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Analysis of Methods for Loss Reserving

A Major Qualifying Project Report
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by

Tim Connor

Xinjia Liu

Greg Lynskey

Ida Rapaj

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Approved by:

Jon Abraham, Project Advisor

Arthur Heinricher, Project Advisor

Abstract

Hanover Insurance uses numerous methods to project total paid claims for all its lines of business. A system was developed to assess the accuracy of the projections, based on data for six accident years and four lines of business. A strictly mathematical forecasting method was sought; however, no model was found to replace the years of experience and knowledge of Hanover's actuaries.

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I. Background

i.) The Fundamentals of Reserving

A loss reserve is "a provision for an insurer's liability for claims" (Wiser 197), namely, the amount of money needed to be put aside to settle unpaid or not fully paid claims. Loss reserving is "the actuarial process of estimating the amount of an insurance company's liabilities for loss and loss adjustment expenses" (Wiser 197).

The starting point is to study losses attributable to one calendar year at a time, the Accident Year, for a single line of business. Once the techniques to develop reserves on this basis are developed, they can be applied to cover all years and all lines of business.

First some terminology: The Ultimate Loss is the total amount needed to settle all the claims for one line of business, for a particular Accident Year. This number does not change over time; however, the work of Hanover's actuaries is to <u>estimate</u> this number – and their estimates <u>will</u> change over time, as more information becomes available. The Ultimate Loss is also known as the Incurred Loss.

Paid Losses represent the portion of the Ultimate (or Incurred) Loss that has been paid to the insured, and will vary by time. For losses associated with a given Accident Year, the insurer has either fully paid each claim or should be holding a loss reserve for any unpaid amounts. This basic relationship among the Incurred Losses, the Paid Losses, and the Loss Reserves is illustrated by:

$$\textit{Incurred Losses} = \textit{Paid Losses} + \textit{Loss Reserves} \quad \text{(Formula 1.1)}$$

The development of a single case will be looked at, to illustrate the above concepts. After an accident occurs on the Accident Date, the insured reports it to the insurer on the Reported Date. The insurance company sends a claim adjuster to assess the loss involved in the case. The claim adjuster's initial assessment is recorded into the company's database on the Recorded Date. The adjuster's estimate of the loss becomes the initial Claim Reserve for the case.

There are a number of reasons why Claim Reserves determined in this fashion may fall short of the total Loss Reserve the insurer should be holding for unpaid losses. First, it is fairly common for the reported amount of loss on a certain case to change over time, as it is very hard to make a perfect estimation in a short given period of time. For this reason, the Hanover actuaries often add a layer of "Supplemental Claim Reserves" to the Claim Reserves set up by the adjusters. Furthermore, claims associated with a particular Accident Year are not always known immediately – it can take months or even years before claims are even reported to the insurer. For these situations, a reserve for Incurred But Not Reported losses, or IBNR, is calculated and held as an additional reserve on the insurer's books. The Incurred Loss can now be presented as:

$$\textit{Incurred Loss} = \textit{Paid Loss} + \textit{Claim Reserves} + \textit{IBNR} \qquad \text{(Formula 1.2)}$$

To further demonstrate how all the pieces of the Incurred Losses come together, consider Figure 1.1. This figure shows the Ultimate Loss for claims from an unspecified line of business for the 2001 Accident Year, broken into sub-components on three measurement dates. At each measurement date, the Ultimate Loss consists of five pieces:

1. Paid Losses, shown in pink
2. Claim Reserves, shown in tan
3. Supplemental Claim Reserves, shown in yellow.
4. Incurred But Not Reported reserves, shown in green.
5. Error, shown in blue.

Notice that as of the initial measurement date, December 31, 2001, a relatively small portion of the eventual Ultimate Loss has been paid (the pink box). The tan and yellow boxes are the estimated amounts which will eventually be paid for claims which have been reported but not yet paid. The green box represents claims which were incurred during the Accident Year 2001 but have not yet been reported to the insurer. The blue box represents the total error in the estimates – it is the balancing item to make sure the pieces sum to the total. The goal of Hanover's actuaries is to minimize the size of the blue box.

The second box shows the development of the same Accident Year four years later, as of December 31, 2005. Paid losses have increased and IBNR has decreased – and the "error" has decreased, too, since more information is available about the 2001 Accident Year at this point. The magnitude of claim reserves at this point is a function of how quickly claims are paid once they have been reported.

Eventually, the illustration of Ultimate Loss reaches the stage as shown below as of December 31, 2036 – at some point, all of the claims are finalized, and the Ultimate Loss consists only of paid losses.

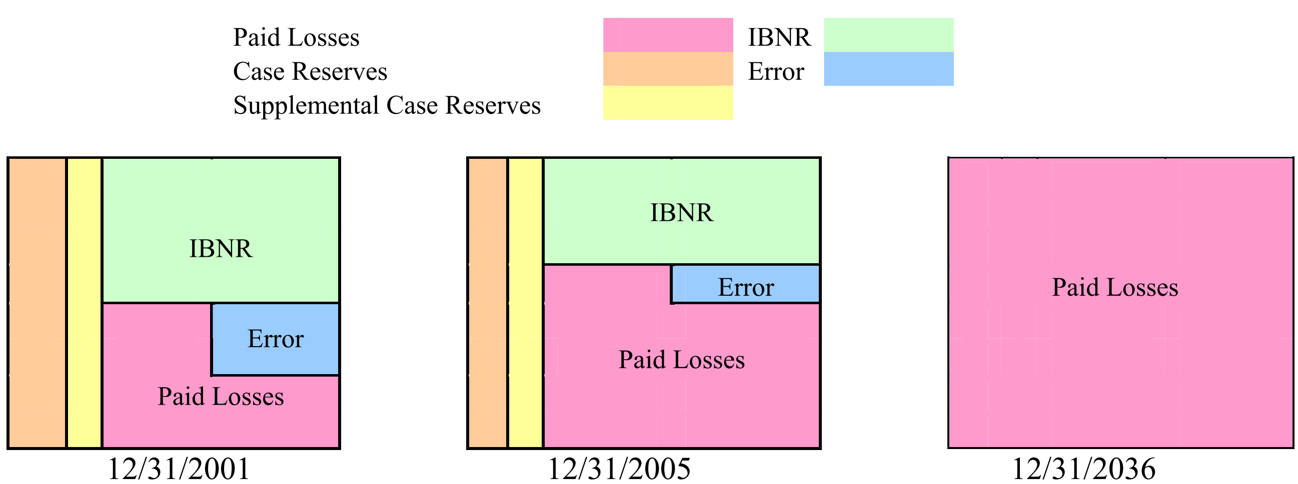

Figure 1.1

Loss Reserving is very important to a property & casualty insurance company. "Insurance companies must plan for the future (Garrell, Lee 3)." Loss reserving is a vital part of that process. "The financial condition of an insurance company can not be adequately assessed without sound loss reserve estimates (Wiser 197)." The concepts introduced in this section are the basics of the loss reserving process.

ii.) Loss Reserve Triangles

The loss reserving triangle is the standard method for maintaining loss data. While the entries vary for different methods, the use of the triangles is always the same.

	12 months	24 months	36 months	48 months	60 months
2001	59,500	70,400	71,700	72,000	71,900
2002	64,200	76,700	77,600	77,700	
2003	58,400	72,800	73,900		
2004	48,000	58,900			
2005	41,300				

Table 1.1

Table 1.1 shows the Incurred losses for a certain line of business for Accident Years 2001 through 2005. As time passes, for a given Accident Year, more claims are reported or their amounts are adjusted. This accounts for the changes in the Net Incurred Losses across each row. The columns show the Net Incurred Losses as of a certain stage in development, in this case every 12 months for each Accident Year. For example, the value 59,500 is the Net Incurred Loss for Accident Year 2001 after one year of development while 71,900 is the Net Incurred Loss for the same Accident Year at five years of development.

Given a loss triangle, one can develop “link ratios”. A link ratio is simply the ratio of the value for development period Z+1 to development period Z for each Accident Year. Table 1.2 shows the link ratios for losses in Table 1.1.

	12-24	24-36	36-48	48-60	60-Ultimate
2001	1.1832	1.0185	1.0042	0.9986	
2002	1.1947	1.0117	1.0013		
2003	1.2466	1.0264			
2004	1.2271				
2005					

Table 1.2

hese link ratios show the change in losses as they develop. Column "12-24" displays the ratios of the 24-month developed losses to the 12-month developed losses, "24-36" for the ratio of the 36-month losses to the 24-month losses, and so on. For example, during the period between 12 months and 24 months of development for the 2003 accident year, there is an increase in the total incurred cost of losses of 124.66%. That is, the value of losses incurred during the accident year 2003 after 24 months of development, 72,800, is 1.2466 times the value after 12 months, 58,400. This table assumes that growth finished shortly after 60 months, likely by the 72-months development period. Different lines of business have different development periods – the example illustrated above has a fairly short "tail". For lines of business with longer tails, the link ratio table will extend further to the right, and the values will converge to 1.000 much later.

Given all of the link ratios for the data, there are a number of ways to choose a link ratio to be used for predicting future loss development. One approach is to take an average of the calculated link ratios. This could be done as a weighted average of the link ratios for all years, an average of the most recent 3 years of link ratios, or a weighted average of the last three years of link ratios. The weighted average takes the sum of the incurred loss values for next period divided by the sum of the incurred loss values for the current period. That is, the weighted average over all years for 12-24 months would be:

$$\frac{\text{24 month developed incurred losses}}{\text{12 month developed incurred losses}}$$

or

$$\frac{70{,}700+76{,}700+72{,}700+58{,}900}{59{,}500+64{,}200+58{,}400+48{,}000}=1.212$$

The following table shows various averages for the data in the example:

	Age in months:				
	12-24	24-36	36-48	48-60	
all yrs wtd avg	1.212	1.015	1.003	1.000	
3 yr avg	1.223	1.015	1.003	1.000	
3 yr wtd avg	1.222	1.015	1.003	1.000	
select	1.220	1.015	1.003	1.000	1.000
ldf	1.242	1.018	1.003	1.000	1.000
cumulative	80.5%	98.2%	99.7%	100.0%	
segmented	80.5%	17.7%	1.5%	0.3%	
all yrs wtd avg	1.212	1.015	1.003	1.000	

Table 1.3

Once the averages have all been calculated, the actuary compares the various link ratio averages and determines which link ratio to use to estimate development in future years. The actuary's choice is known as the "select" link ratio. For the 12-24 month period, the averages are 1.212, 1.223, and 1.222. The "select" ratio in this example was 1.220. For the 24-36, 36-48, and 48-60, the averages are all the same to three decimal points, allowing for easy decision making for the select link ratios to be used for future development. It should be noted that this is not something that usually occurs.

Once the select link ratios are known, they can be used for future loss development. To do this, the link ratio for the development period is applied to the incurred loss as of the prior period. The resulting estimates are shown below in Table 1.4:

Development Age (Months)					
AY	**12**	**24**	**36**	**48**	**60**
2001	59,500	70,400	71,700	72,000	71,900
2002	64,200	76,700	77,600	77,700	*77,768*
2003	58,400	72,000	73,900	*74,182*	*74,199*
2004	48,000	58,900	*59,077*	*59,254*	*59,268*
2005	41,300	*50,368*	*51,142*	*51,295*	*51,307*

Table 1.4

For example, applying the 24-36 month select ratio of 1.015 to the 24-month losses of 58,900 for the 2004 Accident Year leads to a projected 36-month loss of 59,077. All of the numbers highlighted in yellow are projected losses.

Hanover's actuaries use a variety of projection methods to project the Ultimate Loss for a given Accident Year. These methods are discussed in more detail later in this report. All of the methods use the above link ratio approach in some fashion to develop their projections of Ultimate Losses.

iii.) Basic Projection Methods

Hanover uses six methods to project or predict the Ultimate Losses for a given Accident Year for each line of business. These methods are:

1. Incurred Method
2. Paid Method
3. Berquist Sherman Paid Method
4. Berquist Sherman Incurred Method
5. Bornhuetter Ferguson Paid Method
6. Bornhuetter Ferguson Incurred Method

These methods are described in more detail below. As noted above, the goal of each of these methods is to give a projection of the Ultimate Loss for a given Accident Year. Given this projection, as well as values for Paid Losses and Claim Reserves, it is possible to develop estimates for the IBNR – a key element for year-end financial reporting.

a.) Incurred Method

The Incurred Method projects Ultimate Losses based upon losses incurred to date, assuming that historical incurred loss data will be predictive of future incurred losses. This method is commonly used due to its ability to use case reserve estimate, which are developed by the claims adjusters on a case by case basis. However, by using the claims adjuster's estimates, it is possible to obtain inaccurate data if the reserve estimates chosen by the adjusters are not accurate and consistent over time.

The accuracy of the incurred method relies on the accuracy of the case reserve estimates developed by the adjusters, and upon losses occurring in similar patterns for each Accident Year, over time. If estimates are not reliably determined from year to year, or if the pattern of loss emergence changes over time, the incurred method will fail as prior years of incurred data will not accurately predict future years. If the adjusters are accurate in their reserve estimates, and if loss development is consistent, the incurred method will develop accurate estimates of ultimate incurred losses.

The Incurred Loss Development Method utilizes the total incurred losses. Incurred losses, as noted previously, are the total of the paid losses and projected loss reserves. The loss reserve triangle shown below in Table 1.5 is the start of an example of the incurred method:

Development Age (Months)					
AY	**12**	**24**	**36**	**48**	**60**
2001	59,500	70,400	71,700	72,000	71,900
2002	64,200	76,700	77,600	77,700	
2003	58,400	72,000	73,900		
2004	48,000	58,900			
2005	41,300				
Assume that no further losses are paid after 60 months					

Table 1.5

By analyzing this table, some trends can be seen. Although only 5 years worth of data is present, it can be seen that since 2002, the amount of incurred losses in the first 12 months of the accident year has been steadily declining. This could be the result of a decline in the volume of the business. It could also be a result of less conservative loss reserve estimation in the claims department, with adjusters developing lower reserve estimates than in the past. Also, both the 24 and 36 month columns (the cumulative incurred losses after 24 months of the accident year and 36 months of the accident year respectively) show the same trend of a decrease since 2002.

Starting from here, the ultimate losses can be projected by the incurred method using the link ratio approach in the previous section.

b.) Paid Method

The Paid method also projects ultimate losses from losses paid to date based on historical development patterns. The actuary uses the record of *actual loss payments* and disregards the case reserves. The precision of this reserving technique depends on the consistency in loss settlement patterns. Change in the rate of inflation, which affect loss payments, or shifts in can company procedures that influence settlement patterns, can all cause ambiguity in this reserving method.

c.) Berquist Sherman Methods

The Berquist-Sherman Paid and Incurred methods, sometimes simply called the Adjusted Paid and Incurred, account for the changes in the way the firm conducts business from period to period. The methods restate the raw data so that the current Calendar Year remains unaltered, but previous years are adjusted according to the way business is conducted. Once restated losses have been attained, the link ratio method as seen in the regular Paid and Incurred methods is carried out.

In order to find the restated incurred losses, we begin by finding the average loss amount per claim. To do this, we divide the total of outstanding losses by the number of outstanding claims. An example of this is shown in Table 1.6.

	Net Outstanding Losses	Net Outstanding Claims	Average Net Outstanding Losses
1994	50,900	6,700	7,597
1995	48,000	6,940	6,916
1996	53,500	7,160	7,472
1997	59,500	7,140	8,333
1998	52,600	6,850	7,679
1999	44,600	6,540	6,820

Table 1.6

From here, the Average Net Outstanding Claims (ANOC) is adjusted according to the Selected Trend, a value given by the claims department that is meant to reflect a line's inflation. The most recent year's ANOC is left "as is", and all prior years are replaced by this value discounted by the Selected Trend (Table 1.7).

(Trend Factor = 4.0%)	Current ANOC	Discount Factor	Discount Factor	Restated Average OS Claim
1994	6,820	(1.04)^5	1.2167	5605.54
1995	6,820	(1.04)^4	1.1699	5829.76
1996	6,820	(1.04)^3	1.1249	6062.96
1997	6,820	(1.04)^2	1.0816	6305.47
1998	6,820	(1.04)^1	1.0400	6557.69
1999	6,820	(1.04)^0	1.0000	6820.00

Table 1.7

Once the Restated Average Outstanding Claim value is determined, the Restated Outstanding Losses can be obtained by multiplying the Restated Average by the number of outstanding claims (Table 1.8).

	Restated Average Outstanding Claim	Net Outstanding Claims	Restated Outstanding Losses
1994	5605.54	6,700	37,557,137
1995	5829.76	6,940	40,458,566
1996	6062.96	7,160	43,410,759
1997	6305.47	7,140	45,021,080
1998	6557.69	6,850	44,920,192
1999	6820	6,540	44,602,800

Table 1.8

Finally, to get the Restated Incurred Losses, the Restated Outstanding Losses are added to the corresponding Paid Losses (Table 1.9).

	Paid Losses	Restated Outstanding Losses	Restated Incurred Losses
1994	11000.00	37,557,137	37,568,137
1995	11400.00	40,458,566	40,469,966
1996	11000.00	43,410,759	43,421,759
1997	16900.00	45,021,080	45,037,980
1998	14000.00	44,920,192	44,934,192
1999	10100.00	44,602,800	44,612,900

Table 1.9

Using this procedure across all the data in question produces the Restated Incurred Loss triangle. As previously noted, the technique for developing a prediction from a loss triangle using link ratios is known.

For the paid losses, the first step is finding the ratio of the number of Outstanding Claims to the number of Incurred Claims (Table 1.10).

	Number of Outstanding Claims	Number of Incurred Claims	OS Claims / Incurred Claims
1994	6,700	12,823	0.52
1995	6,940	13,949	0.50
1996	7,160	14,772	0.48
1997	7,140	15,474	0.46
1998	6,850	14,893	0.46
1999	6,540	14,049	0.47

Table 1.10

A new set of ratios is obtained, by dividing the difference for the most current Accident Year by all of the differences of the years being adjusted, resulting in a set of Adjustment Ratios (Table 1.11).

	OS Claims / Incurred Claims	1 – OS/Inc Claims	Adjustment Ratio
1994	0.52	0.48	1.12
1995	0.50	0.50	1.06
1996	0.48	0.52	1.04
1997	0.46	0.54	0.99
1998	0.46	0.54	0.99
1999	0.47	0.53	1.00

Table 1.11

Next, Paid Losses are multiplied by the Adjustment Ratios, leading to the Restated Paid Losses (Table 1.12). Repeating this process across the entire Paid Losses triangle gives the Restated Paid Losses triangle, from which link ratios can be developed.

	Adjustment Ratio	Raw Paid Losses	Restated Paid Losses
1994	1.12	11,000	12,313
1995	1.06	11,400	12,126
1996	1.04	11,000	11,410
1997	0.99	16,900	16,772
1998	0.99	14,000	13,856
1999	1.00	10,100	10,100

Table 1.12

The Berquist-Sherman method is useful in situations where there is a discernable shift in losses as a result of changes in how business is conducted. Trends such as a slowdown in settlement or payment rate often indicate a line of business as a candidate for these methods.

d.) Bornheutter Ferguson Method

The Bornheutter-Ferguson Method (BHF) is a method for estimating IBNR claims and, ultimately, loss reserves based on Estimated Ultimate Losses. It is built off of an estimation of expected losses, a value derived from the accident year's earned premium and the line of business's Estimated Loss Ratio for the year to eliminate the effect of irregularly large or small claims. As such, the BHF method is put to best use on lines of business that have a history of being volatile or, in fact, relatively little history at all. As the goal of the BHF method is to stabilize the data, instead of using incurred or paid values, which, depending on the line of business, could very likely fluctuate, the BHF method calculates the Estimated Ultimate Losses as a percentage of the Earned Annual Premium, which is the total premium the company receives for this particular line of business.

There are two ways to arrive at results under BHF: the paid and incurred methods. As the development periods progress, the paid and incurred methods approach the same value. Because the steps for both methods are the same, only the incurred method will be shown in example. The paid method can be developed by applying the same steps to the paid information.

As previously discussed, the BHF method is based upon the line of business's Estimated Ultimate Losses. This value is calculated by multiplying the Estimated Loss Ratio by the respective Annual Earned Premium for the accident year. Both values are inputs depending on the line of business. An example is shown in Table 1.13.

Accident Year	Estimated Loss Ratio	Annual Earned Premium	Estimated Ultimate Losses
2001	75.0%	90,900	68,175
2002	80.0%	102,400	81,920
2003	70.0%	107,200	75,040
2004	70.0%	102,800	71,960
2005	60.0%	94,700	56,820

Table 1.13

The next step is to determine the IBNR Factors. These values are used to take the Estimated Ultimate Loss value and determine the Expected IBNR value for each method. for the incurred method, it is necessary to refer to the Loss Development Factor (LDF) for the unadjusted incurred method. The LDF is the ratio that transforms the current incurred value in the loss development triangle to the ultimate value. The IBNR Factor is determined by Formula 1.3 and an example is given in Table 1.14.

$$IBNR\_Factor = 1 - \frac{1}{LDF} \qquad \text{(Formula 1.3)}$$

Accident Year	Estimated Loss Ratio	Annual Earned Premium	Estimated Ultimate Losses	Incurred IBNR Factor
2001	75.0%	90,900	68,175	0.000
2002	80.0%	102,400	81,920	0.000
2003	70.0%	107,200	75,040	0.003
2004	70.0%	102,800	71,960	0.018
2005	60.0%	94,700	56,820	0.195

Table 1.14

The same steps are undertaken to develop the IBNR Factor for the paid method with the LDF from the unadjusted paid loss development triangle. In order to determine the Estimated IBNR for the incurred method, the IBNR Factor (Incurred) is multiplied by the Estimate Ultimate Loss value, as shown in Table 1.15. Similarly, for the Estimated IBNR for the paid method, the IBNR Factor (Paid) is used.

Accident Year	Estimated Loss ratio Ratio	Annual Earned Premium	Estimated Ultimate Losses	Incurred IBNR Factor	Expected Incurred IBNR
2001	75.0%	90,900	68,175	0.000	0
2002	80.0%	102,400	81,920	0.000	19
2003	70.0%	107,200	75,040	0.003	242
2004	70.0%	102,800	71,960	0.018	1,292
2005	60.0%	94,700	56,820	0.195	11,083

Table 1.15

iv.) The Four Lines of Business

Hanover Insurance is primarily a property and casualty insurer, meaning that most of their business is in automobile and property insurance, both for individuals and for businesses. The four lines of business included in this report are: Business Owners Policy Liability, Commercial Auto Liability, Personal Auto Bodily Injury, and Personal Auto Property Damage Liability. These are discussed briefly below.

a.) Business Owners Policy Liability

This kind of insurance contract is offered to small or medium-sized businesses to protect their buildings and personal property. The policies may also include business income, extra expense coverage, and other common coverages, such as employee dishonesty, money and securities, and equipment.

b.) Commercial Auto Liability

This kind of insurance is offered to small or large businesses providing protection for losses resulting from operating an auto. The insurance covers bodily injury and property damage for which the insured is liable as a result of operating vehicles.

c.) Personal Auto Bodily Injury

This kind of insurance covers bodily injuries or death for which the insured is responsible. It pays for medical bills, loss of income or pain and suffering of another party, and legal defense for the insured if necessary.

d.) Personal Auto Property Damage Liability

This kind of insurance covers the property damages the insured caused to another party in a car accident, and it also includes legal defense.

v.) Paid Losses Development Analysis

Hanover provided data for the four line of business mentioned previously for year-end loss reserve analysis for calendar years 1996 through 2001. In total, there are twenty-four Excel files, one for each of six calendar years for each of the four lines of business. The data in these files range from Accident Year 1981 through 2001. After review of the data, the Accident Years 1981 to 1988 were considered fully developed by the end of 2001, and the following discussion focuses on these Accident Years.

a.) Basic Analysis

First some basic properties of the four lines of businesses were studied. Table 1.16 and Figure 1.2 show the Ultimate Losses for these years, for each of the four lines of business. It is clear that PABI > CAL > PAPD > BOP for the Ultimate Losses across almost all these accident years. This shows the magnitude of the various lines in terms of incurred losses. Further, BOP > CAL > PABI > PAPD in terms of percentage growth over 1981-88, with a minimum of 83% increase for PAPD, and up to 1660% increase for BOP. This indicates the lines of business are growing most rapidly over the study period.

Accident Year	BOP	CAL	PABI	PAPD
1981	750	11,600	23,500	12,600
1982	1,500	16,500	30,600	13,600
1983	1,800	16,100	35,400	14,600
1984	3,200	27,500	42,100	17,000
1985	7,200	31,100	45,000	20,000
1986	4,800	32,700	47,800	21,300
1987	7,800	38,800	49,400	22,500
1988	13,300	37,100	58,100	23,100

Table 1.16

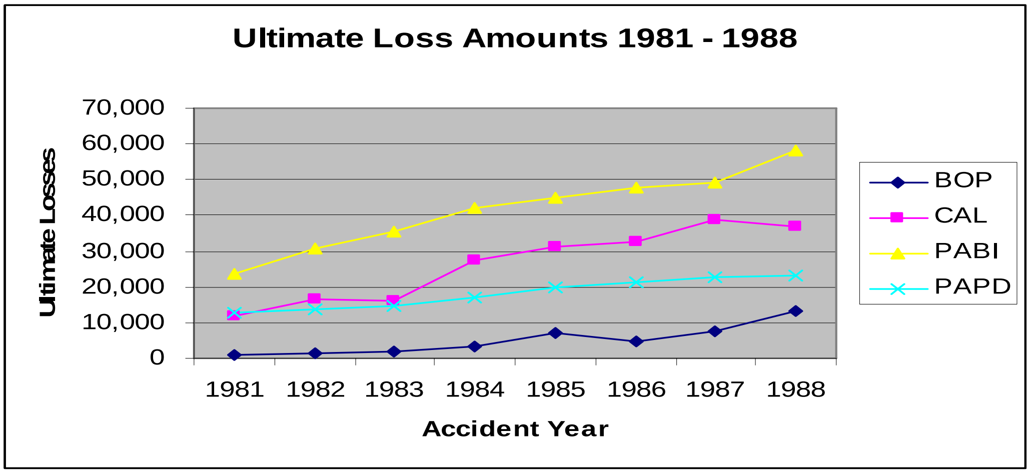

Figure 1.2

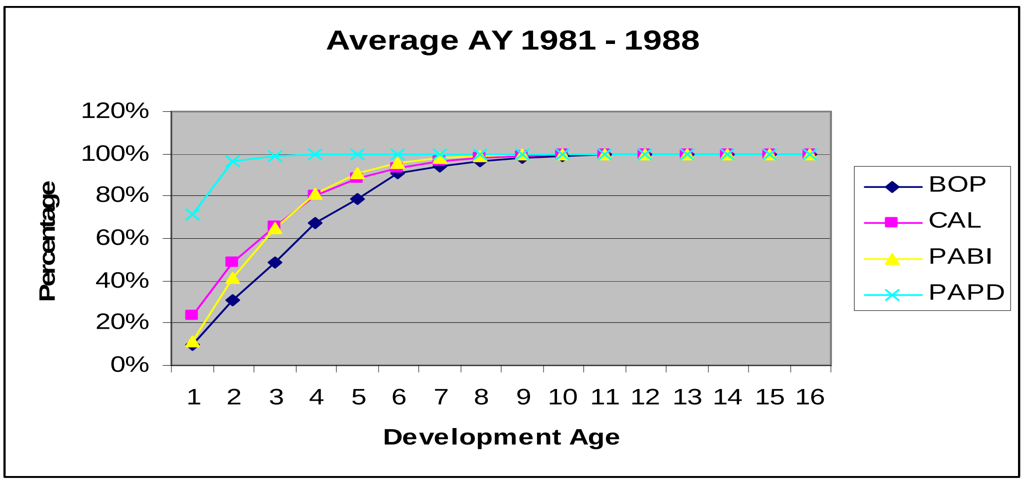

Figure 1.3

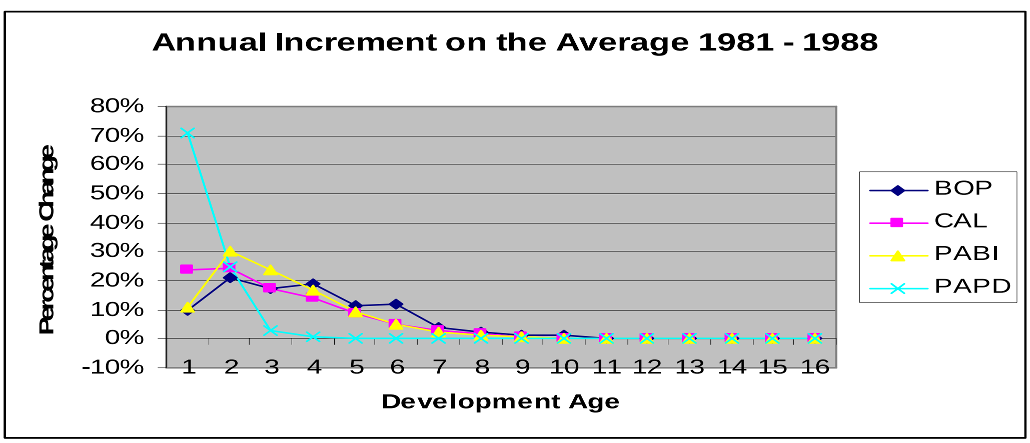

Figure 1.4

Figure 1.3 and Figure 1.4 show a comparison of the average amount of these eight accident years, indicated by “development age”. From the annual increment chart it can be seen that PAPD had a large first year paid loss amount, an increment of about one-third of that in the second year, and finished off soon after that. Both PABI and BOP had peak(s) of increments. PABI had a single-mode graph, of which the peak resides around 2 years after the accident year, where BOP was of multi-mode, year 2, 4, 6 of the development are all little peaks. The increasing rate of the CAL line slowed down over time as PAPD, but with a much flatter slope.

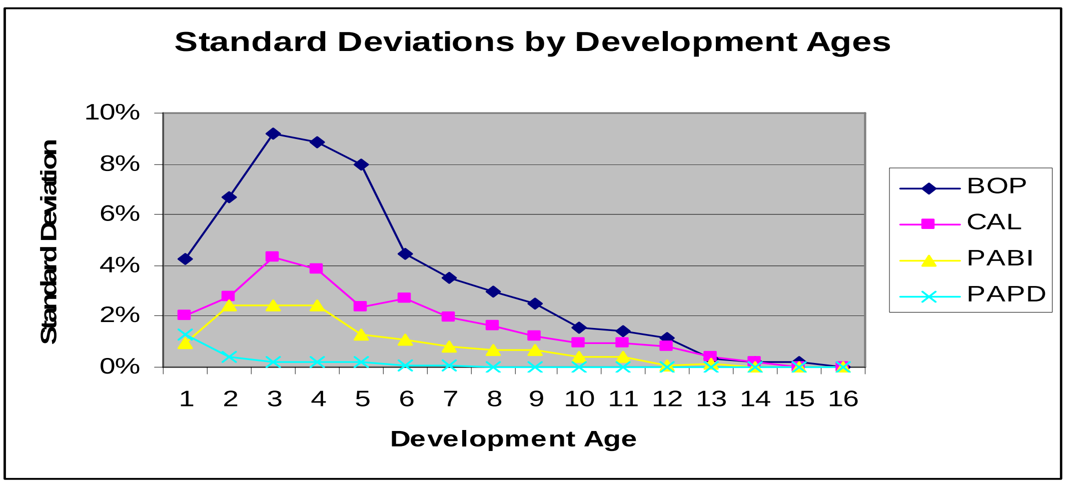

Figure 1.5

Standard deviations were calculated for each development age of these accident years (Figure 1.5). As a percentage of development, the standard deviations follow the pattern BOP > CAL > PABI > PAPD, especially for the first 7 years of development. BOP was the most unstable (highest standard deviation), and PAPD was the most stable.

Another important item to look at is the correlation between different lines of business. Figure 1.6 shows the correlation of different lines of business in different accident years. CAL and PABI are the most related. The lines BPL, CAL, and PABI are fairly closely related. Correlations with PAPD are not as strong as with the other lines.

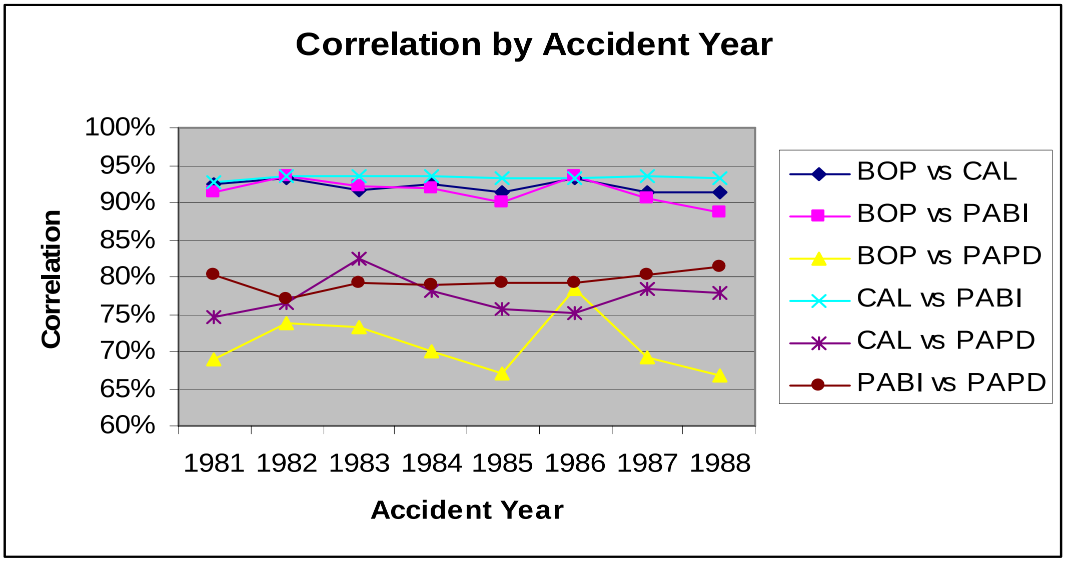

Figure 1.6

Table 1.17 summarizes what was stated above, giving a rank of 1 to the strongest condition and 4 to the weakest.

	BOP	CAL	PABI	PAPD
Ultimate Loss	4	2	1	3
Increment of Ultimate Loss	1	2	3	4
Variance	1	2	3	4
Years to Ultimate	13 - 14	14 - 15	11 – 12	5 - 6

Table 1.17

b.) Analysis by Lines of Business

Another way to look at the data is by line of business. The data provides the development through a number of years. The Ultimate Loss is taken as Hanover's recorded or projected Ultimate Loss as of the end of the 2001 accident year. From this, a percentage is calculated by dividing the known losses (paid losses plus claim reserves) as of a certain development period by the Ultimate Loss. Percentages are used instead of dollar amounts since each line of business experiences different magnitudes of claims, and over the years the magnitude of claims for a certain line of business can vary. This approach results in a data set that gives the cumulative "percentage of ultimate" for each line of business by development age.

As can be seen in Figure 1.7 and Table 1.18, paid losses for BOP did not develop a consistent pattern over time. The graphs for different accident years stay at a distance from each other, and the standard deviations in the table are large. The paid losses start from a low point (around 10% or 12%), and develop very slowly for 13 to 14 years before reaching ultimate.

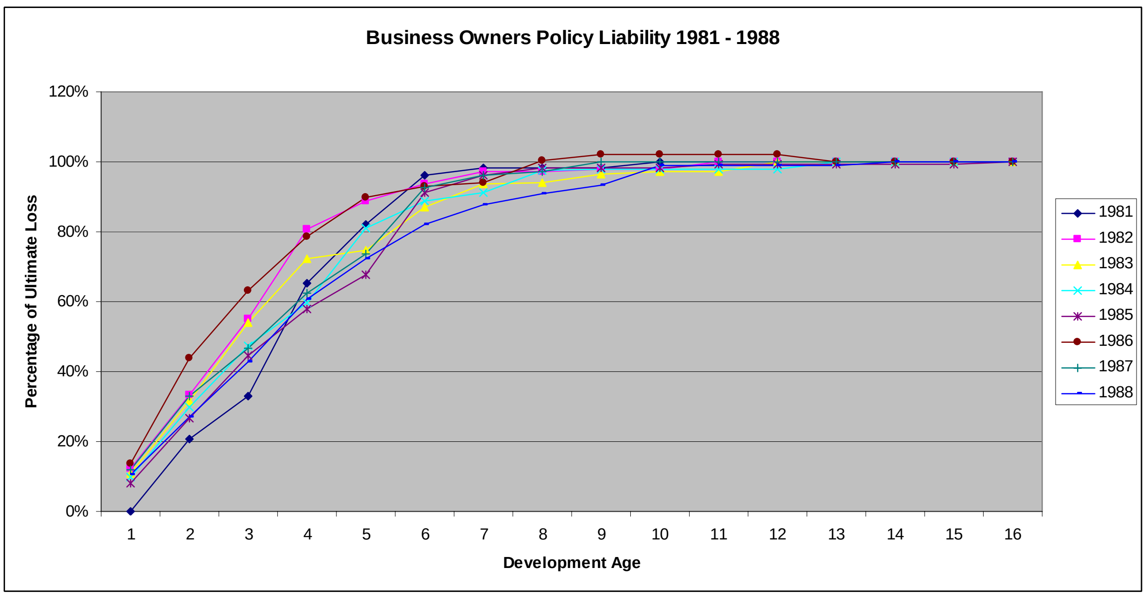

Figure 1.7

BOP	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mean	10%	31%	48%	67%	79%	91%	94%	97%	98%	99%	99%	100%	100%	100%
SD	4%	7%	9%	9%	8%	4%	4%	3%	2%	2%	1%	1%	0%	0%

Table 1.18

Figure 1.8 and Table 1.19 shows that CAL is more consistent than BOP and that standard deviations are also smaller. The line took longer reach ultimate, approximately 14 to 15 years, despite the higher starting point of 24%.

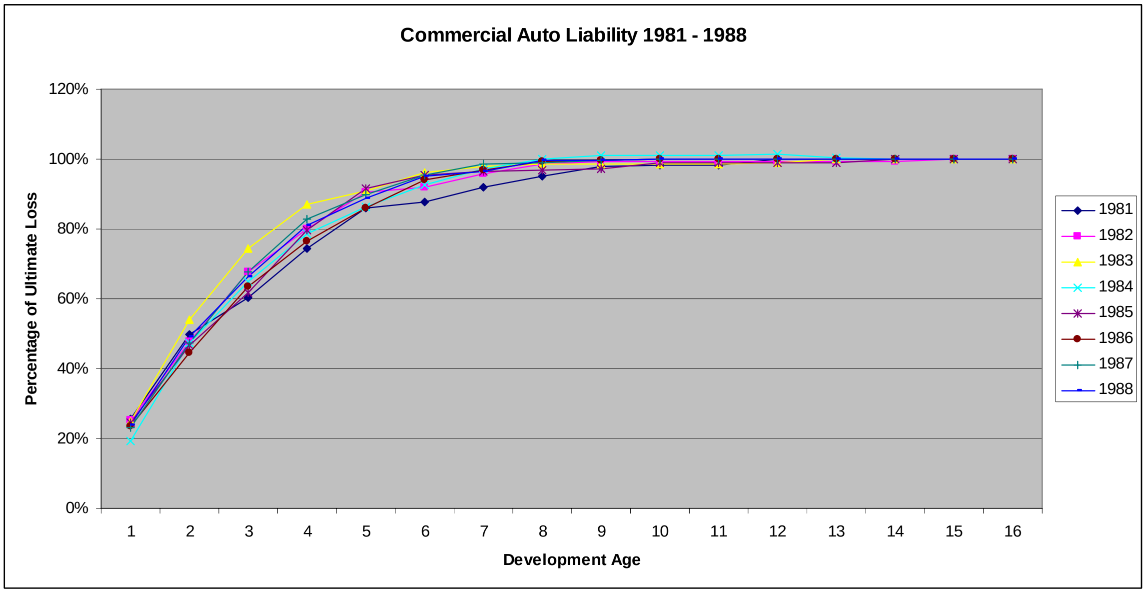

Figure 1.8

CAL	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mean	24%	48%	66%	80%	89%	94%	96%	98%	99%	99%	100%	100%	100%	100%
SD	2%	3%	4%	4%	2%	3%	2%	2%	1%	1%	1%	1%	0%	0%

Table 1.19

PABI has much smaller standard deviations; from Figure 1.9 and Table 1.20 it is very hard to tell the graphs of different years apart. Starting off at 11% development, less than half of CAL's starting point, the PABI develops more quickly, taking approximately 12 years for the paid losses to reach the ultimate.

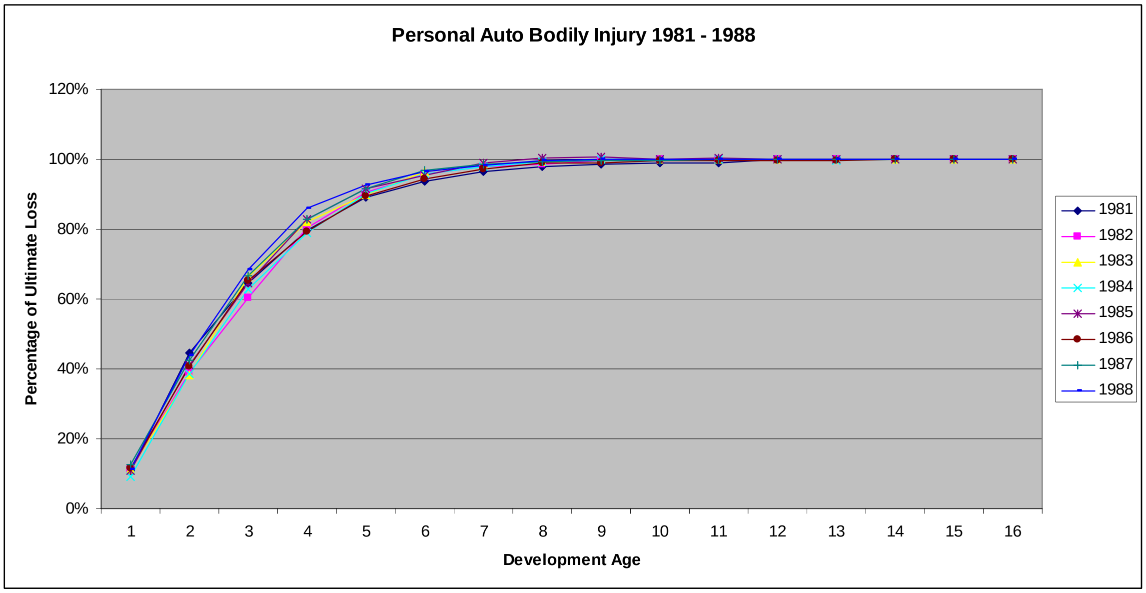

Figure 1.9

PABI	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mean	11%	41%	65%	81%	91%	96%	98%	99%	100%	100%	100%	100%	100%	100%
SD	1%	2%	2%	2%	1%	1%	1%	1%	1%	0%	0%	0%	0%	0%

Table 1.20

PAPD clearly stands out among the four lines of business. With very small standard deviations, the differences between graphs are almost invisible. This line fully develops within about six years, much more quickly than any of the other lines.

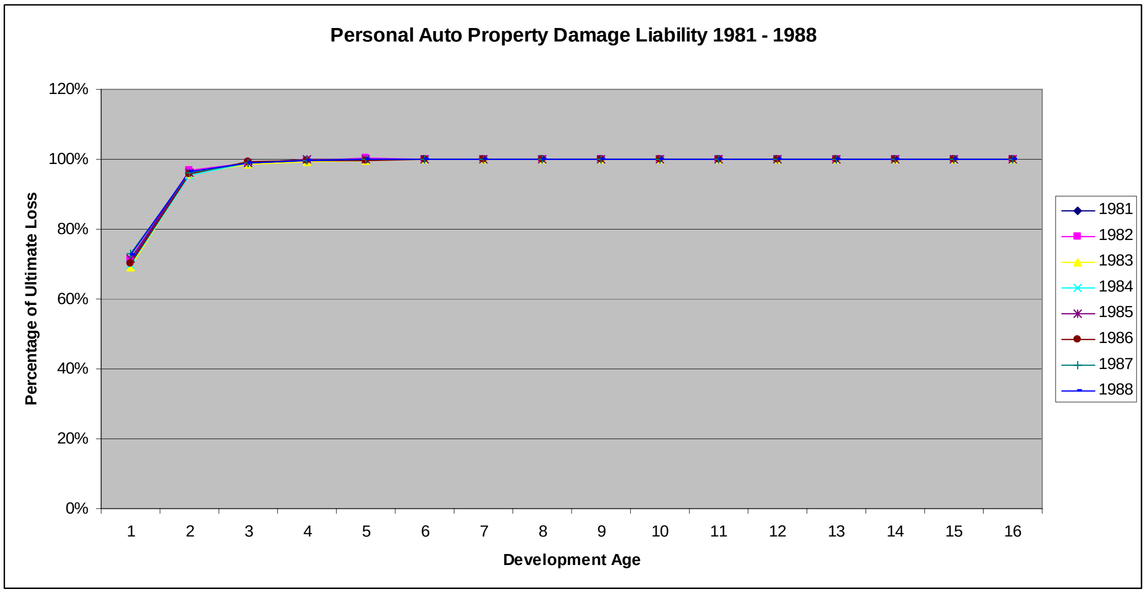

Figure 1.10

PAPD	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mean	71%	96%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
SD	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Table 1.21

II. Analysis of Estimated Ultimate Losses

i.) Analysis of Ultimate Losses

The estimated Ultimate Losses from each of the six methods are recalculated annually as new data comes in. Because this new data is closer to the true Ultimate Loss over time, it would seem that recalculating estimated values based on this new data would result in more accurate estimates. This, however, is not always the case. Trends appearing in the data suggest there are outside factors influencing the changes as well.

Following is a comparative analysis for the four lines of business for the accident years of 1996 and 1997. A percentage error of the estimated ultimate losses against the actual ultimate loss value as of 2006 is calculated.

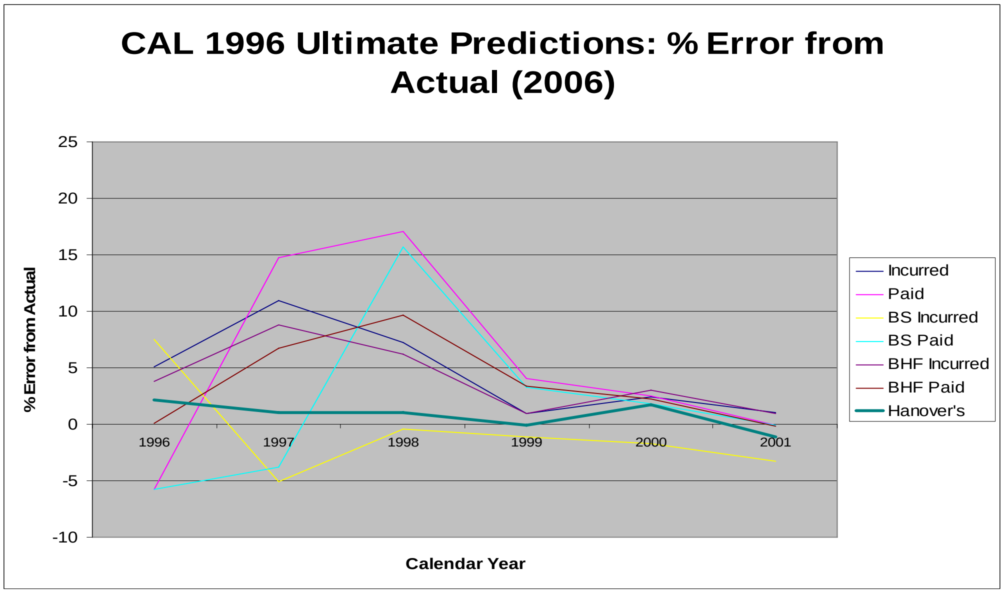

Figure 2.1

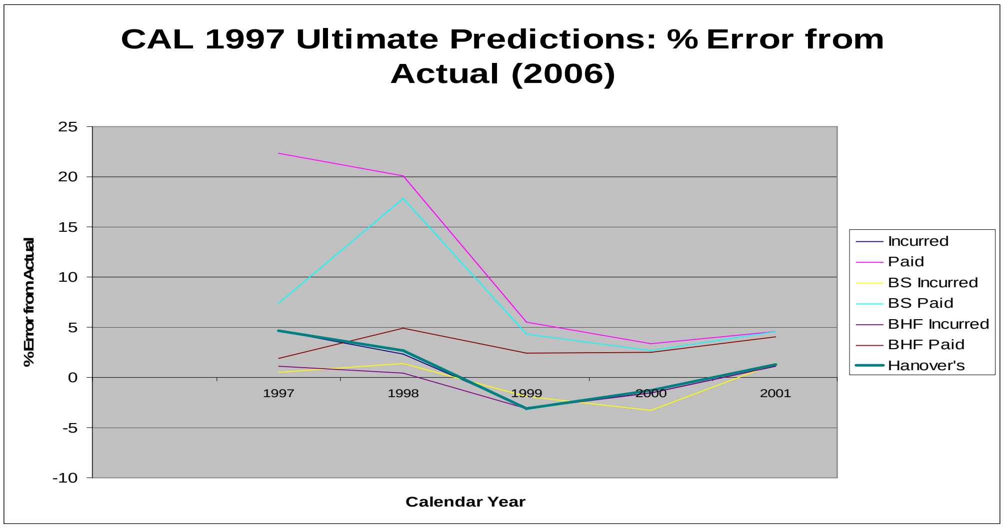

Figure 2.2

In Figure 2.1 and Figure 2.2, it can be seen that for CAL the paid methods (Paid, BS Paid, and BHF Paid) consistently predict ultimate values larger than the actual and, quite frequently, larger than the incurred methods. Calendar years 1997 and 1998 are stand-out years for this trend. Hanover's estimate very closely matches the Incurred method in 1997, implying that Hanover may have been aware of the Paid methods' current inaccuracy and thus put more weights on other methods.

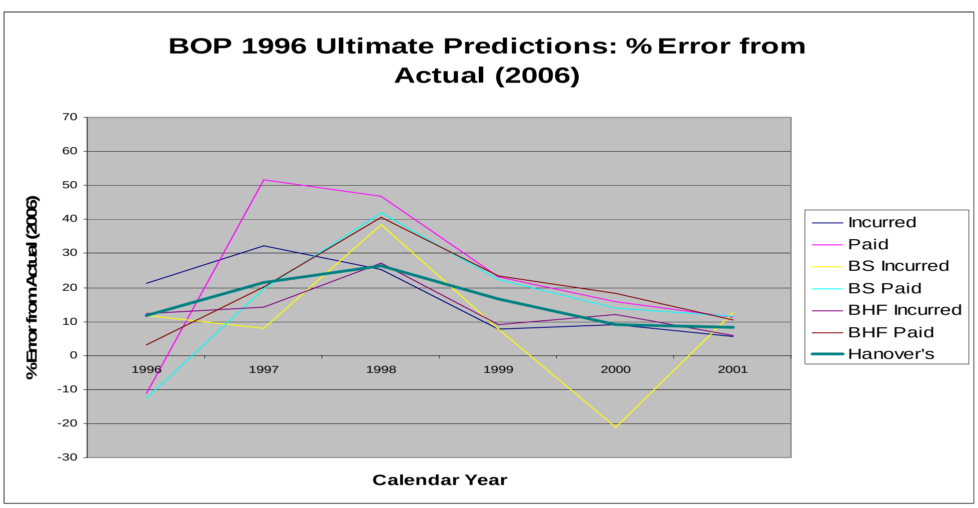

Figure 2.3

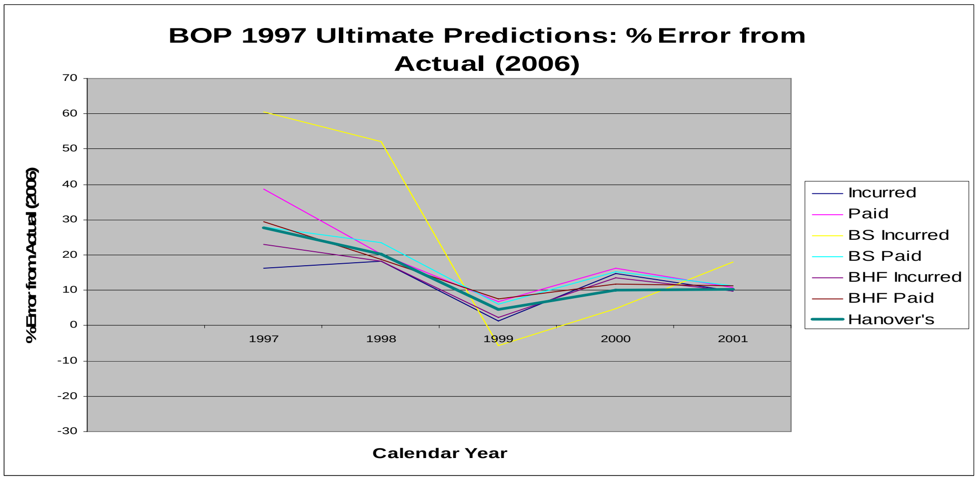

Figure 2.4

In Figure 2.3 and Figure 2.4, the severe overestimation made by the Paid methods that was seen in the CAL graphs continues to be apparent for BOP. The incurred methods also show a significant amount of error, particularly the Berquist-Sherman Incurred method. Hanover's "select" predictions do not seem to follow any particular method, but rather are among the various different methods, indicating that there might not be any historical trends for the accuracy of any of the methods for the BOP line.

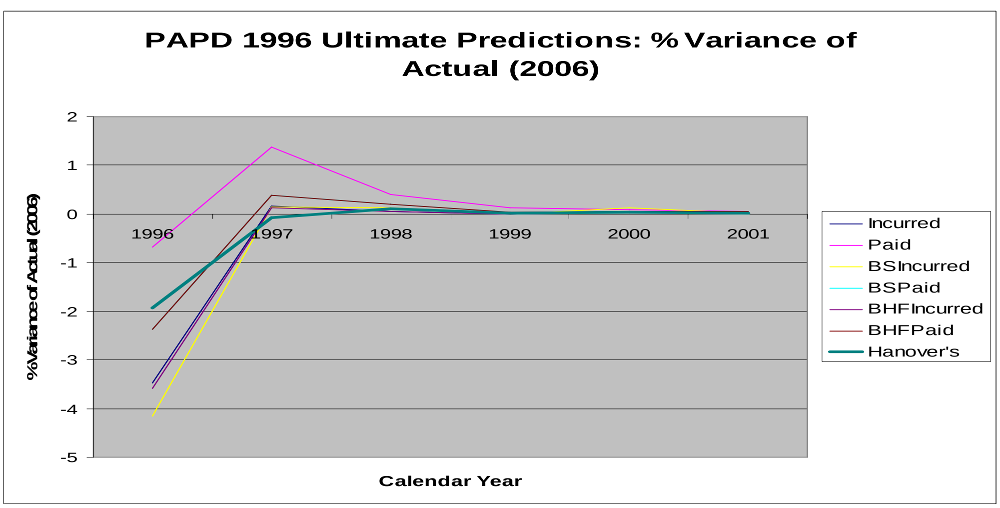

Figure 2.5

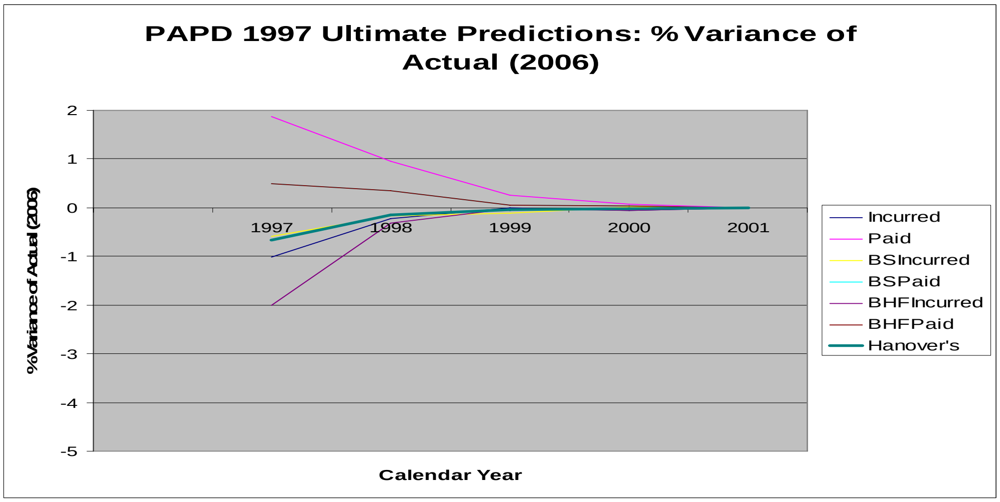

Figure 2.6

Compared with the other three lines of business, PAPD has a relatively short tail that approaches full development very quickly – reasonable, since the nature of this line of business is non-bodily injury with minimal litigation. Within 3-5 years of development age, all the methods and Hanover's choice approach 0% error. The Paid method continues its trend of over-estimating the ultimate, and Hanover chose estimates more in line with the Incurred methods. However, the short-tail of the line reduces the effect of any poor estimates as can be seen in Figure 2.5 and Figure 2.6.

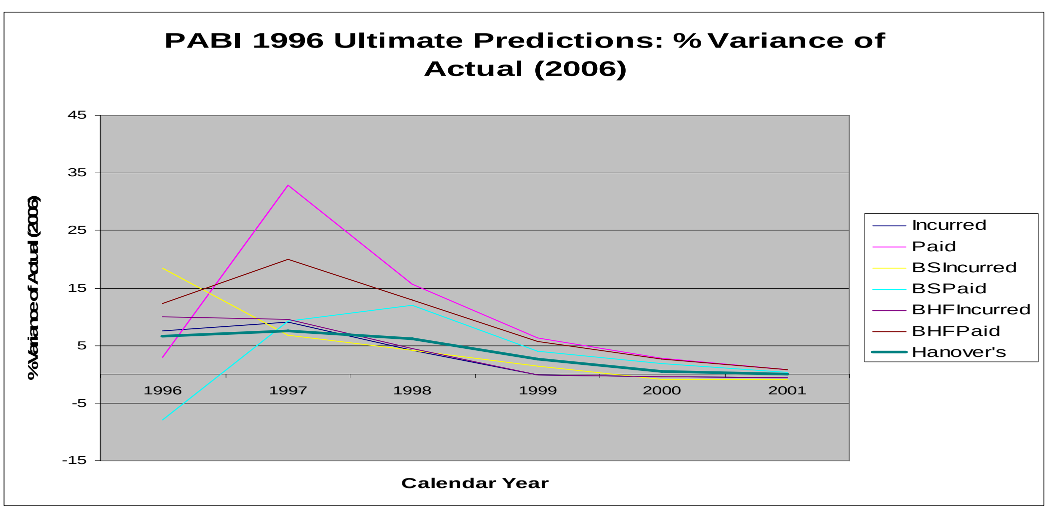

Figure 2.7

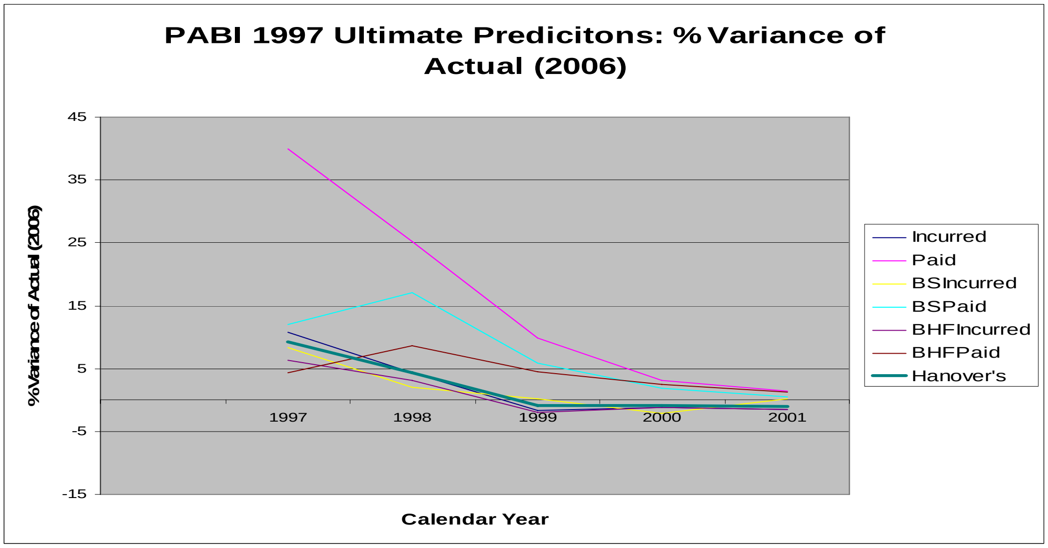

Figure 2.8

The PABI graphs, Figures 2.7 and 2.8, show the same overarching trends that can be seen in the other three lines. For one, the Paid methods tend to consistently overestimate the ultimate losses. Hanover appeared to be aware of this as they chose their estimates to be closer to the Incurred methods. At 5-6 years of development, all the methods are approaching 0% error, indicating that near this age, the losses approach full development.

ii.) The Scoring System

An evaluation system was developed to assess the accuracy of the outcomes from different estimation methods, comparing the Estimated Ultimate Losses (EUL) to the Actual Ultimate Loss (AUL) of each line for each Accident Year (AY).

First the percentage error of each EUL was calculated per evaluation period (one Calendar Year (CY)) for each line of business for each AY. A negative percentage error indicates that the funds set aside for this line in for this AY were insufficient, and a positive one implies an over-estimation of ultimate losses.

The next step is to calculate the "score" for each method per AY per line of business. It is the square root of a weighted average of the squared percentage errors (per CY) within each method for each AY/line of business. The "score" represents the accuracy of the method for that particular AY/line of business. It would be necessary to emphasize the early EULs, as it is important to know how much money to set aside as soon as possible. Several weighting systems were considered: equal weights, linear weights, and geometric weights. The score formula and the three weighting systems can be seen below in Formula 2.1.

$$R_i = 100 * \frac{Est_i - Act_i}{Act_i} \qquad Score = \sqrt{\frac{\sum_i \alpha_i R_i^2}{\sum_i \alpha_i}}$$

Equal weighting: $\alpha_i = 1, i = 1,...,n$

Linear weighting: $\alpha_i = n - i + 1, i = 1,...,n$

Geometric weighting: $\alpha_i = 2^{n-i}, i = 1,...,n$

Est_i : Estimated Ultimate Loss (EUL) for development age i.

Act_i : Actual Ultimate Loss (AUL) for development age i.

R_i : Percentage Error for the EUL of development age i.
α_i : Weighting for the percentage error of development age i in the final score.

Formula 2.1

The effects of different weighting systems were experimented with the data for the 1996 AY. The graphs of the results under equal, linear, and geometric weightings are in Figures 2.9, 2.10, and 2.11 respectively.

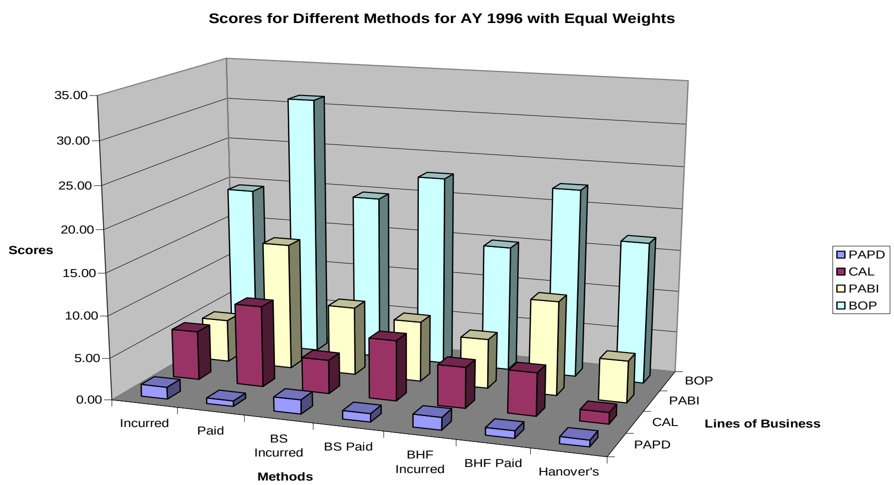

Figure 2.9

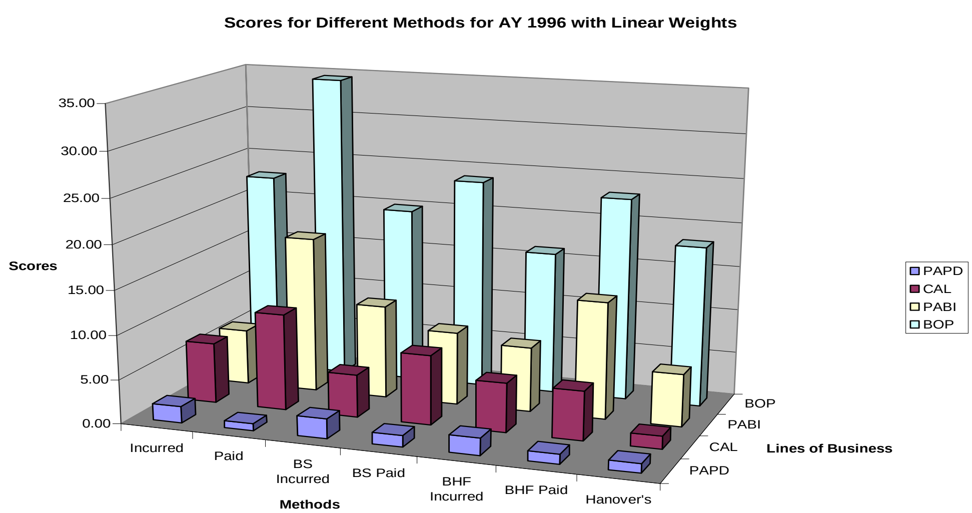

Figure 2.10

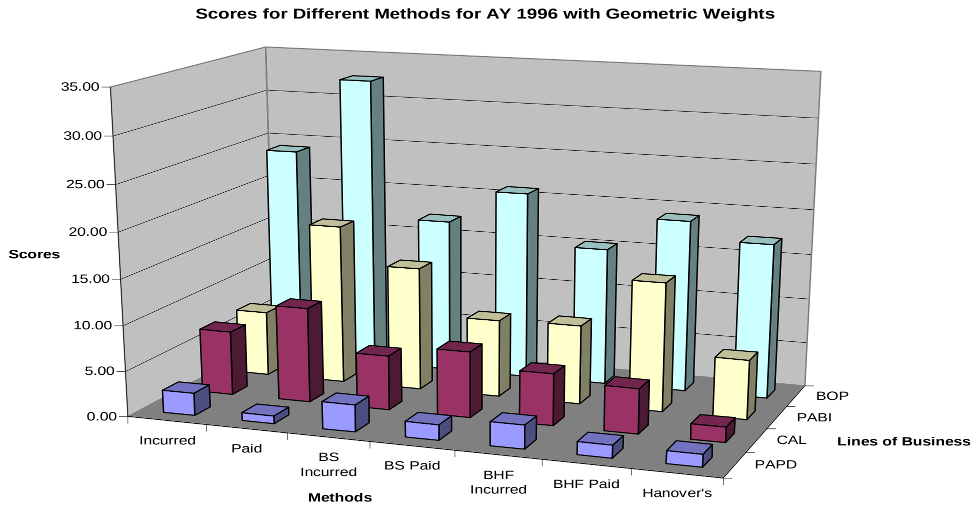

Figure 2.11

The geometric weighting system was considered to be most suitable here. It puts twice as much emphasis on the N-th evaluation period as on the N+1-st. Thus it results in low scores for estimations that were accurate in early years and high scores for estimations that were inaccurate in early years. For example, in the figures above, for the PAPD line, which is very consistent in its growth patterns, the change in score from equal weighting to linear to geometric is very little. Alternatively, for the BOP line, which has a larger fluctuation range, the scores tend to grow from equal to linear to geometric weighting. Table 2.1 below shows the scores for AY 1996, and Table 2.2 shows those for the PABI line.

AY 1996	CAL	BOP	PAPD	PABI
Incurred	7.12	24.11	2.48	7.24
Paid	10.48	32.55	0.86	17.70
BS Incurred	5.98	17.19	2.96	13.66
BS Paid	7.25	20.95	1.70	8.56
BHF Incurred	5.68	15.30	2.56	8.73
BHF Paid	4.92	19.09	1.37	14.17
Hanover's	1.68	17.17	1.37	6.53

Table 2.1

PABI	1996	1997	1998	1999	2000	2001
Incurred	7.24	8.08	2.05	9.26	4.88	9.37
Paid	17.70	31.60	16.80	6.98	5.24	4.55
BS Incurred	13.66	6.15	5.72	6.29	2.98	7.62
BS Paid	8.56	12.44	6.35	12.40	3.93	7.78
BHF Incurred	8.73	4.94	2.90	5.24	3.99	7.70
BHF Paid	14.17	5.66	7.09	1.61	2.38	4.21
Hanover's	6.53	7.00	5.71	1.61	0.80	5.95

Table 2.2

From Table 2.1 it is possible to tell that for AY 1996, the Hanover made the best prediction for the CAL and PABI lines, but the information in Table 2.2 is less clear. Thus the "rankings" were introduced as a more direct way to evaluate the methods. The scores for the seven methods per accident year per line of business were extracted and sorted from low to high, assigning each method a rank from 1 to 7. A "1" corresponds to the lowest score, indicating the best method for a certain year and line. An example is shown in Table 2.3 below, rankings for AY 1996 for all four lines.

AY 96	CAL		BOP		PAPD		PABI	
	Score	Ranking	Score	Ranking	Score	Ranking	Score	Ranking
Incurred	7.12	5	24.11	6	2.48	5	7.24	2
Paid	10.48	7	32.55	7	0.86	1	17.70	7
BS Incurred	5.98	4	17.19	3	2.96	7	13.66	5
BS Paid	7.25	6	20.95	5	1.70	4	8.56	3
BHF Incurred	5.68	3	15.30	1	2.56	6	8.73	4
BHF Paid	4.92	2	19.09	4	1.37	2	14.17	6
Hanover's	1.68	1	17.17	2	1.37	3	6.53	1

Table 2.3

It is still not clear from these individual rankings if there is a method that works constantly better for a certain line of business. In Table 2.4 below are the sums of the rankings across the six accident years for each method, and just to further illustrate, in Table 2.5 these combined rankings were sorted again. It can be seen that the Hanover made the best prediction 2 out of 4 lines of business, with BHF Paid standing out in the other two. The Paid method was doing poorly in general, except for in PAPD, and the Incurred method was not much more accurate either most of the time.

Line	CAL	BOP	PAPD	PABI
Incurred	21	27	25	27
Paid	34	30	18	35
BS Incurred	23	25	33	23
BS Paid	27	23	20	29
BHF Incurred	22	20	33	23
BHF Paid	24	18	18	19
Hanover's	16	25	21	12

Table 2.4

Line	CAL	BOP	PAPD	PABI
Incurred	2	6	5	5
Paid	7	7	1	7
BS Incurred	4	4	6	3
BS Paid	6	3	3	6
BHF Incurred	3	2	7	4
BHF Paid	5	1	1	2
Hanover's	1	5	4	1

Table 2.5

Next the individual rankings were summed by accident years. Table 2.6 shows the sums, and again to illustrate, after having them sorted again, Table 2.7 has a new set of ranks. The Hanover worked with a certain level of precision all accident years except for AY 1997. The BHF methods came out to be better than the BS methods. The Paid method was the worst performer in 4 out of 6 years, but it was the best in AY2001.

AY	1996	1997	1998	1999	2000	2001
Incurred	18	14	11	18	19	20
Paid	22	26	23	16	20	10
BS Incurred	19	13	15	20	19	18
BS Paid	18	18	13	17	17	16
BHF Incurred	14	12	22	16	13	21
BHF Paid	14	14	20	9	8	14
Hanover's	7	15	8	16	16	12

Table 2.6

AY	1996	1997	1998	1999	2000	2001
Incurred	4	4	2	6	6	6
Paid	7	7	7	4	7	1
BS Incurred	6	2	4	7	5	5
BS Paid	5	6	3	5	4	4
BHF Incurred	2	1	6	2	2	7
BHF Paid	3	3	5	1	1	3
Hanover's	1	5	1	3	3	2

Table 2.7

To have a general idea of which method performed best across the six accident years and four lines of business, the individual rankings were summed to have a total ranking for each method, as is shown below in Table 2.8. The Hanover gave the best results overall and the Paid method the poorest. The BHF methods ranked the 2nd and the 3rd, and the Incurred method outperformed the BS Incurred, which was not readily apparent in previous tables.

	Incurred	Paid	BS Incurred	BS Paid	BHF Incurred	BHF Paid	Hanover's
Total	100	117	104	99	98	79	74
Rank	5	7	6	4	3	2	1

Table 2.8

III. Developing Forecasting Methods

After analyzing the past data, the next step was to develop a mathematical model that could be used to predict future development for certain accident years. There were two approaches to this: fitting data into a known function and taking a weighted total of the basic methods.

i.) Fitting into Functions

The Commercial Auto Liability and Personal Auto Bodily Injury were the first lines of business used to develop the model, as their data has relative consistency with the development. Approaches used include straight line analysis, curve fitting, and minimizing residuals. The exponential function, the logistic function and the Weibull function were each considered, due to the shape of the development curve.

a.) Straight Line Regression

The Exponential Model

Starting with the exponential model in the form of $1-\alpha e^{-\beta t}$,the data being processed were the percentages of ultimate losses. The first step was determine a “line of best fit” for the data, graphed on a logarithmic scale. This line would be used to show the potential accuracy of a model while also generating the α and β parameters for the exponential model. The data points for each development period of a given accident year were plotted on a graph with months of development on the x-axis and the LN(1-% of Ultimate) value on the y-axis. A line of best fit was then determined for this plot. For the line of best fit, y=mx+b, the parameters m and b were determined. To transform back to the exponential model, $\alpha = e^{b}$, and $\beta = -m$.

Given α and β, the exponential model is dependent solely upon time. As an example, the 1989 accident year for Commercial Auto Liability paid data had a line of best fit of y=-0.5177x - 0.0691, resulting in $f(t)=1-1.0715e^{-0.5177t}$. The following table shows actual percentage development side by side with the projected development.

Months	CAL	Ultimate	AY1989 %	Model %
12	9,582	40,200	23.8%	36.3%
24	19,203	40,200	47.8%	62.0%
36	29,298	40,200	72.9%	77.4%
48	34,830	40,200	86.6%	86.5%
60	37,200	40,200	92.5%	92.0%
72	38,277	40,200	95.2%	95.2%
84	39,325	40,200	97.8%	97.1%
96	39,675	40,200	98.7%	98.3%
108	39,981	40,200	99.5%	99.0%
120	40,046	40,200	99.6%	99.4%
132	40,079	40,200	99.7%	99.6%
144	40,079	40,200	99.7%	99.8%
156	40,080	40,200	99.7%	99.9%

Table 3.1

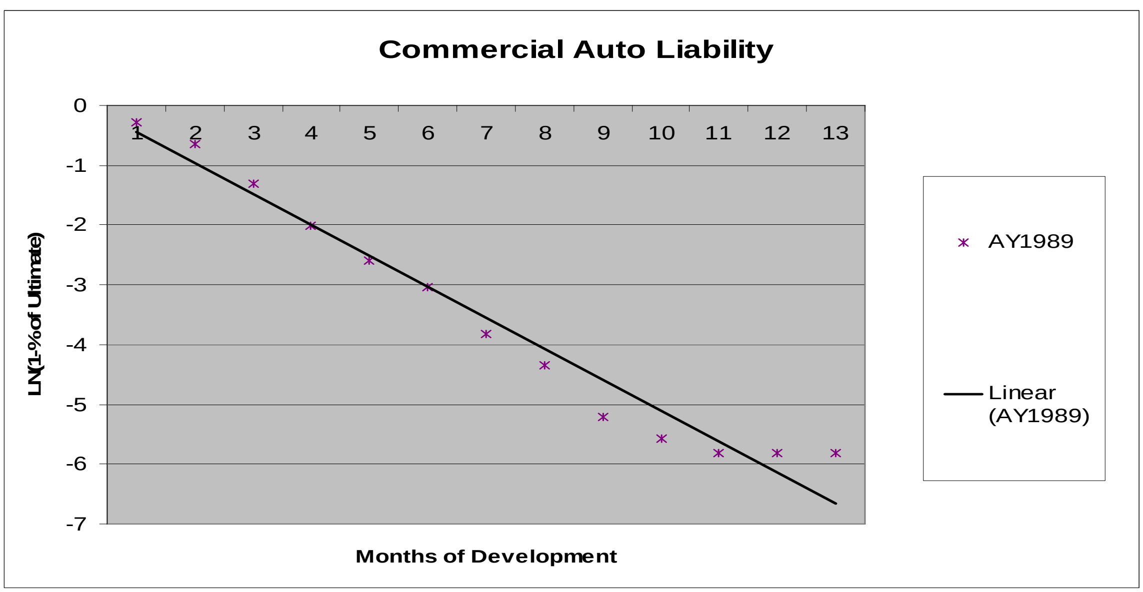

Figure 3.1

In Figure 3.1, the data from the column AY1989 % of Ultimate is shown with the purple stars. The line of best fit is then fit based on this data. Once the Model %'s of Ultimate are known, a graph can be produced to show the actual % of ultimate for the accident year as well as the model's projection. Figure 3.2 below displays AY 1989 CAL paid development. Actual data is shown by a dark red dot and the model is shown with a purple line.

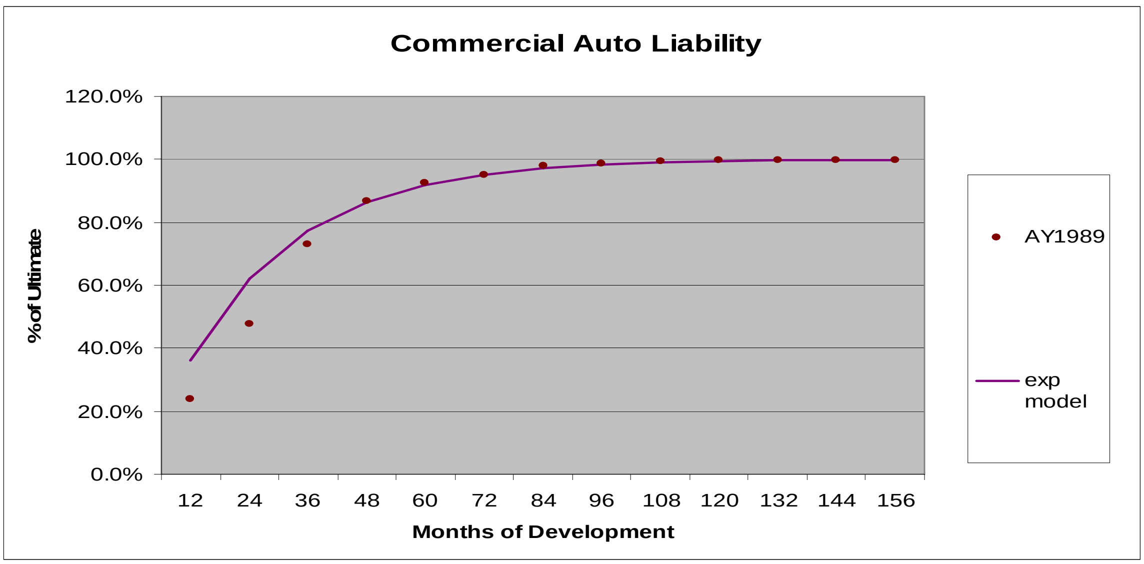

Figure 3.2

There appears to be a very good fit to the data, suggesting that a mathematical approach to loss reserve estimation may be possible. The model projects that the 12 month development is approximately 36% of the eventual ultimate, which is a very good fit to the actual numbers. However, the majority of the years are not this consistent. Shown below in Figure 3.3 is the exponential model for the accident year 1991 paid data for Commercial Auto Liability. The model predicts that the 12 month development will be approximately 13% of the ultimate. However, the actual development shows that the

12 month development is approximately 27% of the ultimate. This poor result eliminates the model's usefulness.

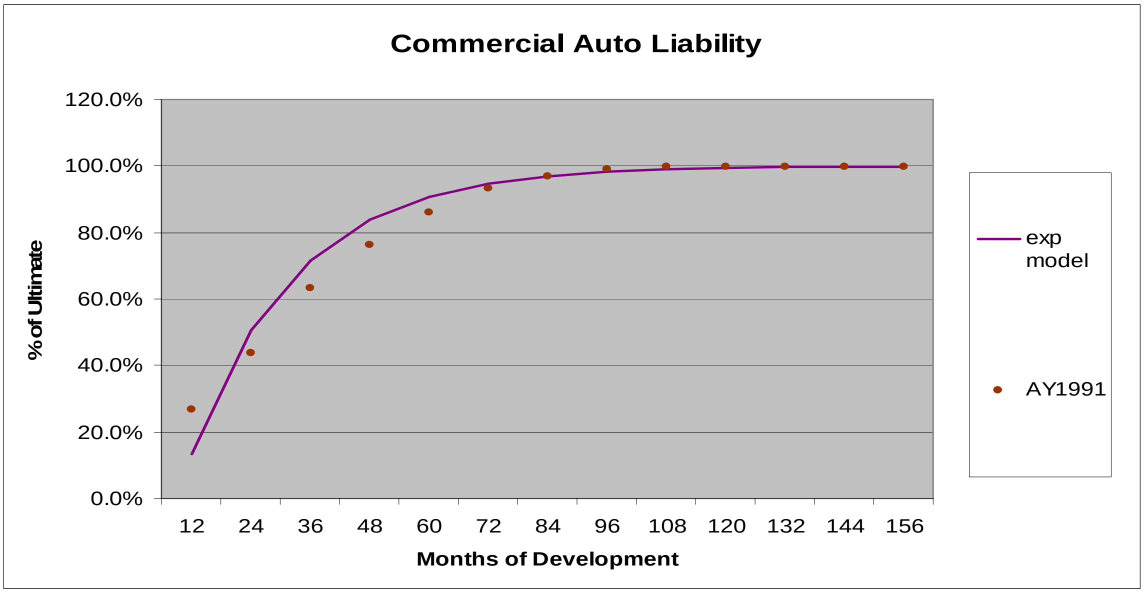

Figure 3.3

If the above model were to be used, we would actually over project our ultimate losses by nearly 200%. This inconsistency in the model's forecasting ability exists across all lines of business.

An additional question that arises with the development of the models is that when applied to 13 different accident years, it results 13 different α value parameters and 13 different β value parameters. The problem then becomes one of deciding which α and β values to use to generate a consistent model for the future.

The Logistic Model

The next model was the logistic model, $f(t) = \dfrac{1}{1+\beta e^{-kt}}$.

LN((1/% of Ultimate)-1) was plotted against months of development. The line of best fit was determined as y = mx+b. To convert from the linear model back to the logistic model, $\beta = e^b$, and k = -m.

For example, the 1989 accident year for Commercial Auto Liability paid data had a line of best fit of y=-0.5767x-0.0165, leading to $f(t) = \dfrac{1}{1+1.0167e^{-.5767t}}$. The following table shows actual percentage development side by side with the projected development.

Months	CAL	Ultimate	AY1989 %	Model %
12	9,582	40,200	23.8%	36.3%
24	19,203	40,200	47.8%	62.0%
36	29,298	40,200	72.9%	77.4%
48	34,830	40,200	86.6%	86.5%
60	37,200	40,200	92.5%	92.0%
72	38,277	40,200	95.2%	95.2%
84	39,325	40,200	97.8%	97.1%
96	39,675	40,200	98.7%	98.3%
108	39,981	40,200	99.5%	99.0%
120	40,046	40,200	99.6%	99.4%
132	40,079	40,200	99.7%	99.6%
144	40,079	40,200	99.7%	99.8%
156	40,080	40,200	99.7%	99.9%

Table 3.2

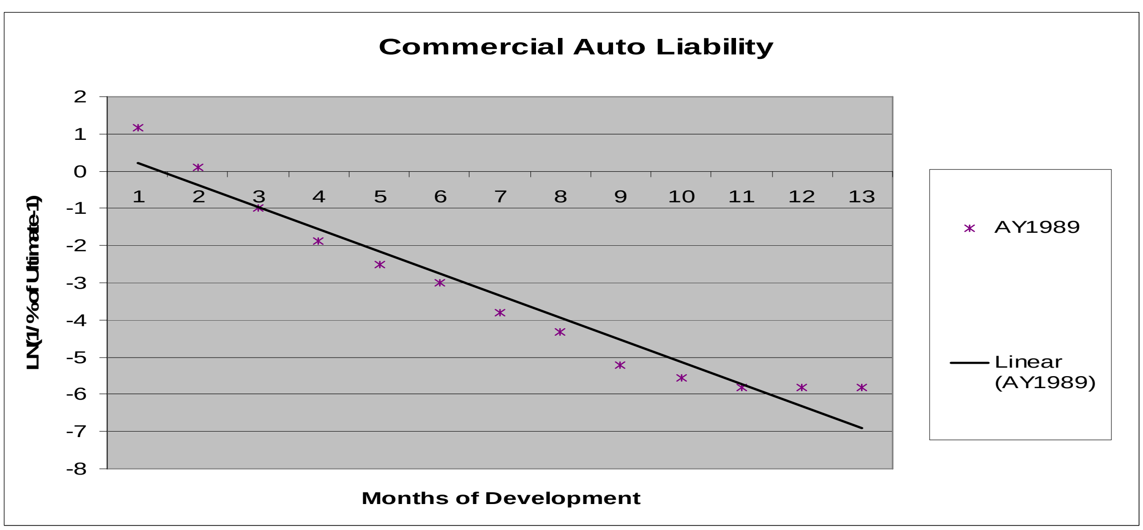

Figure 3.4

In Figure 3.4, the data from the column AY1989 % of Ultimate is shown with the purple stars. The line of best fit is then fit based on this data. In Figure 3.5 below, the % of ultimate for actual data is shown by a dark red dot and the model is shown with a purple line.

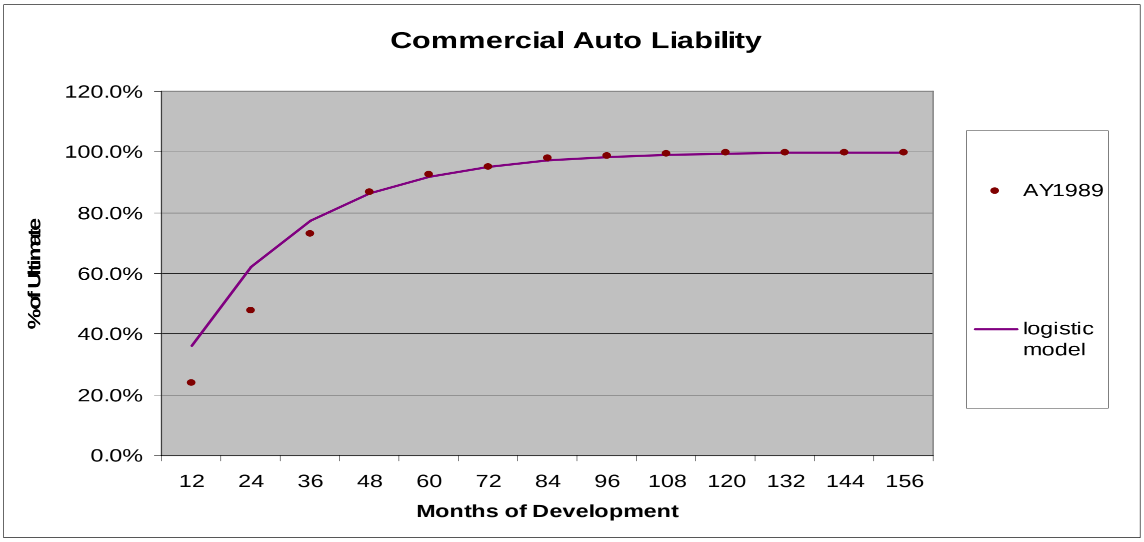

Figure 3.5

This model predicts that the development at 12 months is approximately 36% of the ultimate. The actual data shows it is approximately 23% of the ultimate. However, the majority of the years in the paid loss data could be more consistent. Shown below in Figure 3.6 is the logistic model for the accident year 1992 paid data for Commercial Auto Liability. The model predicts that the 12 month development will be approximately 25% of the ultimate. However, the actual development shows that the 12 month development is approximately 25.4% of the ultimate. Since the goal is to accurately project the ultimate after 12 months of development, this is not a good or useable model.

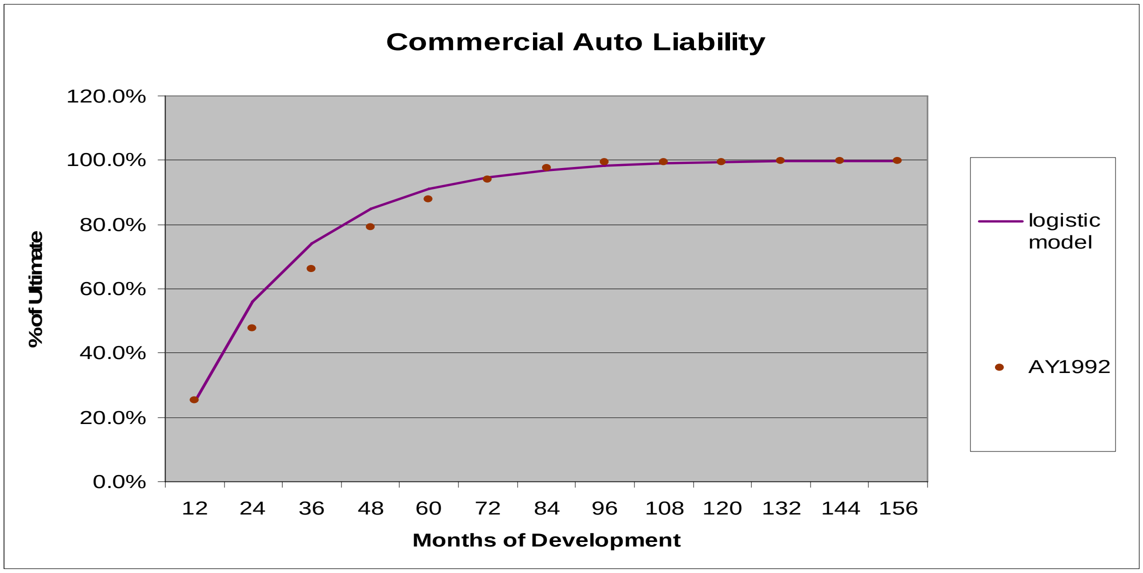

Figure 3.6

b.) Curve Fitting

Another approach was to build a model by looking for a function whose graph was closest to the actual data from the past, and testing it with years that were not fully developed. The formulas of the 3 basic functions, Exponential, Logistic, and Weibull are as follows,

$$f(t) = 1 - a \cdot e^{-bt} \text{ --- Exponential}$$

$$f(t) = \frac{1}{1 + e^{-\frac{t-m}{b}}} \text{ --- Logistic}$$

$$f(t) = 1 - e^{-b \cdot t^r} \text{ --- Weibull}$$

where the t is the development age of the business for a particular Accident Year.

First, the percentages of the paid losses to the ultimate losses the first 12 months and 24 months for CAL were used to calculate a set of values for the two parameters for each function, and then the formulas were used to extrapolate the percentages for later development ages.

CAL	1	2		A	b		m	1/b		b	r
1981	25%	50%		1.1045	0.3933		2.0112	1.0620		0.2939	1.2254
1982	25%	48%		1.0619	0.3529		2.0991	0.9810		0.2928	1.1409
1983	25%	54%		1.2242	0.4890		1.8742	1.2613		0.2867	1.4360
1984	19%	48%		1.2407	0.4301		2.0751	1.3309		0.2144	1.5878
1985	25%	47%		1.0694	0.3489		2.1299	0.9933		0.2818	1.1623
1986	23%	45%		1.0586	0.3238		2.2238	0.9680		0.2669	1.1463
1987	23%	47%		1.1180	0.3760		2.0995	1.0866		0.2645	1.2760
1988	24%	49%		1.1399	0.4027		2.0335	1.1262		0.2718	1.3114

Table 3.3

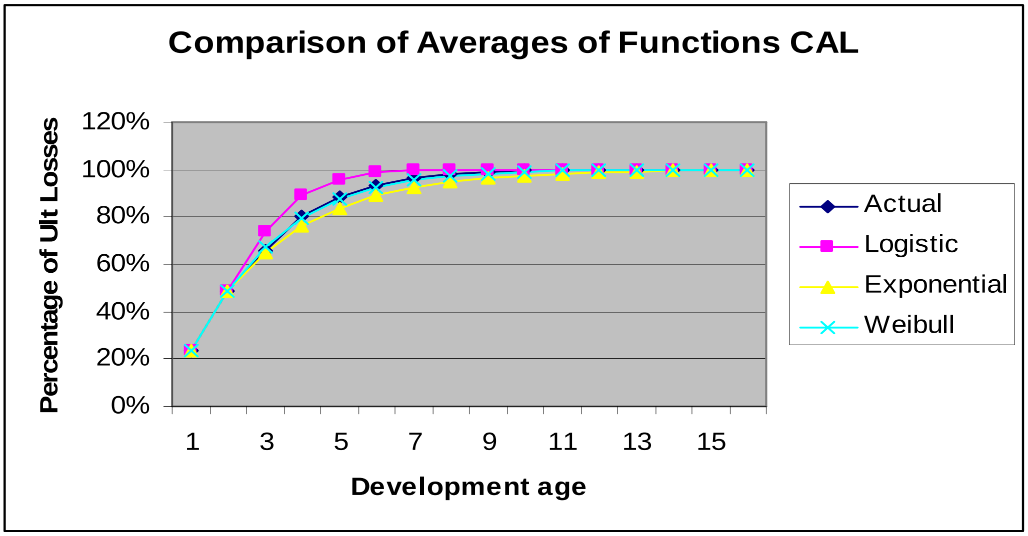

Figure 3.7

The above chart, Figure 3.7 is the average for each development age over eight accident years. From the graph it is clear that the Weibull function fits almost perfectly with the data. Therefore Weibull was selected as a model for CAL.

It was obvious that a new set of parameters was needed for each subsequent year. Further, because paid losses numbers are expressed in real dollars, not in percentages as when they were plotted, a "standard" percentage number of the paid loss to the ultimate loss for the first 12 months is needed to generate forecasts. Looking at the first 12 months data for each year 1981 to 1988, the numbers had a mean 24% with a standard deviation of 2%. Thus 24% was selected as the "standard" first year percentage. The second year percentage number could be obtained once the paid loss for the first 24 months in real dollars was available, $year2\,perc = \frac{\$year2}{\$year1} \cdot 24\%$. From here b, r and the percentage numbers for all the development ages of this accident year could be developed, including the paid losses developing to ultimate, with $\$yearT = \frac{\$year1}{24\%} \cdot yearT\%$. Examples for certain Accident Years are shown in Figure 3.8 below.

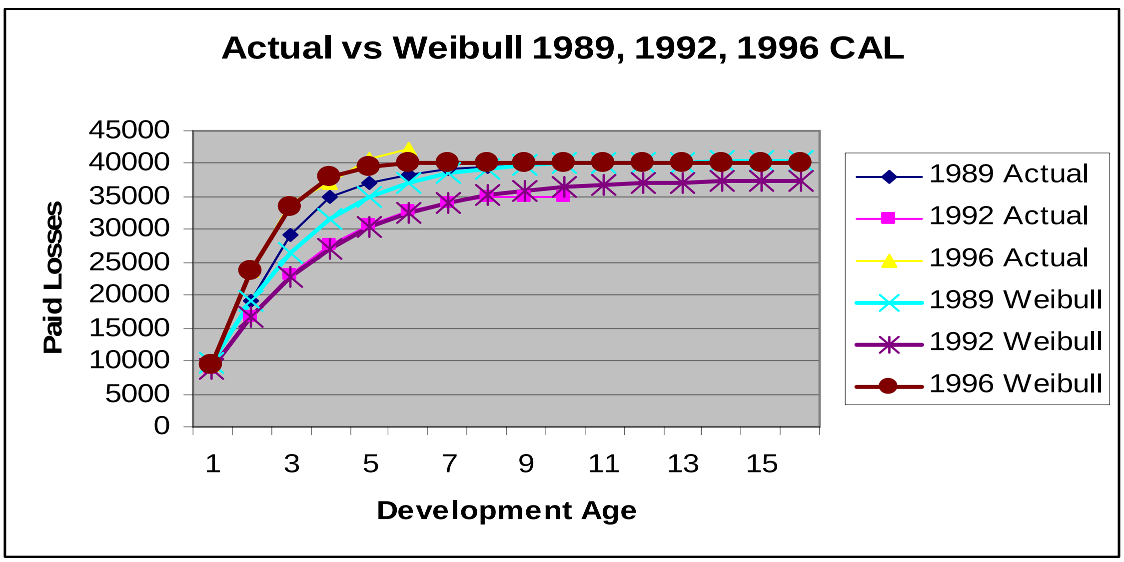

Figure 3.8

The same strategy is used for PABI as with CAL. The parameters were developed, then the averages were extrapolated and compared. However, no one of the three functions worked well for PABI. After taking the average of Logistic and

Exponential, which appeared to be larger than the actual data, the average of Weibull and Exponential seemed to be a good fit for PABI.

PABI	1	2		a	b		m	1/b		b	r
1981	11%	44%		1.4302	0.4731		2.1179	1.8804		0.1153	2.3516
1982	11%	39%		1.2964	0.3768		2.2738	1.6368		0.1172	2.0757
1983	11%	38%		1.2738	0.3628		2.3012	1.5780		0.1207	2.0018
1984	9%	39%		1.3425	0.3904		2.2559	1.8284		0.0959	2.3425
1985	11%	41%		1.3471	0.4139		2.2063	1.7374		0.1160	2.1920
1986	12%	41%		1.3208	0.4007		2.2255	1.6636		0.1224	2.0955
1987	13%	42%		1.3213	0.4143		2.1917	1.6186		0.1357	2.0192
1988	11%	44%		1.4061	0.4605		2.1319	1.8221		0.1197	2.2773

Table 3.4

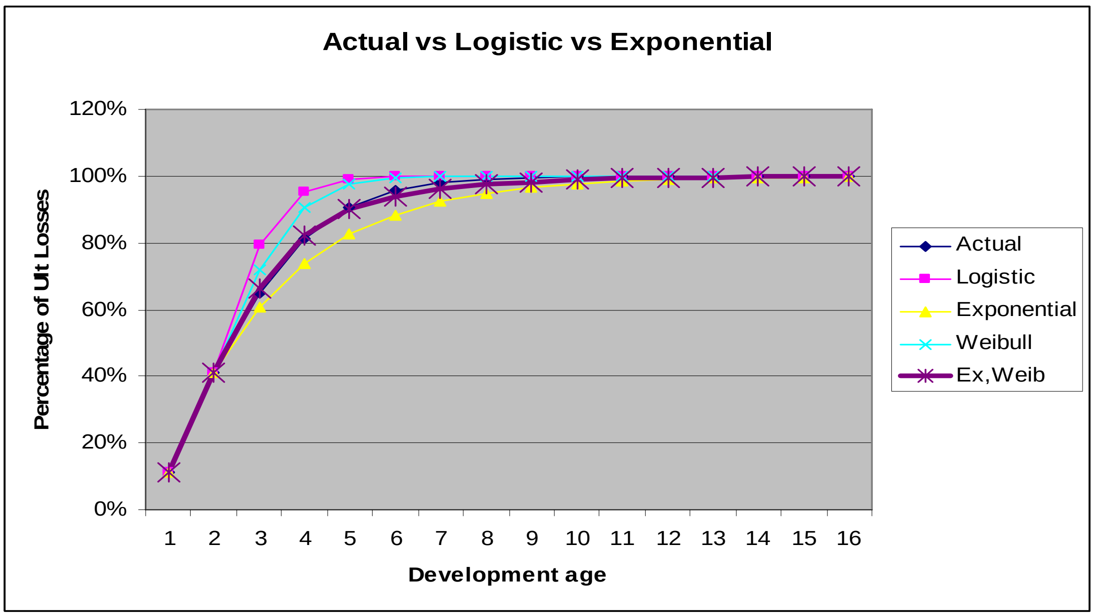

Figure 3.9

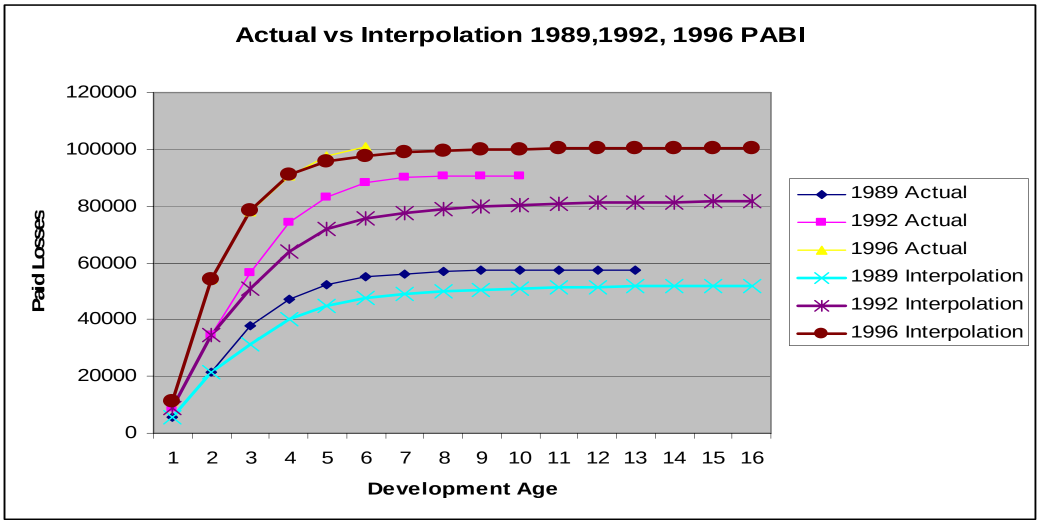

Figure 3.10

The first 12 months percentages had an average of 11% and standard deviation of less than 1%. However, 11% seemed too large for Accident Years 1989 to 1996, as actual paid losses for later years appeared to be much larger than the extrapolated ones. Thus the first year percentage was changed to 10%, leading to better results.

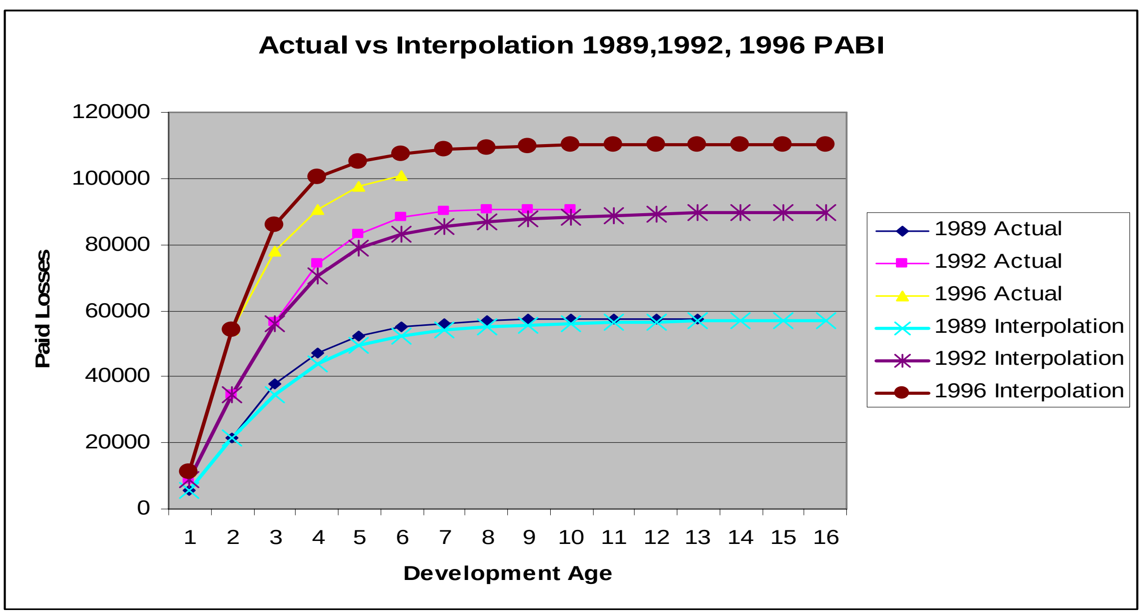

Figure 3.11

After considering three different functions, models whose graphs have physical similarity to the charts for our real data were developed. These models were further adjusted and modified, as discussed in the following section.

c.) Minimizing Residuals

To make the model more accurate, the technique of minimizing residuals was applied to get the “best fit” parameters. A Weibull curve was used for CAL as before, and the problem came down to solving for b, r from

$$\min_{b,r} \sum_{t=1}^{n} \left[y_t - (1 - e^{-b*t^r}) \right]^2$$

Formula 3.1

From Figure 3.12 it can be seen that the extrapolation fits very closely to the actual data. This is an improvement from the extrapolation depending only on the first 2 years of data, which is shown in Figure 3.13. For accident year 1981, the model in the previous section was overstated for most of the development ages, while the current one fits the actual data with the least residuals.

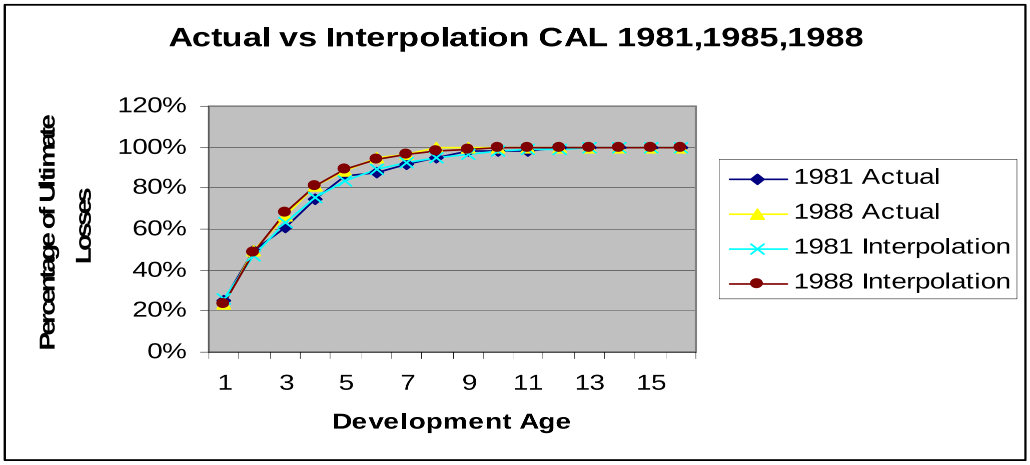

Figure 3.12

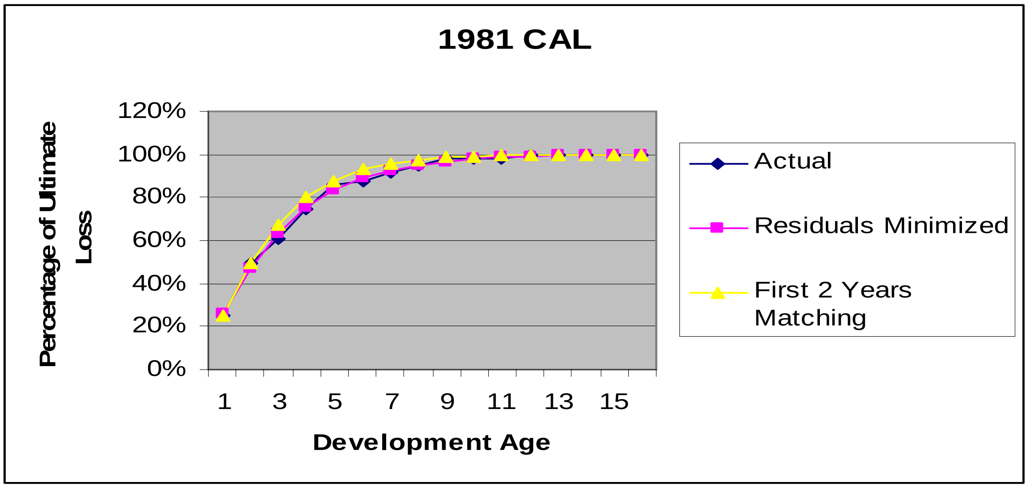

Figure 3.13

The model was then applied to the "not fully developed accident years", 1989 to 2001. The first 12 months paid loss dollar amounts was given, and the residuals for paid loss dollar amounts of ages that have already developed were minimized. The formula is as follows, and in Figure 3.14 this model works better when there are more data available.

$$\min_{b,r} \sum_{t=1}^{k} \left[y_t - \frac{y_1}{(1-e^{-b})} * (1-e^{-b*t^r}) \right]^2$$

Formula 3.2

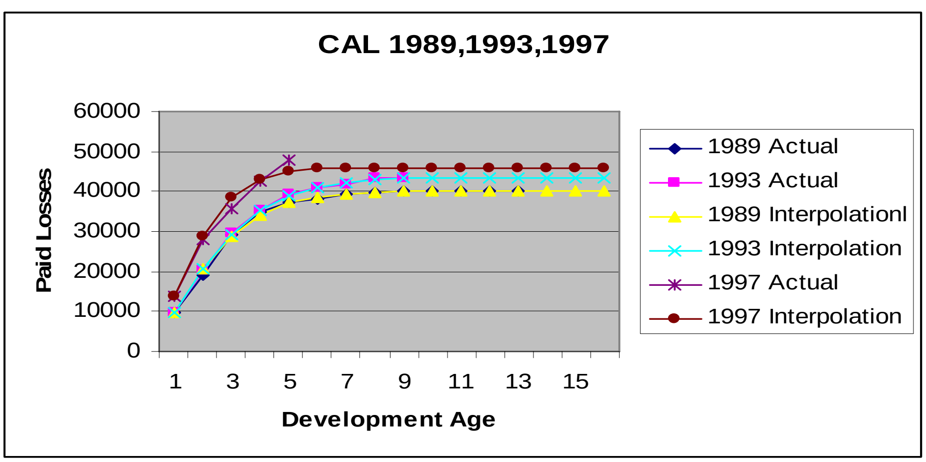

Figure 3.14

The procedure for PABI is similar, as shown in Formula 3.3.

$$\min_{a,b,c,r} \sum_{t=1}^{n} \left[y_t - \frac{(1 - a * e^{-ct}) + (1 - e^{-b*t^r})}{2} \right]^2$$

Formula 3.3

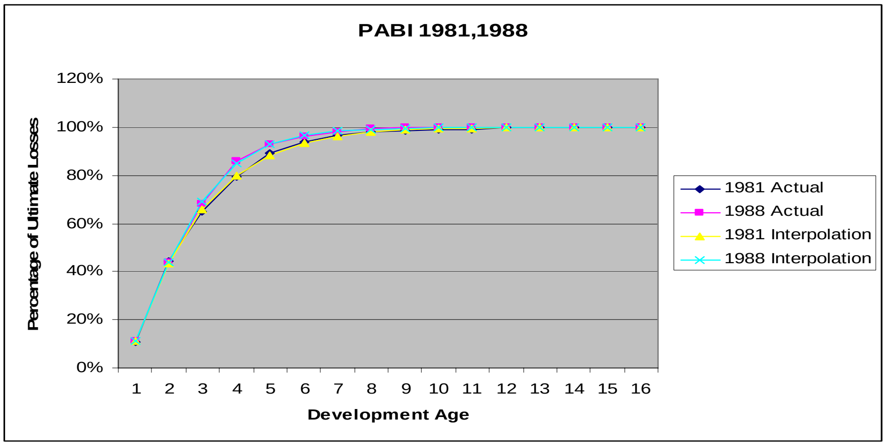

Figure 3.15

Unlike the old model which either stays above or crosses the real data, the new version approximates the actual data very well, which we can see from these graphs.

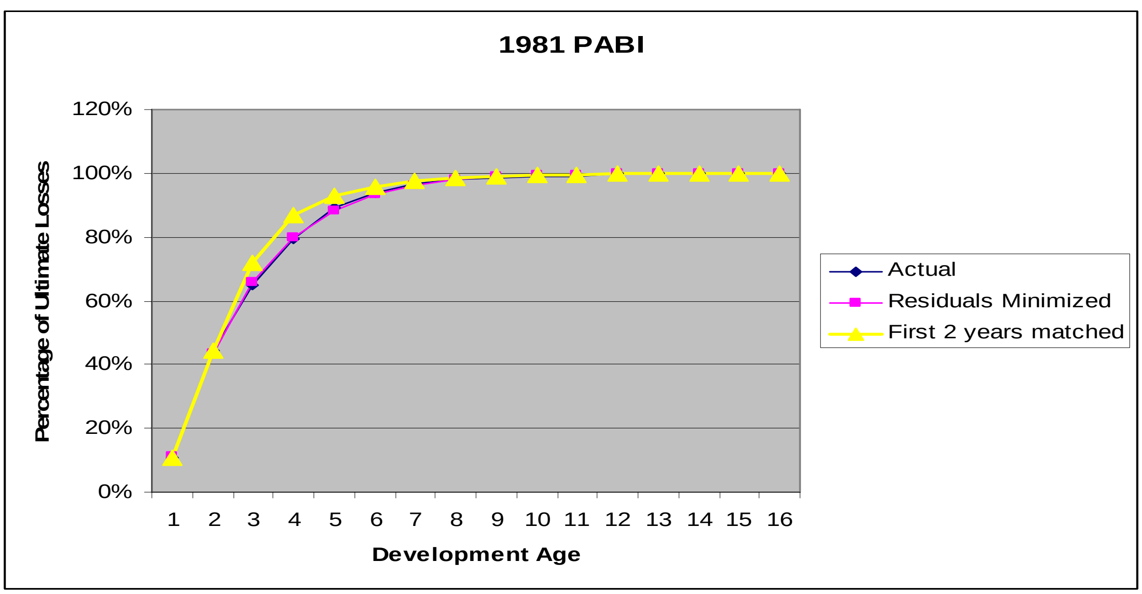

Figure 3.16

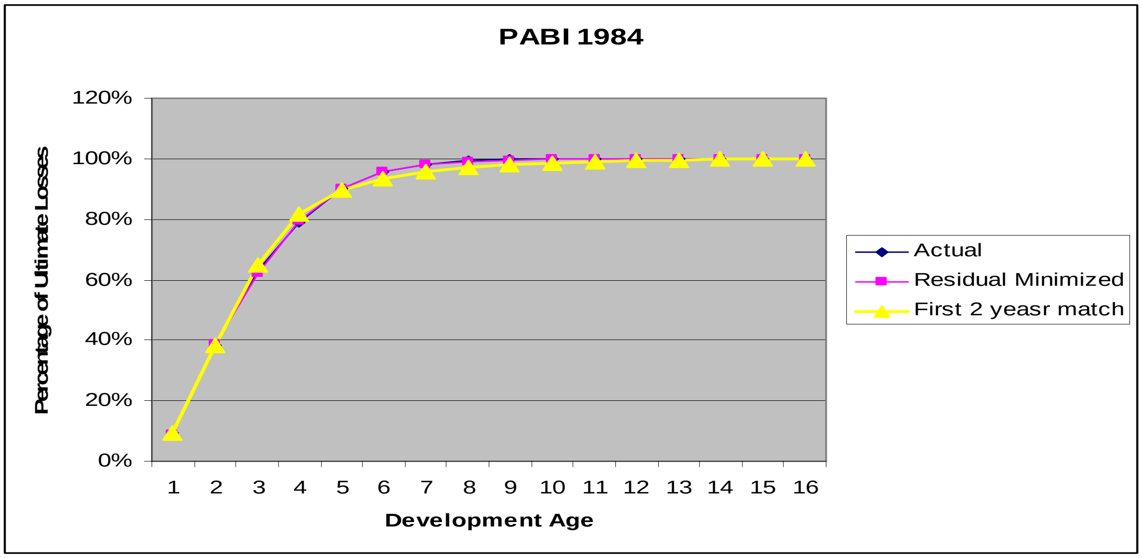

Figure 3.17

Again, the formula for the not fully developed years follows, and the results are very satisfactory as shown in Figure 3.18.

$$\min_{a,b,c,r} \sum_{t=1}^{k} \left[y_t - \frac{y_1}{\frac{(1-a*e^{-c})+(1-e^{-b})}{2}} * \frac{(1-a*e^{-ct})+(1-e^{-b*t^r})}{2} \right]^2$$

Formula 3.4

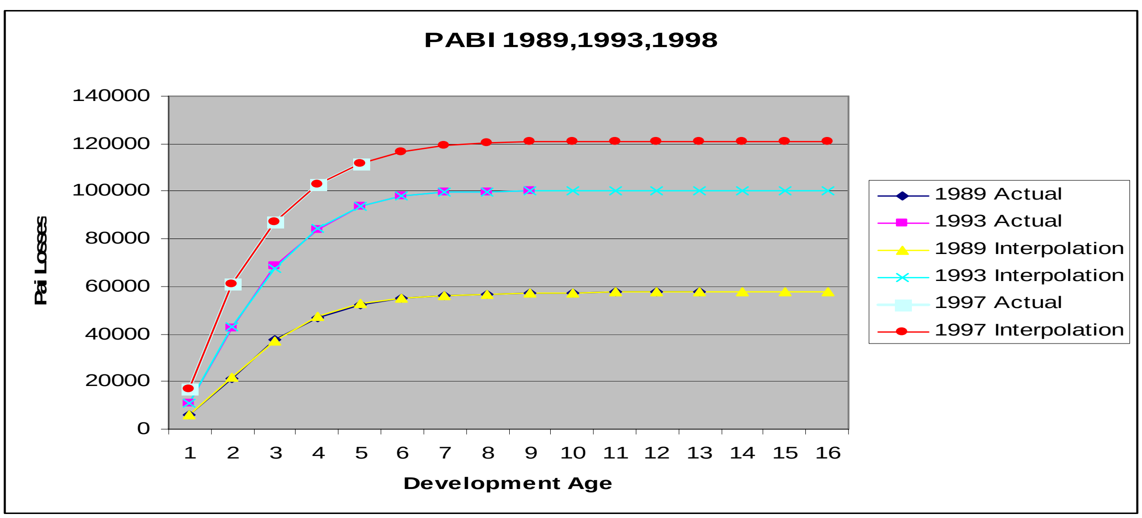

Figure 3.18

The new model greatly improves the accuracy of the interpolation process. However, for accident years that only have 12 or 24 months' development, the model does not produce good results. For this reason, the models in this section would not be useful "real world" tools for projecting Ultimate Loss Reserves.

ii.) The WPI Method

Another approach to developing a new forecasting model is to see if there is a specific combination or adjustment of methods that would result in more accurate predictions, which would be referred to as "The WPI Method".

Accident Year 1997 for the CAL line serves as an example here. The WPI Method is based on each Calendar Year's (CY) Estimated Ultimate Losses for a given Accident Year (AY). These values change with each evaluation period and generally trend toward the Actual Ultimate Losses as of 2006.

The approach was to create a portfolio value from the six basic methods that most closely approximate the actual value as in Formula 1.2. Table 3.5 shows the weights to be applied to the EULs from each methods for each Calendar Year. It indicates that the best combination of the methods includes the Bornheutter-Ferguson Incurred and Paid methods with the weights of about 0.622 and 0.378.

$$\min_{\hat{\beta}} \sum_{k=AY}^{2001} \left(\frac{\sum_{i=1}^{6} {}^{AY}\beta_i \, {}^{AY}_{k}X_i - A_{AY}}{A_{AY}} \right)^2$$

With ${}^{AY}_{k}X_i$ *representing the prediction made by method i for Accident Year AY at a given calendar year k,* A_{AY} *representing the actual losses for Accident Year AY, and* ${}^{AY}\beta_i$ *representing the weight given to a method i for Accident year AY.*

Formula 3.5

Calendar Year	Incurred	Paid	BS Incurred	BS Paid	BHF Incurred	BHF Paid	The WPI Method	Actual (2006)
1997	52700	61600	50600	54100	50900	51300	51100	50700
1998	51500	60500	51000	59400	50500	52800	51400	50700
1999	48800	53100	49400	52500	48800	51500	49800	50700
2000	49500	52000	48700	51700	49500	51600	50300	50700
2001	50900	52600	51000	52600	50900	52400	51500	50700
weights	***0***	***0***	***0***	***0***	***0.622418***	***0.377582***		

Table 3.5

The Scoring System developed earlier was used to compare the WPI Method to the basic methods and the result from the Hanover. Table 3.6 shows the scores for the EULs of Accident Year 1996. The WPI Method outperformed all 6 of the basic methods in the CAL and PABI lines and 5 of 6 in the BOP and PAPD lines. It also gave more accurate predictions than Hanover for all but the CAL line. The WPI Method also worked better than most of the other methods for AY 1997-2001 for most of the four lines. For the 1996 AY, 6 Calendar Years are taken into account when computing the weights given to the basic methods, 1996 through 2001, while for AY 1997, only 5 Calendar Years were being considered. Therefore, the predictions for earlier years are more reliable, as more Calendar Years are taken into account.

	Incurred	Paid	BS Incurred	BS Paid	BHF Incurred	BHF Paid	Hanover's	WPI
CAL	7.12	10.48	5.98	7.25	5.68	4.92	1.68	3.49
BOP	24.11	32.55	17.19	20.95	*15.30*	19.09	17.17	15.33
PAPD	2.48	*0.86*	2.96	1.70	2.56	1.37	1.37	0.88
PABI	7.24	17.70	13.66	8.56	8.73	14.17	6.53	5.70

Table 3.6

The next step was to remove the WPI Method's reliance on knowing the AUL so that it could be used for forecasting. Attempts were carried out to find a pattern in the weights from the developed Accident Years to be applied to future years. Shown in Figure 3.7 are the means and standard deviations of the weights across the Accident Years for each line of business. It is clear that there is very little consistency in the weights. The standard deviations are relatively large, indicating that the mean is not a reliable predictor.

		Incurred	Paid	BS Incurred	BS Paid	BHF Incurred	BHF Paid
BOP	**Mean**	17%	8%	19%	20%	33%	7%
	SD	41%	21%	41%	34%	48%	12%
CAL	**Mean**	11%	14%	22%	4%	36%	14%
	SD	17%	19%	26%	7%	28%	15%
PABI	**Mean**	26%	8%	20%	9%	17%	20%
	SD	40%	15%	23%	12%	33%	20%
PAPD	**Mean**	3%	63%	10%	10%	11%	4%
	SD	4%	40%	25%	9%	27%	8%

Table 3.7

Given the lack of consistency seen in Table 3.7, patterns in the basic methods that were given the highest weights were explored. Results are displayed below in Table 3.8 and Table 3.9 which have the method and the corresponding scores of the highest weighted method.

	CAL	BOP	PAPD	PABI
1996	BSI	BHFI	Paid	Incurred
1997	BHFI	Incurred	BS Paid	BHFI
1998	BHFI	BS Paid	Incurred	Incurred
1999	BSI	BHFP	Paid	BHFP
2000	BHFI	BHFP	Paid	BHFP
2001	Incurred	BSI	Paid	BHFP

Table 3.8

	CAL	BOP	PAPD	PABI
1996	4.92	15.30	0.86	7.24
1997	1.51	15.48	0.39	4.94
1998	4.50	3.22	0.22	4.94
1999	4.94	2.00	0.39	1.61
2000	2.93	14.49	3.11	2.38
2001	0.20	1.33	0.47	4.21

Table 3.9

Two 3-year streaks of consistency, PABI and PAPD 1999-2001 can be seen in the above tables. However, aside from these streaks, there appeared to be no other pattern and the scores for both of the streaks vary greatly. This form of consistency seemed to be coincidental and not sufficient for a conclusion.

The WPI Method is a reasonable way to model existing data as a weighted combination of the six basic methods. Comparisons of the outputs show that the WPI method is an improvement upon the six basic methods, and in many cases, it even outperforms Hanover's prediction. However, the WPI Method's accuracy relies on knowing data that is unavailable when the prediction is being made, and it cannot take into consideration all the information that is available to the actuaries. It did not prove to be a good forecasting method.

IV. Conclusion

The goal of this project was to measure the accuracy the different reserving methods and to develop a method that out-performs all of the existing methods. Development data were provided by the Hanover Insurance Group for accident years 1996 through 2001 for four lines of business.

The percentage errors of the estimations per calendar year by each method were calculated. The weighted average of the percentage errors of all calendar years produced the score for each Accident Year. Scores for the seven methods were ranked for each Accident Year for each line of business. The Hanover method ranks in the top three 60% of the time. By summing the rankings, Hanover's method is shown to be among the best three for each line of business over all accident years. The total rankings for each method over all six accident years and four lines of business indicate that Hanover's selections were the best of the seven.

A weighted average of the estimates from the six basic methods, after minimizing the sum of the errors of all the calendar years for each fully developed accident year for each line of business, may provide a set of weights that could be applied to future years. However, the weights were not consistent enough to make any conclusions. Even for the best line of business, the standard deviations of the weights over the six accident years are as high as 40% (while the weights themselves are between 0% and 100%). The new method does not use all the information that is available to the actuaries. There does not appear to be a way to capture "knowledge and experience" in a mathematical formula.

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