

Irish Flute Design Optimization

A Major Qualifying Project submitted to the faculty
of Worcester Polytechnic Institute in partial fulfillment of the
requirements for the Degree of Bachelor of Science.

Submitted By:

Ying Bao

Kiara Gravel

Ian Jutras

David Knight

Submitted To:

Advisors: Professors John Sullivan and David Planchard

April 23, 2013

Abstract

The goal of the project was to improve the intonation and ergonomics of an Irish flute as well as assess the feasibility of rapid prototyping a flute. SolidWorks was used to model existing Delrin and PVC flutes. A rapid prototyping machine was used to create a physical model of the six holed Pratten-Style Delrin polymer flute based on the SolidWorks data. Using the rapid-prototyped Irish flute and PVC flute, the group collected frequency-based data. Frequencies were collected in the first and second octaves and compared to the A440 Hz standard of tuning musical instruments. Through studying acoustic principles of an open tube, the group developed equations to predict the distance from each of the tone holes to the embouchure hole based on standard frequencies as well as various dimensions within an Irish flute. Parameters of the flute such as the bore diameter, tone hole diameter, cork position, chimney height, and length of the flute were isolated and analyzed. These equations were then linked through a design table in SolidWorks to the 3D model and used to design and create a new, optimized, rapid prototyped flute. The design table facilitated the process of making dimensional modifications to the flute without having to manually calculate the values and input them into a model. The final result was an optimized Irish flute that required minimal tuning. The group gained a deep understanding of the relationships that exist between parameters of the Irish flute and the acoustics through the instrument. Recommendations for further studies on the Irish flute were included.

Executive Summary

The project goal was to optimize the existing Irish flute from the engineering perspective. This was accomplished by developing and applying mathematical modeling as well as modern manufacturing technology, such as 3D rapid prototyping. The team began the project by studying the fundamentals of flute acoustics together with the factors that could affect overall flute performance. These factors included materials, embouchure hole shape and placement, bore shape and diameter, tone hole placement, tone hole size, and chimney heights.

Six Irish flutes were used in the project. The Delrin keyless flute, was modeled in SolidWorks 2012. To create the initial rapid prototype, this SolidWorks model was printed on an Objet260 Connex Rapid Prototype machine. The team used 220-grit sandpaper to smooth out the surface and remove any remaining support material. Two types of flute playing devices were designed for the flute testing in order to ensure the consistency of the airflow.

Existing mathematical models failed to match actual measured values within reasonable error. Consequently, the team developed its own mathematical models. Equations predicting a flute's dimensional parameters based on desired frequency were successfully developed. To facilitate implementation of these equations into a solid model of the flute, a user-friendly SolidWorks Design Table was created. Using the table, flute dimensions could be easily changed and the corresponding locations of tone holes adjusted automatically to the changes. Design tables like this can allow the manufacturer to input information about the player, such as finger spacing, which can allow well-intonated flutes to be custom-designed and rapid-prototyped for individual players.

The second prototype was designed using the SolidWorks design table. This prototyped flute was easily playable across the entire note range while producing a clear tone. A tuning slide with a fully-extended length of 30 mm was implemented to ensure that the flute could be tuned to concert pitch regardless of temperature. The flute was designed so that under standard temperature, the slide would be extended to 10 mm, which allowed the flute's frequency to be raised as well as lowered. The second prototype also featured an extended chimney height on the tone hole farthest from the embouchure hole. In order to raise the

volume of the E-note played by the last tone hole, the hole diameter was increased. The raised chimney height was implemented to counter the raised frequency of the E-note which would have resulted from the increased E-hole diameter.

Due to limited time, the team was not able to acquire sufficient testing data to develop a complete mathematical model for chimney height prediction. Future research could focus on creating this model, as well as the modification of the embouchure hole, bore profile, and bore roughness.

Acknowledgements

This project would not have been possible without help and funding from the ME department. In addition, special thanks to Erika Stults, who supervised the rapid prototyping process and performed extensive cleaning. Peter Hefti donated time and materials to the group by assembling the regulator for the flute playing device. Windward Flutes loaned the group two Irish flutes for testing. Jeffrey Peters acted as a test player. Finally, our thanks go out to our advisors, Professors John Sullivan and David Planchard, who provided invaluable insight and guidance.

Table of Contents

Abstract.....	i
Executive Summary.....	ii
Acknowledgements.....	iv
List of Figures	vii
List of Tables	ix
Nomenclature	x
Introduction	1
Background	3
History.....	3
Acoustics	3
Material.....	5
Embouchure.....	6
Bore.....	9
Tone Hole, Placement, Size and Chimney Heights.	12
Methods and Procedure	14
Irish Flutes Used.....	14
Initial Rapid Prototyping	16
Design and Fabrication of the Flute Playing Device	18
Testing Previously Developed Equation Models	21
Initial Mathematical Modeling.....	21
Frequency Gathering of PVC, Delrin [®] , RP, Olive, and Mopane Material Flutes.....	27
Human Player Frequency Testing	27
Flute Playing Device	28
Chimney Height.....	28
Bore Extension Testing	29
Temperature	30
Cork Displacement	30
Human Player Averaging.....	30
Final Rapid Prototyping.....	30
Results.....	33
Results of Initial Prototype	33
Previously Developed Equations	34
Uncertainty Calculations.....	35
General Frequency.....	36

Cork Displacement	38
Bore Extension	40
Temperature	44
Human Player Averaging.....	45
Chimney Height Results	46
Predicting Frequency	46
Predicting Chimney Height	47
Flow Inside the Bore	48
Implementation of the Equations for Manufacturing	49
Results of Final Prototype.....	51
Conclusions	54
Final Prototype Assessment.....	54
Manufacturing Feasibility	54
Recommendations and Future Work.....	55
Chimney Height Prediction	55
Embouchure Hole	55
Bore Profile Modification.....	56
Future Work on Bore Extension.....	58
Future Work Based on Final Prototype	58
Work Cited	59
Appendix A.....	61

List of Figures

Figure 1: Nomenclature Visualization.....	xi
Figure 2: Basic Parts of the Flute	1
Figure 3 Notes and Fingering Chart for an Irish D Flute	2
Figure 4: Sine Wave	3
Figure 5: Idealized Model of a Standing Wave in a Simplified Flute	4
Figure 6: First Three Modes of Vibration.....	5
Figure 7: Two Semicircles embouchure hole (McGee).....	7
Figure 8: Rounded Rectangle embouchure hole (McGee)	7
Figure 9: 19th Century Flute (<i>Transverse Flute in F</i>).....	8
Figure 10: Cylindrical vs. Conical Bore	10
Figure 11: Pressure Waves through Cylindrical and Conical Bores (Ayers).....	11
Figure 12: PVC D Flute Manufactured by repurposeeverything.org	14
Figure 13: Delrin D Flute Manufactured by David Copley in Ohio.....	14
Figure 14: African Blackwood D Flute Manufactured by Bryan Byrne in Vermont	14
Figure 15: Mopane Wood D Flute Manufactured by Windward Flutes	14
Figure 16: Olive Wood D Flute Manufactured by Windward Flutes	15
Figure 17: Olive Wood D Flute Modified to Play in the Key of Eb	15
Figure 18: Method of Determining Angle of Undercut	16
Figure 19: SolidWorks Models of Delrin® (top) and Initial Rapid Prototype (bottom)	17
Figure 20: Copper Pipe Constant Playing Apparatus.....	19
Figure 21: Completed Flute Playing Device on a PVC Flute.....	20
Figure 22: Correction Length used in Determining Tone Hole Location	22
Figure 23: Change in Wavelength Node due to Reduction in Diameter	23
Figure 24: gString Tuner Application	27
Figure 25: Set up using Cardboard to Increase Chimney Heights	29
Figure 26: Final Rapid Prototype Cross Section with Tone Hole Locations	31
Figure 27: Close of End Joint in Final Flute	32
Figure 28: Magnification of Layers Created by Rapid Prototyping Vertically.....	33
Figure 29: Difference in Frequency vs. Actual Frequency for the Mopane Wood D Flute.....	37
Figure 30: Difference in Frequency vs. Actual Frequency for the Olive Wood D Flute	37
Figure 31: Difference in Frequency vs. Actual Frequency for the Olive Wood Eb Flute	38
Figure 32: Change in Frequency Due to Cork Position	40
Figure 33: Change in Frequency over the Base Frequency (F_{bt}) with Respect to the Base Frequency (F_{bt}) for a Bore Extension of 3.82 mm.....	42
Figure 34: Change in Frequency over the Base Frequency (F_{bt}) with Respect to the Base Frequency (F_{bt}) for a Bore Extension of 7.03 mm.....	42
Figure 35: Change in Frequency over the Base Frequency (F_{bt}) with Respect to the Base Frequency (F_{bt}) for a Bore Extension of 10.52 mm.....	43
Figure 36: Temperature Variations within the Bore of the Flute while being Played	44
Figure 37: Difference between Played Frequency and the Actual Frequency in Average Human Player Test on the Initial Rapid Prototyped Flute.....	45

Figure 38: Difference between Played Frequency and the Actual Frequency in Average Human Player Test on the Delrin® Flute	46
Figure 39: 3-D graph showing relationship among distance, chimney height and frequency	47
Figure 40: Frequency Change due to Chimney Height Increase in the E4 Tone Hole	48
Figure 41: Calculations Inserted into Design Table	50
Figure 42: Final Rapid Prototype.....	51
Figure 43: Magnification of Layers Created by Rapid Prototyping Horizontally	52
Figure 44: Frequency Data for the Final Rapid Prototype	53
Figure 45: Bore Profiles with Varying Tapers (Hopkin).....	57
Figure 46: All required values and equations needed to compute tone hole placement in an Excel spreadsheet	61
Figure 47: Mopane Wood D Flute Frequency Test.....	62
Figure 48: Olive Wood D Flute Frequency Test	62
Figure 49: Olive Wood Eb Flute Frequency Test.....	63
Figure 50: Difference between Predicted Change and Actual Change in Frequency due to Bore Extension.....	63
Figure 51: Frequency Data Collected for Chimney Height Increases	63

List of Tables

Table 1: Testing of Forster's Effective Length of Tone Hole Equations	34
Table 2: Calculation of Distance from Embouchure Hole to the Left of the Cork	35
Table 3: Relative Uncertainty in Tone Hole Placement Calculations.....	36
Table 4: Frequency Played Relative to Cork Position	39
Table 5: Change in Frequency with Respect to Cork Position	39
Table 6: Results from Bore Extension Test	41
Table 7: Difference in Frequency from Base.....	41

Nomenclature

A_{be} (mm²) = Area of the bore at the embouchure hole

A_e (mm²) = Area of the embouchure hole

A_{est} (mm²) = Estimated area of the bore at the active tone hole using the value d_{est}

A_h (mm²) = Area of the active tone hole

d_{be} (mm) = Diameter of the bore at the embouchure hole

d_e (mm) = Diameter of embouchure hole

d_{end} (mm) = Diameter of the end of the bore

d_{est} (mm) = Estimated diameter of the active tone hole

d_h (mm) = Diameter of the active tone hole

E_x (mm) = Extension distance of the tuning slide

F_b (Hz) = Base frequency of the flute: frequency played when all tone holes are closed

F_{bt} (Hz) = In tune base frequency corresponding to each tone hole

F_d (Hz) = Desired frequency for the active tone hole

h_c (mm) = Height of the chimney above the outer bore

h_o (mm) = Wall thickness at embouchure

h_{end} (mm) = Wall thickness at end of flute

h_{est} (mm) = Estimated wall thickness at the active tone hole

Δl_e (mm) = Length correction for the embouchure

Δl_h (mm) = Length correction for the active tone hole

Δl_{hemb} (mm) = Length correction for the end of the flute with all tone holes closed

L_a (mm) = Length between embouchure hole and active tone hole

L_s (mm) = Length of the actual wavelength

c (m/s²) = Speed of sound

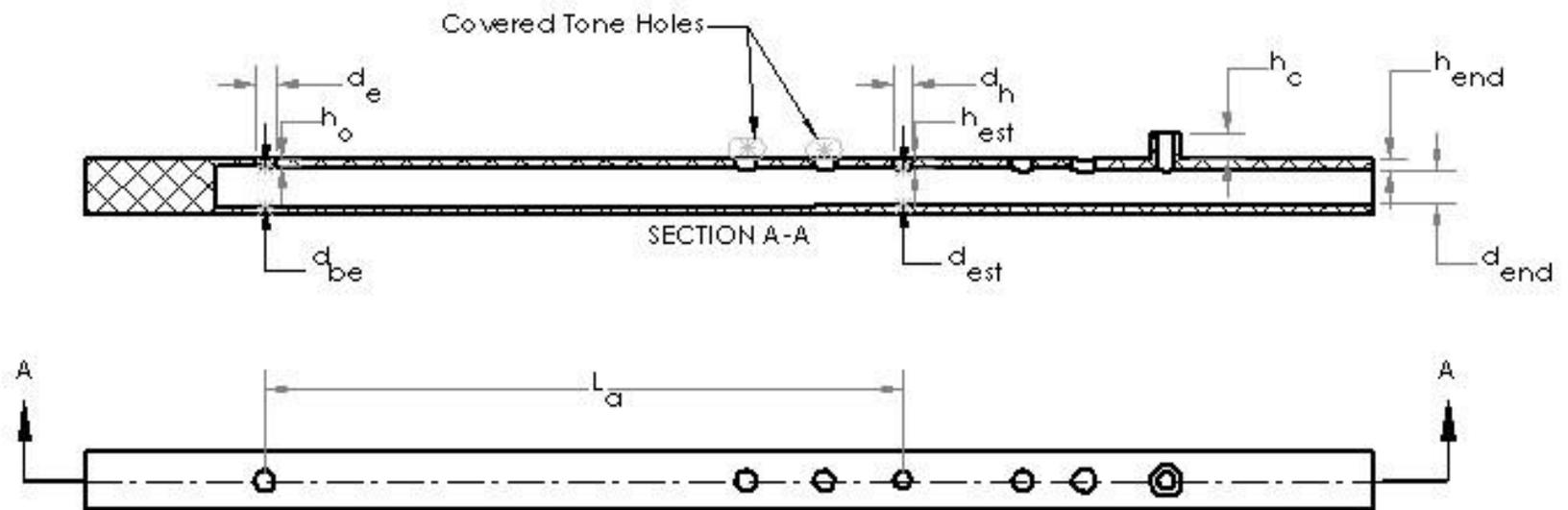


Figure 1: Nomenclature Visualization

Introduction

The Irish flute is a woodwind instrument that is traditionally used to play Irish music. The flute consists of an embouchure hole where a player's lips are placed and six tone holes which can be covered by the player to produce different musical notes. The Irish flute is similar to an open tube; one end remains open while the other "open" end is at the embouchure hole. The basic parts of the flute are shown in Figure 2.

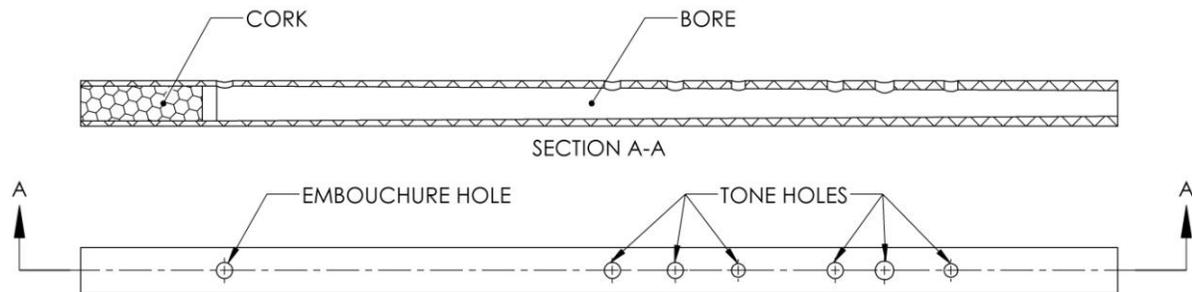


Figure 2: Basic Parts of the Flute

The design of the Irish flute had potential for intonation improvement. While keeping the Irish flute within its true definition as a keyless instrument, the group aimed to optimize the flute's intonation. Intonation is the relation in pitch of tones to their accepted standard values. The group decided to alter the different aspects of the design to improve the intonation with respect to accepted pitch frequencies based upon the A440 Hz standard.

The basic notes that were tested were D, E, F#, G, A, B, and C#. Each of these notes can be produced by covering the tone holes and blowing over the embouchure hole. Figure 3 illustrates the tone hole combinations used to produce each note. The dark circles represent tone holes that are covered while the white circles with a black outline represent uncovered tone holes.

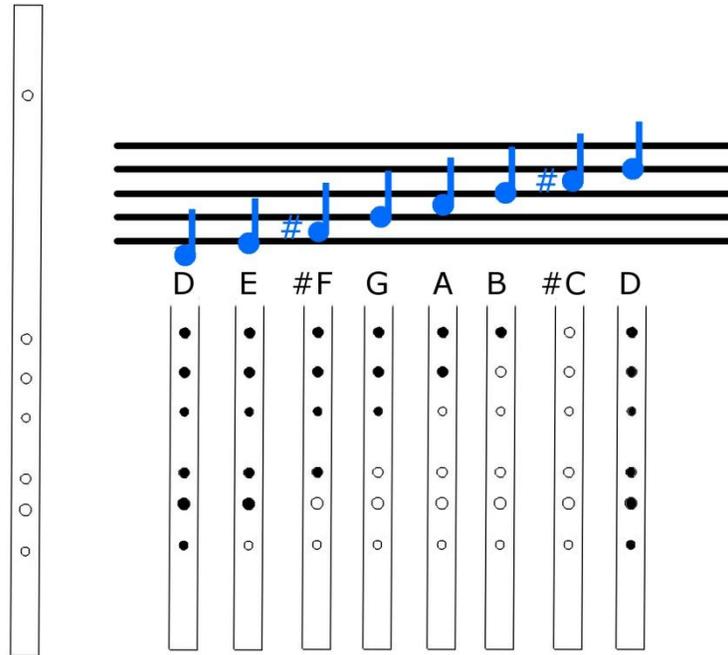


Figure 3 Notes and Fingering Chart for an Irish D Flute

By increasing the air pressure blown across and into the flute, the second octave for each of these notes can be produced. Various dimensional parameters were isolated for the first and second octaves of the flute, and the effects of their modifications were studied. Parameters studied included changing the cork location and the length of the tube. In addition, the embouchure hole and tone holes were tested by changing their profile and location. The chimney height, which is the thickness of the flute at the tone hole, for each note was increased to study the frequency effects based on the thickness.

To begin the study, a flute was modeled in SolidWorks. The model was based on a six holed David Copley Pratten-Style Delrin® polymer flute. A rapid prototype (RP) was created from this model. Frequency data were collected through testing of the prototype, and mathematical relationships between the parameters were identified. Relationships were also analyzed for the Delrin® and PVC flutes. The final RP was created using the identified mathematical relationships. This flute was designed to be played in tune with minimal embouchure adjustment or tuning, and to fit the player’s hand more comfortably.

Background

History

The design of the Irish flute is reminiscent of early flutes from the Renaissance Period (14th to 17th century). During that time period, flutes were typically used for chamber music and military bands. Since the bore was cylindrical, the upper and lower octaves were frequently not in tune (McGee). To improve the tuning, these Irish flutes were replaced by the “Böhm-style” keyed flutes in 1847. The Böhm flute became popular, and the prices of older-style flutes drastically decreased. As a result, less affluent musicians began purchasing the older-style flutes in large numbers. Poor economic conditions in Ireland caused a heavy concentration of these flutes to be bought and used by Irish musicians. This economic situation caused this simple flute style to be referred to as the Irish Flute. The modern Irish flute is typically made of wood, metal, or heavy plastic (such as Delrin®). It has six tone holes and no keys. While the bore is usually conical, less expensive flutes may have cylindrical bores (Xorlys).

Acoustics

Sound is produced when vibrations travel through air. These vibrations can be represented as a sine wave which can be seen in Figure 4 below.

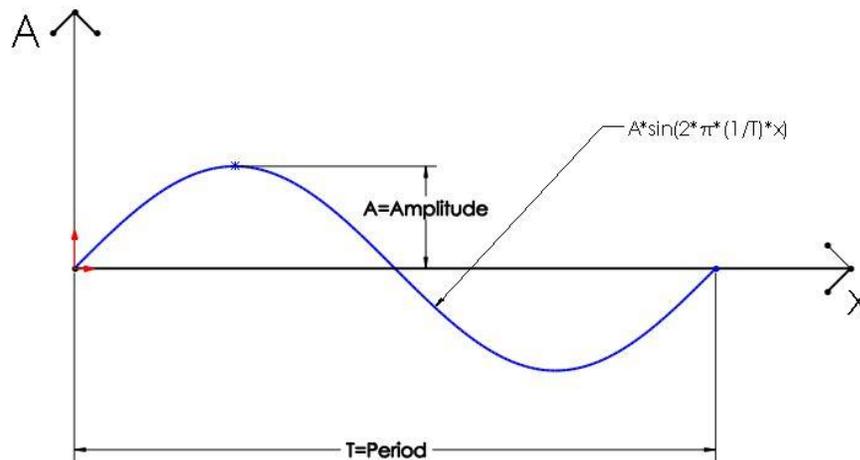


Figure 4: Sine Wave

This wave has an amplitude “A” which correlates to volume of the sound. The period “T” is length that one sine oscillation takes to complete a cycle. The inverse of the period is the frequency of the wave. This frequency is what determines the actual note or sound that is heard. A longer sine wave cycle denotes a lower frequency resulting in a lower-pitched sound. Two notes that are an octave apart will differ in frequency by a factor of two.

In a flute the vibration of the air is caused by a traveling pressure wave created when the player blows air into a flute. A rise in the wave will signify when the air molecules are forced together (“compressions”) while the decrease in the wave will signify when the air molecules are forced apart (“rarefactions”). During flute intonation optimization, the measured frequencies of the notes of the flute must stay in tune relative to the standard tuning of A=440 Hz. In air columns such as what is found in the flute, a traveling wave will reflect back on itself every time it reaches a boundary (Hopkin). Boundaries within the flute include the walls of the tube, the cork located near the embouchure hole and the end of the flute open to atmospheric pressure. This open end acts as a boundary because the air column is no longer constrained to the inside of the flute. Despite the presence of the cork, the flute is analyzed as an open tube on both ends. The embouchure hole acts as the open end adjacent to the cork. Steady-state vibrations exist as result of traveling waves reflecting back and forth in the air column. To create a standing waveform, interacting wave fronts reinforce or cancel each other within the bore (Hopkin). An example of a standing wave can be seen in Figure 5. This standing wave creates the fundamental pitch as well as overtones that are heard.



Figure 5: Idealized Model of a Standing Wave in a Simplified Flute

The different modes of vibration occur because multiple tones arise from reflecting patterns. Fundamental tones and overtones occur together, but the fundamental tone is the most audible. This fundamental tone creates the pitch that is heard. Several modes of vibration are possible and each mode is a pattern of vibratory movement in the standing wave. The simplest and most prominent mode is the first mode, which is responsible for the pitch.

The second mode is an octave above the fundamental. The third mode of vibration occurs at the next-highest overtone, which is a 12th, or 12 musical steps, above the fundamental. These three modes of vibration can be seen in Figure 6.

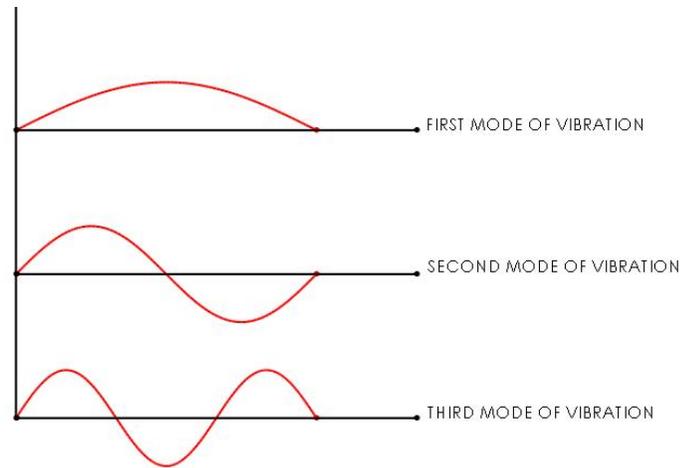


Figure 6: First Three Modes of Vibration

In standing wave patterns, there are points of maximum movement called antinodes, as well as points of minimum or no movement called nodes. Longitudinal modes are the most important in air columns. For an air column to vibrate freely in a particular mode of vibration, it is best for the points located along the antinodes to not be disrupted (Hopkin). Pinpointing the areas where nodes and antinodes occur can allow the researcher to alter the inside of flute and avoid disturbing the waveform.

Material

The material of woodwind instruments presents a point of contention between musicians and scientists. While scientists claim that material choice offers very little to instrument tone, musicians swear that the difference is perceivable by both feel and sound.

In a 2001 study, Gregor Widholm matched seven identical flutes of different materials with seven professional flautists. Each flautist played all seven flutes, and the sound samples were analyzed by a panel of 15 professional flautists, including the seven test players themselves. After analysis of flutes made of solid silver, plated silver, 9 karat gold, 14 karat gold, 24 karat gold, solid platinum, and plated platinum, it was determined that while “sound

analysis pointed out big differences in the sound level and sound color of played tones caused by the player," the material caused tone color differences that were "just measurable but not perceivable" (Linortner).

Other professional flute makers have their opinions on material versus player. Terry McGee, an experienced and prolific flute maker from New South Wales, Australia, agrees with the scientists, but with a small caveat. McGee states that "the performance of a flute is going to be principally determined by its shape...providing it's smooth, doesn't leak, and it's strong enough not to vibrate and rob energy from the vibrating air column." (McGee) On the other hand, Malcolm Tattersall indicates in a 2007 article that while material may not affect tone, it may affect the process by which the flute is made. For example, some materials yield a more precisely manufactured result which in turn may affect tone. Furthermore, a player's "preconceptions and wishful thinking" about the instrument will have an effect on both the quality of the playing and on the player's perception of tonal quality (Tattersall).

In 1998, Cocchi and Tronchin performed a sound analysis of two flutes: one made of "light alloy" and one made of silver. Using homemade software, the researchers found that "the silver flute contains more high frequency and shorter transient than the light-alloy flute" (Cocchi, Tronchin). Through the study, material was found to have some effect on the measured frequencies in the flute. Since the flute material appears to only slightly affect tone, where it is minimally perceptible to the human ear and difficult to collect data with the group's measuring devices, the analysis for the flute was reduced to not focus on material selection as an area of potential improvement.

Embouchure

According to the fifth edition of *The Concise Oxford dictionary of music*, "embouchure" is defined as the mode of application of the lips, or their relation to the mouthpiece in a brass and woodwind playing instrument (Kennedy). The embouchure hole is where energy is imparted to the bore of the flute through blowing air. The energy is used to create musical pitches. There are several aspects related to the embouchure that have great influence on the tone production of a flute. These aspects can be divided into two major categories based on whether or not the tone production involves "human factors".

Without adjusting the shape of the embouchure hole, there are a number of player-specific factors that influence the tone of the flute:

- How high or low the lip is placed on the flute relative to the embouchure hole
- The amount of the embouchure hole that is covered by the lip
- Relation of the lips to the embouchure hole: positioning the embouchure hole slightly to the left or right of the center of the lips
- The angle of the lips on the flute's bore
- The width and the depth of the gap between the lips
- Whether the flute is played with a "smiling," "straight," or "sad" embouchure
- The angle of the air jet. This can have an impact on tone production, not only in changing octaves, but also in the resonance of a note (Wilcocks, 2006).

Without any human factors, the design of the embouchure hole itself can have great influences on the overall intonation of the flute. The classic elliptical embouchure found on 19th century flutes tends to be too noisy and unresponsive, even though it can produce an attractive dark sound when played in the Irish style. The modern-designed embouchure hole is a bit louder, much easier to play, and more responsive with faster articulation compared to the traditional version.

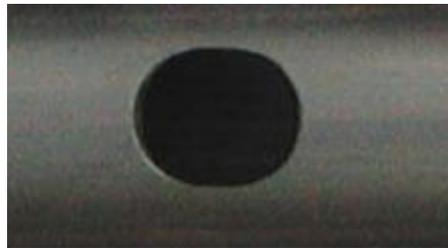


Figure 7: Two Semicircles embouchure hole (McGee)



Figure 8: Rounded Rectangle embouchure hole (McGee)

Figure 7 and Figure 8 show two typical types of modern-cut embouchure holes in modern Irish flutes. The “two semicircles” embouchure hole provides a useful increase in area over the traditional elliptical hole, increased width of the “edge”, and better dissociation between edge and sides. The edge is defined as the opposite side of the hole from the player’s lips. The rounded rectangle embouchure hole provides a further increase in area, an even wider "edge", and even better dissociation between edge and sides which increased the ease of playing the flute and volume. When modern embouchure holes are compared with 19th century embouchure holes, slight variations to the outer shape of the embouchure hole exist with an increased area.



Figure 9: 19th Century Flute (*Transverse Flute in F*)

For example, the smaller embouchure hole dimensions make focusing of the air stream easier. A deeper chimney produces more desirable tone. The undercutting compensates for the smaller dimensions and deeper chimney. The undercut edge sharpens the edge angle. A more rectangular hole maximizes the cross sectional area and widens the edge. The increased cross sectional area improves the volume of the sound and the overall easiness of playing. In addition, when the player's side of the hole is thinned and contoured to get lips closer to edge, focusing becomes easier and lip support is increased. The rounded sides of the embouchure hole, both inside and out, reduce wind noise (Mcgee).

The location of the embouchure hole relative to the cork also contributes to tone production. Based on the book *Fundamentals of Musical Acoustic*, the distance from the embouchure hole to the inside face of the cork, which is the theoretical starting point of the sine curve for each note, is defined as the embouchure hole length correction C_{emb} . The formula for calculating the distance is as follows:

$$C_{emb} = \left(\frac{d_{be}^2}{D * W} \right) * H_e$$

The embouchure hole dimensions are defined by a rectangular shape of width “W”, breadth “D” the chimney height, “H”. The radius of the air column is defined by “ d_{be} ”. “ H_e ” is the effective height of the chimney which includes the chimney height of the flute plus the player’s lower lip thickness

The magnitude of C_{emb} varies with frequency. The most important function of the head joint cork, along with the player’s lip position, can be adjusted to provide a suitable value for C_{emb} which is necessary for better alignment of the nodes. Most flutes have a C_{emb} value of around 50 mm (Benade).

Bore

The inside of the pipe, or “bore,” was another important consideration in the redesign process. The bore is the route that much of the air takes when a person blows through the embouchure hole. In reality, the bore could take any shape from cyclical to random twists and cuts, but outlandish shapes are not normally used. Besides the difficulty in manufacturing, these outlandish shapes are a problem because any sharp edges inside a tube can cause disruptions to the sound waves traveling through the bore. Because a harmonic overtone series is desired, most Irish flutes have been produced with cylindrical or conical bores with some alterations in the taper at strategic points. Outlandish bore shapes are difficult to manufacture (Hopkin).

Surprisingly, cylindrical and conical bores like the one seen below in Figure 10 exhibit very similar behaviors.

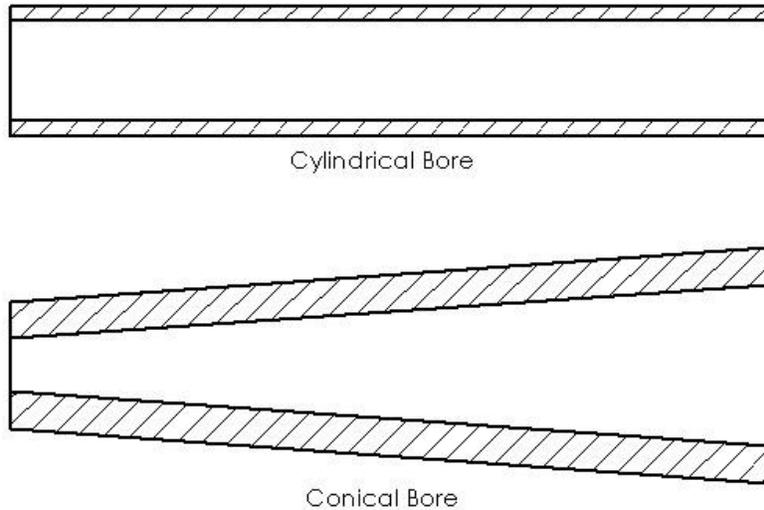


Figure 10: Cylindrical vs. Conical Bore

When calculating the frequencies for both cylindrical and conical bores in their modes of vibration, one may observe that both carry the same general equation:

$$f(n) = \frac{nv}{2L}$$

where “n” is the number of the mode being evaluated, “v” is the speed of sound at 25 °C (346 m/s), and “L” is the tube length. A true conical bore would also have a characteristic of a closed end, but in wind instruments such as the Irish flute, the frequency is determined in the above manner.

The cross-sectional area of the bore is far more important than its shape. As long as the cross-sectional area of a cylindrical bore matches that of a square bore, there will be little acoustic effects. For example, consider two uniformly-increasing or -decreasing bores, one being square-shaped and the other being circular. As long as the cross sectional areas at corresponding ends of the two bores are equal and these bores taper at the same rate, the same tones will be produced (Hopkin).

Conical and cylindrical bores are the most efficient means of creating the fundamental tones in a wind instrument with no keys. Slight differences exist between these two bore profiles as the ability to manufacture different-shaped bores is difficult. These differences can be attributed to the resonant modes or excitement in the overtones. The different bore shapes

have slightly different effects on the pressure waves that will travel through the bore. These pressure variations are illustrated in Figure 11. The trend for resonant modes in an open tube is a sine wave with the function $f(x) = \sin(x)$. For a true conical bore, one end is closed, and pressure variations are depicted as following a pattern with the function $f(x) = \sin(x)/x$. Although the Irish flute was considered open at both ends, it is important to note the pressure waves for the conical bore shown in the illustration below. If the closed end in the illustration was no longer depicted, the resulting pressure variations would be very similar to those of a conical bore flute open at both ends. The difference would be in the amplitudes while both profiles would follow similar sine waves (Ayers).

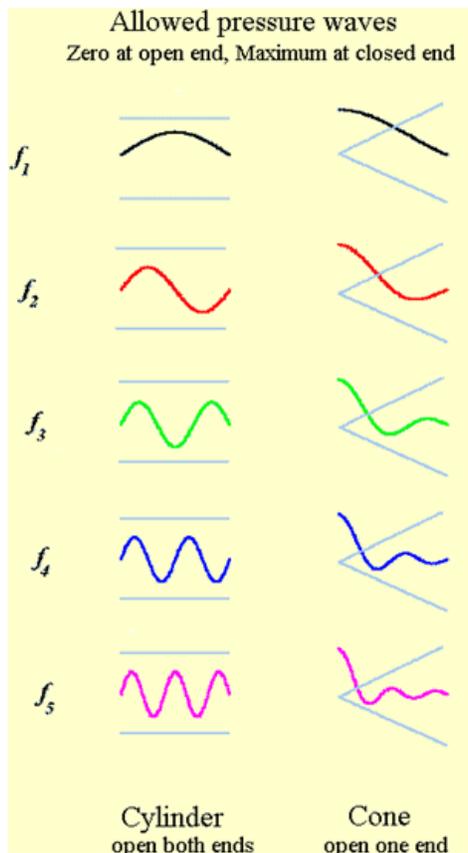


Figure 11: Pressure Waves through Cylindrical and Conical Bores (Ayers)

While changes to the bore profile were design consideration, some argue that the bore has no effect on the sound produced by the flute. The “ideal” flute tone, known as the “son plein,” has a quality of tone resembling that of a clarinet. The tone can be more difficult to achieve with a cylindrical bore than with a conical bore, yet it is not impossible (Welch). Welch

claims that the sound depends more on the skill of the flute player, and that the strength of the lip and size of the holes are more important than the bore profile (Welch).

A conical bore can be thought-of as a cylindrical bore with a taper. A conical bore is observed to be more desirable in flute design, as it seems to improve intonation (McGee). A common design utilizes a cylindrical bore from the embouchure hole to the first finger hole, where the bore becomes and remains conical but is still open at one end (Healy).

When looking at the diameter of the bore, Mark Shepard determined that an ideal length to diameter ratio is 23:1. This is due in part to the fact that pipes with shorter lengths and larger diameters will produce poor overtones, while a flute with a longer length and shorter inside diameter will produce a clearer, higher volume note. However, a tube that is too long and has too small a diameter will produce breaks in the harmonics. Most flutes alter the bore at about 1/5 of the length from the embouchure hole with a taper that ultimately reduces the diameter by 10%. By providing such a taper, the pitch of the lower tones can be lowered without a significant difference in the higher tones (Hopkin).

Tone Hole, Placement, Size and Chimney Heights.

Even though flutes follow the same principles as open tubes they still exhibit some different behaviors. The holes are located along the cylindrical face of the tube, which changes the way the sound waves travel through the tube. The actual distance from the embouchure hole to the next open tone hole is less than the distance that would correspond to a tube with no holes that plays the same note. There is a theoretical end correction factor which must be added to the actual distance in order to determine the note that will be played (Hopkin).

The tone holes that are covered by the player's fingers determine the note that will be played when air is passed over the embouchure hole. The location of the tone hole is the major factor that determines the note that is heard. The closer the hole is to the embouchure hole, the higher the frequency that will be heard.

Another way to vary the pitch is by varying the diameter of the tone hole. A hole with a larger diameter will sound a higher pitch. If the diameter of the tone hole is equal to the internal diameter of the flute at the tone hole, the sounded note will have a frequency equal to the frequency played by a tube of a length equal to the distance between the embouchure and

tone holes (Hopkin). The closer the tone hole is to the internal diameter of the flute, the lower the theoretical end correction factor. With a larger tone hole, a louder volume and a richer tone are produced. However, changes in tone hole size can affect the comfort of the player.

The pitch can also be raised or lowered depending on the height of the material around the hole. This wall thickness can also be referred to as the chimney height. In a typical flute this will be equal to the wall thickness of the tube of the flute. However, if the chimney height is reduced, it will raise the pitch of that note. The reverse is also true. If the height of the material is increased around the hole, it will lower the pitch of the note (Hopkin).

Methods and Procedure

Irish Flutes Used

Throughout the project, six main keyless flutes were analyzed and used to record data. One of the primary flutes used was a D flute made out of a $\frac{3}{4}$ " diameter piece of PVC pipe. This flute was purchased from repurposeeverything.org. Other flutes were also created by the group using $\frac{3}{4}$ " diameter PVC pipe. A Pratten-style D flute made out of a Delrin® polymer was also extensively used in testing. Additional flutes included D flutes made out of African Blackwood, Mopane, and Olive wood. The Olive wood flute had separate segments and could be modified to become an Eb flute. These flutes can be seen in Figure 12, Figure 13, Figure 14, Figure 15, Figure 16, and Figure 17, respectively.



Figure 12: PVC D Flute Manufactured by repurposeeverything.org



Figure 13: Delrin D Flute Manufactured by David Copley in Ohio



Figure 14: African Blackwood D Flute Manufactured by Bryan Byrne in Vermont



Figure 15: Mopane Wood D Flute Manufactured by Windward Flutes



Figure 16: Olive Wood D Flute Manufactured by Windward Flutes



Figure 17: Olive Wood D Flute Modified to Play in the Key of Eb

Throughout the project, it was essential to create accurate 3D CAD models of the Irish Flute. The group used SolidWorks 2012 to create these models of each flute. The models were a convenient way to keep track of the dimensions of each flute. While the group measured each of the flutes as accurately as possible, some dimensions were difficult to obtain. Measuring the undercutting found at the embouchure hole of the Delrin® flute was one challenge. Other small dimensions such as angles within the bore were measured to the group's best ability. A technique was developed in order to determine the angle of undercut for the embouchure hole. This process used a rigid card to translate the angle of undercut to an angle that could be measured on a flat surface. The angle α , seen in Figure 18, was determined by measuring the distances "A" and "B" and using trigonometry to determine the value of the angle.

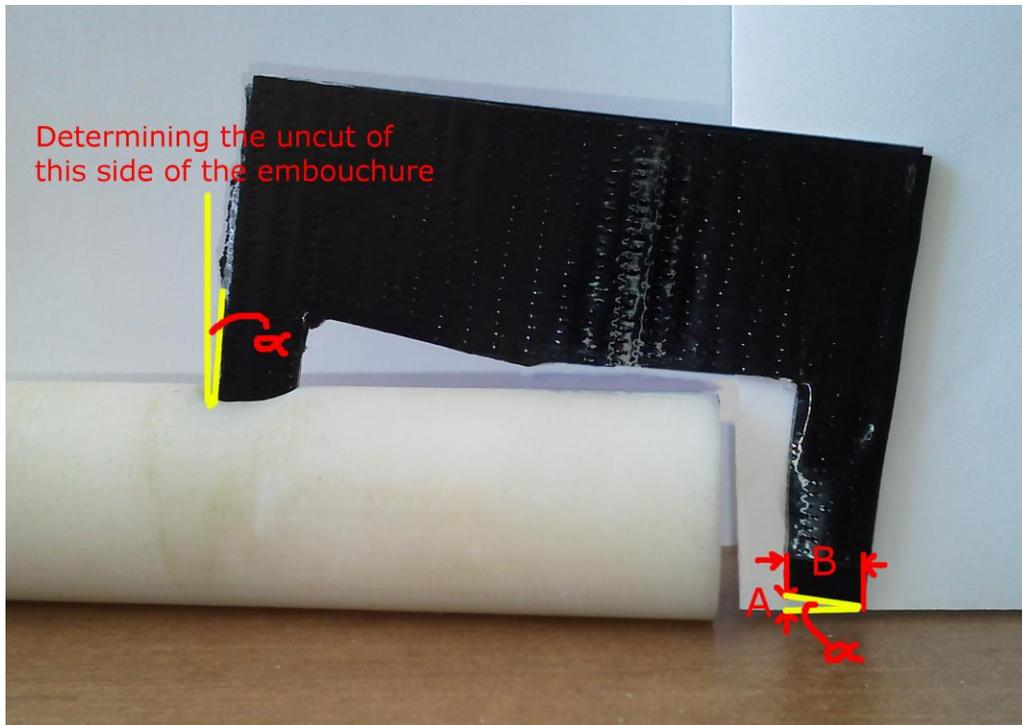


Figure 18: Method of Determining Angle of Undercut

Initial Rapid Prototyping

An important goal of the project was to assess the feasibility of rapid prototyping an Irish flute. A main concern was that the rapid prototype material would allow too much air to escape through the joints of the flute. Another issue was whether the material would be too fragile to be viable for flute production.

The advantage of rapid prototyping is that features such as extended chimney heights on tone holes can be made very easily. Any type of variable bore profile can be used because machining considerations do not need to be made. Using a rapid prototype machine also allows flutes to be created with higher accuracy than traditional methods such as turning on a lathe. Rapid prototyping can result in a more consistent and customizable design.

The group based its initial prototype design on a David Copley Delrin® flute. The CAD model of this flute can be seen in Figure 19.



Figure 19: SolidWorks Models of Delrin® (top) and Initial Rapid Prototype (bottom)

The Delrin® flute seemed like a common design, yet it was more complex than the simple flutes made out of PVC tubing. This design was modeled as precisely as possible before it was rapid prototyped. In the model, a gap of 0.0254 mm was used between all the overlapping joints. This was done to compensate for the tolerance allowance of the rapid prototype machine. The flute model was divided into six smaller parts before it was printed. This was done so that all the parts could be printed vertically, and so that it would be possible to create replacement parts without having to reprint larger sections. Furthermore, when printed vertically the prototype was symmetric about the axis of rotation. This is because the layers created by the printer were perpendicular to the axis of rotation. It was feared that layers parallel to the axis of rotation may cause grooving in the bore. The flute was printed with a cork built into the flute at a fixed location.

The group's rapid prototype was created using an Objet260 Connex Rapid Prototype machine. The material used was called VeroWhite. The .stl files were saved with a tolerance of 0.1524 mm, which corresponded to the tolerance of the RP machine. The group chose to use the finest possible tolerance in an attempt to reduce any effects of surface roughness on the model.

There were a number of different finishing techniques that were applied to the rapid prototype. The first technique was to use an automotive buffing compound to smooth out the flute. This did not provide a significant amount of refinement. The next method was to use 220-grit sandpaper. This resulted in a much smoother finish than the printer was able to produce. This finishing did not remove too much of the material on the flute, so the parts still fit together fairly well, and the flute played as expected.

Design and Fabrication of the Flute Playing Device

In order to ensure consistent testing, a method to provide a steady note on the flute was needed. The first attempt was to take ¼" copper pipe and bend it around the flute. An air hose would then be connected to the end. The end that was above the embouchure hole was bent slightly to angle the air into the embouchure hole. This can be seen in Figure 20.



Figure 20: Copper Pipe Constant Playing Apparatus

This apparatus functioned in the upper octave, but it did not produce very consistent results and it could not play the lower octave. The range also tended to be higher than the normal range of the flute.

The flute playing device was conceived out of the need for easily measurable and repeatable flute frequencies. It was determined that in order to reproduce the human ability to blow into the flute, the machine would need a pressure source, a perturbation chamber, a nozzle, and a simulated lower lip. The upper lip was omitted as it only serves to direct the air stream downward, and this effect could be otherwise simulated. The lower lip was included because it acts as a vertical spacer between the air stream and the flute's embouchure hole.

The completed flute-playing device is shown in Figure 21.



Figure 21: Completed Flute Playing Device on a PVC Flute

Air is blown into the system using a pressure source in the lab (Higgins Labs Room 031). Pressure and volume are controlled using a regulator and two cutoff valves: one located on the regulator and the other on the lab's air nozzle. Air then enters a tube that connects to the perturbation chamber, which is crafted from a prescription pill container. In the chamber, the air stream is disturbed in a manner comparable to the human mouth. Another tube then passes the air over a rolled piece of paper, which acts as the simulated lower lip. Finally, the air passes over the embouchure hole and sounds the desired note.

The flute playing device was hooked up to the regulator and the pressure source at the beginning of each testing period. The flute was then attached to flute playing device's tension clamp. The flute's tone holes were taped until the proper fingering for the desired note was attained. The position of the clamp on the flute was adjusted until the flute sounded the desired note.

Testing Previously Developed Equation Models

At the very beginning of the mathematical modeling, the group tested out the equation models that were developed by previous musical scholars and researchers. Various mathematical equations were found for musical instruments focusing on the simple flutes. The group tested two mathematical equations that were applicable to this particular project.

Chris Forster, a musical instrument builder, composer and scholar, developed a series of mathematical equations for flute construction based on Nederveen's book *Acoustical Aspects of Woodwind Instruments* in 1969. The equations listed below were tested on the initial rapid prototyped flute. The flute was played both by the blowing rig and human players, and frequencies were recorded for calculations of the tone hole locations with all the tuning slides fully shortened.

$$L_H = l_H + d_H - \frac{0.45 * (d_H)^2}{d_1}$$

$$C_{emb} = \left(\frac{4d_{be}^2}{D * W} \right) * H_e$$

The embouchure correction value, which is the distance from embouchure hole to the left of the cork, was also calculated by using the equation given in the book *Fundamentals of Musical Acoustic* for three different types of flutes. Listed parameters of the flute were measured as well as the lip thickness of the player for this calculation. The results of the testing of these equations are reported in the previously developed equations section under Results.

Initial Mathematical Modeling

Equations that the group developed for determining the hole placements for the Irish flute were based on the principle of a tube with two open ends. Correction lengths for both ends were applied in order to determine the tone hole placement.

The first correction was at the embouchure. The embouchure correction length (Δl_e) accounted for that end of the tube being stopped, as well as the existence of the embouchure hole on the cylindrical surface of the flute. This correction was used in all of the tone hole calculations.

At the other end of the flute, there were two different corrections depending on the number of the tone holes that were covered. A correction length for the end of the flute (Δl_{hemb}) was used when all the tone holes were closed on the flute. This correction accounted for the sound wave leaving the end of the tube. When any of the tone holes were open, the correction length for the tone hole (Δl_h) was used. This correction adjusted for the sound wave leaving through a hole along the cylindrical face of the flute as well as the size of that hole.

The wavelength is equivalent to the length of the tube, open at both ends, needed to produce the desired frequency. In the equations to follow, this length is referred to as L_s . Correction factors are subtracted from the wavelength. With these corrections removed, the distance from the center of the embouchure hole to center of the tone hole (L_a) is determined. In the case where all the tone holes are closed, this length is equal to the distance from the embouchure hole to the end of the flute. A simple visual of these correction lengths can be seen in Figure 22.

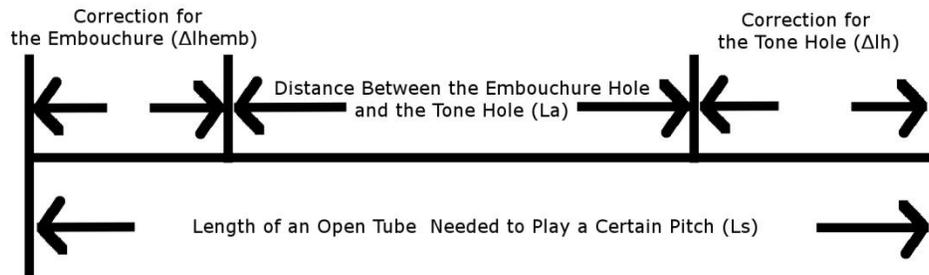


Figure 22: Correction Length used in Determining Tone Hole Location

The equation for the correction lengths that the group created were based on five basic observed relationships. The first relation states that if the diameter of the embouchure hole (d_e) is equal to that of the diameter of the bore (d_{be}) at the embouchure, then the correction length at the embouchure (Δl_e) is equal to zero. If the two diameters are the same size, this is equivalent to cutting the bore of the flute off at this point.

1. If $d_e = d_{be}$, $\Delta l_e = 0$

This same principle is seen in the second relationship. The relationship states that if the diameter of the tone hole (d_h) is equal to the diameter of the bore at the tone hole (d_{bh}) then

the correction length at the tone hole (Δl_h) is equal to zero. The reasoning for this is the same as was the reasoning for the embouchure hole.

2. If $d_h = d_{bh}$, $\Delta l_h = 0$

The third and fourth relations state that if the wall thickness at the tone hole (h_1) increases or the wall thickness at the embouchure hole increases, then the frequency will decrease. The longer the pipe used to produce a sound, the lower the pitch. Therefore, increasing the wall thickness will result in a lower pitch.

3. If h_1 increases, frequency decreases

4. If h_0 increases, frequency decreases

The final relation states that if the diameter of a tone hole (d_h) decreases, then the frequency produced will also decrease. The inverse of this relation is also true.

5. If d_h decreases, frequency decreases

As stated in the second relation, the correction length at the tone hole becomes zero once the tone hole diameter equals that of the bore diameter at the tone hole. The smaller the tone hole, the further the sound wave node will form past that tone hole. This creates a longer wave which produces a lower frequency. This change in the node placement due to a decrease in hole diameter can be seen in Figure 23.

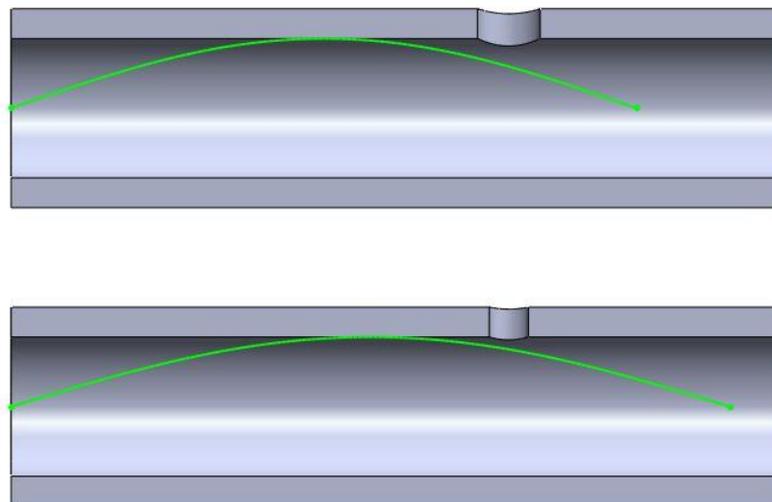


Figure 23: Change in Wavelength Node due to Reduction in Diameter

This causes the theoretical length of the pipe to be longer than the actual distance from embouchure hole to the tone hole, thus lowering the frequency.

The equations that the group developed were based off the five relations stated above. Dimensional analysis was also used to ensure that the units worked out. The equations took the wavelength of a desired frequency and subtracted corrective lengths from both ends of the flute in order to compensate for the flute's embouchure and tone holes. The following describes the equations that the group developed.

When determining hole placements for a flute, it was difficult to know the diameter of bore at the tone hole without first knowing the location of the tone hole. This situation required a method of estimating the diameter of the bore. This estimate for the bore diameter at the tone hole (d_{est}) was calculated using the formula seen below.

$$d_{est} = \left(\left(\frac{d_{be}}{2} \right) - \left(\frac{d_{end}}{2} \right) \right) * \left(\left(\frac{F_d}{F_b} \right) - 1 \right) * 2 + d_{end}$$

This equation uses the principle of similar triangles in order to estimate the diameter of the bore. Using half of the diameter of the bore at the embouchure hole (d_{be}) and half of the diameter of the bore at the end of the flute (d_{end}) will result in a triangle that can be used to determine d_{est} . In order to find the rough length of where the hole should be, the ratio between the desired frequency (F_d) and the base frequency (F_b) is used. Because the angle of taper of the bore is not large this approximation produces fairly accurate results.

This same "similar triangle" technique was used to estimate the wall thickness (h_{est}). This results in the equation as well as the dimensional analysis is seen below.

$$h_{est} = \left(\left(\frac{h_0}{2} \right) - \left(\frac{h_{end}}{2} \right) \right) * \left(\left(\frac{F_d}{F_b} \right) - 1 \right) * 2 + h_{end}$$

$$mm = \left(\left(\frac{mm}{2} \right) - \left(\frac{mm}{2} \right) \right) * \left(\left(\frac{Hz}{Hz} \right) - 1 \right) * 2 + mm$$

The equation uses the wall thickness at the embouchure hole and the wall thickness at the end of the flute to estimate the wall thickness at the tone hole in question.

The correction length at the embouchure hole (Δl_e) was calculated using the ratio of the differences between the area of the bore of embouchure (A_{be}) and the area of the embouchure

hole (A_b) over the diameter of the bore at the embouchure hole (d_{be}) minus the diameter of the embouchure hole (d_e). This value was then added to height of the wall thickness at the embouchure (h_0). This equation is shown below as well the dimension analysis for this equation.

$$\Delta l_e = \frac{(A_{be} - A_e)}{(d_{be} - d_e)} + h_0$$

$$mm = \frac{(mm^2 - mm^2)}{(mm - mm)} - mm$$

An equation was needed to determine the correction length at the end of the flute when all of the tone holes were closed. This equation uses the ratio between the base frequency (F_b) and the desired frequency (F_d) added to 5 times the wall thickness at the end of the flute (h_{end}) and that same ratio times the bore diameter at the end of the flute (d_{end}). The equation as well as the dimensional analysis can be seen below.

$$\Delta l_{hemb} = \left(\frac{F_b}{F_d} \right) + 5 * h_{end} + \left(\frac{F_b}{F_d} \right) * d_{end}$$

$$mm = \left(\frac{Hz}{Hz} \right) + 5 * mm + \left(\frac{Hz}{Hz} \right) * mm$$

The equation to determine the correction length for the tone holes was determined using the ratio between the difference of the estimated area of the bore (A_{est}) and the area of the active tone hole (A_e) over the estimated diameter of the bore at the active tone hole (d_{est}) minus the diameter of the tone hole (d_e). This ratio was multiplied by the ratio of the base frequency (F_b) and the desired frequency (F_d) as well as the ratio of the estimated diameter of the bore at the active tone hole (d_{est}) over the diameter of the tone hole (d_h). Added to this was 5 times the estimated wall thickness (h_{est}) and 0.5 times the ratio of the base frequency (F_b) and the desired frequency (F_d) times the estimated diameter of the bore at the active tone hole (d_{est}). The estimate area of the bore was the calculated using the estimated bore diameter. This equation for the calculation of the correction fact at an active tone hole as well as the dimensional analysis of the equation can be seen below.

$$\Delta l_h = \frac{(A_{est} - A_h)}{(d_{est} - d_h)} * \frac{F_b}{F_d} * \frac{d_{est}}{d_h} + 5 * h_{est} + 0.5 * \frac{F_b}{F_d} * d_{est}$$

$$mm = \frac{(mm^2 - mm^2)}{(mm - mm)} * \frac{Hz}{Hz} * \frac{mm}{mm} + 5 * mm + 0.5 * \frac{Hz}{Hz} * mm$$

The distance from the embouchure to the active tone hole (L_a) was determined by taking the actual length of the desired frequency wavelength (L_s) and subtracting the correction length for the embouchure (Δl_e) as well as the corrective length for the active tone hole (Δl_h).

The distance from the embouchure to the end of the flute (L_{aemb}) was determined by taking the actual length of the desired frequency wavelength (L_s) and subtracting the correction length for the embouchure (Δl_e) as well as the corrective length for the end of the flute (Δl_{hemb}).

All of the equations that the group developed and discussed above can be collectively below.

$$d_{est} = \left(\left(\frac{d_{be}}{2} \right) - \left(\frac{d_{end}}{2} \right) \right) * \left(\left(\frac{F_d}{F_b} \right) - 1 \right) * 2 + d_{end}$$

$$h_{est} = \left(\left(\frac{h_0}{2} \right) - \left(\frac{h_{end}}{2} \right) \right) * \left(\left(\frac{F_d}{F_b} \right) - 1 \right) * 2 + h_{end}$$

$$\Delta l_e = \frac{(A_{be} - A_e)}{(d_{be} - d_e)} + h_0$$

$$\Delta l_{hemb} = \left(\frac{F_b}{F_d} \right) + 5 * h_{end} + \left(\frac{F_b}{F_d} \right) * d_{end}$$

$$\Delta l_h = \frac{(A_{est} - A_h)}{(d_{est} - d_h)} * \frac{F_b}{F_d} * \frac{d_{est}}{d_h} + 5 * h_{est} + 0.5 * \frac{F_b}{F_d} * d_{est}$$

$$L_a = L_s - \Delta l_e - \Delta l_h$$

$$L_{aemb} = L_s - \Delta l_e - \Delta l_{hemb}$$

The equations above do not take into account the chimney height above the outer bore (h_c). This was considered after these equations were applied to determine the hole placement. If the distances between the tone holes were too far for fingers to fit comfortably, then the distance between them could be decreased and the chimney height of the affected holes above the bore could be increased.

In order to easily compute values for the placement of tone holes, all the necessary values and equations were entered into a spreadsheet which can be seen Figure 46 in Appendix A. The values highlighted in orange are the input values needed for the calculations. The values highlighted in green are the final calculated values for the tone hole placement. This allowed all the values for each tone hole to be computed simultaneously. The spreadsheet enabled data from different tests to be easily entered and analyzed.

Frequency Gathering of PVC, Delrin®, RP, Olive, and Mopane Material Flutes

In order to refine and confirm the equations that were developed, a number of different tests were performed to obtain experimental data. Some of these were used in order to determine whether or not certain changes to the flute would greatly alter the frequencies. Six different flutes were used in these tests. The primary flutes used were a D flute that was made out of ¾ inch PVC pipe, a D flute that was made out of Delrin®, and the group’s initial rapid prototyped flute that was based on the Delrin® Flute.

Human Player Frequency Testing

Each note of the flute was played in the two octaves in question. The upper octave of the flute was played with the same fingerings as the lower octave, but overblown. While one group member played the flute, another group member measured the frequencies of the notes played. Frequency was measured using the smart phone tuner application “gStrings”. A screen shot of this application can be seen below in Figure 24.



Figure 24: gString Tuner Application

The frequencies were then recorded either manually or in Excel. Human frequency testing was performed on the Delrin®, PVC, RP, Mopane, and Olive flutes. All flute testing was done with the tuning slides pushed fully in. This was to ensure consistency throughout the group's testing.

Flute Playing Device

The flute playing device was hooked up to the regulator and the pressure source at the beginning of each testing period. The flute was then attached to flute playing device's tension clamp. The flute's tone holes were taped until the proper fingering for the desired note was attained. The position of the clamp on the flute was adjusted until the flute sounded the desired note. Frequency was then measured and recorded. Frequency testing with the flute playing device was performed on the rapid prototyped flute only.

Chimney Height

To vary the chimney height, the group used a series of 15 different cardboard sheets, each with holes cut to fit the size of the flute's tone holes. This set up can be seen below in Figure 25.



Figure 25: Set up using Cardboard to Increase Chimney Heights

During testing a clamp was used to apply pressure to the cardboard to hold it against the flute. Another technique that was used was applying layers of tape to increase the chimney height and then drilling through to open up the tone hole. Frequencies were measured and recorded in the same manner as for human frequency measurements. Instead of one set of data, however, five sets of data were taken: each with a different number of cardboard sheets wrapped around the bore at each tone hole. The height of the cardboard sheets above the tone hole was measured for each trial using a caliper. Each note-chimney height combination was played three times and the average of the frequencies was taken. Chimney height testing was performed on the rapid prototyped flute only.

Bore Extension Testing

Bore extension testing was performed in much the same manner as was the frequency testing. For this test the bore was extended by certain increments and frequency data was taken for two octaves. There were a total of four different extension amounts including a base

set of data with all the tuning slides pushed in. Bore extension testing was performed on the rapid prototyped flute only.

Temperature

Temperature data was taken before the frequency and chimney height tests were performed. A digital thermometer was used to record ambient temperature, as well as the temperature at the far end of the bore, inside each tone hole, and inside the embouchure hole. The flute was warmed up by playing before these tests were performed, and all temperature values were measured while the flute was being played.

Cork Displacement

Cork displacement testing was performed in much the same manner as was the human frequency testing, except that five sets of data were gathered, and each of these data sets corresponded to a different tuning cork position. Before each data set was taken, the distance from the cork to the end of the bore at the first joint was measured by inserting a wooden dowel into the flute and drawing a line on the dowel at the first joint. The distance between the end of the dowel and the line was then measured using a tape measure. Cork displacement testing was performed on the Delrin® flute only.

Human Player Averaging

In human player averaging, frequency data was taken over two octaves for four different human players, one of whom was not member of the group. The frequency data points of these players were then averaged to ensure that the group's flute players played in-tune.

Final Rapid Prototyping

For the final rapid prototype the major change was to increase the E tone hole and increase the chimney height so that the hole could be moved closer to its neighboring tone holes. The decision was also made not to print a cork into the flute, but to insert a cork after the flute was printed. This gave the flute more versatility when it came to tuning the second octave. The tone holes, the locations of which were determined by the set of equations, were

moved in closer to the embouchure hole by 10 mm. This was done so that the flute would be in tune with the tuning slide pulled out 10mm. This allows the player to push the slide in if atmospheric conditions are causing the flute to play flat. A cross section of the flute along with the locations of the tone holes can be seen in Figure 26.

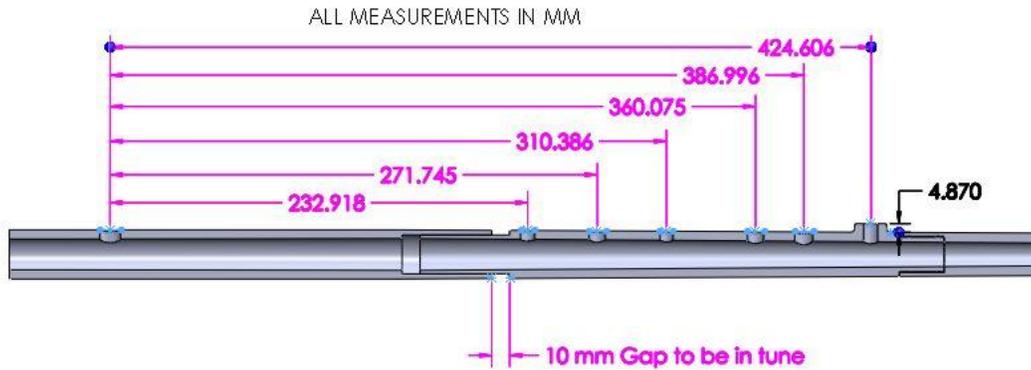


Figure 26: Final Rapid Prototype Cross Section with Tone Hole Locations

The chimney height on the last tone hole was increased by 4.87mm. This was done to compensate for increasing the diameter of that hole to 8mm. If the chimney extension was not added that hole would have been placed an additional 15mm away from the previous hole making it difficult to play.

The flute was printed horizontally to reduce the number of pieces in the flute to three. All six tone holes were placed on one piece to eliminate any shear loads on a joint that was in between sets of tone holes. A joint was included after the tone holes where a small section of the flute attaches to complete the flute's length. This overlapping joint contains two small rings around the smaller joint section. These allow the overlying section to have only two points of contact. This was done in an effort to eliminate any rocking in the joint. A close-up of this joint can be seen below Figure 27.

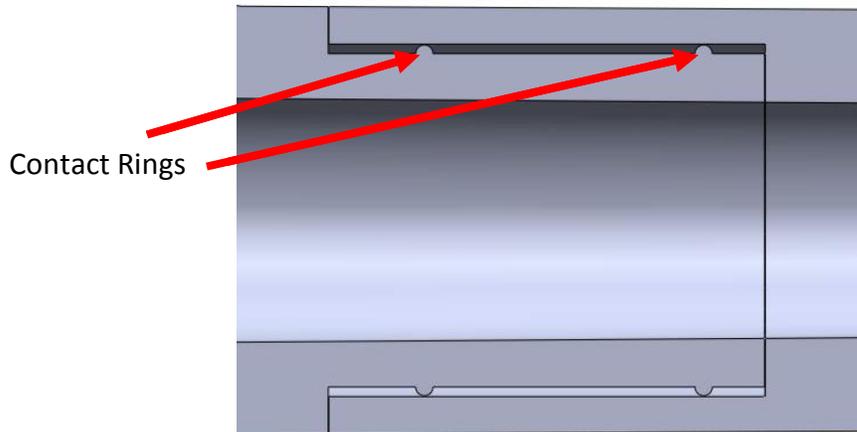


Figure 27: Close of End Joint in Final Flute

The joint closest to the embouchure hole was extended to overlap by 50 mm to accommodate for tuning. This joint was left smooth without any contact rings so that the joint can be easily extended. This removed the possibility that the upper part of the joint will fail to make contact with a contact ring and become unstable.

The final rapid prototype was also created using an Objet260 Connex Rapid Prototype machine. The material used was VeroWhite. The .stl files were saved with a tolerance of 0.1524 mm. The group chose to use the finest tolerance in order to reduce any effects of surface roughness on the model. A gap of 0.0254 mm was used between the joints.

Results

Results of Initial Prototype

The group's initial rapid-prototyped flute was playable with relative ease. In terms of the finish, this flute was noticeably rougher than any of the wood or Delrin® flutes that were encountered. For the most part the 0.0254 mm gap that was designed into the joints of the flutes resulted in a tight fit. There were a couple of joints that had a loose fit. In order to seal these joints the group used Teflon tape. This not only helped to seal the joints but added more rigidity to the flute as a whole. The decision to print a cork in a fixed position was one that limited some of the desired tests. One test was the cork displacement so not being able to move the cork position to see the influences was a limitation for the analysis in the rapid prototype.

Upon close examination of rapid prototyped flute it was seen that the layering effects from the printer are very small and are not very noticeable or intrusive. These layers can be clearly seen in Figure 28.

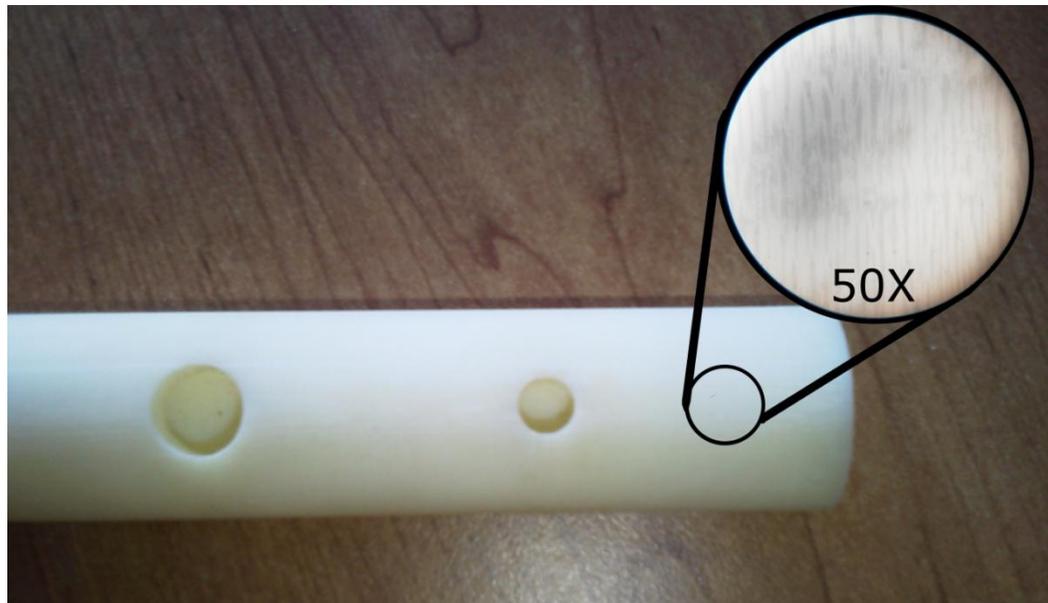


Figure 28: Magnification of Layers Created by Rapid Prototyping Vertically

The decision to make the flute in six pieces turned out to have an effect that the group was not expecting: the multiple joints resulted in a flute that was not as rigid as the flutes that the group used for a comparison.

A final concern of the rapid prototyping process was the cost of manufacturing. The group's first flute cost \$378.43 in materials. If manufacturing processes were considered the total price would increase.

Previously Developed Equations

Based on Forster's formula, effective length of the tone hole could be calculated by the formula shown below. In this equation l_H is the bore thickness at the tone hole, d_H is the tone hole diameter, and d_1 is the bore diameter at the tone hole.

$$L_H = l_H + d_H - \frac{0.45 * (d_H)^2}{d_1}$$

Table 1: Testing of Forster's Effective Length of Tone Hole Equations

Note	Calculated Distance from Tone Hole to Embouchure (mm)	Measured Distance from Tone Hole to Embouchure (mm)	Difference (mm)	% Error
D4	553.2	517.9	35.3	-6.38
E4	459.22	424.89	34.32	-7.48
F#4	414.65	386.64	28.01	-6.76
G4	382.79	358.02	24.77	-6.47
A4	315.87	298.02	17.85	-5.65
B4	285.83	263.93	21.9	-7.66

The formula was tested on the RP flute and a human player played both octaves. As shown in the result chart, the difference between the calculated distance from tone hole to embouchure hole and the actual measured length was considerably large, ranging from 17.8 mm to 35.3 mm in the first octave and even greater in the second octave. Several attempts were done to re-derive the original equation and adjust it by modifying the coefficients. However, all the attempts only resulted in minor changes. Therefore, the team concluded that the Forster equation was not able to correctly predict the RP flute tone hole distance from the embouchure hole based on the given parameters.

As previously indicated, based on the book *Fundamentals of Musical Acoustics*, the distance from embouchure hole to the left of the cork can be determined from the following formula:

$$C_{emb} = \left(\frac{4d_{be}^2}{D*W}\right) * H_e$$

Table 2: Calculation of Distance from Embouchure Hole to the Left of the Cork

Flute	C _{emb} (mm)
RP	32.70
PVC	217.29
African black wood	37.05

The book clearly indicates that most flutes have a C_{emb} value around 50 mm. The equation was put to the test and yielded results of 33 mm and 37 mm for the RP flute and the African black wood flute accordingly. The PVC flute yielded a very large value for C_{emb} based on this equation due to its relatively large radius of air column and small chimney height. Even though there could be measurement uncertainty, especially on the lip thickness of the human player, the differences were quite large considering the magnitude of the total distance of the flute.

Uncertainty Calculations

The final uncertainty of the group's equations was determined for each hole location on the flute. The uncertainties for each of the parameters were based on the accuracy of the group's measurements. For the group's diameter measurements the group's calipers operated with an uncertainty of 0.01 mm. The group's length measurements had an uncertainty of 0.1 mm because a tape measure was required to measure longer distances. This resulted in a range in relative uncertainty from 0.02% to 0.24% in calculating each hole location relative to the embouchure hole. This can be seen in Table 3. The higher-percent uncertainties were found in the upper octave. This is because the higher frequencies have a greater influence on the uncertainty in the equation.

Table 3: Relative Uncertainty in Tone Hole Placement Calculations

Note	Frequency Desired	Tone Hole Placement Uncertainty
D4	293.66	0.02%
E4	329.63	0.04%
F#4	369.99	0.04%
G4	392	0.04%
A4	440	0.06%
B4	493.88	0.07%
C#5	554.37	0.10%
D5	587.33	0.02%
E5	659.26	0.07%
F#5	739.99	0.08%
G5	783.99	0.10%
A5	880	0.13%
B5	987.77	0.18%
C#6	1108.73	0.24%
D6	1174.66	0.02%

General Frequency

The results of the general frequency test were in keeping with the group's expectations. This test was performed with all the tuning slides pushed in. As seen in Figure 29, Figure 30, and Figure 31, in general, the flutes were sharp which can be seen in the positive difference value. The figures below show the difference between the played frequency and the actual frequency in relation to the actual frequency. All of the data for these tests can be seen in Figure 47, Figure 48, and Figure 49 in Appendix A.

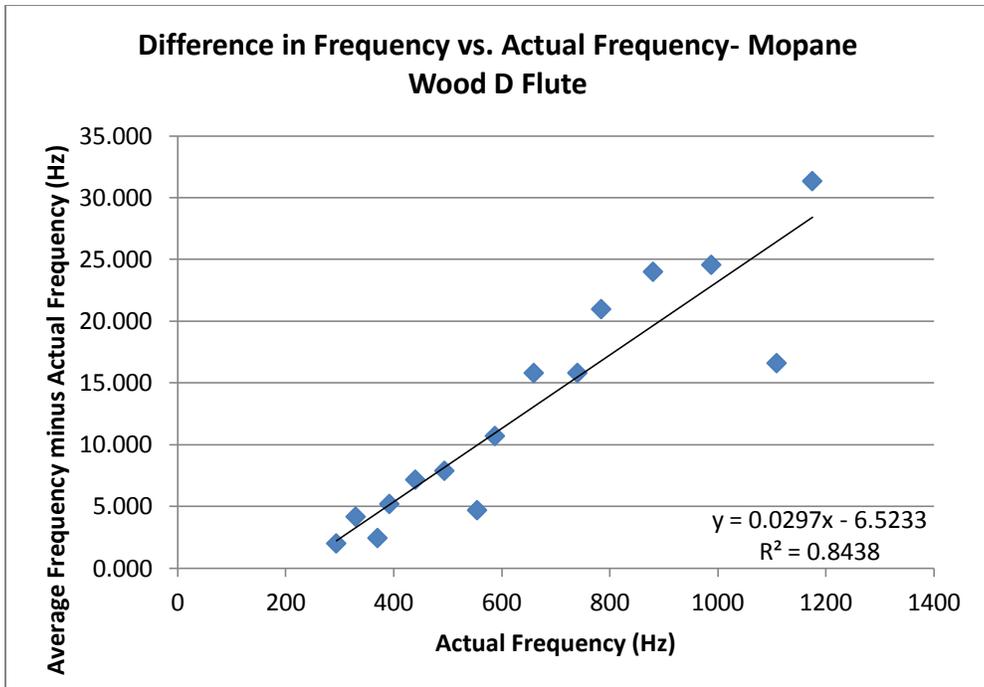


Figure 29: Difference in Frequency vs. Actual Frequency for the Mopane Wood D Flute

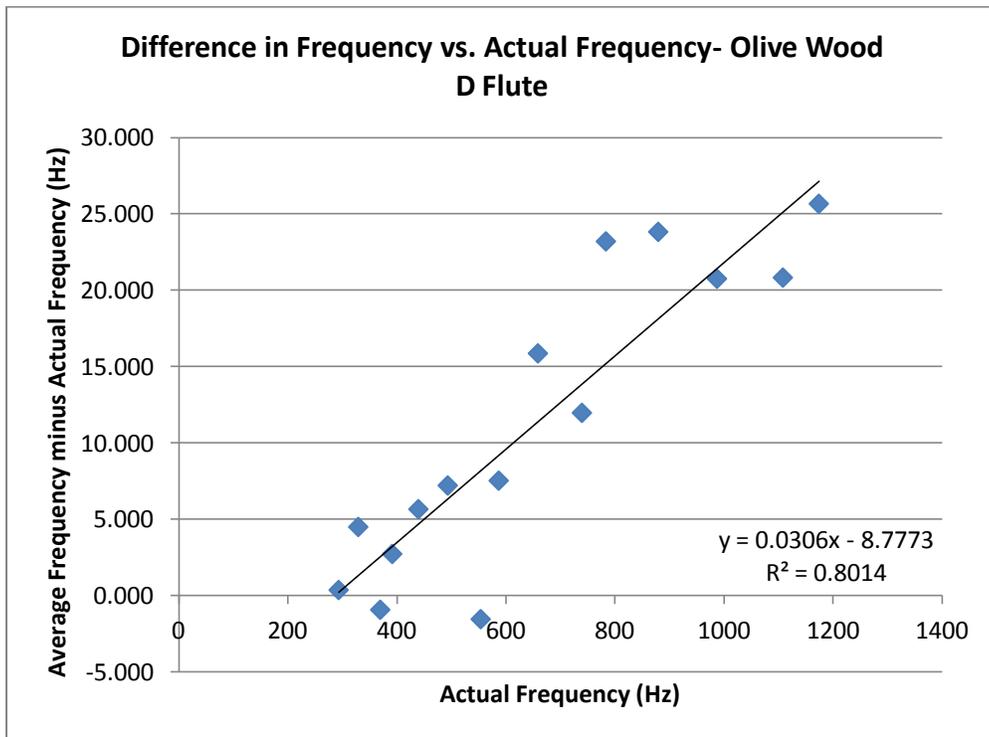


Figure 30: Difference in Frequency vs. Actual Frequency for the Olive Wood D Flute

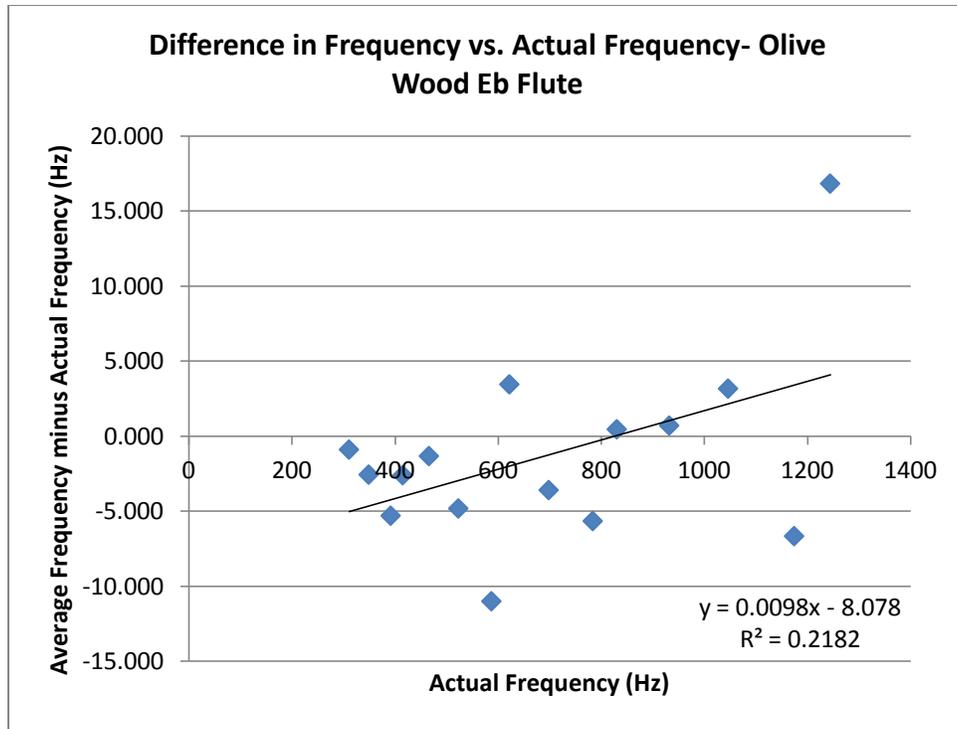


Figure 31: Difference in Frequency vs. Actual Frequency for the Olive Wood Eb Flute

Cork Displacement

The results of the cork displacement test can be seen in Table 4. The effect of the cork displacement appears to be negligible in the first octave. However in the upper octave the influence was much more prevalent. This is very noticeable in Table 5 which shows the change in frequency based on the position of the cork. Values that appear negative in Table 5 mean that the frequency actually increased when the cork was moved further away from the embouchure hole. However, these variations from the trend could account for in the fact that the change is very small. This deviation could be attributed to variations in the player’s embouchure during testing. Before testing the cork was at 16.78 mm away from the edge of the embouchure hole. A graph of the change in frequency from the base frequency for different cork positions can also be seen in Figure 32.

Table 4: Frequency Played Relative to Cork Position

		Distance from Edge of Embouchure Hole to Cork (mm)				
		0	10.3	16.78	31.42	36.61
Frequencies played(Hz)	D4	296.7	297.7	294.2	294.7	292.5
	E4	335.8	337.9	334.6	336	333.3
	F#4	377.5	377.7	373.3	377.2	371.6
	G4	403.2	404.2	396.7	402.6	395.5
	A4	456.6	456.7	451.2	454.4	448.8
	B4	510.7	514.9	501.3	510.7	499.7
	C#5	565.3	566.6	554.3	566.3	547.1
	D5	599.2	600.3	593.2	596.4	588.8
	E5	680	682.4	672.1	670.4	660.2
	F#5	764.6	762.2	751.2	748.2	737
	G5	815	811.4	799.9	800.1	784.2
	A5	919.2	912.3	896.6	897.3	872.1
	B5	1042	1033	1006	995.6	964
	C#6	1151	1146	1111	1080	1040
	D6	1234	1224	1202	1182	1144

Table 5: Change in Frequency with Respect to Cork Position

		Distance from Edge of Embouchure Hole to Cork (mm)			
		10.3	16.78	31.42	36.61
Change in Frequencies from Base Test (Hz)	D4	-1	2.5	2	4.2
	E4	-2.1	1.2	-0.2	2.5
	F#4	-0.2	4.2	0.3	5.9
	G4	-1	6.5	0.6	7.7
	A4	-0.1	5.4	2.2	7.8
	B4	-4.2	9.4	0	11
	C#5	-1.3	11	-1	18.2
	D5	-1.1	6	2.8	10.4
	E5	-2.4	7.9	9.6	19.8
	F#5	2.4	13.4	16.4	27.6
	G5	3.6	15.1	14.9	30.8
	A5	6.9	22.6	21.9	47.1
	B5	9	36	46.4	78
	C#6	5	40	71	111
	D6	10	32	52	90

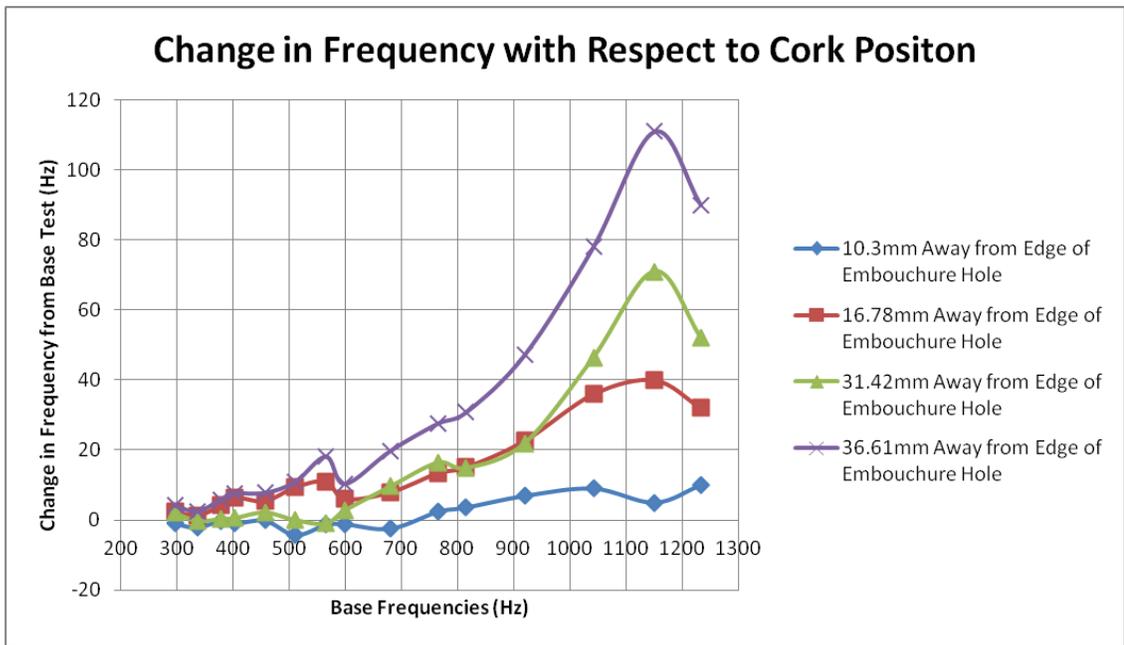


Figure 32: Change in Frequency Due to Cork Position

Bore Extension

The summarized results of the bore extension tests can be seen in Table 6 below. This testing was performed on the rapid prototype flute only. The change in frequency between the extended frequency and the base frequency of no extension was calculated and can be seen in Table 7.

Table 6: Results from Bore Extension Test

	Distances (mm)			
	0	3.82	7.03	10.52
D4	289.1	288.4	285.4	284.5
E4	328.6	326.6	322.8	320.9
F#4	369.9	364.7	362.8	361.9
G4	396.5	391.2	386.9	385.4
A4	447.3	442	440	434.4
B4	505.3	500.1	493.6	486.7
C#5	570	555.5	550.1	544.4
D5	598.3	591.3	588	586.1
E5	673.3	667.7	659.2	658.6
F#5	759.4	751.3	742.5	741.2
G5	812.2	802.7	795.5	797
A5	911.1	900.1	888.8	888.7
B5	1020	1010	998.7	997.2
C#6	1122	1110	1095	1091
D6	1203	1188	1182	1178

Table 7: Difference in Frequency from Base

	Distances (mm)		
	3.82	7.03	10.52
D4	0.7	3.7	4.6
E4	2	5.8	7.7
F#4	5.2	7.1	8
G4	5.3	9.6	11.1
A4	5.3	7.3	12.9
B4	5.2	11.7	18.6
C#5	14.5	19.9	25.6
D5	7	10.3	12.2
E5	5.6	14.1	14.7
F#5	8.1	16.9	18.2
G5	9.5	16.7	15.2
A5	11	22.3	22.4
B5	10	21.3	22.8
C#6	12	27	31
D6	15	21	25

This change in frequency was then divided by the base frequency of each tone hole (F_{bt}) and plotted against the same base frequency. A trend line was then found for this set of points.

This trend line was found to be approximately a linear function with the exception of B and C#. The graphs along with their associated trend lines can be seen in Figure 33, Figure 34, and Figure 35.

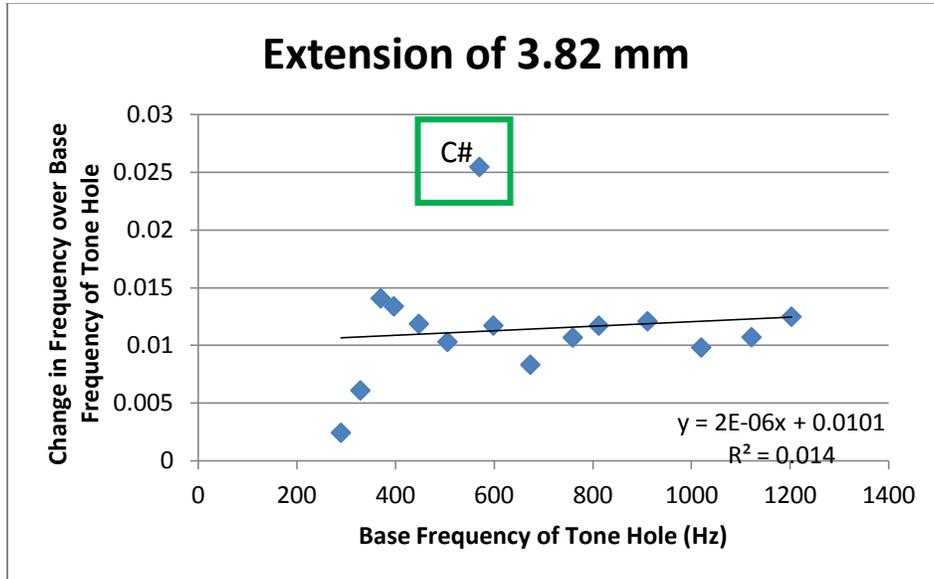


Figure 33: Change in Frequency over the Base Frequency (F_{bt}) with Respect to the Base Frequency (F_{bt}) for a Bore Extension of 3.82 mm

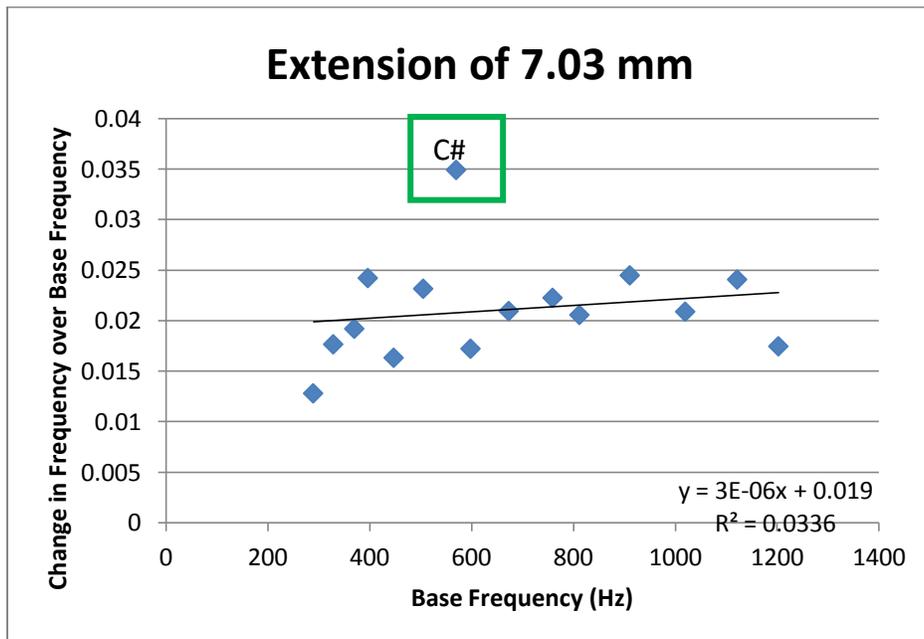


Figure 34: Change in Frequency over the Base Frequency (F_{bt}) with Respect to the Base Frequency (F_{bt}) for a Bore Extension of 7.03 mm

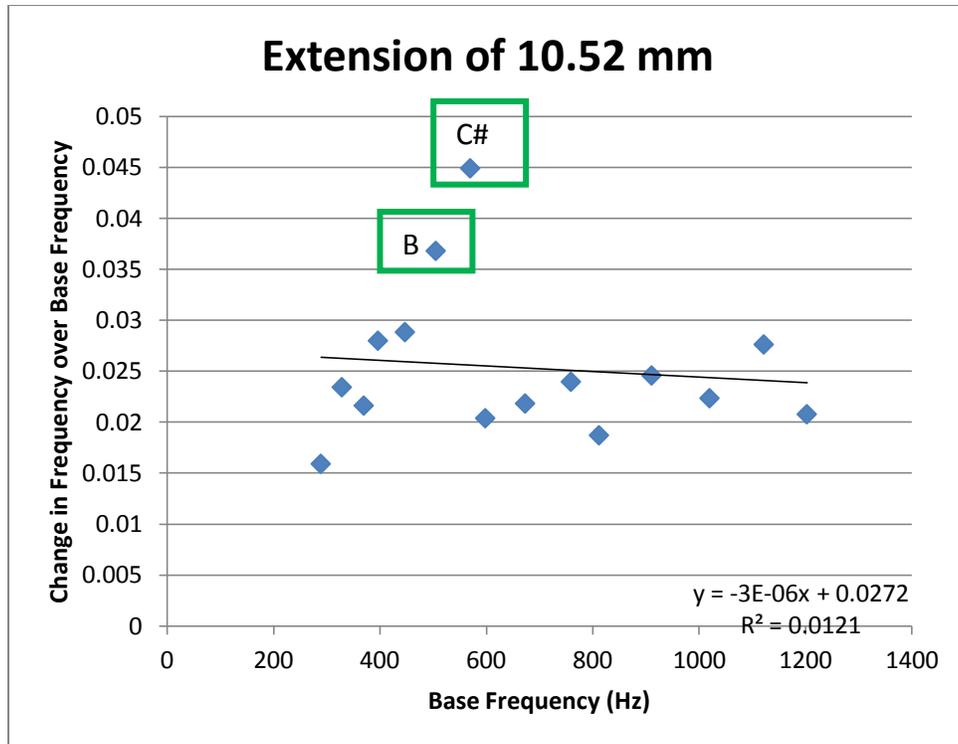


Figure 35: Change in Frequency over the Base Frequency (F_{bt}) with Respect to the Base Frequency (F_{bt}) for a Bore Extension of 10.52 mm

The slope of this line was very close to zero and therefore was approximated as such. The equation was solved for the change in frequency. This equation was then repeated for the other two bore extensions. These equations are the following:

$$\Delta f = 0.0101 f_{bt}$$

$$\Delta f = 0.019 f_{bt}$$

$$\Delta f = 0.0272 f_{bt}$$

The coefficients in these equations were plotted against the extension distance (E_x). This linear equation was then plugged back into the starting equation which resulted in:

$$\Delta f = (0.0024 * E_x + 0.00009) * f_{bt}$$

This equation holds fairly true to all the scale degrees except for B and C#. To correct for this, the previously-stated equation was turned into a step function. This function corrects for the fact that these two tone holes resulted in a greater change in the frequencies. This step function can be seen below.

$$\Delta f = (0.0024 * E_x + 0.00009) * f_{bt} * \begin{cases} 1, & \text{if } 480 < f_{bt} > 580 \\ 2, & \text{if } 480 > f_{bt} < 580 \end{cases}$$

This equation states that if the base frequency of the tone holes is in between 480 Hz and 580 Hz, then multiply the equation by a factor of 2. If not, then the equation will be multiplied by a factor of 1. This compensates for the B and C# having a greater change in frequency.

There are small differences between what was predicted and the actual frequencies played. These can be seen in Figure 50 in Appendix A. Some of this error may be explainable due to inconsistencies in playing and with further testing these errors could be reduced. The equation stated above is independent of the equations used in determining the design of the final model.

Temperature

Figure 36 shows the results for temperature testing. Temperature testing was performed on the rapid prototyped flute only. The temperature probe was inserted into each of the specified holes as the flute was being played. As expected, the temperature increases as the reading point approaches the flute’s embouchure hole. The temperature did drop off quite rapidly after the first few tone holes. Using this data, an average temperature of 25 °C was used to determine the speed of sound, and this temperature is used for tone hole placement.

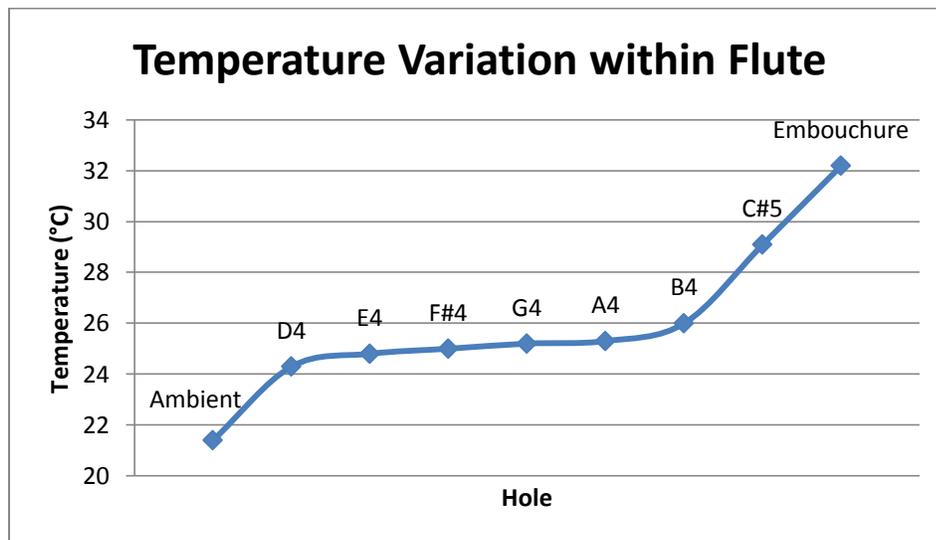


Figure 36: Temperature Variations within the Bore of the Flute while being Played

Human Player Averaging

Human player averaging tests were performed on the rapid prototyped and Delrin® flutes. The summarized results are shown below in Figure 37 and Figure 38. Ideally, the difference between the played frequency and the actual frequency should be zero. However, the all of the tuning slides where pushed in, which resulted in the flute being sharp. This causes the negative difference between the two frequencies. For the rapid prototype, the average played frequencies continued to get sharper as frequencies increased. The Delrin® Flute, on the other hand, stayed relatively sharp the through the entire range of the scale. In the rapid prototyped flute the higher frequencies could be brought more in tune by moving the cork further away from the embouchure hole. The entire flute can then be tuned by adjusting the flute's main tuning slide.

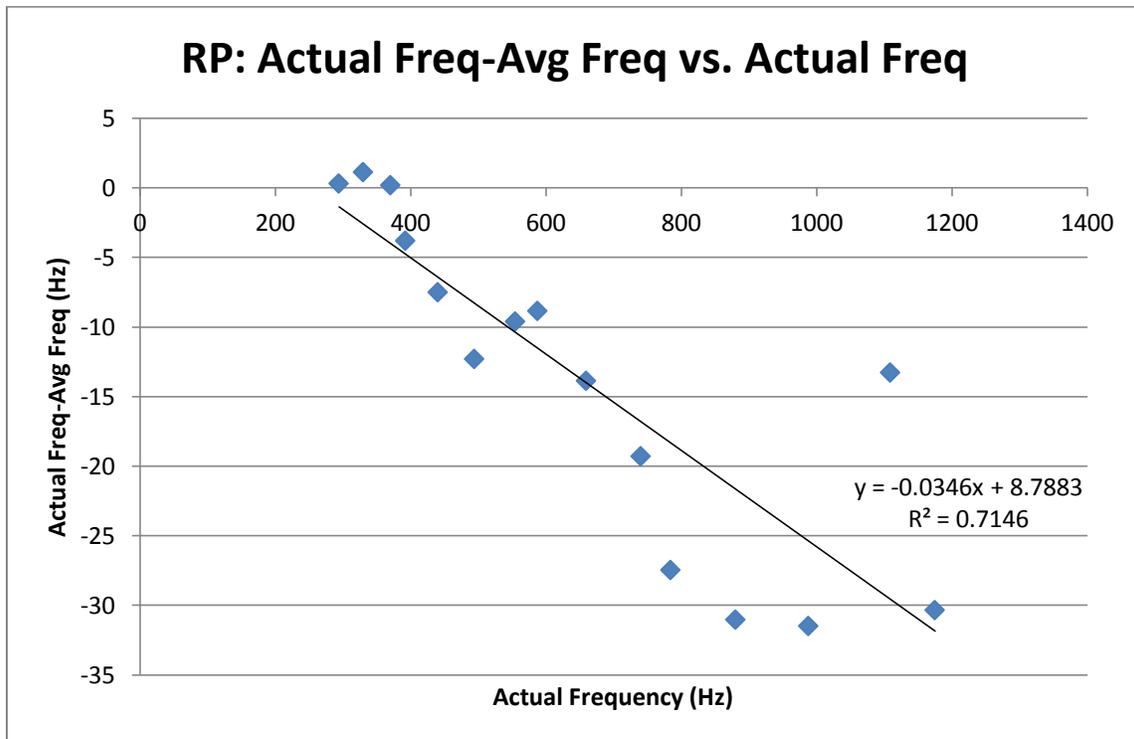


Figure 37: Difference between Played Frequency and the Actual Frequency in Average Human Player Test on the Initial Rapid Prototyped Flute

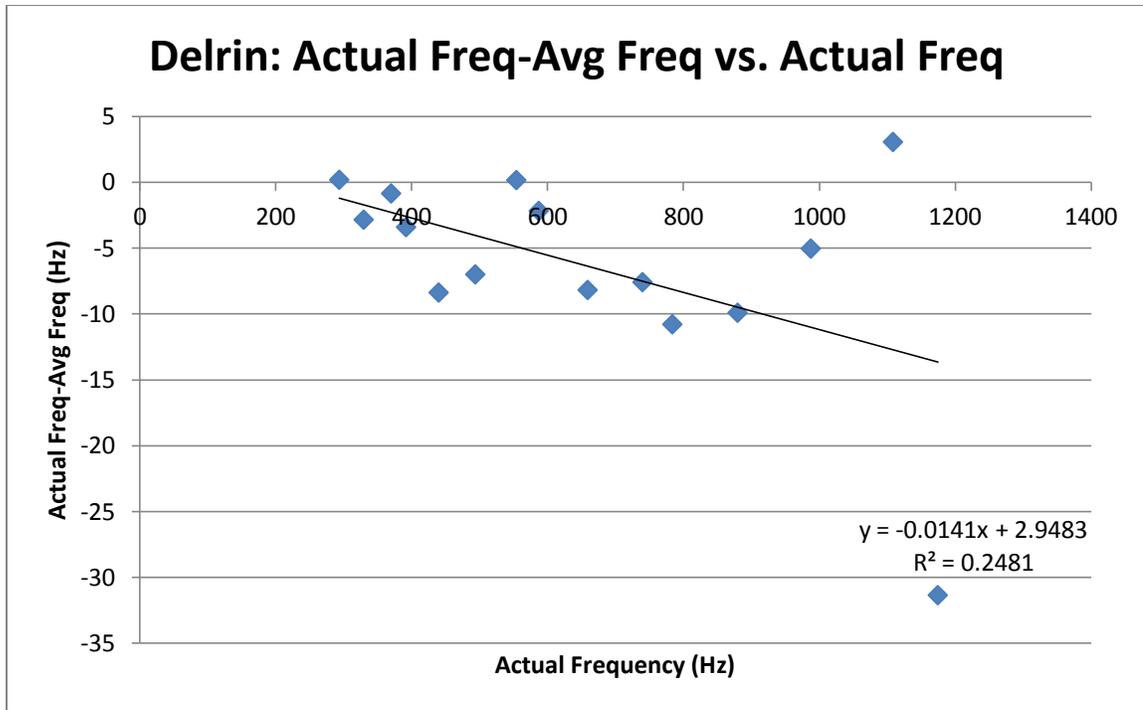


Figure 38: Difference between Played Frequency and the Actual Frequency in Average Human Player Test on the Delrin® Flute

Chimney Height Results

Predicting Frequency

Based on the data collected during experiments by using cardboard to increase the chimney height, a fairly accurate frequency prediction function was found. Matlab was used to draw a 3-D graph for determining the relationship among three values: tone hole distance to embouchure, increased chimney height, and frequency played. The 3-D graph (Figure 39) shows a surface with a nearly linear relationship among the values. Therefore, by using a least-squares and linear regression approach, a linear function was found:

$$Frequency = -1.1546 * Distance * -1.3175 * ChimneyHeight + 809.0704$$

The average difference between predicted frequency and calculated frequency was 8.57 Hz and the average percent error was 1.98%.

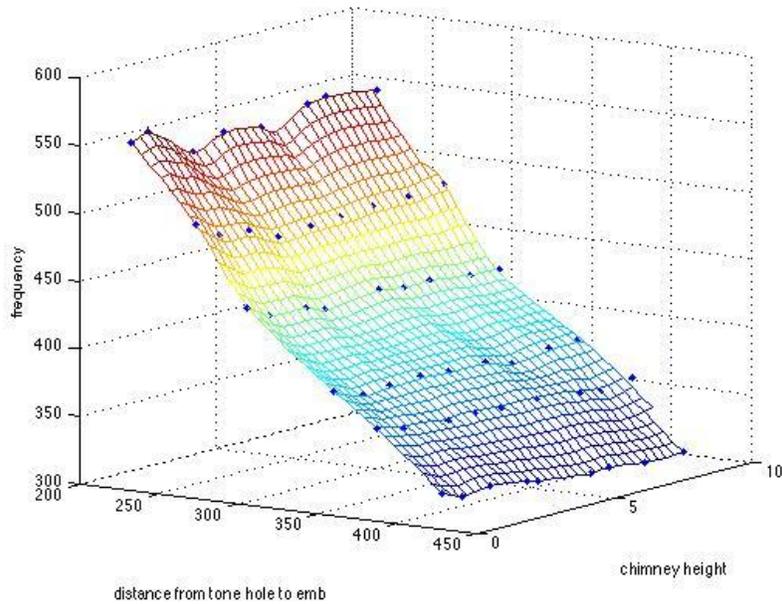


Figure 39: 3-D graph showing relationship among distance, chimney height and frequency

The frequency data tested at zero chimney height (no cardboard added) could be eliminated since this value was already calculated using the frequency equation that the group developed. In doing so, the final function achieved a higher accuracy prediction rate, which lowered the average difference between predicted frequency and calculated frequency to 8.40 Hz and the average percent error to 1.95%. The final frequency-predicting equation is shown below:

$$Frequency = -1.152 * Distance * -0.8875 * ChimneyHeight + 805.463$$

Predicting Chimney Height

A similar linear regression approach was used to predict chimney height based on a given frequency and the distance between each tone hole and the embouchure hole. However, the results had a large error and did not yield predicted chimney height within expectations. The relationship among the three parameters did not demonstrate linear behavior or a predictable pattern.

Several factors could have contributed to this unsuccessful prediction. First of all, due to the time constraint, not enough data was taken. This made it difficult to observe a trend in the data. Secondly, there may have been inaccuracies in the collected data. This could be

improved by using both a human player and the flute playing device, and averaging the frequencies from the two otherwise-identical tests.

As there was not enough time to determine an equation for predicting the effects on all the chimney heights, it was decided to use the results from our E tone hole to determine a suitable chimney height for our final model because it demonstrated a consistent linear trend. The graph of the results for the E tone hole as well as a trend line used to calculate the chimney height on the final rapid prototype can be seen in Figure 40.

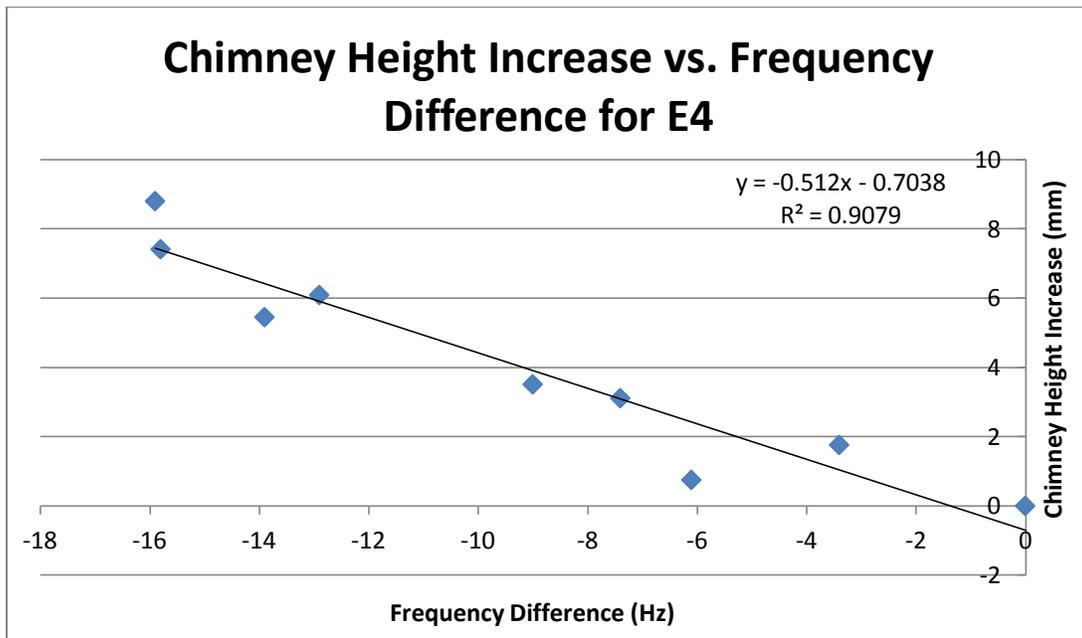


Figure 40: Frequency Change due to Chimney Height Increase in the E4 Tone Hole

Flow Inside the Bore

An important consideration was the degree of turbulence of the flow in the bore. To ascertain a turbulence reading, the Reynolds number of the flow through the bore was calculated according to the following equation:

$$Re = \frac{\rho v D}{\mu}$$

Although the group did not have the equipment to measure the velocity inside the bore, the group was able to use an anemometer to measure airstream velocity at the embouchure hole. Since the velocity inside the bore was known to be lower than the velocity at the embouchure hole, the measurement at the embouchure hole presented a valuable “worst case

scenario" velocity. The velocity was found to be 4.47 m/s. This, paired with other known values, produced a Reynolds number value of:

$$Re = \frac{\left(1.184 \frac{kg}{m^3}\right) \left(4.47 \frac{m}{s}\right) (0.01718 m)}{0.00001849 \frac{kg}{m * s}}$$
$$Re = 4917.95$$

This value placed the flow just inside the turbulent range of values. Laminar flows occur when $Re < 2300$ and turbulent flow occurs when $Re > 4000$. The range in between this is defined as transitional flow, where both laminar and turbulent flows are possible. However, since a large value for velocity was chosen, the flow could realistically be placed in the transition range.

Implementation of the Equations for Manufacturing

The group's equations were implemented in an Excel document to facilitate the computing of the tone hole locations. The equations that the group used were difficult to use in the equation editor within SolidWorks as they required a set of approximately 16 different variables for each tone hole. This made it necessary to find another way to create a link between the calculation sheet and the SolidWorks model. This was achieved by creating a design table in SolidWorks, which is also Excel based, and linking the values between the two different spreadsheets. To aid in the linking of the values, the dimensions within SolidWorks were renamed for easy identification within the design table. When this method is used to link the spreadsheets, only the final dimensions needed for the model are in the design table. Whenever the calculation spreadsheet was updated, all that was necessary to update the model was to open the design table within SolidWorks and then close it.

It was later discovered that the entire calculation spread sheet could be copied and inserted into the design table after making sure to skip a few lines in the design table. This was to ensure that SolidWorks did not try and assign the calculations as new configurations. A small sample of this can be seen in Figure 41.

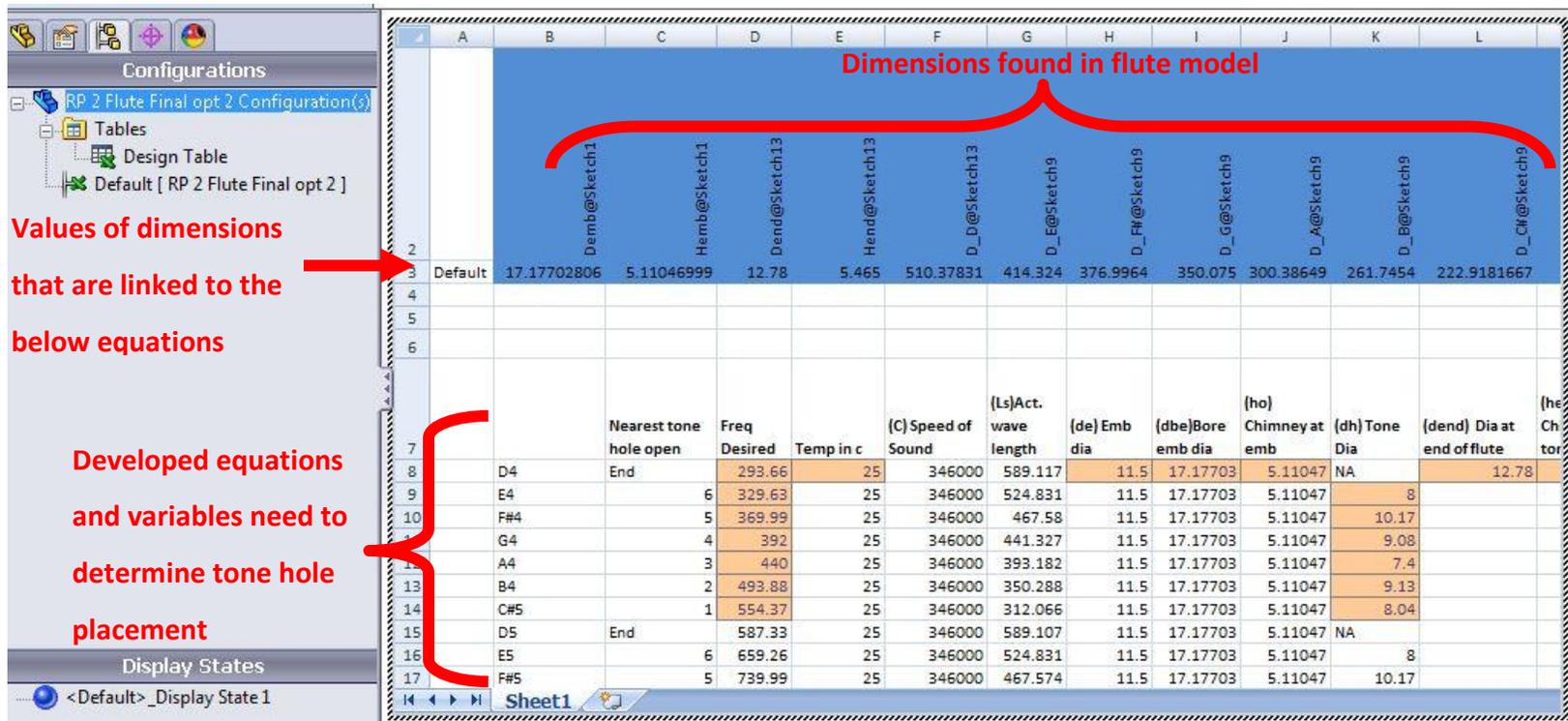


Figure 41: Calculations Inserted into Design Table

The final results of the calculations were referenced in the default configuration within the design table. The input values could then be changed directly in the design table and the updated values would propagate directly into the model.

In terms of manufacturing, a link between the equations governing a model and the model itself was beneficial. This strategy facilitated modifications to the input dimensions, such as hole diameters, and the model could be easily updated to reflect this change. This process streamlined custom fabrications without the user having to manually change values or create a whole new model. In a work environment, since this model already contains the equations, it would be very easy to share the model without having to worry about losing the calculation worksheet or breaking any links.

Results of Final Prototype

The final prototype fulfilled the goals of the project in that the measured frequencies obeyed the A440 standard closely. After extensive cleaning, the flute proved to be easily playable, and produced a clear tone. The hole spacing left some room for improvement, as fingering the last two tone holes proved more difficult than expected. This difficulty would have been corrected if the holes had been angled on the bore to better suit average human finger length ratios. The final model of the flute can be seen below.



Figure 42: Final Rapid Prototype

Printing the flute horizontally did not negatively affect playability. The layers created by rapid prototyping can be seen in Figure 43.

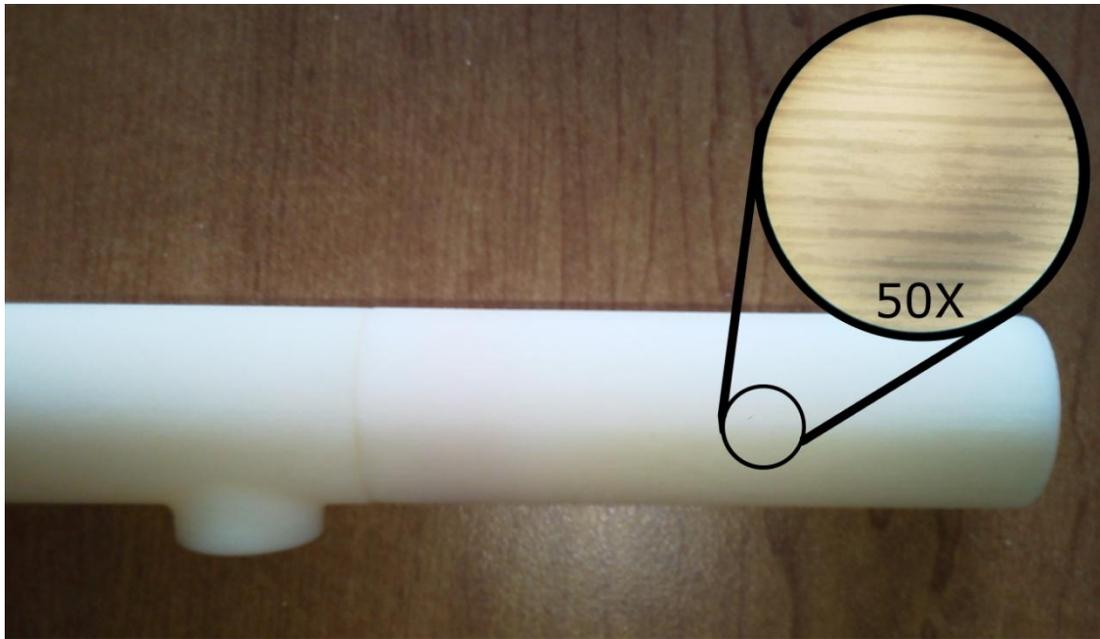


Figure 43: Magnification of Layers Created by Rapid Prototyping Horizontally

The cost of the material for this flute was \$535.35. This would increase if the cost to produce the flute was accounted for. The increased expense was a consequence of more support material needed in the bore to print the flute horizontally. In the initial prototype there was no need for support material in the bore as the pieces were printed vertically. In the final rapid prototype the support material was very difficult to remove from the bore of the flute due to the lengths of the pieces as well as the water nozzle used to dissolve the support material did not fit into the bore.

Frequency data were taken using this flute. Initially the flute tuning slide was pulled out to 10 mm, per the design. Data from this test showed that the flute was relatively uniformly flat in the lower octave. The tuning slide was pushed in slightly and 3 sets of data were taken. Also data was taken with the tuning slide pushed all the way in so there would be a comparison against the data taken for the other flutes. The average of these sets was taken and compared to the theoretical frequencies. This data can be seen below in Figure 44.

RP2 D Flute Frequency Test

Note	Actual Freq	Frequency (Hz)							Average Frequency		Difference between Average Played Frequency and Actual Frequency (Hz)		% deviation	
		Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7						
D4	293.66	287.6	289	291.3	289.4	292.6	293.3	294.3	289.9	293.4	3.8	0.3	1.28	0.09
E4	329.63	324.5	328.2	328.3	326.7	332.1	333.3	333.6	327.7	333.0	1.9	-3.4	0.58	-1.02
F#4	369.99	365.6	370.2	370.2	367.7	374.2	376.3	377.8	369.4	376.1	0.6	-6.1	0.17	-1.65
G4	392	389.3	392.3	395.4	391.1	401.8	402.9	404.3	392.9	403.0	-0.9	-11.0	-0.24	-2.81
A4	440	433.1	435.6	440.2	435.6	447.6	446.3	449.6	437.1	447.8	2.9	-7.8	0.65	-1.78
B4	493.88	490	493.4	500	492.2	510.6	510.1	515	495.2	511.9	-1.3	-18.0	-0.27	-3.65
C#5	554.37	551.4	561.2	561.2	554	575.5	573.8	579.4	558.8	576.2	-4.4	-21.9	-0.80	-3.94
D5	587.33	597.7	600.2	604	595.4	605.4	604.8	611.3	599.9	607.2	-12.5	-19.8	-2.13	-3.38
E5	659.26	674.2	677.3	682.3	674.4	682.2	683.2	686.9	678.0	684.1	-18.7	-24.8	-2.84	-3.77
F#5	739.99	764.6	765.3	766.6	765.5	768.4	774.4	777.5	765.8	773.4	-25.8	-33.4	-3.49	-4.52
G5	783.99	814	811.5	820.3	816.7	817.8	822.3	827.6	816.2	822.6	-32.2	-38.6	-4.10	-4.92
A5	880	886.5	888.6	897.2	889	898.5	903.2	905.6	891.6	902.4	-11.6	-22.4	-1.32	-2.55
B5	987.77	986.4	985.6	989.2	982.4	1007	1010	1011	985.7	1009.3	2.0	-21.6	0.21	-2.18
C#6	1108.73	1082	1087	1087	1079	1101	1124	1117	1084.3	1114.0	24.4	-5.3	2.20	-0.48
D6	1174.66	1174	1177	1174	1176	1193	2108	1199	1175.7	1500.0	-1.0	-325.3	-0.09	-27.70
												Average Deviation	1.36	4.30

gap= 10 mm, cork= 17 mm from center of embouchure hole
gap= 8.81 mm, cork= 20.2 mm from center of embouchure hole
gap= 0 mm, cork= 20.2 mm from center of embouchure hole

Figure 44: Frequency Data for the Final Rapid Prototype

The first octave proved to be in-tune with the theoretical frequencies. The second octave contained some sharp notes. The frequencies of the final rapid prototype differed from A440 by a maximum of 4.1%. When another group member played the flute, the sharp notes were mostly corrected.

Conclusions

Final Prototype Assessment

Testing of the final prototype revealed that the flute was largely in tune. Therefore, it could be concluded that the derived mathematical relationships were accurate in predicting dimensional parameters based on frequency. The flute was optimal in that it obeyed the A440 standard as closely as possible despite player subjectivity and manufacturing error.

Manufacturing Feasibility

In the assessment of the feasibility to rapid prototype flutes the group ran across some concerns with this method. For one thing, the price to print these flutes was upwards of \$400 per flute. There were also time costs for this procedure. The time to grow these flutes using the machine that the group used was more than eight hours. This does not include the time it took to clean out the support material from the inner bore as well as the finishing of the surfaces. These add up to a lot of time spent on after-printing procedures.

Until there is a quicker and less expensive way of rapid prototyping that results in a more finished product, it would appear that rapid prototyping might not be a cost effective means of producing flutes. Rapid prototyping does insure constancy among flutes created as well as leaving the option open to create customized flutes. Such customized flutes could have different hole spacing based upon hand sizes or even bore diameters depending on the comfort levels required by the user.

Recommendations and Future Work

Chimney Height Prediction

The chimney heights prediction based on distance from each tone hole to the embouchure and the desired frequency was not very successful due to the group's limited time. There are several things on which future research could focus for developing a better functional equation.

Firstly, more sample data should be collected from various flutes. The lack of data was one of the main disadvantages. Without enough data points, patterns were hard to follow and trends became hard to find. Curve fits were very difficult with such few data points. Use of the neural network method was attempted for developing the chimney height model. Seventy percent of the data were used to do the "training" while 30% of the data were used for testing. Despite the better prediction that the neural network produced compared to the linear regression method, the results were still not within the desired precision. If enough data were collected, the neural network may have been able to accurately predict the chimney height based on given parameters. Secondly, in order to ensure the accuracy of data collection and to eliminate the human errors as much as possible, it is recommended that data be taken using both the flute playing device and human players as much as possible. Both parties should play the same flute and the same note should be played more than 3 times, and averages should be taken. There seemed to be some outliers in the data that the team collected. However, by using Chauvenet's Criterion, the team was not able to remove the outlier. Therefore, it would be the best to use the above average in an attempt to avoid the outliers in the first place.

Embouchure Hole

Future research may focus on the different possible shapes and sizes of embouchure holes. The group explored elliptical embouchure holes as well as rounded rectangular embouchure holes. While the African Blackwood, Mopane, and Olive wood flutes dealt with elliptical shapes, the Delrin® and Rapid Prototyped had modern rectangular cuts which were maintained in the final design because of the relative ease of playing the instrument. However,

the Mopane and Olive were also favorable to play. Future research on different cuts and variations of the already-investigated holes could be explored.

Further research may also shed more light on the chimney height of the embouchure hole, as well as the undercutting of the embouchure hole. According to the chimney height data of the tone holes, increasing the chimney height of the embouchure hole could decrease the frequency of the notes played in the flute. Altering the embouchure hole chimney height could make the flute more sharp or flat depending on the configurations of other parts in the flute such as length, bore size, and hole spacing. Experimenting with the undercut edge could also be beneficial to the strength and sound of the notes produced. Contouring or rounding the edges could produce different effects on tone.

Within the studies using the flute playing device, lip size and placement had a great effect on whether or not a note could even be produced. A person with thicker lips could essentially be increasing the chimney height and produce a different sound than a person with thinner lips. An embouchure hole specific to the player him- or herself based on lip size and thickness could be applied to a flute. Future studies could also look at the area of the embouchure hole with respect to a person's lips and determine if a certain cut is more preferable based solely on lip size.

Bore Profile Modification

The bore profile is another area that could further be researched. The group worked with cylindrical and conical bore types, but small variations could have substantial effects on tone, pitch, and volume. Changing the angle of the taper is one area that can be explored. The taper could be increased or decreased, and combinations of cylindrical and conical bores could exist in the same profile. In Figure 45 below, models a few configurations of bore profiles.

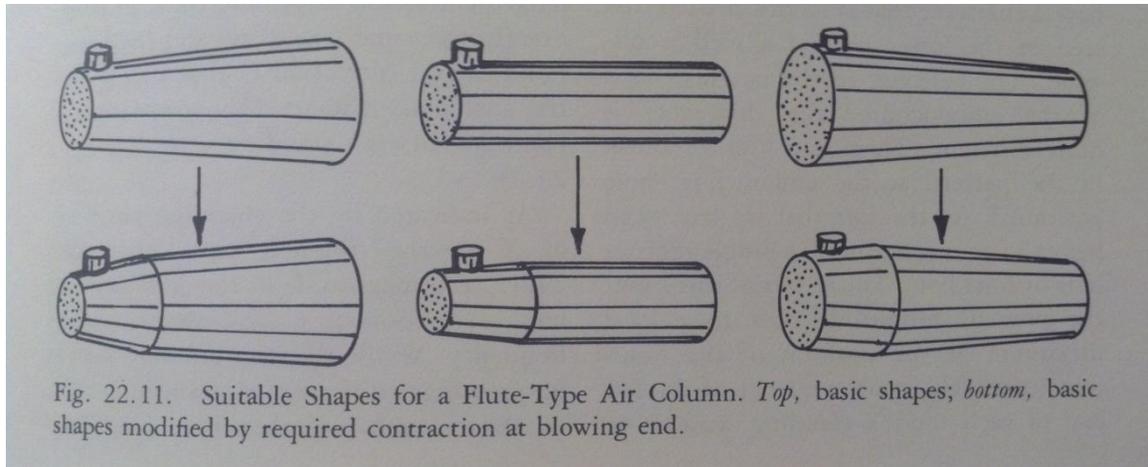


Figure 45: Bore Profiles with Varying Tapers (Hopkin)

Each profile has a tapered distance from the cork, while the taper of the rest of the bore varies. Some even have an increasing taper where the inside diameter of the bore at the end of the flute is greater than the inside diameter of where the taper changes near the embouchure hole. Such a design would mimic that of a plosive aerophone. This is used to improve octave tuning.

Using different lubricants and polishing material on the flute could help improve the intonation. The inside of the bore could be polished in this respect. An exploration of how a “scoop” (increase in bore diameter) before each tone hole may change frequency and/or volume would be beneficial.

Studies of pressure waves in the embouchure and the bore may also be valuable. There may be a generalization concerning the different amplitudes produced and those could be compared to the modes of vibration. With such information, a bore could be created that mimicked the wave itself. Certain areas could alter and optimize the wave to produce the desired sounds or frequencies at each note. Since people apply different amounts of pressure through the flute when playing, a bore profile specific to each person based on his or her “pressure profile” could be produced.

The surface roughness of the bore could also be investigated further. By creating two identical flutes made from PVC the effects of surface roughness could be tested. Using a wire brush, the bore of one of the flutes could be roughened. Leaving the other flute untouched the

two flutes could be played and compared. If there was a significant difference in the intonation then more research could be done to determine the best way of coating the flute to have a very smooth finish.

Future Work on Bore Extension

The equation that was developed for the change in frequency of each tone hole based on the extension of the bore is a large area for future work. More data would need to be taken to refine the equation and determine if this generalization is accurate. The explanation of why the B and C# tone holes have a different impact on the change in frequency as well as all the other tests done need to be explored as well. The benefits of refining this equation would be the ability to predict how much the bore needs to be extended in order to change the frequencies. The use of this equation to predict the location of the tone holes might be possible with future developments.

Future Work Based on Final Prototype

Possible improvements to the final flute design are mostly centered on player comfort. The lower tone holes of the flute were somewhat difficult to finger. Future work could use ergonomic principles to increase the comfort level of the right hand, possible through rotation of the tone holes about the flute's center axis and the rounding of the top surface of the last tone hole.

Furthermore, the end joint of the flute required modification by the group to tighten the fit. Future researchers may wish to investigate different fitting techniques for the joints, such as machined metal inserts.

Work Cited

- Ayers, R. D. "The Conical Bore in Musical Acoustics." *Conical vs Cylindrical Bores*. American Journal of Physics, Vol. 53, No. 6, pgs 528-537, 1985.
<<http://www.phy.mtu.edu/~suits/conecyl.html>>.
- Benade, A. H. (1990). *Fundamentals of Musical Acoustics*, 2nd ed., New York: Dover Publications Inc.
- Burns, Casey (2006). "Irish Wooden Flute FAQ." *Irish Wooden Flute FAQ*. N.p., 28 Apr. 2006. Web. 30 Jan. 2013. <<http://www.caseyburnsflutes.com/faq.php>>.
- Cocchi, Alessandro & Tronchin, Lamberto (1998). "Material and Obsolescence on Flute Tone Quality." *ASA Publications*, Vol.103 (5). N.p., 1998. Web. Sept. 2012.
<http://asadl.org/jasa/resource/1/jasman/v103/i5/p2835_s4?bypassSSO=1>.
- Healy, Skip. "Feel the Bore." Healy Flute Company, n.d. Web.
<<http://www.skiphealy.com/music/tips/24.htm>>.
- Hopkin, Bart (1996). *Musical Instrument Design: Practical Information for Instrument Making*. Tucson, AZ: See Sharp, 1996. Print.
- Linortner, R. (2001). *Silver, Gold, Platinum- and the Sound of the Flute*. Universitat fur Musik und Darstellende Kunst Wien. Web. Sept. 2012
www.xmarks.com/site/iwk.mdw.ac.at/Forschung/english/linortner/e.htm.
- McGee, T (2004). "Flute Myths Exploded!" *McGee Flutes*. N.p., 11 Mar. 2004. Web. Sept. 2012.
<<http://www.mcgee-flutes.com/FluteMyths.htm>>.

Kennedy, M. (2007). *The Concise Oxford Dictionary of Music*. New York: Oxford University Press.

Tattersall, Malcolm (2008). "Does It Matter What It's Made Of?" *Woodwind Materials*. N.p., May 2008. Web. Sept. 2012.

<http://members.iinet.net.au/~mtattersall/Articles/Woodwind_Materials.htm>.

Transverse Flute in F. 1978. Photograph. Metropolitan Museum of Art, New York. *Metropolitan Museum of Art*. Metropolitan Museum of Art. Web. 25 Mar. 2013.

<http://www.metmuseum.org/Collections/search-the-collections/180016501?rpp=60&pg=1&rndkey=20130325&ft=*&what=Wood%7cFlutes&pos=58#fullscreen>.

Welch, Christopher. "Boehm, Nicholson, and the English Flute. Sept. 2012.

<<http://www.standingstones.com/irflute2.html>>

Wilcocks, Gerda R (2006). "Improving Tone Production on the Flute with Regards to Embouchure, Lip Flexibility, Vibrato and Tone Colour, as Seen from a Classical Music Perspective." University of Pretoria, Dec. 2006. Web.

<<http://upetd.up.ac.za/thesis/available/etd-09132007-163345/unrestricted/dissertation.pdf>>.

Xorys. "What Is an Irish Flute? (Part 1)." *What Is an Irish Flute? (Pt. 1)*. N.p., n.d. Web. Sept. 2012. <<http://www.standingstones.com/irflute.html>>.

Appendix A

Flute		Nearest tone hole open	Freq Desired	Temp in c	(C) Speed of Sound	(Ls)Act. wave length	(Lmeas) Dis. From emb to tone	(de) Emb dia	(dbe)Bor e emb dia	(ho) Chimney at emb	(dh) Tone Dia	(dend) Dia at end of flute	(hend) Chimney at tone	hest	dest	(dbe-de)	(dest-dh)	Abe-Ae	Aest-Ah	delle	dellh	La (mm)	cal-meas (mm)
RP	D4	End	293.66	25	346000	589.1167	517.8985	11.5	17.17703	5.11047	NA	12.78	5.465			5.677028		127.8631		27.63336	41.105	520.3783	2.479765
RP	E4		329.63	25	346000	524.8309	427.891	11.5	17.17703	5.11047	6.23			5.421574	13.31859	5.677028	7.088586	127.8631	108.8341	27.63336	62.28156	434.916	7.024909
RP	F#4		369.99	25	346000	467.5802	386.641	11.5	17.17703	5.11047	10.17			5.372848	13.9229	5.677028	3.752904	127.8631	71.01441	27.63336	52.95045	386.9964	0.355351
RP	G4		392	25	346000	441.3265	359.811	11.5	17.17703	5.11047	9.08			5.346276	14.25246	5.677028	5.172464	127.8631	94.78682	27.63336	53.61819	360.075	0.263942
RP	A4		440	25	346000	393.1818	305.8177	11.5	17.17703	5.11047	7.4			5.288327	14.97118	5.677028	7.571177	127.8631	133.0277	27.63336	55.16197	310.3865	4.568745
RP	B4		493.88	25	346000	350.2875	272.9277	11.5	17.17703	5.11047	9.13			5.223278	15.77793	5.677028	6.647933	127.8631	130.0511	27.63336	50.90874	271.7454	-1.18232
RP	C#5		554.37	25	346000	312.0659	232.7527	11.5	17.17703	5.11047	8.04			5.15025	16.68366	5.677028	8.643661	127.8631	167.8419	27.63336	51.51443	232.9182	0.165422
RP	D5	End	587.33	25	346000	589.1066	517.8985	11.5	17.17703	5.11047	NA			5.110458	17.17718	5.677028		127.8631		27.63336	34.21488	527.2584	9.359852
RP	E5		659.26	25	346000	524.8309	427.891	11.5	17.17703	5.11047	6.23			5.023618	18.2542	5.677028	12.0242	127.8631	231.2235	27.63336	54.28163	442.9159	15.02484
RP	F#5		739.99	25	346000	467.5739	386.641	11.5	17.17703	5.11047	10.17			4.926154	19.46299	5.677028	9.292986	127.8631	216.2821	27.63336	46.16819	393.7723	7.131295
RP	G5		783.99	25	346000	441.3322	359.811	11.5	17.17703	5.11047	9.08			4.873034	20.12181	5.677028	11.04181	127.8631	253.2443	27.63336	47.17142	366.5274	6.716344
RP	A5		880	25	346000	393.1818	305.8177	11.5	17.17703	5.11047	7.4			4.757123	21.55938	5.677028	14.15938	127.8631	322.0501	27.63336	49.49576	316.0527	10.23496
RP	B5		987.77	25	346000	350.284	272.9277	11.5	17.17703	5.11047	9.13			4.627014	23.17304	5.677028	14.04304	127.8631	356.2826	27.63336	45.72378	276.9268	3.999091
RP	C#6		1108.73	25	346000	312.0688	232.7527	11.5	17.17703	5.11047	8.04			4.480982	24.9842	5.677028	16.9442	127.8631	439.4842	27.63336	47.06126	237.3741	4.6214
RP	D6	End	1174.66	25	346000	589.1066	517.8985	11.5	17.17703	5.11047	NA			4.401386	25.97138	5.677028		127.8631		27.63336	30.76994	530.7033	12.80479

Figure 46: All required values and equations needed to compute tone hole placement in an Excel spreadsheet

Mopane Wood D Flute Frequency Test

Note	Frequency (Hz)					Difference between Played Frequency and Actual Frequency (Hz)			
	Actual Freq	Test 1	Test 2	Test3	Average	Test 1	Test 2	Test3	Average
D4	293.66	294.6	296	296.4	295.6667	0.94	2.34	2.74	2.007
E4	329.63	333.5	331.2	336.7	333.8	3.87	1.57	7.07	4.170
F#4	369.99	372	370.8	374.5	372.4333	2.01	0.81	4.51	2.443
G4	392	395.7	397.5	398.4	397.2	3.7	5.5	6.4	5.200
A4	440	448	444.5	449	447.1667	8	4.5	9	7.167
B4	493.88	500.8	500.1	504.4	501.7667	6.92	6.22	10.52	7.887
C#5	554.37	558.2	557.3	561.7	559.0667	3.83	2.93	7.33	4.697
D5	587.33	598.1	596.6	599.4	598.0333	10.77	9.27	12.07	10.703
E5	659.26	672.8	672.1	680.3	675.0667	13.54	12.84	21.04	15.807
F#5	739.99	750.5	754.2	762.7	755.8	10.51	14.21	22.71	15.810
G5	783.99	804.2	804.2	806.5	804.9667	20.21	20.21	22.51	20.977
A5	880	905.4	897.2	909.4	904	25.4	17.2	29.4	24.000
B5	987.77	1014	1003	1020	1012.333	26.23	15.23	32.23	24.563
C#6	1108.73	1127	1116	1133	1125.333	18.27	7.27	24.27	16.603
D6	1174.66	1207	1202	1209	1206	32.34	27.34	34.34	31.340

Figure 47: Mopane Wood D Flute Frequency Test

Olive Wood D Flute Frequency Test

Note	Frequency (Hz)					Difference between Played Frequency and Actual Frequency (Hz)			
	Actual Freq	Test 1	Test 2	Test3	Average	Test 1	Test 2	Test3	Average
D4	293.66	293.6	294.1	294.4	294.0333	-0.06	0.44	0.74	0.373
E4	329.63	333.6	336	332.8	334.1333	3.97	6.37	3.17	4.503
F#4	369.99	370.4	369.3	367.5	369.0667	0.41	-0.69	-2.49	-0.923
G4	392	396.5	394.2	393.5	394.7333	4.5	2.2	1.5	2.733
A4	440	447.2	445.2	444.6	445.6667	7.2	5.2	4.6	5.667
B4	493.88	500.9	500.8	501.6	501.1	7.02	6.92	7.72	7.220
C#5	554.37	556.7	551.5	550.3	552.8333	2.33	-2.87	-4.07	-1.537
D5	587.33	598.6	592	594	594.8667	11.27	4.67	6.67	7.537
E5	659.26	677.3	675.7	672.4	675.1333	18.04	16.44	13.14	15.873
F#5	739.99	751.2	754.7	750	751.9667	11.21	14.71	10.01	11.977
G5	783.99	807.4	808.7	805.5	807.2	23.41	24.71	21.51	23.210
A5	880	903.6	908.2	899.7	903.8333	23.6	28.2	19.7	23.833
B5	987.77	1013	1020	992.6	1008.533	25.23	32.23	4.83	20.763
C#6	1108.73	1125	1128	1135.7	1129.567	16.27	19.27	26.97	20.837
D6	1174.66	1200	1209	1192	1200.333	25.34	34.34	17.34	25.673

Figure 48: Olive Wood D Flute Frequency Test

Olive Wood Eb Flute Frequency Test

Note	Frequency (Hz)					Difference between Played Frequency and Actual Frequency (Hz)				
	Actual Freq	Test 1	Test 2	Test3	Average	Test 1	Test 2	Test3	Average	
Eb4	311.13	310.5	310.2	310	310.2333	-0.63	-0.93	-1.13	-0.897	
F4	349.23	346.8	347.8	345.4	346.6667	-2.43	-1.43	-3.83	-2.563	
G4	392	385.2	385.4	389.5	386.7	-6.8	-6.6	-2.5	-5.300	
Ab4	415.3	412.6	409.4	416.1	412.7	-2.7	-5.9	0.8	-2.600	
Bb4	466.16	468.5	462.4	463.6	464.8333	2.34	-3.76	-2.56	-1.327	
C5	523.25	521.2	515.8	518.3	518.4333	-2.05	-7.45	-4.95	-4.817	
D5	587.33	579.2	572.2	577.6	576.3333	-8.13	-15.13	-9.73	-10.997	
Eb5	622.25	625.4	627.5	624.2	625.7	3.15	5.25	1.95	3.450	
F5	698.46	693.2	697.7	693.7	694.8667	-5.26	-0.76	-4.76	-3.593	
G5	783.99	777.6	777.7	779.7	778.3333	-6.39	-6.29	-4.29	-5.657	
Ab5	830.61	827.7	828.6	836.9	831.0667	-2.91	-2.01	6.29	0.457	
Bb5	932.33	925.6	931.2	942.3	933.0333	-6.73	-1.13	9.97	0.703	
C6	1046.5	1040	1054	1055	1049.667	-6.5	7.5	8.5	3.167	
D6	1174.66	1162	1172	1170	1168	-12.66	-2.66	-4.66	-6.660	
Eb6	1244.51	1255	1261	1268	1261.333	10.49	16.49	23.49	16.823	

Figure 49: Olive Wood Eb Flute Frequency Test

Frequency Difference from Base (Hz)	Distances (mm)			Predicted Frequency Difference from Base (Hz)	Distances (mm)			Difference of Predicted Change and Actual (Hz)	Distances (mm)		
	3.82	7.03	10.52		3.82	7.03	10.52		3.82	7.03	10.52
D4	0.7	3.7	4.6	2.57935	4.528173	6.646987	1.87935	0.828173	2.046987		
E4	2	5.8	7.7	2.931769	5.146862	7.555171	0.931769	-0.65314	-0.14483		
F#4	5.2	7.1	8	3.300248	5.793744	8.504741	-1.89975	-1.30626	0.504741		
G4	5.3	9.6	11.1	3.537573	6.21038	9.116328	-1.76243	-3.38962	-1.98367		
A4	5.3	7.3	12.9	3.990811	7.00606	10.28432	-1.30919	-0.29394	-2.61568		
B4	5.2	11.7	18.6	9.016573	15.82903	23.23572	3.816573	4.129028	4.635715		
C#5	14.5	19.9	25.6	10.17108	17.85582	26.21088	-4.32892	-2.04418	0.61088		
D5	7	10.3	12.2	5.338033	9.371173	13.75611	-1.66197	-0.92883	1.556114		
E5	5.6	14.1	14.7	6.007183	10.5459	15.48051	0.407183	-3.5541	0.780514		
F#5	8.1	16.9	18.2	6.775367	11.89448	17.46012	-1.32463	-5.00552	-0.73988		
G5	9.5	16.7	15.2	7.246448	12.72149	18.6741	-2.25355	-3.97851	3.474102		
A5	11	22.3	22.4	8.128834	14.27056	20.94801	-2.87117	-8.02944	-1.45199		
B5	10	21.3	22.8	9.10044	15.97626	23.45184	-0.89956	-5.32374	0.65184		
C#6	12	27	31	10.01048	17.57389	25.79702	-1.98952	-9.42611	-5.20298		
D6	15	21	25	10.73317	18.84259	27.65938	-4.26683	-2.15741	2.659376		

Figure 50: Difference between Predicted Change and Actual Change in Frequency due to Bore Extension

Number of Cards used and Frequencies Played																	
0		1		3		5		7		9		11		13		15	
Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)	Frequency Played (Hz)	Chimney Height (mm)
287.4	N/A	N/A	N/A														
326.2	0	320.1	0.75	322.8	1.76	318.8	3.11	317.2	3.51	312.3	5.45	313.3	6.09	310.4	7.41	310.3	8.8
368.1	0	363.3	0.96	359.9	2.63	360.6	3.61	359.2	4.53	358.7	5.88	355.7	7.44	352.6	8.2	356.9	9.36
391.4	0	383.7	1.06	385.2	2.03	385.8	3.18	384.8	4.18	384	5.54	378.3	6.49	382.1	7.87	382.5	8.89
446	0	436.1	0.79	434.3	2.13	429.1	2.83	433.5	4.79	430.6	5.68	430.8	6.63	427.2	8.13	425.6	9.22
502.3	0	489.7	0.89	487.6	1.95	477.5	3	479.4	4.21	479.8	5.3	482.3	6.47	482.1	7.8	485.1	9.07
557.3	0	561.8	0.61	538.1	2.25	547.1	3.36	544.2	4.76	536.3	5.14	551.6	6.42	553.6	7.13	549.1	8.98
596.9	N/A	N/A	N/A														
670.9	0	664.2	0.75	662.4	1.76	665.9	3.11	648.1	3.51	642.5	5.45	638.5	6.09	632.7	7.41	636.7	8.8
754.3	0	747.4	0.96	741	2.63	738.2	3.61	741.2	4.53	727.1	5.88	727.2	7.44	725.3	8.2	730.1	9.36
804.5	0	797.7	1.06	798.6	2.03	784.2	3.18	794.1	4.18	785.3	5.54	776.2	6.49	780.6	7.87	784.1	8.89
908.6	0	884.3	0.79	879.6	2.13	868	2.83	873.1	4.79	864.7	5.68	864.8	6.63	860.7	8.13	895.7	9.22
1025	0	976.6	0.89	978.2	1.95	967.2	3	962.3	4.21	948.6	5.3	962.6	6.47	965.6	7.8	962	9.07
1117	0	1077	0.61	1074	2.25	1091	3.36	1076	4.76	1063	5.14	1083	6.42	1081	7.13	1076	8.98
1207	N/A	N/A	N/A														

Figure 51: Frequency Data Collected for Chimney Height Increases