RISK MEASUREMENT OF MORTGAGE-BACKED SECURITY PORTFOLIOS VIA PRINCIPAL COMPONENTS AND REGRESSION ANALYSES

by

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Abstract

Risk measurement of mortgage-backed security portfolios presents a very involved task for analysts and portfolio managers of such investments. A strong predictive econometric model that can account for the variability of these securities in the future would prove a very useful tool for anyone in this financial market sector due to the difficulty of evaluating the risk of mortgage cash flows and prepayment options at the same time.

This project presents two linear regression methods that attempt to explain the risk within these portfolios. The first study involves a principal components analysis on absolute changes in market data to form new sets of uncorrelated variables based on the variability of original data. These principal components then serve as the predictor variables in a principal components regression, where the response variables are the day-to-day changes in the net asset values of three agency mortgage-backed security mutual funds. The independence of each principal component would allow an analyst to reduce the number of observable sets in capturing the risk of these portfolios of fixed income instruments.

The second idea revolves around a simple ordinary least squares regression of the three mortgage funds on the sets of the changes in original daily, weekly and monthly variables. While the correlation among such predictor variables may be very high, the simplicity of utilizing observable market variables is a clear advantage.

The goal of either method was to capture the largest amount of variance in the mortgage-backed portfolios through these econometric models. The main purpose was to reduce the residual variance to less than 10 percent, or to produce at least 90 percent

explanatory power of the original fund variances. The remaining risk could then be attributed to the nonlinear dependence in the changes in these net asset values on the explanatory variables. The primary cause of this nonlinearity is due to the prepayment put option inherent in these securities.

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Chapter 1

Introduction

The risk measurement of mortgage-backed securities and mutual funds in the financial markets present a challenge to every analyst or portfolio manager of these investments. The variability of these debt contracts strays away at times from the usual risks of the government and corporate bond market due to the dependence upon prepayments and curtailments of the collateral backing these securities.

While the typical mortgage-backed security appears to be just a collection of principal and interest cash flows, the ability of the owner of a mortgage to prepay or refinance at current rates has some monetary value, as the holder has the right but not the obligation to exercise this put option. From the viewpoint of the investor of the mortgage-backed security, this option can be considered as a liability, as its exercise would force the prepayment of the loan and an end to a portion of principal included in the investment.

There are five main driving forces behind the pricing of mortgage-backed securities. The first factor that influences the modeling of these investments is seasonality. Prepayments and refinancing occurrences tend to increase during the spring and summer due to the weather and school year. Most agency mortgage-backed deals exhibit the slowest exercise of the prepayment option during February, while the months of May through September initiate higher housing turnover due to refinancing.

The second major influence of prepayments is called burnout, or the decrease in marginal utility to refinance for a homeowner. For example, if one has refinanced a number of times prior to the level of present mortgage rates, the desire to capitalize on new lower rates decreases. The reasons for this sort of behavior include the paperwork and costs involved in each mortgage refinancing and the overall willingness of each homeowner to re-enter the prepayment process.

Seasoning, or aging, is a third factor that moves the mortgage-backed securities' market. After living in a home for a number of years, a family is usually less inclined to move to another home because of stability at the workplace and at local schools. Thus, the age of each mortgage loan plays a dramatic part in the valuation of these investments. The Public Securities Association, or PSA, benchmark reflects this assumption concerning the stability of homeowners in the long run, as the seasoning ramp levels off after the initial 30 months of the mortgage contract. While each mortgage-backed security is a percentage of this PSA index, the relative flattening of this ramp illustrates the normal behavior of homeowners after living in the same shelter for a considerable period of time.

The most important feature in the pricing of securities in the secondary mortgage market is interest rates. The dynamics of these rates allow each analyst to determine the value of these securities under various volatility schemes and possible yield curve paths. Most prepayment and pricing models for mortgage-backed securities are rather sophisticated, as they attempt to consider both the principal and interest payments of the mortgage loan as well as the prepayment option embedded within the contract. This option is not like other options in the fixed income arena, as analysts often try to incorporate the other factors mentioned above and include an OAS, or option-adjusted spread calculation. The main idea is that mortgage rates have a greater bearing than any other market factor, and the other influences are not independent of the present level of these rates.

The fifth force behind the pricing of mortgage-backed securities involves macroeconomic factors. These indicators, normally released on a monthly basis, are sometimes assumed to be already incorporated in mortgage rates, a fair assumption considering a prepayment model must then make a forecast of macroeconomic data 360 months forward. In all, however, these monthly announcements would have some sort of influence on the market for such loans and could have an effect on the valuation process.

As one can observe, mortgage-backed securities are very complicated fixed income instruments. There is no simple implied volatility that can be interpolated as with most derivative contracts and investments, and moreover, the correlation between the volatilities of the mortgage cash flows and this refinancing put option cannot be ignored. This project presents a possible method of measuring the risk related to these securities, a linear approximation toward understanding the movements in mortgage-backed mutual fund portfolios. The principal component and regression analyses contained within this paper can provide a framework for any risky investment, which makes this project very worthwhile.

Chapter 2

The Technical Background

The methodology of this project entails a great deal of theory from linear algebra, multivariate statistics and economic interpretation. As with most applications of quantitative finance, the ideas underlying this thesis can be expressed in a bare mathematical form. One must understand and appreciate the mathematics behind the project to get a real sense of how concepts are determined and to gain a visual image of the process at hand. The following chapter can be considered the crux of the practical implementation of principal components analysis and will be referred to often when dealing with the results of this particular project.

The Financial Framework and Market Data

The financial markets have an enormous amount of indices, benchmarks, time series and other values utilized to measure economic conditions and movements in various sectors. In the fixed income setting, U.S. Treasury securities are seen as the most liquid and risk-free investments of all securities throughout the world and thus most markets use them as the measuring stick for excess returns. The London Inter Bank Offer Rates (LIBOR) are the benchmarks for portfolios of swaps, caps, floors and other debt derivative securities. These two standards are only a small set of the various market variables that drive the forces of interest rate and credit risk management.

The financial time series in this project were selected to reflect the measurement of risk in the agency mortgage-backed securities' market. The variables are broken down into daily, weekly and monthly clusters based on the frequency of availability of the data.

Daily Variables

Nine different financial time series in this project are the daily prices of the U.S. Constant Maturity Treasury Bonds. These offerings, denoted as $P(T)$, have maturities in years denoted as T of 0.25 years, 0.50 years, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years and 20 years. The bonds have no coupon payments, meaning they only have one principal outflow of $100 at the maturity date, and are dependent on the U.S. Treasury rates $r(T)$ in decimal form according to the formula

$$P(T) = 100 * \exp(-r(T) \cdot T) \qquad (2.1).$$

The day-to-day changes in such $P(T)$ are the actual variables used in this thesis.

Two other data sets available with this daily frequency are the Moody's Aaa and Baa Corporate Bond indices. Aaa bonds are of a high credit rating, whereas Baa bonds are still investment grade but less creditworthy. The indices are weighted averages of the rates of all Aaa or Baa loans and are converted into bond prices via *(2.1)* as well, where T is 5 years due to the fact that most corporate securities have maturities of intermediate length. Again, the absolute changes in these samples represent the variables in this project.

The last set of daily data deals with the equity markets, specifically the Standard and Poor's 500 Composite Index. The reason behind including this was to determine if stocks had some sort of influence in the fixed income arena. Because the S&P 500 has no associated rate, the index was scaled down so the maximum value of the vector had a value of 100, and the changes in these transformed values comprised the variable in the study.

The three mortgage-backed security portfolios analyzed in this project were the ING GNMA Income Fund A (LEXNX), the Vanguard GNMA Fund (VFIIX) and the Target Prudential Mortgage-Backed Fund (TGMBX). The net asset values of these funds were released on a daily basis and all had primarily the same investment style, that is, low risk mortgage-backed securities issued by Ginnie Mae, Fannie Mae or Freddie Mac.

Weekly Variables

Certain market indicators are published, and hence only available, on a weekly basis. In the secondary mortgage market, such data are the Freddie Mac Conventional 30-Year Fixed, 15-Year Fixed and 1-Year Adjustable Mortgage Rates and the Fannie Mae Conventional 30-Year Fixed Mortgage Rates. The difference between Freddie Mac and Fannie Mae rates can be attributed to the information sources used to gather and average mortgage rates based on geographic location and cost of living. After converting each of the rates into basis points by multiplying by 100, the absolute changes among these values comprised the weekly portion of the variables in this analysis.

Monthly Variables

A wide range of economic indicators and other monthly released data affect the overall outlook on the economy, far too many for all to be included in a master's thesis. This project looks at seven such vectors which would appear to exert influence on the agency mortgage-backed securities' market.

Total Housing Starts reflect the number of groundbreaking contracts for new homes nationwide, whereas New One Family Houses Sold reflect the figure of original homes sold excluding rental properties, condominium properties and community housing. The financial time series of Commercial Bank Real Estate Loans indicate the dollar value of loans created by banks, credit unions and other savings and loan institutions for living properties.

Other economic factors have a direct effect on the entire economy. Civilian Employment is a count of the number of U.S. citizens of 16-years and older which make up the labor force. The Unemployment Rate exhibits the ratio of the citizens in this age group which do not have jobs to the entire population. The Producer Price Index (PPI) describes the inflationary aspect of the economy as a function of other market variables. Finally, the number of Total Pools of Securitized Assets gives the composite market value of collateral involved with mortgage-backed securities, asset-backed securities and other such debt obligations in billions.

Variable Changes

Principal components analysis is deemed to be most suitable when measuring absolute or relative changes in market variables. Thus, if one considers *n* variables and denotes each variable as $\boldsymbol{X_j}$ for $j = 1,\ldots,n$, one can compute the absolute change in value i of data set j as

$$(\Delta X_i)_j = (X_i - X_{i-1})_j \qquad (2.2)$$

and the relative change as

$$\frac{(\Delta X_i)_j}{(X_{i-1})_j} = \frac{(X_i - X_{i-1})_j}{(X_{i-1})_j} \qquad (2.3).$$

These two figures are more comparable when looking at data with different units, and create a more stable source for estimation and drawing conclusions from the numerous market factors. For the purposes of this project, the absolute changes are the primary concern.

<u>The Statistical Framework</u>

For this project, one can assume the market factors $\widetilde{X}_{ij}$ to be identically distributed normal random variables, thus making the $(\Delta\widetilde{X}_i)_j$ independent and identically distributed normal random variables for any fixed j with zero mean.

Suppose each X_{ij} is an observation of a random variable $\widetilde{X}_{ij}$. Now, the $(\Delta X_i)_j$ are $i = 1,\ldots,N$ observations of $j = 1,\ldots,n$ sample sets. The population mean μ_{ij}, or expected value, of each $(\Delta X_i)_j$ would be computed as

$$E((\Delta X_i)_j) = E((X_i - X_{i-1})_j) = \mu - \mu = 0 \qquad (2.4).$$

In addition, the population mean μ_j of the sample set $(\Delta \mathbf{X})_j$ is recognized as

$$E((\Delta \mathbf{X})_j) = \left(\frac{1}{N}\right)\sum_{i=1}^{N} E((\Delta X_i)_j) = 0 \qquad (2.5).$$

However, the sample parameter estimates are not always the same. For instance, the sample mean denoted as $\overline{(\Delta X_i)_j}$ of each $(\Delta X_i)_j$ follows the equation

$$E((\Delta X_i)_j) = E((X_i - X_{i-1})_j) = \overline{X_{ij}} - \overline{X_{i-1j}} \qquad (2.6)$$

which need not equal zero, but is undoubtedly very small. Thus, the sample mean $\overline{(\Delta X)_j}$ of the set $(\Delta \mathbf{X})_j$ may vary from its population value and is calculated as

$$E((\Delta \mathbf{X})_j) = \left(\frac{1}{N}\right)\sum_{i=1}^{N} E((\Delta X_i)_j) = \left(\frac{1}{N}\right)\sum_{i=1}^{N} (\overline{X_{ij}} - \overline{X_{i-1j}}) \qquad (2.7).$$

Next, from elementary multivariate statistical analysis, the sample covariance of two such vectors is equal to

$$Cov\,[(\Delta \mathbf{X})_k(\Delta \mathbf{X})_l] = \left(\frac{1}{N-1}\right)\sum_{i=1}^{N} E((\Delta \mathbf{X})_k - \overline{(\Delta X)_k}))((\Delta \mathbf{X})_l - \overline{(\Delta X)_l})) = s_{kl} \qquad (2.8).$$

The *nxn* sample covariance matrix is denoted as $\sum = (\Delta \mathbf{X})'(\Delta \mathbf{X})$, where $(\Delta \mathbf{X})$ is an *Nxn* matrix consisting of all data set columns $(\Delta \mathbf{X})_j$.

Another well-known fact in probability states that the sample standard deviation of any market data set is

$$Sd\,[(\Delta \mathbf{X})_j] = \sqrt{\left(\frac{1}{N-1}\right)\sum_{i=1}^{N} E((\Delta X_j) - \overline{(\Delta X_j)})^2} = s_j \qquad (2.9),$$

and the squared standard deviation, or sample variance, is

$$Var\,[(\Delta \mathbf{X})_j] = \left(\frac{1}{N-1}\right)\sum_{i=1}^{N} E((\Delta \mathbf{X})_j - \overline{(\Delta X)_j}))^2 = s_j^2 = s_{jj} \qquad (2.10).$$

The sample correlation coefficient between any two data vectors *k* and *l* equals

$$Corr[(\Delta \mathbf{X})_k(\Delta \mathbf{X})_l] = \left(\frac{s_{kl}}{s_k s_l}\right) = r_{kl} \qquad (2.11).$$

The statistical definitions in this section shall serve as a guideline for the computations done to perform principal components analysis. The values of these parameters give quantitative and analytical parameters to economic and financial time series data.

The Geometric Framework

Assume the variables described above are equivalent to n column vectors $(\Delta \boldsymbol{X})_j$, each with N components. The entries of each of the j vectors will be denoted as $(\Delta X_i)_j = (X_i - X_{i-1})_j$ for $i = 1,\ldots,N$ and $j = 1,\ldots,n$ in the same manner as the previous section. A well-known definition from linear algebra states that the dot product of two column vectors k and l is given by

$$(\Delta \boldsymbol{X})_k \cdot (\Delta \boldsymbol{X})_l = \sum_{i=1}^{N} (\Delta X_i)_k (\Delta X_i)_l \qquad (2.12)$$

Again, from elementary notions, the norm, or length, of any column vector is computed as

$$\| (\Delta \boldsymbol{X})_j \| = \sqrt{\sum_{i=1}^{N} (\Delta X_i)_j^{\,2}} \qquad (2.13)$$

A result of these theorems illustrate that the angle θ between two vectors k and l is defined by

$$COS\, \theta = \left(\frac{(\Delta X)_k (\Delta X)_l}{\|(\Delta X)_k\| \|(\Delta X)_l\|} \right) \qquad (2.14).$$

The prior equations indicate that the statistical measures have a direct relationship with mathematical derivations. The covariance matrix $\sum$ holds entries of all sample covariances s_{kl} from *(2.8)*, which are algebraic dot products by way of *(2.12)*. From

(2.13), the norm of any vector can be realized as the standard deviation s_j and the squared length as the variance $s_j^2 = s_{jj}$ as illustrated in *(2.9)* and *(2.10)*. Finally, the $COS\ \theta$ under the circumstances given by *(2.14)* is the identical to the correlation coefficient r_{kl} between any two vectors from *(2.11)*.

The Geometric Interpretation of Principal Components Analysis

In linear algebraic terms, the first principal component is defined as an $Nx1$ column vector $\boldsymbol{y_1} = \boldsymbol{(\Delta X)e_1}$, where $\boldsymbol{e_1}$ is the eigenvector corresponding to the first most dominant, or largest, eigenvalue λ_1 of the nxn matrix $\sum$ containing the dot products of each of the original $\boldsymbol{(\Delta X)_j}$. It is a new vector calculated as the sum of a set of original vectors scaled by the components of the eigenvector corresponding to the largest eigenvalue of $\sum$, where the eigenvector $\boldsymbol{e_1}$ has unit length. The second principal component $\boldsymbol{y_2}$ deals with the unit-length eigenvector based on the next largest eigenvalue such that the two principal component vectors are orthogonal, and so on.

In addition, for any principal component j, one can identify that

$$Var(\boldsymbol{y_j}) = Var((\boldsymbol{\Delta X})\ \boldsymbol{e_j}) = \boldsymbol{e_j}' \sum \boldsymbol{e_j} = \lambda_j \qquad (2.15).$$

The jth principal component is an $Nx1$ column vector

$$\boldsymbol{y_j} = \boldsymbol{(\Delta X)e_j} \qquad (2.16),$$

where $\boldsymbol{e_j}$ is the jth eigenvector corresponding to the jth most dominant, or largest, eigenvalue λ_j of $\sum$. Again, the $nx1$ eigenvector $\boldsymbol{e_j}$ has unit length. In all, there are $j = 1,\ldots,n$ eigenvalue-eigenvector pairs. Principal components analysis holds the characteristic that

$$\sum_{j=1}^{n} \lambda_j = trace(\Sigma) \qquad (2.17),$$

that is, the sum of the eigenvalues exactly equal the sum of the entries on the diagonal of the dot product matrix.

In addition, another result from linear algebra illustrates that the angle between any two principal component vectors $\boldsymbol{y_k}$ and $\boldsymbol{y_l}$ is defined by

$$COS\,\theta = \left(\frac{y_k \cdot y_l}{\|y_k\| \|y_l\|} \right) = 0 \qquad (2.18),$$

and so $\theta = 90°$. Thus, any two principal component vectors $\boldsymbol{y_k}$ and $\boldsymbol{y_l}$ are orthogonal.

In all, the space Y spanned by all of the $\boldsymbol{y_j}$ contains an ellipsoid with axes equal to

$$\pm c\sqrt{\lambda_j}\,e_j \qquad (2.19),$$

where the equation of this ellipsoid is

$$c^2 = \sum_{j=1}^{n} \left(\frac{1}{\lambda_j} \right) \boldsymbol{y_j}^2 \qquad (2.20).$$

Hence, the major axes of this ellipsoid are determined by the most dominant or largest principal component vectors.

<u>The Statistical Interpretation of Principal Components Analysis</u>

The *j*th principal component denoted as $\boldsymbol{y_j}$ is the single random variable vector of *N* components which explains the *j*th most variance of any covariance matrix $\sum$. The components of the associated eigenvector of each $\boldsymbol{y_j}$ are called the factor scores multiplied on each of the original data sets. A theorem from multivariate statistics states that the variance of any principal component $\boldsymbol{y_j}$ equals λ_j, and from *(2.17)*, the sum of the diagonal terms along $\sum$ equal the sum of the variances of the principal components. It also follows that for any two vectors $\boldsymbol{y_k}$ and $\boldsymbol{y_l}$, the *COS θ* between them is zero as exhibited by *(2.18)*, and hence

$$s_{kl} = r_{kl} = 0 \qquad (2.21),$$

showing that all principal components have zero covariance and are thus uncorrelated.

Next, assuming the $\widetilde{X}_{ij}$ are indeed identically distributed $N_n\ (\boldsymbol{\mu},\sum)$, a definition from multivariate statistics states that the density of these normal random variables is constant on $\boldsymbol{\mu}$-centered ellipsoids of the form

$$c^2 = ((\Delta X)_i - \mu') \textstyle\sum^{-1}((\Delta X)_i' - \mu) \qquad (2.22),$$

where $(\Delta X)_i$ is a $1xn$ row vector containing the ith observation of all n sample sets. The case when μ is an $nx1$ zero vector can easily be applied. The lengths of the axes of these constant density ellipsoids are given by *(2.19)*.

The Financial Interpretation of Principal Components Analysis

The main concern with principal components analysis in every realm remains the problem of interpreting the vectors in the real world. Since the effect of the statistical technique is a transformation of the changes of original observed variables, the understanding of the results is unclear.

Certainly, one can identify the first principal component as the most volatile source of uncertainty in a collection of data sets. In doing so, one can claim that the first principal component contains the most risk. Since the variance of the first principal component exactly equals its eigenvalue, one can deduce the volatility of this vector of changes as the square root of the eigenvalue. By calculating all principal components and their variances, one has captured the total variability of the financial time series.

When dealing with the changes of specific market securities, the interpretation of the principal component vectors is somewhat easier to understand. For n original market investments, the first principal component y_1 is the weighted portfolio of the changes in these n securities that captures the most risk such that the sum of the squares of these weights equal one. The following principal components are again portfolios of proportions of the changes in the original securities with the constraint that each $\boldsymbol{y_j}$ portfolio remains uncorrelated with its predecessors.

Another advantage of principal components analysis deals with data reduction. One can specify in advance the percentage of variation desired to be captured by an unknown $p \leq n$ principal components and focus on this subset of data. In doing so, one has determined p variables which reflect the greatest source of volatility of a more extensive set of original observations. The proportion of overall variance of an original data set that can be explained by $p \leq n$ principal components is governed by

$$\textit{percentage of variability captured} = \frac{\sum_{j=1}^{p} \lambda_j}{\sum_{j=1}^{n} \lambda_j} \qquad (2.23).$$

However, the greatest benefit of performing this statistical method lies within the uncorrelated variables. The problem at the root of least-squares regression techniques remains the explanatory power of predictor variables that exhibit multicollinearity. Multicollinearity is a term used to describe the case of highly correlated independent variables. This phenomenon is mostly associated with large variances and standard errors for ordinary least-squares estimates, and consequently higher confidence intervals and insignificant t-ratios. By finding the eigenvalue-eigenvector pairs of an original sample of observations, one can then run principal components regression on the new set of transformed, uncorrelated variables as predictors for an outside response variable. In this project, the dependent data will consist of the absolute changes in net asset values of agency mortgage-backed mutual funds, which will be denoted as ***m***. If a subset $p \leq n$ of principal components claims a great deal of the total variability in ***m***, one may then utilize these ***p*** predictors instead of the entire set of ***n*** variables.

The Financial Interpretation of Principal Components Regression Coefficients

The principal components regression mentioned in the previous paragraph deals with figuring out the betas, denoted by β_j, of the mortgage-backed fund as the response variable and the principal components as the predictor variables. The estimates can be calculated via the formula

$$\beta_j = \frac{Cov(m, y_j)}{s_j^2} \qquad (2.24),$$

for $j = 1,\ldots, p \leq n$ principal components.

In financial literature, betas are usually evaluated in relation to some reference index portfolios, such as the S&P 500 or Russell 2000. In the current context, β_1 is the absolute linear sensitivity of the changes in a mortgage-backed security fund with respect to the riskiest possible portfolio that can be composed of the original market variables. In other words, the first principal component portfolio plays the role of an index. The later principal components serve as indices as well with the added requirement that these portfolios be uncorrelated to all previous ones.

Running the principal components regression shall depict which portfolios of the changes in the original data have the strongest influence on the movement of the mortgage-backed portfolio. And with a little luck, the majority of the predictor variables will have significant least-squares estimates and a high value of explanatory power.

The Statistical Interpretation of Principal Components Regression Coefficients

The prior formula for β_j is part of the regression equation

$$\boldsymbol{m} = \beta_1 \boldsymbol{y_1} + \beta_2 \boldsymbol{y_2} + \ldots + \beta_n \boldsymbol{y_n} + \boldsymbol{r}$$

$$= \sum_{j=1}^{n} \beta_j y_j + \boldsymbol{r} \qquad (2.25)$$

where r is the residual data set or unexplained portion of the response variable $\boldsymbol{m}$. In the case of any one principal component, it can be verified that

$$\boldsymbol{m} = \beta_1 \boldsymbol{y_1} + \boldsymbol{r} \qquad (2.26),$$

and the regression equation on the standardized principal component vectors as the predictor variables is

$$\boldsymbol{m} = \sum_{j=1}^{n} \beta_j y_j + \boldsymbol{r} = \sum_{j=1}^{n} \frac{Cov(m, y_j)}{s_j^2} y_j + \boldsymbol{r} = \sum_{j=1}^{n} \left(\frac{Corr(m, y_j) s_m}{s_j} \right) \left(\frac{y_j}{s_j} \right) + \boldsymbol{r} \qquad (2.27).$$

The residual data set of the principal components regression can be calculated from the regression equation as

$$\boldsymbol{r} = \boldsymbol{m} - \sum_{j=1}^{n} \beta_j y_j \qquad (2.28).$$

The residual has an important place in the study, as one wants to find the R-Squared value of the mortgage fund on the predictor variables. The R-Squared value of the mortgage fund is denoted as R^2 and can be computed by

$$R^2 = \left(\frac{s_m^2 - s_r^2}{s_m^2} \right) = 1 - \left(\frac{s_r^2}{s_m^2} \right) \qquad (2.29),$$

which is the amount of variance of the data set $\boldsymbol{m}$ explained by the principal component predictor variables, where s_r and s_m are the standard deviations of the residual data set and response variable, respectively. It is always true that using all n principal components in a study will yield the same R-Squared value as that of an ordinary least-squares regression. The advantage of principal components regression lies within the fact that $p \leq n$ principal components may indeed claim most of this variability, and hence reduce the number of predictor variables in any analysis.

The ordinary least-squares regression gamma coefficients can easily be computed after having prior knowledge of the principal components regression parameters. The regression equation on the original variable changes is

$$\boldsymbol{m} = \gamma_1(\boldsymbol{\Delta X})_1 + \gamma_2(\boldsymbol{\Delta X})_2 + \ldots + \gamma_n(\boldsymbol{\Delta X})_n + \boldsymbol{r}$$

$$= \sum_{j=1}^{n} \gamma_j (\Delta X)_j + \boldsymbol{r} \qquad (2.30).$$

One can obtain this *nx1* set of best linear unbiased estimates by taking

$$\boldsymbol{\gamma} = \boldsymbol{e\beta} \qquad (2.31),$$

where $\boldsymbol{e}$ is the *nxn* matrix whose columns contain the factor scores dealing with each principal component and $\boldsymbol{\beta}$ is the *nx1* set of principal components regression coefficients defined earlier. While this equation does not omit the multicollinearity of the changes in the original variables, it does form a linear model that incorporates variables that are present in the financial markets.

The Geometric Interpretation of Principal Components Regression Coefficients

Because the $\boldsymbol{y_j}$ form an orthogonal basis for a space Y of the larger space N, linear algebra defines the vector projection of any other vector $\boldsymbol{m}$ in N-space onto any standardized $\boldsymbol{y_j}$ is defined by

$$proj_j\, \boldsymbol{m} = \left(\frac{m \cdot y_j}{\|y_j\|}\right)\left(\frac{y_j}{\|y_j\|}\right) \qquad (2.32)$$

Since the standardized $\boldsymbol{y_j}$ are independent, the vector projection of any vector m in N-space onto the subspace Y is given by

$$proj_{\,Y}\,\boldsymbol{m} = \left(\frac{m \cdot y_1}{\|y_1\|}\right)\left(\frac{y_1}{\|y_1\|}\right) + \ldots + \left(\frac{m \cdot y_n}{\|y_n\|}\right)\left(\frac{y_n}{\|y_n\|}\right)$$

$$= \sum_{j=1}^{n}\left(\frac{m \cdot y_j}{\|y_j\|}\right)\left(\frac{y_j}{\|y_j\|}\right) \qquad \textit{(2.33)}$$

The scalar projection of ***m*** onto any one standardized principal component vector can be calculated as

$$sproj_{\,\boldsymbol{j}}\,\boldsymbol{m} = \left(\frac{m \cdot y_j}{\|y_j\|}\right) \qquad \textit{(2.34)}$$

or simply the dot product between the mortgage-backed portfolio ***m*** and principal component vector $\boldsymbol{y_j}$ divided by the length of the latter. Thus, from previous results *(2.33), (2.27),* and *(2.24),* one can notice that

$$proj_{\,Y}\,\boldsymbol{m} - \sum_{j=1}^{n}\left(\frac{m \cdot y_j}{\|y_j\|}\right)\left(\frac{y_j}{\|y_j\|}\right) = proj_{\,Y}\,\boldsymbol{m} - \sum_{j=1}^{n}\left(\frac{m \cdot y_j}{\|y_j\|^2}\right)(y_j)$$

$$= \boldsymbol{m} - \sum_{j=1}^{n}\frac{Cov(m, y_j)}{s_j^2}y_j = \boldsymbol{m} - \sum_{j=1}^{n}\beta_j y_j = \boldsymbol{r} \qquad \textit{(2.35),}$$

or the residual vector after running a principal components regression. This residual vector can be viewed upon as the orthogonal complement to the projection of ***m*** onto the principal component subspace Y. From *(2.29)*,

$$R^2 = \left(\frac{s_m^2 - s_r^2}{s_m^2}\right) = 1 - \left(\frac{s_r^2}{s_m^2}\right) = 1 - \left(\frac{\|r\|^2}{\|m\|^2}\right) = \left(\frac{\|m\|^2 - \|r\|^2}{\|m\|^2}\right) \qquad (2.36),$$

a ratio dealing with the squared lengths of vectors ***m*** and ***r***.

The major concern with principal components regression draws upon the fact that using all of these uncorrelated transformed variables can only explain the same amount of residual variance. However, using a subset $p \leq n$ of principal components could produce strong approximation to the R-Squared value given by the total number of these orthogonal vectors.

Chapter 3

The Principal Components and Regression Analyses Results

The determination of which factors explain the variability of net asset values of agency mortgage-backed funds is not straightforward due to the prepayment option embedded in these securities from homeowner refinancing behavior. Thus, since mortgage funds are actually portfolios of coupon bonds and short put options, a linear econometric model is only an approximation. Nonetheless, the results given by the principal components analysis and regression are noteworthy and serve as a strong guideline to risk measurement of such portfolios.

The Variable Definitions

As described previously, the daily, weekly and monthly data were separated into three corresponding groups. The variables will be defined in this study as indicators influencing the variance of the LEXNX, VFIIX and TGMBX mutual funds. The chart below displays the market variables and its associated variable indicator in order to simplify the analysis.

Variables and Variable Indicators

Variable	Indicator
3-Month Treasury Constant Maturity Bond	1
6-Month Treasury Constant Maturity Bond	2
1-Year Treasury Constant Maturity Bond	3
2-Year Treasury Constant Maturity Bond	4
3-Year Treasury Constant Maturity Bond	5
5-Year Treasury Constant Maturity Bond	6
7-Year Treasury Constant Maturity Bond	7
10-Year Treasury Constant Maturity Bond	8
20-Year Treasury Constant Maturity Bond	9
Moody's Aaa Corporate Index Bond	10
Moody's Baa Corporate Index Bond	11
S&P 500 Index Daily Close	12
Freddie Mac 30-Year Fixed Conventional Mortgage Rate	13
Freddie Mac 15-Year Fixed Conventional Mortgage Rate	14
Freddie Mac 1-Year Adjustable Conventional Mortgage Rate	15
Fannie Mae 30-Year Fixed Conventional Mortgage Rate	16
Housing Starts: Total	17
New One Family Houses Sold	18
Commercial Bank Real Estate Loans	19
Civilian Employment	20
Unemployment Rate	21
Producer Price Index	22
Total Pools of Securitized Assets	23

From here on, one can refer to the indicator number instead of the variable name so as to make it easier to refer to them. In all, indicators 1-9 represent the Treasury Bonds, 10-11 the Corporate Index Bonds, 12 the S&P 500, 13-16 the Mortgage Rates, and 17-23 the Monthly Economic Indicators. Thus, indicators 1-12 represent all of the daily data.

The next step of the project was to decipher the variability explained by three groups of daily data. First of all, one wants to calculate the explanation of variance from indicators 1-9, which could be characterized as a Treasury market risk effect. The second partition involved indicators 1-11, and the additional explanatory power given by the two new variables could be interpreted as a corporate credit risk effect. Finally, by observing the results from indicators 1-12, one could discover if equity securities moved the mortgage-backed funds to some degree.

The residual variance that was left over from the daily data could be explained by the weekly variables. Because of this, one could use the residual data and engage in another analysis to reduce variability even further. If at any time during the analysis the residual variance was smaller than 10 percent of the original agency mortgage-backed fund variance, the analysis would be complete, as the short put options embedded in this portfolio would hold the remaining risk.

The subroutine written in Visual Basic for Applications in Microsoft Excel took a number of outside inputs into account as illustrated in the following chart.

Inputs

Number Of Daily Variables (9=Treasuries, 11=+Corporates, 12=+S&P 500)	**9**
Final Period Ending Date	**Wednesday, February 09, 2000**
Final Period Ending Date Serial Number	**36565**
Data Sample Size In Days (20=monthly, 60=quarterly, 120=semiannually)	**60**
Error Tolerance	**1.00E-20**
Weekly To Daily Variance Conversion Factor In Days	**10**

One could enter the number of daily variables to be used in the analysis in the top input cell, whether indicators 1-9, 1-11 or 1-12. The final period ending date is the ending date of the last time window used in the analysis. The subroutine runs on six consecutive time periods on sample sizes given by the user in the fourth input cell. Hence, the six time periods used in this analysis as well as the indicators for each time frame are displayed in the chart below.

Six Consecutive Time Periods

Beginning Window Date	Period Indicator	Ending Window Date
Wednesday, September 23, 1998	A	Wednesday, December 16, 1998
Wednesday, December 16, 1998	B	Wednesday, March 10, 1999
Wednesday, March 10, 1999	C	Wednesday, June 02, 1999
Wednesday, June 02, 1999	D	Wednesday, August 25, 1999
Wednesday, August 25, 1999	E	Wednesday, November 17, 1999
Wednesday, November 17, 1999	F	Wednesday, February 09, 2000

The error tolerance figure is the approximation figure used as an input into the eigenvalues and eigenvectors function, which uses a Jacobi rotation algorithm. The last input cell indicates the number of days used to convert the weekly variance into a coherent daily figure compatible with the left over residual data from the daily study.

The Covariance and Correlation Matrices

The initial calculation to be handled concerning the daily data dealt with creating covariance and correlation matrices of indicators 1-12, the mortgage-backed funds and an equally weighted Treasury vector for every time frame. This problem included computing sample covariance and sample correlation numbers for every pair of these variables using data sample sizes of 60 trading days. The values of these covariances and correlations can be seen in Figures 1-6 in Appendix B at the end of this paper.

The two matrices yield results compatible with intuition about the mortgage-backed securities market. The funds tend to have high covariances with intermediate maturity Treasury securities as proposed in the portfolio profile and are negatively correlated with the S&P 500. As a whole, however, the funds are strongly in sync with each other, and so one could assume the variance procedure for one fund as similar for another agency portfolio.

Moreover, the study of the entries of the correlation matrix illustrates a common assumption concerning spot rates of Treasury bonds. Most fixed income analysts intuitively believe that correlations between Treasury securities of similar maturities are nearly perfectly positively related. Hence, the high positive correlations among neighboring Treasuries give credibility to arguments of dependence of movements in spot interest rates.

The Principal Components Analysis

The high correlation amongst the Treasury securities creates a problem when dealing with ordinary least squares regression techniques, as these variables would exhibit multicollinearity in the analysis. Thus, the beta coefficients of these data sets may be statistically insignificant, or not significantly different from zero. In doing this, one may discard data sets from the linear model that provide strong explanatory power of the response variable by themselves but not in conjunction with other predictor variables. Eliminating these sets would also reduce the overall explanatory value of the entire model.

As mentioned in prior sections, principal components analysis serves as a multivariate statistical technique in order to avoid getting rid of predictor variables in an ordinary least squares regression. The fact that the principal components are independent of one another allows for utilization of a reduced number of data sets to capture the overall reduction of variance in an outside dependent variable. The analysis for the six time periods in this project was run on three groups of daily variables, namely indicators

1-9, 1-11 and 1-12. The values of the principal component variances can be viewed in Figures 7-9 in Appendix B.

For the study on indicators 1-9, one can discover that the first principal component on every 60-day period accounts for over 90 percent of the variability of the entire time series, and the first two principal components contribute to about 97 percent of the total variance. Since the first principal component can be described as the weighted portfolio of the original indicators 1-9, the Treasury bond changes, this compilation holds the most risk. The second principal component is the next riskiest proportionally allocated portfolio of the Treasury security changes such that the two new portfolios have zero correlation.

After including the indicators 10-11 in the analysis, the results are nearly the same. Again, the first principal component explains as much as 95 percent of the total variability in the daily variables used, and the combined explanatory power of the first two principal components approaches 97 percent for all six time frames. However, the interpretation of the first principal component changes slightly, as the portfolio with highest risk is now a weighted average of the changes in the Treasury bonds and corporate bonds. In essence, the principal components now bear credit risk as the corporate indices contain some probability of default. In addition, the risk of the company's short put option embedded in these bonds is inherent in these portfolios.

The last study included indicators 1-12, or simply all of the daily data at hand in this project. A drastic change in explanatory power of the first principal component occurred, since the variance of the S&P 500 was high in comparison to the bonds. The riskiest portfolio of the daily variables now accounts for anywhere between 62 to 80 percent of the variance of the entire set, a noticeable decrease from the principal components containing only proportions of Treasury and corporate bonds. The second principal component makes up for this decline, now capturing around 25 percent of the total variance on average over the six time periods. One can deduce that the equity index has a large influence on the weighting schemes of the principal component portfolios on daily data variables.

Another method of illustrating the various principal component variances comes by way of a scree plot. A scree plot is a useful visual aid in determining which principal components to retain if one is looking to use the analysis for dimension reduction. The scree plots for the six time periods of this study exhibiting the decline in the variance of the original data set through the new independent transformed sets can be seen in Figures 10-12. Since $p \leq n$ principal components may provide a large enough percentage of explanatory power in a regression model, a scree plot may help identify the exact number of variables to keep in reducing dimension.

Since the principal component factor scores provide the same information concerning its variance regardless of whether the scalar is negative or positive, an adjustment had to be made to ensure that the principal components had positive correlation with an equally weighted Treasury vector. If the correlation coefficient was less than zero, the signs of the factor scores corresponding to that particular principal component multiplied on the original data sets would be reversed to force positive correlation. This symmetric property deals with the fact that the factor scores are squared as part of carrying out any principal component variance calculation.

The Principal Components and Multiple Regressions: LEXNX Fund

The method of carrying out a principal components analysis on the original data sets served the purpose of discovering new transformed independent variables which contain no more yet no less of the variability of the daily sample. In this manner, each agency mortgage-backed mutual fund could be regressed on the principal components, and the explanatory power of each principal component could be decomposed from the study. In doing so, one could see how each principal component incrementally reduces the total variability due to the uncorrelated nature of the new data sets.

Because of this zero correlation amongst the principal components, the beta coefficients for the principal components regression could be computed individually and the explanatory power of each new variable could be recognized immediately. The following regression table and equations involving the LEXNX fund give the beta coefficients for the principal components regression on the first daily data set of indicators 1-9 for all six time periods.

LEXNX Beta Coefficients 1-9 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9
A	0.15	0.02	-0.07	-0.15	0.18	0.70	0.73	2.65	0.28
B	0.16	0.07	-0.09	0.21	0.33	-0.23	1.79	0.17	-2.45
C	0.18	0.05	0.18	-0.44	0.23	-0.01	-0.32	0.03	3.22
D	0.44	-0.27	-0.79	-0.87	-0.51	0.25	0.35	1.27	1.31
E	0.44	0.06	-0.31	-0.46	0.48	0.08	2.92	3.09	-2.89
F	0.39	0.19	0.02	0.62	1.79	-0.49	0.10	2.18	0.23

$$\mathbf{m} = .147\mathbf{y}_1 + .019\mathbf{y}_2 - .072\mathbf{y}_3 - .147\mathbf{y}_4 + .184\mathbf{y}_5 + .699\mathbf{y}_6 + .730\mathbf{y}_7 + 2.645\mathbf{y}_8 + .285\mathbf{y}_9 + \mathbf{r} \qquad (3.1A)$$

$$\mathbf{m} = .159\mathbf{y}_1 + .071\mathbf{y}_2 - .094\mathbf{y}_3 + .210\mathbf{y}_4 + .335\mathbf{y}_5 - .230\mathbf{y}_6 + 1.791\mathbf{y}_7 + .172\mathbf{y}_8 - 2.448\mathbf{y}_9 + \mathbf{r} \qquad (3.1B)$$

$$\boldsymbol{m} = .181\boldsymbol{y_1} + .047\boldsymbol{y_2} + .179\boldsymbol{y_3} - .436\boldsymbol{y_4} + .227\boldsymbol{y_5} - .012\boldsymbol{y_6} - .325\boldsymbol{y_7} - .030\boldsymbol{y_8} + 3.218\boldsymbol{y_9} + \boldsymbol{r} \quad (3.1C)$$

$$\boldsymbol{m} = .445\boldsymbol{y_1} - .267\boldsymbol{y_2} - .795\boldsymbol{y_3} - .868\boldsymbol{y_4} - .507\boldsymbol{y_5} + .255\boldsymbol{y_6} + .348\boldsymbol{y_7} + 1.275\boldsymbol{y_8} + 1.313\boldsymbol{y_9} + \boldsymbol{r} \quad (3.1D)$$

$$\boldsymbol{m} = .444\boldsymbol{y_1} + .064\boldsymbol{y_2} - .313\boldsymbol{y_3} - .458\boldsymbol{y_4} + .483\boldsymbol{y_5} + .080\boldsymbol{y_6} + 2.922\boldsymbol{y_7} + 3.086\boldsymbol{y_8} - 2.893\boldsymbol{y_9} + \boldsymbol{r} \quad (3.1E)$$

$$\boldsymbol{m} = .387\boldsymbol{y_1} + .193\boldsymbol{y_2} + .023\boldsymbol{y_3} + .619\boldsymbol{y_4} + 1.786\boldsymbol{y_5} - .488\boldsymbol{y_6} + .010\boldsymbol{y_7} + 2.180\boldsymbol{y_8} + .234\boldsymbol{y_9} + \boldsymbol{r} \quad (3.1F)$$

These equations differ very much from period to period, and so one can deduce that these risky portfolios of Treasury bonds play varying parts in explaining the variability of the mortgage fund.

The R-Squared measure of how much of the LEXNX fund variance is accounted for by the principal components of the Treasury bonds is displayed in the chart below.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.768006
Wednesday, March 10, 1999	0.724368
Wednesday, June 02, 1999	0.759696
Wednesday, August 25, 1999	0.796040
Wednesday, November 17, 1999	0.846915
Wednesday, February 09, 2000	0.870239

Average R-Squared Value
0.794211

These figures show that the average percentage of variance captured by the indicators 1-9 is over 79 percent, and moreover always greater than 72 percent. These results are intuitively true as well, as one would expect mortgage-backed portfolios to be very closely related to weighted allocations to various Treasury securities. Since most mortgage funds are relatively high in credit quality to corporate and junk bonds, the majority of the risk is due to fluctuations in the government debt market.

The biggest advantage of principal components regression, although, is the fact that one can see the residual variance reduction take place after each principal component is captured in the model. Figure 13 depicts this decrease, and one can observe that the largest principal component captures about 70 percent of the variability of the LEXNX portfolio by itself. The next eight principal components barely account for any movements in the fund. Figure 14 reflects the percentage of the fund's variance accounted for by indicators 1-9 for all six time frames.

One can also calculate the gamma coefficients of a principal components regression on the original data set to find the sensitivity to indicators 1-9. The table and linear equations for each time period are written below.

LEXNX Gamma Coefficients 1-9 From Principal Components Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9
A	2.23	0.36	-1.54	0.79	0.05	0.18	0.07	0.11	0.09
B	-0.33	2.19	-1.41	1.55	0.29	-0.09	0.14	0.21	0.16
C	1.49	-2.64	0.73	0.83	-0.12	-0.33	0.45	0.10	0.06
D	1.28	-1.27	-0.63	-0.01	-0.91	-0.58	-0.25	0.49	0.58
E	1.67	2.52	-3.76	-1.30	0.72	-1.23	0.07	0.33	0.22
F	1.66	0.72	0.02	-1.16	1.79	-0.45	0.85	-0.09	0.21

$$\boldsymbol{m} = 2.234(\Delta X)_1 + .361(\Delta X)_2 - 1.543(\Delta X)_3 + .790(\Delta X)_4 + .052(\Delta X)_5 + .177(\Delta X)_6 + .072(\Delta X)_7 + .106(\Delta X)_8 + .085(\Delta X)_9 + \boldsymbol{r} \qquad (3.2A)$$

$$\boldsymbol{m} = -.331(\Delta X)_1 + 2.190(\Delta X)_2 - 1.407(\Delta X)_3 + 1.553(\Delta X)_4 + .291(\Delta X)_5 - .094(\Delta X)_6 + .144(\Delta X)_7 + .206(\Delta X)_8 + .156(\Delta X)_9 + \boldsymbol{r} \qquad (3.2B)$$

$$\boldsymbol{m} = 1.494(\Delta X)_1 - 2.643(\Delta X)_2 + .728(\Delta X)_3 + .828(\Delta X)_4 - .123(\Delta X)_5 - .330(\Delta X)_6 + .449(\Delta X)_7 + .101(\Delta X)_8 + .063(\Delta X)_9 + \boldsymbol{r} \qquad (3.2C)$$

$$\boldsymbol{m} = 1.279(\Delta \boldsymbol{X})_1 - 1.266(\Delta \boldsymbol{X})_2 - .633(\Delta \boldsymbol{X})_3 - .012(\Delta \boldsymbol{X})_4 - .907(\Delta \boldsymbol{X})_5 - .579(\Delta \boldsymbol{X})_6 - .250(\Delta \boldsymbol{X})_7 + .495(\Delta \boldsymbol{X})_8 + .583(\Delta \boldsymbol{X})_9 + \boldsymbol{r} \qquad (3.2D)$$

$$\boldsymbol{m} = 1.669(\Delta \boldsymbol{X})_1 + 2.519(\Delta \boldsymbol{X})_2 - 3.765(\Delta \boldsymbol{X})_3 - 1.297(\Delta \boldsymbol{X})_4 + .723(\Delta \boldsymbol{X})_5 - 1.228(\Delta \boldsymbol{X})_6 + .068(\Delta \boldsymbol{X})_7 + .331(\Delta \boldsymbol{X})_8 + .219(\Delta \boldsymbol{X})_9 + \boldsymbol{r} \qquad (3.2E)$$

$$\boldsymbol{m} = 1.662(\Delta \boldsymbol{X})_1 + .718(\Delta \boldsymbol{X})_2 + .024(\Delta \boldsymbol{X})_3 - 1.162(\Delta \boldsymbol{X})_4 + 1.789(\Delta \boldsymbol{X})_5 - .453(\Delta \boldsymbol{X})_6 + .854(\Delta \boldsymbol{X})_7 - .089(\Delta \boldsymbol{X})_8 + .214(\Delta \boldsymbol{X})_9 + \boldsymbol{r} \qquad (3.2F)$$

Thus, if only indicators 1-9 were available, one could use any one of these equations as an index to represent the great majority of variability in the LEXNX fund as a function of the Treasury bonds. The graphical representation of these gamma coefficients in all six time periods is exhibited in Figure 15.

A similar type of analysis could be performed on the second grouping of daily variables, namely indicators 1-11. The LEXNX fund principal components regression table and equations for the six time periods utilizing the beta coefficients are listed below.

LEXNX Beta Coefficients 1-11 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9	Beta 10	Beta 11
A	0.14	0.02	-0.10	-0.22	0.02	0.14	0.67	0.88	0.14	2.57	0.32
B	0.15	0.07	-0.02	-0.09	-0.31	0.32	-0.36	0.24	2.04	-0.25	-2.14
C	0.17	0.05	0.17	-0.12	0.47	0.15	0.00	0.10	-0.40	0.03	3.26
D	0.42	-0.07	-0.38	-0.74	-0.87	-0.47	0.26	-0.58	0.84	1.16	2.28
E	0.42	-0.19	0.36	-0.81	0.42	0.37	-0.38	0.10	2.24	1.50	-3.02
F	0.35	0.19	-0.18	0.05	0.60	0.65	1.83	-0.76	-0.28	1.94	0.63

$$\boldsymbol{m} = .141\boldsymbol{y}_1 + .016\boldsymbol{y}_2 - .103\boldsymbol{y}_3 - .219\boldsymbol{y}_4 + .019\boldsymbol{y}_5 + .138\boldsymbol{y}_6 + .672\boldsymbol{y}_7 + .878\boldsymbol{y}_8 + .138\boldsymbol{y}_9 + 2.566\boldsymbol{y}_{10} + .322\boldsymbol{y}_{11} + \boldsymbol{r} \qquad (3.3A)$$

$$\boldsymbol{m} = .154\boldsymbol{y}_1 + .072\boldsymbol{y}_2 - .021\boldsymbol{y}_3 - .092\boldsymbol{y}_4 - .315\boldsymbol{y}_5 + .322\boldsymbol{y}_6 - .363\boldsymbol{y}_7 + .238\boldsymbol{y}_8 + 2.037\boldsymbol{y}_9 - .252\boldsymbol{y}_{10} - 2.141\boldsymbol{y}_{11} + \boldsymbol{r} \qquad (3.3B)$$

$$\boldsymbol{m} = .171\boldsymbol{y}_1 + .051\boldsymbol{y}_2 + .170\boldsymbol{y}_3 - .120\boldsymbol{y}_4 + .474\boldsymbol{y}_5 + .154\boldsymbol{y}_6 + .002\boldsymbol{y}_7 + .101\boldsymbol{y}_8 - .398\boldsymbol{y}_9 + .032\boldsymbol{y}_{10} + 3.265\boldsymbol{y}_{11} + \boldsymbol{r} \qquad (3.3C)$$

$$\boldsymbol{m} = .419\boldsymbol{y}_1 - .067\boldsymbol{y}_2 - .382\boldsymbol{y}_3 - .739\boldsymbol{y}_4 - .868\boldsymbol{y}_5 - .472\boldsymbol{y}_6 + .261\boldsymbol{y}_7 - .584\boldsymbol{y}_8 + .836\boldsymbol{y}_9 + 1.156\boldsymbol{y}_{10} + 2.284\boldsymbol{y}_{11} + \boldsymbol{r} \qquad (3.3D)$$

$$\boldsymbol{m} = .418\boldsymbol{y}_1 - .187\boldsymbol{y}_2 + .362\boldsymbol{y}_3 - .812\boldsymbol{y}_4 + .417\boldsymbol{y}_5 + .374\boldsymbol{y}_6 - .378\boldsymbol{y}_7 + .010\boldsymbol{y}_8 + 2.242\boldsymbol{y}_9 + 1.502\boldsymbol{y}_{10} - 3.024\boldsymbol{y}_{11} + \boldsymbol{r} \qquad (3.3E)$$

$$\boldsymbol{m} = .354\boldsymbol{y}_1 + .189\boldsymbol{y}_2 - .178\boldsymbol{y}_3 + .048\boldsymbol{y}_4 + .603\boldsymbol{y}_5 + .654\boldsymbol{y}_6 + 1.828\boldsymbol{y}_7 - .764\boldsymbol{y}_8 - .277\boldsymbol{y}_9 + 1.940\boldsymbol{y}_{10} + .630\boldsymbol{y}_{11} + \boldsymbol{r} \qquad (3.3F)$$

Once again, these equations vary from period to period.

The explanatory power of these linear models is depicted in the R-Squared value chart below.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.787251
Wednesday, March 10, 1999	0.749290
Wednesday, June 02, 1999	0.761055
Wednesday, August 25, 1999	0.800824
Wednesday, November 17, 1999	0.863363
Wednesday, February 09, 2000	0.881149

Average R-Squared Value
0.807155

One can notice that the overall accountability of the variance of the mortgage-backed portfolio has increased due to the addition of the corporate bonds, where now indicators 1-11 claim on average nearly 81 percent of the LEXNX variance. This growth is undoubtedly due to the small amount of corporate credit risk inherent in agency mortgage-backed securities, namely those issued by Ginnie Mae, Fannie Mae and Freddie Mac. Thus, one can decompose the average variability due to the corporate bonds alone as 80.7155 – 79.4211 = 1.2944 percent. This excess variance over the Treasury bonds is exposed in the small area over the R-squared values for indicators 1-9 in Figure 16, while the individual contribution of each principal component to the accountability of the variance can be seen in Figure 17.

As with the first group of daily data, the interpretation of the principal components leads one to represent the regression equations in terms of the original data sets which can be observed in the financial markets. The table formulas for indicators 1-11 are listed below.

LEXNX Gamma Coefficients 1-11 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9	Gamma 10	Gamma 11
A	1.46	-0.93	-1.11	-0.91	0.89	0.08	0.23	-0.14	-1.44	0.07	0.18
B	0.61	-1.93	1.04	0.49	1.19	-0.03	1.33	-0.74	-0.05	0.18	0.17
C	0.72	-0.50	0.68	-1.38	-1.12	0.42	-0.28	-2.46	0.69	-0.08	0.07
D	0.49	-2.65	0.44	0.18	-0.64	-0.30	0.06	-0.93	0.55	0.27	0.51
E	0.84	0.16	-3.07	1.21	1.28	0.55	1.17	-1.60	0.40	0.22	0.48
F	1.32	-0.14	-1.45	0.02	1.80	0.89	-0.48	-0.95	0.22	-0.09	0.22

$$m = 1.456(\Delta X)_1 - .927(\Delta X)_2 - 1.114(\Delta X)_3 - .906(\Delta X)_4 + .890(\Delta X)_5 + .083(\Delta X)_6 + .235(\Delta X)_7 - .193(\Delta X)_8 - 1.441(\Delta X)_9 + .075(\Delta X)_{10} + .176(\Delta X)_{11} + r \qquad (3.4A)$$

$$m = .608(\Delta X)_1 - 1.930(\Delta X)_2 + 1.044(\Delta X)_3 + .485(\Delta X)_4 + 1.191(\Delta X)_5 - .027(\Delta X)_6 + 1.335(\Delta X)_7 - .736(\Delta X)_8 - .053(\Delta X)_9 + .182(\Delta X)_{10} + .168(\Delta X)_{11} + r \qquad (3.4B)$$

$$m = .715(\Delta X)_1 - .505(\Delta X)_2 + .681(\Delta X)_3 - 1.377(\Delta X)_4 - 1.120(\Delta X)_5 + .417(\Delta X)_6 - .277(\Delta X)_7 - 2.455(\Delta X)_8 + .695(\Delta X)_9 - .076(\Delta X)_{10} + .068(\Delta X)_{11} + r \qquad (3.4C)$$

$$m = .494(\Delta X)_1 - 2.653(\Delta X)_2 + .440(\Delta X)_3 + .176(\Delta X)_4 - .638(\Delta X)_5 - .303(\Delta X)_6 + .059(\Delta X)_7 - .933(\Delta X)_8 + .547(\Delta X)_9 + .269(\Delta X)_{10} + .507(\Delta X)_{11} + r \quad (3.4D)$$

$$m = .839(\Delta X)_1 + .158(\Delta X)_2 - 3.069(\Delta X)_3 + 1.208(\Delta X)_4 + 1.279(\Delta X)_5 + .546(\Delta X)_6 + 1.172(\Delta X)_7 - 1.596(\Delta X)_8 + .401(\Delta X)_9 + .216(\Delta X)_{10} + .476(\Delta X)_{11} + r \qquad (3.4E)$$

$$\boldsymbol{m} = 1.323(\Delta \boldsymbol{X})_1 - .145(\Delta \boldsymbol{X})_2 - 1.447(\Delta \boldsymbol{X})_3 + .017(\Delta \boldsymbol{X})_4 + 1.803(\Delta \boldsymbol{X})_5 + .894(\Delta \boldsymbol{X})_6 - .483(\Delta \boldsymbol{X})_7 - .947(\Delta \boldsymbol{X})_8 + .220(\Delta \boldsymbol{X})_9 - .091(\Delta \boldsymbol{X})_{10} + .222(\Delta \boldsymbol{X})_{11} + \boldsymbol{r} \quad (3.4F)$$

A pictorial display of these regression coefficients is presented in Figure 18.

The third and final group of daily data encompassed all indicators 1-12 and incorporated the S&P 500 index. The aim of this analysis was to discover if the risk in stocks had a strong influence on the variability of the LEXNX portfolio. The following table and equations state the principal component regression formulas for this study.

LEXNX Beta Coefficients 1-12 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9	Beta 10	Beta 11	Beta 12
A	0.07	0.13	0.02	-0.07	-0.22	0.04	0.20	0.55	0.89	0.19	2.57	0.46
B	-0.01	0.16	0.13	0.03	0.00	-0.27	0.33	-0.36	0.20	2.11	-0.10	-1.29
C	0.03	0.17	0.05	0.17	-0.13	0.47	-0.19	0.06	0.11	-0.40	0.03	3.25
D	0.21	0.32	-0.01	0.21	-0.74	-0.88	-0.48	0.25	0.94	0.02	1.32	1.26
E	0.14	0.37	0.10	-0.29	-0.69	0.41	0.30	-0.20	0.25	2.07	1.45	-3.56
F	0.03	0.35	0.17	-0.12	0.05	0.56	0.48	1.82	-0.75	-0.28	2.04	0.36

$$\boldsymbol{m} = .073\boldsymbol{y}_1 + .129\boldsymbol{y}_2 + .021\boldsymbol{y}_3 - .069\boldsymbol{y}_4 - .217\boldsymbol{y}_5 + .036\boldsymbol{y}_6 + .199\boldsymbol{y}_7 + .554\boldsymbol{y}_8 + .894\boldsymbol{y}_9 + .185\boldsymbol{y}_{10} + 2.571\boldsymbol{y}_{11} + .462\boldsymbol{y}_{12} + \boldsymbol{r} \qquad (3.5A)$$

$$\boldsymbol{m} = -.006\boldsymbol{y}_1 + .155\boldsymbol{y}_2 + .134\boldsymbol{y}_3 + .032\boldsymbol{y}_4 + .003\boldsymbol{y}_5 - .270\boldsymbol{y}_6 + .330\boldsymbol{y}_7 - .363\boldsymbol{y}_8 + .198\boldsymbol{y}_9 + 2.107\boldsymbol{y}_{10} - .097\boldsymbol{y}_{11} - 1.294\boldsymbol{y}_{12} + \boldsymbol{r} \qquad (3.5B)$$

$$\boldsymbol{m} = .028\boldsymbol{y}_1 + .168\boldsymbol{y}_2 + .051\boldsymbol{y}_3 + .172\boldsymbol{y}_4 - .135\boldsymbol{y}_5 + .467\boldsymbol{y}_6 - .188\boldsymbol{y}_7 + .057\boldsymbol{y}_8 + .113\boldsymbol{y}_9 - .402\boldsymbol{y}_{10} + .033\boldsymbol{y}_{11} + 3.246\boldsymbol{y}_{12} + \boldsymbol{r} \qquad (3.5C)$$

$$\boldsymbol{m} = .215\boldsymbol{y}_1 + .320\boldsymbol{y}_2 - .014\boldsymbol{y}_3 + .205\boldsymbol{y}_4 - .740\boldsymbol{y}_5 - .875\boldsymbol{y}_6 - .477\boldsymbol{y}_7 + .254\boldsymbol{y}_8 + .945\boldsymbol{y}_9 + .023\boldsymbol{y}_{10} + 1.321\boldsymbol{y}_{11} + 1.260\boldsymbol{y}_{12} + \boldsymbol{r} \qquad (3.5D)$$

$$\boldsymbol{m} = .140\boldsymbol{y}_1 + .366\boldsymbol{y}_2 + .105\boldsymbol{y}_3 - .289\boldsymbol{y}_4 - .690\boldsymbol{y}_5 + .412\boldsymbol{y}_6 + .297\boldsymbol{y}_7 - .196\boldsymbol{y}_8 + .252\boldsymbol{y}_9 + 2.067\boldsymbol{y}_{10} + 1.449\boldsymbol{y}_{11} - 3.561\boldsymbol{y}_{12} + \boldsymbol{r} \qquad (3.5E)$$

$$\boldsymbol{m} = .030\boldsymbol{y_1} + .353\boldsymbol{y_2} + .174\boldsymbol{y_3} - .116\boldsymbol{y_4} + .050\boldsymbol{y_5} + .561\boldsymbol{y_6} + .483\boldsymbol{y_7} + 1.818\boldsymbol{y_8} - .754\boldsymbol{y_9} - .279\boldsymbol{y_{10}} + 2.042\boldsymbol{y_{11}} + .360\boldsymbol{y_{12}} + \boldsymbol{r} \qquad (3.5F)$$

In this third daily analysis, the beta coefficients are not the same for each period just as with the research of indicators 1-9 and indicators 1-11.

In looking at the R-Squared values of this principal components regression over all of the time frames, one can acquire knowledge concerning the gain in explanatory power by observing the effect of the equity markets. The following table gives the impact of this additional variable.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.788674
Wednesday, March 10, 1999	0.760909
Wednesday, June 02, 1999	0.761267
Wednesday, August 25, 1999	0.807939
Wednesday, November 17, 1999	0.875803
Wednesday, February 09, 2000	0.885682

Average R-Squared Value
0.813379

The separated amount of variance captured through each principal component is depicted in Figure 19. Because the average R-Squared value increased due to the S&P 500, one can compute the excess explanatory power as 81.3379 – 80.7155 = 0.6224 percent, that is, less than 1 percent. Hence, the ups and downs of the stock market do not appear to alter the movements of the LEXNX mortgage fund value. The effect of the equity variable is illustrated in the area graph of Figure 20.

Just as in the prior two analyses dealing with this specific mortgage portfolio, gamma regression coefficients can be obtained which correspond to the original daily data set. The table and regression equations below reflect the sensitivities to these variables over all six time periods.

LEXNX Gamma Coefficients 1-12 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9	Gamma 10	Gamma 11	Gamma 12
A	0.38	1.35	-1.26	-0.30	-0.70	0.72	0.02	-0.15	0.47	1.77	0.12	-0.12
B	1.29	0.35	0.23	-1.27	-0.07	-0.88	0.49	1.32	-0.43	0.36	0.08	-0.01
C	3.21	-0.46	0.24	0.05	0.39	0.08	-0.52	0.08	-0.18	0.05	0.15	-0.04
D	1.22	-0.45	-1.42	0.56	-0.24	0.53	0.10	0.45	1.17	-0.25	0.10	0.07
E	-3.12	1.83	-1.83	-1.03	-1.11	0.22	0.75	0.80	-0.33	0.20	0.17	-0.03
F	0.40	1.42	0.23	-1.71	-0.95	-0.78	-0.98	0.32	0.98	0.47	0.17	-0.09

$$\mathbf{m} = .384(\Delta\mathbf{X})_1 + 1.355(\Delta\mathbf{X})_2 - 1.259(\Delta\mathbf{X})_3 - .305(\Delta\mathbf{X})_4 - .696(\Delta\mathbf{X})_5 + .723(\Delta\mathbf{X})_6 + .016(\Delta\mathbf{X})_7 - .150(\Delta\mathbf{X})_8 + .468(\Delta\mathbf{X})_9 + 1.772(\Delta\mathbf{X})_{10} + .124(\Delta\mathbf{X})_{11} - .116(\Delta\mathbf{X})_{12} + \mathbf{r} \quad (3.6A)$$

$$\mathbf{m} = 1.292(\Delta\mathbf{X})_1 + .354(\Delta\mathbf{X})_2 + .226(\Delta\mathbf{X})_3 - 1.268(\Delta\mathbf{X})_4 - .067(\Delta\mathbf{X})_5 - .876(\Delta\mathbf{X})_6 + .490(\Delta\mathbf{X})_7 + 1.316(\Delta\mathbf{X})_8 - .429(\Delta\mathbf{X})_9 + .364(\Delta\mathbf{X})_{10} + .082(\Delta\mathbf{X})_{11} - .007(\Delta\mathbf{X})_{12} + \mathbf{r} \quad (3.6B)$$

$$\mathbf{m} = 3.207(\Delta\mathbf{X})_1 - .457(\Delta\mathbf{X})_2 + .244(\Delta\mathbf{X})_3 + .054(\Delta\mathbf{X})_4 + .388(\Delta\mathbf{X})_5 + .077(\Delta\mathbf{X})_6 - .519(\Delta\mathbf{X})_7 + .076(\Delta\mathbf{X})_8 - .180(\Delta\mathbf{X})_9 + .053(\Delta\mathbf{X})_{10} + .151(\Delta\mathbf{X})_{11} - .035(\Delta\mathbf{X})_{12} + \mathbf{r} \quad (3.6C)$$

$$\mathbf{m} = 1.222(\Delta\mathbf{X})_1 - .452(\Delta\mathbf{X})_2 - 1.424(\Delta\mathbf{X})_3 + .556(\Delta\mathbf{X})_4 - .243(\Delta\mathbf{X})_5 + .532(\Delta\mathbf{X})_6 + .097(\Delta\mathbf{X})_7 + .447(\Delta\mathbf{X})_8 + 1.166(\Delta\mathbf{X})_9 - .254(\Delta\mathbf{X})_{10} + .097(\Delta\mathbf{X})_{11} + .066(\Delta\mathbf{X})_{12} + \mathbf{r} \quad (3.6D)$$

$$\mathbf{m} = -3.120(\Delta\mathbf{X})_1 + 1.834(\Delta\mathbf{X})_2 - 1.826(\Delta\mathbf{X})_3 - 1.030(\Delta\mathbf{X})_4 - 1.106(\Delta\mathbf{X})_5 + .216(\Delta\mathbf{X})_6 + .752(\Delta\mathbf{X})_7 + .798(\Delta\mathbf{X})_8 - .329(\Delta\mathbf{X})_9 + .199(\Delta\mathbf{X})_{10} + .173(\Delta\mathbf{X})_{11} - .026(\Delta\mathbf{X})_{12} + \mathbf{r} \quad (3.6E)$$

$$\mathbf{m} = .402(\Delta\mathbf{X})_1 + 1.422(\Delta\mathbf{X})_2 + .230(\Delta\mathbf{X})_3 - 1.714(\Delta\mathbf{X})_4 - .955(\Delta\mathbf{X})_5 - .779(\Delta\mathbf{X})_6 - .983(\Delta\mathbf{X})_7 + .317(\Delta\mathbf{X})_8 + .978(\Delta\mathbf{X})_9 + .466(\Delta\mathbf{X})_{10} + .172(\Delta\mathbf{X})_{11} - .091(\Delta\mathbf{X})_{12} + \mathbf{r} \quad (3.6F)$$

Figure 21 shows the gamma coefficients for each 60-day period utilizing the principal components regression of indicators 1-12. In all, the analysis of the total daily data set has provided linear equations which explain on average over 81 percent of the variability in the LEXNX agency mortgage-backed fund portfolio.

From the outset, however, the goal of the study was to claim at least 90 percent of the variance of the fund and assume the other 10 percent was due to other factors or the nonlinear nature of mortgage-backed securities. The residual data set from the daily analysis could be determined from indicators 1-12, and then could be used as the response variable for a principal components regression on the weekly variables. The beginning step of this process was to convert the daily residual mortgage data into weekly values by adding five net asset value changes together and naming it as one weekly value change, since there are five trading days per week in the financial markets. As a result, the residual LEXNX weekly fund had 60/5 = 12 entries in all six time frames. This reduction of time series sample size did not affect the estimation of statistical parameters. However, the variances estimated by the weekly data were made into daily variances by dividing by a factor of 7. This conversion factor was used due to a lag in information known by the mutual fund portfolio manager but not recognized by the data sources. Since the residual variances of the initial weekly mortgage vector could not exceed the final residual variances of the study of indicators 1-12, a necessary adjustment was made.

The weekly data dealt with mortgage rates in basis points and were recognized as indicators 13-16. The table and regression equations for the weekly residual mortgage data of LEXNX on the principal components of these loan rates is presented below.

LEXNX Beta Coefficients 13-16 From Principal Components Regression

Period	Beta 13	Beta 14	Beta 15	Beta 16
A	0.0012	-0.0048	0.0049	0.0258
B	0.0033	0.0004	0.0007	-0.0518
C	0.0013	0.0059	0.0057	0.1195
D	0.0014	-0.0022	-0.0003	-0.0088
E	-0.0031	0.0019	-0.0007	0.0558
F	-0.0007	-0.0004	-0.0068	-0.0481

$$\boldsymbol{m} = .0012\boldsymbol{y}_{13} - .0048\boldsymbol{y}_{14} + .0049\boldsymbol{y}_{15} + .0258\boldsymbol{y}_{16} + \boldsymbol{r} \qquad (3.7A)$$

$$\boldsymbol{m} = .0033\boldsymbol{y}_{13} + .0004\boldsymbol{y}_{14} + .0007\boldsymbol{y}_{15} - .0518\boldsymbol{y}_{16} + \boldsymbol{r} \qquad (3.7B)$$

$$\boldsymbol{m} = .0013\boldsymbol{y}_{13} + .0059\boldsymbol{y}_{14} + .0057\boldsymbol{y}_{15} + .1195\boldsymbol{y}_{16} + \boldsymbol{r} \qquad (3.7C)$$

$$\boldsymbol{m} = .0014\boldsymbol{y}_{13} - .0022\boldsymbol{y}_{14} - .0003\boldsymbol{y}_{15} - .0088\boldsymbol{y}_{16} + \boldsymbol{r} \qquad (3.7D)$$

$$\boldsymbol{m} = -.0031\boldsymbol{y}_{13} + .0019\boldsymbol{y}_{14} - .0007\boldsymbol{y}_{15} + .0558\boldsymbol{y}_{16} + \boldsymbol{r} \qquad (3.7E)$$

$$\boldsymbol{m} = -.0007\boldsymbol{y}_{13} - .0004\boldsymbol{y}_{14} - .0068\boldsymbol{y}_{15} - .0481\boldsymbol{y}_{16} + \boldsymbol{r} \qquad (3.7F)$$

For every 60-day period, the regression coefficients differ, showing how these econometric models must be dynamically adjusted in order to maintain stability and accountability.

The explanatory power of these weekly data sets negated the possibility of working with the monthly economic variables. The following table conveys the R-Squared values for the analysis considering all indicators 1-16.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.864404
Wednesday, March 10, 1999	0.945540
Wednesday, June 02, 1999	0.822709
Wednesday, August 25, 1999	0.958836
Wednesday, November 17, 1999	0.992320
Wednesday, February 09, 2000	0.985908

Average R-Squared Value
0.928286

One can see the large spike in variance captured by the mortgage rates from the study. The average R-Squared value of over 92 percent shows a significant portion of movement in the fund due to the mortgage-backed securities sector of the market. Specifically, 92.8286 – 81.3379 = 11.4907 percent of the original LEXNX variance is explained by these weekly rates, which could be considered as the risk attributed to the secondary mortgage market. Since the principal components obtained from the weekly analysis are not necessarily uncorrelated with those from the daily study, it is possible that this sector risk inherently holds some market, credit and equity variance as well. This additional explanatory power can be visually seen in Figure 22.

The linear sensitivities of the weekly residual mortgage portfolio based on the original indicators 13-16 are included in the following principal components regression table and equations.

LEXNX Gamma Coefficients 13-16 From Ordinary Least Squares Regression

Period	Gamma 13	Gamma 14	Gamma 15	Gamma 16
A	0.0186	0.0014	-0.0188	0.0040
B	-0.0512	-0.0045	0.0047	0.0048
C	0.1180	-0.0189	0.0053	-0.0064
D	-0.0087	0.0018	0.0004	-0.0023
E	0.0434	-0.0325	0.0124	0.0054
F	-0.0476	-0.0080	-0.0050	-0.0011

$$m = .0186(\Delta X)_{13} + .0014(\Delta X)_{14} - .0188(\Delta X)_{15} + .0040(\Delta X)_{16} + r \qquad (3.8A)$$

$$m = -.0512(\Delta X)_{13} - .0045(\Delta X)_{14} + .0047(\Delta X)_{15} + .0048(\Delta X)_{16} + r \qquad (3.8B)$$

$$m = .1180(\Delta X)_{13} - .0189(\Delta X)_{14} + .0053(\Delta X)_{15} - .0064(\Delta X)_{16} + r \qquad (3.8C)$$

$$m = -.0087(\Delta X)_{13} + .0018(\Delta X)_{14} + .0004(\Delta X)_{15} - .0023(\Delta X)_{16} + r \qquad (3.8D)$$

$$m = .0434(\Delta X)_{13} - .0325(\Delta X)_{14} + .0124(\Delta X)_{15} + .0054(\Delta X)_{16} + r \qquad (3.8E)$$

$$m = -.0476(\Delta X)_{13} - .0080(\Delta X)_{14} - .0050(\Delta X)_{15} - .0011(\Delta X)_{16} + r \qquad (3.8F)$$

Thus, combining two econometric models based on the original daily and weekly data could serve as a replicating portfolio reflecting a strong majority of variance of the changes in the LEXNX fund portfolio going forward into the future.

The Principal Components and Multiple Regressions: VFIIX Fund

Looking at the dynamics of principal components regression on solely one fund does not justify its advantages. The VFIIX agency mortgage-backed mutual fund had strong correlations with the LEXNX portfolio over the six time periods between September 23, 1998 and February 9, 2000. It would prove useful to determine if a principal components regression model could decompose the risks of any mortgage fund. Instead of writing out each equation as with the previous fund, the beta coefficients in the following table represent the linear sensitivity of the changes in the VFIIX portfolio to the changes in the daily principal components of indicators 1-9 in all six time frames.

VFIIX Beta Coefficients 1-9 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9
A	0.17	-0.01	-0.39	-0.31	0.10	-0.05	0.31	3.00	-3.14
B	0.14	0.21	-0.62	0.12	0.40	-0.35	0.72	0.79	-2.29
C	0.18	0.33	-0.02	-0.40	-0.06	0.23	0.12	-1.25	4.29
D	0.46	-0.13	-0.73	-0.98	-0.19	-0.17	0.37	1.43	5.30
E	0.43	0.25	-1.00	0.70	0.65	0.57	1.85	2.82	-1.55
F	0.36	-0.04	-0.07	0.91	1.12	-0.24	0.54	2.01	-0.20

In comparing the beta coefficients of the VFIIX and LEXNX funds, one can notice that the signs of the scalars are nearly the same in every equation. This seems to indicate that a relative pattern amongst agency mortgage backed security portfolios may be evident when explaining their variances in linear models containing principal components data sets of Treasury bonds. For instance, because the regression coefficient on the first principal component was positive in the study of both funds, one could assume that this portfolio of indicators 1-9 gives a strong representation of each portfolio's risk.

A numerical example of the explanatory power of the principal components of the first set of daily variables can be viewed in the following R-Squared table for all six time periods.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.706599
Wednesday, March 10, 1999	0.655820
Wednesday, June 02, 1999	0.616919
Wednesday, August 25, 1999	0.805150
Wednesday, November 17, 1999	0.802477
Wednesday, February 09, 2000	0.806684

Average R-Squared Value
0.732275

These calculations tell that the average accountability of variance in the VFIIX mutual fund determined by indicators 1-9 was above 73% for any 60-day period. Just as before, one can attribute this percentage of variance captured by the Treasury bond changes as the effect of Treasury market risk on movements in the mortgage fund. Figure 23 in Appendix B reveals that the first principal component of this first daily data set claims over 63 percent of the variability in the agency portfolio on average. In addition, Figure 24 plots the R-Squared values given by the Treasury bond changes over each time frame.

Putting this principal components regression in terms of the original daily variables, the gamma coefficients in the equations below depict the absolute sensitivities of the mortgage fund with respect to a change in one of indicators 1-9.

VFIIX Gamma Coefficients 1-9 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9
A	0.14	2.43	-3.48	0.88	0.61	0.14	-0.05	0.29	0.11
B	-0.47	2.08	-1.25	0.35	0.57	-0.58	0.06	0.44	0.15
C	1.55	-3.76	1.75	0.04	0.21	-0.54	0.36	0.47	-0.02
D	3.08	-4.22	1.02	1.18	-0.89	-0.63	-0.39	0.82	0.59
E	2.04	1.97	-2.12	-1.41	0.45	-0.30	-0.80	0.84	0.21
F	1.51	1.00	-0.45	-0.76	1.53	-0.11	0.27	0.08	0.36

In congruence with the regression equations for first fund, these formulas could serve as a benchmarking index for variability in the VFIIX portfolio as a function of the Treasury bonds. Figure 25 displays a graph of these gamma coefficients for each of the six periods in this analysis.

Indicators 1-9 only can capture the Treasury market risk present in the mortgage-backed fund, and so the addition of the corporate bonds would serve as a good measure of the variability in changes in VFIIX due to corporate credit risk. The regression equations that follow relate the estimators for the principal components of the entire set of bond prices, both at the Treasury and corporate level.

VFIIX Beta Coefficients 1-11 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9	Beta 10	Beta 11
A	0.17	-0.01	-0.41	-0.26	-0.15	0.06	-0.11	0.55	0.32	2.91	-3.34
B	0.13	0.22	-0.13	-0.62	-0.33	0.43	-0.13	-0.09	1.18	0.35	-1.46
C	0.17	0.32	0.06	-0.22	0.38	-0.07	0.32	-0.53	0.16	-1.41	3.92
D	0.43	0.13	-0.43	-0.65	-0.98	-0.18	-0.24	-0.32	0.67	1.33	6.01
E	0.41	-0.03	0.28	-1.28	-0.31	1.53	0.33	-0.08	0.72	1.11	-1.23
F	0.33	0.01	-0.03	-0.05	0.89	0.52	1.12	-0.61	0.28	1.81	0.05

Thus, one can discern that while the beta coefficients on a time series basis are dissimilar, on a cross-sectional basis with the LEXNX fund, the scalars have almost always the same sign and magnitude. This appears to verify the fact that the two agency security funds have strong correlation with one another, as they should due to the comparable nature of their profiles and holdings in Ginnie Mae, Freddie Mac and Fannie Mae mortgage-backed investments.

The explanatory power of this linear principal components model can be seen in the R-Squared values in the table below.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.726505
Wednesday, March 10, 1999	0.699307
Wednesday, June 02, 1999	0.623433
Wednesday, August 25, 1999	0.809374
Wednesday, November 17, 1999	0.835119
Wednesday, February 09, 2000	0.812753

Average R-Squared Value
0.751082

According to these values, the addition of the corporate bonds into the study explains close to 84 percent of the total variance of the mortgage fund in the fifth time period, and on average, the R-Squared value shows that the principal components of indicators 1-11 capture over 75 percent of the variability in changes in the VFIIX fund. Hence, the incremental amount of explanatory power gained by including the two new daily variables is 75.1082 – 73.2275 = 1.8807 percent. This growth in the accountability of variance can be described as the corporate credit risk portion of the total fund variance, and its value is slightly greater than that of the first fund in the study. As a whole, however, this figure is extremely low, portraying the relatively high creditworthiness of the agency mortgage-backed securities' market. In Appendix B, Figure 26 renders the decrease in left over variance in the mortgage fund after each principal component is taken into account, and Figure 27 expresses the additional variability captured by the corporate bonds in the study.

Due to the fact that the principal components of indicators 1-11 are not readily present as securities in the financial markets, the least-squares regression equations below display the linear models in terms of the Treasury and corporate bonds alone.

VFIIX Gamma Coefficients 1-11 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9	Gamma 10	Gamma 11
A	0.31	-0.41	-0.35	0.85	0.12	-0.11	0.31	-0.80	-4.28	0.17	0.31
B	0.30	-1.15	0.35	0.19	0.52	-0.48	1.22	-0.83	0.24	0.31	0.10
C	0.45	-0.71	0.70	-0.90	-0.14	0.36	-0.15	-3.88	0.93	0.14	-0.08
D	1.32	-4.34	2.12	-0.42	-0.19	-0.25	0.17	-3.14	2.16	0.12	0.69
E	0.79	0.12	-0.98	0.14	-0.25	0.09	1.70	-1.67	0.39	0.32	0.33
F	1.18	-0.16	-0.66	0.47	1.19	0.97	-0.36	-1.15	0.26	-0.38	0.22

These formulas for all six periods could be used to approximate the fund variance due to the Treasury and corporate bond markets. The R-Squared values of these equations shall exactly equal the explanatory power of the linear models using principal components data sets, so no change in accountability is present. The gamma coefficients for these equations are illustrated in the bar graph of Figure 28 in Appendix B.

The third daily data study consisted of all bonds and the S&P 500 index. A new set of regression equations on the principal components of indicators 1-12 was developed and these formulas are stated below.

VFIIX Beta Coefficients 1-12 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9	Beta 10	Beta 11	Beta 12
A	0.08	0.17	0.00	-0.35	-0.26	-0.14	0.11	-0.24	0.36	0.31	2.95	-2.93
B	-0.01	0.13	0.33	-0.03	-0.52	-0.35	0.43	-0.13	-0.13	1.24	0.49	-0.56
C	0.04	0.17	0.33	0.03	-0.21	0.39	-0.02	0.38	-0.50	0.15	-1.40	3.82
D	0.23	0.32	0.20	0.23	-0.63	-0.99	-0.19	-0.25	0.68	0.45	1.63	4.55
E	0.12	0.37	0.17	-0.21	-1.24	-0.27	1.55	0.28	-0.02	0.65	1.10	-1.52
F	0.03	0.33	-0.01	0.04	-0.05	0.86	0.37	1.10	-0.59	0.27	1.92	-0.25

As in the prior analyses, the regression equations are very different over each 60-day time frame, yet are fairly consistent with the formulas for the principal components of the entire daily data set for the LEXNX fund. This is an obvious result of the high correlation between changes in net asset values for both portfolios.

The incorporation of the stock index into the study brings another source of risk into the picture. The R-Squared values associated with the above equations for the VFIIX fund in each period are posted in the following table.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.733360
Wednesday, March 10, 1999	0.712962
Wednesday, June 02, 1999	0.628363
Wednesday, August 25, 1999	0.822811
Wednesday, November 17, 1999	0.837863
Wednesday, February 09, 2000	0.818665

Average R-Squared Value
0.759004

One may compute the incremental average percentage of variance captured by the addition of the S&P 500 as 75.9004 – 75.1082 = 0.7922 percent, meaning action in the stock market accounts for less than 1 percent of the variability in the mortgage-backed fund. The area graph in Figure 29 of Appendix B conveys this additional accountability. This gain in explanatory power can be embodied as equity risk, and in this case is negligible for the VFIIX agency portfolio. Figure 30 reflects the marginal decrease in residual variance after every principal component data set is measured. One can realize that the first principal component on average accounts for less than 20 percent of the total variance of the mutual fund, whereas the second principal component contains over 43 percent of the overall variance. In essence, it seems as if the volatility of the S&P 500 forces the second principal component to have more meaning.

The regression equations of the indicators 1-12 using gamma coefficients serve as a more useful proxy for reducing residual variance of the mortgage fund. The linear models for each time period are printed below.

VFIIX Gamma Coefficients 1-12 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9	Gamma 10	Gamma 11	Gamma 12
A	2.68	-1.23	-1.47	-0.47	-1.44	0.72	-0.12	0.05	0.48	1.95	0.14	-0.15
B	0.55	0.15	0.35	-1.00	-0.49	-0.75	0.09	0.67	0.33	0.06	0.03	-0.08
C	3.73	-0.80	0.89	-0.33	0.25	0.17	-0.23	-0.05	1.30	0.06	0.01	0.01
D	4.12	-1.98	-1.74	0.02	0.06	0.14	0.40	0.65	1.05	-0.22	0.04	0.06
E	-1.20	1.21	-1.08	-0.26	-1.50	-1.17	0.28	-0.16	0.32	0.48	0.17	-0.14
F	-0.21	1.32	-0.06	-1.80	-0.50	-0.45	-0.63	-0.14	0.57	0.17	0.32	-0.04

A pictorial view of these regression coefficients in each 60-day window can be found in Figure 31. These econometric models could be utilized to interpret the source of variance in the VFIIX fund due to daily market variables.

As was the case with the LEXNX mortgage-backed portfolio, the remaining residual variance did not eclipse the 90 percent mark. Therefore, a principal components regression on the weekly data sets proved to be necessary. The residual of the VFIIX portfolio from the daily procedure was formed into a weekly set of fund changes, and this would become the response variable in a regression on the principal components of indicators 13-16, the changes of four various mortgage rates in basis points. The following equations specify the linear sensitivities of the residual agency fund to the principal components of the changes in these mortgage rates.

VFIIX Beta Coefficients 13-16 From Principal Components Regression

Period	Beta 13	Beta 14	Beta 15	Beta 16
A	-0.0001	-0.0044	0.0118	-0.0070
B	-0.0047	-0.0030	-0.0114	-0.0213
C	-0.0001	0.0063	0.0118	0.0655
D	-0.0021	-0.0014	-0.0003	-0.0119
E	-0.0034	0.0042	0.0043	-0.0007
F	-0.0015	0.0011	-0.0007	0.0209

Once again, these regression equations are analogous to the principal component regression equations from the previous fund in that they have nearly the same signs in their beta coefficients. This phenomenon portrays the idea that the LEXNX and VFIIX mutual funds consist of primarily the same mortgage-backed securities, or at least investments of the same risk.

One would want to check to see if the residual variance reduction from the weekly data is comparable from fund to fund as well. In the table below, the R-Squared values show the rise in explanatory power created by involving indicators 13-16.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.820619
Wednesday, March 10, 1999	0.951906
Wednesday, June 02, 1999	0.901535
Wednesday, August 25, 1999	0.988435
Wednesday, November 17, 1999	0.990013
Wednesday, February 09, 2000	0.974897

Average R-Squared Value
0.937901

One can quickly acknowledge that the additional data in the model contribute to a large decrease in the residual variance of the VFIIX mortgage fund, even greater than that of the LEXNX fund. The extra amount of variance accounted for by the weekly data is exactly equal to 93.7901 – 75.9004 = 17.8897 percent. This new portion of risk is illustrated in Figure 32, and can be interpreted as the degree of risk within the mortgage backed securities sector. While this risk factor is not completely uncorrelated with the risks determined by the daily data, it profoundly reduces the residual variance of the mortgage portfolio.

The gamma regression estimates indicate the best fit line through the indicators 13-16. The equalities below describe the relationship between the fund and each set of changes in mortgage rates.

VFIIX Gamma Coefficients 13-16 From Ordinary Least Squares Regression

Period	Gamma 13	Gamma 14	Gamma 15	Gamma 16
A	-0.0078	-0.0005	0.0003	0.0121
B	-0.0208	-0.0111	-0.0076	-0.0008
C	0.0658	-0.0041	0.0089	-0.0066
D	-0.0121	0.0007	0.0009	0.0007
E	-0.0005	0.0022	0.0030	0.0058
F	0.0209	-0.0002	0.0017	-0.0014

Any one of the equations could be used in conjunction with one of the formulas from the regression on the total daily data set as a replicating portfolio to measure the overall variance of VFIIX agency portfolio going into the future. This benchmark would need to be updated periodically to give accurate results based on regression parameter estimates. As a consequence of the two sets of equations explaining over 90 percent of the variability in this portfolio, the monthly data could be omitted and one could make the assumption that the left over residual risk is nonlinear in nature because of the option to prepay within mortgage-backed security contracts.

The Principal Components and Multiple Regressions: TGMBX Fund

The third and final fund observed in this report was the TGMBX agency mortgage-backed security portfolio. The table that follows outlines the econometric model used if working with only the principal components of the changes in the indicators 1-9.

TGMBX Beta Coefficients 1-9 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9
A	0.17	0.10	-0.12	-0.15	0.25	0.47	0.69	1.01	0.16
B	0.18	0.12	-0.06	0.16	-0.15	0.29	0.86	0.37	-0.48
C	0.21	0.21	0.09	-0.16	0.13	0.12	0.32	-1.51	2.87
D	0.36	-0.09	-0.45	-0.78	0.02	-0.01	-0.13	1.02	0.72
E	0.36	-0.02	-0.11	0.33	-0.33	0.56	0.52	1.60	-1.24
F	0.06	0.01	0.65	1.38	2.61	0.26	0.25	-2.73	-1.92

In comparison with the LEXNX and VFIIX mortgage funds, the beta coefficients have nearly always the same sign in their coefficients being multiplied by each principal component of the Treasury bonds. For example, the positive nature of the coefficient on the first principal component portrays the positive correlation between changes in the mortgage-backed portfolio and the riskiest allocation of assets in the changes of indicators 1-9. This consistency among all three of the funds reveals a trend regarding agency mortgage portfolios.

The measure of accountability by these models is calculated from the R-Squared values for each time frame. The percentage of variance of the TGMBX fund gained from the principal components regressions is depicted in the following table.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.817526
Wednesday, March 10, 1999	0.793551
Wednesday, June 02, 1999	0.761729
Wednesday, August 25, 1999	0.901769
Wednesday, November 17, 1999	0.867465
Wednesday, February 09, 2000	0.257966

Average R-Squared Value
0.733334

The last period appears to be an outlier, yet still the average R-Squared value utilizing the principal components of the Treasury bonds has a value over 73 percent. Figure 33 in Appendix B highlights the explanatory power for all six periods for the TGMBX fund. One can observe Figure 34, which demonstrates that a principal components regression on the first principal component alone would on average account for nearly 66 percent of this reduction in the fund variance. Hence, it may be worthwhile to reduce the dimension of the original indicators 1-9 and watch over the riskiest portfolio of Treasury bonds by itself.

Once again, working with principal components may be a shortcoming of this type of analysis. Converting the model to an ordinary least squares regression gives the following coefficients based on the government securities.

TGMBX Gamma Coefficients 1-9 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9
A	1.17	0.25	-0.58	-0.02	-0.04	0.23	-0.06	0.21	0.06
B	0.44	0.65	-0.55	0.45	-0.15	0.12	-0.20	0.25	0.17
C	0.82	-2.57	1.74	-0.39	0.03	-0.35	0.32	0.30	0.08
D	0.69	-0.73	-0.62	0.37	-0.37	-0.44	-0.49	0.52	0.40
E	0.67	1.09	-1.37	-1.04	-0.49	0.12	-0.22	0.12	0.34
F	-1.56	2.64	2.37	1.96	0.91	-0.73	0.02	-0.36	-0.15

The bar graph of Figure 35 details the values of these coefficients for every 60-day time frame. Besides the oddity in the last period, these gamma scalars have just about the same signs when related to the first two funds, demonstrating the strong positive correlation among the mortgage-backed portfolios.

The situation for TGMBX with indicators 1-11 takes on the same form as that of the two other funds. The beta coefficients for the principal components regression of this agency fund are specified in the chart below.

TGMBX Beta Coefficients 1-11 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9	Beta 10	Beta 11
A	0.16	0.10	-0.13	-0.11	-0.11	0.24	0.49	0.73	0.26	1.04	-0.05
B	0.17	0.11	0.09	-0.06	-0.17	-0.14	0.12	0.37	0.97	0.21	-0.28
C	0.20	0.18	0.15	-0.21	0.02	0.19	0.06	0.18	0.37	-1.49	3.10
D	0.34	0.06	-0.23	-0.41	-0.78	0.04	-0.03	-0.63	0.29	0.93	1.30
E	0.34	-0.06	0.13	-0.14	-0.18	0.27	0.61	0.34	0.16	0.94	-1.30
F	0.05	0.14	0.37	0.59	1.41	0.79	2.30	1.17	0.56	-2.40	-2.39

As before, ignoring the final period, the coefficients exhibit the relative togetherness of the changes in the net asset values of the TGMBX mutual fund and those of the first principal component. Thus, this risky portfolio of the indicators 1-11 appears to be closely in tune with movements in this mortgage portfolio.

The explanatory value of these regressions expose just how much of the variance of the changes in the fund can be claimed by the entire set of the principal components of the Treasury and corporate bonds. The R-Squared figures for each of the six time periods are listed below.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.822166
Wednesday, March 10, 1999	0.796691
Wednesday, June 02, 1999	0.766488
Wednesday, August 25, 1999	0.905512
Wednesday, November 17, 1999	0.871300
Wednesday, February 09, 2000	0.276083

Average R-Squared Value
0.739707

A graphical view of these accountability numbers is shown in Figure 36 of Appendix B at the end of this paper, in addition to the additional explanatory power given by the corporate debt. The average variance captured by the changes in Moody's bonds alone equals 73.9707 – 73.3334 = 0.6373 percent. The numbers in Figure 37 disclose the individual reduction in the residual variance of the TGMBX mortgage fund after taking each principal component into account. In direct relationship with the study of indicators 1-9, the first principal component of the changes in the Treasury bond plus the corporate bonds can be attributed with claiming an average of over 65 percent of the total variability in the mortgage-backed portfolio. Because the average R-Square from the entire set of principal components is just under 74 percent, the first one tends to serve as a good proxy for the whole.

Changing this analysis to be put in terms of the Treasury and corporate bonds would prove a better approach to deciphering variances, as these original daily data are more distinct in the market. The gamma coefficients of the ordinary least squares regression on the changes in indicators 1-11 are stated in the following table.

TGMBX Gamma Coefficients 1-11 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9	Gamma 10	Gamma 11
A	0.99	-0.32	-0.55	-0.11	0.19	0.03	0.20	0.07	-0.73	0.18	0.09
B	0.64	-0.77	-0.07	-0.14	0.32	0.11	0.36	0.01	0.11	0.11	0.11
C	0.63	-0.33	0.06	-0.27	-0.21	0.65	-0.15	-3.24	0.79	0.00	0.00
D	0.08	-1.62	0.62	0.17	-0.42	-0.59	0.19	-0.37	0.34	0.22	0.31
E	0.55	0.39	-1.25	-0.60	0.17	0.37	0.63	-0.53	0.09	0.15	0.31
F	1.06	3.51	2.36	-0.29	1.13	-0.55	0.68	0.42	0.43	-0.12	-0.13

These coefficients indicate the linear sensitivity of a change in the net asset value of the TGMBX portfolio for a movement in one of changes in the bonds. Figure 38 renders a pictorial representation of these scalar figures for each 60-day time frame. The regression equations determined by these gamma values could be considered an index to measure variability in the mortgage fund from Treasury and corporate bonds.

The last data set involved indicators 1-12, or every market variable measured on a daily basis used in this project. The addition of the S&P 500 to the analysis for the

TGMBX fund yielded a new calculation of beta coefficients in its principal components regression detailed below.

TGMBX Beta Coefficients 1-12 From Principal Components Regression

Period	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8	Beta 9	Beta 10	Beta 11	Beta 12
A	0.09	0.14	0.11	-0.11	-0.10	-0.11	0.27	0.42	0.79	0.31	1.03	-0.13
B	0.00	0.17	0.13	0.13	0.00	-0.14	-0.13	0.12	0.35	1.00	0.29	0.30
C	0.04	0.19	0.18	0.14	-0.20	0.02	-0.23	0.14	0.20	0.36	-1.49	3.05
D	0.17	0.27	0.10	0.12	-0.40	-0.79	0.04	-0.03	0.46	-0.31	1.02	0.60
E	0.10	0.31	0.03	-0.12	-0.12	-0.18	0.27	0.72	0.48	-0.01	1.02	-1.72
F	0.02	0.05	0.14	0.45	0.60	1.36	0.61	2.30	1.20	0.54	-2.31	-2.63

Considering the last time frame as an outlier, the study gives analogous results to the LEXNX and VFIIX mortgage funds. The correlations with the first principal component in each analysis are positive as exhibited in the first beta value being greater than zero.

As with the other daily variable examples, the R-Squared value measures the amount of variance explained by the indicators 1-12. The following figures for this parameter reflect the linear accountability of variance of these data sets.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.822435
Wednesday, March 10, 1999	0.801860
Wednesday, June 02, 1999	0.767847
Wednesday, August 25, 1999	0.911520
Wednesday, November 17, 1999	0.880868
Wednesday, February 09, 2000	0.282089

Average R-Squared Value
0.744436

One can observe that the S&P 500 brings about a new 74.4436 – 73.9707 = 0.4729 percent of variability in the model. This equity risk proves that the stock market does not appear to influence the variance of the TGMBX mortgage-backed fund. Figure 39 contains an area graph depicting the excess accountability brought about by the new market variable. For this particular portfolio, however, the first principal component eliminates only about 20 percent of its total variance, whereas the top two principal component data sets account for over 66 percent of the entire variability. This incremental drop in residual variance can be viewed in the table of Figure 40.

The ordinary least-squares regression gamma coefficients can be easily computed from the principal components regression for the TGMBX mortgage-backed fund. The following table establishes the regression coefficients in terms of the original indicators 1-12.

TGMBX Gamma Coefficients 1-12 From Ordinary Least Squares Regression

Period	Gamma 1	Gamma 2	Gamma 3	Gamma 4	Gamma 5	Gamma 6	Gamma 7	Gamma 8	Gamma 9	Gamma 10	Gamma 11	Gamma 12
A	0.64	0.69	-0.82	-0.06	-0.21	0.27	0.21	0.08	0.15	0.58	0.06	-0.04
B	-0.29	0.44	0.11	-0.90	0.04	-0.07	0.31	0.42	0.02	0.13	0.09	-0.05
C	2.98	-0.65	0.17	-0.29	0.62	0.20	0.02	0.15	1.43	-0.02	0.14	0.00
D	0.54	-0.29	-0.81	0.40	-0.08	-0.02	0.41	0.40	0.96	-0.37	0.05	0.04
E	-1.42	1.23	-0.66	-0.49	-0.18	-0.15	-0.47	-0.56	0.20	0.33	0.18	-0.11
F	-2.61	2.18	1.98	1.84	-0.27	-1.03	1.21	-0.53	-0.53	0.05	-0.11	-0.01

Because these econometric models provide the highest R-Squared values for any of the daily data analyses, one could make these equations into hedge portfolios capturing the greatest amount of variance in the agency portfolio. Figure 41 of Appendix B contains a bar chart showing the values of each gamma coefficient during the course of every 60-day period. These formulas depict how absolute changes in Treasury bonds, corporate bonds and the S&P 500 tend to claim the most variability of movements in the mortgage-backed mutual fund market.

The figure for R-Squared did not satisfy the goal of the project however, and so the residual fund changes data set had to be made into weekly observances and regressed on the principal components of the mortgage rates. The beta coefficients displaying the linear sensitivities of the residual fund data set to moves in the Freddie Mac, Fannie Mae and Ginnie Mae conventional loan rates are specified below.

TGMBX Beta Coefficients 13-16 From Principal Components Regression

Period	Beta 13	Beta 14	Beta 15	Beta 16
A	-0.0012	-0.0028	0.0078	0.0318
B	0.0005	-0.0045	0.0102	0.0241
C	0.0021	0.0043	0.0095	0.0968
D	-0.0034	-0.0023	0.0225	-0.0236
E	-0.0006	0.0001	0.0000	0.0653
F	-0.0013	0.0070	0.0020	0.0399

These beta coefficients are relatively in line with the corresponding beta coefficients of the previous two agency mortgage portfolios. The correlations between the first principal component of the mortgage rates and the funds vary over each of the six time periods.

The real gain of using both the daily and weekly data comes from the increase in explanatory power regarding the linear model. The R-Squared values after running two principal component regressions on indicators 1-12 and indicators 13-16 simultaneously are rendered in the table below.

Ending Window Date	R-Squared Values After PC Regression
Wednesday, December 16, 1998	0.943826
Wednesday, March 10, 1999	0.881522
Wednesday, June 02, 1999	0.814830
Wednesday, August 25, 1999	0.909471
Wednesday, November 17, 1999	0.903055
Wednesday, February 09, 2000	0.939248

Average R-Squared Value
0.898659

The mortgage rates for the changes in the TGMBX fund are responsible on average for 89.8659 – 74.4436 = 15.4223 percent of the overall variability by itself. This additional accountability is illustrated in Figure 42. The excess variance above that captured by the weekly variables can be attributed to mortgage sector risk, which need not be uncorrelated with the Treasury market risk, corporate credit risk and equity market risk presented in the regression on indicators 1-12.

In order to devise two linear models to evaluate this risk over time, one must have the gamma coefficients for the regression in terms of the actual mortgage rates, which are presented in the table below.

TGMBX Gamma Coefficients 13-16 From Ordinary Least Squares Regression

Period	Gamma 13	Gamma 14	Gamma 15	Gamma 16
A	0.0252	0.0012	-0.0200	0.0069
B	0.0249	0.0032	0.0081	0.0027
C	0.0961	-0.0122	0.0081	-0.0043
D	-0.0212	0.0089	0.0233	0.0030
E	0.0515	-0.0371	0.0154	0.0029
F	0.0394	0.0030	0.0077	-0.0051

Now, utilizing these equations in combination with those regression formulas from the daily variables could be worked to provide a good approximation to the overall variance of changes in the TGMBX mortgage-backed mutual fund. These types of benchmarks would give a fairly accurate description of the variability in the portfolio without taking the nonlinear nature of the securities into account.

Chapter 4

Executive Summary

To sum up the project, principal components and regression analyses can aid in understanding the risks of mortgage-backed security portfolios and identify the areas of the market from where these risks can be determined.

First, one can build a covariance matrix of absolute or relative changes in market variables, rates or indicators. The data can encompass security prices, company fundamentals, rates of return, sector indices, economic indicators, or any other related figures that are capable of explaining the variability of changes in net asset values of agency mortgage-backed mutual funds. This covariance matrix will contain the variances of each of these variables along its diagonal and most likely some strong positive and negative correlations between indicators.

The second step would be to extract the principal component variables and the variances of each principal component. This process involves the computation of the eigenvectors and eigenvalues of the covariance matrix of the original market variables and ordering them according in decreasing order corresponding to their eigenvalues. The principal components would form an orthogonal basis for a subspace of a much larger investment security vector space. In addition, this orthogonal characteristic would create new independent and uncorrelated variables derived from the original set.

Then, one could either run an ordinary least squares regression on the original changes in market variables or run a principal components regression on the new set of variables transformed from the market data.

The advantage of utilizing principal components regression would be the ability to regress only on a subset of the principal component variables, that is, the new variables that explain the greatest portion of variance of the original data set. Since the sum of the diagonal entries along both the covariance matrix of the market variables and those of the principal components are equivalent, the proportion of variance captured by each principal component can easily be identified. Thus, one could reduce dimension of the econometric model by utilizing solely this subset of principal components as predictor variables for the mortgage fund. The uncorrelated trait of these principal components would allow for one to observe the incremental accountability of each predictor variable as well. The explanatory power of the linear model could be computed to place a value on how well the regression equation would match action in the mortgage-backed security portfolio.

The disadvantage of using the principal components as predictor variables would be the lack of interpretability of the transformed data sets. If the principal components analysis was performed on a set of changes of market securities, the first principal component would be considered the weighted portfolio of the original securities with maximum risk such that the sums of the squares of the weights equal one. Every other principal component would be a compilation of the securities with the next highest risk with the constraint that the portfolios be uncorrelated with the previous ones. These portfolios are not directly observable in the financial markets, and so utilizing principal components may cause computational complexities.

An alternative method would be to undertake a simple ordinary least squares regression basing the response variable as the changes in the net asset values of the mortgage fund and the predictor variables as the changes in the market variables. The best linear unbiased estimates for such a regression model would minimize the total sum of squares of the response variable to a line. This sum of squares would then consist of two factors; one being the amount of variance described by the ordinary least squares relationship and the other as an estimate of residual variance.

This type of multiple regression has the advantage of being very simple. The regression coefficients are unbiased estimates, and thus no outside source is causing any dependence upon these values. The method takes into account any number of predictor variables without the need for any scaling procedures or variable transformations. Most of all, this econometric model can yield a great deal of explanatory power, and the addition of more and more predictor variables can reduce the residual variance down to a very small figure.

The drawback of ordinary least squares regression can be found in the possible high correlations among the predictor variables. This behavior, called multicollinearity, can skew the interpretation of the gamma estimates in that the standard errors of these parameters could be very large. As a consequence, the confidence intervals for the gamma values would be wide, and statistically these estimates may not be different from zero. Hence, the interpretative ability of this regression would not be clear, as the variance accountability would be vague.

Either linear model could be utilized to measure the variance of a mortgage-backed security mutual fund. A portfolio manager may wish to track the risk of such a portfolio and the investments held within it, as long as the explanatory power of the line had a high value. An analyst would need to update this regression equation with fresh coefficient estimates on a routine basis to capture the largest portion of variability in the fund on a consistent basis, but nonetheless, the model would serve as a good approximation to the overall variance of the mutual fund.

The risk measurement technique above could also be implemented on an individual mortgage-backed security level. Instead of using the changes in the net asset value of a fund as the response variable, one could study the effect of the predictor variables on a set of price changes in some particular agency debt investment. In doing so, one could derive a much stronger estimate for the variance of a mortgage-backed fund by accounting for the individual variances of each security as well as the covariances between them. This sort of procedure, while rather computationally more extensive, would detail the effect of movements in an entire portfolio based on the risk within one of its investments.

In all, risk measurement of mortgage-backed portfolios and securities is a highly researched topic. The difficulty in calculating the risk of such investments comes as a result of the complicated characteristics of the cash flows. Because every security in the secondary mortgage market inherently has coupon payments and short put options due to homeowner prepayments and refinancing behavior, the implied variances are not easily derived. The linear econometric model presented in this thesis attends to such a problem, and the estimate of variability defined in this project can be considered as a guideline for accounting for mortgage-backed mutual fund and security movements.

Appendix A

The Data Sources

Daily Data

Constant Maturity Treasury Rates

http://research.stlouisfed.org/fred/data/wkly.html

Corporate Bond Yield Indices

http://research.stlouisfed.org/fred/data/wkly.html

S&P 500

http://finance.yahoo.com/q?d=t&s=^GSPC

Weekly Data

Freddie Mac 30-Year Conventional Mortgage Rate

http://research.stlouisfed.org/fred/data/wkly/wmortg

Freddie Mac 15-Year Conventional Mortgage Rate

http://www.freddiemac.com/corporate/pmms/docs/historicalweeklydata.xls

Freddie Mac 1-Year Adjustable Mortgage Rate

http://www.freddiemac.com/corporate/pmms/docs/historicalweeklydata.xls

Fannie Mae 30-Year Conventional Mortgage Rate

http://www.freelunch.com

Monthly Data

Total Housing Starts, New One Family Houses Sold and Real Estate Loans at All Commercial Banks

http://research.stlouisfed.org/fred/data/business.html

Civilian Employment and Civilian Unemployment Rate

http://www.research.stlouisfed.org/fred/data/employ.html

Producer Price Index on Finished Goods

http://www.research.stlouisfed.org/fred/data/ppi/ppifgs

Total Pools of Securitized Assets

http://www.freelunch.com

Mutual Funds

LEXNX, VFIIX and TGMBX Net Asset Values http://finance.yahoo.com/

Appendix B

The Results Figures

Figure 1 **Covariance Matrices**

Beginning Date **9/23/98**
Ending Date **12/16/98**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	0.0007	0.0011	0.0016	0.0021	0.0024	0.0028	0.0022	0.0029	0.0021	0.0003	0.0003	-0.0064	0.0013	0.0002	0.0012	0.0020
2	0.0011	0.0019	0.0030	0.0045	0.0052	0.0066	0.0062	0.0072	0.0050	0.0011	0.0011	-0.0101	0.0026	0.0015	0.0029	0.0045
3	0.0016	0.0030	0.0060	0.0102	0.0129	0.0180	0.0203	0.0229	0.0186	0.0059	0.0056	-0.0254	0.0063	0.0052	0.0077	0.0126
4	0.0021	0.0045	0.0102	0.0225	0.0284	0.0415	0.0487	0.0543	0.0429	0.0143	0.0137	-0.0540	0.0142	0.0152	0.0173	0.0283
5	0.0024	0.0052	0.0129	0.0284	0.0417	0.0629	0.0777	0.0876	0.0719	0.0274	0.0255	-0.0724	0.0227	0.0260	0.0269	0.0434
6	0.0028	0.0066	0.0180	0.0415	0.0629	0.1024	0.1287	0.1467	0.1255	0.0490	0.0450	-0.0919	0.0389	0.0455	0.0451	0.0706
7	0.0022	0.0062	0.0203	0.0487	0.0777	0.1287	0.1773	0.1963	0.1777	0.0730	0.0666	-0.1168	0.0536	0.0649	0.0611	0.0928
8	0.0029	0.0072	0.0229	0.0543	0.0876	0.1467	0.1963	0.2357	0.2051	0.0859	0.0770	-0.1343	0.0603	0.0739	0.0690	0.1065
9	0.0021	0.0050	0.0186	0.0429	0.0719	0.1255	0.1777	0.2051	0.2196	0.0903	0.0800	-0.1422	0.0544	0.0641	0.0591	0.0965
10	0.0003	0.0011	0.0059	0.0143	0.0274	0.0490	0.0730	0.0859	0.0903	0.0451	0.0391	-0.0504	0.0240	0.0295	0.0244	0.0386
11	0.0003	0.0011	0.0056	0.0137	0.0255	0.0450	0.0666	0.0770	0.0800	0.0391	0.0354	-0.0429	0.0220	0.0272	0.0226	0.0350
12	-0.0064	-0.0101	-0.0254	-0.0540	-0.0724	-0.0919	-0.1168	-0.1343	-0.1422	-0.0504	-0.0429	0.9717	-0.0229	-0.0121	-0.0357	-0.0726
LEXNX	0.0013	0.0026	0.0063	0.0142	0.0227	0.0389	0.0536	0.0603	0.0544	0.0240	0.0220	-0.0229	0.0244	0.0269	0.0214	0.0283
VFIIX	0.0002	0.0015	0.0052	0.0152	0.0260	0.0455	0.0649	0.0739	0.0641	0.0295	0.0272	-0.0121	0.0269	0.0393	0.0249	0.0330
TGMBX	0.0012	0.0029	0.0077	0.0173	0.0269	0.0451	0.0611	0.0690	0.0591	0.0244	0.0226	-0.0357	0.0214	0.0249	0.0273	0.0322
EqWtTr	0.0020	0.0045	0.0126	0.0283	0.0434	0.0706	0.0928	0.1065	0.0965	0.0386	0.0350	-0.0726	0.0283	0.0330	0.0322	0.0508

Beginning Date **12/16/98**
Ending Date **3/10/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	0.0001	0.0001	0.0002	0.0002	0.0004	0.0006	0.0006	0.0008	0.0007	0.0002	0.0002	-0.0025	0.0002	0.0001	0.0003	0.0004
2	0.0001	0.0003	0.0005	0.0013	0.0020	0.0032	0.0037	0.0045	0.0041	0.0019	0.0014	-0.0025	0.0014	0.0011	0.0016	0.0022
3	0.0002	0.0005	0.0015	0.0033	0.0049	0.0078	0.0090	0.0108	0.0093	0.0042	0.0030	-0.0064	0.0032	0.0025	0.0038	0.0053
4	0.0002	0.0013	0.0033	0.0103	0.0150	0.0222	0.0271	0.0321	0.0291	0.0131	0.0097	-0.0146	0.0095	0.0080	0.0106	0.0156
5	0.0004	0.0020	0.0049	0.0150	0.0233	0.0348	0.0421	0.0500	0.0452	0.0202	0.0154	-0.0232	0.0152	0.0126	0.0168	0.0242
6	0.0006	0.0032	0.0078	0.0222	0.0348	0.0555	0.0659	0.0787	0.0713	0.0321	0.0247	-0.0299	0.0225	0.0189	0.0263	0.0378
7	0.0006	0.0037	0.0090	0.0271	0.0421	0.0659	0.0864	0.0986	0.0934	0.0392	0.0299	-0.0066	0.0295	0.0260	0.0332	0.0474
8	0.0008	0.0045	0.0108	0.0321	0.0500	0.0787	0.0986	0.1196	0.1083	0.0483	0.0367	-0.0250	0.0341	0.0306	0.0387	0.0559
9	0.0007	0.0041	0.0093	0.0291	0.0452	0.0713	0.0934	0.1083	0.1150	0.0440	0.0339	0.0185	0.0317	0.0251	0.0354	0.0529
10	0.0002	0.0019	0.0042	0.0131	0.0202	0.0321	0.0392	0.0483	0.0440	0.0239	0.0180	-0.0047	0.0144	0.0136	0.0157	0.0226
11	0.0002	0.0014	0.0030	0.0097	0.0154	0.0247	0.0299	0.0367	0.0339	0.0180	0.0147	-0.0011	0.0112	0.0102	0.0120	0.0172
12	-0.0025	-0.0025	-0.0064	-0.0146	-0.0232	-0.0299	-0.0066	-0.0250	0.0185	-0.0047	-0.0011	0.9647	0.0088	0.0098	-0.0005	-0.0102
LEXNX	0.0002	0.0014	0.0032	0.0095	0.0152	0.0225	0.0295	0.0341	0.0317	0.0144	0.0112	0.0088	0.0155	0.0124	0.0128	0.0164
VFIIX	0.0001	0.0011	0.0025	0.0080	0.0126	0.0189	0.0260	0.0306	0.0251	0.0136	0.0102	0.0098	0.0124	0.0148	0.0115	0.0139
TGMBX	0.0003	0.0016	0.0038	0.0106	0.0168	0.0263	0.0332	0.0387	0.0354	0.0157	0.0120	-0.0005	0.0128	0.0115	0.0167	0.0185
EqWtTr	0.0004	0.0022	0.0053	0.0156	0.0242	0.0378	0.0474	0.0559	0.0529	0.0226	0.0172	-0.0102	0.0164	0.0139	0.0185	0.0269

Figure 2 **Covariance Matrices**

Beginning Date **3/10/99**
Ending Date **6/2/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	0.0001	0.0001	0.0001	0.0001	0.0003	0.0005	0.0007	0.0009	0.0007	0.0003	0.0003	0.0004	0.0003	0.0004	0.0003	0.0004
2	0.0001	0.0002	0.0002	0.0005	0.0007	0.0013	0.0019	0.0021	0.0015	0.0008	0.0007	-0.0002	0.0006	0.0007	0.0007	0.0009
3	0.0001	0.0002	0.0010	0.0024	0.0037	0.0058	0.0074	0.0084	0.0074	0.0039	0.0033	0.0033	0.0029	0.0031	0.0035	0.0040
4	0.0001	0.0005	0.0024	0.0075	0.0110	0.0171	0.0220	0.0251	0.0223	0.0121	0.0102	0.0118	0.0087	0.0090	0.0100	0.0120
5	0.0003	0.0007	0.0037	0.0110	0.0186	0.0275	0.0354	0.0410	0.0362	0.0195	0.0164	0.0338	0.0136	0.0142	0.0159	0.0194
6	0.0005	0.0013	0.0058	0.0171	0.0275	0.0446	0.0580	0.0677	0.0597	0.0317	0.0267	0.0327	0.0223	0.0229	0.0258	0.0313
7	0.0007	0.0019	0.0074	0.0220	0.0354	0.0580	0.0805	0.0925	0.0825	0.0444	0.0372	0.0496	0.0305	0.0313	0.0346	0.0423
8	0.0009	0.0021	0.0084	0.0251	0.0410	0.0677	0.0925	0.1120	0.0961	0.0508	0.0426	0.0499	0.0342	0.0360	0.0401	0.0496
9	0.0007	0.0015	0.0074	0.0223	0.0362	0.0597	0.0825	0.0961	0.0939	0.0497	0.0411	0.0481	0.0313	0.0298	0.0349	0.0445
10	0.0003	0.0008	0.0039	0.0121	0.0195	0.0317	0.0444	0.0508	0.0497	0.0286	0.0232	0.0329	0.0168	0.0157	0.0191	0.0237
11	0.0003	0.0007	0.0033	0.0102	0.0164	0.0267	0.0372	0.0426	0.0411	0.0232	0.0196	0.0261	0.0140	0.0136	0.0160	0.0198
12	0.0004	-0.0002	0.0033	0.0118	0.0338	0.0327	0.0496	0.0499	0.0481	0.0329	0.0261	1.0252	0.0193	0.0289	0.0263	0.0255
LEXNX	0.0003	0.0006	0.0029	0.0087	0.0136	0.0223	0.0305	0.0342	0.0313	0.0168	0.0140	0.0193	0.0160	0.0136	0.0142	0.0160
VFIIX	0.0004	0.0007	0.0031	0.0090	0.0142	0.0229	0.0313	0.0360	0.0298	0.0157	0.0136	0.0289	0.0136	0.0218	0.0138	0.0164
TGMBX	0.0003	0.0007	0.0035	0.0100	0.0159	0.0258	0.0346	0.0401	0.0349	0.0191	0.0160	0.0263	0.0142	0.0138	0.0206	0.0184
EqWtTr	0.0004	0.0009	0.0040	0.0120	0.0194	0.0313	0.0423	0.0496	0.0445	0.0237	0.0198	0.0255	0.0160	0.0164	0.0184	0.0227

Beginning Date **6/2/99**
Ending Date **8/25/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	0.0002	0.0002	0.0003	0.0004	0.0006	0.0009	0.0011	0.0012	0.0008	0.0004	0.0003	0.0013	0.0012	0.0013	0.0009	0.0006
2	0.0002	0.0004	0.0005	0.0012	0.0017	0.0029	0.0033	0.0038	0.0029	0.0017	0.0015	0.0035	0.0031	0.0031	0.0025	0.0019
3	0.0003	0.0005	0.0016	0.0029	0.0045	0.0067	0.0082	0.0093	0.0075	0.0042	0.0038	0.0137	0.0067	0.0068	0.0056	0.0046
4	0.0004	0.0012	0.0029	0.0084	0.0127	0.0194	0.0238	0.0270	0.0214	0.0126	0.0113	0.0329	0.0203	0.0213	0.0168	0.0130
5	0.0006	0.0017	0.0045	0.0127	0.0208	0.0307	0.0379	0.0426	0.0339	0.0201	0.0180	0.0561	0.0317	0.0337	0.0269	0.0206
6	0.0009	0.0029	0.0067	0.0194	0.0307	0.0503	0.0605	0.0680	0.0538	0.0322	0.0285	0.0894	0.0532	0.0554	0.0437	0.0326
7	0.0011	0.0033	0.0082	0.0238	0.0379	0.0605	0.0796	0.0878	0.0707	0.0422	0.0373	0.1238	0.0713	0.0742	0.0585	0.0414
8	0.0012	0.0038	0.0093	0.0270	0.0426	0.0680	0.0878	0.1031	0.0796	0.0479	0.0422	0.1475	0.0815	0.0842	0.0658	0.0469
9	0.0008	0.0029	0.0075	0.0214	0.0339	0.0538	0.0707	0.0796	0.0688	0.0397	0.0348	0.1186	0.0631	0.0649	0.0512	0.0377
10	0.0004	0.0017	0.0042	0.0126	0.0201	0.0322	0.0422	0.0479	0.0397	0.0277	0.0239	0.0694	0.0386	0.0388	0.0309	0.0224
11	0.0003	0.0015	0.0038	0.0113	0.0180	0.0285	0.0373	0.0422	0.0348	0.0239	0.0213	0.0645	0.0342	0.0344	0.0276	0.0198
12	0.0013	0.0035	0.0137	0.0329	0.0561	0.0894	0.1238	0.1475	0.1186	0.0694	0.0645	0.7578	0.1392	0.1512	0.1092	0.0652
LEXNX	0.0012	0.0031	0.0067	0.0203	0.0317	0.0532	0.0713	0.0815	0.0631	0.0386	0.0342	0.1392	0.0861	0.0858	0.0609	0.0369
VFIIX	0.0013	0.0031	0.0068	0.0213	0.0337	0.0554	0.0742	0.0842	0.0649	0.0388	0.0344	0.1512	0.0858	0.0919	0.0620	0.0383
TGMBX	0.0009	0.0025	0.0056	0.0168	0.0269	0.0437	0.0585	0.0658	0.0512	0.0309	0.0276	0.1092	0.0609	0.0620	0.0493	0.0302
EqWtTr	0.0006	0.0019	0.0046	0.0130	0.0206	0.0326	0.0414	0.0469	0.0377	0.0224	0.0198	0.0652	0.0369	0.0383	0.0302	0.0222

Figure 3 **Covariance Matrices**

Beginning Date **8/25/99**
Ending Date **11/17/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	0.0001	0.0001	0.0002	0.0004	0.0007	0.0010	0.0012	0.0014	0.0011	0.0007	0.0006	0.0021	0.0012	0.0013	0.0009	0.0007
2	0.0001	0.0002	0.0003	0.0008	0.0011	0.0017	0.0021	0.0026	0.0021	0.0015	0.0011	0.0020	0.0027	0.0024	0.0019	0.0012
3	0.0002	0.0003	0.0012	0.0027	0.0038	0.0062	0.0080	0.0091	0.0083	0.0048	0.0036	0.0096	0.0075	0.0073	0.0060	0.0044
4	0.0004	0.0008	0.0027	0.0075	0.0104	0.0164	0.0216	0.0246	0.0224	0.0131	0.0097	0.0241	0.0199	0.0195	0.0161	0.0119
5	0.0007	0.0011	0.0038	0.0104	0.0162	0.0241	0.0308	0.0352	0.0319	0.0182	0.0144	0.0374	0.0287	0.0276	0.0227	0.0171
6	0.0010	0.0017	0.0062	0.0164	0.0241	0.0402	0.0515	0.0584	0.0537	0.0309	0.0245	0.0610	0.0469	0.0459	0.0386	0.0281
7	0.0012	0.0021	0.0080	0.0216	0.0308	0.0515	0.0702	0.0781	0.0726	0.0424	0.0323	0.1001	0.0637	0.0635	0.0515	0.0373
8	0.0014	0.0026	0.0091	0.0246	0.0352	0.0584	0.0781	0.0901	0.0828	0.0486	0.0369	0.1156	0.0728	0.0696	0.0579	0.0425
9	0.0011	0.0021	0.0083	0.0224	0.0319	0.0537	0.0726	0.0828	0.0807	0.0463	0.0355	0.1204	0.0669	0.0639	0.0545	0.0395
10	0.0007	0.0015	0.0048	0.0131	0.0182	0.0309	0.0424	0.0486	0.0463	0.0290	0.0216	0.0830	0.0413	0.0402	0.0324	0.0230
11	0.0006	0.0011	0.0036	0.0097	0.0144	0.0245	0.0323	0.0369	0.0355	0.0216	0.0200	0.0730	0.0324	0.0307	0.0253	0.0176
12	0.0021	0.0020	0.0096	0.0241	0.0374	0.0610	0.1001	0.1156	0.1204	0.0830	0.0730	1.1502	0.1383	0.1168	0.0929	0.0525
LEXNX	0.0012	0.0027	0.0075	0.0199	0.0287	0.0469	0.0637	0.0728	0.0669	0.0413	0.0324	0.1383	0.0738	0.0665	0.0543	0.0345
VFIIX	0.0013	0.0024	0.0073	0.0195	0.0276	0.0459	0.0635	0.0696	0.0639	0.0402	0.0307	0.1168	0.0665	0.0749	0.0507	0.0334
TGMBX	0.0009	0.0019	0.0060	0.0161	0.0227	0.0386	0.0515	0.0579	0.0545	0.0324	0.0253	0.0929	0.0543	0.0507	0.0449	0.0278
EqWtTr	0.0007	0.0012	0.0044	0.0119	0.0171	0.0281	0.0373	0.0425	0.0395	0.0230	0.0176	0.0525	0.0345	0.0334	0.0278	0.0203

Beginning Date **11/17/99**
Ending Date **2/9/00**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	0.0002	0.0002	0.0003	0.0005	0.0007	0.0011	0.0012	0.0014	0.0016	0.0010	0.0009	0.0018	0.0012	0.0011	0.0005	0.0008
2	0.0002	0.0004	0.0005	0.0010	0.0015	0.0026	0.0029	0.0035	0.0034	0.0023	0.0021	0.0011	0.0027	0.0026	0.0006	0.0018
3	0.0003	0.0005	0.0013	0.0024	0.0035	0.0059	0.0066	0.0080	0.0077	0.0046	0.0042	0.0027	0.0056	0.0051	0.0020	0.0041
4	0.0005	0.0010	0.0024	0.0060	0.0088	0.0141	0.0160	0.0193	0.0167	0.0100	0.0090	0.0069	0.0148	0.0127	0.0035	0.0094
5	0.0007	0.0015	0.0035	0.0088	0.0139	0.0220	0.0248	0.0299	0.0261	0.0153	0.0142	0.0085	0.0224	0.0195	0.0049	0.0146
6	0.0011	0.0026	0.0059	0.0141	0.0220	0.0377	0.0436	0.0529	0.0474	0.0284	0.0264	0.0079	0.0369	0.0331	0.0049	0.0253
7	0.0012	0.0029	0.0066	0.0160	0.0248	0.0436	0.0554	0.0652	0.0618	0.0368	0.0342	0.0085	0.0468	0.0434	0.0085	0.0309
8	0.0014	0.0035	0.0080	0.0193	0.0299	0.0529	0.0652	0.0806	0.0756	0.0448	0.0415	0.0048	0.0553	0.0506	0.0072	0.0374
9	0.0016	0.0034	0.0077	0.0167	0.0261	0.0474	0.0618	0.0756	0.0824	0.0467	0.0434	-0.0124	0.0523	0.0503	0.0092	0.0359
10	0.0010	0.0023	0.0046	0.0100	0.0153	0.0284	0.0368	0.0448	0.0467	0.0306	0.0295	0.0105	0.0321	0.0305	0.0032	0.0211
11	0.0009	0.0021	0.0042	0.0090	0.0142	0.0264	0.0342	0.0415	0.0434	0.0295	0.0306	0.0074	0.0299	0.0284	0.0020	0.0195
12	0.0018	0.0011	0.0027	0.0069	0.0085	0.0079	0.0085	0.0048	-0.0124	0.0105	0.0074	1.0757	0.0309	0.0259	0.0215	0.0033
LEXNX	0.0012	0.0027	0.0056	0.0148	0.0224	0.0369	0.0468	0.0553	0.0523	0.0321	0.0299	0.0309	0.0493	0.0414	0.0092	0.0264
VFIIX	0.0011	0.0026	0.0051	0.0127	0.0195	0.0331	0.0434	0.0506	0.0503	0.0305	0.0284	0.0259	0.0414	0.0449	0.0093	0.0243
TGMBX	0.0005	0.0006	0.0020	0.0035	0.0049	0.0049	0.0085	0.0072	0.0092	0.0032	0.0020	0.0215	0.0092	0.0093	0.0428	0.0046
EqWtTr	0.0008	0.0018	0.0041	0.0094	0.0146	0.0253	0.0309	0.0374	0.0359	0.0211	0.0195	0.0033	0.0264	0.0243	0.0046	0.0178

Figure 4 **Correlation Matrices**

Beginning Date **9/23/98**
Ending Date **12/16/98**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	1.0000	0.9170	0.7807	0.5328	0.4380	0.3300	0.1936	0.2236	0.1655	0.0474	0.0565	-0.2458	0.3071	0.0430	0.2829	0.3298
2	0.9170	1.0000	0.8933	0.6981	0.5912	0.4783	0.3370	0.3407	0.2474	0.1214	0.1326	-0.2351	0.3877	0.1755	0.3985	0.4626
3	0.7807	0.8933	1.0000	0.8810	0.8196	0.7290	0.6228	0.6094	0.5136	0.3572	0.3820	-0.3331	0.5250	0.3377	0.6000	0.7237
4	0.5328	0.6981	0.8810	1.0000	0.9285	0.8658	0.7713	0.7454	0.6102	0.4490	0.4841	-0.3652	0.6075	0.5124	0.6999	0.8388
5	0.4380	0.5912	0.8196	0.9285	1.0000	0.9623	0.9037	0.8842	0.7521	0.6333	0.6633	-0.3599	0.7116	0.6429	0.7972	0.9436
6	0.3300	0.4783	0.7290	0.8658	0.9623	1.0000	0.9551	0.9445	0.8368	0.7216	0.7478	-0.2913	0.7796	0.7167	0.8530	0.9785
7	0.1936	0.3370	0.6228	0.7713	0.9037	0.9551	1.0000	0.9604	0.9010	0.8173	0.8409	-0.2815	0.8157	0.7774	0.8784	0.9776
8	0.2236	0.3407	0.6094	0.7454	0.8842	0.9445	0.9604	1.0000	0.9018	0.8338	0.8432	-0.2805	0.7960	0.7681	0.8604	0.9735
9	0.1655	0.2474	0.5136	0.6102	0.7521	0.8368	0.9010	0.9018	1.0000	0.9076	0.9070	-0.3078	0.7438	0.6901	0.7636	0.9136
10	0.0474	0.1214	0.3572	0.4490	0.6333	0.7216	0.8173	0.8338	0.9076	1.0000	0.9782	-0.2410	0.7239	0.7011	0.6960	0.8064
11	0.0565	0.1326	0.3820	0.4841	0.6633	0.7478	0.8409	0.8432	0.9070	0.9782	1.0000	-0.2315	0.7494	0.7289	0.7287	0.8245
12	-0.2458	-0.2351	-0.3331	-0.3652	-0.3599	-0.2913	-0.2815	-0.2805	-0.3078	-0.2410	-0.2315	1.0000	-0.1490	-0.0620	-0.2193	-0.3268
LEXNX	0.3071	0.3877	0.5250	0.6075	0.7116	0.7796	0.8157	0.7960	0.7438	0.7239	0.7494	-0.1490	1.0000	0.8701	0.8315	0.8034
VFIIX	0.0430	0.1755	0.3377	0.5124	0.6429	0.7167	0.7774	0.7681	0.6901	0.7011	0.7289	-0.0620	0.8701	1.0000	0.7595	0.7374
TGMBX	0.2829	0.3985	0.6000	0.6999	0.7972	0.8530	0.8784	0.8604	0.7636	0.6960	0.7287	-0.2193	0.8315	0.7595	1.0000	0.8662
EqWtTr	0.3298	0.4626	0.7237	0.8388	0.9436	0.9785	0.9776	0.9735	0.9136	0.8064	0.8245	-0.3268	0.8034	0.7374	0.8662	1.0000

Beginning Date **12/16/98**
Ending Date **3/10/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	1.0000	0.5629	0.5485	0.2879	0.3003	0.3155	0.2448	0.2874	0.2462	0.1853	0.1916	-0.2979	0.2233	0.1173	0.3105	0.3009
2	0.5629	1.0000	0.7895	0.7725	0.7838	0.7877	0.7273	0.7561	0.6986	0.7026	0.6741	-0.1461	0.6531	0.5385	0.7164	0.7787
3	0.5485	0.7895	1.0000	0.8248	0.8180	0.8443	0.7797	0.7998	0.7015	0.6897	0.6344	-0.1669	0.6568	0.5310	0.7608	0.8193
4	0.2879	0.7725	0.8248	1.0000	0.9700	0.9258	0.9083	0.9144	0.8455	0.8321	0.7891	-0.1461	0.7535	0.6504	0.8106	0.9393
5	0.3003	0.7838	0.8180	0.9700	1.0000	0.9698	0.9388	0.9484	0.8738	0.8564	0.8345	-0.1550	0.7999	0.6813	0.8515	0.9679
6	0.3155	0.7877	0.8443	0.9258	0.9698	1.0000	0.9515	0.9657	0.8926	0.8797	0.8619	-0.1291	0.7661	0.6606	0.8654	0.9784
7	0.2448	0.7273	0.7797	0.9083	0.9388	0.9515	1.0000	0.9695	0.9365	0.8616	0.8371	-0.0229	0.8049	0.7259	0.8754	0.9840
8	0.2874	0.7561	0.7998	0.9144	0.9484	0.9657	0.9695	1.0000	0.9239	0.9026	0.8746	-0.0737	0.7920	0.7265	0.8662	0.9870
9	0.2462	0.6986	0.7015	0.8455	0.8738	0.8926	0.9365	0.9239	1.0000	0.8398	0.8240	0.0555	0.7491	0.6087	0.8093	0.9524
10	0.1853	0.7026	0.6897	0.8321	0.8564	0.8797	0.8616	0.9026	0.8398	1.0000	0.9582	-0.0310	0.7477	0.7198	0.7878	0.8903
11	0.1916	0.6741	0.6344	0.7891	0.8345	0.8619	0.8371	0.8746	0.8240	0.9582	1.0000	-0.0093	0.7420	0.6894	0.7677	0.8654
12	-0.2979	-0.1461	-0.1669	-0.1461	-0.1550	-0.1291	-0.0229	-0.0737	0.0555	-0.0310	-0.0093	1.0000	0.0716	0.0822	-0.0036	-0.0636
LEXNX	0.2233	0.6531	0.6568	0.7535	0.7999	0.7661	0.8049	0.7920	0.7491	0.7477	0.7420	0.0716	1.0000	0.8168	0.7953	0.8016
VFIIX	0.1173	0.5385	0.5310	0.6504	0.6813	0.6606	0.7259	0.7265	0.6087	0.7198	0.6894	0.0822	0.8168	1.0000	0.7335	0.6967
TGMBX	0.3105	0.7164	0.7608	0.8106	0.8515	0.8654	0.8754	0.8662	0.8093	0.7878	0.7677	-0.0036	0.7953	0.7335	1.0000	0.8760
EqWtTr	0.3009	0.7787	0.8193	0.9393	0.9679	0.9784	0.9840	0.9870	0.9524	0.8903	0.8654	-0.0636	0.8016	0.6967	0.8760	1.0000

Figure 5 **Correlation Matrices**

Beginning Date **3/10/99**
Ending Date **6/2/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	1.0000	0.5522	0.3540	0.2048	0.2595	0.3136	0.3496	0.3702	0.3208	0.2614	0.2784	0.0573	0.3483	0.3396	0.3387	0.3439
2	0.5522	1.0000	0.5532	0.4130	0.3743	0.4589	0.4953	0.4823	0.3625	0.3445	0.3644	-0.0144	0.3829	0.3690	0.3877	0.4655
3	0.3540	0.5532	1.0000	0.8799	0.8519	0.8719	0.8221	0.7959	0.7617	0.7304	0.7451	0.1023	0.7174	0.6632	0.7723	0.8485
4	0.2048	0.4130	0.8799	1.0000	0.9344	0.9353	0.8944	0.8675	0.8420	0.8251	0.8453	0.1341	0.7920	0.7034	0.8084	0.9203
5	0.2595	0.3743	0.8519	0.9344	1.0000	0.9564	0.9152	0.8992	0.8661	0.8459	0.8594	0.2450	0.7874	0.7070	0.8159	0.9429
6	0.3136	0.4589	0.8719	0.9353	0.9564	1.0000	0.9685	0.9582	0.9223	0.8880	0.9056	0.1531	0.8377	0.7353	0.8520	0.9853
7	0.3496	0.4953	0.8221	0.8944	0.9152	0.9685	1.0000	0.9743	0.9491	0.9254	0.9370	0.1728	0.8504	0.7466	0.8506	0.9897
8	0.3702	0.4823	0.7959	0.8675	0.8992	0.9582	0.9743	1.0000	0.9371	0.8973	0.9108	0.1473	0.8082	0.7283	0.8364	0.9824
9	0.3208	0.3625	0.7617	0.8420	0.8661	0.9223	0.9491	0.9371	1.0000	0.9592	0.9583	0.1549	0.8085	0.6591	0.7939	0.9630
10	0.2614	0.3445	0.7304	0.8251	0.8459	0.8880	0.9254	0.8973	0.9592	1.0000	0.9821	0.1922	0.7856	0.6272	0.7861	0.9294
11	0.2784	0.3644	0.7451	0.8453	0.8594	0.9056	0.9370	0.9108	0.9583	0.9821	1.0000	0.1842	0.7939	0.6579	0.7980	0.9409
12	0.0573	-0.0144	0.1023	0.1341	0.2450	0.1531	0.1728	0.1473	0.1549	0.1922	0.1842	1.0000	0.1510	0.1933	0.1809	0.1670
LEXNX	0.3483	0.3829	0.7174	0.7920	0.7874	0.8377	0.8504	0.8082	0.8085	0.7856	0.7939	0.1510	1.0000	0.7278	0.7837	0.8423
VFIIX	0.3396	0.3690	0.6632	0.7034	0.7070	0.7353	0.7466	0.7283	0.6591	0.6272	0.6579	0.1933	0.7278	1.0000	0.6527	0.7360
TGMBX	0.3387	0.3877	0.7723	0.8084	0.8159	0.8520	0.8506	0.8364	0.7939	0.7861	0.7980	0.1809	0.7837	0.6527	1.0000	0.8535
EqWtTr	0.3439	0.4655	0.8485	0.9203	0.9429	0.9853	0.9897	0.9824	0.9630	0.9294	0.9409	0.1670	0.8423	0.7360	0.8535	1.0000

Beginning Date **6/2/99**
Ending Date **8/25/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	1.0000	0.7620	0.4457	0.3387	0.2765	0.2970	0.2736	0.2681	0.2122	0.1644	0.1622	0.1022	0.2864	0.3099	0.2958	0.3017
2	0.7620	1.0000	0.6514	0.6207	0.5855	0.6243	0.5756	0.5750	0.5416	0.4910	0.5110	0.1960	0.5190	0.4928	0.5533	0.6179
3	0.4457	0.6514	1.0000	0.7786	0.7678	0.7446	0.7235	0.7190	0.7102	0.6330	0.6465	0.3899	0.5648	0.5579	0.6271	0.7689
4	0.3387	0.6207	0.7786	1.0000	0.9627	0.9470	0.9240	0.9209	0.8910	0.8311	0.8456	0.4138	0.7573	0.7699	0.8299	0.9566
5	0.2765	0.5855	0.7678	0.9627	1.0000	0.9510	0.9321	0.9209	0.8966	0.8376	0.8556	0.4467	0.7502	0.7714	0.8404	0.9600
6	0.2970	0.6243	0.7446	0.9470	0.9510	1.0000	0.9561	0.9452	0.9144	0.8639	0.8712	0.4583	0.8087	0.8154	0.8770	0.9763
7	0.2736	0.5756	0.7235	0.9240	0.9321	0.9561	1.0000	0.9695	0.9550	0.8997	0.9063	0.5040	0.8613	0.8675	0.9345	0.9868
8	0.2681	0.5750	0.7190	0.9209	0.9209	0.9452	0.9695	1.0000	0.9445	0.8973	0.9012	0.5278	0.8648	0.8645	0.9234	0.9822
9	0.2122	0.5416	0.7102	0.8910	0.8966	0.9144	0.9550	0.9445	1.0000	0.9103	0.9101	0.5192	0.8192	0.8157	0.8789	0.9655
10	0.1644	0.4910	0.6330	0.8311	0.8376	0.8639	0.8997	0.8973	0.9103	1.0000	0.9820	0.4790	0.7901	0.7696	0.8367	0.9025
11	0.1622	0.5110	0.6465	0.8456	0.8556	0.8712	0.9063	0.9012	0.9101	0.9820	1.0000	0.5077	0.7988	0.7785	0.8505	0.9096
12	0.1022	0.1960	0.3899	0.4138	0.4467	0.4583	0.5040	0.5278	0.5192	0.4790	0.5077	1.0000	0.5450	0.5730	0.5648	0.5031
LEXNX	0.2864	0.5190	0.5648	0.7573	0.7502	0.8087	0.8613	0.8648	0.8192	0.7901	0.7988	0.5450	1.0000	0.9646	0.9344	0.8448
VFIIX	0.3099	0.4928	0.5579	0.7699	0.7714	0.8154	0.8675	0.8645	0.8157	0.7696	0.7785	0.5730	0.9646	1.0000	0.9209	0.8492
TGMBX	0.2958	0.5533	0.6271	0.8299	0.8404	0.8770	0.9345	0.9234	0.8789	0.8367	0.8505	0.5648	0.9344	0.9209	1.0000	0.9145
EqWtTr	0.3017	0.6179	0.7689	0.9566	0.9600	0.9763	0.9868	0.9822	0.9655	0.9025	0.9096	0.5031	0.8448	0.8492	0.9145	1.0000

Figure 6 **Correlation Matrices**

Beginning Date **8/25/99**
Ending Date **11/17/99**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	1.0000	0.4324	0.4544	0.4505	0.4533	0.4246	0.4106	0.4072	0.3533	0.3799	0.3722	0.1729	0.3888	0.4051	0.3741	0.4263
2	0.4324	1.0000	0.6145	0.6101	0.5710	0.5519	0.5034	0.5414	0.4753	0.5483	0.5045	0.1200	0.6292	0.5515	0.5572	0.5524
3	0.4544	0.6145	1.0000	0.9112	0.8859	0.9139	0.8895	0.8910	0.8562	0.8353	0.7447	0.2642	0.8159	0.7849	0.8318	0.9121
4	0.4505	0.6101	0.9112	1.0000	0.9399	0.9479	0.9413	0.9464	0.9126	0.8916	0.7955	0.2603	0.8469	0.8249	0.8793	0.9625
5	0.4533	0.5710	0.8859	0.9399	1.0000	0.9437	0.9138	0.9194	0.8803	0.8407	0.7976	0.2740	0.8302	0.7911	0.8403	0.9435
6	0.4246	0.5519	0.9139	0.9479	0.9437	1.0000	0.9699	0.9705	0.9436	0.9056	0.8655	0.2839	0.8621	0.8377	0.9096	0.9850
7	0.4106	0.5034	0.8895	0.9413	0.9138	0.9699	1.0000	0.9823	0.9651	0.9393	0.8621	0.3523	0.8856	0.8763	0.9166	0.9893
8	0.4072	0.5414	0.8910	0.9464	0.9194	0.9705	0.9823	1.0000	0.9713	0.9515	0.8693	0.3592	0.8922	0.8476	0.9101	0.9926
9	0.3533	0.4753	0.8562	0.9126	0.8803	0.9436	0.9651	0.9713	1.0000	0.9582	0.8826	0.3952	0.8674	0.8222	0.9054	0.9761
10	0.3799	0.5483	0.8353	0.8916	0.8407	0.9056	0.9393	0.9515	0.9582	1.0000	0.8953	0.4548	0.8922	0.8638	0.8976	0.9462
11	0.3722	0.5045	0.7447	0.7955	0.7976	0.8655	0.8621	0.8693	0.8826	0.8953	1.0000	0.4812	0.8441	0.7934	0.8439	0.8743
12	0.1729	0.1200	0.2642	0.2603	0.2740	0.2839	0.3523	0.3592	0.3952	0.4548	0.4812	1.0000	0.4748	0.3981	0.4085	0.3435
LEXNX	0.3888	0.6292	0.8159	0.8469	0.8302	0.8621	0.8856	0.8922	0.8674	0.8922	0.8441	0.4748	1.0000	0.8948	0.9424	0.8908
VFIIX	0.4051	0.5515	0.7849	0.8249	0.7911	0.8377	0.8763	0.8476	0.8222	0.8638	0.7934	0.3981	0.8948	1.0000	0.8737	0.8577
TGMBX	0.3741	0.5572	0.8318	0.8793	0.8403	0.9096	0.9166	0.9101	0.9054	0.8976	0.8439	0.4085	0.9424	0.8737	1.0000	0.9198
EqWtTr	0.4263	0.5524	0.9121	0.9625	0.9435	0.9850	0.9893	0.9926	0.9761	0.9462	0.8743	0.3435	0.8908	0.8577	0.9198	1.0000

Beginning Date **11/17/99**
Ending Date **2/9/00**

	1	2	3	4	5	6	7	8	9	10	11	12	LEXNX	VFIIX	TGMBX	EqWtTr
1	1.0000	0.7224	0.6291	0.4441	0.4229	0.3983	0.3744	0.3501	0.3988	0.4056	0.3563	0.1230	0.3757	0.3803	0.1783	0.4300
2	0.7224	1.0000	0.6880	0.6250	0.6232	0.6286	0.5972	0.5828	0.5674	0.6144	0.5728	0.0492	0.5858	0.5865	0.1384	0.6411
3	0.6291	0.6880	1.0000	0.8599	0.8229	0.8274	0.7719	0.7720	0.7361	0.7202	0.6623	0.0703	0.6936	0.6543	0.2632	0.8305
4	0.4441	0.6250	0.8599	1.0000	0.9582	0.9341	0.8757	0.8725	0.7491	0.7339	0.6610	0.0854	0.8561	0.7729	0.2175	0.9098
5	0.4229	0.6232	0.8229	0.9582	1.0000	0.9595	0.8956	0.8945	0.7709	0.7433	0.6878	0.0692	0.8548	0.7789	0.1990	0.9282
6	0.3983	0.6286	0.8274	0.9341	0.9595	1.0000	0.9547	0.9583	0.8506	0.8341	0.7754	0.0390	0.8559	0.8050	0.1220	0.9749
7	0.3744	0.5972	0.7719	0.8757	0.8956	0.9547	1.0000	0.9755	0.9153	0.8938	0.8302	0.0347	0.8963	0.8711	0.1749	0.9834
8	0.3501	0.5828	0.7720	0.8725	0.8945	0.9583	0.9755	1.0000	0.9275	0.9023	0.8354	0.0164	0.8779	0.8413	0.1229	0.9871
9	0.3988	0.5674	0.7361	0.7491	0.7709	0.8506	0.9153	0.9275	1.0000	0.9288	0.8649	-0.0416	0.8202	0.8275	0.1549	0.9371
10	0.4056	0.6144	0.7202	0.7339	0.7433	0.8341	0.8938	0.9023	0.9288	1.0000	0.9617	0.0578	0.8260	0.8229	0.0885	0.9039
11	0.3563	0.5728	0.6623	0.6610	0.6878	0.7754	0.8302	0.8354	0.8649	0.9617	1.0000	0.0409	0.7700	0.7672	0.0543	0.8376
12	0.1230	0.0492	0.0703	0.0854	0.0692	0.0390	0.0347	0.0164	-0.0416	0.0578	0.0409	1.0000	0.1341	0.1178	0.1004	0.0238
LEXNX	0.3757	0.5858	0.6936	0.8561	0.8548	0.8559	0.8963	0.8779	0.8202	0.8260	0.7700	0.1341	1.0000	0.8804	0.2000	0.8934
VFIIX	0.3803	0.5865	0.6543	0.7729	0.7789	0.8050	0.8711	0.8413	0.8275	0.8229	0.7672	0.1178	0.8804	1.0000	0.2119	0.8593
TGMBX	0.1783	0.1384	0.2632	0.2175	0.1990	0.1220	0.1749	0.1229	0.1549	0.0885	0.0543	0.1004	0.2000	0.2119	1.0000	0.1663
EqWtTr	0.4300	0.6411	0.8305	0.9098	0.9282	0.9749	0.9834	0.9871	0.9371	0.9039	0.8376	0.0238	0.8934	0.8593	0.1663	1.0000

Figure 7 **Principal Component Variances For Indicators 1-9**

Time Period	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9
A	0.73984	0.04184	0.01314	0.00760	0.00259	0.00137	0.00101	0.00018	0.00006
B	0.39082	0.01246	0.00333	0.00292	0.00158	0.00052	0.00023	0.00009	0.00003
C	0.34194	0.00781	0.00482	0.00193	0.00083	0.00063	0.00023	0.00007	0.00003
D	0.31789	0.00596	0.00406	0.00223	0.00155	0.00076	0.00040	0.00026	0.00004
E	0.29645	0.00468	0.00194	0.00139	0.00101	0.00052	0.00017	0.00012	0.00009
F	0.25996	0.01225	0.00257	0.00161	0.00065	0.00050	0.00029	0.00016	0.00006

Figure 8 **Principal Component Variances For Indicators 1-11**

Time Period	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11
A	0.80026	0.05284	0.01373	0.00811	0.00726	0.00256	0.00136	0.00097	0.00076	0.00018	0.00006
B	0.42151	0.01284	0.00735	0.00333	0.00246	0.00163	0.00079	0.00041	0.00021	0.00009	0.00003
C	0.38505	0.01010	0.00508	0.00241	0.00167	0.00081	0.00062	0.00038	0.00022	0.00007	0.00003
D	0.35955	0.00834	0.00459	0.00403	0.00223	0.00157	0.00074	0.00041	0.00040	0.00026	0.00004
E	0.33877	0.00627	0.00383	0.00201	0.00172	0.00116	0.00084	0.00042	0.00016	0.00010	0.00009
F	0.30986	0.01618	0.00675	0.00255	0.00160	0.00087	0.00061	0.00039	0.00028	0.00015	0.00005

Figure 9 **Principal Component Variances For Indicators 1-12**

Time Period	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
A	1.16924	0.60408	0.05265	0.01289	0.0081	0.0071	0.00251	0.00133	0.00091	0.00076	0.00018	0.00006
B	0.9681	0.42025	0.01115	0.00712	0.00318	0.00239	0.00163	0.00079	0.00041	0.00021	0.00009	0.00002
C	1.04239	0.36812	0.0101	0.00498	0.00238	0.00166	0.00071	0.00061	0.00038	0.00022	0.00007	0.00003
D	0.8935	0.22416	0.00823	0.00448	0.00396	0.00223	0.00157	0.00074	0.0004	0.00038	0.00026	0.00004
E	1.21371	0.27671	0.00503	0.00378	0.00197	0.00171	0.00115	0.00078	0.00041	0.00016	0.00010	0.00008
F	1.07627	0.30973	0.0161	0.00641	0.00255	0.0016	0.00082	0.00061	0.00039	0.00028	0.00015	0.00005

Figure 10

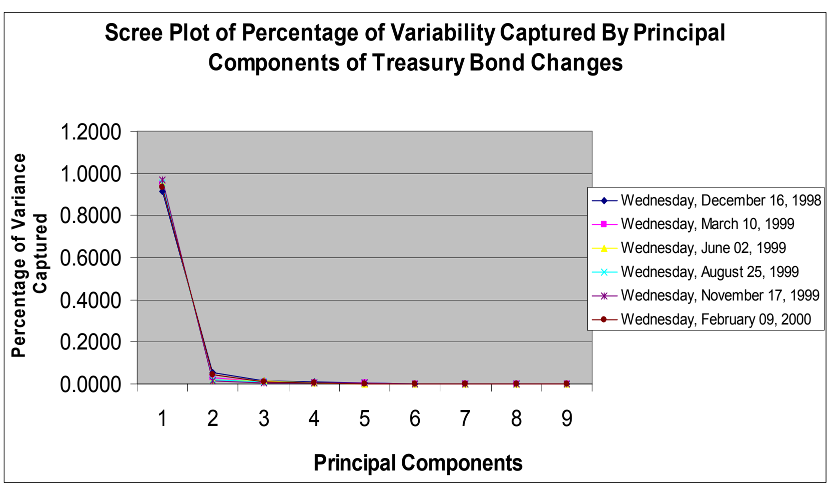


Figure 11

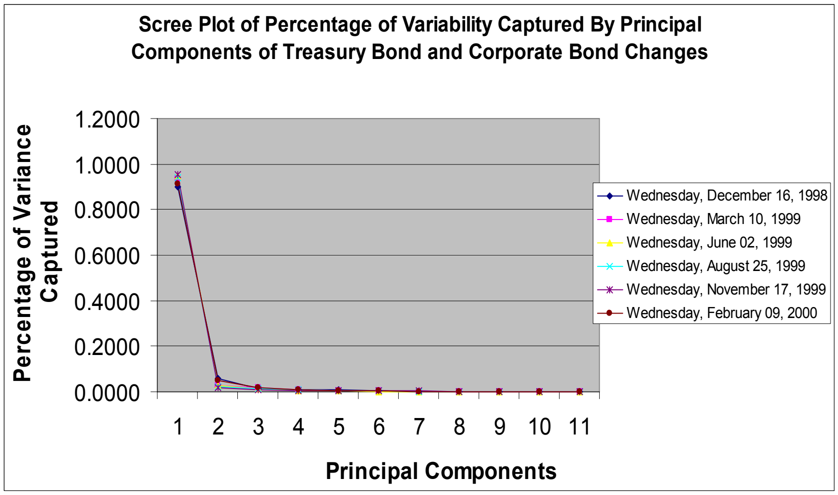


Figure 12

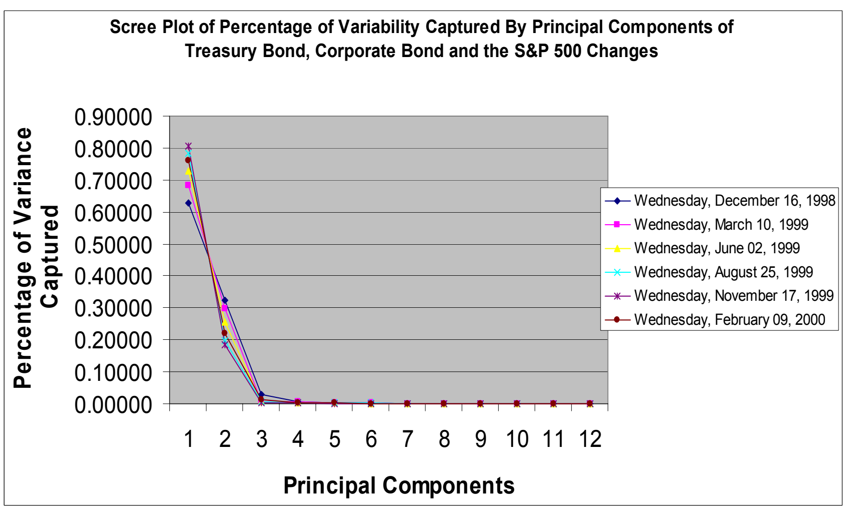


Figure 13 **LEXNX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9
A	0.024	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.006	0.006
B	0.016	0.006	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.004
C	0.016	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004
D	0.086	0.023	0.023	0.020	0.019	0.018	0.018	0.018	0.018	0.018
E	0.074	0.015	0.015	0.015	0.015	0.015	0.015	0.013	0.012	0.011
F	0.049	0.010	0.010	0.010	0.009	0.007	0.007	0.007	0.006	0.006

Figure 14

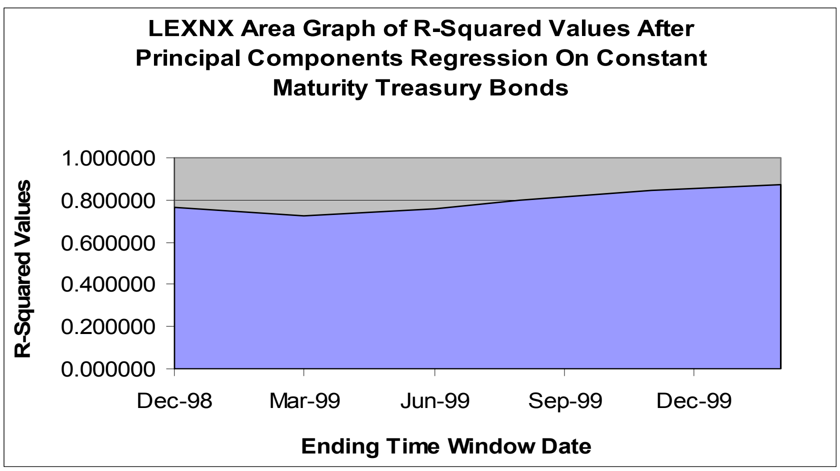


Figure 15

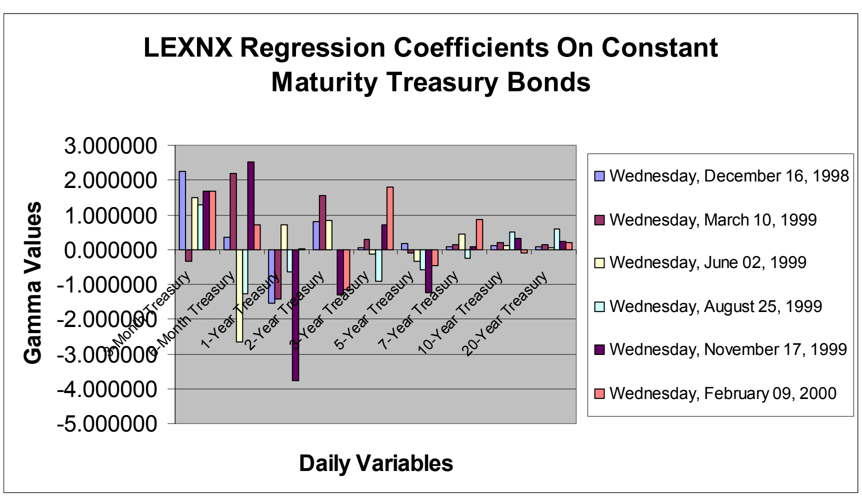


Figure 16

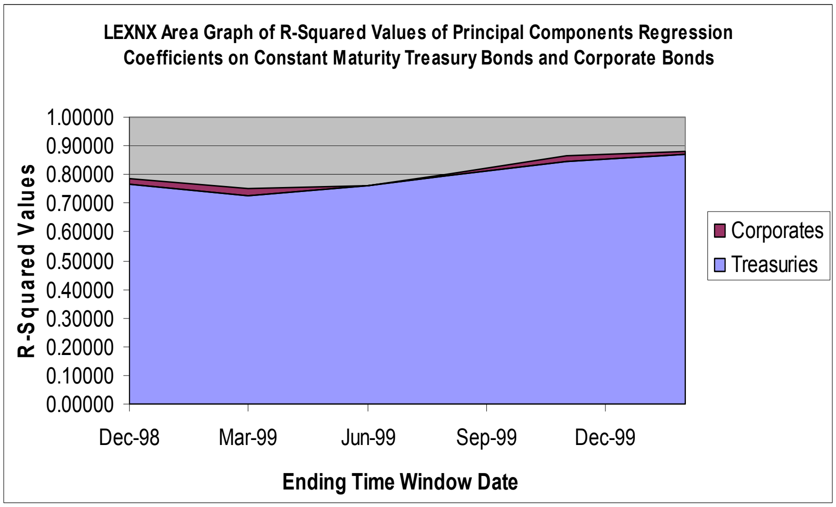


Figure 17 **LEXNX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11
A	0.024	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.006	0.006	0.005	0.005
B	0.016	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.004
C	0.016	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004
D	0.086	0.023	0.023	0.022	0.020	0.019	0.018	0.018	0.018	0.018	0.017	0.017
E	0.074	0.015	0.014	0.014	0.012	0.012	0.012	0.012	0.012	0.011	0.011	0.010
F	0.049	0.010	0.010	0.010	0.010	0.009	0.009	0.007	0.006	0.006	0.006	0.006

Figure 18

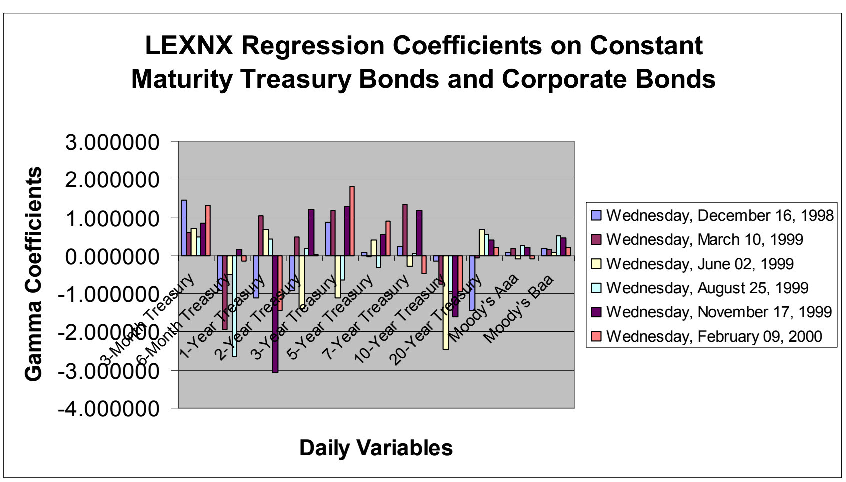


Figure 19 **LEXNX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
A	0.024	0.018	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.006	0.006	0.005	0.005
B	0.016	0.016	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.004
C	0.016	0.015	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004
D	0.086	0.045	0.022	0.022	0.022	0.020	0.018	0.017	0.017	0.017	0.017	0.017	0.017
E	0.074	0.050	0.013	0.013	0.013	0.012	0.011	0.011	0.011	0.011	0.010	0.010	0.009
F	0.049	0.048	0.010	0.009	0.009	0.009	0.009	0.009	0.007	0.006	0.006	0.006	0.006

Figure 20

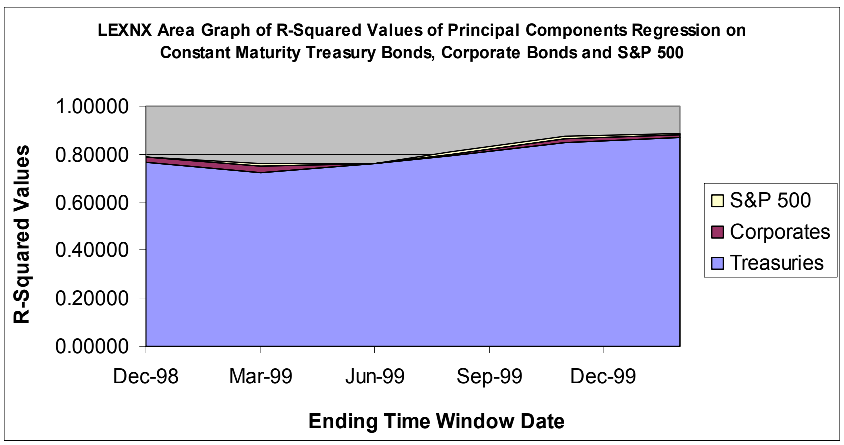


Figure 21

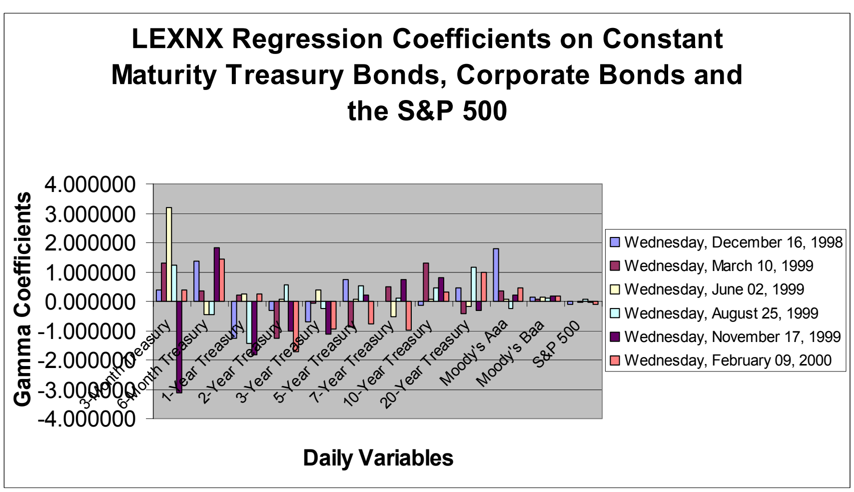


Figure 22

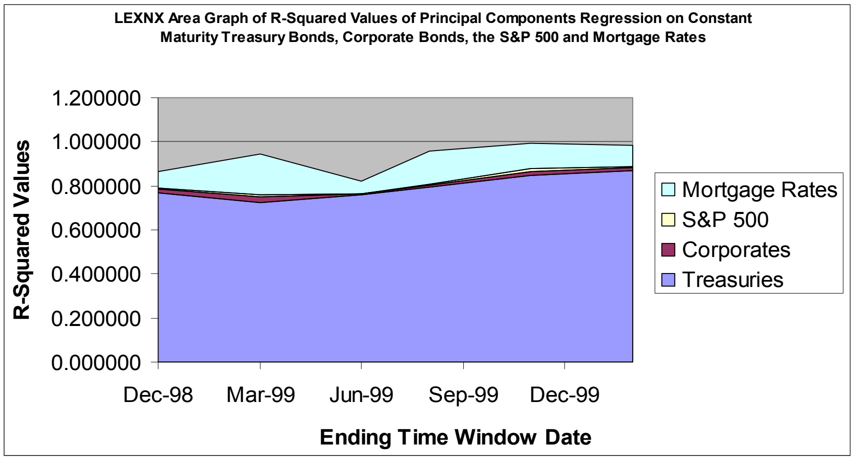


Figure 23 **VFIIX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9
A	0.039	0.017	0.017	0.015	0.014	0.014	0.014	0.014	0.012	0.012
B	0.015	0.008	0.007	0.006	0.006	0.005	0.005	0.005	0.005	0.005
C	0.022	0.010	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.008
D	0.092	0.024	0.024	0.022	0.020	0.020	0.020	0.020	0.019	0.018
E	0.075	0.020	0.020	0.018	0.017	0.017	0.017	0.016	0.015	0.015
F	0.045	0.012	0.012	0.012	0.010	0.009	0.009	0.009	0.009	0.009

Figure 24

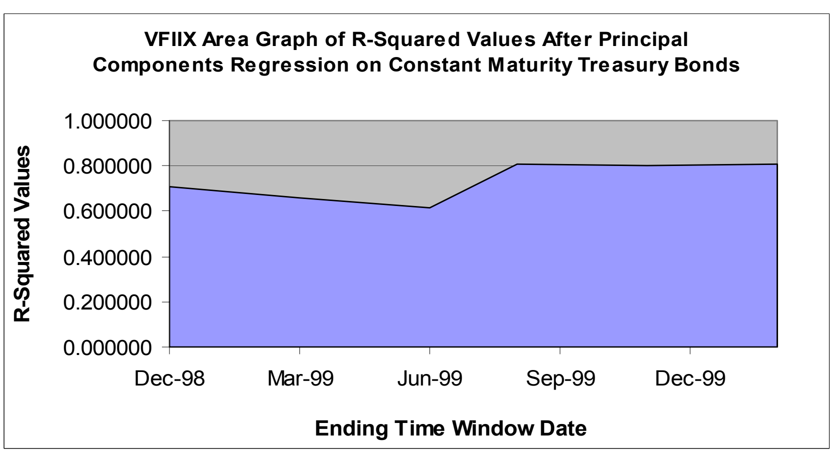


Figure 25

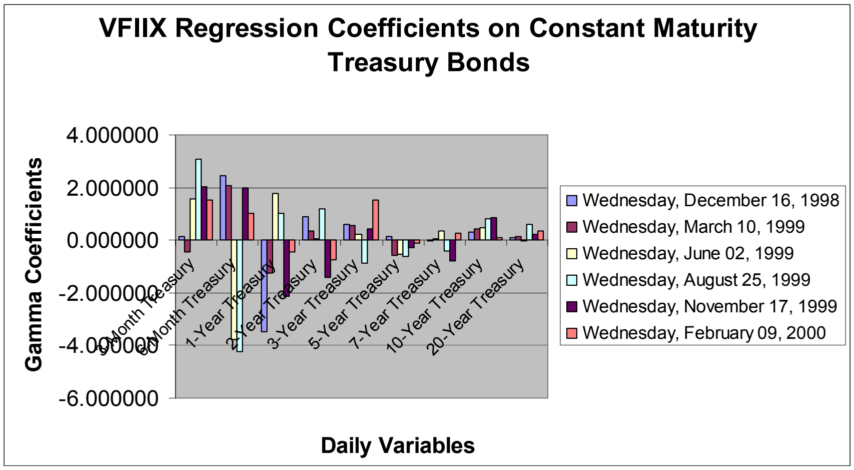


Figure 26 **VFIIX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11
A	0.039	0.016	0.016	0.014	0.014	0.013	0.013	0.013	0.013	0.013	0.011	0.011
B	0.015	0.007	0.007	0.007	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.004
C	0.022	0.010	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.008
D	0.092	0.025	0.024	0.024	0.022	0.020	0.020	0.020	0.020	0.019	0.019	0.018
E	0.075	0.019	0.019	0.019	0.016	0.016	0.013	0.013	0.013	0.013	0.012	0.012
F	0.045	0.011	0.011	0.011	0.011	0.010	0.010	0.009	0.009	0.009	0.008	0.008

Figure 27

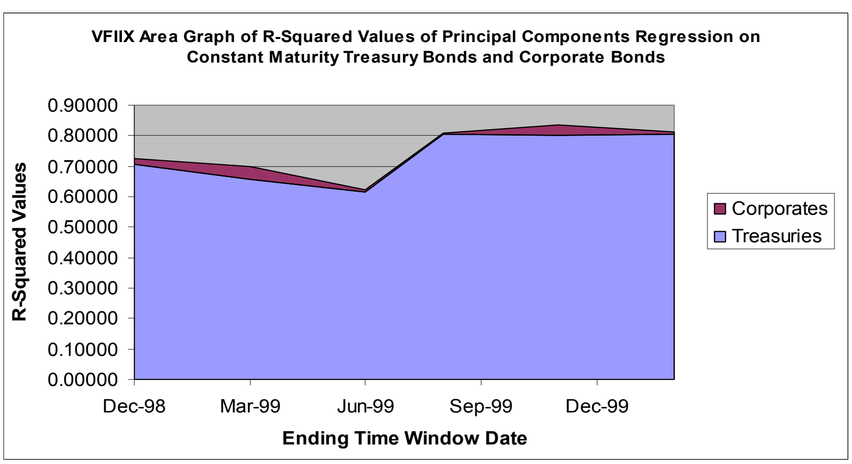


Figure 28

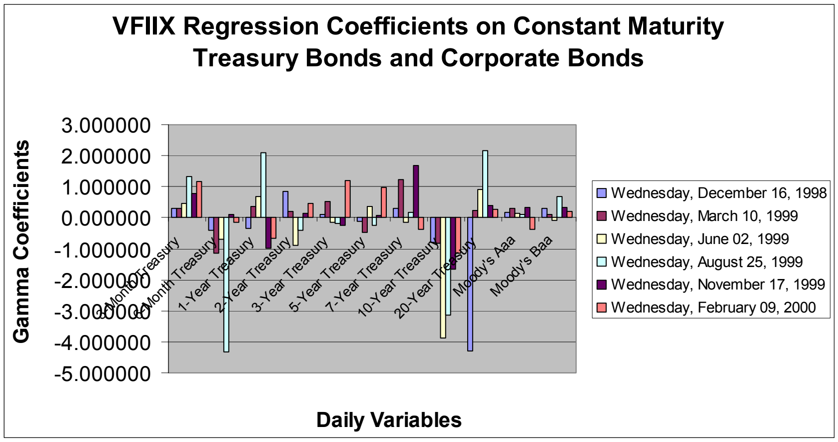


Figure 29

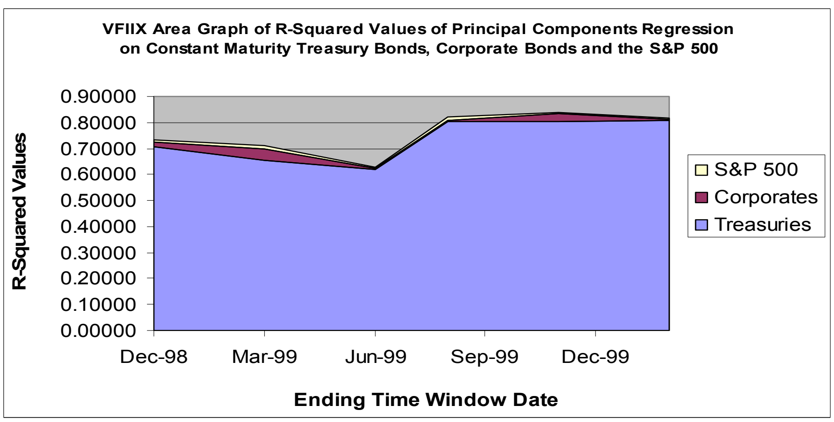


Figure 30 **VFIIX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
A	0.039	0.032	0.015	0.015	0.014	0.013	0.013	0.013	0.013	0.013	0.013	0.011	0.010
B	0.015	0.015	0.007	0.006	0.006	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.004
C	0.022	0.020	0.010	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.008	0.008
D	0.092	0.045	0.022	0.022	0.022	0.020	0.018	0.018	0.018	0.018	0.018	0.017	0.016
E	0.075	0.057	0.019	0.019	0.018	0.015	0.015	0.013	0.013	0.013	0.012	0.012	0.012
F	0.045	0.044	0.011	0.011	0.011	0.011	0.010	0.010	0.009	0.009	0.009	0.008	0.008

Figure 31

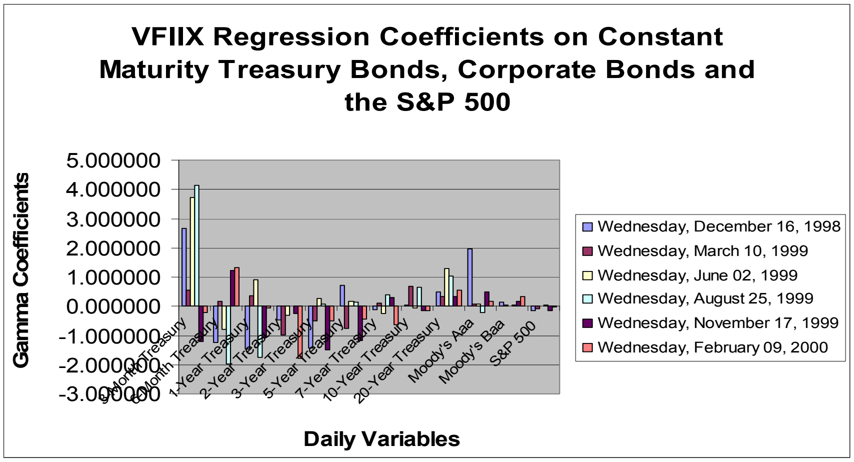


Figure 32

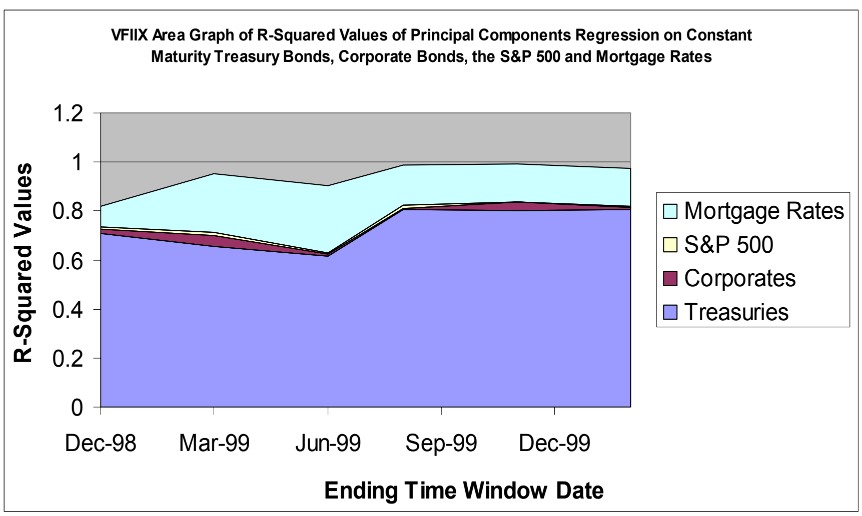


Figure 33

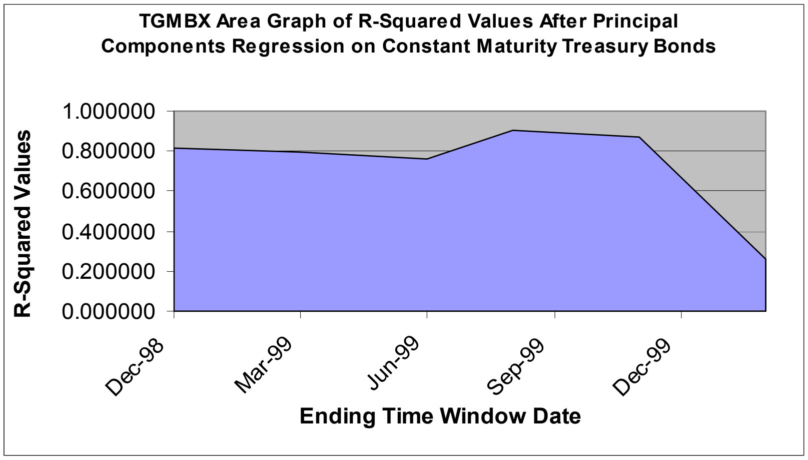


Figure 34 **TGMBX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9
A	0.027	0.007	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005
B	0.017	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.003
C	0.021	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
D	0.049	0.007	0.007	0.007	0.005	0.005	0.005	0.005	0.005	0.005
E	0.045	0.007	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.006
F	0.043	0.042	0.042	0.041	0.038	0.033	0.033	0.033	0.032	0.032

Figure 35

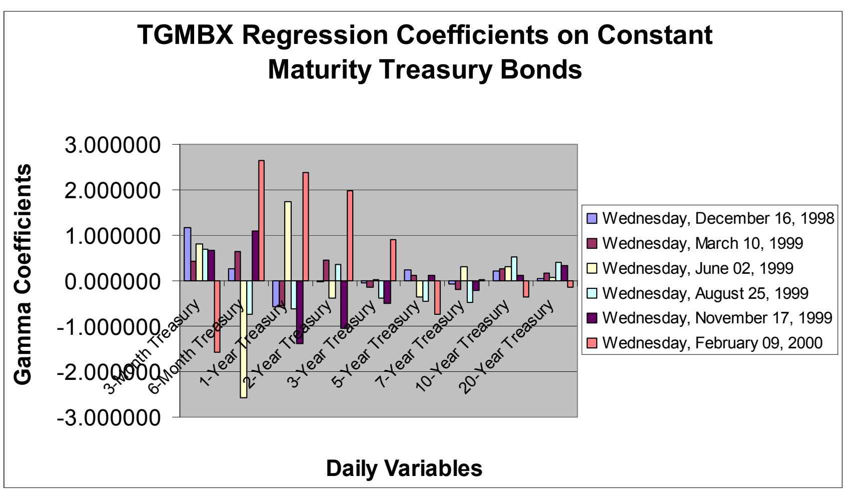


Figure 36

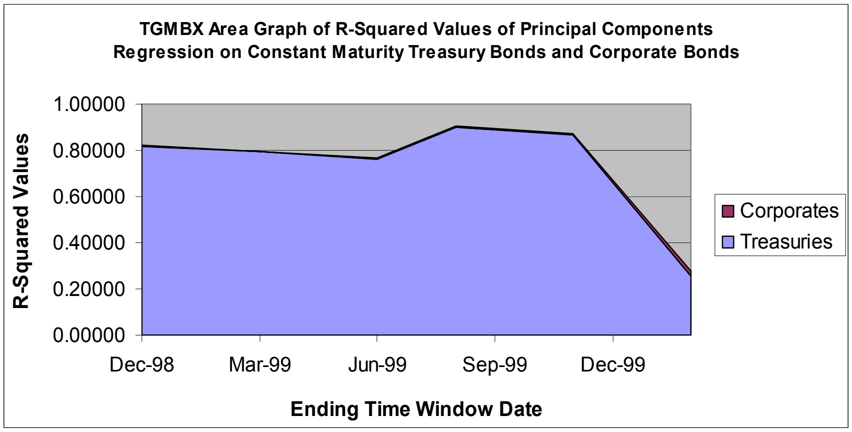


Figure 37 **TGMBX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11
A	0.027	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005
B	0.017	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.003
C	0.021	0.006	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
D	0.049	0.007	0.007	0.007	0.007	0.005	0.005	0.005	0.005	0.005	0.005	0.005
E	0.045	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.006
F	0.043	0.042	0.042	0.041	0.040	0.037	0.036	0.033	0.032	0.032	0.031	0.031

Figure 38

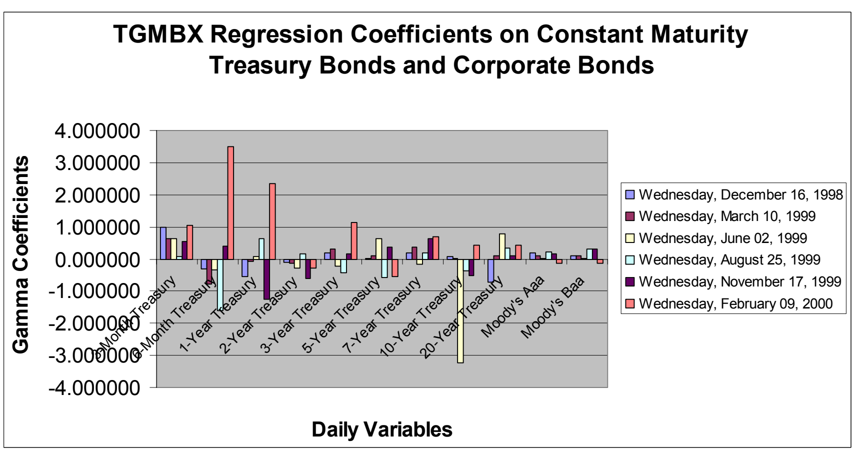


Figure 39

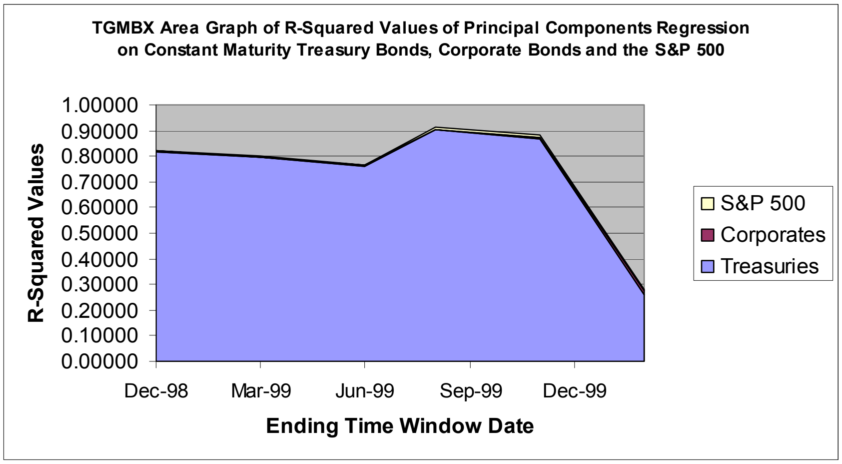


Figure 40 **TGMBX Residual Variance After Each PC**

Time Period	Original MBS Variances	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
A	0.027	0.018	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005
B	0.017	0.017	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.003
C	0.021	0.019	0.006	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
D	0.049	0.023	0.007	0.007	0.007	0.006	0.005	0.005	0.005	0.005	0.005	0.004	0.004
E	0.045	0.033	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.005
F	0.043	0.042	0.042	0.041	0.040	0.039	0.036	0.036	0.033	0.032	0.032	0.031	0.031

Figure 41

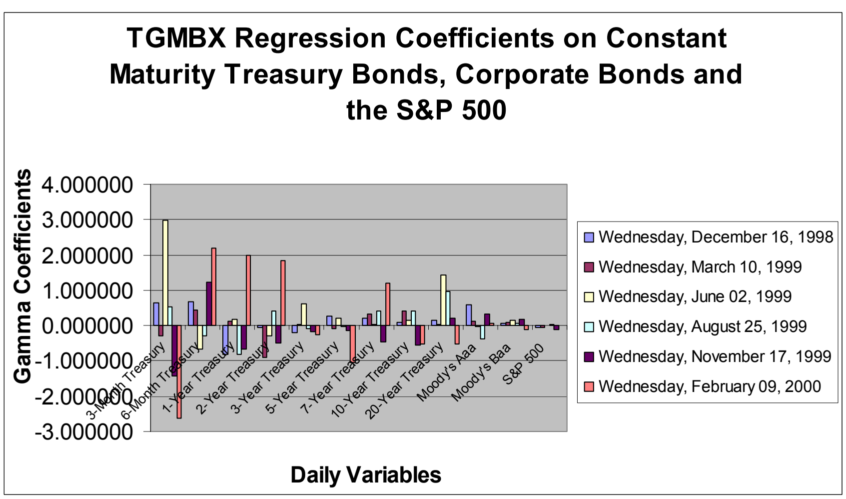
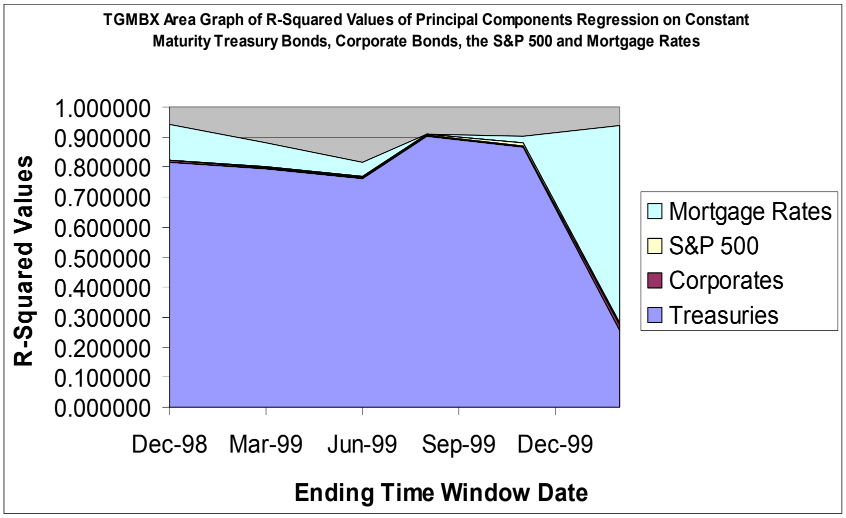


Figure 42


TGMBX Area Graph of R-Squared Values of Principal Components Regression on Constant Maturity Treasury Bonds, Corporate Bonds, the S&P 500 and Mortgage Rates
R-Squared Values
1.000000
0.900000
0.800000
0.700000
0.600000
0.500000
0.400000
0.300000
0.200000
0.100000
0.000000
Dec-98
Mar-99
Jun-99
Sep-99
Dec-99
Ending Time Window Date
Mortgage Rates
S&P 500
Corporates
Treasuries

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