The Math of Fantasy Football

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Abstract

The team, with the help of Advanced Sports Logic (ASL), researched and developed a quantitative method for rating projections of professional Football in order to easily compare the quality of projection systems. We used real life data and projections provided by ASL to gain insight on how to develop the method. The team decided to evaluate the projections for an entire football season to generate an annual score for projection systems. What we created was a rating system that would help developers of projection systems by providing them with a quantitative measure that they could use to value the quality of their projection systems.

Authorship

This report was developed through the collaborative efforts of John Lucker, Patrick Maynard and Matthew Poppa. All group members contributed equally to the completion of this project.

Executive Summary

Advanced Sports Logic (ASL) is a company founded in 2009 by Worcester Polytechnic Institute (WPI) alumni Leonard LaPadula. The company works to provide its customers with recommendations leading to a competitive advantage in their fantasy football leagues. ASL's product, software called "The Machine", uses mathematical formulas to which players to draft as well as recommending starting line-ups and trade opportunities. The overall goal of "The Machine" and its recommendations is to provide the customer with highest chance of winning the league.

In order to provide its customers with the best recommendations possible and get ahead of its competitors, ASL is continually looking to improve its products. ASL brought the challenge of improving "The Machine" to our team of three senior actuarial mathematic students at WPI. Before further adjustments could be made directly to the software, the team's goal was to develop a quantitative rating system for evaluating the quality of the projections used by "The Machine". In doing so, any future adjustments could be evaluated by the rating system where an increase or decrease in the quality of the projections could be seen directly. This rating system can also be used for rating any projection system.

The team set itself four objectives to achieve the final goal of a fully functioning projection measurement system. The first was to develop a better understanding of the different aspects of the projection system that should be considered in the accuracy measurement. The second was to create a mathematical system that is able to distinguish how effective a given projection system is. The third was to adjust the system's output into a single

number or score. Lastly, the team needed to define the method behind the rating system so that ASL could reproduce it for any projection set.

With the aim of completing our task the team first needed to understand how "The Machine" develops its projections. To do this we explored ASL's developer tool, which is software on their website that employees can use to explore different functionalities of "The Machine". To even further understand the projections, the team used data from the developer tool to create basic projections based on ASL's projection method.

After educating ourselves on the workings of "The Machine", we were then ready to begin the development of a projection rating system. While creating the basic projections, the team found that there were two major measures that would affect the quality of a projection. These being: how close the projection is to the actual data and how significantly the projection system would change its projections over time. The team decided to use correlation and variation to measure these two aspects. The team next determined that the most important times in the football season to evaluate the correlation were after the draft, before each preseason week, and at the beginning of each week in the regular season. The variation was decided to be evaluated for each week of the season.

The final two objectives were to combine the correlations and variations all into a single score and to provide the method behind the projection rating system in an easily understandable form so that it could be reused by ASL. To produce a single score the team decided to give each correlation and variation a weight based on their importance. The weights allowed for the values to be summed and then become a rating out of 100.

The team created a rating system which will take projections for a season of football and return a score based on a 100 point scale. We have recommendations for how ASL can easily implement this system. The first recommendation is that they store the projection and actual data in a SQL database and develop a program to calculate the projection scores from that database. This will make the calculation of the projection scores faster and remove the need of formatting the data. A second recommendation would be to explore the weighting system and create different weighting systems for the non-traditional fantasy football leagues.

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Acknowledgements

The team would like to thank Advanced Sports Logic for providing and sponsoring this interesting and academically challenging project. We would like to thank Professor Abraham for his guidance throughout the project.

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Introduction

Fantasy football was originally created in 1962, but has rapidly expanded in recent years due to leagues becoming easily available, through companies like ESPN and CBSSports, on the internet. In 2012, Fantasy football had over 25 million users in the United States alone, 2 million more users than in 2011. The fantasy football market currently has profits over $1 billion dollars. Due to this accelerated growth many businesses have been started to provide information and services to these millions of users. One such company is Advanced Sports Logic.

Advanced Sports Logic (ASL) sells a product called "The Machine" to fantasy football users. "The Machine" makes recommendations to fantasy football participants during their draft to help them acquire the best possible team. It considers weekly player projections and uses them to project the amount of points that each fantasy team in the league will score. Then it recommends to the user a team with the highest probability of making it to the fantasy football playoffs and winning the league. The machine also provides projection based recommendations during the season for favorable waiver wire pickups and trades. ASL's machine relies heavily on projections to make its recommendations and as a result they have sponsored an MQP project for actuarial students to measure the accuracy of projections.

The overall goal of this project is to develop a measurement system for the accuracy of projections, something that does not currently exist in the open market. This will allow ASL to compare the projections they are using in the machine to projections being used by other companies. It also has the potential to be used as a tool in helping to see if adjustments in their

projection system will make the projections better or worse. ASL's hope is that one day this

metric will be used as the industry standard in measuring projections. In order to achieve ASL's

desires the project team set for itself several major goals. The first was to develop a better

understanding of the different aspects of the projection system that should be considered in

the accuracy measurement. The second was to create a mathematical system that is able to

distinguish how effective a given projection system is. The third was to adjust the system's

output into a single number or score. Lastly, the team needed to define the method behind the

measurement system so that ASL could reproduce it for any projection set. The team achieved

these goals through many discussions with ASL's founder and chief executive Leonard LaPadula,

project advisor Jon Abraham, and other ASL employees as well as extensive testing of both

fabricated and real projection sets.

Background

Fantasy Football:

Fantasy football is the competition between a league of fantasy teams, where each week the teams "play" against each other. Like in the National Football League, there is a regular season schedule that is followed by playoffs to determine the league champion. The goal is simple in each matchup; score more fantasy points than your opponent. Fantasy point scoring rules can vary from league to league. The most standard scoring rules are the following:

Standard Scoring Rules:

Passing:
Every 25 passing yards = 1 point
TD pass = 4 points
Two point conversion = 2 points
Interception thrown = -2 points
Fumble Lost = -2 points

Receiving:
Every 10 receiving yards = 1 point
TD catch = 6 points
Two point conversion = 2 points
Fumble lost = -2 points

Kicking:
Extra point made= 1 point
Field goal made (0-39 yards) = 3 points
Field goal made (40-49 yards) = 4 points
Field goal made (50 plus yards) = 5 points
Any field goal missed= -1 point
Any extra point missed= -1 point

Defense & Special Teams:
Interception = 2 points
Fumble recovery = 2 points
Blocked punt, field goal, or extra point = 2 points
Safety = 2 points
Sack = 1 point
Interception returned for a touchdown = additional 3 points
Fumble recovery for a touchdown = additional 3 points
Blocked punt or field goal return for a touchdown = additional 3 points
Kickoff or Punt return for touchdown = 3 points

Less than 2 points allowed = 10 points
2-6 points allowed = 7 points
7-13 points allowed = 4 points
14-17 points allowed = 1 point
18-21 points allowed = 0 points
22-27 points allowed = -1 point
28-34 points allowed = -4 points
35-45 points allowed = -7 points
46-plus = -10 points
(All points scored against the team count as points allowed)

Fantasy Football players pick their teams from a pool of NFL players; this is usually done in a draft format. The most common lineup setup is ten active players and six bench players. The bench players can be moved in and out of the lineup as the owner pleases, up until the start of the player's game that week. Once the game starts the player is locked into their position or on the bench. Team rosters are commonly created under the following format:

Common Roster Format:

Active Roster:
1 Quarterback (QB)
2 Running backs (RB)
2 Wide receivers (WR)
2 Tight ends (TE)
1 Flex (can be a running back, wide receiver, or tight end)
1 Kicker (K)
1 Defense/ Special team (D/ST)
Bench:
6 Reserve players (can be any assortment of the types of players)

Outside of the normal "annual" leagues there are multiple types of leagues, such as keeper & dynasty leagues. These leagues have become very popular; in 2012 over 24 million Americans played fantasy football. Some of these leagues are played for "bragging rights" and others are played for monetary prizes. In addition to the leagues that are being played for money, in 2012 the NFL offered a prize of 1 million dollars to any person who could pick a perfect fantasy lineup in a given week. The "perfect" fantasy lineup would be the lineup that scores the maximum possible points in that week. The monetary prizes give fantasy owners the incentive to have the "best" projected statistics available come draft day for their leagues, and this has created a billion dollar market for companies who supply information based on fantasy relevant news and statistics.

Correlation:

The Pearson correlation is a coefficient used to measure the linear association of two variables. To measure a linear association, begin with two sets of data acquired from the variables. Then the mean and standard deviation is calculated for both sets of data according to the following equations:

$$Mean: \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$Standard\ Deviation: S_X = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2}$$

Where X_i denotes the ith term in the set and n denotes the total number of terms in the set.

Using the two means and standard deviations the data sets can be standardized. Standardizing the data creates new data sets that are void of units of measurement. First, subtract the mean and then divide by the standard deviation accordingly for each value in the data sets.

$$X_i^1 = \frac{X_i - \bar{X}}{S_X} \qquad Y_i^1 = \frac{Y_i - \bar{Y}}{S_Y}$$

$Where\ X_i^1\ and\ Y_i^1\ denote\ the\ ith\ standardized\ value\ in\ the\ corresponding\ set.$

After the data has been standardized, the formula for the Pearson correlation coefficient can be applied. The formula summates the product of each standardized data value and then divides the sum by the degrees of freedom.

$$r = \frac{1}{n-1} \sum_{i=1}^{n} X_i^1 Y_i^1$$

The resulting value will be between -1 and 1. Values close to 1 explain that there is a strong positive linear association between the two variables, and for values close to -1 there is a strong negative association. Either of those two conditions shows that there is some association, but for values close to zero there is almost no association at all between the two variables.

Methodology

The first step in the project was to study and understand ASL's (Advanced Sports Logic) expectations of the team and any information and data that they had to offer. In our first meetings with Leonard LaPadula the team decided that of the several objectives presented we focused on helping ASL by creating a system for rating projections. During these meetings we discussed gaining access to ASL's developer tool and the projection data they were using in "The Machine" from a company named AccuScore. After we gained access to the information we were able to begin analyzing the data and proceeding with the project.

Developer Tool

ASL's developer tool is software on their website that employees can use to explore different functionalities of "The Machine". The most important data that the developer tool provided was the historic data for the 2010 and 2011 NFL seasons. The developer tool can also be used to find other historic data and to project box scores for games between any two teams in the league, based on the user's choice of historic games.

Creating a Basic Projection

After exploring the developer tool the team decided to use excel to create a basic projection for the number of plays in a NFL game, in order to evaluate ASL's method of projecting. The projection was created using the historic data from the 2010 NFL season to project data for the 2011 NFL season. ASL's method was to use the season averages, for a particular statistic, of a team's offense and the opposing team's defense and average them to

generate projections for that game. We had access to many different statistics from the 2010

NFL season, but, in order to understand what was going on in more detail, we decided to focus

on nine specific statistics; total plays, passing plays, rushing plays, passing yards, rushing yards,

field goals attempted, field goals made, extra points attempted, and extra points made. We

used the projections of those nine statistics to build graphs in excel that displayed the

correlation between the historic data and the projection data. These graphs can be found in

Appendix A. At this point, we determined that correlation would be a worthwhile evaluation

tool to be incorporated into our projection rating system.

During the following meeting with Leonard we discussed these basic projections and

graphs. With his help and intuition we altered the methods used in ASL's projection system.

Instead of taking the season averages, for a particular statistic, of a team's offense and the

opposing team's defense and averaging them, the new method used the sum of the two team's

seasonal averages and subtracted the league's average.

Projection Rating System

In order to construct a projection rating system we had to decide on the components

that would be used in calculating a final score. We decided to use correlation as a measure of

how accurate the projections are relative to each other, and variation as a measure of how

consistent the projections are. Having chosen the components of the projection rating system,

we discussed how we would calculate each of them, and what points in the season we would

use to rate the projections. After discussion with Leonard, it was decided that for fantasy

football there are a series of points where the quality of the projections matter most. The first

time of measurement is after the NFL draft, because that is when the rookie players have been

drafted by NFL teams and also the earliest that most fantasy leagues will schedule their fantasy

drafts. Additionally, the other important measurement times are before each pre-season week

and before each regular season week. The pre-season projections are important to many

fantasy leagues because drafts can occur up until the regular season starts. The projections

before each regular season week are important because they are the most up to date

projections made before the fantasy participants can set their lineups for that week.

Correlation

It was determined that correlation would be a fair assessment of the accuracy of

projections because it measures the linear association between two variables. The team

determined that it was appropriate to have 6 different scores calculated with correlation. Five

of the scores are determined from the projections made before each of pre-season weeks 1

through 4 and the post draft projection. The sixth correlation score comes from the projection

made each regular season week before the game is played.

In fantasy football, the draft is an important part of the game. If a fantasy user makes

poor choices during the draft their team will suffer. This is why it is important to have the best

possible projections for preseason weeks 1 through 4. We calculated these 5 scores using the

correlation between each preseason week's projections and the corresponding actual data. The

correlations are then averaged within their preseason week to create 4 different scores

between -1 and 1.

Lineups and benched players in fantasy football get locked during each user's week so

fantasy football users cannot switch around their team while the NFL games are being played.

Fantasy users try to set their lineups each week to play their combination of players that will score the most fantasy points. In order to accomplish this, the most important projections are those before each week. The team decided to use the correlations of these projections and averaged all 16 to create a single score between -1 and 1, called "final accuracy".

Variation

The best projection system would project the actual results at every time projections are made, meaning it would have no variation. If projections vary from week to week then that projection system should not be rated as highly as a projection system which has less variation from week to week. To reflect this consistency in the rating system we decided that the variance of the projections should be accounted for. We calculate the variation for each week of projections, enter that value into a function that converts it into scores from 0 to 1, and then average those scores to obtain a single "variation score" that ranges from 0 to 1.

Weighting

The team discussed how to weight the different scores with Leonard and together came up with a way to incorporate the variation and correlation components of the final score. We developed multiple sets of projections to help demonstrate the influence of changes to the weights. The different projection sets were created to produce a range of ratings from poor to excellent for the correlation and variation components. This allowed the team to analyze the effect of changes in the weights and develop a proper weighting scheme.

Calibration

Once the team had developed the initial concept for evaluating a projection system, it was necessary to calibrate the system. The purpose of the calibration was to figure out what qualifies as a good score and what qualifies as a poor score. In order accomplish this we generated test projections of predictable quality. The team decided that the best method for this was to start with the actual statistics and add random error to them.

Initially we achieved this effect, by taking the actual value and multiplying it by a randomly generated number between 0.95 and 1.05. We proceeded by increasing the ranges of error, with the final range being from .7 to 1.3. This generated the following graph of projection scores.
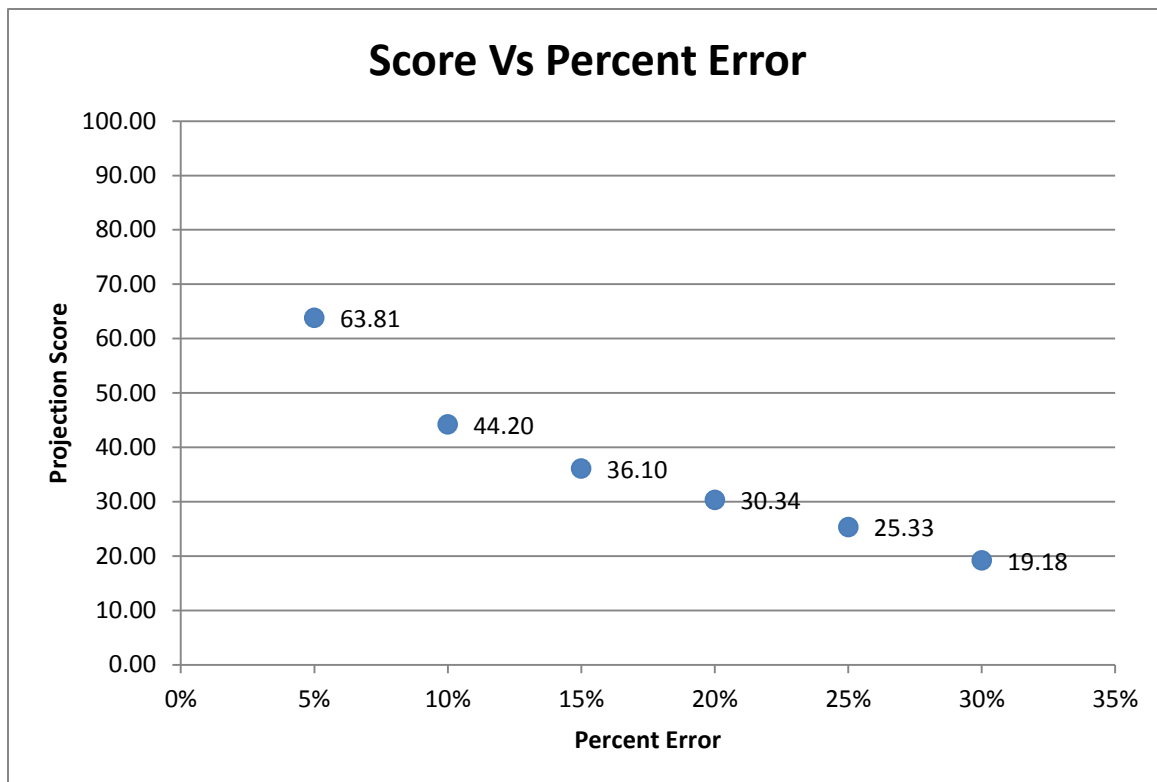
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The first calibration showed that our system, as it stood gave low scores, compared to our expectations, giving a set of projections with a maximum of 5% error a score of 63.81. Re-examination of our projection rating system found the reason being that our variation scores were lower than expected, giving the projections with a 5% error band a score of approximately -2. In essence, the projection system was scoring well on the correlation component, but had been losing points on their variation scores.

Having identified the issue, the team adjusted the function for converting raw variations into variation scores. We also realized that our method of generating test projections could be improved. The projections were altered in a manner that would decrease the amount of error as the season progressed towards the week being projected. We decided on maximum and minimum error amounts and used linear extrapolation to generate the error bands for the projections between the first and last projections. Below is a graph showing the range of possible test projections for a set of five test projections with error bands going from 60% to 40%.
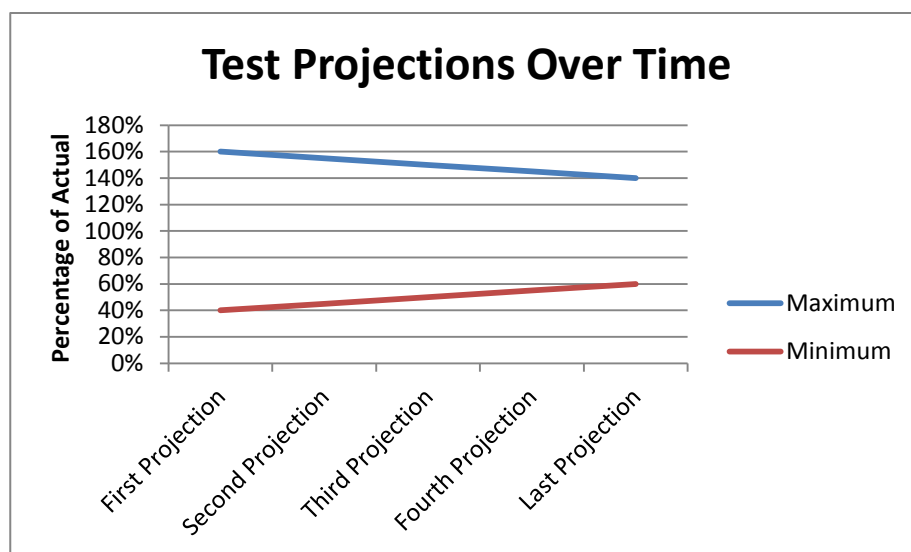

Figure 2: Error Bands Example

The idea behind this method is that projections, in general, should improve as the time remaining until the game being projected decreases. The adjustments in the projections generated the graph seen below.
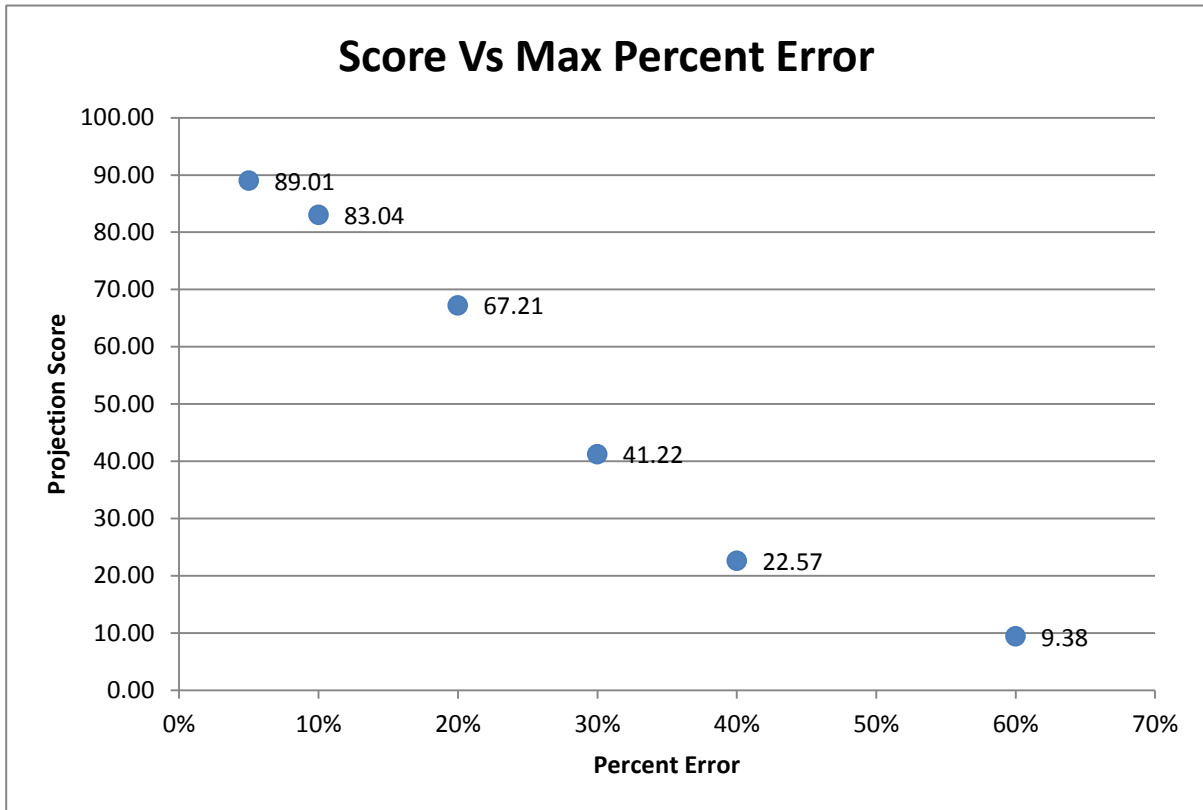
Score Vs Max Percent Error

Figure 3: Second Calibration Graph

This distribution of scores is closer to the expected scores, based on the error introduced to the test projections. The team decided that our projection rating system was now producing scores that would appropriately differentiate the quality of projection systems.

Results & Conclusions

Variation Conversion Function

The team explored many different functions before arriving at:

$$f(x) = K^{-|x|}$$

Where K is a constant such that f(league average) = 0.5, and x is the variation.

This specific function was created to convert the variation numbers into a score bound by 0 and 1. The function could then be calibrated for a statistic, such that a variation equal to the league average generates a score of 0.5. This ensures that the function is scaled to a slope based on the statistic being evaluated.

Weighting

Each of the six correlation scores and the variation score were given individual weights, the scores are multiplied by their weights, summed together, divided by the sum of all of the weights, multiplied by 50, and added to 50. Correlations from the pre-seasons, post draft, and the final accuracy were each given a weight of 1 for a total of 6. To incorporate the variation the team decided to weight it by multiplying it by the final accuracy and then giving it a weight of 3. Although, at first the team wanted to then multiply this rating by 100 to give a rating out of 100, a meeting with Leonard changed our minds. During the meeting it was mentioned that, unless a projection system was randomly chosen numbers, it should not receive a rating lower

than if the projections were chosen by the toss of a coin, or on our scale, a 50 out of 100. In order to account for this, the team decided that instead of multiplying by 100 we would multiply by 50 and then add 50. This still provides a rating out of 100, but now the rating would only score less than a 50 if the projection system had negatively correlated projections.

Rating ASL's & Accuscore's Projections

Correlation	Correlations between all projections for each week vs the actual values for that week.															
Projection Week	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14	Week 15	Week 16
Post Draft	0.662005	0.527584	0.356097	0.785458	0.693526	0.473408	0.724756	0.625134	0.45983	-0.24381	0.262174	0.366643	0.218654	0.471373	0.087359	0.466075
PreSeason 1	0.700832	0.563559	0.449555	0.764523	0.62891	0.426352	0.670102	0.554064	0.419821	-0.24924	0.241864	0.398551	0.22943	0.514777	0.132546	0.494542
PreSeason 2	0.706001	0.527092	0.416764	0.710526	0.593413	0.396945	0.713028	0.557344	0.494622	-0.25555	0.242515	0.428074	0.251329	0.518698	0.165009	0.533977
PreSeason 3	0.705468	0.54374	0.402319	0.711652	0.575834	0.385778	0.714745	0.558389	0.543989	-0.30328	0.197728	0.406242	0.247816	0.525896	0.186997	0.522872
PreSeason 4	0.744424	0.574451	0.486243	0.725182	0.630127	0.524549	0.64632	0.60884	0.607359	-0.31267	0.188237	0.471896	0.345408	0.530713	0.324559	0.616119
Week 1	0.713019															
Week 2		0.649242														
Week 3			0.61392													
Week 4				0.754824												
Week 5					0.675832											
Week 6						0.641717										
Week 7							0.773687									
Week 8								0.669488								
Week 9									0.693346							
Week 10										-0.26807						
Week 11											0.448728					
Week 12												0.774795				
Week 13													0.687383			
Week 14														0.756926		
Week 15															0.662082	
Week 16																0.825374

Figure 4: Triangle of Correlation Scores

The first step in using our projection rating system was to organize 8000+ projections so we could calculate 96 initial correlation scores, shown in the figure above (there are 5 weeks with 16 weeks of projections before the season begins, shown in the white boxes above. We also included 1 additional week of projections at the beginning of each of the 16 weeks through the season, shown in the green boxes above. Thus, 5x16 plus 16 equals 96). Each of the 96 numbers represent the correlation between a set of projections, made at a specific point in time, for a single week of football and the actual values for that week. The next step in the process is to calculate the average variation for the projections made for each week. These average variations are run through our variation conversion function, generating 16 variation

scores. We average the correlation scores for the first five rows and the green cells, in the figure above, average the variation scores and take the weighted average of these numbers. This weighted average will be a number between -1 and 1 which we multiply by 50 and add 50 to arrive at a score out of 100.

Projection Score	Weighted Average of Scores*50+50	
	Score	Weight
Post Draft	AVERAGE(B16:R16)	X1
PreSeason 1	AVERAGE(B17:R17)	X2
PreSeason 2	AVERAGE(B18:R18)	X3
PreSeason 3	AVERAGE(B19:R19)	X4
PreSeason 4	AVERAGE(B20:R20)	X5
Variation	AVERAGE(C40:C56)	
Final Accuracy	AVERAGE(B21,C22,D23,E24,F25,G26,H27,I28,J29,K30,L31,M32,N33,O34,P35,Q36)	X6
FA and Variation	Variation x Final Accuracy	X7

Figure 5: Final Score Calculation in Excel

The team used the rating system to rate ASL's projections of 2011's passing yards, which provided a rating of 61.41. Similarly Accuscore's projection for fantasy points for QB received a rating of 73.4 and their projection for fantasy points for D/ST received a 58.2. This shows that there is room for improvement in ASL's current projection system.

Recommendations

We recommend that ASL use an SQL database to store the projections and actual data. They should then develop a program to calculate the projection scores from that database. This will allow for a timely evaluation of the projections, making it more feasible to evaluate all combinations of fantasy football statistics. A second recommendation would be to explore different weighting systems for different types of fantasy leagues. The different league types value projections at different times in separate ways. We also recommend using the rating system on statistics that are components of fantasy points to determine which statistics are being projected accurately and which have the most room for improvement. When comparing projection systems it is important to consider past performance scores for evaluating the potential value of the future projections from that system.

While testing our projection rating system we found that it encountered errors when someone projects the same values for an entire week. This is because correlation cannot be calculated when there is no change in either data set. In response to this we decided that if a projection that has the same value for an entire week, the projection gets a correlation of zero for that week. While we do not anticipate there to be further errors, we recommend that ASL look out for potential situations that can cause errors because of calculations.

A final recommendation from the team is to determine how much data is required when projecting at the player level for each player's projections to be considered in the rating of the projection system. As an example, when rating Accuscore's quarterback projections there were certain players whose projections did not exist in most weeks.

Glossary of Acronyms

ASL – Advanced Sports Logic

D/ST- Defense/ Special Teams

K- Kicker

NFL- National Football League

QB- Quarterback

RB- Running back

TD- Touchdown

TE- Tight End

WPI- Worcester Polytechnic Institute

WR- Wide Receiver

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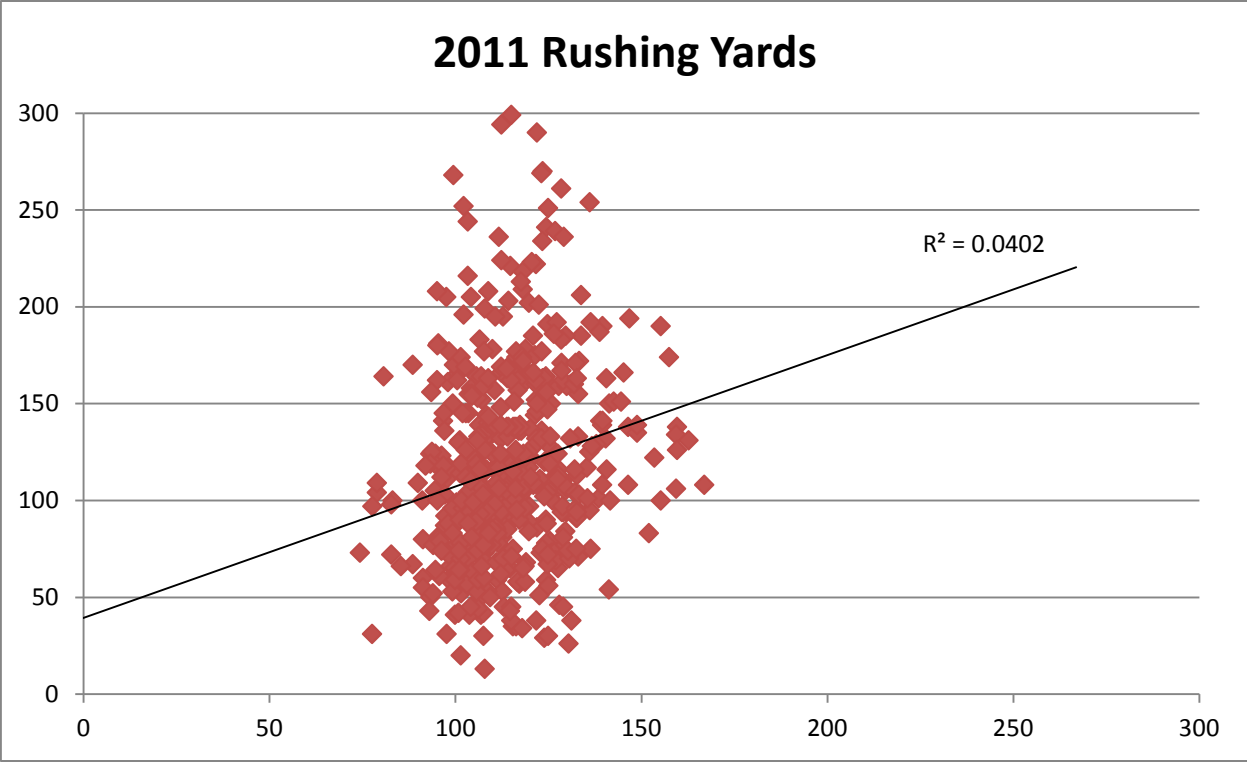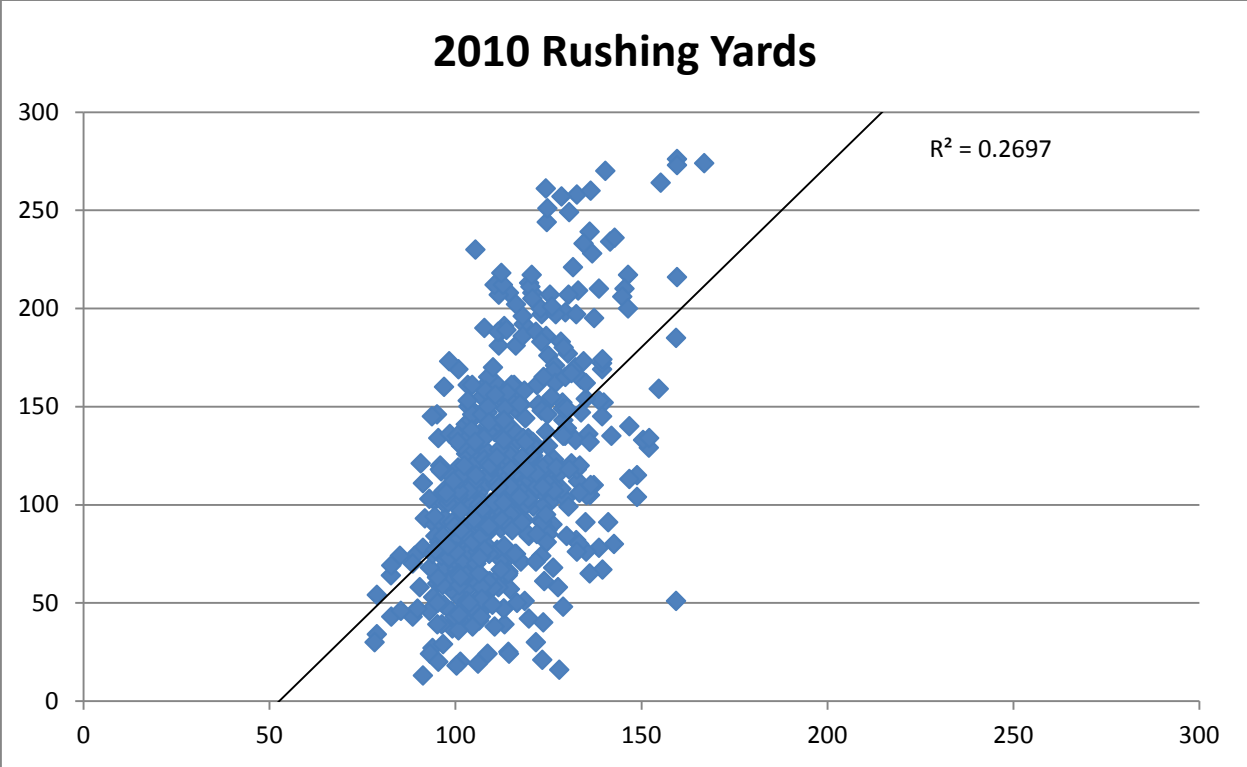
Drew.

Smith, Seana. "The Business of Fantasy Football." *Fox Business*. N.p., 31 Aug. 2012. Web. 18

Apr. 2013.

Appendix A

2010 Passing Yards

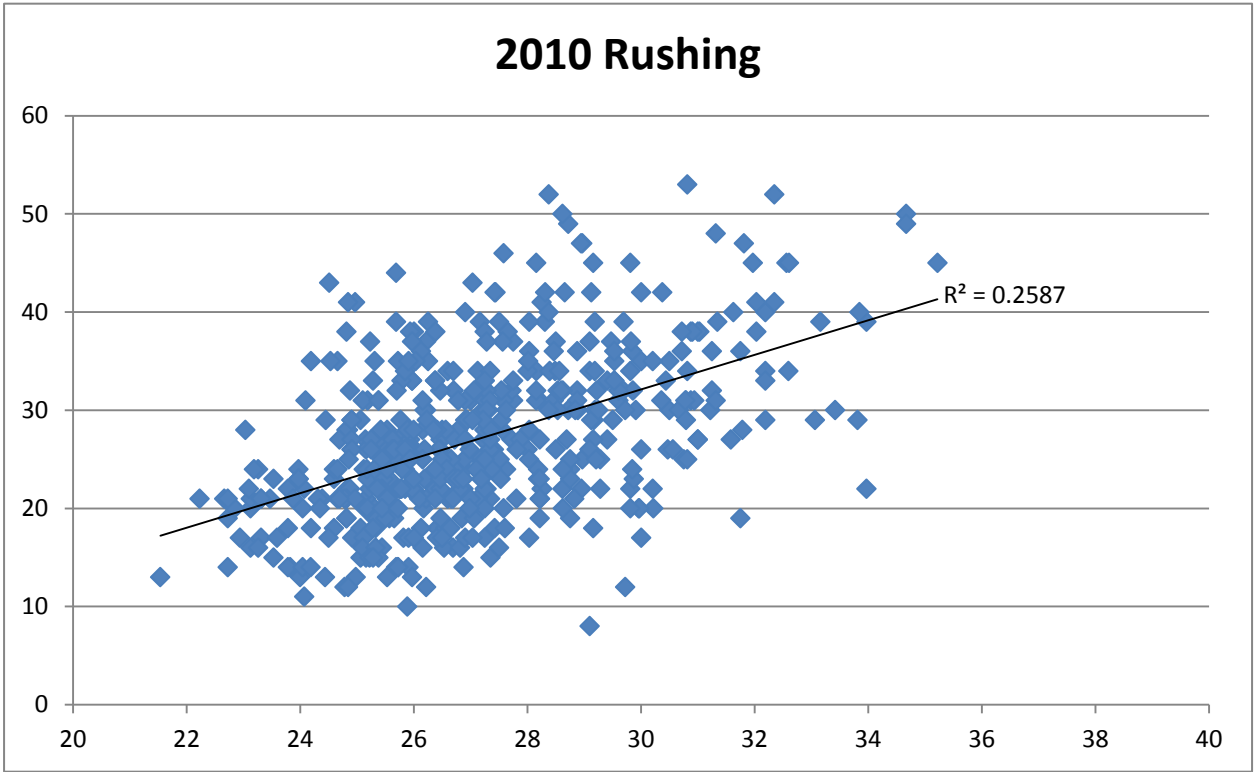R² = 0.2318

2011 Passing Yards

R² = 0.0284

2010 Rushing Yards

R² = 0.2697

2011 Rushing Yards

R² = 0.0402

2010 Passing

$R^2 = 0.2404$

2011 Passing

$R^2 = 0.0193$

2010 Rushing

$R^2 = 0.2587$

2011 Rushing

$R^2 = 0.0441$
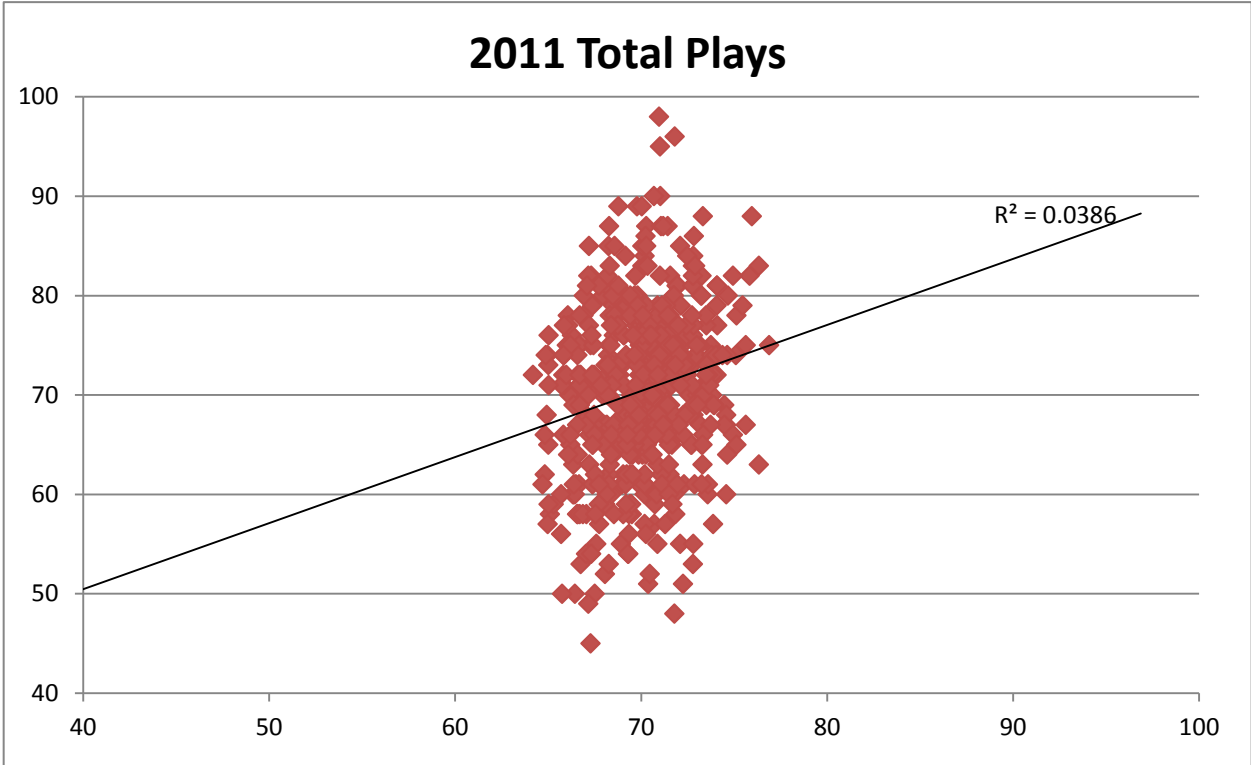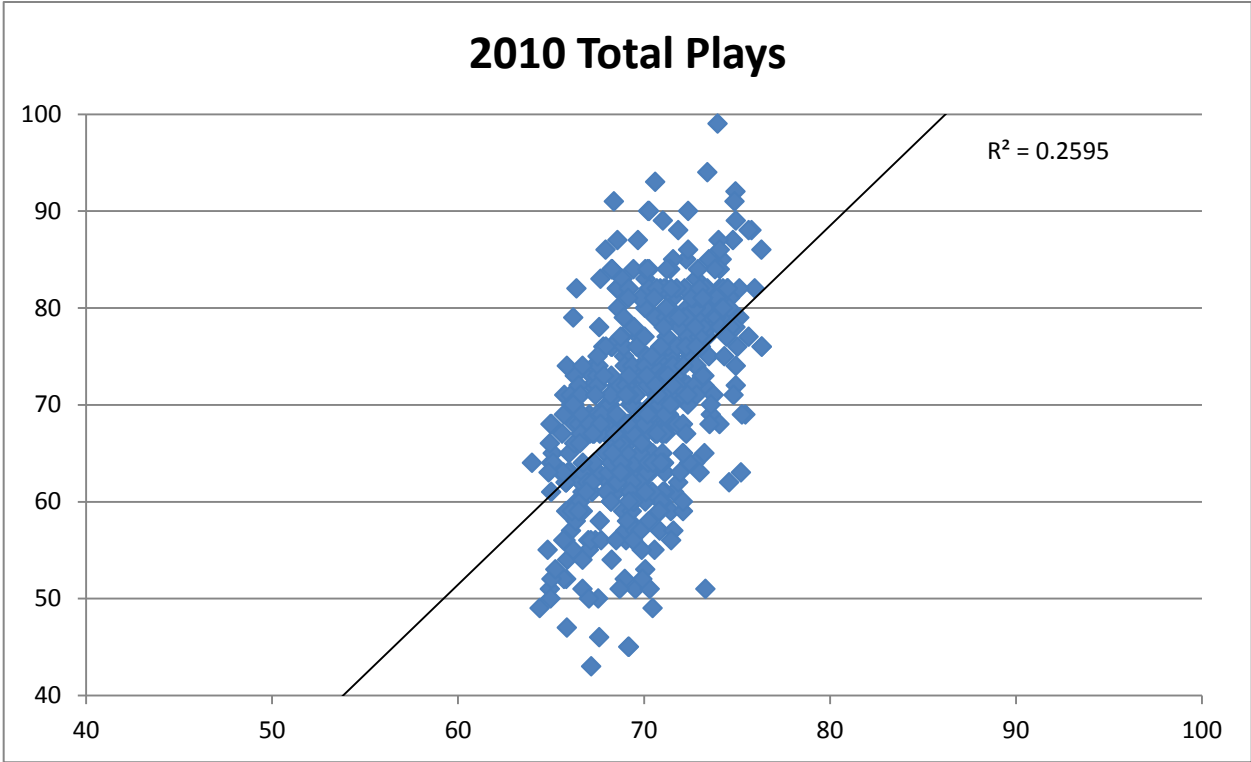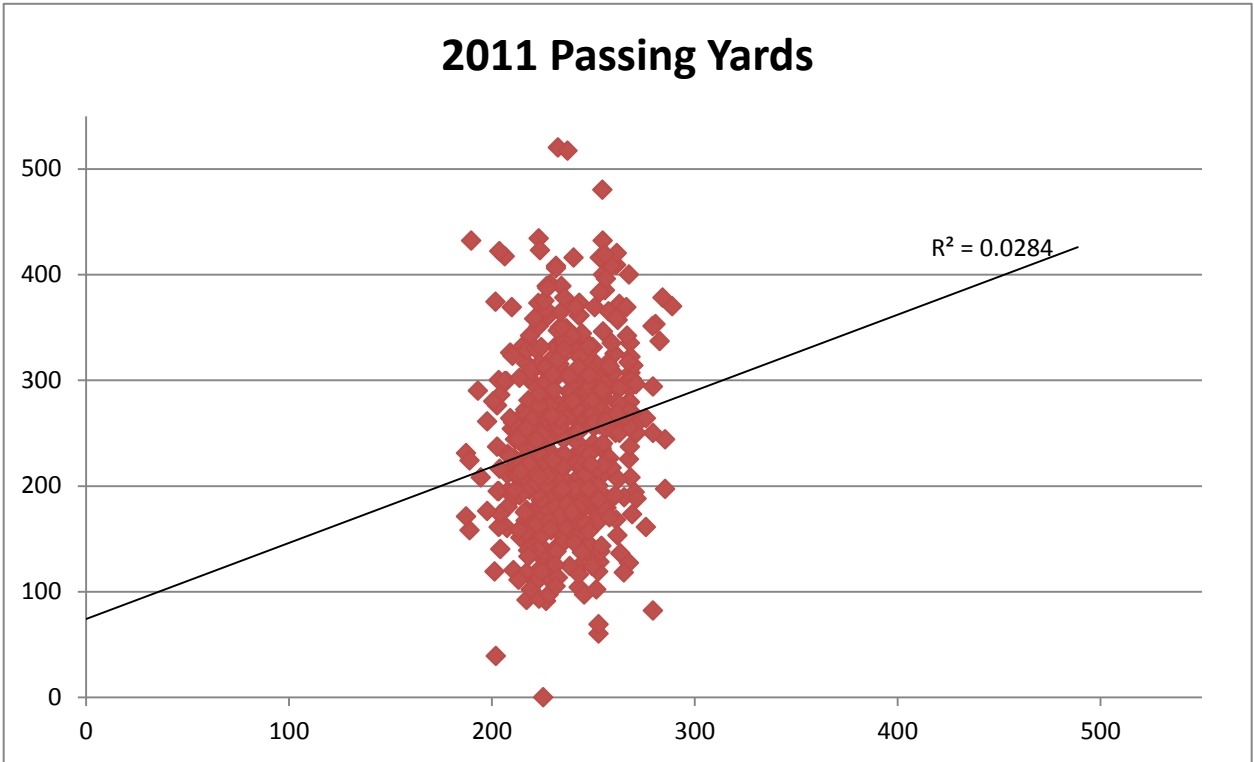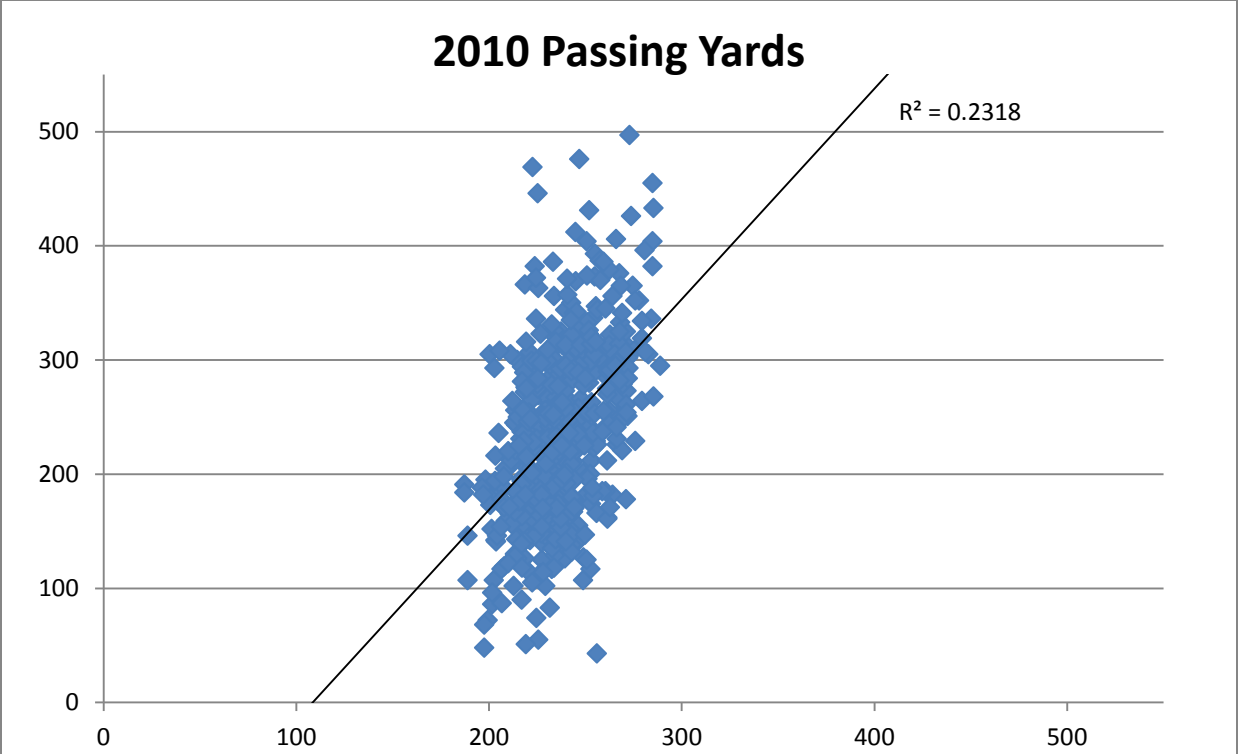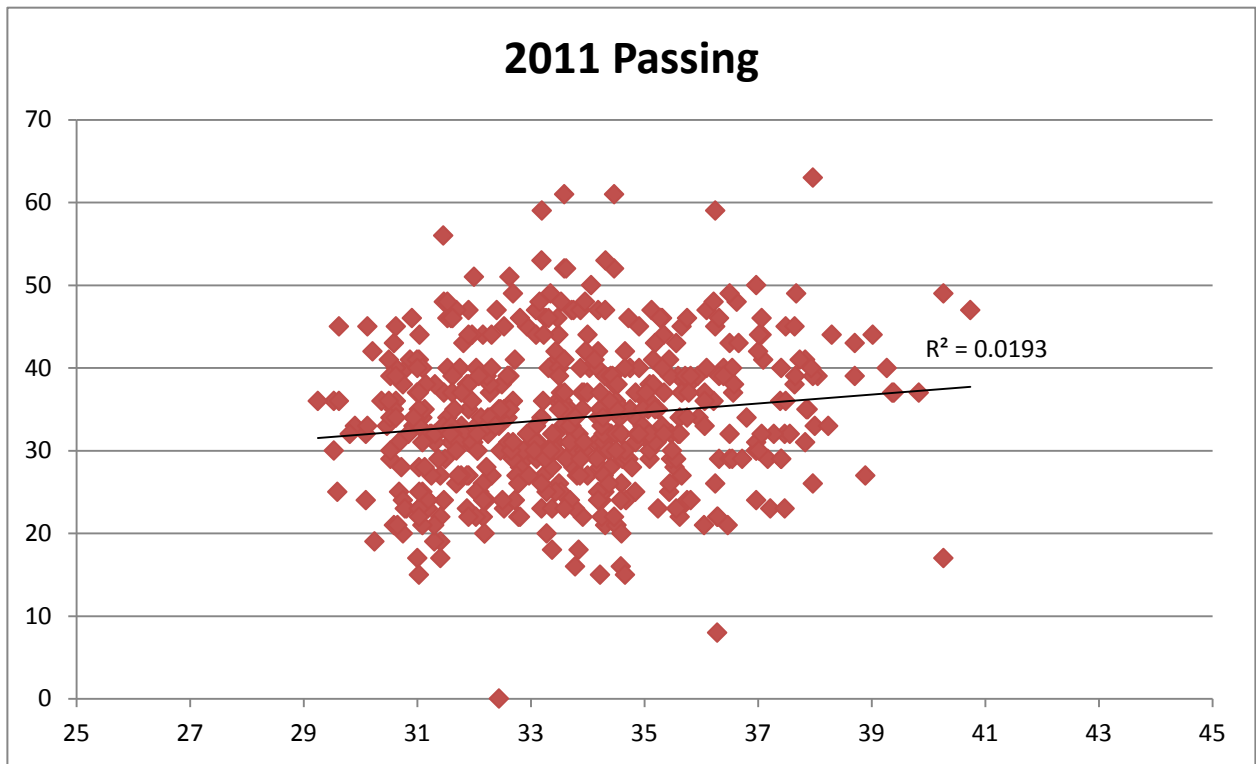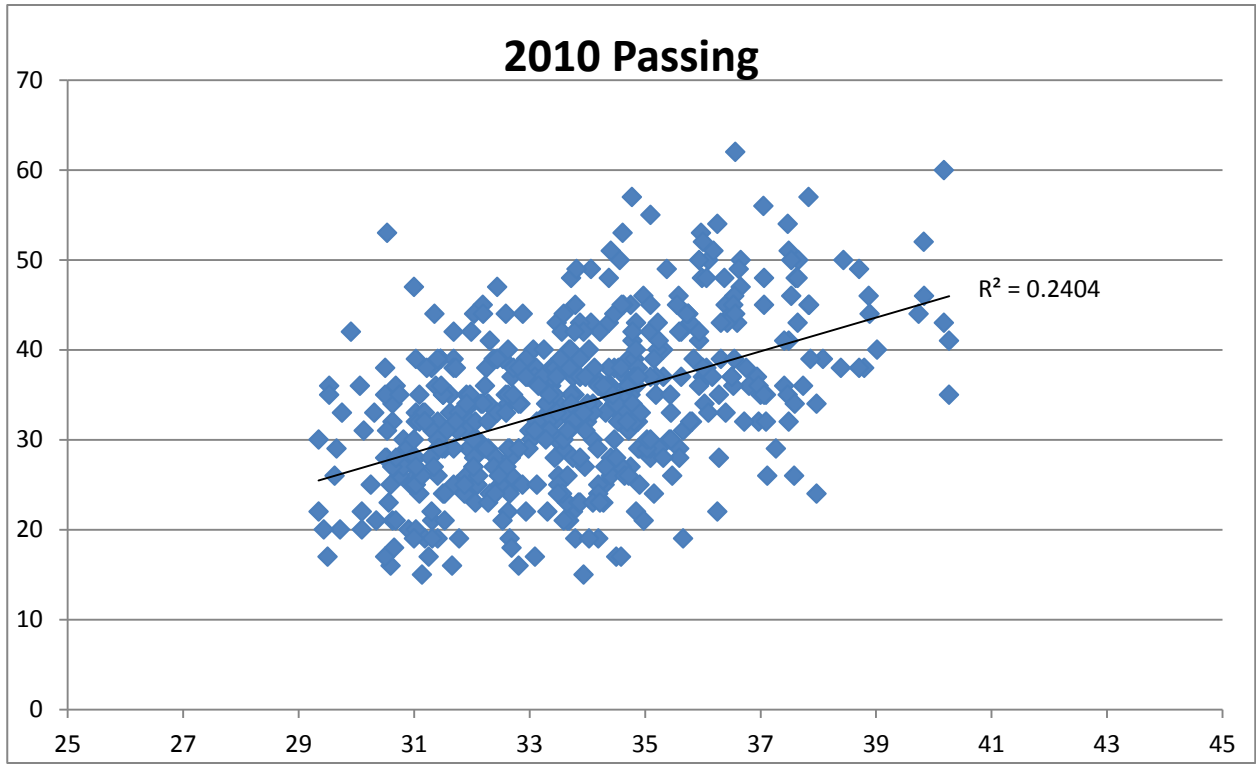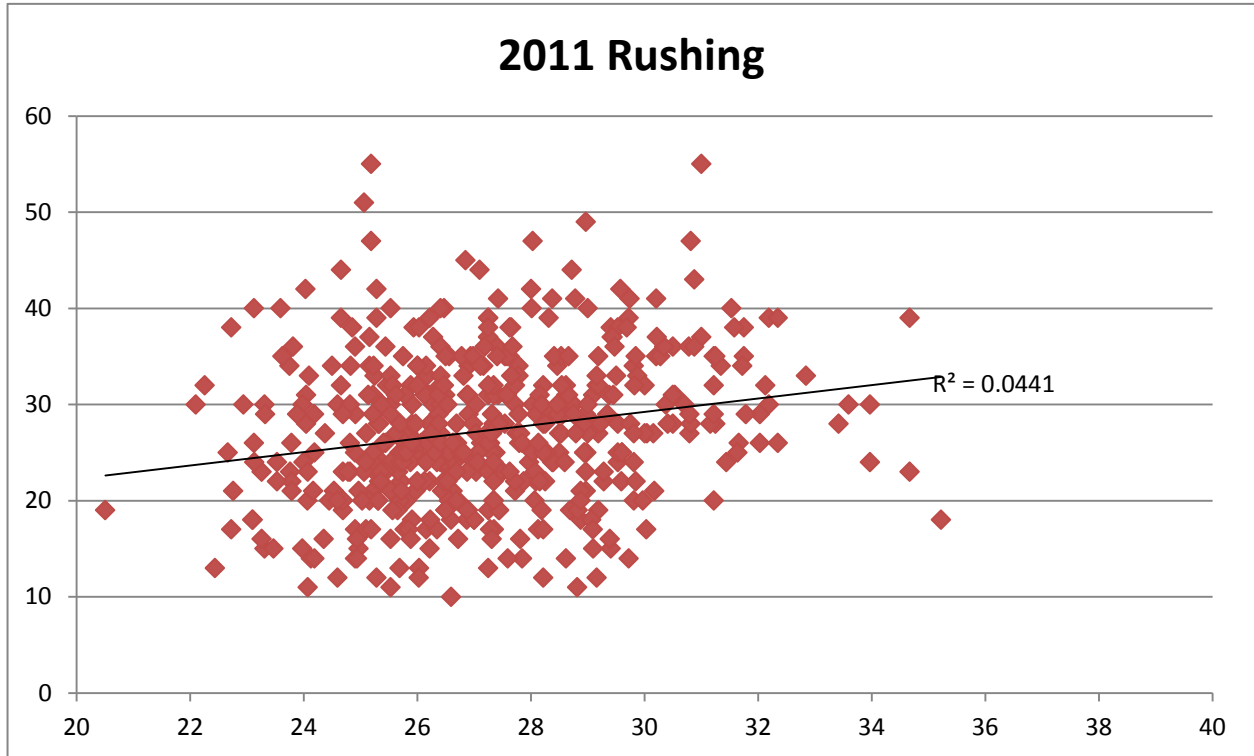
2010 Extra Points Attempted

R² = 0.2261

Extra Points Attempted 2011

R² = 0.0475

2010 Extra Points Made

R² = 0.2262

2011 Extra Points Made

R² = 0.0482

2010 Field Goal

R² = 0.1841

2011 Field Goal

R² = 0.0152

2010 Field Goals Made

$R^2 = 0.1417$

Field Goals Made 2011

$R^2 = 0.009$