MATHEMATICS IN THE ART OF SALVADOR DALÍ

An Interactive Qualifying Project Report

submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

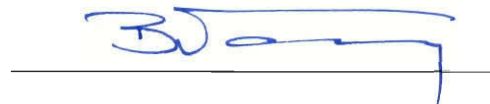Degree of Bachelor of Science

by

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Date: May 30, 2000

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Professor Suzanne L. Weekes, Major Advisor

1. **Dalí**
2. **mathematics**
3. **art**

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Abstract

Salvador Dalí (1904-1989) was one of the most famous artists of the twentieth century and one of the most well known artists of the Surrealist movement. He was an experimental artist whose works were influenced by mathematics, physics, and psychology. In this project, we explore the ideas in geometry, topology, differential equations, and optics that evoked and are evoked by Dalí's art.

Table of Contents

Table of Figures

Introduction

Salvador Dalí (1904-1989) was one of the most famous artists of the twentieth century and one of the most well known artists of the Surrealist movement. He was an experimental artist who worked, not only as a painter, but also as a jewelry designer, set designer, sculptor, cinematographer, and author. Dalí's work was sustained by his curiosity in: pop culture, the psychology of Sigmund Freud, the natural world, past artists, and contemporary science (including genetics, physics, and mathematics, among other things). The science and mathematics that inspired Dalí and his art inspired me to seek out the extent in which ideas behind math and science appear in his work. After studying many paintings, four areas of mathematics appeared to show influence on his works. Those areas were topology, geometry, optics, and differential equations, and these four areas will be discussed over the next five chapters.

Chapter 1 is dedicated to topology inspired by Dalí's *Topological Contortion of a Female Figure*. We seek to determine whether or not the painted female is a topological equivalence to a human female. Topics that will be discussed in this section include topological properties of objects and their genus, which will then be applied to the human body, showing that it is topologically equivalent to a three-holed donut.

In Chapter 2, I explore the role of geometry as it relates to perspective. In his paintings, Dalí constructs an image that appears to have depth, even though it was painted on a flat, 2-dimensional canvas. By the use of parallel lines, varying the sizes of objects, and overlapping different subjects, Dalí is able to add a feeling of depth to such paintings as *The Path of Enigmas*, *The Invisible Man*, *The Angelus of Gala*, and *Suburbs of a Paranoiac-Critical Town*.

While perspective simulates depth, stereoscopy actually achieves it. Chapter 3 investigates Dalí's use of stereoscopy by explaining what stereoscopy is, why two images are necessary to create depth, and different ways stereoscopic images are formed. It concludes with an explanation of how a stereo painting such as *Gala's Christ* is set up in a museum exhibit to see 3D, and then we describe how to create stereoscopic pictures by using 35mm cameras.

Optics and anamorphic art are the subjects of Chapter 4. Anamorphic art is the name given to distorted images that, when viewed at an angle or in the reflection of a special viewer, reforms into a more natural image. Dalí's anamorphic pieces were erotic pictures requiring a reflective cylinder to view the picture. We discus the optics behind seeing a viewable image and creating the distorted image, and in Appendix 4, we include a Maple V code with which we can experiment with anamorphic images.

Finally, in Chapter 5, differential equations are used to mathematically explain the workings of Dalí's *Gala Contemplating the Mediterranean Sea Which at Twenty Meters Becomes the Portrait of Abraham Lincoln – Homage to Rothko*. When viewed at a close distance, the painting is of Gala looking out a window and at a body of water. However, if we step away from the painting, the image of Gala fades out to a portrait of Abraham Lincoln. To mathematically describe the double-imagery of this painting, I pose a simple partial differential equation problem and use homogenization, a scaling-up method, to determine the macroscopic properties of the material from the microscopic properties of its makeup.

Background on the Project

This IQP was initially started with one question: depending on where we stand, why do we see two images in Dalí's *Gala Contemplating the Mediterranean Sea Which at Twenty Meters Becomes the Portrait of Abraham Lincoln – Homage to Rothko* ? For the next twenty-one plus weeks, many of Dalí's paintings that we found in both books and at the museum, were studied and analyzed to see whether there was a perceptible mathematical influence. The primary source for paintings was a book by Robert Descharnes and Gilles Néret titled Dalí. It was chosen over many other Dalí books because of its large quantity of reproductions, and proved to be invaluable, since most of its reproductions were not available in other books or local museums.

The Wadsworth Atheneum Museum of Art in Hartford, Connecticut, also served as a source for studying Dalí's paintings. The museum was the first in the world to purchase a painting by Dalí, so it housed his paintings on a general basis. However, from January 21 – March 26, 2000, the Atheneum hosted an exhibition entitled Salvador Dalí's Optical Illusions. The traveling exhibition traces the evolution of Dalí's interest in modes of perception and various aspects of illusionism from his time with the Surrealists in 1920s to the late 1970s, at which time he began to experiment with holography and stereoscopy. Paintings that were on exhibit included *Topological Contortion of a Female Figure, The Invisible Man, Gala's Christ, Disappearing Bust of Voltaire,* and the sculpture *Venus de Milo With Drawers.* However, perhaps the most fascinating part of the exhibit were the two tables that housed books owned by Dalí. Traite de Perspective a L'usage des Artistes (Treatise on Perspective for the Use of Artists), 1750 by E.S. Jeaurat was one of several classical treatises on perspective Dalí used. Other books that were on

display were: <u>Geometry of Art and Life</u> (Matila Ghyka, 1946), <u>Paraboles et Catastrophes</u> (René Thom) and <u>Anamorphes ou Perspectives Curiures</u> (J. Baltrusaitis, 1952). These books were the most fascinating part of the exhibit because they provided concrete evidence that Dalí read and familiarized himself with mathematics.

To research the various aspects of mathematics found in Dalí's work, I used a combination of books, the Internet, and interviews with various professors on campus at WPI. The books used in research hailed from the Gordon Library, Worcester Public Library, Holyoke Community College Library, or from the personal libraries of Professor Suzanne Weekes and Professor Bogdan Vernescu. The books ranged from college and graduate-level texts to mathematics written for common people to books with lots of pictures and illustrations, designed for enjoyment. In addition, the Internet was used to research such topics as topology and stereoscopy, with careful attention paid to the information's source. Interviews with various professors at WPI played a crucial role in the completion of this IQP. By speaking with professors in physics, mathematics, and biology, I was able to work through areas of question and gain a commanding grasp of the material at hand faster than if I worked solely out of a book or web page.

Because of time and group constraints, not every painting could be analyzed for traces of mathematics. Furthermore, not every mathematical influence could be discussed. The areas of math that were researched and written about were areas that I happened to find interesting. During research, I found other aspects of mathematics that appear in Dalí's work: René Thom's catastrophe theory, geometry, linear and nonlinear equations, the fourth dimension, logarithms, Golden Ratio, and hyper cubes. If one expresses an interest in paintings that have some sort of mathematical inspiration to them

that I did not cover, I would suggest referring to Dalí's *The Swallow's Tail*, *The Last Supper*, *Young Virgin Auto-Sodomized by her Own Chastity*, *Corpus Hypercubus*, *Galatea of the Spheres*, or *Nuclear Cross* as a starting point.

Background on Dalí

Salvador Dalí y Domenech was born in Figueras, Spain on May 11, 1904, the son of a prosperous notary. Dalí received formal training from the Madrid School of Fine Arts in the early 1920s. It was in Madrid where Dalí began to break away from the norm and experiment with an avant-garde style. In 1929 that Dalí officially became a Surrealist – a group of artists centered around André Breton whose style was a combination of styles that fused together the every day world with the world of fantasy and dream – only to be "expelled" by 1948. To this day, Dalí is the best-known artist of the Surrealist movement.

Dalí's career spanned several decades, during which time he not only painted but worked in other mediums as well. Dalí attempted to make rational the irrational and opened himself to the science and art of the past as well as that of his time. He succeeded in shocking and delighting the public of his time, and, though no longer with us, will continue to shock and delight people of the future thanks to museums, video, books, and prints.

Acknowledgments

1 - Topology

Fig 1.1 *Topological Contortion of a Female Figure*, 1983 Salvador Dalí [3]

Topological Contortion of a Female Figure, Fig 1.1, was painted in 1983 during Dalí's final period. This painting is one of several that are influenced by ideas in topology. Nevertheless, how true did Dalí stay to those ideas? Is the female figure in the painting a true topological contortion, or did Dalí use his artistic license and simply incorporate principles of topology in his work?

1.1 Topology - What is it?

Topology is a branch of geometry that deals with the properties of an object's points in relation to each other. It does not deal with rigid properties, such as length or angles, but rather the properties that no amount of twisting, bending, stretching, or shrinking can change. For this reason, it has been known as the "rubber-sheet geometry".

Topology is a creation of approximately the last 100 years of work. Even so, there were some discoveries prior to the nineteenth century that helped lay the

groundwork for topology. Around 1640, the French mathematician René Descartes first noted a relationship between the number of vertices, faces, and edges of a simple polyhedron, i.e. a polyhedron without any holes through it. In 1752, Leonard Euler rediscovered this relationship and proved that the number of vertices minus the number of edges plus the number of faces equals 2, or V-E+F=2 [2]. Euler's proof works under the assumption that the polyhedron has no holes; the more general formula, V-E+F+2N=2, takes into account the number of holes N the polyhedron may have. "Euler's formula", as it came to be known, and its generalizations are one of the central theorems of topology[2].

1.2 Topological Properties

1.2.1 Transformations, Deformations, and Equivalence

A topological transformation is the result of morphing one geometric object A into another object A′. There must be a correspondence p↔p′ between all points p of A and all points p′ of A′, and they must comply with the following properties:

1. Each point p of A corresponds to only one point p′ of A′, and each point p′ of A′ corresponds to only one point p of A.

2. If any two points p, q of A are chosen, and p is moved so the distance between p and q approaches zero, then the distance between the corresponding points p′, q′ of A′ also approach zero. The converse is also true [2].

A topological deformation is the act of deforming or morphing one object A into another object A′. Consider the object that is called a "torus" in topology, or doughnut in everyday terms, in Fig. 1.2. Twisting, bending, and stretching are valid forms of

deformation, but tearing and joining points are not. See Fig 1.3. Tearing or cutting any portion of the object violates transformation condition 2. Joining points, as illustrated in Fig. 1.4, violates condition 1 because the newly joined point can be represented on the original surface by two points.

Fig 1.2 A torus.
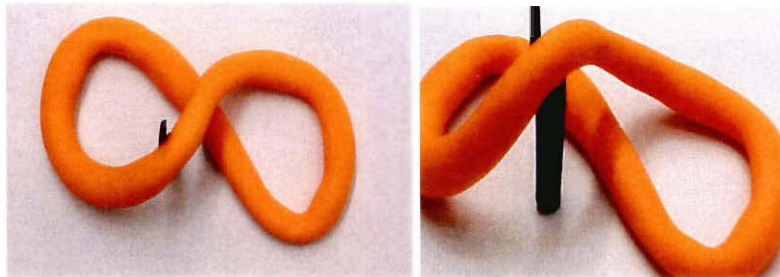
Fig 1.3 Twisting and bending the torus are valid acts of deformation.
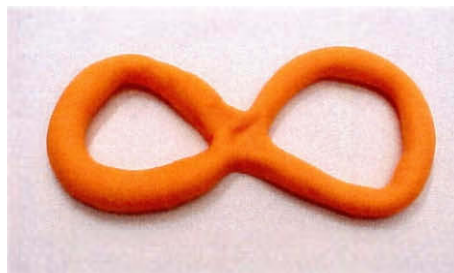
Fig 1.4 A figure-8 violating condition 1. The joint in the center is one point on the figure-8 that now corresponds to two points on the torus.

Figures 1.5a and 1.5b illustrate how making a cut violates condition 2. Figure 1.5a illustrates a torus with 2 points, P and Q, which are very close to each other. Starting

with point P, if we move clockwise through points 1-4, we will end up at Q. The distance between P and Q is almost zero. If the torus were cut between points P and Q, the resulting object would resemble Figure 1.5b. Again, by starting at P and passing through points 1-4, we'll end up at Q. However, the distance between P and Q does not approach zero, but approaches a fixed distance L. Thus the objects in Figure 1.5a and 1.5b are not equivalent.
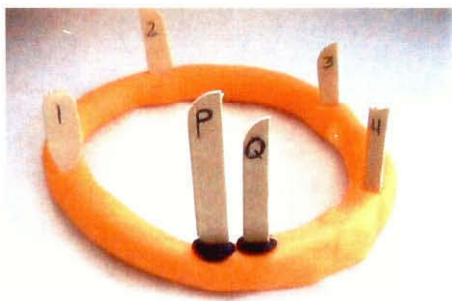
Fig 1.5a   Fig 1.5b

Fig 1.5 How a cut violates property 2.

Objects that comply with the rules of transformation and deformation are said to be equivalent and share topological properties with one another. Figure 1.6 illustrates the fact that a circle is topologically equivalent to a triangle. Three points on the circle can be stretched out to form the vertices of the triangle, and the curves between them flattened to form the legs. In addition, a sphere is topologically equivalent to a bowl. See Fig 1.7. To create a bowl, an indentation is created on the sphere, and because indenting an object does not violate conditions 1 or 2, they are equivalent. Protrusions, too, are a valid form of deformation. They stretch a portion of the surface out away from the object, but don't detach from the object in any way, as shown in Fig 1.8.
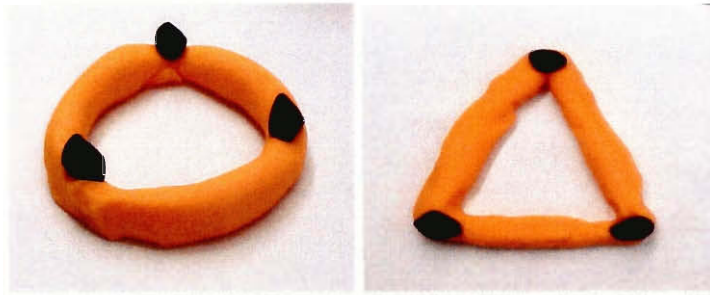
Fig 1.6 A circle is topologically equivalent to a triangle. The black points on the circle correspond to the vertices of the triangle.
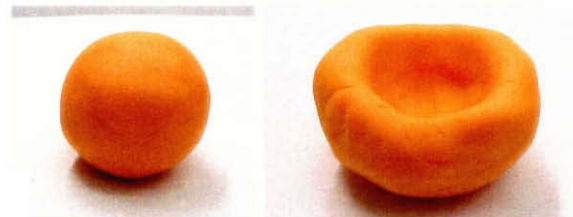
Fig 1.7 On the left, a sphere. On the right, a bowl-shaped object formed by indenting the sphere. These two objects are topologically equivalent to the other.
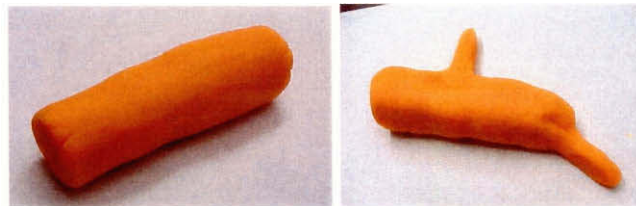
Fig 1.8 A cylinder is topologically equivalent to a cylinder with a protrusion.

A torus and a coffee mug are also equivalent. Consider Figure 1.9. If an indentation is made on one piece of the torus, the already present hole resembles the handle of a coffee mug, and the indentation starts to form the container holding the coffee, like Fig 1.9.2. Making the indentation wider, as in Fig 1.9.3, and then deeper, as in Fig 1.9.4, will manipulate the torus such that it resembles a coffee mug. Since the handle is now horizontal, we can slide one end towards the bottom, making the handle vertical and easier to grip. See Fig 1.9.5. Now, our torus has completed its

transformation into a coffee mug. With some minor cosmetic touching up, our torus will look like Fig 1.9.6.
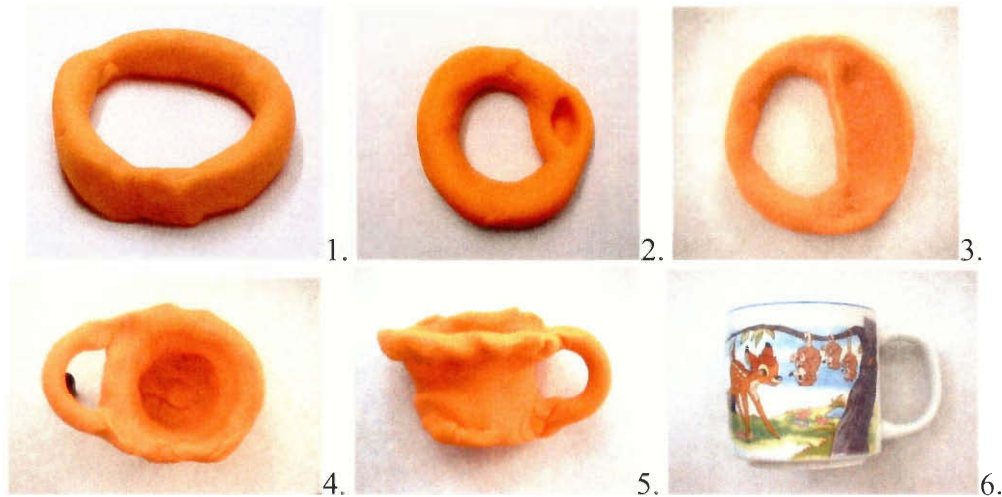
Fig 1.9 A torus transforming into a coffee mug.

However, a circle and a line segment are not equivalent, as aren't a sphere and a torus, because their topological properties are not the same [2]. The circle and sphere require a cut and a puncture, respectively, to make the transformation into a line segment or torus. Since cutting or tearing any part of an object during transformation violates condition 2, the transformation is not possible and the two objects are not topologically equivalent.

1.2.2 Genus

A way of classifying objects as topologically similar to or different from one another is by comparing their genus. The genus is the maximum number of closed curves that can possibly be drawn on a surface that does not separate one region from another. The number of such closed curves an object can have is a topological property distinct to its genus. All topologically equivalent objects have the same genus, and all objects of the

same genus are topologically equivalent. For example, any number of closed curves can be drawn on a sphere, as seen in Fig 1.10, but each curve separates the sphere into two independent surfaces. Points on one surface cannot be connected to points on the other surface unless the closed curve is crossed. Because there are no closed curves that do not separate its surface into two, the genus of a sphere is zero.

Fig 1.10 A sphere with a randomly placed closed curve.

On a torus, we can find one closed curve that does not divide the surface into two parts, as seen in Fig 1.11a and Fig 1.11b. Any point on the surface can be connected to another point by a curve that does not cross the closed curve. However, implementing a second closed curve will separate the surface into two pieces. Refer to Fig 1.11c and Fig 1.11d below to see this. "This difference between the sphere and the torus marks the two types of surfaces as topologically distinct… it is impossible to deform one into the other in a continuous way."[2]

Fig 1.11a A torus with one closed curve.

Fig 1.11b A torus cut along the closed curve. Cutting helps visualize how the closed curve sections off the torus.

Fig 1.11c A torus with a second closed curve.

Fig 1.11d A torus cut along the two closed curves. Note the division of the surface into two pieces, stressing that the two points on either side of the original closed curve cannot be joined together.

For an object with two holes, as in Fig 1.12a, there can be at most two closed curves so that points on the surface can be connected by a curve that does not cross the closed curves. Three closed curves will separate the surface into two parts. Figures 1.12b-1.12d illustrate obtaining the genus by way of the closed curve method. In general, a surface with n-holes can have n-closed curves and still be connected. So for a sphere, torus, and a 2-holed donut, the genus would be 0, 1, and 2, respectively.
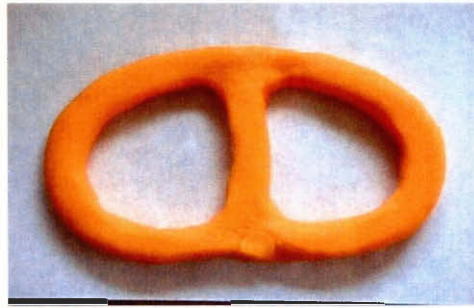
Fig 1.12a A 2-holed donut.
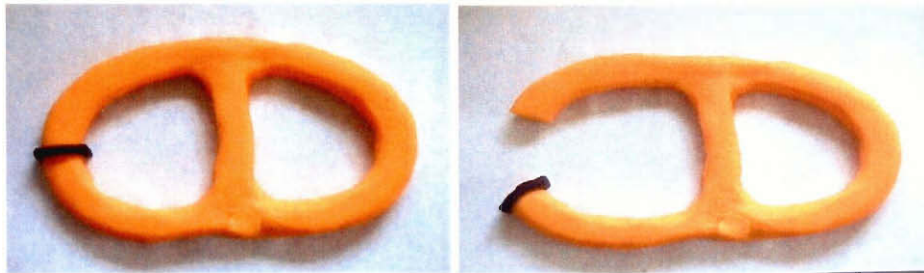
Fig 1.12b One closed curve was found, represented by the black line. The 2-holed donut is cut along the first closed curve.
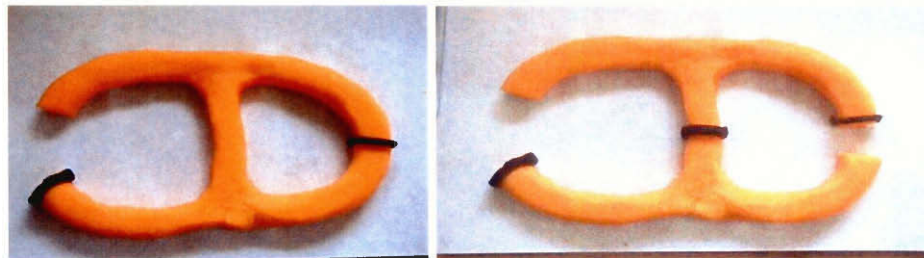
Fig 1.12c A second closed curve was found, shown by the black line. A cut was made along the second closed curve, and a third closed curve was found.
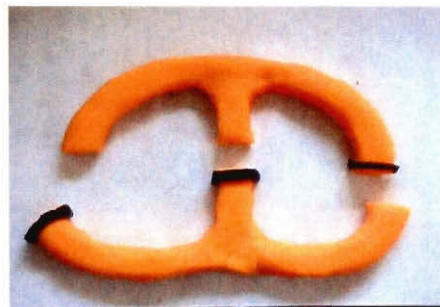
Fig 1.12d Cutting along the third closed curve separates the surface into two parts. Therefore, it has a genus of two.

14

1.3 The Human Body - a 3-Holed Donut?

Topologically speaking, the human body is a cylinder with three holes at one end, one hole at the other end, and all connected by a common passageway. Our digestive system is like a long tube, starting at the mouth and becoming the esophagus, stomach, small intestine, large intestine, and terminating at the anus. [1] Because they are protrusions, the liver, gall bladder, bile duct, and pancreas are considered part of the digestive "tube" and do not change its topology. See Figure 1.8. The mouth is one of the holes at the top of the cylinder, and the anus is the cylinder's bottom hole. The other two holes are the nostrils. Our digestive and respiratory systems are connected at the throat, for the esophagus and the trachea share a common tube. Thus, we have a continuous passageway from the mouth and nostrils to the anus. Appendages such as arms, legs, head, eyes, hair, nose, neck and penis are protrusions of the body and can be removed by pushing them back into the body. The reproductive, transport, nervous, and skeletal systems in humans are closed systems and do not effect the transformation of a human body to a cylinder. Cavities, like the vagina and ears, are "indentations", similar to the bowl in Fig 1.7, and can be pushed out to form a smooth surface. Once all the appendages and cavities are taken care of, the body resembles a cylinder with three holes at the top and one at the bottom. By topologically deforming it into a three-holed donut and finding the genus of the latter, we see that the human body has a genus of three. Alternatively, by applying the transitive property to the human body, multi-holed cylinder, and 3-holed doughnut, we conclude that the human body is topologically equivalent to a three-holed doughnut.

1.4 A Possible Rendering of a Human Transforming into a 3-Holed Donut

As stated in the previous section, the human body is topologically equivalent to a three-holed donut. Since it is not physically possible for a human to transform into the donut, we follow steps 1-9[1], as shown in Fig 1.12-1.20, and attempt to illustrate what would happen if the transformation could take place.
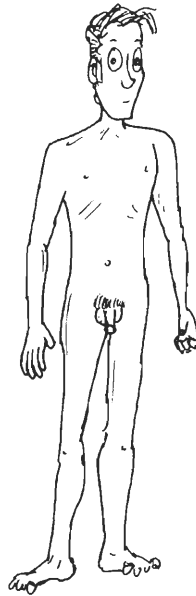
Fig 1.12 Step 1: Start with a human. Our subject here is male.

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[1] Steps 1-9 by Luke Goljan
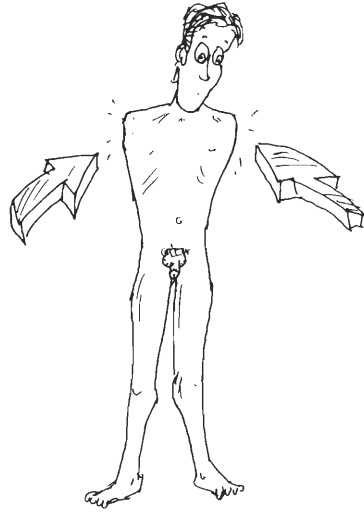
Fig 1.13 Step 2: Arms compress into body.

Fig 1.14 Step 3: Legs compress into body.
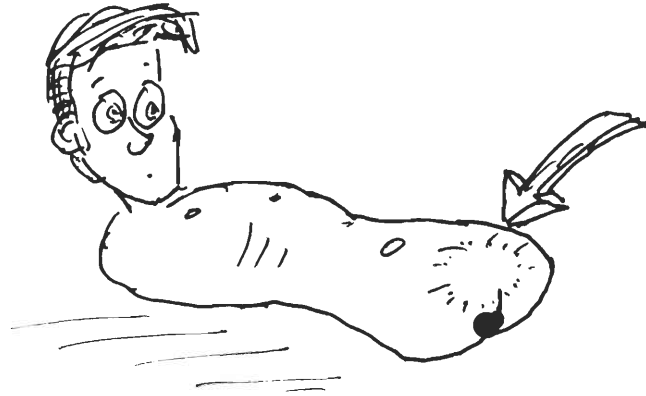
Fig 1.15 Step 4: Genitalia compress. The only indication that this was the man's bottom is the presence of the anus.

Fig 1.16 Step 5: Neck compresses into body.
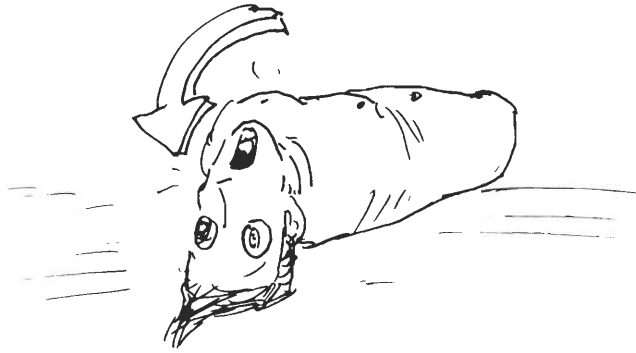
Fig 1.17 Step 6: Head rotates up from loss of neck.
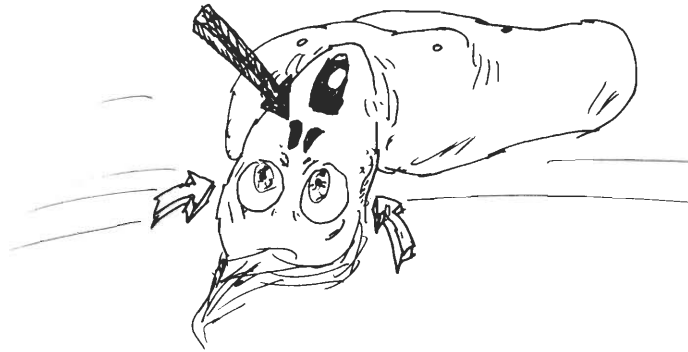
Fig 1.18 Step 7: Ears and nose compress into head. Nostrils remain.
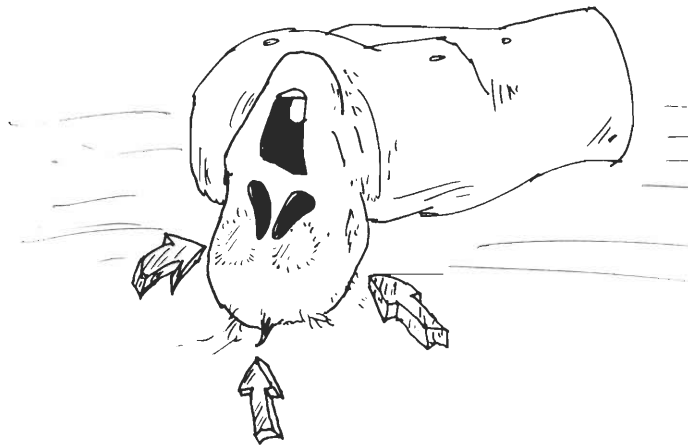
Fig 1.19 Step 8: Eyes and hair disappear.
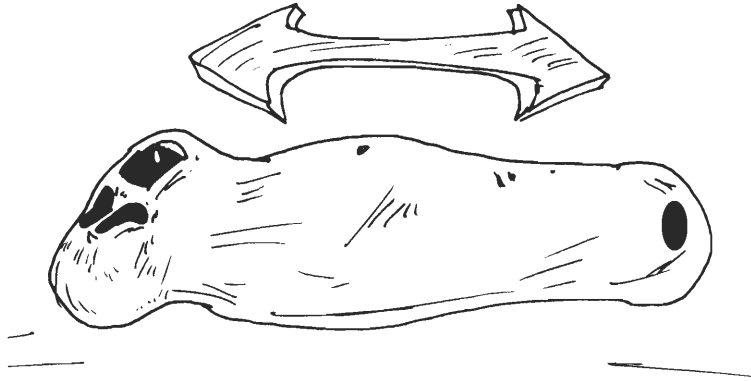
Fig 1.20 Step 9: Body stretches and takes on tubular shape.

At Step 9 in our transformation, the man's body is a cylinder with one hole (the anus) at one end and three holes (nostrils and mouth) at the other end. All the intestines have been topologically transformed into a tube that connects the ends of the cylinder together. Topologically speaking, this is equivalent to a 3-holed donut. To demonstrate the validity of this statement, the following illustrations, Fig 1.21-1.25, will show how to take a 3-holed donut and transform it into a cylinder with three holes at one end all connected to one hole at the other end.
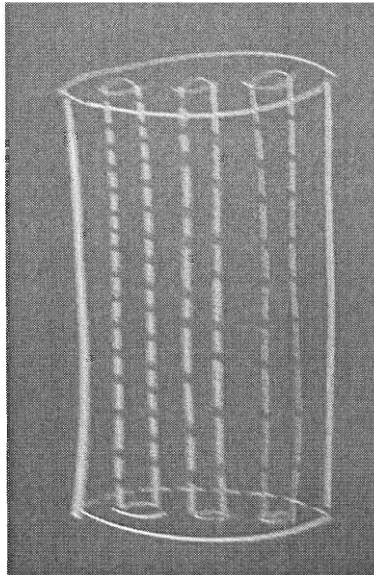
Fig 1.21 We start with a 3-holed cylinder. It is of genus 3.
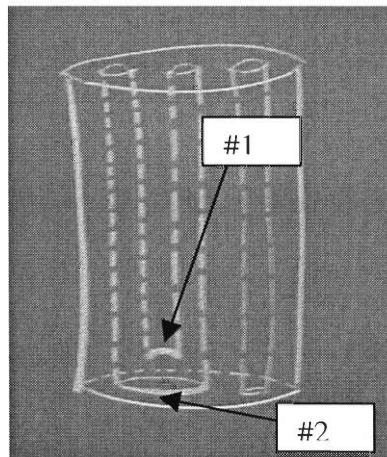
Fig 1.22 The section between two of the holes is pushed upwards.

Next, the section between two of the holes is pushed upwards (#1 in Fig 1.22). This causes the bottoms of the two leftmost holes to fuse together, creating a large fused hole (#2 in Fig 1.22). Imagine the result to look like an upside down pair of jeans, where the waist of the pants is the fused hole, the legs of the jeans are the two holes, and the crotch of the pants to be the section pushed up.
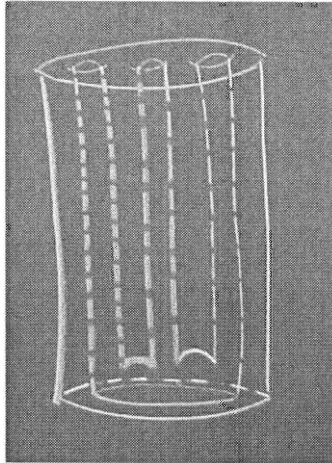
Fig 1.23 Push the section between the new fused hole and the rightmost hole upwards. The result is one big hole at the bottom that forks into three smaller holes.
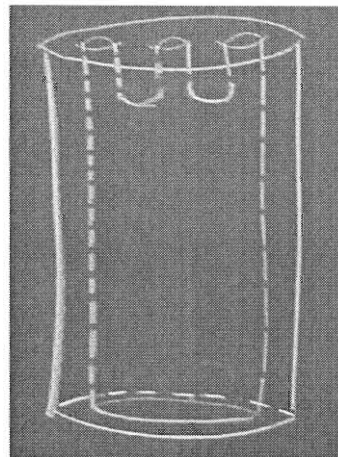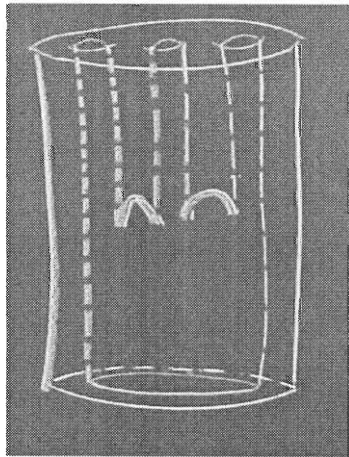
Fig 1.24 Continue to push up on the sections that were between the holes.
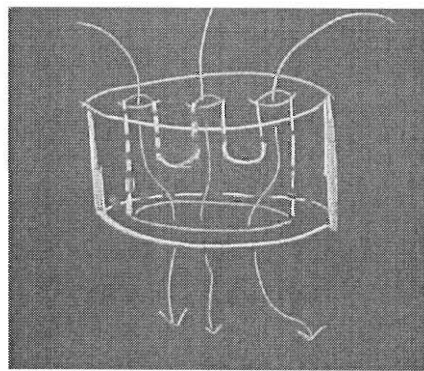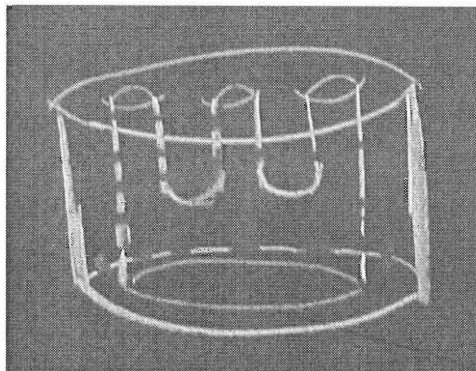
Fig 1.25 Compress the elongated cylinder into a shorter tube.

Finally, we compress the elongated cylinder, forming the left picture in Fig 1.25. The picture to the right illustrates the three separate holes meeting together and forming one shared exit hole. The three holes at the top of the cylinder correspond to the two nostrils and mouth of the man. The three openings are connected via the digestive system, where the opening at the end is the anus. We have thus shown that a human is topologically equivalent to a three-holed donut, thereby having a genus of three.

1.5 Thoughts on Dali's Painting

Is the woman in Dali's *Topological Contortion of a Female Figure* (Fig 1.1) really topologically equivalent to a woman? Truthfully, it cannot be known because we only have one view of the woman. If this were a three-dimensional piece, we could thoroughly examine all around the woman and make a decision. However, based on the painting, I do not believe it is truly topologically equivalent to a woman but instead borrows topological concepts.
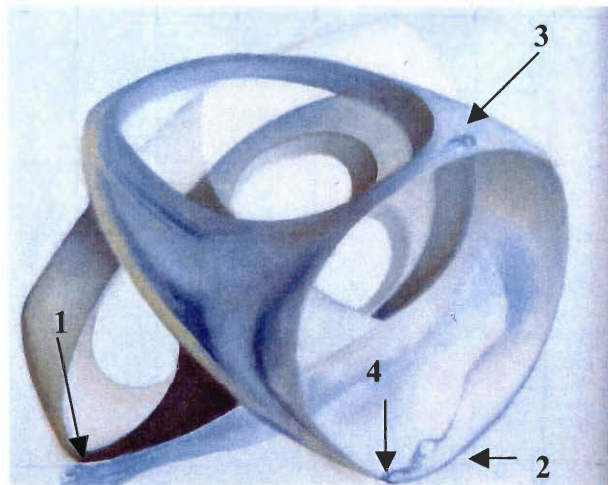
Fig 1.26 Identifying four of the 5 observances found after studying *Topological Contortion of a Female Figure*.

On studying this painting, there are five features that I have noted. The first two features are the presence of a hand and arm (1) and a foot and leg (2). If the painting is turned upside down, what was the foot and leg also resembles a muscular arm and a hand making a pointing gesture. Third, there is also the presence of a small bump near the top of the female figure (3). If the foot and leg is indeed another arm, I believe the bump is the nipple of a breast. Towards the bottom of the painting (4), it appears as if the foot touches, but does not connect, another part of the figure, creating a circle-like opening in the figure. Refer to Fig 1.26 to see the first four observances. Finally, the female is contorted in such a way as to resemble a Roman Surface, one of three surfaces that can be formed by sewing a Möbius strip to the edge of a disk [15]. The result is an image of the real projective plane formed when a function transforms the projective plane, abbreviated $\mathfrak{R}P^2$, into $\mathfrak{R}^3$ [6]. Dalí was known to have been interested in the fourth-dimension and incorporate a purely geometric representation of it in several paintings, such as *Search of the 4ᵗʰ Dimension* and *Pentagonal Sardana*.
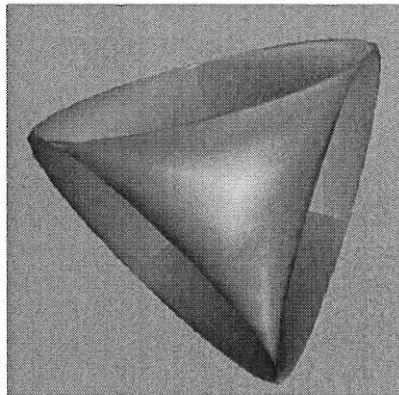
Fig 1.27 A Roman Surface [6].
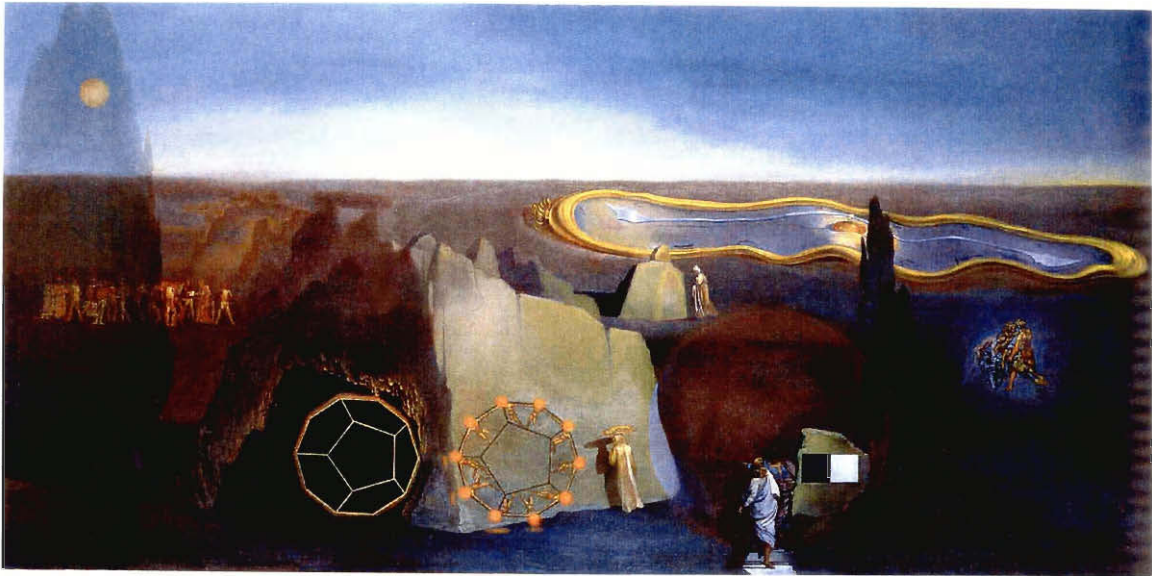
Fig 1.28 Dali's *Search of the 4th Dimension* (1979) [3]
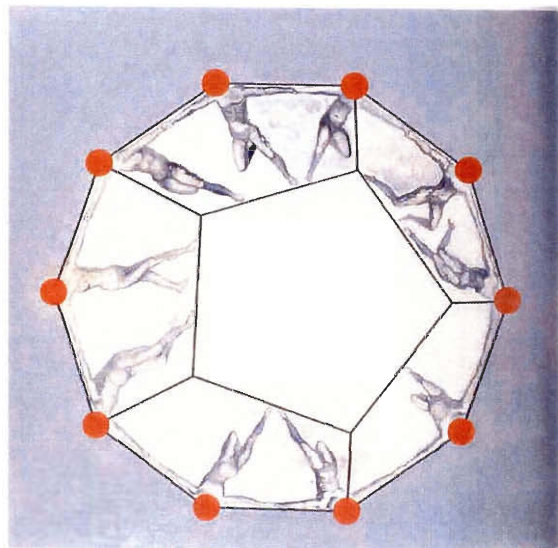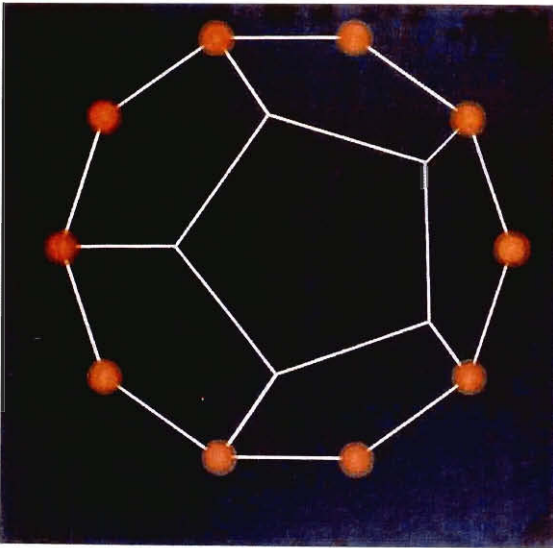
Fig 1.29 Pentagonal Sardana (1979, Stereoscopic work, left and right component) [3]

2 – Perspective

2.1 What is It?

The Merriam-Webster dictionary defines perspective as "the technique or process of representing on a plane or curved surface the spatial relation of objects as they might appear to the eye; specifically: representation in a drawing or painting of parallel lines as converging in order to give the illusion of depth and distance [11]." While today we may take linear perspective, or simply perspective, for granted, it was not until 600 years ago that man was able to illustrate a sense of depth and distance on a flat canvas. It was during the Renaissance that painters began to develop and use this technique. Three mathematical ideas play a role in the use of perspective: parallel lines, diminution, and overlapping.

2.2 Parallel Lines

The representation of parallel lines in a drawing or painting is used to convey a sense of distance. Parallel lines will never intersect each other, regardless of how long the lines are, but they appear to converge as we look at them into the distance. As an example of this phenomena, consider railroad tracks. The rails upon which the trains ride are parallel to one another. However, if you stand between the rails and look into the distance, the rails appear to meet at a point in the distance. In painting, the illusion of distance or infinity is indicated by "lines that are moving away from the viewer at an angle parallel to the direction that the viewer is looking [9]."

Figure 2.1a, *The Path of Enigmas* (second version), is a good example of how Dali used converging lines to indicate distance. The objects assemble to form lines

which seem to converge somewhere in the horizon, thus giving the sense that the path goes on forever. Figure 2.1b shows the lines the objects form.

Fig 2.1a Dali's *The Path of Enigmas* (second version, 1981)[3].
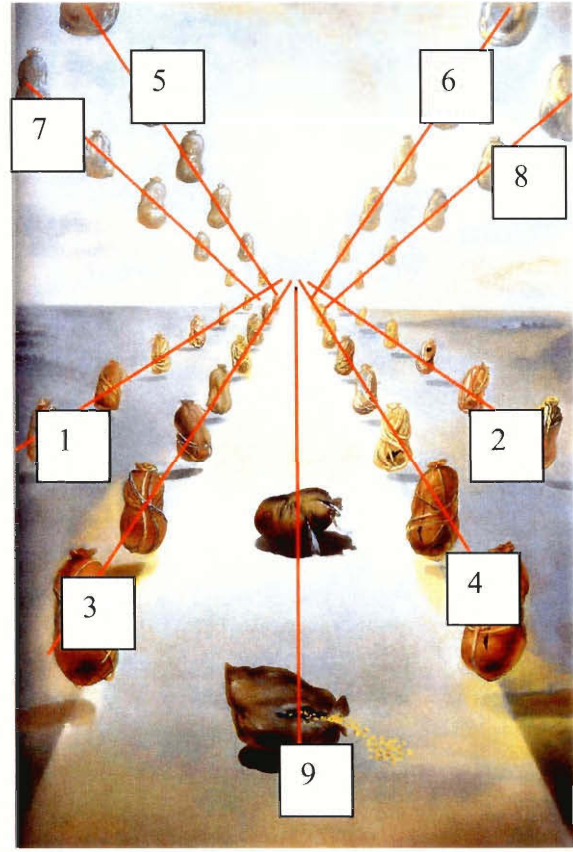
Fig 2.1b Dali's *The Path of Enigmas* (second version, 1981). The red lines illustrate the converging lines the objects form. Each pair of lines, 1&2, 3&4, 5&6, 7&8, are parallel to each other and converge in the distance. Line 9 is parallel to 3&4 and 1&2.

The converging lines discussed above are called orthogonals. The orthogonals converge to a point in the distance called the vanishing point, located on the horizon line. The horizon line is a horizontal line that runs across the canvas at eye level. It is where the sky appears to meet the ground. Usually, but not always, the vanishing point is located in the center of the horizon line.

Dali's *The Invisible Man*, Fig 2.2a, is a prime piece to illustrate the use of orthagonals, horizon lines, and vanishing points. The orthagonal lines are not as obvious here as in *The Path of Enigmas*. Instead, the architecture filling the painting contains the orthagonal lines, helping to draw the eye's attention to the upper left-center of the painting. The architecture along the right, notably the arcades and molding along the wall, form converging lines. Likewise, on the left, we see a horse standing on an elevated platform. The edges of the platform, along with its shadow and edges of the wall just left of the horse, form another set of orthagonal lines. To the left of the invisible man's head, behind the broken wall, lie rows of pillars, which are an extension of the lines starting in the wall on the right. The horizon line is easy to locate, and the vanishing point is situated on the left half of the horizon line. Illustrated in Fig 2.2b are the orthagonal lines, horizon line, and vanishing point. Figure 2.2c is a study for *The Invisible Man.* Here, we get to see first hand Dali using converging lines, a vanishing point, and a horizon line while planning his painting.

Fig 2.2a Dali's *The Invisible Man* (1929) [3].

Fig 2.2b Dali's *The Invisible Man* (1929), with orthagonal lines (pink), horizon line (blue), and vanishing point (box) illustrated.
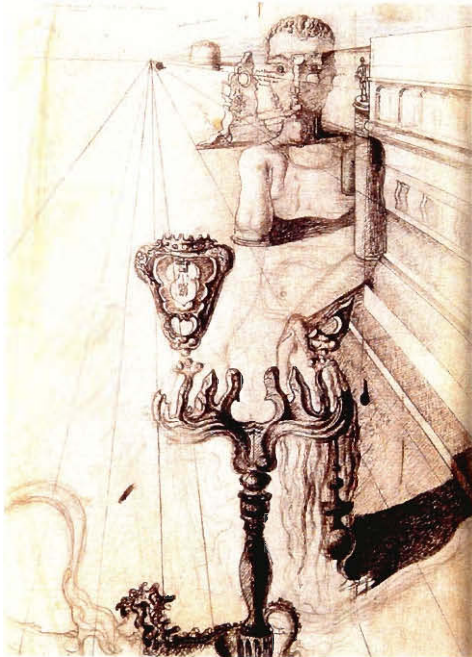
Fig 2.2c Study for *The Invisible Man* (1929) [3].

29

2.3 Diminution

Another technique for adding depth to a painting is by diminution, or making objects that are in the distance smaller than those closer to the foreground. In *The Path of Enigmas*, Fig 2.1a, as the objects recede into the distance, they are drawn slightly smaller than the previous object. The larger object appears to be closer to the viewer, while the smaller object appears to be further out. Another example can be seen in Fig 2.3, *The Angelus of Gala*. Here, the Gala with her back facing the viewer is larger in size than the Gala facing the viewer. Thus, we get the impression that the Gala with her back to us is closer than the Gala that is facing us.
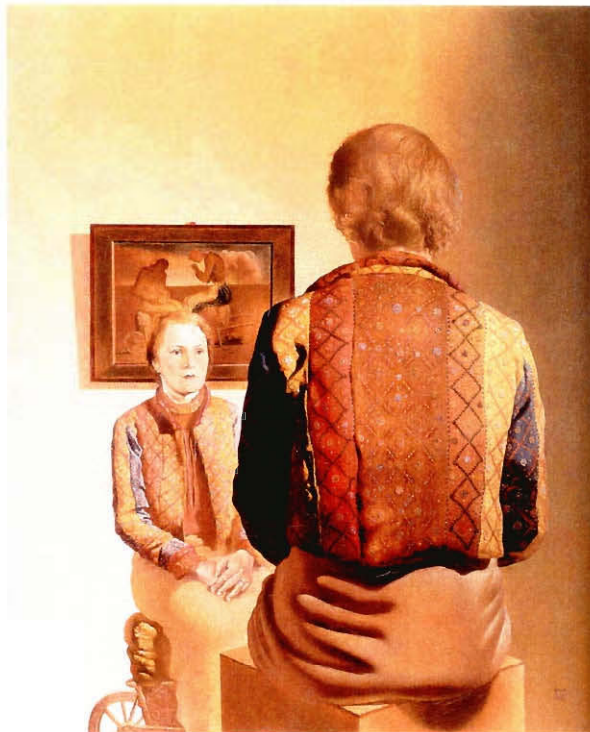
Fig 2.3 *The Angelus of Gala* (1935) [3].

2.4 Overlapping

A third method of obtaining perspective on a canvas is by overlapping one or more objects over another object. The object that is partially covered appears to be in the distance, while the object on top seems closer to the viewer. For example, Fig 2.4's *Suburbs of a Paranoiac-Critical Town: Afternoon on the Outskirts of European History* (1936) has several layers of overlapping that brilliantly portray depth. There is a Spanish-style church in the background, which is partially covered by a large black structure. When seen through the archway of the black structure, the church appears to be at quite a distant from the structure. The smallness of the church compared to the large structure could account for its appearance in the distance, but size doesn't always indicate distance. Overlapping the black structure is Gala holding up a grape bunch. Gala, even though smaller than the black structure, comes across as being closer to the viewer because she is painted over the structure. The combined effect is Gala in the foreground, a large black structure perhaps several yards behind her, and a church further in the distance.
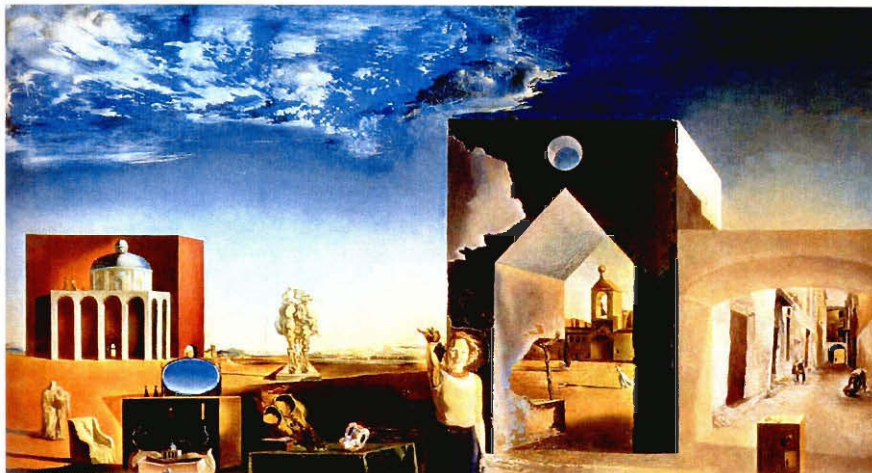
Fig 2.4 *Suburbs of a Paranoiac-Critical Town: Afternoon on the Outskirts of European History* (1936) [3].

3 - Stereoscopy

3.1 What Is Stereoscopy?

"[We live] in a three-dimensional, spatial, environment. Without a feeling for space, we cannot move within it. Our perception of space is created almost exclusively by our eyes. There are many ways to orient oneself in space, e.g., by perspective, gradation of color, contrast and movement [14]." Perspective is often confused with 3D. As we discussed in Chapter 2, perspective only simulates the third dimension, depth. Stereoscopic imaging, or "real 3D", however, requires a minimum of two pictures, simulating our two eyes, to achieve the third dimension.

3.2 Do You See What I See?

The most popular example of stereoscopic images are the Magic Eye pictures. These are the pictures usually found at mall kiosks and in the back of the Sunday comics at which one stares until an image appears. Other sources of stereoscopy include the children's toy Viewmaster (Fig 3.1a), magazines (Fig 3.1b), and Dalí's painting *Gala's Christ*.

Fig 3.1a The Viewmaster toy.
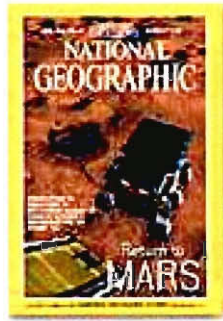
Fig 3.1b The cover of the August, 1998 National Geographic magazine, highlighting the Mars expedition and containing 3D pictures of the Martian landscape [12].
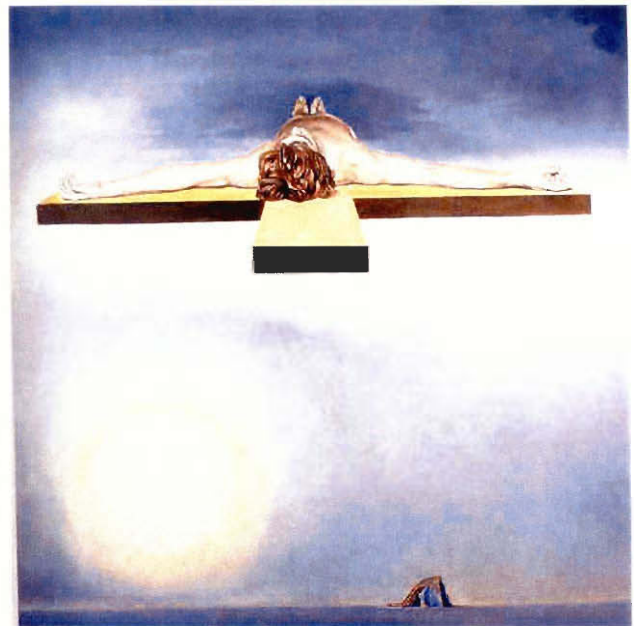
Left   Right

Fig 3.2 *Gala's Christ*, 1978, left and right components [3].

3.3 Stereo Pairs

Stereo pairs are "…image[s] portrayed from different angles, simulating what each eye would see alone [4]." The two images that are drawn or painted can be illustrated by the following example. Take your hand and place it one foot from your nose, with the index finger pointed straight up. Close your left eye and remember your finger's position and the amount of your fingernail you can see. Switch eyes (open your left eye and close your right). Your finger should have shifted its position slightly and you will now see a different amount of your fingernail. The two different positions your fingernails were in are exactly like the two images in a stereo pair.

33

3.4 Why Two Different Pictures?

Human beings have two eyes that sit side-by-side in the front of their head. This results in viewing a single object from two points of view. Each eye gathers an image from its own view and sends it to the brain. When the two images arrive simultaneously in the back of the brain, they are united into one picture. The brain does more than simply overlay the two images, as in Fig 3.3. Our brain has been trained over time to fuse the two images together, matching up the similarities and adding in the small differences. The resulting image is a combination of what each eye sees.

Under normal viewing conditions, the world appears to us as seen from a virtual eye placed midway between the left and right eye positions. This virtual eye is called the cyclopean eye, and its geometry is depicted in Fig 3.4. "This simple geometrical arrangement has an important consequence: since the world appears different from any of these viewpoints, the image we perceive of the world is never recorded directly by any sensory array, but constructed by our neural hardware! [8] "

3.5 How To See 3D

3.5.1 Anaglyphic Method

We can attempt to make a 3D reconstruction of the original object in one of two ways: either the anaglyphic or stereo method. The anaglyphic method produces an anaglyph, a composite image in which the right component, usually red in color, is superposed on the left component in a contrasting color, usually blue. When viewed through correspondingly colored filters, such as the glasses in Fig 3.5, the red filter will make the red ink "disappear" and turn the blue into black. The opposite is true for the blue filter. This will allow your eyes to see only one image each and your brain then takes over to process them into 3D. Examples of anaglyphic images today include the Sports Illustrated Swimsuit Edition 2000 and the 8/98 issue of National Geographic, which contains several pictures of Mars.

Left Image   Right Image
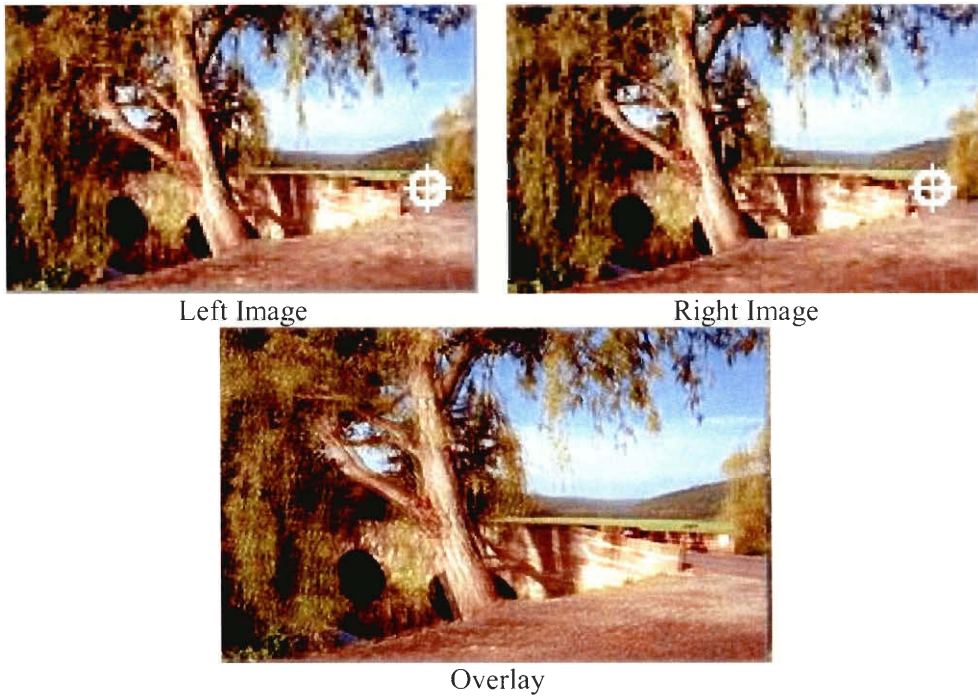
Overlay

Fig 3.3 What happens if left and right eye images were overlaid. "The fixation marks in the stereo images above indicate the current fixation point; the simple overlay of the two images shows [a blurry double image] in most image areas - except for the current fixation point [8] ."
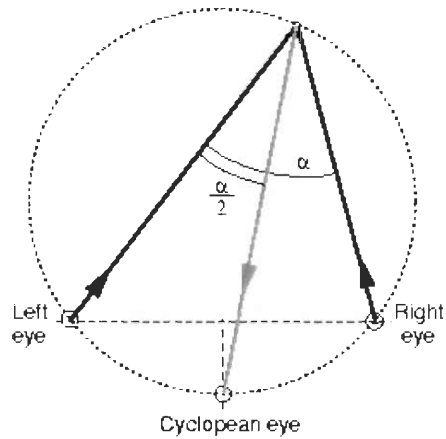
Fig 3.4 The geometry of the cyclopean eye [8].

Fig 3.5 A pair of 3D glasses commonly found in comic books and magazines.
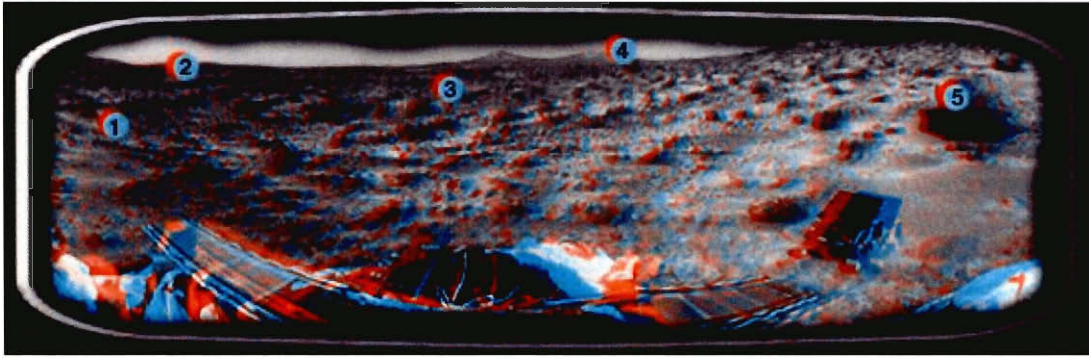
Fig 3.6 A 3D view of the Martian Landscape, from 8/98 issue of National Geographic [12]

3.5.2 Stereo Method

The second way to see stereo pairs in 3D is by the stereo method. Viewing the 3D image can be done by one of several methods, but each requires that the left and right components be placed side by side. One way to see the 3D image is by using a special viewer. One such viewer is a Holmes viewer. It is a handheld device with a stereo card (a card with left and right components of an image) on one end and a viewpiece, with small angled mirrors and a divider so each eye sees only one image, at the other. See Fig 3.7. Devised during the early 1800s, Oliver Wendell Holmes and Joseph Bates designed the viewer, which allowed for the viewing of inexpensive, mass-produced paper prints. A modern descendant of the Holmes Viewer is the Berezin Stereo Photography Products' Viewmaster viewer, as seen in Fig 3.1a. Like the Holmes Viewer, the Viewmaster has an eyepiece that allows the user to see only one image with each eye, but instead of having one image per card, the Viewmaster uses multiple pictures presented on circular reels that are inserted into the viewer.
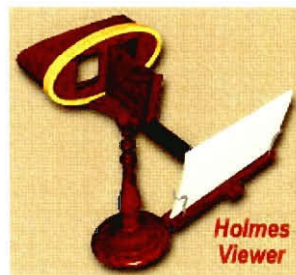
Fig 3.7 A Holmes viewer.

Free viewing is an alternative way to see the stereo pictures in 3D. Free viewing is just seeing the 3D combination of the stereo pictures without any outside help, i.e. no Holmes viewer or mirror. There are two ways to free view: parallel method or cross-eyed method. In the parallel method, lines of sight of your eyes move outward toward parallel and meet in the distance at a point behind and beyond the image. To parallel-view, look deep beyond the canvas as if you were looking at something way in the distance. As you look further into the distance, the two images will converge to form a third, 3D image. See Fig 3.8. If the image is hard to see, try taking a piece of cardboard and place one edge between the two pictures and the other to the bridge of your nose, forcing each eye to see an independent image. Then, just stare straight at the images until a third combined image is formed [13].

Fig 3.8 What happens in parallel viewing [7].

Another method for 3D viewing is called the cross-eyed method. In this method, you aim your eyes so that the lines of sight of your eyes cross in front of the image. Cross your eyes so that either the left or right component converges onto the other. See Fig 3.9. Holding your focus on this third image should make the image appear 3D. If not, hold a finger between the pictures and your eyes. Focus on the finger tip, and move it toward and away from the screen, staying focused on the fingertip until the two images merge into a third image that is 3D [13].

Fig 3.9 What happens when cross-viewing [7].

Here is a practice exercise in free viewing. Below, there is an O next to an X. By using either the parallel or cross-eyed method, try to combine the O and X so that the X is inside the O. The combined image should look like Fig 3.10.
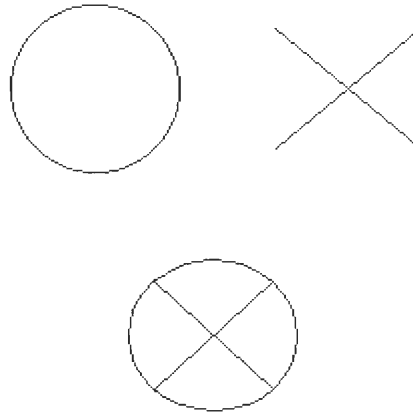
Fig 3.10 The desired image, after using the parallel or cross-eyed method to combine X and O.

3.6 Dalí In 3D

Dalí painted several stereoscopic paintings during his career. One of those paintings is *Gala's Christ*, seen in Fig 3.2. How the pieces were painted is irrelevant; it is the way that the pictures are viewed that is important. Painted in 1978, the two stereoscopic sections are 100cm long x 100cm high and are now part of a private collection. From mid January to late March of 2000, the Wadsworth Atheneum, located in Hartford, Connecticut, hosted a traveling exhibit on the optical illusions of Salvador Dali. Among the paintings present at the exhibit was *Gala's Christ*. Situated in a corner of a room, the two pieces were hung on either side of the walls, with angled mirrors located between them. To see the combined picture, patrons had to gaze into the mirrors and shift back and forth until a combined image was seen in front of them. Appendix A1 illustrates the setup of *Gala's Christ*.

Here is how the illusion is done. When a patron steps up to the mirror, he or she places their nose to where the two mirrors meet. In doing so, the left and right eyes of the patron are focused only on the left and right mirrors, respectfully. The mirrors reflect the painting's image,

and the patron sees the left and right components reflected in the mirror. After fine-tuning the position of the head, the patron should see a combined image directly in front of them. The image, which seems to be behind the two mirrors, appears in 3D, with the top of the cross and Christ's head popping out towards the viewer.

3.7 How To Make Your Own 3D Pictures

Three-dimensional stereo pair pictures can be made with the use of a special 3D camera, but these cameras cost a lot of money. One way of taking stereo pictures is by using a conventional camera, be it the film-loading or disposable kind. First, place your legs about 18 inches apart, keeping your weight evenly distributed. Next, frame your subject in the camera, lightly bend your left knee, and take a picture. Straighten up, lightly bend your right knee this time, and take another picture. This creates two stereo pairs, optically separated by about 3 inches (roughly the same distance between your eyes pupil-to-pupil).

Another way to make 3D pictures is to tape side-by-side two disposable cameras. To take the picture, frame the subject and snap the two shutters at the same time.

There are a few things to keep in mind when making homemade stereo pairs. First, choose a subject with lots of layers (foreground, middle, and background), leading lines (similar to the orthagonal lines discussed in Chapter 2), or more importantly, objects at an angle. The second thing to keep in mind is to remember which photo is the left or right component of the subject. Mixing up the components can lead to an inability to see the combined image. Finally, remember that the pictures cannot combine by themselves into a 3D image; they have to be viewed either by parallel or cross-eyed method. Please refer to A2 for samples of photos using this method.

4 - Anamorphic Art

4.1 What is Anamorphic Art?

Anamorphic art is the art of distorting images. From the Greek words *ana* (again) and *morphe* (form), it refers to art distorted from a projective transformation. The distorted image can be reformed into viewable form by viewing it on a slant or in the reflection of a special mirror. Little information exists on any of Dalí's anamorphic pieces, except that he made a set of erotic paintings that were sold in Switzerland. Because we could not find a reprint of Dalí's set, similar paintings by other artists will be used as examples of the various styles of anamorphic art.

4.2 Two Kinds of Anamorphic Art

4.2.1 Slant Art

The simplest kind of anamorphic art are stretched pictures that, when viewed at an angle, appear correct. Figures 4.1 and 4.2a show examples of slant art. This technique was first used during the early part of the Renaissance, approximately 1590, when artists started to master perspective. "When the Renaissance painters discovered how to transform flat shapes to give a depth illusion to the canvas, they discovered simultaneously how to do it in reverse. A picture stretched according to the same rules of perspective becomes a distorted form [5]." For example, a typical college textbook, rectangle in shape, resembles a trapezoid when viewed from a certain angle and is painted as a trapezoid. However, because of experience, our brain perceives the textbook as a slanted rectangle [5].

Fig 4.1 *The Ambassador*, by Hans Holbein [5].
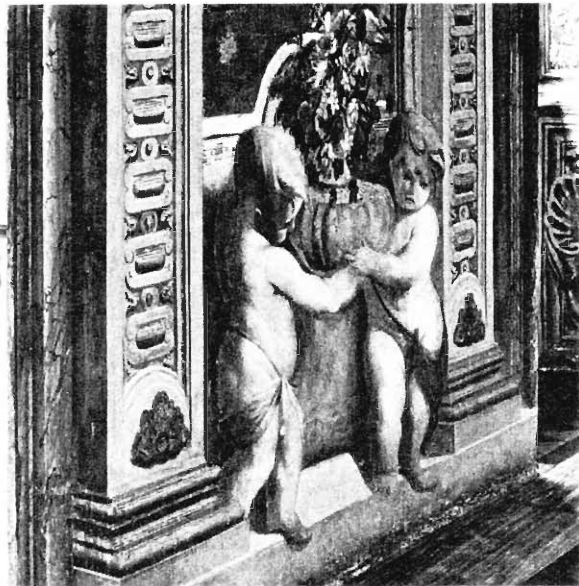
Fig 2.2a A slant picture when viewed at an angle. Notice how things are in proportion [10].
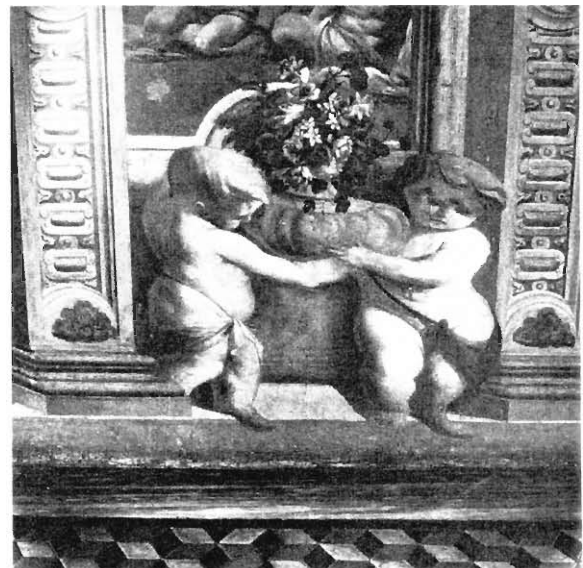
Fig 2.2b How Fig 2.2a looks when viewed straight on – distorted [10].

4.2.2 Cylindrical/Conical Art

The other types of anamorphic art are those that require a cylindrical or conical mirror for viewing the drawing. The viewers, called anamorphoscopes, were made of a

41

reflective substance and usually sold together with the anamorphic pictures. When placed in a certain spot on the art piece, a viewable image appears in the anamorphoscope's reflection. Figure 4.3 shows an anamorphoscope and anamorphic drawing set up for viewing. These pictures were popular during seventeenth and eighteenth century in Europe and the Orient, where they were used as toys and occasionally as means of carrying messages of political protest.
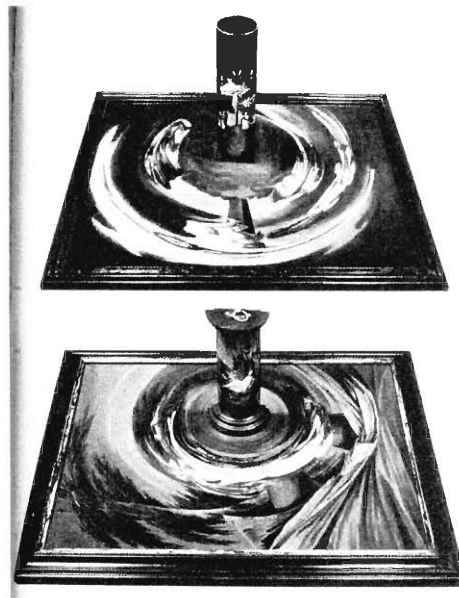
Fig 4.3 Two anamorphic drawings set up for viewing with an anamorphoscope [10].

4.3 Anamorphic Art Today

Anamorphic art is not a common art form anymore, but it does live on in museum collections and books. When purchasing a book that contains examples of cylindrical or conical anamorphic art, from now on referred to as cylindrical or conical pictures respectfully, an anamorphoscope is not always provided. If this is the case, it is possible to fashion our own anamorphoscope. To view Fig 4.4 and Fig 4.5, a cylindrical anamorphoscope is needed. Build your own anamorphoscope by following one of the

subsequent descriptions for a homemade viewer in Appendix A3. Figures 4.4 and 4.5 are examples of cylindrical and conical pictures reproduced from Fred Leeman's book Hidden Images.

4.4 How'd He Do That?

When Dalí made a set of cylindrical paintings, he looked into the mirror while painting on the surface under it [5]. In order to see how an anamorphic image may have been painted, we conducted several simple experiments involving aluminum foil, a can of shaving cream, paper and a pencil. The aluminum foil was smoothed out and taped around a cylindrical can of shaving cream, and then placed on top of a piece of paper. The first experiment consisted of drawing different objects, such as lines, circles, and triangles, on the paper and observing their reflections. It was noted that, in general, the reflected images were curved and distorted. The amount of curve and distortion varied, depending on the viewing angle, or where the eye is relative to the drawing. The second experiment dealt with drawing straight lines in the reflection. The results showed that, to draw a straight line in the reflection of the cylinder, the line had to be curved, with the amount of curvature depending on the viewing angle. When drawing lines normal to the cylinder, little to no curving of the line was necessary. From these results, subsequent experiments were conducted where circles, triangles, and other geometric shapes were drawn in the reflection, which reinforced the findings in the original experiments on straight lines.
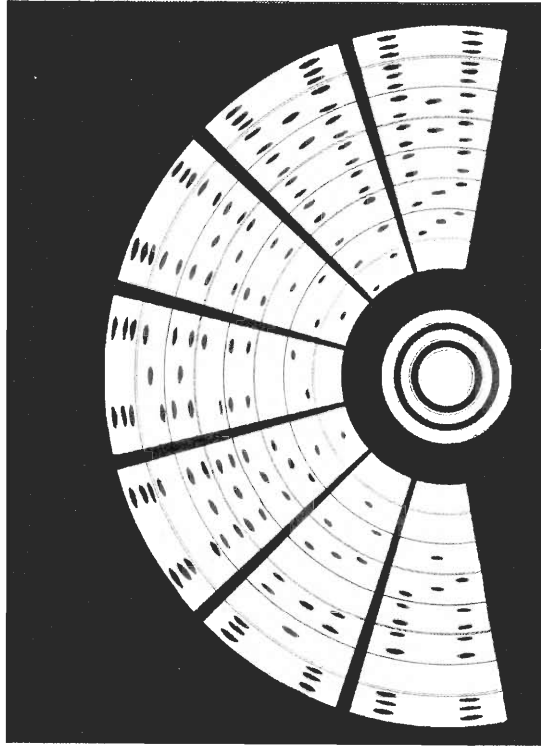
Fig 4.4 *Dominoes*

Fig 4.5 Crucifixion

4.4.1 Physics Behind the Art

To understand how a viewable image is formed from a distorted picture, we will show how to locate the reflection of a point, and then work backwards. To do this, we first assume two things: 1) the eye is located in a fixed position, and 2) the reflection rays and their normal are in the same plane. A cylindrical anamorphoscope is placed on a sheet of paper. On that paper, a point P_1 is drawn. There are infinitely many rays emanating from P_1 that are reflected off of the cylinder. However, there is only one unique ray that starts from P_1, is reflected off the cylinder at point P, and terminates at the eye, P_2. Figure 4.6 illustrates this phenomenon. We know the location of P_1 and P_2 and need to find P. To find P, we apply Fermat's Theorem, which states that P is the point on the cylinder such that the sum of the length of P_1P and PP_2 is the minimum. If we take the ray P_2P, and extend it until it intersects the paper, we will find the location of Q_1, the reflected image of P_1.

We can work backwards from these steps to find what the distorted picture would be if we have the viewable reflection. We start with a cylinder on a sheet of paper, with a point Q_1 located within the cylinder and will find P_1, the location of the original object. Assuming the eye is stationary, the first step is to find the ray that passes through Q_1 and P_2, the eye. The intersection of ray Q_1P_2 and the cylinder is at point P. Next, find the vector normal to the cylinder at point P. The angle between the normal at P and the ray PP_2 is the same as the angle between ray P_1P, because the angle of incidence is equal to the angle of reflection. The points that satisfy this condition lie on a cone. By applying the assumption that P_1P and PP_2 are in the same plane, we select the ray on the cone on which P_1 lies. The intersection of this ray with the paper is the location of P_1. By

repeating these steps for all the points on the reflected image, we can create an anamorphic picture like Dalí.
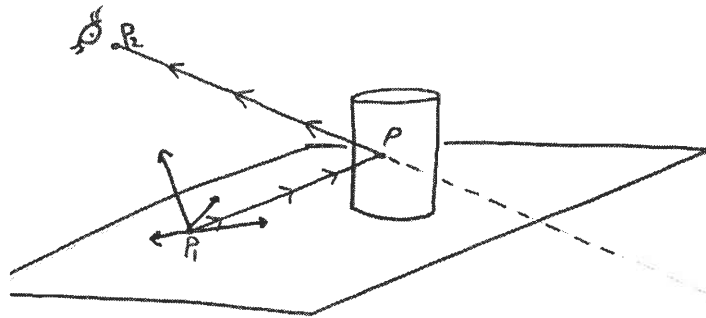
Fig 4.6 Many rays emminate from P_1 and reflect off the cylinder, but only one ray starts at P_1, reflects off P, and terminates at P_2, the eye.
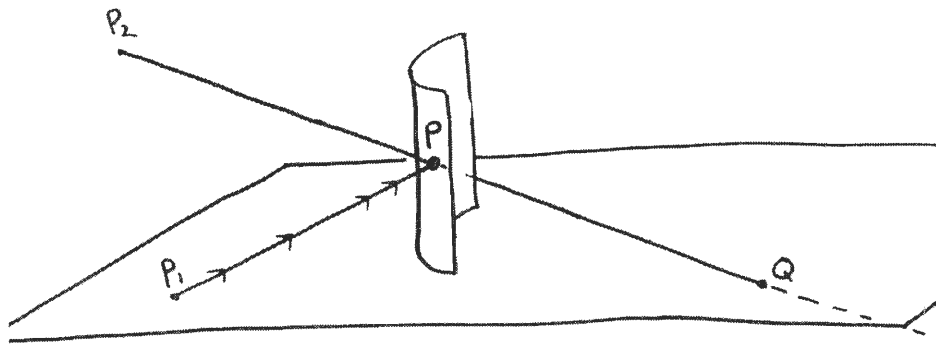
Fig 4.7 Extending the ray P_2P until it intersects with the paper will give us the location of Q.

To show that it is possible to find the distorted image from a cylinder and an image within the cylinder, we wrote a Maple V worksheet that creates the distorted image when given a drawing located within a cylinder. The worksheet and test image can be found in Appendix A4.

5 – Homogenization

5.1 Dalí's Double Painting

Dalí's *Gala Contemplating the Mediterranean Sea which at Twenty Meters Becomes the Portrait of Abraham Lincoln-Homage to Rothko* (Fig 5.1) is an example of his double image painting. Upon viewing the painting from a close distance, we notice a nude Gala looking out a window into the Mediterranean Sea, all surrounded by a ring of dark color. A striking feature of *Gala Contemplating the Mediterranean Sea…*is that it appears to be painted mostly in cubes of color, similar to a patchwork quilt or a tile floor. If we view the painting twenty meters back, as it states in its title, the painting now resembles a portrait of Abraham Lincoln.

Each cube that makes up this painting has a specific property: the color or colors it contains. On the small scale, that is, when the painting is viewed from a close distance, the individual cubes are set up such that we see an image of Gala overlooking the sea. As we move away from the painting, the difference between any two cubes seems to be less distinguishable, and the painting begins to be blurry. When we view the painting on a large scale, that is, when viewed from afar, the individual cubes of color are undistinguishable from one another; now, the cubes appear to form one single color, leading to a new image.
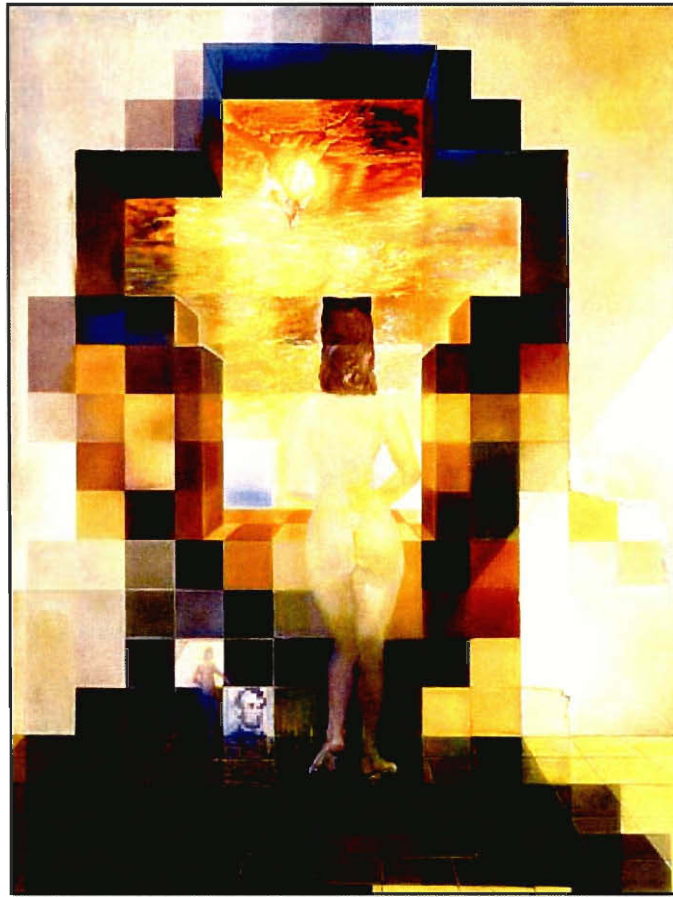
Fig 5.1 *Gala Contemplating the Mediterranean Sea which at Twenty Meters Becomes the Portrait of Abraham Lincoln-Homage to Rothko* (1974-75).

5.2 Explaining the Painting Mathematically

The portrait of Abraham Lincoln is a result of the painting of Gala being "homogenized." Homogenization is a scaling-up process that determines the macroscopic properties of a material from the microscopic properties of its makeup. It is commonly used in describing properties of composite materials. For example, the material to create a pair of skis is actually a composite of two different materials. Separately, neither material would be good in making skis. When their properties are combined they form a new material, which is desirable to make skis.

5.2.1 Heat Conductivity Example

Homogenization can help determine the heat conductivity of a composite material. By blending together the properties of each material, the temperature at a point in the material can be calculated by using differential equations. For example, the equation

$$\frac{d}{dx}(k(x)\frac{dT}{dx}) = f(x)$$

describes a heat equation for a material made up of two distinct heterogeneous materials, M_1 and M_2, that, when combined in the manufacturing process, have heat conductivities of both k_1 and k_2. The large-scale temperature is dependent on the temperature conductivities of the individual materials that make up the composite on the small scale. To calculate the temperature of a point on the combined, or homogenized, material, the heat properties of M_1 and M_2 need to be taken into account. A length L of the homogenized material consists of alternating strips of M_1 and M_2 which form a repeating pattern with period ε unit throughout L. Each period contains a strip of M_1 and M_2 whose lengths are $\varepsilon \Theta_1$ and $\varepsilon \Theta_2 = (1-\Theta_1) \varepsilon$, respectively. See Fig 5.2 for an illustration.
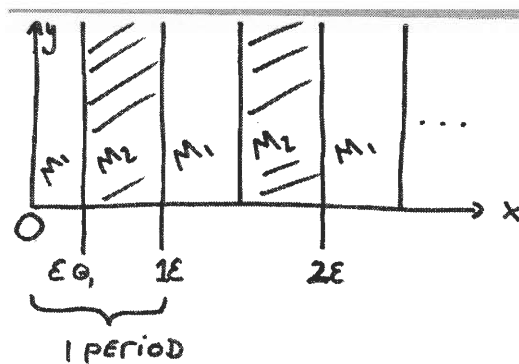
Fig 5.2 Breakdown of the composite on the small scale.

Because of periodicity, the temperature of a point is almost the same in each

succeeding ε as long as the location of a point within ε is the same. So assume that a

period is on the interval [0,1] and test the temperature of point y within those bounds.

The temperature of a point is directly related to which strip of material it is located in,

and the strip's conductivity. The conductivity of heat for each strip is determined by the

material's coefficient of conductivity, or k, where $k(y) = \begin{cases} k_1 : 0 < y < \Theta_1 \\ k_2 : \Theta_1 < y < 1 \end{cases}$. If we

multiply the period by ε, where $\varepsilon \ll 1$, we now get a material that, on the small scale,

resembles Fig 5.3.

$\square = M_1$

$\boxtimes = M_2$
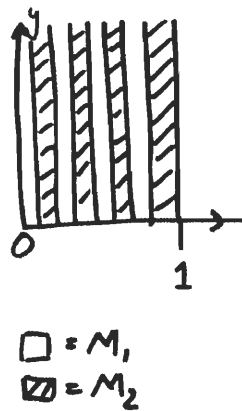
Fig 5.3 same length L, same period interval [0,1], but now illustrate the effects of $\varepsilon \ll 1$

The conductivity coefficient of a point x on the macro scale is determined by its

position on the macro scale, $n\varepsilon$, and its location within the period, εy. Thus, the

equation looks like: $x = n\varepsilon + \varepsilon y$. Next, divide both sides by ε and isolate y. Because

of periodicity, we can ignore n, resulting in $y = \dfrac{x}{\varepsilon}$. Through substitution, we now have k

in terms of x and ε. So, now the original heat equation can be written as

$$\frac{d}{dx}\left(k\left(\frac{x}{\varepsilon}\right)\frac{dT^{\varepsilon}}{dx}\right) = f(x)$$

where T^{ε} = temperature, and f(x) represents heat sources. Now we will work towards solving for the temperature at a random point.

5.2.1.1 Solving for Temperature

Step 1: We will first expand T^{ε} by the multiples scale method. T^{ε} describes the temperature at a point. The temperature depends on the point's location on the macro scale (represented in $\varepsilon^n T_n(x, y)$ by x) and also on the micro scale (represented by y). Because y is periodic, T_n is periodic in $y = \frac{x}{\varepsilon}$, having a period of length1. We consider a two-scale expansion of T^{ε}:

$$T^{\varepsilon}(x) = \varepsilon^0 T_0(x, y) + \varepsilon^1 T_1(x, y) + \varepsilon^2 T_2(x, y) + \ldots + \varepsilon^n T_n(x, y) + \ldots$$

Step 2: Next, take the total derivative of $T^{\varepsilon}(x)$, noting that $y = \frac{x}{\varepsilon}$

$$\frac{d}{dx}T^{\varepsilon}(x) = \varepsilon^0\left(\frac{\partial T_0}{\partial x} + \frac{\partial T_0}{\partial y}\frac{1}{\varepsilon}\right) + \varepsilon^1\left(\frac{\partial T_1}{\partial x} + \frac{\partial T_1}{\partial y}\frac{1}{\varepsilon}\right) + \varepsilon^2\left(\frac{\partial T_2}{\partial x} + \frac{\partial T_2}{\partial y}\frac{1}{\varepsilon}\right) + \ldots + \varepsilon^n\left(\frac{\partial T_n}{\partial x} + \frac{\partial T_n}{\partial y}\frac{1}{\varepsilon}\right) + \ldots$$

Step 3: Multiply the total derivative obtained in Step 2 by k, the conductivity k(y). The result is equal to the inner parenthesis of the heat conductivity equation.

$$k\left(\frac{dT^{\varepsilon}}{dx}\right) = k\left[\frac{1}{\varepsilon}\frac{\partial T_0}{\partial y} + \frac{\partial T_0}{\partial x} + \frac{\partial T_1}{\partial y} + \varepsilon\left(\frac{\partial T_1}{\partial x} + \frac{\partial T_2}{\partial y}\right) + \varepsilon^2\left(\frac{\partial T_2}{\partial x} + \ldots\right) + \ldots\right]$$

Step 4: Now take the total derivative of Step 3.

$$f = \frac{d}{dx}(k(\frac{dT^{\varepsilon}}{dx})) = \frac{1}{\varepsilon}\frac{\partial}{\partial x}(k\frac{\partial T_0}{\partial y}) + \frac{1}{\varepsilon^2}\frac{\partial}{\partial y}(k\frac{\partial T_0}{\partial y}) + \frac{\partial}{\partial x}(k(\frac{\partial T_0}{\partial x} + \frac{\partial T_1}{\partial y})) +$$

$$\frac{1}{\varepsilon}\frac{\partial}{\partial y}(k(\frac{\partial T_0}{\partial x} + \frac{\partial T_1}{\partial y})) + \varepsilon\frac{\partial}{\partial x}(k(\frac{\partial T_1}{\partial x} + \frac{\partial T_2}{\partial y})) + \frac{\partial}{\partial y}(k(\frac{\partial T_1}{\partial x} + \frac{\partial T_2}{\partial y})) + ...$$

Step 5: From the expanded equation obtained in Step 4, group together powers of

ε. The first three equations are:

$$1. \frac{\partial}{\partial y}(k\frac{\partial T_0}{\partial y}) = 0$$

$$2. \frac{\partial}{\partial x}(k\frac{\partial T_0}{\partial y}) + \frac{\partial}{\partial y}(k\frac{\partial T_0}{\partial x} + \frac{\partial T_1}{\partial y}) = 0$$

$$3. \frac{\partial}{\partial x}(k(\frac{\partial T_0}{\partial x} + \frac{\partial T_1}{\partial y})) + \frac{\partial}{\partial y}(k(\frac{\partial T_1}{\partial x} + \frac{\partial T_2}{\partial y})) = f$$

Equation (1) is of power $\frac{1}{\varepsilon^2}$ and equation (2) is of power $\frac{1}{\varepsilon}$. Both have partial

derivatives zero. Equation (3) is of power 1, and is equal to f. From these three equations

we can obtain three results that help solve the heat conductivity problem:

$$1. T_0 = T_0(x)$$

$$2. T_1(x, y) = w(y)\frac{\partial T_0(x)}{\partial x}$$

$$3. \frac{\partial}{\partial x}(\bar{k}\frac{\partial T_0}{\partial x}) = f$$

We'll now solve Equations (1), (2), and (3) and prove results 1,2, and 3.

5.2.1.2 Solving Equation 1

Step 1: Because $\frac{\partial}{\partial y}(k\frac{\partial T_0}{\partial y}) = 0$, $k\frac{\partial T_0}{\partial y}$ is only dependent on values of x, so

$$k\frac{\partial T_0}{\partial y} = s(x)$$

Step 2: After integrating both sides of Step 1 with respect to y (to remove the derivative), both sides are divided by k, so that only the temperature equation is on the left.

$$T_0(x, y) = \frac{1}{k} ys(x) + c$$
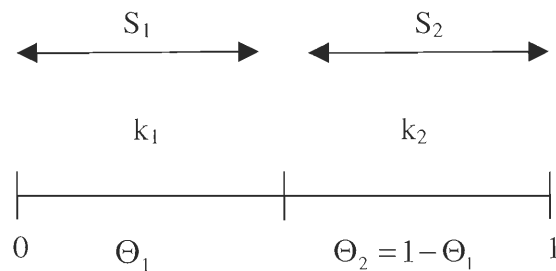
Fig 5.4 A Micro scale model of the material in question.

Here is a micro scale model of the material in question. Material 1, with conductivity k_1, is on the interval $[0, \Theta_1]$. Material 2, with conductivity k_2, is on the interval $[\Theta_1, 1]$. S_1 and S_2 represent T_0 in material M_1 and M_2 respectively. They are equal to each other at P because, when approaching from the left, the temperature must equal the temperature as approached from the right.

Step 3: We find T_0 for S_1 and S_2 by applying the equation in Step 3 to each material.

$$S_1 = \frac{1}{k_1}(ys(x) + c_1) \quad , \quad S_2 = \frac{1}{k_2}(ys(x) + c_2)$$

Next, substitute $y = \Theta_1$ into the respected equations above, and because S_1 and S_2 are equal, we get the following:

$$\frac{\Theta_1 s(x)}{k_1} + \frac{c_1}{k_1} = \frac{\Theta_2 s(x)}{k_2} + \frac{c_2}{k_2}$$

Step 4: Now we take T_0 at the extreme points:

$$T_0(0) = \frac{c_1}{k_1}$$

$$T_0(1) = \frac{s(x) + c_2}{k_2}$$

And because of periodicity, $\dfrac{c_1}{k_1} = \dfrac{s(x) + c_2}{k_2}$

Step 5: Subtract the equation in Step 4 from the final equation in Step 3 to obtain:

$$s(x)\left[\frac{\Theta_1}{k_1} + \frac{1 - \Theta_2}{k_2}\right] = 0$$

Because $\Theta_1, \Theta_2 \neq 0$ and $k_1, k_2 \neq 0$, s(x) must equal zero.

Step 6: Finally, substitute s(x)=0 into Step 4 to get $\dfrac{c_1}{k_1} = \dfrac{c_2}{k_2}$. Do the same for Step 5 and

you will also get $\dfrac{c_1}{k_1} = \dfrac{c_2}{k_2}$. Therefore, $T_0 = T_0(x)$.

5.2.1.3 Solving Equation 2

Step 1: We start with $\dfrac{\partial}{\partial x}(k\dfrac{\partial T_0}{\partial y}) + \dfrac{\partial}{\partial y}(k\dfrac{\partial T_0}{\partial x} + \dfrac{\partial T_1}{\partial y}) = 0$. Because of Equation 2,

we simplify down to $\dfrac{\partial}{\partial y}(k\dfrac{\partial T_0}{\partial x} + \dfrac{\partial T_1}{\partial y}) = 0$

Step 2: $T_1(x, y) = \dfrac{\partial T_0}{\partial x} w(y)$, where w, a periodic function, can be solved for in

$$\frac{\partial}{\partial y}\left(k(1 + \frac{\partial w(y)}{\partial y})\right) = 0$$

5.2.1.4 Solving Equation 3

Step 1: Begin with $\frac{\partial}{\partial x}(k(\frac{\partial T_0}{\partial x}+\frac{\partial T_1}{\partial y}))+\frac{\partial}{\partial y}(k(\frac{\partial T_1}{\partial x}+\frac{\partial T_2}{\partial y}))=f$. First, integrate,

from 0 to 1, with respect to y: $\int_0^1 \frac{\partial}{\partial x}\left(k(\frac{\partial T_0}{\partial x}+\frac{\partial T_1}{\partial y})\right)dy=f$

Step 2: Since we integrate with respect to y, $\frac{\partial}{\partial x}$ can be moved outside the

integral. Substitute for T_1 (from Equation 1) and factor out a $\frac{\partial T_0}{\partial x}$ to get

$$\frac{\partial}{\partial x}\left(\frac{\partial T_0}{\partial x}\left(\int_0^1 k(1+\frac{\partial w}{\partial y})dy\right)\right)=0.$$

Step 3: Denote $\int_0^1 k(1+\frac{\partial w}{\partial y})dy=\overline{k}$. We obtain $k=\dfrac{1}{\dfrac{\Theta_1}{k_1}+\dfrac{\Theta_2}{k_2}}$ by solving the

problem for w. Substitute back into Step 2 so that $\frac{\partial}{\partial x}\left(\overline{k}\frac{\partial T_0}{\partial x}\right)=f$.

5.3 From Equations to Painting

It is doubtful that Dalí was familiar with differential equations and homogenization at the time of him painting Fig 5.1. However, we were inspired by the painting to find a mathematical reason why there appear two different images at different distances. In doing so, we were led to a problem involving finding the temperature of a point in a composite material. On the microscopic level, the temperature of a point is dependent on the conductivity $k_1,..,k_n$ of the material that the point resides in. However, the composite material comes together and forms a homogenized material on the macroscopic level. This homogeneous material has a conductivity $\overline{k}$, whose value is a

function of the conductivity values of the material that make up the homogenized material.

Why we see two images at different distances can be explained in a similar fashion. On the microscopic level, the different colored cubes are like the conductivities for the different materials. The heterogeneous image at this level is Gala overlooking the sea. As we step further away from the painting, the painting takes on a new form. The image of Abraham Lincoln comes together because Lincoln's colors, like $\overline{k}$, are dependent on the colors on the microscopic level. At twenty meters, the painting now takes on new properties, like the homogenized material, and an image of Lincoln can be seen.

Conclusion

Dalí created new designs from his artistic ideas. From 1945, with the drop of the atomic bomb, to 1985, when his last painting was painted, Salvador Dalí was known to have been interested in contemporary math and science. Principles in geometry and optics allowed Dalí to both simulate and achieve a third-dimension in such paintings as *Gala's Christ* and *The Invisible Man*. Optics, mainly properties of reflection, also explains the workings of anamorphic art. Even though the female figure in *Topological Contortion of a Female Figure* is not truly topologically equivalent to a human female, Dalí borrowed the deformation ideas of topology, a branch of geometry. It is highly unlikely that Dalí was familiar with differential equations and homogenization, but they help explain why two different images are visible at different distances in *Gala Contemplating the Mediterranean Sea Which at Twenty Meters Becomes the Portrait of Abraham Lincoln – Homage to Rothko*. While it may not be apparent at first glance, the influence of mathematics is present in the art of Dalí.

A1 - Setup of Gala's Christ at the Wadsworth Atheneum
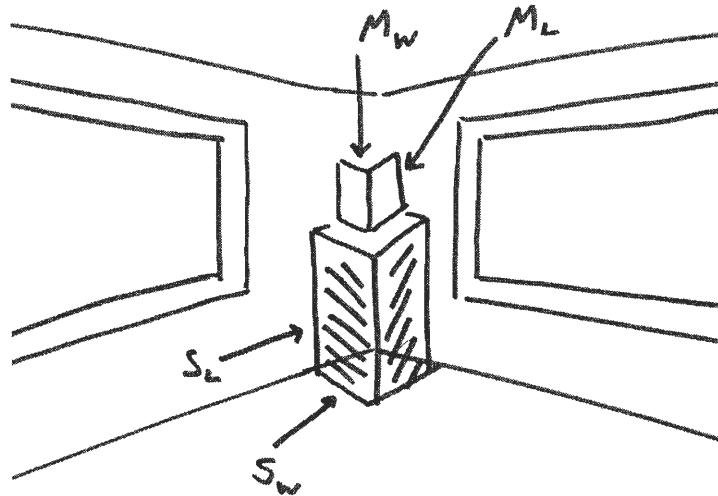
Fig A1.1 *Gala's Christ* from a front view. M_w and M_L are measurements for the width and length of the mirror.
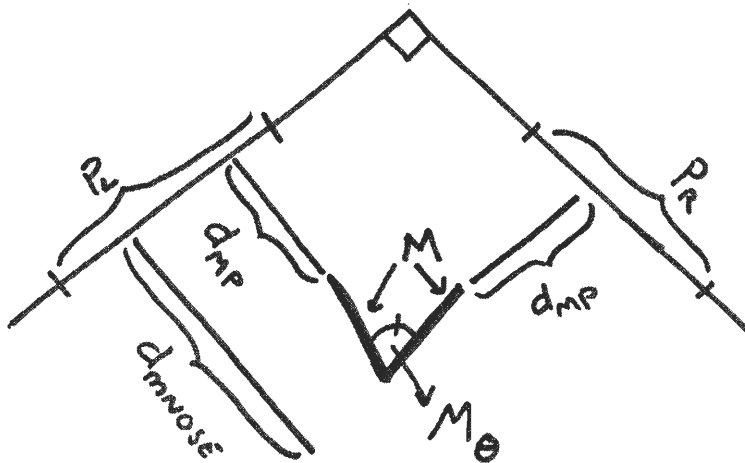
Fig A1.2 *Gala's Christ* setup, top view. P_L and P_R are the left and right components of the painting. M = mirror, and M_θ is the angle the two mirrors form. d_{MP} is the distance from the edge of the mirror to the painting, while d_{MNose} is the distance from the painting to the corner of the two mirrors, where one places their nose to see the illusion.

P_L	100cm
P_R	100cm
M_L	89cm
M_W	107cm
M_θ	45°
S_L	152cm
S_W	
d_{MP}	20cm
d_{Mnose}	152cm

Table A1.1 Dimensions for the setup of *Gala's Painting*. All measurements are approximate.

A3 – Homemade Stereoscopic Photos

Fig A3.1 Jeep situated in the woods (left & right components).

Fig A3.2 Alden Hall, WPI (left & right components).

Fig A3.3 The Duque family van and garage (left & right components).

A3 - How to Make a Homemade Anamorphoscope

The homemade viewer should have a base that fits within the circle in the picture, but one with a larger or smaller base will still work. A soup can with the label and glue removed is one possible viewer. Other viable viewers include chromium-plated tubing and Mylar paper taped around a cylinder. Mylar paper is a good choice to make a conical viewer. To do this, cut out a circular piece of Mylar paper and then cut into it a radius. Overlap the cut edges until desired base size is achieved, and then tape or glue the edges. The viewable image for a conical anamorphic picture is seen from above with one eye.

A4 – Anamorphic Maple Code

```
> with(plots):
(a,b,0) is the reflected image location
Given:   (xI,yI,zI)  the location of the eye
Goal:    Find the location of the object (xO,yO,0)


> a:='a': b:='b': R:=1; xI:=-1: yI:=1:zI:=15:
  image:=[(a,b,0)]; eye:=[(xI,yI,zI)];curvature:=1/R;
```
Must have (a^2 + b^2)<R^2 and b<0
```
> p1:=implicitplot3d(x^2+(y+R)^2=R^2,x=-R..R,
  y=-2*R..0,z=0..zt+0.5,axes=none,color=blue,
  scaling=constrained,thickness=1):Draw the
```
curved mirror
```
> tt:=(a,b)->max(fsolve((a+t*(xI-a))^2 + (b+t*(yI-b) +
  R)^2 = R^2,t));
```
$$tt := (a, b) \rightarrow \max(\text{fsolve}((a + t\,(xI - a))^2 + (b + t\,(yI - b) + R)^2 = R^2, t))$$
```
> xT:=(a,b)->a+tt(a,b)*(xI-a);
  yT:=(a,b)->b+tt(a,b)*(yI-b);zT:=(a,b)->0+tt(a,b)*(zI
  -0);yTT:=(a,b)->yT(a,b)+R;
```
Point of incidence with the   mirror
(x^2 +(y+R)^2)=R^2   is the   point (xT,yT,zT)
```
> C:=(a,b)-> (xT(a,b)*(xI-xT(a,b)) +
  yTT(a,b)*(yI-yT(a,b)))/sqrt((xI-xT(a,b))^2 +
  (yI-yT(a,b))^2 + (zI-zT(a,b))^2);
> #cond1:=(a,b)->C(a,b)*(XO^2 + YO^2 + zT(a,b)^2) =
  (xT(a,b)*XO + yT(a,b)*YO)^2;
```
(Condition 1) The angle between
TI and N is the same as the angle between TO and N
```
> XOO:=(a,b)->(yTT(a,b)*(xI-xT(a,b)) -
  xT(a,b)*(yI-yT(a,b)) -
  xT(a,b)*(zI-zT(a,b))*YO/zT(a,b))/(-yTT(a,b)*(zI-zT(a
  ,b))/zT(a,b));
```
(Condition 2) Vectors TI, N and TO are in the same
plane.
```
> condO:=(a,b)->C(a,b)*sqrt(XOO(a,b)^2 + YO^2 +
  zT(a,b)^2) = (xT(a,b)*XOO(a,b) + yTT(a,b)*YO);
```
Solve
conditions (1) and (2) = condO to find YO=yo-yT ---- TO=(XO,YO,-zT) =
(xo-xT,yo-yT,0-zT)) # XO=(xO-xT); YO=(yO-yT);

```
> rp:=(a,b)->fsolve(condO(a,b),YO);rp(-0.5,-0.2);
```
```
> yO:=(a,b)->rp(a,b);  Pick yO > 0, i.e. yo>yT
```
```
> #xO:=(a,b)-> (-xT(a,b)*(zI-yT(a,b))*yO(a,b) +
  zT(a,b)*(xI*yT(a,b)-xT(a,b)*yI))/(-yT(a,b)*(zI-zT(a,
  b)));
```

```
> xO:=(a,b)->(yTT(a,b)*(xI-xT(a,b)) -
  xT(a,b)*(yI-yT(a,b)) -
  xT(a,b)*(zI-zT(a,b))*yO(a,b)/zT(a,b))/(-yTT
  (a,b)*(zI-zT(a,b))/zT(a,b));
```
```
> yo:=(a,b)->yO(a,b)+yT(a,b);xo:=(a,b)->xO(a,b)+xT(a,b
  );
```

```
> c:=-0.5; d:=-0.2;  (c^2 + (d+R)^2);R^2;Must
  have (c^2 + (d+R)^2)<R^2 and must have d<0 and must have c
  between -R/2 and R/2
```

```
> xt:=xT(c,d);yt:=yT(c,d);zt:=zT(c,d);xx:=xo(
  c,d);yy:=yo(c,d);(xx^2 + (yy+R)^2);
```
```
> p2:=spacecurve([c+t*(xt-c),d+t*(yt-d),t*zt,
  t=0..1],color=green,thickness=1):Draw the ray
  connecting Image to lens
```
```
> p3:=spacecurve([xI+t*(xt-xI),yI+t*(yt-yI),z
  I+t*(zt-zI),t=0..1],color=green,thickness=1
  ):Draw the ray connecting Lens to Eye = TI
```
```
> p4:=spacecurve([xx+t*(xt-xx),yy+t*(yt-yy),t
  *zt,t=0..1],thickness=1,color=red,axes=norm
  al): Draw the ray connecting Lens to Object = TO
```
```
> pn:=spacecurve([xt+t*xt,yt+t*(yt+R),zt,t=0.
  .0.2],thickness=1,color=black):
```

```
> #display(p1,p2,p3,pn,p4);   Draw lens and rays

> (xt*(xI-xt)+(yt+R)*(yI-yt))/sqrt((xI-xt)^2+
  (yI-yt)^2+(zI-zt)^2);
> (xt*(xx-xt)+(yt+R)*(yy-yt))/sqrt((xx-xt)^2+
  (yy-yt)^2+(0-zt)^2);
> c1:=0; d1:=-0.85; (c1^2+(d1+R)^2);
  c2:=-.20; d2:=-1.05;(c2^2+(d2+R)^2);
  c3:=0.20; d3:=-1.05;(c3^2+(d3+R)^2);
> m:=25; for j from 0 to m do

  reflectFaceX[j]:=0.75*R*cos(2*Pi*j/m):
  reflectFaceY[j]:=0.75*R*sin(2*Pi*j/m)-R:
  origFaceX[j]:=xo(reflectFaceX[j],reflectFac
  eY[j]):
  origFaceY[j]:=yo(reflectFaceX[j],reflectFac
  eY[j]):

  reflectLeftEyeX[j]:=(1/8)*R*cos(2*Pi*j/m)-(
  1/3):
  reflectLeftEyeY[j]:=(1/8)*R*sin(2*Pi*j/m)-(
  R-(1/3)):
  origLeftEyeX[j]:=xo(reflectLeftEyeX[j],refl
  ectLeftEyeY[j]):
  origLeftEyeY[j]:=yo(reflectLeftEyeX[j],refl
  ectLeftEyeY[j]):

  reflectRightEyeX[j]:=(1/8)*R*cos(Pi*j/m)+(1
  /3):
  reflectRightEyeY[j]:=(1/8)*R*sin(Pi*j/m)-(R
```

```
-(1/4)):
origRightEyeX[j]:=xo(reflectRightEyeX[j],re
flectRightEyeY[j]):
origRightEyeY[j]:=yo(reflectRightEyeX[j],re
flectRightEyeY[j]):

XR1[j]:=c1+(j/m)*(c2-c1):
YR1[j]:=d1+(j/m)*(d2-d1):
XJ1[j]:=xo(XR1[j],YR1[j]):
YJ1[j]:=yo(XR1[j],YR1[j]):

XR2[j]:=c1+(j/m)*(c3-c1):
YR2[j]:=d1+(j/m)*(d3-d1):
XJ2[j]:=xo(XR2[j],YR2[j]):
YJ2[j]:=yo(XR2[j],YR2[j]):

XR3[j]:=c2+(j/m)*(c3-c2):
YR3[j]:=d2+(j/m)*(d3-d2):
XJ3[j]:=xo(XR3[j],YR3[j]):
YJ3[j]:=yo(XR3[j],YR3[j]):

od:
> faceOrig:=plot([[ origFaceX[q],
  origFaceY[q]]$q=0..m],style=line,
  color=red,thickness=2): # actual object
> faceRef:=plot([[ reflectFaceX[q],
  reflectFaceY[q]]$q=0..m],style=line,
  color=black): # mirror image
> leftEyeOrig:=plot([[ origLeftEyeX[q],
  origLeftEyeY[q]]$q=0..m],style=line,
```

```
     color=red,thickness=2):
>  leftEyeRef:=plot([[ reflectLeftEyeX[q],
   reflectLeftEyeY[q]]$q=0..m],style=line,
   color=black):
>  rightEyeOrig:=plot([[ origRightEyeX[q],
   origRightEyeY[q]]$q=0..m],style=line,
   color=red,thickness=2):
>  rightEyeRef:=plot([[ reflectRightEyeX[q],
   reflectRightEyeY[q]]$q=0..m],style=line,
   color=black):
>  smileOrig:=plot([[ origSmileX[q],
   -origSmileY[q]-2.2]$q=0..m],style=line,
   color=red,thickness=2):
>  smileRef:=plot([[ reflectSmileX[q],
   -reflectSmileY[q]-2.2]$q=0..m],style=line,
   color=black):
>
>  v1:=plot([[ XJ1[q],
   YJ1[q]]$q=0..m],style=line, color=red): #
   actual object
>  v1r:=plot([[ XR1[q],
   YR1[q]]$q=0..m],style=point, color=black):
   # mirror image
>  v2:=plot([[ XJ2[q],
   YJ2[q]]$q=0..m],style=line, color=red): #
   actual object
>  v2r:=plot([[ XR2[q],
   YR2[q]]$q=0..m],style=point, color=black):
   # mirror image
>  v3:=plot([[ XJ3[q],
```

```
  YJ3[q]]$q=0..m],style=line, color=red): #
  actual object
> v3r:=plot([[ XR3[q],
  YR3[q]]$q=0..m],style=point, color=black):
  # mirror image
>

>

> lens2d:=implicitplot(x^2+(y+R)^2=R^2,x=-R..
  R,y=-2*R..0,axes=normal,color=blue,scaling=
  constrained,thickness=1): #Draw the curved
  mirror
> display(faceOrig,faceRef,leftEyeOrig,leftEy
  eRef,rightEyeOrig,rightEyeRef,v1r,v1,v2r,v2
  ,v3r,v3, lens2d); #eye at (-1,1,15)
>

>
```
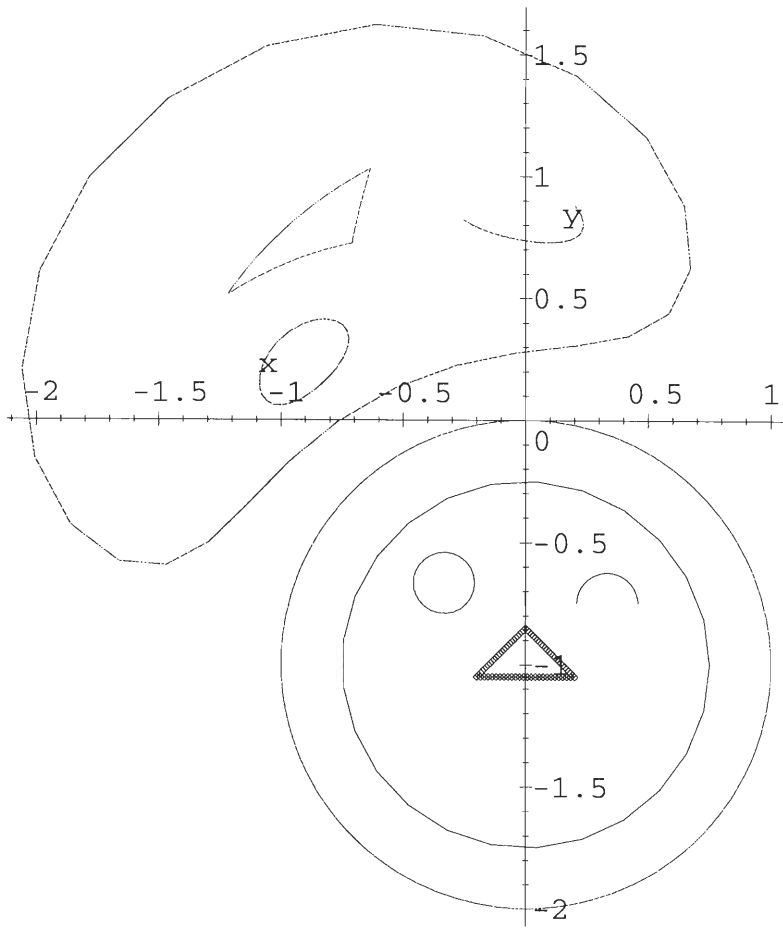
Fig A4.1 Anamorphic Maple Face. Place the viewer
on the circle centered at (0,1) to view the image.

>
>

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