## Covered Calls

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## Contents

Chapter 1: Introduction ..... 2
Chapter 2: Background .....  3
Chapter 3: Methodology ..... 21
Chapter 4: Results and Analysis ..... 45
Chapter 5: Conclusion ..... 62

## Chapter 1: Introduction

Many people invest in the market with expectations of it going up and that leads to many investment strategies being built for success only in that situation. The team wanted to investigate investment strategies that are still profitable whether or not the market is anticipated to move in a certain direction, as well as being low-effort to manage.

The overall purpose of this project is to test different investment strategies by running them in 10,000 Monte Carlo simulations to see how they perform. This paper revolves around selling covered calls and we looked at several alterations to this strategy. The results of each investment strategy considered will be compared to the results of the simple strategy of buying and holding shares in an asset to see if any of them are expected to consistently yield higher returns.

## Chapter 2: Background

## Call Options

A call option entitles the buyer to purchase a fixed amount of the underlying asset at a strike price at or before a predetermined future date known as the expiration date. For this type of option, the buyer pays a premium for the right, but not the obligation, to exercise the agreement. The other party, known as the seller of the option, is obligated to sell the asset at the predetermined strike price. The right to exercise an option is also known as the long position and the obligation to, in this case, sell to the party in the long position is called the short position. For the sake of this project the asset will be stocks. In the market, call options on stocks are sold in lots of 100 shares of that stock.

## Examples

Payoff and Profit diagrams; The examples below will illustrate both the payoff and the profit for different situations. The currency used will always be in US dollars. The payoff of an option is the amount, disregarding price, the option is worth, as if you have just found the contract on the street. Therefore, the payoff will never be below 0 . Meanwhile the profit of an option reflects the initial cost of the option as well as any profit generated, and will always be a shifted down payoff line. In the following visuals, the blue lines show the payoff.

On November 1st, 2019, a stock is currently valued at $\$ 49.50$ per share. A call option on the stock is purchased, with an expiration of seven days and strike price of $\$ 50$, for $\$ 0.25$ per share of stock. This contract allows the holder of the option to purchase 1 share of XYZ Company stock at expiration November 8th, 2019.


Scenario 1: Long Position: Stock Price > Strike Price at Expiration

At expiration, the XYZ Company stock is worth $\$ 70$ per share. The holder of the option has the right, but not the obligation to buy a share of XYZ Company stock which has a current market value of $\$ 70$ for only $\$ 50$ per share. Since the call is "in-the-money", meaning the stock price is greater than the strike price, the holder will exercise the call option.

Payoff is $\$ 70-\$ 50=\$ 20.00$ per share.
Profit is $\$ 70-\$ 50-\$ 0.25=\$ 19.75$ per share, because the holder of the option paid $\$ 0.25$ to enter the agreement

## Scenario 2: Long Position: Stock Price < Strike Price at Expiration

At expiration, the XYZ Company stock is worth $\$ 30$ per share, the holder of the option chooses not to exercise the option since it is "out-of-the-money", or stock price lower than the strike place. Obviously, the holder of the option would not want to purchase the stock for a higher price than what it is currently worth. Which means there is no further transaction, as the option has expired.

Payoff is $\$ 0$ per share
Profit is $-\$ 0.25$ per share, the price paid to enter the contract.

Scenario 3: Long Position: Stock Price $=$ Strike Price at Expiration

At expiration, the XYZ company stock is worth $\$ 50$ per share. Since the option is "at-themoney", meaning the strike price and the stock price are equal, not exercising the call option gives the holder of the option the same payoff.

Payoff per share if not exercised is $\$ 0$ and if exercised $\$ 50-\$ 50=\$ 0.00$
Profit is $-\$ 0.25$ per share, the price paid to enter the contract.

## Long Position and Short Position

The graph from the previous example illustrates profit from the long position arising from various stock prices at expiration. This position refers to the buyer who has the right to purchase the underlying asset at strike price. From the examples above, it is clear to see that the party in this position profits when the stock price at expiration is greater than the sum of the strike price and the initial cost.

When purchasing call options the buyer always has a capped loss, which is the premium paid to enter the agreement. Meanwhile, the buyer's potential profit is unlimited as the underlying asset price can skyrocket. The party in the short position profits when the stock price at expiration is less than the sum of the strike price and the initial cost of the option (seen in Scenario 2). Similarly, the party entering the long position profits when the stock price at expiration is greater than the sum of the strike price and the initial cost of the option. Therefore, the potential for profit when selling calls is capped at the premium received, while potential losses are uncapped; The opposite is true when buying calls. The strategies analyzed in this project involve writing calls and therefore holding the short position.

See the next page for the payoff graph of the same example now from the short position.

## Examples

On November 1st, 2019, a stock is currently valued at $\$ 49.50$ per share. A call option on the stock is purchased, with an expiration of seven days and strike price of $\$ 50$, for $\$ 0.25$ per share of stock. This contract allows the holder of the option to purchase 1 share of XYZ Company stock at expiration November 8th, 2019.


## Scenario 1: Short Position: Stock Price > Strike Price at Expiration

At expiration, the XYZ Company stock is worth $\$ 70$ per share, since the option is "in-themoney" therefore it will be exercised, which means the buyer will pay you $\$ 50$ per share. The seller of the option is obligated to sell one share of XYZ Company stock to the buyer. Since the seller doesn't already own the stock, they must purchase the share at the current market price of $\$ 70$. In this scenario, the seller receives a total payment of $\$ 50.25$ from the sale of the asset and call premium, and must pay $\$ 70$ to purchase the stock in the market.

Payoff per share is $\$ 50-\$ 70=-\$ 20$
Profit per share is $\$ 50-\$ 70+\$ 0.25=-\$ 19.75$

## Scenario 2: Short Position: Stock Price < Strike Price at Expiration

At expiration, the XYZ Company stock is worth $\$ 30$ per share, the call option will not be exercised by the buyer since they don't want to pay more than market price, which means no further transaction as the option has expired.

Payoff is $\$ 0.00$ per share
Profit is $\$ 0.25$ per share from selling the call option

Scenario 3: Short Position: Stock Price $=$ Strike Price at Expiration
At expiration, the XYZ Company stock is worth $\$ 50$ per share. If the buyer exercises their option, they would pay the seller of the option $\$ 50$ for a share of XYZ company stock and the seller would have to provide it to them by purchasing it from the market for $\$ 50$. Similar to the short position, when the call is "at-the-money"

Payoff is $\$ 0.00$ per share
Profit is $\$ 0.25$ per share from selling the call option

## American Style Option vs. European Style Option

Options come in different styles, notably American and European. They are fairly similar, but the key difference is when the buyer is allowed to exercise their right. American options can be exercised any time before expiration, while European options are only allowed to be exercised on the date of expiration. The investment scenarios analyzed in our project will involve writing European options rather than American options so it can easily be determined whether or not they have been exercised using only the stock price at expiration. American options may be more popularly traded in the market, but over such a short time period they are about equal in value to European options. European options were chosen for this project so the Black-Scholes formula can be used to calculate call prices. Since the time until expiration is only one week, a $0 \%$ dividend and $0 \%$ risk-free rate were chosen as inputs.

## Pricing

## Black-Scholes Formula

A traditional approach used to price an option is the Black-Scholes Formula. This formula was developed by economists Fischer Black and Myron Scholes and assumes a price "without transaction costs, taxes, arbitrage opportunities, and short selling constraints". Therefore, the formula calculates the price of an option (call premium) from the current stock price, time until expiration, option strike price, stock volatility, risk free interest rate, and dividend rate. For the purpose of this paper these variables will be denoted as the following. For the reference of the reader also find below the formula for Black-Scholes calculation.

## Notation:

C: call premium
S: current stock price
t : time until expiration
K : option strike price $\sigma$ : volatility
r: risk free interest rate q : dividend rate

$$
\begin{gathered}
C=N\left(d_{1}\right) S e^{-q t}-N\left(d_{2}\right) K e^{-r t} \\
d_{1}=\frac{\ln \left(\frac{s}{K}\right)+\left(r-q+\frac{\sigma^{2}}{2}\right) t}{\sigma \sqrt{t}} \\
d_{2}=d_{1}-\sigma \sqrt{t}
\end{gathered}
$$

## Pricing Example

On November 1st, 2019, Sally sold a call option for 100 shares of XYZ Company stock to Zach. The price Zach paid to enter the contract is derived from the current stock price of $\$ 49.5$ per share, strike price of $\$ 50$ per share, volatility of XYZ Company stock of $20 \%$, dividend of $2 \%$, risk-free interest rate of $1 \%$, and expiration of 7 days. What is the premium Zach paid for this option?

Inputs:
Current Stock Price: $\mathrm{S}=49.5$
Time Until Expiration: $\mathrm{t}=7 / 365=0.0192$
Option Strike Price: $\mathrm{K}=50$
Volatility: $\boldsymbol{\sigma}=20 \%$
Risk Free Interest Rate: $r=1 \%$
Dividend Rate: $\mathrm{q}=2 \%$

$$
\begin{aligned}
& \text { Calculations: } \\
& d_{1}=\frac{\ln \left(\frac{49.5}{50}\right)+\left(1 \%-2 \%+\frac{20 \%^{2}}{2}\right)(0.0192)}{20 \% \sqrt{0.0192}}=0.0051 \\
& d_{2}=0.0051-20 \% \sqrt{0.0192}=-0.0226 \\
& N\left(d_{1}\right)=N(0.0051)=0.5020 \\
& \begin{array}{l}
N\left(d_{2}\right)=N(-0.0226)=0.4910 \\
\begin{array}{l}
C=(0.5020)(49.5) e^{-(2 \%)(0.0192)} \\
\quad-(0.4910)(50) e^{-(1 \%)(0.0192)} \\
=
\end{array}
\end{array} . \begin{array}{l}
\quad .30
\end{array}
\end{aligned}
$$

Plugging these values into the Black-Scholes equation, we find that the premium paid by Zach for this option is $\$ 0.30$ per share. Therefore, Zach pays $100 \cdot \$ 0.30=\$ 30.00$ to enter the contract.

## Black Scholes Examples

Each of the variables included in the pricing formula influences the premium. How impactful each individual variable is on the final calculation can be better visualized below. The following control variables will be used as inputs and will stay constant in each graphed relationship.

## Inputs

Stock Price: $\mathrm{S}=\$ 77$
Time Until Expiration: $\mathrm{t}=1 / 365=0 \%$
Strike Price: $\mathrm{K}=\$ 78$
Volatility: $\boldsymbol{\sigma}=10 \%$
Interest Rate: $\mathrm{I}=0 \%$
Dividend Rate: $\mathrm{q}=0 \%$


Time:
The above graph shows the relationship between expiration (measured in days) and premium. Value of an option increases as time until expiration increases. This makes sense because there is more time for the market value of the underlying asset to change, therefore making the option more valuable.


## Strike Price:

Value of an option decreases as the strike increases. The price of a call comes from its intrinsic value and time value. The intrinsic value is equal to the amount of premium the option holder would get if the call were to expire right now, with the current stock price and strike price. The time value accounts for the potential for the stock price to move before expiration. Options with strike prices below the current asset price are worth at least the difference between the strike price and asset price. Higher strike prices lead to lower call option values, all else being equal.

Call Premium vs. Stock Price


Stock Price:
Value of an option increases as current market price of the asset increases. This is similar to the previous case where decreasing strike price relates to increased call premium. The higher above the strike price the current market price of the asset is, the higher the intrinsic value of the call and the higher the call premium is overall. The value of a call option decreases as the stock price decreases, all else being equal.


## Volatility:

The value of a call option increases as volatility increases. This is because a higher volatility means there is an increased chance of the asset price changing, giving the option a higher time value, all else being equal.

## Call Strategies

## Naked or Covered Calls

We describe two situations: naked calls and covered calls.
Naked calls (also called uncovered calls) are calls where the party in the short position does not own the underlying asset. Thus, they would have to purchase the assets from the market if the call is exercised against them. In the case of selling calls in (Scenario 1-Short Position), the assets are stocks and the seller of the option is obligated to sell 1 share to the buyer when the call option is exercised. Selling naked calls is risky because they have unlimited downside if the stock price greatly exceeds the strike price at expiration.

When a party owns the underlying asset and sells calls based on it, those options are called covered calls. Covered calls are less risky than naked calls because the party in the short position is always able to supply the assets if they are called away. The party in the short position would only ever miss out on potential returns gained by stock price increasing, but they would not be out additional money if the stock skyrockets.

## Examples

On November 1st, 2019, you charge $\$ 0.25$ for a call option for 100 shares of XYZ Company stock which is currently valued at $\$ 49.50$ per share with strike price $\$ 50$ and expiration of 7 days. On November 8th, 2019, XYZ Company stock has risen to $\$ 51$ per share. The buyer exercises the option and purchases 100 shares of XYZ Company stock at $\$ 50$ per share.

## Naked Calls

When you sold this call option on November 1st, 2019, you did not have 100 shares of XYZ Company stock in your portfolio, so this option is a naked call. When the option is exercised against you on November 8th, 2019, to fulfill your obligation you must purchase 100 shares of XYZ Company stock from the stock market at $\$ 51$ per share, and sell this 100 at the strike price $\$ 50$ per share.

You pay $\$ 51$ per stock while receiving $\$ 50$,
Profit $\$ 50-\$ 51=-\$ 1$ per stock plus premium
Total Loss $\$ 50 \cdot \$ 100-\$ 51 \cdot \$ 100+\$ 0.25 \cdot \$ 100=-\$ 75$ from receiving premium and strike.

## Covered Calls

When you sold this call option on November 1st, 2019, you already had the 100 shares in your portfolio, so your obligation is covered. The buyer exercises the option on November 8th because the current market value of the stock is $\$ 51$, which is greater than the strike price. When the buyer exercises the option, you sell your 100 stocks originally worth $\$ 49.50$ per share for the strike price of $\$ 50$ per share.

You paid $\$ 49.50$ per stock while receiving $\$ 50$,
Profit $\$ 50-\$ 49.50=\$ 0.50$ per stock plus premium
Total Profit of $\$ 50 \cdot \$ 100+\$ 0.25 \cdot \$ 100-\$ 49.50 \cdot \$ 100=\$ 75$
In this scenario, you would have a higher profit if you had just held onto your shares, but you still profit overall from selling the calls.

The difference in profit when selling naked versus covered calls can be better visualized on the next page.

If the price of the stock never goes beyond the strike price, both selling a naked call and a covered call would result in the same profit, equal to the purchase price of the call. However, if the stock price goes up, the seller of a naked call can potentially lose money as they must buy the asset at market price. Whereas the seller of a covered call would not face such a problem. Selling naked calls is so risky that it was not chosen to be the main strategy to focus on because the team did not expect it to have high returns.

## Profit of Naked and Covered Call


$\longrightarrow$ Naked Call $\longrightarrow$ Covered Call

## Terminology Review

## American Option

An option that can be exercised any time before expiration

## Call Option

An agreement that gives the holder the right, not the obligation, to purchase an agreed upon asset for an agreed upon price at an agreed upon time

## Covered Call

A call where the seller also owns the underlying asset from which the call derives its value

## European Options

An option that can only be exercised at expiration.

## Expiration

The date by which an option must be exercised.

## Long Position

A position in an option agreement which possesses the right, but not obligation, to exercise the contact.

## Naked Call

A call where the seller does not own the underlying asset from which the call derives its value.

## Payoff

The monies gained from entering an option contract, not taking into account the cost to enter the agreement.

## Profit

The monies gained from entering an option contract, accounting for the cost of entering the agreement.

## Short Position

A position in an option agreement which is obligated to provide an underlying asset if buyer exercises.

## Strike Price

Price the holder may purchase an underlying asset for, specified in the call option.

## Chapter 3: Methodology

This project analyzes the returns of five different trading strategies involving stocks and call options. The strategies are compared to a simple buy and hold strategy.
For our calculations, we are choosing the underlying asset for the calls to be SPY ETF. Every strategy has the same amount of starting capital, which is enough to purchase 100 shares of the SPY ETF at the starting date. For consistency, we chose the initial value of the underlying asset in each scenario to be $\$ 52.56$, the price of a share of SPY from May 26,1995 .

Strategy 1 consists of selling covered calls. The initial capital will first be used to buy and maintain 100 shares and each week, a contract for 100 call options with expirations of 7-days will be written. All funds collected from selling calls will be held as cash.

Strategy 2 will start with the same investment in SPY ETF. Call options will be written in lots of 100 . However, if call prices are too low, an additional 100 shares of SPY will be purchased, with call options written on them as well. Purchasing additional shares leads to a total of 200 shares being held and 200 call options being written.

Likewise, Strategy 3 will start with an investment of 100 shares of SPY ETF and call options on lots of 100 will be sold. The premium collected from selling calls is reinvested in the market, rather than just being held in cash.

While starting capital is fully invested into the market for the above strategies, Strategies 4 and 5 involve holding the starting $\$ 5,256$ in cash and investing a small portion each week.

Strategy 4 is from the long position. Every 7 days, a contract for 100 options is purchased using the cash balance and held till expiration. Strategy 5 is from the short position and involves selling 100 calls each week without holding any shares of SPY.

Each strategy was first backtested in the R programming language using a historical set of stock prices. The historical data used in all strategies come from the SPY prices every 7 days from May 26th, 1995 to May 29th, 2021, which were collected from Bloomberg. After backtesting with historical data, each strategy was run with 10,000 simulations of stock prices and volatilities.

## Base Case

To compare the performance of each strategy, it is important to set up the Base Case. The Base Case is the traditional buy and hold strategy. For this project, we are holding 100 shares of the S\&P 500 ETF (SPY) for the entire period of time. Therefore, in this strategy the portfolio value is directly proportional to current stock value. For instance, if SPY were valued at $\$ 70$ per share at the end of the period, the portfolio would have an ending value of $\$ 7,000$.

## Strategy 1: Selling Covered Calls

The trading method used in Strategy 1 is to purchase 100 shares of SPY and then sell covered calls on lots of 100 shares each week. 100 shares will be held at all times. The premiums made from selling the covered calls will be a part of the cash balance. These covered calls have expiration of 7 days from the day they were sold. The options written are European style ${ }^{1}$, so they are only exercisable on the expiration date. The strike price used for the call is chosen to be the next whole dollar above the stock price at purchase. For example, if the current stock price is $\$ 99.55$, the strike price would be set at $\$ 100$. When using historical data, the call prices are calculated using the Black-Scholes formula with a constant $15 \%$ volatility, $0 \%$ risk free interest rate, and $0 \%$ dividend rate ${ }^{2}$.

At the beginning of the week, a contract for 100 call options is sold and call premium is added to the cash balance. At the end of the week one of two things would happen:

First, the stock price could rise above the strike price, which means the calls would be exercised and 100 shares will be sold at the strike price. 100 shares will then be purchased at the current price to replace the shares called away. The existing cash balance, along with the payment for 100 shares at strike is often enough to purchase the replacement shares. If the cash balance is not sufficient, a loan would be taken out to cover the difference. Taking out loans happens very rarely in this scenario, for example only two small (under \$50) loans would have been taken out over the historical period. The strike price will be updated based on the current share price of SPY, and another lot of 100 calls with 7 -day expiration will be written.

Otherwise, at expiration the stock price remains below or equal to strike price. Meaning the calls expire worthless, which would relieve us of any obligation to the call buyer. So 100 shares will continue to be held and another lot of 100 calls with the previous strike price and 7day expiration will be written. The strike price remains the integer above the purchase price because we never want to sell our shares for less than we paid for.

[^0]
## Strategy 1 Example

Week 7: Say we are looking at the beginning of the seventh week of a scenario using Strategy 1 and holding 100 shares of SPY ETF, currently valued at $\$ 59.00$. We then sell a lot of 100 calls which are priced at $\$ 0.40$ per share. The call price is calculated by Black-Scholes, using a strike price of $\$ 60$ and a volatility of $15 \%$. At this point we are left with the below conditions.

$$
\text { Portfolio Value }=\text { Number of Shares } \cdot S+\text { Cash Balance }-C \cdot \text { Calls Sold }
$$

Number of Shares $=100$
Cash Balance $=\$ 260+\$ 40=\$ 300$
Stock Price (S) = \$59
Current Strike Price (K) = \$60
Number of Calls Sold $=100$
Call Price (C) = \$0.40
Portfolio Value $=\$ 5,900+\$ 300-\$ 40=\$ 6,160$

Week 8: At expiration end of the week, if stock price is $\$ 63$ (stock price ( $\mathbf{S}$ ) $\geq$ strike price (K)) we would be obligated to provide the 100 shares of SPY ETF. For these shares we would receive $\$ 6,000$, or $\$ 60$ per share, which would be added to the cash balance.

Number of Shares $=100-100=0$
Cash Balance $=\$ 300+\$ 6,000=\$ 6,300$

We would then purchase 100 new shares at the current price of $\$ 63$ per share using total cash balance. Then write calls at a new strike price, for an estimated call price $\$ 0.50$ per share (also assuming a $15 \%$ volatility) to start the next week.

Number of Shares $=100$
Cash Balance $=\$ 6,300+\$ 50=\$ 6,350-\$ 6,300=\$ 50$
Stock Price $(\mathrm{S})=\$ 63$
Current Strike Price (K) = \$64
Number of Calls Sold = 100
Call Price (C) $=\$ 0.50$
Portfolio Value $=\$ 6,300+\$ 50-\$ 50=\$ 6,300$

## Strategy 1 Flow Chart



## Strategy 2: Selling Covered Calls, Double-Down

The trading method used in Strategy 2 also consists of writing covered calls on 100 shares of SPY with an expiration of 7-days and strike price the next integer above initial purchase price of SPY. At the beginning of every week call premiums are collected, and at expiration the calls are either worthless or we are obligated to provide shares of the underlying assets. However, if the ratio of the call price to the strike price drops below 25 basis points, we attempt to generate more cash flow by taking out a loan to purchase 100 additional shares. Buying shares at the new, lower market price allows us to sell calls with a lower strike price for a higher call premium. The strike price will always be set at the next integer above purchase price which does not change when the stock prices decrease. Not lowering the strike price means that if the stock price dropped drastically, the strike would be set significantly higher than the stock price. If the strike price is much higher than the stock price, the value of the call would be very small, in some cases the call is almost worthless. When calls are worthless, there is no income being generated until the stock price bounces back.

Rather than waiting for the stock price to bounce back up, we will purchase another 100 shares of SPY ETF, potentially taking out a loan to do so. For this strategy, we will hold at most 200 shares at any given time.

Calls will be sold on this new 100 shares with a strike price of the next whole dollar above the new, lower, purchase price. Thus, the calls sold on this new 100 shares will now have higher value than those with higher strike prices. When the stock price reaches this new strike price, these additional shares are called away and any revenue added to the cash balance. The additional shares are not necessarily replaced. The decision to purchase an additional 100 shares is triggered when the call price drops below $0.25 \%$ of the stock price. When the stock bounces back to a point where the call price is above $0.25 \%$ of the stock price, no shares are purchased to replace the additional 100 when they are called away.

## Strategy 2 Example

Week 7: We are looking at the beginning of the seventh week of a scenario using Strategy 2 and are holding 100 shares of SPY ETF, currently valued at $\$ 59.00$. We then sell a lot of 100 calls which are priced at $\$ 0.40$ per share. The call price is calculated by Black-Scholes, using a strike price of $\$ 60$ and a volatility of $15 \%$. At this point we are left with the below conditions.

$$
\text { Portfolio Value }=\text { Number of Shares } \cdot S+\text { Cash Balance }- \text { Loans }-C \cdot 100-\text { C2 } \cdot 100
$$

Number of Shares $=100$
Cash Balance $=\$ 40$
Stock Price $(\mathrm{S})=\$ 59$
Current Strike Price (K) = \$60
Number of Calls Sold $=100$
Call Price (C) $=\$ 0.40$
Second Lot Strike Price (K2) = \$0
Second Lot Call Price (C2) = \$0
Portfolio Value $=\$ 5,900+\$ 40-\$ 40=\$ 5,900$

Week 8: At expiration, if stock price $=\$ 49$ (stock price $(\mathbf{S})<$ strike price (K)) we would be relieved of our obligation as the calls will expire worthless. To start the next week, we write calls at the same strike price $\$ 60$ for an estimated call price $\$ 0.000012$ per share (also assuming a $15 \%$ volatility) to start the next week. Before doing so we will check if the ratio of the call price to the strike price is below 25 basis points:

$$
\frac{C}{K}=\frac{\$ 0.000012}{\$ 60}<0.25 \% .
$$

Since the ratio is below 25 basis points, another 100 shares will be bought for $\$ 49$ per share, and additional calls will be written at a lower strike price. Also note that if the stock price continues to decrease, we will not buy a third lot of 100 shares.

Number of Shares $=200$
Cash Balance $=\$ 40+\$ 0.0012+\$ 7 \approx \$ 47-\$ 47=0$
Loan $=\$ 4,900-\$ 47=\$ 4,853$
Stock Price (S) = \$49
Current Strike Price (K) = \$60
Call Price (C) = \$0.000012
Number of Calls Sold $=200$
Second Lot Strike Price (K2) = \$50
Second Lot Call Price (C2) $=\$ 0.07$
Portfolio Value $=\$ 9,800+\$ 0-\$ 4,853-\$ 7.0012 \approx \$ 4,940$

## Strategy 2 Flow Chart



## Strategy 3: Fully Invested

The trading method used in Strategy 3 consists of writing covered calls on 100 shares of SPY with an expiration of 7-days and strike price the next integer above initial purchase price of SPY. When a call option is sold at the beginning of the week, the premium is immediately used to purchase more shares. At the end of the week, the option is either exercised against us, and shares called away at strike price or it isn't and it expires worthless. Any money collected is immediately invested into buying more shares. No cash balance will be held for this strategy, all capital will be reinvested in the market and there is no limit on the amount of shares held. Strategy 3 is the only strategy where it is assumed fractional stocks can be purchased.

## Strategy 3 Example

Week 7: We are looking at the beginning of the seventh week of a scenario using Strategy 3 and are holding 100 shares of SPY ETF, currently valued at $\$ 59.00$. We then sell a lot of 100 calls which are priced at $\$ 0.40$ per share. The call price is calculated by Black-Scholes, using a strike price of $\$ 60$ and a volatility of $15 \%$. The call premium received is immediately reinvested in SPY ETF shares. At this point we are left with the below conditions.

## Portfolio Value $=$ Number of Shares $\cdot S-C \cdot$ Calls Sold

Number of Shares $=100.68$
Cash Balance $=\$ 0$
Stock Price (S) = \$59
Current Strike Price (K) = \$60
Number of Calls Sold $=100$
Call Price (C) $=\$ 0.40$
Portfolio Value $=\$ 5,940.12-\$ 40=\$ 5,900$

Week 8: At expiration end of the week, if stock price $=\$ 59$ (stock price $(\mathbf{S})=$ strike price $(\mathbf{K})$ ) we would end up in the same position regardless of whether or not the option is exercised against us. The option is not exercised against us, it expires, leaving us with our 100 shares. To start the next week 100 calls of the same price and strike price will be sold since the conditions did not change. We would use the premium from selling those calls to purchase 0.68 more shares of SPY, leaving us with the following:

Number of Shares $=101.36$
Cash Balance $=\$ 0$
Stock Price $(\mathrm{S})=\$ 59$
Current Strike Price (K) = \$60
Number of Calls Sold $=100$
Call Price (C) $=\$ 0.40$
Portfolio Value $=\$ 5,980-\$ 40=\$ 5,940$

## Strategy 3 Flow Chart



## Strategy 4: Buying Calls

All trading strategies discussed previously take the perspective of selling covered calls or being short the call. Strategy 4 is now from the long position or purchasing calls. The starting cash balance, like all other strategies, is $\$ 5,256$ and rather than purchasing shares of SPY ETF and selling call options, 1007 -day call options are bought. At the end of each week the calls will either expire worthless and the price we paid to purchase the call will be lost. Otherwise. the strike would be met and we would be given an amount equal to $S$ (stock price) - $K$ (strike price) in return for our position. ${ }^{3}$ Regardless, 100 more calls are bought every 7 days so a contract for 100 calls is always held. Most other assumptions remain the same as the other strategies, except loans are not considered. Because loans are not being taken out, the cash balance is dipped into every week to purchase new calls and once this fund is empty or there is too little cash to purchase another 100 calls, the strategy is terminated.

[^1]
## Strategy 4 Example

Week 7: We are looking at the beginning of the seventh week of a scenario using Strategy 4 and there is $\$ 3,500$ of starting capital left. The initial capital will be used to purchase a contract of 100 options for $\$ 0.40$ per share. The call price is calculated by Black-Scholes, using a strike price of $\$ 60$ and a volatility of $15 \%$.

$$
\text { Portfolio Value }=\text { Previous Week Cash Balance }+\max (S-K, 0) \cdot 100
$$

Current Stock Price $(\mathbf{S})=\$ 59$
Current Strike Price (K) = \$60
Call Price (C) $=\$ 0.40$
Number of Calls Bought $=100$
Cash Balance $=$ Portfolio Value $=\$ 3,500$

Week 8: If the current stock price is $\$ 63$ (stock price $(\mathbf{S}) \geq$ strike price (K)) we will theoretically purchase 100 shares of SPY ETF for $\$ 60$ per share. In practice we are likely to be bought out of our position. Meaning we would receive a payment of the difference between the current stock price and the strike price per share, therefore increasing our portfolio value. To start the next week another 100 calls will be purchased.

Current Stock Price $(S)=\$ 63$
Current Strike Price (K) = \$64
Call Price (C) = \$0.60
Number of Calls Bought $=100$
Cash Balance $=$ Portfolio Value $=\$ 3,460-\$ 60+\$ 300=\$ 3,700$

## Strategy 4 Flow Chart



## Strategy 5: Selling Naked Calls

Strategy 1-3 are based on purchasing and holding shares or being invested in the stock market. Strategy 4 is based on buying calls, with no investment in market shares and rather holding cash. Strategy 5 consists of selling naked calls. The starting capital, $\$ 5,256$, will simply be the cash balance used to close out 7-day call options, and will not be used to purchase shares of SPY to hold. At the beginning of the week, premium is immediately collected and if the option expires worthless a profit has been made. If the current stock price is greater than the strike price at expiration, the call will be exercised and we will have to pay out the difference between the current stock price and the strike price for each share to the buyer in order to close the call. Similarly to Strategy 4, no loans are being considered. If the cash balance goes to zero or lower, the strategy is terminated.

## Strategy 5 Example

Week 7: We are looking at the beginning of the seventh week of a scenario using Strategy 5 there is $\$ 3,500$ of starting capital left. This will be used to sell a contract of 100 call options for $\$ 0.40$ per share. The call price is calculated by Black-Scholes, using a strike price of $\$ 60$ and a volatility of $15 \%$.

$$
\text { Portfolio Value }=\text { Cash Balance }-C \cdot 100-\max (S-K, 0) \cdot 100
$$

Current Stock Price $(\mathrm{S})=\$ 59$
Current Strike Price $(\mathrm{K})=\$ 60$
Call Price (C) = \$0.40
Number of Calls Sold $=100$
Cash Balance $=\$ 3,500+\$ 40=\$ 3,540$
Portfolio Value $=\$ 3,540-\$ 40=\$ 3,500$

Below, we demonstrate what would happen in two different scenarios: asset price being above strike at expiration and asset price being below strike at expiration.

Week 8 (stock price $(\mathbf{S})<$ strike price $(\mathbf{K})$ ): If the current stock price is $\$ 55$ the calls have expired worthless. Therefore, to start the next week another 100 calls will be sold.

Current Stock Price $(S)=\$ 55$
Current Strike Price (K) = \$56
Call Price (C) = \$0.15
Number of Calls Sold $=100$
Cash Balance $=\$ 3,540+\$ 15=\$ 3,555$
Portfolio Value $=\$ 3,555-\$ 15=\$ 3,540$

Week 8 (stock price $(\mathbf{S})>$ strike price $(\mathbf{K})$ ): If the current stock price is $\$ 100$ we will be obligated to close our position by buying the calls from the buyer for S-K per each share.

Current Stock Price $(\mathrm{S})=\$ 100$
Current Strike Price (K) = \$101
Call Price (C) $=\$ 0.40$
$S-K=\$ 10,000-\$ 6,000=\$ 4,000$
Cash Balance $=\$ 3,540+\$ 40=\$ 3,595$
Portfolio Value $=\$ 3,595-\$ 40-\$ 4,000=-\$ 445$
If the strike price was to skyrocket the strategy would run out of the money entirely.

## Strategy 5 Flow Chart



In a 5-week demonstration, the portfolio value of each strategy has been modeled. There are three different scenarios for stock price trends: increasing 5\% weekly, decreasing 5\% weekly, and staying constant. The starting investment in each case is $\$ 5,000$.

| Stock Price | Increasing 5\% <br> weekly | Decreasing 5\% <br> weekly | Constant 0\% <br> weekly |
| :--- | :--- | :--- | :--- |
|  | Final Portfolio |  |  |
| Base Case: Buy <br> and Hold Shares | $\$ 6,078$ | $\$ 4,073$ | $\$ 5,000$ |
| Strategy 1: Selling <br> Covered Calls | $\$ 5,460$ | $\$ 4,082$ | $\$ 5,038$ |
| Strategy 2: Selling <br> Covered Calls, <br> Double-Down | $\$ 5,640$ | $\$ 3,424$ | $\$ 5,038$ |
| Strategy 3: Fully <br> Invested | $\$ 5,340$ | $\$ 4,080$ | $\$ 5,038$ |
| Strategy 4: <br> Buying Calls | $\$ 5,738$ | $\$ 4,990$ | $\$ 4,962$ |
| Strategy 5: Selling <br> Naked Calls | $\$ 4,262$ | $\$ 5,010$ | $\$ 5,038$ |


| Stock Price | Increasing 5\% <br> weekly | Decreasing 5\% <br> weekly | Constant 0\% <br> weekly |
| :--- | :--- | :--- | :--- |
|  | Weekly Return |  |  |
| Base Case: Buy <br> and Hold Shares | $3.98 \%$ | $-4.02 \%$ | $0 \%$ |
| Strategy 1: Selling <br> Covered Calls | $1.78 \%$ | $-3.98 \%$ | $0.15 \%$ |
| Strategy 2: Selling <br> Covered Calls, <br> Double-Down | $2.44 \%$ | $-7.29 \%$ | $0.15 \%$ |
| Strategy 3: Fully <br> Invested | $1.32 \%$ | $-3.98 \%$ | $0.15 \%$ |
| Strategy 4: Buying <br> Calls | $2.79 \%$ | $-0.04 \%$ | $-0.15 \%$ |
| Strategy 5: Selling <br> Naked Calls | $-3.14 \%$ | $0.04 \%$ | $0.15 \%$ |

In the circumstance where the stock market is consistently decreasing over 5 weeks, Strategy 5 proves to be the most profitable as there is no investment in SPY, which would be decreasing, and the premium from selling calls is an income to the portfolio. Strategy 4 yields the next highest returns in a decreasing market, however the returns are slightly negative. Strategy 4 does not lose as much as Strategies 1-3 since there is no market investment. The loss of the price to purchase a contract of 100 options is a minimal hit to portfolio value compared to having at least 100 shares decrease in value weekly. Strategy 3 , 1, and the Base Case are all very similar in movement considering the third strategy is fully investing all cash into the market therefore the portfolio is tanking with the market but premium was still collected from selling calls. Strategy 1 is not far behind since most of the money is invested in SPY ETF. Strategy 2 is the least profitable. Since the parameter for buying an additional 100 shares has been triggered resulting in the portfolio having 200 shares in decreasing stock, there is double the investment in the movement of SPY ETF shares.


In the circumstance where the stock market is consistently increasing over 5 weeks, the Base Case proves most profitable while Strategies 1-3 perform almost identical to each other. For these strategies premium was collected and shares called away. When writing covered calls we collect call premium but "miss out" on the opportunity to receive returns from skyrocketing stock. Therefore, if we were to just hold shares we would have made more, but the tradeoff for selling calls means we will always get some sort of return, regardless of how the market moves throughout the week. Strategy 5 tanks since we are always having to purchase SPY ETF shares higher than we are selling them for causing us to run out of cash quickly. While Strategy 4 performs second best since the options are always expiring in the money and we are getting SPY ETF shares for less than they are worth, reselling them would make us an immediate profit.

Increasing Stock Price


In a steady market it is assumed being in the short position is best. When selling naked calls, premium is immediately collected and if the stock price remains constant, all calls essentially expire worthless. Therefore, you'd always be collecting a premium. Similar to this, Strategies 1-3 make about the same profit as most of the portfolio value comes from holding shares. Premium is collected from selling options while SPY ETF shares only provide a constant value to the portfolio. Strategy 4 performs the worst in a constant market. Strategy 4 performs poorly because in a constant market calls purchased with a strike above the current stock price will always expire worthless, causing you to lose money each week.


## Simulations

To quantify the difference between the strategies, they are compared in different economic scenarios. The historical data is only one market scenario, and simulating different stock prices and trends can show that these strategies differ in returns. This can provide a basis to the analysis which will determine just how different the portfolios of each strategy are. Two different methods were considered for generating different scenarios: utilizing an existing Economic Scenario Generator and bootstrapping weekly growth factors.

The Economic Scenario Generator (ESG) was a major qualifying project for Actuarial Mathematics back in 2020 by Erica Lee, Hanyi Jiang, Rahul Kumar. This tool generates a given number of stock market scenarios for a given number of years. Its major advantage is that the correlation between consecutive prices are taken into consideration by using a regime switching model. However, the downside of the ESG, for this project, is that scenarios are generated on an annual basis while the data we are handling is on a weekly basis. Therefore, a large modification of this tool would be necessary if we were to use this Economic Scenario Generator. So, another approach for simulated markets was chosen.

Bootstrapping is a commonly used technique to generate a list from a set of data which is relatively small. This method involves sampling from the dataset with replacement. Meaning once this sample is chosen it is put back into the original dataset. Therefore, the same variable can be chosen more than once. Regardless of how many data points are needed in the new dataset, it can always be generated from the sample set. The example below shows 2 different bootstrapped scenarios which can be generated from the original data set.

| Data: |
| :--- |
| A B C D E F |
| Bootstrapped Scenario 1: |
| A C F E A A B C A |
| Bootstrapped Scenario 2: |
| B D C A F E D B A |

For our project, we wanted to generate 10,000 scenarios for 250 weeks. Scenarios were generated by calculating the weekly growth factors of the historical data. Each weekly growth factor is found by dividing the SPY price by the previous week's price. We looked at 10,000 scenarios of 250 weeks of simulated growth factors. These growth rates were then bootstrapped to create a list of 250 rates, which were then used to generate a new list of weekly stock prices and volatilities.

To generate a list of values, each weekly price was calculated by multiplying the previous week's price by the corresponding bootstrapped growth factor. ${ }^{4}$ The below formula was used to calculate the list of weekly volatilities ${ }^{5}$ :

## Volatility $=\sigma_{\text {Past } 15 \text { weeks growth factors }} \cdot \sqrt{52}$

The strategies were then run using the newly generated 10,000 sets of prices and volatilities, and the growth of each portfolio was recorded. From this we were able to use the annualized return formula (below) to determine the growth of each strategy for each scenario.

Annualized Return $=\sqrt[5]{\frac{\text { Beginning Portfolio }}{\text { Ending Portfolio }}}-1$
Histograms were made using the 10,000 annualized returns for each scenario and the mean, standard deviation, skewness, and kurtosis were calculated. Lastly, the annualized return of each strategy was compared to the Base Case, and significance of differences between strategies were analyzed using t-tests.

[^2]
## Chapter 4: Results and Analysis

The returns of each strategy when run for 25 years with historical prices of the SPY ETF starting May 26th, 1995 to May 28th, 2021 only illustrate one scenario. The performance of strategies for only one scenario is not enough to base conclusions on because there are many potential ways the stock market could move. Therefore, 10,000 additional scenarios, each with 5 years' worth of simulated prices, were generated and used to run each strategy. The resulting lists of returns were then compared using a t-test to show significance of difference of the expected returns for each strategy.

## Historical Results

Each strategy was first backtested using the historical prices of SPY.


For the Base Case of buying and holding 100 shares of SPY for 25 years, the portfolio would grow from $\$ 5,256$ to $\$ 42,004$, making the annual effective growth rate $8.67 \%$. This strategy was used as a basis for comparison.

## Strategy 1: Selling Covered Calls Results

The portfolio value of Strategy 1 using historical data is graphed alongside the Base Case. This graph shows selling covered calls on 100 shares of SPY performs comparably to and, after May 2013, substantially better than buying and holding 100 shares. When selling covered calls, the portfolio would grow from $\$ 5,256$ to $\$ 60,373$, making the annual effective growth rate $10.26 \%$. Strategy 1 showed a higher return than the Base Case by $\$ 18,369$.

## Strategy 1 (Blue), and Base Case (Red) over Time



## Strategy 2: Selling Covered Calls, Double-Down Results

Next, the portfolio value of Strategy 2 using historical data is graphed alongside the Base Case. This graph shows selling covered calls on 100 shares, and sometimes 200 shares, of SPY outperformed the Base Case by $\$ 48,532$. When following this strategy, the portfolio would grow from $\$ 5,256$ to $\$ 90,536$, making the annual effective growth rate $12.06 \%$. This strategy outperformed Strategy 1 by $\$ 30,163$.


## Strategy 3: Fully Invested Results

The portfolio value of Strategy 3 using historical data is graphed alongside the Base Case. Similar to the previous strategies, selling covered calls on the 100 shares of SPY ETF performs better than holding 100 shares. The portfolio for Strategy 3 would grow from $\$ 5,256$ to $\$ 79,454$, making the annual effective growth rate $11.48 \%$. This strategy outperformed the Base Case by $\$ 37,450$, Strategy 1 by $\$ 19,080$, and underperformed Strategy 2 by $\$ 11,083$.

## Strategy 3 (Purple), and Base Case (Red) over Time



## Strategy 4: Buying Calls

Using historical data, the portfolio of Strategy 4 is graphed against the portfolio of the Base Case. In this case, the Base Case actually performs better than buying calls. The value had grown from $\$ 5,256$ to $\$ 20,897$, making the annual effective growth $5.68 \%$ per year. This strategy performed worse than the Base Case by $\$ 21,106$, Strategy 1 by $\$ 39,525$, Strategy 2 $\$ 69,429$, and Strategy 3 by $\$ 58,794$.

Strategy 4 (Orange), and Base Case (Red) over Time


## Strategy 5: Selling Naked Calls

The portfolio of strategy 5 using historical data is graphed alongside the Base Case. This graph shows Strategy 5 ran out of investment capital in the year 2000. By February of 2000, all of the cash that was held had been paid out to cover any losses incurred from selling naked calls. Strategy 5 performed the worst when backtested with historical data because none of the other strategies led to losing all of the investment capital in the historical scenario.

## Strategy 5 (Pink), and Base Case (Red) over Time




To summarize these results, ending portfolio values were ranked by highest to lowest.

1. Strategy 2 (Green): Selling Covered Calls, Double-Down
2. Strategy 3 (Purple): Fully Invested
3. Strategy 1 (Blue): Selling Covered Calls
4. Base Case (Red): Holding 100 Shares
5. Strategy 4 (Yellow): Buying Calls
6. Strategy 5 (Pink): Selling Naked Calls

Strategy 2 had the highest portfolio value most of the time in the historical scenario. A possible explanation for this would be that the secondary lots of shares are only ever called away for a profit. When the second lot of shares is called away, it does not cause a loss in portfolio value. This is because the shares aren't necessarily replaced when called away and are called away for a strike that's higher than the purchase price. Strategy 3, being fully invested, is the second most profitable strategy in the historical scenario, which makes sense considering call premium is invested in shares of an asset whose price is increasing. Strategy 1, selling covered calls performs worse than Strategy 3 in the historical scenario because call premium is held as cash rather than having potential to grow by being invested in the market.
All strategies involving selling covered calls perform better than the Base Case of holding 100 shares. Strategy 4, buying calls, is significantly worse than just investing capital in market shares. The last strategy, selling naked calls, performed the worst, as all investment capital was depleted in about 5 years.

| Historical Scenario Result Summary |  |  |  |
| :--- | :--- | :--- | :--- |
|  | Beginning Portfolio | Ending Portfolio | Annual Growth |
| Holding 100 Shares | $\$ 5,256$ | $\$ 42,004$ | $8.67 \%$ |
| 1: Selling Covered <br> Calls | $\$ 5,256$ | $\$ 60,373$ | $10.28 \%$ |
| 2: Selling Covered <br> Calls, Double- <br> Down | $\$ 5,256$ | $\$ 90,536$ | $12.06 \%$ |
| 3:Fully Invested | $\$ 5,256$ | $\$ 79,454$ | $11.48 \%$ |
| 4:Buying Calls | $\$ 5,256$ | $\$ 20,898$ | $5.68 \%$ |
| 5:Selling Naked <br> Calls | $\$ 5,256$ | $\$ 0$ | Failed in January <br> 2000 |

## Bootstrapped Scenarios Result

We demonstrate the results of the 10,000 different scenarios using histograms of annual returns. The total growth was calculated and used to find the annual effective growth rate for each scenario. The distribution of which are in the graph below:

## Distribution of Annual Return of S\&P



| Mean | 0.0832 |
| :--- | :--- |
| Standard Deviation | 0.0846 |
| Skewness | 0.2215 |
| Kurtosis | 3.0892 |
| Sharpe Ratio (Mean/Standard Deviation) | 0.9835 |

The histogram of the 10,000 annual returns from running Strategy 1 with the bootstrapped scenarios can be found below:

## Distribution of Annual Return of Strategy 1



| Mean | 0.0735 |
| :--- | :--- |
| Standard Deviation | 0.0768 |
| Skewness | 0.0365 |
| Kurtosis | 2.9709 |
| Sharpe Ratio (Mean/Standard Deviation) | 0.9570 |

The histogram of the 10,000 annual returns from running Strategy 2 with the bootstrapped scenarios can be found below:

Distribution of Annual Return of Strategy 2


| Mean | 0.1174 |
| :--- | :--- |
| Standard Deviation | 0.1622 |
| Skewness | -4.6986 |
| Kurtosis | 49.4568 |
| Sharpe Ratio (Mean/Standard Deviation) | 0.7238 |

The histogram of the 10,000 annual returns from running Strategy 3 with the bootstrapped scenarios can be found below:

## Distribution of Annual Return of Strategy 3



| Mean | 0.0844 |
| :--- | :--- |
| Standard Deviation | 0.0841 |
| Skewness | 0.0774 |
| Kurtosis | 2.9264 |
| Sharpe Ratio (Mean/Standard Deviation) | 1.0036 |

The histogram of the 10,000 annual returns from running Strategy 4 with the bootstrapped scenarios can be found below:

## Distribution of Annual Return of Strategy 4



| Mean | -0.0300 |
| :--- | :--- |
| Standard Deviation | 0.1070 |
| Skewness | -4.4038 |
| Kurtosis | 39.0120 |

The histogram of the 10,000 annual returns from running Strategy 5 with the bootstrapped scenarios can be found below:

Distribution of Annual Return of Strategy 5 BT


| Mean | -0.0060 |
| :--- | :--- |
| Standard Deviation | 0.1349 |
| Skewness | -5.5465 |
| Kurtosis | 39.2725 |

Upon obtaining the annual returns of each scenario for all strategies, the difference between returns for each strategy, including the Base Case, was calculated. Hypothesis testing was used to determine if the differences ( $d$ ) between all strategies are statistically significant. We used paired t-tests to examine how significant the difference is. The null hypothesis is that the mean of the differences $(\bar{d})$, is 0 , meaning that there is no difference between two strategies. The alternative hypothesis is that the mean of the differences $(\bar{d})$ is not 0 , meaning there is a difference between two strategies.

$$
\begin{aligned}
& H_{0}: \bar{d}=0 \\
& H_{a}: \bar{d} \neq 0
\end{aligned}
$$

For all the t-tests performed, all the p-values are very small, meaning we accept the alternative hypothesis that the mean difference is not 0 . Accepting the alternative hypothesis implies that the difference in each strategy is statistically significant. The p-values of each pair of t -tests and the sample mean of the differences are in the tables below:

|  | Base <br> Case | 1: Selling <br> Covered <br> Calls | 2: Double-Down | 3: Fully <br> Invested | 4: Buying <br> Calls | 5: Selling <br> Naked <br> Calls |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Base Case |  |  |  |  |  |  |
| 1: Selling <br> Covered Calls | 0 |  |  |  |  |  |
| 2: Double-Down | $1.9 \mathrm{E}-210$ | 0 |  |  |  |  |
| 3: Fully Invested | $1.7 \mathrm{E}-09$ | 0 | $1.5 \mathrm{E}-213$ |  |  |  |
| 4: Buying Calls | 0 | 0 | 0 | 0 |  |  |
| 5: Selling Naked <br> Calls | 0 | 0 | 0 | 0 | $4.28 \mathrm{E}-29$ |  |

p-values of the paired t-tests

|  | Base <br> Case | 1: Selling <br> Covered <br> Calls | 2: Double-Down | 3: Fully <br> Invested | 4: Buying <br> Calls | 5: Selling <br> Naked <br> Calls |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Base Case | 0 |  |  |  |  |  |
| 1: Selling <br> Covered Calls | -97 | 0 |  |  |  |  |
| 2: Double-Down | 342 | 439 | 0 | 0 |  |  |
| 3: Fully Invested | 13 | 109 | -330 | -1144 | 0 |  |
| 4: Buying Calls | -1131 | -1035 | -1474 | -904 | 240 | 0 |
| 5: Selling Naked <br> Calls | -892 | -795 | -1234 |  |  |  |

Estimated mean difference (in basis points)
The value of each cell is calculated by subtracting the mean of the strategy in the column from the mean of the strategy in the row. All the results are consistent with our findings in the historical data with one exception, that Strategy 1 did not outperform the Base Case like in the historical scenario. As with the historical data Strategy 2 had the highest average return and Strategies 4 and 5 had the lowest.

We investigated the returns of Strategy 1 and the Base Case to understand why Strategy 1 performed worse overall than Base Case in the simulated scenarios, when it outperformed the Base Case in the historical backtesting. We separated the scenarios where Strategy 1 outperformed the Base Case by 10 basis points and vice versa from the overall 10,000. Out of the 10,000 scenarios generated, Strategy 1 outperformed the Base Case by at least 10 basis points in 3,136 scenarios. The Base Case outperformed Strategy 1 by at least 10 or more basis points in 6,368 scenarios.


Distribution of annual returns when Strategy 1 (left) outperformed Base Case (right)


Distribution of annual returns when Strategy 1 (left) underperformed Base Case (right)

The mean difference was calculated for the situations with significant difference between Strategy 1 and the Base Case. For the 3,136 scenarios where Strategy 1 has a higher return than the Base Case, the estimated mean difference is 90 basis points. For the 6,368 scenarios where Strategy 1 has a lower return than the Base Case, the estimated mean difference is -196 basis points. One-third of the time, Strategy 1 performs better, and the other two thirds of the time worse than the Base Case. For the historical scenario Strategy 1 has an ending portfolio 1.5\% higher than the Base Case. Strategy 1 only performs that much better 524 times out of the 10,000 generated scenarios out, only about $5 \%$ of the time.

It is worth noting that the historical scenario is one representation of how the market will perform. The prices pulled from the years 1995 to 2021 reflect events like the 2008 housing crisis and the following unprecedented bullish market. The difference between the expected outcome for Strategy 1 based off the historical data and how it performed in the simulations exemplifies the potential danger of drawing conclusions after backtesting only using one scenario.

## Chapter 5: Conclusion

The goal of this project was to analyze different investment strategies which require minimal speculation and account management while yielding a higher risk adjusted return than buying and holding shares of an asset. Each of the strategies outlined only require prices to be checked weekly and that outlined actions be taken based on those values; this process can be easily automated. Conversely, selling naked calls and buying calls not only yielded lower returns than the base case of buying and holding in our simulations, but they lose money. In fact, selling naked calls often ends in absolute financial ruin. Therefore these strategies are not recommended. Most strategies involving selling covered calls had an annual return higher than the Base Case, the only exception being Strategy 1 whose returns were $1 \%$ lower on average than the Base Case in our simulations.

Strategy 2: selling covered calls and potentially buying another lot of shares, would be recommended to investors because the average returns from the simulation and the historical data were so high. This strategy sometimes requires doubling the number of shares of an asset held by purchasing more when the market significantly goes down. Therefore, the investor must be prepared to potentially invest extra capital in hopes of continuing to generate some sort of income from selling covered calls until the market goes up again. While a loan was sometimes used to purchase the additional shares, it is not required if the investor has enough capital. It is also worth noting that on a risk-adjusted basis, Strategy 2 does have a lower Sharpe ratio than the base case so the higher returns are related to the higher risk.

Strategy 3: selling covered calls and investing additional income, is another strategy that the team recommends. Strategy 3 also outperforms the traditional buy and hold method but often has lower annual returns than Strategy 2 in the simulation. While Strategy 2 has a lower Sharpe ratio than the Base Case, Strategy 3 has a higher Sharpe ratio. This higher Sharpe ratio means that Strategy 3 also has higher risk-adjusted returns when compared to the traditional

While Strategies 1-3 all involve selling covered calls, Strategies 2 and 3 outperformed Strategy 1 in most simulated scenarios. This is because Strategies 2 and 3 both have modifications which more aggressively invest in the market and as a result add more value to the portfolio. For instance, Strategy 2 continues to collect call premium after the price of the asset decreases significantly. Strategy 3 is fully invested in the market, so when asset value increases, portfolio value is increasing maximally, while premium in Strategy 1 stays in a cash account with no potential to change in value.

Based on the returns and volatilities, we conclude that selling covered calls, with aggressive modification close to full market investment, will likely produce higher returns than traditional buy and hold strategies.

## Chapter 6: Recommendations

In addition to exploring other trading strategies, possibly involving covered calls, certain criteria of existing concepts can be further developed. Further analysis could make conclusions and recommendations more accurate for each strategy. Some possible avenues to consider for the future follow.

## Changing Strike Price

The strike price for all strategies is set at the next whole dollar above the purchase price. If this were different i.e. if the strike was set to be greater than, or less than, the next whole dollar above purchase price, the returns could look very different. For the specific scenario of selling in-the-money calls, the benefits of having a higher call premium would have to be weighed against the risk of shares being called away at a strike below original purchase price.

## Autocorrelation Simulation vs. Bootstrapping

Before running the 10,000 economic scenarios using bootstrapped growth factors there was some consideration about utilizing different methods. An auto-correlated list of growth factors to generate new returns and/or volatilities are better for more accurately simulating stock prices, because they take into account that the market often stays up/down for a while. The fault in the bootstrapping method utilized in this paper is that the results can be almost too randomized. However, all proposed strategies in this paper base decisions on weekly market performance and never have to take into account past data when making said decisions. This makes autocorrelation between the weekly prices less necessary. If an investment method involved basing decisions off more than two weeks of data at a time, the success of the strategy would rely more on market trends, making an economic scenario generator almost necessary.


[^0]:    ${ }^{1}$ With such a short duration, American and European options are essentially interchangeable. While American calls are typically written in practice, we are assuming European calls for ease of calculating prices.
    ${ }^{2}$ Because the duration of the call is only one week, we are assuming the interest generated would be nearly 0 , and no dividends will be paid.

[^1]:    ${ }^{3}$ Being given the difference between the current asset price and strike price is equivalent to exercising the call and purchasing the asset for the strike and then selling it.

[^2]:    ${ }^{4}$ The first week's price was set as $\$ 52.56$ per share for all scenarios.
    ${ }^{5}$ The first 15 weeks volatilities were set to $15 \%$.

