# The Economic Benefits of Battery Energy Storage System in Electric Distribution System

by

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# Abstract

The goal of this study was to determine the economic feasibility of battery energy storage system (BESS). Three major economic benefits derived from BESS using were studied:

1. **Energy Purchase Shifting,**
2. **Distribution Feeder Deferral,**
3. **Outage Avoidance.**

The economic analysis was based on theoretical modeling of the BESS and distribution system. Three simulation models were developed to quantify the effects of different parameters, such as: BESS round-trip efficiency, life span, rated power, rated discharge time, marginal cost of electric energy, 24 h feeder load profile, annual load variation, feeder load growth rate and feeder length.

An optimal battery charging/discharging method was presented to determine the differential cost of energy (DCE). The annual maximum DCE was calculated using stochastic probability analysis on seasonal load variation. The net present value was evaluated as the present value difference between two investments: first, the distribution feeder upgrade without BESS deferral, and second, with BESS deferral. Furthermore, the BESS's contributions under different outage strategies were compared.

It was determined that feeder length is the most significant parameter. The economics of the studied system becomes favorable when the feeder length exceeds a critical value.

# Acknowledgements

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# List of Symbols

$A$: Annual cash flow

$A_{PB}$: The annual payment for the BESS

$A_{PF}$: Annual loan payment for feeder construction

$C$: Total loan money (Capital cost)

$C_B$: BESS capital cost

$C_F$: Feeder capital cost

$C_O$: BESS operating cost

CL: BESS life span, measured in cycles

$d$: Discount rate

DCE: Differential Cost of Energy

$F$: Future value

$i$: Interest rate

$i_R$: Bank interest rate

$K_B$: BESS price

$K_F$: Price of feeder construction

$K_W$: Marginal cost of electric energy

$K_{WC}$: The extra cost of charging energy

$K_{WD}$: The reduced cost of the energy delivered to feeder's loads by the BESS discharging energy that shifts the on-peak power

$l$: Feeder length

$N_{bn}$: The total number of installations for the nth battery considering the replacement time of each BESS

$P$: Present value

$P_B$: BESS charging/discharging power

| | |
|---|---|
| $P_{Bmax}$: | BESS rated charging/discharging power |
| $P_{max}$: | The maximum load demand value |
| $P_{max}^{max}(n)$: | The upper boundary of $P_{max}(n)$ |
| $P_{min}$: | The minimum load demand value |
| $P_{max}^{min}(n)$: | The upper boundary of $P_{min}(n)$ |
| $P_M$: | The maximum value of $P_{max}^{max}$ |
| $P(t)$: | 24 hours Load Variation in time |
| $PV_{t_{S_1}}$: | The total present value of the additional feeder (without deferral) at the time $t_{S_1}$ |
| $PV_{t_{S_1}f_1}$: | $PV_{t_{S_1}} = PV_{t_{S_1}f_1}$ |
| $PV'_{t_{s_1}}$: | Total present value of additional feeder construction with BESSs deferral at the time $t_{S_1}$ |
| $PV'_{t_{s_1}b_i}$: | The $i^{th}$ BESS present value at the time $t_{S_1}$ |
| $PV'_{t_{s_1}f_1}$: | The additional feeder's present value at the time $t_{s_1}$with deferral |
| $Q_{B}$: | BESS maximum discharging capacity |
| $r_a$%: | Load growth rate |
| $S_a$: | The maximum feeder apparent power demand (MVA) at the year 0 |
| $S_L$: | Feeder apparent power thermal limit |
| $S_{max}$: | The maximum feeder demand |
| $T_B$: | BESS maximum discharging time, under the rated power $P_{Bmax}$ |
| $t_{b_k}$: | $k^{th}$ BESS commission starting time |
| $t_{BP_i}$: | The deferral time contributed by the $i^{th}$ BESS |
| $t_C$: | The feeder installation duration |
| $t_{F1}$: | The first additional feeder commissioning time (without deferral) |
| $t'_{F1}$: | The first additional feeder commissioning time (with deferral) |

| | |
|---|---|
| $t_{F2}$: | The second additional feeder commissioning time (without deferral) |
| $t_{L}$: | Loan time |
| $t_{Lb}$: | The loan time of the BESS investment |
| $t_{Lf}$: | The loan time for the feeder construction investment |
| $t_{P}$: | The total feeder deferral time |
| $t_{S_1}$: | The first additional feeder installation start time |
| $W_{BCh}$: | Energy supplied to the BESS during the charging time |
| $W_{BDCh}$: | Energy delivered during the discharging while eventually returning to the initial battery charge |
| $W_{\underset{max}{BDCh}}$: | The maximum energy that can be delivered from a full charge, without damaging the unit. $Q_B = W_{\underset{max}{BDCh}}$ |
| $W_{ch(available)}$: | The energy available to be charged in 24 hours (MWh) |
| $W_{dch(need)}$: | The extra peak load energy needed to be supplied by the BESS |
| $W_{T}$: | The total 24 h energy supplied to the loads |
| $\eta_{B}$: | BESS round-trip efficiency |

# 1. Introduction

## *1.1 Scope*

The development and application of energy storage systems are crucial issues for the future Smart Grid. Their ability to store energy has changed the traditional power system's definition. It is critical to evaluate the economic feasibility of energy storage systems. However, their benefits are hard to quantify because of the complexity of different application cases and system characteristics. Hence, there is a need to develop econometric models that help to judge the feasibility of energy storage systems.

The scope of this thesis will focus on evaluating economic benefits of battery energy storage system (BESS). There are three major battery energy storage system application topics considered in this research: energy purchases shifting, distribution feeder deferral and outage avoidance. The first problem involves purchasing inexpensive electric energy, available during periods when demand is low, and selling the energy when the demand is high. The BESS performance is quantified as a function of the differential cost of energy (DCE) representing the difference between the cost of energy purchased to charge the BESS (buy) and the cost of energy delivered (sold). The second problem involves developing an econometric model to quantify the potential benefits using BESSs to defer the feeder construction. The approach is using cash flow diagram to do the net present value (NPV) analysis. The last problem deals with the outage case where the load demand is randomly above the feeder thermal limit. The economic benefit can be evaluated by comparing the outage reparation costs with/without BESS peak shaving support. Three problems are related and their benefits can be summed up. To detail the numerous factors which control the eventual benefit, the following BESS's charging/discharging operation parameters are evaluated: energy capacity, maximum charging/discharging power, life span and "round-trip" efficiency. Also, it is essential to study the following system parameters: marginal cost (MC) of electric energy, feeder 24 hour load profile, annual load variation, feeder length, and the overall system load growth rate. This research follows the policies of distributed system operation and electricity market. The effects of inflation are ignored and the interest rate is annualized in this study. Although this thesis did not focus on the distribution generation by the renewable sources, the optimal charging/discharging method can be addressed to this issue as an extension. This thesis will focus on quantifying the different parameters' effects on the BESS's economic benefit under three major applications.

## *1.2 Literature Review*

This literature review covers following six major topics that deals with Battery Energy Storage System (BESS) study:

1. Energy Storage System Research History
2. Energy Market Economics and BESS Economics
3. Early BESS Research & Applications
4. BESS Research & Applications on Energy Purchases Shifting
5. BESS Research & Applications on Distribution Feeder Deferral
6. BESS Research & Applications on Outage Avoidance

### 1.2.1 Energy Storage System Research History

Over decades, numbers of utilities and research organizations have done a lot of work on developing and evaluating new technologies of grid connected energy storage systems [1-3]. This type of study can be tracked back to as early as 1940's. The researchers such as Arlie

Graham Sterling, JR. from MIT, had begun the potential energy storage devices study. In Arlie's thesis, he dedicated to design a flywheel energy storage system for the grid peaking shaving application [4]. He, as a pioneer, stated that the flywheel storage system was not economical and technically feasible. As time went by the middle 80th, The Sandia National Laboratories had started their energy storage system research [5]. One of the major topics in that series of study and also in this thesis is the Battery Energy Storage System (BESS). For example, they had a detailed technique report related to the Zinc/Bromine battery model development. In addition, they compared different techniques parameters with other flow batteries, even through the overall techniques were not mature at that time [6]. Since then, the U.S. Department of Energy as well as Electric Power Research Institute (EPRI) had been contracted Energy Research Corporation (ERC) [6] to do the battery energy storage study. The technique of battery storage devices was improving gradually and hence more and more institutes were getting involved. In the 90th, some papers have been published dealing with the issues related to the battery design as well as battery early application cases [7-11]. For the recent 15 years, the energy storage techniques have been developed rapidly and many new application areas have been recognized [12-14]. The BESS is becoming more and more attractive especially after the innovation of electric vehicle. In the late 2008, MIT electric vehicle team published a guide to understanding the battery specification [15] and hence improved the comprehension of the BESS's performance. With the proliferation of Smart Grid technologies and the mature electricity market, the companies like NYISO and EOS have played a role of pushing the energy storage applications [16, 17]. It can be predicted that the energy storage system applications' area and techniques will keep increasing through the year [18-21].

### 1.2.2 Energy Market and BESS Economics

Energy Market is the key to drive the economical use of BESS. Paper [8] did the cost analysis where the author used the present value calculation to determine the economical feasibility of using BESS/PV/Wind system. His study considered the discount rate, maintenance fee, life span of BESS as well as the feeder length effects on the cost. But that paper did not consider the load growth rate since during the expected project life, the system electric consumption will increase. This thesis considered this condition in Ch. 4. As for the general market report, the [22, 23] gave the excellent guidance for the energy market economical analysis in the distribution system. Besides, paper [24] detailed the steps to calculate the power systems marginal cost curve and its potential applications. Similarly, the [25] used actual measurement of the electricity price to predict the one in the future. The ISO-NE annual report [26] detailed overall New England energy market which serves as the important information of this thesis study. The BESS market reports [27-29] evaluated each BESS's applications market potential as well as the BESS costs. The thesis economic analysis is based on that.

### 1.2.3 Early BESS Research & Applications

Before moving into the three major thesis topics, some of the early BESS research studies are especially interesting and worth to highlight. In the 1996, the University of Massachusetts Lowell's Bogdan S. Borowy and Ziyad M. Salameh developed a methodology to determine the optimizing BESS and PV array size in the Wind/PV Hybrid System [7]. They calculated the average power outputs of both the wind turbine and the PV module. The idea was to operate the system by the purely renewable energy with the BESS backed up. The BESS will supply power when the power output from the renewable sources is insufficient to supply the system load and charge when the generated renewable energy exceeds the load demand. Then he introduced the term loss of power supply probability (LPSP) and used the algorithm to determine the number of BESS and PV panel. The cost function is to minimize the PV/BESS combination cost. Their study was innovative at that time because they used the BESS to achieve the isolated operation and the approach is reasonable if the case study is ideal. However, the problem was oversimplified in their study since first, the cost function should consider multiple BESS and PV array's variables constraints and then second, they ignored the potential BESS economic benefit which this thesis will address. Instead of optimization problem, the Dr. Salameh also published another paper in the year 2001 where he focused most on the BESS performance [30]. The seasonal load variation which was introduced in that paper makes it more practical. Similar work had been done in [31]. As an example of battery design research, GE had reported its design example of a 5 MVA, 2.5 MWh battery energy storage system located in California [10]. It had all the details of design like BESS control system, filter, relay as well as various response curves under different operation conditions (on utility, isolated as well as resynchronizing). Another GE's paper [9] detailed more about the battery potential using cases, including back-up generation, power control, demand charge management as well as voltage support. Some notations such as power application and energy application of BESS as well as the distinctions between BESS and uninterruptable power supplies (UPS) were also presented.

### 1.2.4 BESS Research & Applications on Energy Purchases Shifting

For the first major thesis topic: Energy Purchases Shifting, there are numbers of papers which have dedicated in this issue [32, 33]. One early research on this problem was done by Kyung-Hee Jung from Korea [11]. He tried to find the optimal BESS installation and capacity for loading leveling. In order to estimate the system load, he first classified the hourly load pattern for different substations second side of main transformer to determine the possibility of load factor improvement, and then used BESS to achieve the constant power operation as considered to be the optimal operation condition. It is interesting to see his approach to estimate the load curve in the real regional system and his study does present an interesting inside of BESS load shifting. However, his strategy to determine the optimal operation condition can not be guaranteed to be proper since he did not consider the economic benefit which is the one important consideration in this thesis work. For the issue of optimal charging/discharging method, the paper [34] considered it as a multi-step optimization problem to achieve the real-time control of BESS. [7] used the similar strategy as the one in the Ch. 3, but unlike this thesis, the author did not consider the rated charge/discharge power. Some papers used the dynamic programming to achieve the optimal BESS charge control, like [35-37], this approach is elegant but may take a long time to find the optimal solution. And they did not study the multiple BESS's parameters effects on the benefit as this thesis did in Ch. 3.

### 1.2.5 BESS Research & Applications on Distribution Feeder Deferral

The amount of research had been done on BESS distribution feeder deferral is not as much as the work on the energy purchases shifting. But there are still some papers addressed this important topic. As stated before, paper [34] did point out the feeder deferral issue in its model; however, it did not do the economic analysis as this thesis did in Ch. 4. Another excellent paper which tackled the deferral problem is [38]. The importance of deferral was introduced and different upgrade strategies were presented. The cost of upgrade in that paper was a function of the feeder length. Although the paper was using distribution generation to defer the feeder construction, similar approach can be addressed in BESS. It is also important to notice that Sandia National Laboratories had several reports focused on the deferral problem [39-41]. In those reports, the BESSs were used to supply the peak demand and both power constraint and energy capacity constraint were considered in the reports. They stated the need to develop a computer tool to quick calculate the deferral benefit and the developed simulation model in this thesis Ch. 4 can serve as the solution.

### 1.2.6 BESS Research & Applications on Outage Avoidance

The third thesis topic is outage avoidance. Papers [42-45] addressed the importance of reliability issue in the electric energy supply. They gave the valuable data for the outage cost which will be used in the Ch. 5. Two papers specifically focused on the energy storage system in the power system reliability issue [46, 47]. The authors in [46] developed a hybrid control strategy which was combined the outage avoidance and demand charge application. This approach is similar to what is done in Ch. 5, except that in this thesis, the outage avoidance application is combined with energy purchase shifting application. The [47] used the optimization approach to achieve the maximum benefit. It was a non-linear and non-convex constrained integer problem.

## *1.3 Thesis Goals and Overview*

The Goals of this thesis can be summarized as following:

1. Determine the economic feasibility of the BESS.
2. Expose the distribution system characteristics' effects on BESS applications and benefits.
3. Evaluate the BESS parameters' effects on its economic benefits.
4. Understand the energy market economics and its role on BESS economic benefits.
5. Discover the optimal BESS charging/discharging methodology.
6. Develop the simulation tools to quantify the BESS multiple economic benefits.
7. Demonstrate the three major applications of BESS: Energy Purchases Shifting, Distribution Feeder Deferral and Outage Avoidance.

The following Ch. 2 will serve as the background. It reports the BESS characteristics and its parameters. In addition, the distribution system's operational policies and model are introduced. The energy market economics is presented in the end. This chapter's information will be used in the later chapters' simulation models.

Ch. 3 deals with the first BESS application topic. The purpose of this chapter is to evaluate the BESS energy purchase shifting application's maximum economic benefit. To achieve that goal, the optimal BESS charging/discharging methodology is developed and the annual load variations are simulated. And then, different BESS and system parameters' effects on BESS's benefits are evaluated. Finally, the maximum annual benefits are qualified.

Distribution feeder deferral application is presented in Ch. 4. The method to evaluate the feeder deferral economic benefit is presented first. Then the simulation model is developed to quantify this benefit. Sensitivity study is reported and different parameters' effects are analyzed.

Ch. 5 determines the BESS outage avoidance economic benefits by comparing different outage operation strategies. Thesis conclusions and suggestions for future work are detailed in Ch. 6.

# 2. Essential Study of Energy Storage System Issues in Distribution System

This chapter addresses three significant sections of this background research works:

- Energy Storage Technologies
- Distribution System Consideration
- Energy Market Economics

## *2.1 Electricity Energy Storage Technologies*

### 2.1.1 Technology Options and Applications

There are seven major storage system types among the available energy storage technologies [1]:

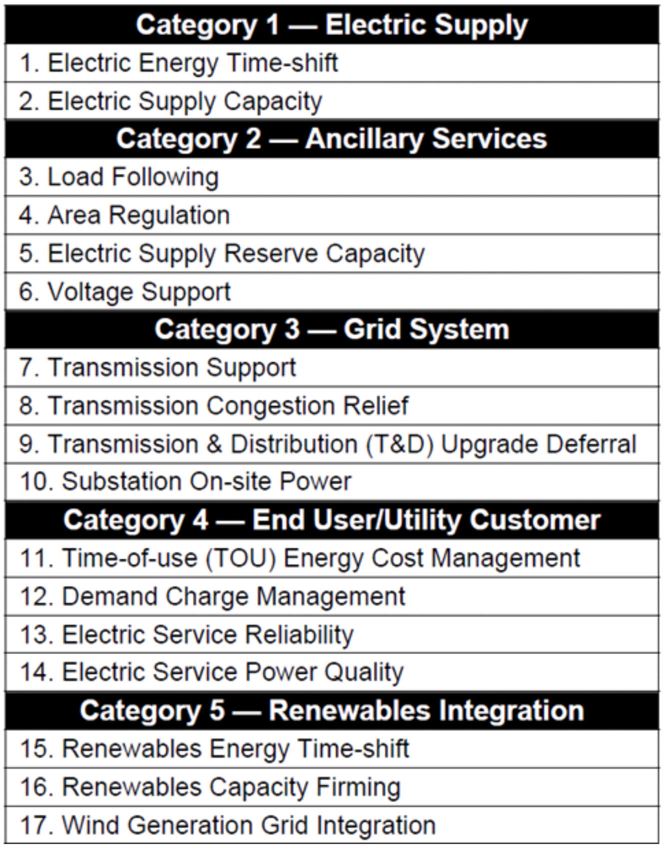
- Electrochemical Batteries
- Capacitors
- Compressed Air Energy Storage (CAES)
- Flywheel Energy Storage
- Pumped Hydroelectric
- Superconducting Magnetic Energy Storage (SMES)
- Thermal Energy Storage

Each type of energy storage system has its own characteristics which should match the corresponding applications. For example, the capacitors energy storage systems are attractive for high power applications that require short or very short discharge durations [1]. Some electrochemical batteries like flow batteries are used in the cases where more discharge duration is needed [1]. In this case, it is relatively easy to increase the flow battery's discharge time by designing units with larger volume of electrolyte.

Overall, different energy storage technologies are classified into two major categories. The first group is best suited for **power applications** and the other group is desirable for large **energy applications** [1]. Mitigation of transient power quality disturbances requires high power output, usually for relatively short period of time from seconds to minutes. The energy storage systems like SMES, capacitors and flywheel energy storage are the choices for **power applications** since they have the capacity to store fairly modest amounts of energy per rated MW output power, but have only relatively short period of discharge time. On the contrary, **energy applications** require relatively large amount of energy, often for discharge durations of many minutes to hours [1]. Therefore, the storage systems like CAES, pumped hydro, thermal energy storage as well as most of batteries are usually the right devices for these applications due to the fact that they have fairly long discharge times. The three major applications studied in this thesis are in the category of large energy applications, thus the Battery Energy Storage System (BESS) is desirable.

Recent researchers have summarized the seventeen electric grid-related energy storage applications into five categories as shown in Table 2-1.

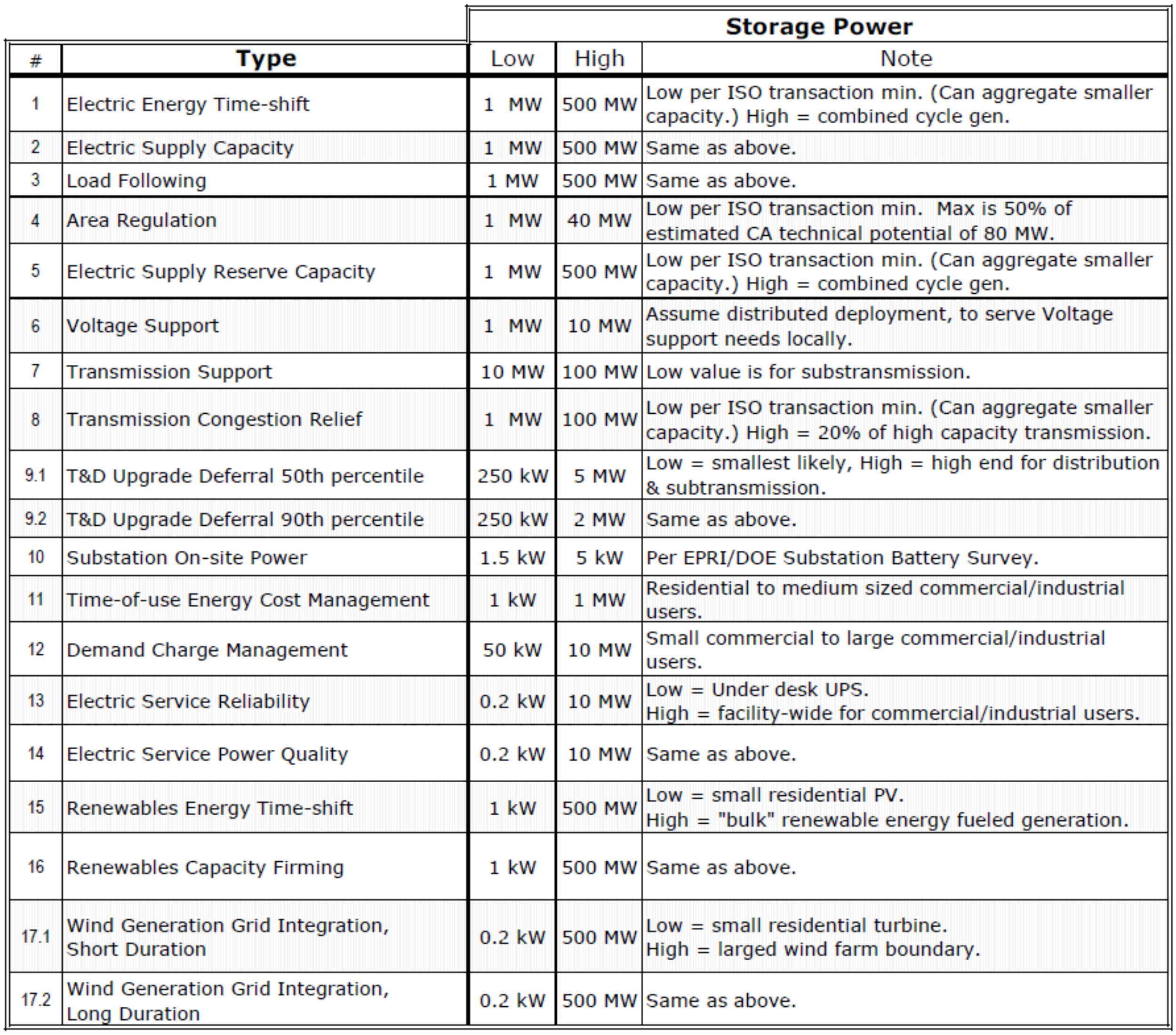
**Table 2-1:** The Five Categories of Energy Storage Applications [1].



| **Category 1 — Electric Supply** |
|---|
| 1. Electric Energy Time-shift |
| 2. Electric Supply Capacity |
| **Category 2 — Ancillary Services** |
| 3. Load Following |
| 4. Area Regulation |
| 5. Electric Supply Reserve Capacity |
| 6. Voltage Support |
| **Category 3 — Grid System** |
| 7. Transmission Support |
| 8. Transmission Congestion Relief |
| 9. Transmission & Distribution (T&D) Upgrade Deferral |
| 10. Substation On-site Power |
| **Category 4 — End User/Utility Customer** |
| 11. Time-of-use (TOU) Energy Cost Management |
| 12. Demand Charge Management |
| 13. Electric Service Reliability |
| 14. Electric Service Power Quality |
| **Category 5 — Renewables Integration** |
| 15. Renewables Energy Time-shift |
| 16. Renewables Capacity Firming |
| 17. Wind Generation Grid Integration |

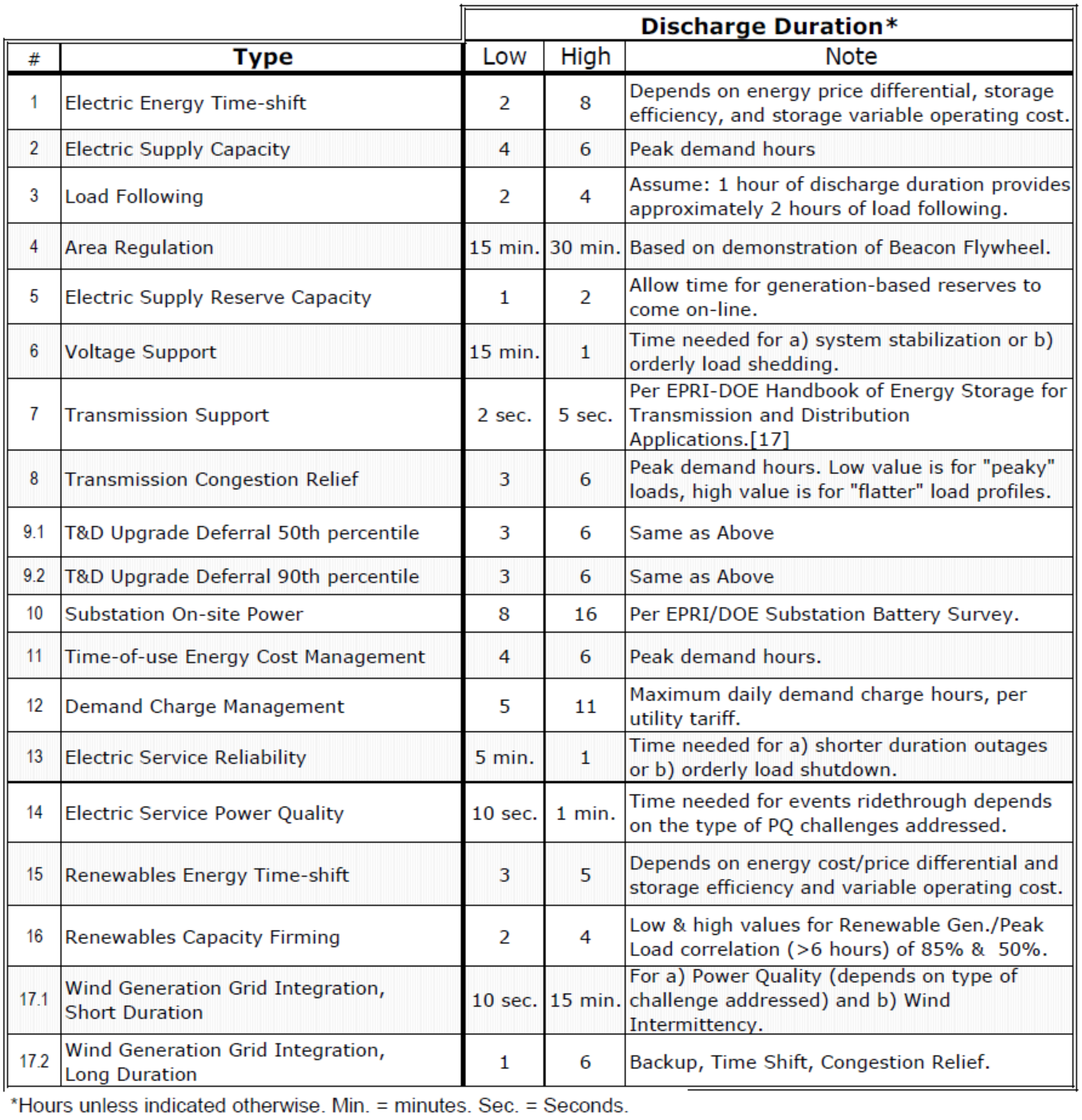
Table 2-2 and Table 2-3 summarized different recommended ranges of energy storage system discharging power (Storage Power) and discharge duration for various types of applications. And it exhibits that the energy-shifting, feeder deferral as well as the outage avoidance (Electric Service Reliability) applications need relatively low discharging power but long discharging duration.

**Table 2-2:** Application vs. Storage Power [1].



| | | Storage Power | | |
|---|---|---|---|---|
| # | Type | Low | High | Note |
| 1 | Electric Energy Time-shift | 1 MW | 500 MW | Low per ISO transaction min. (Can aggregate smaller capacity.) High = combined cycle gen. |
| 2 | Electric Supply Capacity | 1 MW | 500 MW | Same as above. |
| 3 | Load Following | 1 MW | 500 MW | Same as above. |
| 4 | Area Regulation | 1 MW | 40 MW | Low per ISO transaction min. Max is 50% of estimated CA technical potential of 80 MW. |
| 5 | Electric Supply Reserve Capacity | 1 MW | 500 MW | Low per ISO transaction min. (Can aggregate smaller capacity.) High = combined cycle gen. |
| 6 | Voltage Support | 1 MW | 10 MW | Assume distributed deployment, to serve Voltage support needs locally. |
| 7 | Transmission Support | 10 MW | 100 MW | Low value is for substransmission. |
| 8 | Transmission Congestion Relief | 1 MW | 100 MW | Low per ISO transaction min. (Can aggregate smaller capacity.) High = 20% of high capacity transmission. |
| 9.1 | T&D Upgrade Deferral 50th percentile | 250 kW | 5 MW | Low = smallest likely, High = high end for distribution & subtransmission. |
| 9.2 | T&D Upgrade Deferral 90th percentile | 250 kW | 2 MW | Same as above. |
| 10 | Substation On-site Power | 1.5 kW | 5 kW | Per EPRI/DOE Substation Battery Survey. |
| 11 | Time-of-use Energy Cost Management | 1 kW | 1 MW | Residential to medium sized commercial/industrial users. |
| 12 | Demand Charge Management | 50 kW | 10 MW | Small commercial to large commercial/industrial users. |
| 13 | Electric Service Reliability | 0.2 kW | 10 MW | Low = Under desk UPS.<br>High = facility-wide for commercial/industrial users. |
| 14 | Electric Service Power Quality | 0.2 kW | 10 MW | Same as above. |
| 15 | Renewables Energy Time-shift | 1 kW | 500 MW | Low = small residential PV.<br>High = "bulk" renewable energy fueled generation. |
| 16 | Renewables Capacity Firming | 1 kW | 500 MW | Same as above. |
| 17.1 | Wind Generation Grid Integration, Short Duration | 0.2 kW | 500 MW | Low = small residential turbine.<br>High = larged wind farm boundary. |
| 17.2 | Wind Generation Grid Integration, Long Duration | 0.2 kW | 500 MW | Same as above. |

**Table 2-3:** Application vs. Discharge Duration [1].



| | | **Discharge Duration*** | | |
|---|---|---|---|---|
| # | **Type** | Low | High | Note |
| 1 | Electric Energy Time-shift | 2 | 8 | Depends on energy price differential, storage efficiency, and storage variable operating cost. |
| 2 | Electric Supply Capacity | 4 | 6 | Peak demand hours |
| 3 | Load Following | 2 | 4 | Assume: 1 hour of discharge duration provides approximately 2 hours of load following. |
| 4 | Area Regulation | 15 min. | 30 min. | Based on demonstration of Beacon Flywheel. |
| 5 | Electric Supply Reserve Capacity | 1 | 2 | Allow time for generation-based reserves to come on-line. |
| 6 | Voltage Support | 15 min. | 1 | Time needed for a) system stabilization or b) orderly load shedding. |
| 7 | Transmission Support | 2 sec. | 5 sec. | Per EPRI-DOE Handbook of Energy Storage for Transmission and Distribution Applications.[17] |
| 8 | Transmission Congestion Relief | 3 | 6 | Peak demand hours. Low value is for "peaky" loads, high value is for "flatter" load profiles. |
| 9.1 | T&D Upgrade Deferral 50th percentile | 3 | 6 | Same as Above |
| 9.2 | T&D Upgrade Deferral 90th percentile | 3 | 6 | Same as Above |
| 10 | Substation On-site Power | 8 | 16 | Per EPRI/DOE Substation Battery Survey. |
| 11 | Time-of-use Energy Cost Management | 4 | 6 | Peak demand hours. |
| 12 | Demand Charge Management | 5 | 11 | Maximum daily demand charge hours, per utility tariff. |
| 13 | Electric Service Reliability | 5 min. | 1 | Time needed for a) shorter duration outages or b) orderly load shutdown. |
| 14 | Electric Service Power Quality | 10 sec. | 1 min. | Time needed for events ridethrough depends on the type of PQ challenges addressed. |
| 15 | Renewables Energy Time-shift | 3 | 5 | Depends on energy cost/price differential and storage efficiency and variable operating cost. |
| 16 | Renewables Capacity Firming | 2 | 4 | Low & high values for Renewable Gen./Peak Load correlation (>6 hours) of 85% & 50%. |
| 17.1 | Wind Generation Grid Integration, Short Duration | 10 sec. | 15 min. | For a) Power Quality (depends on type of challenge addressed) and b) Wind Intermittency. |
| 17.2 | Wind Generation Grid Integration, Long Duration | 1 | 6 | Backup, Time Shift, Congestion Relief. |

*Hours unless indicated otherwise. Min. = minutes. Sec. = Seconds.

One of the advantages of energy storage system is that their applications can be combined to have multiple benefits. There will be application synergies matrix as shown in Table 2-4.

**Table 2-4:** Application Synergies Matrix [1].

● Excellent ◉ Good ⭘ Fair ○ Poor ⊗ Incompatible

| Application | Electric Energy Time-shift | Electric Supply Capacity | Load Following | Area Regulation | Electric Supply Reserve Capacity | Voltage Support[1] | Transmission Congestion Relief[1] | T&D Upgrade Deferral[1] | Time-of-Use Energy Cost Management[1] | Demand Charge Management[1] | Electric Service Reliability[1] | Electric Service Power Quality[1] | Renewables Energy Time-shift | Renewables Capacity Firming | Wind Generation Grid Integration |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Electric Energy Time-shift |  | ● | ◉ | ⭘* | ◉ | ● | ●† | ●† | ⊗ | ⊗ | ⊗ | ⊗ | ◉ | ◉ | ⭘ |
| Electric Supply Capacity | ● |  | ◉* | ⭘* | ◉* | ● | ◉† | ●† | ⊗ | ⊗ | ⊗ | ⊗ | ⭘X* | ⭘X* | ⊗ |
| Load Following | ◉ | ◉* |  | ⭘* | ◉* | ◉ | ⭘X | ◉X* | ⭘*‡ | ⭘*‡ | ⊗ | ⊗ | ◉ | ⊗ | ⊗ |
| Area Regulation | ⭘* | ⭘* | ⭘* |  | ⭘* | ⊗ | ⭘X* | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | ○ | ○ | ⊗ |
| Electric Supply Reserve Capacity | ◉ | ◉* | ◉* | ⭘* |  | ● | ⭘* | ◉* | ◉*‡ | ◉*‡ | ⊗ | ⊗ | ◉* | ◉* | ○* |
| Voltage Support[1] | ● | ● | ◉ | ⊗ | ● |  | ◉ | ● | ◉‡ | ◉‡ | ◉‡ | ◉‡ | ◉#‡ | ◉#‡ | ⊗ |
| Transmission Congestion Relief[1] | ●† | ◉† | ⭘X | ⭘X* | ⭘* | ◉ |  | ◉X† | ◉† | ◉† | ○ | ⊗ | ◉# | ◉† | ⊗ |
| T&D Upgrade Deferral[1] | ●† | ●† | ◉X* | ⊗ | ◉* | ● | ◉X† |  | ◉† | ◉† | ○ | ⊗ | ◉# | ◉† | ⊗ |
| Time-of-Use Energy Cost Management[1] | ⊗ | ⊗ | ◉*‡ | ⊗ | ◉*‡ | ◉‡ | ◉† | ◉† |  | ●† | ● | ● | ◉# | ◉†# | ⊗ |
| Demand Charge Management[1] | ⊗ | ⊗ | ◉*‡ | ⊗ | ◉*‡ | ◉‡ | ◉† | ◉† | ●† |  | ● | ● | ◉# | ●†# | ⊗ |
| Electric Service Reliability[1] | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | ◉‡ | ○ | ○ | ● | ● |  | ● | ◉# | ◉# | ⊗ |
| Electric Service Power Quality[1] | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | ◉‡ | ⊗ | ⊗ | ● | ● | ● |  | ⊗ | ⊗ | ⊗ |
| Renewables Energy Time-shift | ◉ | ⭘X* | ◉ | ○ | ⭘* | ◉#‡ | ◉# | ◉# | ◉# | ◉# | ◉# | ⊗ |  | ● | ⭘X |
| Renewables Capacity Firming | ◉ | ⭘X* | ⊗ | ○ | ◉* | ◉#‡ | ◉† | ◉† | ◉†# | ●†# | ◉# | ⊗ | ● |  | ⭘X |
| Wind Generation Grid Integration | ◉ | ⊗ | ⊗ | ⊗ | ○* | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | ⊗ | ⭘X | ⭘X |  |

As one will notice, there are many overlaps between different applications. Some of them are matching very well others are not. It is fortunate that three major BESS applications used in this thesis are synergic. For example, two major thesis problems: energy time shifting as well as distribution feeder deferral can be implemented at the same time without conflict. In the sense that if the plant used for energy time shifting in the right place as well as at the right times, it can also help to defer the distribution feeder construction. More specifically, the energy storage system will be used almost everyday to shift the energy purchase which will also include few days shifting the peak demand to achieve deferral.

### 2.1.2 Battery Energy Storage System (BESS) Characteristics

This thesis will focus on the Battery Energy Storage System (BESS) applications. Although there are many batteries parameters that help describe its performances, five of them deserve special attention for their importance:

- BESS rated charging/discharging power, $\mathbf{P_{Bmax}}$,
- BESS maximum discharging time $\mathbf{T_B}$, under the rated power $P_{Bmax}$,
- BESS maximum discharging capacity, $\mathbf{Q_B}$,
- BESS round-trip efficiency, $\boldsymbol{\eta_B}$,
- BESS life span, measured in cycles, **CL**.

The $\mathbf{P_{Bmax}}$ is the maximum power the BESS can be operated without damage. And its maximum discharging capacity has the expression:

$$\boldsymbol{Q_B = P_{Bmax} T_B} \tag{2-1}$$

The $\boldsymbol{\eta_B}$ reflects the amount of energy that comes out of storage relative to the amount has to be putted into the storage and is given by:

$$\boldsymbol{\eta_B = \frac{W_{BDCh}}{W_{BCh}}} \tag{2-2}$$

where

$\boldsymbol{W_{BCh}}$=energy supplied to the BESS during the charging time.

$\boldsymbol{W_{BDCh}}$=energy delivered during the discharging while eventually returning to the initial battery charge.

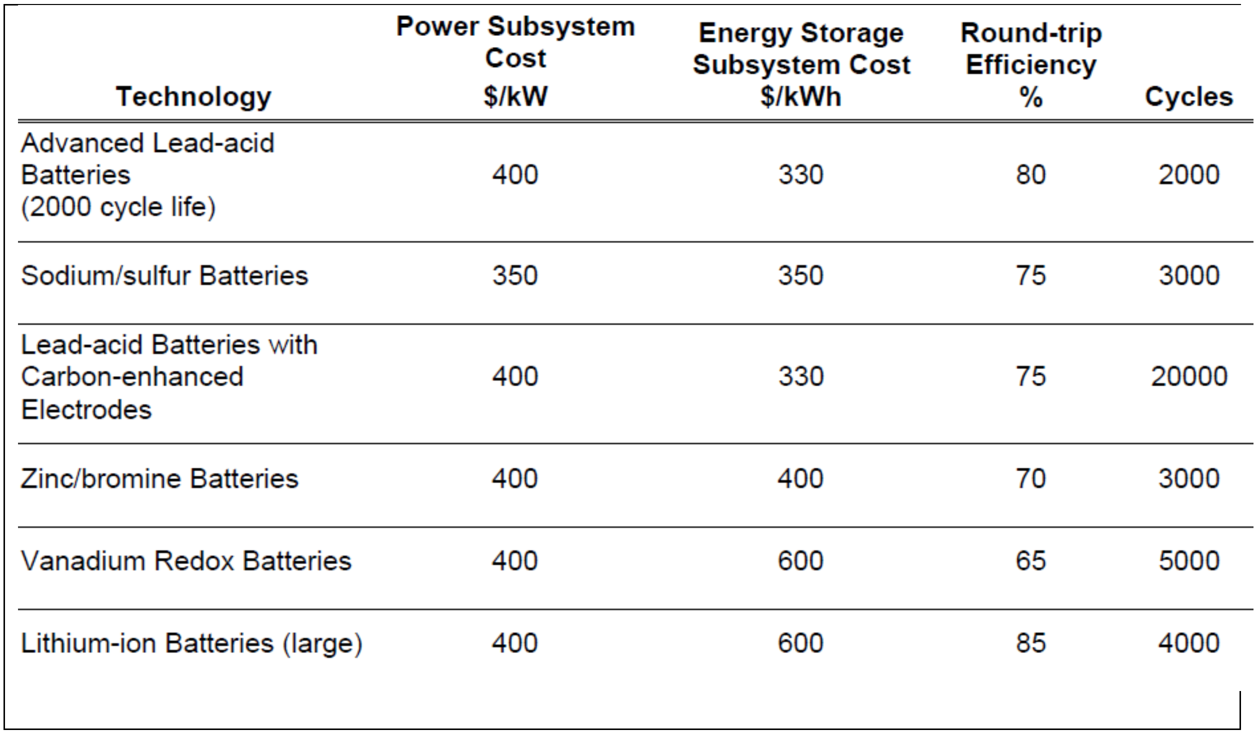
Finally, the **CL** is defined as the number of discharge-charge cycles the BESS can experience before it fails to meet specific performance criteria [15]. The **CL** is given under the assumption that the BESS is operated in a recommended mode. In reality, many other conditions such as temperature and humidity as well as the depth of discharge will affect the number of **CL**. This research ignored these side-effects. Nevertheless, the BESS still needs to be replaced after it reaches the maximum recommended **CL**. One of the advantages of BESS is that it is relatively easy to replace its electrolyte when it degrades.

### 2.1.3 Battery Energy Storage System (BESS) Costs

There are two major types of costs for BESS. One is called the capital cost: $\mathbf{C_B}$ **($)**; the other is called the operating cost, $\mathbf{C_O}$ **($)**. The $\mathbf{C_B}$ is the one-time investment which brings the BESS into an operable status. On the other side, the $\mathbf{C_O}$ is a continuous expense through the project life span having two key components: 1) **energy-related operating costs** and 2) **operating costs not related to energy** [1].

Table 2-5 summarized the $\mathbf{C_B}$ for typical types of BESS. It contains two subsystems. One is called power subsystem; the other is energy storage subsystem. The cost of the two subsystems should be added together to get the overall $\mathbf{C_B}$ [28].

**Table 2-5:** BESS capital cost [28].



| Technology | Power Subsystem Cost $/kW | Energy Storage Subsystem Cost $/kWh | Round-trip Efficiency % | Cycles |
|---|---|---|---|---|
| Advanced Lead-acid Batteries (2000 cycle life) | 400 | 330 | 80 | 2000 |
| Sodium/sulfur Batteries | 350 | 350 | 75 | 3000 |
| Lead-acid Batteries with Carbon-enhanced Electrodes | 400 | 330 | 75 | 20000 |
| Zinc/bromine Batteries | 400 | 400 | 70 | 3000 |
| Vanadium Redox Batteries | 400 | 600 | 65 | 5000 |
| Lithium-ion Batteries (large) | 400 | 600 | 85 | 4000 |

To simplify the analysis, the $\mathbf{C_B}$ has the expression in this thesis:

$$\boldsymbol{C_B = K_B Q_B} \quad (\$) \tag{2-3}$$

This work assumes that BESS's price to be $\boldsymbol{K_B = 0.333 \times 10^6}$ $\boldsymbol{\$/MWh}$**.** This specific price was considered independent of the $\mathbf{Q_B}$. This approach is used in the Ch. 4 cost analysis.

For the operating costs, the first part of it is an **energy-related operating costs**, which are directly related to the energy losses due to the BESS's inefficiency. In the other word, part of the charging energy will be lost during charging/discharging. This part of costs is also addressed in the Ch. 4 while dealing with the energy losses issues.

The second part of the operating cost consists of **non-energy operating costs**. According to the Sandia reports [1], there are at least four elements: 1) labor associated with plant operation, 2) plant maintenance and replacement, 3) equipment wear leading to its loss-of-life, and 4) decommissioning and disposal cost. The replacement cost is the biggest part which will dominate the non-energy operating cost. The BESS will need to be replaced after a few years of use, usually around 3 to 10 years [27]. The cost for one time replacement is about equal to the $\mathbf{C_B}$ of BESS. This part of investment is considered in Ch. 4 as part of simulation development process. The other **non-energy operating costs** are ignored in this study.

## *2.2 Distribution System Consideration*

### 2.2.1 Distribution System Operational Policies

- **Electric Energy Generation Options**

Different types of plants with different efficiency and capital costs are used to generate the electric energy. The utilities are trying to use the most efficient and cheap energy sources like hydro electric, unclear and modern fossil fuels power plants as the first options for electricity generation. If the energy generated by high efficiency sources is insufficient to supply the system load, the less effective and more expensive energy sources will be used. The change of the generation options will affect the generation cost also called fuel cost. The use of low efficiency energy sources will increase the electric energy price.

It is possible to reduce the generation cost by energy time-shift application using BESS. The BESS has the ability to charge its batteries during the night hours, when the demand is low and the energy sources efficiency is high. The stored energy can be used to offset the high peak generation in order to avoid the use of less efficiency energy sources.

- **Renewable Energy Generation**

Modern distribution systems are using renewable energy sources. Two of the most popular one are the Photovoltaic (PV) and the wind power systems. However, the quality of generated energy is not always repetitive or predictable. The wind speed is usually high when the system demand is low such as late night or early morning (Figure 2-1). During that period, the cost of energy is low. In addition, the randomness of wind generation will require specialized BESS's or active compensators that help to eliminate the power spikes. Evidently this condition will transcend to an extra cost [1].

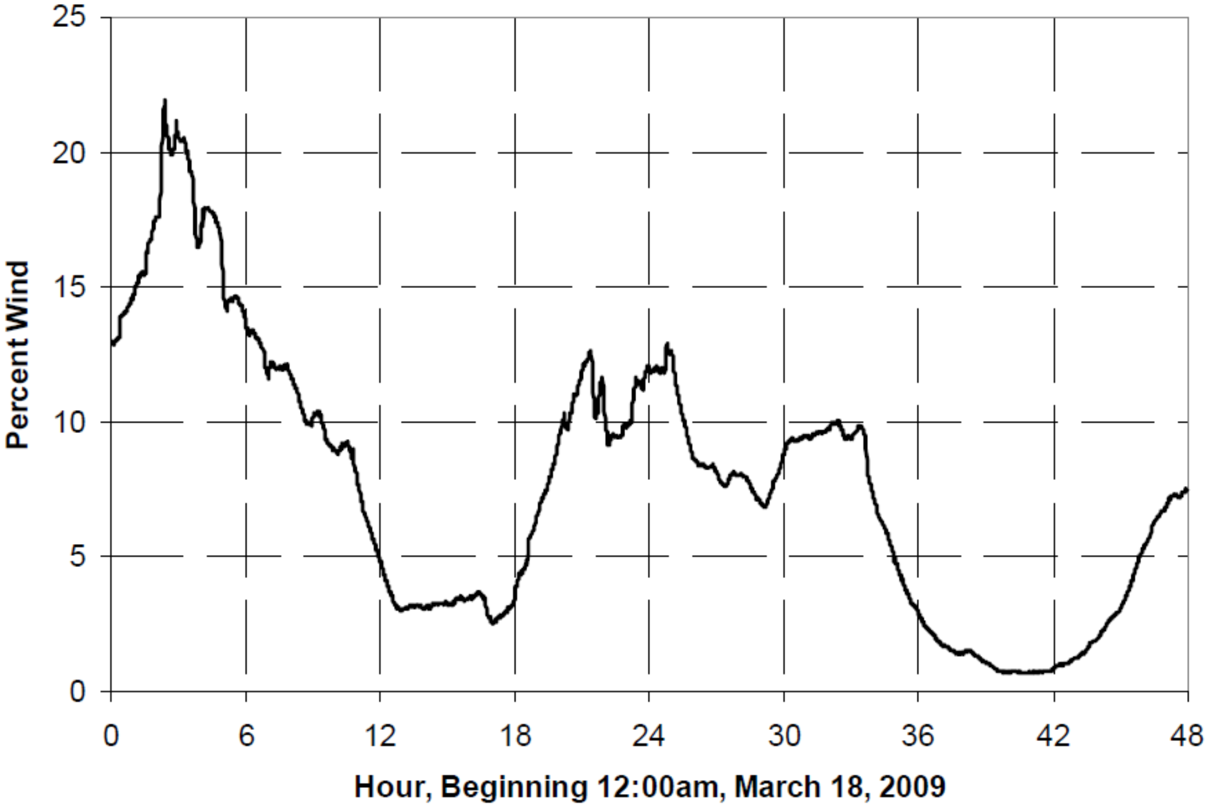


**Figure 2-1** Wind Generation in Electric Reliability Council of Texas (ERCOT) Variation [48].

Similarly, different weather conditions will change the solar radiation dramatically. There are some days when the sky is cloudy and the insolation is minute. In addition, the insolation may change rapidly during the day causing the significant power variation. Figure 2-2 shows one week's real output power generated by PV system.

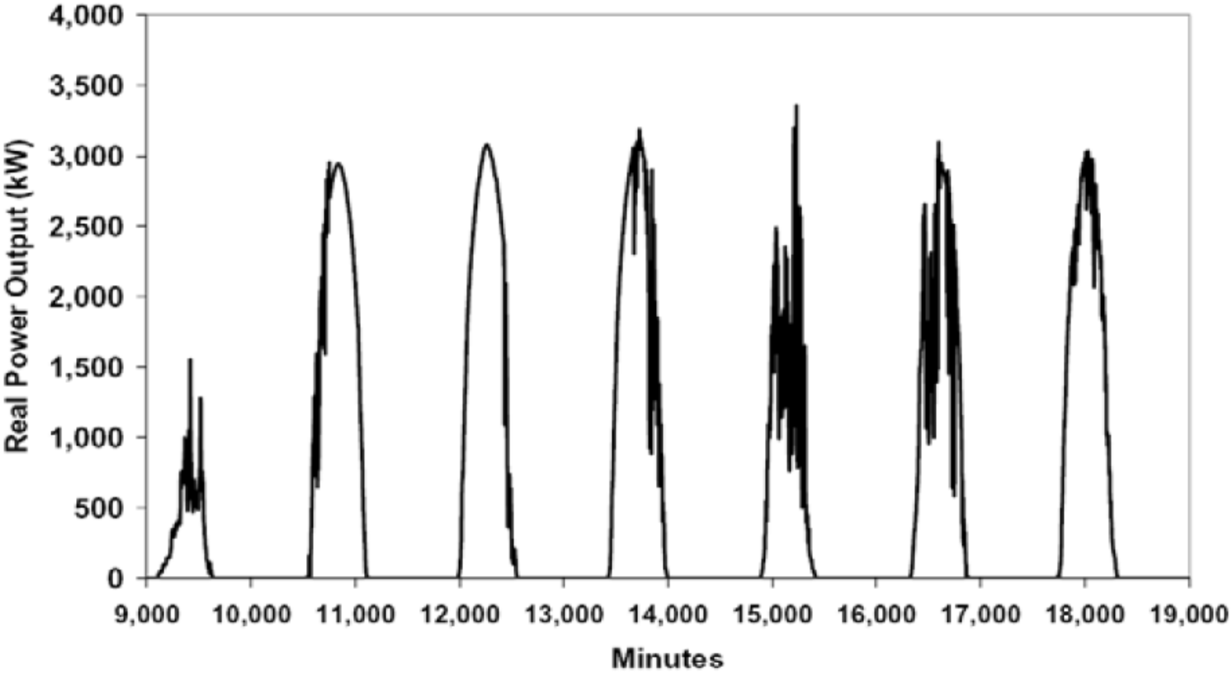


**Figure 2-2** One week's PV output variation (Springerville, AZ) [48].

The PV output power can be simulated using a probability approach as shown in Figure 2-3 [Appendix A for MATLAB code]. One will notice that the output power varies randomly. For example, the 3rd day's power output is small (Figure 2-3b) to a cloudy sky while the 5th day's power output is smooth and high due to a sunny sky.

One of the BESS important applications is shifting the renewable energy needed to achieve its economical dispatch. It is possible to charge the BESS with the random renewable PV output energy and discharge the BESS during the high demand. Moreover, this feature enables randomness elimination of the renewable power delivered by PV and wind turbines.

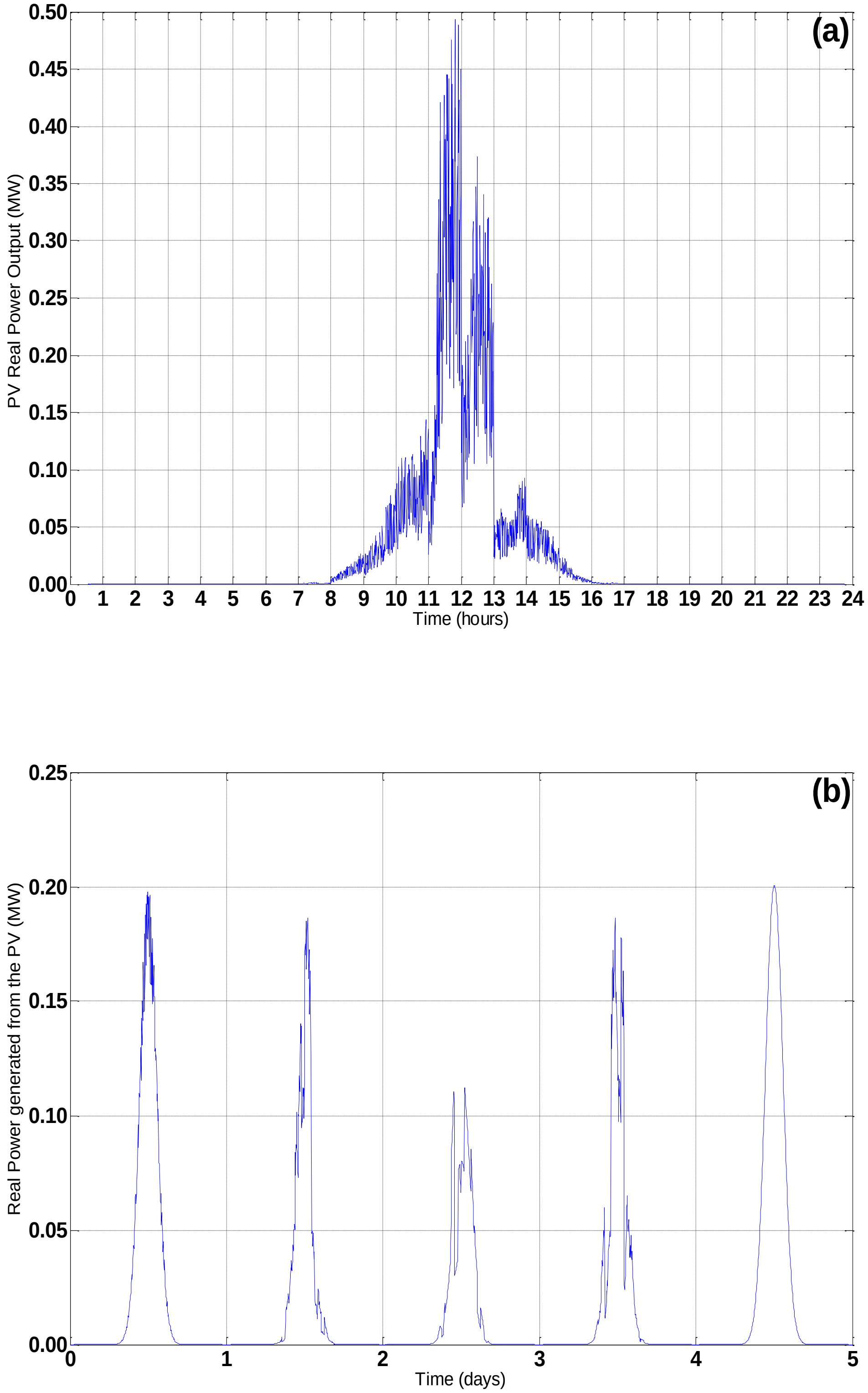


**Figure 2-3** PV Output Power Simulation Results. (a) one day (0.5MW). (b) five days (0.2MW), [49, 50].

- **Feeder Thermal Limit**

In the distribution system, the power is delivered from substation to the end user through a dedicated feeder. Each feeder has its recommend ampacity limitation due to thermal effects. That affects substation equipment and cables and overhead conductors. It was assumed a typical 15 KV level class feeder that has the thermal limit of $\mathbf{S_L}$=10 MVA (400 A) in this thesis. Potential damages to the substation equipment can be caused if the feeder's load exceeds this value. To protect the system, a planned outage will be implemented if the feeder load is exceeding the thermal limit.

### 2.2.2 Distribution System Model

When modeling the distribution system, one must consider the following five parameters:

1. Load Growth Rate, $\mathbf{r_a\%}$,
2. 24 hours Load Variation in time, **P(t)**,
3. Feeder Length, $\boldsymbol{l}$, and Capital Cost, $\mathbf{C_F}$,
4. Annual Load Variation

Each parameter is detailed as following:

1. **Load Growth Rate, $\mathbf{r_a\%}$.**

The overall load demand monitored at the substation bus is increasing year by year. The electricity demand is driven by various factors like the number of households served by the considered feeder, industrial productivity and the consumer confidence. The economic condition will drive the system load increase in the future. The load Growth rate $r_a\%$ has the following expression:

$$\boldsymbol{r_a}\% = \left(\frac{\boldsymbol{S}_{\mathbf{max}(\boldsymbol{n})}}{\boldsymbol{S}_{\mathbf{max}(\boldsymbol{n-1})}} - \mathbf{1}\right) \times \mathbf{100}\% \tag{2-4}$$

where

$\boldsymbol{S}_{\mathbf{max}(\boldsymbol{n})}$=the maximum feeder demand (MVA) at the year n.

$\boldsymbol{S}_{\mathbf{max}(\boldsymbol{n-1})}$=the maximum feeder demand (MVA) at the year n-1.

## 2. 24 hours Load Variation in time, P(t).

The feeder’s load demand curve is time varying. For the most cases, the power will peak during the early afternoon. Sometimes the load curve will also have multiples peaks.

Figure 2-4 and Figure 2-5 exhibit weekly load demand variations. Figure 2-4 shows the case when there is only one peak demand/day while Figure 2-5 shows the double peak situation for the daily load curve.

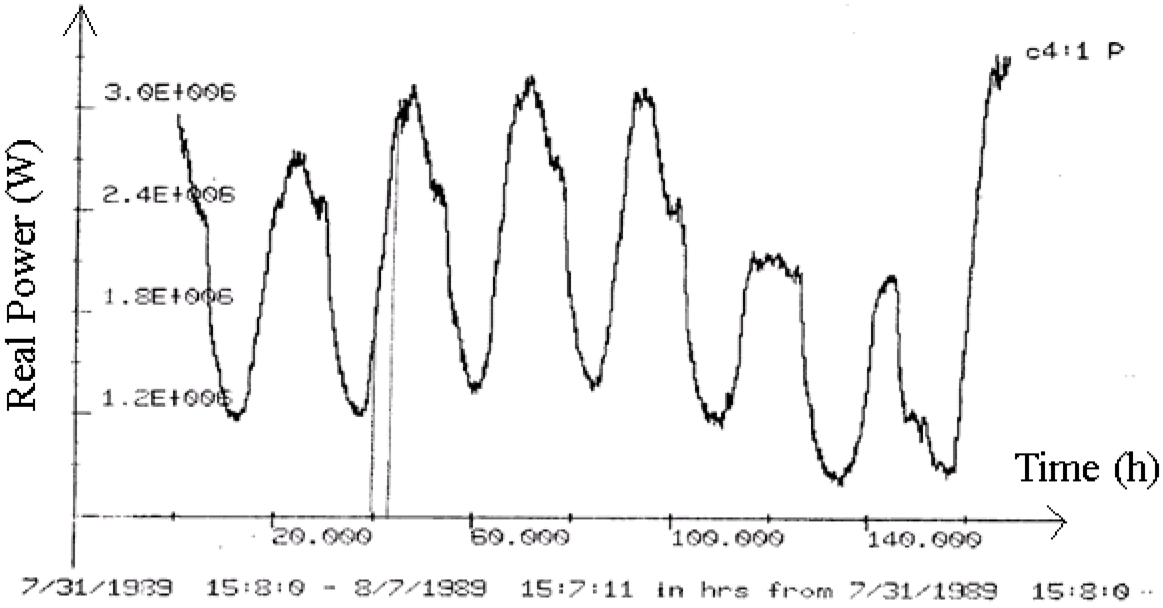


**Figure 2-4** Weekly Load Demand Curve (Single Peak for Daily Load Curve) [51].

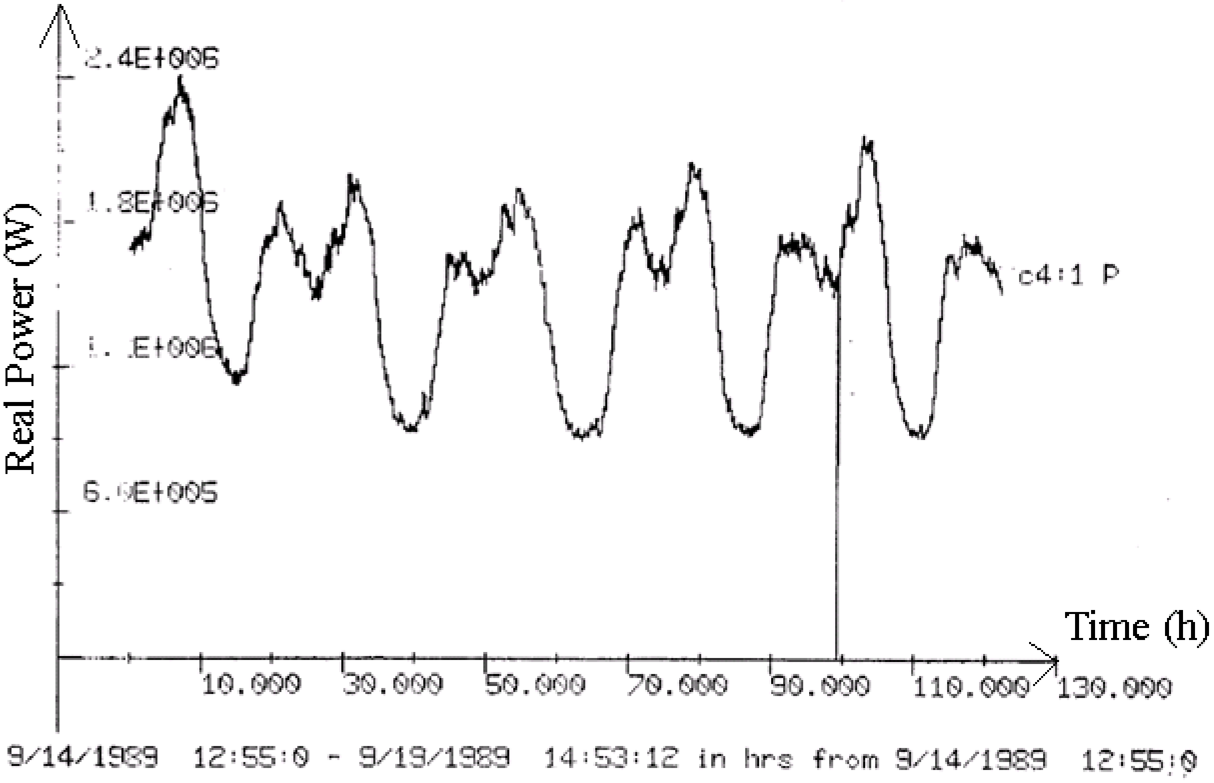


**Figure 2-5** Weekly Load Demand Curve (Double Peak for Daily Load Curve) [52].

In both cases, it should be realized that the peak demand is not allowed to go above the feeder's thermal limit $\mathbf{S_L}$. If such a situation developed, a planned outage will be implemented.

### 3. Feeder Length, $l$, and Capital Cost, $C_F$.

Giving the feeder length: $\boldsymbol{l}$ (mi), assuming the price of constructing feeder is $\boldsymbol{K_F = 6 \times 10^5}$ $\$/\boldsymbol{mile}$, the feeder capital cost $\boldsymbol{C_F}$ has the expression:

$$\boldsymbol{C_F = K_F l} \quad (\$) \qquad (2\text{-}5)$$

### 4. Annual Load Variation

Similar to the daily load demand variation, the load curve will also vary through the year. The maximum peak demand usually happens during the summer. The spring and fall times will have lower demand. The winter depending on the geographies region will be in the middle range of the demand for most of the time.

The detailed expression for annual load variation is developed in the Ch. 3. The annual load demand variation will also cause electricity price difference between the seasons.

## *2.3 Energy Market Economics*

### 2.3.1 The Electricity Market

The electric energy is a product. “Over the long run, competitive and efficient electricity markets provide the incentives to maintain an adequate supply of electric energy at prices consistent with the cost of providing it.” [26]. The **Marginal Cost** and **Locational Marginal Prices** need to be considered in order to quantify the market outcome.

In economics, the **Marginal Cost** is the tool used to evaluate the changing of the cost when the quantity of a product is changed. As was stated in the Ch. 2.2.1, the demand increasing will cause the electric energy price to increase due to the inefficiency of the generation sources. In this thesis, the **Marginal Cost** gives the relation between the electric energy price and demand.

The load demand is varying daily and annually. Therefore the price of the electric energy will also change daily and with the seasons. One example of annual electric energy price variation is presented in Figure 2-6.

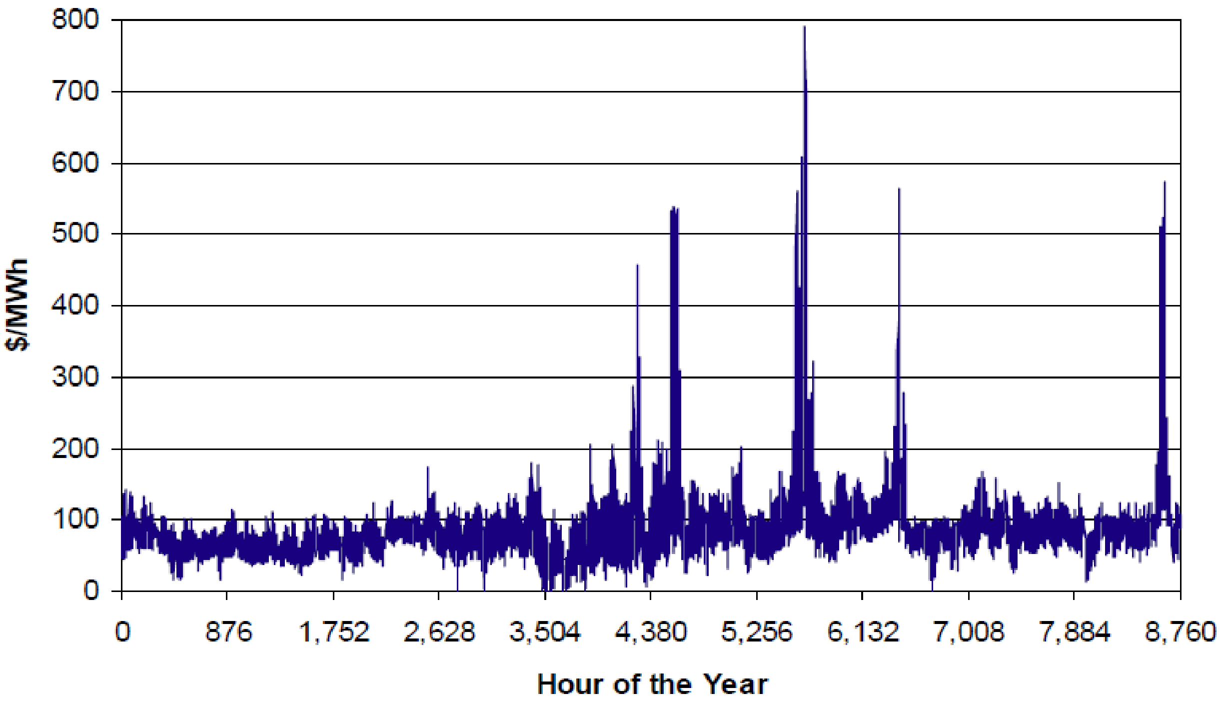


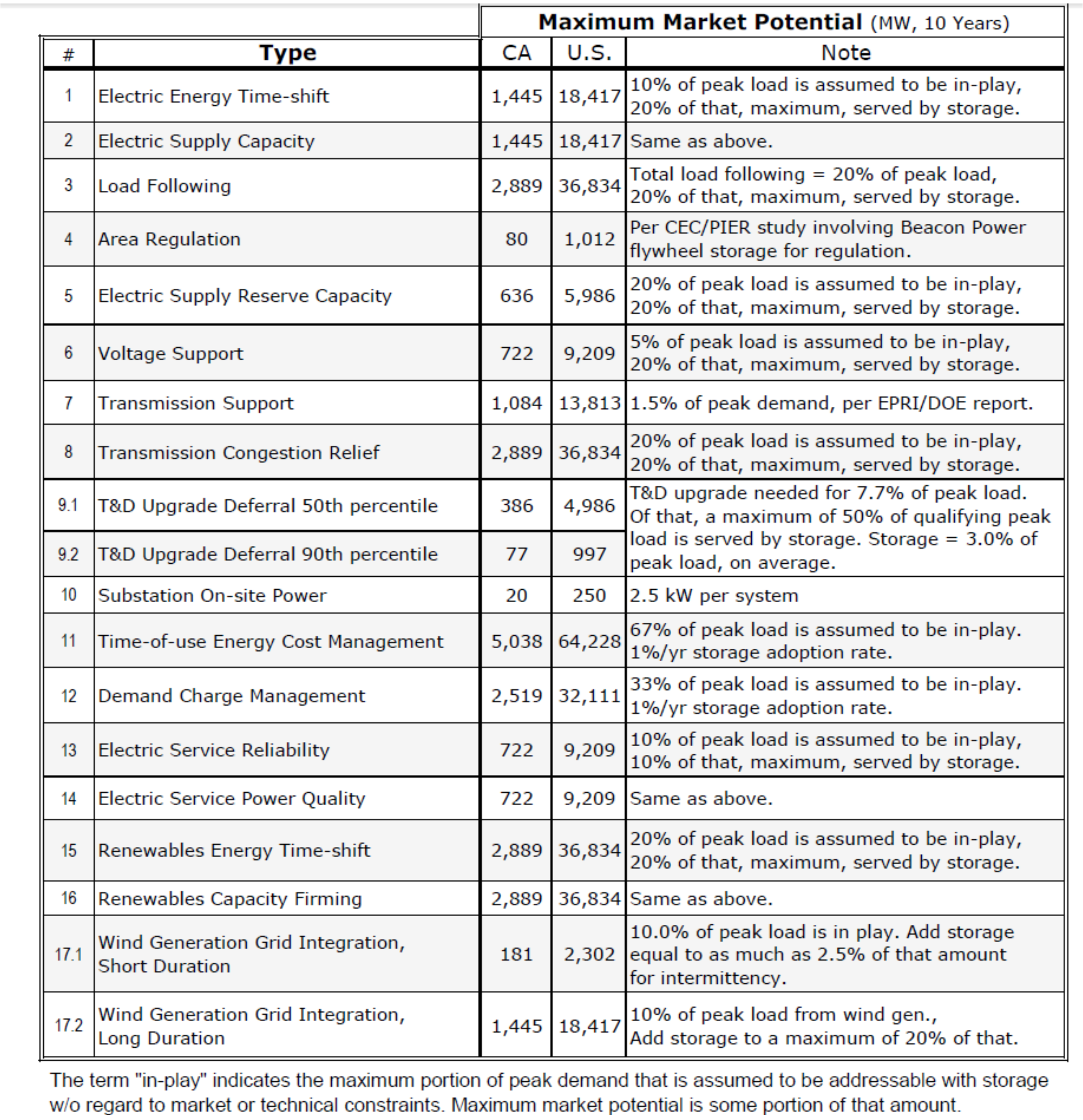
**Figure 2-6** The Forecasted Electric Energy Spot Prices for California [1].

The price in Figure 2-6 is called **Locational Marginal Prices (LMPs)**. As one will notice that the LMPs have several peaks through the year. The reason for these peaks is related to the marginal cost. The LMPs will goes up if the system overall load demand increases. It should be pointed out that for a particular feeder, its load demand may not have the exact load curve shape as the overall region load. Nevertheless, the feeder load curve will have the similar shape as the overall distribution system load demand which will bring the interesting insight of energy shifting problem in later Ch. 3.

### 2.3.2 The Energy Storage System Market

The energy storage system applications have been listed before. The Sandia Report [1] has also estimated each application's maximum market potential as was listed in Table 2-6.

**Table 2-6:** Maximum Market Potential Estimates [1].



| | | **Maximum Market Potential** (MW, 10 Years) | | |
|---|---|---|---|---|
| # | **Type** | CA | U.S. | Note |
| 1 | Electric Energy Time-shift | 1,445 | 18,417 | 10% of peak load is assumed to be in-play, 20% of that, maximum, served by storage. |
| 2 | Electric Supply Capacity | 1,445 | 18,417 | Same as above. |
| 3 | Load Following | 2,889 | 36,834 | Total load following = 20% of peak load, 20% of that, maximum, served by storage. |
| 4 | Area Regulation | 80 | 1,012 | Per CEC/PIER study involving Beacon Power flywheel storage for regulation. |
| 5 | Electric Supply Reserve Capacity | 636 | 5,986 | 20% of peak load is assumed to be in-play, 20% of that, maximum, served by storage. |
| 6 | Voltage Support | 722 | 9,209 | 5% of peak load is assumed to be in-play, 20% of that, maximum, served by storage. |
| 7 | Transmission Support | 1,084 | 13,813 | 1.5% of peak demand, per EPRI/DOE report. |
| 8 | Transmission Congestion Relief | 2,889 | 36,834 | 20% of peak load is assumed to be in-play, 20% of that, maximum, served by storage. |
| 9.1 | T&D Upgrade Deferral 50th percentile | 386 | 4,986 | T&D upgrade needed for 7.7% of peak load. Of that, a maximum of 50% of qualifying peak load is served by storage. Storage = 3.0% of peak load, on average. |
| 9.2 | T&D Upgrade Deferral 90th percentile | 77 | 997 | |
| 10 | Substation On-site Power | 20 | 250 | 2.5 kW per system |
| 11 | Time-of-use Energy Cost Management | 5,038 | 64,228 | 67% of peak load is assumed to be in-play. 1%/yr storage adoption rate. |
| 12 | Demand Charge Management | 2,519 | 32,111 | 33% of peak load is assumed to be in-play. 1%/yr storage adoption rate. |
| 13 | Electric Service Reliability | 722 | 9,209 | 10% of peak load is assumed to be in-play, 10% of that, maximum, served by storage. |
| 14 | Electric Service Power Quality | 722 | 9,209 | Same as above. |
| 15 | Renewables Energy Time-shift | 2,889 | 36,834 | 20% of peak load is assumed to be in-play, 20% of that, maximum, served by storage. |
| 16 | Renewables Capacity Firming | 2,889 | 36,834 | Same as above. |
| 17.1 | Wind Generation Grid Integration, Short Duration | 181 | 2,302 | 10.0% of peak load is in play. Add storage equal to as much as 2.5% of that amount for intermittency. |
| 17.2 | Wind Generation Grid Integration, Long Duration | 1,445 | 18,417 | 10% of peak load from wind gen., Add storage to a maximum of 20% of that. |

The term "in-play" indicates the maximum portion of peak demand that is assumed to be addressable with storage w/o regard to market or technical constraints. Maximum market potential is some portion of that amount.

One will notice that the applications No. 1, 9 and 13 are three major items that will be addressed in later chapters. The market potentials among the three applications will have the following order: electric energy time shift application > reliability (outage avoidance) application > distribution feeder deferral application.

Although in the Ch. 2.1.1 is stated that the different applications can be combined resulting in optimal use of the BESS, thus the energy storage system's benefit will increase. However, it is inappropriate to add all the different applications' market potential together due to the fact that many applications are overlapping. The concept of application and benefit intersection is sketched in Figure 2-7.

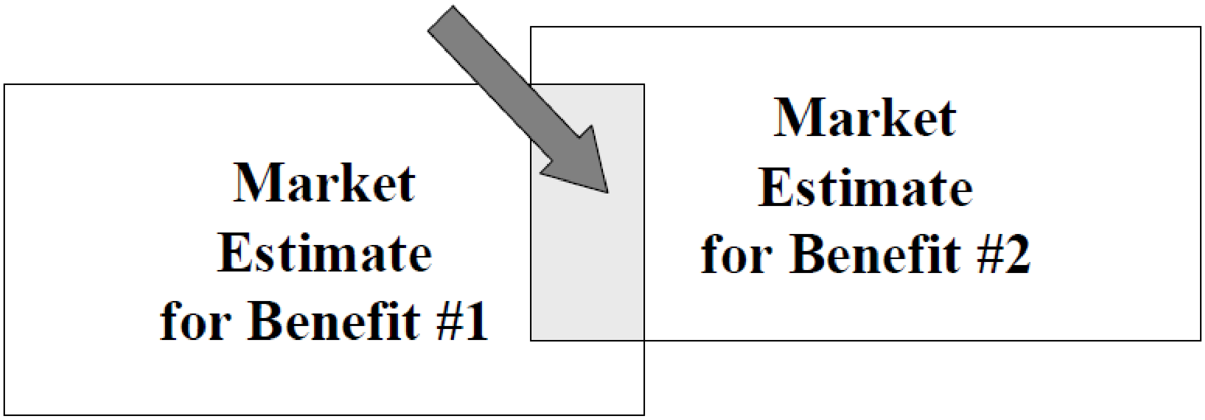


**Figure 2-7** Energy Storage System Applications Market Intersection [1].

In the following example, the utility customers are using energy storage systems to shift energy purchase. At the same time, the high demand which is above the thermal limit may also be shifted so that a planned outage can be avoided. As the maximum load demand is increasing through the year, the distribution feeder deferral application may also be involved. Thus their benefit can be added so that the energy storage systems will increase their overall benefit, but at the same time, their market potential will overlap which means their total market potential will decrease.

### 2.3.3 The Economic Analysis Principle

1. **Present value (P)**

The value of money is a function of time. The same amount of money is less valuable in the future than at present. To account for the differences, the **present value** notion is introduced. Under the current interest rate $\boldsymbol{i}$, the relation between the present worth **P** and the equivalent $\boldsymbol{n^{th}}$ year later future value **F** is as follows [53]:

$$\boldsymbol{P = \frac{F}{(1+i)^n}} \tag{2-6}$$

The interest rate ($\boldsymbol{i}$) is usually referred to as a discount rate (**d**)**.** The definition of discount rate (**d**) is shown below:

**Discount Rate (d)** is the interest rate that could have been earned if the money had been put in the best alternative investment.

Therefore, (2-6) can be also written as [53]:

$$\boldsymbol{P = \frac{F}{(1+d)^n}} \tag{2-7}$$

In some of the cases, it will be given a stream of annual cash flow **A** (i.e. annual loan payment), for **n** years into the future, with a discount rate (**d**). If that was a case, the present value **P** can be expressed as [53]:

$$\boldsymbol{P = A\left(\sum_{i=1}^{N}\frac{1}{(1+d)^i}\right) = A\frac{(1+d)^n-1}{d(1+d)^n}} \tag{2-8}$$

## 2. Annual Payment (A)

Assuming the investors will borrow money (in an amount of **C**) from a lending company under an interest rate ($\boldsymbol{i_R}$), the economic analysis could be considered as a loan that converts the extra capital cost into a series of equal annual payment **A** that will eventually pay off the loan with interest ($\boldsymbol{i_R}$). The loan time is $\boldsymbol{t_L}$ (years).

The relation between annual payment **A** and borrowed money **C** can be shown as [53]:

$$\boldsymbol{A = C\,\frac{i_R(1+i_R)^{t_L}}{(1+i_R)^{t_L}-1}} \tag{2-9}$$

# 3. Energy Purchases Shifting Problem

This chapter reports results that detail the effects of the parameters that control the charging/discharging operation of a Battery Energy Storage System (BESS) located on a distribution feeder.

This chapter has five sections:

1. Energy Purchases Shifting Problem Definition,
2. BESS and Distribution System Data Preparation,
3. Optimal Battery Charging/Discharging Methodology,
4. Simulation Results and Discussion,
5. Energy Purchases Shifting Problem Conclusions.

The effects of the following BESS parameters are evaluated: energy capacity, rated charging/discharging power and “round-trip” efficiency. Also, the following system parameters are studied: marginal cost (MC) of electric energy and the 24 hour load profile of the feeder. An optimal battery charging/discharging methodology is described and the BESS performance is quantified as a function of the differential cost of energy (DCE), representing the difference between the cost of energy purchased to charge the BESS and the cost of energy delivered, (sold). The simulation results prove that the use of BESS can be beneficial and yielding significant savings, if the battery size is sufficiently large and the battery efficiency is high enough.

## *3.1 Energy Purchases Shifting Problem Definition*

The price of the electric energy varies widely from hour to hour. Regionally, the energy market allows real-time dynamic pricing, also called **locational marginal price (LMP)** as was stated in Ch. 2.3.1, the price characteristic for each location reflects the cost of the resources needed to meet the next increment of load in that location. From the Figure 3-1, one learns that the price of the electricity and the demand are correlated [26]. The higher demand will cause higher price. Many factors influence the LMP. One of the major influences is the fuel cost. As stated in the Ch. 2.2.1, different energy sources command different fuel costs. This study is based on the assumption that the curve describing the Marginal Cost (MC) of the energy consumed by the studied feeder loads is a reduced scale replica of the MC that corresponds to a region large enough to include power plants, substations and a complex transmission and distribution system with multiple feeders. The energy purchases shifting problem can be defined as using BESSs to charge energy during low demand (low price) and discharge during the high demand (high price) in order to achieve the economic benefit. To demonstrate the optimal battery charging/discharging methodology, it is necessary to know the parameters that characterize the BESS as well as the 24 hours load curve and the MC that applies to the studied feeder.

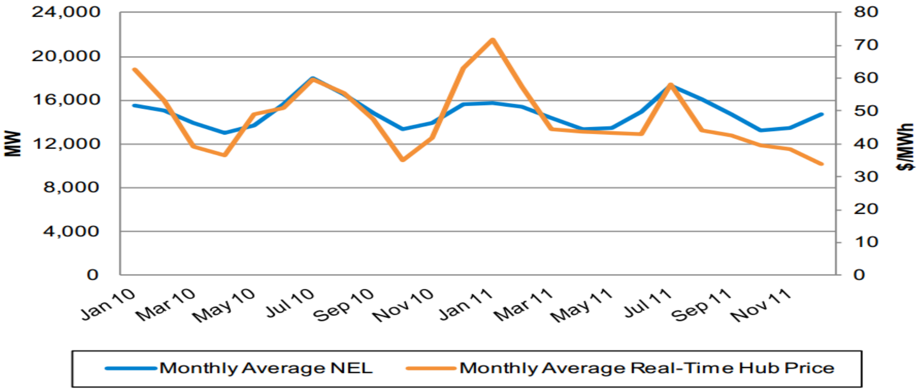


**Figure 3-1** The monthly average net energy for load (NEL) and real-time Hub prices [26].

## *3.2 BESS and Distribution System Data Preparation*

The economic feasibility of a BESS is a function of:

- BESS rated charging/discharging power, $\mathbf{P_{Bmax}}$,
- BESS maximum discharging time $\mathbf{T_B}$, under the rated power $P_{Bmax}$,
- BESS maximum discharging capacity, $\mathbf{Q_B}$,
- BESS round-trip efficiency, $\boldsymbol{\eta_B}$,
- BESS life span, measured in cycles, **CL**.
- Marginal Cost of Electric Energy
- Load Profile

This work presents an organized method for evaluating the combined impact of those quantities on cost savings. The analysis of electrical and economic quantities that govern energy storage and charging/discharging operation is presented.

### A. Marginal Cost of Electric Energy

In Figure 3-2a are presented actual measurements of electric energy covering a region with a total load that varies from 1600 MW to 7250 MW [54]. The curves given in Figure 3-2b summarize the three MC curves used in this work as representative of the MC of electric energy, ($/MWh), versus the total power, (MW), delivered to the feeder loads [Appendix B for MATLAB code]. These three curves are the best fit curves, representing three conditions: A - low demand, B - medium, and C - very high demand, described by equation

$$\boldsymbol{K_W = \alpha + \beta P^{\gamma}} \qquad \textbf{(\$/MWh)} \tag{3-1}$$

where P is the total active power (MW) supplied to the end users served by the BESS and the parameters $\alpha$ , $\beta$ and $\gamma$ are given in Table 3-1. This study is focused on a single feeder, 15 kV class with a maximum loading limited to $\mathbf{S_L}$=10 MVA.

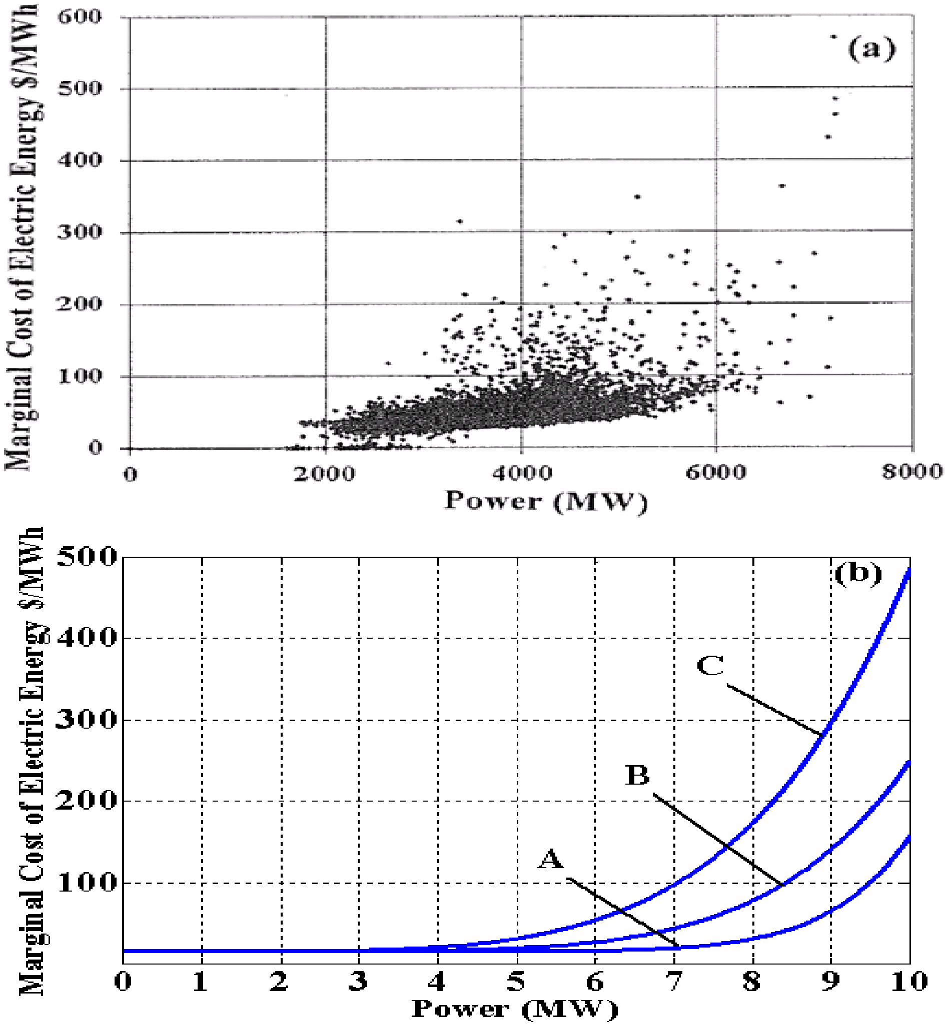


**Figure 3-2** Electric energy marginal cost vs. supplied power: (a) actual measurements [54], (b) best fit curves.

**Table 3-1:** Marginal Cost Parameters

| Curve | $\alpha$ ($\$/MWh$) | $\beta$ ($\$/(MW)^{\gamma+1}h$) | $\gamma$ |
|---|---|---|---|
| A | 15.00 | $1.58 \cdot 10^{-8}$ | 9.95 |
| B | 15.00 | $2.75 \cdot 10^{-4}$ | 5.93 |
| C | 15.00 | $5.89 \cdot 10^{-3}$ | 4.90 |

This approach to the feeder MC has a limitation. Actually, there is not a strong correlation between the regional MC and the MC that corresponds to instantaneous demand on a particular feeder. The available data on MC is limited to regions that include hundreds of thousands of customers. However, if the studied feeder belongs to a region where relatively homogenous clusters of customers constitute the majority of the loads (advanced proliferation of electric vehicles represents such a case) the results of this work may provide valuable insight in the benefits derived from the use of BESS.

## B. The 24 hour Load Curve

The range of possible shapes of the time variation of total feeder power P(t) in hours (h) is shown in Figure 3-3b [Appendix B for MATLAB code]. These shapes closely represent actual feeder load profiles, Figure 3-3a. In this study the shape is parametrically defined using the expression:

$$\boldsymbol{P(t) = P_{min} + a\left\{1 - \exp\left(1 - \frac{(t-b)^2}{2c^2}\right)\right\}} \ \textbf{(MW)} \tag{3-2}$$

where the parameters a, b and c are given in Table 3-2 for the three curves depicted in Figure 3-3b. The curves are constrained within the extremes $P_{min}$ and $P_{max}$.

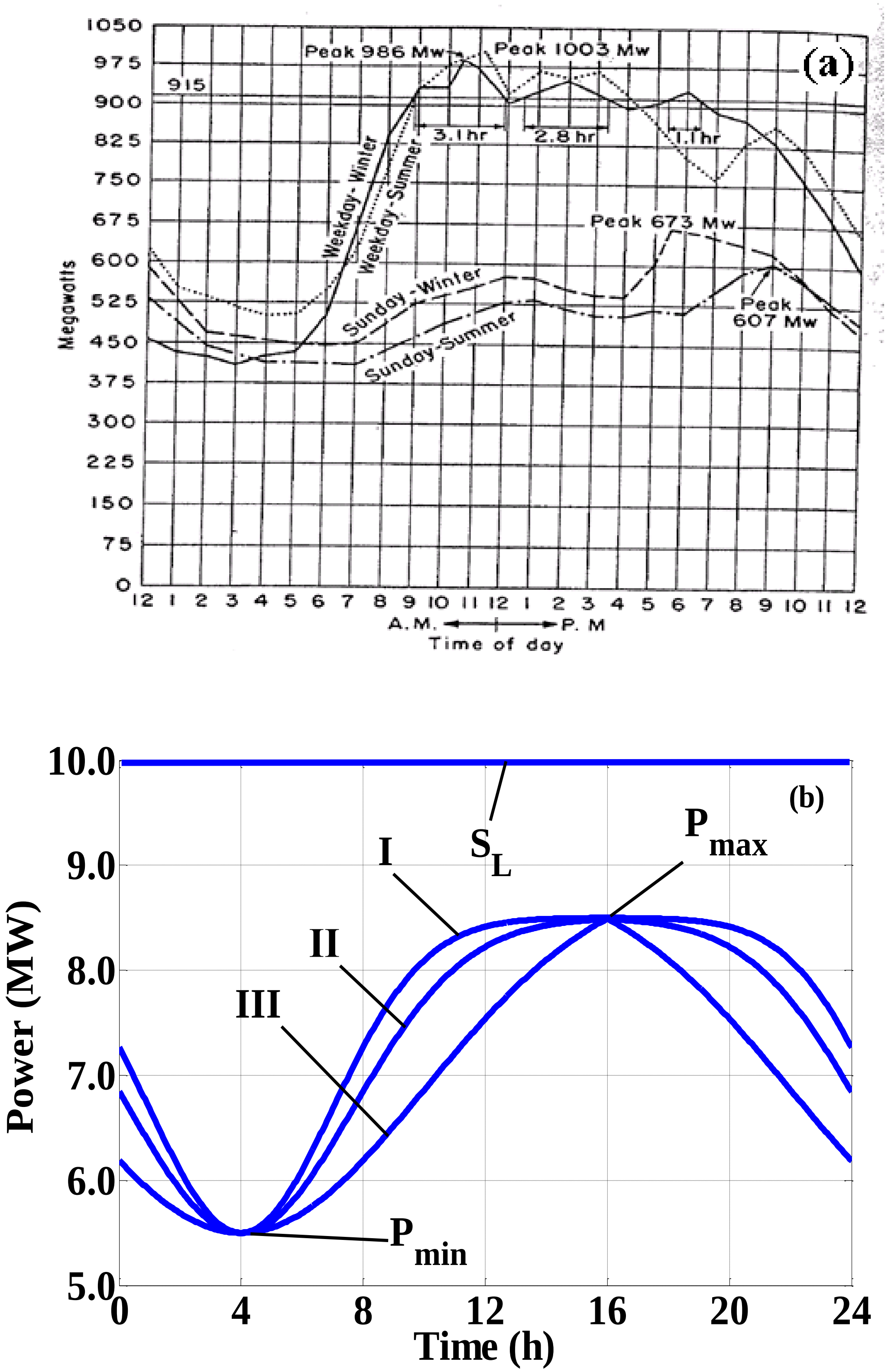


**Figure 3-3** (a)Actual Load demand (24 hours) [55], (b) 24 hour Load Curves, SL=10MVA,$P_{max}/S_L$ =0.85 W/VA, $P_{min}/S_L$ =0.55 W/VA.

**Table 3-2:** Parameters Characterizing the 24 h Load Curves.

| Curve | a (MW) | b (h) | | c (h) |
|---|---|---|---|---|
| | | $0 \le t \le 16$ | $16 \le t \le 24$ | |
| I | 3.00 | 4.00 | 28.00 | 3.00 |
| II | 3.01 | 4.00 | 28.00 | 3.67 |
| III | 3.46 | 4.00 | 28.00 | 6.00 |

## C. Battery characteristics

The **capacity** $\boldsymbol{Q_B = \underset{max}{W_{BDCh}}}$ (MWh), is the maximum energy that can be delivered from a full charge, without damaging the unit. This energy can be supplied over the time $\boldsymbol{T_B = \underset{max}{W_{BDCh}} / P_{Bmax}}$.

where the power $\boldsymbol{P_{Bmax}}$ (MW) is the **maximum output power** that can be continuously sustained for the time $\boldsymbol{T_B}$. The battery life will be limited to a certain number of cycles **CL** determined by the type of battery. The efficiency is given by $\boldsymbol{\eta_B = W_{BDCh}/W_{BCh}}$, (round-trip efficiency) where $\boldsymbol{W_{BCh}}$ is the energy supplied to the battery during the charging time and $\boldsymbol{W_{BDCh}}$ is the energy delivered while returning to the initial battery charge.

Three major variables were considered in this study: $\boldsymbol{\eta_B, T_B}$ and $\boldsymbol{W_{BCh}}$. The remaining parameters were normalized using a base values $\boldsymbol{W_T}$ (MWh), the total 24 h energy supplied to the loads that benefit from the BESS, and $\mathbf{S_L}$ (MVA), the maximum apparent power that can be continuously supplied by the feeder under normal conditions (see Figure 3-3).

## *3.3 Optimal Battery Charging/Discharging Methodology*

A typical 24 h load curve is presented in Figure 3-4. The impact of the charging/discharging process is illustrated by the shaded areas that represent the charging/discharging energy. In the absence of the BESS the feeder load would be represented by the bold blue line. During the charge process the feeder load increases with the energy transfer to the BESS indicated by the shaded area between $t_0$ and $t_e$. Note that $P(t = t_0) = P(t = t_e)$. During $t_0' \leq t \leq t_e'$ the battery is supplied with the power $\mathbf{P_{Bmax}}$. During $t_0 \leq t \leq t_0'$ and $t_e' \leq t \leq t_e$, $\mathbf{P_B} < \mathbf{P_{Bmax}}$. The charging strategy is such that the area $A_1 = W_{BCh}$ occupies a region as close as possible to $\mathbf{P_{min}}$ in order to minimize the cost of energy purchased. The discharge takes place for $t_b \leq t \leq t_f$ The discharge area $A_2 = W_{BDCh} = \eta_B W_{BCh}$is located as close as possible to $\mathbf{P_{max}}$ in order to maximize the price of the energy delivered.

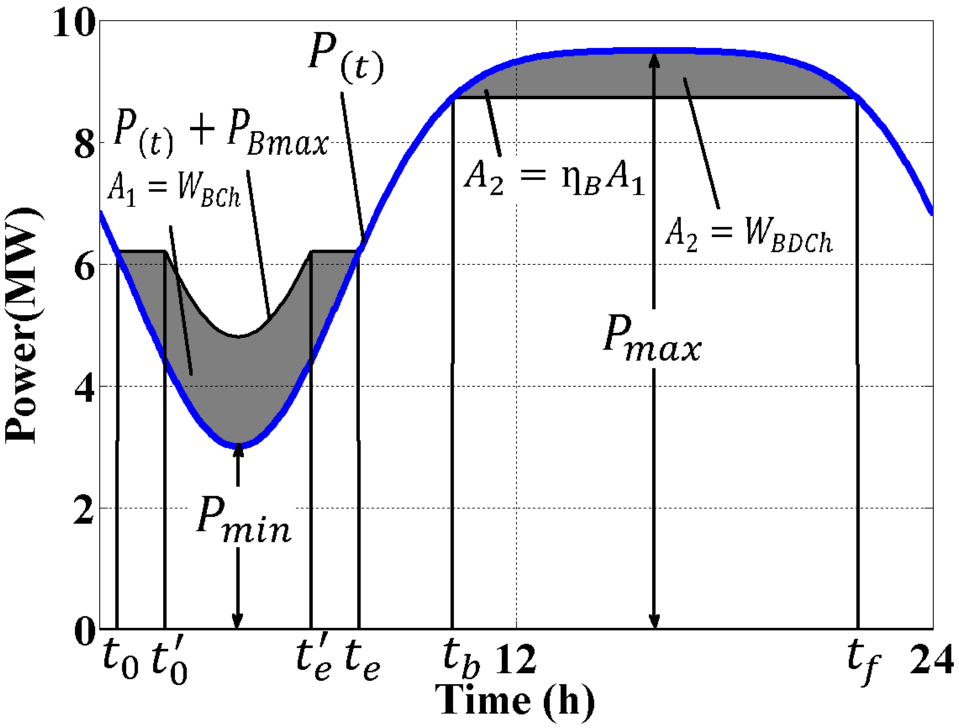


**Figure 3-4:** The optimal charging/discharging methodology.

The benefit of energy shifting was evaluated using the DCE (Differential Cost of Energy):

$$\mathbf{DCE}=\boldsymbol{K_{WD}-K_{WC}}\ \mathbf{(\$)} \tag{3-3}$$

where

$$\boldsymbol{K_{WC}=\int_{t_0}^{t_0'}P(t_0)K_W\big(P(t_0)\big)dt+\int_{t_0'}^{t_e'}[P(t)+P_{Bmax}]K_W(P(t)+P_{Bmax})dt}$$

$$\boldsymbol{+\int_{t_e'}^{t_e}P(t_0)K_W(P(t_0))dt-\int_{t_0}^{t_e}P(t)K_W(P(t))dt} \tag{3-4}$$

is the extra cost of charging energy, and

$$\boldsymbol{K_{WD}=\int_{t_b}^{t_f}P(t)K_W(P(t))dt-\int_{t_b}^{t_f}P(t_b)K_W\big(P(t_b)\big)dt} \tag{3-5}$$

is the reduced cost of the energy delivered to feeder's loads by the BESS discharging energy that shifts the on-peak power.

The actual battery charging/discharging process takes place in real time, but the start and stop times are planned in advance based on the 24 hour load curve prediction. This same approach may be used for a load curve with multiple peaks as shown in Figure 3-5[Appendix B for MATLAB code]. The algorithm operates by identifying the times of relative minimum energy cost for charging and of maximal energy cost for discharging.

Minimum MC occurs at $P_{min}$ and maximal MC occurs at $P_{max}$. From these starting points the charging and discharging times are expanded as shown in the shaded areas of Figure 3-4, subject to the constraints of BESS energy capacity $\underset{\boldsymbol{max}}{\boldsymbol{W_{BDCh}}}$, $A_2=\eta_B A_1$, $P(t_o)=P(t_e)$ and $P(t_b)=P(t_f)$ The power $P(t_0)$ is increased incrementally, (meaning an earlier start of the battery charging) and the charging/discharging periods are expanding. At each increment the DCE is calculated and plotted vs. the variables and parameters as illustrated in the results. With unlimited BESS capacity there is an optimum time $t_0$, which provides maximum economic benefit. Thus if the battery total charging time is increased to allow the storage of more energy, the DCE will peak at some point, after which the DCE will start to decrease. Theoretically it is possible, if the battery capacity is large enough, to reach the situation where $\mathrm{t_e} = \mathrm{t_b}$. One may call this condition saturation. Under this condition, any additional battery capacity will not be used.

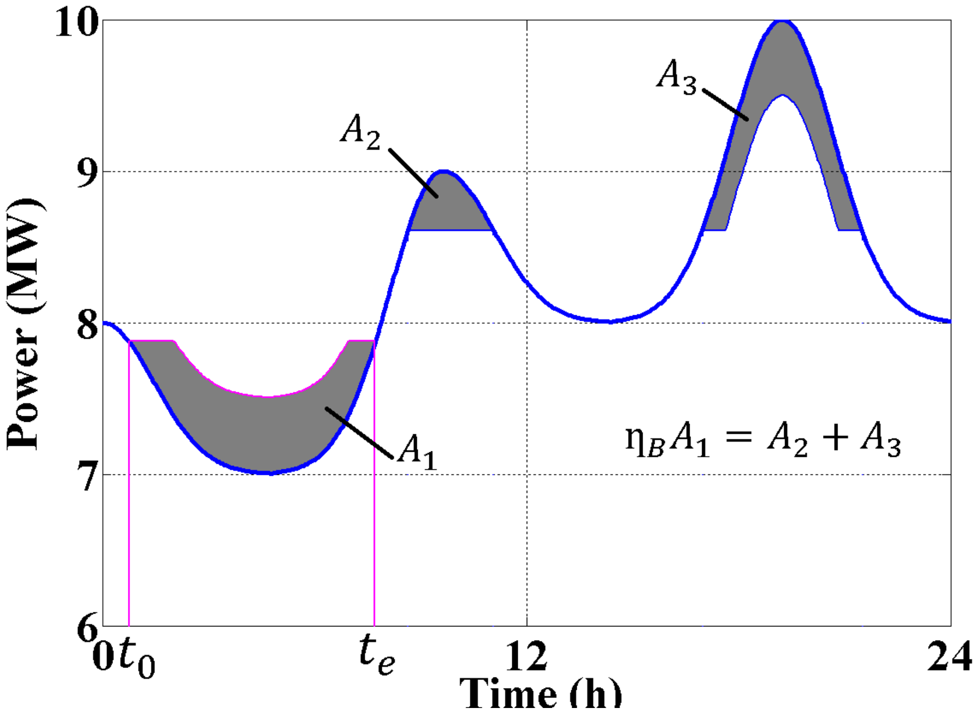


**Figure 3-5** The charge/discharge method for a load curve with two peaks.

The charging process is finished at $t_e$ before the discharging starts. This approach assumes perfect day-ahead knowledge of the load and MC curves, so that the charging/discharging commands may be determined in advance. In practice updated estimates of the load and MC curves may be used to adjust the BESS control in real time.

## 3.4 Simulation Results and Discussion

This section presents DCE results as a function of the variables $\boldsymbol{\eta_B}, \boldsymbol{T_B}$ and $\boldsymbol{W_{BCh}}$ while $P_{Bmax}/S_L = 0.05$ W/VA, $\mathbf{S_L}$=10 MVA.

In Figure 3-6 are summarized the DCE curves as functions of the discharging time $\boldsymbol{T_B}$, with the efficiency $\boldsymbol{\eta_B}$ as a parameter[Appendix B for MATLAB code]. The BESS is assumed to be fully charged during the low demand and discharged to a reasonable, non-distractive limit during the peak demand. From these curves one learns that for a given 24 h load curve and battery round trip efficiency, there is a battery size, characterized by a $\boldsymbol{T_B}$, that provides the highest DCE. The DCE increases as the MC describes a more extreme variation of price with demand. For example, if $\boldsymbol{\eta_B} = 70\%$ and $\boldsymbol{T_B}$=5 h, the DCE = \$ 520/day for curve A, and \$ 1,700/day for curve C. In addition, Figure 3-6c tells the BESS will reach the peak DCE when $T_B$=4 h under $\eta_B$=55% while $T_B$=7 h under $\eta_B$=85%. This illustrates that the BESS will reach the peak DCE much easier when the $\eta_B$ is low than the $\eta_B$ is high due to the inefficiency of BESS. Similarly, comparing with a high efficiency condition, the BESS is easier to go into the saturation when there is a low efficiency. In Figure 3-6c, one can see that the BESS will go into saturation when $T_B$=5.5 h under $\eta_B$=55% but $T_B$=8 h under $\eta_B$=85%.

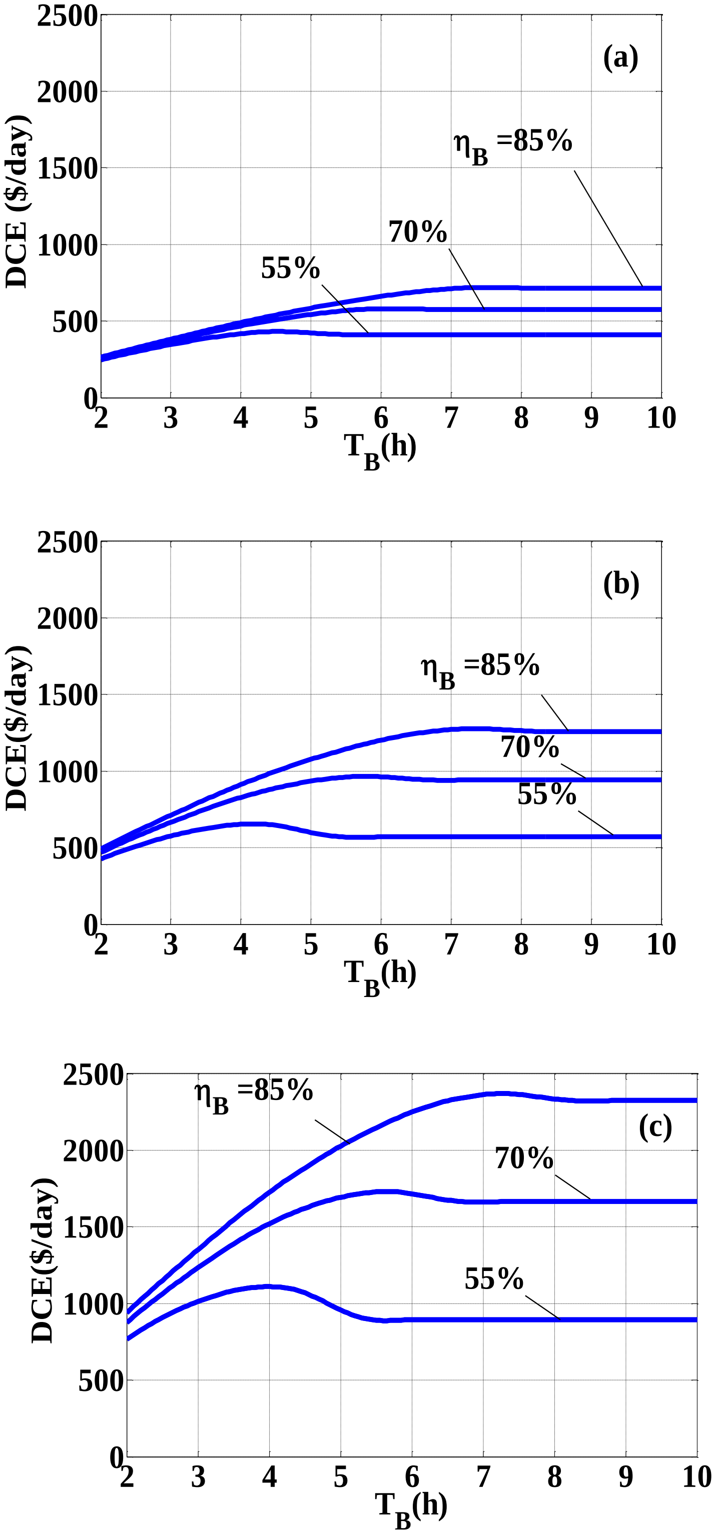


**Figure 3-6** Differential cost of energy (DCE) vs. $T_B$ with round-trip efficiency $\eta_B$ as parameter, 24 hour Load Curve Type I, $\boldsymbol{P_{max}/S_L = 0.85}$W/VA1, $\boldsymbol{P_{min}/S_L = 0.55}$W/VA, $\boldsymbol{P_{Bmax}/S_L = 0.05}$W/VA, $S_L$=10 MVA, $\boldsymbol{W_T = 181.8MWh}$, (a) marginal cost curve A, (b) marginal cost curve B, (c) marginal cost curve C.

The graphs presented in Figure 3-7 provide information on the effects of the load curve's maximum power $P_{max}$, as well as the effect of the marginal cost[Appendix B for MATLAB code]. Similar to the Figure 3-6, the BESS is assumed to be fully charged during the low demand and discharged to a reasonable, non-distractive limit during the peak demand. For the marginal cost curve A, the DCE is as low as \$1800/day for $\eta_B = 85\%$ and $P_{max}/S_L = 0.95$. For the same parameters a jump to \$3900/day is found for the marginal cost curve C. A high $P_{max}$ value will also help to contribute a high DCE.

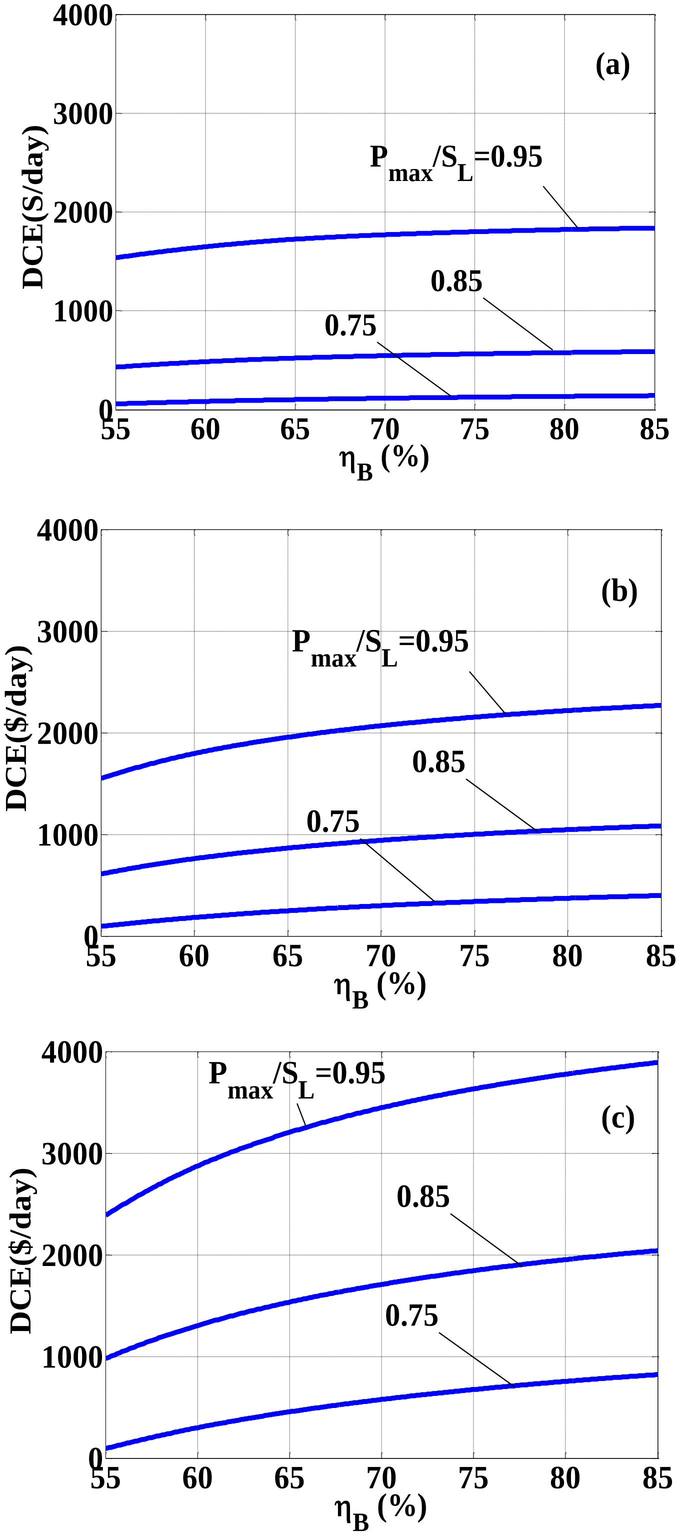


**Figure 3-7** Differential cost of energy (DCE) vs. the round trip efficiency $\boldsymbol{\eta_B}$ with feeder loading ($\boldsymbol{0.75 \le P_{max}/S_L \le 0.95}$W/VA) as parameter, 24 hour Load Curve Type I, $\boldsymbol{P_{min}/S_L = 0.55}$W/VA, $\boldsymbol{P_{Bmax}/S_L = 0.05}$W/VA t, $\boldsymbol{T_B = 5.0}$h, $S_L$= 10 MVA, $\boldsymbol{W_T = 181.8\,MWh}$. (a) marginal cost, curve A, (b) marginal cost, curve B, (c) marginal cost, curve C.

It is to be expected that lower efficiency will lower the DCE. In Figure 3-8 is presented the DCE versus the BESS input energy $W_{BCh}$, normalized to the total 24 h energy demand $W_T$ under load condition I with $\eta_B$ a parameter[Appendix B for MATLAB code]. The effect of $\eta_B$ is dramatic: for $\eta_B$= 70%, the peak savings occurs for BESS input energy representing 1% of the total feeder energy and for a really low value of $\eta_B$ (55% in the Figure 3-8) the DCE may always be negative due to the energy losses cost during charging/discharging.

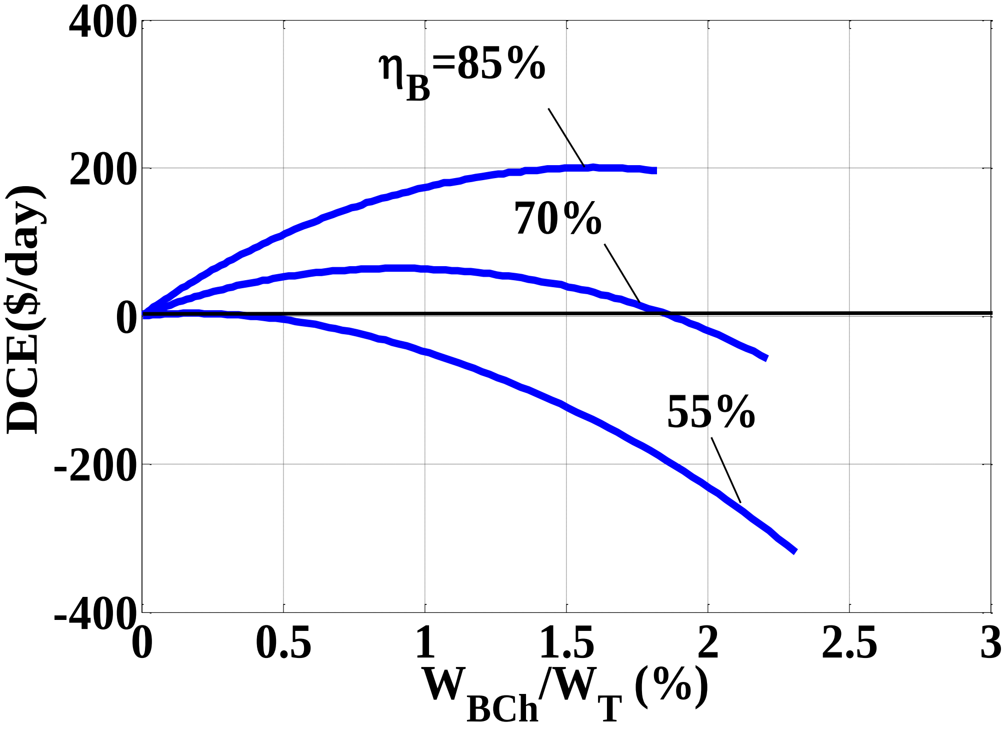


**Figure 3-8** DCE vs. $\boldsymbol{W_{BCh}/W_T}$, round trip efficiency $\boldsymbol{\eta_B}$ as parameter. 24 hour Load Curve Type I, $\boldsymbol{P_{max}/S_L}$=0.70 W/VA, $\boldsymbol{P_{min}/S_L}$ =0.60 W/VA, $\boldsymbol{P_{Bmax}/S_L}$=0.05 W/VA, $\boldsymbol{T_B = 5.0}$h, $S_L$=10 MVA, $\boldsymbol{W_T = 160.8MWh,}$ marginal cost curve C.

Figure 3-9 and Figure 3-10 reveal the effect of battery charging energy and the effect of the 24 hour load curve. It is seen that curve III, that requires less energy during the on-peak hours, yields lower DCE, while a load curve that requires higher energy during on-peak hours (curve I) will produce a higher DCE.

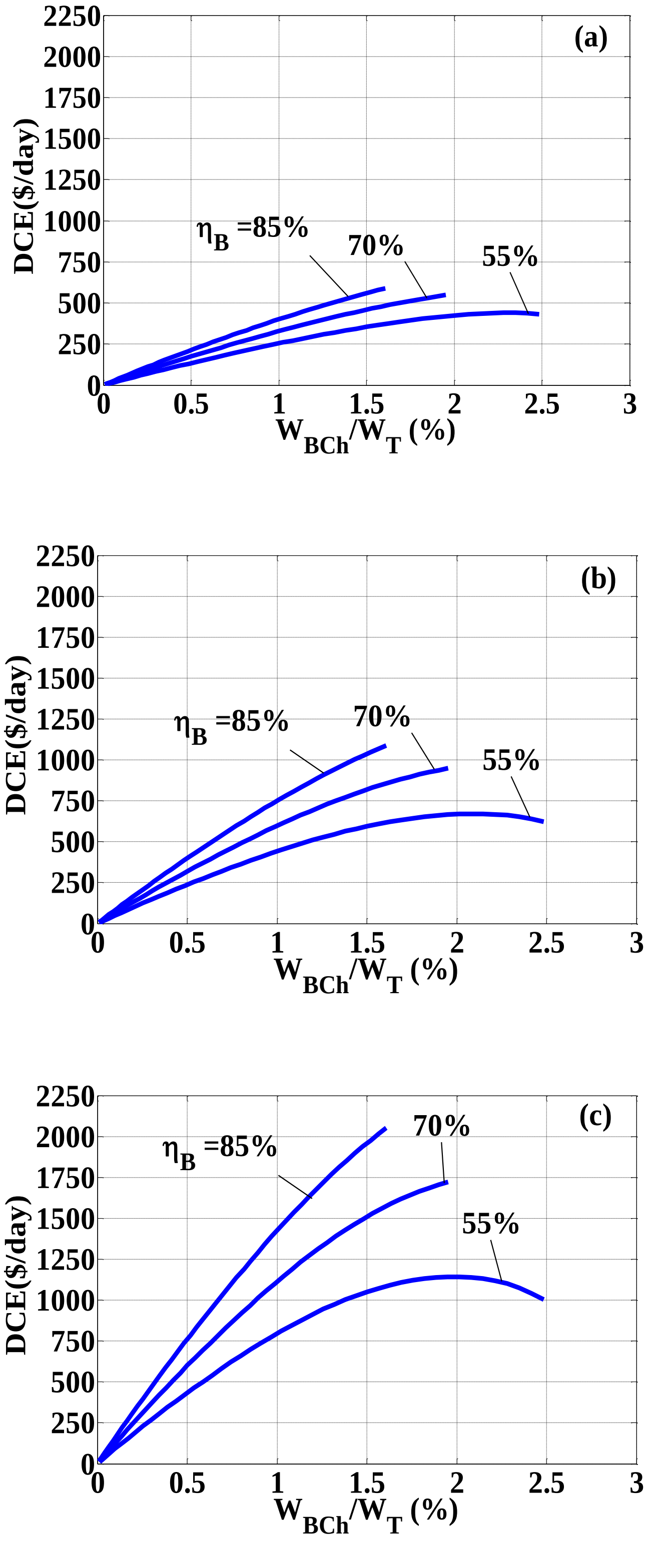


**Figure 3-9** DCE vs. $W_{BCh}/W_T$. The 24 hour Load Curve Type I, Round Trip Efficiency $\eta_B$ is parameter. $P_{max}/S_L = 0.85$ W/VA, $P_{min}/S_L = 0.55$ W/VA, $P_{Bmax}/S_L$= 0.05W/VA, $T_B = 5.0$h, $S_L$=10 MVA, $W_T = 181.8$ MWh. (a) marginal cost, curve A, (b) marginal cost, curve B, (c) marginal cost, curve C.

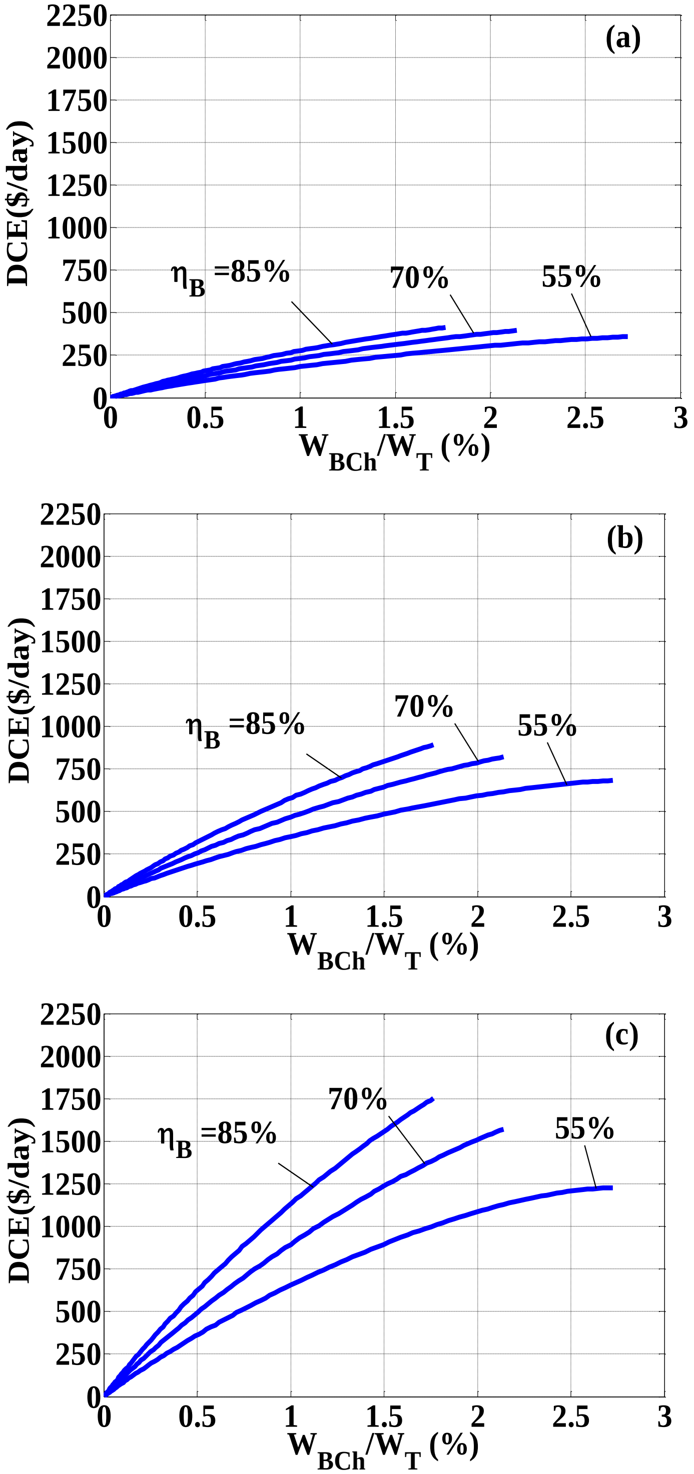


**Figure 3-10** DCE vs. $W_{BCh}/W_T$. The 24 hour Load Curve Type III. Round trip efficiency $\boldsymbol{\eta_B}$ is parameter. $\boldsymbol{P_{max}/S_L = 0.85}$ W/VA, $\boldsymbol{P_{min}/S_L = 0.55}$ W/VA, $\boldsymbol{P_{Bmax}/S_L}$= 0.05W/VA, $\boldsymbol{T_B = 5.0}$h, $S_L$=10 MVA, $\mathbf{W_T = 165.7MWh}$. (a) marginal cost, curve A, (b) marginal cost, curve B, (c) marginal cost, curve C.

The DCE (Differential Cost of Energy) was calculated also for one year using a probabilistic approach. The instantaneous load profile for a typical year is shown in Figure 3-11 [see Appendix B for MATLAB code].

The 365 days load variation assumed a 24 hour Load Curve Type I. Each day has a Load Curve characterized by $\mathrm{P_{min}}$ and $\mathrm{P_{max}} = \mathrm{P_{min}} + a$, as is shown in (3-2). The peak load for each day *n* was assumed to follow a uniform distribution based on the expression

$$\boldsymbol{P_{max}(n) = P_{max}^{max}(n)[0.65 + 0.35rand(0,1)]} \tag{3-6}$$

where $P_{max}^{max}(n)$ defines the upper boundary of $P_{max}(n)$; it is a curve that peaks during the summer and winter times. One can see the maximum peak value for the 24 hour load curve is varying randomly in a range: $0.65P_{max}^{max}(n) \leq P_{max}(n) \leq P_{max}^{max}(n)$. The yearly maximum value of $P_{max}^{max}(n)$ occurs in July and is termed $P_M$.

$P_{min}(n)$ is computed in the same manner:

$$\boldsymbol{P_{min}(n) = P_{max}^{min}(n)[0.65 + 0.35rand(0,1)]} \tag{3-7}$$

where $P_{max}^{min}(n)$ defines the upper boundary of $P_{min}(n)$. $W_{BCh}/W_T$ was adjusted daily for maximum DCE.

The results for the annual maximum DCE are summarized in Table 3-3 for the three MC curves, Figure 3-2, and the power $\mathrm{P_M}$.

Table 3-3 results show the annual maximum DCE is depended on MC curve as well as $\mathrm{P_M}$ , the higher $\mathrm{P_M}$ will give higher DCE. For example, the annual maximum DCE is about 11 times more for $\mathrm{P_M}$ =10MW under MC curve A than for $\mathrm{P_M}$ =8MW under MC curve A. The MC which describes a more extreme variation of price with demand will also contribute to the DCE. For instance, the annual maximum DCE is about 9 times more for $\mathrm{P_M}$ =9MW under MC curve C than for $\mathrm{P_M}$ =9MW under MC curve A.

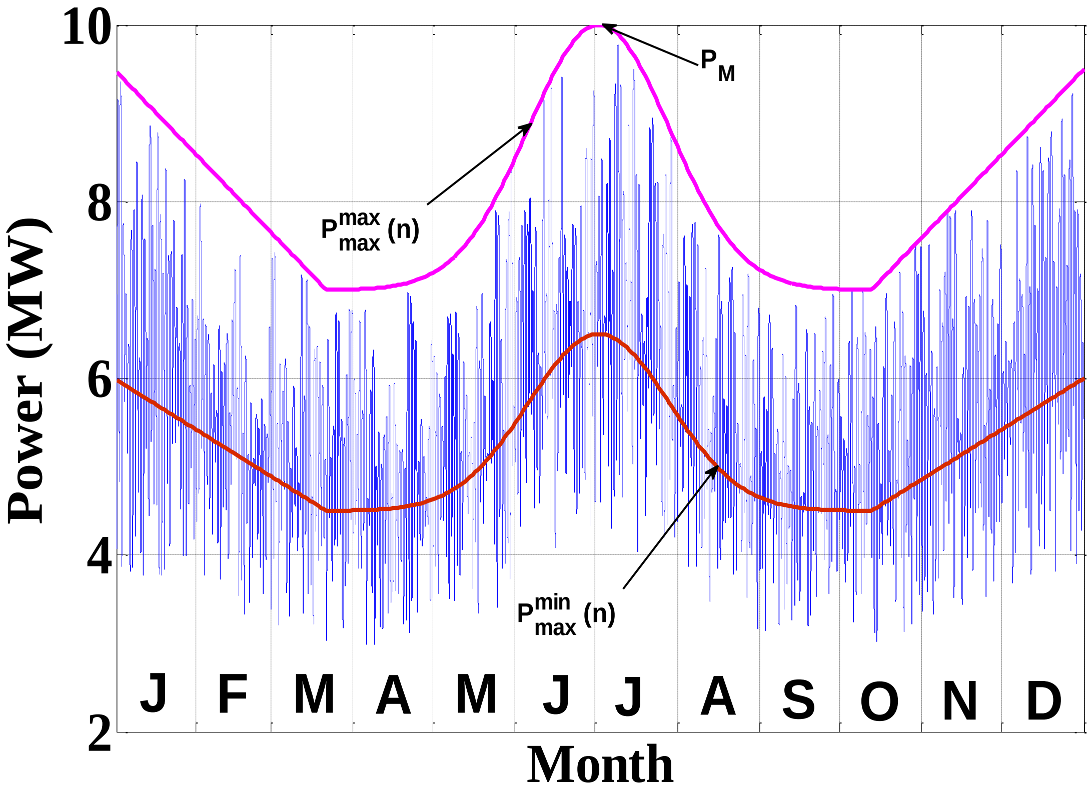


**Figure 3-11** Annual load variation.

**Table 3-3:** Annual Maximum DCE

| MC Curve | Annual Maximum DCE ($) | | |
|---|---|---|---|
| | $P_M$=10MW | $P_M$=9MW | $P_M$=8MW |
| A | 57,702 | 15,389 | 4,688 |
| B | 139,260 | 62,951 | 33,230 |
| C | 296,000 | 154,860 | 92,545 |

### 3.5 Conclusions

This chapter reports the results of a study of the differential cost of energy obtained by shifting the energy purchases from on-peak times using a BESS. The study considered a 15 kV class feeder, rated at 400 A maximum current. The optimal charging/discharging methodology helps to get the maximum DCE, the annual maximum DCE given certain conditions can be calculated to determine the feasibility of BESS under the consideration of the capital cost of BESS and distribution systems characteristics. The overall results show a large amount of potential benefit can be got ($ 296,000 in table III under MC curve C and $P_M$=10MW) and illustrate impacts of characteristics of BESS ($P_{Bmax}$, $T_B$, $\eta_B$, CL) , load profile ($P_{max}$, $P_{min}$, $P(t)$). The conclusions can be summarized as following:

1. If the marginal cost of energy is directly related to the instantaneous load demand of the feeder, the annual benefit may be substantial.
2. A high $\eta_B$ BESS is especially economical attractive under the high $P_{max}$ and low $P_{min}$ with an extreme variation of MC.
3. A high DCE can be obtained if the peak energy is large.

# 4. Distribution Feeder Deferral

This chapter deals with the economics of distribution feeder deferral. The scope of the presentation submitted in this chapter is to determine the conditions that lead to the maximum economic benefit which can be provided by the BESSs. The potential economic benefit of feeder deferral is evaluated using the method explained in Ch. 2.3.3. Sensitivity studies that reveal the effects of BESS parameters such as BESS cycle life, capacity and round-trip efficiency combined with the feeder length, as well as the feeder load growth rate, are reported.

## *4.1 Distribution Feeder Deferral Problem Definition*

As the apparent power demanded by the costumers increase, its peak value reaches the point where the thermal stresses caused to distribution transformers, voltage regulators and conductors exceed the recommended levels and the life-span of the involved devices can be compromised.

Nearing such conditions can be avoided by using one of two methods:

1. Installation of additional feeders as shown in Figure 4-1 and eventual expansion of the involved substation.

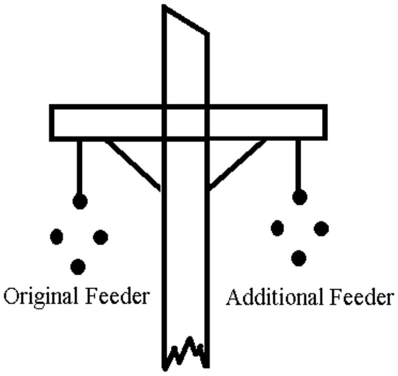


**Figure 4-1** Additional Feeder Installation in the Distribution System, (Spaced Cable).

2. Incorporation of storage devices such as BESSs, pumped hydro energy storage or compressed air storage systems located near the areas of load growth. The storage device will supply a portion of the feeder load during peak times, thereby keeping the feeder apparent power within normal operating limits. The energy storage program can also include renewable energy sources such as wind farms and photovoltaic generation. This method makes the feeder's installation deferral possible.

## 4.2 Distribution Feeder Deferral: Economic Benefit

Knowing the present values of the cost of installation of the additional feeder with and without deferral, as well as the present value of the BESSs, it is possible to compute the Net Present Value at a given point in time, which is the benefit derived from feeder deferral. The detailed analysis and procedures are described below.

### 4.2.1 Distribution System Load History

Figure 4-2, presents the system loading history of a radial distribution feeder supplying a load with the maximum apparent power characterized by the annual load growth rate $\boldsymbol{r_a}$%. The maximum thermal stress is correlated to the maximum apparent power demand $\boldsymbol{S_{max}}$ (MVA), whose analytical expression is:

$$\boldsymbol{S_{max} = S_a e^{ln(1+\frac{r_a}{100})t}} \qquad \textit{\textbf{(MVA)}} \qquad (4\text{-}1)$$

where

$\boldsymbol{S_a}$= the maximum apparent power demand (MVA) at the year 0 ($\boldsymbol{t}$=0).

$\boldsymbol{t}$= time (year).

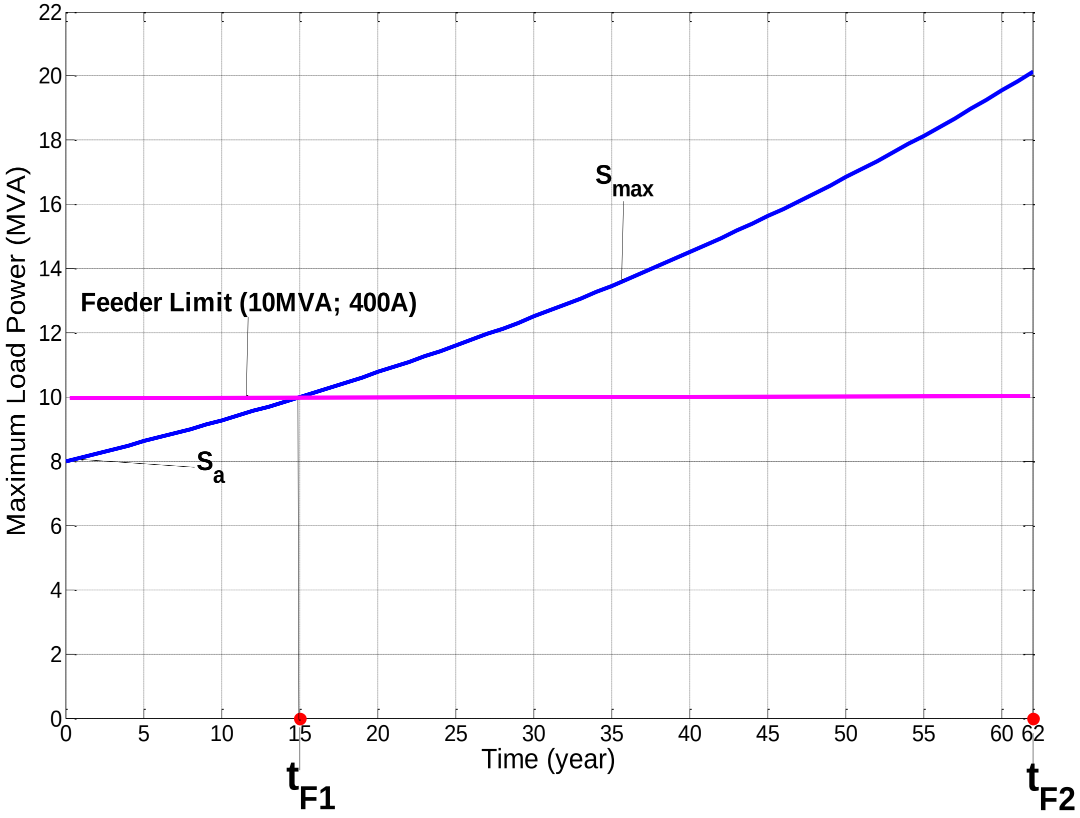


**Figure 4-2** Load History: $r_a\% = 1.5\%/year$; $S_a = 8MVA$.

One will notice that at $t$=0, $S_a$=8 MVA and after 15 years, at the time $t_{F1}$, $S_{max}$=$S_L$=10 MVA, which is the critical level when the thermal stress will start causing excessive damage to the equipment.

The two proposed mitigation methods will lower the new $S_{max}$ level. Ideally, after adding an additional feeder, the feeders will be able to share the same amount of load so that the maximum demand withstood by the system may be doubled. However, in the year $t_{F2}$, $S_{max}$=2$S_L$=20 MVA, and the two feeders reach their apparent power limit. Hence, another (3rd) additional feeder must be ready to operate at $t_{F2}$.

In reality, the two feeders will supply different customers hence never share the exact same amount of load. Therefore, the third additional feeder commission starting time $\boldsymbol{t_{F2}}$ may need to be moved to an earlier time.

Assuming $\boldsymbol{S_a}$=8 MVA; $\boldsymbol{r_a}$%=1.5%/year and that the feeder thermal limit can not exceed $\boldsymbol{S_{max}}$=$S_L$=10 MVA, the time $\boldsymbol{t_{F1}}$ is calculated using the expression:

$$\boldsymbol{t_{F1}} = \left[\frac{\boldsymbol{ln(S_L/S_a)}}{\boldsymbol{ln(1+r_a/100)}}\right] = \left[\frac{\boldsymbol{ln(10/8)}}{\boldsymbol{ln(1+0.015)}}\right] = \boldsymbol{15\ years} \tag{4-2}$$

## 4.2.2 Feeder Installation Model

As stated in the previous sub-section, an additional feeder construction in the year $\boldsymbol{t_{F1}}$ will upgrade the system. Therefore, at the year $\boldsymbol{t_{F1}}$**,** the two feeders will sustain a maximum demand of 10MVA with a capacity of 20MVA. The new system maximum load evolution is presented in Figure 4-3.

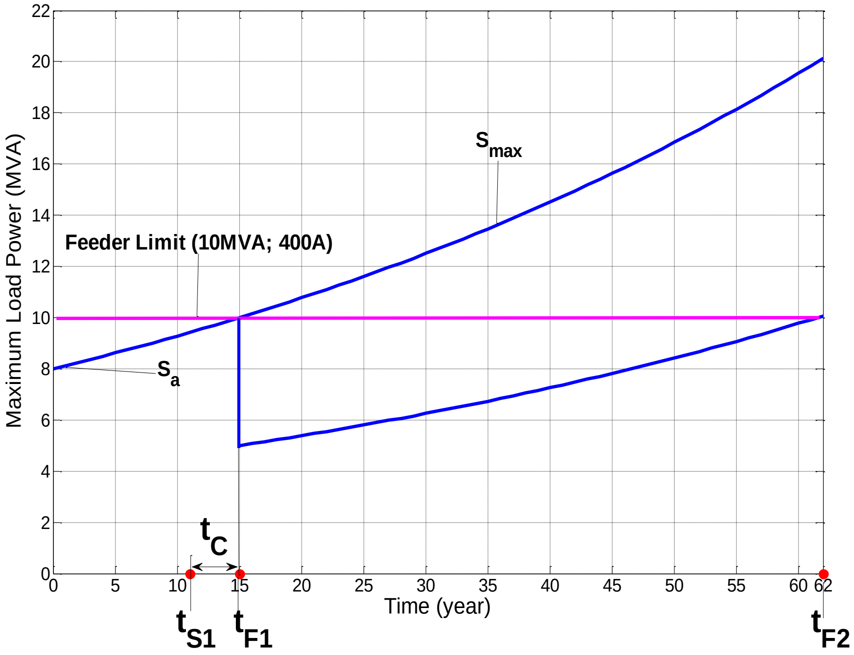


**Figure 4-3** Load History with an Additional Feeder Construction.

It can be noticed that the feeder requires the installation time $\boldsymbol{t_C}$ given by the expression:

$$\boldsymbol{t_C = t_{F_1} - t_{S_1}} \qquad \textbf{(year)} \qquad (4\text{-}3)$$

where

$\boldsymbol{t_{S_1}}$=the additional feeder installation start time.

$\boldsymbol{t_{F_1}}$=the additional feeder construction finish time (without deferral).

From Figure 4-3 it is learned that the additional feeder doubles the maximum ampacity of the supplying system. However, later at the year $\boldsymbol{t_{F2}}$, one of the two feeders will experience the demand limitation due to the load growth rate $\boldsymbol{r_a}$% and a$n$ additional feeder's construction is needed, for a total of three feeders..

Assuming the discount rate ($\boldsymbol{d}$), the loan interest rate ($\boldsymbol{i_R}$) and the capital cost of the feeder ($\boldsymbol{C_F}$), the annual payment for the feeder ($\boldsymbol{A_{PF}}$) can be computed using the expression [22]:

$$\boldsymbol{A_{PF} = C_F \frac{i_R(1+i_R)^{t_{Lf}}}{(1+i_R)^{t_{Lf}}-1}} \qquad \textit{\textbf{(\$/year)}} \qquad (4\text{-}4)$$

where

$\boldsymbol{t_{Lf}}$= the loan time for the feeder construction investment.

From (2-8), the present value for the $1^{st}$ additional feeder at the time $\boldsymbol{t_{S1}}$ is calculated from the following expression:

$$\boldsymbol{PV_{t_{S_1}} = PV_{t_{S_1}f_1} = \frac{(1+d)^{t_{Lf}}-1}{d(1+d)^{t_{Lf}}} A_{PF}} \qquad \textit{\textbf{(\$)}} \qquad (4\text{-}5)$$

where

$PV_{t_{S_1}}$=the total present value of the additional feeder (without deferral) at the time $t_{S_1}$.

$t_{Lf}$=the loan time for the feeder construction investment.

$t_{S_1}$=the additional feeder construction starting time.

The cash flow diagram is shown in Figure 4-4. It considers the feeder construction time ($t_C$) as well as annual loan payment ($A_{PF}$).

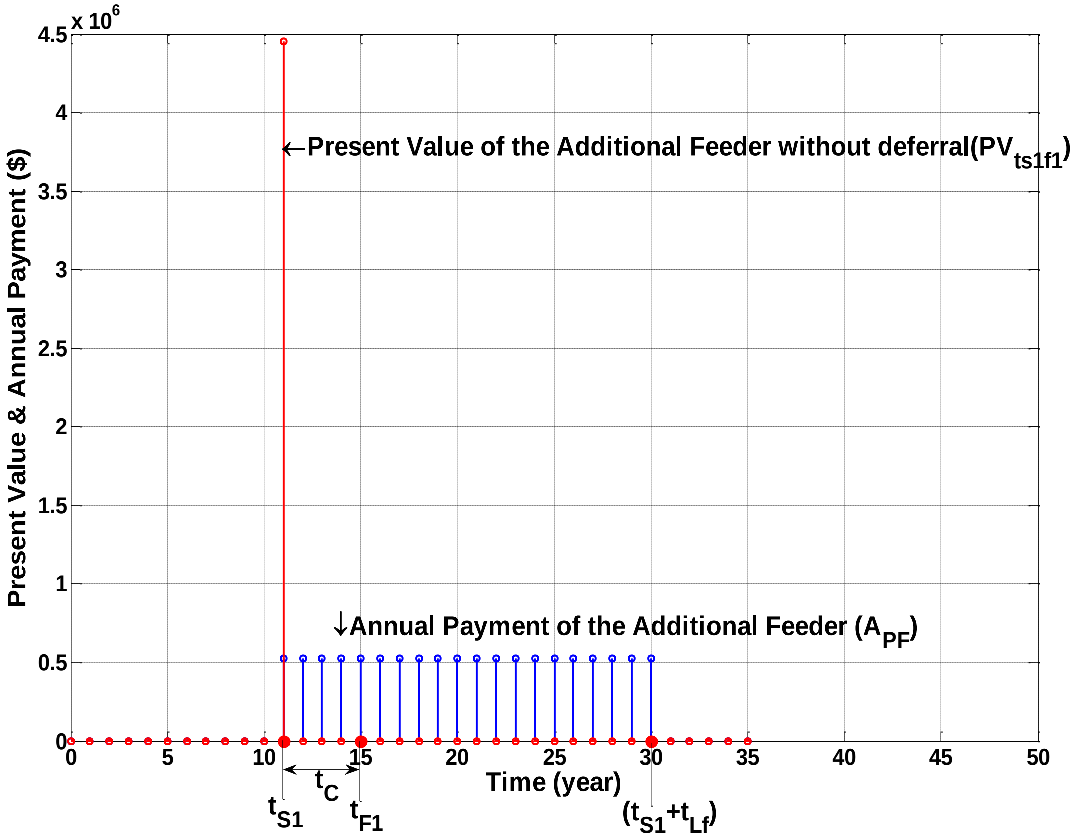


**Figure 4-4** Cash Flow Diagram for the Additional Feeder Construction.

### 4.2.3 Battery Energy Storage Model—Deferral Distribution Feeder implementation

Instead of building a new feeder at $\boldsymbol{t_{F_1}}$, a BESS is installed to defer new feeder construction. The concept is presented in Figure 4-5. This  ignores the fact that a BESS's lifetime is limited to a certain number of cycles.

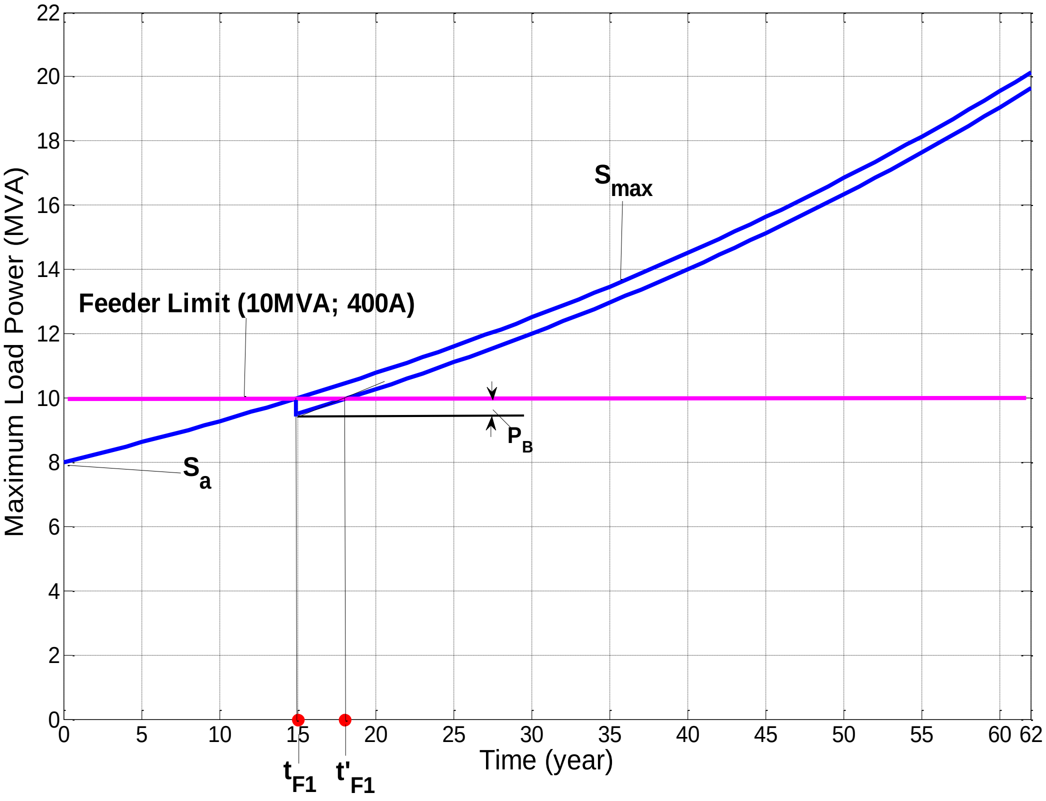


**Figure 4-5** Load History with one BESS Installation.

The BESS can be discharged at the rated power $\boldsymbol{P_B}$ during peak times, so that the critical time $\boldsymbol{t_{F1}}$ will be transferred to $\boldsymbol{t'_{F1}}$. The way to find $\boldsymbol{t'_{F1}}$ is demonstrated in Ch. 4.3. The present value of the deferred additional feeder as well as the BESS should be converted to a common timeline

$t_{S1}$ in order to compare the two investments. There are two terms in the total deferred present value expression in the year $t_{S1}$:

$$PV'_{t_{s_1}} = PV'_{t_{s_1}b_1} + PV'_{t_{s_1}f_1} \qquad (\$) \tag{4-6}$$

where

$PV'_{t_{s_1}b_1}$= the BESS's present value at the time $t_{S1}$.

$PV'_{t_{s_1}f_1}$= the additional feeder's present value with deferral at the time $t_{S1}$.

In the case described in Figure 4-5, only one BESS is used to defer one additional feeder installation. The cash flow diagram for BESS and deferred feeder relating uniform series (annual payment) to its present equivalent values is shown in Figure 4-6.

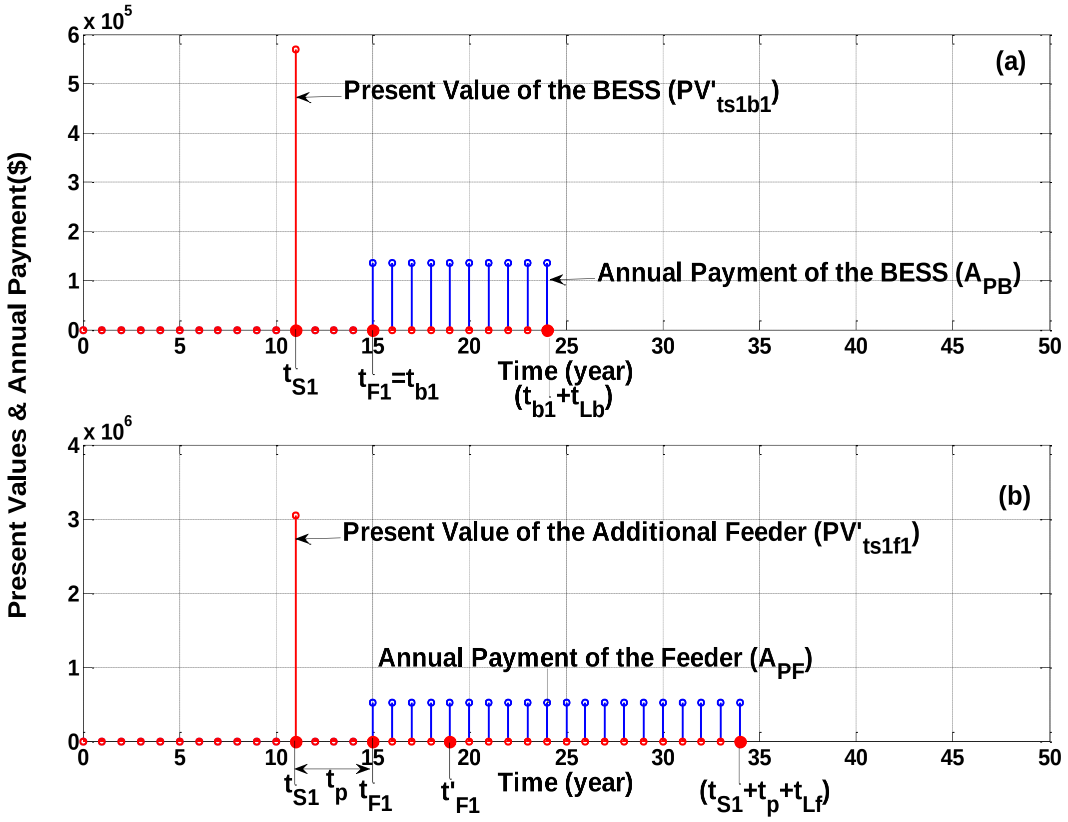


**Figure 4-6** Present Value analysis diagram (Cash Flow). (a) Present Value of BESS at the time $t_{S1}$ ($PV'_{t_{s_1}b_1}$) and its Annual Payment ($A_{PB}$), (b) Present Value of Additional Feeder (with $t_P$ years deferral) at the time $t_{S1}$ ($PV'_{t_{s_1}f_1}$) and its Annual Payment ($A_{PF}$).

The annual payment for the BESS is first calculated by:

$$A_{PB} = C_B \frac{i_R(1+i_R)^{t_{Lb}}}{(1+i_R)^{t_{Lb}-1}} \qquad (\$)/year \qquad (4\text{-}7)$$

where

$C_B$= the capital cost of the BESS.

$t_{Lb}$= the loan time of the BESS investment.

Then for (4-6), the first part ($PV'_{t_{s_1}b_1}$) is the present value of BESS at the year $t_{S1}$ with the equation:

$$PV'_{t_{s_1}b_1} = \frac{(1+d)^{t_{Lb}}-1}{d(1+d)^{t_{Lb}}} A_{PB} \frac{1}{(1+d)^{(t_C-1)}} \quad (\$) \tag{4-8}$$

where

$t_C$= the feeder construction time.

The second part ($PV'_{t_{s_1}f_1}$) of (4-6) is the present value of the additional feeder considering deferral, given by equation:

$$PV'_{t_{s_1}f_1} = PV_{t_{S_1}f_1} \frac{1}{(1+d)^{t_p}} \quad (\$) \tag{4-9}$$

where

$t_P$ =the total feeder deferral time.

If only one BESS was built as is shown in Figure 4-5, then:

$$t_P = t_{BP_1} = t'_{F_1} - t_{F_1} \quad (year) \tag{4-10}$$

where

$t_{F_1}$=the additional feeder construction finish time (without deferral).

$t'_{F_1}$=the additional feeder construction finish time (deferred).

In reality, in order to maximize the benefit of feeder deferral, more than one BESS may be installed at successive times. The expression for the total feeder deferral time has the expression:

$$t_P = \sum_{i=1}^{N} t_{BP_i} = t'_{F_1} - t_{F_1} \qquad \textit{(year)} \tag{4-11}$$

where

$t_{BP_i}$=the deferral time contributed by the $i^{th}$ BESS.

For the $i^{th}$ BESS, when $i \geq 2$, the present value of the $i^{th}$ BESS is:

$$PV'_{t_{s_1} b_i} = \frac{(1+d)^{t_{Lb}}-1}{d(1+d)^{t_{Lb}}} A_{PB} \frac{1}{(1+d)^{\left(t_C - 1 + \sum_{k=1}^{i-1} t_{BP_k}\right)}} \quad \textit{(\$)} \tag{4-12}$$

where

$t_{BP_k}$= the deferral time contributed by the $k^{th}$ BESS ($k < i$).

Then the (4-6) will be rewritten as:

$$PV'_{t_{s_1}} = \sum_{i=1}^{N} PV'_{t_{s_1} b_i} + PV'_{t_{s_1} f_1} \qquad \textit{(\$)} \tag{4-13}$$

where

$PV'_{t_{s_1} b_i}$=the $i^{th}$ additional BESS's present value at the time $t_{s_1}$with deferral.

$PV'_{t_{s_1} f_1}$=the additional feeder's present value at the time $t_{s_1}$with deferral.

Finally, the Net present Value (NPV) can be computed by substituting (4-5), (4-8), (4-12) and (4-13) in (4-14).

$$NPV = PV_{t_{S_1}} - PV'_{t_{s_1}} \qquad \textit{(\$)} \tag{4-14}$$

where

$\boldsymbol{PV}_{t_{s_1}}$=the total present value at the time $t_{S_1}$ (without deferral)

$\boldsymbol{PV}'_{t_{s_1}}$= the total present value at the time $t_{S_1}$ (with deferral)

Equation (4-14) gives the net saving obtained by deferring the feeder construction (benefit).

### 4.2.4 Distribution Feeder Deferral Benefit—A Numerical Example

This example is used to illustrate the economic benefit by deferring the additional feeder construction. Three BESSs are installed to achieve the feeder deferral for 8 years. The Net Present Value (**NPV**) calculation parameters used in this example are summarized in Table 4-1.

**Table 4-1:** BESS Economics Data [56]

| Load Growth Rate (b) (%/year) | Construction Time (year) | | Capital Cost ($) | | Loan Term (years) | | Interest ($i_R$) (%/year) | Discount Rate (d) (%/year) | BESS Characteristics (3MWh) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.5 | Feeder ($t_C$) | BESS | 15 kV class Feeder (10 mi) ($C_F$) | BESS ($C_B$) | Feeder ($t_{Lf}$) | BESS ($t_{Lb}$) | 6 | 10 | Rated Power ($P_{Bmax}$) (MW) | Round-trip Efficiency ($\eta_B$) (%) | Discharge Time ($T_B$) (h) |
| | 5 | 1 | $6 \times 10^6$ | $1 \times 10^6$ | 20 | 10 | | | 0.5 | 60 | 6 |

Given the system load history shown in Figure 4-3, three BESSs (**N=3**) can be installed to achieve the maximum 8 years' feeder deferral shown in Figure 4-7. The strategy that helps to determine the BESS commission starting time ($\boldsymbol{t_{b_h}}$) and feeder deferral time is presented in the next section as a part of the simulation model developing process. It shows that BESSs will be installed in the years 15, 19 and 21 respectively.

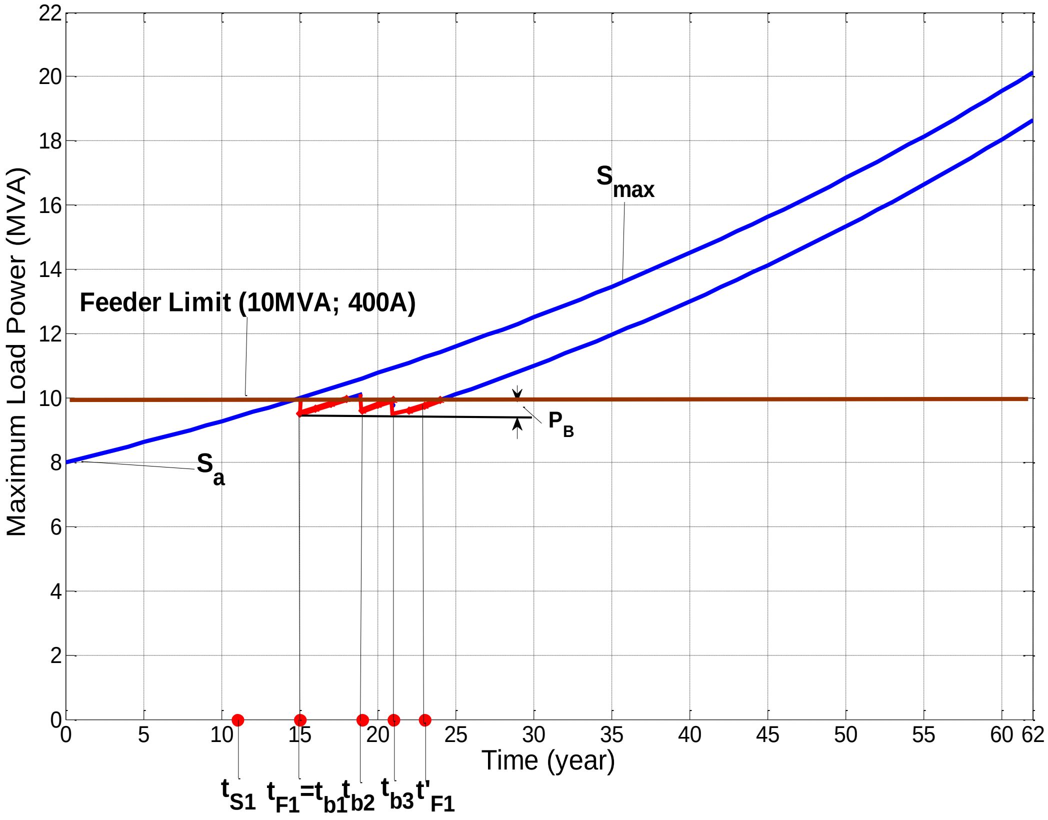


**Figure 4-7** Load History with three BESS Installation.

The first investment's total present value $\boldsymbol{PV_{t_{s_1}}}$ is the feeder upgrade investment at year 15 without deferral. The second investment's total present value $\boldsymbol{PV'_{t_{s_1}}}$ has the similar feeder upgrade investment but with 8 year's deferral plus the three BESSs' investment at the year 15, 19 and 21. The two investment's total present value can be subtracted to get the NPV which is the eventual deferral benefit. The feeder original upgrade starting time ($t_{S1}$=11) is used as the common time. Each present value has to be referred to this common time in order to be compared. A flowchart diagram for evaluating those two investments is shown in Figure 4-8.

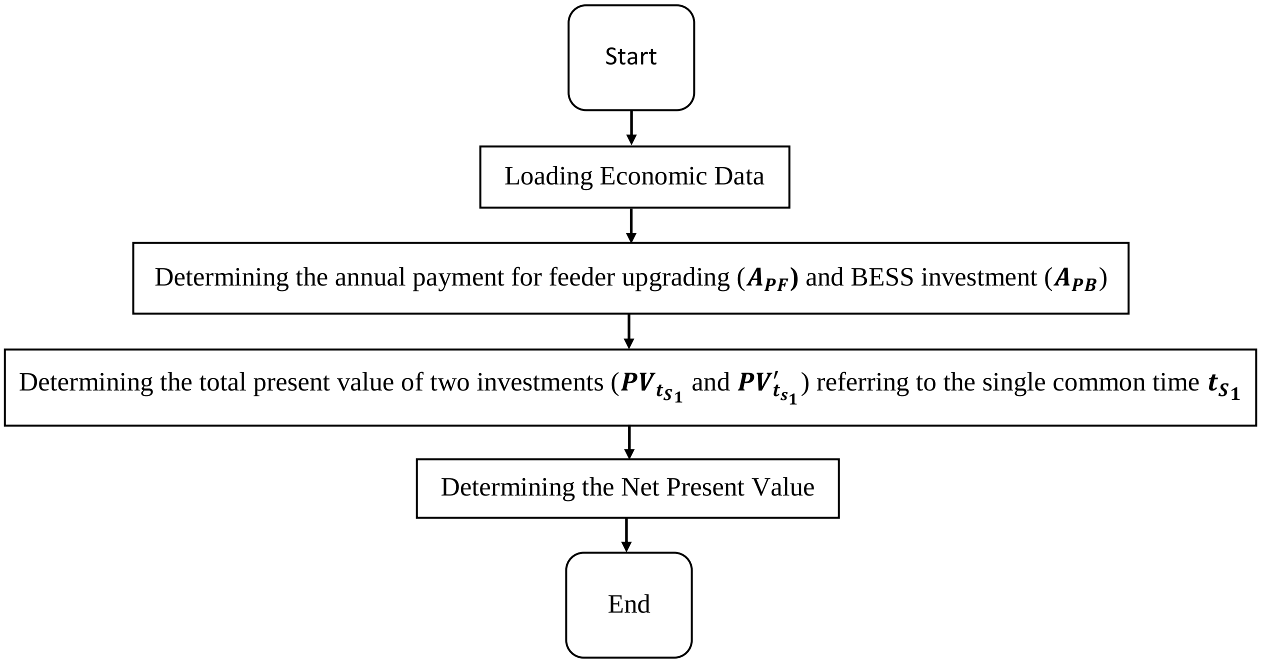


**Figure 4-8** Flowchart Diagram for NPV calculation.

Equation (4-4) and (4-5) yield the value of the $A_{PF}$ (the annual payment for the added feeder) and $PV_{t_{s_1}}$(the total present value at the time $t_{S_1}$, without deferral):

$$A_{PF} = 6 \times 10^6 \times \frac{0.06(1+0.06)^{20}}{(1+0.06)^{20}-1} = \$5.23 \times 10^5 \tag{4-15}$$

$$PV_{t_{s_1}} = \frac{(1+0.1)^{20}-1}{0.1(1+0.1)^{20}} A_{PF} = \frac{(1+0.1)^{20}-1}{0.1(1+0.1)^{20}} \times 5.23 \times 10^5 = \$4.45 \times 10^6 \tag{4-16}$$

The result in (4-16) gives the present value of the additional feeder if there is no deferral.

The feeder present value ($PV'_{t_{s_1}f_1}$), considering 8 years' deferral, can be calculated as follows:

$$PV'_{t_{s_1}f_1} = 4.45 \times 10^6 \times \frac{1}{(1+0.1)^8} = \$2.08 \times 10^6 \tag{4-17}$$

Similarly, (4-7) allows the computation of the annual payment ($A_{PB}$) for the BESS:

$$\boldsymbol{A_{PB} = 1 \times 10^{6} \times \frac{0.06\times(1+0.06)^{10}}{(1+0.06)^{10}-1} = \$1.36 \times 10^{5}} \tag{4-18}$$

The three BESSs installed will have the total present value:

$$\boldsymbol{\sum_{i=1}^{3} PV'_{t_{s_1}b_i}} = \frac{(1+0.1)^{10}-1}{0.1\times(1+0.1)^{10}} \times \boldsymbol{1.36 \times 10^{5}} \times \left(\frac{1}{(1+0.1)^{4}} + \frac{1}{(1+0.1)^{8}} + \frac{1}{(1+0.1)^{10}}\right) = \$\boldsymbol{1.28 \times 10^{6}} \tag{4-19}$$

Therefore, (4-13) tells the total present value $\boldsymbol{PV'_{t_{s_1}}}$ at the time $\boldsymbol{t_{S_1}}$ with deferral is

$$\boldsymbol{PV'_{t_{s_1}} = PV'_{t_{s_1}f_1} + \sum_{i=1}^{3} PV'_{t_{s_1}b_i} = 2.08 \times 10^{6} + 1.28 \times 10^{6} = \$3.36 \times 10^{6}} \tag{4-20}$$

The two investments' present values are summarized in Table 4-2. One is for feeder upgrade without deferral, the other is for feeder upgrade with three BESSs deferral.

**Table 4-2:** Economic Analysis Summary—Present Value (referred to the year 11 ($\boldsymbol{t_{S_1}}$))

| Case1: (Feeder Upgrade **without** deferral) | | Case2: (Feeder Upgrade **with 3 BESS** deferral) | |
|---|---|---|---|
| Upgrade Feeder | $ $4.45 \times 10^{6}$ | BESS—1 | $ $5.71 \times 10^{5}$ |
| | | BESS—2 | $ $3.90 \times 10^{5}$ |
| | | BESS—3 | $ $3.22 \times 10^{5}$ |
| | | Deferred Feeder | $ $2.08 \times 10^{6}$ |
| Total Present Value ($\boldsymbol{PV_{t_{s_1}}}$) | $ $4.45 \times 10^{6}$ | Total Present Value ($\boldsymbol{PV'_{t_{s_1}}}$) | $ $3.36 \times 10^{6}$ |

Table 4-2 above summarized the present values of two different investment scenarios.

Finally, (4-14) helps to compute this feeder deferral economic benefit:

$$\boldsymbol{NPV = PV_{t_{s_1}} - PV'_{t_{s_1}} = 4.45 \times 10^{6} - 3.36 \times 10^{6} = \$1.09 \times 10^{6}} \tag{4-21}$$

## *4.3 Simulation Model Development*

By the feeder deferral economic principle evaluated in the Ch. 4.2, the simulation model can be developed to calculate NPV giving parameters. In this section, the economic model considers the following five constrains:

- BESS commission starting time,
- Feeder deferral time,
- BESS cycle life,
- Energy losses,
- Capital cost.

This simulation model will be used in the next section's sensitivity study. The Net Present Value will be determined under these five constraints. A maximum value of NPV can be achieved. In the last part of this section, an example is given to detail the simulation procedures as the section 4.2.4 numerical example's counterpart. This method pivots around the feeder energy constraints and leads to the maximum deferral time. For simplicity, this model ignores the inflation effects.

### A. BESS commission starting time consideration

The BESS's commission starting time should be carefully determined in order to optimize its use. The BESSs need to have two features in order to defer the feeder installation. These two constraints determine the BESS commission starting time. The first one requires that the total rated power of the BESSs should be able to supply all the power above the feeder's thermal limit. For instance, if there are h BESS installed, the total rated power of the BESSs will be $\boldsymbol{hP_{Bmax}}$ MW. Assuming the maximum load demand at the year n is $\boldsymbol{P_{maxn}}$, thermal limit $\mathbf{S_L}$=10

MW, the maximum power above the thermal limit will be $(\boldsymbol{P_{maxn}-10})$ MW. The mathematical expression for this constraint is:

$$\boldsymbol{hP_{Bmax}-(P_{maxn}-10)\geq 0} \tag{4-22}$$

The second constraint on the BESSs insures that their total capacity should be large enough to supply the peak load energy which exceeds the thermal limit of the system. In the other words, when the h$^{th}$ BESS is connected, the total capacity $(\boldsymbol{hP_BT_B})$ should be larger than the extra peak load energy needed to be supplied by the BESS, namely: $\boldsymbol{W_{dch(need)}}$. The mathematical expression of this second constraint is:

$$\boldsymbol{hP_BT_B\geq W_{dch(need)}} \tag{4-23}$$

Considering the load growth rate, the h$^{th}$ BESSs commission starting time $(\boldsymbol{t_{B_h}})$ can be found by considering both constraints as stated in (4-22) and (4-23). To maximize the benefit of using BESSs, the commission starting time should be decided when either of the two constraints does not hold. Therefore, the present value of the total BESSs will be minimized and the potential benefit will be maximized.

### B. Feeder deferral time consideration

The total feeder deferral time has a limitation: the maximum load demand is continuously increasing. This means the energy that is available to charge under the thermal limit is decreasing while the total energy which is above the thermal limit is increasing. Considering the BESS efficiency, the total amount of energy available to charge should be sufficient to balance the energy which needs to be supplied. The mathematical expression for this constraint is:

$$\boldsymbol{W_{ch(available)}}\eta_{\boldsymbol{B}} \geq \boldsymbol{W_{dch(need)}} \tag{4-24}$$

where

$\boldsymbol{W_{ch(available)}}$= the energy available to be charged in 24 hours (MWh).

$\eta_{\boldsymbol{B}}$= the round-trip efficiency of the BESS.

The maximum feeder deferral time ($\boldsymbol{t_P}$) can be determined when the (4-24) constraint does not apply and the $\boldsymbol{t'_{F_1}}$will have the expression:

$$\boldsymbol{t'_{F_1} = t_{F1} + t_P} \tag{4-25}$$

### C. BESS life consideration

The BESS's life span is limited. It is determined by the number of charge/discharge cycles and it can be assumed that BESS performs one cycle/day, because the BESSs will operate daily to shift peak energy as stated in chapter 3. The BESS will need to be replaced after some number of years of use. Hence, one or more additional BESS replacement investments should be taken into account while doing the present value calculation. Depending on the type of BESS, the number of cycles in the lifespan ($\boldsymbol{CL}$) will be different.

Considering the replacement time of each BESS, the total number of installations for the n$^{th}$ battery can be named $\boldsymbol{N_{bn}}$. The equation for $\boldsymbol{N_{bn}}$ is:

$$\boldsymbol{N_{bn}} = \left\lceil \frac{\boldsymbol{t_p - (t_{b_n} - t_{F1})}}{\boldsymbol{CL/365}} \right\rceil \tag{4-26}$$

Therefore, (4-8) and (4-12) can be rewritten as:

$$PV'_{ts1b1} = \sum_{n=0}^{N_{b1}-1} \frac{(1+d)^{t_{lb}}-1}{d(1+d)^{t_{lb}}} A_{PB} \frac{1}{(1+d)^{\left(t_c-1+\left\lfloor \frac{CL}{365} \right\rfloor n\right)}} \tag{4-27}$$

$$PV'_{ts1bi} = \sum_{n=0}^{N_{bi}-1} \frac{(1+d)^{t_{lb}}-1}{d(1+d)^{t_{lb}}} A_{PB} \frac{1}{(1+d)^{\left(t_c-1+\sum_{k=1}^{i-1} t_{BPK}+\left\lfloor \frac{CL}{365} \right\rfloor n\right)}} \tag{4-28}$$

The modified BESS present value calculation will be used in the simulation model to determine the total present value for the feeder installation with BESSs deferral ($PV'_{t_{s_1}}$).

## D. Energy losses consideration

Electric energy will be lost during the charging/discharging process. In chapter 3, the losses have already been considered when determining the maximum DCE. However, in the case of deferring feeder construction, the losses should be addressed again. The main reason for this reconsideration is the following. Ideally, if the BESS has 100% round-trip efficiency, all the charged energy can be used to discharge during the high demand. However, in reality, the efficiency of the BESS is less than 100%, which means the energy will be lost during charging/discharging process and therefore should be considered as the energy related operational cost. These energy losses due to BESS's inefficiency have to be accounted for financially in the simulation model. In that way, the daily money losses will is addressed in the present value calculation with the following mathematical expression:

$$K_{loss} = K_{avg}(W_{ch}(1-\eta_B)) \tag{4-29}$$

where

$K_{loss}$=wasted capital during charging/discharging process due to BESS inefficiency.

$K_{avg}$=the average LMP in the region.

$\boldsymbol{W_{ch}}$=the charging energy

## E. Capital cost consideration

As was addressed in (2-3), the BESS's capital cost is proportional to its capacity, which should be addressed in the BESS model present value calculation. Similarly, the feeder capital cost is proportional with the length of the feeder as was stated in (2-5). The length of the feeder will be used as a parameter in the next section's sensitivity study. The capital cost difference is also considered in the feeder present value calculation.

- ***Simulation Model Implementation Example***

In the previous example (Ch. 4.2.4), the simulation was based on the following assumptions:

- BESS round-trip efficiency ($\boldsymbol{\eta_B}$): 60%
- BESS cycle life ($\boldsymbol{CL}$): 3000 cycles
- BESS rated power ($\boldsymbol{P_{Bmax}}$): 0.5 MW
- BESS discharging time at rated power ($\mathbf{T_B}$): 6 h
- Feeder maximum load demand growth rate ($\mathbf{r_a\%}$): 1.5%
- Feeder length ($\boldsymbol{l}$): 10 mi

The average load profile can be extracted from the National Grid data [55]. The data is scaled for the worst case analysis. Considering the load growth rate and constraints stated in (4-22) and (4-23), the first, second and third BESS commission starting time can be found at the years 15, 19 and 21.

After the year 23, according to (4-24), there will not be enough energy available to charge the BESS in order to shift the entire peak load. The maximum 8 years' deferral will be achieved. In that case, a new feeder construction is inevitable.

Figure 4-9 shows the worst 24 h load curve at the year 0. The entire load profile is below the feeder limit (10 MVA). However, under 1.5%/year load growth rate, the peak load will exceed the feeder thermal limit at year $\boldsymbol{t_{F1} = t_{b1} = 15}$. The worst case 24 hour load curve at the last year of deferral is shown in Figure 4-10.

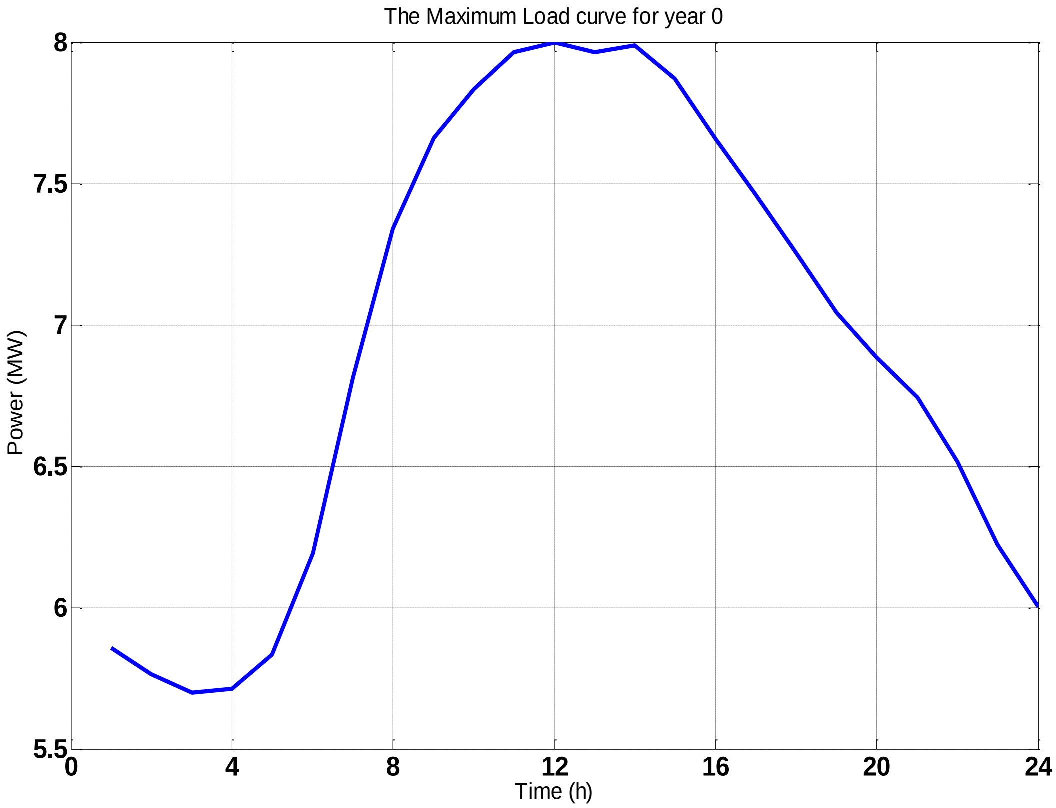


**Figure 4-9** The 24h Worst Load Curve at the year 0 [55].

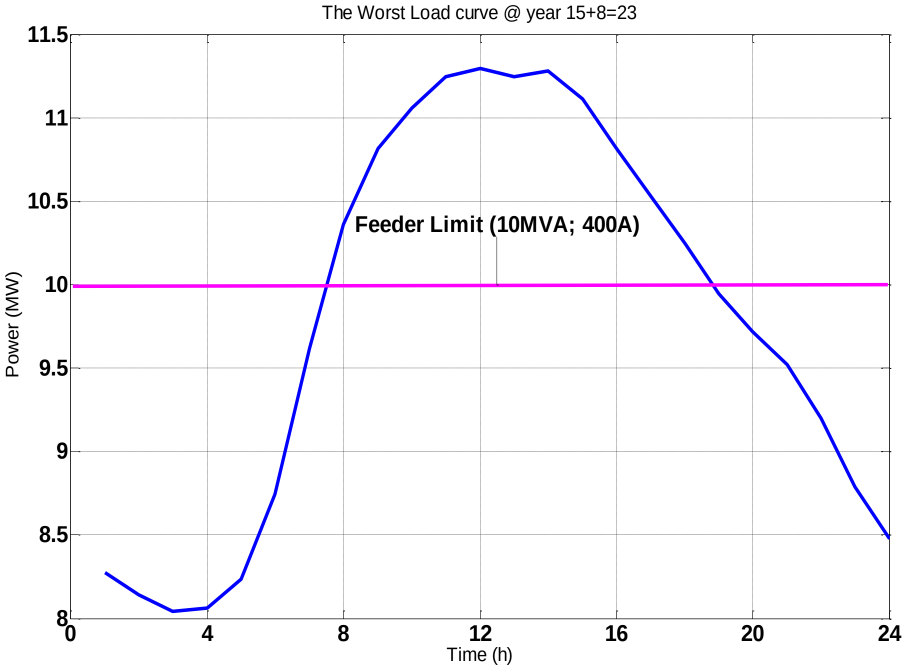


**Figure 4-10** The 24h Worst Load Curve at the last year of Deferral (Total 8 years' deferral).

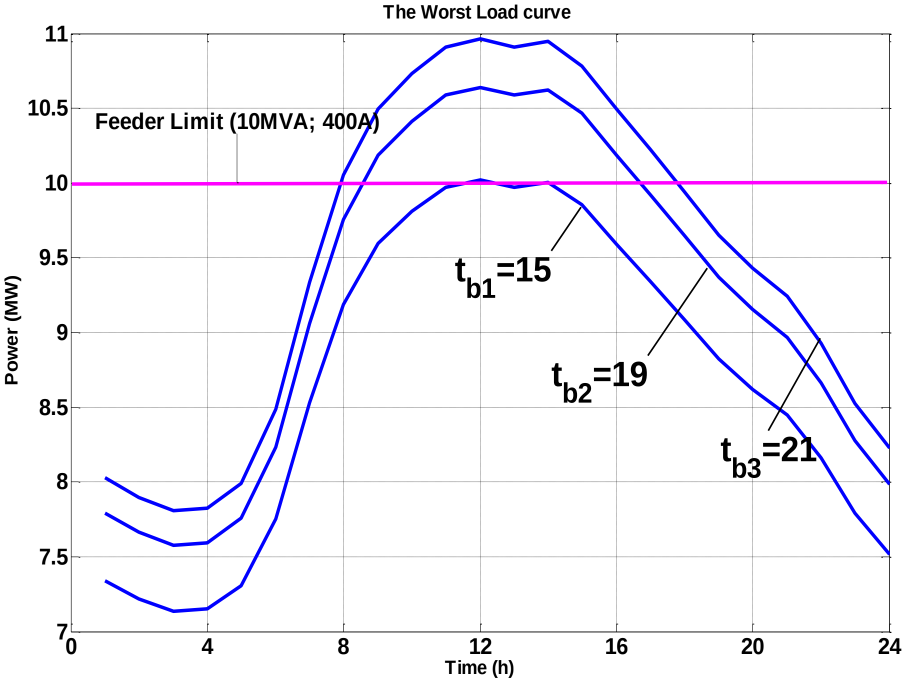


**Figure 4-11** The 24h Worst Load Curve at the each BESS installation year.

Figure 4-11 shows the worst 24 h load curve under each BESS installation year. The first BESS is installed in the year **15** when the maximum load reaches the thermal limit of the existing feeder, This BESS has the capacity needed to supply the peak power demand till the year **19** when a new BESS is installed. It is lucky that even the first BESS does not need to be replaced since the feeder deferral year is 8, and the life span of BESS is considered to be longer ($\frac{3000\ cycle}{365\ cycle/year} = 8.22$ year). The old and the new BESSs are effectively supplying the additional demand power till the year **21** when a third BESS will be installed.

The energy losses are calculated daily. These losses have been taken into account in the present value calculation. After this simulation model development, the sensitivity study is ready to be implemented [Appendix C for MATLAB code].

## *4.4 Simulation Results*

The following six parameters controlled the economics of the BESS's applications:

1. Feeder maximum load demand increasing rate, $\mathbf{r_a\%}$,
2. Feeder length, ***l***,
3. BESS life span, measured in cycles, **CL**,
4. BESS round-trip efficiency, $\boldsymbol{\eta_B}$,
5. BESS rated output/input power, $\mathbf{P_{Bmax}}$,
6. BESS maximum discharging time $\mathbf{T_B}$, under the rated power $P_{Bmax}$.

Multiple BESSs are used to achieve the maximum feeder deferral time $t_P$. The specific cost of the BESS, \$/MWh, is a function of the round-trip efficiency $\eta_B$, the life span CL, as well as the BESS's capacity $Q_B$. In this chapter it was assumed that the BESS's specific cost is independent of the CL, $\eta_B$ and $Q_B$.

The graph of NPV, as a function of the feeder load increasing rate $r_a\%$, with the BESS's cycle life as a parameter is shown in Figure 4-12. The range of the cycles is from 1000 to 5000. The BESS is assumed to have the rated values of: $P_{Bmax}$= 0.5 MW, $T_B$=6 h ($Q_B=P_{Bmax}T_B$=3 MWh) and the round-trip efficiency $\eta_B$=60%. The feeder length is 10 mi.

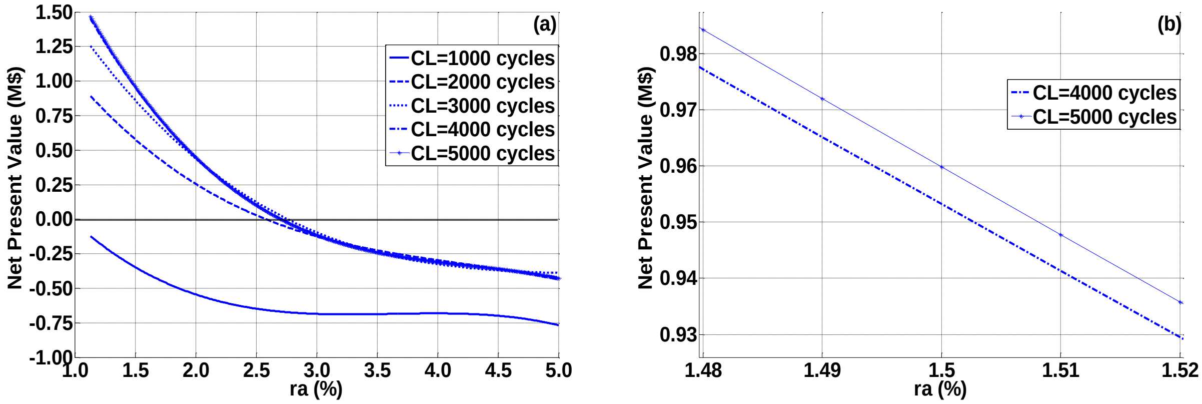


**Figure 4-12** NPV vs. ra% with CL is parameter. $P_B$=0.5 MW, $T_B$=6 h, $\eta_B$=60%, $\boldsymbol{l}$=10 mi. (a) CL from 1000 cycles to 5000 cycles, (b) Zoom in for CL=4000 cycles and 5000 cycles.

In the Figure 4-12a, the NPV is decreasing while ra% is increasing. For a certain range of $r_a$%, there is a boundary value of $r_a$% that separates the positive and negative NPV. If the CL is very small (e.g., CL≤1000 cycles), the NPV will always be negative, thus, increasing CL will increase the NPV. If the $r_a$%=1.5% and the CL is increased from 1000 cycles to 2000 cycles, the NPV will leap from negative $\$\ 0.35 \times 10^6$ to positive $\$\ 0.58 \times 10^6$. On the contrary, when CL is increased from 4000 to 5000 cycles (the same 1000 cycles step), the NPV will increase from $\$\ 0.953 \times 10^6$ to $\$\ 0.960 \times 10^6$ with only $\$\ 0.007 \times 10^6$ difference (Figure 4-12b). The following is the major reason for these results: increasing CL helps to reduce the battery replacement cost. However, when the CL is high, the BESS's replacement is less often and the NPV increase is small. Even if some early installed BESSs may need to be replaced under a long deferral time, the present value of replaced BESS will still be minute when compared with the situation when CL is low. Finally, one can observe that as the $r_a$% increasing to a certain high value ($r_a$%=3% in the Figure 4-12a), there is not much difference among the NPV curves (except for CL≤2000 cycles). The explanation for this result is the fact that if a short period of deferral is implemented, most of the BESSs do not need to be replaced.

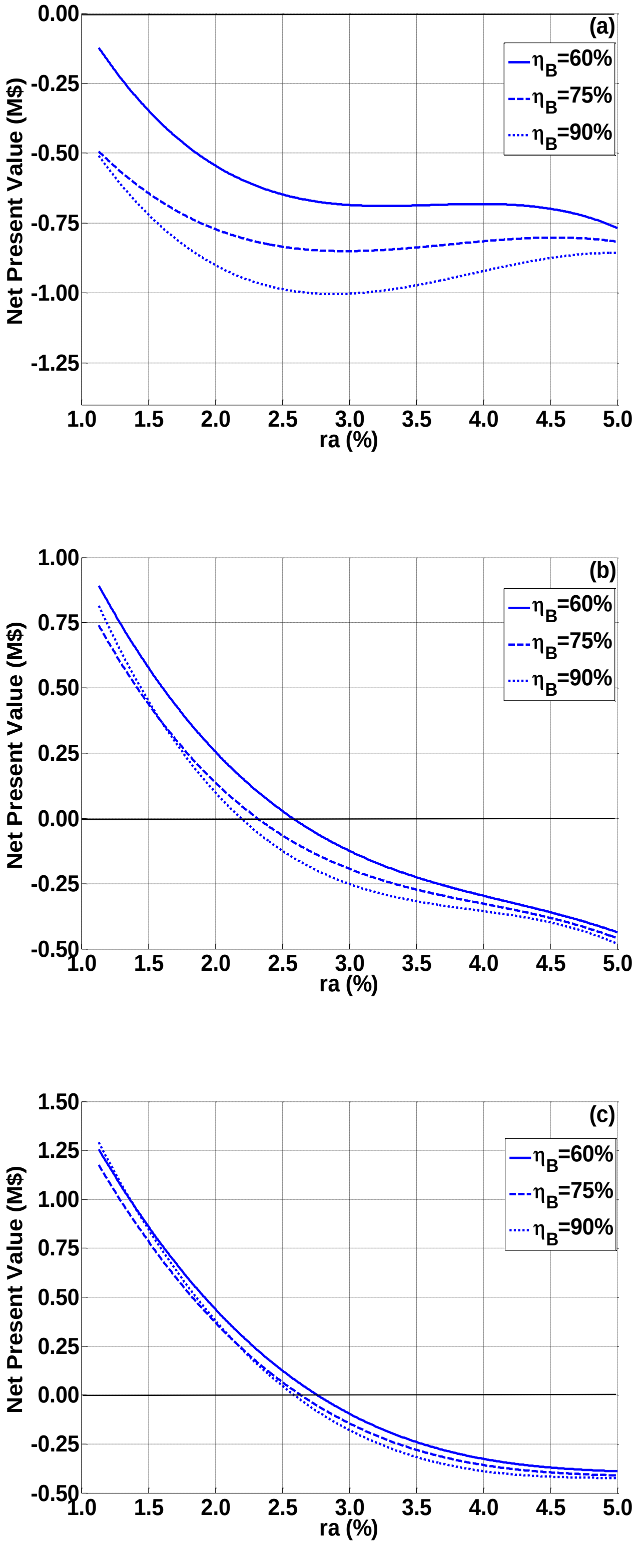


**Figure 4-13** NPV vs. ra%, $\eta_B$ is parameter. $P_B$=0.5 MW, $T_B$=6 h, ***l***=10 mi. (a) CL = 1000 cycles, (b) CL = 2000 cycles, (c) CL = 3000 cycles.

Both BESS round-trip efficiency and life span effects on NPV are presented in Figure 4-13. The round-trip efficiency is parameter and each one of the exhibits is characterized by a different value of CL (Figure 4-13a CL=1000 cycles, Figure 4-13b CL=2000 cycles and Figure 4-13c 3000 cycles). The BESS is assumed to have the nominal values of: $P_B$=0.5 MW and $T_B$=6h. The feeder length is 10 mi.

One will observe that the overall NPV is increasing while the CL is increasing from 1000 to 3000 cycles. For a BESS with high round-trip efficiency $\eta_B$, the NPV will be higher when the CL is higher and lower when CL is lower. However, the deferral time can be extended by using a higher $\eta_B$, which will increase the NPV. As the deferral time gets longer, the number of BESSs as well as the need for BESS replacement may also increase. Equations (4-13), (4-27) and (4-28) show that the $PV'_{t_{s_1}f_1}$ will decrease while $\sum_{i=1}^{N} PV'_{t_{s_1}b_i}$will increase. Under this condition, when CL is low, Figure 4-13a, the NPV will have a negative value in spite of the fact the fact that $\eta_B$ is increasing. This means it is not worth to defer the feeder for a long time because it will require investing more capital over a short life span. But if the CL is high (Figure 4-13c), the NPV will increase due to the dominating contribution of the long feeder deferral time. It can also be noticed that the round-trip efficiency $\eta_B$ tends to have a stronger impact on NPV when CL is small than when CL is high.

Two extreme CL conditions are presented in Figure 4-14: First is shown in Figure 4-14a where the CL is low (500 cycles).The other condition is provided in Figure 4-14b where the CL is high (5000 cycles). Both graphs show the NPV vs. $r_a$% with BESS round-trip efficiency $\eta_B$% as the parameter. The BESS is assumed to have rated power $P_B$=0.5 MW and discharging time $T_B$=6 h. The feeder length is 10 mi.

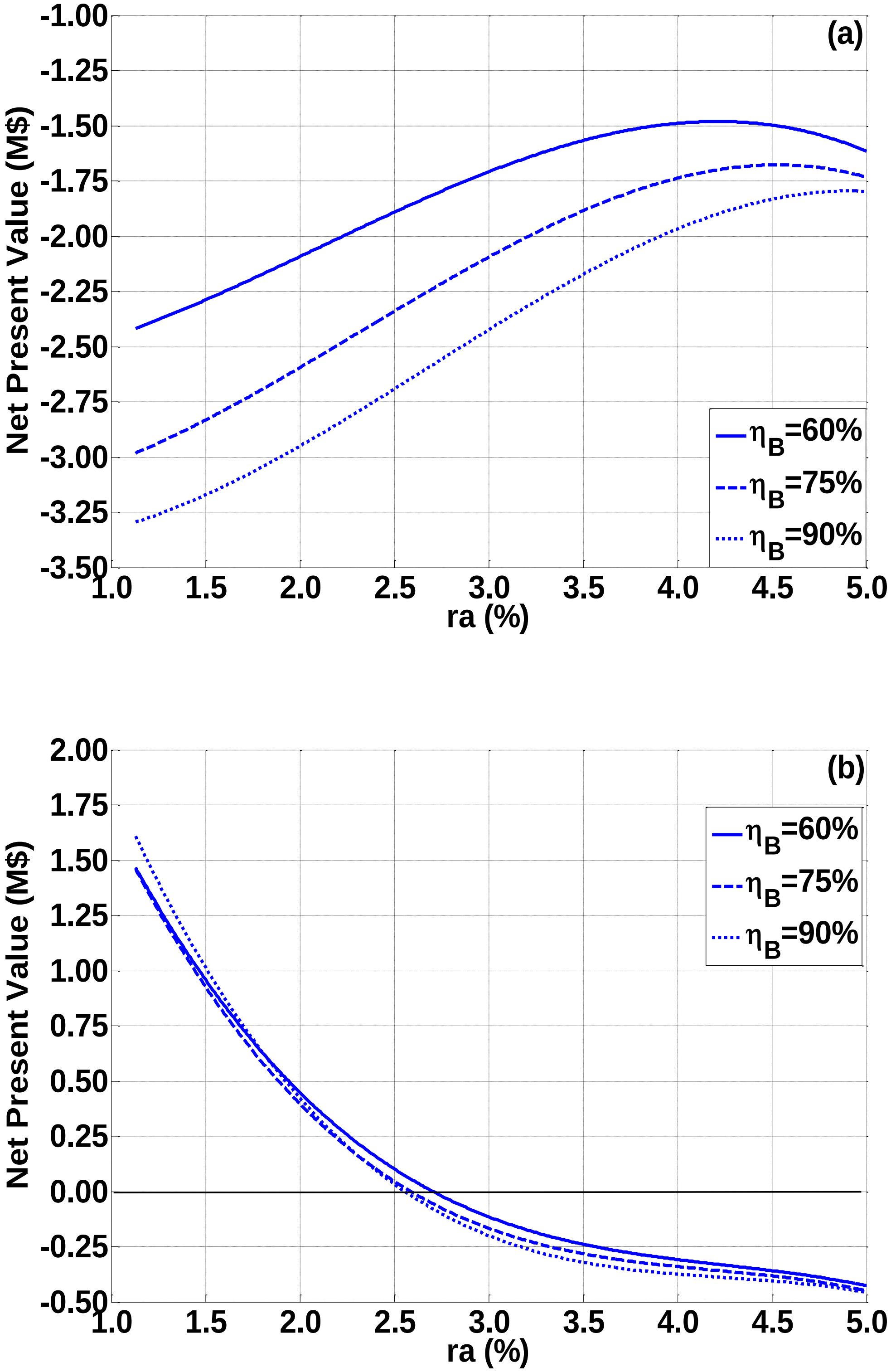


**Figure 4-14** NPV vs. ra%, $\eta_B$ is parameter. $P_B$=0.5 MW, $T_B$=6 h, $l$=10 mi. (a) CL = 500 cycles, (b) CL = 5000 cycles.

In the case when CL=500 cycles, the NPV will always be negative. One can learn from Figure 4-14 that the NPV will become even lower as $r_a$% is decreasing. This result is expected since the BESS with low life span will need to be replaced at short intervals. As the $r_a$% is increasing, there is need for a less often replacements of BESSs, thus, the NPV will increase. However, since the overall deferral time is also decreasing, the overall BESSs deferral benefit will still remain negative. On the contrary, if the CL is high as shows in the Figure 4-14b (CL=5000 cycles), the NPV will become significant if the $r_a$% is low. Also, since the BESSs replacements will be more seldom due to the high value of CL, the higher efficiency BESSs will have the higher NPV.

The effects of the feeder length on the NPV are presented in Figure 4-15. The BESS is assumed to have the characteristics of: $P_B$=0.5 MW, $T_B$=6h. CL= 3000 cycles with $\eta_B$=60%. The length of the feeder is the main parameter varying from 2 mi to 16 mi.

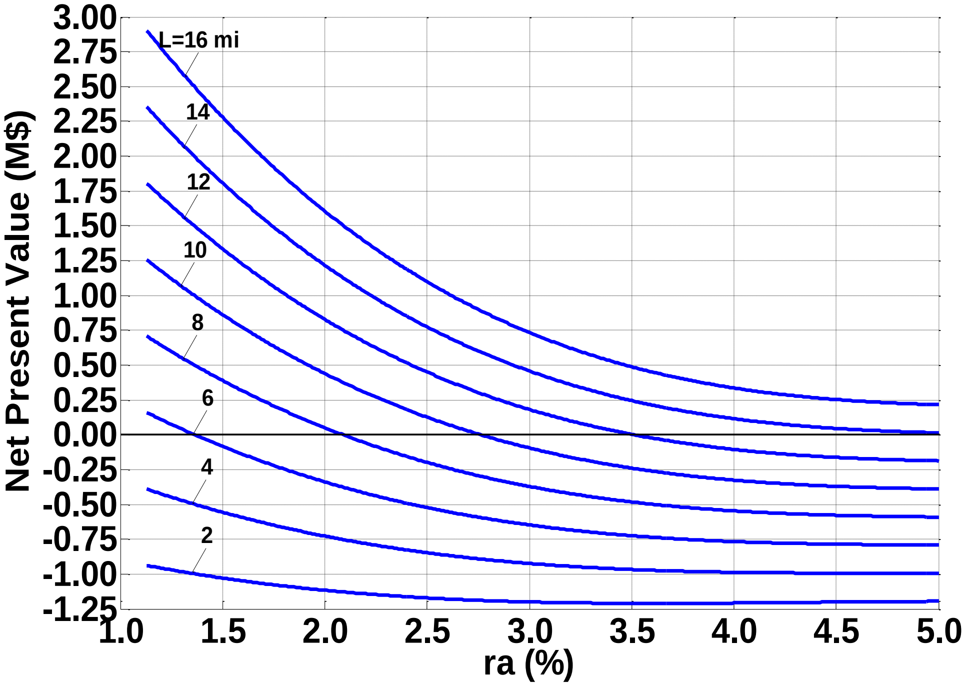


**Figure 4-15** NPV vs. ra%, “*l*” is parameter. $P_B$=0.5 MW, $T_B$=6h. CL= 3000 cycles, $\eta_B$=60%.

The obtained results show that the NPV is highly dependent on the length of the feeder. Since the length of the feeder is proportional to its capital cost, there will be a critical feeder length, which will separate between positive value and the negative value of the NPV. The impact of the feeder length is higher when the $r_a$% is low. For example, the NPV will increase $\$ 0.47 \times 10^6$ from $l$=8 mi to $l$=10 mi under the load increasing rate of 1.5%. A lower value of $\$ 0.39 \times 10^6$ increasing will be obtained from $l$=8 mi to $l$=10 mi, while the load increasing rate is increased from 1.5% to 2.0%. Since the NPV increases as the length of the feeder increases, it is more economically to defer a long feeder than a short feeder.

The Figure 4-16 explores the relation between NPV and BESS round trip efficiency $\eta_B$%, when the load increasing rate $r_a$% as the parameter. The BESS is assumed to have: $P_B$=0.5 MW, $T_B$=6 h and CL=3000 cycles. The feeder length is 10 mi.

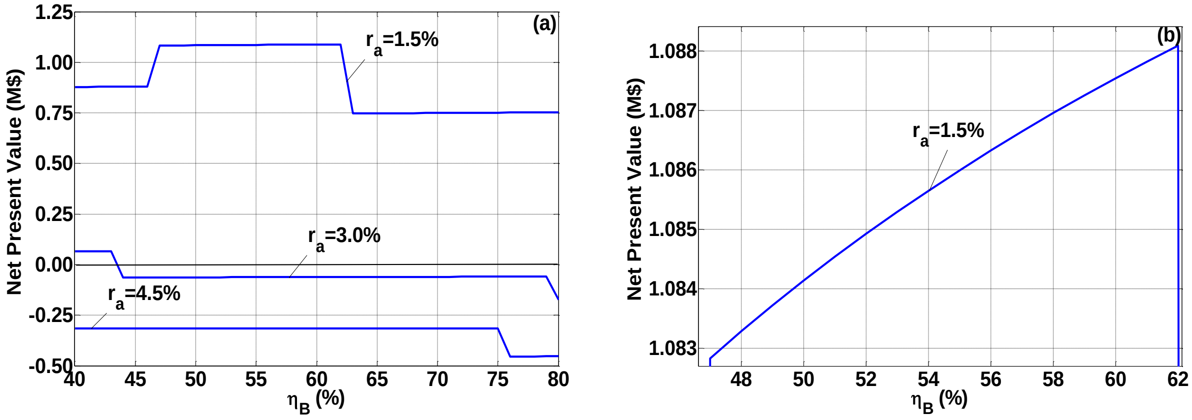


**Figure 4-16** NPV vs. battery efficiency ($\eta_B$%), feeder load demand increasing rate $r_a$% is parameter. $P_B$=0.5 MW, $T_B$=6 h, CL=3000 cycles, $\boldsymbol{l}$=10 mi. (a) $r_a$% from 1.5% to 4.5%, (b) Zoom in for $r_a$%=1.5%.

The resulting curves show that the lower $r_a$% will have higher NPV. Changing $\eta_B$% may change the number of BESSs as well as the feeder total deferral time, thus causing the sudden NPV leaps. For example, the NPV will step-up from $\$ 0.88 \times 10^6$ to $\$ 1.08 \times 10^6$ if the $\eta_B$% is increased from 46% to 47% when $r_a$%=1.5%. The reason is that one more year's of deferral is gained by the same number of BESSs. However, the NPV will also step-down from $\$ 1.09 \times 10^6$ to $\$ 0.75 \times 10^6$ if the $\eta_B$% is increased from 62% to 63% under the $r_a$%=1.5%. The reason is that one additional BESS is added in the simulation model under this condition. However, the contribution of this additional BESS is minute since it can only help to defer the feeder installation for one more year. Similar explanation can also be applied to the case for $r_a$%=3.0% and 4.5%.

The results shown in Figure 4-16b illustrates the effect of energy lost while charging/discharging the BESS. As one can see, if the $\eta_B$% is increasing, the NPV will increase because less energy will be lost. However, if one go back to Figure 4-16a, it tells the increasing amount is almost negligible when compared with the NPV leaps caused by the changes of the number of BESS as well as the deferral year.

The NPV versus the BESS rated power $P_B$ with the load increasing rate $r_a$% as a parameter is presented in Figure 4-17. It is assumed the BESS has the following parameters: $T_B$= 6 h, CL=3000 cycles and $\eta_B$%=60%. The feeder length is 10mi.

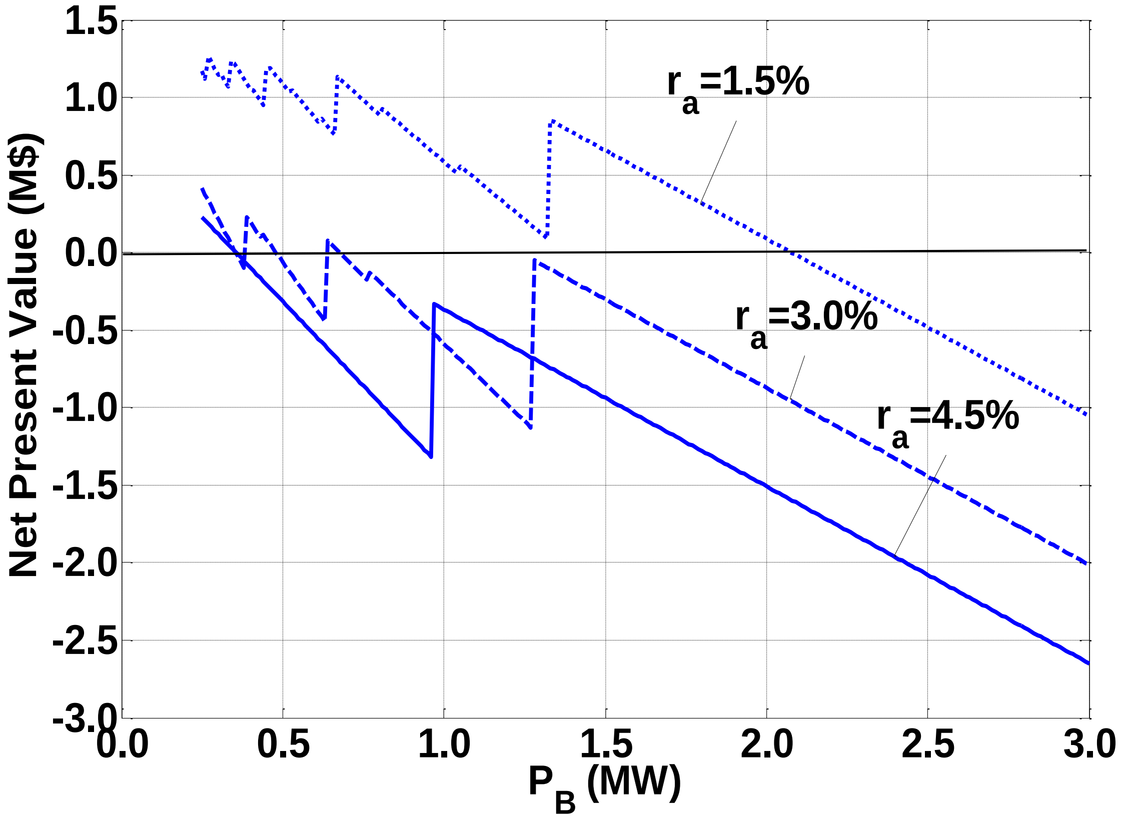


**Figure 4-17** NPV vs. BESS rated power ($P_B$), feeder load demand increasing rate $r_a$% is parameter. $T_B$= 6 h, CL=3000 cycles, $\eta_B$%=60%, ***l***=10 mi.

In general, the NPV will decrease if the original BESS is replaced with a higher $P_B$ one while keeping the same $T_B$. The reason is rooted in the fact that the capital cost of BESS is going up (2-3). However, one will notice that there are present a few leaps through the curves. Bigger jumps are due to the sudden decrease in the BESSs numbers. For example, under $r_a$%=1.5%, the NPV will step up from $\$ 0.1 \times 10^6$ to $\$ 0.85 \times 10^6$ when the $P_B$ is increased from 1.32 MW to 1.33 MW since the number of BESS needed is decreased from two to one, while still have the same total deferral time. The small jumps are due to the changes of $n^{th}$ BESS installation time $t_{Bn}$. For example, when $P_B$ changes from 0.61 MW to 0.62 MW, the $3^{rd}$ BESS installation time $t_{B3}$ will move from year 19 to year 20, which will slightly increase NPV.

Similarly, the NPV as a function of BESS discharging time $T_B$ with the load increasing rate $r_a$% as a parameter is shown in Figure 4-18. It is assumed the BESS has the following parameters: $P_B$= 0.5 MW, CL=3000 cycles and $\eta_B$%=60%. The feeder length is 10mi.

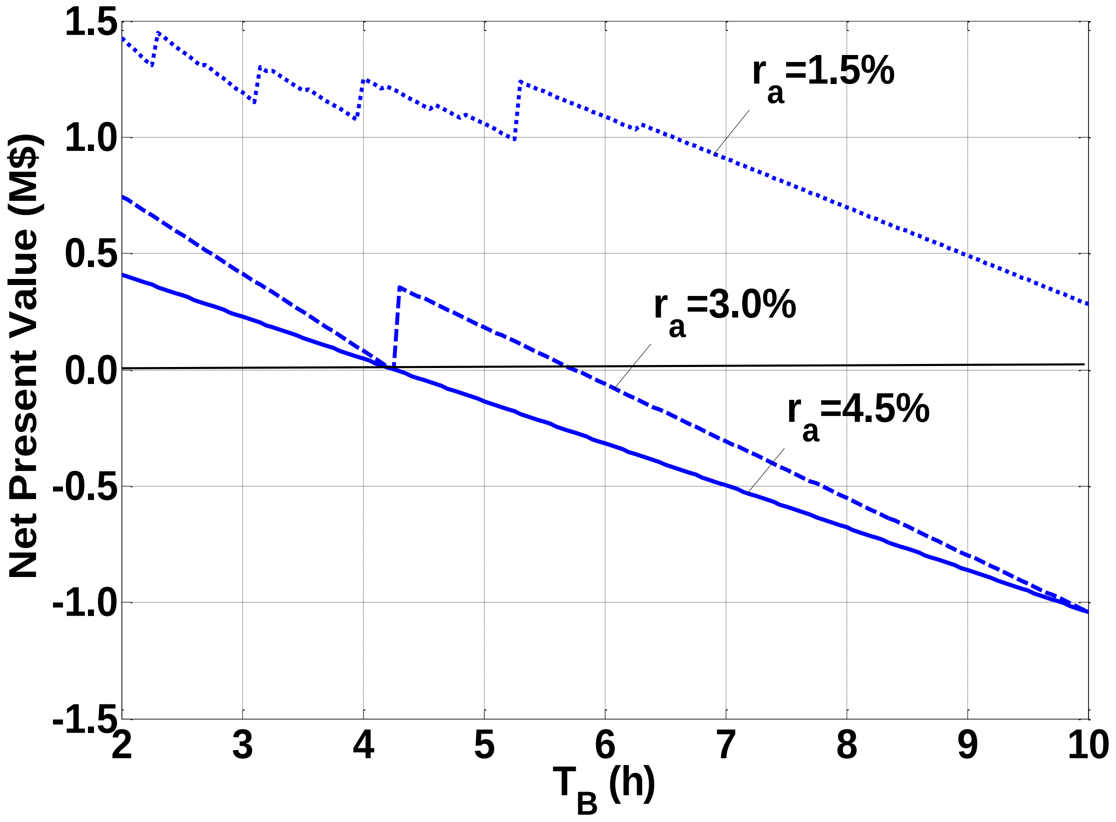


**Figure 4-18** NPV vs. BESS discharging time ($T_B$), feeder load demand increasing rate $r_a$% is parameter. $P_B$= 0.5 MW, CL=3000 cycles, $\eta_B$%=60%, ***l***=10 mi.

Similar to the Figure 4-17, it is expected that the increase of $T_B$ will decrease the NPV. There are also leaps along the curves due to the same explanations that were presented in the Figure 4-17.

## *4.4 Discussion & Conclusion*

The NPV is a function of a host of parameters which need to be carefully analyzed to achieve the maximum deferral benefit. Each parameter's effects are discussed below:

### 1. Feeder maximum load demand increasing rate, $r_a$%,

The feeder installation could be deferred for a long term if the system's $r_a$% is small ($r_a\% \leq$ 1.5%). Ideally, the higher NPV will be obtained under the longer deferral time. However, the optimum may not occurred in the longest deferral time considering the additional expenses caused by BESSs capital cost, maintenances and replacements.

### 2. Feeder length, *l*,

This is a key parameter that governs the economical feasibility of the feeder deferral. For the same deferral time, a longer feeder will have a higher NPV which is economically more attractive.

### 3. BESS life span, measured in cycles, CL,

A higher value CL means less or no replacements, which will increase the NPV. Given the total deferral time and the number of BESSs, the CL will affect the NPV significantly.

### 4. BESS round-trip efficiency, $\eta_B$,

High $\eta_B$ means the BESSs can use the energy more effectively which will help increase feeder deferral time using the same number of BESSs. Besides, the daily energy losses will also decrease. Those factors will boost the NPV. Furthermore, with higher $\eta_B$, a larger number of

BESSs can be added to the system. However, this is a double-edged sword since there will be more capital as well as replacements cost for BESSs.

### 5. BESS rated output/input power, $P_{Bmax}$,

If the $T_B$ has a fixed value, a BESS with higher $P_B$ will provide a larger $Q_B$ require a larger $C_B$, thus decreasing the NPV. However, a higher $Q_B$ BESS can defer the feeder for a longer term when compared with a lower $Q_B$ BESS. Therefore, the NPV will boost up when the number of year deferral is increased with the same number but higher $Q_B$ BESSs.

### 6. BESS maximum discharging time $T_B$, under the rated power $P_{Bmax}$.

Similar to $P_{Bmax}$, the $T_B$ will have the same effects on NPV using the same explanation.

Through the previous discussion, it is obvious that multiple parameters/variables will affect the NPV at the same time. Given the system condition of $r_a$%=1.5% with a 10 mi 15 KV (400A) feeder and 80% round-trip efficiency BESSs (maximum nine years' deferral), the best option of BESS deferral option can be obtained by using optimization [see Appendix E for the optimization problem introduction].

# 5. Outage Avoidance

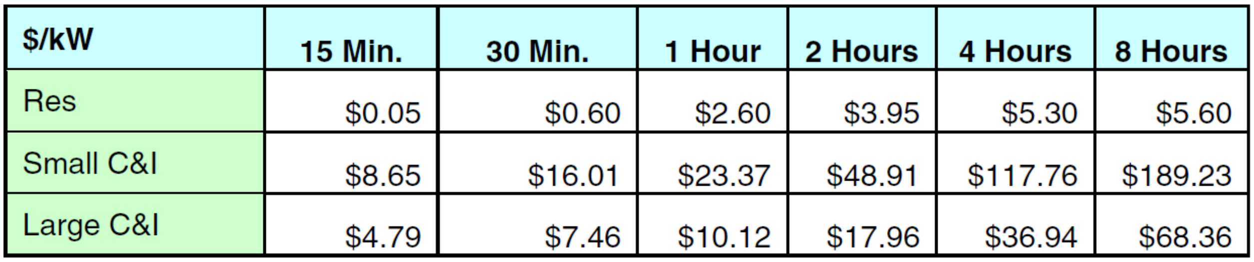
## *5.1 Problem Definition*

The scope of this chapter is to evaluate the economical benefit that can be derived from installation of BESS. Power reliability is a critical issue in power system operation and maintenance. Momentary or long term outage may cause power interruption to thousands of customers. There are three options that respond to outage conditions:

1. The conventional approach: the entire feeder is disconnected by the main circuit breaker located at the substation bus. In this case, all the customers supplied by the feeder will experience the outage.
2. If more than one circuit breaker is included in the radial feeder topology, then the outage will affect only a limited group of customers.
3. With the proliferation of Smart Grid technologies, it will be possible to limit the outage conditions only to the residential loads, or to other low priority loads.

This chapter deals with the economic benefit of outage avoidance using the BESS. The average outage reparation costs are presented in Table 5-1. The BESS can be used to help ride through the outage and achieve the economic benefit by avoiding all or partial outage reparation costs.

**Table 5-1:** The Average Outage Reparation Costs [2]



| $/kW | 15 Min. | 30 Min. | 1 Hour | 2 Hours | 4 Hours | 8 Hours |
|---|---|---|---|---|---|---|
| Res | $0.05 | $0.60 | $2.60 | $3.95 | $5.30 | $5.60 |
| Small C&I | $8.65 | $16.01 | $23.37 | $48.91 | $117.76 | $189.23 |
| Large C&I | $4.79 | $7.46 | $10.12 | $17.96 | $36.94 | $68.36 |

## *5.2 Outage Avoidance Case Study: Methodology*

The annual load daily variation can be computed and displayed as shown in Figure 5-1 using (3-6) and (3-7). It was assumed $P_M$=10.25 MW, and the $max(P_{max}^{min}(n))$=8.5 MW. As one can see, the weeks with the highest demand occurs during the end of June and beginning of July. A planned outage will be implemented while the feeder peak load demand is above the feeder's thermal limit.

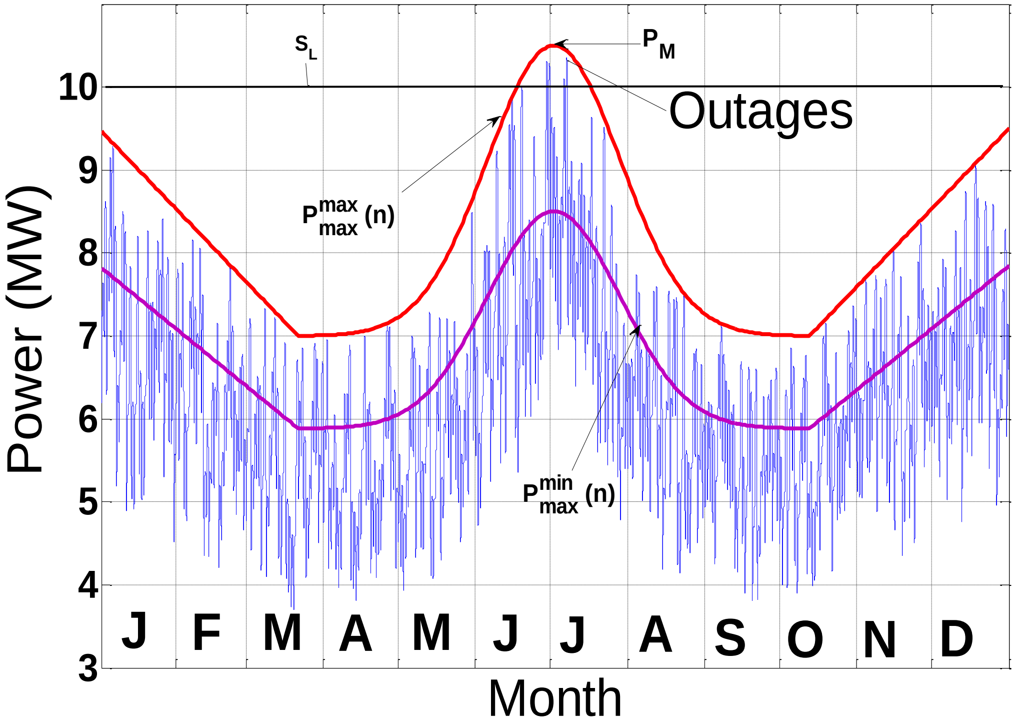


**Figure 5-1** Annual Load Variation with Outages.

After zooming in the week of July 1st (Figure 5-2), one will observe that for two days in row the demand exceeds $S_L$=10MVA. In this example, it was assumed that the feeder's load is mixed: 50% residential, 25% small commercial and industrial and 25% large commercial and industrial loads. As it is shown in the Table 5-1, the outage reparation costs for the residential customer are relatively low comparing to the outage reparation costs for large or small C&I customers [2].

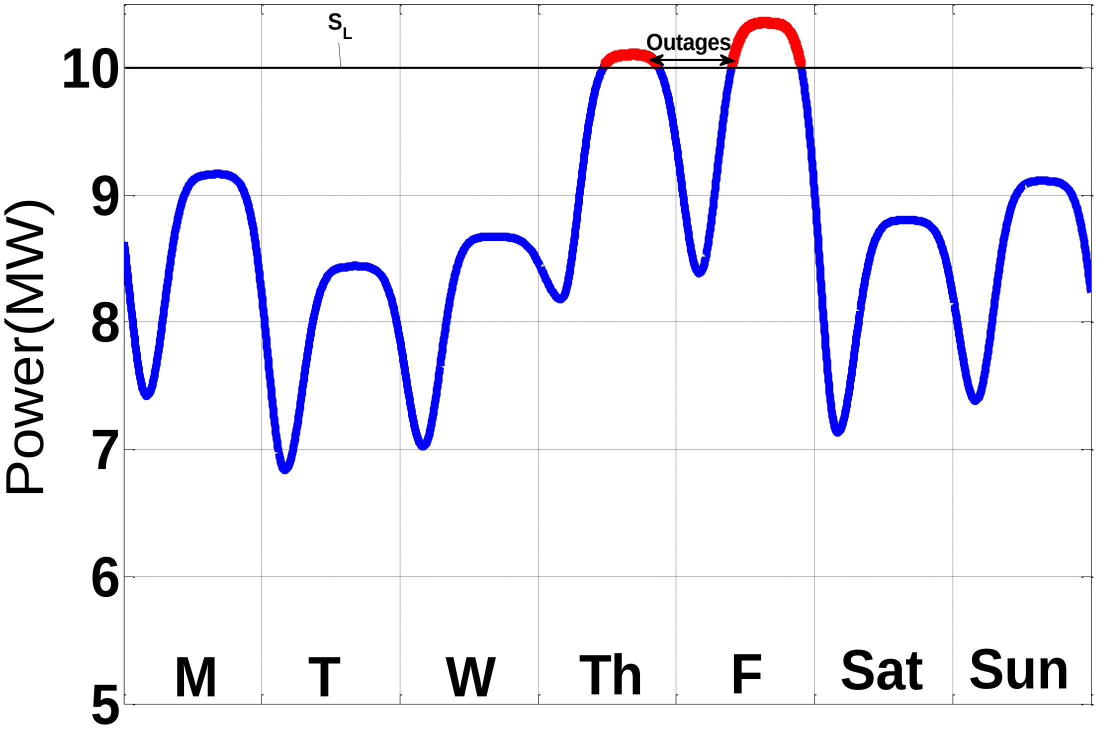


**Figure 5-2** One Week Load for the beginning of July.

The BESS charges its batteries during the low demand and discharge during the peak. By correctly using a BESS dedicated to a group of affected loads, depending on the size of the BESS, it is possible to ride through all or part of the high demand times.

Assuming that the BESS rated value are $P_{Bmax}$=0.5 MW, $T_B$=5 h and $\eta_B$=85%, it is feasible to “ride through” during the entire Thursday outage period with the estimated energy of 0.5 MWh exceeds the limit of 10 MVA. In this case there is enough energy stored in the BESS to supply the needed additional energy. The details are provided in Figure 5-3 [Appendix D for MATLAB code].

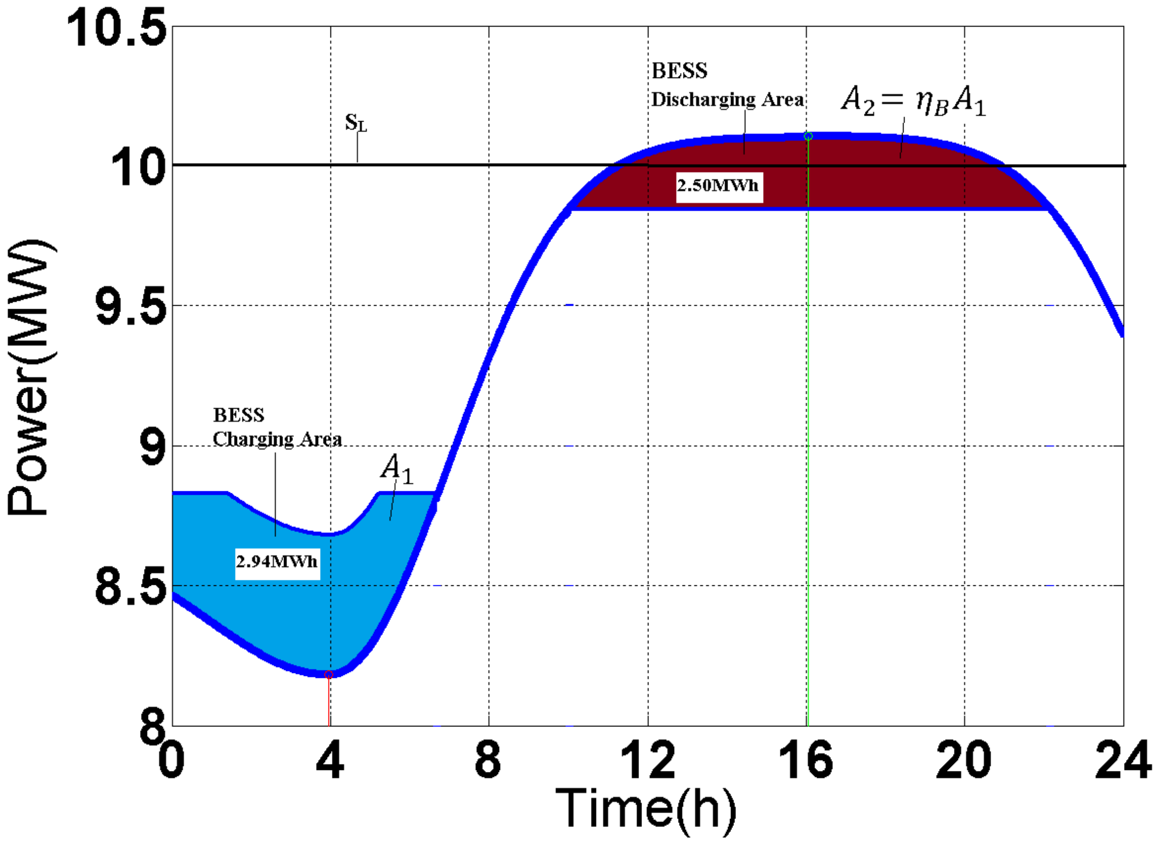


**Figure 5-3** Thursday Outage Avoidance Case.

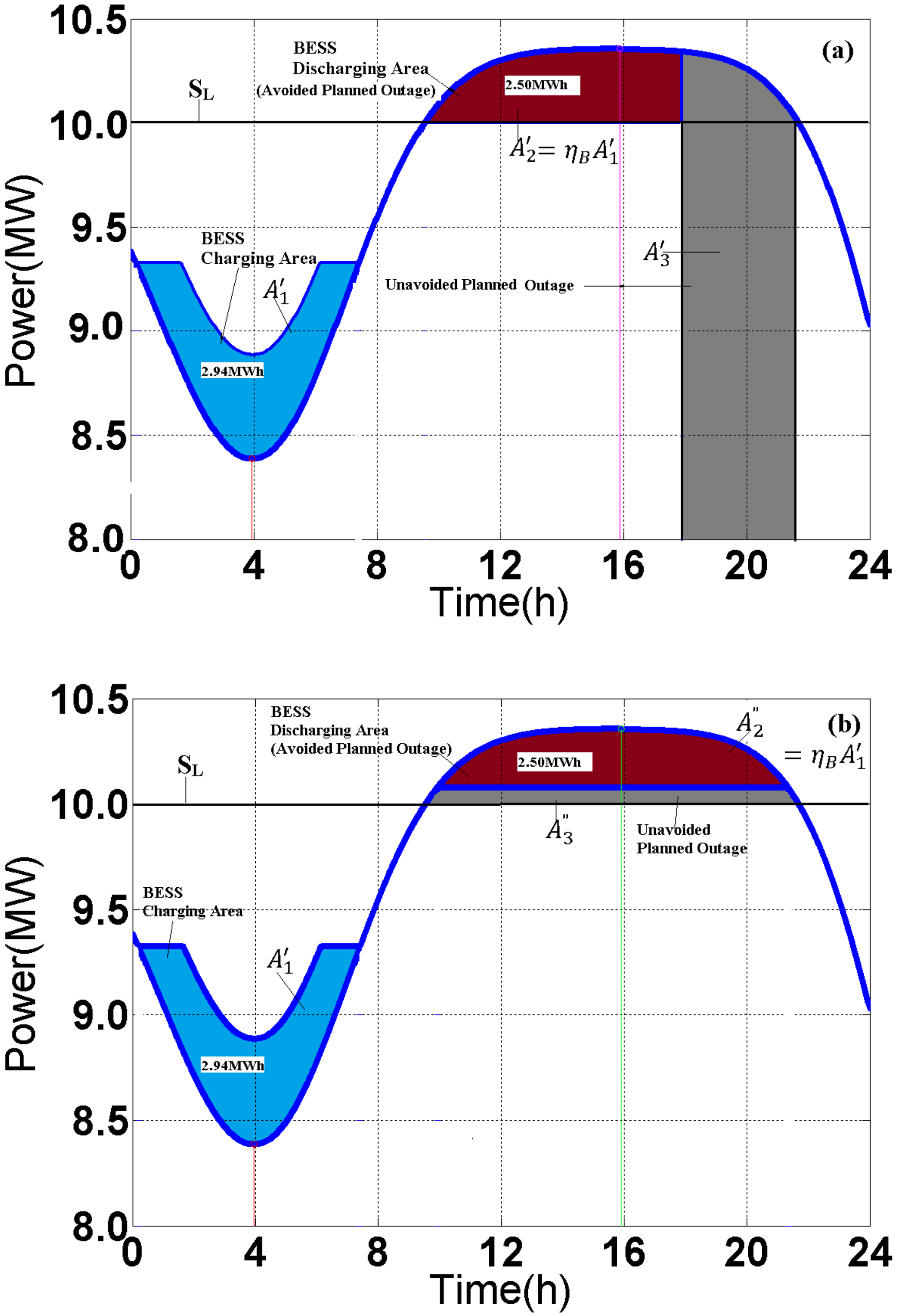


**Figure 5-4** Friday Partial Outage Avoidance Cases. (a) Traditional Way. (b) Smart Grid Method.

For the Friday's load demand, however, the BESS's energy is insufficient to cover all the exceeding demand. Depending on the type of feeder's energy management strategy, there are two possible ways that lead to a planned partial outage (Figure 5-4a, b) [Appendix D for MATLAB code].

1. For the conventional way, the BESS will charge with the energy $A'_1 = 2.94\ MWh$, and discharge $A'_2 = \eta_B A'_1 = 2.50\ MWh$ avoiding outage and riding through during the time 9:30 A.M to 5:50 P.M, Figure 5-4a. A complete outage will be experienced for the remaining time 5:50 P.M to 9:40 P.M when BESS can not supply the demand $A'_3$.
2. If a modern, Smart Feeder is involved, the BESS will charge with $A'_1 = 2.94\ MWh$ and discharge $A_2^{''} = \eta_B A'_1 = 2.50\ MWh$ over the entire peak demand (10:00 A.M to 9:20 P.M). The utility that operates a Smart Grid has the outstanding ability to remote control many loads such as compressors, welders, heaters, as well as major residential loads like air-conditioners devices, and pumps. This condition helps ride through the planned outages while minimizing the reparation costs.

Two methods were considered: first was called Smart I. It was assumed that it is fair to impose on all customers the same burdens of bearing the inconveniences of the outage. For example if the loads are 50% residential, 25% small commercial and industrial and 25% large commercial and industrial, then the outage will disconnect loads belonging to all these groups in proportion with their proliferation. The second method, Smart II, limits the outage only to residential customers.

## 5.3 Outage Avoidance Case Study: Results and Discussion

Giving the method described above, the outage reparation costs can be calculated and compared using Table 5-1. The following equation helps compute the total costs of the outage:

$$\boldsymbol{K_{outage} = P_{avg}K_O} \qquad (\$) \qquad \text{(5-1)}$$

where

$P_{avg}$= average outage power (kW).

$K_O$=average outage reparation costs ($/kW) for the unavoided outage time (h).

The avoided/unavoided outage reparation costs as well as the daily Differential Cost of Energy (DCE) are summarized in Tables 5-2, 3, 4 assuming that the outage reparation costs remain constant for 8 h or a longer period of time [2].

**Table 5-2:** Avoided Outage Reparation Costs (Thursday)

| Feeder Type | Resid. 50% ($) | Small Com./Ind 25% ($) | Large Com./Ind. 25% ($) | DCE ($) | TOTAL ($) |
|---|---|---|---|---|---|
| Conventiona | 28,353 | 479,036 | 173,053 | 3,000 | 683,442 |
| Smart I | 227 | 3,832 | 1,384 | 3,000 | 8,443 |
| Smart II | 454 | -- | -- | 3,000 | 3,454 |

**Table 5-3:** Avoided Outage Reparation Costs (Friday)

| Feeder Type | Resid. 50% ($) | Small Com./Ind 25% ($) | Large Com./Ind. 25% ($) | DCE ($) | TOTAL ($) |
|---|---|---|---|---|---|
| Conventiona | 1,758 | 186,736 | 81,801 | 2,928 | 273,223 |
| Smart I | 286 | 4,825 | 1,743 | 3,304 | 10,158 |
| Smart II | 571 | -- | -- | 3,304 | 3,875 |

**Table 5-4:** Unavoided Outage Reparation Costs (Friday)

| Feeder Type | Resid. 50% ($) | Small Com./Ind 25% ($) | Large Com./Ind. 25% ($) | TOTAL ($) |
|---|---|---|---|---|
| Conventiona (No BESS) | -28,899 | -488,261 | -176,386 | -693,546 |
| Conventiona (With BESS) | -27,141 | -301,524 | -94,585 | -423,250 |
| Smart I (No BESS) | -504 | -8,515 | -3,076 | -12,095 |
| Smart I | -218 | -3,690 | -1,333 | -5,241 |
| Smart II (No BESS) | -1,008 | -- | -- | -1,008 |
| Smart II | -437 | -- | -- | -437 |

In conclusion, if the general outage can not be avoided (as happened Thursday), and the substation main circuit breaker is tripped then the reparation costs will be significant, $ 680442. If the feeder and the significant loads are equipped with smart meters and remote controlled switches outage can be limited to residential loads as well as the low priority customers, in which case the Smart I method will avoid the reparation costs of $ 5443. The Smart II method helps avoid $ 454. In addition, $ 3000 DCE can be obtained by shifting energy purchase for both methods.

Friday's outage can only be partially avoided due to the BESS capacity limitation. The avoided reparation costs are summarized in Table 5-3 and the unavoided costs are summarized in Table 5-4. From Table 5-4 one can realize that there will be $ 693,546 costs by traditional outage method. From Table 5-3, however, it shows that $ 270,295 can be saved in addition to $ 2,928 DCE by using BESS. If the outage happened to the modern smart feeder, there will also be $ 6,854 and $ 571 money saved by methods "Smart I" and "II", respectively. A DCE of $ 3,304 will be achieved in each case.

The obtained results indicate that under outage conditions the economic benefit obtained from BESS shifting energy may be substantial. Although due to the capacity limitation the BESS may only shift part of the peak energy avoiding only partial outage reparation costs, nevertheless, the use of storage energy still helps to avoid significant reparation costs. This approach is especially attractive for the case of a conventional feeder since from Table 5-2 and 3, one can learn that the BESS can help avoid about one million dollars reparation costs in only two outage days.

## *5.4 Conclusion*

From the results discussed in this chapter, it is learned the following:

1. Modern control and smart metering instrumentation allows the application of different strategies for outage avoidance. The traditional method, tripping the main circuit breaker, may be more than hundreds times expensive than the outage avoidance method offered by the Smart Grid implementation.
2. The BESS can help ride through all or a partial outage which is especially an attractive approach under the traditional outage operation option.
3. A substantial economic benefit can be obtained by combining outage avoidance application with the energy purchase shifting application.

# 6. Conclusions and Suggestions for Future Work

## 6.1 Conclusions

This thesis justified the economical feasibility of the Battery Energy Storage System (BESS) by exploring three major application topics: Energy Purchases Shifting, Distribution Feeder Deferral and Outage Avoidance. After introduce of BESS and distribution system characteristics, the relations among different applications are evaluated. Applied energy market economics, the developed simulation tools ensured the optimal BESS charging/discharging, quantified three major applications benefits. The overall results yield a significant amount of benefit can be obtained by combining three applications. The effects of different BESS's and distribution system's characteristics on the three applications are exposed. Given system condition, the maximum benefit can be found. The proposed simulation models and finding regarding the three applications and the different parameters effects can be very helpful to various organizations, including utilities companies, distribution system operators and end-user customers.

### On Energy Purchases Shifting

Under the assumption of the direct relation between marginal cost of energy and the instantaneous load demand of the feeder, the optimal BESS charging/discharging method is developed. The various simulation results indicate the high round-trip efficiency BESS energy purchases shifting economical potential will be substantial with high peak demand and extreme variation of marginal cost. Annual load variation was generated and the maximum annual differential cost of energy (DCE) is calculated. The annual DCE will still fairly attractive even through the conditions may not be ideal.

## On Distribution Feeder Deferral

Following the guidance of energy market economic, the potential benefit of BESS distribution deferral benefit can be quantified as Net Present Value (NPV). The NPV is the present value difference between two investments: the distribution feeder upgrade without/with BESS deferral. The NPV is a function of feeder load growth rate, feeder length, BESS round-trip efficiency, life span, rated power as well as rated discharge time. The sensitivity study results indicate a long feeder with proportional large capital cost will obtain a significant benefit. The importance of long life span BESS is also proven. It indicates the maximum benefit may not occur under the longest deferral scenario. The optimal BESS operation strategy should be determined by solving nonlinear optimization problem under the given system constraints.

## On Outage Avoidance

The outage will be implemented when the feeder load power is above the thermal limit. The different outage operation strategies and their corresponding benefits are compared. The results shows the BESS is much more economic attractive under the conventional outage operation option than the Smart Grid implementation. It also demonstrates the importance for combining the different BESS applications to get higher benefit.

## *6.2 Suggestions for Future Work*

This future research work can be done in the following topics;

1. More complex market economic can be considered. For example, instead of annualized interest calculations, the interest rate of the bank can be considered in a shorter period to get the more accurate results. Besides, the inflation effects should also be studied.
2. More BESS characteristic parameters could be considered. The relations and effects between different parameters should also be addressed. For instance, the life span of the BESS should be a function of temperature and deep of battery discharge.
3. This research considered the price of BESS and feeder construction to be constant in order to simplify the calculation. In the future, they should be considered as a function of different system parameters. The price of the BESS, for example, should be a function of BESS capacity as well as life span. The price of the feeder construction is also dependent on the length of the feeder and the overall market conditions.
4. The optimization should be implemented to obtain the optimal BESSs choice and operating strategy in feeder deferral application.
5. The more complex distribution system network should be implemented. The relation between system apparent power, active power and reactive power should be studied and addressed in the study.

# References

[1] G. C. Jim Eyer. Energy storage for the electricity grid: Benefits and market potential assessment guide. SAND2010-0815 2010. Available: http://prod.sandia.gov/techlib/access-control.cgi/2010/100815.pdf.

[2] Dan Rastler. Electricity energy storage technology options: A white paper primer on applications, costs, and benefits. EPRI, Palo Alto, CA, 2010, 102676.

[3] Zhenguo Yang, Jianlu Zhang, Michael C. W. Kintner-Meyer, Xiaochuan Lu, Daiwon Choi, John P. Lemmon and Jun Liu. Electrochemical energy storage for green grid. Chem Rev 2011.

[4] A. G. Sterling. Flywheel energy storage for carrying the daily peak loads of electric power systems. 1949.

[5] DOE Energy Storage Systems Program - Sandia National Laboratories (SNL) Publications. Available: http://www.sandia.gov/ess/publications/pubslist_06.html.

[6] Nicholas J. Magnani, Robert P. Clark, Jeffrey W. Braithwaite, Donald M. Bush, Paul C. Butler, James M. Freese, Kenneth R. Grothaus, Kevin D. Murphy, Paul E. Shoemaker. Exploratory battery technology development and testing report for 1985. Sandia National Laboratories. 1987.

[7] B. S. Borowy and Z. M. Salameh. Methodology for optimally sizing the combination of a battery bank and PV array in a wind/PV hybrid system. Energy Conversion, IEEE Transactions On 11(2), pp. 367-375. 1996. . DOI: 10.1109/60.507648.

[8] W. D. Kellogg, M. H. Nehrir, G. Venkataramanan and V. Gerez. Generation unit sizing and cost analysis for stand-alone wind, photovoltaic, and hybrid wind/PV systems. Energy Conversion, IEEE Transactions On 13(1), pp. 70-75. 1998. . DOI: 10.1109/60.658206.

[9] N. W. Miller, R. S. Zrebiec, R. W. Delmerico and G. Hunt. Battery energy storage systems for electric utility, industrial and commercial applications. Battery Conference on Applications and Advances, 1996. , Eleventh Annual pp. 235-240. 1996. . DOI: 10.1109/BCAA.1996.485001.

[10] N. W. Miller, R. S. Zrebiec, G. Hunt and R. W. Deimerico. Design and commissioning of a 5 MVA, 2.5 MWh battery energy storage system. Transmission and Distribution Conference, 1996. Proceedings. , 1996 IEEE pp. 339-345. 1996. . DOI: 10.1109/TDC.1996.545957.

[11] Kyung-Hee Jung, Hoyong Kim and Daeseok Rho. Determination of the installation site and optimal capacity of the battery energy storage system for load leveling. Energy Conversion, IEEE Transactions On 11(1), pp. 162-167. 1996. . DOI: 10.1109/60.486591.

[12] P. F. Ribeiro, B. K. Johnson, M. L. Crow, A. Arsoy and Y. Liu. Energy storage systems for advanced power applications. Proceedings of the IEEE 89(12), pp. 1744-1756. 2001. . DOI: 10.1109/5.975900.

[13] J. P. Barton and D. G. Infield. Energy storage and its use with intermittent renewable energy. Energy Conversion, IEEE Transactions On 19(2), pp. 441-448. 2004. . DOI: 10.1109/TEC.2003.822305.

[14] Z. Yang, C. Shen, L. Zhang, M. L. Crow and S. Atcitty. Integration of a StatCom and battery energy storage. Power Systems, IEEE Transactions On 16(2), pp. 254-260. 2001. . DOI: 10.1109/59.918295.

[15] MIT Electric Vehicle Team. A guide to understanding battery specifications. 2008.

[16] David Allen, Chris Brown, John Hickey, Vinh Le, Robert Safuto. Energy storage in the new york electricity market. 2009.

[17] EOS. EOS webpage presentation-energy storage. EOS 2013. Available: http://www.eosenergystorage.com/wp-content/uploads/2013/04/Eos-Public-Presentation-2013-02-11.pdf.

[18] S. Vazquez, S. M. Lukic, E. Galvan, L. G. Franquelo and J. M. Carrasco. Energy storage systems for transport and grid applications. Industrial Electronics, IEEE Transactions On 57(12), pp. 3881-3895. 2010. . DOI: 10.1109/TIE.2010.2076414.

[19] S. C. Smith, P. K. Sen and B. Kroposki. Advancement of energy storage devices and applications in electrical power system. Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE pp. 1-8. 2008. . DOI: 10.1109/PES.2008.4596436.

[20] K. C. Divya and J. Østergaard. Battery energy storage technology for power systems—An overview. Electr.  Power Syst.  Res. 79(4), pp. 511-520. 2009. . DOI: 10.1016/j.epsr.2008.09.017.

[21] A. Oudalov, T. Buehler and D. Chartouni. Utility scale applications of energy storage. Energy 2030 Conference, 2008.   ENERGY 2008.   IEEE pp. 1-7. 2008. . DOI: 10.1109/ENERGY.2008.4780999.

[22] W. Shirley, R. Cowart, R. Sedano, F. Weston, C. Harrington, D. Moskovitz. State electricity regulatory policy and distributed resources: Distribution system cost methodologies for distributed generation. NREL/SR-560-32500 2002.

[23] Michelle Michot Foss. Economics of the energy industries. 2006.

[24] S. J. Wang, S. M. Shahidehpour and N. -. Xiang. Power systems marginal cost curve and its applications. Power Systems, IEEE Transactions On 10(3), pp. 1321-1328. 1995. . DOI: 10.1109/59.466521.

[25] C. P. Rodriguez and G. J. Anders. Energy price forecasting in the ontario competitive power system market. Power Systems, IEEE Transactions On 19(1), pp. 366-374. 2004. . DOI: 10.1109/TPWRS.2003.821470.

[26] Internal Market Monitor. 2011 annual markets report. ISO New England 2012.

[27] Susan M. Schoenung and William V. Hassenzahl. Long- vs. short-term energy storage technologies analysis-A life-cycle cost study. SAND2003-2783 2003.

[28] Susan Schoenung. Energy storage systems cost update. SAND2011-2730 2011.

[29] Rahul Walawalkar and Jay Apt. Market analysis of emerging electric energy storage systems. DOE/NETL-2008/1330 2008.

[30] F. Giraud and Z. M. Salameh. Steady-state performance of a grid-connected rooftop hybrid wind-photovoltaic power system with battery storage. Power Engineering Review, IEEE 21(2), pp. 54-54. 2001. . DOI: 10.1109/MPER.2001.4311278.

[31] D. B. Nelson, M. H. Nehrir and C. Wang. Unit sizing of stand-alone hybrid wind/PV/fuel cell power generation systems. Power Engineering Society General Meeting, 2005. IEEE pp. 2116-2122 Vol. 3. 2005. . DOI: 10.1109/PES.2005.1489286.

[32] A. Oudalov, R. Cherkaoui and A. Beguin. Sizing and optimal operation of battery energy storage system for peak shaving application. Power Tech, 2007 IEEE Lausanne pp. 621-625. 2007. . DOI: 10.1109/PCT.2007.4538388.

[33] Weihao Hu, Zhe Chen and B. Bak-Jensen. Optimal operation strategy of battery energy storage system to real-time electricity price in denmark. Power and Energy Society General Meeting, 2010 IEEE pp. 1-7. 2010. . DOI: 10.1109/PES.2010.5590194.

[34] Dinghuan Zhu and G. Hug-Glanzmann. Real-time control of energy storage devices in future electric power systems. PowerTech, 2011 IEEE Trondheim pp. 1-7. 2011. . DOI: 10.1109/PTC.2011.6019262.

[35] Xuzhu Dong, Guannan Bao, Zhigang Lu, Zhichang Yuan and Chao Lu. Optimal battery energy storage system charge scheduling for peak shaving application considering battery lifetime. Informatics in Control, Automation and Robotics 2pp. 211-218. 2012. . DOI: 10.1007/978-3-642-25992-0_30.

[36] D. K. Maly and K. S. Kwan. Optimal battery energy storage system (BESS) charge scheduling with dynamic programming. Science, Measurement and Technology, IEE Proceedings - 142(6), pp. 453-458. 1995. . DOI: 10.1049/ip-smt:19951929.

[37] C. H. Lo and M. D. Anderson. Economic dispatch and optimal sizing of battery energy storage systems in utility load-leveling operations. Energy Conversion, IEEE Transactions On 14(3), pp. 824-829. 1999. . DOI: 10.1109/60.790960.

[38] H. A. Gil and G. Joos. On the quantification of the network capacity deferral value of distributed generation. Power Systems, IEEE Transactions On 21(4), pp. 1592-1599. 2006. . DOI: 10.1109/TPWRS.2006.881158.

[39] Jim Eyer, Joe Iannucci, Paul C. Butler. Estimating electricity storage power rating and discharge duration for utility transmission and distribution deferral. SAND2005-7069 2005.

[40] Jim Eyer. Electric utility transmission and distribution upgrade deferral benefits from modular electricity storage. SAND2009-4070 2009.

[41] M. Ralph, A. Ellis, D. Borneo, G. Corey and S. Baldwin. Transmission and distribution deferment using PV and energy storage. Photovoltaic Specialists Conference (PVSC), 2011 37th IEEE pp. 002415-002419. 2011. . DOI: 10.1109/PVSC.2011.6186436.

[42] M. J. Sullivan, B. Noland Suddeth, T. Vardell and A. Vojdani. Interruption costs, customer satisfaction and expectations for service reliability. Power Systems, IEEE Transactions On 11(2), pp. 989-995. 1996. . DOI: 10.1109/59.496185.

[43] R. Ghajar, R. Billinton and E. Chan. Distributed nature of residential customer outage costs. Power Systems, IEEE Transactions On 11(3), pp. 1236-1244. 1996. . DOI: 10.1109/59.535595.

[44] Kristina Hamachi LaCommare, Joseph H. Eto. Understanding the cost of power interruptions to U.S. electricity consumers. Energy Analysis Department Ernest Orlando Lawrence Berkeley National Laboratory. University of California Berkeley Berkeley, California 94720. 2004Available: http://certs.lbl.gov/pdf/55718.pdf.

[45] Michael J. Sullivan, Matthew Mercurio, Josh Schellenberg, Sullivan & Co., "Estimated Value of Service Reliability for Electric Utility Customers in the United States," 2009.

[46] Yixing Xu and C. Singh. Distribution systems reliability and economic improvement with different electric energy storage control strategies. Power and Energy Society General Meeting, 2011 IEEE pp. 1-8. 2011. . DOI: 10.1109/PES.2011.6039058.

[47] J. M. Gantz, S. M. Amin and A. M. Giacomoni. Optimal mix and placement of energy storage systems in power distribution networks for reduced outage costs. Energy Conversion Congress and Exposition (ECCE), 2012 IEEE pp. 2447-2453. 2012. . DOI: 10.1109/ECCE.2012.6342550.

[48] Dan Rastler. Electric energy storage an essential asset in the smart grid. EPRI 2009.

[49] Jie Shi, Wei-jen Lee, Yongqian Liu, Yongping Yang and P. Wang. Forecasting power output of photovoltaic system based on weather classification and support vector machine.

Industry Applications Society Annual Meeting (IAS), 2011 IEEE pp. 1-6. 2011. . DOI: 10.1109/IAS.2011.6074294.

[50] L. Cristaldi, M. Faifer, M. Rossi and F. Ponci. Monitoring of a PV system: The role of the panel model. Applied Measurements for Power Systems (AMPS), 2011 IEEE International Workshop On pp. 90-95. 2011. . DOI: 10.1109/AMPS.2011.6090437.

[51] A. E. Emanuel, J. A. Orr, D. Cyganski and E. M. Gulachenski. A survey of harmonic voltages and currents at distribution substations. Power Delivery, IEEE Transactions On 6(4), pp. 1883-1890. 1991. . DOI: 10.1109/61.97736.

[52] Alexander Eigeles Emanuel, Power Definitions and the Physical Mechanism of Power Flow. Wiley-IEEE Press, 2010.

[53] Gilbert M. Masters. Renewable and Efficient Electric Power Systems 2004.

[54] ISO-New England: Hourly Zonal Information. Available: http://www.iso-ne.com/markets/hstdata/znl_info/hourly/index.html.

[55] J. W. R. Zaborsky, Electric Power Transmission. Rensse;aer Bookstore, 1969.

[56] H. Lee Willis. Power Distribution Planning Reference Book (2nd ed.) 2004.

# Appendix A: MATLAB Code for Chapter 2

The following code was used to generate PV output power shown in Figure 2-3.

- **Code for PV output power**

```
clear all;clc;close all;
%% Play with mother curve
time_step=1/20;
t=0:time_step:24;
time_max=12;
time_diff=4;
Smax_PV=0.5e6;
Smin_PV=0.2e6;
year_day=1:1:365;
year_max=182.5;
year_diff=150;
Pmax_PV_possible=(Smax_PV-Smin_PV)*exp(-(year_day-
year_max).^2/(2.*(year_diff./3).^2))+Smin_PV;
time_yearold=0:time_step:24*365;
time_year=[time_yearold 1:1:364];
Power_yearold=0:time_step:24*365;
Power_year=[Power_yearold 1:1:364];
day=zeros(1,365);
for f=1:1:365;
    weather=rand(1,1);
    if weather>=0.5
for a=1:10:(length(t)-20)
        Pmax1=Pmax_PV_possible(f).*round(5*(0.2+0.8*rand))/5;
        mother_S_R1=Pmax1.*exp(-(t-time_max).^2/(2.*(time_diff./3).^2));
        Pmax2=Pmax_PV_possible(f).*round(5*(0.2+0.8*rand))/5;
        mother_S_R2=Pmax2.*exp(-(t-time_max).^2/(2.*(time_diff./3).^2));
    jump=randi(19,1,2);
    c=min(jump);
    d=max(jump);
    for b=a:1:a+19
        if b<(c+a)
        magni_S_R(b)=mother_S_R1(b);
        elseif (c+a)<=b&&b<=(d+a)
            k=(mother_S_R2(d+a)-mother_S_R1(c+a))./(d-c);
            q=mother_S_R1(c+a)-k*(c+a);
        magni_S_R(b)=k*b+q;
        else
        magni_S_R(b)=mother_S_R2(b);
        end
    end
end
    else
        Pmax=Pmax_PV_possible(f);
        magni_S_R=Pmax.*exp(-(t-time_max).^2/(2.*(time_diff./3).^2));
    end
magni_S_R(481)=0;
```

```
magni_S_R;
r = 0.8 + (1-0.8).*rand(1,481);
weather_2=rand(1,1);
if  weather_2<=0.5
    S_R_rreal=magni_S_R;
else S_R_rreal=magni_S_R.*r;
end
%give the n day's value to a one big matrix
Power_year((f+((f-1)*24/time_step)):1:(f+(f*24/time_step)))=S_R_rreal;
end
year=(1/(1+(20*24))):(1/(1+(20*24))):365;
Power_year;
%plot
figure(1)
str_arr = char( 'x', '-', '.', '+','*', '--','<',   ':', 'd','>' );
incr2=5;
plot(year,1e-6*abs(Power_year),'b');
hold on
%% make the lable
xlabel('Time (days)','Fontsize',16);
ylabel('Active Power generated from the PV (MW)','Fontsize',16);
title('The Active Power generated from the PV  (MW) for a
year','Fontsize',16);
%text(22,3,'Day-1','FontSize',11,'FontWeight','bold');
xlim([0 4]);
%set(gca,'XTick',[0:1:120]);
set(gca,'FontSize',20,'FontWeight','bold');
grid on;
%% legend
hleg2=legend('Active Power generated from the PV (MW) over the year');
set(hleg2,'Location','NorthEast','FontSize',10);
```

# Appendix B: MATLAB Code for Chapter 3

There are seven code scripts for Ch. 3. The list of the scripts is summarized:

1. Code for marginal cost—generates the marginal cost curves in Figure 3-2
2. Code for 24 hour load curves—generates the load demand in Figure 3-3b
3. Code for double peaks load demand—generates the load demand in Figure 3-5
4. Code for DCE vs. $T_B$—generates the results in Figure 3-6
5. Code for DCE vs. $\eta_B$—generates the results in Figure 3-7
6. Code for DCE vs. $W_{Ch}/W_T$—generates the results in Figure 3-8, 3-9, 3-10.
7. Code for annual load variation—generates the annual load variation shown in Figure 3-11

- **Code for marginal cost**

```
clear all;clc;close all;
%% money
a1=15;b1=0.1;c1=9.955;d1=6.8;%Curve A (Bottom)
a2=15;b2=0.1;c2=5.931;d2=2.56;% Curve B (MIddle)
a3=15;b3=0.1;c3=4.904;d3=1.23; % Curve C (Top)
power=0:0.001:10;
price1= a1+(b1*abs(power).^c1).*10^(-d1);
price2= a2+(b2*abs(power).^c2).*10^(-d2);
price3= a3+(b3*abs(power).^c3).*10^(-d3);
figure(1)
axes1 = axes('Parent',figure(1),...
    'XTick',[0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76
77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101
102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119
120],...
    'Position',[0.12609375 0.0908988764044947 0.775 0.815],...
    'FontWeight','bold',...
    'FontSize',20);
str_arr = char( '*', '-', '.', '+','x', '--','<',   ':', 'd','>' );
incr2=100;
plot(power(1:incr2:end),price1(1:incr2:end),str_arr(1),'LineWidth',1,'Display
Name','a=15 b=1/10^8 c=9.955 ');
hold on
plot(power(1:incr2:end),price2(1:incr2:end),str_arr(2),'LineWidth',4,'Display
Name','a=15 b=1/10^4 c=5.931');
plot(power(1:incr2:end),price3(1:incr2:end),str_arr(3),'LineWidth',1,'Display
Name','a=15 b=1/10^2 c=3.904');
xlabel('Power (MW)','Fontsize',16);
```

```
ylabel('Price of the electricity $/MWh','Fontsize',16);
title('The relation between price of the electricity and the power
(MW)','Fontsize',16);
xlim([0 10]);
ylim([0 500]);
set(gca,'XTick',[0:1:120]);
set(gca,'FontSize',20,'FontWeight','bold');
grid on;
```

- **Code for 24 hour load curves**

```
clear all;close all;clc
%% generate 4 graphs, each contain 3 curves
time_step=1/20;
t=-16:time_step:24;
t_day=0:time_step:24;
time_max1=-4;
time_max2=20;
S_real=zeros(36,481);
time_diff1=[9 11 18];
for Smax=7e6:0.5e6:10e6
for Smin=5.5e6:0.5e6:6.5e6;
Smax_delta=Smax-Smin;
    for time_diff=[9 11 18]
if time_diff>11
min_error = realmax('double');
    for a=0.5e6:0.01e6:5.5e6
        error_D=Smax-(-a.*exp(-(16-8-
time_max1).^2/(2.*(time_diff./3).^2))+(Smin+a));
        if error_D < min_error && error_D>=0 %Make sure no energy above the
limit and no negative energy
        min_error_D = error_D;
        a_best=a;%Always find the min error point by given efficiency

        end

    end
           if Smax==10e6&&Smin==5.5e6
    display('here we go')
    a_best
             end
        magni_S=((t <= 8+8) .*-a_best.*exp(-(t-8-
time_max1).^2/(2.*(time_diff./3).^2))+(t >8+8) .*-a_best.*exp(-(t-8-
time_max2).^2/(2.*(time_diff./3).^2)))+ones(1,801)*(Smin+a_best);
else magni_S=((t <= 8+8) .*-Smax_delta.*exp(-(t-8-
time_max1).^2/(2.*(time_diff./3).^2))+(t >8+8) .*-Smax_delta.*exp(-(t-8-
time_max2).^2/(2.*(time_diff./3).^2)))+ones(1,801)*(Smax);
end
S_real(9*((Smax-7e6)/0.5e6)+3*((Smin-
5.5e6)/0.5e6)+find(time_diff1==time_diff),:)=magni_S(
find(t==0):1:find(t==24));
plot(t_day,S_real(9*((Smax-7e6)/0.5e6)+3*((Smin-
5.5e6)/0.5e6)+find(time_diff1==time_diff),:)./1e6,'LineWidth',3);
hold on
hold on
```

```
        xlabel('Time (h)','Fontsize',16);
ylabel('Power (MW)','Fontsize',16);
title('The Load curve','Fontsize',16);
set(gca,'FontSize',20,'FontWeight','bold');
grid on;
xlim([0,24]);
set(gca,'XTick',[-4:4:120]);
ylim([5,10]);
end
end
end
hold off
for a=28:1:30
plot(t_day,S_real(a,:)./1e6,'LineWidth',6);
hold on
xlabel('Time (h)','Fontsize',40);
ylabel('Power (MW)','Fontsize',16);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;
xlim([0,24]);
set(gca,'XTick',[-4:4:120]);
ylim([5,10]);
end
load_best=S_real(28,:)
```

- **Code for double peaks load demand**

```
%% generate multiple peak load curve

clear all;close all;clc

time_step=1/20;
t=0:time_step:24;
time_max=12;
time_diff=11;
case_day=1:1:3;
time_caseold=0:time_step:24*3;
time_case=[time_caseold 48:1/20:48.05];
Power_yearold=0:time_step:24*3;
Power_year=[Power_yearold 1:1:2];
Smin=zeros(1,3);
Power=zeros(1,length(t));

for a=1:1:3
    Smax=6.5e+6+a*1e6;
    Smin_o=5e6+a*1e6;
    Smax_delta=Smax-Smin_o;
    if a<3
    Smin(a+1)=5e6+(a+1)*1e6;
    magni_S_1=Smax_delta.*exp(-(t-
time_max).^2/(2.*(time_diff./3).^2))+ones(1,481)*Smin_o;
    magni_S_2=(Smax_delta+Smin_o-Smin(a+1)).*exp(-(t-
time_max).^2/(2.*(time_diff./3).^2))+ones(1,481)*Smin(a+1);
    magni_S=[magni_S_1(1:240) magni_S_2(241:481)];
    elseif a==3
```

```
    magni_S=Smax_delta.*exp(-(t-
time_max).^2/(2.*(time_diff./3).^2))+ones(1,481)*Smin_o;




    end
    Power_year((a+((a-1)*24/time_step)):1:(a+(a*24/time_step)))=magni_S;
end
Power_3day=Power_year+0.5e6;
S_multipeak=Power_3day(241:2.5:1443);
time=0:1/20:24;
plot(time,S_multipeak./1e6,'LineWidth',3);
hold on
hold on
        xlabel('Time (h)','Fontsize',16);
ylabel('Power (MW)','Fontsize',16);
title('The Double-Peak Load curve','Fontsize',16);
set(gca,'FontSize',20,'FontWeight','bold');
grid on;
xlim([0,24]);
set(gca,'XTick',[0:4:120]);
```

- **Code for DCE vs. $T_B$**

```
%This code need 24 hour load curve data named 'load_best'

%One can get the 24 hour load data by run the code number 2
load_best;
for b=1:1:1
       for efficiency=0.55:0.15:0.85

%W_aveP_worstrage=154.54; %<W>= average power
%% Money saving
time_step=1/20;
t=0:time_step:24;
 efficiency1=[0.55 0.7 0.85];
    Discharging_bar=zeros(length(t),length(t));
    Charging_bar=zeros(length(t),length(t));
    Charging_bar_try=zeros(4*length(t),length(t));%Find the matching charging
point given the discharging condition, four times accuracy

    W_charging_real=zeros(length(t),length(t));
    W_discharging_real=zeros(length(t),length(t));
    TB1=2:0.05:10;
    Benefit=zeros(1,36*length(TB1));

    energy_charging=zeros(1,length(t));

    Power_D=zeros(1,length(t));
    Power_C=zeros(1,length(t));

    Money_saving_D=zeros(1,length(t));
    Money_waste_C=zeros(1,length(t));
```

```
    Money_waste_C_new=zeros(1,length(TB1));
    Money_saving_D_new=zeros(1,length(TB1));
        Money_before_D=zeros(1,length(TB1));
    Money_after_D=zeros(1,length(TB1));
    Money_before_C=zeros(1,length(TB1));
    Money_after_C=zeros(1,length(TB1));
    S_charging1_ok_real=zeros(1,length(t));
    S_discharging_real=zeros(1,length(t));
    S_discharging_real_ok=zeros(1,length(t));
    Price_waste_C=zeros(1*length(t),length(t));
    Price_Saving_D=zeros(1*length(t),length(t));
    Price_waste_C_new=zeros(1*length(TB1),length(t));
    Price_Saving_D_new=zeros(1*length(TB1),length(t));

    Pmax=5e5;%Power Limit


% Since only on curve, No outside loop needed
    Power_B_D=zeros(1,length(t));
    Power_A_D=zeros(1,length(t));
    Power_B_C=zeros(1,length(t));
    Power_A_C=zeros(1,length(t));

    [M,TM] = max(load_best(b,:));%Find the maximum power point by the given
load
    Smax=M;
    time_max=TM;

    [m,Tm] = min(load_best(b,1:TM));% Find the minimum power point occured
before the maximum power point by the given load
    Smin=m;
    time_min=Tm;

    step_discharging=(abs(Smax)-abs(Smin))/(length(t));%The step for
discharging
    step_charging=(abs(Smax)-abs(Smin))/(4*length(t));%The step for
charging(Notice the step is four times smaller than charging step, meaning we
have more accuracy)
          for TB=2:0.05:10
    Wmax=Pmax*TB;%Energy Limit
    WBch=Wmax/efficiency;
%% Find the Charging point where fully charged
min_error = realmax('double');
    for a=1:1:4*length(t)%This inner loop help us find the charging point


Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(t));%let's
try from beginning (Charging bar from bottom to the top)
        S_charging=Charging_bar_try(a,:)-abs(load_best(b,:));
        S_charging(S_charging<0)=0;
        S_charging(S_charging>=Pmax)=Pmax;%Address the power limit
        S_charging(TM:end)=0;% find the charging power(Consider the case
time)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
   %Power together (Charging positive & Discharging negative)
    Power_all_try=zeros(1,length(t));
    %point_D_try=find(S_discharging_real(1,:));%find the discharging time
    point_C_try=find(S_charging(1,:));%find the charging time
    %Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try);%get update information of
energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>WBch);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
         error=WBch-max(energy_try);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        pointc=a;%Always find the min error point by given efficiency
        energy_full=time_step.*cumsum(Power_all_try);
                S_charging1_ok_real=Charging_bar_try(a,:)-
abs(load_best(b,:));
        S_charging1_ok_real(S_charging1_ok_real<0)=0;
        S_charging1_ok_real(S_charging1_ok_real>=Pmax)=Pmax;%Address the
power limit
        S_charging1_ok_real(TM:end)=0;% find the charging power(Consider the
case time)
        end
    end

 %% find the discharging power by given efficiency

     W_D=Wmax;

 min_error_D = realmax('double');
    for d=1:1:length(t)%we move our discheging bar down from top to the
bottom

        Power_D(1,d)=(abs(Smax)-step_discharging*d);%For each 'd', this power
is for the discharge point power
        Discharging_bar(d,:)=Power_D(1,d).*ones(1,length(t));% Discharging
bar was created and it is moving down through the loop
 % Address the discharging power and the power limit (Consider the case time)
        S_discharging_real(1,:)=abs(load_best(b,:))-Discharging_bar(d,:);
        S_discharging_real(1,S_discharging_real(1,:)>=Pmax)=Pmax;%Address the
power limit
        S_discharging(1:Tm)=0;% I can discharge nothing before the first
charging point(Consider the case time)
```

```
        S_discharging_real(1,S_discharging_real(1,:)<0)=0;% we got the 
discharging power
Energy_D=time_step*cumsum(S_discharging_real);
    point_negative_energy_D=find(Energy_D<0);
    point_energylimit_D=find(Energy_D>W_D);
error_D=W_D-max(Energy_D);
        if error_D < 
min_error_D&&isempty(point_negative_energy_D)==1&&isempty(point_energylimit_D
)==1 %Make sure no energy above the limit and no negative energy
        min_error_D = error_D;
         pointd=d;
        energy_full_D=time_step.*cumsum(S_discharging_real);
        % Address the discharging power and the power limit (Consider the 
case time)
        S_discharging_real_ok(1,:)=abs(load_best(b,:))-Discharging_bar(d,:);
        
S_discharging_real_ok(1,S_discharging_real_ok(1,:)>=Pmax)=Pmax;%Address the 
power limit
        S_discharging(1:Tm)=0;% I can discharge nothing before the first 
charging point(Consider the case time)
        S_discharging_real_ok(1,S_discharging_real_ok(1,:)<0)=0;% we got the 
discharging power
        end
    end
if abs(Smax)-step_discharging*pointd<=abs(Smin)+step_charging*pointc
    Benefit(1,(b-1)*length(TB1)+round(1+((TB-
2)/0.05)):b*length(TB1))=Benefit(1,(b-1)*length(TB1)+round(1+((TB-2)/0.05))-
1);
 break
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%
else
    %% Benefit
    a1=15;b1=0.1;c1=9.955;d1=6.8;% A
    %a1=15;b1=0.1;c1=5.931;d1=2.56;% B
    %a1=15;b1=0.1;c1=4.904;d1=1.23;% C
    %find the power before&after discharge (NEED TO ADDRESS THE MULTIPLE PEAK 
SITUATION for loop)
    point_D=find(S_discharging_real_ok(1,:));%find the discharging time
    
    Power_B_D(1,point_D)=abs(load_best(b,point_D));
    Power_A_D(1,point_D)=abs(load_best(b,point_D))-
S_discharging_real_ok(1,point_D);% we get the power after we discharge(NOT 
consider the multible peak yet)
    
    %find the power before&after charge (NEED TO ADDRESS THE MULTIPLE VALLEY 
BOTTOM SITUATION for loop)
    point_C=find(S_charging1_ok_real(1,:));%find the charging time
    
    Power_B_C(1,point_C)=abs(load_best(b,point_C));
    
Power_A_C(1,point_C)=abs(load_best(b,point_C))+S_charging1_ok_real(1,point_C)
;% we get the power after we charge(NOT consider the multible peak yet)
    
   %find the price&money saving by discharging
    Pricebefore_D=a1+(b1*abs(Power_B_D./10^6).^c1).*10^(-d1);
```

```
    Priceafter_D=a1+(b1*abs(Power_A_D./10^6).^c1).*10^(-d1);
    Price_Saving_D_new(round(1+((TB-2)/0.05)),:)=Pricebefore_D-
Priceafter_D;%positive because of saving
    Money_before_D(1,round(1+((TB-
2)/0.05)))=time_step.*sum(Pricebefore_D.*Power_B_D./10^6);
    Money_after_D(1,round(1+((TB-
2)/0.05)))=time_step.*sum(Priceafter_D.*Power_A_D./10^6);
    Money_saving_D_new(1,round(1+((TB-
2)/0.05)))=Money_before_D(1,round(1+((TB-2)/0.05)))-
Money_after_D(1,round(1+((TB-2)/0.05)));%positive value and it is increasing
as the bar moving down
    %find the extra price&money cost by charging
    Pricebefore_C=a1+(b1*abs(Power_B_C./10^6).^c1).*10^(-d1);
    Priceafter_C=a1+(b1*abs(Power_A_C./10^6).^c1).*10^(-d1);
    Price_waste_C_new(round(1+((TB-2)/0.05)),:)=Pricebefore_C-Priceafter_C;
%negative price because of waste
    Money_before_C(1,round(1+((TB-
2)/0.05)))=time_step.*sum(Pricebefore_C.*Power_B_C./10^6);
    Money_after_C(1,round(1+((TB-
2)/0.05)))=time_step.*sum(Priceafter_C.*Power_A_C./10^6);
    Money_waste_C_new(1,round(1+((TB-
2)/0.05)))=Money_before_C(1,round(1+((TB-2)/0.05)))-
Money_after_C(1,round(1+((TB-2)/0.05)));%negative


    %% Total Bebefit
    Benefit(1,(b-1)*length(TB1)+round(1+((TB-
2)/0.05)))=Money_saving_D_new(1,round(1+((TB-
2)/0.05)))+Money_waste_C_new(1,round(1+((TB-2)/0.05)));%for each loop,
calculate the total benefit
end
       end
    %% Plot
figure(1)
yy2 = smooth(TB1,Benefit((b-1)*length(TB1)+1:1:b*length(TB1)),0.3,'loess');
plot(TB1,yy2,'LineWidth',6);
hold on
xlabel('T_B(h)','Fontsize',40);
ylabel('Differential Benefit ($)','Fontsize',40);
xlim([2,10]);
set(gca,'XTick',[0:1:120]);
%title('The Total Benefit by using the Battery','Fontsize',16);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;

    end
end
```

- **Code for DCE vs. $\eta_B$**

```
%This code need 24 hour load curve data with different Pmax named 'P_diffmax'

%One can get the 24 hour load data by run the code number 2 and varying
different Pmax
```

```
P_diffmax;
for b=1:1:3
%W_aveP_diffmaxrage=154.54; %<W>= average power
%% Money saving
time_step=1/20;
t=0:time_step:24;
 efficiency1=0.55:0.001:0.85;
    Discharging_bar=zeros(length(t),length(t));
    Charging_bar=zeros(length(t),length(t));
    Charging_bar_try=zeros(4*length(t),length(t));%Find the matching charging
point given the discharging condition, four times accuracy

    W_charging_real=zeros(length(t),length(t));
    W_discharging_real=zeros(length(t),length(t));

    Benefit=zeros(1,36*length(efficiency1));

    energy_charging=zeros(1,length(t));

    Power_D=zeros(1,length(t));
    Power_C=zeros(1,length(t));

    Money_saving_D=zeros(1,length(t));
    Money_waste_C=zeros(1,length(t));
    Money_waste_C_new=zeros(1,length(efficiency1));
    Money_saving_D_new=zeros(1,length(efficiency1));

    S_charging1_ok_real=zeros(1,length(t));
    S_discharging_real=zeros(1,length(t));
    S_discharging_real_ok=zeros(1,length(t));
    Price_waste_C=zeros(1*length(t),length(t));
    Price_Saving_D=zeros(1*length(t),length(t));
    Price_waste_C_new=zeros(1*length(efficiency1),length(t));
    Price_Saving_D_new=zeros(1*length(efficiency1),length(t));

    Pmax=5e5;%Power Limit
    TB=5;
    Wmax=Pmax*TB;%Energy Limit

% Since only on curve, No outside loop needed
    Power_B_D=zeros(1,length(t));
    Power_A_D=zeros(1,length(t));
    Power_B_C=zeros(1,length(t));
    Power_A_C=zeros(1,length(t));
    Money_before_D=zeros(1,length(efficiency1));
    Money_after_D=zeros(1,length(efficiency1));
    Money_before_C=zeros(1,length(efficiency1));
    Money_after_C=zeros(1,length(efficiency1));
    [M,TM] = max(P_diffmax(b,:));%Find the maximum power point by the given
load
    Smax=M;
    time_max=TM;

    [m,Tm] = min(P_diffmax(b,1:TM));% Find the minimum power point occured
before the maximum power point by the given load
```

```
    Smin=m;
    time_min=Tm;

    step_discharging=(abs(Smax)-abs(Smin))/(length(t));%The step for
discharging
    step_charging=(abs(Smax)-abs(Smin))/(4*length(t));%The step for
charging(Notice the step is four times smaller than charging step, meaning we
have more accuracy)

%% Find the fully discharging point
W_D=Wmax;

 min_error_D = realmax('double');
    for d=1:1:length(t)%we move our discheging bar down from top to the
bottom

        Power_D(1,d)=(abs(Smax)-step_discharging*d);%For each 'd', this power
is for the discharge point power
        Discharging_bar(d,:)=Power_D(1,d).*ones(1,length(t));% Discharging
bar was created and it is moving down through the loop
 % Address the discharging power and the power limit (Consider the case time)
        S_discharging_real(1,:)=abs(P_diffmax(b,:))-Discharging_bar(d,:);
        S_discharging_real(1,S_discharging_real(1,:)>=Pmax)=Pmax;%Address the
power limit
        S_discharging(1:Tm)=0;% I can discharge nothing before the first
charging point(Consider the case time)
        S_discharging_real(1,S_discharging_real(1,:)<0)=0;% we got the
discharging power
Energy_D=time_step*cumsum(S_discharging_real);
    point_negative_energy_D=find(Energy_D<0);
    point_energylimit_D=find(Energy_D>W_D);
error_D=W_D-max(Energy_D);
        if error_D <
min_error_D&&isempty(point_negative_energy_D)==1&&isempty(point_energylimit_D
)==1 %Make sure no energy above the limit and no negative energy
        min_error_D = error_D;
        pointd=d;
        energy_full_D=time_step.*cumsum(S_discharging_real);
        % Address the discharging power and the power limit (Consider the
case time)
        S_discharging_real_ok(1,:)=abs(P_diffmax(b,:))-Discharging_bar(d,:);

S_discharging_real_ok(1,S_discharging_real_ok(1,:)>=Pmax)=Pmax;%Address the
power limit
        S_discharging(1:Tm)=0;% I can discharge nothing before the first
charging point(Consider the case time)
        S_discharging_real_ok(1,S_discharging_real_ok(1,:)<0)=0;% we got the
discharging power
        end
    end

 %% find the charging power by given efficiency
 for efficiency=0.55:0.001:0.85
     WBch=Wmax/efficiency;
min_error = realmax('double');
    for a=1:1:4*length(t)%This inner loop help us find the charging point
```

```
Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(t));%let's
try from beginning (Charging bar from bottom to the top)
        S_charging=Charging_bar_try(a,:)-abs(P_diffmax(b,:));
        S_charging(S_charging<0)=0;
        S_charging(S_charging>=Pmax)=Pmax;%Address the power limit
        S_charging(TM:end)=0;% find the charging power(Consider the case
time)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
   %Power together (Charging positive & Discharging negative)
    Power_all_try=zeros(1,length(t));
    %point_D_try=find(S_discharging_real(1,:));%find the discharging time
    point_C_try=find(S_charging(1,:));%find the charging time
    %Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try);%get update information of
energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>WBch);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
         error=WBch-max(energy_try);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        point=a;%Always find the min error point by given efficiency
        energy_full=time_step.*cumsum(Power_all_try);
                S_charging1_ok_real=Charging_bar_try(a,:)-
abs(P_diffmax(b,:));
        S_charging1_ok_real(S_charging1_ok_real<0)=0;
        S_charging1_ok_real(S_charging1_ok_real>=Pmax)=Pmax;%Address the
power limit
        S_charging1_ok_real(TM:end)=0;% find the charging power(Consider the
case time)
        end
    end
if abs(Smax)-step_discharging*pointd<=abs(Smin)+step_charging*point
    Benefit(1,(b-1)*length(efficiency1)+round(1+((efficiency-
0.55)/0.001)))=0;
else

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%
    %% Benefit
    %a1=15;b1=0.1;c1=9.955;d1=6.8;% A
    %a1=15;b1=0.1;c1=5.931;d1=2.56;% B
    a1=15;b1=0.1;c1=4.904;d1=1.23;% C
```

```
    %find the power before&after discharge (NEED TO ADDRESS THE MULTIPLE PEAK
SITUATION for loop)
    point_D=find(S_discharging_real_ok(1,:));%find the discharging time

    Power_B_D(1,point_D)=abs(P_diffmax(b,point_D));
    Power_A_D(1,point_D)=abs(P_diffmax(b,point_D))-
S_discharging_real_ok(1,point_D);% we get the power after we discharge(NOT
consider the multible peak yet)

    %find the power before&after charge (NEED TO ADDRESS THE MULTIPLE VALLEY
BOTTOM SITUATION for loop)
    point_C=find(S_charging1_ok_real(1,:));%find the charging time

    Power_B_C(1,point_C)=abs(P_diffmax(b,point_C));

Power_A_C(1,point_C)=abs(P_diffmax(b,point_C))+S_charging1_ok_real(1,point_C)
;% we get the power after we charge(NOT consider the multible peak yet)

    %find the price&money saving by discharging
    Pricebefore_D=a1+(b1*abs(Power_B_D./10^6).^c1).*10^(-d1);
    Priceafter_D=a1+(b1*abs(Power_A_D./10^6).^c1).*10^(-d1);
    Price_Saving_D_new(round(1+((efficiency-0.55)/0.001)),:)=Pricebefore_D-
Priceafter_D;%positive because of saving

    Money_before_D(1,round(1+((efficiency-
0.55)/0.001)))=time_step.*sum(Pricebefore_D.*Power_B_D./10^6);
    Money_after_D(1,round(1+((efficiency-
0.55)/0.001)))=time_step.*sum(Priceafter_D.*Power_A_D./10^6);

    Money_saving_D_new(1,(round(1+(efficiency-
0.55)/0.001)))=Money_before_D(1,round(1+((efficiency-0.55)/0.001)))-
Money_after_D(1,round(1+((efficiency-0.55)/0.001)));
    %find the extra price&money cost by charging
    Pricebefore_C=a1+(b1*abs(Power_B_C./10^6).^c1).*10^(-d1);
    Priceafter_C=a1+(b1*abs(Power_A_C./10^6).^c1).*10^(-d1);
    Price_waste_C_new(round(1+((efficiency-0.55)/0.001)),:)=Pricebefore_C-
Priceafter_C;     %negative price because of waste

    Money_before_C(1,round(1+((efficiency-
0.55)/0.001)))=time_step.*sum(Pricebefore_C.*Power_B_C./10^6);
    Money_after_C(1,round(1+((efficiency-
0.55)/0.001)))=time_step.*sum(Priceafter_C.*Power_A_C./10^6);

    Money_waste_C_new(1,round(1+((efficiency-
0.55)/0.001)))=Money_before_C(1,round(1+((efficiency-0.55)/0.001)))-
Money_after_C(1,round(1+((efficiency-0.55)/0.001)));    %negative value

    %% Total Bebefit
    Benefit(1,(b-1)*length(efficiency1)+round(1+((efficiency-
0.55)/0.001)))=Money_saving_D_new(1,round(1+((efficiency-
0.55)/0.001)))+Money_waste_C_new(1,round(1+((efficiency-0.55)/0.001)));%for
each loop, calculate the total benefit
end
end
%% Plot
```

```
figure(1)
yy2 = smooth(efficiency1,Benefit((b-
1)*length(efficiency1)+1:1:b*length(efficiency1)),0.3,'rloess');
plot(efficiency1*100,yy2,'LineWidth',6);
hold on
xlabel('Efficiency (%)','Fontsize',40);
ylabel('Differential Benefit ($)','Fontsize',40);
xlim([55,85]);
%set(gca,'XTick',[0:1:120]);
%title('The Total Benefit by using the Battery','Fontsize',16);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;
end
```

- **Code for DCE vs. W$_{Ch}$/W$_{T}$**

```
%This code need 24 hour load curve data named 'load_7'

%one can get the 24 hour load data by run the code number 2 and change
%Pmax, Pmin respectively.
load_7;
time_step=1/20;
t=0:time_step:24;
day=length(load_7)/481;
efficiency1=[0.55 0.7 0.85];
best_benefit=zeros(length(efficiency1),length(day));
%% Big Loop for days
for g=1:1:day
    load_2=load_7((g-1)*481+1:1:g*481); % Address one day power
    if g==6
        disp('find day 6');%Tell me day 6
    end
    %% Address Battery information
for   efficiency=[0.55 0.7 0.85];%Define the battery efficiency
    PB=5e5;%Power Limit
    TB=5;
    WB=PB*TB;%Energy Limit
    WBch=WB/efficiency;

    %% creat Matrix
    W_total=zeros(1,1);

    Money_before_D=zeros(1,length(t));
    Money_after_D=zeros(1,length(t));
    Money_before_C=zeros(1,length(t));
    Money_after_C=zeros(1,length(t));

    Benefit=zeros(1,length(t));
    energy_charging_all=zeros(1,length(t));

    Discharging_bar=zeros(length(t),length(load_2));
    Charging_bar=zeros(length(t),length(load_2));
    Charging_bar_try=zeros(4*length(t),length(load_2));%Find the matching
charging point given the discharging condition, four times accuracy
```

```
    W_charging_real=zeros(length(t),length(load_2));
    W_discharging_real=zeros(length(t),length(load_2));

    S_charging_real=zeros(1,length(load_2));
    S_discharging_real=zeros(1,length(load_2));

    Power_D=zeros(1,length(t));
    Power_C=zeros(1,length(t));

    Power_B_D=zeros(1,length(load_2));
    Power_A_D=zeros(1,length(load_2));
    Power_B_C=zeros(1,length(load_2));
    Power_A_C=zeros(1,length(load_2));

    Money_saving_D=zeros(1,length(t));
    Money_waste_C=zeros(1,length(t));

    Price_waste_C=zeros(1*length(t),length(t));
    Price_Saving_D=zeros(1*length(t),length(t));
%% Small loop for one day Charging & Discharging
for p=1:1:1

W_total(p)=time_step*sum(load_2((p-1)*481+1:1:p*481))*1e-6 %W total power

%% Find Max and Min point
    [m,Tm] = min(load_2(1,((p-1)*481+1:1:p*481)));% Find the minimum power
point occured before the maximum power point by the given load
    Smin=m;
    time_min=Tm+(p-1)*481;
    [M,TM] = max(load_2(1,((p-1)*481+time_min:1:p*481)));%Find the maximum
power point by the given load
    Smax=M;
    time_max=TM+(p-1)*481+time_min-1;

    if time_max<=time_min

           [m,Tm] = min(load_2(1,((p-1)*481+1:1:(p-1)*481+time_max-1)));%
Find the minimum power point occured before the maximum power point by the
given load

    if isempty(Tm)==1
        for j=1:1:481
            [M,TM] = max(load_2(1,((p-1)*481+j:1:p*481)));
            if TM>j
                Smax=M;
                time_max=TM+(p-1)*481+j-1;
            [m,Tm] = min(load_2(1,((p-1)*481+1:1:(p-1)*481+time_max)));% Find
the minimum power point occured before the maximum power point by the given
load
                Smin=m;
                time_min=Tm+(p-1)*481;
            end
        end
    else
```

```
        Smin=m;
        time_min=Tm+(p-1)*481;
    end
    end
    minpoint=zeros(length(load_2),1);
    minpoint(time_min)=Smin;
    maxpoint=zeros(length(load_2),1);
    maxpoint(time_max)=Smax;
%% Define steps for charging and didcharging
    step_discharging=(abs(Smax)-abs(Smin))/(1*length(t));%The step for
discharging
    step_charging=(abs(Smax)-abs(Smin))/(4*length(t));%The step for
charging(Notice the step is four times smaller than charging step, meaning we
have more accuracy)
 %% Discharging && Charging steps
    for d=1:1:length(t)%we move our discheging bar down from top to the
bottom

        Power_D(1,d)=(abs(Smax)-step_discharging*d);%For each 'd', this power
is for the discharge point power
        Discharging_bar(d,:)=Power_D(1,d).*ones(1,length(load_2));%
Discharging bar was created and it is moving down through the loop
 % Address the discharging power and the power limit (Consider the case time)
        S_discharging_real(1,:)=abs(load_2(1,:))-Discharging_bar(d,:);
        S_discharging_real(1:(p-1)*481+Tm)=0;% I can discharge nothing before
the first charging point(Consider the case time)
        S_discharging_real(1,S_discharging_real(1,:)>=PB)=PB;%Address the
power limit
        S_discharging_real(1,S_discharging_real(1,:)<0)=0;% we got the
discharging power
 %% Find a charging point for each discharging point(no problem with case
time issue; bar hitting; multi charge&discharge; energy&power limit)Awesome!
min_error = realmax('double');
    for a=1:1:4*length(t)%This inner loop help us find the charging point
   % Avoid two bars hit
        if (abs(Smin)+step_charging*a)>=Power_D(1,d)
            break %out of the loop 'a'
        end

Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(load_2));%let
's try from beginning (Charging bar from bottom to the top)
        S_charging_try=Charging_bar_try(a,:)-abs(load_2(1,:));
        S_charging_try(1:(p-1)*481)=0;
        S_charging_try(S_charging_try(1,:)<0)=0;
        S_charging_try(S_charging_try(1,:)>=PB)=PB;%Address the power limit
        S_charging_try((p-1)*481+TM:end)=0;% find the charging power(Consider
the case time)

   %Power_all_try together (Charging positive & Discharging negative)
    Power_all_try=zeros(1,length(load_2)); % creat a matrix to have all the
power together
    point_D_try=find(S_discharging_real(1,:));%find the discharging time
    point_C_try=find(S_charging_try(1,:));%find the charging time
seeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeecheck
```

```
    Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging_try(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try(((p-1)*481+1:1:p*481)));%get
update information of energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>WBch);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
        error=energy_try(1,end);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        point=a; %Always find the min error point by given efficiency
        end
    end
 %% Address the real charging power and the power limit
    Power_C(1,d)=(abs(Smin)+step_charging*point);%For each 'd', this power is
for the charge point power
    Charging_bar(d,:)=Power_C(1,d).*ones(1,length(load_2));%Charging bar was
created and it is moving up through the 2nd loop
    if d==2
        disp('Tenacious');%Tell me how many big loop have been run(Max=1)
    end
    S_charging_real(1,:)=Charging_bar(d,:)-abs(load_2(1,:));
    S_charging_real(1:(p-1)*481)=0;
    S_charging_real(1,S_charging_real(1,:)<0)=0;
    S_charging_real(1,S_charging_real(1,:)>=PB)=PB;%Address the power limit
%we got the charging power
    S_charging_real(1,(p-1)*481+TM:end)=0;
    %% Battery Energy update
    Power_all=zeros(1,length(t));
    point_D=find(S_discharging_real(1,:));%find the real discharging time
    point_C=find(S_charging_real(1,:));%find the real charging time
    Power_all(1,point_D)=(-
1/efficiency).*(S_discharging_real(1,point_D));%Discharging power is
negative(larger due to efficiency)
    Power_all(1,point_C)=S_charging_real(1,point_C);% charging is positive
    energy_charging_all(1,d)=time_step.*sum(S_charging_real(((p-
1)*481+1:1:p*481)));
    W_charging_real(d,((p-1)*481+1:1:p*481))=time_step.*cumsum(Power_all(((p-
1)*481+1:1:p*481)));
    %% Address stop situation
    if d>1&&Power_C(1,d)==Power_C(1,d-1)%We at somepoint stop charging (can
not find a new point)
        %Stop Charging&Discharging(More energy-power address)
        %Benefit stop grow or decrease
        Benefit(1,d:end)=Benefit(1,d-1);
        Money_saving_D(1,d:end)=Money_saving_D(1,d-1);
        Money_waste_C(1,d:end)=Money_waste_C(1,d-1);
        energy_charging_all(1,d:end)=energy_charging_all(1,d-1);
```

```
        %Charging bar& Discharging bar stopped
        for e=d:1:length(t)
        W_discharging_real(e,((p-1)*481+1:1:p*481))=W_discharging_real(d-
1,((p-1)*481+1:1:p*481));
        W_charging_real(e,((p-1)*481+1:1:p*481))=W_charging_real(d-1,((p-
1)*481+1:1:p*481));
        Charging_bar(e,:)=Power_C(1,d-1).*ones(1,length(load_2));
        Discharging_bar(e,:)=Power_D(1,d-1).*ones(1,length(load_2));
        Price_Saving_D(e,:)=Price_Saving_D(d-1,:);
        Price_waste_C(e,:)=Price_waste_C(d-1,:);
        end
        break %out of the loop 'd'
    end
    %% Benefit
    %Choose one Marginal Cost Curve
    %a1=15;b1=0.1;c1=9.955;d1=6.8;% A
    %a1=15;b1=0.1;c1=5.931;d1=2.56;% B
    a1=15;b1=0.1;c1=4.904;d1=1.23;% C
    %find the power before&after discharge (NEED TO ADDRESS THE MULTIPLE PEAK
SITUATION for loop)
    Power_B_D(1,point_D)=abs(load_2(1,point_D));
    Power_A_D(1,point_D)=abs(load_2(1,point_D))-
S_discharging_real(1,point_D);% we get the power after we discharge(NOT
consider the multible peak yet)
    %find the power before&after charge (NEED TO ADDRESS THE MULTIPLE VALLEY
BOTTOM SITUATION for loop)
    point_C=find(S_charging_real(1,:));%find the charging time
    Power_B_C(1,point_C)=abs(load_2(1,point_C));
    Power_A_C(1,point_C)=abs(load_2(1,point_C))+S_charging_real(1,point_C);%
we get the power after we charge(NOT consider the multible peak yet)
    %find the price&money saving by discharging
    Pricebefore_D=a1+(b1*abs(Power_B_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_D=a1+(b1*abs(Power_A_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Price_Saving_D(d,:)=Pricebefore_D-Priceafter_D;%positive because of
saving
    Money_before_D(1,d)=time_step.*sum(Pricebefore_D.*Power_B_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_D(1,d)=time_step.*sum(Priceafter_D.*Power_A_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_saving_D(1,d)=Money_before_D(1,d)-Money_after_D(1,d);%positive
value and it is increasing as the bar moving down
    %find the extra price&money cost by charging
    Pricebefore_C=a1+(b1*abs(Power_B_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_C=a1+(b1*abs(Power_A_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Price_waste_C(d,:)=Pricebefore_C-Priceafter_C;     %negative price
because of waste
    Money_before_C(1,d)=time_step.*sum(Pricebefore_C.*Power_B_C(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_C(1,d)=time_step.*sum(Priceafter_C.*Power_A_C(((p-
1)*481+1:1:p*481))./10^6);
    Money_waste_C(1,d)= Money_before_C(1,d)-Money_after_C(1,d);    %negative
value
    %% Total Bebefit
```

```
    Benefit(1,d)=Money_saving_D(1,d)+Money_waste_C(1,d);%for each loop,
calculate the total benefit
  end
end
%% Plot
best_benefit(find(efficiency1==efficiency),g)=max(Benefit);
figure(g)
plot(energy_charging_all*1e-4/W_total(p),Benefit(1,:),'LineWidth',6);
hold on
xlabel('W_B_c_h/W_T (%)','Fontsize',40);
ylabel('Differential Benefit ($)','Fontsize',40);
%title('The Total Benefit by using the Battery','Fontsize',16);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;
figure(g+find(efficiency1==efficiency)) %% Should we plot power_all instead?
plot(t,Power_B_C/1e6,'LineWidth',6);
hold on
plot(t,Power_A_C/1e6,'LineWidth',6);
plot(t,Power_A_D/1e6,'LineWidth',6);
plot(t,Power_B_D/1e6,'LineWidth',6);
plot(t,load_2/1e6,'LineWidth',6);
stem(t,minpoint/1e6,'r');
stem(t,maxpoint/1e6,'g');
xlabel('Time(h)','Fontsize',40);
ylabel('Power(MW)','Fontsize',40);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;

best_shot=best_benefit(find(efficiency1==efficiency),g)
end
end
```

- **Code for annual load variation**

```
clear all;clc;close all;
%% Smax possible year characteristic
Smax_load_1_3=7.5e6;
Smax_load_2=8e6;
Smin_load=5.5e6;
year_day_1=1:1:80;%section 1
year_day_2=81:1:285;%section 2
year_day_3=286:1:365;%section 3
year_max=182.5;
year_diff=80;
Smax_load_possible_1=Smax_load_1_3-(Smax_load_1_3-
Smin_load)*year_day_1/length(year_day_1);%day 1 to day 80
Smax_load_possible_2=(Smax_load_2-Smin_load)*exp(-(year_day_2-
year_max).^2/(2.*(year_diff./3).^2))+Smin_load;%day 81 to day 285
Smax_load_possible_3=Smin_load+(Smax_load_1_3-Smin_load)*(year_day_3-
285)/length(year_day_3);%day 286 to day 365
Smax_load_possible_year=[Smax_load_possible_1 Smax_load_possible_2
Smax_load_possible_3];
Smax_load_possible_year;
%% Generate the load characteristic
time_step=1/20;
```

```
t=-16:time_step:28;
t_day=0:time_step:24;

year_day=1:1:365;
time_max1=-4;
time_max2=20;
S_real=zeros(1,481*365);
Smin=zeros(1,length(year_day));

Smin_base=8.5e+06;

Smax=Smax_load_possible_year.*(0.65 + (1-0.65).*rand(1,365));
Smax_delta=zeros(1,length(year_day));
day_min_base=find(Smax==min(min(Smax)));
Smin(day_min_base)=Smin_base*(min(Smax)/10.5e6);
time_diff=9;
for d=1:1:365
  Smin(d)=Smin(day_min_base)*(1+((Smax(d)-min(Smax))/min(Smax)));
  Smax_delta(d)=Smax(d)-Smin(d);

 if d==1
     Smin(d+1)=Smin(day_min_base)*(1+((Smax(d+1)-min(Smax))/min(Smax)));
     min_error = realmax('double');
    for a=0:0.01e6:5.5e6
        error_D=Smax(d)-(-a.*exp(-(16-8-
time_max1).^2/(2.*(time_diff./3).^2))+(Smin(d)+a));
        if error_D < min_error && error_D>=0 %Make sure no energy above the
limit and no negative energy
        min_error_D = error_D;
        a_best1=a;%Always find the min error point by given efficiency
        end
    end
     for e=0:0.01e6:6.5e6
        error_D2=Smax(d)-(-e.*exp(-(16-8-
time_max1).^2/(2.*(time_diff./3).^2))+(Smin(d+1)+e));
        if abs(error_D2) < min_error&& error_D2>=0%Make sure no energy above
the limit and no negative energy
        min_error_D2 = error_D2;
        a_best2=e;%Always find the min error point by given efficiency
        end
     end
     extra=zeros(1,881);
    extra(1:1:640)=Smin(d)+a_best1;
    extra(640:1:881)=Smin(d+1)+a_best2;

    magni_S=((t <= 8+8) .*-(a_best1).*exp(-(t-8-
time_max1).^2/(2.*(time_diff./3).^2))+(t >8+8) .*-(a_best2).*exp(-(t-8-
time_max2).^2/(2.*(time_diff./3).^2)))+extra;

   S_real(1:1:561)=magni_S( find(t==0):1:find(t==28));

  elseif d>1&&d<365
Smin(d+1)=Smin(day_min_base)*(1+((Smax(d+1)-min(Smax))/min(Smax)));
     min_error = realmax('double');
     for a=0.5e6:0.01e6:5.5e6
```

```
        error_D=Smax(d)-(-a.*exp(-(16-8-
time_max1).^2/(2.*(time_diff./3).^2))+(Smin(d)+a));
        if error_D < min_error && error_D>=0 %Make sure no energy above the
limit and no negative energy
        min_error_D = error_D;
        a_best1=a;%Always find the min error point by given efficiency
        end
    end
     for e=-6.5e6:0.01e6:6.5e6
        error_D2=Smax(d)-(-e.*exp(-(16-8-
time_max1).^2/(2.*(time_diff./3).^2))+(Smin(d+1)+e));
        if abs(error_D2) < min_error&& error_D2>=0%Make sure no energy above
the limit and no negative energy
        min_error_D2 = error_D2;
        a_best2=e;%Always find the min error point by given efficiency
        end
     end

    extra=zeros(1,881);
    extra(1:1:640)=Smin(d)+a_best1;
    extra(640:1:881)=Smin(d+1)+a_best2;

    magni_S=((t <= 8+8) .*-(a_best1).*exp(-(t-8-
time_max1).^2/(2.*(time_diff./3).^2))+(t >8+8) .*-(a_best2).*exp(-(t-8-
time_max2).^2/(2.*(time_diff./3).^2)))+extra;
   S_real((80+d+((d-1)*24/time_step)):1:80+(d+(d*24/time_step)))=magni_S(
find(t==4):1:find(t==28));
 else
     for a=0.5e6:0.01e6:5.5e6
        error_D=Smax(d)-(-a.*exp(-(16-8-
time_max1).^2/(2.*(time_diff./3).^2))+(Smin(d)+a));
        if error_D < min_error && error_D>=0 %Make sure no energy above the
limit and no negative energy
        min_error_D = error_D;
        a_best1=a;%Always find the min error point by given efficiency
        end
    end
        magni_S=((t <= 8+8) .*-a_best1.*exp(-(t-8-
time_max1).^2/(2.*(time_diff./3).^2))+(t >8+8) .*-a_best1.*exp(-(t-8-
time_max2).^2/(2.*(time_diff./3).^2)))+ones(1,881)*(Smin(d)+a_best1);
      S_real((80+d+((d-1)*24/time_step)):1:end)= magni_S(
find(t==4):1:find(t==24));
 end



end
    year=(1/(1+(20*24))):(1/(1+(20*24))):365;
 plot(year,S_real./1e6,'LineWidth',1);
hold on
        xlabel('Time (day)','Fontsize',16);
ylabel('Power (MW)','Fontsize',16);
title('The Load curve','Fontsize',16);
set(gca,'FontSize',20,'FontWeight','bold');
grid on;
xlim([1 365]);
%set(gca,'XTick',[0:1:365]);
```

# Appendix C: MATLAB Code for Chapter 4

The code below is used in the Ch. 4, Figure 4-15. It can also be used to simulate different sensitivity study results by varying different parameters & variables.

- **Code for sensitivity study (NVP vs. ra%, "*l*" is parameter)**

Below are the functions that will be used in the main program.

- **Function: 'loadhis'**

```
function [tf1, tf2,loadyear]=loadhis(Sa,ra)
load1=zeros(1,10000);
load1(1)=Sa;
for a=2:1:10000
    load1(a)=load1(a-1)*ra+load1(a-1);
    if load1(a)>=10&&load1(a-1)<10
        tf1=a-1;
    end
    if load1(a)>=20
        tf2=a-1;
        break ,end
end
loadyear=load1(1:tf2+1);
```

- **Function: 'load_f'**

```
function [load_feeder_nodeferral]=load_f(load1,tf1,tf2)
load_feeder_nodeferral=load1;
load_feeder_nodeferral(tf1+1:1:tf2+1)=load1(tf1+1:1:tf2+1)./2;
```

- **Function: 'loan'**

```
function [h,A,PV_f,anual_pay,value_now]=loan(x,y,z,cf,d,ir)% h: feeder
construction start time x: feeder construction "on" time. y: loan term; z:
feeder construction time
anual_pay=zeros(1,x+y+1);
value_now=zeros(1,x+y+1);
A=cf*((ir*(1+ir)^y)/((1+ir)^y-1));
anual_pay(x-z+2:1:x+y-z+1)=A;
value_now(x-z+2)=A*(((1+d)^y-1)/(d*(1+d)^y));
PV_f=value_now(x-z+2);
h=x-z+1;
```

- **Function: 'load_b1'**

```
%% load history after adding battery
function [load_battery_1,tp_1]=load_b1(load1,tf1,tf2,PB)
load_battery_1=load1;
load_battery_1(tf1+1:end)=load1(tf1+1:end)-PB*ones(1,tf2-tf1+1);
for a=1:1:tf2+1
    if load_battery_1(a)>=10;
        tfd1=a-1;
        break,end
end
tp_1=tfd1-tf1;
```

- **Function: ‘cashhb’**

```
%% cash flow for one battery bank deferral
function
[PV_fd1,PV_b1,anual_pay,value_now,anual_pay_fp,value_now_fp,A_b]=cashb(tlb,ti
me_loan,Apf,tp1,tf1,tc,PV_f,C_b,d,ir)%input: battery loan time:tlb; postpone
time: tp1, capital cost of the battery C_b,tc:construction time of feeder
anual_pay=zeros(1,tf1+tlb);
value_now=zeros(1,tf1+tlb);
A_b=C_b*((ir*(1+ir)^tlb)/((1+ir)^tlb-1));
anual_pay(tf1+1:1:tf1+tlb)=A_b;
value_now(tf1+2-tc)=(1/(1+d)^(tc-1))*A_b*(((1+d)^tlb-1)/(d*(1+d)^tlb));
PV_b1=value_now(tf1+1-tc);
PV_fd1=(1/(1+d)^tp1)*PV_f;
value_now_fp=zeros(1,tf1+time_loan);
value_now_fp(tf1+1+1-tc)=PV_fd1;
anual_pay_fp=zeros(1,tf1+time_loan);
anual_pay_fp(tf1+1+1+tp1-tc:1:1+tf1+tp1-tc+time_loan)=Apf;
```

- **Function: ‘defertime’**

```
%% worst case ESS analysis%% find deferring time

function [y,z,number_b,yearcons] = defertime(x,h,ra,ef,tf1,PB,tb)%x: worst
load at year 0. h: second feeder construction time. ra: increasing rate
worstload=zeros(h+1,24);
incr_rate=ra;
efficiency=ef;
limit=10*ones(1,24);
for a=1:1:h+1
    worstload(a,:)=x*exp((a-1)*log(1+incr_rate));
    %% Energy need to be discharged
    power_dis=limit-worstload(a,:);
    power_dis(power_dis>0)=0;
    energy_dis=-sum(power_dis);
    %% Energy can be charged
    power_ch=limit-worstload(a,:);
    power_ch(power_ch<0)=0;
    energy_ch=sum(power_ch);
    %% determine deferring
    if energy_ch*efficiency<energy_dis
        y=a-1-tf1;
        z=worstload;
```

```
        break
    end
end
%% find each battery construction time
number_b=0;
n=0;
yearcons=zeros(1,h+1);
for b=1:1:a-1
    worstload(b,:)=x*exp((b-1)*log(1+incr_rate));
    %% Energy need to be discharged
    power_dis=limit-worstload(b,:);
    power_dis(power_dis>0)=0;
    energy_dis=-sum(power_dis);
        if b==1360
        disp('find ra');%Tell me day 6
    end
    % make a decision
    if -min(power_dis)>PB*n ||energy_dis>n*PB*tb
        n=n+1;
        number_b=n;
        yearcons(n)=b-1;
    end
end
```

- **Function: ‘load_bn’**

```
function [load_battery_n]=load_bn(load1,tf2,PB,number_b,yearcons)
load_battery_n=load1;
for a=1:1:number_b
load_battery_n(yearcons(a)+1:1:end)=load_battery_n(yearcons(a)+1:end)-
PB*ones(1,tf2-yearcons(a)+1);
end
```

- **Function: ‘lifebess’**

```
function [cost_bess,number_extra] =
lifebess(total_dftime,life,tp,A_b,d,tlb,number_b)
cost_bess=zeros(1,number_b);
number_extra=zeros(1,number_b);
for a=1:1:number_b
    number_extra(a)=ceil((total_dftime-tp(a)+4)/life);
    year=tp(a):life:tp(a)+life*(number_extra(a)-1);
    PVB=zeros(1,number_extra(a));
    for b=1:1:number_extra(a)
        PVB(b)=A_b*(((1+d)^tlb-1)/(d*(1+d)^tlb))*(1/((1+d)^year(b)));
    end
    cost_bess(a)=sum(PVB);
end
```

- **Function: ‘energy’**

```
function [energyloss]=energy(worst_load,total_dftime,LMP,ra,d,efficiency);
energyloss=zeros(1,total_dftime);
```

```
limit=10*ones(1,24);
for a=1:1:total_dftime
    load0=(10/8).*worst_load.*exp((a-1)*log(1+ra));
    load1=0.8*load0;
    load2=0.95*load0;
    load3=0.9*load0;
    power_dis0=limit-load0;
    power_dis0(power_dis0>0)=0;
    energy_dis0=-sum(power_dis0)*((1/efficiency)-1);
    money0=LMP*energy_dis0*2;
    power_dis1=limit-load1;
    power_dis1(power_dis1>0)=0;
    energy_dis1=-sum(power_dis1)*((1/efficiency)-1);
    money1=LMP*energy_dis1*91*2;
    power_dis2=limit-load2;
    power_dis2(power_dis2>0)=0;
    energy_dis2=-sum(power_dis2)*((1/efficiency)-1);
    money2=LMP*energy_dis2*91;
    power_dis3=limit-load3;
    power_dis3(power_dis3>0)=0;
    energy_dis3=-sum(power_dis3)*((1/efficiency)-1);
    money3=LMP*energy_dis3*91;
    energyloss(a)=(money0+money1+money2+money3)*(1/((1+d)^(4+a)));
end
```

- **Main program**

```
%% main programming
clear all;clc
step=0.0001;
ra1=0.0113:step:0.05;
%% define parameter's value
cl=3000;% BESS cycle life
life=cl/365;% define battery life (year)
Sa=8;
time_loan=20;
for length_f=2:2:16
C_f=length_f*0.6*10^6;%capital cost of the feeder
d=0.1;%discount rate
I_r=0.06;%interest rate
tc=5;
PB=0.5;
C_b=1*10^6;
tlb=10;
tb=6;
LMP=50;
load worst_load;
NPV=zeros(1,length(0.0113:step:0.05));
efficiency=0.6
for ra=0.0113:step:0.05;
    C_b=(1*10^6)*(PB*tb/3);
%% Deferral analysis
%figure(1)%plot original feeder history
[tf1,tf2,load1]=loadhis(Sa,ra);%find tf1, tf2,loadyear
```

```
%figure(2)%plot feeder construction load history
[load_f_nd]=load_f(load1,tf1,tf2);%find load history after feeder
construction
    if tf1<=20
        disp('find tf1');%Tell me day 6
    end
% ts1: feeder construction start time. tf1: feeder construction "on" time.
time_loan: loan term; tc: feeder construction time
% Apf: annual payment of the feeder. PV_f: present value of the feeder. I_r:
interesting rate.
%figure(3)% plot cash follow diagram after feeder
[ts1,Apf,PV_f,annual_cashflow_f,value_now_f]=loan(tf1,time_loan,tc,C_f,d,I_r)
;

%figure(4)% Plot load profile after one battery bank
[load_battery_1,tp1]=load_b1(load1,tf1,tf2,PB);%one battery added (load
history)

%figure(5)%plot cash follow diagram after one BESS
[PV_fd1,PV_b1,anual_pay_defer1,value_now_defer1,anual_pay_fp,value_now_fp,A_b
]=cashb(tlb,time_loan,Apf,tp1,tf1,tc,PV_f,C_b,d,I_r);

[total_dftime,lastload,number_b,yearcons] =
defertime(worst_load,tf2,ra,efficiency,tf1,PB,tb);
%figure(6)%plot load history after multiple battery bank
[load_battery_n]=load_bn(load1,tf2,PB,number_b,yearcons);%multiple battery
added (load history)

%% NPV calculation
tp=yearcons-ts1*ones(1,tf2+1);
tp=tp(1:1:number_b);
PVB=zeros(1,number_b);
PV_fd=PV_f*(1/(1+d)^total_dftime);
[cost_bess,number_extra] = lifebess(total_dftime,life,tp,A_b,d,tlb,number_b);
[energyloss]=energy(worst_load,total_dftime,LMP,ra,d,efficiency);
NPV(round((ra/step))-112)=PV_f-PV_fd-sum(cost_bess)-sum(energyloss);
    if ra==0.012
        disp('find ra');%Tell me day 6
    end
end
figure(1)%plot ra vs NPV
%plot(ra1.*100,NPV./(10^6),'LineWidth',3);
hold on
p=polyfit(ra1.*100,NPV./(10^6),3);
f = polyval(p,ra1.*100);
plot(ra1.*100,f,'LineWidth',3);
set(gca,'FontSize',30,'FontWeight','bold');
title('Load Increasing rate (ra) vs NPV','FontSize',30,'FontWeight','bold');
xlabel('ra (%)','FontSize',30,'FontWeight','bold');
%xlim([0 time(tf2+1)]);
ylabel('Net Present Value (M$)','FontSize',30,'FontWeight','bold');
grid on
end
```

# Appendix D: MATLAB Code for Chapter 5

There are two Matlab code files for Ch. 5. One is for conventional outage strategy; the other is for Smart Grid strategies. Both code need the 24 hour load profile with outage, the load profile can be obtained from annual load variation.

- **Code for conventional outage strategy**

```
load_7;
time_step=1/20;
t=0:time_step:24;
day=length(load_7)/481;
best_benefit=zeros(1,length(day));
%% Address Battery information
    efficiency=0.85;%Define the battery efficiency
    Pmax=5e5;%Power Limit
    TB=5;
    Wmax=Pmax*TB;%Energy Limit
    Wchmax=Wmax/efficiency;
%% Big Loop for days
for g=1:1:day
    if g==6
        disp('find day 6');%Tell me day 6
    end

    load_2=load_7((g-1)*481+1:1:g*481); % Address one day power
    %% creat Matrix
    W_total=zeros(1,1);
    Money_before_D=zeros(1,length(t));
    Money_after_D=zeros(1,length(t));
    Money_before_C=zeros(1,length(t));
    Money_after_C=zeros(1,length(t));
    Benefit=zeros(1,length(t));
    energy_charging_all=zeros(1,length(t));

    Discharging_bar=zeros(length(t),length(load_2));
    Charging_bar=zeros(length(t),length(load_2));
    Charging_bar_try=zeros(8*length(t),length(load_2));%Find the matching
charging point given the discharging condition, four times accuracy

    W_charging_real=zeros(length(t),length(load_2));
    W_discharging_real=zeros(length(t),length(load_2));

    S_charging_real=zeros(1,length(load_2));
    S_discharging_real=zeros(1,length(load_2));

    Power_D=zeros(1,length(t));
    Power_C=zeros(1,length(t));
```

```
    Power_B_D=zeros(1,length(load_2));
    Power_A_D=zeros(1,length(load_2));
    Power_B_C=zeros(1,length(load_2));
    Power_A_C=zeros(1,length(load_2));

    Money_saving_D=zeros(1,length(t));
    Money_waste_C=zeros(1,length(t));

    Price_waste_C=zeros(1*length(t),length(t));
    Price_Saving_D=zeros(1*length(t),length(t));
%% Small loop for one day Charging & Discharging
for p=1:1:1

W_total(p)=time_step*sum(load_2((p-1)*481+1:1:p*481))*1e-6 %W total power

%% Find Max and Min point
    [m,Tm] = min(load_2(1,((p-1)*481+1:1:p*481)));% Find the minimum power
point occured before the maximum power point by the given load
    Smin=m;
    time_min=Tm+(p-1)*481;
    [M,TM] = max(load_2(1,((p-1)*481+time_min:1:p*481)));%Find the maximum
power point by the given load
    Smax=M;
    time_max=TM+(p-1)*481+time_min-1;

    if time_max<=time_min

           [m,Tm] = min(load_2(1,((p-1)*481+1:1:(p-1)*481+time_max-1)));%
Find the minimum power point occured before the maximum power point by the
given load

    if isempty(Tm)==1
        for j=1:1:481
            [M,TM] = max(load_2(1,((p-1)*481+j:1:p*481)));
            if TM>j
                Smax=M;
                time_max=TM+(p-1)*481+j-1;
            [m,Tm] = min(load_2(1,((p-1)*481+1:1:(p-1)*481+time_max)));% Find
the minimum power point occured before the maximum power point by the given
load
                Smin=m;
                time_min=Tm+(p-1)*481;
            end
        end
    else
        Smin=m;
        time_min=Tm+(p-1)*481;
    end
    end

    minpoint=zeros(length(load_2),1);
    minpoint(time_min)=Smin;
    maxpoint=zeros(length(load_2),1);
    maxpoint(time_max)=Smax;
```

```
%% Define steps for charging and didcharging
    step_discharging=(abs(Smax)-abs(Smin))/(1*length(t));%The step for
discharging
    step_charging=(abs(Smax)-abs(Smin))/(8*length(t));%The step for
charging(Notice the step is four times smaller than charging step, meaning we
have more accuracy)
 %% Discharging && Charging steps
    for d=1:1:length(t)%we move our discheging bar down from top to the
bottom

        Power_D(1,d)=(abs(Smax)-step_discharging*d);%For each 'd', this power
is for the discharge point power
        Discharging_bar(d,:)=Power_D(1,d).*ones(1,length(load_2));%
Discharging bar was created and it is moving down through the loop
 % Address the discharging power and the power limit (Consider the case time)
        S_discharging_real(1,:)=abs(load_2(1,:))-Discharging_bar(d,:);
        S_discharging_real(1:(p-1)*481+Tm)=0;% I can discharge nothing before
the first charging point(Consider the case time)
        S_discharging_real(1,S_discharging_real(1,:)>=Pmax)=Pmax;%Address the
power limit
        S_discharging_real(1,S_discharging_real(1,:)<0)=0;% we got the
discharging power
 %% Find a charging point for each discharging point(no problem with case
time issue; bar hitting; multi charge&discharge; energy&power limit)Awesome!
min_error = realmax('double');
    for a=1:1:8*length(t)%This inner loop help us find the charging point
   % Avoid two bars hit
        if (abs(Smin)+step_charging*a)>=Power_D(1,d)
            break %out of the loop 'a'
        end

Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(load_2));%let
's try from beginning (Charging bar from bottom to the top)
        S_charging_try=Charging_bar_try(a,:)-abs(load_2(1,:));
        S_charging_try(1:(p-1)*481)=0;
        S_charging_try(S_charging_try(1,:)<0)=0;
        S_charging_try(S_charging_try(1,:)>=Pmax)=Pmax;%Address the power
limit
        S_charging_try((p-1)*481+TM:end)=0;% find the charging power(Consider
the case time)

   %Power_all_try together (Charging positive & Discharging negative)
    Power_all_try=zeros(1,length(load_2)); % creat a matrix to have all the
power together
    point_D_try=find(S_discharging_real(1,:));%find the discharging time
    point_C_try=find(S_charging_try(1,:));%find the charging time
seeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeecheck
    Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging_try(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try(((p-1)*481+1:1:p*481)));%get
update information of energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>Wchmax);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
        error=energy_try(1,end);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        point=a; %Always find the min error point by given efficiency
        end
    end
 %% Address the real charging power and the power limit
    Power_C(1,d)=(abs(Smin)+step_charging*point);%For each 'd', this power is
for the charge point power
    Charging_bar(d,:)=Power_C(1,d).*ones(1,length(load_2));%Charging bar was
created and it is moving up through the 2nd loop
    if d==2
        disp('Tenacious');%Tell me how many big loop have been run(Max=1)
    end
    S_charging_real(1,:)=Charging_bar(d,:)-abs(load_2(1,:));
    S_charging_real(1:(p-1)*481)=0;
    S_charging_real(1,S_charging_real(1,:)<0)=0;
    S_charging_real(1,S_charging_real(1,:)>=Pmax)=Pmax;%Address the power
limit %we got the charging power
    S_charging_real(1,(p-1)*481+TM:end)=0;
    %% Battery Energy update
    Power_all=zeros(1,length(t));
    point_D=find(S_discharging_real(1,:));%find the real discharging time
    point_C=find(S_charging_real(1,:));%find the real charging time
    Power_all(1,point_D)=(-
1/efficiency).*(S_discharging_real(1,point_D));%Discharging power is
negative(larger due to efficiency)
    Power_all(1,point_C)=S_charging_real(1,point_C);% charging is positive
    energy_charging_all(1,d)=time_step.*sum(S_charging_real(((p-
1)*481+1:1:p*481)));
    W_charging_real(d,((p-1)*481+1:1:p*481))=time_step.*cumsum(Power_all(((p-
1)*481+1:1:p*481)));
    %% Address stop situation
    if d>1&&Power_C(1,d)==Power_C(1,d-1)%We at somepoint stop charging (can
not find a new point)
        %Stop Charging&Discharging(More energy-power address)
        %Benefit stop grow or decrease
        Benefit(1,d:end)=Benefit(1,d-1);
        Money_saving_D(1,d:end)=Money_saving_D(1,d-1);
        Money_waste_C(1,d:end)=Money_waste_C(1,d-1);
        energy_charging_all(1,d:end)=energy_charging_all(1,d-1);
        %Charging bar& Discharging bar stopped
        for e=d:1:length(t)
        W_discharging_real(e,((p-1)*481+1:1:p*481))=W_discharging_real(d-
1,((p-1)*481+1:1:p*481));
        W_charging_real(e,((p-1)*481+1:1:p*481))=W_charging_real(d-1,((p-
1)*481+1:1:p*481));
        Charging_bar(e,:)=Power_C(1,d-1).*ones(1,length(load_2));
        Discharging_bar(e,:)=Power_D(1,d-1).*ones(1,length(load_2));
        Price_Saving_D(e,:)=Price_Saving_D(d-1,:);
```

```
        Price_waste_C(e,:)=Price_waste_C(d-1,:);
        end
        break_d=d;
        break %out of the loop 'd'
    end
    %% Benefit
    %Choose one Marginal Cost Curve
    %a1=15;b1=0.1;c1=9.955;d1=6.8;% A
    %a1=15;b1=0.1;c1=5.931;d1=2.56;% B
    a1=15;b1=0.1;c1=4.904;d1=1.23;% C
    %find the power before&after discharge (NEED TO ADDRESS THE MULTIPLE PEAK
SITUATION for loop)
    Power_B_D(1,point_D)=abs(load_2(1,point_D));
    Power_A_D(1,point_D)=abs(load_2(1,point_D))-
S_discharging_real(1,point_D);% we get the power after we discharge(NOT
consider the multible peak yet)
    %find the power before&after charge (NEED TO ADDRESS THE MULTIPLE VALLEY
BOTTOM SITUATION for loop)
    point_C=find(S_charging_real(1,:));%find the charging time
    Power_B_C(1,point_C)=abs(load_2(1,point_C));
    Power_A_C(1,point_C)=abs(load_2(1,point_C))+S_charging_real(1,point_C);%
we get the power after we charge(NOT consider the multible peak yet)
    %find the price&money saving by discharging
    Pricebefore_D=a1+(b1*abs(Power_B_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_D=a1+(b1*abs(Power_A_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Price_Saving_D(d,:)=Pricebefore_D-Priceafter_D;%positive because of
saving
    Money_before_D(1,d)=time_step.*sum(Pricebefore_D.*Power_B_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_D(1,d)=time_step.*sum(Priceafter_D.*Power_A_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_saving_D(1,d)=Money_before_D(1,d)-Money_after_D(1,d);%positive
value and it is increasing as the bar moving down
    %find the extra price&money cost by charging
    Pricebefore_C=a1+(b1*abs(Power_B_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_C=a1+(b1*abs(Power_A_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Price_waste_C(d,:)=Pricebefore_C-Priceafter_C;     %negative price
because of waste
    Money_before_C(1,d)=time_step.*sum(Pricebefore_C.*Power_B_C(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_C(1,d)=time_step.*sum(Priceafter_C.*Power_A_C(((p-
1)*481+1:1:p*481))./10^6);
    Money_waste_C(1,d)= Money_before_C(1,d)-Money_after_C(1,d);    %negative
value
    %% Total Bebefit
    Benefit(1,d)=Money_saving_D(1,d)+Money_waste_C(1,d);%for each loop,
calculate the total benefit
    end
  %% Address a special outage case
  if d==break_d&&max(Power_A_D)>=10e6
   %% creat Matrix

    Money_before_D=zeros(1,length(t));
```

```
    Money_after_D=zeros(1,length(t));
    Money_before_C=zeros(1,length(t));
    Money_after_C=zeros(1,length(t));
    Benefit=zeros(1,length(t));
    energy_charging_all=zeros(1,length(t));

    Discharging_bar=zeros(length(t),length(load_2));
    Charging_bar=zeros(length(t),length(load_2));
    Charging_bar_try=zeros(8*length(t),length(load_2));%Find the matching
charging point given the discharging condition, four times accuracy

    W_charging_real=zeros(length(t),length(load_2));
    W_discharging_real=zeros(length(t),length(load_2));

    S_charging_real=zeros(1,length(load_2));
    S_discharging_real=zeros(1,length(load_2));

    Power_D=zeros(1,length(t));
    Power_C=zeros(1,length(t));

    Power_B_D=zeros(1,length(load_2));
    Power_A_D=zeros(1,length(load_2));
    Power_B_C=zeros(1,length(load_2));
    Power_A_C=zeros(1,length(load_2));

    Money_saving_D=zeros(1,length(t));
    Money_waste_C=zeros(1,length(t));

    Price_waste_C=zeros(1*length(t),length(t));
    Price_Saving_D=zeros(1*length(t),length(t));
    %% new discharging method
    Power_D=10e6;
    SC=load_2-ones(1,481).*10e6;
    SC(1,SC(1,:)<0)=0;
    SC(1,SC(1,:)>=Pmax)=Pmax;
    point_c_first=find(SC, 1, 'first');
    point_c_last=find(SC, 1, 'last');
    for h=1:1:point_c_last+2-point_c_first
        S_discharging_real(point_c_first-1:1:point_c_first-
1+h)=SC(point_c_first-1:1:point_c_first-1+h);



        %% Find a charging point for each discharging point(no problem with
case time issue; bar hitting; multi charge&discharge; energy&power
limit)Awesome!
min_error = realmax('double');
    for a=1:1:8*length(t)%This inner loop help us find the charging point
   % Avoid two bars hit
        if (abs(Smin)+step_charging*a)>=Power_D
            break %out of the loop 'a'
        end

Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(load_2));%let
's try from beginning (Charging bar from bottom to the top)
```

```
        S_charging_try=Charging_bar_try(a,:)-abs(load_2(1,:));
        S_charging_try(1:(p-1)*481)=0;
        S_charging_try(S_charging_try(1,:)<0)=0;
        S_charging_try(S_charging_try(1,:)>=Pmax)=Pmax;%Address the power
limit
        S_charging_try((p-1)*481+TM:end)=0;% find the charging power(Consider
the case time)

   %Power_all_try together (Charging positive & Discharging negative)
    Power_all_try=zeros(1,length(load_2)); % creat a matrix to have all the
power together
    point_D_try=find(S_discharging_real(1,:));%find the discharging time
    point_C_try=find(S_charging_try(1,:));%find the charging time
seeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeecheck
    Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging_try(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try(((p-1)*481+1:1:p*481)));%get
update information of energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>Wchmax);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
        error=energy_try(1,end);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        point=a; %Always find the min error point by given efficiency
        end
    end
 %% Address the real charging power and the power limit
    Power_C(1,h)=(abs(Smin)+step_charging*point);%For each 'h', this power is
for the charge point power
    Charging_bar(h,:)=Power_C(1,h).*ones(1,length(load_2));%Charging bar was
created and it is moving up through the 2nd loop
    if h==2
        disp('Tenacious');%Tell me how many big loop have been run(Max=1)
    end
    S_charging_real(1,:)=Charging_bar(h,:)-abs(load_2(1,:));
    S_charging_real(1:(p-1)*481)=0;
    S_charging_real(1,S_charging_real(1,:)<0)=0;
    S_charging_real(1,S_charging_real(1,:)>=Pmax)=Pmax;%Address the power
limit %we got the charging power
    S_charging_real(1,(p-1)*481+TM:end)=0;
    %% Battery Energy update
    Power_all=zeros(1,length(t));
    point_D=find(S_discharging_real(1,:));%find the real discharging time
    point_C=find(S_charging_real(1,:));%find the real charging time
    Power_all(1,point_D)=(-
1/efficiency).*(S_discharging_real(1,point_D));%Discharging power is
negative(larger due to efficiency)
```

```
    Power_all(1,point_C)=S_charging_real(1,point_C);% charging is positive
    energy_charging_all(1,h)=time_step.*sum(S_charging_real(((p-
1)*481+1:1:p*481)));
    W_charging_real(h,((p-1)*481+1:1:p*481))=time_step.*cumsum(Power_all(((p-
1)*481+1:1:p*481)));
    %% Address stop situation
    if h>1&&Power_C(1,h)==Power_C(1,h-1)%We at somepoint stop charging (can
not find a new point)
        %Stop Charging&Discharging(More energy-power address)
        %Benefit stop grow or decrease
        Benefit(1,h:end)=Benefit(1,h-1);
        Money_saving_D(1,h:end)=Money_saving_D(1,h-1);
        Money_waste_C(1,h:end)=Money_waste_C(1,h-1);
        energy_charging_all(1,h:end)=energy_charging_all(1,h-1);
        %Charging bar& Discharging bar stopped
        for e=h:1:length(t)
        W_discharging_real(e,((p-1)*481+1:1:p*481))=W_discharging_real(h-
1,((p-1)*481+1:1:p*481));
        W_charging_real(e,((p-1)*481+1:1:p*481))=W_charging_real(h-1,((p-
1)*481+1:1:p*481));
        Charging_bar(e,:)=Power_C(1,h-1).*ones(1,length(load_2));

        Price_Saving_D(e,:)=Price_Saving_D(h-1,:);
        Price_waste_C(e,:)=Price_waste_C(h-1,:);
        end
        break %out of the loop 'h'
    end
        %% Benefit
    %Choose one Marginal Cost Curve
    %a1=15;b1=0.1;c1=9.955;d1=6.8;% A
    %a1=15;b1=0.1;c1=5.931;d1=2.56;% B
    a1=15;b1=0.1;c1=4.904;d1=1.23;% C
    %find the power before&after discharge (NEED TO ADDRESS THE MULTIPLE PEAK
SITUATION for loop)
    Power_B_D(1,point_D)=abs(load_2(1,point_D));
    Power_A_D(1,point_D)=abs(load_2(1,point_D))-
S_discharging_real(1,point_D);% we get the power after we discharge(NOT
consider the multible peak yet)
    %find the power before&after charge (NEED TO ADDRESS THE MULTIPLE VALLEY
BOTTOM SITUATION for loop)
    point_C=find(S_charging_real(1,:));%find the charging time
    Power_B_C(1,point_C)=abs(load_2(1,point_C));
    Power_A_C(1,point_C)=abs(load_2(1,point_C))+S_charging_real(1,point_C);%
we get the power after we charge(NOT consider the multible peak yet)
    %find the price&money saving by discharging
    Pricebefore_D=a1+(b1*abs(Power_B_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_D=a1+(b1*abs(Power_A_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Price_Saving_D(h,:)=Pricebefore_D-Priceafter_D;%positive because of
saving
    Money_before_D(1,h)=time_step.*sum(Pricebefore_D.*Power_B_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_D(1,h)=time_step.*sum(Priceafter_D.*Power_A_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_saving_D(1,h)=Money_before_D(1,h)-Money_after_D(1,h);%positive
value and it is increasing as the bar moving down
```

```
  %find the extra price&money cost by charging
  Pricebefore_C=a1+(b1*abs(Power_B_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
  Priceafter_C=a1+(b1*abs(Power_A_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
  Price_waste_C(h,:)=Pricebefore_C-Priceafter_C;     %negative price
because of waste
  Money_before_C(1,h)=time_step.*sum(Pricebefore_C.*Power_B_C(((p-
1)*481+1:1:p*481))./10^6);
  Money_after_C(1,h)=time_step.*sum(Priceafter_C.*Power_A_C(((p-
1)*481+1:1:p*481))./10^6);
  Money_waste_C(1,h)= Money_before_C(1,h)-Money_after_C(1,h);    %negative
value


  %% Total Bebefit
  Benefit(1,h)=Money_saving_D(1,h)+Money_waste_C(1,h);%for each loop,
calculate the total benefit
  end
 end
[bestshot,time_best]=max(Benefit);
Power_A_D_best=zeros(1,length(load_2));
Power_A_C_best=zeros(1,length(load_2));
if Power_D==10e6
      S_discharging_real(point_c_first-1:1:point_c_first-
1+time_best)=SC(point_c_first-1:1:point_c_first-1+time_best);



    %% Find a charging point for each discharging point(no problem with
case time issue; bar hitting; multi charge&discharge; energy&power
limit)Awesome!
min_error = realmax('double');
  for a=1:1:8*length(t)%This inner loop help us find the charging point
  % Avoid two bars hit
    if (abs(Smin)+step_charging*a)>=Power_D
      break %out of the loop 'a'
    end

Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(load_2));%let
's try from beginning (Charging bar from bottom to the top)
    S_charging_try=Charging_bar_try(a,:)-abs(load_2(1,:));
    S_charging_try(1:(p-1)*481)=0;
    S_charging_try(S_charging_try(1,:)<0)=0;
    S_charging_try(S_charging_try(1,:)>=Pmax)=Pmax;%Address the power
limit
    S_charging_try((p-1)*481+TM:end)=0;% find the charging power(Consider
the case time)

  %Power_all_try together (Charging positive & Discharging negative)
  Power_all_try=zeros(1,length(load_2)); % creat a matrix to have all the
power together
  point_D_try=find(S_discharging_real(1,:));%find the discharging time
  point_C_try=find(S_charging_try(1,:));%find the charging time
seeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeecheck
```

```
    Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging_try(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try(((p-1)*481+1:1:p*481)));%get
update information of energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>Wchmax);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
        error=energy_try(1,end);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        point=a; %Always find the min error point by given efficiency
        end
    end
 %% Address the real charging power and the power limit
    Power_C(1,time_best)=(abs(Smin)+step_charging*point);%For each 'h', this
power is for the charge point power

Charging_bar(time_best,:)=Power_C(1,time_best).*ones(1,length(load_2));%Charg
ing bar was created and it is moving up through the 2nd loop
    if h==2
        disp('Tenacious');%Tell me how many big loop have been run(Max=1)
    end
    S_charging_real(1,:)=Charging_bar(time_best,:)-abs(load_2(1,:));
    S_charging_real(1:(p-1)*481)=0;
    S_charging_real(1,S_charging_real(1,:)<0)=0;
    S_charging_real(1,S_charging_real(1,:)>=Pmax)=Pmax;%Address the power
limit %we got the charging power
    S_charging_real(1,(p-1)*481+TM:end)=0;
  %% Battery Energy update
    Power_all=zeros(1,length(t));
    point_D=find(S_discharging_real(1,:));%find the real discharging time
    point_C=find(S_charging_real(1,:));%find the real charging time
    Power_all(1,point_D)=(-
1/efficiency).*(S_discharging_real(1,point_D));%Discharging power is
negative(larger due to efficiency)
    Power_all(1,point_C)=S_charging_real(1,point_C);% charging is positive

    W_charging_real(time_best,((p-
1)*481+1:1:p*481))=time_step.*cumsum(Power_all(((p-1)*481+1:1:p*481)));
    Power_A_D_best(1,point_D)=abs(load_2(1,point_D))-
S_discharging_real(1,point_D);

Power_A_C_best(1,point_C)=abs(load_2(1,point_C))+S_charging_real(1,point_C);
end
end

%% Plot
```

```
best_benefit(g)=max(Benefit)
figure(g)
plot(energy_charging_all*1e-4/W_total(p),Benefit(1,:),'LineWidth',6);
hold on
xlabel('W_c_h_a_r_g_i_n_g/W_t_o_t_a_l (%)','Fontsize',40);
ylabel('Benefit ($)','Fontsize',40);
%title('The Total Benefit by using the Battery','Fontsize',16);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;
figure(g+day) %% Should we plot power_all instead?
plot(t,Power_B_C/1e6,'LineWidth',6);
hold on
plot(t,Power_A_C_best/1e6,'LineWidth',6);
plot(t,Power_A_D_best/1e6,'LineWidth',6);
plot(t,Power_B_D/1e6,'LineWidth',6);
plot(t,load_2/1e6,'LineWidth',6);
stem(t,minpoint/1e6,'r');
stem(t,maxpoint/1e6,'g');
xlabel('Time(h)','Fontsize',40);
ylabel('Power(MW)','Fontsize',40);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;

end
figure(2*day+1)
plot(best_benefit,'LineWidth',6);
```

- **Code for Smart Grid outage strategies**

```
load_7;
time_step=1/20;
t=0:time_step:24;
day=length(load_7)/481;
best_benefit=zeros(1,length(day));
%% Address Battery information
    efficiency=0.85;%Define the battery efficiency
    Pmax=5e5;%Power Limit
    TB=5;
    Wmax=Pmax*TB;%Energy Limit
    Wchmax=Wmax/efficiency;
%% Big Loop for days
for g=1:1:day
    if g==6
        disp('find day 6');%Tell me day 6
    end

    load_2=load_7((g-1)*481+1:1:g*481); % Address one day power
    %% creat Matrix
    W_total=zeros(1,1);
    Money_before_D=zeros(1,length(t));
    Money_after_D=zeros(1,length(t));
    Money_before_C=zeros(1,length(t));
    Money_after_C=zeros(1,length(t));
    Benefit=zeros(1,length(t));
    energy_charging_all=zeros(1,length(t));
```

```
    Discharging_bar=zeros(length(t),length(load_2));
    Charging_bar=zeros(length(t),length(load_2));
    Charging_bar_try=zeros(8*length(t),length(load_2));%Find the matching
charging point given the discharging condition, four times accuracy

    W_charging_real=zeros(length(t),length(load_2));
    W_discharging_real=zeros(length(t),length(load_2));

    S_charging_real=zeros(1,length(load_2));
    S_discharging_real=zeros(1,length(load_2));

    Power_D=zeros(1,length(t));
    Power_C=zeros(1,length(t));

    Power_B_D=zeros(1,length(load_2));
    Power_A_D=zeros(1,length(load_2));
    Power_B_C=zeros(1,length(load_2));
    Power_A_C=zeros(1,length(load_2));

    Money_saving_D=zeros(1,length(t));
    Money_waste_C=zeros(1,length(t));

    Price_waste_C=zeros(1*length(t),length(t));
    Price_Saving_D=zeros(1*length(t),length(t));
%% Small loop for one day Charging & Discharging
for p=1:1:1

W_total(p)=time_step*sum(load_2((p-1)*481+1:1:p*481))*1e-6 %W total power

%% Find Max and Min point
    [m,Tm] = min(load_2(1,((p-1)*481+1:1:p*481)));% Find the minimum power
point occured before the maximum power point by the given load
    Smin=m;
    time_min=Tm+(p-1)*481;
    [M,TM] = max(load_2(1,((p-1)*481+time_min:1:p*481)));%Find the maximum
power point by the given load
    Smax=M;
    time_max=TM+(p-1)*481+time_min-1;

    if time_max<=time_min

           [m,Tm] = min(load_2(1,((p-1)*481+1:1:(p-1)*481+time_max-1)));%
Find the minimum power point occured before the maximum power point by the
given load

    if isempty(Tm)==1
        for j=1:1:481
            [M,TM] = max(load_2(1,((p-1)*481+j:1:p*481)));
            if TM>j
                Smax=M;
                time_max=TM+(p-1)*481+j-1;
```

```
            [m,Tm] = min(load_2(1,((p-1)*481+1:1:(p-1)*481+time_max)));% Find
the minimum power point occured before the maximum power point by the given
load
                Smin=m;
                time_min=Tm+(p-1)*481;
            end
        end
    else
        Smin=m;
        time_min=Tm+(p-1)*481;
    end
    end

    minpoint=zeros(length(load_2),1);
    minpoint(time_min)=Smin;
    maxpoint=zeros(length(load_2),1);
    maxpoint(time_max)=Smax;
%% Define steps for charging and didcharging
    step_discharging=(abs(Smax)-abs(Smin))/(1*length(t));%The step for
discharging
    step_charging=(abs(Smax)-abs(Smin))/(8*length(t));%The step for
charging(Notice the step is four times smaller than charging step, meaning we
have more accuracy)
 %% Discharging && Charging steps
    for d=1:1:length(t)%we move our discheging bar down from top to the
bottom

        Power_D(1,d)=(abs(Smax)-step_discharging*d);%For each 'd', this power
is for the discharge point power
        Discharging_bar(d,:)=Power_D(1,d).*ones(1,length(load_2));%
Discharging bar was created and it is moving down through the loop
 % Address the discharging power and the power limit (Consider the case time)
        S_discharging_real(1,:)=abs(load_2(1,:))-Discharging_bar(d,:);
        S_discharging_real(1:(p-1)*481+Tm)=0;% I can discharge nothing before
the first charging point(Consider the case time)
        S_discharging_real(1,S_discharging_real(1,:)>=Pmax)=Pmax;%Address the
power limit
        S_discharging_real(1,S_discharging_real(1,:)<0)=0;% we got the
discharging power
 %% Find a charging point for each discharging point(no problem with case
time issue; bar hitting; multi charge&discharge; energy&power limit)Awesome!
min_error = realmax('double');
    for a=1:1:8*length(t)%This inner loop help us find the charging point
   % Avoid two bars hit
        if (abs(Smin)+step_charging*a)>=Power_D(1,d)
            break %out of the loop 'a'
        end

Charging_bar_try(a,:)=(abs(Smin)+step_charging*a)*ones(1,length(load_2));%let
's try from beginning (Charging bar from bottom to the top)
        S_charging_try=Charging_bar_try(a,:)-abs(load_2(1,:));
        S_charging_try(1:(p-1)*481)=0;
        S_charging_try(S_charging_try(1,:)<0)=0;
        S_charging_try(S_charging_try(1,:)>=Pmax)=Pmax;%Address the power
limit
```

```
        S_charging_try((p-1)*481+TM:end)=0;% find the charging power(Consider
the case time)

   %Power_all_try together (Charging positive & Discharging negative)
    Power_all_try=zeros(1,length(load_2)); % creat a matrix to have all the
power together
    point_D_try=find(S_discharging_real(1,:));%find the discharging time
    point_C_try=find(S_charging_try(1,:));%find the charging time
seeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeecheck
    Power_all_try(1,point_D_try)=(-
1/efficiency).*(S_discharging_real(1,point_D_try));%Discharging power is
negative(larger due to efficiency)
    Power_all_try(1,point_C_try)=S_charging_try(1,point_C_try);% charging is
positive
    energy_try=time_step.*cumsum(Power_all_try(((p-1)*481+1:1:p*481)));%get
update information of energy in the battery
    point_negative_energy=find(energy_try<0);
    point_energylimit=find(energy_try>Wchmax);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
   %The algorithm of finding the best charging point (Already make sure no
problem with multi charge&discharge energy&power limit)
        error=energy_try(1,end);
        if error <
min_error&&isempty(point_negative_energy)==1&&isempty(point_energylimit)==1
%Make sure no energy above the limit and no negative energy
        min_error = error;
        point=a; %Always find the min error point by given efficiency
        end
    end
 %% Address the real charging power and the power limit
    Power_C(1,d)=(abs(Smin)+step_charging*point);%For each 'd', this power is
for the charge point power
    Charging_bar(d,:)=Power_C(1,d).*ones(1,length(load_2));%Charging bar was
created and it is moving up through the 2nd loop
    if d==2
        disp('Tenacious');%Tell me how many big loop have been run(Max=1)
    end
    S_charging_real(1,:)=Charging_bar(d,:)-abs(load_2(1,:));
    S_charging_real(1:(p-1)*481)=0;
    S_charging_real(1,S_charging_real(1,:)<0)=0;
    S_charging_real(1,S_charging_real(1,:)>=Pmax)=Pmax;%Address the power
limit %we got the charging power
    S_charging_real(1,(p-1)*481+TM:end)=0;
    %% Battery Energy update
    Power_all=zeros(1,length(t));
    point_D=find(S_discharging_real(1,:));%find the real discharging time
    point_C=find(S_charging_real(1,:));%find the real charging time
    Power_all(1,point_D)=(-
1/efficiency).*(S_discharging_real(1,point_D));%Discharging power is
negative(larger due to efficiency)
    Power_all(1,point_C)=S_charging_real(1,point_C);% charging is positive
    energy_charging_all(1,d)=time_step.*sum(S_charging_real(((p-
1)*481+1:1:p*481)));
    W_charging_real(d,((p-1)*481+1:1:p*481))=time_step.*cumsum(Power_all(((p-
1)*481+1:1:p*481)));
```

```
    %% Address stop situation
    if d>1&&Power_C(1,d)==Power_C(1,d-1)%We at somepoint stop charging (can 
not find a new point)
        %Stop Charging&Discharging(More energy-power address)
        %Benefit stop grow or decrease
        Benefit(1,d:end)=Benefit(1,d-1);
        Money_saving_D(1,d:end)=Money_saving_D(1,d-1);
        Money_waste_C(1,d:end)=Money_waste_C(1,d-1);
        energy_charging_all(1,d:end)=energy_charging_all(1,d-1);
        %Charging bar& Discharging bar stopped
        for e=d:1:length(t)
        W_discharging_real(e,((p-1)*481+1:1:p*481))=W_discharging_real(d-
1,((p-1)*481+1:1:p*481));
        W_charging_real(e,((p-1)*481+1:1:p*481))=W_charging_real(d-1,((p-
1)*481+1:1:p*481));
        Charging_bar(e,:)=Power_C(1,d-1).*ones(1,length(load_2));
        Discharging_bar(e,:)=Power_D(1,d-1).*ones(1,length(load_2));
        Price_Saving_D(e,:)=Price_Saving_D(d-1,:);
        Price_waste_C(e,:)=Price_waste_C(d-1,:);
        end
        break %out of the loop 'd'
    end
    %% Benefit
    %Choose one Marginal Cost Curve
    a1=15;b1=0.1;c1=9.955;d1=6.8;% A
    %a1=15;b1=0.1;c1=5.931;d1=2.56;% B
    %a1=15;b1=0.1;c1=4.904;d1=1.23;% C
    %find the power before&after discharge (NEED TO ADDRESS THE MULTIPLE PEAK 
SITUATION for loop)
    Power_B_D(1,point_D)=abs(load_2(1,point_D));
    Power_A_D(1,point_D)=abs(load_2(1,point_D))-
S_discharging_real(1,point_D);% we get the power after we discharge(NOT 
consider the multible peak yet)
    %find the power before&after charge (NEED TO ADDRESS THE MULTIPLE VALLEY 
BOTTOM SITUATION for loop)
    point_C=find(S_charging_real(1,:));%find the charging time
    Power_B_C(1,point_C)=abs(load_2(1,point_C));
    Power_A_C(1,point_C)=abs(load_2(1,point_C))+S_charging_real(1,point_C);%
we get the power after we charge(NOT consider the multible peak yet)
    %find the price&money saving by discharging
    Pricebefore_D=a1+(b1*abs(Power_B_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_D=a1+(b1*abs(Power_A_D(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Price_Saving_D(d,:)=Pricebefore_D-Priceafter_D;%positive because of 
saving
    Money_before_D(1,d)=time_step.*sum(Pricebefore_D.*Power_B_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_D(1,d)=time_step.*sum(Priceafter_D.*Power_A_D(((p-
1)*481+1:1:p*481))./10^6);
    Money_saving_D(1,d)=Money_before_D(1,d)-Money_after_D(1,d);%positive 
value and it is increasing as the bar moving down
    %find the extra price&money cost by charging
    Pricebefore_C=a1+(b1*abs(Power_B_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
    Priceafter_C=a1+(b1*abs(Power_A_C(((p-
1)*481+1:1:p*481))./10^6).^c1).*10^(-d1);
```

```
    Price_waste_C(d,:)=Pricebefore_C-Priceafter_C;      %negative price
because of waste
    Money_before_C(1,d)=time_step.*sum(Pricebefore_C.*Power_B_C(((p-
1)*481+1:1:p*481))./10^6);
    Money_after_C(1,d)=time_step.*sum(Priceafter_C.*Power_A_C(((p-
1)*481+1:1:p*481))./10^6);
    Money_waste_C(1,d)= Money_before_C(1,d)-Money_after_C(1,d);    %negative
value
    %% Total Bebefit
    Benefit(1,d)=Money_saving_D(1,d)+Money_waste_C(1,d);%for each loop,
calculate the total benefit
  end
end

%% Plot
best_benefit(g)=max(Benefit)
figure(g)
plot(energy_charging_all*1e-4/W_total(p),Benefit(1,:),'LineWidth',6);
hold on
xlabel('W_c_h_a_r_g_i_n_g/W_t_o_t_a_l (%)','Fontsize',40);
ylabel('Benefit ($)','Fontsize',40);
%title('The Total Benefit by using the Battery','Fontsize',16);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;
figure(g+day) %% Should we plot power_all instead?
plot(t,Power_B_C/1e6,'LineWidth',6);
hold on
plot(t,Power_A_C/1e6,'LineWidth',6);
plot(t,Power_A_D/1e6,'LineWidth',6);
plot(t,Power_B_D/1e6,'LineWidth',6);
plot(t,load_2/1e6,'LineWidth',6);
stem(t,minpoint/1e6,'r');
stem(t,maxpoint/1e6,'g');
xlabel('Time(h)','Fontsize',40);
ylabel('Power(MW)','Fontsize',40);
set(gca,'FontSize',40,'FontWeight','bold');
grid on;

end
figure(2*day+1)
plot(best_benefit,'LineWidth',6);
```

# Appendix E: Optimization Problem for Feeder Deferral

This appendix shows the form of optimization problem in Ch. 4. The purpose is to introduce the Lagrange method that helps to find the optimal BESS operation strategy that yields the maximum Net Present Value (NPV). Given the system condition of $r_a$%=1.5% with a 10 mi 15 KV (400A) feeder and 80% round-trip efficiency BESSs (maximum nine years' deferral), the optimization problem can be formatted as:

$$\begin{aligned} \text{minimize} \quad & PV'_{t_{s_1}} = \sum_{i=1}^{N} PV'_{t_{s_1}b_i} + PV'_{t_{s_1}f_1} \\ \text{subject to} \quad & 500 \leq CL \leq 5000 \\ & 0.25 \leq P_{Bmax} \leq 3 \\ & 2 \leq T_B \leq 10 \\ & 0 \leq t_P \leq 9 \end{aligned} \tag{E-1}$$

it defines

$$PV'_{t_{s_1}f_1} = 4.45 \times 10^6 \times \frac{1}{1.1^{t_P}}$$

$$PV'_{t_{s_1}b_i} = \begin{cases} \sum_{n=0}^{N_{b1}-1} \dfrac{2.78 \cdot 10^5 P_{Bmax} T_B}{1.1^{\left(4+\left\lfloor \frac{CL}{365} \right\rfloor n\right)}}, & i = 1 \\ \sum_{n=0}^{N_{bi}-1} \dfrac{2.78 \cdot 10^5 P_{Bmax} T_B}{1.1^{\left(4+\left\lfloor \frac{CL}{365} \right\rfloor n + \sum_{k=1}^{i-1} t_{BP_k}\right)}}, & i \geq 2 \end{cases} \tag{E-2}$$

where

$$N_{bn} = \left\lceil \frac{t_p - (t_{b_n} - t_{F1})}{CL/365} \right\rceil$$

$$t_{BP_k} = t_{b_k} - t_{b_{k-1}}$$

There are two algorithms that helps to determine $\boldsymbol{t_{b_\gamma}}$ and $\boldsymbol{N}$.

- Algorithm 1: Determine $\boldsymbol{t_{b_\gamma}}$:

```
    Input:        $P_{Bmax}$, P(t), $T_B$
    Initialize:   $\gamma = 0, t = 0, t_{b_\gamma} = 0$
    While:        $t - 15 < 9$     do
        compute:      $M = \gamma P_{Bmax} - (8e^{ln(1.015)t} - 10)$
        and
                      $W = \gamma P_{Bmax} T_B - \int_0^{24} (P(t')1.015^t - 10)^+ dt'$
        where         $P(t') = 5.5 + 3\left\{1 - \exp\left(1 - \frac{(t'-b)^2}{36}\right)\right\}$
        and           b=4 when $0 \leq t' \leq 16$; b=28 when $16 \leq t' \leq 24$
     If     $M > 0$ & $W \geq 0$
        then $t = t + 1$
     else   $t_{b_k} = t$;
        $\gamma = \gamma + 1$;
        $t = t + 1$;
end    if
end    while
Output matrix: $t_b = [t_{b_1},\ t_{b_2},\ \cdots t_{b_N} \cdots t_{b_{\gamma-1}}], \gamma - 1$
```

- Algorithm 2: Determine $\boldsymbol{N}$**:**

```
Input:  $\boldsymbol{t_b} = [\boldsymbol{t_{b_1}},\ \boldsymbol{t_{b_2}},\ \cdots \boldsymbol{t_{b_N}} \cdots \boldsymbol{t_{b_{\gamma-1}}}], \boldsymbol{t_P}, \boldsymbol{\gamma}$
for     $\boldsymbol{\beta} = \mathbf{1}:\mathbf{1}:(\boldsymbol{\gamma} - \mathbf{1})$
    If      $\boldsymbol{t_P} > \boldsymbol{t_{b_{\beta-1}}}$ & $\boldsymbol{t_P} < \boldsymbol{t_{b_\beta}}$
            $\boldsymbol{N} = \boldsymbol{\beta} - \mathbf{1}$
    break
    end     if
    end     for
Output: $\boldsymbol{N}$
```

One can realize for $\boldsymbol{t_b}$ and $\boldsymbol{N}$, one can introduce two equalities constraints from two algorithms:

$$\boldsymbol{h_1}(\boldsymbol{P_{Bmax}}, \boldsymbol{T_B}) = \boldsymbol{t_b} - \boldsymbol{F}(\boldsymbol{P_{Bmax}}, \boldsymbol{T_B}) = \mathbf{0} \tag{E-3}$$

$$\boldsymbol{h_2}(\boldsymbol{P_{Bmax}}, \boldsymbol{T_B}, \boldsymbol{t_P}) = \boldsymbol{N} - \boldsymbol{G}(\boldsymbol{F}(\boldsymbol{P_{Bmax}}, \boldsymbol{T_B}), \boldsymbol{t_P}) = \mathbf{0} \tag{E-4}$$

The $\boldsymbol{h_1}$ is a function of $\boldsymbol{P_{Bmax}}, \boldsymbol{T_B}$ and $\boldsymbol{h_2}$ is a function of $\boldsymbol{P_{Bmax}}, \boldsymbol{T_B}$ and $\boldsymbol{t_P}$**.**

Noticing (E-1) are four box constraints which can be expressed into eight inequality constraints, in a standard form:

$$\text{minimize} \quad \boldsymbol{PV'_{t_{s_1}}(CL, P_{Bmax}, T_B, t_P)} \tag{E-5}$$

$$\begin{aligned}
\text{subject to} \quad & \boldsymbol{g_1 = CL - 5000 \le 0} \\
& \boldsymbol{g_2 = 500 - CL \le 0} \\
& \boldsymbol{g_3 = P_{Bmax} - 3 \le 0} \\
& \boldsymbol{g_4 = 0.25 - P_{Bmax} \le 0} \\
& \boldsymbol{g_5 = T_B - 10 \le 0} \\
& \boldsymbol{g_6 = 2 - T_B \le 0} \\
& \boldsymbol{g_7 = t_P - 9 \le 0} \\
& \boldsymbol{g_8 = -t_P \le 0} \\
& \boldsymbol{h_1(P_{Bmax}, T_B) = t_b - F(P_{Bmax}, T_B) = 0} \\
& \boldsymbol{h_2(P_{Bmax}, T_B, t_P) = N - G(F(P_{Bmax}, T_B), t_P) = 0}
\end{aligned} \tag{E-6}$$

By introducing Lagrange multipliers, this optimization problem can be reformatted:

$$\begin{gathered}
\boldsymbol{\nabla PV'_{t_{s_1}}} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{8}} \mu_{\boldsymbol{i}} \boldsymbol{\nabla g_i} + \sum_{\boldsymbol{j=1}}^{\boldsymbol{2}} \boldsymbol{\lambda_j \nabla h_j = 0} \\
\mu_{\boldsymbol{i}} \boldsymbol{\le 0} \\
\boldsymbol{g_i \le 0} \\
\boldsymbol{h_j = 0}
\end{gathered} \tag{E-7}$$

As stated in the Ch. 2.1.3 and Ch. 6.2, this study ignored the effects of the variation of BESS's capacity and cycle life on the price of BESS. Under this assumption, the optimal solution must be get under the lower boundary of variables $\boldsymbol{CL}$**,** $\boldsymbol{P_{Bmax}}$ and $\boldsymbol{T_B}$ (**CL=5000,** $\boldsymbol{P_{Bmax}}$**=0.25 MW and** $\boldsymbol{T_B = 2}$ **h).** This statement illustrates "the comparison between adding fewer, larger increments of capacity versus smaller, but more frequent, increment. Smaller increments, such as smaller BESS, better track the changing load and results in less idle capacity on line over time. [53]" Therefore, $\sum_{i=1}^{N} \boldsymbol{PV'_{t_{s_1}b_i}}$ in (E-2) will be minimized.

The critical point will then happen when the $\boldsymbol{t_P}$ changes**.** The value of $\boldsymbol{t_P}$ has the range of $\boldsymbol{0 \leq t_P \leq 9}$**,** by iteration of $\boldsymbol{PV'_{t_{s_1}}}$ calculation under the different value of $\boldsymbol{t_P}$. The minimum $\boldsymbol{PV'_{t_{s_1}}}$ can be obtained under $\boldsymbol{t_P = 9}$ **year.** The maximum NPV=\$$2.045 \times 10^6$. There will be eight BESSs (**N=8**) installed in the year 15, 17, 18, 19, 20, 21, 22 and 23 respectively.

To sum up:

**Optimal NPV=\$**$\boldsymbol{2.045 \times 10^6}$**,**

**CL=5000 cycles,**

$\boldsymbol{P_{Bmax}}$**=0.25 MW,**

$\boldsymbol{T_B = 2}$ **h,**

$\boldsymbol{t_P = 9}$ **year,**

**N=8 units,**

$\boldsymbol{t_b = [15, 17, 18, 19, 20, 21, 22, 23]}$**.**

The solution for this optimization problem under the more complex conditions is left for the future work.