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WIDEBAND MATCHING NETWORK FOR A BLADE MONOPOLE

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Daniel David Harty

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Professor Sergey N. Makarov, Major Advisor

1. Impedance matching
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Abstract

This project investigates a novel technique for wideband impedance matching of short blade monopoles in the VHF-UHF bands using a simple network of five discrete components. This network is of one fixed topology consisting of an inductive L-section cascaded with a high-pass T-section and is effectively used with monopoles of differing shapes and matching bandwidths. A matching network of minimal complexity (and loss) such as this, is desirable for its practical realization and straightforward design.

Introduction

Applications involving wideband impedance matching to non-resonant monopole or dipole antennas are steadily increasing. While current impedance matching techniques involve modifying the antenna structure, this can be difficult to design, build, and analyze. It would be beneficial to solve the problem of wideband matching without any modifications involving the antenna geometry. Therefore, the use of a simple lumped element matching network to achieve the same desired matching performance is advantageous. Additionally, it could serve as the basis for further development of adaptive matching networks that modify the antenna response dynamically.

This project investigates a novel technique for wideband impedance matching of short blade monopoles in the VHF-UHF bands using a simple network of five discrete components. This network is of one fixed topology consisting of an inductive L-section cascaded with a high-pass T-section and is effectively used with monopoles of differing shapes and matching bandwidths. A matching network of minimal complexity (and loss) such as this, is desirable for its practical realization and straightforward design.

Theoretical background

Impedance matching

The standard impedance matching techniques in the VHF-UHF bands often utilize L, T, Π sections of reactive lumped circuit elements to match to a generator with a fixed generator resistance of $50\,\Omega$; this is preferable since lumped elements have a smaller size. Unfortunately by themselves, these circuits are only useful for narrowband matching and are often non-applicable for 20 % bandwidth or greater. An alternative approach to the conventional narrowband matching technique is to cascade two stages of lumped element sections. The first stage is an L-section with two inductors, shown to have excellent double-tuning performance for impedance matching at a specific frequency over a wide range of frequencies. The second stage is a high-pass T-section with a shunt inductor and two capacitors, effectively broadening the narrowband response of the L-section. In total, the matching circuit has five lumped elements: three inductors and two capacitors.

Antenna impedance model

For a wire or strip dipole, the input impedance, Z_A can be approximated with a high degree of accuracy [16] as

$$
\begin{aligned}
Z_A &= R(z) - j\left[120\left(\ln\frac{l_A}{2a} - 1\right)\cot z - X(z)\right] \\
R(z) &\approx -0.4787 + 7.3246z + 0.3963z^2 + 15.6131z^3 \\
X(z) &\approx -0.4456 + 17.00826z - 8.6793z^2 + 9.6031z^3
\end{aligned}
\tag{1a}
$$

In Eq. (1a), l_A is the dipole length, a is the dipole radius, $z = kl_A/2$ with $k = 2\pi/\lambda$ being the wavenumber. The accuracy of Eq. (1a) quickly degrades above the first resonance [16]; thus, at the high-frequency end, very small dipoles cannot be considered. At the lower end, Eq. (1a) is only valid when the dipole radiation resistance is positive and does not approach zero. This gives

$$z = kl_A / 2 > 0.07 \text{ or } \frac{l_A}{0.5\lambda} > 0.05 \tag{1b}$$

If a strip or blade dipole of width t, *is* considered, then $a_{\text{eq}} = t/4$ [17]. We note here that *a*, is the radius of a cylindrical dipole, and a_{eq} is the equivalent radius of a wire approximation to the strip dipole. Eq. (1) holds for relatively small non-resonant dipoles and for half-wave dipoles, i.e. in the frequency domain approximately given by

$$0.05 \leq f_C / f_{\text{res}} \leq 1.2 \tag{1c}$$

where $f_{\text{res}} \equiv c_0 /(2l_A)$ is the resonant frequency of an idealized dipole having exactly a half-wave resonance (c_0 is the speed of light) and f_C is the center frequency. When a monopole over an infinite ground plane is studied, the impedance is half. Therefore, Eq. 1 for the case studied becomes

$$Z_A^{blade} = 0.5R(z) - j\left[60\left(\ln\frac{2l_A}{t_{blade}} - 1\right)\cot z - 0.5X(z)\right] \tag{1d}$$

with the blade width, $t_{blade} = 4a_{cyl.dipole}$ or $a_{cyl.dipole} = t_{blade}/4$

Wideband impedance matching – the reflective equalizer

The reactive matching network is shown in Fig. 1a [11]. The generator resistance is fixed at 50Ω. This network does not include transformers. Following Ref. [11], the reactive matching network is included into the Thévenin impedance of the circuit as viewed from the antenna, see Fig. 1b.

In fact, the network in Fig. 1 is not a matching network in the exact sense since it does not match the impedance exactly, even at a single frequency. Rather, it is a reflective (but lossless) equalizer familiar to amplifier designers, which matches the impedance equally well (or equally "badly") over the entire frequency band. The equalizer network is reflective since a portion of the power flow is always being reflected back to generator and absorbed. Following Ref. [11], we can consider the generator or transducer gain in the form

$$T(\omega^2) = \frac{\text{Power to load}}{\text{Power to conjugate - matched load } Z_T^*} = \frac{4R_T(\omega)R(\omega)}{\left|Z_A(\omega) + Z_T(\omega)\right|^2} = 1 - \left|\Gamma(\omega)\right|^2 \qquad (2)$$

The gain T is the quantity to be uniformly maximized over the bandwidth, B. In practice, the minimum gain over the bandwidth is usually maximized [11], [14]. The problem may be also formulated in terms of the power reflection coefficient $\left|\Gamma(\omega)\right|^2$, viewing from the generator with the equalizer into the antenna. Obviously, the power reflection coefficient needs to be minimized. Note that the transducer gain is none other than the square magnitude of the microwave voltage transmission coefficient. In this text, we follow the "generator gain" terminology in order to be consistent with the background research in this area.

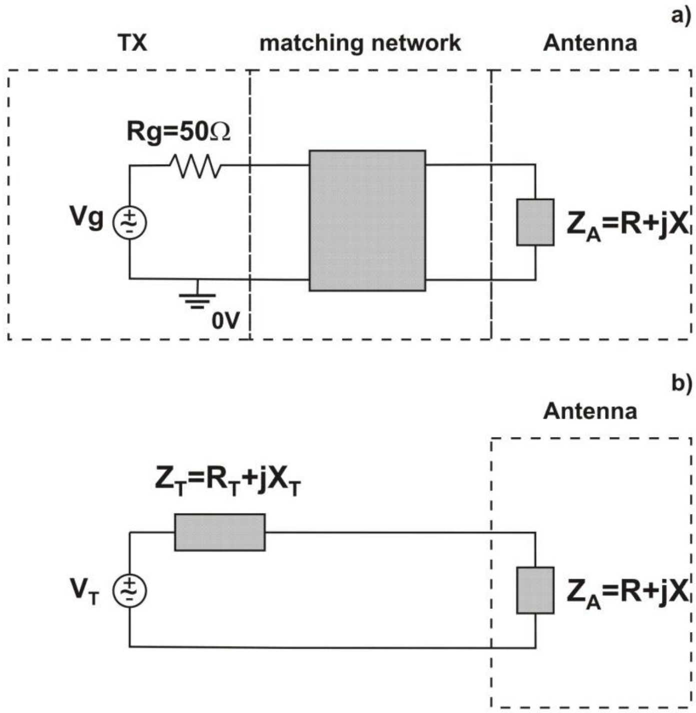

Fig. 1. Transformation of the matching network: a) reactive matching network representation, b) Thévenin-equivalent circuit representation. The matching network does not include transformers.

Bode-Fano bandwidth limit

The Bode-Fano bandwidth limit of broadband impedance matching ([4], [2]) only requires knowledge of the antenna's input impedance; it approximates this impedance by one of the canonic *RC*, *RL*, or *RLC* loads ([4], and [2], p. 262). The input impedance of a small- to moderate-size dipole or monopole is usually very similar to a series *RC* circuit, as seen from Eq. (1a). When $f_C / f_{\mathrm{res}} \le 0.5$ or $z = kl_A / 2 < 0.75$ (a small antenna or an antenna operated below the first resonance), the antenna resistance is usually a slowly-varying function of z (almost a constant) over the limited frequency band of interest whereas the antenna's reactance is almost a pure capacitance. This observation is valid at a common geometry condition: $\ln\left(l_A /(2a)\right) > 1.5$. The Bode-Fano bandwidth limit for such a *RC* circuit is written in the form [2]

$$\int_0^\infty \frac{1}{\omega^2} \ln\left[\frac{1}{|\Gamma(\omega)|}\right] d\omega < \pi RC \qquad (3)$$

For a rectangular band-pass frequency window $[f_C - B/2, f_C + B/2]$ of bandwidth B and centered at f_C, with $T = T_0$ within the window and $T = 0$ otherwise, Eq. (3) and Eq. (1a) allow us to estimate approximately the theoretical limit to the gain-bandwidth product as long as the dipole or monopole size remains smaller than approximately one quarter or one eight wavelength, respectively.

Small fractional bandwidth and small transducer gain

Let us first obtain the simple closed-form estimate for the gain bandwidth product. Using the expression for gain, T, in terms of power reflection coefficient $|\Gamma(\omega)|^2$, in (1) we rewrite (3) as,

$$\int_0^\infty \frac{1}{\omega^2} \ln\left[\frac{1}{\sqrt{1-T}}\right] d\omega < \pi RC \qquad (4)$$

Substituting $T = T_0$ over the bandwidth B and applying the appropriate limits for ω to the integral in Eq. (4) we arrive at

$$\ln\left[\frac{1}{\sqrt{1-T_0}}\right] \int_{\omega_L}^{\omega_U} \frac{1}{\omega^2} d\omega < \pi RC \quad , \ \omega_L = 2\pi f_C - \pi B, \omega_U = 2\pi f_C + \pi B \qquad (5)$$

Solving the integral in (5) yields

$$-\frac{1}{2}\ln(1-T_0) > \frac{\pi^2 RC\left(4f_C^2 - B^2\right)}{2B} \tag{6}$$

Next, we assume the fractional bandwidth $\overline{B} = B / f_C$ to be small, i.e. $\overline{B} < 0.1$; we also assume that $T_0 << 1$. We then simplify the inequality in Eq. (6) and rewrite the resulting expression in terms of $\overline{B}$ as follows:

$$-\frac{1}{2}\ln(1-T_0) \approx \frac{T_0}{2} > \frac{2\pi^2 f_C RC}{\overline{B}}, \quad f_C RC \approx \frac{R(z)}{480\Omega}\frac{f_C}{f_{\text{res}}}\frac{1}{\left(\ln\frac{l_A}{2a} - 1\right)}, \quad z = \frac{\pi}{2}\frac{f_C}{f_{\text{res}}} \tag{7}$$

In Eq. (7), we have replaced the geometric mean of the upper and lower band frequencies by its center frequency, which is valid when i) $\overline{B} < 0.25$; ii) the half-wavelength approximation for dipole's resonant frequency is used; and iii) dipole's capacitance in the form $C^{-1} \approx 480\Omega \times f_{\text{res}}\left(\ln\frac{l_A}{2a} - 1\right)$ is chosen. The last approximation follows from Eq. (1) when *z* is at least less than one half. Thus, from Eq. (7) one obtains the upper estimate for the gain-bandwidth product in the form

$$T_0 \overline{B} < 4\pi^2 \frac{R(z)}{480\Omega} \frac{f_C}{f_{\text{res}}} \frac{1}{\left(\ln \frac{l_A}{2a} - 1\right)}, \quad z = \frac{\pi}{2} \frac{f_C}{f_{\text{res}}} \tag{8a}$$

The value of this simple equation is in the fact that the gain-bandwidth product is obtained and estimated explicitly. Unfortunately, Eq. (8a) is limited to small transducer gains.

Arbitrary fractional bandwidth and arbitrary transducer gain

The only condition we will exploit here is $z = 0.5\pi f_C / f_{\text{res}} < 0.5$. Then the dipole capacitance is still approximately described by the formula from subsection 2.4. The analysis of subsection 2.4 also remains the same until Eq. (6). However, we now discard the assumption on small transducer gain. We define the fractional bandwidth $\overline{B} = B / f_C$ as before. After some manipulations Eq. (6) yields

$$\frac{\overline{B}}{\left(1 - \overline{B}^2 / 4\right)} \ln \frac{1}{1 - T_0} < 4\pi^2 \frac{R(z)}{480\Omega} \frac{f_C}{f_{\text{res}}} \frac{1}{\left(\ln \frac{l_A}{2a} - 1\right)}, \quad z = \frac{\pi}{2} \frac{f_C}{f_{\text{res}}} \tag{8b}$$

This estimate does not contain the gain-bandwidth product $T_0 \overline{B}$ explicitly, but rather individual contributions of T_0 and $\overline{B}$. It is valid below the first dipole resonance, and it is a function of two parameters: the dimensionless antenna geometry parameter $l_A / (2a)$ and the ratio of the matching frequency to the antenna's resonant frequency, f_C / f_{res}.

Frequently, the fractional bandwidth is given, and the maximum gain T_0 over this bandwidth is desired. In this case, Eq. (8b) can be transformed into

$$T_0 < 1 - \exp\left(-4\pi^2 \frac{R(z)}{480\Omega} \frac{f_C}{f_{\mathrm{res}}} \frac{\left(1 - \bar{B}^2/4\right)}{\bar{B}\left(\ln\frac{l_A}{2a} - 1\right)} \right), \quad z = \frac{\pi}{2}\frac{f_C}{f_{\mathrm{res}}} \tag{8c}$$

We note that this result does not depend on particular value of the generator's resistance, R_g. Fig. 2 gives the maximum realizable gain according to Eq. (8c) obtained at different desired bandwidths as a function of matching frequency.

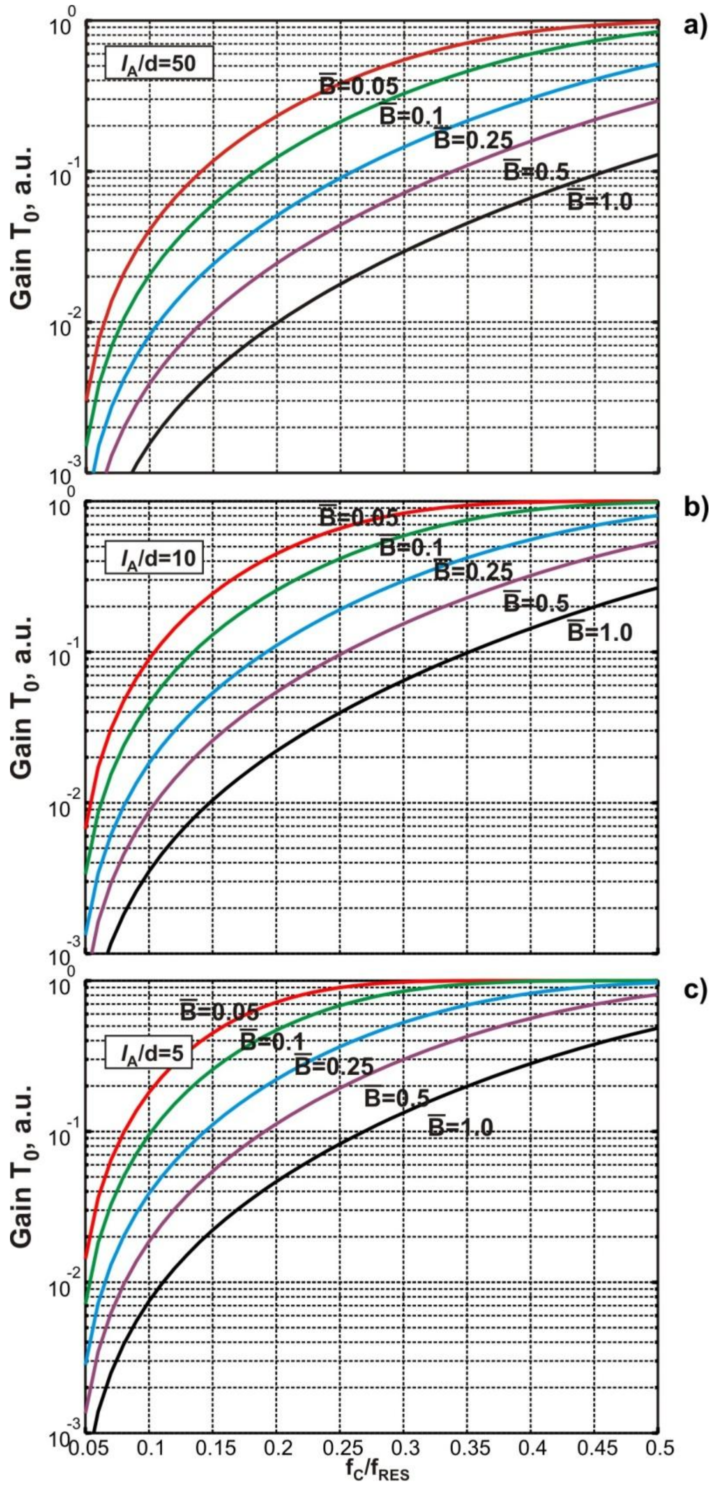

Fig. 2. Upper transducer gain limit for three dipoles (from a to c) of diameter d and length l_A as a function of matching frequency vs. resonant frequency of the infinitesimally thin dipole of the

same length. The five curves correspond to five fractional bandwidth values 0.05, 0.1, 0.25, 0.5, and 1.0, as labeled in the figure, and have been generated by using Eq. (8c).

In Fig. 2 a-c we have considered three different dipoles, with $l_A / 2a = l_A / d = 5, 10, 50$, where $d = 2a$ is the dipole diameter. Also, we observe that the condition $\ln(l_A / 2a) > 1.5$ is satisfied for every case in Fig. 2.

The monopole's impedance is half of the dipole's impedance. At first glance, it might therefore appear that one should halve the argument of the exponent in Eq. (8c). This is not true because the argument in fact contains the product *R*C. The capacitance *C* is hidden in other terms of this expression. While the resistance *R* decreases by 0.5, the capacitance *C* increases by 0.5, and thus the estimate for the gain remains unchanged. Consequently, the dipole estimate for the bandwidth is always applicable to the equivalent monopole of half length, assuming an infinite ground plane.

Comparison with Chu's bandwidth limit

It is instructive to compare the above results with Chu's antenna bandwidth limit [19] conveniently rewritten in Refs. [18], [20] in terms of tolerable output VSWR of the antenna and the antenna *ka*, where *a* is the radius of the enclosing sphere. We consider, for example, a short thick dipole of total length l_A=23 cm and $l_A / d = 5$ shown in Fig. 2c. The dipole is designed to have a passband from 250 to 400 MHz, and a center frequency of 325 MHz. The resonant frequency of the corresponding infinitesimally thin dipole is found as $f_{\mathrm{res}} = c_0 /(2l_A) = 650$ MHz ; thus, $f_C / f_{\mathrm{res}} = 0.5$. The fractional bandwidth is approximately $\overline{B} \approx 0.5$ or the bandwidth is 50%. According to Fig. 2c, this case leads to a significant generator gain of $T_0 \approx 0.8$ over the frequency band.

Now, this gain corresponds to the squared reflection coefficient $|\Gamma|^2 = 0.2$, and yields a return loss of -7dB ($\mathrm{VSWR} = (1+|\Gamma|)/(1-|\Gamma|) = 2.6$) that is uniform over the operating frequency band. For this dipole example with VSWR=2.6 and $ka = kl_A / 2 = 0.78$, the Chu's bandwidth limit is about 34% [18]-[20]. Note that this estimate is less optimistic than the Bode-Fano model discussed above, but it includes an uncertainty in relating the antenna *Q*-factor to the antenna's circuit parameters [18].

Matching circuit development

L-section impedance matching

A small relatively-thin monopole (whip monopole) or a small dipole is frequently matched with a simple *L*-matching double-tuning section [15]. This section is shown in Fig. 3. Ohmic losses of the matching circuit, R_o, are mostly due to losses in the series inductor, which may be the larger one for very short antennas. Namely, L_1 might be on the order of 0.1-1.0 mH for HF and VHF antennas. In this UHF-related study, we will neglect those losses.

Qualitatively, the series inductor L_1 cancels the (large) capacitance of the whip antenna whereas the shunt inductor L_2 matches the (small) resistance of the whip antenna to the generator resistance of 50 Ω. Quantitatively, referring to Fig. 3, the analytical result for the tuning inductances has the form for $R_o = 0$, see for example Ref. [15].

$$L_2 = \frac{1}{\omega_C}\sqrt{\frac{R_g R}{1 - R/R_g}}, \quad L_1 = -\frac{X_A}{\omega_C} - \frac{L_2}{2} - \sqrt{\frac{L_2^2}{4} - \frac{R^2}{\omega_C^2}} \tag{2}$$

where ω_C is the angular matching frequency.

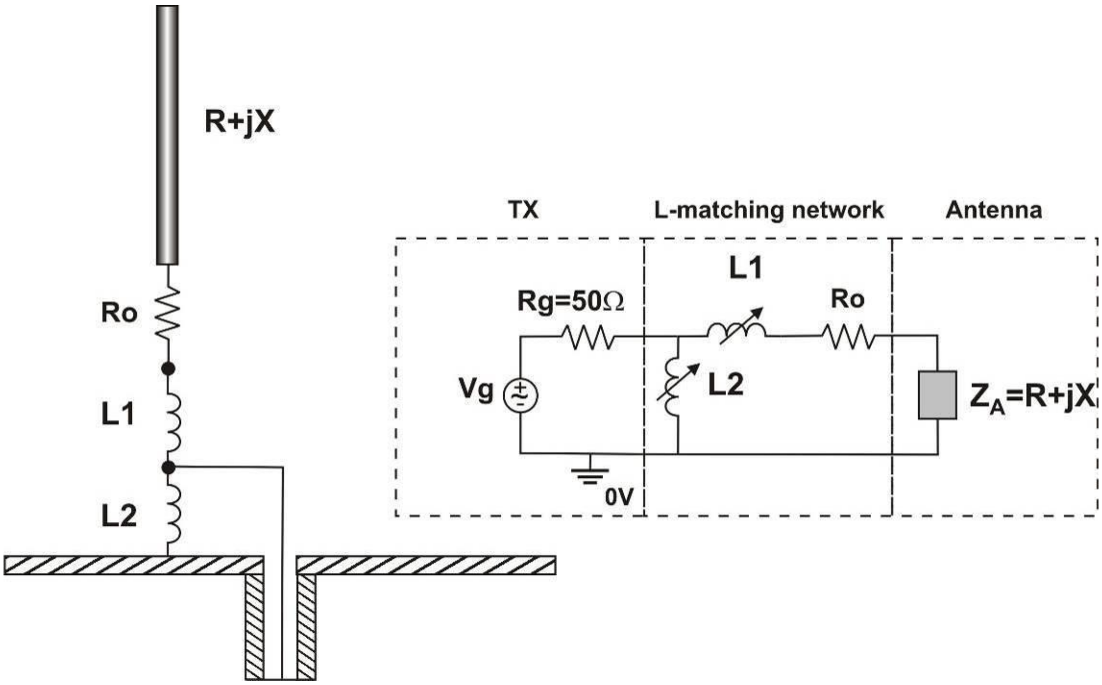

Fig. 3. A whip-monopole *L*-tuning network [15] used in the present study (monopole version). Ohmic resistance of the series inductor, R_o, will be neglected. The matching network does not show the DC blocking capacitor in series with L_2.

Although the *L*-tuning section is very versatile and can be tuned to any frequency by varying L_1, L_2, its bandwidth is extremely small since impedance matching, when done analytically, is carried out for a single frequency.

Extension of the L-section matching network

To increase the bandwidth of the *L*-tuning section at some fixed values of L_1, L_2, we suggest to consider the matching circuit shown in Fig. 4. It is seen from Fig. 4 that we can simply add a high-pass *T*-network with three lumped components (a shunt inductor and two series capacitors) to the *L*-section or, equivalently, use two sections of the high-pass *LC* ladder and investigate the bandwidth improvement. The Thévenin impedance of the equalizer, as seen from the antenna, is given by

$$Z_T = \frac{sL_2 Z_g}{sL_2 + Z_g} + sL_1, \quad \begin{cases} \text{case I} & Z_g = R_g \\ \text{case II} \quad Z_g = \dfrac{sL_4 (R_g + 1/(sC_5))}{sL_4 + (R_g + 1/(sC_5))} + 1/(sC_3) \end{cases} \tag{3}$$

where $s = j\omega$. The default values of the circuit parameters for the sole *L*-tuning section read $C_3 = L_4 = C_5 = \infty$. Thus, we introduce three new lumped circuit elements, but avoid using transformers. Instead of using impedances, an ABCD matrix approach would be more beneficial when using transformers.

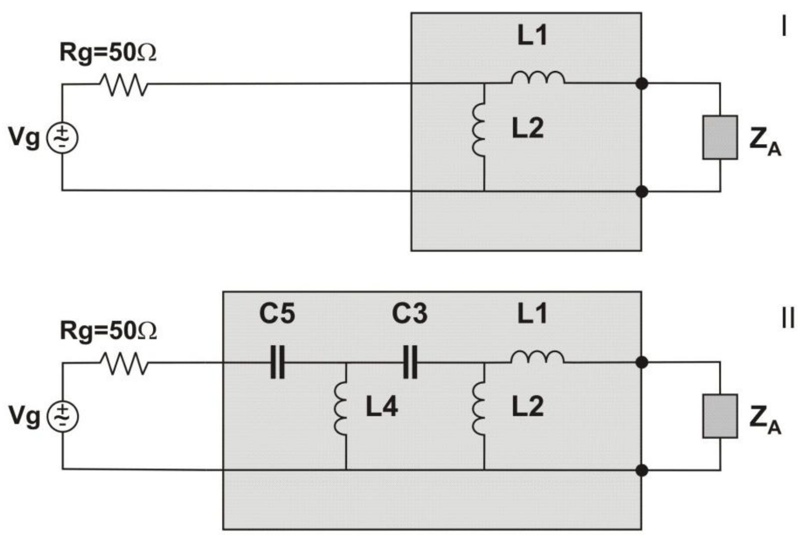

Fig. 4. An extension of the *L*-tuning network for certain fixed values of L_1, L_2 by the *T*-match.

Reduction of the 5-element network was also investigated. Shown below is a possible implementation of a 4-element equalizer:

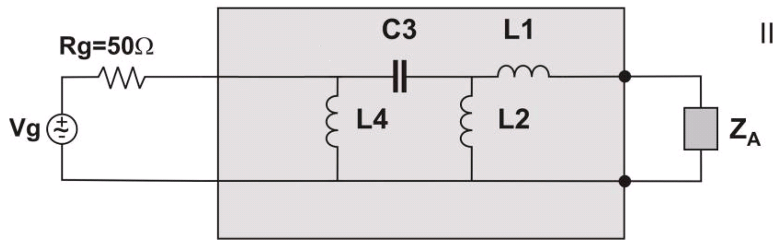

Fig. 5. 4-element equalizer

$$Z_T = \frac{sL_2 Z_g}{sL_2 + Z_g} + sL_1 \text{ , Case I: } Z_g = R_g \text{ , } Case\ IV: Z_g = \frac{sL_4 R_g}{sL_4 + R_g} + \frac{1}{sC_3} \qquad (4)$$

Circuit optimization task

The antenna is assume to be matched over a certain band *B* centered at f_C, and that the gain variation in Eq. (2) does not exceed ±25% over the band. If, for this given equalizer circuit, such small variations at any values of the circuit parameters cannot be acheived, the equalizer is not considered capable of wideband impedance matching over the bandwidth *B*. It is known that a low-order equalizer (the *L*-matching section) alone is not able to provide a nearly uniform gain over a wider band. However, increasing the circuit order helps. Thus, two practical questions need to be answered:

A. For a given center frequency f_C and bandwidth *B*, or for a given fractional bandwidth $\overline{B} = B / f_C$, what are the (normalized) circuit parameters that give the required bandwidth?
B. What is the gain-bandwidth product and how does it relate to the upper estimate given by Eq. (8c)?

Yet another important question is the phase linearity over the band; this question will not be considered in the present study.

Numerical simulation results

Task table and the numerical method

We will consider the dipole case and assume monopole equivalency. The set of tested antenna parameters includes:

$$l_A / d = [50, 10, 5], \quad \bar{B} = [0.1, 0.5], \quad f_C / f_{\mathrm{res}} = [0.05 : 0.05 : 0.50] \qquad (11)$$

To optimize the matching circuit with 5 lumped elements we employ a direct global numerical search in the space of circuit parameters. The grid in $\Re^5$ space includes up to 100^5 nodes. The vector implementation of the direct search is fast and simple, but it requires a large (64 Gbytes or higher) amount of RAM on a local machine.

For every set of circuit parameters, the minimum gain over the bandwidth is first calculated [11]. The results are converted into integer form and sorted in a linear array, in descending order, using fast sorting routines on integer numbers. Then, starting with the first array element, every result is tested with regard to ±25% acceptable gain variation. Among those that pass the test, the result with the highest average gain is finally retained. After the global maximum position found on a coarse mesh, the process is repeated several times on finer meshes in the vicinity of the anticipated circuit solution.

A viable alternative to the direct global numerical search used in this study, which is also a derivative free and a global method, is the genetic algorithm [21]. The genetic algorithm (GA) belongs to the class of stochastic optimization algorithms. GA's have been widely used in many fields including antenna array design [22] and electromagnetics [23]. A particularly interesting application of the GA was reported in Ref. [24] wherein the authors have demonstrated its use in optimizing lumped component networks for an antenna synthesis application as well as the matching network.

The *MATLAB* © Genetic Algorithm and Direct Search Toolbox™ [25] provided us with another numerical platform to optimize the matching circuit in this study. This toolbox features a vast array of choices with which we were able to tailor the GA solver for our requirement. Yet another alternative is a combination of the GA and the direct search: the direct search technique known as "Pattern search" can be used along with the GA to improve its performance. This is known as a hybrid GA [25] and it works by taking the best solution arrived at by the GA as the initial point and proceeds to refine the result. The pattern here refers to a set of vectors that define the parameter space (in our case the circuit parameters) for the current iteration over which the search is performed.

To use this method we first generated a population of random candidate solutions with a uniform distribution, for the circuit parameters. The GA solver then tests these candidate solutions based on a specified criterion, which in this particular case, is to maximize the minimum gain over the band. After assigning scores to the various candidate solutions, it then creates the next generation of solutions, referred to as the '*children*', by pairing candidate solutions from the previous generation, referred as '*parents*'. To ensure diversity in the next generation, mutations, or random changes to one of the parents in a pair are introduced. These new solutions replace the current population and the process repeats. Several options for stopping criterion can be used such as time limit, no. of successive generations or even simply the change in the value objective function between two generations. During this study, the results obtained with the GA toolbox were found to be close to the results obtained with the direct global numerical search in most of the cases. In particular, the results for Fig. 8 nearly coincide for both methods, including the circuit parameter values.

Realized gain – wideband matching for $\overline{B} = 0.5$

Fig. 5 shows the realized average generator gain over the passband based on the ±25% gain variation rule at different matching center frequencies. Three dipole geometries with $l_A / d = 50, 10, 5$ are considered. The bandwidth is fixed at $\overline{B} = 0.5$; we again consider three dipoles of different radii/widths. The realized values are shown by circles; the ideal upper estimate from Fig. 2 is given by solid curves. One can see that the 5-element equalizer performs rather closely to the upper theoretical limit T_0 when the average gain over the band, $\overline{T}$, is substituted instead. For the majority of cases, the difference between T_0 and $\overline{T}$ is within 30% of T_0. The sole *L*-section was not able to satisfy the ±25% gain variation rule in all cases except the very last center frequency for the thickest dipole.

Gain and circuit parameters – wideband matching for $\overline{B} = 0.5$, $f_C / f_{\text{res}} = 0.5$.

Table 1a reports circuit parameters of the equalizer for three dipoles with $l_A / d = 50, 10, 5$. In every case, matching is done for $f_C / f_{\text{res}} = 0.5$, $\overline{B} \approx 0.5$. Fig. 6 shows the corresponding gain variation with frequency within the passband. In Table 1a, we have presented all circuit parameters for a 23 cm long dipole.

To scale parameters to other antenna lengths one needs to multiply them by the factor $l_A / 0.23\,\text{m}$. Table 1a also shows the anticipated gain tolerance error. Whilst the average gain itself does not significantly change when changing capacitor/inductor values, the gain uniformity may require extra attention for a thin dipole (second row in Table 1a). For thicker dipoles (third and fourth row of the table) one solution to the potential tolerance problem is to slightly overestimate the circuit parameters for a better tolerance. Generally, the usual uncertainty in low-cost chip capacitors and chip inductors seems to be acceptable.

Table 1a also indicates that the equalizer for a wideband matching of the dipole does not involve very large inductors (and large capacitors) and is thus potentially low-loss.

Table 1b presents the same data for the equivalent monopole of length 11.5 cm over the infinite ground plane. It is worth noting that nearly the same gain as in Table 1a is achieved over the matching bandwidth, which confirms our early theoretical predictions. However, the components values appear to be quite different. The most important difference is related to a considerably smaller value of inductance L_1. This is a positive tendency since the loss also decreases in such a case.

Table 1a. Circuit parameters and gain tolerance for a short dipole with the total length l_A=23 cm. Matching is done for $f_C / f_{\text{res}} = 0.5$, $\overline{B} \approx 0.5$ based on the ±25% gain variation rule.

Antenna geometry l_A / d	Circuit parameters		Gain/Variance over the band	Gain/Variance over the band at +5% parameter variation	Gain/Variance over the band at -5% parameter variation
50	L1 = 176 nH L2 = 70 nH C3 = 4.9 pF	L4 = 80 nH C5 = 15.3 pF	$\overline{T} = 0.20$ $\lvert \Delta T / \overline{T} \rvert < 25\%$	$\overline{T} = 0.19$ $\lvert \Delta T / \overline{T} \rvert < 27\%$	$\overline{T} = 0.20$ $\lvert \Delta T / \overline{T} \rvert < 38\%$
10	L1 = 72.4 nH L2 = 48.7 nH C3 = 39.6 pF	L4 = 102 nH C5 = 10.2 pF	$\overline{T} = 0.36$ $\lvert \Delta T / \overline{T} \rvert < 24\%$	$\overline{T} = 0.35$ $\lvert \Delta T / \overline{T} \rvert < 19\%$	$\overline{T} = 0.38$ $\lvert \Delta T / \overline{T} \rvert < 35\%$
5	L1 = 21.5 nH L2 = 24.6 nH C3 = 61.9 pF	L4 = 537 nH C5 = 15.3 pF	$\overline{T} = 0.60$ $\lvert \Delta T / \overline{T} \rvert < 25\%$	$\overline{T} = 0.59$ $\lvert \Delta T / \overline{T} \rvert < 19\%$	$\overline{T} = 0.61$ $\lvert \Delta T / \overline{T} \rvert < 34\%$

Table 1b. Circuit parameters and gain tolerance for a short monopole with the total length l_A =11.5 cm. Matching is done for $f_C / f_{\text{res}} = 0.5$, $\overline{B} \approx 0.5$ based on the ±25% gain variation rule.

Antenna geometry l_A / d	Circuit parameters		Gain/Variance over the band
50	L1 = 0.2 fH L2 = 24.33 nH	C3 = 153.33 pF L4 = 102.22 nH C5 = 10.2 pF	$\overline{T} = 0.17$ $\lvert \Delta T / \overline{T} \rvert < 25\%$
10	L1 = 38.2 nH L2 = 36.5 nH C3 = 26.2 pF	L4 = 58 nH C5 = 30.6 pF	$\overline{T} = 0.31$ $\lvert \Delta T / \overline{T} \rvert < 25\%$
5	L1 = 96.5 nH L2 = 48.6 nH C3 = 153.3 pF	L4 = 306.6 nH C5 = 10.2 pF	$\overline{T} = 0.52$ $\lvert \Delta T / \overline{T} \rvert < 22\%$

Considering the antenna length to width ratio $\frac{l_A}{t} = [\,50, 10, 5\,]$, Percent fractional bandwidth $\overline{B} = 0.5$, and ratio of center frequency to resonant frequency of $f_c/f_{res} = 0.50$, a direct global parameter search is employed in the space of circuit parameters using MATLAB to optimize the matching circuit for both 5 and 4 lumped elements with a $\pm 25\%$ allowed gain variation. The approximate parameter values generated are shown in the tables below

Table 2a: 5-element network, $L, highpassT$

Antenna geometry l_A/t	Circuit parameters		Gain/Tolerance over the band
50	L1 = 176nH L2 = 70nH C3 = 4.9pF	L4 =80nH C5 =15.3pF	$\overline{T} = 0.20$ $\lvert \Delta T / \overline{T} \rvert < 25\%$
10	L1 = 72.4nH L2 = 48.7nH C3 = 39.6pF	L4 = 102nH C5 = 10.2pF	$\overline{T} = 0.36$ $\lvert \Delta T / \overline{T} \rvert < 24\%$
5	L1 = 21.5nH L2 = 24.6nH C3 = 61.9pF	L4 = 537nH C5 =15.3pF	$\overline{T} = 0.60$ $\lvert \Delta T / \overline{T} \rvert < 25\%$

Table 2b: 4-element network

Antenna Geometry l_A/t	Circuit Parameters		Gain/Tolerance over the band
50	L1 = 0 L2 = 0	C3 = 0 L4 =0	$\bar{T}=0.20$ $\lvert \Delta T/\bar{T} \rvert < 25\%$
10	L1 = 0 L2 = 0	C3 = 0 L4 =0	$\bar{T}=0.36$ $\lvert \Delta T/\bar{T} \rvert < 24\%$
5	L1 = 11.8 nH L2 = 15.2 nH	C3 = 5.2 pF L4 = 50 nH	$\bar{T}=0.60$ $\lvert \Delta T/\bar{T} \rvert < 25\%$

Table 2c: 5-element network, $L, lowpassT$

Antenna Geometry l_A/t	Circuit Parameters		Gain/Tolerance over the band
50	L1 = 0 L2 = 174.6 nH L3 = 0	C4 = 6.7 pF L5 = 66.7 nH	$\bar{T}=0.20$ $\lvert \Delta T/\bar{T} \rvert < 25\%$
10	L1 = 0 L2 = 127 nH L3 = 0	C4 = 6.7 pF L5 = 66.7 nH	$\bar{T}=0.36$ $\lvert \Delta T/\bar{T} \rvert < 24\%$
5	L1 = 21.8 nH L2 = 14.7 nH L3 = 0	C4 = 4.1 pF L5 = 0	$\bar{T}=0.60$ $\lvert \Delta T/\bar{T} \rvert < 25\%$

The parameter values obtained are shown above in table 3. From the results obtained, zero values are given for 2 of the five parameters in each case. This suggests that the 5-element network could be reduced to 3, while maintaining the necessary gain tolerances.

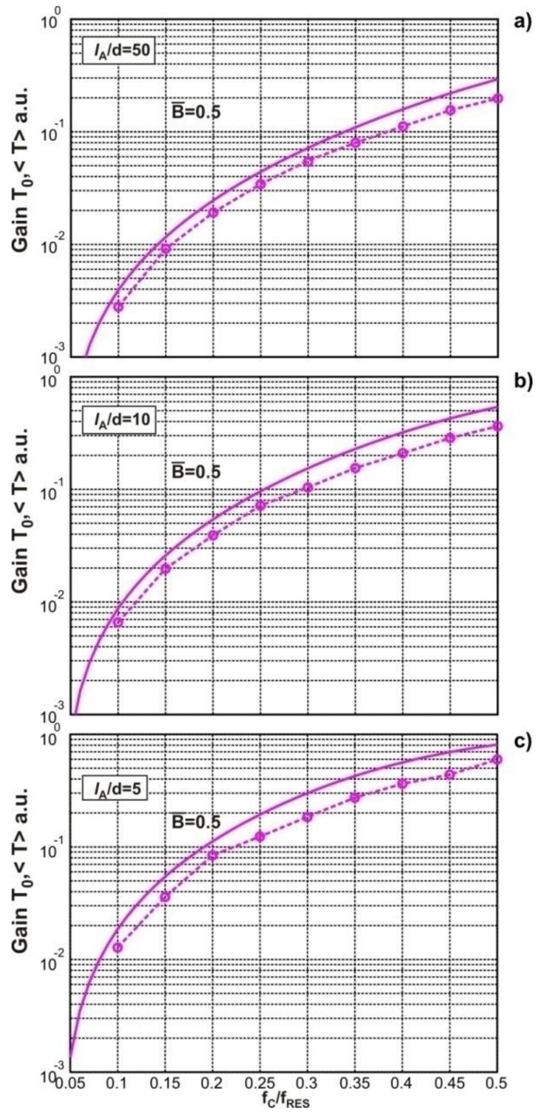

Fig 5. Realized average generator gain $\overline{T}$ over the band (circles) based on the ±25% gain variation rule at different matching center frequencies and $\overline{B} = 0.5$ for three different dipoles, obtained through numerical simulation. The realized values are shown by circles; the ideal upper estimates of T_0 from Fig. 2 are given by solid curves, which are realized by using eqn. (8c).

Gain and circuit parameters – narrowband matching for $\overline{B} = 0.1$

It is not the subject of this study to discuss the narrowband matching results; however, they have been obtained and may be discussed briefly. When the two-element *L*-section network is able to provide us with the required match, its performance is not really distinguishable from that of the full 5-element equalizer. However, it does not always happen that the reduced *L*-section equalizer is able to do so. The full equalizer is the only solution at smaller resonant frequencies and for thinner dipoles.

Unfortunately, the deviation from the Bode-Fano maximum gain may be higher for narrowband matching than for the wideband matching; in certain cases it reaches 100%. It is not clear whether this high degree of deviation is due to the numerical method or if it has a physical nature.

Gain and circuit parameters – wideband matching for $\overline{B} = 0.5$, $f_C / f_{\mathrm{res}} = 0.15$.

A more challenging case is a smaller wideband dipole; we consider here the case when $f_C / f_{\mathrm{res}} = 0.15$ and refer to the corresponding theory data in Fig. 2. Fig. 7 shows the transducer gain variation with frequency within the passband, after the equalizer has been applied based on the ±25% gain variation rule. The circuit parameters indicate a higher value of $L_1 = 2.46\mu\mathrm{H}$ for $l_A / d = 50$ and $L_1 = 1.06\mu\mathrm{H}$ for $l_A / d = 10$. For $l_A / d = 5$, inductance L_4 attains a larger value of $1.40\mu\mathrm{H}$.

Comparison with the results of Ref. [14]

In Ref. [14] a similar matching problem was solved for a thin dipole of length l_A =0.5 m and the radius *a* of 0.001m. Matching is carried out for $f_C / f_{\mathrm{res}} = 0.416$, $\overline{B} = 0.4$. A Carlin's equalizer with an extra LC section has been considered. Fig. 8 reports the performance of our equalizer for this problem (dashed curve). The thick solid curve within the passband is the corresponding result of Ref. [14] (and copied from Fig. 6). In our case, the optimization was done based on the ±25% gain variation rule. The difference between the two average band gains was found to be 5%. The circuit components for our circuit are 1.06μH, 0.21 μH, 20 pF, 0.95μH, and 17 pF. Note that without the extra LC section, the Carlin's equalizer may lead to a considerably lower passband gain than the gain shown in Fig. 8 [14]. Without any equalizer, the performance is expectedly far worse. The plot indicates that a 20 dB improvement is achieved at the lower edge of the band and approximately 10 dB at the upper band edge, when the equalizer is used.

Effect of impedance transformer

A set of numerical simulations for the same dipoles with a 4:1 ideal transformer has shown that the wideband matching results (achievable gain) are hardly affected by the presence of a transformer, even though the parameters of the matching circuit change considerably. For example, in the case of $\overline{B} = 0.5$, $f_C / f_{\mathrm{res}} = 0.15$ and discussed above, the average gain without and with transformer is 0.0092/0.0092, 0.020/0.020, and 0.036/0.040 for the three dipoles with $l_A / d = 50, 10, 5$.

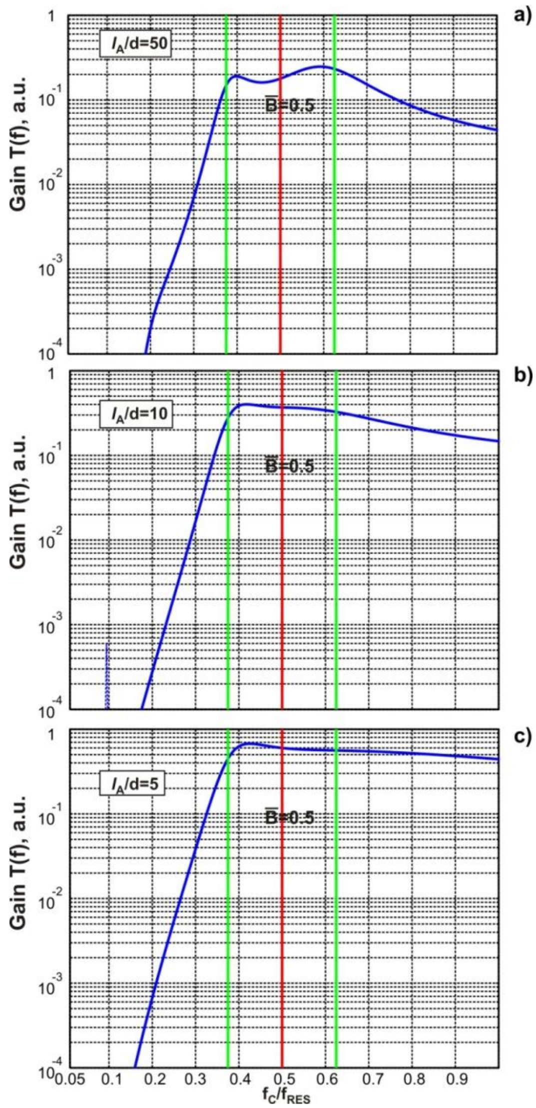

Fig. 6. Gain variation with frequency for a short dipole or for an equivalent monopole at different thicknesses/widths obtained by numerical simulation which uses Eq. (2). Matching is done for $f_C / f_{\text{res}} = 0.5$, $\overline{B} \approx 0.5$ based on the ±25% gain variation rule. Vertical lines show the center frequency and the passband.

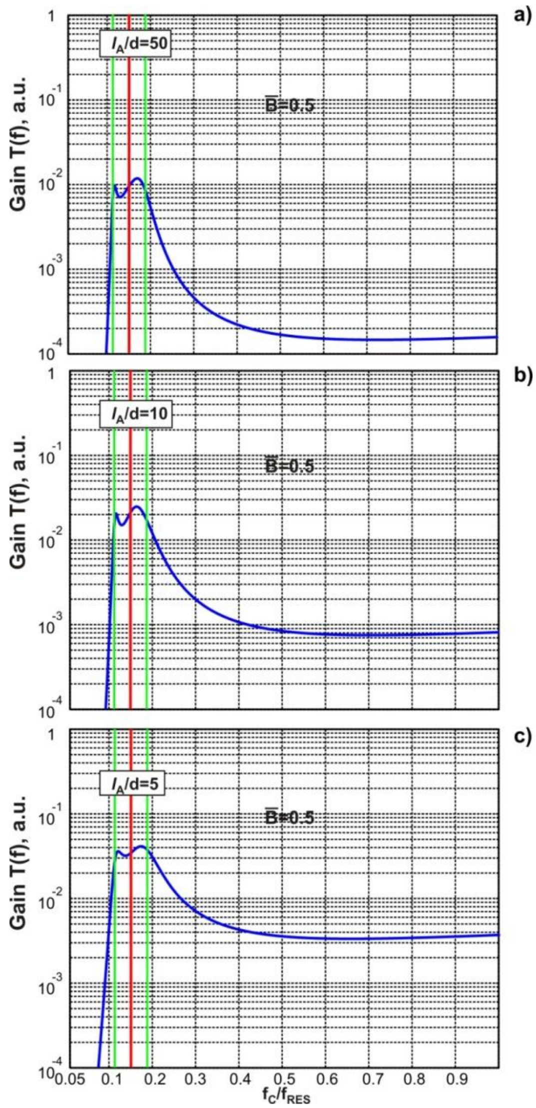

Fig. 7. Gain variation with frequency for a short dipole or for an equivalent monopole at different thicknesses/widths obtained by numerical simulation which uses Eq. (2). Matching is done for $f_C / f_{\rm res} = 0.15$, $\overline{B} \approx 0.5$ based on the ±25% gain variation rule. Vertical lines show the center frequency and the passband.

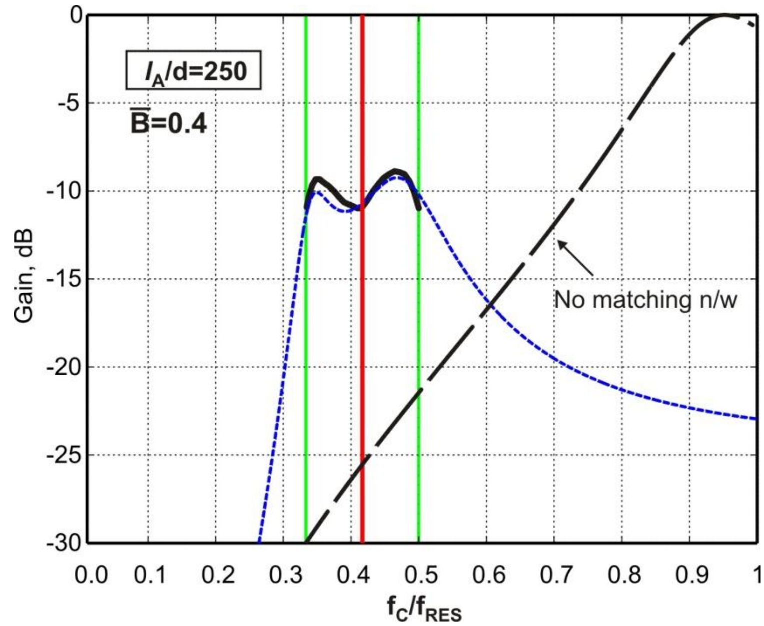

Fig. 8. Gain variation with frequency for a short dipole or for an equivalent monopole of length 0.5 m and radius of 0.001m, by numerical simulation of associated matching network. Matching is done for $f_C\ /\ f_{\mathrm{res}} = 0.416$, $\overline{B} = 0.4$ based on the ±25% gain variation rule (dashed curve). The thick solid curve is the result of Ref. [14] with the modified Carlin's equalizer, which was optimized over the same passband for the same dipole. Vertical lines show the center frequency and the passband. Transducer gain, in the absence of a matching network, is also shown by a dashed curve following Eq. (2).

Matching circuit design

The pcb layout for the 5-element matching circuit design is shown below

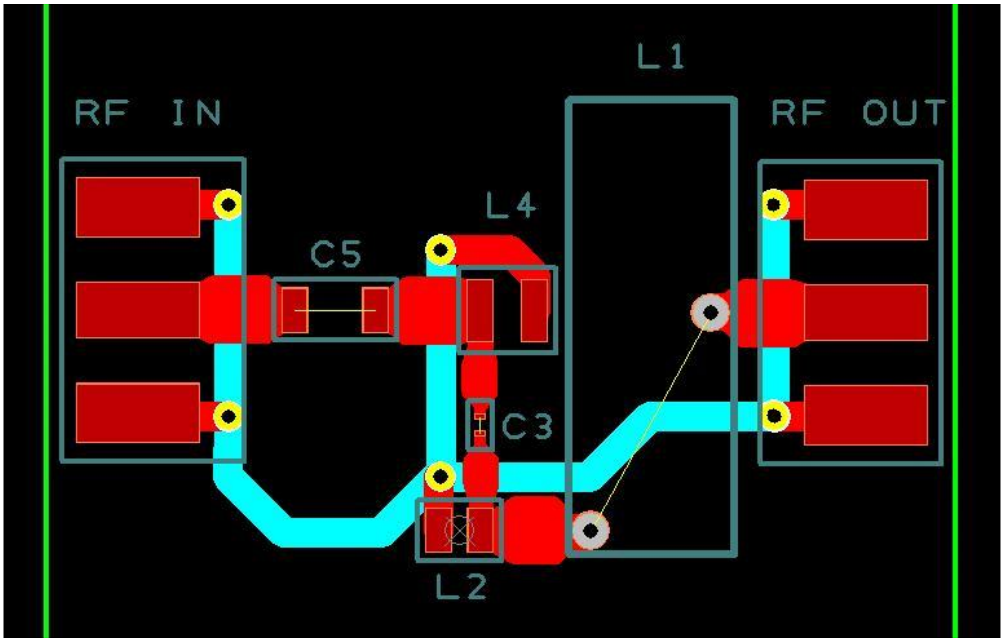

Fig. 9. 5-element matching circuit pcb layout

Shown below in Fig. 10 and 11. are the top and bottom views of the bare PCB for the 5 element matching circuit. The RF input and output footprints are for SMA connectors and the routed traces connecting components have a width of 105 mils which has been calculated for the board's 62 mil thickness, FR4 dielectric, and operating frequency of 325 MHz. The thick traces have been tapered down at the connections to the pads relative to their respective pad dimension.

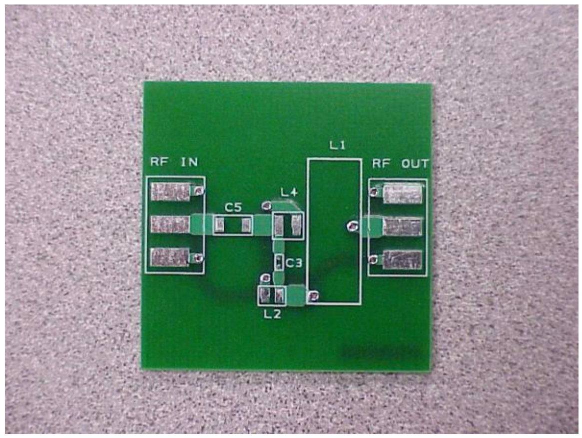

Fig. 10. PCB board, top view

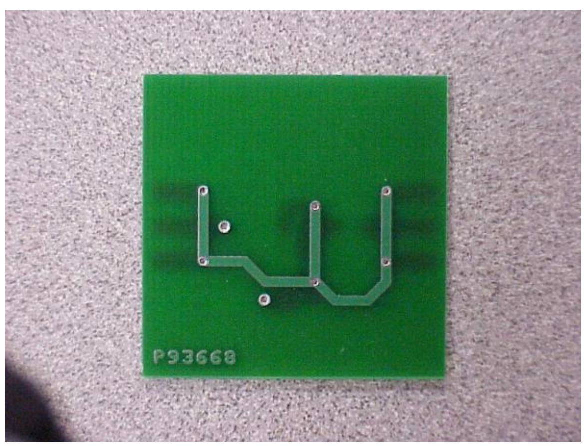

Fig. 11. PCB board, bottom view

Fig. 12 and 13 show the completed board: C5 is a surface mount trimmer capacitor, C3, L2, and L4 are surface mount design chip capacitors and inductors, respectively; the L1 tuning inductor is the only leaded component. Leaded tuning inductors were readily available, so the layout was modified to accommodate one. The most significant compromise was that the bottom of the PCB could no longer be one continuous ground plane. In future designs the layout could be modified so that there is a partial ground plane on the underside of the area of the board containing the chip components only.

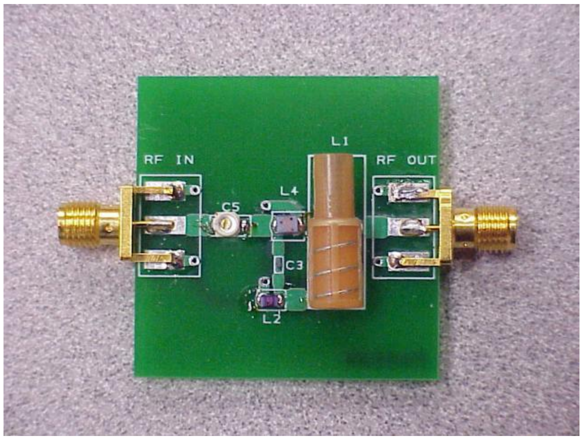

Fig. 12. Completed board, top view

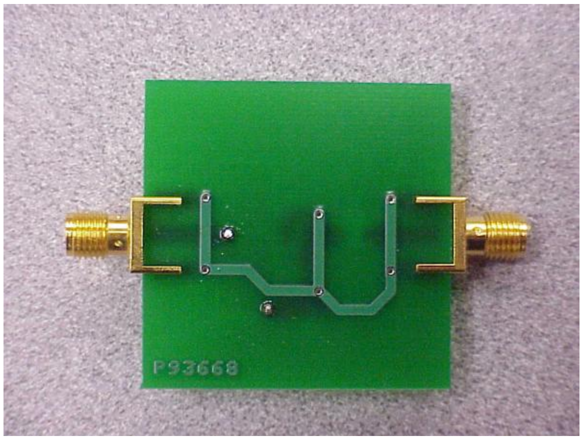

Fig. 13. Completed board, bottom view

Monopole antenna construction

Shown below in Fig. 14 is the blade monopole and 1x1 meter ground plane:

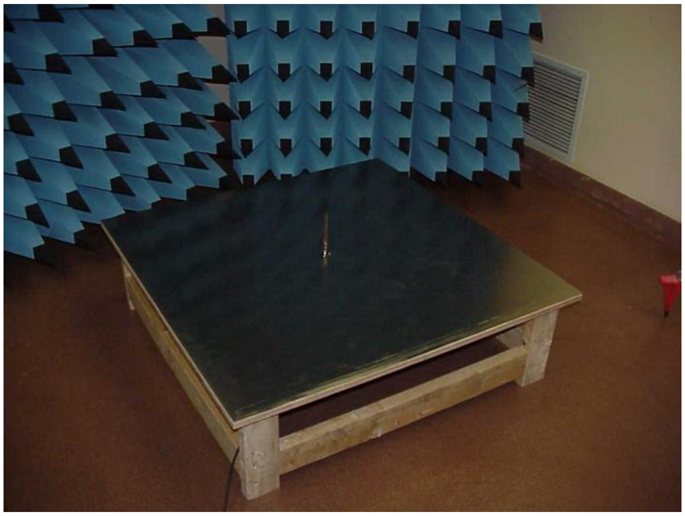

Fig. 14. Blade monopole and 1x1(m) ground plane

The blade monopole is designed for a length/thickness= 10; for length of 11.5cm, the thickness of the blade is 1.15cm. The blade constructed is shown in Fig. 15:

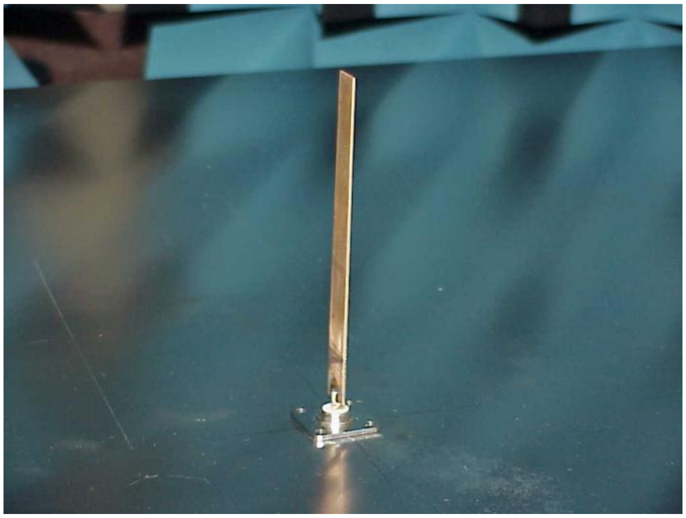

Fig. 15. Blade Monopole, length = 11.5cm, width = 1.15cm

In Fig. 16, the connection between the blade base and the central pin of the N-type connector is shown. The blade has been soldered directly to the central pin. The ground plane is clamped between the N-type connector and an N-to-SMA type adapter on the underside of the aluminum sheet. The underside is shown in Fig. 17.

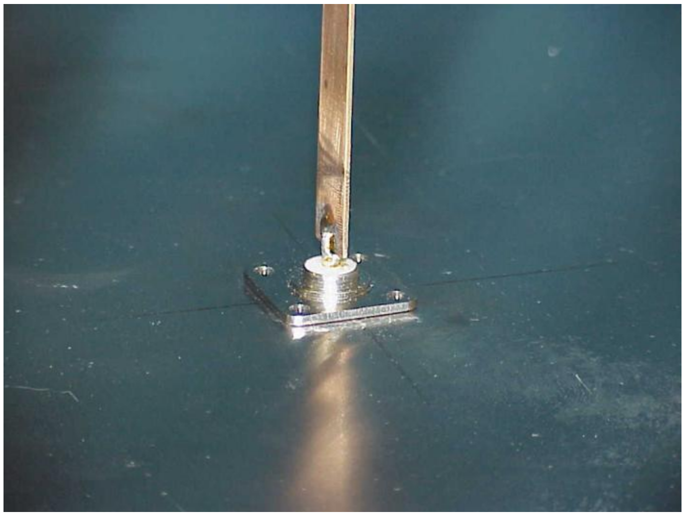

Fig. 16. Monopole base soldered directly to central pin N-type connector mounted to ground plane.

A coaxial cable with an sma connector is joined to the SMA-to- N-type adapter, the adapter's casing is connected to the ground plane between 2 metal washers by hand-tightening the male to female N-type connection.

Fig. 17. Antenna feed underneath the ground plane, SMA-to-N-type adapter.

The resonant frequency of the constructed antenna is known to be 650 MHz. The operating frequency has been chosen as fres/2 =325MHz. It is predicted that this antenne will show similar results when tested with the spectrum analyzer. There will most likely be some deviation from the prediction due to the 2 metal washers used in the antenna feed, and the additional height above the ground plane added to the blade by the N-type connector.

Experimental results

Short blade monopole

We have designed, constructed, and tested a number of short blade monopole test antennas and the corresponding matching networks. The antenna's first resonant frequency is in the range 550-650 MHz. The matching is to be done over a wide, lower frequency band of 250-400 MHz, with the center frequency of 325 MHz.

For every monopole, the ratio, l_A / d, equal to 10 has been used in the experiment. The brass monopole antennas have been centered in the middle of the 1×1 m aluminum ground plane. We then investigated the matching performance for two specific cases: i) the monopole is resonant at 650 MHz and; ii) monopole is resonant at a slightly lower frequency of 600 MHz. The results for both cases are reported in this section. Fig. 18 shows the generic monopole setup.

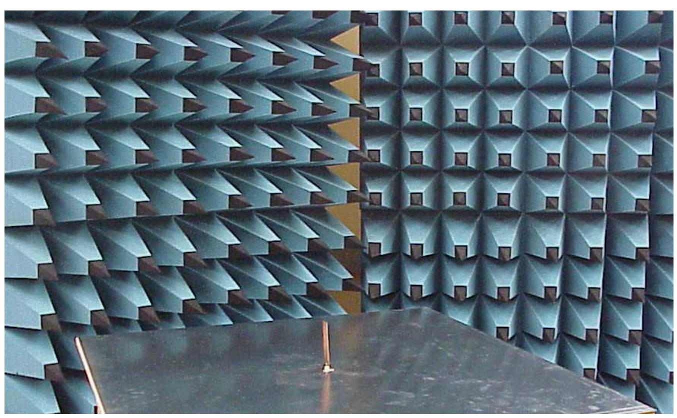

Fig. 18. A 10.1cm long and 2.3cm wide blade monopole over a 1×1 m ground plane used as a test antenna for wideband impedance matching.

Wideband equalizer

The ubiquitous FR-4 substrate has been used for the equalizer. We have chosen two tunable high-*Q* components among the five to compensate for parasitic effects due to the board assembly, the finite *Q* of the discrete components and the manufacturing uncertainties. These tunable components were L_1 and C_5, respectively. L_1 is a tunable RF inductor from Coilcraft's series 148 with a tuning range of 56nH - 86nH, and a nominal value of 73nH. This inductor has a *Q* of 106 at 50MHz. C_5 is a Voltronics series JR ceramic chip trimmer capacitor with a tuning range of 4.5pF - 20pF within a half turn. This capacitor has a minimum Q of 1500 at 1MHz. Apart from L_1 which is a leaded component; all the other components are the high-*Q* surface mount devices. Table 2 lists the parameter values. The designed wideband equalizer is shown in Fig. 19.

Table 3. Practical component values used in the monopole equalizer for $l_A / d = 10$ and $t = 2.3$ cm.

Component	Value
L_1	56 nH - 86 nH
L_2	48 nH
C_3	39.1 pF
L_4	100 nH
C_5	4.5 pF - 20 pF

Gain comparison

We compare the gain performance for two different modifications of the blade monopole dimensions in Fig. 20. The first modification involves a 10.1 cm long an 2.3 cm wide blade monopole, which is resonant at 650 MHz. In Fig. 20a the gain achieved by the unmatched monopole antenna (thin solid curve) and the gain of the monopole antenna with the designed wideband equalizer (thick dotted curve), respectively are shown. The average transducer gain achieved in experiment is 0.262 over the bandwidth 250MHz - 400MHz with a gain variation of 40 %. The L_1 and C_5 values for this particular result are 86 nH and 7.63 pF respectively.

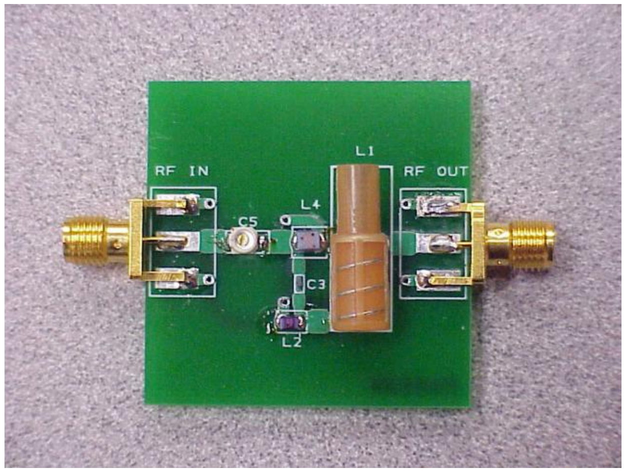

Fig. 19. Practical realization of the wideband equalizer for the blade monopole antenna following Table 3.

Next, we consider a blade monopole antenna of length 10.8 cm and width 2.3 cm. This blade monopole resonates at 600MHz. Fig. 20b shows the matching performance with (thick dotted curve) and without (thin solid curve) the wideband equalizer. We see that the equalizer performs rather well even under this scenario and achieves a gain of 0.259 within the bandwidth of interest. The gain variation over the band is 28.8 %. In this case the value of L_1 is changed to 73nH while the capacitance C_5 is unchanged. Here, we also notice an approximate 10dB improvement over the unmatched antenna, provided by the equalizer at the lower edge of the band. During the experiments we have noticed that a resonance may appear at lower frequencies below 200 MHz.

The theoretically predicted gain is shown by thick solid curves in Fig. 20a and b. Generally the experiment follows the theory. In the case of Fig. 20a, the average experimental gain over the band is 0.262 and is slightly higher than the corresponding theoretical value of 0.245. In the case of Fig. 20b, the average experimental gain over the band is 0.259 versus the theoretical value of 0.245. We believe that the average gain difference is within the experimental uncertainty. This statement can be further confirmed by the results from the fourth and fifth column of Table 1a, where we observe the quite similar variation when the component values of the matching circuit are varied by ±5%.

However, for the local gain behavior, we observe somewhat larger variations. The experimental gain is higher in the middle of the band, but is lowered at the band edges. We explain these variations by the associated tuning procedure and by the inability to exactly follow the requested values of inductance L_1 and capacitance C_5. Additional important mechanisms are lumped-element losses at the higher band end.

Yet one more uncertainty factor is due to a relatively small size of the measurement chamber. This effect becomes apparent at low frequencies as Fig. 20 indicates. The present results are preliminary and have a very significant room for improvement.

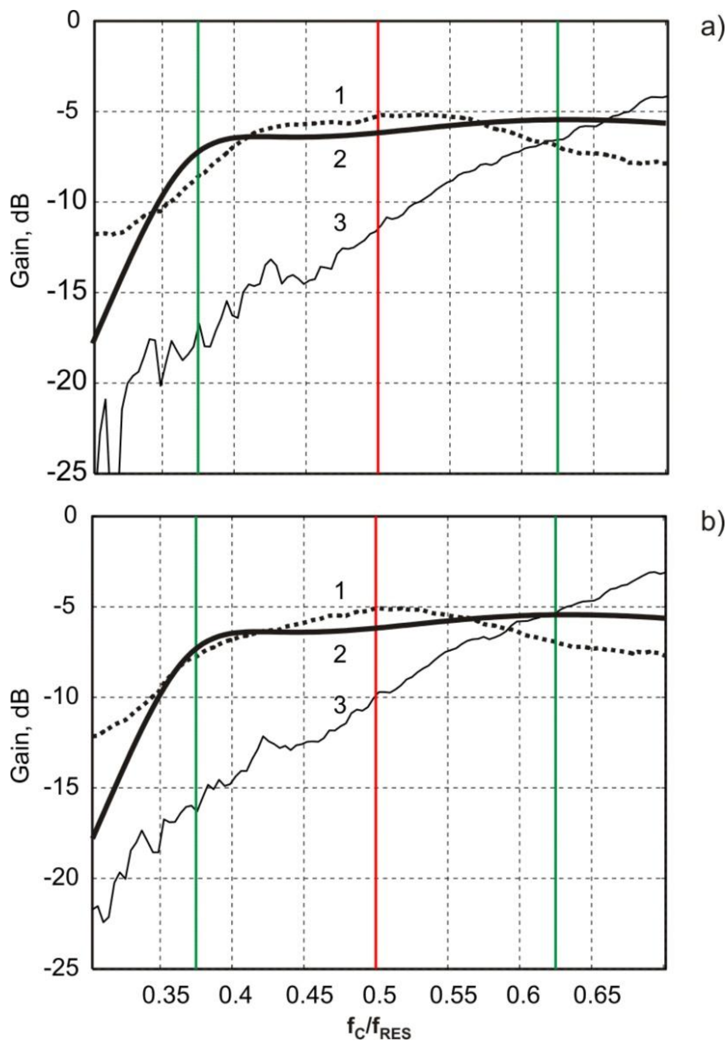

Fig. 20. The experimental gain data (dotted curve 1) in comparison with the theoretical result (thick curve 2) for two blade monopoles with the matching network from Table 3: a) - the blade length is 10.1 cm and the width is 2.3 cm; b) - the blade length is 10.8 cm and the width is 2.3 cm. The thin solid curve 2 in this graph corresponds to the antenna gain (based on the measured return loss) without the matching network.

Conclusions

In this study a new technique of wideband impedance matching using a lumped circuit for relatively short non-resonant dipoles or monopoles of different thickness has been presented, investigated, and experimentally proven for relatively short non-resonant monopoles of different thicknesses/widths. The belief is that one simple fixed-topology network can be used to perform wideband impedance matching for a variety of dipole-like antennas, thereby simplifying the matching process.

The particular circuit suggested in the present paper includes five lumped components with common manufacturing values at VHF and UHF center frequencies. It is found that the circuit's performance deviates on average by 30% (maximally by 40%) from the theoretical impedance matching limit when $0.05 < f_C / f_{\mathrm{res}} < 0.5$ and $\overline{B} = 0.5$, where f_C is antenna's center matching frequency and $\overline{B}$ is the desired fractional bandwidth. Experimental results based on wideband impedance matching of a short monopole antenna over the frequency range 250-400 MHz indicate that an average improvement of 10 dB can be expected at the lower edge of the band. The circuit is not intended to be applied to resonant dipoles/monopoles or to dipoles/monopoles above the first resonance. Its phase characteristics and the noise figure need to be optimized separately.

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Appendix

Matlab scripts

```
function [MeanGain VarGain CP] = optimizer02_function(dipole_y, dipole_lt, Rg, DCF, FB, Ls, Us);
%   Matching circuit optimizer for a dipole - a reflective equalizer
%   Direct search on 1D arrays; the search here is limited to 5 circuit
%   elements - see Fig. 3 of the text.
%
%   dipole_y  - dipole length
%   dipole_lt - length-to-width or length-to-radius ratio
%   DCF       - relative center frequency fc/f_res
%   FB        - fractional bandwidth vs. center frequency
%   Ls        - lower bound of search parameters
%   Us        - upper bound of search parameters
%   There are no output arguments; the function saves all data in a mat
%   file.
%
%   1D Array assembly in parameter space:
%   (M(1) = 3; M(2) = 2; M(3) = 2; M(4) = 2; M(5) = 1)
%   [1 2 3][1 2 3][1 2 3][1 2 3][1 2 3][1 2 3][1 2 3][1 2 3]    M(1) A(1, :)
%   [1 1 1][2 2 2][1 1 1][2 2 2][1 1 1][2 2 2][1 1 1][2 2 2]    M(2) A(2, :)
%   [1 1 1][1 1 1][2 2 2][2 2 2][1 1 1][1 1 1][2 2 2][2 2 2]    M(3) A(3, :)
%   [1 1 1][1 1 1][1 1 1][1 1 1][2 2 2][2 2 2][2 2 2][2 2 2]    M(4) A(4, :)
%   [1 1 1][1 1 1][1 1 1][1 1 1][1 1 1][1 1 1][1 1 1][1 1 1]    M(5) A(5, :)
%   S. Makarov and V. Iyer, ECE Dept., WPI Aug. 2008

%   Antenna/generator/bandwidth
dipole_x        = dipole_y/dipole_lt;   %   dipole width (m)
f_res           = 3e8/(2*dipole_y);     %   resonant frequency (ideal; half-wave resonance)
B               = 4;                    %   number of frequency observation points over the bandwidth
f_center        = DCF*f_res;            %   absolute center frequency vs. f_res
bandwidth       = FB*f_center;          %   absolute bandwidth

%   Bandwidth  discretization for initial and final search
f     = linspace(f_center-bandwidth/2, f_center+bandwidth/2, B);    %    initial search
s     = j*2*pi*f;                                                   %    initial search
ZA    = dipole(f, dipole_x, dipole_y); RL = real(ZA);               %    initial search
f1    = linspace(f_center-bandwidth/2, f_center+bandwidth/2, 16*B); %    final search
s1    = j*2*pi*f1;                                                  %    final search
ZA1   = dipole(f1, dipole_x, dipole_y); RL1 = real(ZA1);            %    final search

%   Equalizer circuit
M(1) = 128;                            %   first parameter search space
M(2) = 64;                            %   second parameter search space
M(3) = 32;                            %   third parameter search space
M(4) = 16;                            %   fourth parameter search space
M(5) = 16;                            %   fifth parameter search space
A    = complex(zeros(5, prod(M)));     %   all parameter values assembled in linear arrays
Gain = zeros(1, prod(M));              %   the generator gain
Par_min = Ls;                          %   lower initial bounds for the parameter search
Par_max = Us;                          %   upper initial bounds for the parameter search

%   main loop over the seach domains (the domain is refined at every step)
VarGain         = 0;                    %   controls gain variation over the band (global)
MeanGain        = 0;                    %   controls mean gain over the band (global)
TGain           = 0;                    %   controls max mean gain over the band (loop)
TGain_plot      = TGain;                %   visualize gain improvement over the seach
CP              = zeros(length(M), 1);  %   circuit parameters to be found (global)
search_domains  = 7;                    %   number of domain iterations
```

```
for search_domains_ind = 1:search_domains
    StopGain = 0; tic                   %   controls the current iteration
    %   Circuit parameters (cell arrays)
    par(1)   = {linspace(Par_min(1), Par_max(1),  M(1))};  %   L1 here (0)
    par(2)   = {linspace(Par_min(2), Par_max(2),  M(2))};  %   L2 here (inf)
    par(3)   = {linspace(Par_min(3), Par_max(3),  M(3))};  %   C3 here (inf)
    par(4)   = {linspace(Par_min(4), Par_max(4),  M(4))};  %   L4 here (inf)
    par(5)   = {linspace(Par_min(5), Par_max(5),  M(5))};  %   C5 here (inf)
    %   Fill out the 1D arrays
    temp  = 1;
    for m = 1:length(M)
        if m >1
            temp = prod(M(1:m-1));
        end
        block_length  = temp;
        no_of_blocks  = prod(M(m+1:end));
        A(m, :)       = reshape(repmat(par(m), block_length, no_of_blocks), 1, prod(M));
    end
    %   Find min power over the entire frequency band for every particular parameter set
    for m = 1:length(s)
        %-------------------------IMPEDANCE CALCULATOR------------------------------------
                    % !!! CHANGE Zg if you change network ( assuming L1 and L2
                    % are still present in changed network) !!!!
        Zg  = s(m)*A(4, :).*(Rg + 1./(s(m)*A(5, :)))./( s(m)*A(4, :) + (Rg+1./(s(m)*A(5, :))) ) +...
                                          1./(s(m)*A(3, :));
        ZT  = s(m)*A(2, :).*Zg./(s(m)*A(2, :) + Zg) + s(m)*A(1, :);
        %-----------------------------------------------------------------
        if m ==1
            Gain = (4*RL(m))*real(ZT)./((abs(ZA(m)+ZT)).^2);
        else
            Gain = min(Gain, (4*RL(m))*real(ZT)./((abs(ZA(m)+ZT)).^2));
        end
    end
    %   Sort that power (most CPU time-involved step)
    [dummy index] = sort(uint16(1e6*Gain), 'descend');
    %   Check if the +/-25% criterion of gain variation is really satisfied on a finer grid
    for p = 1:round(prod(M)/128)    %   a critical point: only initial values are examined
        temp    = index(p);         %   index into arrays
        a       = A(:, temp);       %   obtain particular circuit parameters
        %------------------------IMPEDANCE CALCULATOR------------------------------------
                                    % !!!! Change Zg !!!!! %
        Zg_      = s1*a(4).*(Rg + 1./(s1*a(5)))./( s1*a(4) + (Rg+1./(s1*a(5))) ) +...
                                      1./(s1*a(3));
        ZT_      = s1*a(2).*Zg_./(s1*a(2) + Zg_) + s1*a(1);
        %-----------------------------------------------------------------
        Gain_ = (4*RL1).*real(ZT_)./((abs(ZA1+ZT_)).^2);
        MGain = mean(Gain_);                                %   local mean gain
        VGain = 100*max(abs(Gain_-MGain)/MGain);            %   local gain variation
        if  (VGain <= 25)                                   %   +/-25% satisfied
            if (MGain> TGain)                               %   search for a higher average gain
                VarGain = VGain; MeanGain = MGain; CP = a;
                TGain      = MGain;                         %   TGain controls the loop
                StopGain   = MGain;                         %   StopGain controls the loop iteration
                TGain_plot = [TGain_plot TGain];            %   Convergence history
                plot(TGain_plot, '-bs',...;                 %   Plot converegence history
                'MarkerEdgeColor', 'k', 'MarkerFaceColor', 'g', 'MarkerSize',10);
```

```
                title('Gain convergence'); hold on; grid on; drawnow;
            end
        end
    end
    if TGain    == 0 return; end;                       %   found none - exit this function
    if StopGain == 0 break; end;                        %   found none for this iteration (no more iterations)
    %   Find the bettter search range based on the found circuit parameters (+/- 50%)
    Par_min    = 0.75*CP;
    Par_max    = 1.25*CP; toc
end
```

```
function [ZA] = dipole(f, dipole_x, dipole_y);
%   Dipole self-impedance: analytical solution for the cylindrical or strip dipole
%   C.-T. Tai and S. A. Long, "Dipoles and monopoles," in: Antenna Engineering Handbook,
%   John L. Volakis, Ed., Mc Graw Hill, 2007, fourth edition, pp. 4-3 to
%   4-32.

%   EM data
const.epsilon       = 8.85418782e-012;  %   ANSOFT HFSS value
const.mu            = 1.25663706e-006;  %   ANSOFT HFSS value
const.c             = 1/sqrt(const.epsilon*const.mu);
const.eta           = sqrt(const.mu/const.epsilon);

k   = 2*pi*f/const.c;
kl  = k*dipole_y/2;
a   = dipole_x/2;       %   equivalent radius
l   = dipole_y/2;
R   = -0.4787 + 7.3246*kl   + 0.3963*kl.^2 + 15.6131*kl.^3;
R(find(R < 0)) = 0;
X   = -0.4456 + 17.00826*kl - 8.6793*kl.^2 + 9.6031*kl.^3;
ZA  = R - j*(120*(log(l/a)-1)*cot(kl)-X);   %   Antenna impedance
```

```
clear all
%   A loop for matching circuit estimation for different center frequencies
%   fc and bandwidths B. Saves the matching circuit parameters and optimized
%   performance dataset for every parameter set.
%   S. Makarov and V. Iyer, ECE Dept., WPI Aug. 2008

dipole_y        = 150e-3;                   %   dipole length (m)
Rg              = 50;                       %   generator resistance
f_res           = 3e8/(2*dipole_y);         %   resonant frequency (ideal; half-wave resonance)
Ls = [ 1 1 1 1 1]*eps;                      %   lower initial bounds for the parameter search
Us = [2e-6 1e-6 100e-12 1e-6 100e-12];     %   upper initial bounds for the parameter search
                                            %   (optimized for 150mm length)
Us = Us*(dipole_y/150e-3);                  %   good for any length

dipole_lt   = [5 10 50];                    %   length to width ratio
DCF         = [0.05:0.05:0.5];              %   relative center frequency vs. f_res
FB          = [0.1 0.5];                    %   fractional bandwidth

MeanGain    = zeros(   length(dipole_lt), length(DCF), length(FB));
VarGain     = zeros(   length(dipole_lt), length(DCF), length(FB));
CP_nHpF     = zeros(5, length(dipole_lt), length(DCF), length(FB));

for ilt = 1:length(dipole_lt)
    for iDCF = 1:length(DCF)
        for iFB = 1:length(FB)
            dcf = DCF(iDCF); fb  = FB(iFB);
            run.center_freq     = dcf;
            run.bandwidth       = fb;
            run.dipole_lt       = dipole_lt(ilt);
            %--------------------OPTIMIZER---------------------------------------
            [temp1  temp2 CP] = ...
            optimizer02_function(dipole_y, dipole_lt(ilt), Rg, dcf, fb, Ls, Us);
            MeanGain(ilt, iDCF, iFB)    = temp1;
            VarGain (ilt, iDCF, iFB)    = temp2;
            CP_nHpF(:, ilt, iDCF, iFB)  = CP.*[1e9 1e9 1e12 1e9 1e12]';
            run.gain = temp1; run
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
            %   Post-processing (plot)
            thisfile = strcat('fig_lt_', num2str(dipole_lt(ilt)), ...
                             '_FB_', num2str(fb), '_DCF_' , num2str(dcf));
            if temp1 >0
                dipole_x    = dipole_y/dipole_lt(ilt);               %   dipole width (m)
                F           = linspace(0.01*f_res, 1.0*f_res, 2e5); %   full spectrum
                ZA          = dipole(F, dipole_x, dipole_y); RL = real(ZA);
                s           = j*2*pi*F;
                %-----------------------NW IMPEDANCE-------------------------------------------
                                    % !!! CHANGE Zg !!!
                Zg  = s*CP(4).*(Rg + 1./(s*CP(5)))./(s*CP(4) + (Rg + 1./(s*CP(5)))) +...
                                                      1./(s*CP(3));
                ZT  = s*CP(2).*Zg./(s*CP(2) + Zg) + s*CP(1);
                %-----------------------------------------------------------------
                Gain = (4*RL).*real(ZT)./((abs(ZA+ZT)).^2);
                ind  = find(abs(F/f_res-dcf)<fb*dcf/2 + 0.5*(F(2)-F(1))/f_res);
                GM   = mean(Gain(ind)); GV = 100*max(abs(Gain(ind)-GM)/GM);  %  just cheking on a finer grid
                h = figure; semilogy(F/f_res, Gain, 'LineWidth', 2); grid on; hold on;
                line([dcf-fb*dcf/2    dcf-fb*dcf/2], [1e-4 1], 'Color', 'g');
```

```
            line([dcf+fb*dcf/2    dcf+fb*dcf/2], [1e-4 1], 'Color', 'g');
            line([dcf,            dcf],          [1e-4 1], 'LineWidth', 2, 'Color', 'r');
            title(strcat('Average gain, a.u. = ', num2str(GM), '; Variation(%) =', num2str(GV)));
            xlabel('f/f_{res}'); ylabel('Gain, a.u.'); axis([min(F/f_res) max(F/f_res) 1e-4, 1]);
            saveas(h, strcat(thisfile, '.fig'));
        end
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        close all;
      end
    end
end
save;
```