Worcester Polytechnic Institute

ASSISTments Interactive Qualifying Project

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Abstract

ASSISTments is an online tool for teachers around the world to assign schoolwork and assist in student learning in multiple fields. We built content for six Common Core skills new to ASSISTments: absolute value equation solving, logarithmic equation solving, matrix multiplication, matrix addition, complex numbers, and exponential equations. We were able to run 4 different experiments with these skills: adjusting the number of hints provided, varying the number of problems students are required to complete, presenting struggling students with prerequisite content, and examining the difference between theoretical and contextual examples. Our goal in running these experiments is to determine strategies which improve students' understanding of the material and transference of the skill.

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Introduction

General Terminology

For the purposes of this experiment we will be using the ASSISTments educational software. It is a way for teachers to provide their students with problem sets on the taught topics. We would like to define the following terms that will be used throughout this paper:

- Variabilized Template (Template): It is a specific way to write a problem where variables are used instead of actual numbers or words. Writing problems in this format allows the software to create multiple versions of the problem.
- Instantiated Problem: It is one specific problem that has been created by Assistments from one variabilized template. One template will be usually able to produce multiple instantiated problems.
- Problem Set: It is a combination of selected problems that will appear to the student in a specific way. There are many options for building problems sets including if-then-else statements, random choose commands, complete all problems, Skill builders and others.
- Skill builder: This type of problem set (or section in a problem set) asks for students to succeed in a selected number of questions in a row. We make a list of multiple (50 or more problems are usually created) problems (instantiated problems coming from the same or similar templates), and the skill builder option will display questions to the student until they solve a number X in row. When they manage to solve the required number of questions in a row, the skill builder is completed.
- Experiment: Assistments gives us the chance to make complex problem sets which allow for results to be extracted regarding the student learning process. A simple example is to randomly give students questions from two different groups of problems, and then compare which question group was more successful.

Building our Experiments

During our time with ASSISTments, we built out six problem sets of new content in order to conduct experiments on students' learning strategies. All of our content filled gaps in current material required for ASSISTments' full coverage of the Common Core standard. The skills we

built out were as follows: absolute value equation solving, logarithmic equation solving, matrix multiplication, matrix addition, complex numbers, and exponential equations.

For each skill, we created multiple variablized templates to capture the entirety of the skill and add variety. We could then generate as many problems as necessary to conduct each experiment. When creating the experimental problem sets, we used these generated problems in different ways to test differing learning strategies.

In absolute value and logarithmic equation solving, we wanted to discover how many problems in a row would demonstrate proficiency in a skill. For matrix multiplication and complex numbers, we wanted to see if presenting struggling students with prerequisite skills would improve their ability to complete the content. For exponential equation solving we wanted to see if splitting up large hints helped or hindered student learning. Finally, for matrix addition we wanted to test whether contextual examples helped students understand content better than classical mathematically-styled examples.

Experiments

We conducted multiple experiments during our IQP Term, and were able to collect data on and analyze two in particular: The effectiveness of presenting prerequisite content to struggling students, and exploring the implications of presenting students with variable amounts of problems. We created four unique studies overall, but two of them have not yet collected enough data for us to analyze. The bulk of this paper will analyze two out the four experiments that we created which gathered data. Experimental designs of the two experiments that did not collect data are given below.

Generalized vs Contextual Learning
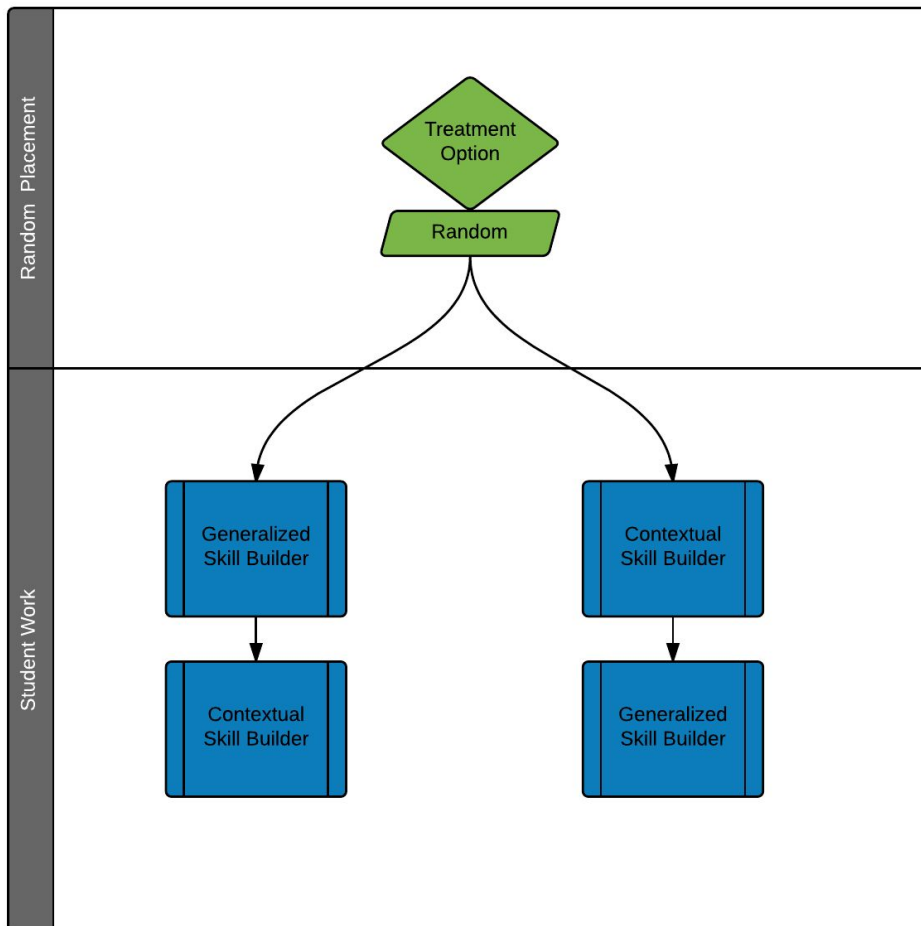
Introduction to this Experiment

Purpose

The goal of this experiment is to test the ordering effect of presenting a student with generalized or contextual examples in skill builders. Students will either be presented with real world matrix addition questions or more typical textbook problems. Generalized problems use standard matrix notation and terminology, whereas contextual examples populate matrices with data and give it meaning labels. For example, a matrix of cab fares per day and time or basketball scores per team and game. After completing the first set of problems, the student will be entered into a skill builder of the remaining skill; if the student completed the theoretical

problems they will receive the real world problems, and if they completed the real world problems they will receive the theoretical problems. This is an attempt to see which teaching strategy offers the most transferable skill. We hypothesize that, when this experiment gathers data, presenting the more difficult real world examples will transfer better than the theoretical examples.

Content Selection

Matrix addition was chosen as a skill to test this hypothesis because of it's unary nature. Once students understand that adding or subtracting two matrices is just adding or subtracting their corresponding positions, they are likely to not make many mistakes. Because of this, we will examine how the skill's presentation effects its transfer by comparing it with a different presentation of the same skill. Five skill builders were selected to accomplish this task, some with contextual matrices, and some with generalized matrices. Below are the templates we created for this skill:

Template 1 (theory) - PRA7UJW	
Template 2 (theory) - PRA8CEW	

Template 3 (theory) - PRA8CE6

Problem ID: **PRA8CE6**   *Comment on this problem*

Submit C_{41} from the difference of the following matrices.

$$\begin{vmatrix} 3 & 17 \\ 12 & 14 \\ 6 & 16 \\ 1 & 7 \end{vmatrix} - \begin{vmatrix} 10 & 7 \\ 5 & 1 \\ 17 & 16 \\ 3 & 14 \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{vmatrix}$$

Type your answer below (mathematical expression):

[   ]   100% ⑦

[Submit Answer]   [Show hint 1 of 2]

Template 4 (real-world) - PRA8VZK

Problem ID: **PRA8VZK**   *Comment on this problem*

In total, how much did Sam and Nick spend on Breakfast on Monday?

Sam

	Monday	Tuesday	Wednesday	Thursday	Friday
Breakfast	$10.00	$14.00	$1.00	$2.00	$2.00
Lunch	$6.00	$8.00	$7.00	$12.00	$15.00
Dinner	$13.00	$8.00	$11.00	$6.00	$11.00

Nick

	Monday	Tuesday	Wednesday	Thursday	Friday
Breakfast	$8.00	$9.00	$1.00	$5.00	$12.00
Lunch	$12.00	$6.00	$8.00	$15.00	$3.00
Dinner	$3.00	$11.00	$5.00	$7.00	$13.00

Type your answer below (mathematical expression):

[   ]   100% ⑦

[Submit Answer]   [Show hint 1 of 2]

Template 5 (real-world) - PRA8XSX

Problem ID: *PRA8XSX*   Comment on this problem

How many more points did Team A score in Game 2's third quarter than Team B in Game 1's third quarter?

Game 1

	Team A	Team B
First Quarter	21 pts	9 pts
Second Quarter	14 pts	3 pts
Third Quarter	15 pts	7 pts
Fourth Quarter	15 pts	7 pts

Game 2

	Team A	Team B
First Quarter	18 pts	7 pts
Second Quarter	13 pts	4 pts
Third Quarter	19 pts	3 pts
Fourth Quarter	12 pts	2 pts

Type your answer below (mathematical expression):

[   ]   100% ?

Submit Answer   Show hint 1 of 2

Template 6 (real-world) - PRA85CZ

Problem ID: *PRA85CZ*   Comment on this problem

How much does it cost to take the Subway 10 miles to work in the morning and the Cab 10 miles back in the afternoon on Thursday?

Cab Fare for a 10 Mile Trip

	Monday	Tuesday	Wednesday	Thursday	Friday
Morning	$16.00	$19.00	$19.00	$18.00	$15.00
Afternoon	$15.00	$17.00	$15.00	$15.00	$16.00

Subway Fare for a 10 Mile Trip

	Monday	Tuesday	Wednesday	Thursday	Friday
Morning	$4.00	$4.00	$6.00	$4.00	$5.00
Afternoon	$4.00	$6.00	$4.00	$2.00	$5.00

Type your answer below (mathematical expression):

[   ]   100% ?

Submit Answer   Show hint 1 of 2

Variable Number of Hints

This experiment dealt with the number of hints given in the skill builder questions. Complicated problems were selected, that could either be dealt with in two long hints, or in five more analytical ones. The hypothesis was that fewer hints will be less discouraging for the students that look at the number of hints, in order to judge the complexity of the problem, thus leading to lower dropout rates.

Decision	Choose Condition
Student Work	Skill builder with 2 Hints Skill builder with 5 Hints
Post Test	Post test
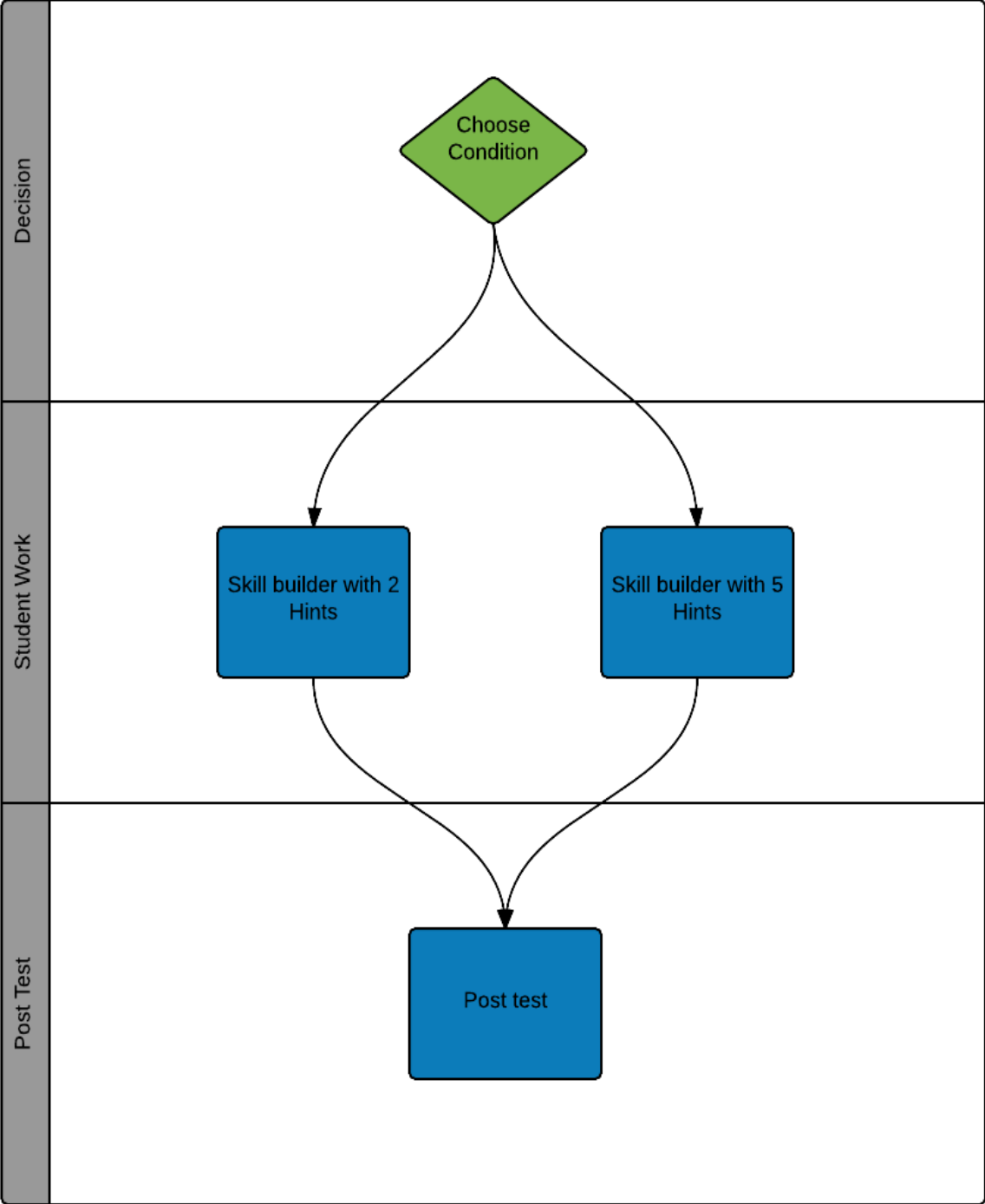
Introduction to this experiment

This study is an attempt to evaluate how the number of hints influences the student's results on a particular problem set. The problem set selected in order to carry out this study is a skill builder on exponential equations. In order to help the students navigate through the problems, hints are added, explaining the solving procedure behind the skill builder. The number of hints that appear in each problem, however, can be affecting the student's performance by itself. Inclusion of multiple hints gives the teacher the possibility to explain the solving process in a more analytical way, thus providing a better understanding to the student. However there can be a case made for the higher number of hints to be having a negative effect on performance; as it can be a factor that is discouraging the students from trying to complete the problem set overall. In this study we will create two versions of the exponential equations problem set mentioned above: the first will include two hints, and the second will include five hints. The selection of this specific skill builder was made based on the flexibility that it gives regarding the number of hints that could be written, as the solution involves manipulation of the exponents and solving of the linear equation that will arise afterwards, offering enough content for the use of five but can also be funneled into two distinct hints. Students that have been assigned this problem set will be randomly given one version or the other. The results for both versions will be gathered and compared in order to formulate results on the effect of the number of hints.

Fixed variables

This experiment is dealing with the effect that the number of hints of a problem might have on the effectiveness of assisting the student. It is important in such research to indicate the following factors which must be kept fixed in order for the results to be affected only by the number of hints.
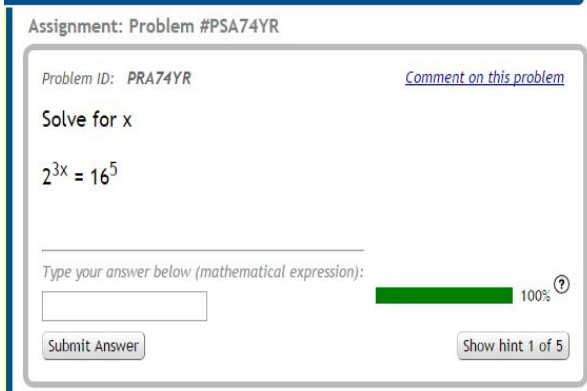
1.Content of the hints
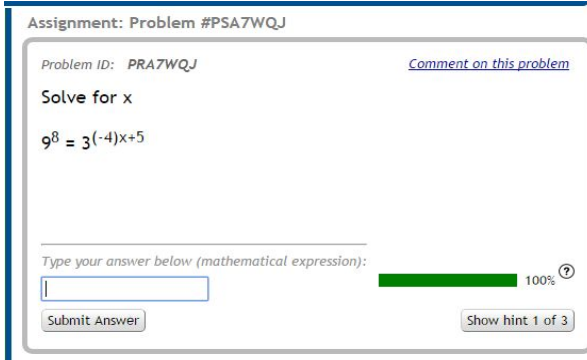
Explaining the same problem in two or five hints could cause the tutoring of the five hints to be more in-depth. This was eliminated from our experiment. In both cases we are using the exact same instructions and equations. The order is also unchanged. The only factor that is allowed to change is grouping of these instructions, creating 2-hint and 5-hint content.

2.Ordering effects

In this experiment the students are required to solve three problems in a row in order to successfully pass the problem set. All the problems that will be given are generated from templates written with variables. This means that Assistments randomly generates the problems by inputting random numbers. This way a multitude of problems is produced. This means that

each student is randomly assigned some of the 50 problems that we asked assistments to generate. This allows for some inconsistency regarding the difficulty of work. Some problems might come up to have numbers that are easier to manipulate and the problems could come in "easier" to "harder" order, which would be helpful for some students. However, the significance of these effects are minimized by the randomization process, where there are more than enough available problems. We are confident that all students will share similar difficulty from the problem set and they will need to attain a specific mastery of the skill in order to succeed.

Content selection:

Template 1	Assignment: Problem #PSA74YR Problem ID: **PRA74YR** *Comment on this problem* Solve for x $2^{3x} = 16^5$ Type your answer below (mathematical expression): [ ] 100% ? Submit Answer Show hint 1 of 5
Template 2	Assignment: Problem #PSA7WQJ Problem ID: **PRA7WQJ** *Comment on this problem* Solve for x $9^8 = 3^{(-4)x+5}$ Type your answer below (mathematical expression): [ ] 100% ? Submit Answer Show hint 1 of 3

Template 3	

In order to solve this problem students have to go through two different stages. Stage 1 is to change the base of the left hand side to a base of 3 (using the fact that 27 is a power of 3). Once they get through that step, students are required to apply the property of equality which states that the exponent of each side is equal to one another, since they have the same base. Stage 2 requires solving of the remaining linear equation. The way the **tutoring** was broken down in the two cases (five against two hints) is shown below.

Two hints

Temlpate 1

Assignment: Problem #PSA828P

Problem ID: *PRA828P*

Solve for x

$2^{7x} = 16^1$

Express 16 as a power of 2.

$16^1 = (2^4)^1$

Set the exponents to be equal for both sides and solve the equation:

$2^{7x} = 2^{4*1}$

Then,

$7x = 4*1$

Comment on this hint

$7x = 4$

$\dfrac{7x}{7} = \dfrac{4}{7}$

$x = \dfrac{4}{7}$

Type in 4 / 7

Comment on this hint

Type your answer below (mathematical expression):

Submit Answer

Template 2

Problem ID: PRA83AF

Solve for x

$$9^2 = 3^{(-2)x+7}$$

$4 = (-2)x+7$

$4-7 = (-2)x$

$(-2)x = 4-7$

$$\frac{(-2)x}{(-2)} = \frac{-3}{(-2)}$$

$$x = \frac{-3}{(-2)}$$

Type in 3 / 2.

Comment on this hint

Express 9 as a power of 3.

$$9^2 = (3^2)^2$$

Set the exponents to be equal for both sides and solve the equation:

$$3^{2*2} = 3^{(-2)x+7}$$

Then,

$$2*2 = (-2)x+7$$

Comment on this hint

Type your answer below (mathematical expression):

Submit Answer

Template 3

Problem ID: PRA829F

Solve for x

$5^{3x-5}=125^2$

Express 125 as a power of 5.

$125^2 = (5^3)^2$

Set the exponents to be equal for both sides and solve the equation:

$5^{3x-5}=5^{3*2}$

Then,

$3x-5=3*2$

Comment on this hint

$3x-5 = 6$

$3x = 6+5$

$\dfrac{3x}{3} = \dfrac{11}{3}$

$x = \dfrac{11}{3}$

Type in 11 / 3

Comment on this hint

Type your answer below (mathematical expression):

Submit Answer

Five hints

Template 1

Template 2

Problem ID: PRA83AK

Solve for x

$$81^2 = 3^{(-7)x+5}$$

Express 81 as a power of 3.

Comment on this hint

$81^2 = (3^4)^2$

Comment on this hint

Set the exponents to be equal for both sides and solve the equation:
$3^{4 \cdot 2} = 3^{(-7)x+5}$

Comment on this hint

Then,
$4 \cdot 2 = (-7)x+5$

Comment on this hint

$$8 = (-7)x+5$$

$$8-5 = (-7)x$$

$$(-7)x = 8-5$$

$$\frac{(-7)x}{(-7)} = \frac{3}{(-7)}$$

$$x = \frac{3}{(-7)}$$

Type in -3 / 7.

Comment on this hint

Type your answer below (mathematical expression):

Submit Answer

| **Template 3** | Assignment: Problem #PSA83BF |
| | |

Problem ID: PRA83BF

Solve for x

$$5^{3x-9}=625^4$$

Express 625 as a power of 5.

Comment on this hint

$625^4 = (5^4)^4$
Comment on this hint

Set the exponents to be equal for both sides and solve the equation:
$5^{3x-9}=5^{16}$

Comment on this hint

Then,
$3x-9=16$
Comment on this hint

$$3x-9 = 16$$
$$3x = 16+9$$
$$\frac{3x}{3} = \frac{25}{3}$$
$$x = \frac{25}{3}$$

Type in 25 / 3

Comment on this hint

Type your answer below (mathematical expression):

Submit Answer

This paper goes on to include further analysis of past studies that are similar to the ones that are analyzing (variable assignment length, prerequisite questions), in order to make use of newer results that were not available when the summaries were written.

Experiment 1: Variable assignment length

When students rely on ASSISTments to reinforce a skill, we use a structure called a skill builder. Skill builders provide the student with as many problems in a row as they need to get a certain number correct in a row, giving them practice and ample chances to achieve mastery of the skill. The goal of this experiment is to determine whether the amount of questions needed to escape a skill builder affects how well a student learns the material. This study is important because of the difference between high-knowledge students and low-knowledge students; high

knowledge students may become frustrated by too many questions after quickly mastering the content, where low knowledge students may be able to guess their way through two questions in a row, thus not demonstrating mastery. Our hypothesis is that the dropout rate will increase when the number of problems to demonstrate mastery increases. The most common number in ASSISTments and the default option is three, but we will look into the differences between offering two, three, four, or five questions to demonstrate mastery (escape the skill builder). We will use an absolute value equation skill builder to test this because it has many variations to eliminate guessing, and stands well on its own(doesn't have many prerequisite skills). Students will be randomly put into skill builders requiring 2,3,4, or 5 questions needed to complete, containing the same problems. Data can then be compared between each type of skill builder to determine its effect on the students.
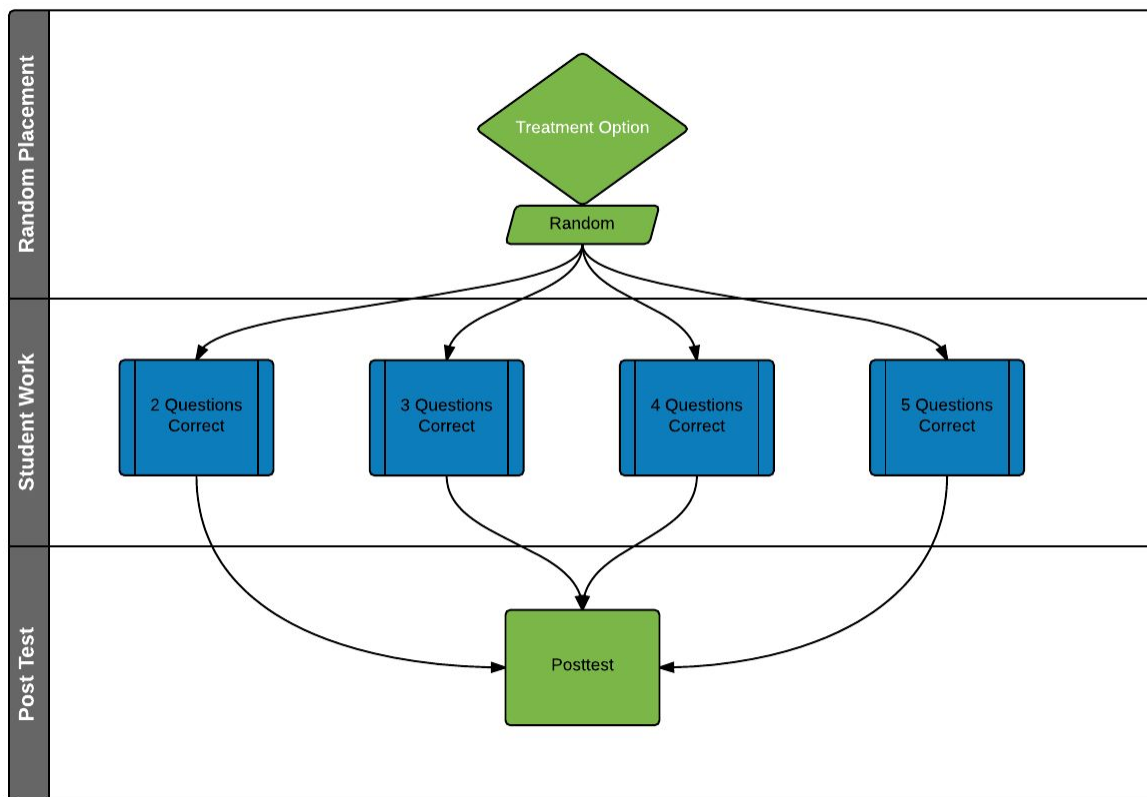
Fixed Variables

In order to ensure our results are not a confounded effect, we control many variables between the skill builders. The questions for each skill builder set are selected randomly from an identical pool, generated via templates to ensure their difficulty is the same. The presentation of the questions are identical between the skill builders, and the student does not know which pool they are in. All instructions read "This equation has two solutions [problem] Submit only the larger/smaller one" to prevent any effect of the text.

Ordering Effects

Because we are only interested in the effect of the number of questions to escape the skill builder, we control other possibly confounding variables that we cannot fix through randomness. The skill builders present questions in a random order, minimizing the impact of any ordering effects that would arise with a linear presentation. The problems themselves are also generated randomly via templates to ensure their difficulty is consistent.

Content Selection

Initial Notice

Because the students will have to answer a varying amount of problems in a row to complete the assignment, we show the following message to students before beginning the problem set:

Absolute Value

Absolute Value equations were selected for this experiment because of two reasons. The many templates that create the problems in the skill builders provide enough variety that

students will not get bored and complete the transfer item, and also similar and focused enough that only the target skill is being exercised. There are nearly identical hints for every problem detailing its solution, eliminating any chance that a student would get completely stuck. The absolute value equation sets fulfill these requirements. The templates are in a table below.

Template 1 - PRA7KZJ	
	Assignment: Problem #PSA7KZJ *Problem ID:* **PRA7KZJ** *Comment on this problem* This equation has two solutions. | 4x + 12 | = 24 Submit only the larger one. *Type your answer below (mathematical expression):* [ ] 100% ⑦ Submit Answer Show hint 1 of 2
Template 2 - PRA7PEU	
	Assignment: Problem #PSA7PEU *Problem ID:* **PRA7PEU** *Comment on this problem* This equation has two solutions. 4| x + 3 | = 20 Submit only the smaller one. *Type your answer below (mathematical expression):* [ ] 100% ⑦ Submit Answer Show hint 1 of 2
Template 3 - PRA7QSY	
	Assignment: Problem #PSA7QSY *Problem ID:* **PRA7QSY** *Comment on this problem* This equation has two solutions. -3| x + 3 | = -27 Submit only the larger one. *Type your answer below (mathematical expression):* [ ] 100% ⑦ Submit Answer Show hint 1 of 2

Template 4 - PRA7RV3	
	Assignment: Problem #PSA7RV3
	Problem ID: **PRA7RV3** *Comment on this problem*
	This equation has two solutions.
	| -3x + 12 | = 24
	Submit only the larger one.
	Type your answer below (mathematical expression): 100%
	[Submit Answer] Show hint 1 of 2

To solve these problems, students must understand when to split an absolute value equation and when to simplify it; order of operations is a key element in this skill. In this example, the equation is split immediately, and the student will then solve both sides of the equation to determine which side yields a larger value for x.

Congratulations Message

Assignment: Problem #PSA85DK

Problem ID: **PRA85DK**   *Comment on this problem*

You finished the Skill Builder. Now you will be asked to solve one follow up question.

Select one:

◯ I am ready to do my best   100%

[Submit Answer]

Transfer Item / Post-Test

The transfer question chosen as the post-test was chosen because it brings the students one step past what the skill builders exposed them to. By adding an additional constant to an

absolute value equation, we are able to test how well the content helped them learn the skill by asking them to apply it in a slightly different scenario.

Assignment: Problem #PSA8Q9C

Problem ID: PRA8Q9C   Comment on this problem

This equation has two solutions.

$3| x - 5 | + 6 = 21$

Submit only the larger.

_____

Type your answer below (mathematical expression):

[   ]   [████████████] 100% ?

[Submit Answer]   [Show hint 1 of 3]

Data Analysis

Considerations

There are a few considerations we had to make when analyzing the data. Firstly, there was a dropout effect of the five-in-a-row treatment as seen in the following table:

Conditions	Students who started the problem set	Students who finished the problem set	Percent of students completed
2 correct	44	38	86
3 correct	60	55	92
4 correct	46	45	98
5 correct	46	34	74
N	196		

Degrees Of Freedom	3		
Chi-Squared	13.4777		
P-Value	0.0037		

When analyzing the data, we had to be careful not to compare the 5 group on equal terms with the others because of the differential dropout. The dropout could have led to low-skill or time-limited students dropping the course, and skewing the results.

Additionally, on the post-test data collected for this experiment, the wrong answer and the correct answer were swapped. This meant anyone answering correctly had their score labeled incorrect, and everyone answering incorrectly answered correctly. I attempted to account for this by switching everyone's post-test scores (newscore = (1 - score)) and then performing our analysis. This may have had strange interactions with people who left the test without eventually inputting the "wrong answer", as they would not have been marked as correct.

Results

To work around these considerations, we decided to analyze the data by comparing each group to each other in order to determine whether the treatment significantly helped the students' learning. We performed a two-tailed T-Test on each treatment option to determine whether there was any effect. The data we used were the post-test scores of each completed student, taking into account partial credit.

Group	Post-test Average
2	.550
3	.470
4	.630
5	.763

T-Test Results	2	3	4	5
2	0	.3531	.3622	.0113
3	.3531	0	.0462	.0001

4	.3622	.0462	0	.0827
5	.0113	.0001	.0827	0

As seen above, between the three and four groups we did establish significance.

When not taking into account partial credit, the results look as such:

Group	Post-test Average
2	.400
3	.321
4	.478
5	.552

T-Test Results	2	3	4	5
2	0	.437	.4714	.1819
3	.437	0	.1112	.028
4	.4714	.1112	0	.5032
5	.1819	.028	.5032	0

When viewing it as a binary distribution, we can observe no significant difference between any of the two groups. In a much larger study done by Kim Kelley with over 1700 students participating, we can observe similar results without the worry of the problem having an incorrect answer, and with access to a much larger pool of data.

Conditions	Students who started the problem set	Students who finished the problem set	Percent of students completed
2 correct	421	356	85
3 correct	473	337	71

4 correct	415	293	71
5 correct	417	289	69
N	1726		
Degrees Of Freedom	3		

A difference in drop-out rate was detected for this study, with a Chi-Squared of 33.4116.

Group	Post-test Average
2	.427
3	.479
4	.481
5	.491

T-Test Results	2	3	4	5
2	0	.0829	.0886	.0442
3	.0829	0	.9531	.7036
4	.886	.9531	0	.7583
5	.0442	.7036	.7583	0

Similar to our data, the study shows that as students complete more problems their understanding of the material also increases. Specifically, 5 questions compared to 2 questions improved student post-test score by an average of .064 with a significance of p=0.0442. We can also observe from this that 3 and 4 questions in a row yielded extremely similar results, with neither of them being significantly different from 5 in a row. 2 Questions in a row yielded the lowest average score, which is likely the result of lower-skill students being able to guess through two questions in a row without having a complete understanding of the skill.

The errors in our study which could have corrupted our results have since been fixed, and we believe that when more data is available the results should reflect the results shown in Kim's study.

Experiment 2: Prerequisite questions

The goal of this experiment is to test the effectiveness of using problem sets that include problems with prerequisite material. This means that when the student seems to struggle with the questions that he is given he will be presented with some easier questions, designed for re-teaching him the necessary prerequisite material. After this inclusion, the student returns to working on the initial problems. This is an attempt to help the student succeed in the problem set and enhance his understanding of the material.

In this experiment we are trying to test this assumption. We have created two problem sets. One is dealing with content on solving equations with complex numbers, and one dealing with matrix multiplication. Both problems sets are built with the same structure. This structure is analyzed below.

Figure 1: Experiment flow diagram
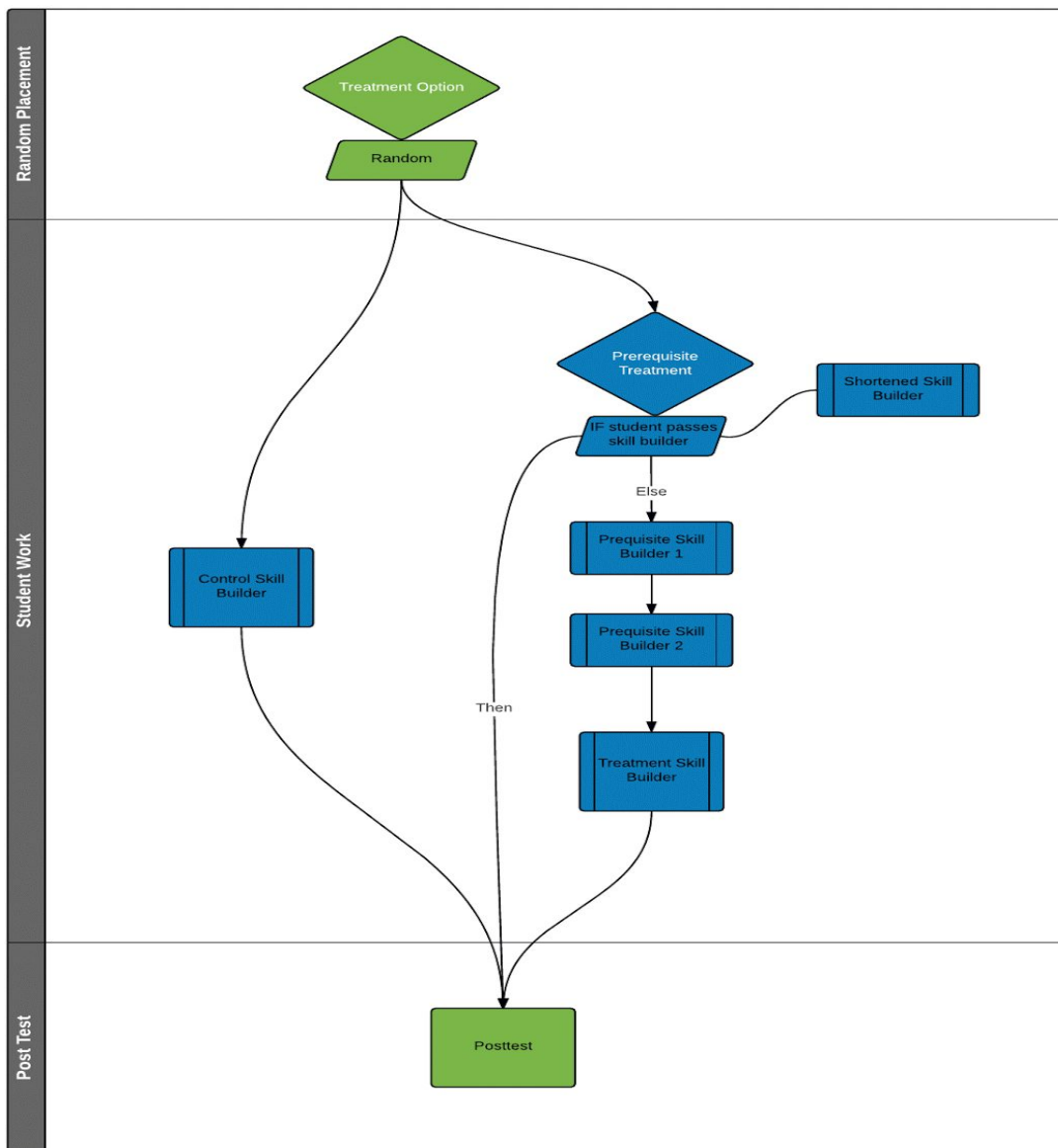
Hypotheses

1) Prerequisite questions will be helpful for the students to complete the skill builder. This would be the case for any relatively complex skill builder as the one selected, since the prerequisites are helpful to clarify the concepts that are required. This would be clearly seen in the post test scores, where the students understanding is tested more directly.

2) Prerequisite questions will prove to be more beneficial for students with weaker background. For many students it might be easy to solve the overall skill builder without need for prerequisites (where these might end up confusing or annoying students who are well prepared). For the students who seem to be struggling

though, we expect prerequisite questions to make a great difference in their performance.

Problem Set Structure

The problem set requires the students to complete all of the following sections in linear order:
1. Choose Condition Section
2. Post Test section

Choose Condition Section:

This section will randomly choose if the student will be sent to the control or the treatment group.

Students in the control group will face a normal skill builder problem set, composed of at least 50 questions on the topics mentioned above (complex equation solving or matrix multiplication). After completing the skill builder problem set (they need to get three correct questions in a row) they will move on to the post test section.

Students in the treatment group will be presented with a short skill builder problem set. It is composed of 5 problems, meaning that the student will have to solve the skill builder with a small number of tries (he will be allowed a mistake in the first two questions). This is where an if-then-else statement has been incorporated in the problem set structure. The IF condition is the completion of the short skill builder. If the student manages to solve it he goes to the THEN statement, which sends him to the post test. If he does not complete the skill builder, he is led to the ELSE statement. This is a section that requires him to complete all the following:
1. *A linear order skill builder on prerequisite material
2. The 'normal' skill builder (identical to the control skill builder).

*This linear order skill builder is composed of problems that examine the prerequisite skills. They come in alternating order depending on which skill they focus on (for example skill 1, skill 2, skill 1, skill 2, skill 1, and so forth). Students are required to get 2 correct answers in a row in order to show understanding of both skills.

Once these steps are completed the student will be sent to the post test.

Post Test

This section is designed to test the level of mastery that the student demonstrates after going through the problem set. It is composed of two problems that must be solved in linear order. These problems present the student with questions that will challenge him (on the topic that the problem set is designed for). Results from this section are important for the analysis part of this experiment.
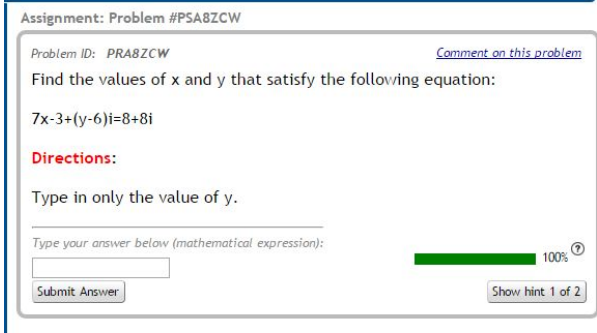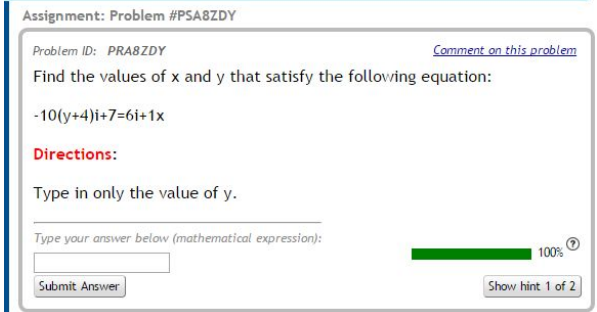
Content Selection

Complex numbers
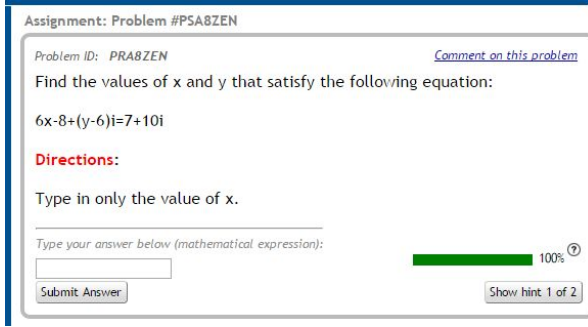
For the purposes of the experiment we used problems that require skill in at least two different sectors. We chose to use a skill builder problem set on complex number equation solving. For this case, the students will have to make use of two different skills:

1. Linear equation solving,
2. grouping of the real and imaginary parts.

In this way we were able to set up questions that allow the student to practice on the specific prerequisite skills. The types of problems that were used are shown below.

Figure 2: Complex Number Equation Solving (control skill builder)

Templates ~PRA7SBH	
~PRA8Y4N	

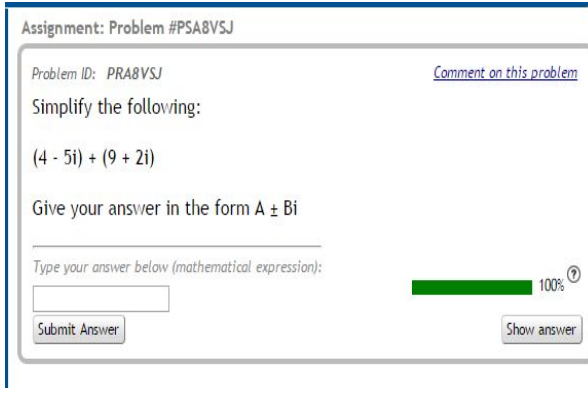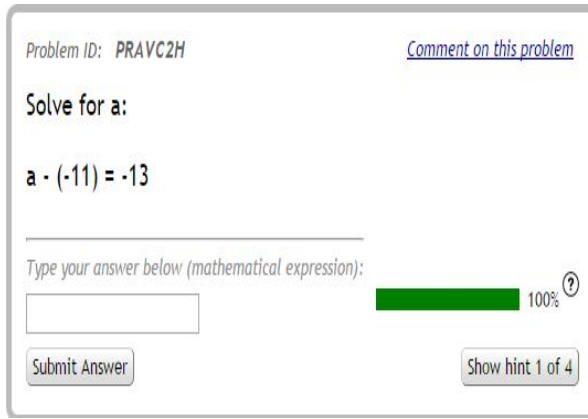| ~PRA8Y4Q | Assignment: Problem #PSA8ZEN

Problem ID: PRA8ZEN _Comment on this problem_
Find the values of x and y that satisfy the following equation:

6x-8+(y-6)i=7+10i

Directions:

Type in only the value of x.

Type your answer below (mathematical expression):

Submit Answer — 100% ⑦ — Show hint 1 of 2 |

Selected questions from this set were found in the normal skill builder, the short skill builder, and the control skill builder.

Figure 3: Prerequisite Skills for Complex Numbers (Templates)

| **Grouping~PRA8TF3**

The picture on the right displays an instantiated problem for exercising grouping of real and imaginary parts. Students are asked to complete an addition of complex, for which they need to perform the grouping of the two parts. | Assignment: Problem #PSA8VSJ

Problem ID: PRA8VSJ _Comment on this problem_
Simplify the following:

(4 - 5i) + (9 + 2i)

Give your answer in the form A ± Bi

Type your answer below (mathematical expression):

Submit Answer — 100% ⑦ — Show answer |
| **Linear Equation~PRA8ZE8**

In this case the students are asked to solve a simple linear equation. Mastery of this skill, combined with mastery of the grouping, will give them the ability to complete the control skill builder, | Problem ID: PRAVC2H _Comment on this problem_
Solve for a:

a · (-11) = -13

Type your answer below (mathematical expression):

Submit Answer — 100% ⑦ — Show hint 1 of 4 |

These questions concentrate on the two basic skills that are required for the rest of the skill builder.

Figure 4: Intermediate Messages

PRA8WAJ	
PRA8WAJ	Assignment: Problem #PSA8WAJ *Problem ID: PRA8WAJ* Comment on this problem Congratulations, you finished the skill builder. Now you will be asked to solve two follow-up questions! Select one: ○ I am ready to do my best. Submit Answer
PRA8WAK	Assignment: Problem #PSA8WAK *Problem ID: PRA8WAK* Comment on this problem Let us try to break the problem into two different components. Please complete the following skill builder with simplified questions. You need to get 2 right in a row for these questions. Select one: ○ I am ready to start! Submit Answer
PRA86C7	Assignment: Problem #PSA86C7 *Problem ID: PRA86C7* Comment on this problem Good job! Now let's try the complex number equations again. For these you need to get three right in a row. Select one: ○ I'm ready to return to the complex number equations Submit Answer

These messages are used to help students move around the problem set, informing them of what they are about to do next, as they switch from normal skill builder to prerequisite skill builder as well as the opposite, and also let them know when they have reached the post test section.

Figure 5: Post Test

Table of Post Test Questions used

PRA8US2	
PRA8US2	Assignment: Problem #PSA8US2 Problem ID: PRA8US2 Comment on this problem Solve the following equation: $5y + 12(2x - 3)i + 2 = 0$ Type in the value of x Type your answer below (mathematical expression): Submit Answer Break this problem into steps

PRA84HB	Assignment: Problem #PSA84HB
	Problem ID: PRA84HB Comment on this problem
	The position of two different objects is defined using complex numbers. The real part represents the **horizontal** displacement, while the imaginary part represents the **vertical** displacement (height).
	The positions are given below in terms of time t:
	Obj. 1: S = 12 + (2t - 1)i
	Obj. 2: S = 5t + 6i
	Find the time at which the objects are going to be at the same height (represented by the imaginary part).
	Type your answer below (mathematical expression):
	Submit Answer Break this problem into steps

These two were the problems shown on the post test section. They were the same for all students.

Data Analysis

The first step in the analysis of the results is to compare the results between control and treatment conditions. The following tables address the completion rate, the post test scores, and the average number of problems attempted in both conditions.

Table 1: Completion rate for the two conditions

Conditions	Students who started the problem set	Students who finished the problem set	Percent of students completed
control	54	37	69
treatment	61	41	67

This statistic gives 1 to the students who completed the skill builder, and 0 to those who did not.

Table 2: Post Test Scores for the two conditions (counting dropouts as 0)

	Students who completed the problem set	Percent of students completing the problem set	PostTestScore Average
control	37	69	0.87037
treatment	41	67	0.86885

The students answered two post test questions. They get 0 for answering 0 correct questions, 1 for answering one correct question, 2 for answering two correct questions. Students who did not reach the post test due to dropping out earlier received a 0 in this statistic.

Table 3: Post Test Scores for the two conditions (discarding dropouts)

	Students who completed the problem set	Percent of students completing the problem set	PostTestScore Average
control	37	69	1.2368
treatment	41	67	1.1777

The students answered two post test questions. They get 0 for answering 0 correct questions, 1 for answering one correct question, 2 for answering two correct questions. Students who did not reach the post test due to dropping out, were not included in this statistic.

Table 4: Average Number of problems attempted for both conditions

	Average Number of Problems Attempted	Average Number of Hard Problems Attempted
control	5.00	5.00
treatment	6.88	5.61

This statistic compares the number of problems that students in each condition faced. In this statistic students that did not make any attempts to answer any problems were not included. This can be done since students that made no attempts could not have been influenced by condition (as the first problem is similar for both conditions), and were evenly split out between the two conditions.

Hard problems are all skill builder problems, except from the prerequisite problems.

The TTest for all problems has a value of 0.0029

The TTest for hard problems has a value of 0.1839

A deeper look in the treatment condition gives the following.

Table 5: Dropout point during the skill builder (treatment only)

Dropout point	Short Skill Builder	Prerequisite Skill Builder	Normal Skill Builder
Number of students	1	1	4

This statistic does not include the students that did not make any attempts on any of the questions.

Data Analysis for Students with Weaker Background

We also tried to examine the hypothesis that prerequisite skill builders would prove beneficial for students with weaker background. The following data come from students with weaker background only. In order to select which students belong in this category, we looked at their Assistments record (percentage of correct answers on previous questions). We found the median for the entire sample, and selected all students that were below the median.

Table 6: Completion rate for the two conditions

Conditions	Students who started the problem set	Students who finished the problem set	Percent of students completed
control	24	12	50
treatment	33	19	58

The TTest for the Completion Rate shows a value of 0.5801

Table 7: Post Test Scores for the two conditions (counting dropouts as 0)

	Students who completed the problem set	Percent of students completing the problem set	PostTestScore Average
control	24	12	0.41667
treatment	33	19	0.57576

The TTest for the Post test scores shows a value of 0.4106

Conclusions

A few points can be made about the results:

- The first four tables reveal that there is no significant difference between the control and the treatment overall. A small increase in the number of problems attempted by treatment students is noticed, but the p-value of the TTest is not significant. This could however be analyzed further when more data points are collected.
- Table 5 gives a useful inquiry about the point where most students might drop out. It seems that the transition back to a hard skill builder after an easy one is a good cause for dropping out. It seems that either the students are reluctant to continue after having been asked to complete two different skill builders, or they cannot really put into use what they learned in the prerequisite problem sets. However, only six students are included in this table, which means that results can really be formed only with addition of new students.
- The second section is the analysis of the students who have done worse in past Assistments studies.Their results seem to be more diverse with regards to control against treatment conditions. In particular, treatment students now seem to have an advantage, with higher completion rate and better post test results. This is in contrast with the overall student population, where this difference was not observed. This would agree with our hypothesis that prerequisite questions are more beneficial for students with weaker background, who need this in depth review before they master the new skill. The TTests however, are still not providing high enough significance levels, which could be attributed to the few data points collected.
- Our first hypothesis does not seem supported well by the data, while there hints towards supporting our second hypothesis. More data should be analyzed in the future, so that the accuracy of the TTest is also increased.

Expanding Previous Studies

Adaptive vs Non-Adaptive-Multiplication,Division

A study that is similar to the prerequisites study was made some years earlier with only a few students participating. This study is based on the idea of using one pretest question that will decide if the students in the treatment will get prerequisite questions or not. We will provide a renewed analysis for this study with the new students that participated. This study uses two problem sets: PSASDZY and PSAR9Y9 on multiplication and division skills.

Figure 1: Design diagram of the experiment
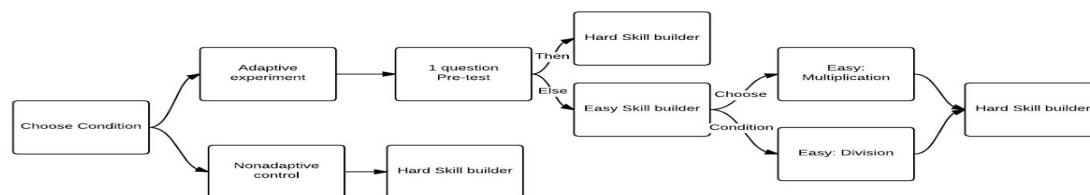
Table 8: Data results for Problem set PSASDZY

Condition	Pretest Question	Number of Students	Average Number of Hard Problems
Control		75	3.50
	Correct	56	2.91
	Wrong	19	5.23
Experiment		78	3.38
	Correct	70	3.22
	Wrong	8	4.26
Total		153	3.45

Table 9: Data results for Problem set PSAR9Y9

Case	Pretest Question	Number of Students	Average Number of Hard Problems
Control		80	3.01
	Correct	72	2.81
	Wrong	8	4.84

Experiment		75	3.00
	Correct	63	2.69
	Wrong	12	4.60
Total		155	3.01

Conclusion:

This study seems to suggest that control and experiment are not heavily differentiated in terms of overall performance. Slightly more effort seems to be required for the control students, which is close to the initial hypothesis. We can also see that looking at students who got the question wrong, there is an greater advantage for the experimental condition group. In that case the prerequisite questions seem to be helpful. With more data we draw better conclusions in the future.

It should be noted that the part of the analysis that separates between multiplication and division is still not available due to the small amount of students that reached that category.

Adaptive vs Non-Adaptive Study- Coin Quiz

A study that is similar to the prerequisites study was made some years earlier with only a few students participating. This study is based on the idea of using one pretest question that will decide if the students in the treatment will get prerequisite questions or not. We will provide a renewed analysis for this study with the new students that participated. This study uses the problem set PSASA4B, with questions on handling and understanding coin values.
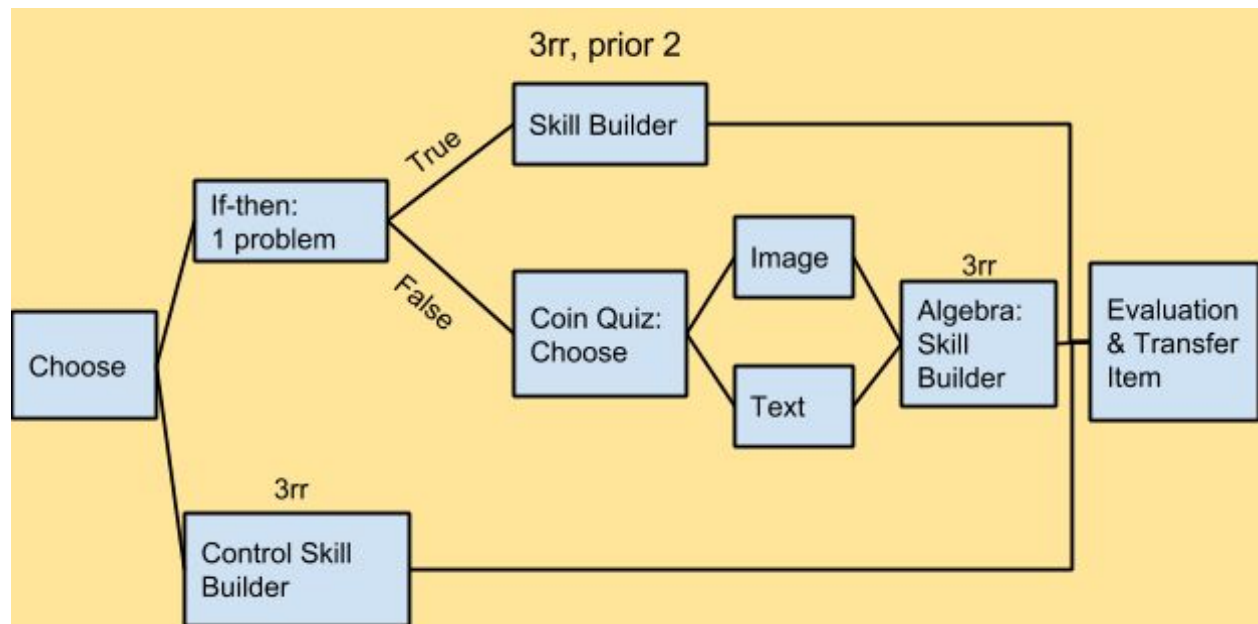
Figure 2: Design Diagram of the coin quiz study

3rr, prior 2

Skill Builder

If-then:
1 problem

True

False

Coin Quiz:
Choose

Image

Text

3rr

Algebra:
Skill
Builder

Evaluation
& Transfer
Item

Choose

3rr

Control Skill
Builder

Table 10: Data results for problem set PSASA4B

Group [DE]	First question	Second question		Droup outs	
Experimental: 37	14	13		5	Total: 9
		1			
	23	Text quiz: 12	9	3	
			0		
		Visual quiz: 11	7	3	
			1		
Control: 34	12	11		2	Total: 12
		1			
	22	14		10	
		8			

*In the above table blue color means that the students answered the question correctly, and red color means that they answered incorrectly.

Comments:

This experiment seems to add to our previous results, showing no significant difference between control and treatment. Comparing the students who got the first question wrong in both conditions however, again shows a small imbalance with the number of dropouts being 10 for those in the control and 6 six for those in treatment. Again with 71 students in total, the results are inconclusive and therefore this problem set requires more time to collect data.

Appendix

Problem Set "All templates" id:[PSAZZ9F]

☐ **Select All**

☐ **1) Problem #PRA85CZ "PRA85CZ - TEMPLATE - Add or Subtract Matrices Calendar 3"**

A couple is trying to get %v{times[morning]} on %v{weekdays[day]}, how much would it cost for one of them to take the cab and one of them to take the subway?

Cab Fare

	Monday	Tuesday	Wednesday	Thursday	Friday
Morning	$%v{dollars1[0]}.00	$%v{dollars1[1]}.00	$%v{dollars1[2]}.00	$%v{dollars1[3]}.00	$%v{dollars1[4]}.00
Afternoon	$%v{dollars1[5]}.00	$%v{dollars1[6]}.00	$%v{dollars1[7]}.00	$%v{dollars1[8]}.00	$%v{dollars1[9]}.00

Subway Fare

	Monday	Tuesday	Wednesday	Thursday	Friday
Morning	$%v{dollars1[10]}.00	$%v{dollars1[11]}.00	$%v{dollars1[12]}.00	$%v{dollars1[13]}.00	$%v{dollars1[14]}.00
Afternoon	$%v{dollars1[15]}.00	$%v{dollars1[16]}.00	$%v{dollars1[17]}.00	$%v{dollars1[18]}.00	$%v{dollars1[19]}.00

Algebraic Expression:

✓ %v{dollars1[5*morning+day]+dollars1[10+5*morning+day]}

Hints:

- Add the values across the corresponding cells.

So you have:

%v{weekdays[day]} %v{mornoon[morning]} Cab fare + %v{weekdays[day]} %v{mornoon[morning]} Subway fare

-
$$\text{%v\{weekdays[day]\} %v\{mornoon[morning]\} Cab fare + %v\{weekdays[day]\} %v\{mornoon[morning]\} Subway fare} = \text{Total Cost}$$

$$\text{%v\{dollars1[5*morning+day]\} + %v\{dollars1[10+5*morning+day]\}} = \text{Total Cost}$$

$$\text{%v\{dollars1[5*morning+day]+dollars1[10+5*morning+day]\}} = \text{Total Cost}$$

Type in %v{dollars1[5*morning+day]+dollars1[10+5*morning+day]}

☐ **2) Problem #PRA8XSX "PRA8XSX - TEMPLATE - Add or Subtract Matrices Calendar 2"**

How many more points did Team A score in Game %v{game+1}'s %v{fourths[quarter]} quarter than Team B in Game %v{gametwo+1}'s %v{fourths[quarter]} quarter?

Game 1

	Team A	Team B
First Quarter	%v{score[0]} pts	%v{score[1]} pts
Second Quarter	%v{score[2]} pts	%v{score[3]} pts
Third Quarter	%v{score[4]} pts	%v{score[5]} pts
Fourth Quarter	%v{score[6]} pts	%v{score[7]} pts

Game 2

	Team A	Team B
First Quarter	%v{score[8]} pts	%v{score[9]} pts
Second Quarter	%v{score[10]} pts	%v{score[11]} pts
Third Quarter	%v{score[12]} pts	%v{score[13]} pts
Fourth Quarter	%v{score[14]} pts	%v{score[15]} pts

Algebraic Expression:

✔️ %v{score[game*8+quarter*2]-score[gametwo*8+quarter*2+1]}

Hints:

- Subtract the values across the corresponding cells.

So you have:

Team A's Game %v{game+1} %v{fourths[quarter]} quarter score - Team B's Game %v{gametwo+1} %v{fourths[quarter]} quarter score

-

Team A's Game %v{game+1} %v{fourths[quarter]} quarter score - Team B's Game %v{gametwo+1} %v{fourths[quarter]} quarter score = Point Difference

%v{score[game*8+quarter*2]} - %v{score[gametwo*8+quarter*2+1]} = Point Difference

%v{score[game*8+quarter*2]-score[gametwo*8+quarter*2+1]} = Point Difference

Type in %v{score[game*8+quarter*2]-score[gametwo*8+quarter*2+1]}

☐ **3) Problem #PRA8VZK "PRA8VZK - TEMPLATE - Add or Subtract Matrices Calendar 1"**

In total, how much did Sam and Nick spend on %v{meals[meal]} on %v{weekdays[day]}?

Sam

	Monday	Tuesday	Wednesday	Thursday	Friday
Breakfast	$%v{dollars1[0]}.00	$%v{dollars1[1]}.00	$%v{dollars1[2]}.00	$%v{dollars1[3]}.00	$%v{dollars1[4]}.00
Lunch	$%v{dollars1[5]}.00	$%v{dollars1[6]}.00	$%v{dollars1[7]}.00	$%v{dollars1[8]}.00	$%v{dollars1[9]}.00
Dinner	$%v{dollars1[10]}.00	$%v{dollars1[11]}.00	$%v{dollars1[12]}.00	$%v{dollars1[13]}.00	$%v{dollars1[14]}.00

Nick

	Monday	Tuesday	Wednesday	Thursday	Friday
Breakfast	$%v{dollars2[0]}.00	$%v{dollars2[1]}.00	$%v{dollars2[2]}.00	$%v{dollars2[3]}.00	$%v{dollars2[4]}.00
Lunch	$%v{dollars2[5]}.00	$%v{dollars2[6]}.00	$%v{dollars2[7]}.00	$%v{dollars2[8]}.00	$%v{dollars2[9]}.00
Dinner	$%v{dollars2[10]}.00	$%v{dollars2[11]}.00	$%v{dollars2[12]}.00	$%v{dollars2[13]}.00	$%v{dollars2[14]}.00

Algebraic Expression:

✔️ %v{dollars1[meal*5+day]+dollars2[meal*5+day]}

Hints:

- Add the values across the corresponding cells.

So you have:

Sam's %v{weekdays[day]} %v{meals[meal]} + Nick's %v{weekdays[day]} %v{meals[meal]} = Total Cost

-

$$\text{Sam's } \%v\{weekdays[day]\}\ \%v\{meals[meal]\} + \text{Nick's } \%v\{weekdays[day]\}\ \%v\{meals[meal]\} = \frac{\text{Total}}{\text{Cost}}$$

$$\$\%v\{dollars1[meal*5+day]\}.00 + \$\%v\{dollars2[meal*5+day]\}.00 = \frac{\text{Total}}{\text{Cost}}$$

$$\$\%v\{dollars1[meal*5+day]+dollars2[meal*5+day]\} = \frac{\text{Total}}{\text{Cost}}$$

Type in $\%v\{dollars1[meal*5+day]+dollars2[meal*5+day]\}$

---

4) Problem #PRA8EHN "PRA8EHN - TEMPLATE - Matrix Rows vs Columns 6"

How many columns does the following matrix have?

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} \\ \%v\{rows[3]\} & \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} & \%v\{rows[8]\} \\ \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{vmatrix}$$

Algebraic Expression:

✓ $\%v\{ans\}$

✗ $\%v\{wans\}$

- You confused columns with rows.

$$a_{rc}$$

One Column

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

One Row

$$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$$

Hints:

- Columns are vertical pillars, so because there are three vertical columns, the answer is 3.

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} \\ \%v\{rows[3]\} & \%v\{rows[4]\} & \%v\{rows[5]\} \end{vmatrix}$$

$$\left| \begin{array}{ccc} \%v\{rows[6]\} & \%v\{rows[7]\} & \%v\{rows[8]\} \\ \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{array} \right|$$

Type in 3.

---

5) Problem #PRA8EHM "PRA8EHM - TEMPLATE - Matrix Rows vs Columns 5"

How many columns does the following matrix have?

$$\left| \begin{array}{cc} \%v\{rows[0]\} & \%v\{rows[1]\} \\ \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} \end{array} \right|$$

Algebraic Expression:

✓ %v{ans}

✗ %v{wans}

- You confused columns with rows.

$$a_{rc}$$

One Column

$$\left| \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \right|$$

One Row

$$\left| \begin{array}{ccc} 1 & 2 & 3 \end{array} \right|$$

Hints:

- Columns are vertical pillars, so because there are two vertical columns, the answer is 2.

$$\left| \begin{array}{cc} \%v\{rows[0]\} & \%v\{rows[1]\} \\ \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} \end{array} \right|$$

Type in 2.

---

6) Problem #PRA8EHK "PRA8EHK - TEMPLATE - Matrix Rows vs Columns 4"

How many columns does the following matrix have?

$$\left| \qquad\qquad \right|$$

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \end{vmatrix}$$

Algebraic Expression:

✓ %v{ans}

✗ %v{wans}

- You confused columns with rows.

$$a_{rc}$$

One Column

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

One Row

$$\begin{vmatrix} 1\ 2\ 3 \end{vmatrix}$$

Hints:

- Columns are vertical pillars, so because there are four vertical columns, the answer is 4.

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \end{vmatrix}$$

Type in 4.

☐ **7) Problem #PRA8EHJ "PRA8EHJ - TEMPLATE - Matrix Rows vs Columns 3"**

How many rows does the following matrix have?

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} \\ \%v\{rows[3]\} & \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} & \%v\{rows[8]\} \\ \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{vmatrix}$$

Algebraic Expression:

✓ %v{ans}

✗ %v{wans}

- You confused rows with columns.

$$a_{rc}$$

One Row

$$\begin{vmatrix} \quad & \quad \end{vmatrix}$$

$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$

One Column

$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$

Hints:

- Rows are horizontal bars, so because there are four horizontal rows, the answer is 4.

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} \\ \%v\{rows[3]\} & \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} & \%v\{rows[8]\} \\ \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{vmatrix}$$

Type in 4.

---

8) **Problem #PRA8EHH "PRA8EHH - TEMPLATE - Matrix Rows vs Columns 2"**

How many rows does the following matrix have?

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} \\ \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} \end{vmatrix}$$

Algebraic Expression:

✓ %v{ans}

✗ %v{wans}

- You confused rows with columns.

a_{rc}

One Row

$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$

One Column

$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$

Hints:

- Rows are horizontal bars, so because there are three horizontal rows, the answer is 3.

%v{rows[0]} %v{rows[1]}

%v{rows[2]} %v{rows[3]}

%v{rows[4]} %v{rows[5]}

Type in 3.

☐ **9) Problem #PRA8CE6 "PRA8CE6 - TEMPLATE - Add or Subtract Matrices 3"**

Submit C$_{\%v\{arow\}\%v\{acol\}}$ from the %v{operation[choice]} of the following matrices.

%v{mat1[0]} %v{mat1[1]}

%v{mat1[2]} %v{mat1[3]}

                              %v{sign[choice]}

%v{mat1[4]} %v{mat1[5]}

%v{mat1[6]} %v{mat1[7]}

%v{mat2[0]} %v{mat2[1]}

%v{mat2[2]} %v{mat2[3]}

%v{mat2[4]} %v{mat2[5]}

%v{mat2[6]} %v{mat2[7]}

C_{11} C_{12}

C_{21} C_{22}

=

C_{31} C_{32}

C_{41} C_{42}

Algebraic Expression:

✔ %v{mat3[(arow-1)*2 + (acol-1)]}

Hints:

- %v{operationtwo[choice]} across the corresponding cells.

So you have:

A_{rc} %v{sign[choice]} B_{rc} = C_{rc}

-

$A_{\%v\{arow\}\%v\{acol\}}$ %v{sign[choice]} $B_{\%v\{arow\}\%v\{acol\}}$ = $C_{\%v\{arow\}\%v\{acol\}}$

%v{mat1[(arow-1)*2 + (acol-1)]} %v{sign[choice]} %v{mat2[(arow-1)*2 + (acol-1)]}   = $C_{\%v\{arow\}\%v\{acol\}}$

%v{mat3[(arow-1)*2 + (acol-1)]}   = $C_{\%v\{arow\}\%v\{acol\}}$

Type in %v{mat3[(arow-1)*2 + (acol-1)]}.

☐ **10) Problem #PRA8CEW "PRA8CEW - TEMPLATE - Add or Subtract Matrices 2"**

Submit C$_{\%v\{arow\}\%v\{acol\}}$ from the %v{operation[choice]} of the following matrices.

%v{mat1[0]} %v{mat1[1]} %v{mat1[2]}

                              %v{sign[choice]}

%v{mat1[3]} %v{mat1[4]} %v{mat1[5]}

%v{mat2[0]} %v{mat2[1]} %v{mat2[2]}

%v{mat2[3]} %v{mat2[4]} %v{mat2[5]}

C_{11} C_{12} C_{13}

=

C_{21} C_{22} C_{23}

Algebraic Expression:

✔ %v{mat3[(arow-1)*3 + (acol-1)]}

Hints:

- %v{operationtwo[choice]} across the corresponding cells.

So you have:

A_{rc} %v{sign[choice]} $B_{rc} = C_{rc}$

-

$A_{\%v\{arow\}\%v\{acol\}}$ %v{sign[choice]} $B_{\%v\{arow\}\%v\{acol\}}$ $= C_{\%v\{arow\}\%v\{acol\}}$

%v{mat1[(arow-1)*3 + (acol-1)]} %v{sign[choice]} %v{mat2[(arow-1)*3 + (acol-1)]} $= C_{\%v\{arow\}\%v\{acol\}}$

%v{mat3[(arow-1)*3 + (acol-1)]} $= C_{\%v\{arow\}\%v\{acol\}}$

Type in %v{mat3[(arow-1)*3 + (acol-1)]}.

☐ **11) Problem #PRA8CEK "PRA8CEK - TEMPLATE - Intro to Matrices 3"**

What is the value at $a_{\%v\{arow\}\%v\{acol\}}$?

$$a = \begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} \\ \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} \end{vmatrix}$$

Algebraic Expression:

✔ %v{ans}

✘ %v{wrong_ans}

- You confused rows with columns.

a_{rc}

One Row

$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$

One Column

$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$

Hints:

- Go down %v{arow} ($a_{\%v\{arow\}\%v\{acol\}}$) rows

$$\left| \begin{array}{cc} \%v\{rows[0]\} & \%v\{rows[1]\} \\ \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} \end{array} \right.$$

Go right %v{acol} (a$_{\%v\{arow\}}$%v{acol}) columns

$$\left| \begin{array}{cc} \%v\{rows[0]\} & \%v\{rows[1]\} \\ \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} \end{array} \right.$$

- Type in %v{ans}

☐ **12) Problem #PRA8B8R "PRA8B8R - TEMPLATE - Multiply Matrices 6"**
Submit C$_{\%v\{arow\}\%v\{acol\}}$ from the following matrix multiplication.

$$\left| \begin{array}{ccc} \%v\{mat1[0]\} & \%v\{mat1[1]\} & \%v\{mat1[2]\} \\ \%v\{mat1[3]\} & \%v\{mat1[4]\} & \%v\{mat1[5]\} \end{array} \right| \; x \; \left| \begin{array}{cc} \%v\{mat2[0]\} & \%v\{mat2[1]\} \\ \%v\{mat2[2]\} & \%v\{mat2[3]\} \\ \%v\{mat2[4]\} & \%v\{mat2[5]\} \end{array} \right| \; = \; \left| \begin{array}{cc} C_{11} & C_{12} \\ C_{21} & C_{22} \end{array} \right|$$

Algebraic Expression:

✔ %v{mat3[(arow-1)*2+(acol-1)]}

Hints:

- Multiply row %v{arow} by column %v{acol}.

$$\left| \begin{array}{ccc} \%v\{mat1[0]\} & \%v\{mat1[1]\} & \%v\{mat1[2]\} \\ \%v\{mat1[3]\} & \%v\{mat1[4]\} & \%v\{mat1[5]\} \end{array} \right| \; x \; \left| \begin{array}{cc} \%v\{mat2[0]\} & \%v\{mat2[1]\} \\ \%v\{mat2[2]\} & \%v\{mat2[3]\} \\ \%v\{mat2[4]\} & \%v\{mat2[5]\} \end{array} \right|$$

- A$_{22}$ = %v{mat1[0]} * %v{mat2[1]} + %v{mat1[1]} * %v{mat2[3]} + %v{mat1[2]} * %v{mat2[5]}

A$_{22}$ = %v{mat1[0] * mat2[1]} + %v{mat1[1] * mat2[3]} + %v{mat1[2] * mat2[5]}

A$_{22}$ = %v{mat3[(arow-1)*2+(acol-1)]}.
Type in %v{mat3[(arow-1)*2+(acol-1)]}.

☐ **13) Problem #PRA74QN "PRA74QN - TEMPLATE - Multiply Matrices 5"**
Submit C$_{\%v\{arow\}\%v\{acol\}}$ from the following matrix multiplication.

$$\begin{vmatrix} \%v\{mat1[0]\} \ \%v\{mat1[1]\} \ \%v\{mat1[2]\} \\ \\ \%v\{mat1[3]\} \ \%v\{mat1[4]\} \ \%v\{mat1[5]\} \end{vmatrix} x \begin{vmatrix} \%v\{mat2[0]\} \ \%v\{mat2[1]\} \\ \\ \%v\{mat2[2]\} \ \%v\{mat2[3]\} \\ \\ \%v\{mat2[4]\} \ \%v\{mat2[5]\} \end{vmatrix} = \begin{vmatrix} C_{11} \quad C_{12} \\ \\ C_{21} \quad C_{22} \end{vmatrix}$$

Algebraic Expression:

✔ %v{mat3[(arow-1)*2+(acol-1)]}

Hints:

- Multiply row %v{arow} by column %v{acol}.

$$\begin{vmatrix} \%v\{mat1[0]\} \ \%v\{mat1[1]\} \ \%v\{mat1[2]\} \\ \\ \%v\{mat1[3]\} \ \%v\{mat1[4]\} \ \%v\{mat1[5]\} \end{vmatrix} x \begin{vmatrix} \%v\{mat2[0]\} \ \%v\{mat2[1]\} \\ \\ \%v\{mat2[2]\} \ \%v\{mat2[3]\} \\ \\ \%v\{mat2[4]\} \ \%v\{mat2[5]\} \end{vmatrix}$$

- A_{22} = %v{mat1[0]} * %v{mat2[0]} + %v{mat1[1]} * %v{mat2[2]} + %v{mat1[2]} * %v{mat2[4]}

A_{22} = %v{mat1[0] * mat2[0]} + %v{mat1[1] * mat2[2]} + %v{mat1[2] * mat2[4]}

A_{22} = %v{mat3[(arow-1)*2+(acol-1)]}.
Type in %v{mat3[(arow-1)*2+(acol-1)]}.

☐ **14) Problem #PRA74PS "PRA74PS - TEMPLATE - Multiply Matrices 4"**

Submit $C_{\%v\{arow\}\%v\{acol\}}$ from the following matrix multiplication.

$$\begin{vmatrix} \%v\{mat1[0]\} \ \%v\{mat1[1]\} \ \%v\{mat1[2]\} \\ \\ \%v\{mat1[3]\} \ \%v\{mat1[4]\} \ \%v\{mat1[5]\} \\ \\ \%v\{mat1[6]\} \ \%v\{mat1[7]\} \ \%v\{mat1[8]\} \end{vmatrix} x \begin{vmatrix} \%v\{mat2[0]\} \ \%v\{mat2[1]\} \ \%v\{mat2[2]\} \\ \\ \%v\{mat2[3]\} \ \%v\{mat2[4]\} \ \%v\{mat2[5]\} \\ \\ \%v\{mat2[6]\} \ \%v\{mat2[7]\} \ \%v\{mat2[8]\} \end{vmatrix} = \begin{vmatrix} C_{11} \quad C_{12} \quad C_{13} \\ \\ C_{21} \quad C_{22} \quad C_{23} \\ \\ C_{31} \quad C_{32} \quad C_{33} \end{vmatrix}$$

Algebraic Expression:

✔ %v{mat3[(arow-1)*3+(acol-1)]}

Hints:

- Multiply row %v{arow} by column %v{acol}.

$$\begin{vmatrix} \%v\{mat1[0]\} \ \%v\{mat1[1]\} \ \%v\{mat1[2]\} \\ \\ \%v\{mat1[3]\} \ \%v\{mat1[4]\} \ \%v\{mat1[5]\} \\ \\ \%v\{mat1[6]\} \ \%v\{mat1[7]\} \ \%v\{mat1[8]\} \end{vmatrix} x \begin{vmatrix} \%v\{mat2[0]\} \ \%v\{mat2[1]\} \ \%v\{mat2[2]\} \\ \\ \%v\{mat2[3]\} \ \%v\{mat2[4]\} \ \%v\{mat2[5]\} \\ \\ \%v\{mat2[6]\} \ \%v\{mat2[7]\} \ \%v\{mat2[8]\} \end{vmatrix}$$

- A_{22} = %v{mat1[3]} * %v{mat2[0]} + %v{mat1[4]} * %v{mat2[3]} + %v{mat1[5]} * %v{mat2[6]}

A_{22} = %v{mat1[3] * mat2[0]} + %v{mat1[4] * mat2[3]} + %v{mat1[5] * mat2[6]}

A_{22} = %v{mat3[3]}.
Type in %v{mat3[3]}.

☐ **15) Problem #PRA74PP "PRA74PP - TEMPLATE - Multiply Matrices 3"**

Submit C$_{%v\{arow\}%v\{acol\}}$ from the following matrix multiplication.

$$\begin{vmatrix} \text{\%v\{mat1[0]\} \%v\{mat1[1]\} \%v\{mat1[2]\}} \\ \text{\%v\{mat1[3]\} \%v\{mat1[4]\} \%v\{mat1[5]\}} \\ \text{\%v\{mat1[6]\} \%v\{mat1[7]\} \%v\{mat1[8]\}} \end{vmatrix} \text{ x } \begin{vmatrix} \text{\%v\{mat2[0]\} \%v\{mat2[1]\} \%v\{mat2[2]\}} \\ \text{\%v\{mat2[3]\} \%v\{mat2[4]\} \%v\{mat2[5]\}} \\ \text{\%v\{mat2[6]\} \%v\{mat2[7]\} \%v\{mat2[8]\}} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

Algebraic Expression:

✓ %v{mat3[(arow-1)*3+(acol-1)]}

Hints:

- Multiply row %v{arow} by column %v{acol}.

$$\begin{vmatrix} \text{\%v\{mat1[0]\} \%v\{mat1[1]\} \%v\{mat1[2]\}} \\ \text{\%v\{mat1[3]\} \%v\{mat1[4]\} \%v\{mat1[5]\}} \\ \text{\%v\{mat1[6]\} \%v\{mat1[7]\} \%v\{mat1[8]\}} \end{vmatrix} \text{ x } \begin{vmatrix} \text{\%v\{mat2[0]\} \%v\{mat2[1]\} \%v\{mat2[2]\}} \\ \text{\%v\{mat2[3]\} \%v\{mat2[4]\} \%v\{mat2[5]\}} \\ \text{\%v\{mat2[6]\} \%v\{mat2[7]\} \%v\{mat2[8]\}} \end{vmatrix}$$

- A$_{22}$ = %v{mat1[6]} * %v{mat2[1]} + %v{mat1[7]} * %v{mat2[4]} + %v{mat1[8]} * %v{mat2[7]}

A$_{22}$ = %v{mat1[6] * mat2[1]} + %v{mat1[7] * mat2[4]} + %v{mat1[8] * mat2[7]}

A$_{22}$ = %v{mat3[7]}.
Type in %v{mat3[7]}.

☐ **16) Problem #PRA74PN "PRA74PN - TEMPLATE - Multiply Matrices 2"**

Submit C$_{%v\{arow\}%v\{acol\}}$ from the following matrix multiplication.

$$\begin{vmatrix} \text{\%v\{mat1[0]\} \%v\{mat1[1]\} \%v\{mat1[2]\}} \\ \text{\%v\{mat1[3]\} \%v\{mat1[4]\} \%v\{mat1[5]\}} \\ \text{\%v\{mat1[6]\} \%v\{mat1[7]\} \%v\{mat1[8]\}} \end{vmatrix} \text{ x } \begin{vmatrix} \text{\%v\{mat2[0]\} \%v\{mat2[1]\} \%v\{mat2[2]\}} \\ \text{\%v\{mat2[3]\} \%v\{mat2[4]\} \%v\{mat2[5]\}} \\ \text{\%v\{mat2[6]\} \%v\{mat2[7]\} \%v\{mat2[8]\}} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

Algebraic Expression:

✓ %v{mat3[(arow-1)*3+(acol-1)]}

Hints:

- Multiply row %v{arow} by column %v{acol}.

$$\begin{vmatrix} \text{\%v\{mat1[0]\} \%v\{mat1[1]\} \%v\{mat1[2]\}} \\ \text{\%v\{mat1[3]\} \%v\{mat1[4]\} \%v\{mat1[5]\}} \\ \text{\%v\{mat1[6]\} \%v\{mat1[7]\} \%v\{mat1[8]\}} \end{vmatrix} \text{ x } \begin{vmatrix} \text{\%v\{mat2[0]\} \%v\{mat2[1]\} \%v\{mat2[2]\}} \\ \text{\%v\{mat2[3]\} \%v\{mat2[4]\} \%v\{mat2[5]\}} \\ \text{\%v\{mat2[6]\} \%v\{mat2[7]\} \%v\{mat2[8]\}} \end{vmatrix}$$

- A$_{22}$ = %v{mat1[0]} * %v{mat2[2]} + %v{mat1[1]} * %v{mat2[5]} + %v{mat1[2]} * %v{mat2[8]}

A$_{22}$ = %v{mat1[0] * mat2[2]} + %v{mat1[1] * mat2[5]} + %v{mat1[2] * mat2[8]}

A$_{22}$ = %v{mat3[2]}.
Type in %v{mat3[2]}.

☐ **17) Problem #PRA7Z7T "PRA7Z7T - TEMPLATE - Multiply Matrices"**

Submit the number missing from the %v{operation[choice]}.

$$\begin{vmatrix} \%v\{mat1[0]\} & \%v\{mat1[1]\} & \%v\{mat1[2]\} \\ \%v\{mat1[3]\} & \%v\{mat1[4]\} & \%v\{mat1[5]\} \\ \%v\{mat1[6]\} & \%v\{mat1[7]\} & \%v\{mat1[8]\} \end{vmatrix} \%v\{sign[choice]\} \begin{vmatrix} \%v\{mat2[0]\} & \%v\{mat2[1]\} & \%v\{mat2[2]\} \\ \%v\{mat2[3]\} & \%v\{mat2[4]\} & \%v\{mat2[5]\} \\ \%v\{mat2[6]\} & \%v\{mat2[7]\} & \%v\{mat2[8]\} \end{vmatrix} = \begin{vmatrix} \%v\{mat3[0]\} & \%v\{mat3[1]\} & \%v\{mat3[2]\} \\ \%v\{mat3[3]\} & ? & \%v\{mat3[5]\} \\ \%v\{mat3[6]\} & \%v\{mat3[7]\} & \%v\{mat3[8]\} \end{vmatrix}$$

Algebraic Expression:

✓ %v{mat3[4]}

Hints:

- %v{operationtwo[choice]} according to
A_{rc} %v{sign[choice]} $B_{rc} = C_{rc}$

- A_{22} %v{sign[choice]} $B_{22} = C_{22}$
%v{mat1[4]} %v{sign[choice]} %v{mat2[4]} = C_{22}
%v{mat3[4]} = C_{22}
Type in %v{mat3[4]}.

☐ **18) Problem #PRA7UJW "PRA7UJW - TEMPLATE - Add or Subtract Matrices 1"**

Submit $C_{\%v\{arow\}\%v\{acol\}}$ from the %v{operation[choice]} of the following matrices.

$$\begin{vmatrix} \%v\{mat1[0]\} & \%v\{mat1[1]\} & \%v\{mat1[2]\} \\ \%v\{mat1[3]\} & \%v\{mat1[4]\} & \%v\{mat1[5]\} \\ \%v\{mat1[6]\} & \%v\{mat1[7]\} & \%v\{mat1[8]\} \end{vmatrix} \%v\{sign[choice]\} \begin{vmatrix} \%v\{mat2[0]\} & \%v\{mat2[1]\} & \%v\{mat2[2]\} \\ \%v\{mat2[3]\} & \%v\{mat2[4]\} & \%v\{mat2[5]\} \\ \%v\{mat2[6]\} & \%v\{mat2[7]\} & \%v\{mat2[8]\} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

Algebraic Expression:

✓ %v{mat3[(arow-1)*3 + (acol-1)]}

Hints:

- %v{operationtwo[choice]} across the corresponding cells.

So you have:

A_{rc} %v{sign[choice]} $B_{rc} = C_{rc}$

-
$A_{\%v\{arow\}\%v\{acol\}}$ %v{sign[choice]} $B_{\%v\{arow\}\%v\{acol\}}$ = $C_{\%v\{arow\}\%v\{acol\}}$

%v{mat1[(arow-1)*3 + (acol-1)]} %v{sign[choice]} %v{mat2[(arow-1)*3 + (acol-1)]}   = $C_{\%v\{arow\}\%v\{acol\}}$

%v{mat3[(arow-1)*3 + (acol-1)]}   = $C_{\%v\{arow\}\%v\{acol\}}$

Type in %v{mat3[(arow-1)*3 + (acol-1)]}.

19) Problem #PRA7UJV "PRA7UJV - TEMPLATE - Matrix Rows vs Columns"

How many rows does the following matrix have?

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \end{vmatrix}$$

Algebraic Expression:

✓ %v{ans}

✗ %v{wans}

- You confused rows with columns.

a_{rc}

One Row

$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$

One Column

$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$

Hints:

- Rows are horizontal bars, so because there are two horizontal rows, the answer is 2.

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \end{vmatrix}$$

Type in 2.

---

20) Problem #PRA7RV3 "PRA7RV3 - TEMPLATE- Absolute Value Equations 7"

This equation has two solutions.

$| \%v\{c\}x + \%v\{a\} | = \%v\{b\}$

Submit only the %v{size[choice]} one.

Algebraic Expression:

✓ %v{ans[choice]}

✗ %v{ans[wchoice]}

- You input the smaller value, solve for the larger one.

Hints:

-

$| \%v\{c\}x + \%v\{a\} | = \%v\{b\}$

$\%v\{c\}x + \%v\{a\} = \%v\{b\}$ or $\%v\{c\}x + \%v\{a\} = -\%v\{b\}$

Continue Solving for x.

-
%v{c}x + %v{a} - %v{a} = %v{b} - %v{a}   %v{c}x + %v{a} - %v{a} = %v{(-1*b)} - %v{a}

%v{c}x = %v{(1)*b - a}   %v{c}x = %v{(-1)*b - a}

$$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{(1)*b - a\}}{\%v\{c\}}$$ $$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{(-1)*b - a\}}{\%v\{c\}}$$

x = %v{((1)*b - a) / c}   x = %v{((-1)*b - a) / c}

%v{ans[choice]} is %v{size[choice]}. Type in %v{ans[choice]}.

---

21) Problem #PRA7QSY "PRA7QSY - TEMPLATE- Absolute Value Equations 5"
This equation has two solutions.

%v{c}| x + %v{a} | = %v{b}

Submit only the %v{size[choice]} one.
Algebraic Expression:
✔ %v{ans[choice]}
✘ %v{ans[wchoice]}

- You solved for the larger value of X, try solving for the other side of the absolute value.

Hints:

- Isolate the absolute value on one side of the equal sign.

    %v{c}| x + %v{a} | = %v{b}

    $$\frac{\%v\{c\}| x + \%v\{a\} |}{\%v\{c\}} = \frac{\%v\{b\}}{\%v\{c\}}$$

    | x + %v{a} | = %v{b / c}

x + %v{a} = %v{b / c} or x + %v{a} = %v{(-1*b) / c}

Continue Solving for x.

-
x + %v{a} - %v{a} = %v{b / c} - %v{a}   x + %v{a} - %v{a} = %v{(-1*b) / c} - %v{a}

    x = %v{((1)*b / c) - a}   x = %v{((-1)*b / c) - a}

%v{ans[choice]} is %v{size[choice]}. Type in %v{ans[choice]}.

---

22) Problem #PRA7QB9 "PRA7QB9 - TEMPLATE- Absolute Value Equations 3"
This equation has two solutions.

%v{c}| x + %v{a} | = %v{b}

Submit only the larger one.
Algebraic Expression:
✔ %v{ans}
✘ %v{wans}

- You solved for the smaller value of X, try solving for the other side of the absolute value.

Hints:

- Start by solving for the absolute value by dividing the other side
         equation 1
  equation 2   equation 3
  equation 2a equation 3b
         .   .
         .   .
         .   .

%v{c}| x + %v{a} | = %v{b}

| x + %v{a} | = %v{b} / %v{c}

Then undo the absolute value operation

x + %v{a} = %v{b} / %v{c}   Or   x + %v{a} = -%v{b} / %v{c}

Then continue to solve for X

- Then subtract the constant

x = %v{b} / %v{c} - %v{a} Or x = -%v{b} / %v{c} - %v{a}

So the answers are

x = %v{(1)*b} / %v{c} - %v{a} Or x = %v{(-1)*b} / %v{c} - %v{a}
Choose the larger.

- %v{ans}

☐ **23) Problem #PRA7P9T "PRA7P9T - TEMPLATE - Intro to Matrices 2"**
What is the value at $a_{\%v\{arow\}\%v\{acol\}}$?

$$a = \begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \\ \%v\{rows[8]\} & \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{vmatrix}$$

Algebraic Expression:
✓ %v{ans}
✗ %v{wrong_ans}

- You confused rows with columns.

a_{rc}

One Row

$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$

One Column

$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$

Hints:

- Go down %v{arow} (a$_{\%v\{arow\}\%v\{acol\}}$) rows

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \\ \%v\{rows[8]\} & \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{vmatrix}$$

Go right %v{acol} (a$_{\%v\{arow\}\%v\{acol\}}$) columns

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} & \%v\{rows[3]\} \\ \%v\{rows[4]\} & \%v\{rows[5]\} & \%v\{rows[6]\} & \%v\{rows[7]\} \\ \%v\{rows[8]\} & \%v\{rows[9]\} & \%v\{rows[10]\} & \%v\{rows[11]\} \end{vmatrix}$$

- Type in %v{ans}

---

24) Problem #PRA7P9A "PRA7P9A - TEMPLATE - Intro to Matrices 1"

What is the value at a$_{\%v\{arow\}\%v\{acol\}}$?

$$a = \begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} \\ \%v\{rows[3]\} & \%v\{rows[4]\} & \%v\{rows[5]\} \\ \%v\{rows[6]\} & \%v\{rows[7]\} & \%v\{rows[8]\} \end{vmatrix}$$

Algebraic Expression:

✔ %v{ans}

✘ %v{wrong_ans}

- You confused rows with columns.

$$a_{rc}$$

One Row

$$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$$

One Column

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

Hints:

- Go down %v{arow} (a$_{\%v\{arow\}\%v\{acol\}}$) rows

$$\begin{vmatrix} \%v\{rows[0]\} & \%v\{rows[1]\} & \%v\{rows[2]\} \end{vmatrix}$$

%v{rows[3]} %v{rows[4]} %v{rows[5]}

%v{rows[6]} %v{rows[7]} %v{rows[8]}

Go right %v{acol} (a_{%v{arow}%v{acol}}) columns

%v{rows[0]} %v{rows[1]} %v{rows[2]}

%v{rows[3]} %v{rows[4]} %v{rows[5]}

%v{rows[6]} %v{rows[7]} %v{rows[8]}

- Type in %v{ans}

---

25) Problem #PRA7PEU "PRA7PEU - TEMPLATE- Absolute Value Equations 3"

This equation has two solutions.

%v{c}| x + %v{a} | = %v{b}

Submit only the %v{size[choice]} one.

Algebraic Expression:

✔ %v{ans[choice]}

✘ %v{ans[wchoice]}

- You solved for the smaller value of X, try solving for the other side of the absolute value.

Hints:

- Isolate the absolute value on one side of the equal sign.

%v{c}| x + %v{a} | = %v{b}

$$\frac{\%v\{c\}| x + \%v\{a\} |}{\%v\{c\}} = \frac{\%v\{b\}}{\%v\{c\}}$$

| x + %v{a} | = %v{b / c}

x + %v{a} = %v{b / c} or x + %v{a} = -%v{b / c}

Continue Solving for x.

-

x + %v{a} - %v{a} = %v{b / c} - %v{a}   x + %v{a} - %v{a} = -%v{b / c} - %v{a}

x = %v{((1)*b / c) - a}   x = %v{((-1)*b / c) - a}

%v{ans[choice]} is %v{size[choice]}. Type in %v{ans[choice]}.

---

26) Problem #PRA7KZJ "PRA7KZJ - TEMPLATE- Absolute Value Equations 1"

This equation has two solutions.

| %v{c}x + %v{a} | = %v{b}

Submit only the %v{size[choice]} one.

Algebraic Expression:

✓ %v{ans[choice]}

✗ %v{ans[wchoice]}

- You input the %v{size[wchoice]} value, solve for the %v{size[choice]} one.

Hints:

-

$$| \%v\{c\}x + \%v\{a\} | = \%v\{b\}$$

%v{c}x + %v{a} = %v{b}   or   %v{c}x + %v{a} = -%v{b}

Continue Solving for x.

-

%v{c}x + %v{a} - %v{a} = %v{b} - %v{a}   %v{c}x + %v{a} - %v{a} = -%v{b} - %v{a}

%v{c}x = %v{(1)*b - a}   %v{c}x = %v{(-1)*b - a}

$$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{(1)*b - a\}}{\%v\{c\}}$$ $$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{(-1)*b - a\}}{\%v\{c\}}$$

x = %v{((1)*b - a) / c}   x = %v{((-1)*b - a) / c}

%v{ans[choice]} is %v{size[choice]}. Type in %v{ans[choice]}.

Problem Set "Created on 2/5/2016" id:[PSAZ3X9]

☐ **Select All**

☐ **1) Problem #PRA828M "PRA828M - Exponential equations using the Property of Equality"**

Solve for x

$$\%v\{b\}^{\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

Algebraic Expression:

✓ $\%v\{x\}$

Hints:

- Express $\%v\{b\}$ as a power of $\%v\{a\}$.

$$\%v\{b\}^{\%v\{d\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{d\}}$$

Set the exponents to be equal for both sides and solve the equation:

$$\%v\{a\}^{\%v\{e\}*\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

Then,

$$\%v\{e\}*\%v\{d\} = (\%v\{-c\})x+\%v\{f\}$$

-

$$\%v\{e*d\} \quad = (\%v\{-c\})x+\%v\{f\}$$

$$\%v\{e*d\}-\%v\{f\} = (\%v\{-c\})x$$

$$(\%v\{-c\})x = \%v\{e*d\}-\%v\{f\}$$

$$\frac{(\%v\{-c\})x}{(\%v\{-c\})} = \frac{\%v\{e*d-f\}}{(\%v\{-c\})}$$

$$x = \frac{\%v\{e*d-f\}}{(\%v\{-c\})}$$

Type in $\%v\{display[whole]\}$.

☐   **2) Problem #PRA8ZE8 "PRA8ZE8 - Solving 1-Step Subtraction Equations"**

Solve for %v{a}:

%v{a} - (%v{b}) = %v{c}

Algebraic Expression:

✓   %v{b+c}

Hints:

- This is how to solve a problem similar to your problem.

$$m - (-4) = -25$$
$$m + (+4) = -25$$
$$\underline{\quad -4 \quad\quad -4\quad}$$

$$m = -29$$

- The first step to solve is to change the subtraction sign and negative sign in front of %v{b*(-1)} to plus signs.

- Next, you need to do the opposite of the sign in front of %v{b*-1}. Therefore, you must subtract %v{b*-1} from both sides of the equation.

- This is what it should look like:

%v{a} - (%v{b})   = %v{c}
%v{a} + (+ %v{b*(-1)})
        - %v{b*(-1)}   - %v{b*(-1)}

                %v{a}   = %v{c+b}

Type in %v{c+b}

☐   **3) Problem #PRA828K "PRA828K - Exponential equations using the Property of Equality"**

Solve for x

$$\%v\{b\}^{\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

Algebraic Expression:

✓   %v{x}

Hints:

- Express %v{b} as a power of %v{a}.

- %v{b}$^{\%v\{d\}}$ = (%v{a}$^{\%v\{e\}}$)$^{\%v\{d\}}$

- Set the exponents to be equal for both sides and solve the equation:

%v{a}$^{\%v\{e\}*\%v\{d\}}$=%v{a}$^{(\%v\{-c\})x+\%v\{f\}}$

- Then,

%v{e}*%v{d}=(%v{-c})x+%v{f}

-
     %v{e*d}   = (%v{-c})x+%v{f}

%v{e*d}-%v{f} = (%v{-c})x

(%v{-c})x = %v{e*d}-%v{f}

$$\frac{(\%v\{-c\})x}{(\%v\{-c\})} = \frac{\%v\{e*d-f\}}{(\%v\{-c\})}$$

$$x \quad = \frac{\%v\{e*d-f\}}{(\%v\{-c\})}$$

Type in %v{display[whole]}.

☐   **4) Problem #PRA828H "PRA828H - Exponential equations using the Property of Equality"**
Solve for

%v{a}$^{\%v\{c\}x}$ = %v{b}$^{\%v\{d\}}$

Algebraic Expression:
✔️ %v{x}

Hints:

- The bases must be equal in both sides of the equation.

Express %v{b} as a power of %v{a}.

%v{b}$^{\%v\{d\}}$ = (%v{a}$^{\%v\{e\}}$)$^{\%v\{d\}}$

- Now write the complete equation:

$\%v\{a\}^{\%v\{c\}x} = \%v\{a\}^{\%v\{e\}*\%v\{d\}}$

Then apply the Property of Equality to get a linear equation for x.

- $\%v\{a\}^{\%v\{c\}x} = \%v\{a\}^{\%v\{e\}*\%v\{d\}}$

Applying the Property of Equality gives us:

$\%v\{c\}x = \%v\{e\}*\%v\{d\}$

-

$\%v\{c\}x \quad = \quad \%v\{e*d\}$

$$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{e*d\}}{\%v\{c\}}$$

$$x \quad = \quad \frac{\%v\{e*d\}}{\%v\{c\}}$$

- Type in %v{display[whole]}

☐ **5) Problem #PRA8Y4Q "PRA8Y4Q - Equations with complex numbers"**
Find the values of x and y that satisfy the following equation:

$\%v\{a\}x-\%v\{b\}+(y-\%v\{c\})i=\%v\{d\}+\%v\{e\}i$

Directions:

Type in only the value of x.
Algebraic Expression:
✓ %v{(d+b)/a}

 Hints:

- Separate the real and the imaginary parts.

$\%v\{a\}x-\%v\{b\}+(y-\%v\{c\})i=\%v\{d\}+ \%v\{e\}i$
- Real part:

$$\%v\{a\}x - \%v\{b\} = \%v\{d\}$$

$$\%v\{a\}x = \%v\{d\} + \%v\{b\}$$

$$\frac{\%v\{a\}x}{\%v\{a\}} = \frac{\%v\{d+b\}}{\%v\{a\}}$$

$$x = \frac{\%v\{d+b\}}{\%v\{a\}}$$

Imaginary part:

$$y - \%v\{c\} = \%v\{e\}$$

$$y = \%v\{e\} + \%v\{c\}$$

$$y = \%v\{e+c\}$$

Type in only the value of x.

☐   **6) Problem #PRA8Y4N "PRA8Y4N - Equations with complex numbers"**

Find the values of x and y that satisfy the following equation:

$$-\%v\{b\}(y+\%v\{c\})i+\%v\{a\}=\%v\{e\}i+\%v\{d\}x$$

Directions:

Type in only the value of y.

Algebraic Expression:

 ✓ $\%v\{e+b*c\}/\%v\{-b\}$

Hints:

- Separate the real and the imaginary parts.

$$-\%v\{b\}(y+\%v\{c\})i+\%v\{a\}=\%v\{e\}i+\%v\{d\}x$$

- Real part:

$$\%v\{d\}x = \%v\{a\}$$

$$\frac{\%v\{d\}x}{\%v\{d\}} = \frac{\%v\{a\}}{\%v\{d\}}$$

$$x = \frac{\%v\{a\}}{\%v\{d\}}$$

Imaginary part:

$$-\%v\{b\}(y+\%v\{c\}) = \%v\{e\}$$

$$-\%v\{b\}y-\%v\{b\}*\%v\{c\} = \%v\{e\}$$

$$-\%v\{b\}y = \%v\{e+b*c\}$$

$$y = \frac{\%v\{e+b*c\}}{-\%v\{b\}}$$

Type in only the value of y.

☐ **7) Problem #PRA8USZ "PRA8USZ - Equations with complex numbers"**
Find the values of x and y that satisfy the following equation:

$\%v\{a\}x-\%v\{b\}+(y-\%v\{c\})i=\%v\{d\}+\%v\{e\}i$

Directions:

Type in only the value of y.
Algebraic Expression:

✓ $\%v\{y\}$

Hints:

- Separate the real and the imaginary parts.

$\%v\{a\}x-\%v\{b\}+(y-\%v\{c\})i=\%v\{d\}+ \%v\{e\}i$

- Real part:

$\%v\{a\}x-\%v\{b\} = \%v\{d\}$

$$\%v\{a\}x \quad = \%v\{d\}+\%v\{b\}$$

$$\frac{\%v\{a\}x}{\%v\{a\}} = \frac{\%v\{d+b\}}{\%v\{a\}}$$

$$x \quad = \frac{\%v\{d+b\}}{\%v\{a\}}$$

Imaginary part:

$$y-\%v\{c\} = \quad \%v\{e\}$$

$$y \quad = \%v\{e\}+\%v\{c\}$$

$$y \quad = \quad \%v\{e+c\}$$

Type in y=%v{y}

---

8) Problem #PRA8TQD "PRA8TQD - Exponential populations"

A scientist is studying two populations of different types of bacteria. By modeling her data she has come up with the expressions for the population of bacteria for each type:

Type 1: $P = e^{\%v\{a\}t - \%v\{b\}}$

Type 2: $P = e^{\%v\{-c\}t + \%v\{d\}}$

Find the time t, at which the population of bacteria is going to be the same for both types.

Algebraic Expression:

✔ %v{ans}

Hints:

- Set the two population functions to be equal.

$e^{\%v\{a\}t - \%v\{b\}} = e^{\%v\{-c\}t + \%v\{d\}}$

Using the property of equality set the two exponents to be equal:

$\%v\{a\}t - \%v\{b\} = \%v\{-c\}t + \%v\{d\}$

- Solve the linear equation to find the time t at which the populations are equal.

9) Problem #PRA8TF9 "PRA8TF9 - Logarithmic equations (definition of logarithm)"
Simplify the following:

$\sqrt{(-\%v\{b\})}$
Algebraic Expression:

✔️   %v{a}i

  Hints:

- Use imaginary number notation:

$\sqrt{(-1)} = i$

- $\sqrt{(-\%v\{b\})} =$

$\sqrt{(\%v\{b\})} \times \sqrt{(-1)} =$

$(\%v\{a\})i$

Type in %v{a}i

---

10) Problem #PRA8TF3 "PRA8TF3 - Logarithmic equations (definition of logarithm)"
Simplify the following:

$(\%v\{a\} - \%v\{b\}i) + (\%v\{c\} + \%v\{d\}i)$
Algebraic Expression:

✔️   %v{real}+%v{imag}i

  Hints:

- Combine the real parts and combine the imaginary parts:

$(\%v\{a\}$ - $\%v\{b\}i) + (\%v\{c\}$ + $\%v\{d\}i)$

$(\%v\{a\}+\%v\{c\})$ + $(\%v\{-b\}i + \%v\{d\}i)$

$\%v\{real\}$ + $\%v\{imag\}i$

Type in %v{real} + %v{imag}i

---

11) Problem #PRA8S7E "PRA8S7E - Exponential equations using the Property of Equality"
Solve for x in terms of y:

$\%v\{a\}^{y} \times \%v\{b\}^{\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$

Algebraic Expression:

✓ (%v{e*d-f}+y) / (%v{-c})

Hints:

- Express %v{b} as a power of %v{a}.

$$\%v\{b\}^{\%v\{d\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{d\}}$$

Then, the equation is:

$$\%v\{a\}^{y} * \%v\{a\}^{\%v\{e\}*\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

Now the exponents in the left hand side can be added:

$$\%v\{a\}^{\%v\{e\}*\%v\{d\}+y} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

- Set the exponents to be equal for both sides and solve the equation:

$$\%v\{a\}^{\%v\{e\}*\%v\{d\}+y} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

So,

$$\%v\{e\}*\%v\{d\}+y=(\%v\{-c\})x+\%v\{f\}$$

-

  $$\%v\{e*d\}+y \quad = (\%v\{-c\})x+\%v\{f\}$$

$$\%v\{e*d\}-\%v\{f\}+y = (\%v\{-c\})x$$

$$(\%v\{-c\})x = \%v\{e*d\}-\%v\{f\}+y$$

$$\frac{(\%v\{-c\})x}{(\%v\{-c\})} = \frac{\%v\{(e*d)-f\}+y}{(\%v\{-c\})}$$

$$x = \frac{\%v\{e*d-f\}+y}{(\%v\{-c\})}$$

Type in (%v{e*d-f}+y) / (%v{-c}) .

☐   **12) Problem #PRA8S7D "PRA8S7D - Exponential equations using the Property of Equality"**

Solve for x

$$\%v\{a\}^{\%v\{c\}x} = \%v\{b\}^{\%v\{d\}}$$

Algebraic Expression:

✓ $\%v\{x\}$

  Hints:

- The bases must be equal in both sides of the equation.

Express $\%v\{b\}$ as a power of $\%v\{a\}$.

$$\%v\{b\}^{\%v\{d\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{d\}}$$

- Now write the complete equation:

$$\%v\{a\}^{\%v\{c\}x} = \%v\{a\}^{\%v\{e\}*\%v\{d\}}$$

Then apply the Property of Equality to get a linear equation for x.

- $\%v\{a\}^{\%v\{c\}x} = \%v\{a\}^{\%v\{e\}*\%v\{d\}}$

Applying the Property of Equality gives us:

$\%v\{c\}x = \%v\{e\}*\%v\{d\}$

-

$$\%v\{c\}x \;\; = \;\; \%v\{e*d\}$$

$$\frac{\%v\{c\}x}{\%v\{c\}} \;=\; \frac{\%v\{e*d\}}{\%v\{c\}}$$

$$x \;\; = \;\; \frac{\%v\{e*d\}}{\%v\{c\}}$$

- Type in $\%v\{display[whole]\}$

☐   **13) Problem #PRA8S4A "PRA8S4A - Exponential equations using the Property of Equality"**

Solve for x

$$\%v\{a\}^{\%v\{c\}x} = \%v\{b\}^{\%v\{d\}}$$

Algebraic Expression:

✔   %v{x}

Hints:

- Express %v{b} as a power of %v{a}.

- $\%v\{b\}^{\%v\{d\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{d\}}$

- Set the exponents to be equal for both sides and solve the equation:

$$\%v\{a\}^{\%v\{c\}x} = \%v\{a\}^{\%v\{e\}*\%v\{d\}}$$

- Then,

$$\%v\{c\}x = \%v\{e\}*\%v\{d\}$$

-

$$\%v\{c\}x \;=\; \%v\{e*d\}$$

$$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{e*d\}}{\%v\{c\}}$$

$$x \;=\; \frac{\%v\{e*d\}}{\%v\{c\}}$$

       Type in %v{display[whole]}

☐   **14) Problem #PRA8SS6 "PRA8SS6 - Exponential equations using the Property of Equality"**

Solve for x

$$81^{7} = 3^{(-6)x+8}$$

Algebraic Expression:

✔   -3.33333333333333

Hints:

- Express 81 as a power of 3.

$$81^7 = (3^4)^7$$

Set the exponents to be equal for both sides and solve the equation:

$$3^{4*7} = 3^{(-6)x+8}$$

Then,

$$4*7 = (-6)x + 8$$

-

$$28 = (-6)x + 8$$

$$28 - 8 = (-6)x$$

$$(-6)x = 28 - 8$$

$$\frac{(-6)x}{(-6)} = \frac{20}{(-6)}$$

$$x = \frac{20}{(-6)}$$

Type in -20 / 6.

---

15) Problem #PRA8SSH "PRA8SSH - Exponential equations using the Property of Equality"
Solve for x

$$\%v\{b\}^{\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$$

Algebraic Expression:

✓ %v{x}

Hints:

- Express %v{b} as a power of %v{a}.

- $\%v\{b\}^{\%v\{d\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{d\}}$

- Set the exponents to be equal for both sides and solve the equation:

$\%v\{a\}^{\%v\{e\}*\%v\{d\}} = \%v\{a\}^{(\%v\{-c\})x+\%v\{f\}}$

- Then,

$\%v\{e\}*\%v\{d\} = (\%v\{-c\})x+\%v\{f\}$

-

$\%v\{e*d\} \quad\quad = (\%v\{-c\})x+\%v\{f\}$

$\%v\{e*d\}-\%v\{f\} = (\%v\{-c\})x$

$(\%v\{-c\})x = \%v\{e*d\}-\%v\{f\}$

$$\frac{(\%v\{-c\})x}{(\%v\{-c\})} = \frac{\%v\{e*d-f\}}{(\%v\{-c\})}$$

$$x = \frac{\%v\{e*d-f\}}{(\%v\{-c\})}$$

Type in $\%v\{display[whole]\}$.

☐   **16) Problem #PRA74QP "PRA74QP - Logarithmic equations (definition of logarithm)"**

Solve the following logarithmic equation:

$$\log_{\%v\{base\}} x = \frac{\%v\{b\}}{\%v\{e\}}$$

Algebraic Expression:

✓ $\%v\{x\}$

Hints:

- Definition of logarithm:

$x = \%v\{base\}^{\%v\{b\}/\%v\{e\}}$

- Use the fact that:

$\%v\{base\} = \%v\{a\}^{\%v\{e\}}$

Then:

$$\%v\{base\}^{\%v\{b\}/\%v\{e\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{b\}/\%v\{e\}} = \%v\{a\}^{\%v\{e\}*(\%v\{b\}/\%v\{e\})} = \%v\{a\}^{\%v\{b\}}$$

- $x = \%v\{a\}^{\%v\{b\}}$,

$x = \%v\{a**b\}$,

Type in $\%v\{x\}$

---

17) Problem #PRA73PT "PRA73PT - Logarithmic Equations using Property of Equality"
Solve for x

$\log(\%v\{a\}x+\%v\{b\})=\log(\%v\{c\}x-\%v\{d\})$
Algebraic Expression:

✓ $\%v\{x\}$

Hints:

-

Use the property of equality:

$\log(\%v\{a\}x+\%v\{b\})=\log(\%v\{c\}x-\%v\{d\})$

Set the arguments of the logarithms to be equal:

$(\%v\{a\}x+\%v\{b\})=(\%v\{c\}x-\%v\{d\})$

- Solve the linear equation:

$(\%v\{a\}x+\%v\{b\}) = (\%v\{c\}x-\%v\{d\})$

$\%v\{a\}x-\%v\{c\}x = \%v\{-d\}-\%v\{b\}$

$\%v\{a-c\}x = \%v\{-d-b\}$

$$\frac{\%v\{a-c\}x}{\%v\{a-c\}} = \frac{\%v\{-d-b\}}{\%v\{a-c\}}$$

$$x = \frac{\%v\{-d-b\}}{\%v\{a-c\}}$$

$x = \%v\{display[whole]\}$

For the solution to be correct, the arguments of both logarithms must be positive for this value of x.

- Perform a check:

%v{a}x+%v{b} is always positive for a positive x

%v{c}x-%v{d} = %v{c}(%v{display[whole]})-%v{d} = %v{x*c}-%v{d} = %v{(x*c)-d} which is positive

Then x = %v{display[whole]} is a correct solution

Type in %v{display[whole]}

---

18) Problem #PRA7XRR "PRA7XRR - Exponential equations applying interest rates"
An investment account pays %v{r*100}% annual interest rate compounded %v{frequency[pointer]}. If the investment amount is %v{p}$, find the balance in the account after %v{years} years.
Algebraic Expression:

✔ 1

Hints:

- Hi

---

19) Problem #PRA7WQK "PRA7WQK - Exponential equations using the Property of Equality"
Solve for x

$$\%v\{a\}^{\%v\{c\}x-\%v\{f\}}=\%v\{b\}^{\%v\{d\}}$$

Algebraic Expression:

✔ %v{x}

Hints:

- Express %v{b} as a power of %v{a}.

$$\%v\{b\}^{\%v\{d\}} = (\%v\{a\}^{\%v\{e\}})^{\%v\{d\}}$$

Then, the equation is:

$$\%v\{a\}^{\%v\{+c\}x-\%v\{f\}}=\%v\{a\}^{\%v\{e\}*\%v\{d\}}$$

- Set the exponents to be equal for both sides and solve the equation:

$$\%v\{a\}^{\%v\{+c\}x-\%v\{f\}}=\%v\{a\}^{\%v\{e\}*\%v\{d\}}$$

So,

%v{+c}x-%v{f}=%v{e}*%v{d}

-

%v{c}x-%v{f}   =   %v{e*d}

%v{c}x   =   %v{e*d}+%v{f}

$$\frac{\%v\{c\}x}{\%v\{c\}} = \frac{\%v\{e*d+f\}}{\%v\{c\}}$$

$$x = \frac{\%v\{e*d+f\}}{\%v\{c\}}$$

Type in %v{display[whole]}

---

20) Problem #PRA7SBH "PRA7SBH - Equations with complex numbers"

Find the values of x and y that satisfy the following equation:

%v{a}x-%v{b}+(y-%v{c})i=%v{d}+%v{e}i

Directions:

Type in only the value of y.

Algebraic Expression:

✔ %v{y}

Hints:

- Separate the real and the imaginary parts.

%v{a}x-%v{b}+(y-%v{c})i=%v{d}+ %v{e}i

- Real part:

%v{a}x-%v{b}   =   %v{d}

%v{a}x   =   %v{d}+%v{b}

$$\frac{\%v\{a\}x}{\%v\{a\}} = \frac{\%v\{d+b\}}{\%v\{a\}}$$

$$x = \frac{\%v\{d+b\}}{}$$

%v{a}

Imaginary part:

y-%v{c} =   %v{e}

y   = %v{e}+%v{c}

y   =   %v{e+c}

Type in only the value of y.

☐ **21) Problem #PRA7R4D "PRA7R4D - Remainder theorem"**

Using the remainder theorem, calculate the remainder of the following polynomial division:

$(\%v\{a\}x^2+\%v\{b\}x+\%v\{c\}) \div (x-\%v\{d\})$

Algebraic Expression:

✓ %v{r}

Hints:

- When f(x) is divided by (x-c), the remainder is equal to f(c).

- c = %v{d}

$f(\%v\{d\})=(\%v\{a\}\times\%v\{d\}^2+\%v\{b\}\times\%v\{d\}+\%v\{c\})$

- f(%v{d}) = %v{r}

Type in %v{r}

☐ **22) Problem #PRA7QB8 "PRA7QB8 - Quadratic formula expressions"**

The quadratic expression $\%v\{a\}x^2\%v\{b\}x+\%v\{c\}$ is given. Find the quadratic formula for this expression and write it in the form: a±bi.
The constants a and b should be kept to 2 decimal places.

Give your answer in the form: a and b

Algebraic Expression:

✓ 1

Hints:

- The correct answer is %v{d}±%v{e}i

☐ **23) Problem #PRA7QB7 "PRA7QB7 - Remainder theorem"**

Using the remainder theorem, calculate the remainder of the following polynomial division:

$(-\%v\{a\}x^2+\%v\{b\}x-\%v\{c\}) \div (x+\%v\{d\})$

Algebraic Expression:

✔ %v{r}

Hints:

- When f(x) is divided by (x-c), the remainder is equal to f(c).

- c = %v{-d}

$f(\%v\{-d\}) = ((-\%v\{a\})*(\%v\{-d\})^2+\%v\{b\}*(\%v\{-d\})-\%v\{c\})$

- Then we compute:

$f(\%v\{-d\}) = ((-\%v\{a\})*(\%v\{-d\})^2+\%v\{b\}*(\%v\{-d\})-\%v\{c\})$

$f(\%v\{-d\}) = (\%v\{(-a)*((-d)**2)\}+\%v\{b*(-d)\}-\%v\{c\})$

$f(\%v\{-d\}) = \%v\{r\}$

Type in %v{r}

☐   **24) Problem #PRA7QB6 "PRA7QB6 - Remainder theorem"**

Using the remainder theorem, calculate the remainder of the following polynomial division:

$(\%v\{a\}x^2-\%v\{b\}x+\%v\{c\}) \div (x-\%v\{d\})$

Algebraic Expression:

✔ %v{r}

Hints:

- When f(x) is divided by (x-c), the remainder is equal to f(c).

- c = %v{d}

$f(\%v\{d\})=(\%v\{a\}*\%v\{d\}^2-\%v\{b\}*\%v\{d\}+\%v\{c\})$

- Then we compute:

$f(\%v\{d\}) = (\%v\{a\}*\%v\{d\}^2-\%v\{b\}*\%v\{d\}+\%v\{c\})$

$f(\%v\{d\}) = (\%v\{a*(d**2)\}-\%v\{b*d\}+\%v\{c\})$

$f(\%v\{d\}) = \%v\{r\}$

Type in %v{r}

☐   **25) Problem #PRA7QB5 "PRA7QB5 - Remainder theorem"**

Using the remainder theorem, calculate the remainder of the following polynomial division:

$(\%v\{a\}x^2-\%v\{b\}x+\%v\{c\}) \div (x-\%v\{d\})$

Algebraic Expression:

✓ %v{r}

Hints:

- When f(x) is divided by (x-c), the remainder is equal to f(c).

- The value of f(c) is found by replacing x with the value of c in the first polynomial. In this case,
f(%v{d})=(%v{a}×(%v{d})^2-%v{b}×(%v{d})+%v{c}

- The correct answer is %v{r}.

☐ **26) Problem #PRA7MMP "PRA7MMP - Remainder theorem"**

Using the remainder theorem calculate the remainder of the following polynomial division:

%v{a}*x^2+%v{b}x+%v{c} / %v{d}

Algebraic Expression:

✓ %v{r}

Hints: