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Thermal Analysis of the deCordova Snow House Exhibit

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Abstract

The deCordova Snow House exhibit consists of an underground granite structure which preserves an enormous snowball from winter to summer solstice. The team was tasked with the thermal analysis of the exhibit to recommend viable options for insulation and storage. By performing a heat balance on the proposed design, a finite difference model was created to calculate the resulting diameter of the snowball in summer. By utilizing industrial insulators and preindustrial ice house designs, an appropriate result can be reached.

Acknowledgements

We would like to acknowledge the following individuals for their contribution throughout the completion of our project.

Our initial thanks go out to artist Andy Goldsworthy, who without him this project would not be possible.

We would like to thank the deCordova Museum staff who went above and beyond there call of duty in helping us throughout project. For that alone, we cannot thank them enough.

In addition, we would like to give a special thanks to our MQP Advisor, Brian Savilonis, who guided us throughout our project and helped us find the necessary resources for research.

We would also like to recognize David A. Mark and his tremendous contribution to the project and our report. Not only did he help us discover new ways to insulate and design the structure, but he showed great interest in the success of the exhibit itself.

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Executive Summary

The deCordova Sculpture Park and Museum in Lincoln, Massachusetts has partnered with sculptor Andy Goldsworthy to create a Snow House exhibit that will preserve an 8-foot tall snowball from winter to summer. Based on a preliminary test of a large snowball, the museum decided that a more detailed analysis of the exhibit was necessary. In this project, the team was enlisted to perform the thermal analysis on the deCordova Snow House exhibit, and to provide sufficient results and suggestions to support the construction in the coming years.

Objective

The goal of this project is to design a preindustrial underground snow house that could maintain an 8-foot tall snowball for a four month period.

To accomplish this goal, the team mainly looked into the thermal analysis of the structure in three phases. The first uses a simplified energy balance to determine the change in temperature inside of the granite structure. Secondly a time-based model uses the results from the first phase to calculate the new temperatures through the top layer of soil and the energy change throughout the structure during a four month period. These results can be used to determine the melting rate of the snowball over time. The final phase utilizes ANSYS, a finite element analysis software package, to simulate a 3D analysis of the structure, resulting in a temperature change throughout. The results from the final phase are used to justify the 1 dimensional analysis by comparing their results under similar conditions.

Recommendation

While building the snowball, it is recommended that the dome be kept at a minimum temperature. This can be easily maintained by assuring that cold air is circulated throughout the snow house in the early stages of construction. Maintaining low temperatures early on is crucial for the survival of the snowball.

Proper drainage will be required for the insulated structure in order to minimize the amount of standing water in the snow house. Any passages created for water removal must be large enough to allow for the free transfer of water, but small enough as to minimize any movement of air that may occur. Freely moving air will cause an increase in melting rate of the snowball due to the low insulating capabilities of moving air.

Based on the 500 kg/m^3 density and a 2.4-meter (8 feet) diameter snowball, the weight is estimated at roughly 8500 lbs**. It is recommended that the snowball be built directly in the snow house to minimize the amount of personnel required for construction.**

The results of the analytical MATLAB model do not support the use of natural insulators. To ensure that the snowball is of an acceptable diameter at the end of the four month period, it will be necessary to utilize industrial insulation methods such as expanded polystyrene and polyurethane blocks.

1. Introduction

In order to become a clear leader among the United States sculpture parks, the deCordova Museum in Lincoln, Massachusetts partnered with environmental artist Andy Goldsworthy to create a permanent exhibition that would bring in new faces and new art lovers. This exhibition would qualify the museum as the only institution in New England with a publicly accessible site made by such a renowned artist. They chose to work with Goldsworthy because of his naturalistic, pre-industrial style and awe inspiring work.

Goldsworthy, after visiting the site during the winter, was motivated to recreate a similar project that he had attempted a few years prior. His new project is called “Snow House.” Goldsworthy wants to create a snowball of about eight feet tall and keep it throughout the winter until summer solstice. His plan is to keep the snow ball intact within a granite house, which will be buried underground. Essentially, each winter a snow ball will be made and kept inside the house, until summer solstice when it will be open for people to admire the snowball as it melts away. After the snow ball has completely melted, the granite house will be left open for anyone to walk into. Once winter returns, the process will continue.

Unfortunately when preliminary tests were made, the results were far worse than anticipated. Goldsworthy created a six foot snow ball, and covered it with various insulators: hay, a thermal blanket, and a tarp. The results from this preliminary test prompted the museum to enlist the help of this WPI project team to perform a more in depth analysis of the exhibit.

The goal of this project was to design a pre-industrial underground snow house that could maintain an eight foot snowball for a four month period. To accomplish the goal, the team researched old refrigeration techniques, including the use of natural insulators such as clay and wood chips for ice preservation. Several various insulation materials were considered for the construction of the snow house. Furthermore, the team modeled the heat transfer through the chamber analytically in order to arrive at a logical temperature for the snow ball to remain intact. A one dimensional finite difference heat transfer model was created using MATLAB to find a suitable insulation for the snow house, while ANSYS was used to verify the 1-D calculations with a 3-D analysis.

2. Background

The deCordova museum is well known for its modernistic approach to displaying sculptures and artistic projects. In 2009 the deCordova museum sought out British artist Andy Goldsworthy to propose a site specific project which would be constructed in the winter of 2014 and displayed on the following summer solstice (deCordova, 2012). Goldsworthy's initial design consists of an "ice house" constructed of locally obtained granite which houses a snowball of roughly eight feet in diameter. This snowball would ideally be kept in this underground chamber from the winter until the summer solstice when visitors would be allowed to visit and experience a reminder of the winter past. "According to the artist, 'the work is not an object, but a container—a forum for change, memory, replenishment, season—in which the construction and care of the object, along with its interaction with people, are integral to the work'" {{2 deCordova 2012}}. Andy Goldsworthy, being an esteemed sculptor, is highly interested in preserving natural and preindustrial practices and processes. In order to comply with Goldsworthy's desires, the MQP team was tasked with the thermal analysis of this underground snow house and to assure that, with the assistance of preindustrial methods of cooling, the snowball would be preserved inside of the chamber with minimal loss to its size. A study into historical methods of cooling, therefore, is integral to the completion of the project. Thermal properties and relationships of soil and snow were also considered in order to achieve a result that would satisfy the museum as well as Goldsworthy's vision of a classic memento of the passing seasons.

2.1 Andy Goldsworthy

The British sculptor, photographer and environmentalist, Andy Goldsworthy is one of the most influential contemporary artists in the world. What makes Goldsworthy and his works so popular is the connection with nature.

"At its most successful, my 'touch' looks into the heart of nature; most days I don't even get close. These things are all part of a transient process that I cannot understand unless my touch is also transient; only in this way can the cycle remain unbroken and the process be complete."

-- Andy Goldsworthy

Goldsworthy's connection with nature is apparent in many of his earlier projects, for example, he uses wool to cover a wall in a town that depends on sheep and wool trading; he creates cracks in the "Drawn Stone" project in San Francisco to echo the frequent earthquakes and their effects; and he proposed the Snow House project in the deCordova museum to illustrate the substantial amount of snow precipitation covering New England each year.

With a strong interest in snow in summer, Andy Goldsworthy has carried out two major ephemeral projects. *Snowballs in Summer* took place in Glasgow in 1989 and *Midsummer Snowballs* in London took place in 2000. The difference between these two snow projects and today's *Snow House* project is his focus on the granite house as a permanent installation rather than centering his work on the snowball itself.

In the *Snow House* project, Andy Goldsworthy not only looks into using the house to preserve a large snowball from winter to summer, but also to create an emotional shift for visitors when entering the house with or without the presence of the snowball. The initial idea of the Snow House design is to create a dome-shaped chamber that will be used to store the snow ball while having enough space for visitor access as shown in Figure 1 below.

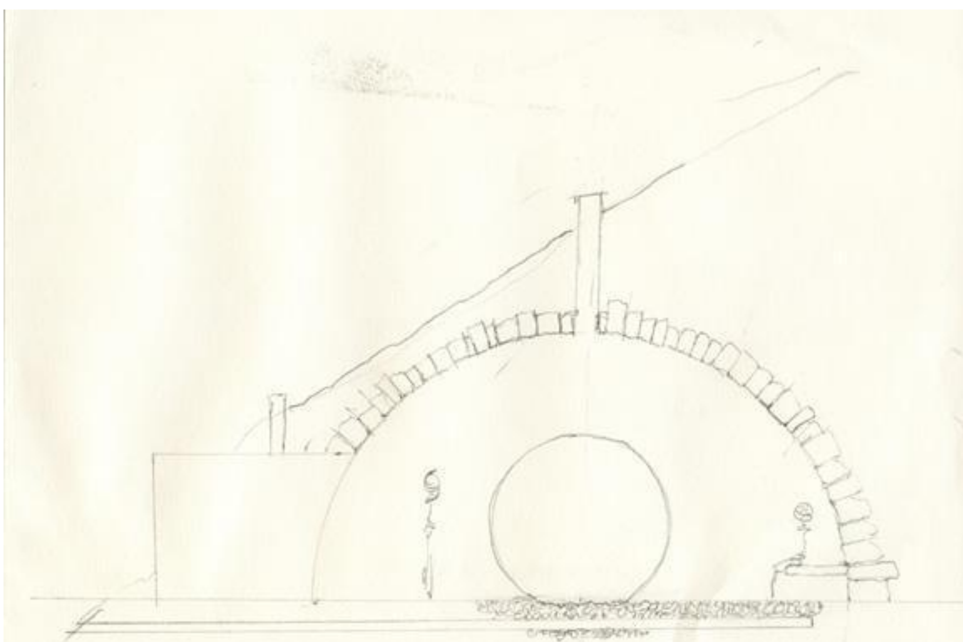


Figure 1: Initial Sketch of Ice House Exhibit (www.decordova.org/snow-house)

Thermal design is a key factor in guaranteeing the presence of the snowball after a half year following its construction. In fact, many of Goldsworthy's previous projects have utilized modern engineering and architecture. Some of his modern engineering projects include the construction of arches built of ice or stones, the balancing of rock on a small point, and the Clay Dome where chemical features of different mineral materials were used to create a curved path on a wall. Andy Goldsworthy has been working with local scientists and students to help realize his artistic ideas, and the Snow House project is no exception. The modeling of an insulation system similar to pre-industrial ice houses will be analyzed and applied to preserve the enclosed snow.

2.2 Ice Houses: Preindustrial Methods to Preserve Ice and Snow

"Ice-houses" have been utilized throughout history for snow storage purposes. From as early as 2000 B.C there is evidence of ice storage and reuse. During the reign of Shulgi in Mesopotamia, "ice-pits" were constructed and insulated with timber for the purpose of storing ice for long periods of time (Forbes, 1958). During the era of the Roman Empire, similar snow pits were covered with straw in order to maintain low temperatures. Regardless of the time period, the most commonly used insulators included wood chips such as sawdust, and straw.

Since the 19th century it was common to replace the use of seasonal ice storage with refrigeration systems in order to cool food and other amenities. Natural thermal insulators became the subject of research for the preservation of snow or ice. For example, Professor Kjell Skogsberg from Lulea University of Technology is interested in snow storage for space cooling. In his doctoral thesis: "Seasonal Snow Storage for Space and Process Cooling," he emphasizes the use of snow and the different types of natural insulators used to preserve low temperatures. Professor Bo Norderll, a colleague of professor Skogsberg, similarly works in the field of thermal energy storage and snow and ice related problems.

2.2.1 Natural Insulators:

Wood chips have become a traditional thermal insulation for snow. Wood chips appear in numerous forms, such as sawdust, wood powder, cutter shavings, and bark. In ancient Greece, ice harvested from lakes and rivers was stored into barns which were thermally insulated by sawdust. (Taylor, 1985) This technique became common in Europe and North America until the beginning of the 20th century. During the 19th century, Frederic Tudor, also known as the Ice

King of New England, implemented the use of wood chips for ice storage purposes. (Felten, 2010). T end?During this same time period, Herbert Thompson, creator of the Thompson Ice House Preservation Corporation, was able to conserve ice by implementing a double-walled storage room using wood chips as insulation.

More recent establishments have utilized and analyzed the feasibility of wood chips as thermal insulation. Since the beginning of 2000's, the hospital in Sundsvall, Sweden has implemented the traditional snow-storage system to cool its facilities. In order to begin the cooling process, snow is stored into a chamber thermally insulated with wood chips in the form of cutter shavings. By using this insulation method, snow is preserved throughout the whole year in the Sundsvall Hospital. The snow can then be used to cool the hospital in the summer.

While wood chips are known to be one of the most common insulators used for ice-houses, not all forms of woodchips are considered viable options. While sawdust has become one of the most common types of wood chips used in ice houses of the early 20th century due to its availability and relatively low cost, it is susceptible to moisture which lowers its performance as an insulator (John T. Bowen, 1992.) Wood shavings generally out-perform sawdust because they are less compact and do not absorb moisture so rapidly. Preliminary tests by Skogsberg, showed that cutter shavings were more efficient insulators than other forms of wood chips. By insulating one snow pile with 0.1 meter thick of cutter shavings and one with 0.2 meter thick of saw dust, he found that the two piles melted at approximately the same rate; indicating that cutter shavings were more efficient due to the high concentration of trapped air (Skogsberg, 2002.)

One major drawback to the use of wood chips as insulators is the rapid rate of decay. Due to deteriorating insulation, the thermal properties of wood chips become less resistant to heat transfer. Material must be added each year, and in some cases (as with moisture) all material must be replaced.

The effect of debris on melting glacial snow can provide some insight into other forms of thermal insulation. Debris is defined as a fine-grained clay-sand mixture that covers many glaciers around the world. Over the past hundred years, many studies have been conducted on its relationship with the ablation, or melting rate. Results show that the glacier melt rate increases for very thin layers of debris (less than .003m); however, the ablation decreases for thicker layers.

For instance, a debris layer of about 0.10 meters reduced the melting rate by 35%-66% and with a 0.40 meter layer the ablation reduced by 59%-85%. (Mattson and Gardner, 1992; Kayastha, 2000) A thin layer of debris increases the ablation rate due to the increase in absorptivity of solar radiation. However, as the layer thickness increases, the ablation rate decreases due to the insulation effect and heat storage.

2.2.2 Ice Houses in the Early 20th Century

The development of ice-houses in the United States was very common in the early 20th century. The construction of these facilities depended greatly upon local condition and the amount and difficulty of obtaining ice. For instance, at a location where ice was hard to obtain, a better constructed and therefore a more expensive ice house was advisable. However, if natural ice were to be stored in areas where it was commonly found, cheaper structures were satisfactory since the loss from melting ice was a small consideration (Bowen, 1920).

After determining certain aspects about the construction such as its location and the environmental effects, an insulation system had to be considered. In cheaper ice houses, sawdust and cutter shavings were used as insulators. However, in some areas of higher outside temperatures other commercial and chemical refrigerants were necessary to assure the safekeeping of ice. Using these insulators had the benefit of being practically fireproof, occupying little space and retaining efficiencies over a longer time.

Efficient drainage and ventilation systems are crucial for satisfactory ice storage. In houses where the floor was below the ground level, drainage usually was obtained through the soil if it was porous. However, if it were clay soil, it became necessary to excavate holes of a foot or two in order to fill them with gravel or cinders. Also, ice-houses were usually constructed with a centered sloped ceiling to assist the circulation and carry warm air to the controlled ventilator that was put into place. The controlled ventilator reduced the amount of moisture in the room, which actually reduced the melting rate. Finally, whether they were constructed out of brick, concrete or wood, the building had to be waterproofed on the inside. Usually brick and concrete were readily waterproofed by painting the walls with a suitable paint or waterproofing compound such as preparations of paraffin and asphalt.

One example of an earlier ice house functions as an underground storage system, which uses 12 inch (0.3 meters) layer wall of sawdust as an insulator to prevent ice from melting. The floor contains a 12 inch layer (0.3 meters) of well tamped cinders which then lead to a sloped drainage system with crushed rock. Walls were made of a mix of cement, sand and crushed stone or gravel, and the ceiling was constructed in a symmetrical manner to circulate heat. Waterproofing the walls was unnecessary, since drainage occurred primarily at the bottom of the pit (Bowen, nd1920).

Overall, the use of ice houses for snow storage purposes has been implemented for hundreds of years. While today they have been replaced by refrigeration systems, several studies use ice houses as a possible way to preserve snow. Since the project entails an underground ice house, soil properties will also be considered.

2.3 Soil Properties

The thermal properties of soil will affect the melting rate of the snow ball enclosed in the ice house and must be taken into account to provide an accurate heat transfer model. By studying the earth's core, scientists have discovered that as the depth of the earth increases temperature becomes more constant. The ground acts as an insulator, therefore it takes longer to heat up and longer to cool down. At about four feet the average temperature is approximately 50 to 55 ℉ (10 to 13 ℃)

2.4 Modeling Heat Transfer in the Snow House

In order to analyze the heat transfer of the snow house, the entire process must be broken down into components. The three forms of heat transfer used in the model are as follows:

1. Phase change (melting rate) of the snowball.
2. One-Dimensional plane wall conduction
3. Transient 1-D finite difference

2.4.1 Free Convection

Free convection is the result of fluid motion driven only by body forces in the fluid and a density gradient. In order to simplify the model it is assumed that the density gradient is a result of a temperature gradient and the body force is simply gravity. The primary equation that governs free convection is the heat flow from the solid which is given by:

$$q = hA(T_s - T_\infty)$$

where A represents the surface area, T_s and T_∞ are the surface temperature and air temperature respectively and h is the convection heat transfer coefficient given by the equation:

$$h = \frac{Nu_D k}{D}$$

To calculate h it is first necessary to calculate Nu_D, or the Nusselt number, which is a function of the Rayleigh number (Ra), Prandtl number (Pr) and the geometry of the solid. The Rayleigh number is simply the product of the Prandtl number and the Grashof number (Gr) which is given as:

$$Gr_l = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

where g is gravity, L is the characteristic length defined by the geometry, ν is the kinematic viscosity and β is the inverse of the average temperature or:

$$\beta = \frac{2}{T_s + T_\infty}$$

By manipulating the above equations it is possible to relate the heat transferred from the air and the amount of snow that has been removed from the snowball. This manipulation as well as calculations will be further pursued in the methodology section {{3 Incropera, Frank P. 2007}}.

2.4.2 Conduction

As a first pass it is beneficial to find the time required for heat to travel through each layer of the snow house. With this information one can calculate the time before heat transfer reaches its maximum rate to the inside air. The team can first calculate the thermal diffusivity of a layer using the equation:

$$\propto = \frac{k}{\rho c_p}$$

Treating the material layers as semi-infinite solids, one can estimate the temperature throughout a layer at a given time. A function defined as the Gaussian Error Function can be used to find this gradient:

$$erf\ (x)\ = \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{(4\alpha t)^5}} * e^{-u^2} du$$

The temperature gradient is then given by:

$$T(x) = T_s + (T_i - T_s) erf(x)$$

While this process shows the time necessary for heat to begin transferring to the chamber air, it can also be used to roughly estimate the temperature of the wall which is in contact with the chamber air.

As a worst case scenario the layers of the icehouse will transfer heat at a maximum rate without any heating time. Each layer of the ice house is treated as parallel walls with infinite lengths. The problem can then be treated as one dimensional heat transfer through a series of thermal resistances. The heat transfer rate for the system can be expressed as:

$$q = \frac{T_{\infty,1} - T_{\infty,n}}{R_{Total}}$$

Where $T_{\infty,1}$ is the temperature of the air outside of the chamber and $T_{\infty,n}$ is the air inside the chamber. R_{Total} is the total thermal resistance of the system and can be written as:

$$R_{Total} = \frac{1}{h_1 A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} \cdots \frac{1}{h_n A}$$

With these simple concepts an equation that solves for the change in temperature inside the chamber as a function of the outside temperature and time can be derived. These derivations will be further investigated in Section 3.0.

2.4.3 Finite Difference Analysis

The Finite Difference Method is used to approximate the varying of a property throughout a simple two- or three-dimensional geometry. The method mainly involves three steps: gridding the geometry into sections, finding formulas governing the difference between these sections or grids, solving the formulas for needed property at all sections or grids. Finite Difference method can be implemented in transient state to involve time change. In this project, this method is applied to the soil around the Snow House to find out how temperature distribution through the soil changes during the period of insulation.

2.5 Snow

Rather than ice that freezes from liquid water, snow is precipitation in the form of ice crystals. Snow originates in clouds at temperatures below the freezing point (0 ℃, or 32 ℉), where water vapor in the atmosphere condenses directly into ice bypassing a liquid phase change.

2.5.1 Snow Melting and Snowpack

Snowpack forms from layers of snow that accumulate in geographic regions and high altitudes where the climate remains cold for extended periods during the year.

Snow water equivalent is measured to account for the amount of water which can be produced from snowmelt for a water reservoir. Researchers often use snow telemetry, or SNOTEL, instruments to measure the water equivalent of the overlying snow.

Using snow energy for air-conditioning has been researched in recent years. In Funagata, Japan, where there is a heavy amount of snow precipitation in winters, a significant quantity of snow is stored in a 120 m^3 storage room in the winter and is used for air cooling in summer. The innovative air-conditioning design takes advantage of stored snow energy. A problem arises during the first month of storage where snow melt rate is especially high.

2.5.2 Snow Density

Compared to liquid water's density of 1000 kg/m^3 and the density of ice (at 4 ℃), 917kg/m^3, snow has a smaller and a wider range of density values:

Typical densities of snow and ice (kg/m^3)	
New snow (immediately after falling in calm)	50-70
Damp new snow	100-200
Settled snow	200-300
Depth hoar	100-300
Wind packed snow	350-400
Firn (Crystallized and Partially Compact)	400-830
Firn and Wet Snow	700-800
Glacier ice	830-917

Source: Paterson, W.S.B. 1994. The Physics of Glaciers

Table 1: Typical densities of snow and ice

Storing snow becomes more difficult due to its relatively low density compared to ice. In order to achieve the best results when storing snow, it is necessary to store snow with a high density. Table 1 shows that it is possible to achieve high densities before storage.

2.6 Average Weather Data for Lincoln, Massachusetts

By using average values for temperature wind and snowfall, it is possible to theoretically discern the transfer of heat through the snow house.

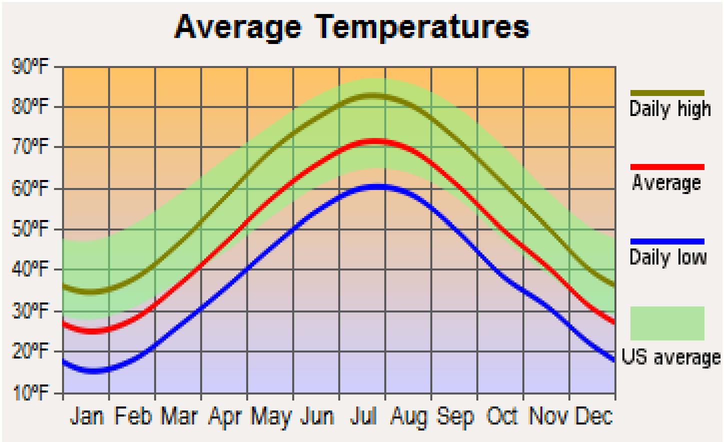


Figure 2: Average Temperature in Lincoln Massachusetts

It is clear by observing Figure 2 that temperatures above freezing will not occur until the month of April. This simplifies the heat transfer model into a four month timeframe.

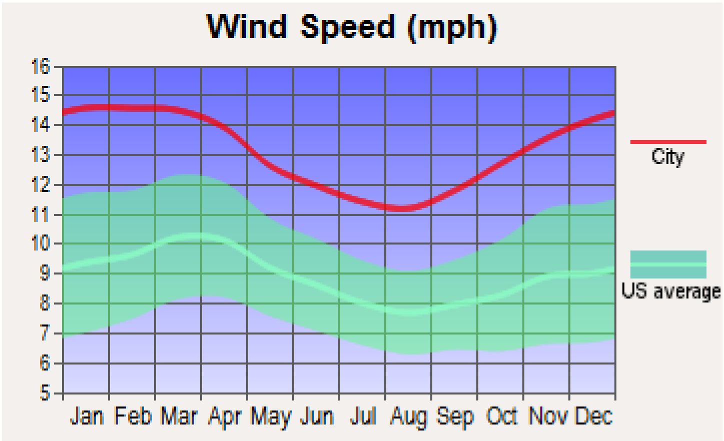


Figure 3: Average Wind speed in Lincoln Massachusetts

By using the wind speed given in Figure 3, it is possible to calculate the forced convection over the snow and soil above the granite ice house.

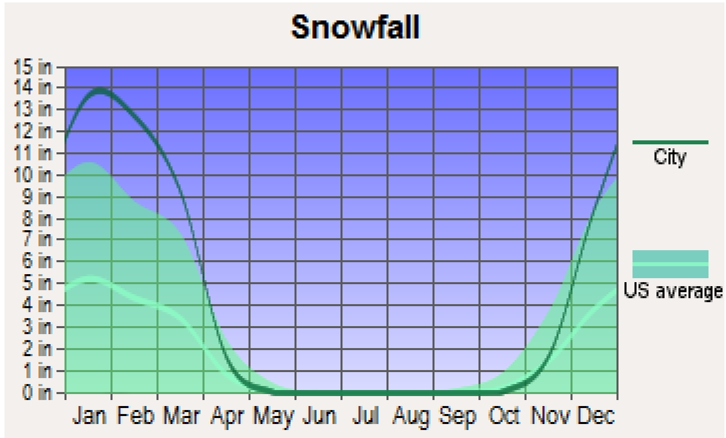


Figure 4: Average Snowfall in Lincoln Massachusetts

Figure 4 demonstrates that there will be a significant amount of snow above the ice house in the months of March and April. This factor will benefit the preservation of the snowball, but was not included in the model due to the unpredictability of snowfall in New England.

2.7 ANSYS

ANSYS Workbench is a common platform from the engineering simulation software, ANSYS, for solving engineering problems. Typical tasks that can be performed can range from heat conduction analysis through a cylinder, to performing a two-dimensional static truss analysis. This platform has become the industry's broadest tool of advanced engineering simulation technology. It contains several features including an innovative project schematic, an integrated parameter management, and an automatic project-level update, among others. Also within this platform several sub-platforms are implemented to develop the geometry, create a mesh, and then simulate the actual analysis. In this project, ANSYS system for Transient Thermal (Figure 5) will be used to simulate the thermal aspects of the snow house and the results of which are used to compare and justify the MALAB finite difference model.

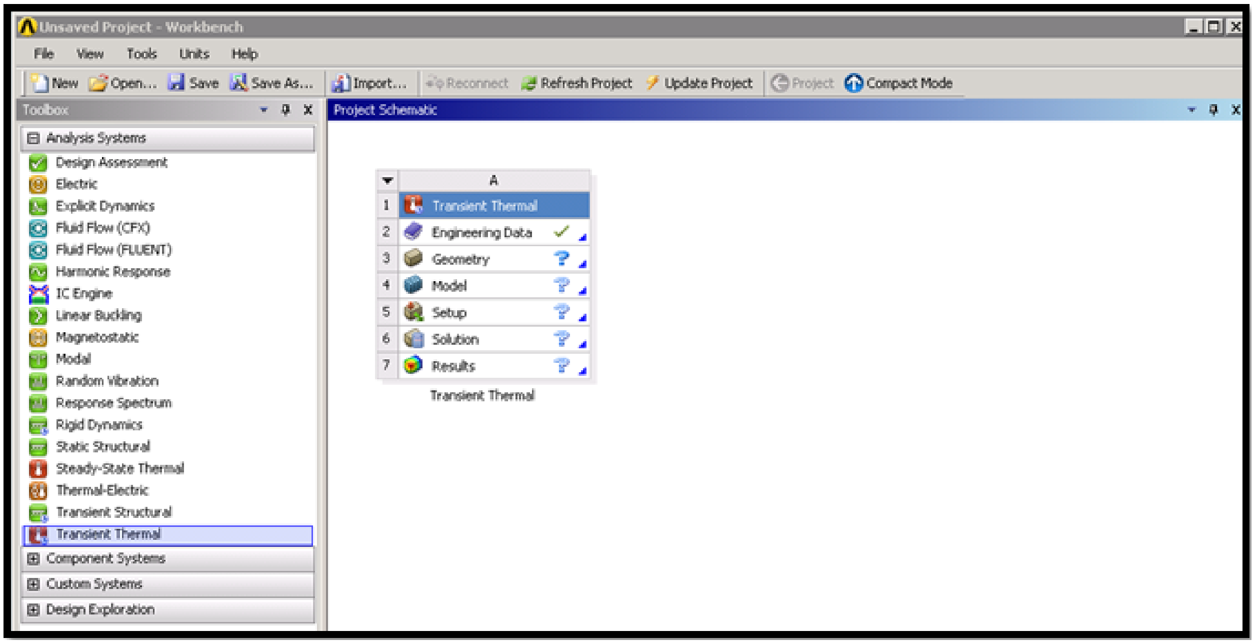


Figure 5: ANSYS Analysis System chosen as Transient Thermal

2.8 Conclusions

The Snow House project is one of the first priorities of staff at the deCordova Sculpture Park and Museum. The museum encourages fundraising for the installation; however, skepticism from investors arises due to the uncertainty of the projects success. With the failure of the trial of preserving a six feet diameter snow ball in previous year, and the artist's requirement of pre-industrial methods of construction, the artist and the museum realized the need of scientific data to ensure the preservation of the snowball. The team was consulted to perform a thermal analysis of the design within the artist's design specifications to preserve the snow ball from winter to summer. The methods and experimental procedure proposed by the team will be discussed in Section 3.0 that follows.

3.0 Methodology

This Section illustrates the two approaches used to analyze the thermal aspects of the snow house and the comparison between them. All models are based on the initial configuration in Figure 6:

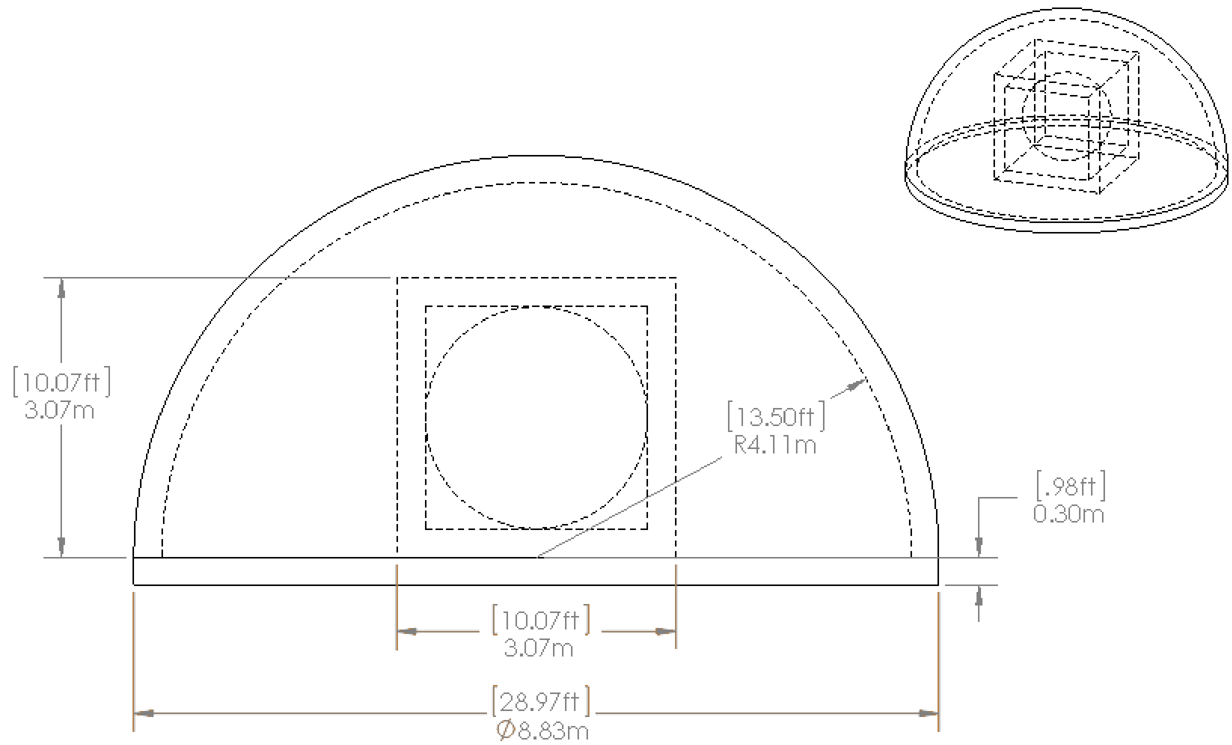


Figure 6: Initial Configuration

3.1 Finite Difference Method using a Time Loop

In the MATLAB code, a time step analysis is used to calculate property change through time. For the whole time span of 4 months, 967980 loops are gone through, each loop is 10 seconds. For each loop, calculation goes through the finite difference method and each of the three steps to calculate temperature change of soil, air outside and inside structure, and diameter of snowball for 10 seconds, and pass the resulting values to the next time step until the end of the 4 month period. The property values used in MATLAB are summarized in Table 2:

Name	**Symbol**	**First Unit**	**Second Unit**
		Dimensions	
Snowball diameter	D	2.44m	8ft
Dome radius, inner	h_dome	4.1148m *different from pic above	13.5ft
Dome radius, outer		4.415m	14.5ft
Ground diameter, inner	D_ground	=h_dome*2=8.23m	27ft
Ground diameter, outer		8.83m	28.97ft
Structure Inner	L_i*L_i*L_i	2.435m*2.435m*2.435m	8ft*8ft*8ft
Structure Outer	L_s*L_s*L_s	3.07m*3.07m*3.07m	10ft*10ft*10ft
		Time Loop Setup	
Total time elapsed	t	3600*24*7*4*4=9679800s	112 days
Time step	dt	10s	
Number of time step	nt	967980 timesteps	
		Air Properties	
Thermal conductivity	k_air	0.024 W/m*K	
Heat capacity	cp_air	1005 J/kg*K	
density	air_density	1.2kg/m	
Initial Ambient air	Tair	274K	
Change in Ambient		273K, first 30 days	

air		273 – 300K, after 30 days	
Initial Air inside structure	T_s	273K	
Initial Air outside structure	Tin	273K	
Snow Properties			
Fusion of melting	h_ls	334000 J/kg	
density	density_snow	400 kg/m^3	
Snowball temperature	T_snow	273K	
Soil Properties			
Thermal conductivity	k_soil	0.18 W/m*K	
Heat capacity	cp_soil	1200J/kg*K	
density	soil_density	1750 kg/m^3	
Soil at 5ft	T_soil	283K	
Soil above 5ft	=Tair	274K	
Soil Depth above dome	L_top	1m	
Soil Depth below dome	L_bot	1.2m	
Granite			
thermal conductivity	k_granite	2 W/m*K	
Heat capacity	cp_granite	780 J/kg*K	
Density	granite_density	2650 kg/m^3	

Table 2: Property Values

As a first step, the finite difference method is used to calculate the change in temperature throughout the top layer of soil. By utilizing an energy balance on the air enclosed in the snow house, the temperature is calculated at each time step. The layer of granite is considered negligible due to its relatively low thermal resistance thus low impact on the rate of heat transfer into the air. Figure 7 shows the first configuration of the snowball which is analyzed without insulation.

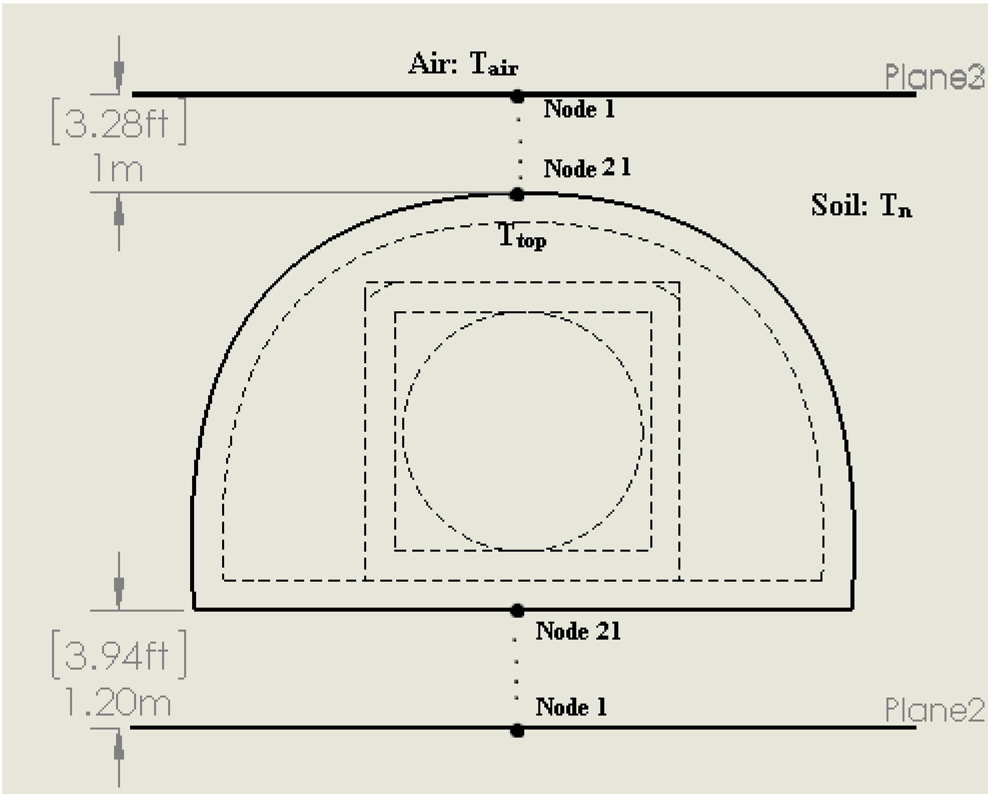


Figure 7: Discretization of Soil around Chamber

The top and bottom planes of soil are each analyzed using finite difference analysis. The top plane, or the layer of soil on top of the chamber, is assumed to have a thickness of 1 m, the bottom layer of soil is assumed to have a thickness of 1.2 m due to the soil's constant thermal properties at this depth. For the sake of finite analysis, 21 nodes are utilized on both the top and the bottom layers of soil.

In order to simplify the heat transfer model, the chamber is treated as a rectangular prism with perfectly insulated sides. The top of the snow house is treated as a flat plane with a surface area equal to that of the physical dome, while the bottom is analyzed normally.

For the top plane, temperature at node 1 is taken from the outside air temperature which follows a linear equation derived from the average ambient air temperature change in Lincoln, MA:

$$T_{air}^{i+1} = T_{air}^{i} + \frac{26}{number\ of\ time\ steps}$$

Where i+1 denotes the new time step and i denotes previous time step. In order to correctly model the temperature change of the year, the first month is considered a constant air temperature of 273 k, therefore the only heat transfer in the first month occurs at the ground boundary layer. The air temperature then follows the equation above.

For the bottom plane the temperature at node 1 is taken from the soil temperature which is a constant temperature of 283K at a depth of 4 feet.

For the internal nodes (2-20) in both the top and bottom planes the temperature is found using the following equations:

$$k_{soil}A\frac{T_{n+1}^{i} - T_{n}^{i}}{\Delta x} + k_{soil}A\frac{T_{n-1}^{i} - T_{n}^{i}}{\Delta x} = \rho_{soil}Acp_{soil}\frac{T_{n}^{i+1} - T_{n}^{i}}{\Delta t}$$

This can be reduced to the following equation

$$T_{n}^{i+1} = F_{o}\left(T_{n+1}^{i} + T_{n-1}^{i}\right) + (1 - 2F_{o}) * T_{n}^{p}$$

Where:

$$F_{o} = \frac{\alpha_{soil} * \Delta t}{\Delta x^{2}} = \frac{k_{soil}}{\rho_{soil} * cp_{soil}} * \frac{\Delta t}{\Delta x^{2}}$$

For the last node that is in between the air and soil which utilizes a convective boundary, the temperature at the node is found using the equation:

$$h_{air}A\left(T_{c} - T_{n}^{i}\right) + k_{soil}A\frac{T_{n-1}^{i} - T_{n}^{i}}{\Delta x} = \rho_{soil}A\frac{\Delta x}{2}cp_{soil}\frac{T_{n}^{i+1} - T_{n}^{i}}{\Delta t}$$

Which can again be reduced to:

$$T_n^{i+1} = 2B_iF_oT_c + 2F_oT_{n-1}^i + (1 - 2B_iF_o - 2F_o)T_n^i$$

Where:

$$B_i = \frac{h*\Delta x}{k_{soil}} \ .$$

For simplicity the initial temperature at all nodes is assumed to be 273K. The finite difference method explained above is used to calculate a new temperature for each node at each time step. For the finite difference method to function appropriately the time step chosen must be relatively small, therefore the team selected a time step of 10 seconds. The finite difference loop is run in MATLAB for the four month period in which the snow ball must be kept cool.

Lastly, by performing a heat balance on the air inside the chamber, the team can calculate the internal temperature and relate it to the melting rate of the snowball. This is done in three steps each with a different control volume:

Step 1: Air outside structure:

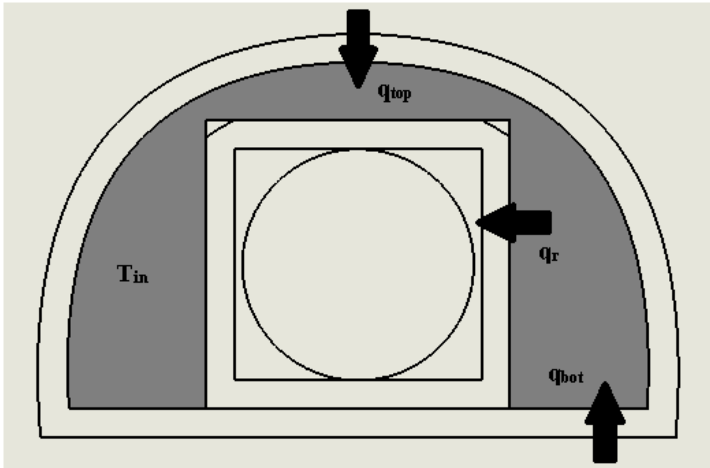


Figure 8: Air outside structure as control volume

For the grey section of air in Figure 8, the control volume is receiving heat by convection (q_{top}, q_{bot}) from the top and bottom surfaces of the dome, whose temperatures have been found in the finite difference method of soil; and the air loses heat (q_r) by convection and conduction to the structure, resulting in the following energy rate balance equation:

$$q_{top} + q_{bot} - q_r = \frac{dT_{in}}{dt} * m_{air} * Cp_{air}$$

where q_{top} and q_{bot} are calculated by h*(T_{node21}-Tin)*$A_{top\ or\ bot}$, where the calculation of h values are explained in Section 2.4.1 Free Convection, and

$q_r = \frac{T_{in}-T_s}{R}$ and $R = \frac{1}{Ast} * \left(\frac{1}{h_{in,top}} + \frac{L_{ins}}{k_{ins}}\right)$,

$m_{air} = air - density * (V_{dome,inner} - V_{structure,outer})$

Solving of the energy rate balance equation will give the temperature change of air outside the structure, T_{in}.

Step 2: Air inside structure:

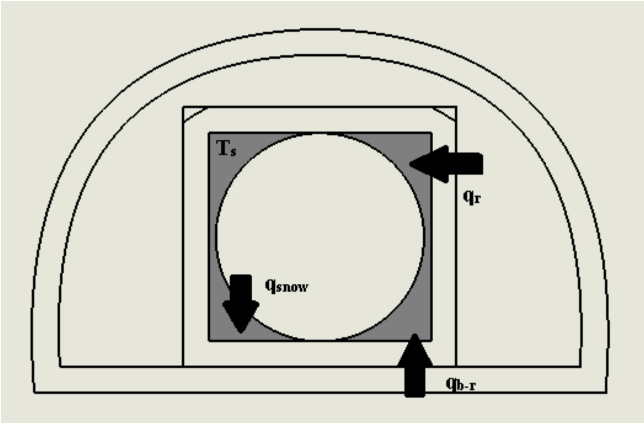


Figure 9: Air inside structure as control volume

In this step, the control volume is the air inside the structure (Figure 9), surrounding the snowball, the temperature of which will directly affect the snowball's melting rate. This section of air receives heat by conduction and convection from 6 sides of the structure (q_r, q_{b_r}), and loses heat to the snowball by convection (q_{snow}), resulting in the following energy rate balance equation:

$$q_r + q_{b-r} - q_{snow} = \frac{dT_s}{dt} * m_{air} * Cp_{air}$$

where $q_{snow} = h_s * (Ts - 273) * A_{sb}$,

$$q_{b-r} = \left(T_{bot,node21} - T_s\right) * A_{ground}/R_{t-b},$$

$$R_{t-b} = L_{ins}/k_{ins}$$

$$m_{air} = air - density * (V_{structure,inner} - V_{snowball})$$

Solving of the energy rate balanceequation will give the temperature change of air inside the structure, T_s.

Step 3: The snowball

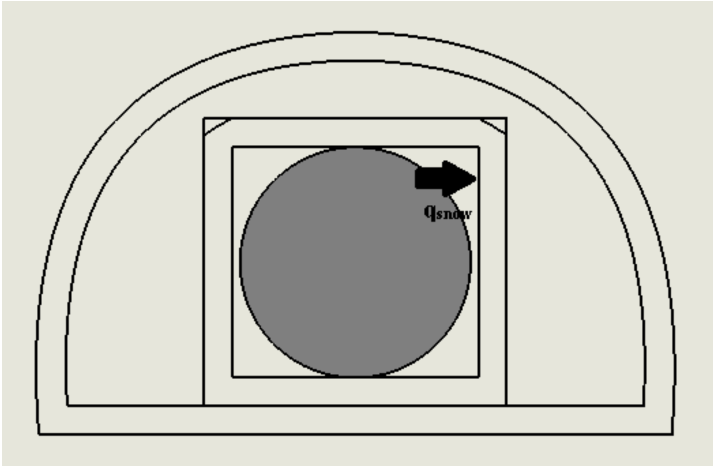


Figure 10: Snowball as control volume

For the snowball seen in Figure 10, the only transfer of heat through the control volume's boundary is the convection heat transfer to the air surrounding it, which results in the melting of the snowball:

$$q_{snow} = h * A_{sb} * (T_s - 273K) = h_{ls} * \dot{m_{melt}}$$

where $m_{melt} = \frac{4}{3}\pi R(t)^3 * \rho_{snow},\ so\ \dot{m_{melt}} = \frac{dm_{melt}}{dt} = 4\pi R(t)^2 * \frac{dR}{dt} * \rho_{snow}$

$$h * 4\pi R(t)^2 * (T_s - 273) = h_{ls} * 4\pi R(t)^2 * \frac{dR}{dt} * \rho_{snow}$$

$$h * (T_s - 273) = h_{ls} * \frac{dR}{dt} * \rho_{snow}$$

$$q_{snow}/A_{sb} = h_{ls} * \frac{dR}{dt} * \rho_{snow}$$

Where h_{ls} is the enthalpy of fusion of water, measuring the heat losing rate of H_2O when it changes state from solid to liquid, or when melting in the unit of J/kg. Solving of the equation will give the diameter change of the snowball with respect to time t, dR/dt.

3.2 Finite Element Analysis

A finite element analysis of the heat transfer in the snow house was performed using ANSYS Workbench in order to support our one dimensional numerical analysis. Within this program, two models were developed to simulate the change in temperature over time. This model represents the underground granite chamber with a double walled structure insulated with sawdust. This analysis can validate our numerical results found in our MATLAB loop.

The same procedure was followed for each model transient thermal model for analysis in ANSYS. Material properties were first entered into the system based on information used in the MATLAB models. By using the integrated ANSYS DesignModeler the geometry for both models were created separately for analysis. The workbench Simulation module was used to set up the FE-Mesh and boundary conditions of the system. For more information about how to use ANSYS for thermal analysis, please refer to Section 2.7.

As previously mentioned a model was created to compare the 3D analysis with the 1D analysis made in MATLAB. This initial model represents the underground chamber with insulation. The system was treated as axisymmetric at the leftmost boundary in order to simulate 3D heat transfer, as seen in Figure 11, and resulting in a cylindrical structure instead of a cube shaped one, but the difference in shape wouldn't affect the heat transfer much thus is assumed negligible.

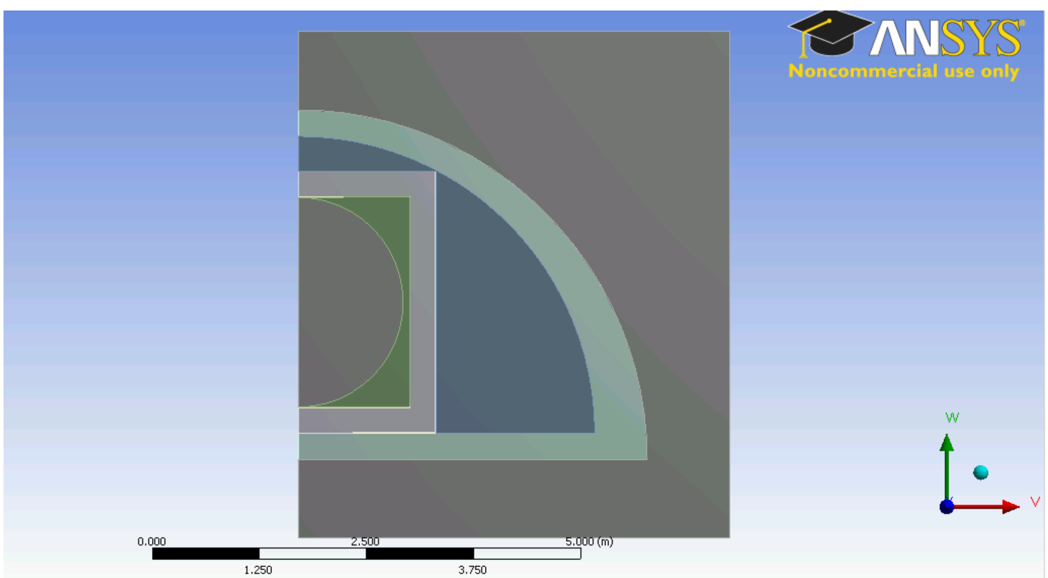


Figure 11: Chamber with Insulation

The dimensions used are the following:

- Snow ball: 2.438 m (≈8 ft.)
- Chamber height: 3.45 m (≈11.3 ft.)
- Granite layer: .3 m (≈1 ft.)
- Soil Layer (from the granite to the atmospheric air): 1 m (≈3.3 ft.)
- Wooden layer: .00635 m (≈0.02 ft.)
- Insulation layer: 0.3 m (≈1 ft.)

In order to simplify the model ́s analysis, few assumptions were made. These are the following:

- An assumed initial temperature of 0 º(32 ℉) was used for the overall model.
- No mass transfer was considered. Instead the snow ball was a treated as a solid object with a constant temperature of 0 ℃ (32 ℉).
- The same assumptions were made for soil temperature as previously mentioned in the MATLAB model.

The following thermal properties were used for the analysis in ANSYS.

Properties	Density kg/m^3	Thermal Conductivity W/m* K	Specific heat J/kg*K
Air	1.614	0.026	1007
Granite	2663	0.29	783
Snow	400	0.1195	2090
Soil	1525	0.29	1140
Wood	700	0.173	2310
Wood Chips	210	0.16	2500

Table 3: Thermal Properties of Different Materials Involved

Boundary conditions were the same for both models. These include:

- Constant Temperature: For both soil boundaries at the right and at the bottom, a constant temperature was given of 10 ℃ (32 ℉).
- A tabular convection and temperature boundary was given at the top of the soil. It was determined that the models had a convection coefficient that began at 1 W/m^2*K and finished in 1.4 W/m^2*K by the end of June. Also, it was determined that the atmospheric temperature began at 0 ℃ (32 ℉) and ended at 19 ℃ (66 ℉) by the end of June.

Finally, it is important to mention that the air inside the chamber and the insulation box was treated as conduction, rather than convection. For more accurate results, other methods may be further investigated such as FLUENT.

4.0 Results and Recommendations

4.1 Verification of MATLAB Results

ANSYS finds a range of final temperatures within the chamber. Both Figure 12 and 13 represent a half of the chamber with an axisymmetric boundary condition in the left. These show range of temperatures inside and outside the insulation box. In the first figure, it can be seen that the inside temperature ranges from 0 ℃ to 6 ℃ (32 ℉ to 42.8 ℉), while in the second one, temperature ranges from 2 ℃ to 9 ℃ (35.6 ℉ to 48.2 ℉); Blue represents the colder temperatures and red represents warmer temperatures. These results correlate with the results found in the MATLAB analysis, where it was predicted that the final temperature inside the insulation box would be 7 ℃ (44.6 ℉) and outside the box was 8 ℃ (46.4 ℉).

It is important to take into account that both of these analyses are quite distinct, and that is why results may slightly vary. The ANSYS model applies different boundary conditions than MATLAB, which can be found in Appendix B. Some of the reasons why results vary include the omission of melting from the snow in the ANSYS analysis, and the distinct boundary conditions used in both analyses. For more information about the distinct boundary condition refer to Appendix A and B.

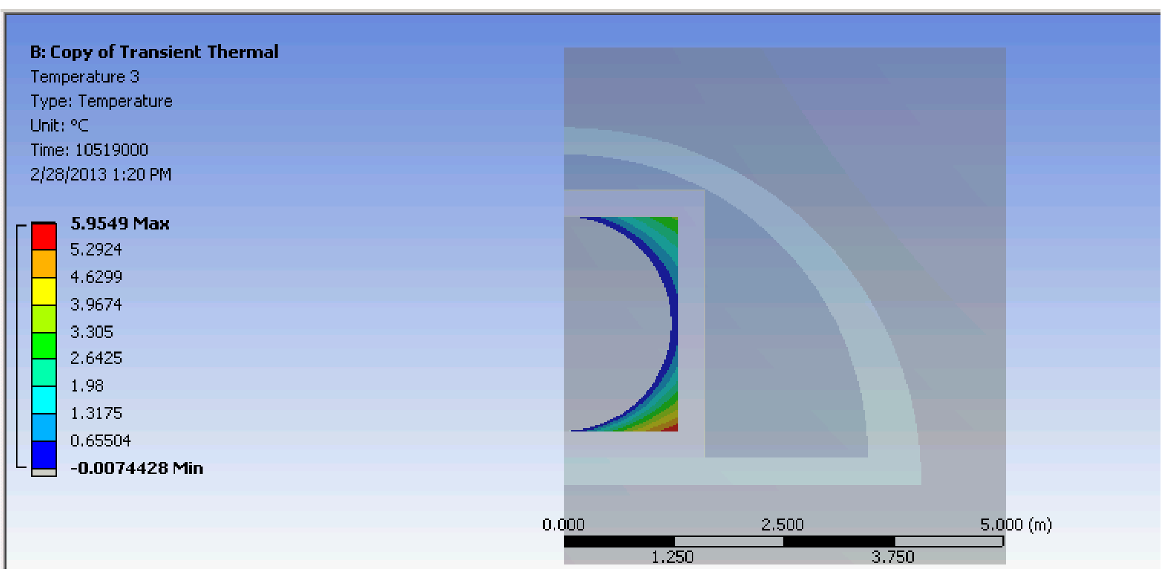
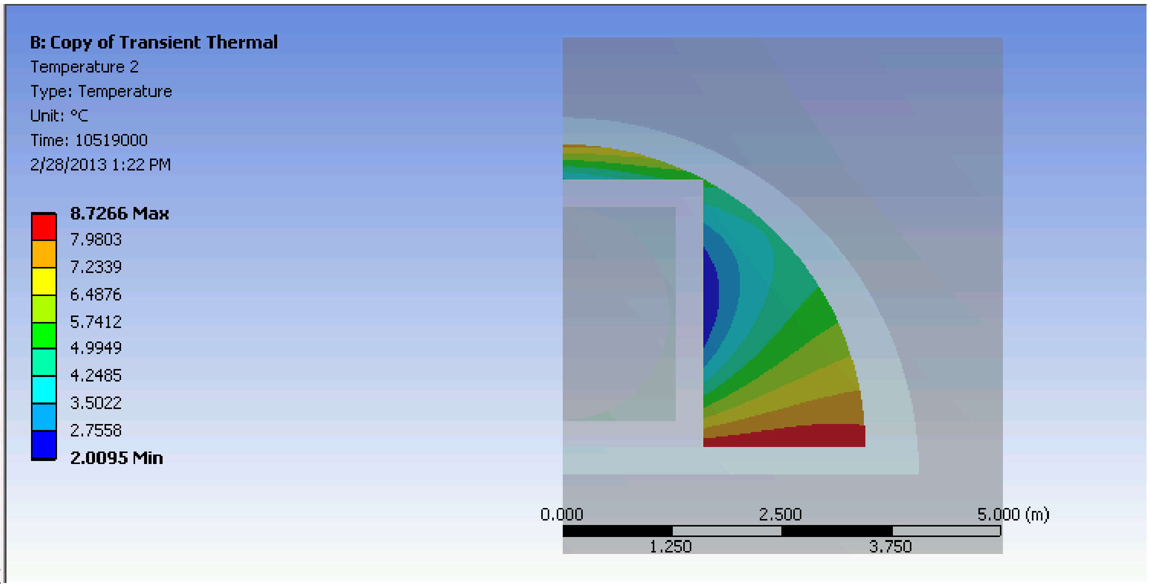


Figure 12: Temperature Range in the Structure

l



t

Figure 13: Temperature Range in the Dome

ANSYS also calculated the final heat flux that into the insulation box. As it can be appreciated, it ranges from 0.4979 W/m^2 to 1.006 W/m^2. Just like for the temperature analysis, the heat flux found in the 1D analysis also falls into the range (0.5 W/m^2). However, in ANSYS, lower heat flux values prevail throughout the insulation box. This may be because similar temperatures prevail within the both the air inside and outside of the insulation box.

So, based on the results from ANSYS under the same initial configuration, the MATLAB model used is verified and trustworthy

4.2 MATLAB Snowmelt Results

The following section shows the results of the MATLAB model for an array of industrial and preindustrial insulation. While the group focused on the methods of preindustrial icehouses, the lower density of snow creates a different atmosphere. In order to properly insulate the exhibit it may be necessary to utilize more advanced insulation systems.

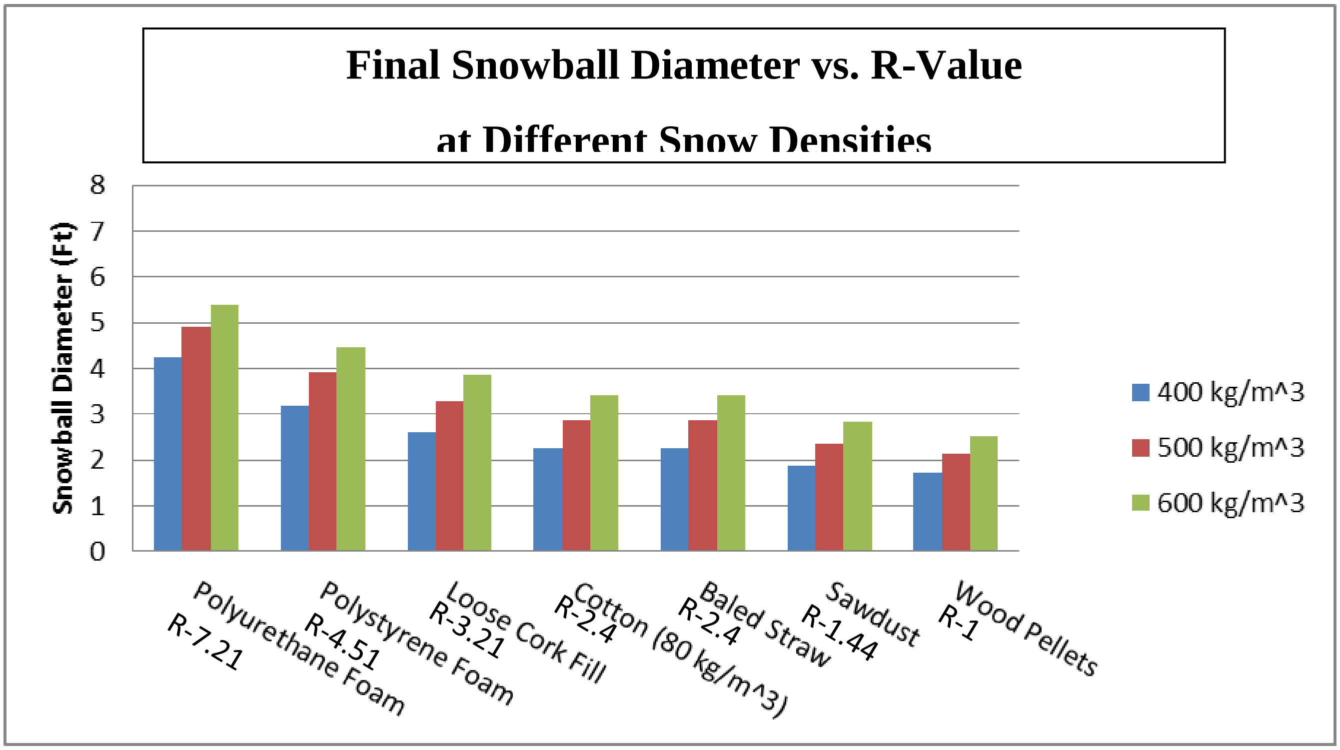


Figure 14: This figure shows the final diameter of the snowball in the snow house after the allotted 112 days based on the density of snow.

Figure 14 shows the final snowball diameter after four months of storage (March, April, May and June.) Common preindustrial insulators cannot keep the snowball at the desired diameter. With the current configuration even industrial insulators struggle to keep the snowball at a reasonable diameter. The chart also shows us that achieving the highest density possible will provide a better end result.

Insulator	Density (kg/m^3)	Thermal Conductivity k (W/m*K)	RSI Value (m^2*k/W) Per Inch	R-Value (h*ft^2*F/Btu) Per Inch	Approximate Mass Needed (kg)
Polyurethane Foam	40	0.02	1.27	7.21	600
Polystyrene Foam	45	0.032	0.79	4.51	480
Loose Cork Fill	120	0.045	0.56	3.21	1800
Cotton (80 kg/m^3)	80	0.06	0.42	2.40	1200
Baled Straw	90	0.06	0.42	2.40	1350
Sawdust	500	0.1	0.25	1.44	7500
Wood Pellets	720	0.15	0.17	0.96	10800

Table 4: Thermal Properties of Insulating Materials Used in the Analysis

By simple geometric analysis of the snow house design the volume of insulation needed for the double walled structure is roughly $15m^3$ and the approximate mass was calculated accordingly. Table 4 shows the R values for the insulation used in the analytical model; polyurethane foam being the most resistant to heat. While insulation with higher R-Values exist, most are used in cryogenic applications and are far less economically

In order to consider other options for a reveal date, the snow-melt over time is illustrated in Figure 15 below based on a 7, 8 and 9-foot diameter. The insulation used is expanded polyurethane.

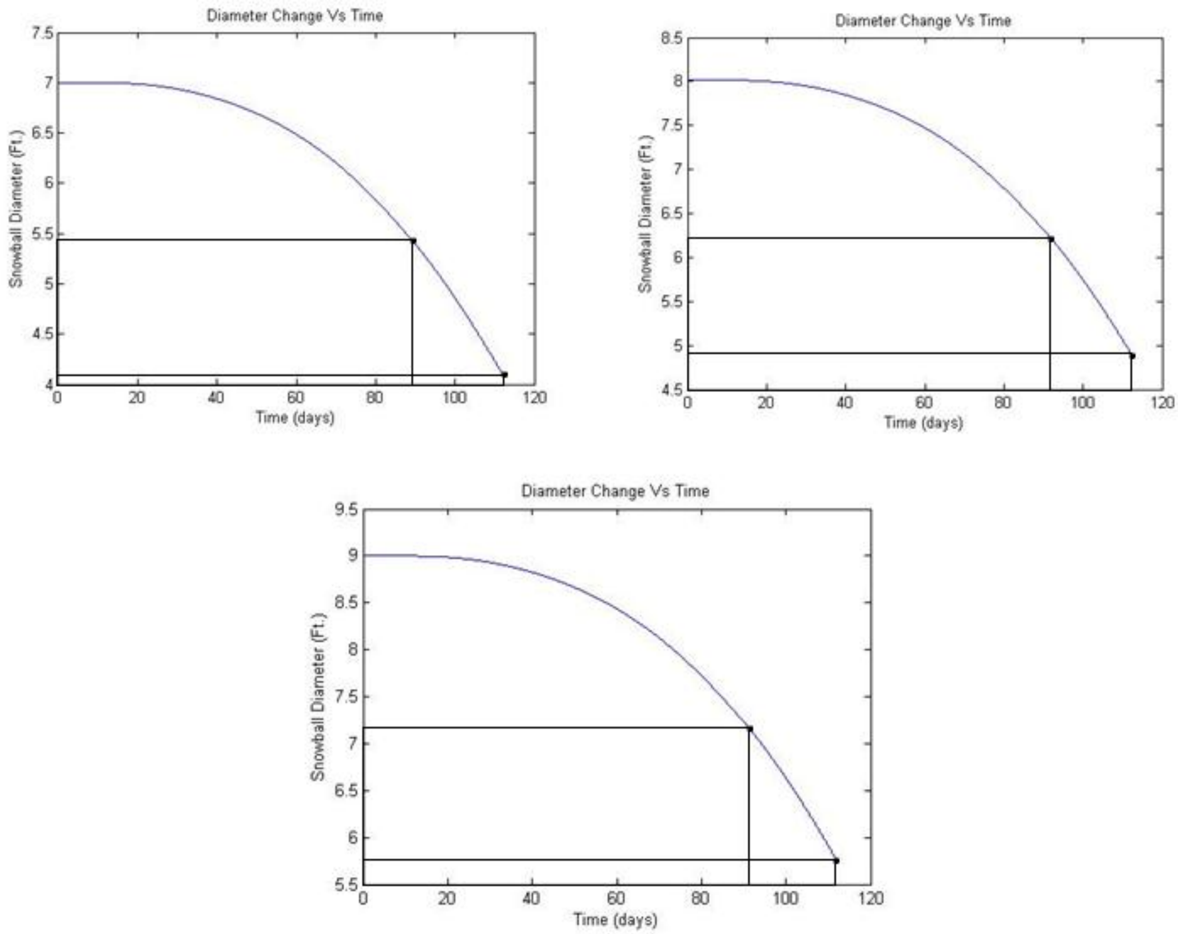


Figure 15: Change in diameter over time based on an initial snowball diameter of 7, 8 and 9 feet.

The MATLAB model simulates the heat transfer from the first day of March until the end of June. During the first 2 months the snow house is heating up and therefore the snowball does not see a large change in diameter. However once the structure reaches steady state the curve becomes essentially linear. This linear slope is approximately the inverse of the r value of the insulation used. It is possible to maximizing the end diameter of the snowball using several different methods: First it may be beneficial to move the unveiling of the exhibit to an earlier date. The black lines in Figure 15 show the snowball diameter if the exhibit was opened at the end of May (90 days). About 7 feet of snow is left over when a 9-foot snowball is stored until May. It may also be possible to increase the amount of time needed to heat up the chamber, thus delaying the drop in diameter caused by the structure reaching steady state. Materials with high thermal storage (high specific heat) can be used in non-insulating applications to maximize this lag period. In addition to the double-walled cube structure design, a structure built out of EPS foam block may provide a few advantages. In this case, expanded polystyrene or polyurethane foam blocks can be stacked and glued together to form a replacement for the double--walled wooden structure. This structure can allow for extra insulation as the space between the snowball and the insulation can be initially eliminated.

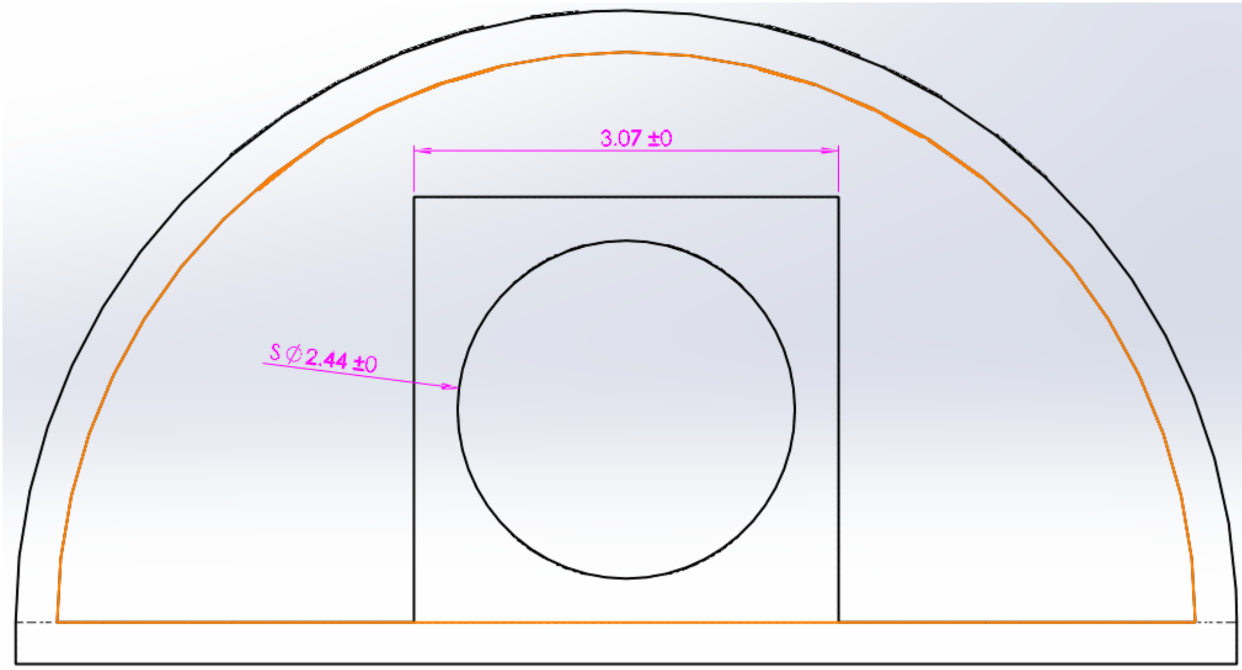


Figure 16: Figure design with EPS foam structure

In figure 16 above the EPS foam insulating structure is seen in place of the original wooden structure. The total volume of the EPS is roughly 23.52m^3 (or 830.72ft^{3}.) The design removes the wooden frame, as the foam itself is strong enough to support the snowball and also eliminates any airspace initially between the snowball and the insulation. As the snowball is left to melt however, air space will begin to grow gradually. Expanded Polystyrene foam is chosen as the insulating material for its low price, light weight, sufficient R-value (R-3.3 to R-4.3), and easy installation or packing. The diameter change of the snowball with this configuration can be seen below in Figure 17:

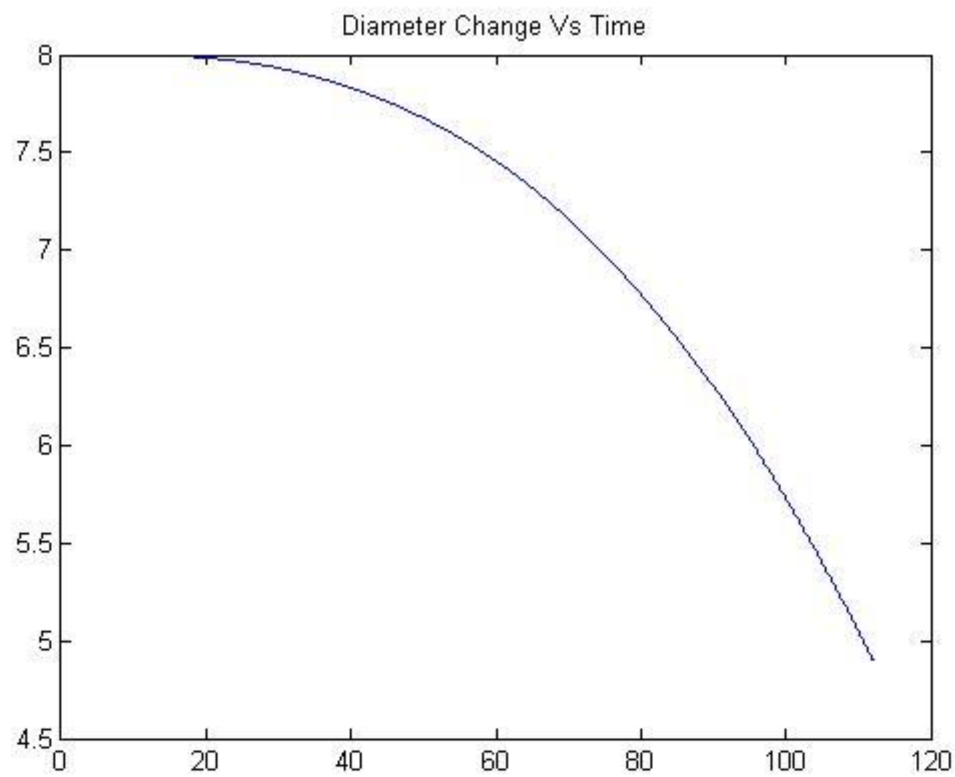


Figure 17: Snowball Diameter Change over Time with Polystyrene Insulation Chamber.

The curve in Figure 17 was generated with the same model as the previous design. The final snowball diameter with this configuration is just under 5 feet whereas the 1st design yielded a snowball of just under 4 feet. This design does provide promising results, however if pursued a second energy balance should be performed on this configuration rather than using the same model as the earlier design.

4.3 Recommendations

The Snow House Exhibit is a fragile project and should be carefully constructed in order for the above results to hold true. In the following section the group will recommend ways to keep the snowball in ideal conditions.

While building the snowball, it is recommended that the dome be kept at a minimum temperature. This can be easily maintained by assuring that cold air is circulated throughout the snow house in the early stages of construction. Maintaining low temperatures early on is crucial for the survival of the snowball.

Proper drainage will be required for the insulated structure in order to minimize the amount of standing water in the snow house. Any passages created for water removal must be large enough to allow for the free transfer of water, but small enough as to minimize any movement of air that may occur. Freely moving air will cause an increase in melting rate of the snowball due to the low insulating capabilities of moving air.

Based on the 500 kg/m^3 density and an 8-foot snowball, the snowball weight is estimated at roughly 8500 lbs. It is recommended that the snowball be built directly in the snow house to minimize the amount of personnel required for construction.

5.0 Conclusions

The deCordova snow house exhibit is a promising spectacle that will enthrall audiences for years. The exhibit hinges on the success or failure of the snowball's well-being. While early ice houses successfully kept large quantities of ice for long periods, the density of snow causes less desirable results.

After researching the successes and failures of early ice houses, an initial design was created that would suit the needs of the deCordova museum. Initially, the team tested various methods of heat transfer to find methods that could be applied to the exhibit. Once preliminary tests were performed, an analytical model was created that accurately depicted the transfer of heat in the snow house. A 3D finite element analysis was performed in order to verify the results of the 1D MATLAB analysis.

The results of the analysis provided insight into what materials should be used to insulate the snowball, and how the chamber should be prepared before being sealed. For the maximum snowball diameter at the reveal, density of the snow should be maximized, and proper insulation should be utilized. In particular, polyurethane insulation has a relatively high R-value and can be used as a replacement for the entire wooden structure if desired. The initial diameter of the snowball greatly influences its melting rate; therefore designs that allow for a large snowball may provide better results than designs that don't allow sufficient room.

There are several outside factors that can affect the final diameter of the snowball at reveal. The initial analytical model does not take into account snow covering the surface of the soil in the first few months. While snow is covering the top surface the temperatures inside the snow house will only heat up due to ground heat and heat from the top of the dome would be minimal. By dumping snow on the ground above the snowball the duration of this effect can be lengthened and the final snowball diameter would be significantly larger than our original model depicts.

References

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[6] Precision Geothermal, "Samplereport Pdf Free Ebook Download from Precisiongeothermal.Com " **2012**(10/19/2012) .

Appendix

MATLAB Code

```
%General Variables
nx       =   21;                          %Number of Points (n-1 nodes)
t        =   3600*24*7*4*4;               %Total Time Elapsed (3 Months)[s]
dt       =   10;                          %Timestep (half hour)[s]
nt       =   t/dt;                        %Number of Timesteps
a        =   ones(1,nx);                  %Array of all ones
Tair     =   274;                         %Initial ambient air Temp [K]
T_s      =   273;                         %Initial structure air Temp [K]
Tin      =   273;                         %Initial inside air Temp [K]
day      =   3600*24;                     %Seconds in a Day [s]
T_soil   =   283;                         %Soil Temp at 5 ft [K]
time     =   0;                           %Sets Initial Time to Zero;

%Snowball, Structure and Dome dimensions
D        =   2.1336;                       %Diameter of Snowball (=8ft)[m]
A_sb     =   4*pi*(D/2)^2;                %Surface Area of Snowball [m^2]
V_sb     =   (4/3)*pi*(D/2)^3;            %Volume of Snowball [m^3]
h_dome   =   4.1148;                      %Height of dome (=13.5ft) [m]
L_s      =   3.07;                        %Length of Base of Structure
L_i      =   2.435;                       %Length of Inside Structure
V_s      =   L_i^3-V_sb;                  %Volume of Air in Structure
V_a      =   ((4/3)*pi*h_dome^3*.5)-L_s^3;  %Volume of Air in Dome[m^3]
A_d      =   4*pi*h_dome^2*.5;            %Surface Area in Dome [m^2]
D_ground=    h_dome*2;                    %Diameter of circular ground[m]
A_g      =   pi*(D_ground/2)^2;           %Area of the Ground [m^2]
A_st     =   L_s^2*5;                     %Surface Area of Structure
V_ins    =   L_s^3-L_i^3;                 %Volume of insulation
%Thermal Properties
h_ls                 =   334*10^3;        %Enthalpy of Fusion (Water)[J/kg]
k_air                =   0.024;           %Thermal Conductivity of Air
[W/m*K]
k_soil               =   0.18;            %Thermal Conductivity of Soil
[W/m*K]
k_granite            =   2;               %Thermal Conductivity of Granite
[W/m*K]
cp_air               =   1005;            %Specific Heat of Air [J/kg*K]
cp_soil              =   1200;            %Specific Heat of Soil [J/kg*K]
cp_granite            =   780;             %Specific Heat of Granite [J/kg*K]
air_density          =   1.2;             %Density of Air [kg/m^3]
soil_density         =   1750;            %Density of Soil [kg/m^3]
granite_density      =   2650;            %Density of Granite [kg/m^3]
snow_density         =   500;             %Density of Snow [kg/m^3]
A_soil      = k_soil/(soil_density*cp_soil); %Thermal Diffusivity of Soil
[m^2/s]
A_granite   = k_granite/...
    (granite_density*cp_granite);         %Thermal Diffusivity of granite
[m^2/s]
k_ins                =    .02;          %Thermal Resistance Value
L_ins                =    .3;             %Depth of Insulation
%Top Properties
T_top     =   a*273;                      %Initial Temperatures [K]
```

```
h_air_top                              %Convection Coeff. Inside Chamber
[W/m^2*K]
L_top      =   1;                      %Depth of the soil layer [m]
dx_top     =   L_top/(nx-1);           %Step Length Through the Soil [m]
x_top      =   0:dx_top:L_top;         %Grids
Fo_top     =   A_soil*(dt/dx_top^2);   %Fourier Number
Bi_in_top =    h_in_top*dx_top/k_soil; %Biot Number for Inside Conv.

%Bottom
T_bot        =   a*273;
h_air_bot
L_bot        =   1.2;
dx_bot       =   L_bot/(nx-1);
x_bot        =   0:dx_bot:L_bot;
Fo_bot       =   A_soil*(dt/dx_bot^2);
Bi_in_bot    =   h_in_bot*dx_bot/k_soil;

%Allocate space for saving variables
T_t = zeros(1,nt);
T_b = zeros(1,nt);
T_d = zeros(1,nt);
T_i = zeros(1,nt);
t   = zeros(1,nt);

for n=(1:nt)   %Timestep Loop
    %Creates an empty array for new temperatures
    T_new_top     =   zeros(1,nx);
    T_new_bot     =   zeros(1,nx);

    %New Internal Node Temperatures
    for i=(2:nx-1);
        T_new_top(i)  =  Fo_top*(T_top(i+1)+T_top(i-1))+...
            (1-2*Fo_top)*T_top(i);
        T_new_bot(i)  =  Fo_bot*(T_bot(i+1)+T_bot(i-1))+...
            (1-2*Fo_bot)*T_bot(i);
    End

    %Boundary Conditions
    T_new_top(1)  =  Tair;
    T_new_bot(1)  =  T_soil;
    T_new_top(nx)=  2*Bi_in_top*Fo_top*Tin+2*Fo_top*T_top(nx-1)+...
        (1-2*Bi_in_top*Fo_top-2*Fo_top)*T_top(nx);
    T_new_bot(nx)=  2*Bi_in_bot*Fo_bot*Tin+2*Fo_bot*T_bot(nx-1)+...
        (1-2*Bi_in_bot*Fo_bot-2*Fo_bot)*T_bot(nx);

    %Update Parameters
    T_top    =   T_new_top;
    T_bot    =   T_new_bot;
    time     =   time +  dt;
    days     =   time/(3600*24);
    h_air_top
    h_air_bot
    h_air_s

    if days > 30                              %No change in Tair 1st Month
```

```
        Tair     =   Tair + 26/(nt-nt*(1/4));
    end

    %Solving for New Inside Air Temperature (C.V. Dome)
    R_t     = 1/A_st*(1/h_in_top+L_ins/k_ins);     %Conduction through
Insulation from Air
    q_top   = h_in_top*(T_top(nx)-Tin)*A_d;       %Top Dome Heatflux
    q_bot   = h_in_bot*(T_bot(nx)-Tin)*A_g;       %Dome Base Heatflux
    q_r     = (Tin - T_s)/R_t;                    %Heatflux Through Insulation
    q       = q_top + q_bot - q_r;                %Total Heatflux (Dome)
    dT_in   = q*dt/(air_density*V_a*cp_air);      %Change in Dome Air Temp.
    Tin     = Tin + dT_in;                        %Updating Air Temp.

    %Solving for New Inside Structure Temperature (C.V. Structure)
    R_t_b   = L_ins/k_ins;                        %Conduction through
Insulation from Ground
    q_snow = h_s*(T_s-273)*A_sb;                  %Heat Flux into Snowball
    q_b_r   = (T_bot(nx)-T_s)/((1/L_s^2)*R_t_b);                %Heat Flux
from Ground Cond.
    q_s     = q_r + q_b_r - q_snow;               %Total Heatflux (Structure)
    dT_s    = q_s*dt/(air_density*V_s*cp_air);    %Change in structure Temp.
    T_s     = T_s + dT_s;                         %Updating Structure Temp.

    %Solving for New Radius (C.V. Snowball)
    dR      = q_snow*dt/(snow_density*h_ls);
    D       = D-2*dR;
    A_sb = 4*pi*(D/2)^2;                          % updating snowball surface
area

    %Parameter Storage for Time Plots (In order of subplots)
    T_t(n)   = h_in_top;
    T_b(n)   = h_in_bot;
    T_i(n)   = h_s;
    T_d(n)   = D*3.28084;

    %Stores Amount of Days for Plots (Don't Change)
    t(n) = days;

end
disp(D*3.28084)
disp(V_ins)
plot(t(1:nt),T_d)
title('Diameter Change Vs Time')
%{
subplot(2,2,1)
plot(t(1:nt),T_t)
title('h_top')
subplot(2,2,2)
plot(t(1:nt),T_b)
title('h_bot')
subplot(2,2,3)
plot(t(1:nt),T_i)
title('h_snowball')
subplot(2,2,4)
plot(t(1:nt),T_d)
title('Change in Snowball Diameter (m)')
```