How to Use Minitab:

Design of Experiments

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Contents

Terminology

- Controlled Experiment: a study where treatments are imposed on experimental units, in order to observe a response
- Factor: a variable that potentially affects the response
  - ex. temperature, time, chemical composition, etc.
- Treatment: a combination of one or more factors
- Levels: the values a factor can take on
- Effect: how much a main factor or interaction between factors influences the mean response

TERMINOLOGY

- Design Space: range of values over which factors are to be varied
- Design Points: the values of the factors at which the experiment is conducted
  - One design point = one treatment
  - Usually, points are coded to more convenient values
  - ex. 1 factor with 2 levels – levels coded as (-1) for low level and (+1) for high level
- Response Surface: unknown; represents the mean response at any given level of the factors in the design space.
- Center Point: used to measure process stability/variability, as well as check for curvature of the response surface.
  - Not necessary, but highly recommended.
  - Level coded as 0 .

When Do You Use a Factorial Design?

- Factorial designs are good preliminary experiments
- A type of factorial design, known as the fractional factorial design, are often used to find the "vital few" significant factors out of a large group of potential factors.
  - This is also known as a screening experiment
- Also used to determine curvature of the response surface

Full Factorial Designs

- Every combination of factor levels (i.e., every possible treatment) is measured.
  - 2^k design = k factors, each with 2 levels, 2^k total runs
  - 3^3 design = 3 factors, each with 3 levels, 3^3 = 27 total runs
- Every factor effect can be estimated
- Can include center points, but not necessary
- 2^k designs are the most popular
  - High Level (+1) and Low Level (-1)

Example: 2^2 design → 4 runs

Run	Factor A Level	Factor B Level
1	-1	-1
2	-1	+1
3	+1	-1
4	+1	+1

Full Factorial Designs

- Full factorials can also allows factors to have different # of levels
  - $2^1 3^2 4^1$ = 4 factors total (sum of exponents)
    One factor has 2 levels, two have 3 levels, one has 4 levels
    Total of 2*3*3*4 = 72 runs

Ex. $2^1 3^1$ design → 6 runs

Run	Factor A Level	Factor B Level
1	1	1
2	1	2
3	1	3
4	2	1
5	2	2
6	2	3

FRACTIONAL FACTORIAL DESIGNS

- Sometimes, there aren't enough resources to run a Full Factorial Design. Instead, you can run a fraction of the total # of treatments.
  - 2^{k-p} design = k factors, each with 2 levels, but run only 2^{k-p} treatments (as opposed to 2^k)
  - 2^{4-1} design = 4 factors, but run only 2^3 = 8 treatments (instead of 16)
    - 8/16 = 1/2 → design known as a "½ replicate" or "half replicate"

- However, not all factor effects can be estimated
  - Factors are aliased with one another. In other words, factors are confounded, and you cannot estimate their effects separately.
    - Ex. Suppose factors A and D are aliased. When you estimate the effect for A, you actually estimate the effect for A and D together. Only further experimentation can separate the two.
  - Main effects and low order interactions are of most interest, and are usually more significant that high order interaction terms.
    - Why? See http://en.wikipedia.org/wiki/Sparsity-of-effects_principle
  - So, by aliasing main effects with high order interactions, you can obtain fairly accurate estimates of the main effects.

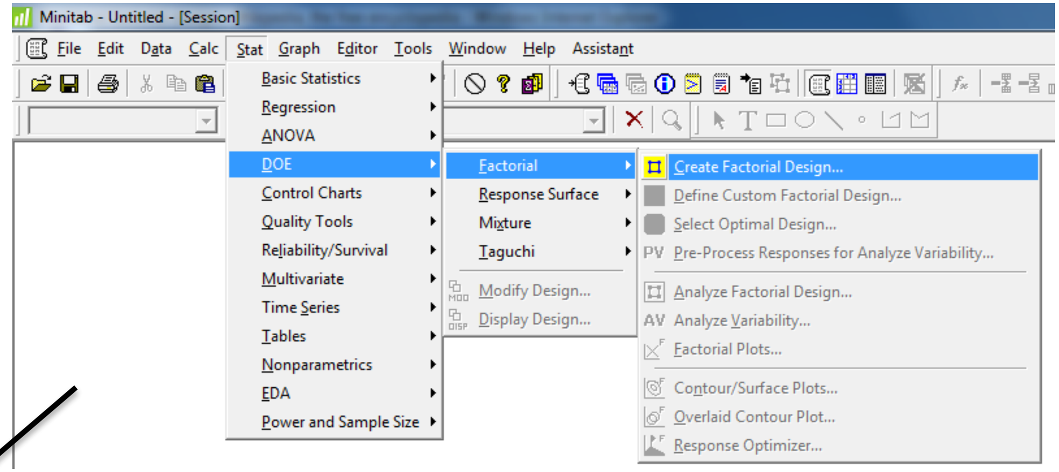
Fractional Factorial Designs

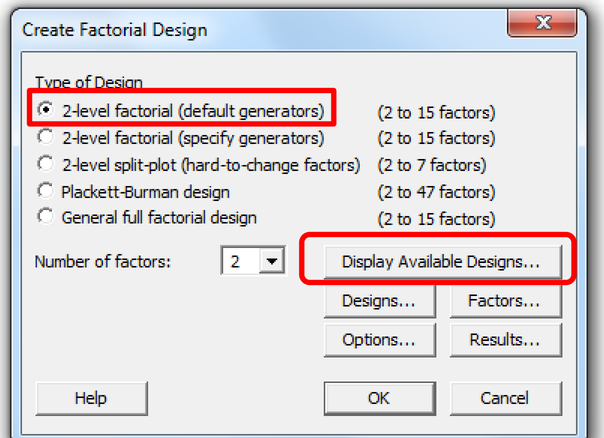
- Certain fractional factorial designs are better than others
  - Determine the best ones based on the design's Resolution
  - Resolution: the ability to separate main effects and low-order interactions from one another
  - The higher the Resolution, the better the design

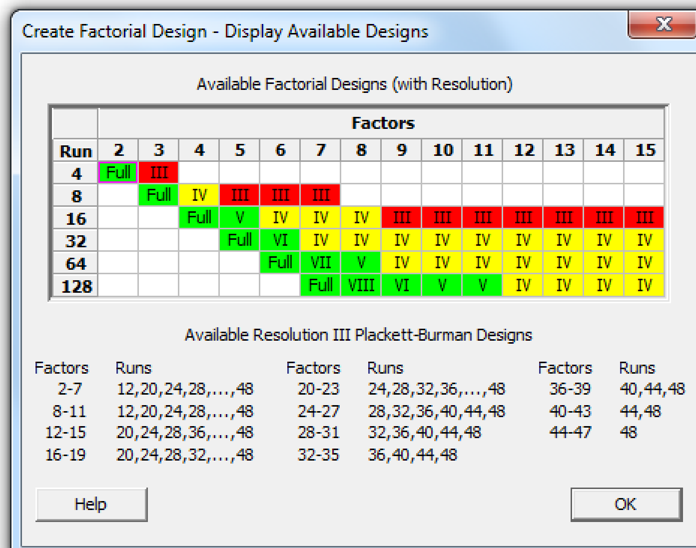
Resolution	Ability
I	Not useful: an experiment of exactly one run only tests one level of a factor and hence can't even distinguish between the high and low levels of that factor
II	Not useful: main effects are confounded with other main effects
III	Can estimate main effects, but these may be confounded with two-factor interactions
IV	Can estimate main effects, and they are unconfounded with two-factor interactions Can estimate two-factor interaction effects, but these may be confounded with other two-factor interactions
V	Can estimate main effects, and they are unconfounded with three-factor (or less) interactions Can estimate two-factor interaction effects, and they are unconfounded with two-factor interactions Can estimate three-factor interaction effects, but these may be confounded with other three-factor interactions
VI	Can estimate main effects, and they are unconfounded with four-factor (or less) interactions Can estimate two-factor interaction effects, and they are unconfounded with three-factor (or less) interactions Can estimate three-factor interaction effects, but these may be confounded with other three-factor interactions

http://en.wikipedia.org/wiki/Fractional_factorial_design#Resolution

Creating a Factorial Design







Run	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	Full	III												
8		Full	IV	III	III	III								
16			Full	V	IV	IV	IV	III	III	III	III	III	III	III
32				Full	VI	IV	IV	IV	IV	IV	IV	IV	IV	IV
64					Full	VII	V	IV	IV	IV	IV	IV	IV	IV
128						Full	VIII	VI	V	V	IV	IV	IV	IV

Available Resolution III Plackett-Burman Designs

Factors	Runs	Factors	Runs	Factors	Runs
2-7	12,20,24,28,...,48	20-23	24,28,32,36,...,48	36-39	40,44,48
8-11	12,20,24,28,...,48	24-27	28,32,36,40,44,48	40-43	44,48
12-15	20,24,28,36,...,48	28-31	32,36,40,44,48	44-47	48
16-19	20,24,28,32,...,48	32-35	36,40,44,48		

Choosing the "default generators" option means that Minitab will select what effects are aliased with one another for you.

You can see what designs are available for a specific # of runs or factors, as well as the corresponding design resolution.

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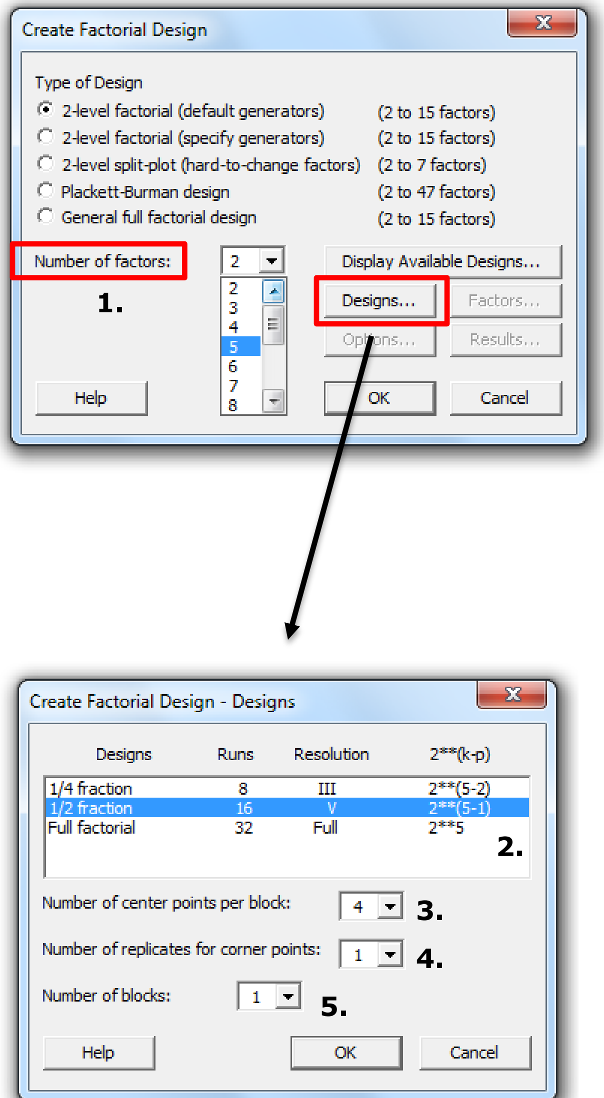
Creating a Factorial Design



1. Select the # of factors

2. Select your design (full or fractional)

3. Select the # of center points (not required, but a good idea)

4. Select how many replicates for each treatment (corner points). See Slide 12

5. Select # of blocks See Slide 13

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REPLICATION

Replicates

- NOT the same as repeated measurements
  - Repeated measurements is when you take multiple measurements on the same unit.
  - Replication is when you repeat your design a 2nd, 3rd, 4th, etc. time.
    - Ex. Say you have a 2^2 design (2 factors, 4 runs) and want 3 replicates. Your experiment will have $3*2^2 = 12$ runs.
- Replication will help give you more accurate effect estimates.
- Replicates should be run at the same time as your original design (to ensure all controlled conditions are the same). If that's not possible, consider blocking

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BLOCKING

Blocking

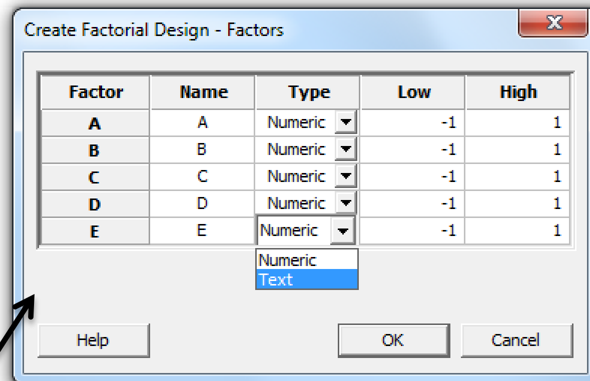
- Grouping together experimental units that are similar to one another – the groups are called blocks

- Blocking "reduces known, but irrelevant sources of variation between units and thus allows greater precision"

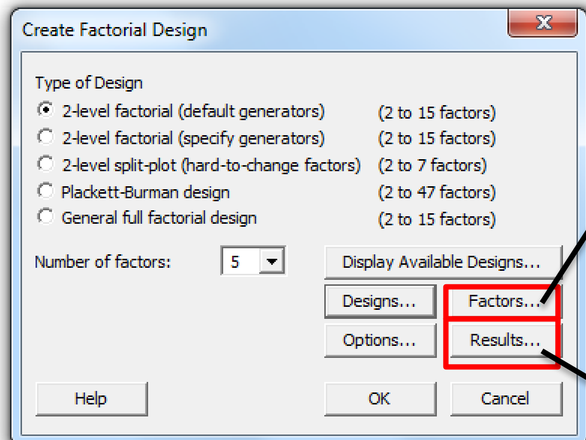
- In Factorial Designs, blocks are confounded with higher order interactions. This means you don't know if an observed relationship between a block and the response variable is due to the block itself, or due to the factor interaction. Assuming the higher order interactions are insignificant (see Slide 7), one canestimate the block effect.

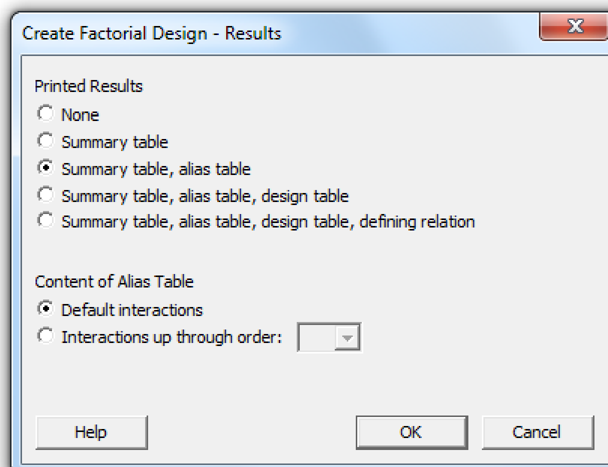
http://en.wikipedia.org/wiki/Design_of_experiments#Principles_of_experimental_design.2C_following_Ronald_A._Fishers

CREATING A FACTORIAL DESIGN (CONTINUED)



You can name factors, select what type, and give what CODED values you want for each level.





You should make sure to have the alias table printed out in the session window. This information is important for interpretation

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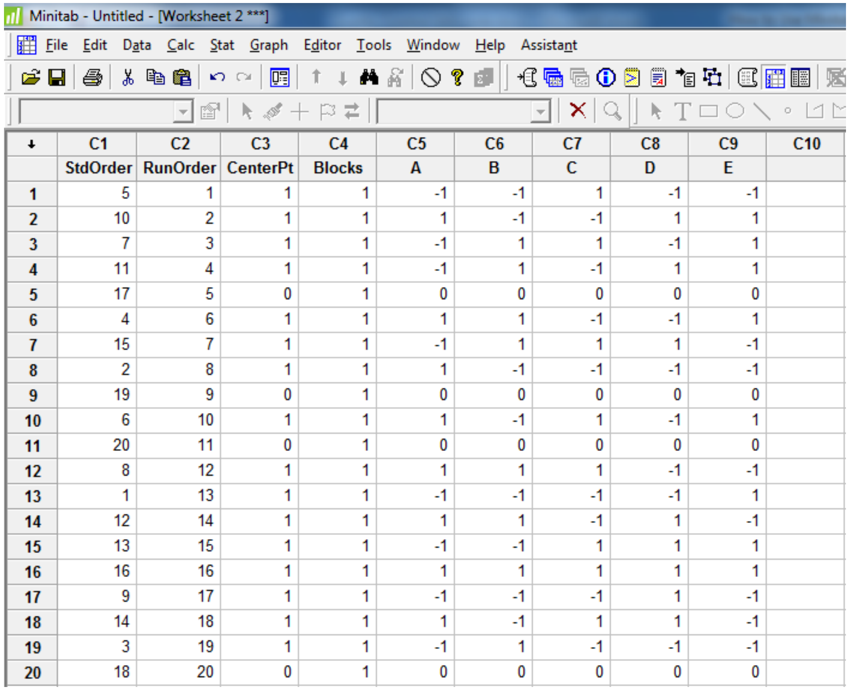
OUTPUT

Just add another column (C10) for your observations.



	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 C	C8 D	C9 E	C10
1	5	1	1	1	-1	-1	1	-1	-1	
2	10	2	1	1	1	-1	-1	1	1	
3	7	3	1	1	-1	1	1	-1	1	
4	11	4	1	1	-1	1	-1	1	1	
5	17	5	0	1	0	0	0	0	0	
6	4	6	1	1	1	1	-1	-1	1	
7	15	7	1	1	-1	1	1	1	-1	
8	2	8	1	1	1	-1	-1	-1	-1	
9	19	9	0	1	0	0	0	0	0	
10	6	10	1	1	1	-1	1	-1	1	
11	20	11	0	1	0	0	0	0	0	
12	8	12	1	1	1	1	1	-1	-1	
13	1	13	1	1	-1	-1	-1	-1	1	
14	12	14	1	1	1	1	-1	1	-1	
15	13	15	1	1	-1	-1	1	1	1	
16	16	16	1	1	1	1	1	1	1	
17	9	17	1	1	-1	-1	-1	1	-1	
18	14	18	1	1	1	-1	1	1	-1	
19	3	19	1	1	-1	1	-1	-1	-1	
20	18	20	0	1	0	0	0	0	0	

```
Fractional Factorial Design

Factors:   5   Base Design:         5, 16   Resolution:    V
Runs:     20   Replicates:              1   Fraction:    1/2
Blocks:    1   Center pts (total):      4


Design Generators: E = ABCD


Alias Structure

I + ABCDE

A + BCDE
B + ACDE
C + ABDE
D + ABCE
E + ABCD
AB + CDE
AC + BDE
AD + BCE
AE + BCD
BC + ADE
BD + ACE
BE + ACD
CD + ABE
CE + ABD
DE + ABC
```

I stands for Identity
Identity is another word for control – i.e. no treatments.

The effect estimate for factor A is actually the effect for A and BCDE

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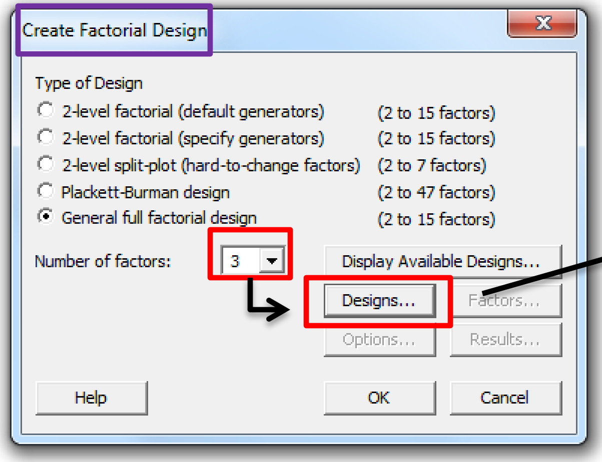
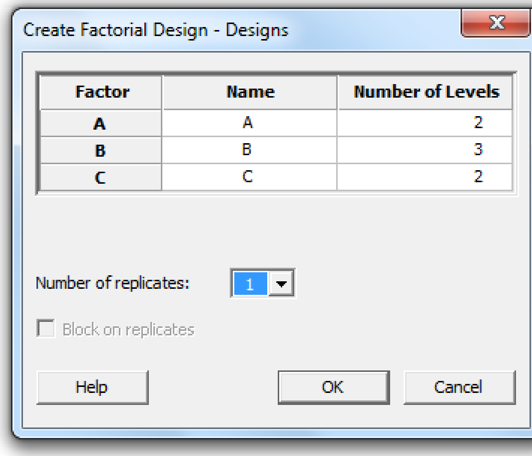
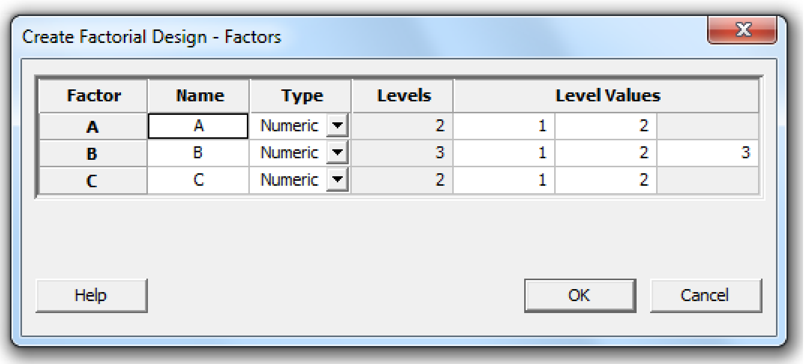
Creating General Factorial Designs





1. Specify # of factors
2. Add # of levels for each factor
3. Select # of replicates

After you enter in the # of levels, the "Factors" tab in the *Create Factorial Design* window should be clickable.



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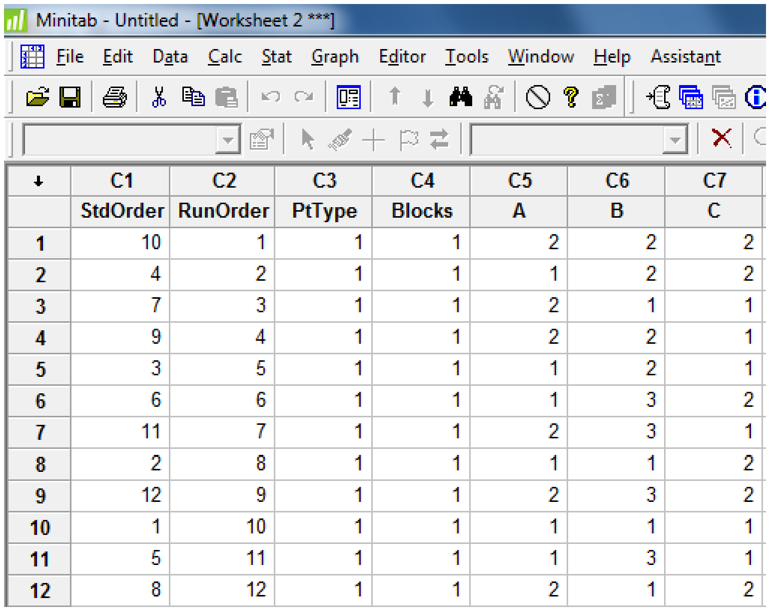
Output



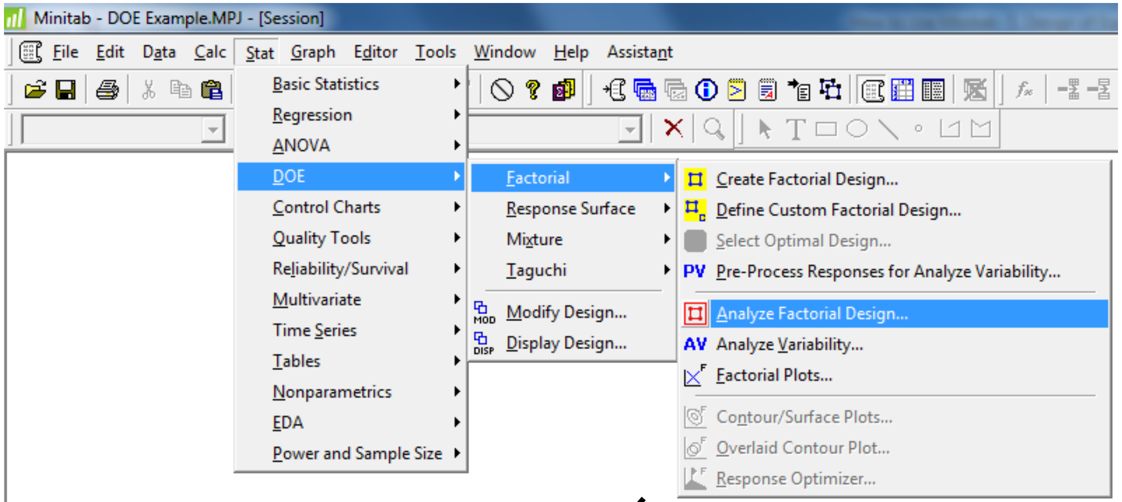
↓	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	PtType	Blocks	A	B	C
1	10	1	1	1	2	2	2
2	4	2	1	1	1	2	2
3	7	3	1	1	2	1	1
4	9	4	1	1	2	2	1
5	3	5	1	1	1	2	1
6	6	6	1	1	1	3	2
7	11	7	1	1	2	3	1
8	2	8	1	1	1	1	2
9	12	9	1	1	2	3	2
10	1	10	1	1	1	1	1
11	5	11	1	1	1	3	1
12	8	12	1	1	2	1	2

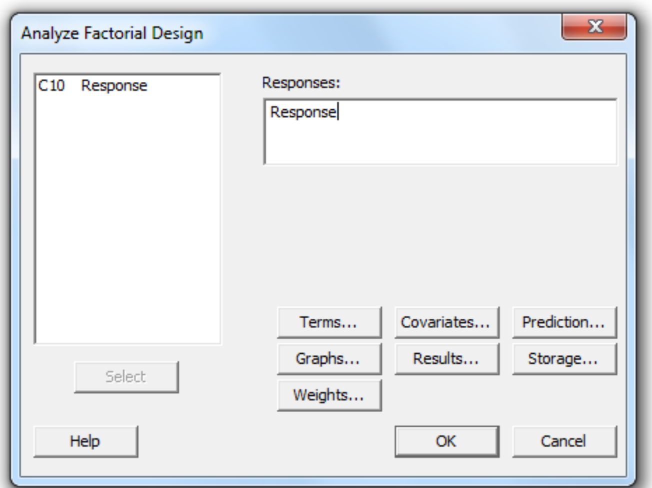
Multilevel Factorial Design

```
Factors:         3     Replicates:      1
Base runs:      12     Total runs:     12
Base blocks:     1     Total blocks:    1

Number of levels: 2, 3, 2
```

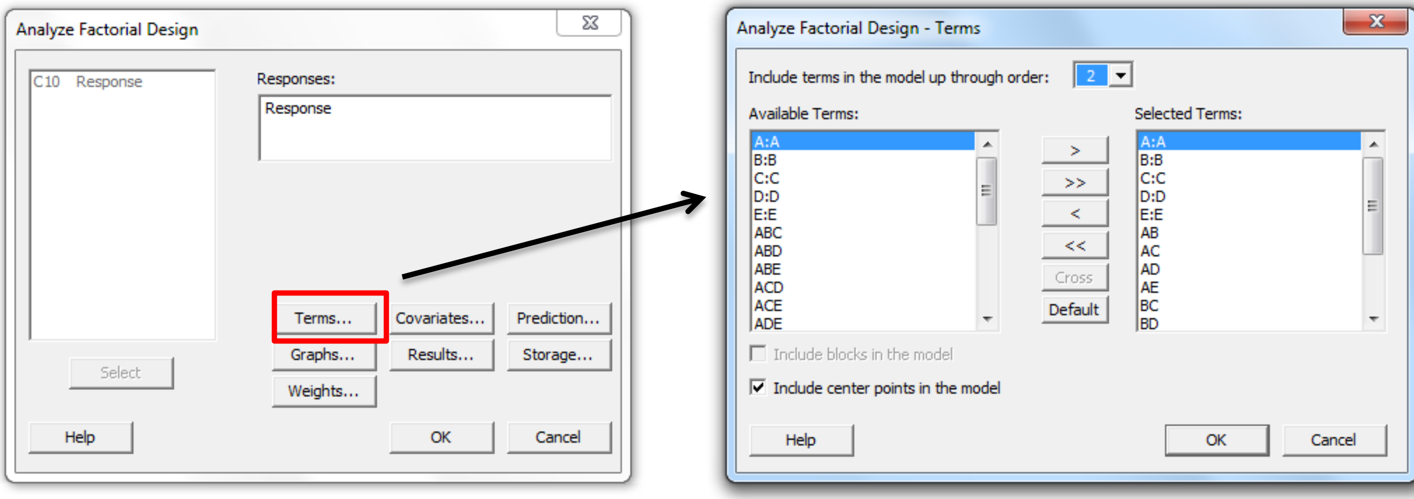
Analyzing a Factorial Design





Enter in your measurement(s) column(s) as responses

Analyzing a Factorial Design



When you analyze an experiment, you are actually fitting a model to the data. You estimate the effects of main factors and interaction terms.

Here, you can choose how high of an interaction term you want to estimate.

Selected Terms are the main factor/interaction effects that will be estimated.

Available Terms are other interaction terms that are not being estimated (but could be).

You can pick and choose specific effects to estimate using the arrow buttons in the middle.

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Analysis Output

```
Factorial Fit: Response versus A, B, C, D, E

Estimated Effects and Coefficients for Response (coded units)

Term        Effect    Coef  SE Coef      T      P
Constant            760.19    45.28  16.79  0.000
A           -21.88  -10.94    45.28  -0.24  0.825
B           125.87   62.94    45.28   1.39  0.259
C            57.12   28.56    45.28   0.63  0.573
D           -10.37   -5.19    45.28  -0.11  0.916
E            -0.37   -0.19    45.28  -0.00  0.997
A*B         -81.88  -40.94    45.28  -0.90  0.433
A*C         -97.62  -48.81    45.28  -1.08  0.360
A*D         -51.13  -25.56    45.28  -0.56  0.612
A*E          38.38   19.19    45.28   0.42  0.700
B*C          64.63   32.31    45.28   0.71  0.527
B*D         -35.87  -17.94    45.28  -0.40  0.719
B*E         156.62   78.31    45.28   1.73  0.182
C*D          43.87   21.94    45.28   0.48  0.661
C*E         -81.13  -40.56    45.28  -0.90  0.436
D*E        -168.63  -84.31    45.28  -1.86  0.160
Ct Pt                56.06   101.26   0.55  0.618

S = 181.130     PRESS = *
R-Sq = 81.65%   R-Sq(pred) = *%   R-Sq(adj) = 0.00%


Analysis of Variance for Response (coded units)

Source              DF  Seq SS  Adj SS  Adj MS     F      P
Main Effects         5   78776   78776   15755  0.48  0.779
  A                  1    1914    1914    1914  0.06  0.825
  B                  1   63378   63378   63378  1.93  0.259
  C                  1   13053   13053   13053  0.40  0.573
  D                  1     431     431     431  0.01  0.916
  E                  1       1       1       1  0.00  0.997
2-Way Interactions  10  349024  349024   34902  1.06  0.543
  A*B                1   26814   26814   26814  0.82  0.433
  A*C                1   38123   38123   38123  1.16  0.360
  A*D                1   10455   10455   10455  0.32  0.612
  A*E                1    5891    5891    5891  0.18  0.700
  B*C                1   16706   16706   16706  0.51  0.527
  B*D                1    5148    5148    5148  0.16  0.719
  B*E                1   98126   98126   98126  2.99  0.182
  C*D                1    7700    7700    7700  0.23  0.661
  C*E                1   26325   26325   26325  0.80  0.436
  D*E                1  113738  113738  113738  3.47  0.160
  Curvature          1   10058   10058   10058  0.31  0.618
Residual Error       3   98425   98425   32808
  Pure Error         3   98425   98425   32808
Total               19  536283
```

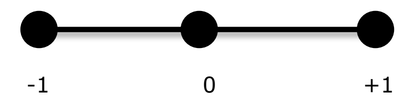
Effect Estimates

Which are the vital few significant effects?
Determine this using p-values.

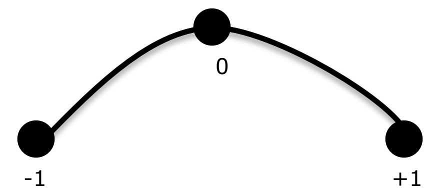
1. Select your confidence level. Usually, L = 0.05.
2. P-values < 0.05 indicate the effect is significant.

There is a little leeway: If you choose L = 0.1, effects with p-values < 0.1 are considered significant. What confidence level you choose depends on how many factors you want to keep.

In this example, no effect is significant at the 0.1 level. I would re-fit the model, removing interaction terms that have large p-values (such as AD, AE, etc.) Then, re-examine p-values.



vs.



Is your response surface simply a multi-dimensional plane? Or does it have curvature?

The p-values < 0.05 indicate significant curvature.

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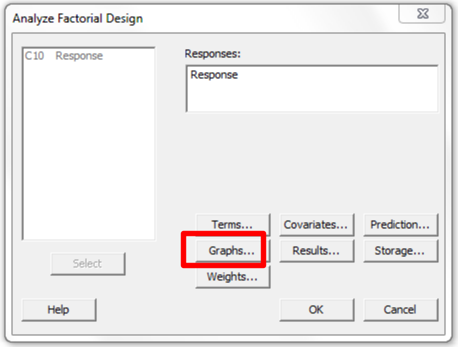
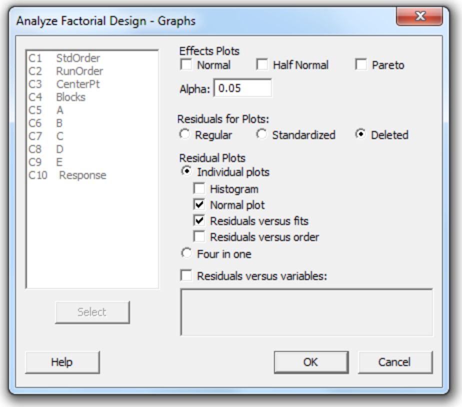
Analysis Output (continued)

```
Unusual Observations for Response

                                                        St
Obs  StdOrder  Response      Fit  SE Fit  Residual   Resid
  1         1    633.00   633.00  181.13      0.00       * X
  2         2    749.00   749.00  181.13      0.00       * X
  3         3    601.00   601.00  181.13      0.00       * X
  4         4   1052.00  1052.00  181.13      0.00       * X
  5         5    706.00   706.00  181.13      0.00       * X
  6         6    650.00   650.00  181.13      0.00       * X
  7         7   1063.00  1063.00  181.13      0.00       * X
  8         8    669.00   669.00  181.13      0.00       * X
  9         9    780.00   780.00  181.13      0.00       * X
 10        10    642.00   642.00  181.13      0.00       * X
 11        11    761.00   761.00  181.13      0.00       * X
 12        12    635.00   635.00  181.13      0.00       * X
 13        13    550.00   550.00  181.13      0.00       * X
 14        14    868.00   868.00  181.13      0.00       * X
 15        15   1075.00  1075.00  181.13      0.00       * X
 16        16    729.00   729.00  181.13      0.00       * X

X denotes an observation whose X value gives it large leverage.


Alias Structure
I + A*B*C*D*E
A + B*C*D*E
B + A*C*D*E
C + A*B*D*E
D + A*B*C*E
E + A*B*C*D
A*B + C*D*E
A*C + B*D*E
A*D + B*C*E
A*E + B*C*D
B*C + A*D*E
B*D + A*C*E
B*E + A*C*D
C*D + A*B*E
C*E + A*B*D
D*E + A*B*C
```

Can also look at R-sq. values and residuals to determine how well the model fits. See Regression Analysis for an explanation on how to interpret residuals

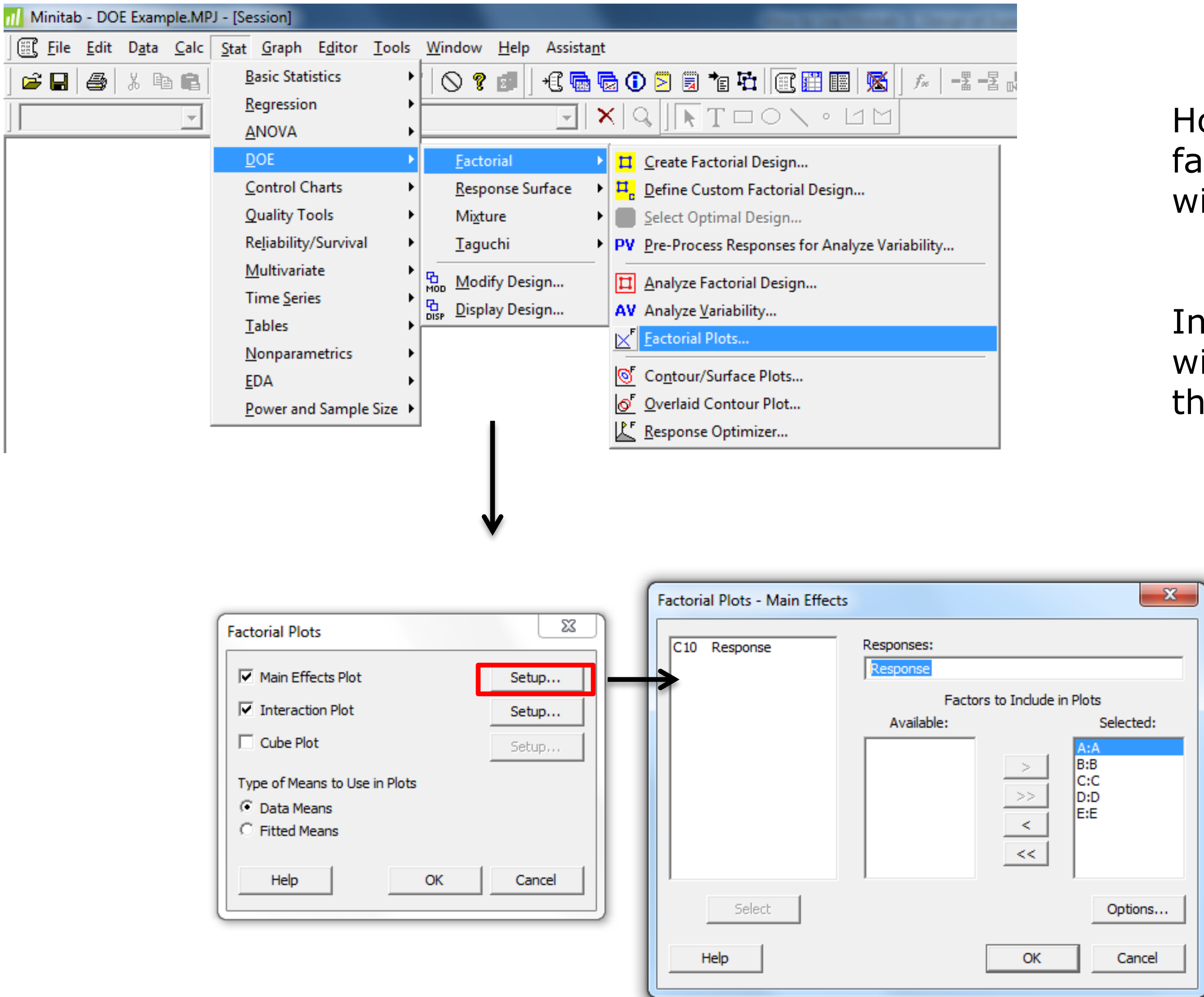




Outliers in the X (independent) variables are called high leverage points.

Remember, in 2^k designs, the independent variables are the factors, and they take on either a high or low level. It makes sense that those runs have large leverage, whereas the center points do not.

Interaction Plots



How do certain factors interact with one another?

Interaction plots will help answer this.

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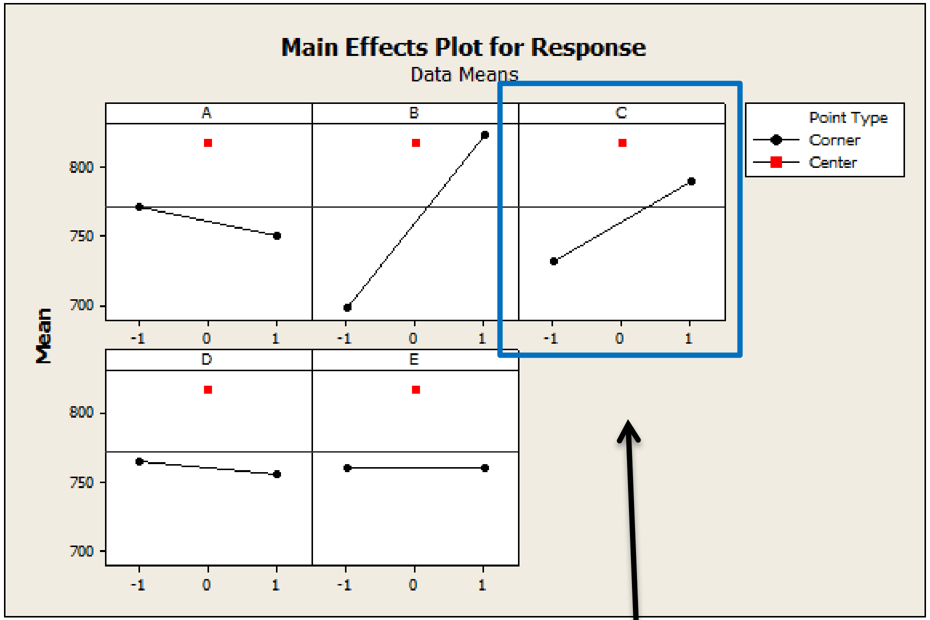
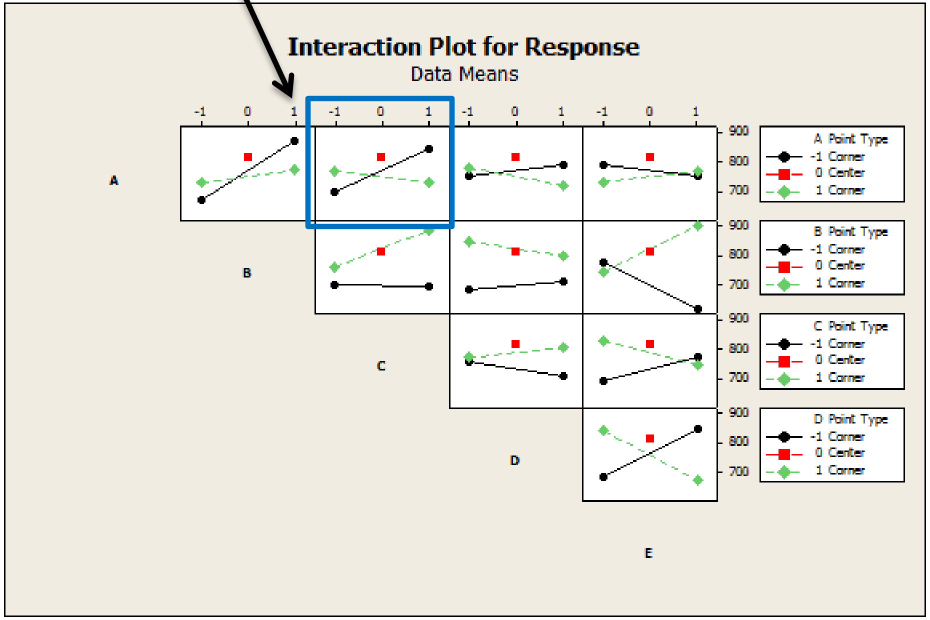
Interaction Plots



Interpretation:

Black Line: When factor A is at it's low level (-1), the mean response increases when factor C changes from it's low level (-1) to it's high level (+1).

Green Line: When factor A is at it's high level (+1), the mean response decreases when factor C changes from it's low level (-1) to it's high level (+1).



Interpretation:

Black Line: The change in mean response when factor C changes from it's low level (-1) to it's high level (+1), assuming all other factors are kept constant

Black and green lines with considerably different slopes indicate an interaction between the two factors.

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Split – Plot Designs

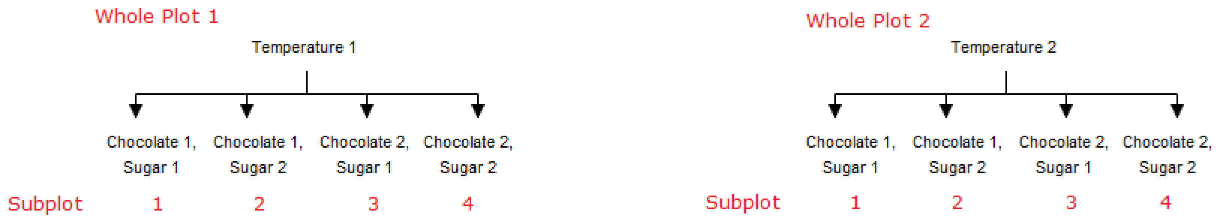
- If one or more of your factors are hard to change, consider using a split – plot design

- "The levels of the hard-to-change factors are held constant for several runs, which are collectively treated as a whole plot, while easy-to-change factors are varied over these runs, each of which is a subplot." – Minitab Help

Split – Plot designs contain an embedded factorial (full or fractional) design.

Example: 3 factors each with 2 levels: Temperature, Chocolate, Sugar

Temperature is the hard to change factor. Run whole plot 1 on day 1, whole plot 2 on day 2



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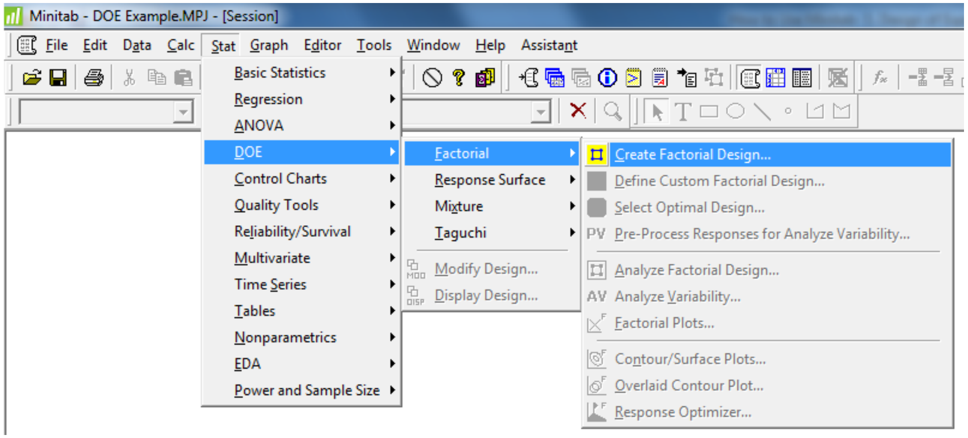
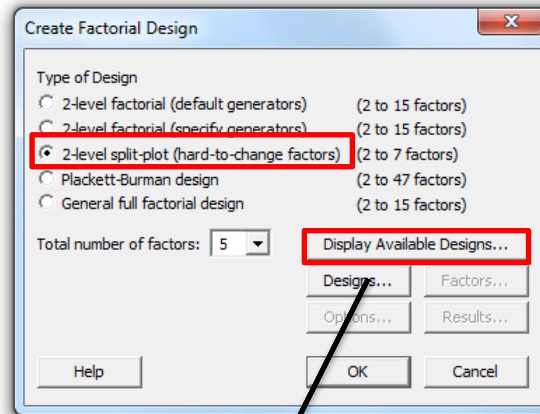
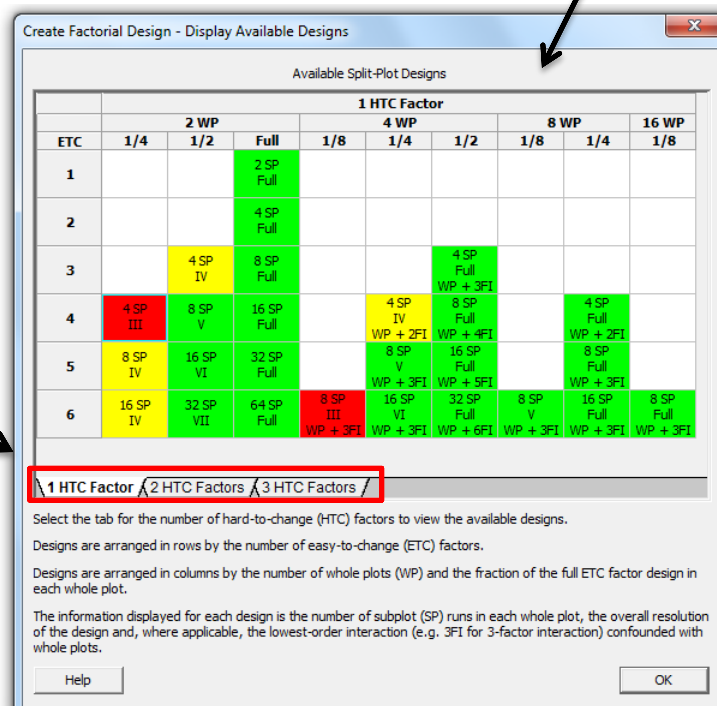
CREATING SPLIT – PLOT DESIGNS







	1 HTC Factor								
	2 WP			4 WP			8 WP		16 WP
ETC	1/4	1/2	Full	1/8	1/4	1/2	1/8	1/4	1/8
1			2 SP Full						
2			4 SP Full						
3		4 SP IV	8 SP Full			4 SP Full WP + 3FI			
4	4 SP III	8 SP V	16 SP Full		4 SP IV WP + 2FI	8 SP Full WP + 4FI		4 SP Full WP + 2FI	
5	8 SP IV	16 SP VI	32 SP Full		8 SP V WP + 3FI	16 SP Full WP + 5FI		8 SP Full WP + 3FI	
6	16 SP IV	32 SP VII	64 SP Full	8 SP III WP + 3FI	16 SP VI WP + 3FI	32 SP Full WP + 6FI	8 SP V WP + 3FI	16 SP Full WP + 3FI	8 SP Full WP + 3FI

You can view all available split – plot designs, as well as the resolution

1 HTC = 1 Hard to Change Factor
2 HTC = 2 Hard to Change Factors
3 HTC = 3 Hard to Change Factors

ETC = Easy to Change Factor
WP = Whole Plot
SP = Subplot

3FI = 3 factor interaction, confounded with whole plots for some designs

1/8, ¼, ½, Full – Refers to the type of factorial design used within the split – plot design

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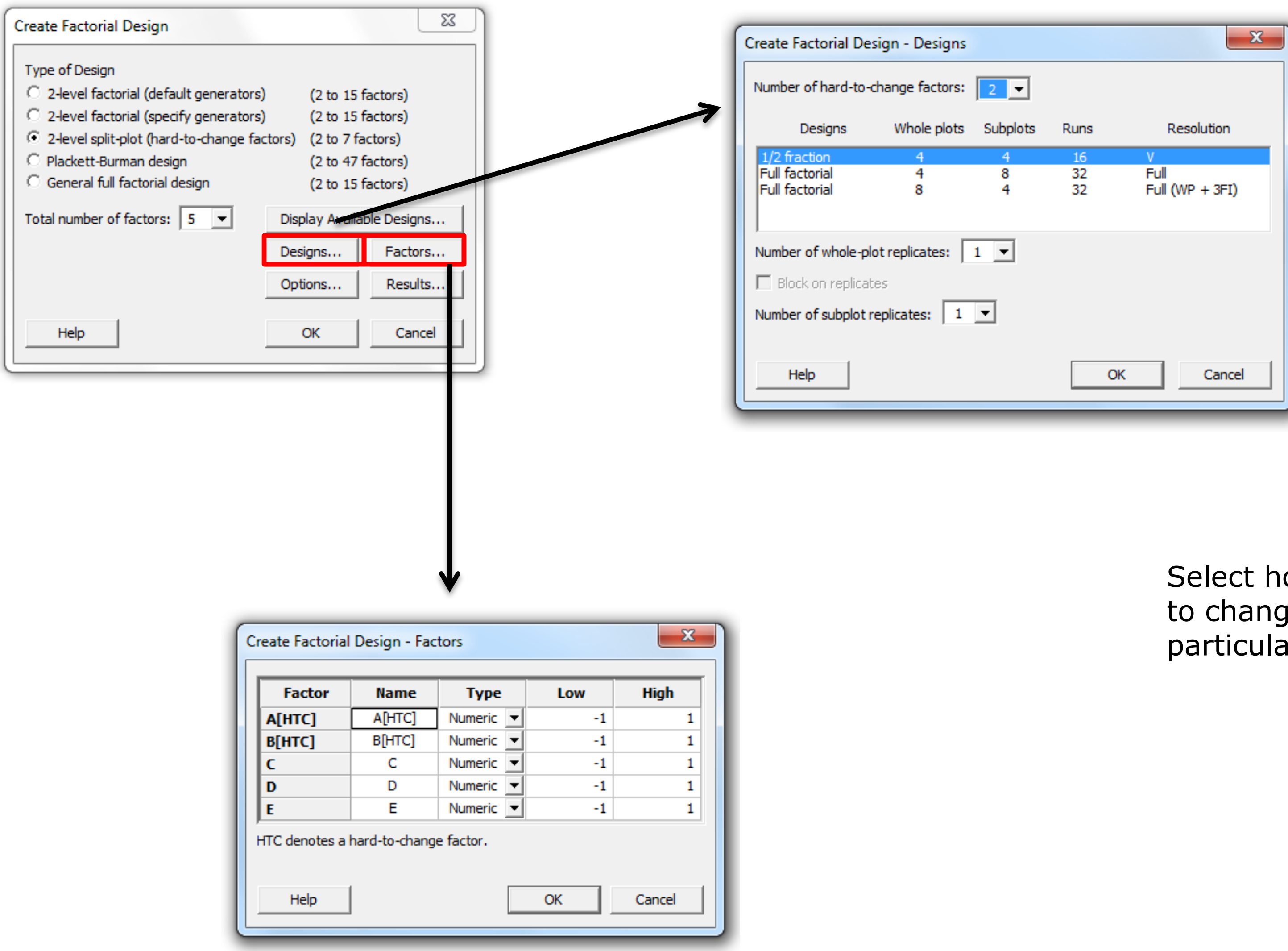
Creating Split – Plot Designs



Select how many hard to change factors, the particular design, etc.

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Output

```
Fractional Factorial Split-Plot Design

Factors:           5   Whole plots:           4   Resolution:    V
Hard-to-change:    2   Runs per whole plot:   4   Fraction:    1/2
Runs:             16   Whole-plot replicates: 1
Blocks:            1   Subplot replicates:    1


Design Generators: E = ABCD


Hard-to-change factors: A, B


Whole Plot Generators: A, B


Alias Structure

I + ABCDE

A + BCDE
B + ACDE
C + ABDE
D + ABCE
E + ABCD
AB + CDE
AC + BDE
AD + BCE
AE + BCD
BC + ADE
BD + ACE
BE + ACD
CD + ABE
CE + ABD
DE + ABC
```

Minitab - DOE EXAMPLE.MPJ - [Worksheet 8 ***]

File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	StdOrder	RunOrder	PtType	Blocks	WP	A[HTC]	B[HTC]	C	D	E
1	7	1	1	1	2	1	-1	-1	1	1
2	6	2	1	1	2	1	-1	1	-1	1
3	5	3	1	1	2	1	-1	-1	-1	-1
4	8	4	1	1	2	1	-1	1	1	-1
5	16	5	1	1	4	1	1	1	1	1
6	14	6	1	1	4	1	1	1	-1	-1
7	15	7	1	1	4	1	1	-1	1	-1
8	13	8	1	1	4	1	1	-1	-1	1
9	12	9	1	1	3	-1	1	1	1	-1
10	11	10	1	1	3	-1	1	-1	1	1
11	10	11	1	1	3	-1	1	1	-1	1
12	9	12	1	1	3	-1	1	-1	-1	-1
13	3	13	1	1	1	-1	-1	-1	1	-1
14	4	14	1	1	1	-1	-1	1	1	1
15	2	15	1	1	1	-1	-1	1	-1	-1
16	1	16	1	1	1	-1	-1	-1	-1	1

Analysis: Follow the same steps as for the factorial design. See Here

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Response Surface Designs

- As mentioned before, factorial designs are useful when determining the "vital few" significant factors

- Once you have determined those vital factors, you may want to map the response surface. Why?

  1. To find the factor settings that optimize the response (max./min. problem, or hitting a specific target)

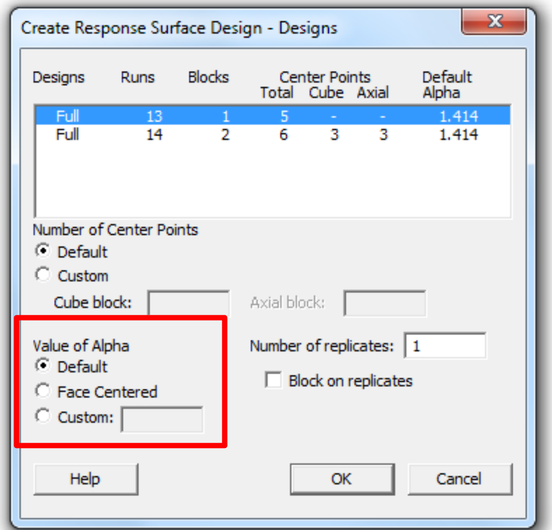
  2. In order to improve a process, you'll need to understand how certain factors influence the response

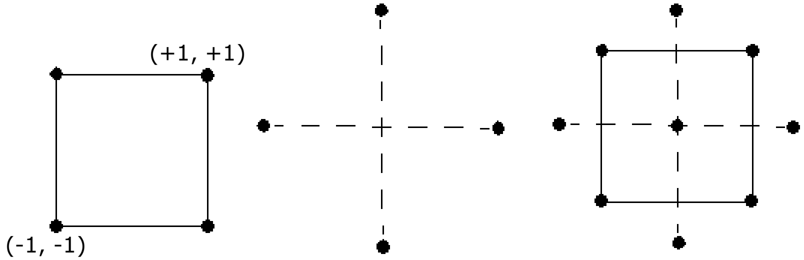
  3. Find out what tradeoffs can be made in factor settings, while staying near the optimal response

- Essentially, you are finding a model that describes the relationship between the vital factors and the response.

Central Composite Designs (CCD)

- A CCD design is one type of response surface design. It is a factorial design (2^k or 2^{k-p}) with 2k additional points.
  - The additional points are known as star points or axial points
  - Axial points have coded values (±a, 0, 0, … 0), (0, ±a, 0, … 0), … (0, 0, 0, … ±a)
- The design is rotatable if
  - All points are the same distance from the center point, so the quality of predication is the same in any direction. See here for more.
- The design is face centered if a =1
  - Only three factor levels (-1, 0, +1) are needed as opposed to five levels (-1, -a , 0, +a, +1)





2^k Design Points

Axial Design Points

CCD = 2^k Design Points + Axial Design Points + Center Points

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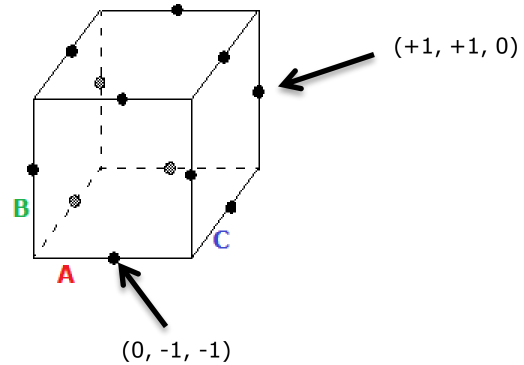
Box Behnken Designs

- Another type of response surface design.
- Does not contain an embedded factorial design like the CCD.
  - Instead, design points are the midpoints
  - Requires 3 levels (-1, 0, +1) for each factor
- Less expensive to run than the CCD (less points)
- Does not contain axial points, so all design points are sure to be within safe operating limits



Box Behnken design for 3 factors

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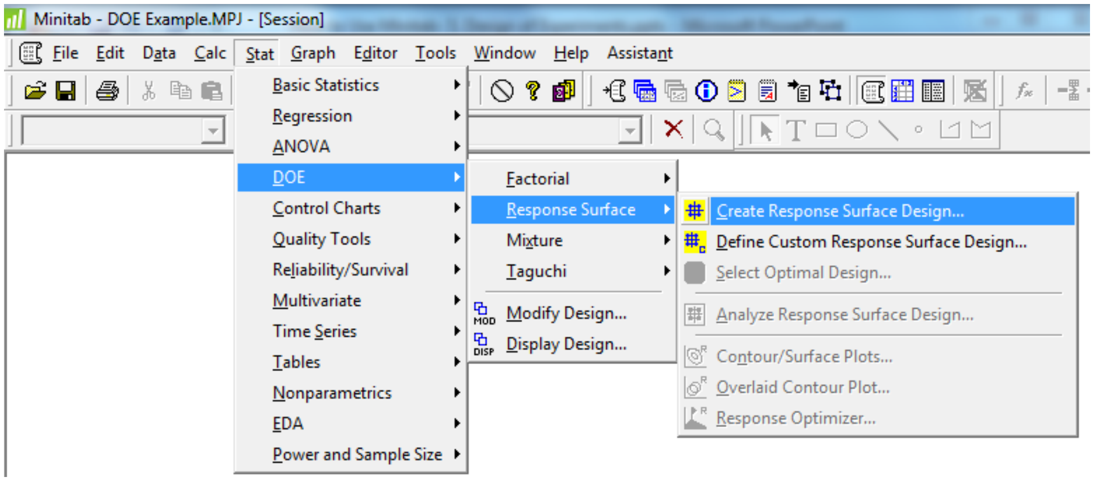
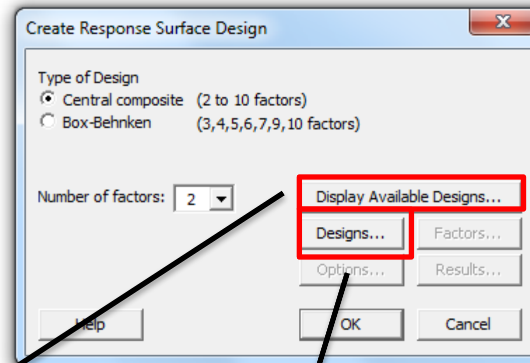
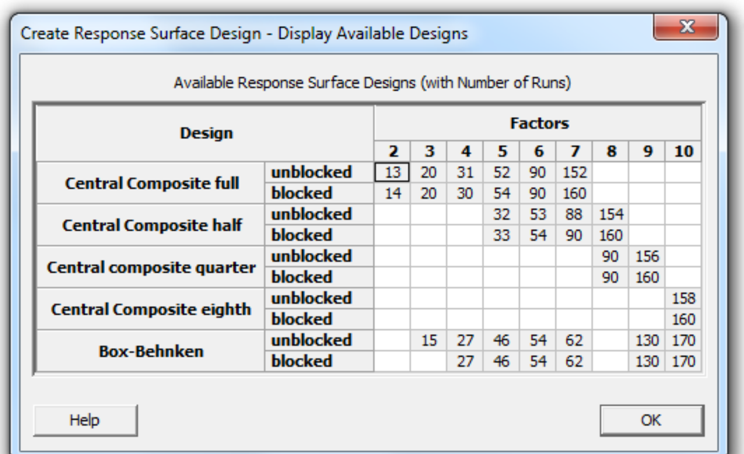
Creating a Response Surface Design





Click on “Designs” to select the specific design you wish to use for a specified number of factors. See Here


Create Response Surface Design - Display Available Designs

Available Response Surface Designs (with Number of Runs)

Design		Factors 2	3	4	5	6	7	8	9	10
Central Composite full	unblocked	13	20	31	52	90	152			
	blocked	14	20	30	54	90	160			
Central Composite half	unblocked				32	53	88	154		
	blocked				33	54	90	160		
Central composite quarter	unblocked							90	156	
	blocked							90	160	
Central Composite eighth	unblocked									158
	blocked									160
Box-Behnken	unblocked		15	27	46	54	62		130	170
	blocked			27	46	54	62		130	170

Help OK

You can view all available designs

“Full”, “Half”, “Quarter”, and “Eighth” – refers to the factorial design piece of the CCD.

Full = 2^k

Half = 2^{k-1}

Quarter = 2^{k-2}

Eighth = 2^{k-3}

Output

Minitab - DOE Example.MPJ - [Worksheet 10 ***]

↓	C1 StdOrder	C2 RunOrder	C3 PtType	C4 Blocks	C5 A	C6 B	C7
1	12	1	0	1	0.00000	0.00000	
2	11	2	0	1	0.00000	0.00000	
3	7	3	-1	1	0.00000	-1.41421	
4	6	4	-1	1	1.41421	0.00000	
5	4	5	1	1	1.00000	1.00000	
6	2	6	1	1	1.00000	-1.00000	
7	5	7	-1	1	-1.41421	0.00000	
8	1	8	1	1	-1.00000	-1.00000	
9	9	9	0	1	0.00000	0.00000	
10	8	10	-1	1	0.00000	1.41421	
11	13	11	0	1	0.00000	0.00000	
12	3	12	1	1	-1.00000	1.00000	
13	10	13	0	1	0.00000	0.00000	

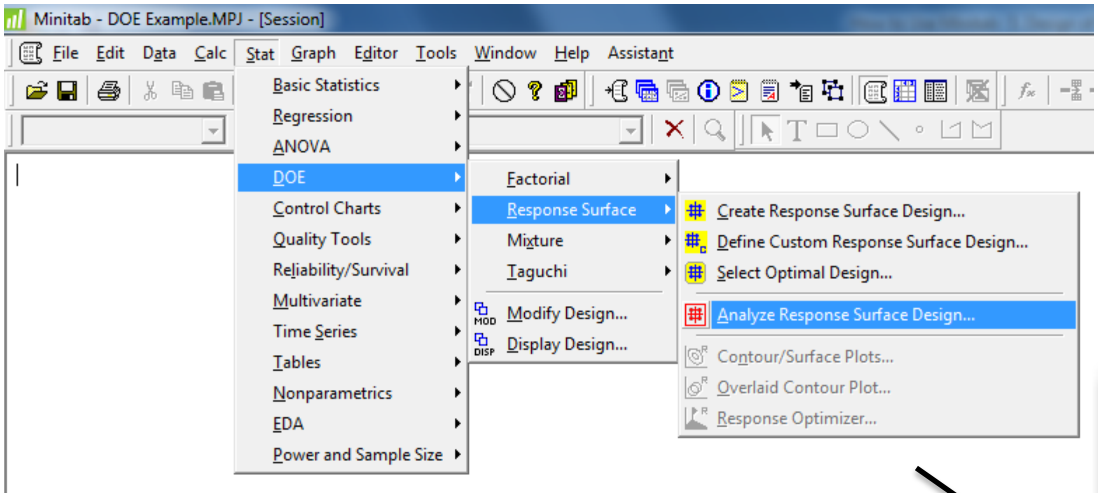
```
Central Composite Design

Factors:         2     Replicates:      1
Base runs:      13     Total runs:     13
Base blocks:     1     Total blocks:    1

Two-level factorial: Full factorial

Cube points:                4
Center points in cube:      5
Axial points:               4
Center points in axial:     0

Alpha: 1.41421
```

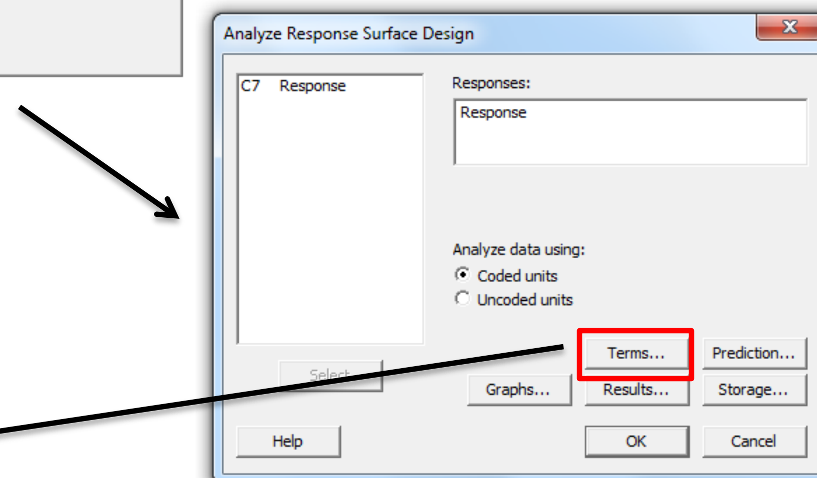
Enter measurements into C7 column

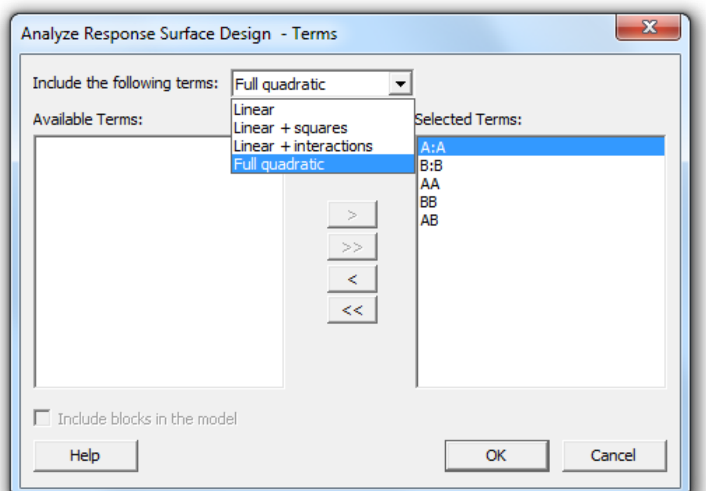
Minitab - DOE Example.MPJ - [Worksheet 10 ***]

↓	C1 StdOrder	C2 RunOrder	C3 PtType	C4 Blocks	C5 A	C6 B	C7 Response
1	12	1	0	1	0.00000	0.00000	0.244
2	11	2	0	1	0.00000	0.00000	0.256
3	7	3	-1	1	0.00000	-1.41421	0.261
4	6	4	-1	1	1.41421	0.00000	0.274
5	4	5	1	1	1.00000	1.00000	0.290
6	2	6	1	1	1.00000	-1.00000	0.270
7	5	7	-1	1	-1.41421	0.00000	0.231
8	1	8	1	1	-1.00000	-1.00000	0.251
9	9	9	0	1	0.00000	0.00000	0.254
10	8	10	-1	1	0.00000	1.41421	0.312
11	13	11	0	1	0.00000	0.00000	0.252
12	3	12	1	1	-1.00000	1.00000	0.263
13	10	13	0	1	0.00000	0.00000	0.251

Analyzing Response Surface Designs







If your preliminary screening experiments indicated curvature, then you should use a quadratic equation. If there was no significant curvature, then try fitting a linear model.

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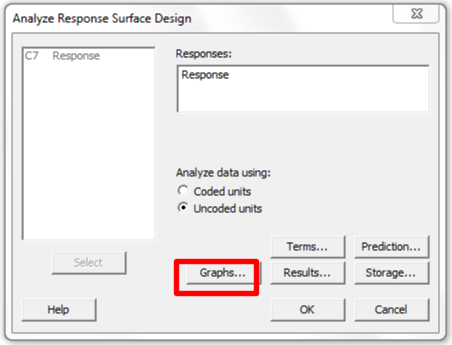
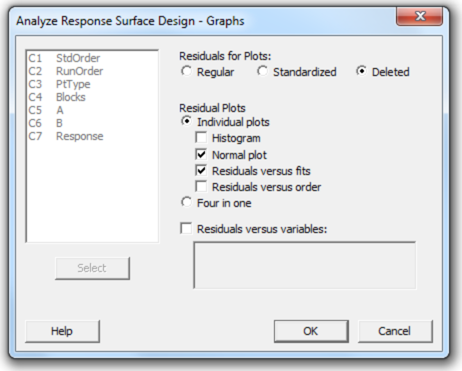
OUTPUT

```
Response Surface Regression: Response versus A, B

The analysis was done using coded units.

Estimated Regression Coefficients for Response

Term          Coef   SE Coef       T      P
Constant  0.251400  0.002995  83.950  0.000
A         0.013351  0.002367   5.640  0.001
B         0.013016  0.002367   5.498  0.001
A*A       0.000300  0.002539   0.118  0.909
B*B       0.017300  0.002539   6.814  0.000
A*B       0.002000  0.003348   0.597  0.569


S = 0.00669619   PRESS = 0.00177034
R-Sq = 93.99%   R-Sq(pred) = 66.09%   R-Sq(adj) = 89.69%


Analysis of Variance for Response

Source          DF    Seq SS    Adj SS    Adj MS      F      P
Regression       5  0.004906  0.004906  0.000981  21.88  0.000
  Linear         2  0.002781  0.002781  0.001391  31.01  0.000
    A            1  0.001426  0.001426  0.001426  31.80  0.001
    B            1  0.001355  0.001355  0.001355  30.22  0.001
  Square         2  0.002109  0.002109  0.001055  23.52  0.001
    A*A          1  0.000027  0.000001  0.000001   0.01  0.909
    B*B          1  0.002082  0.002082  0.002082  46.43  0.000
  Interaction    1  0.000016  0.000016  0.000016   0.36  0.569
    A*B          1  0.000016  0.000016  0.000016   0.36  0.569
Residual Error   7  0.000314  0.000314  0.000045
  Lack-of-Fit    3  0.000231  0.000231  0.000077   3.70  0.119
  Pure Error     4  0.000083  0.000083  0.000021
Total           12  0.005220


Estimated Regression Coefficients for Response using data in uncoded units

Term             Coef
Constant     0.251400
A           0.0133514
B           0.0130156
A*A       0.000300000
B*B         0.0173000
A*B        0.00200000
```

Once again, you can use p-values to determine the significant effects.

AA and AB are not significant.
So, you could reduce your model to only include the A, B, and BB terms.





Can also look at R-sq. values and residuals to determine how well the model fits.
See Regression Analysis for an explanation on how to interpret residuals

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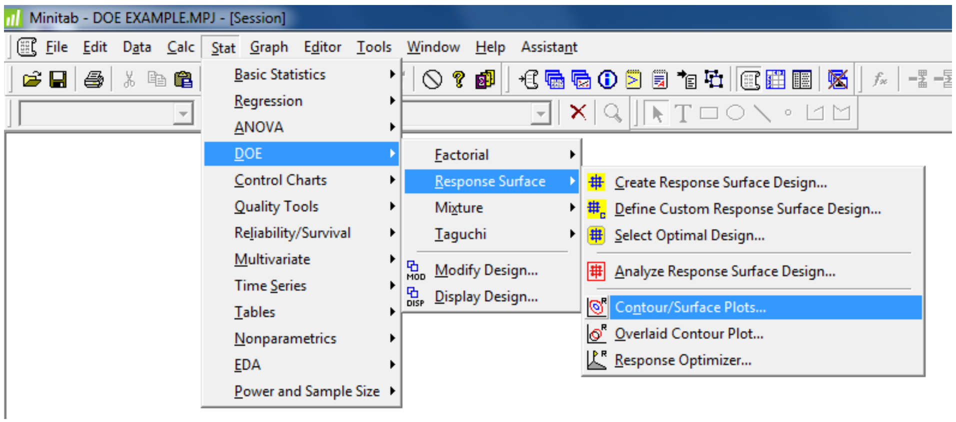
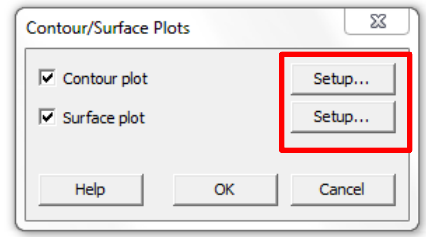
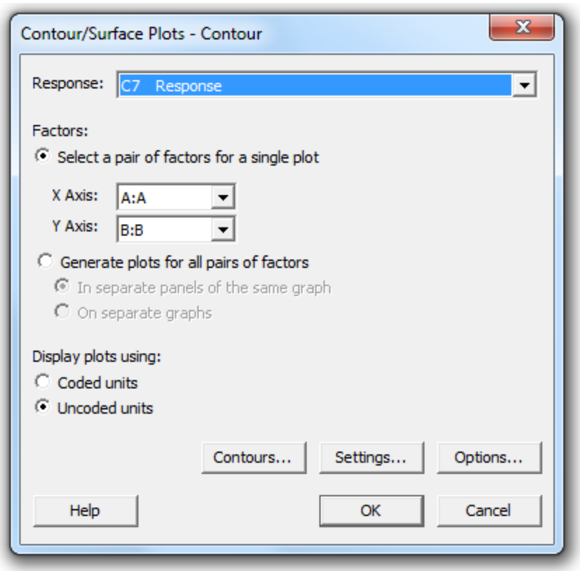
Contour/Surface Plots

Can draw these AFTER you fit a model to the data.

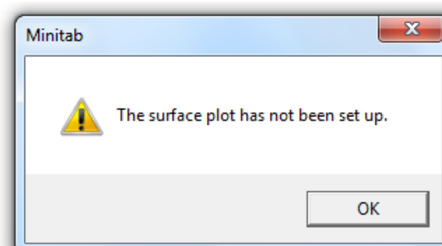


You will need to click setup in order for Minitab to draw the plot(s)

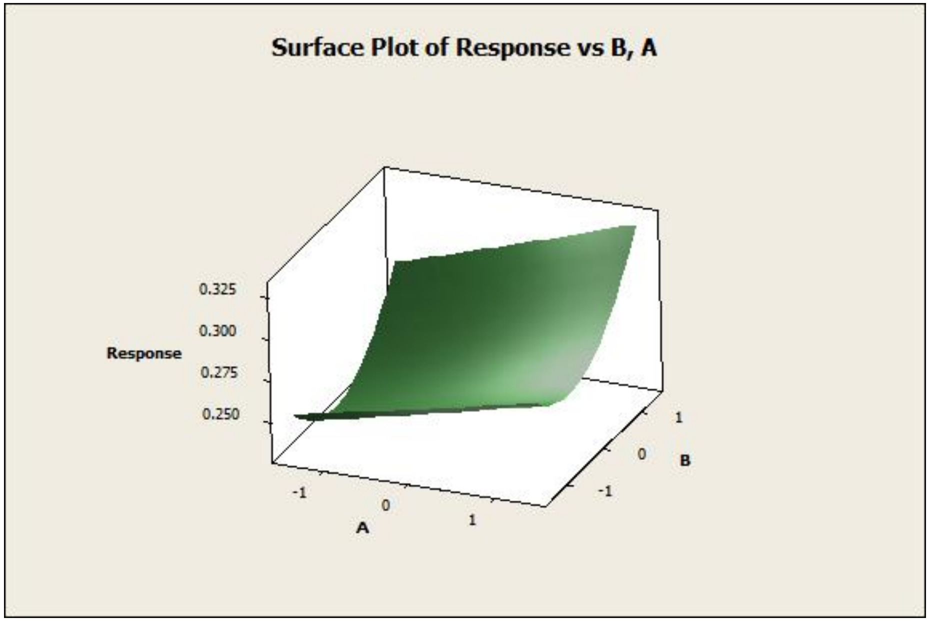


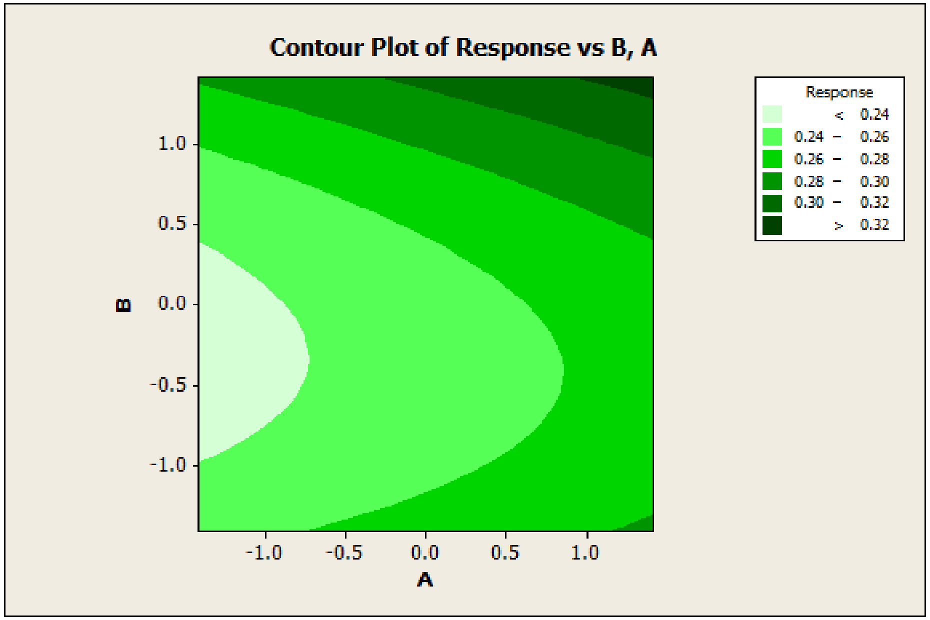


If you do not click on setup, this error will pop up.



OUTPUT


Surface Plot of Response vs B, A
Response
0.325
0.300
0.275
0.250
A
-1
0
1
B
-1
0
1


Contour Plot of Response vs B, A
B
1.0
0.5
0.0
-0.5
-1.0
A
-1.0
-0.5
0.0
0.5
1.0
Response
< 0.24
0.24 – 0.26
0.26 – 0.28
0.28 – 0.30
0.30 – 0.32
> 0.32

OPTIMIZATION

- Response Surface Analysis involves 2 steps
  1. Initial search for the region that contains the optimum (max, min, or target)
  2. Detailed search of the region from #1, to find the optimum

- Basically, you find the region where you believe the optimum to be. Then, you zoom in on that area and model it in more detail

- On page 34, a fitted model was outputted as part of the analysis. This model is an equation that describes the surface in that specific region. From the model, you can find the direction where the surface increases (or decreases) most quickly - the gradient

- A secondary experiment could then be run by using factor settings along the gradient
  1. Find the gradient vector.
  2. Divide the gradient vector by it's length (Euclidean norm) to obtain a unit vector
  3. New Experiment Points = Initial Factor Settings Vector + m * Step Size * Unit Vector   for m = 1, 2, …

  Note: This is all in terms of CODED units

Caution: In step 1, it's possible you will be looking at a region that contains a LOCAL optimum as opposed to the overall GLOBAL optimum. In step 2, you may discover that the region does NOT contain the optimum. Repeating steps 1 and 2 in other regions may be necessary.

References

- Khan, R. M. (2013). *Problem solving and data analysis using minitab: A clear and easy guide to six sigma methodology* (1st ed.). West Sussex, United Kingdom: Wiley.

- http://en.wikipedia.org/wiki/Fractional_factorial_design#Resolution

- http://en.wikipedia.org/wiki/Design_of_experiments#Principles_of_experimental_design.2C_following_Ronald_A._Fisher

- http://www.itl.nist.gov/div898/handbook/pri/pri.htm

- Minitab's Help Section