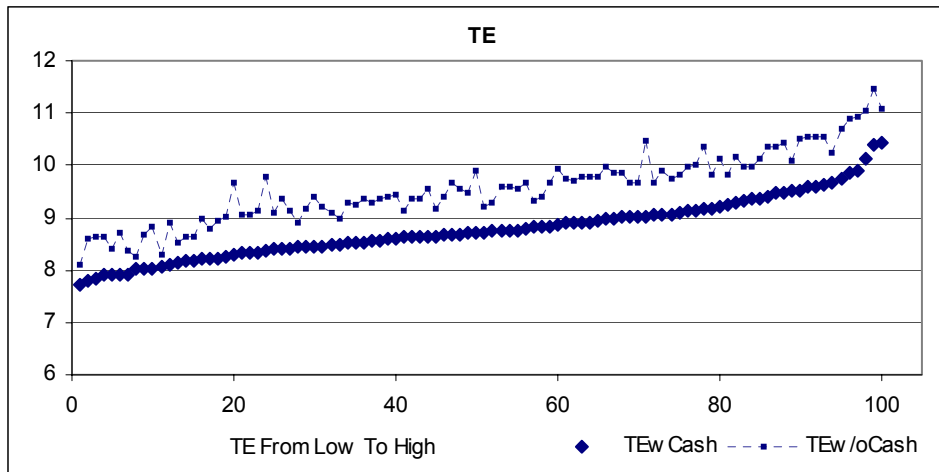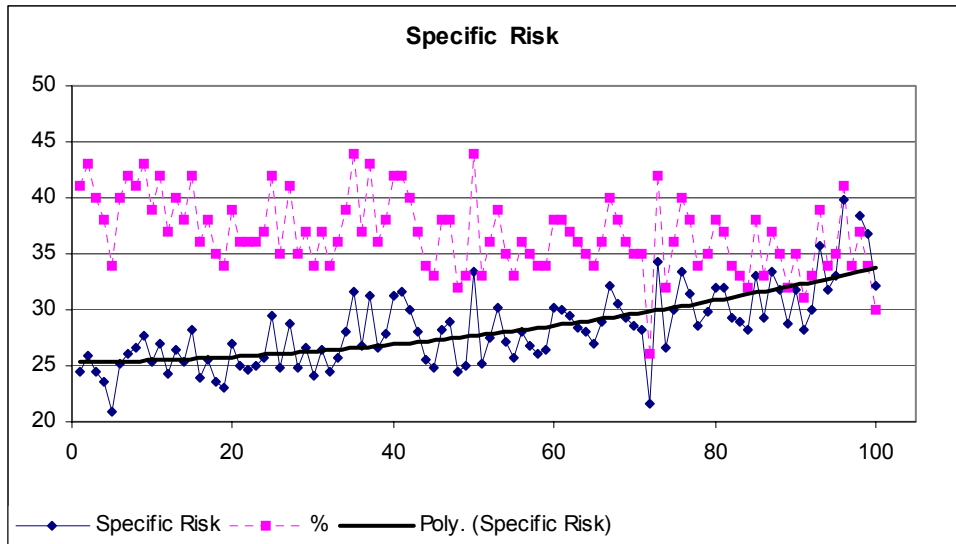
To see why specific risk is in a relatively tight range, examine the following approximation:

$$\text{Portfolio Specific Risk} = \sum_{i=1}^{n} \sigma_{si}^2 w_i^2 = \sum_{i=1}^{n} \sigma_{si}^2 \left(\frac{v_i}{V}\right)^2 \approx \sum_{i=1}^{n} \overline{\sigma}^2 \left(\frac{\overline{v}}{n\overline{v}}\right)^2 \approx n\overline{\sigma}^2 \left(\frac{1}{n}\right)^2 = \frac{1}{n}\overline{\sigma}$$

where $w_i = \dfrac{v_i}{V} = \dfrac{v_i}{n\overline{v}}$

$V =$ portfolio's total market value

$v_i =$ a security's market value

$n =$ number of securities in the portfolio

$\sigma_{si}^2 =$ the stock specific risk of ith security

The first approximation is done assuming the portfolio has a random composition of specific risks. The assumption is appropriate since the risk model is based on the similar assumption that stock specific risk is a normal random variable. To argue against this assumption means a portfolio is intentionally skewed toward specific risk, which can be done through intentional adding or shedding high (or low) specific risk stocks. However, that would not be a good practice to use for specific risk alone.

27

The second approximation is done assuming equal weight for all assets. Noting that we have restricted the security weights in the portfolios to be no more than 2.5%, the assumption is largely valid.

To summarize the relationship of factor risk, specific risk and tracking error:

- Increase the number of securities will likely reduce tracking error. In other words, the more securities in a portfolio, the easier to lower the portfolio's specific risk, resulting in lower tracking error.

- When the number of securities is set, factor risk is the primary variable in managing tracking error.

4.2 Choose the best tracking portfolios

The next step in the analysis is to look in detail at the risk characteristics of the tracking portfolios with the lowest tracking errors. The following table lists the five best (in terms of TE).

nth	Acct	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TEwCash	TE var	Factor Risk	%	Specific Risk	%
1	97	**18793**	21.37	106	**4.37**	**8.1**	7.73	**59.78**	35.3	59	**24.48**	41
2	88	17591	22.25	94	**9.36**	8.59	7.79	60.76	34.92	57	25.84	43
3	27	16605	22.18	99	7.72	8.63	7.85	61.55	37	60	24.55	40
4	47	14607	21.4	114	7.18	8.62	7.9	62.39	38.77	62	23.62	38
5	60	21308	21.99	117	4.37	8.4	7.9	62.38	41.46	66	20.91	34

Consider each portfolio in turn. The important points to notice in reviewing the portfolios are that:

1. Cash will dilute asset weights and give artificially low tracking errors. A good portfolio will have low tracking error and a very small dilution by cash. Portfolio number 97 looks a lot better than the next runner up and it's the best among the top 5 choices.

28

2. As we discussed above, factor risks reflect portfolio's characteristics, and we are looking for a portfolio whose deviation from the benchmark is very small. Among the top 5 choices, Portfolio number 97 has the second lowest tracker error.

Of course, there are more aspects and more specific requirements can be employed as criteria in choosing an initial portfolio. And if we run more simulations, there will be more flexible and perhaps better choices. However, the 100- portfolio simulation suits our purpose of illustration well, and therefore we choose portfolio N. 97 as our initial portfolio.

The next step is to analyze risk characteristics of the initial portfolio. The goal of risk analysis is to assess the current risk level for the portfolio and for the particular factors, understand how and why it obtains this level of risk. The manager must make judgment as to what actions, if any, to take to direct the portfolio to achieve better risk adjusted return.

Figure I: Risk Decomposition Summary

RUSSELL 1000 - GROWTH MIRROR Risk Decomposition

As Of:	12/06/2001	%
TENoCash	8.1	
TEWithCash	7.73	
Total Risk	59.78	
Factor	35.3	0.59
Common	34.95	0.58
Industry	0.35	0.01
Specific	24.48	0.41
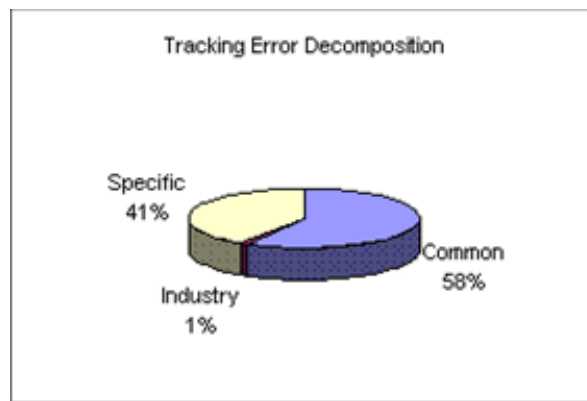
Things to notice in this table:

a. The industry risk only accounts for 1% of tracking error. That is reasonable as we employed a sector neutral strategy, and the strategy reflects the small industry bets.

b. The specific risk looks normal, as compared with other portfolios in the previous analysis; The factor risks are normal but there is room for diversification and we want to exploit opportunities to neutralize factor bets

Figure II: Factor Risk Decomposition

Fundamental Factors (sorted by exposure)

Factor	Exp	Rel	Risk	% OfTE
LEVERAGE	0.4885	-0.0498	-0.0243	0.0%
VOLTILTY	0.342	15.5161	5.3058	**8.9%**
TRADEACT	0.3328	3.8599	1.2844	**2.2%**
EARNVAR	0.2496	1.9487	0.4865	0.8%
VALUE	0.2337	1.6223	0.3792	0.6%
GROWTH	**0.0791**	3.0036	0.2375	0.4%
MOMENTUM	0.0687	2.4096	0.1656	0.3%
NONESTU	0.0159	2.7071	0.0431	0.1%
CURRSEN	-0.0108	1.9977	-0.0216	0.0%
YIELD	-0.1051	-2.3008	0.2417	0.4%
EARNYLD	-0.1078	0.2222	-0.0239	0.0%
SIZENONL	-0.2798	8.9667	-2.5091	-4.2%
SIZE	**-1.5477**	-18.9899	29.3902	**49.2%**
			34.9548	58%

Things to note:

- SIZE is the biggest risk contributor in terms of percentage of tracking error.

- After SIZE, VOL and TRADACT are next largest sources of factor risks.

The report further identifies the sources of the factor risk and examines how, by looking at the factor covariance matrix, these significant factors will affect or be affected by other factors.

Contributors of Top Positive Factor Exposures (sorted by exposure)

Factor	Exposure	Ticker	AWt	Coef	Exp
LEVERAGE	**0.49**	PFE	-0.05	-1.07	0.053
		MSFT	-0.044	-1	0.044
		APH	0.01	3.14	0.032
		WWCA	0.007	4.02	0.029
VOLTILTY	0.34	BVSN	0.01	4.09	0.043
		INRG	0.011	3.38	0.036
		QSFT	0.01	3.08	0.032
		PFE	-0.05	-0.55	0.027

Contributors of Top Negative Factor Exposures (sorted by exposure)

Factor	Exposure	Ticker	AWt	Coef	Exp
SIZE	-1.55	GE	-0.043	1.75	-0.076
		PFE	-0.05	1.52	-0.075
		MSFT	-0.044	1.68	-0.074
		INTC	-0.042	1.36	-0.057
SIZENONL	-0.28	INRG	0.011	-1.46	-0.015
		BVSN	0.01	-1.03	-0.011
		PPDI	0.015	-0.71	-0.01
		SNDK	0.01	-0.92	-0.01

Analysis of Covariance of Top Risk Factors (with or countering)

SIZE

Fundamental:	MOMENTUM	1.99	YIELD	1.13	TRADEACT	0.29
Industry:	INTERNET	25.31	WIRELESS	20.21	AIRLINES	18.71
	SIZENONL	-8.12	EARNYLD	-2.92	VOLTILTY	-1.75
	ENGYRES	-13.72	GOLD	-12.02	TOBACCO	-9.37

VOLTILTY

Fundamental:	TRADEACT	7.11	SIZENONL	3.11	GROWTH	3.07
Industry:	SEMICOND	128.16	CMPTRSW	100.39	CMPTRHW	97.08
	EARNYLD	-6.98	NONESTU	-3.31	MOMENTUM	-3.14
	GOLD	7.34	MEDPROVR	19.66	FOODBEV	35.82

Figure III: Industry Risk

Normally, this would be the third key contributor to risk in the model. For this example, the earlier analysis shows that industry does not make a significant contribution to risk.

5. Summary

The goal of this project was to explore all of the challenges that face a portfolio manager in the real world. Much of academic portfolio theory focuses on the core set of optimization tools that will provide an "optimal" portfolio under the assumption that all of the inputs are available and accurate. In fact, the theory only identifies efficient portfolios and leaves the investor (or the manager) with the task of deciding which of the portfolios best fits the needs of the investor's.

In practice, much of the work of the manager is done before the optimizer enters the picture. Many of the desired inputs are not available and those that are available are estimated with significant error.   In addition, the single number used in the classical analysis (be it variance or standard deviation or some other number) rarely helps the investor make the crucial final decision among the efficient portfolios. Factor models for risk which allow the manager to separate and identify the main contributors to the total risk in the portfolio give information that can help guide the decision.

Risk models also enable portfolio managers to manage portfolio's risk through factor exposure and each factor's risk contribution to portfolio's total risk. As a result, risk analysis goes beyond estimation of portfolio's total risk or the estimation of covariance matrix of factors. Risk analysis becomes a standard tool for portfolio managers to use in quantifying and managing risk.

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Appendix A: Barra Factors

Barra Risk Factors

MINING	DEPTSTOR	MEDPROVR	FINSVCS
GOLD	CONSTRUC	MEDPRODS	INTERNET
FOREST	PUBLISH	DRUGS	EQTYREIT
CHEMICAL	MEDIA	ELECEQP	BIOTECH
ENGYRES	HOTELS	SEMICOND	VOLTILTY
OILREF	RESTRNTS	CMPTRHW	MOMENTUM
OILSVCS	ENTRTAIN	CMPTRSW	SIZE
FOODBEV	LEISURE	DEFAERO	SIZENONL
ALCOHOL	ENVSVCS	TELEPHON	TRADEACT
TOBACCO	HEAVYELC	WIRELESS	GROWTH
HOMEPROD	HEAVYMCH	INFOSVCS	EARNYLD
GROCERY	INDPART	INDSVCS	VALUE
CONSDUR	ELECUTIL	LIFEINS	EARNVAR
MOTORVEH	GASUTIL	PRPTYINS	LEVERAGE
APPAREL	RAILROAD	BANKS	CURRSEN
CLOTHING	AIRLINES	THRIFTS	YIELD
SPLTYRET	TRUCKFRT	SECASSET	NONESTU

Appendix B: Presentation Slides

Portfolio Construction and Risk Measurement:
Practical Issues and Examples

Pam Gao

April 28, 2003

4/28/2003

Overview:

- Quick introduction: Mean-Variance Approach
  - Risk = Variance in Return   (Markowitz)

- Risk analysis based on Risk Models
  - Benchmarks, Tracking Error
  - Factor Models and Factor Decomposition

- Example in Detail
  - Tracking the Russell 1000 Growth

4/28/2003

Background:
Active versus Passive Management

- Measure return relative to a *benchmark*.
  - **Passive:** believe the market is efficient, and the goal is to match the benchmark
  - **Active:** believe that active stock selection will outperform market, and the goal is to beat the benchmark

- Measure *total risk* relative to the same benchmark
  - **Passive**: maintain a tight risk target range and try to match benchmark risk as closely as possible
  - **Active**: manage portfolio risk characteristics to be in line with active stock selection strategy, and target risk within a specific lower and upper bounds

4/28/2003

Classical Modern Portfolio Theory

- Markowitz: Efficient Frontier

- Maximize Utility:

$$U = R_P - \lambda \sigma_P^2$$
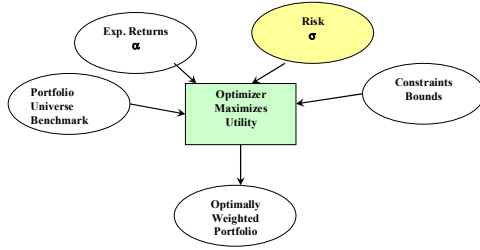
4/28/2003

Some Notation

- Universe of stocks: $i = 1, 2, ..., N$

- Expected returns: $\mu_i = E(R_i)$

- Covariance matrix: $\sigma_{ij} = E\left[(R_i - \mu_i) \cdot (R_j - \mu_j)\right]$

- Portfolio returns: $R_P = \sum w_i u_i$

- Portfolio risk: $\sigma_P^2 = \sum_i \sum_j w_i w_j \sigma_{ij} = w^T V w$

4/28/2003

Slide 1: Portfolio Construction And Risk Analysis

Portfolio Construction And Risk Analysis

- Exp. Returns α
- Risk σ
- Portfolio Universe Benchmark
- Optimizer Maximizes Utility
- Constraints Bounds
- Optimally Weighted Portfolio

4/28/2003

Slide 2: Estimating Risk

Estimating Risk

- Problems with asset covariance matrix
  - Parameter estimation is hard
  - Provides little workable information

- Advantages of risk models
  - Reduce parameter space
  - Provide ways to manage risk through exposures
  - Provides ways to attribute risk to factors by percent of total risk

4/28/2003

Slide 3: Estimating Portfolio's Risk

Estimating Portfolio's Risk

- Tracking Error $\quad \sigma_p \equiv Std[R_p - R_B]$

  Tracking Error is the standard deviation of portfolio's active return

  - Historical Realization
    Based on 36-monthly returns, 120, 60, 20- daily returns
  - Estimated Based On Assets (Modern Portfolio Theory)

$$\sigma_p = \sqrt{w \cdot V \cdot w^T} = \sqrt{\sum w_i^2 \sigma_i^2 + \sum \sum_j w_i w_j \sigma_{ij}} = \sqrt{\sum w_i^2 \sigma_i^2 + \sum \sum_j w_i w_j \rho_{ij} \sigma_i \sigma_j}$$

4/28/2003

Slide 4: Estimating Portfolio's Risk

Estimating Portfolio's Risk

Estimated Based On Factors (Risk Models)
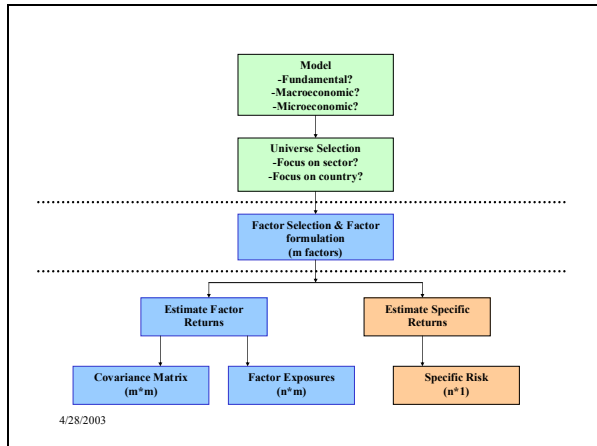
$$\sigma_P = \sqrt{x^T F x + w^T \Delta w}$$

where
  x: M by 1 Factor Exposure Vector
  F: M by M Factor Covariance Matrix
  Δ: N by N diagonal matrix of specific variance

(M is much smaller than N)

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Slide 5: Model flow chart

Model
-Fundamental?
-Macroeconomic?
-Microeconomic?

Universe Selection
-Focus on sector?
-Focus on country?

Factor Selection & Factor formulation
(m factors)

Estimate Factor Returns → Covariance Matrix (m*m), Factor Exposures (n*m)

Estimate Specific Returns → Specific Risk (n*1)

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Slide 6: Factor covariance matrix

	VOLTILTY	MOMEN	SIZE	SIZENONL	TRADEACT	GROWTH	EARNYLD	VALUE	EARNVAR	LEVERAGE	CURRSEN	YIELD	NONESTU
VOLTILTY	37.73	0.54	-2.68	2.88	7.07	3.11	-5.73	0.36	-0.01	-1.92	1.68	-0.04	-2.45
MOMENTUM	0.54	38.17	1.62	-8.40	0.04	1.10	-6.35	0.28	1.17	-1.97	0.95	-0.75	1.93
SIZE	-2.68	1.62	13.46	-8.29	0.40	-1.22	-2.74	-0.34	-0.84	0.22	-0.73	1.12	-1.41
SIZENONL	2.88	-8.40	-8.29	15.87	-0.27	-0.20	5.18	0.61	-0.46	0.34	-0.60	-1.21	0.72
TRADEACT	7.07	0.04	0.40	-0.27	7.02	0.41	-1.27	-0.03	0.15	-0.38	0.08	-1.04	-0.18
GROWTH	3.11	1.10	-1.22	-0.20	0.41	2.87	-0.68	-0.21	-0.07	-0.65	0.42	-0.14	-0.85
EARNYLD	-5.73	-6.35	-2.74	5.18	-1.27	-0.68	8.95	-0.28	-0.08	1.33	-0.85	-0.40	-3.99
VALUE	0.36	0.28	-0.34	0.61	-0.03	-0.21	-0.28	2.64	0.48	0.52	0.16	0.17	2.26
EARNVAR	-0.01	1.17	-0.84	-0.46	0.15	-0.07	-0.08	0.48	3.19	0.08	-0.43	-0.15	1.61
LEVERAGE	-1.92	-1.97	0.22	0.34	-0.38	-0.65	1.33	0.52	0.08	2.53	0.02	-0.05	-0.08
CURRSEN	1.68	0.95	-0.73	-0.60	0.08	0.42	-0.85	0.16	-0.43	0.02	3.66	-0.22	-2.00
YIELD	-0.04	-0.75	1.12	-1.21	-1.04	-0.14	-0.40	0.17	-0.15	-0.05	-0.22	2.67	0.18
NONESTU	-2.45	1.93	-1.41	0.72	-0.18	-0.85	-3.99	2.26	1.61	-0.08	-2.00	0.18	34.77

4/28/2003

C

Estimating Factor Risk

1. Test, and select a pool of descriptors
2. Formulate factors and obtain factor *exposures*
3. Estimate factor returns

$$r_i = \sum_{j=1}^{m} x_{ij} f_j + u_i$$

4. Estimate factor *covariance*

Estimating Specific Risk

- Specific returns are returns unique to each asset (Not accounted for by factors)

$$r_i = \sum_{j=1}^{m} x_{ij} f_j + u_i$$

- Specific risk is the risk of specific returns
  - For most assets, specific risk is not zero
  - Specific risk varies with market overall volatility

Main Example

- Build a portfolio to track the R1000G

- Choose 100 portfolios of 100 assets
  - Constrained by Growth and MktCap
 - Sector neutral

- Calculate the tracking error for each

Analysis of Tracking Portfolios

	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TE	TE Var	Factor Risk	%	Specific Risk	%
Averag	15404	22.3	98.8	6.6	9.6	8.8	77.5	49.2	63.2	28.3	36.8
Min	8311	20.7	73.0	2.2	8.1	7.7	59.8	34.9	56.0	20.9	26.0
Max	24054	24.4	125.0	12.0	11.5	10.4	108.6	76.5	74.0	39.9	44.0
Std	3475	0.8	11.5	1.7	0.7	0.6	10.2	8.0	3.4	3.5	3.4

Lowest 15 TE Portfolios

	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TE	TE Var	Factor Risk	%	Specific Risk	%
Averag	17563	21.7	100.9	5.7	8.5	8.0	63.7	38.2	60.0	25.5	40.0
Min	12346	20.8	85.0	2.2	8.1	7.7	59.8	34.9	57.0	20.9	34.0
Max	21308	22.5	117.0	9.4	8.9	8.2	66.9	41.5	66.0	28.3	43.0
Std	2697	0.6	9.0	2.2	0.2	0.1	2.2	2.1	2.5	1.8	2.5

Highest 15 TE Portfolios

	Avg MktCap	Avg Ltg	N Security	Cash %	TEw/oCash	TE	TE Var	Factor Risk	%	Specific Risk	%
Averag	12694	22.4	92.9	6.5	10.6	9.7	94.8	61.8	65.1	32.9	34.9
Min	8621	20.7	76.0	4.4	10.1	9.4	87.7	54.7	59.0	28.2	30.0
Max	16033	23.5	115.0	7.8	11.5	10.4	108.6	76.5	70.0	39.9	41.0
Std	2149	0.7	8.7	1.1	0.4	0.3	6.6	5.7	2.9	3.5	2.9
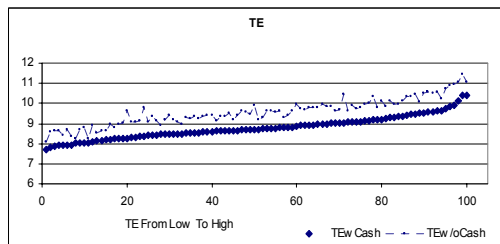
nth	Acct	MktCap	Avg Ltg	N Security	Cash %	w/o Cash	TE	TE var	Factor Risk	%	Specific Risk	%
20	39	21568	22.69	79	11.65	9.66	8.28	68.51	41.49	61	27.02	39
24	15	16114	22.59	101	12.01	9.8	8.36	69.95	44.15	63	25.79	37

D

Specific Risk

Legend: Specific Risk — % — Poly. (Specific Risk)

4/28/2003

RUSSELL 1000 - GROWTH MIRROR Risk Decomposition

As Of:	12/06/2001	
TENoCash	8.1	
TEWithCash	7.73	
Total Risk	59.78	
Factor	35.3	0.59
Common	34.95	0.58
Industry	0.35	0.01
Specific	24.48	0.41

Tracking Error Decomposition — Specific 41%, Common 58%, Industry 1%
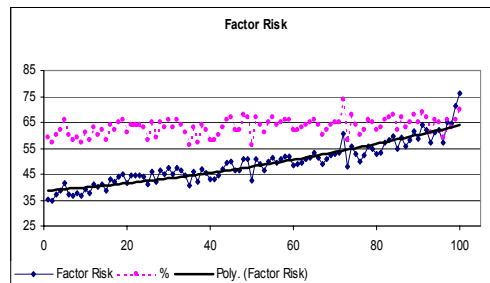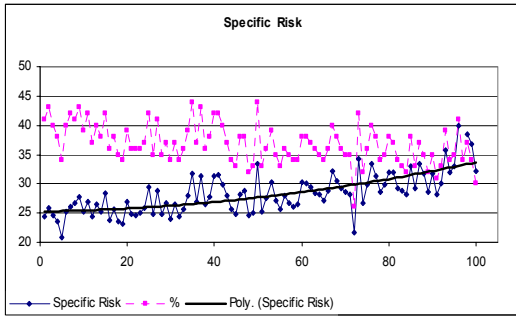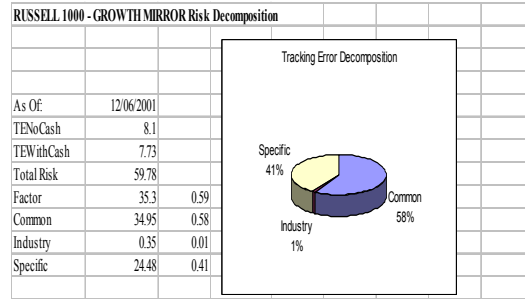
4/28/2003

Analyze a Portfolio Risk

- Manage by exposure
  - Factor exposures too high or too low
  - Scores are all standardizes… comparable

- Monitor percentage of total risk
  - Some factors carry more risk
  - Measure of true diversification

4/28/2003

Fundamental Factors (sorted by exposure)

Factor	Exp	Rel	Risk	% OfTE
LEVERAGE	0.4885	-0.0498	-0.0243	0.0%
VOLTILTY	0.342	15.5161	5.3058	8.9%
TRADEACT	0.3328	3.8599	1.2844	2.2%
EARNVAR	0.2496	1.9487	0.4865	0.8%
VALUE	0.2337	1.6223	0.3792	0.6%
GROWTH	**0.0791**	3.0036	0.2375	0.4%
MOMENTUM	0.0687	2.4096	0.1656	0.3%
NONESTU	0.0159	2.7071	0.0431	0.1%
CURRSEN	-0.0108	1.9977	-0.0216	0.0%
YIELD	-0.1051	-2.3008	0.2417	0.4%
EARNYLD	-0.1078	0.2222	-0.0239	0.0%
SIZENONL	-0.2798	8.9667	-2.5091	-4.2%
SIZE	**-1.5477**	-18.9899	29.3902	**49.2%**
			34.9548	58%

4/28/2003

Contributors of Top Positive Factor Exposures (sorted by exposure)

Factor	Exposure	Ticker	AWt	Coef	Exp
LEVERAGE	**0.49**	PFE	-0.05	-1.07	0.053
		MSFT	-0.044	-1	0.044
		APH	0.01	3.14	0.032
		WWCA	0.007	4.02	0.029
VOLTILTY	0.34	BVSN	0.01	4.09	0.043
		INRG	0.011	3.38	0.036
		QSFT	0.01	3.08	0.032
		PFE	-0.05	-0.55	0.027

Contributors of Top Negative Factor Exposures (sorted by exposure)

Factor	Exposure	Ticker	AWt	Coef	Exp
SIZE	-1.55	GE	-0.043	1.75	-0.076
		PFE	-0.05	1.52	-0.075
		MSFT	-0.044	1.68	-0.074
		INTC	-0.042	1.36	-0.057
SIZENONL	-0.28	INRG	0.011	-1.46	-0.015
		BVSN	0.01	-1.03	-0.011
		PPDI	0.015	-0.71	-0.01
		SNDK	0.01	-0.92	-0.01

4/28/2003

Analysis of Covariance of Top Risk Factors (with or countering)

SIZE						
Fundamental:	MOMENTUM	1.99	YIELD	1.13	TRADEAC	0.29
Industry:	INTERNET	25.31	WIRELESS	20.21	AIRLINES	18.71
	SIZENONL	-8.12	EARNYLD	-2.92	VOLTILTY	-1.75
	ENGYRES	-13.72	GOLD	-12.02	TOBACCO	-9.37
VOLTILTY						
Fundamental:	TRADEACT	7.11	SIZENONL	3.11	GROWTH	3.07
Industry:	SEMICOND	128.16	CMPTRSW	100.39	CMPTRHW	97.08
	EARNYLD	-6.98	NONESTU	-3.31	MOMENTU	-3.14
	GOLD	7.34	MEDPROVR	19.66	FOODBEV	35.82

4/28/2003

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