Planet Formation

A Major Qualifying Project Report:

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Abstract

Scientists have been looking towards the stars recently with the interest of finding Earth-like planets outside of our system, yet we have very little understanding of the planet formation process. A system of partial differential equations modeling the gas around a proto-star can be solved to find steady solutions where off-core local extrema form in the density. The off-core local extrema I found demonstrates the early formation of planets through gas accretion around these extrema in the steady state solution.

Executive Summary

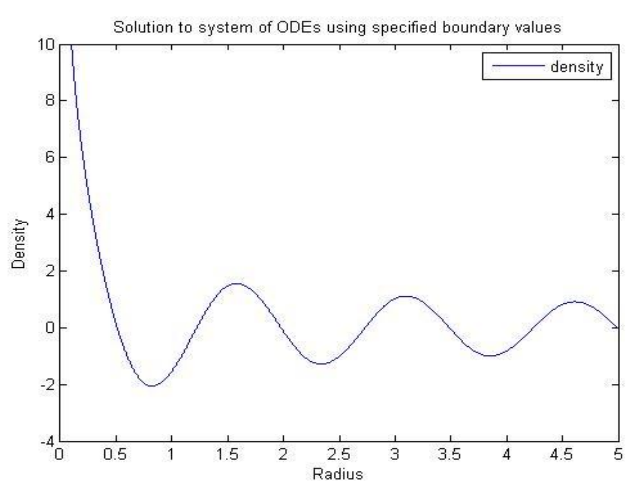
As we look to understand life on our own planet we look for life elsewhere in the universe. The scientific community is currently looking to stars outside of our own system to host planets that may have life on them. While many exoplanets are being found in systems across our galaxy with a wide range of sizes, compositions and proximity to the host star. The diversity of exoplanets clearly indicates that there is a fundamental process in which planets are created that is not understood. To gain an understanding of the process a model needed to be developed to describe the gaseous cloud remaining around the newly formed protostar.

This model is created with the following assumptions: the gaseous cloud surrounding the newly formed protostar is homogenous and radially symmetric and has collapsed into a purely two-dimensional disk. While this is a simplification, it does hold with the Nebula Theory, the currently accepted theory of solar system formation. Two equations can be used to describe the gaseous cloud once simplified and non-dimensionalized.

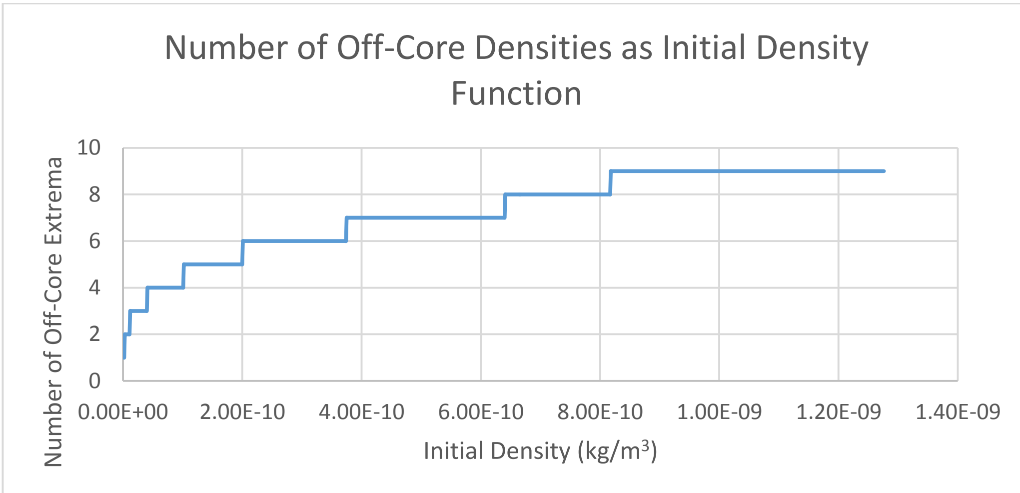
$$\rho'' = -\varphi - \frac{\rho'}{r}$$

$$\varphi'' = c\rho - \frac{\varphi}{r}$$

By solving this system of differential equations for a steady state solution entirely dependent on the initial density, a density curve can be used to demonstrate the possible formation of protoplanets in a solar system. For example with an initial density of 1.8×10^{-11} kg/m^3 three protoplanets are indicated as possibly forming by the number of off-core local maximum points.



The number of off-core densities is shown below to be related to the initial density of the gaseous disk surrounding the star. As the initial density of the disk increases the number of potential protoplanets forming in the system also increases.



While this model is based off a few assumptions, it does provide for a starting point of developing a full model of planetary formation. The equations can be expanded to take into account a non-homogenous disk without radial symmetry to better predict the wide myriad of planetary systems we have found in our galaxy.

Introduction

The basis for this project is the paper 'Steady states of self-gravitating incompressible fluid in two dimensions' published in the Journal of Mathematical Physics authored by Mayer Humi. The goal of that paper was to derive the equations describing a solar system briefly after the protostar has formed and the remaining gasses are still rotating around it. These gasses later form planets and thus the system of equations that describes them can be used to locate and describe the number of planets possible forming in the system. This paper explores a diverse selection of possible initial equations and conditions in the search for a solution that results in multiple densities off core that are local maxima. These results will imply the formation of protoplanets in star systems. This steady state solution can be used to better analyze planetary system development in proto-stars and thus aide in the search for exoplanets.

For the sake of brevity the following terms are used in the Matlab function which can be found at the end:

$$\rho(r) = density\ at\ radius\ r$$

$$\varphi(r) = gravitational\ field\ strength\ at\ radius\ r$$

$$c = 4\pi G$$

Background

The birth of a solar system is a long drawn out process, with many complex variables and large systems of equations describing all of the conditions and parameters. How a star forms is dependent on its location and the material that the star draws from, resulting in the large gamut of stars we see in the night sky. Each one is unique in its composition, processes and life cycle, with literally billions of examples within our sight but far away from our grasp.

Stellar Formation

Stars form in large clouds of molecular hydrogen across the universe. An individual cloud may be anywhere from a few parsecs to a few thousand light-years in diameter. The larger clouds do not form stars light-years in radius, but instead form thousands of stars inside their depths. These large occurrences of star formations are called stellar nurseries because hundreds and thousands of stars are born in these regions. We look to giant molecular clouds like the Orion Molecular Cloud Complex in our local region to observe this process in our relative vicinity.

Jeans Instability

When a cloud reaches sufficient size dependent on its composition and density, it will begin to collapse under self-gravitation. The Jeans length is the radius required for a cloud of specific density and composition to begin this process. To derive the Jeans length, start with the virial theorem that states that the total kinetic energy of a system, multiplied by two plus the potential energy of the system must be zero (Baez, n.d.).

$$2K + U = 0,$$

Where K is the kinetic energy of the gas molecules and U is the gravitational potential energy of the cloud of gas in the situation. Taking the cloud to be spherical, the spheres gravitational potential energy is

$$U = \frac{3GM^2}{5R},$$

Where *G* is the gravitational constant, *M* is the total mass of the cloud, and *R* is the radius of the cloud. Similarly, the kinetic energy is

$$K = \tfrac{3}{2}NkT = \frac{3MkT}{m},$$

Where *N* is the number of molecules, *k* is Boltzmann's constant, *T* is the temperature, and *m* is the molecular mass (Baez, n.d.). Plugging it gives

$$\frac{GM}{5R} = \frac{kT}{m}.$$

But

$$M = \tfrac{4}{3}\pi R^3 \rho,$$

Where ρ is the cloud's mass density, so plugging this in and solving for *R* gives the Jeans length as

$$R = \sqrt{\frac{15kT}{4\pi\rho G m}}.$$

As the cloud collapses, it does so isothermally meaning the Jeans Mass (the mass equivalent to the Jeans Length) decreases as the density increases. This is because the thermal adjustment timescale is much shorter than the free-fall time which is $(G\varrho)^{-1/2}$. When the Jeans Mass is half the original value, the cloud can split into two and so on. As density increases and opacity decreases the processes becomes more and more adiabatic. The conditions for fragmentation during adiabatic collapse can be derived with the ideal gas law.

For adiabatic processes the following is true

$$PV^\gamma = \text{constant} \rightarrow V \sim P^{-1/\gamma}.$$

For any ideal gas by the Ideal Gas Law

$$PV = nRT \rightarrow P.P^{-1/\gamma} \sim T \rightarrow P \sim T^{\frac{\gamma}{\gamma-1}}.$$

Thus by the polytrophic equation of state is reduced to

$$P = K\rho^{\gamma} \rightarrow T \sim \rho^{\gamma-1}.$$

So Jeans mass can be reduced to

$$M_J \sim T^{3/2}\rho^{-1/2} \sim \rho^{\frac{3}{2}(\gamma-1)}\rho^{-1/2}.$$

Thus

$$M_J \sim \rho^{\frac{3}{2}(\gamma-\frac{4}{3})}.$$

If the adiabatic index $\gamma > \frac{4}{3}$ Jeans mass increases with increasing density while if $\gamma < \frac{4}{3}$ Jeans mass decreases with increasing density. The collapse and fragmentation occurs until the fragments are on the order of a solar mass (Kippenhahn and Weigert 1990). The flaw with this model is it requires uniform initial density, constant temperature fixed by radiative processes and does not account for rotation, magnetic fields or turbulence. To account for non-uniform initial density, or more importantly considering the background density as clouds of gas have continuous boundaries. However, this disregarding of the background term is justified by the expansion of the Universe as the two terms surprisingly cancel out (Falco, et al. 2013).

History of Solar System Formation Theories

The first proposition of a model of the origin of the solar system was made by Rene Descartes in 1662. The premise of his model was that the Universe was filled with currents and vortices of particles and our solar system had condensed on a larger vortex that then contracted. However because this was formulated before Newton had published his theory of gravity, Descartes had very little to base it off of other than intuition. Newton's paper and formulation of gravity paved the way for much more rigorous model development over the next couple of centuries.

Tidal Theory

The tidal theory was the first proposed theory after the nebulae hypothesis, in order to resolve the angular momentum problem, by James Jeans in 1917. The theory formulated the planets formed due to another star approaching the Sun, and the tidal forces of the passing star upon the Sun would pull a filament of matter away from the Sun (Woolfson, 1992) . The filament would be pulled around the Sun in an elliptical orbit due to the gravitational attraction of the other star. The matter in the filament coalesced into the planetesimals that over time aggregated enough mass to form the planets of our system today. This theory however was shown to be unlikely as planets formed in this matter would not have had the required angular momentum to avoid being reabsorbed by the Sun nor the arrangement of inner rocky planets and outer gas ones (Woolfson, 1992).

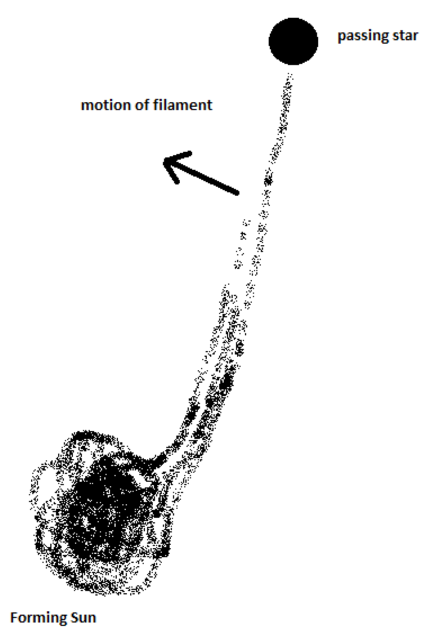


Figure 1 The Tidal Theory as proposed by Jean

The Chamberlin-Moulton model

Forest Moulton in 1900 proposed a model based off of pictures of "spiral nebulas" (Cremin & Williams, 1968). While it was later shown these were galaxies instead of star forming nebulas, the premise was that the protostar would eject filaments due to tidal forces from a passing star. These filaments would could and form the planets however, this model was incompatible with the angular momentum of Jupiter, but it was the first model to propose planetesimals accretion which is widely considered to be an important factor in protoplanet development (Cremin & Williams, 1968).

Lyttleton's model

In 1937 Ray Lyttleton proposed a model similar to the tidal theory where the two stars collided as opposed to just pass by (Cremin & Williams, 1968). The majority of one star was absorbed by the other, with the remaining star mass splitting in two forming Jupiter and Saturn and the filament connecting the two coalescing into the remaining planets. In 1940 Lyttleton refined the model to include a binary star system and our Sun (Cremin & Williams, 1968). The binary stars would merge and then separate with the filament being pulled by our Sun to form the planets. This model however is not likely due to the rarity of occurrence and no evidence of a local binary system in our Sun exists.

Band-Structure model

Hannes Alfven developed a model in 1954 where the nebula around the protostar became banded due to EM effects amplifying the rotational forces. Four distinct bands formed, A-cloud which was mostly helium, B-cloud which was mostly hydrogen, C-cloud which was mostly carbon and D-cloud mostly silicon and iron. Grains of dust in A-cloud led to the creation of Mars and the Moon (pre-capture by Earth) while the B-cloud condensed into Mercury, Venus and Earth. The C-cloud formed the outer planets and Kuiper objects such as Pluto and Triton formed from the D-cloud. This model is unlikely due to the known composition of planets, their locations and centrifugal force dynamics.

Interstellar cloud theory

Soviet astronomer Otto Schmidt devised a theory that the Sun, already formed to near its current form, passed through an interstellar cloud of gas and dust in 1944 (Woolfson, 1992). The Sun then pulled part of the cloud away with it which would form the planets. This process would solve the issue of the angular momentum (Cremin & Williams, 1968) but however it was shown by Victor Safronov that this process would take longer than would be allowed under calculations of the Solar System's determined age and thus this theory was discarded (Cremin & Williams, 1968).

Hoyle's theory

Hoyle developed a hypothesis in 1944 where a companion star to the Sun went either nova or supernova (Cremin & Williams, 1968). This event caused some of the mass of the Sun to break off from the protostar and begin to form the planets. The magnetic couple of the gas ejected and the Sun would act as a transfer for the angular momentum to be sapped away from the Sun and into the majority of the mass for the planets, or Jupiter. This model correctly agrees with the mass and composition of the planets along with the angular momentum distribution but does not explain the belting of the planets and the ratio of mass in the terrestrial planets. Lyttleton concluded that the terrestrial planets must have formed as a result of tidal forces breaking up a larger protoplanet in conjunction with Hoyle's theory (Cremin & Williams, 1968).

Kuiper's theory

Gerard Kuiper put forth a theory in 1944 that the density distribution of the protoplanetary disk would determine if a planetary system formed or a stellar companion. By arguing that large gravitational instabilities would eventually form due to the density distribution the disk would collapse into either multiple gaseous planets or a secondary star. By this theory, the two distinct types of planets formed due to the Roche limit but this theory did not explain the speed (or lack thereof) of the Sun's rotation and thus angular momentum issue.

Whipple's theory

Fred Whipple devised a scenario in 1948 where a single cloud contracted and formed the Sun. This cloud would have had little to no angular momentum and just enough mass to form the protostar. This event however would draw in a secondary cloud, smaller than the first but with a large angular momentum. This second cloud would collapse into the planets we currently have, with the accretion process reducing eccentricity of the orbits. The weakness with this scenario was that the majority of the final results are based heavily off of a priori assumptions and not quantitative calculations.

Protoplanet theory

The Protoplanet Theory was first proposed by W. H. McCrea in 1960 and is centered on the concept that there was no difference between the solar nebula and the protoplanetary one with the planets individually forming at the same time as the Sun and then being captured by gravity (Cremin & Williams, 1968). The model started with a dense interstellar cloud forming a stellar cluster. This collapse would create turbulence and pockets of high pressure termed floccules. McCrea calculated while a large number of these floccules coalescing would create a star, a smaller number could form protoplanets around a star (Cremin & Williams, 1968). By forming from randomly spinning floccules, the star would naturally have little angular momentum while a body formed from just a few would have a better chance of having more angular momentum. This theory does not explain the orbits of the planets all being in the same direction, an unlikely feat if all of the planets formed independently.

Cameron's theory

American astronomer Alastair G. W. Cameron formed a hypothesis in 1962 where a protosun formed of greater mass than our current Sun (Woolfson, 1992). The star becomes unstable and breaks apart into smaller parts causing the magnetic lines of force to twist. This allows for some of the fallen apart star to form a disk and cool down (Cremin & Williams, 1968). This disk then forms the planets that we currently have, but it does not provide a solid argument for the arrangement of planets in our Solar System however.

Capture theory

The capture theory first proposed in 1964 by M. M. Woolfson proposed that tidal forces upon a nearby, low-density protostar would have drawn enough material from it to halt the fusion core and have it collapse to form Jupiter (Woolfson, 1992). The rest of the planets would have formed from the mass drawn away from the neighbor star. However this model proposes a very big difference between the age of the Sun and the rest of the solar system, something we have evidence against (Cremin & Williams, 1968).

Solar Fission theory

The solar fission theory first proposed in 1951 by Louis Jacot reintroduced the ideas of swirling vortices across space time of varying sizes and degrees (Louis, 1981). This meant that the planets formed by being expelled from the Sun one at a time and was dragged outward by these vortices. The asteroid belt formed from a shattered planet possibly due to a collision with Mars. Planetary moons were formed in the same way, except being expelled from their host planet as opposed to the Sun (Louis, 1981). Jacot used the unknown vortex dynamics to explain the differences in the planets. While this does provide an adequate answer to the angular momentum quandary, it is disproven by the known age of the planets with the smaller terrestrial planets forming before the larger Jovian bodies (Louis, 1981).

These theories were all formulated to try to solve in major problems with the widely accepted nebula theory, the largest of which is the distribution of angular momentum. However, since the nebula theory only does not account for the distribution as opposed to contradict it, it is widely accepted since the previous theories all contradicted some observed evidence about the formation of the solar system.

Formation of Solar System by Nebula Theory

The generally accepted method of planet formation is accretion, where tiny amounts of dust began to accumulate and through collisions and gravity slowly became larger and larger. This theory is based off the Nebula Theory. Because of the chaotic nature of the inner Solar System, the rare metals that were present in the nebula were the ones that formed the majority of the planetesimals. Some of the angular momentum of the sun was leeched onto the terrestrial planets due to the drag of the planets through the remaining, slower orbiting gas. Beyond the frost line, or where icy compounds were able to remain solid, the Jovian planets formed. Due to an overabundance of these ices, the Jovian planets grew large enough to swallow up the remaining hydrogen and helium gasses the Sun did not encapsulate. Pressure systems in the gas caused large amounts to be stopped at the frost line, allowing for Jupiter to reach its massive size with Saturn forming a few million years later picking up the leftover gasses. By the time Uranus and Neptune started to form, the Sun reached a period in its life cycle where the stellar winds were strong enough to blow away much of the remaining disc material. Current models predict Uranus and Neptune formed closer to the Sun than they currently reside and slowly migrated outwards. By the time the Sun was 5 million years old most of the gas and dust had been blown away by stellar winds ending the formation of planets.

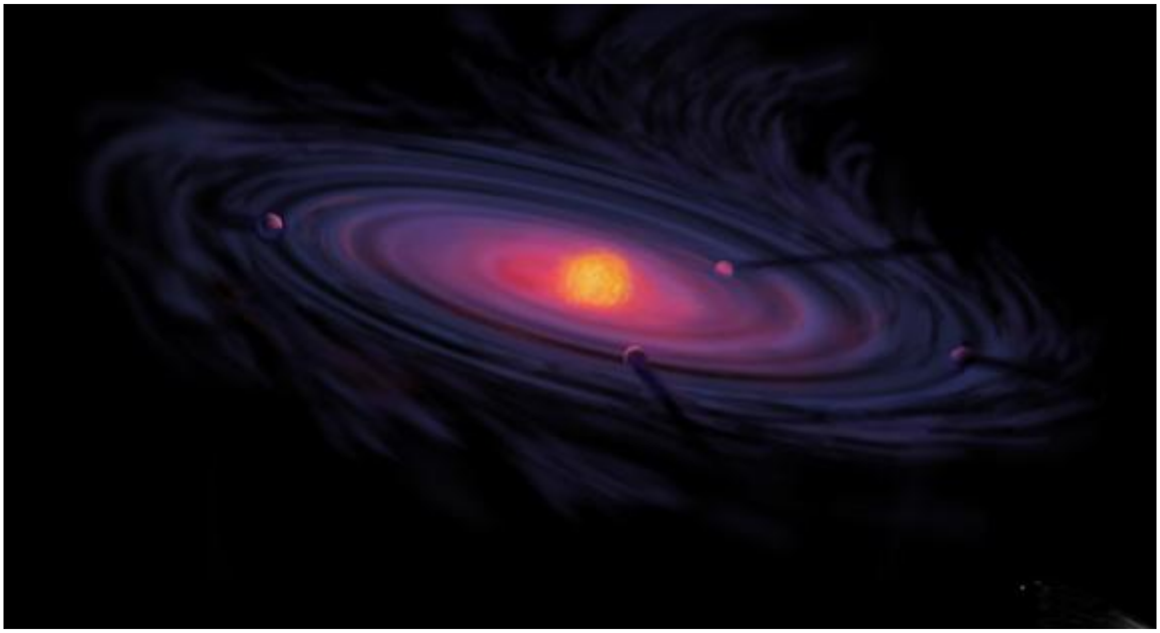


Figure 2 Artist Illustration of proto-planet formation in a Solar System

The inner Solar System ended the formation epoch with 50-100 size planetary embryos which collided and merged until forming the four terrestrial planets and their respective moons. This process would have required eccentric orbits of the large planetary bodies, a stark contrast from the nearly circular orbits of the modern planets. The leading hypothesis behind this is a gravitation wake made of smaller bodies that were caught by the gravitational pull of the large planetoids and formed tails that slowed them down and then either merged with their planet leaders or were slingshot off to form the asteroid belt. Large bodies did not form in the region currently known as the asteroid belt due to tidal forces from Jupiter's gravity which forced the bodies to shatter upon collision as opposed to accrete like the bodies of the inner solar system. Some of the larger bodies were forces out of the asteroid belt and impacted the inner planets. These objects are thought to have delivered the water currently found on Earth and hypothesized on Mars.

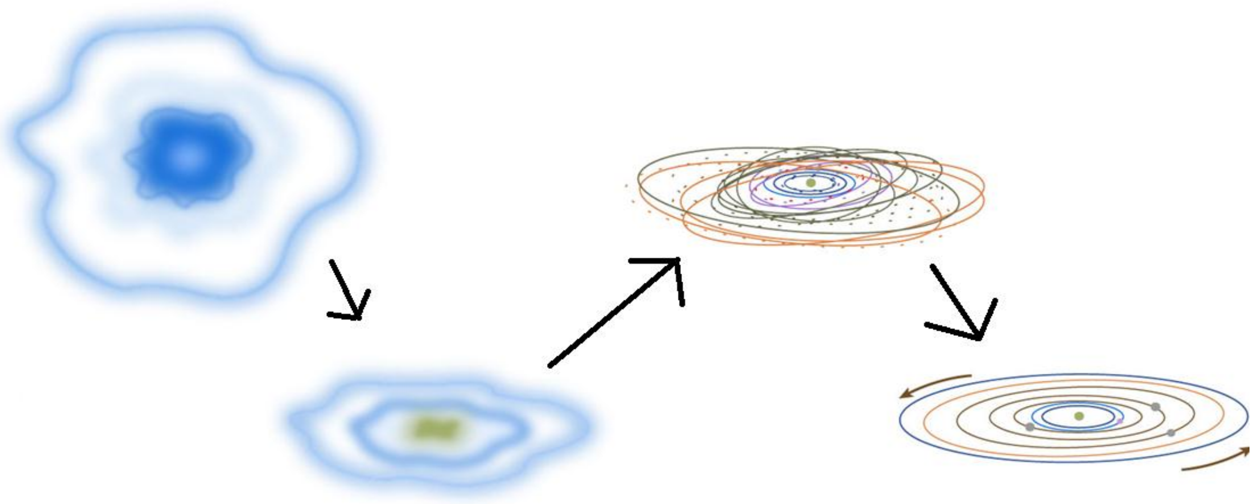


Figure 3 Diagram of the Formation of the Solar System

Constructing the Model

Following the derivation by Humi (Humi, 2006) we start with the hydrodynamic equations of inviscid and incompressible stratified fluid with **u**=(u,v) being the velocity vector, ρ is the density, p is the pressure and ψ is the gravitational field strength:

$$u_x + v_y = 0$$

$$u\rho_x + v\rho_y = 0$$

$$\rho(uu_x + vu_y) = -p_x - \rho\varphi_x$$

$$\rho(uv_x + vv_y) = -p_y - \rho\varphi_y$$

$$\nabla^2\varphi = 4\pi G\rho$$

The first equation describes the incompressibility of the cloud, the second details the conservation of mass. The third and fourth equations are the ones describing the momentum of the gas in the cloud and the final equation is that for the gravitational field of the cloud. All of the equations are nondimensionalized by scaling factors as follows:

$$x = Lx^{\sim}, \qquad y = Ly^{\sim}, \qquad u = U_0 u^{\sim}, \qquad v = U_0 u^{\sim}, \qquad \rho = \rho_0 \rho^{\sim}, \qquad p = \rho_0 U_0^2 p^{\sim},$$

$$\varphi = U_0^2 \varphi^{\sim}, \qquad G^{\sim} = G\rho_0 \frac{L^2}{U_0^2}$$

The very basic case first considered is the one where h =1 and S= 0. S is chosen as a simple case as S can be any function. The first part of the case states that for all ρ the square of the first ρ derivative of the stream function is 1/ρ.

$$h(\rho) = \rho\varphi_\rho^2 = 1$$

$$\varphi_\rho^2 = \frac{1}{\rho}$$

To derive the system of equations to use in Matlab we substitute in the values for h and S to obtain the following.

$$\nabla^2 \rho = -\varphi$$

$$\nabla^2 \varphi = c\rho$$

Converting these from Cartesian coordinates to polar coordinates and considering that both ρ and ψ are independent of the angle theta results in this new system via the following process.

$$\nabla^2 f = \frac{1}{r}\frac{d}{dr}(r\frac{df}{dr}) + \frac{1}{r^2}\frac{d^2 f}{d\varphi^2}$$

This results in the following second order differential equations. All derivatives of the angle cancel out leaving the equations just dependent on the radial derivatives.

$$\rho'' = -\varphi - \frac{\rho'}{r}$$

$$\varphi'' = c\rho - \frac{\varphi}{r}$$

This system of equations can also be solved analytically where α is $\sqrt{4\pi G}$

$$\rho(r) = -\frac{1}{\alpha^2}(\alpha\,(-C_1 * J_0(\sqrt{-\alpha} * r) - C_2 * Y_0(\sqrt{-\alpha} * r) + C_1 * J_0\left((-\alpha^2)^{1/4} * r\right) + C_2 * Y_0((-\alpha^2)^{1/4} * r)$$

The coefficients are alternating C_1 and C_2 only because we are looking for a real solution.

Sturm-Liouville Theory

These two equations need to satisfy some boundary conditions at both the interior of the cloud and the exterior, and we are hoping to find cyclic density of some degree in order to demonstrate multiple planetoids forming. These conditions make our problem and optimal candidate for a Sturm-Liouville problem approach. The classical Sturm-Liouville equation is (Sturm-Liouville theory, n.d.)

$$-\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + q(x)y = \lambda w(x)y$$

An S-L problem is said to be regular if all functions are continuous on [a,b] and p(x), w(x) > 0 and has boundary conditions of the form (Sturm-Liouville theory, n.d.)

$$\alpha_1 y(a) + \alpha_2 y'(a) = 0$$

$$\beta_1 y(b) + \beta_2 y'(b) = 0$$

Under these conditions, the eigenvalues of the problem are all real and can be strictly ordered. Each eigenvalue λ_n also has a unique eigenfunction with exactly n-1 zeroes in the interval (a,b) and these eigenfunctions form an orthonormal basis (Sturm-Liouville theory, n.d.) Therefore in a system in which we hope to find x number of planetoids forming, we look at the eigenfunction for λ_{2x+1} . This helps by providing an abstract foundation to our analytical problem.

Results

This system is solvable using the Matlab function bvp5c as demonstrated in the annotated Matlab function at the conclusion of the paper. The next step was to apply the function using selected boundary conditions.

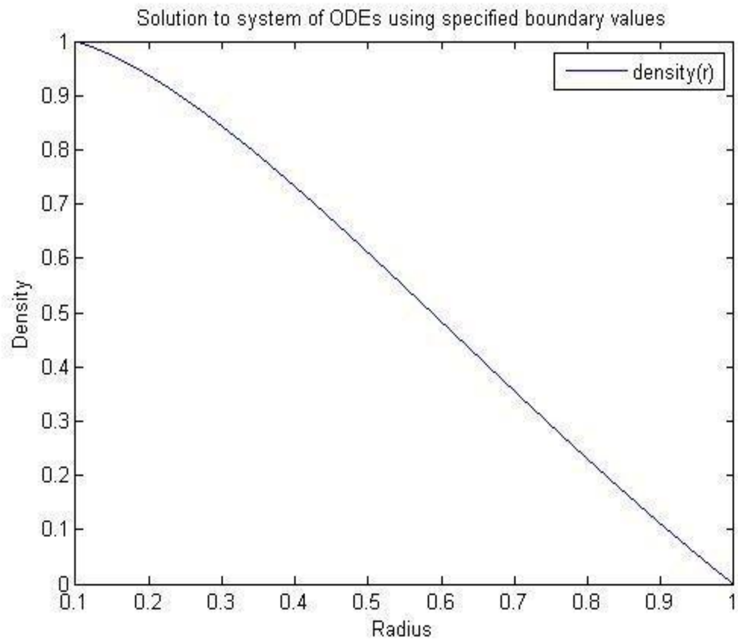
The first few boundary conditions were chosen with normalized values for starting density at the core, no pressure at r tended to 1 (a normalized boundary) and the gravitational field was normalized to small values at the core with no gravity influence at the outer boundary. In figure set one the boundary conditions used are:

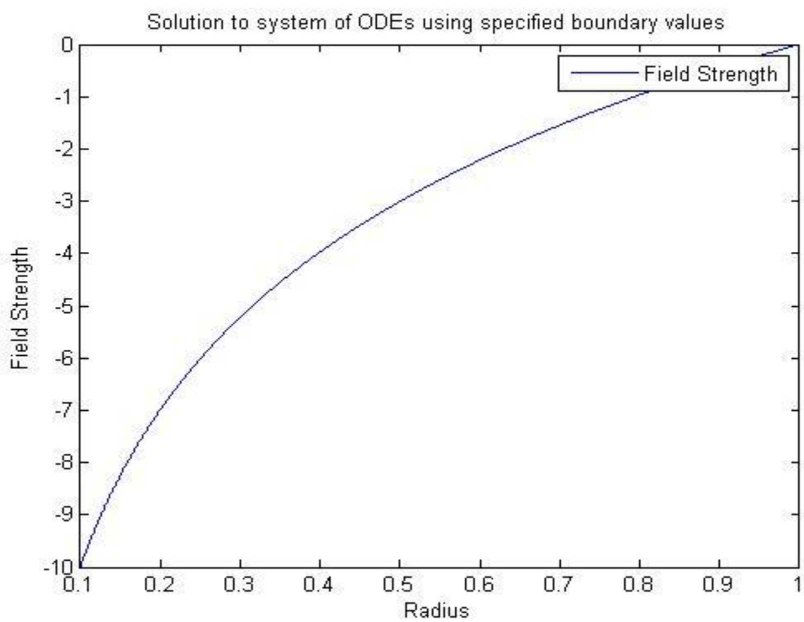
$$\rho(.1) = 1$$

$$\rho(1) = 0$$

$$\varphi\,(.1) = -10$$

$$\varphi\,(1) = 0$$


Solution to system of ODEs using specified boundary values
density(r)
Density
Radius
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Solution to system of ODEs using specified boundary values
Field Strength
Field Strength
Radius
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

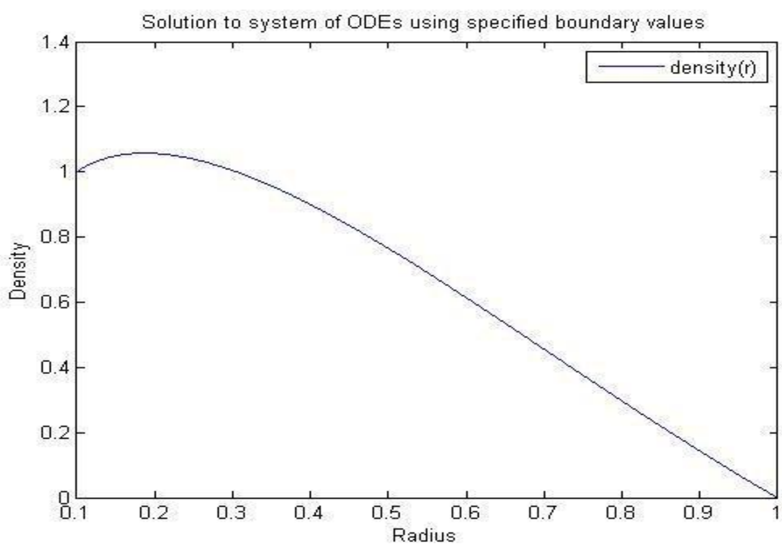
The curve is not quite linear in nature so the thought process behind choosing the next boundary values was to see if the linearity was due to the ratio of density at the core and the gravity field strength being 1 to 10. In figure set two the following boundary conditions were considered:

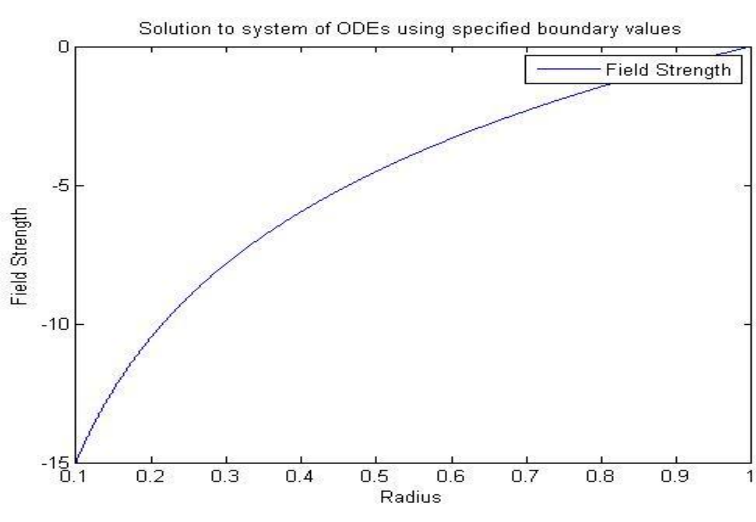
$$\rho(.1) = 1$$

$$\rho(1) = 0$$

$$\varphi(.1) = -15$$

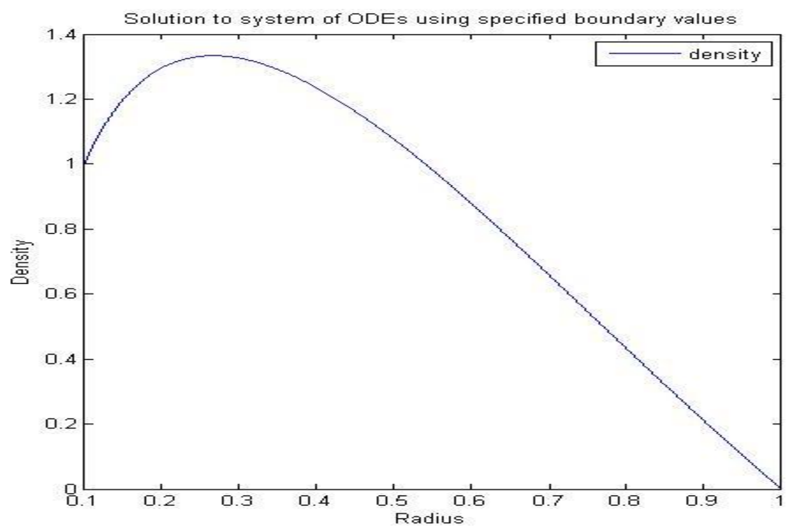
$$\varphi(1) = 0$$

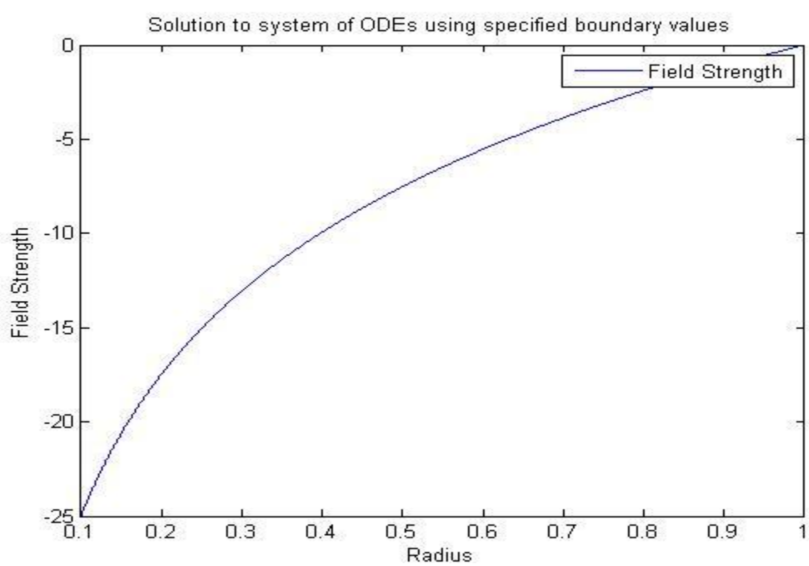




As visible in the first figure, the shape taken by the density does indeed have a local maximum off the core, however it is still too close to the core to be considered an off-core density maximum we are looking for. In an attempt to accentuate this maximum the ratio between the density and the gravitational field strength at the core was increased from 1:15 to 1:25 in figure set three resulting in the following boundary conditions:

$$\rho(.1) = 1$$
$$\rho(1) = 0$$
$$\varphi\,(.1) = -25$$
$$\varphi\,(1) = 0$$





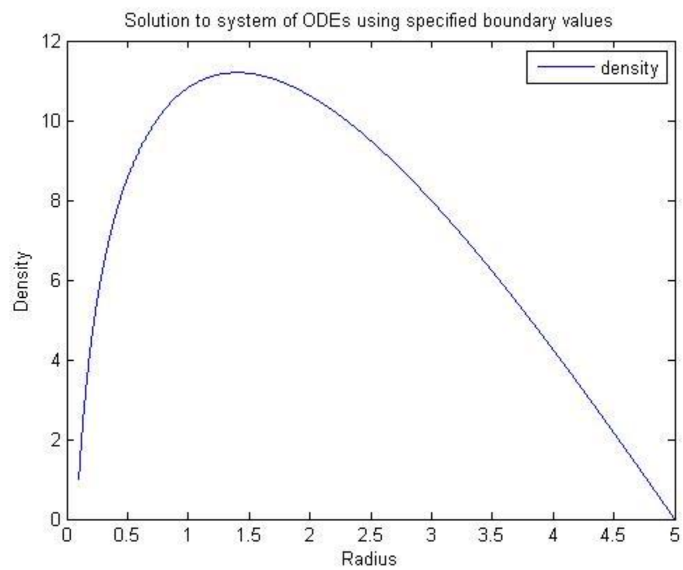
$$\rho(.1) = 1$$

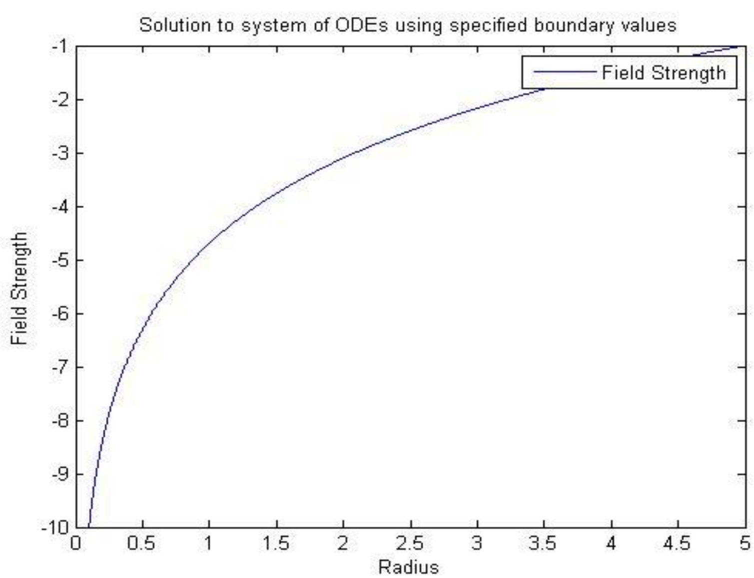
$$\rho(5) = 0$$

$$\varphi\,(.1) = -10$$

$$\varphi\,(5) = -.1$$

$$.1 \leq r \leq 5$$





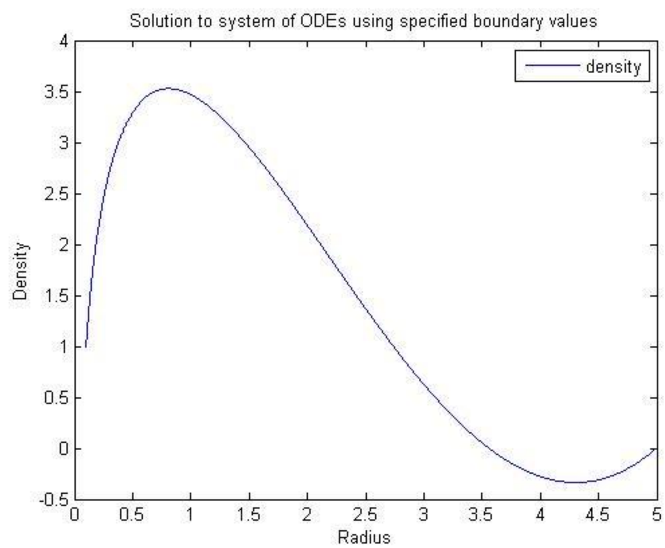
$$\rho(.1) = 1$$

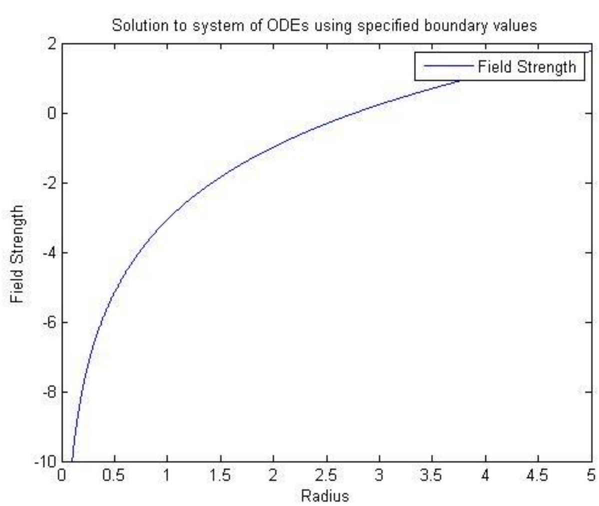
$$\rho'(5) = 1$$

$$\varphi\,(.1) = -10$$

$$\rho(5) = 0$$

$$.1 \le r \le 5$$





The result is a promising description of a more well-defined binary star system than in our past results. Modifying these conditions should produce even more well-defined systems and possibly a system with a smaller proto-star or possibly even planetoid forming.

Now, all four conditions are assumed to be related to the condition of the density while the gravitational field strength is just determined by the system of equations. This still follows logically as we scaled the size of the initial protostar anyways, so the gravity field would be directly related to this scaling. With the ability to now control both the value of the density at both ends but the slope of the density curve, a shape more in line with our target may be reached.

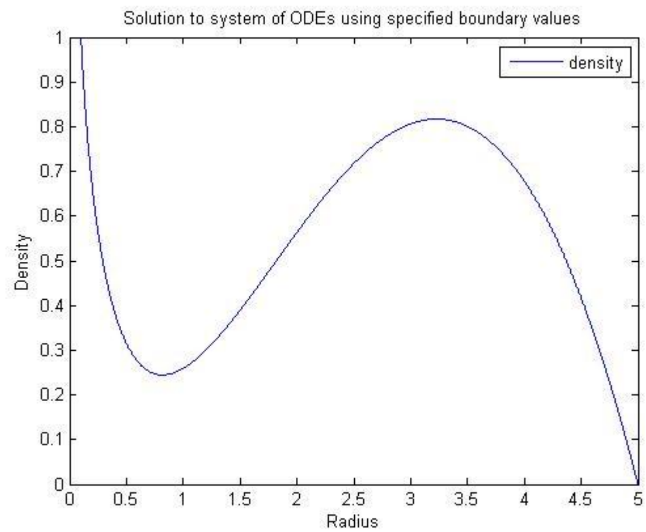
$$\rho(.1) = 1$$

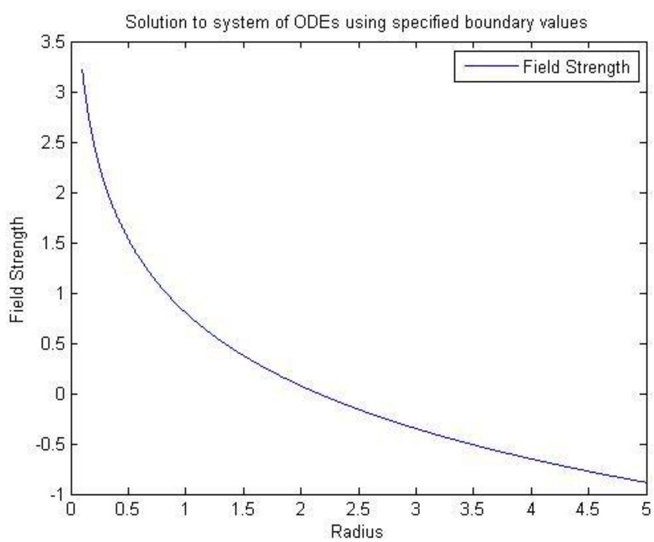
$$\rho'(.1) = -5$$

$$\rho(5) = 0$$

$$\rho'(5) = -1$$

$$.1 \le r \le 5$$





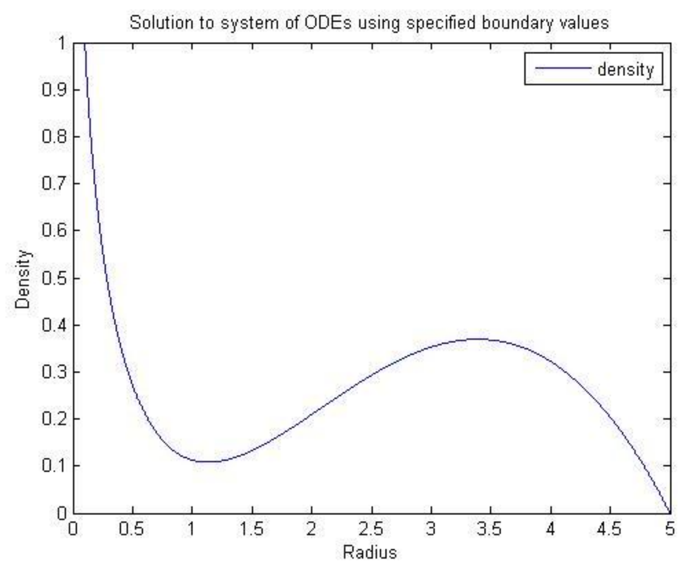
$$\rho(.1) = 1$$

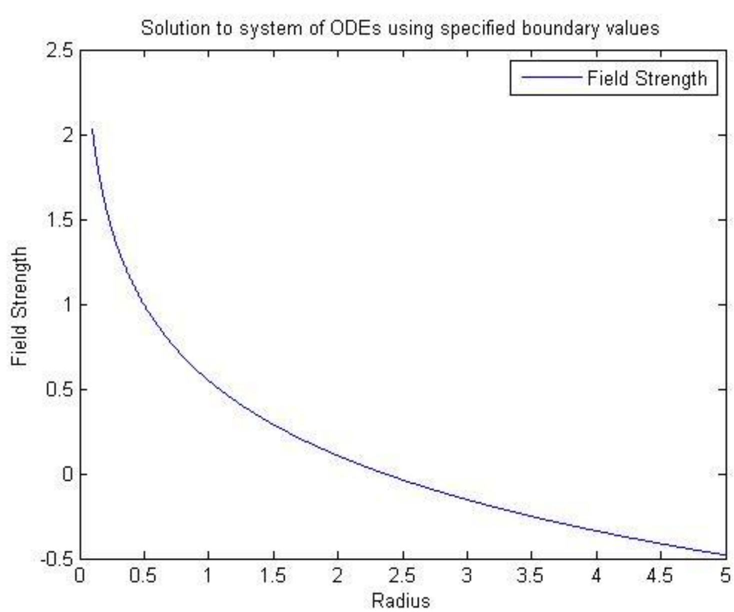
$$\rho'(.1) = -5$$

$$\rho(5) = 0$$

$$\rho'(5) = -.5$$

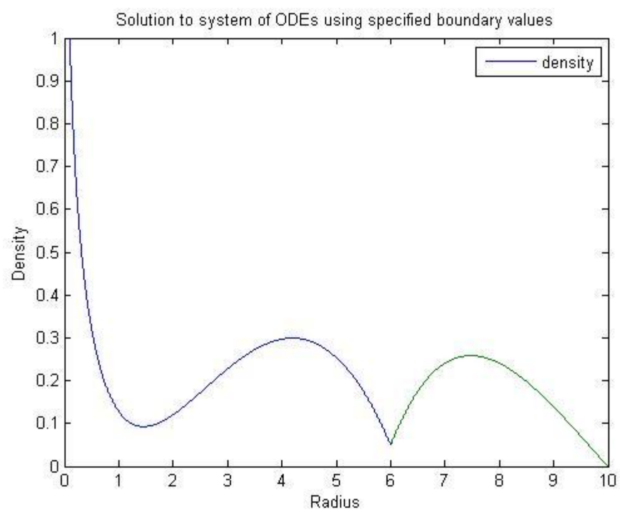
$$.1 \leq r \leq 5$$

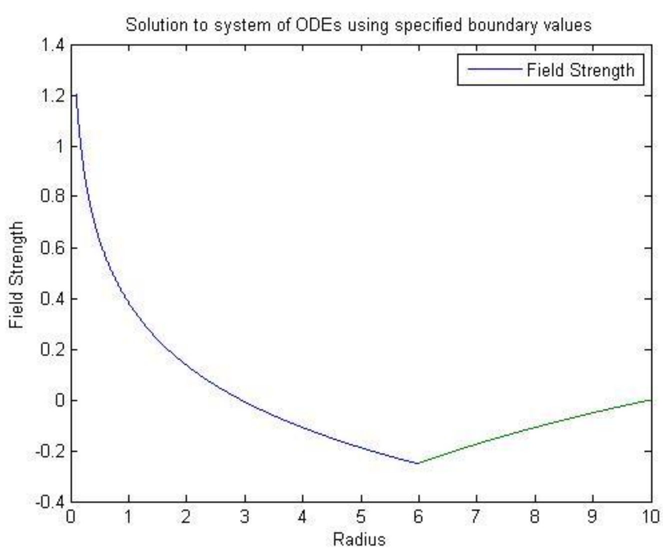




This is where we split into 2 separate portions of the solution

Portion 1	Portion 2
$\rho(.1) = 1$	$\rho(6) = .05$
$\rho'(.1) = -4.5$	$\varphi\,(6) = -.3$
$\rho(6) = .05$	$\rho(10) = 0$
$\rho'(6) = -.3$	$\varphi\,(10) = 0$
$.1 \le r \le 6$	$6 \le r \le 10$

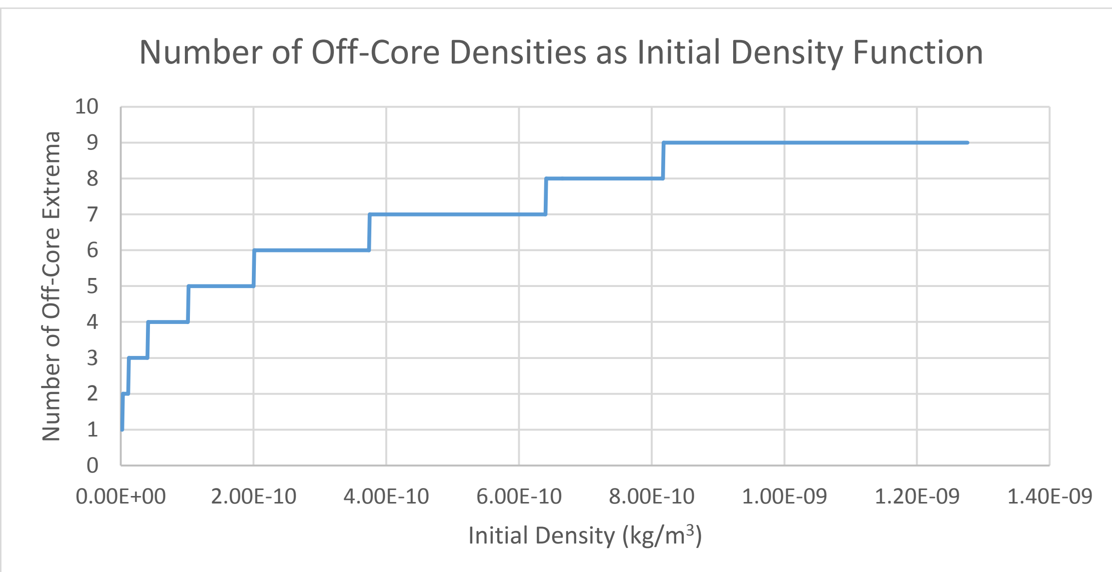




This result, while promising, exhibits this behavior because of our chosen characteristics of the initial cloud of gas. In these cases the initial density of the cloud has been so small that the scaling factor on the density in the equations approached zero, causing the density to be negligible in the solution. To remedy this I looked at the term itself and the constants and variables it depended on.

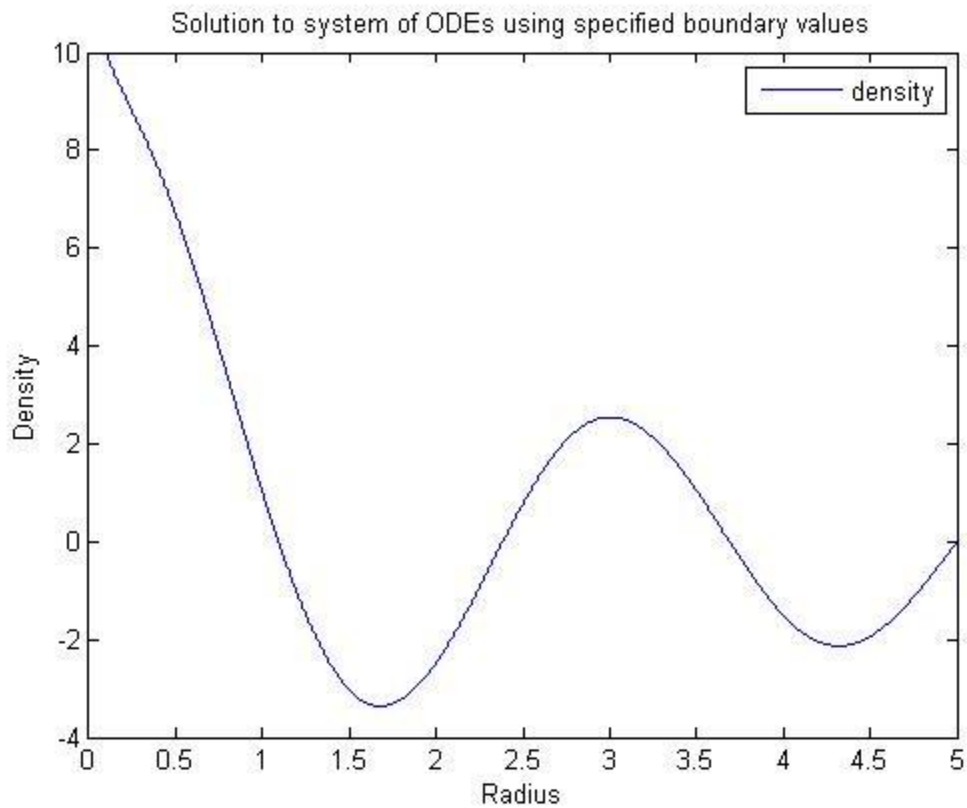
$$G = G_{constant}\rho_0 \frac{L^2}{U^2}$$

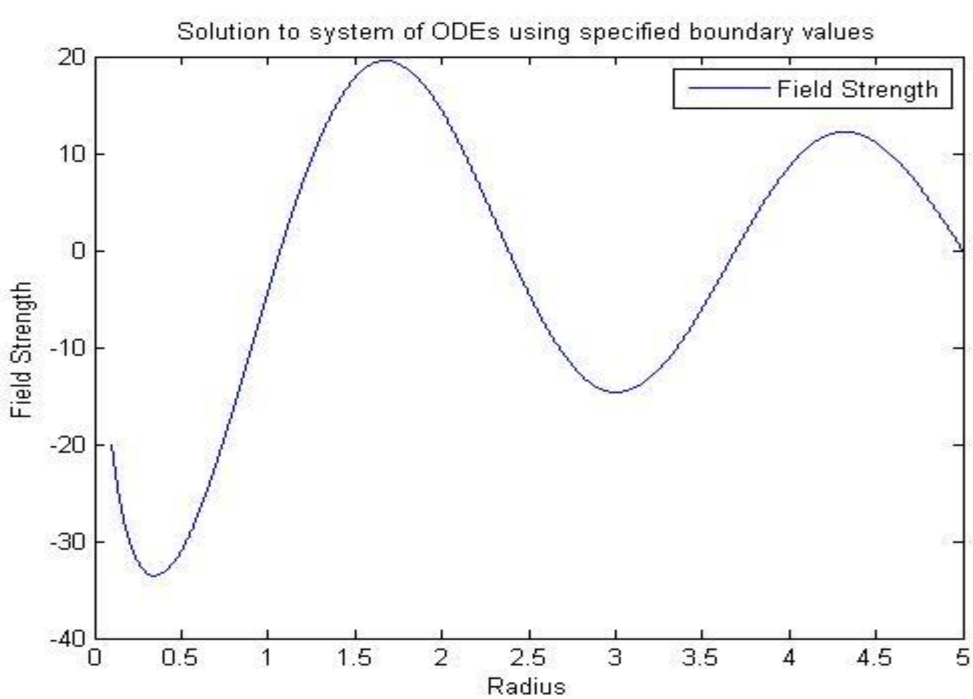
The L value is the diameter of the cloud in m, but obviously the diameter of our solar system is not the same as it was when it was forming. To determine the initial diameter of I looked at studies of stellar nurseries to estimate the average size of a protoplanetary nebula. The vast majority of such nebula were in the range of 50-150 AU (Rost, Eckart, & Ott, 2005) so a size of 100 AU is chosen. The U value is also estimated to be in the range of 30 m/s. I then use these values and the known value of the Gravitational Constant to graph the number of resulting off-core densities resulting from such a G value.



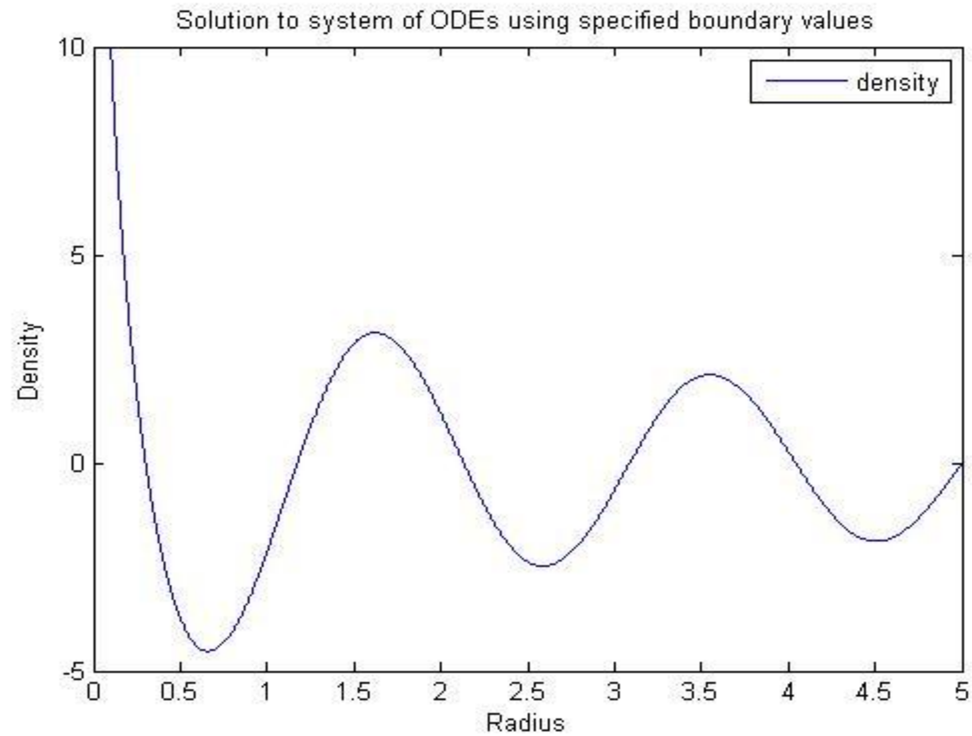
The following graphs are representative of the groups with same number of off-core densities. The scaling is variable and thus can be altered to better fit realistic models, however the shape matches our desired results.

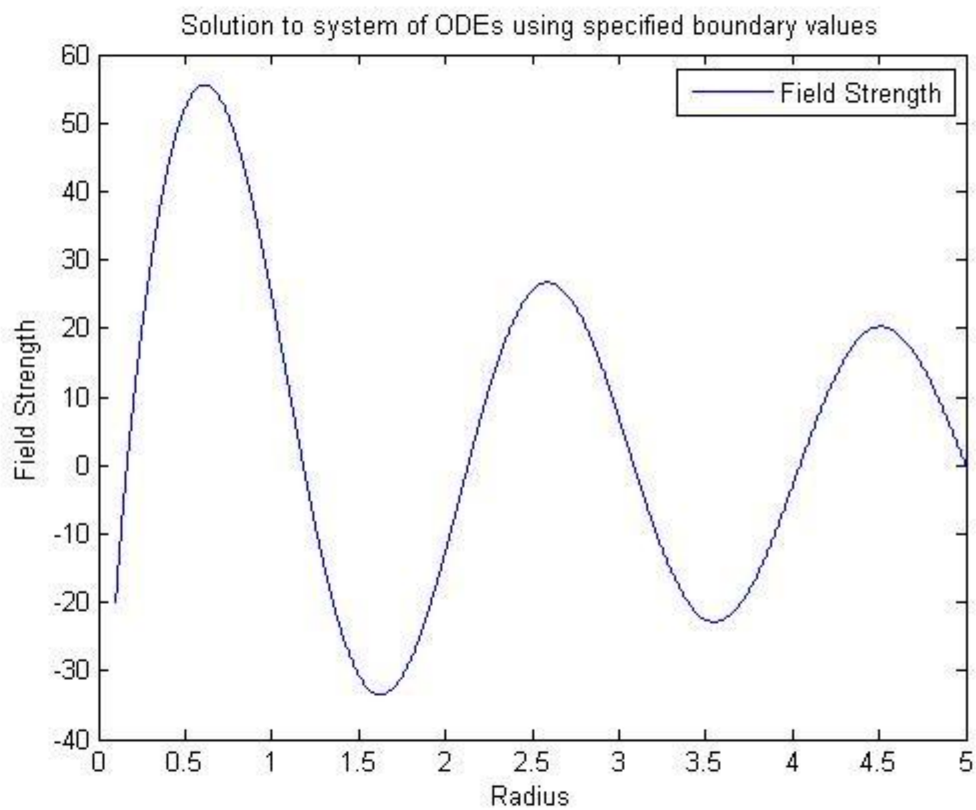
Initial Density = $2*10^{-12}$ kg/m^3



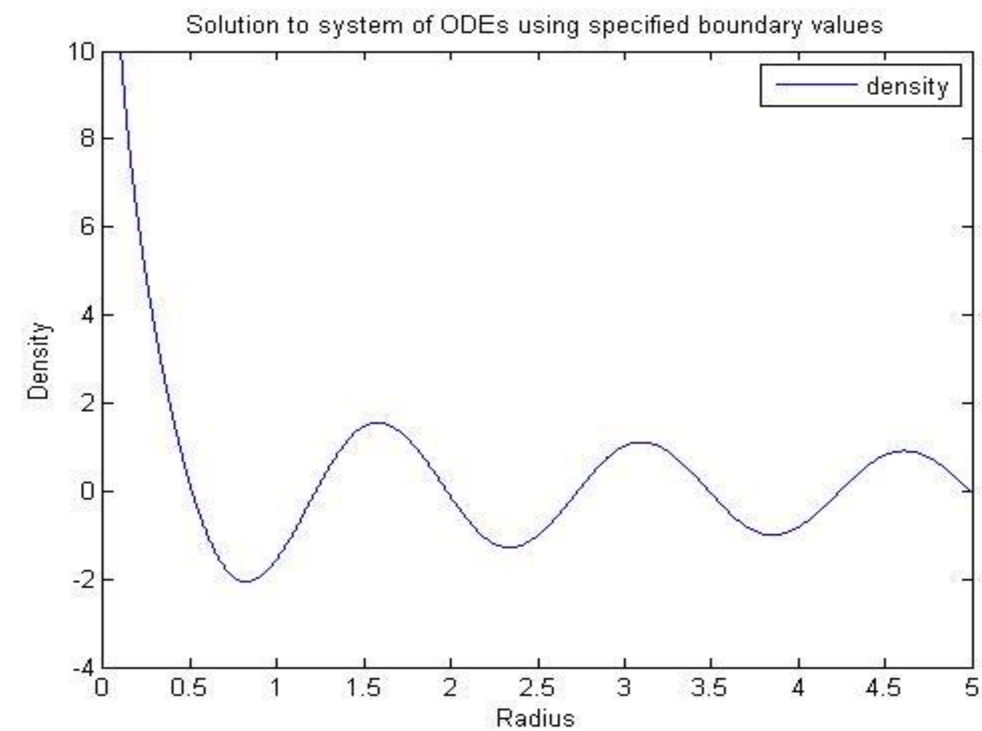


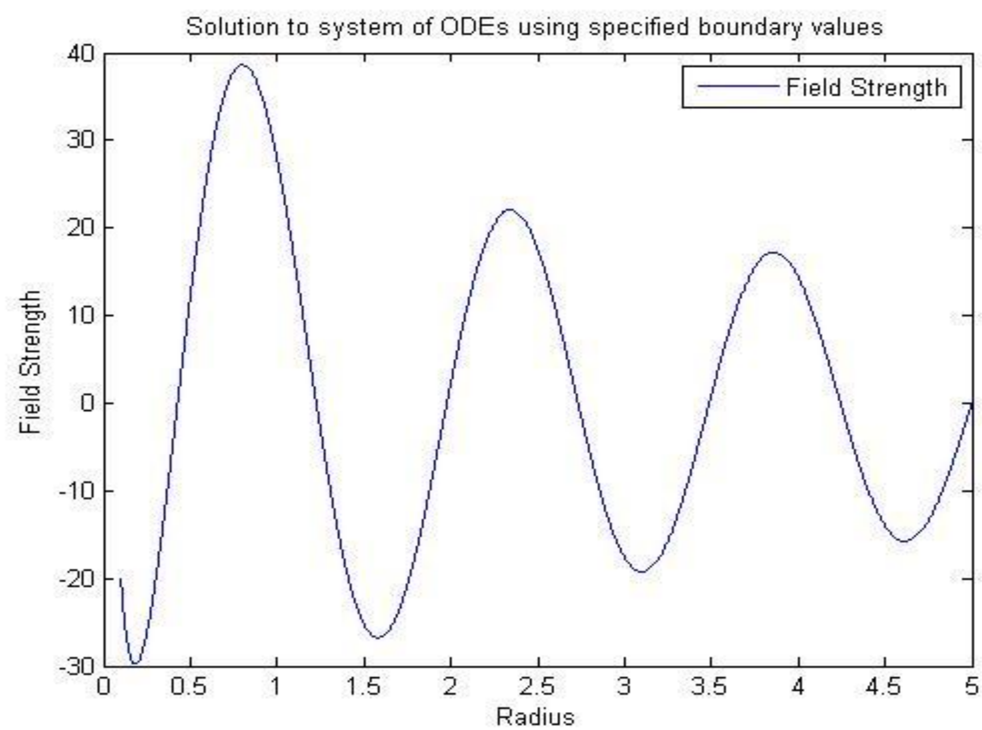
Initial Density = $7*10^{-12}$ kg/m^3



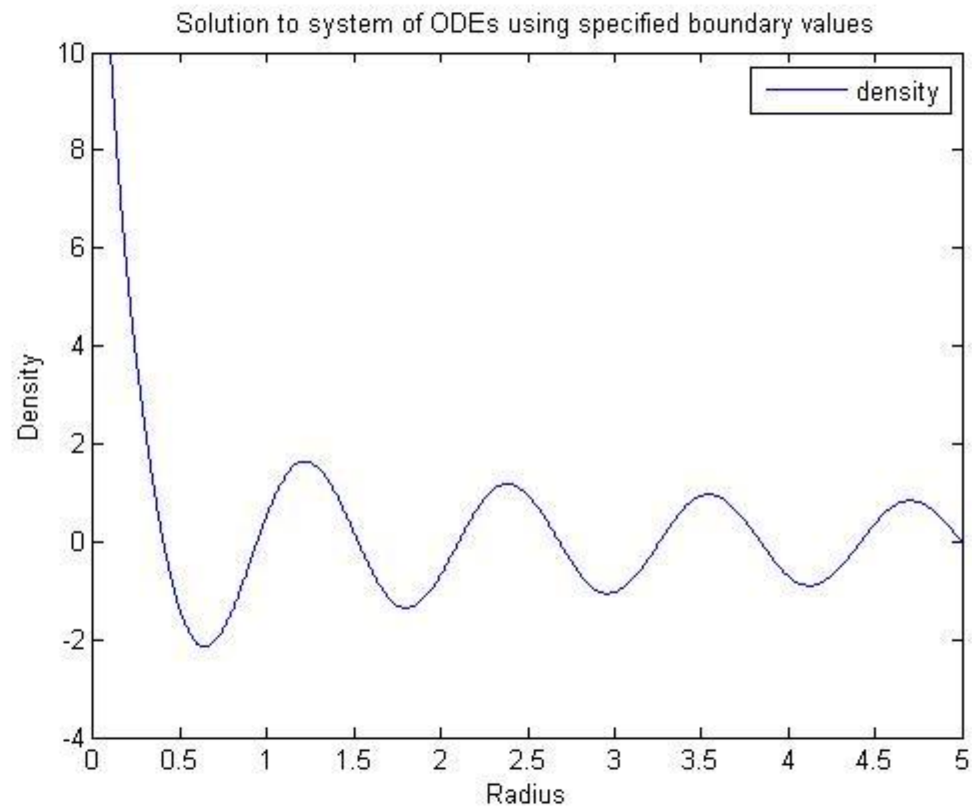


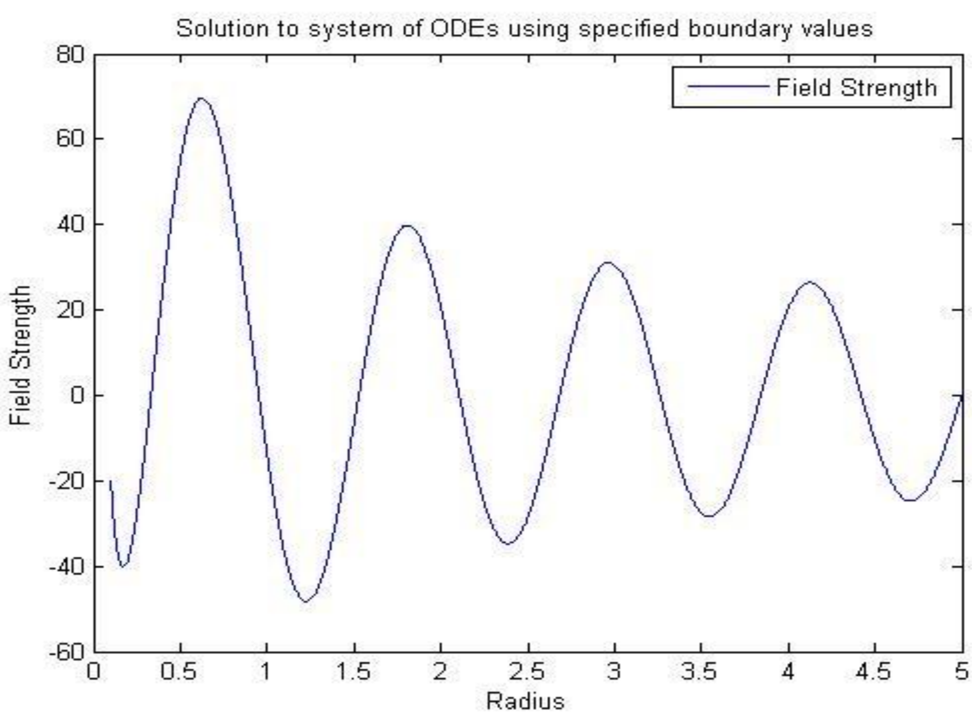
Initial Density = $1.8*10^{-11}$ kg/m^3



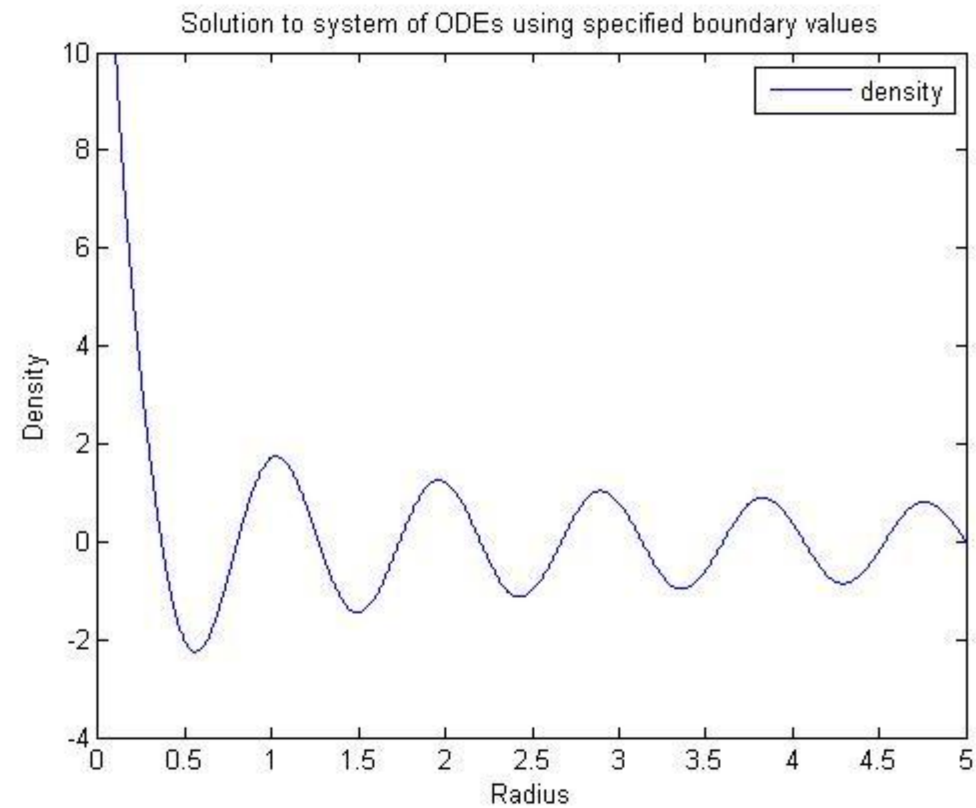


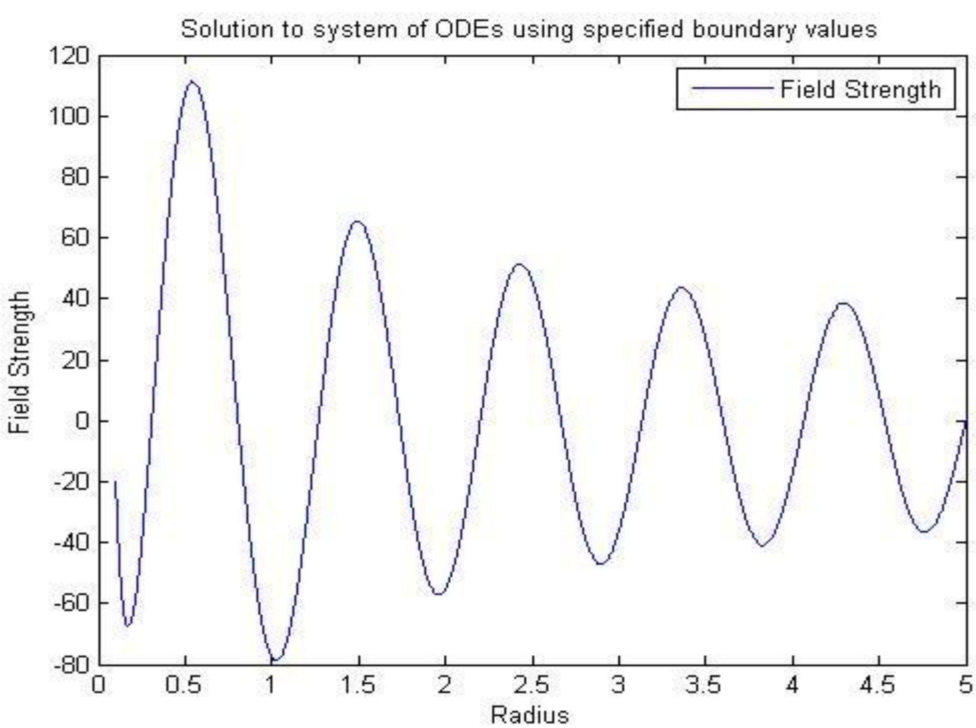
Initial Density = $5.3*10^{-11}$ kg/m^3



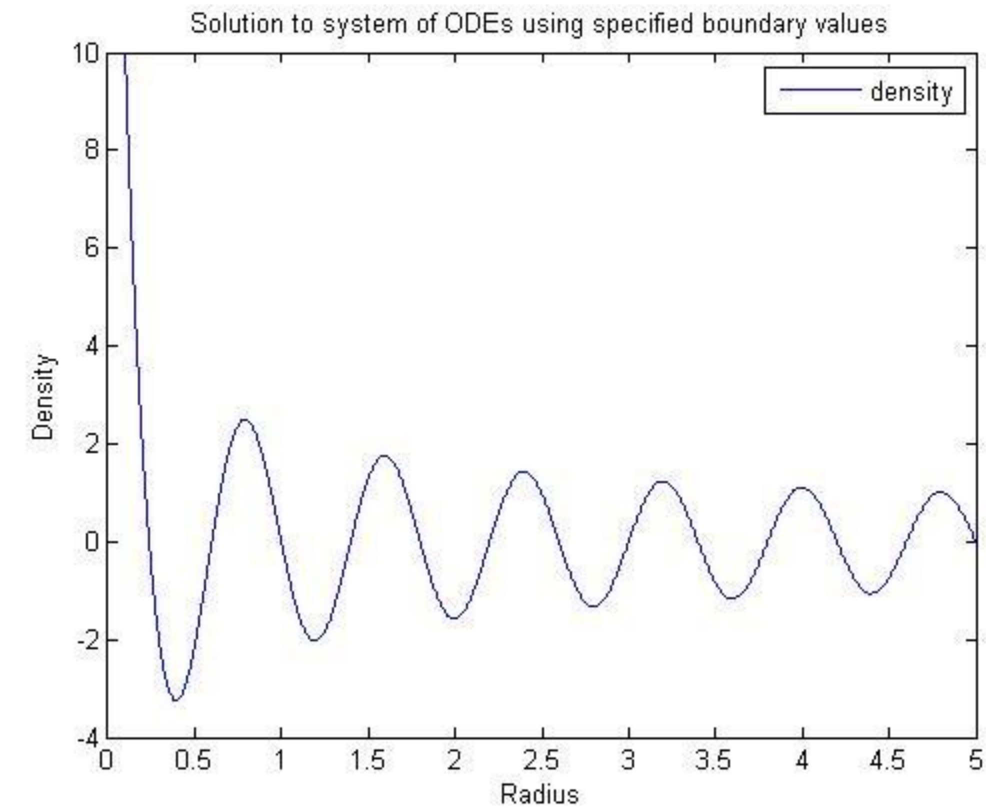


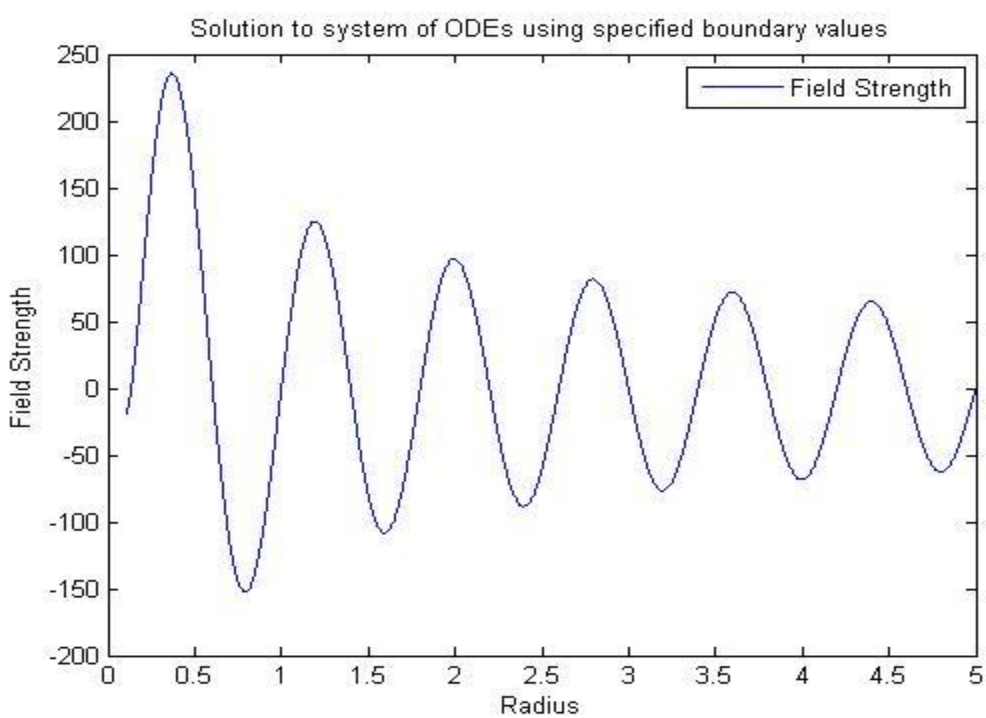
Initial Density = $1.24*10^{-10}$ kg/m^3



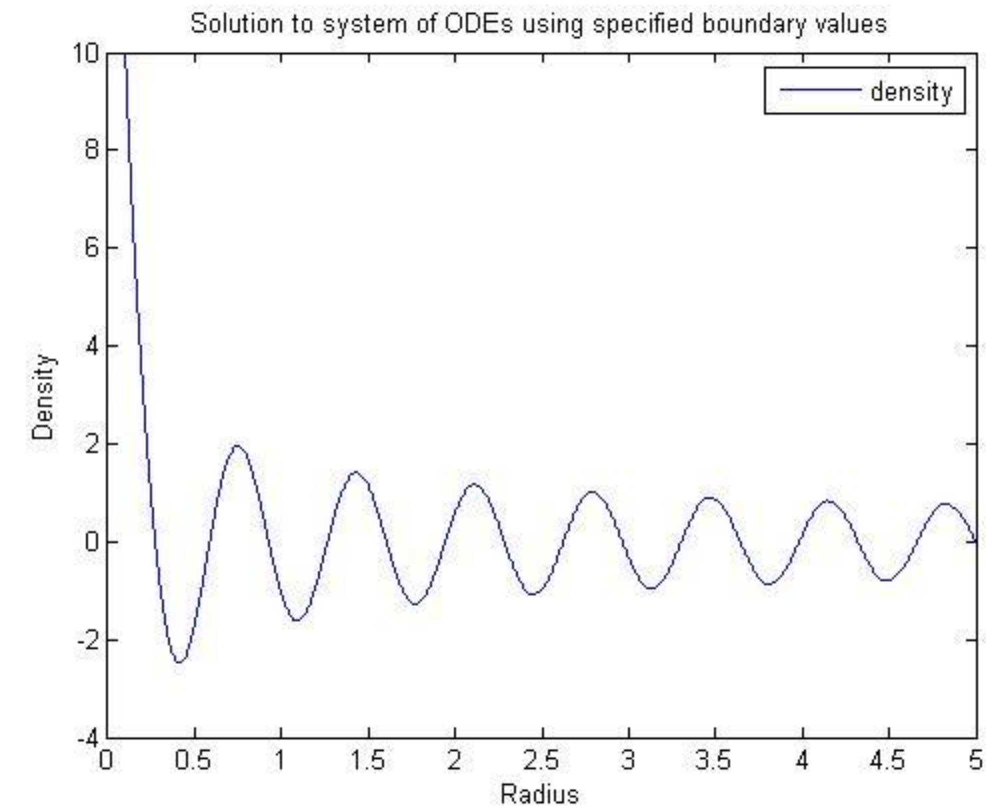


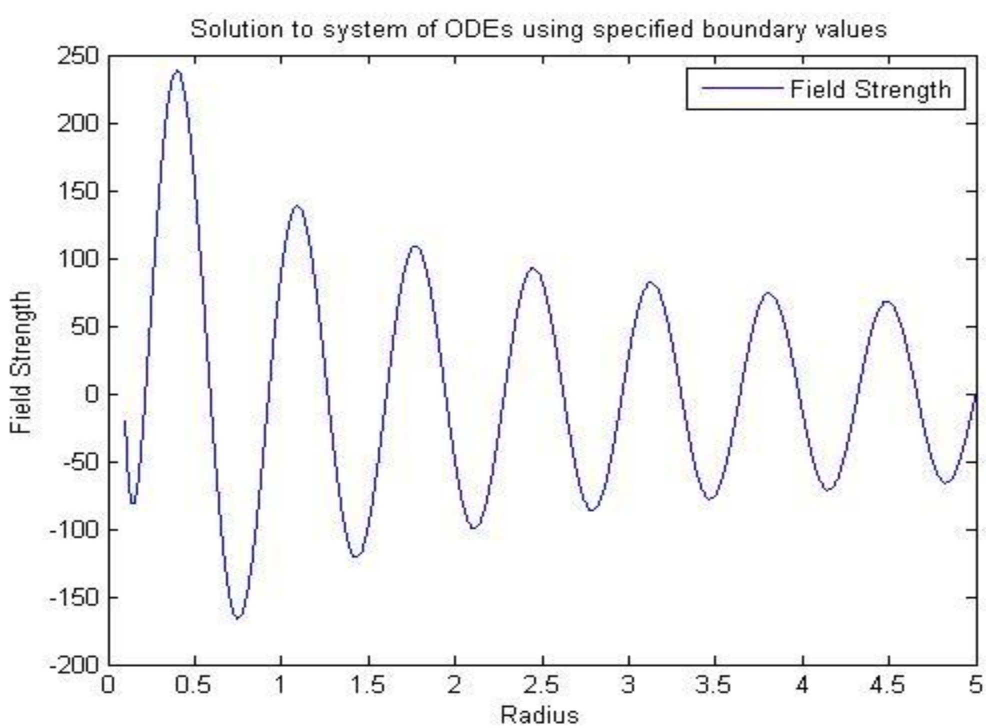
Initial Density = $2.29*10^{-10}$ kg/m^3



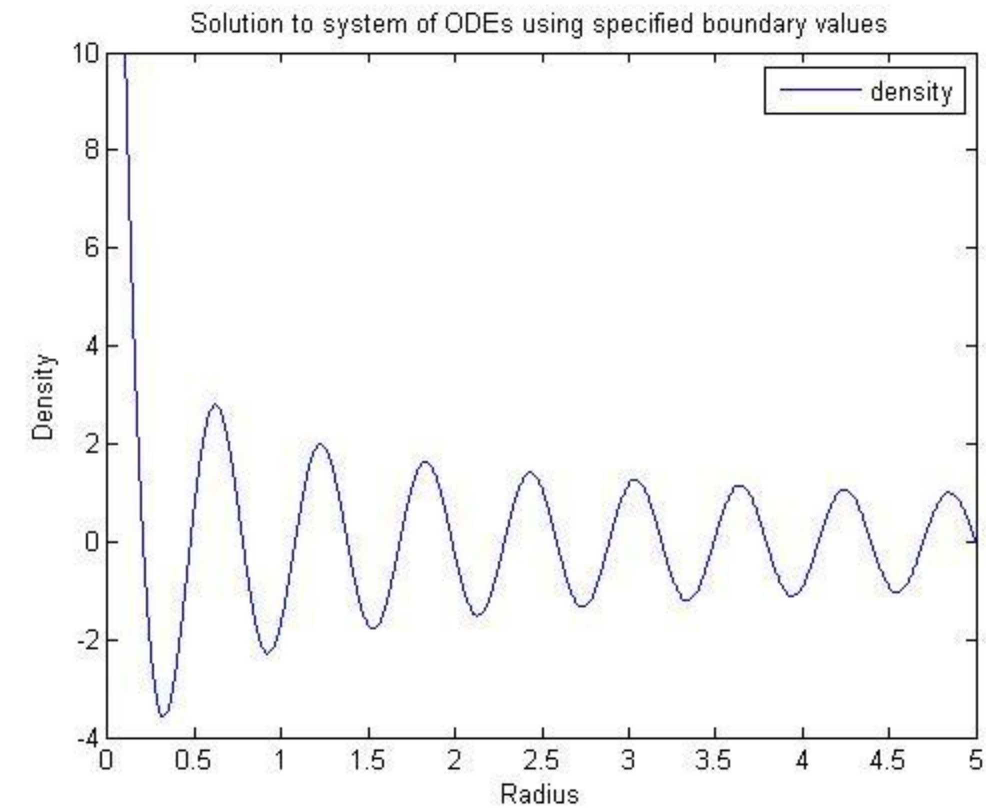


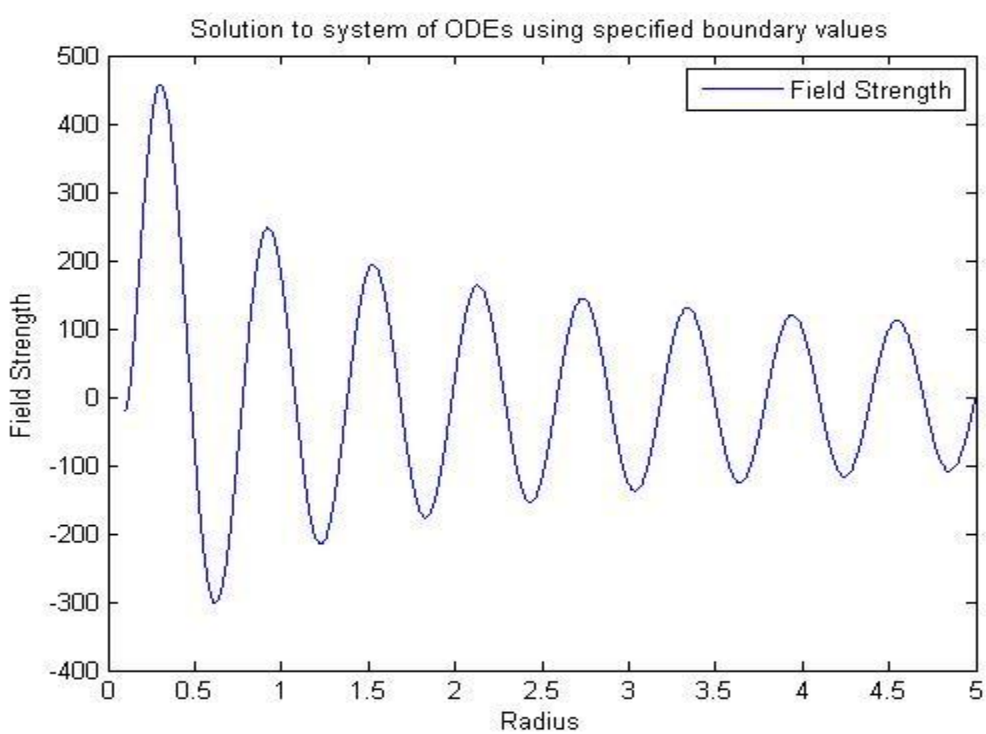
Initial Density = $4.42*10^{-10}$ kg/m^3



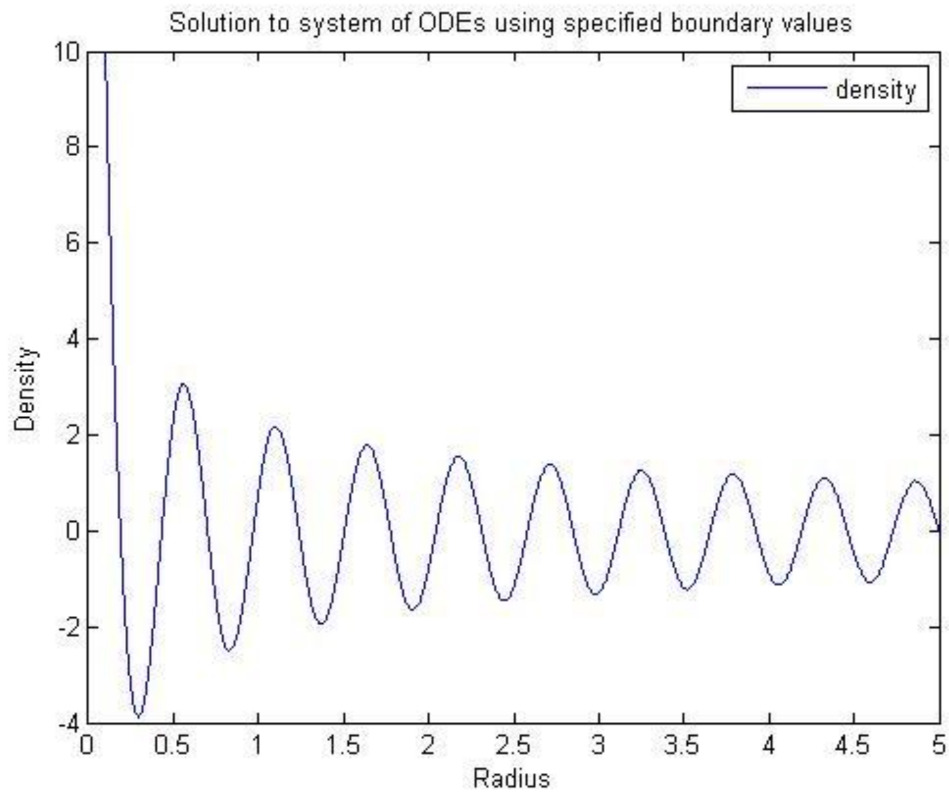


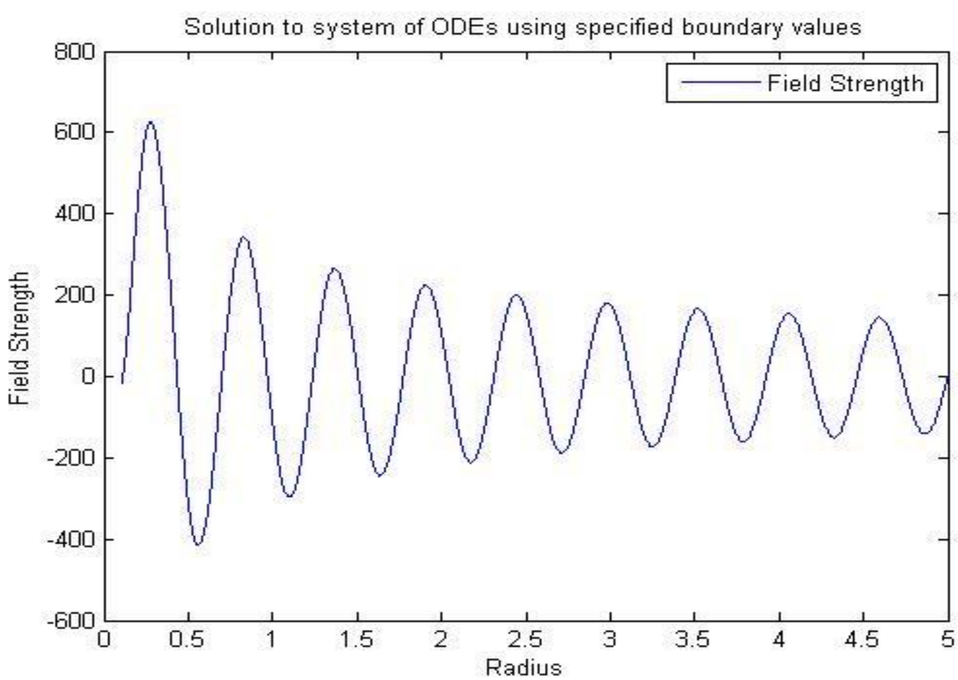
Initial Density = $7.09*10^{-10}$ kg/m^3





Initial Density = $1.125*10^{-9}$ kg/m^3





Conclusion

The model produced depicts the formation of planetoids in a newly formed solar system with a direct variance between the initial density of the gaseous disc and how many planetoids possibly are forming. The scope of the project was to consider a singular, average proto-star with accompanying homogenous disc surrounding it to find how many planetoids could possibly form in such a system. This was accomplished by modeling the disc around the proto-star and finding the steady state solution to the equations describing the gas and looking for points of density off of the core with greater values than the surrounding thus indicating the clumping of gas. This gaseous clumping should theoretically through gravitational accretion form planetoids according to the Nebula Theory.

While the model clearly shows that the number of planets forming around a proto-star is related to the initial density of the gaseous disk surrounding the proto-star, the model is far from perfect. The model is a two dimensional simplification of a process in a three dimensional space. This simplification is why the planetoids formed all in the same plane, a very unlikely result practically and unlike our own Solar System. The consideration of a radially symmetric cloud of homogenous gas also eliminated w wide number of variables present in real-world examples. Very few stars form on their own, and often form in clusters even in binary systems, which this model also does not account for. Future work is planned by Professor Mayer Humi of the Mathematics Department of Worcester Polytechnic Institute to further develop the model and consider more of the variables involved in planetoid formation.

Appendix

Matlab Function

```
function bscattmp2
 %--------------------------------------------------
 % h(p) = 1 ---------------------- S(p)=0
 % r is radius
```

```
 % p is density
 % b is gravitational field
 %--------------------------------------------------


 %--------------------------------------------------
 % Initial solution required by bvp5c
 %--------------------------------------------------
solinit = bvpinit(linspace(0.1,1,2500),@a2init);

 %--------------------------------------------------
 % The solving of the reduced system using bvp5c
 % followed by the plotting of the result  %--------
-----------------------------------------  sol =
bvp5c(@a2ode,@a2bc,solinit);

 x = linspace(0.1,1,2500);
y = deval(sol,x);  v =
y(1,:);  plot(x,v)
 xlabel('Radius');  ylabel('Density');  legend('density(r)');
title('Solution to system of ODEs using specified boundary values');

shg

 %--------------------------------------------------
 % This is the equation vector with the following allocation
 % r = r
 % p = y(1)
 % p' = y(2)
 % p'' = y(3)-y(2)/r
 % b = y(3)
 % b' = y(4)
 % b'' = y(1)-y(4)/r
  %--------------------------------------------------
function dydx = a2ode(r,y,c)          c =
4*pi*(6.67e-11);         dydx = [y(2)
y(3)-y(2)/r                 y(4)
                c*y(1)-y(4)/r];
```

```
  %---------------------------------------------------
  % This function is the boundary value conditions on the interval [a b]
  % p(.1) = x* {gotten from your paper, eqn 4.3}
  % b(.1) = y* {mass of the star normalized to 1}
  % p(1) = 0 {edge of the system}
  % b(1) = 0 {from the paper you sent me this week}
%---------------------------------------------------
function res = a2bc(ya,yb)             res = [ya(1)-
x*                    ya(3)+y*
yb(1)                    yb(3)
                  ];
  %---------------------------------------------------
  % This function is the 'guess' required by bvp5c
  % for simplicty I just used a vector of ones   %--
-----------------------------------------------
function v = a2init(r)
            v = [1
1
                1
                1
                ];
```

Figure Sources

All Figures, images and graphics are either original creations or reused with permission

Figure 1: created by Peter Dowling

Figure 2: http://origins.jpl.nasa.gov/stars-planets/ra4.html

Figure 3: created by Peter Dowling

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