

ANALYSIS OF THE WPI ON-LINE CALCULUS PLACEMENT TESTS

An Interactive Qualifying Project Report

submitted to the Faculty of

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Degree of Bachelor of Science

by

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Date: April 29, 2008

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# Table of Contents

<b>1. Introduction</b> .....	<b>1</b>
<b>2. Background</b> .....	<b>5</b>
2.1. Format and Content of the Calculus Placement Tests .....	5
2.2. Comparison with Former Calculus Readiness Test .....	11
2.3. Review of Literature .....	15
<b>3. Results: Analysis of Student Performance</b> .....	<b>20</b>
3.1. Scores on the Calculus Placement Tests.....	20
3.2. Performance in A-Term Mathematics Courses.....	30
3.2.1. Calculus II Final Grades .....	36
3.2.2. Student Scores on Calculus I Final Exam.....	41
3.3. Performance in B-Term Mathematics Courses.....	48
<b>4. Results: Analysis of Placement Test Structure and Design</b> .....	<b>54</b>
4.1. Internal Consistency.....	54
4.2. Analysis of Placement Test Completion Time .....	65
<b>5. Results: Correlation Analysis for the Placement Tests</b> .....	<b>83</b>
5.1. Using Calculus I Placement Tests to Predict Calculus I Performance .....	83
5.2. Comparing Placement Recommendations with Course Choices.....	90
5.3. Placement Test Cutoff Scores.....	94
<b>6. Conclusions and Recommendations</b> .....	<b>118</b>
<b>7. Future Work</b> .....	<b>122</b>
<b>8. Bibliography</b> .....	<b>124</b>
<b>9. Appendix</b> .....	<b>127</b>
9.1. Outline of Goals.....	127
9.2. Calculus Placement Test Hardcopies.....	127
9.3. Calculus Readiness Test Hardcopy.....	127

# 1. Introduction

During the summer of 2007, a new Internet-based Calculus Placement Test was administered to the incoming Class of 2011 at Worcester Polytechnic Institute (WPI) in Worcester, MA. This optional test generated a non-binding recommendation for a first university mathematics course. The goal of this study is to determine the extent to which the new Calculus Placement Tests communicated course content and made appropriate course recommendations.

Because placement testing can have a large impact on a student's academic program, the notion of an "effective" placement test must be very carefully defined. Simply put, a good placement test should recommend a course that suits the student's academic background. That is, a student should understand all of the necessary prerequisite concepts for the recommended class, but the student still should be challenged at an appropriate level. The goal of the new WPI Calculus Placement Tests was therefore to communicate to students an honest representation of prerequisite material for various mathematics courses and to test their knowledge of that prerequisite material. Having been provided with this information, students were allowed to decide for themselves which mathematics course they would take. Of course, a student's success in their chosen course depends on many more factors than their performance on the placement test; high scores on even the highest-quality placement test would not guarantee high grades in the ensuing course. An effective placement test would therefore provide a valid, self-consistent summary of the prerequisite concepts of a mathematics course, communicate those prerequisites to a student completing the test, and through this accurate communication influence the course choices of students.

One of the most important contributors to the effectiveness of a placement test is simply its content. In the model used for this study, a placement test should be an honest representation of the prerequisite material for the corresponding course. If the problems on the placement test are well-suited to measure what the placement test was designed to measure, then the placement test is said to have a high degree of content validity. Content validity is achieved by carefully examining the prerequisite material, condensing these prerequisites into a short list of critical topics, and writing questions that accurately assess student proficiency in those topics.

Like any other educational measurement, a placement test must also be reliable; it must present a self-consistent set of test items that each contribute meaningfully and reproducibly to the overall score and to the resulting recommendation. One measurement of reliability is internal consistency, or the extent to which test items are correlated with each other and with the overall score. While an extremely high level of internal consistency might indicate redundancy, a reasonable level of consistency ensures that test items are similar enough in difficulty to make the test meaningful. In this way, analysis of internal consistency could identify items that should be edited or replaced for reasons of poor wording or inappropriate level of difficulty.

A more difficult quality to measure, but one with much more far-reaching implications, is that of predictive validity. Broadly speaking, predictive validity for a placement test is the extent to which the test predicts success in the corresponding course. For example, if a placement test is indeed an accurate representation of prerequisite material, then a student earning a high score on this test might also be more likely to obtain a high grade in the resulting mathematics course. Grades are obviously influenced by many more factors than simple proficiency with prerequisites. Nevertheless, it is important to make extremely clear the extent to which placement test results may be used to predict course choice and course grades.

Because the WPI Calculus Placement Tests were optional and provided non-binding course recommendations, the extent to which students actually followed these course recommendations is important. Even if it is assumed that a placement test is well-designed and produces appropriate course recommendations, students still must follow the recommendation in order to reap the full benefit of the placement exercise. To determine whether following the course recommendation was in fact beneficial to students, it would be worthwhile to compare the grade distributions of students who followed the placement test recommendation with those who did not. If this comparison showed that following the recommendation was significantly beneficial to students, special attention would need to be devoted to explaining and publicizing the placement tests to maximize the likelihood of its recommendations being followed.

Throughout this study it is important to be mindful that many other factors affect the placement of students in mathematics courses. Often, students who take Advanced Placement courses and examinations base their course choices on those results. Personal advice from parents, peers, and teachers also has a strong influence on student decision-making. Also, even if a student acknowledges the correctness of a placement test recommendation, he or she may deliberately choose to enroll in a course that is either more or less difficult. This decision might depend on a student's risk tolerance and perception of the rigors of university-level studies. In short, even an excellent placement test is in competition with many other sources of advice, so it is critical that the test be an effective *communicator* of course expectations and content, in addition to providing an accurate recommendation.

The new Calculus Placement Tests were developed in large part because of the advantages of having an Internet-based system; however, as with any other test format, there are both advantages and disadvantages to Internet-based testing. Many logistical difficulties are

overcome with this format, in exchange for a lowered amount of control over the specific manner in which students complete the test. For example, students may complete the tests very quickly or over a period of many days, and they may or may not collaborate with fellow students and others. An honest evaluation of the appropriateness of this open-ended format for calculus placement tests is therefore important. This warrants a search of the literature on placement testing, as well as a comparison of the WPI testing system with those of other institutions. Finally, if the placement tests were found to provide reliable, valid information despite the less-controlled environment of an Internet-based test, then the offering of the test in this form would be justified.

This study addresses the effectiveness of the WPI Calculus Placement Tests from a variety of perspectives. Certainly, it is critical to consider general concepts of test design such as reliability and validity. More specifically, the Placement Tests should be able to communicate the content of the corresponding mathematics courses while also producing a recommendation that is helpful to students. Finally, the Internet-based format of the Placement Tests, and the implications of that format, plays a central role in considerations of effectiveness.

## 2. Background

### 2.1. Format and Content of the Calculus Placement Tests

WeBWorK is an Internet-based application developed by the University of Rochester for “generating and delivering homework problems to students,” usually in mathematics or physics classes (WeBWorKdocs). The following discussion serves to familiarize the reader with WeBWorK in general and with its use in the WPI Calculus Placement Tests in particular.

By making homework assignments Internet-based, WeBWorK streamlines the process of completing and grading homework and implements many additional features that are impossible with traditional homework assignments. Students and professors can access a WeBWorK homework assignment from any computer with an Internet connection. Each student receives similar but unique versions of the problem; typically various numerical constants will differ from student to student. This allows students to collaborate but prevents direct copying of answers from peers. Students may attempt a particular problem more than once, either with a limited total number of attempts or with no limit, at the instructor’s discretion. Upon submission of an answer, students are immediately told whether their answer was correct.

Instructors have access to detailed information about the scores of individual students or aggregate statistics about the overall performance of the class. Instructors may even “act” as a particular student to view that student’s personalized version of the homework set and all previous responses the student has made. Furthermore, WeBWorK facilitates communication between students and instructors; if a student desires assistance on a particular problem, the student can press a “Feedback” button that sends e-mail to a designated instructor.

Beyond these features, which would facilitate online homework in any subject, WeBWorK is particularly suited to courses in mathematics. Whereas many computer-based mathematics tests are restricted to a multiple-choice format, WeBWorK allows for a wide variety of question types, even short-answer questions involving symbolic computations. The programming language of WeBWorK borrows syntax and functionality from the scripting language Perl and the typesetting language LaTeX, making it a powerful tool for creating homework problems. If instructors do not wish to create their own homework problems, the University of Rochester and other universities offer extensive libraries of problems that may be used “as is” or with modifications. Problems may be written to require numeric, symbolic, or string answers, each of which is handled appropriately by WeBWorK. Sophisticated numerical subroutines are in place to recognize numerically equivalent forms of an answer, such as  $\cos(0)$  and 1. Using the existing scripts and subroutines, nearly any conceivable mathematics problem may be implemented in WeBWorK, and the ability to write new subroutines and homework problems allows for functionality that is indeed limitless.

By far the most common use of WeBWorK is for online homework assignments; however, Worcester Polytechnic Institute recognized the potential usefulness of WeBWorK for mathematics placement tests. An Internet-based placement test, offered over an extended timeframe, would not require students to travel to campus simply to take a test. In addition, placement information could be available well in advance of a student’s arrival on campus, allowing more time to select courses appropriately. For these reasons, WPI created a new placement testing system for calculus using WeBWorK.

The testing system was designed so that students could complete the Calculus Placement Tests in the summer before matriculating to WPI. In the summer of 2007, the Placement Tests



were available to students for a period of approximately four weeks, starting in the last week of May. Students could complete the Placement Tests at any time during this period and were allowed to save their work and resume later. Upon completion of a particular test, a student would receive an e-mail message with their score and an interpretation of that score in terms of course selection.

The first mathematics course of a first-year student at WPI may be any of a number of courses. Students who require a review of precalculus material are advised to take Calculus I with Preliminary Topics (MA 1020), which reviews algebra, trigonometry, geometry, and functions before presenting the topics of differential calculus. Students who are prepared to begin calculus courses select either Calculus I (differential calculus), Calculus II (integral calculus), Calculus III (series and vector algebra), or Calculus IV (multivariable calculus) depending on their mathematics background. If a student completed Advanced Placement courses in high school, then the selection of a first mathematics course is often informed by the student's Advanced Placement Exam scores in mathematics. However, all students are ultimately free to choose any mathematics course, and it is therefore important to communicate the content of each course to students as well as to assess the current state of their mathematics knowledge accurately.

The WeBWork placement testing system was divided into three separate placement tests, denoted "Calculus I Placement," "Calculus II Placement," and "Calculus III Placement." A particular placement test determined whether a student was prepared to enroll in the course for which the test is named, and this was done by testing proficiency with appropriate prerequisite material. For example, the Calculus I Placement Test would assess a student's knowledge of precalculus topics, and if a deficiency in precalculus knowledge were found, the student would

be recommended to enroll in Calculus I with Preliminary Topics. (Given that WPI has a Calculus IV course in multivariable calculus, the inclusion of a “Calculus IV Placement” test might seem natural; however, very few students in the typical entering class are prepared to study multivariable calculus as a first mathematics course, and those who are prepared usually have Advanced Placement credit indicating the appropriate course choice.) All three tests took full advantage of the capabilities of WeBWorK and used short-answer questions rather than multiple-choice items. For all three of the Calculus Placement Tests, students were limited to three attempts per problem. In most cases, a hint was displayed after two incorrect attempts.

Because it was assumed that students whose calculus background was uncertain would require the most guidance for course selection, the Calculus I Placement Test was longer and more highly structured than the other WeBWorK tests. The 16 total questions of the Calculus I Placement Test were equally subdivided among four main subject areas: Algebra, Geometry, Trigonometry, and Functions. Students whose total score on the exam was 13 or higher (out of a possible 16) were recommended for Calculus I (MA 1021). Students with scores of 8 or lower were recommended for Calculus I with Preliminary Topics (MA 1020). If a score was higher than 8 but lower than 13, the student was classified as “Borderline” and asked to consult with their faculty advisor before choosing a mathematics course. In addition, students received information about their scores in the four specific subject areas of Algebra, Geometry, Trigonometry, and Functions. Students with a score of 2 or less (out of a possible 4) were recommended to complete a remedial program called “JumpStart” in that particular subject area.

The Calculus II assessed prerequisite knowledge of Calculus I (differential calculus), and the Calculus III Placement Tests tested knowledge of Calculus II (integral calculus). These two Placement Tests were not subdivided by topic as was the Calculus I Placement Test. The

Calculus II Placement Test consisted of 8 questions about differential calculus and a score of 6 or higher resulted in a recommendation to enroll in Calculus II (MA 1022). Students scoring lower than 6 on Calculus II Placement were recommended for Calculus I (MA 1021). There were 9 problems in the Calculus III Placement Test, which tested concepts of integral calculus. Students were recommended for Calculus III (MA 1023) with scores of 7 or higher and were recommended for Calculus II otherwise. With no historical data from previous placement tests, these cutoff scores were set arbitrarily based on the notion that a score of approximately 75% should be considered a passing score. For the summer of 2007, there was no classification of “Borderline” for Calculus II and Calculus III Placement as there was with Calculus I Placement.

It is important to note that none of the recommendations of the Calculus Placement Test were binding; indeed, completion of the tests was optional, albeit highly encouraged. The number of tests that a student chose to complete was also optional; students could complete the Calculus I and Calculus II Placement Tests without an obligation to complete Calculus III Placement. In addition, several students took the Calculus III Placement Test without taking either the Calculus I or Calculus II tests (See *3.1. Scores on the Calculus Placement Tests*).

After taking some or all of the Placement Tests, a student received a recommendation for a first mathematics course. Of course, the course recommendation generated from a higher-level placement test overrode that of a lower-level test in most cases. For example, passing scores on both Calculus I and Calculus II Placement would result in a recommendation for enrollment in Calculus II. However, it was possible for some students to receive contradictory recommendations. In some cases, students received passing scores for all three placement tests (implying that they should begin in Calculus III) while scoring 2 or lower on Algebra, Geometry, Trigonometry, or Functions (implying that remedial work in

some area of pre-calculus was necessary) (See 3.1. *Scores on the Calculus Placement Tests*). Personal consultation with an academic advisor at WPI was typically used to resolve these potentially unclear scenarios.

Because students could pause and later resume their work as desired, it was necessary to have an explicit indicator of when a student was finished with a test. To that end, a question was placed at the end of each test for a student to answer when their test was complete; this question served no other purpose and did not contribute to the total score. Within 24 hours of answering that question, a student would receive an e-mail message with their test score and the corresponding recommendation.

## 2.2. Comparison with Former Calculus Readiness Test

The new Internet-based Calculus Placement Tests are a replacement for an older placement test known as the *Calculus Readiness Test*.

The Calculus Readiness Test consisted of 25 multiple-choice questions, each with five possible answers. Originally, the test was offered during New Student Orientation just before the start of the academic year. Students were assembled in a central location to take the test, calculators were prohibited, and a time limit of 30 minutes was enforced. In 2002, the same test was made available on the Internet during the summer prior to the arrival of new students to campus. Students took the test at home, graded their test themselves, and emailed the score to the Math Department at WPI. No time limit was enforced and students often admitted to using a calculator, even though the instructions asked them not to. (The scores on the Calculus Readiness Test increased significantly when it was first administered in this way.)

The content of the Calculus Readiness Test included various precalculus subjects such as algebra, geometry, trigonometry, and functions. Significantly, the goal of the Calculus Readiness Test was to determine whether or not a student should enroll in MA 1020 (Calculus I with Preliminary Topics). For students considering a more advanced course from the regular calculus sequence, the test provided no information regarding the particular course to choose.

The new Calculus Placement Tests differ significantly in format from the old Calculus Readiness Test. There is no time limit, and students may attempt each question up to three times. The questions themselves have a variety of formats but are generally of the short-answer type; indeed, the new tests contain no multiple choice questions. Also, where the old Calculus Readiness Test included only precalculus material, the new Calculus Placement Tests contain

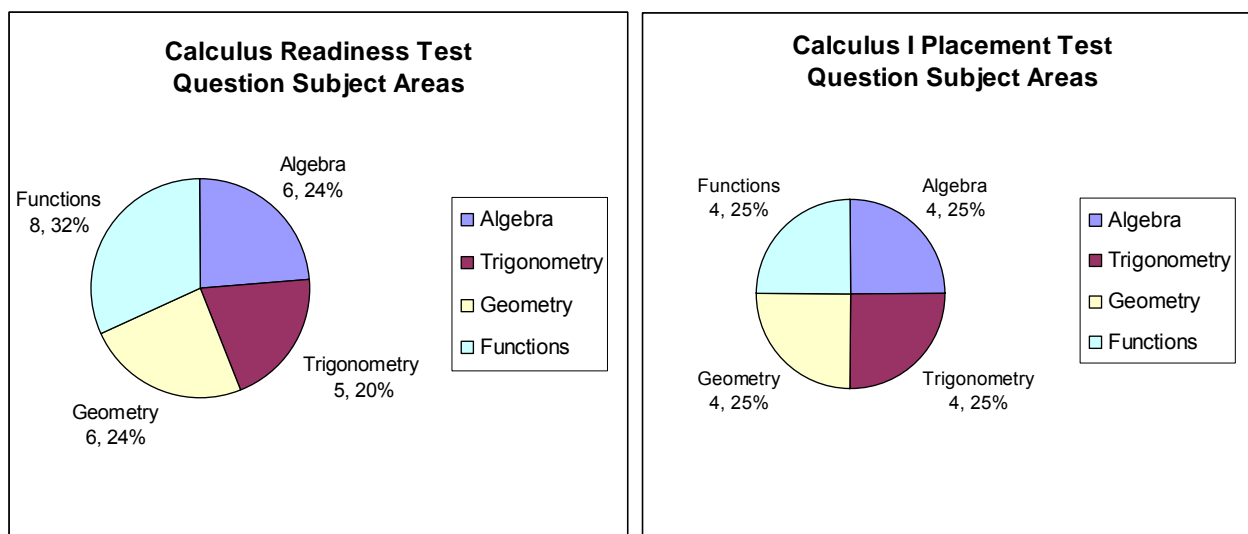
topics from calculus as well. Because of this increase in scope, the new tests have enough information to help a student choose between Calculus I with Preliminary Topics (MA 1020), Calculus I (MA 1021), Calculus II (MA 1022), and Calculus III (MA 1023).

While the new Calculus Placement Tests do indeed differ from the Calculus Readiness Test in many ways, it is believed that these differences constitute improvements over the former system. By allowing students to complete the tests at their leisure over an essentially unlimited time, the logistical difficulties encountered when a test is administered at a fixed place and time are eliminated. Allowing students three attempts per question reduces the possibility of careless errors resulting in incorrect scores, whether those are errors in mathematical computation or simply typographical errors. The WeBWorK system used for the placement tests allows for automatic grading of a very wide variety of question types, allowing the use of questions (such as the symbolic computation of a derivative) that would be difficult or impossible to implement in a multiple-choice test (See 2.1. *Format and Content of the Calculus Placement Tests*).

WeBWorK also allows each student to take a unique, personalized version of the tests with different values for numerical constants and other slight person-to-person differences. Finally, and most importantly, the ability of the Calculus Placement Tests to tell students not only *whether* they are ready to begin the regular calculus sequence, but also *which* course to take in that sequence, is seen as a significant advantage of the new testing scheme.

Although the new Calculus Placement Tests differ significantly in format from the Calculus Readiness Test, it is important for the content of the two tests to be comparable. Of course, the questions on the new tests that pertain to calculus have no counterpart in the former Calculus Readiness Test. On the other hand, the precalculus questions in the two tests should be similar in content. Like those in the new Calculus I Placement Test, the questions in the

Calculus Readiness Test may be classified as pertaining to one of four major pre-calculus mathematical topics: algebra, geometry, trigonometry, or functions. Certainly, there is considerable overlap among these categories (is a question that requires algebraic manipulation of a logarithmic function a question about *algebra* or about *functions*?), but classification along these lines allows for straightforward comparisons of test content. Figure 1 below shows the distribution of questions among the four major subject areas for the old and new placement tests.



**Figure 1: Content comparison of Calculus Readiness Test and Calculus I Placement Test**

Both the Calculus Readiness Test and the Calculus I Placement Test are fairly evenly split between the four subject areas in question, with the Calculus Readiness Test perhaps emphasizing functions slightly at the expense of trigonometry. The Calculus I Placement Test was specifically designed to address those four subject areas to an equal extent. At this level, the content of the two placement tests is certainly comparable.

It could easily be argued that while the basic subject matter of the Calculus I Placement Test is comparable to the Calculus Readiness Test, the new Placement Test is more difficult. For example, the trigonometry questions of the Calculus Readiness Test were confined to the values,

simple identities, and basic graphs of trigonometric functions (See Appendix, Calculus Readiness Test, Questions 21-25). The Calculus I Placement Test, on the other hand, contained a question involving the Law of Sines (Appendix, Calculus I Placement Test, Question 7) and a more difficult and open-ended graphing problem (Question 8). Questions involving functions and their graphs were mostly confined to linear and quadratic functions in the Calculus Readiness Test, whereas the Calculus I Placement Test also addressed implicit equations for a circle (Question 9) and an ellipse (Question 10). Indeed, all three of the new Calculus Placement Tests were designed to be more difficult than the typical multiple-choice placement test. With unlimited time, three attempts per problem, and the potential (however dishonest) for the use of a calculator or other outside help, students had more opportunities to answer a given question correctly with the new Internet-based format. The increased difficulty of the new tests was a deliberate, and arguably successful, attempt to compensate for that tendency.

The former Calculus Readiness Test, it could be said, served as a starting point from which an improved Internet-based placement system could be constructed. The format of the new Calculus Placement Tests allows for greatly increased flexibility in test design and administration. The new tests maintain a similar scope in precalculus material, and through the addition of topics from calculus the usefulness of the placement system has been greatly expanded.



### 2.3. Review of Literature

The purpose of this study was to determine the effectiveness of the WPI Calculus Placement Tests. There is a large literature on the science of test design as well as the analysis of test results (Allen). Placement tests, especially on-line placement tests, have special features that must be considered in evaluating their effectiveness.

The way the test is delivered has an important impact on the meaning of the test results. A test delivered in a controlled environment, monitored and timed without instant feedback to the student, is the “traditional” approach to testing. Perhaps the main advantage of the traditional pencil and paper tests is that a student’s performance on the test is determined almost entirely by how well the student knows the material on the test. The disadvantage is that it is not always practical or possible to bring all students to the controlled environment for the test.

On-line tests have the advantage that they can be available whenever the student has the time to take the test. For on-line tests, where the students are not supervised, there is no guarantee that the student receiving the grade did the work entirely on their own. For example, WPI students are asked to complete the Calculus Placement Tests without using a calculator, but more than half of the students in an informal survey admitted that they had used a calculator. The fact that the time spent on the test is not controlled also has an impact on the scores. For example, a student who spends 40 minutes on the Calculus I Placement Test (with no calculator, no books, and no assistance) and scores 12 out of 16 is probably better prepared for Calculus I than a student who spends 4 days, uses a calculator and a textbook and several friends, to score 15 out of 16.

Until recently, on-line mathematics tests had to consist of only multiple-choice questions. The development of WeBWorK at the University of Rochester has made it possible to deliver short-answer questions in an on-line environment. The key is that WeBWorK recognized equivalent forms of the correct solution to a problem. This has made it possible to deliver problems that are much more like the problems that students will actually have to do in a regular mathematics course. It also eliminates the kind of test-taking strategies encouraged by multiple-choice testing, where the goal is often to eliminate answers that cannot be right rather than to actually find the correct solution.

The many successful placement tests in existence exhibit a variety of designs and formats. The WPI Calculus Placement Tests are online, optional, non-binding placement tests with little time restriction (students have nearly a month-long window to complete the tests). Although some placement tests are still taken with paper and pencil, many universities have switched to an online test for the advantages present in that system. For example, the University of Nebraska currently uses an online placement test with features such as instant feedback and automatic grading (Soh 1). There are still disadvantages inherent in an online test, such as the difficulty of monitoring such a test.

Many universities use some type of standardized test for placement. For example, it is not uncommon for tests such as the ACT or SAT to be used in placement (Marshall 2). The Mathematical Association of America (MAA) developed a Calculus Readiness Test that has been used nationally for many years. The MAA test was used at WPI prior to developing the current online exam. Recently, Maplesoft partnered with the MAA to develop a collection of online Mathematics Readiness Tests supported by the Maple software package. Although these standardized tests are a convenient option for universities, it could be argued that there are

advantages to developing exams that are tailored to the needs of a particular university or even a particular course. The University of California (Web-based Calculus Readiness Test) system has a well-developed Calculus Placement test that is delivered (and graded) automatically on-line. The California test consists of about 40 multiple choice questions exploring areas similar to those included in the WPI Calculus I Placement Test.

Placement tests have a different purpose than end-of-course tests designed to evaluate student performance and assign a grade after a student has completed a specific course. There is certainly a different kind of motivation to do well on a test if the test will determine whether or not a student receives credit for a class. A student who does poorly may have to pay additional tuition to retake the course or may not be allowed to progress to the desired next course.

Placement tests are “low stakes” and generate a recommendation that can be used (or ignored).

Two central requirements of placement tests, and indeed of almost any psychometric measurement, are *reliability* and *validity* (Kane 1). Reliability measures the degree to which a test is consistent with itself. As an example, if a student takes a test and receives a certain score, then the student should get a similar score if he takes the test a second time. The *validity* of a test is the degree to which the test measures what it is intended to measure. An effective test must be both valid and reliable.

Reliability may be measured by a number of statistical methods, most of which focus on the *internal consistency* of the test. For example, it is common to analyze the item-item correlation between different pairs of test items. If two items are highly correlated, then a student who answers one question correctly would be more likely to answer the second question correctly as well. In a sense, the two questions are then measuring the same thing. Item-total correlation also measures whether items are consistent with one another by comparing one item

on the test to the entire test; in other words, how well did success in a particular item on the test predict overall success on the test? Both of these methods are useful in trying to find single questions that may be different from the other questions on the test. Questions that do not correlate well with other questions on the test are candidates for revision or removal. Another way to measure reliability in a test is a statistic called Cronbach's Alpha, which will be studied in Section 4.1. *Internal Consistency*.

Another way to determine the reliability of a test is to see how it compares with existing tests already deemed to be reliable. If a new calculus placement test is consistent with an established calculus placement test already certified to be reliable, then the new test is also reliable.

There are also different ways to determine the validity of a test. One approach is to follow up with the students who took the tests (Frisbie), in our case placement tests, to see if the students performed well in the course recommended. In another study done at the University of Nebraska, students were followed through a particular class (Soh 2) to validate a gateway test. To pass the course, the student must pass a gateway test (much like the Basic Skills Test in Calculus I and Calculus II at WPI). To determine if their gateway exam was measuring how well a student had learned the material, they gave the test to the students twice. They gave it first before students started the course and then again after the students had completed the course. Students, understandably, did not do as well the first time. When they compared the post-test grades with student work from the class, they found that students who did well in the coursework also did well on the post-gateway exam. Students also did better on the post-test than the pre-test, showing that the test provided validation for their test.

Another method of validating a test, especially a non-mandatory placement test, is to record the opinions of the students. Students at the University of Nebraska were asked to fill out a survey regarding their opinions about the usefulness of the online gateway exam (Soh 2).

There are other methods of validation as well. The correlation between scores on the placement test and grades in the course taken provides a measure of validity. Students who do well on the placement exam should also do well in the class. If the placement test is not binding and there is a group of students who chose to ignore the recommendation, then it is also possible to compare the groups who followed the recommendation to the students who did not to see if the students who followed the recommendation did better. In this study, a variety of validation techniques were used to study the Calculus Placement Tests.

### **3. Results: Analysis of Student Performance**

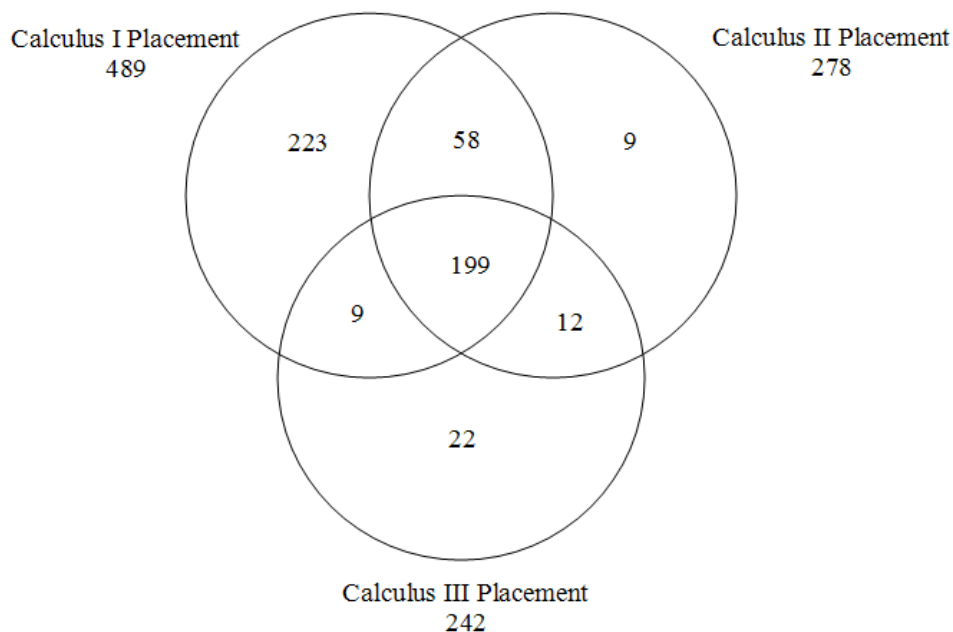
#### **3.1. Scores on the Calculus Placement Tests**

The Calculus Placement Tests consisted of several questions to test whether a student is prepared to succeed in a particular class. There were three separate Placement Tests which tested prerequisite material for Calculus I, II, and III. Students could attempt as many of the tests as they wished and received a recommendation based upon the scores of the tests that they did attempt and complete. This section serves to make general, quantitative statements about the number of students who completed the Placement Tests, the scores students received, and the resulting course recommendations.

Recall that completion of any of the Placement Tests was entirely optional. Nevertheless, it was hoped that a substantial proportion of new students would complete the tests, and they were encouraged to do so by the Office of Academic Advising. During the first year of this placement system, 532 students completed at least one of the Placement Tests, and this represents 66% of the 805 students in the class of 2011. This participation rate is seen as a substantial success, considering that the tests were optional, some students may have based their course choices solely on results of an Advanced Placement Examination, and still other students (particularly international students) may not have had access to a reliable Internet connection to complete the Placement Tests. In future years, it would be desirable to increase participation further, mainly through increased promotion of the Placement Tests to new students.

Of the 532 students who completed at least one Placement Test, some completed all three tests while others only chose to complete one or two. The Venn diagram shown in Figure 2

details how many students attempted and completed each possible combination of Placement Tests.



**Figure 2: Number of students completing each possible combination of the three Placement Tests**

Allowing students to choose how many tests to take has interesting implications both for the data analysis in this study, and for the students' course choices. There were 223 students who completed only the Calculus I Placement Test. There were 21 students from this group who received a perfect score and yet did not complete any additional placement tests. Furthermore, 109 of these students received a score of 13 or higher and did not complete an additional test. These scores were typical of students who completed both Calculus Placement Tests I and II, and were recommended for Calculus II based on the scores on the second test. This group of students represents 20 percent of the population. This is a large number of students who could have potentially received a recommendation for a higher leveled course. These students could

have potentially been in classes that were too easy for them, or they could have chosen not to take tests where the material was completely unfamiliar.

Figure 3 below shows a summary of the course recommendations for the students who did take the tests. Of the group of students who took at least one of the tests, 44% were recommended to attempt Calculus III as their first math course at WPI. This was the most common recommendation, and this could have a few implications. It could imply that many of the students who took the tests were in fact ready for Calculus III. It could also imply that the tests were too easy, or that the conditions by which the students took the test made it easier (i.e. they were at home, comfortable, and perhaps seeking outside help or using a calculator) to earn a high score. The only way to determine the validity of these recommendations is to follow the students who followed the recommendation of the tests and examine their performance in the recommended class. Did these tests properly predict success? (See 5.1. *Using Calculus I Placement Tests to Predict Calculus I Performance* and 5.3. *Placement Test Cutoff Scores*)

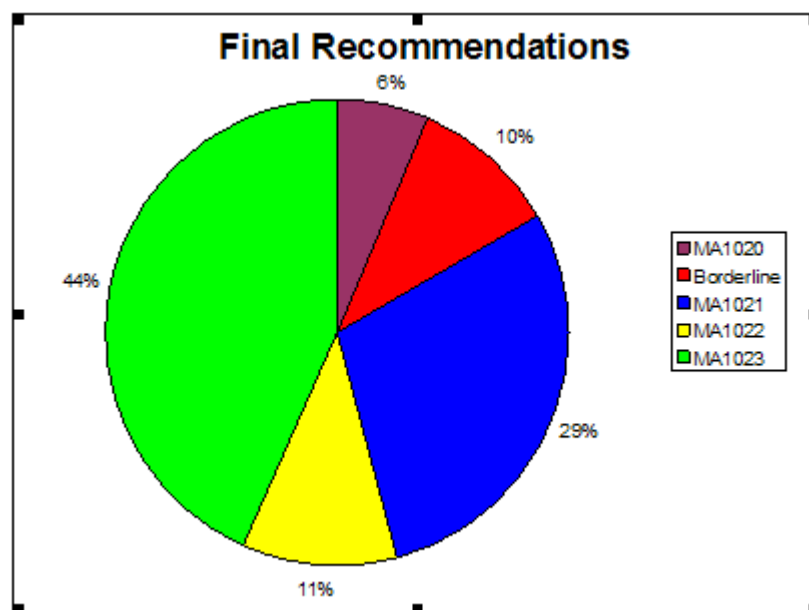


Figure 3: Percentages of students receiving particular course recommendations



Figure 4 shows the distribution of scores for the Calculus 1 placement exam. The most common score was in fact a perfect score of 16; of those who completed the test, 27% received a perfect score. This distribution is certainly not the approximately normal distribution that might be expected from an examination in a typical college course. However, this need not indicate that the Placement Test was unsuccessful. Unlike an examination taken during a course, which seeks to discern slight differences in students' mastery of course topics, a placement test only aims to determine whether a student has achieved *general* mastery of some prerequisite body of knowledge. Such placement tests should be easy to pass if the material is familiar and difficult if not.

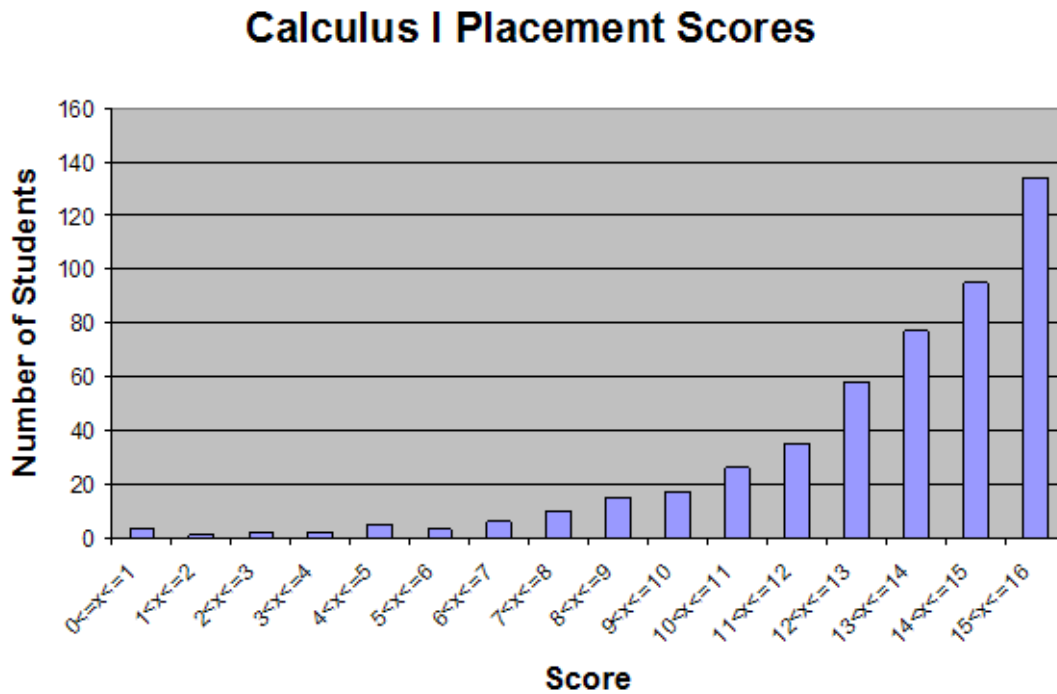


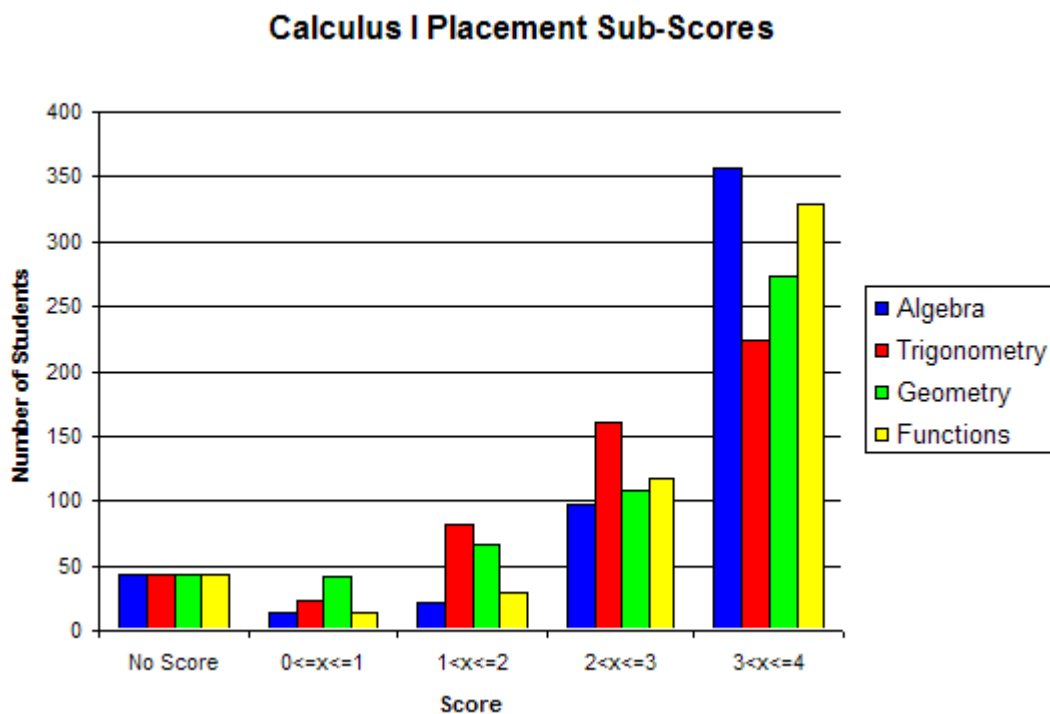
Figure 4: Distribution of Scores on the Calculus I Placement Test

The Calculus I Placement Test tests knowledge of pre-calculus material. To obtain more detailed information about students' strengths and weaknesses, each question on this test was constructed to test skills in Algebra, Geometry, Trigonometry, or Functions. There were four questions of each type in this 16-question test. By subdividing the test in this manner, the test can identify not only a general deficiency in pre-calculus knowledge, but also hopefully the specific nature of that deficiency.

Because there were four questions of each type on the Calculus I Placement Test, a total score of 4 was possible in each of the four subject areas. In an e-mail sent upon completion of the test, students were notified of their scores in each area in addition to their total score. If a student's score in a given area was 2 or less, the student was notified that he or she may have a deficiency in that specific area. It was also suggested that the student complete a set of internet-based practice problems in that area as part of an optional program called Jump Start. Note that it was possible to score 13 points or higher on the Calculus I Placement Test (the score at which a student was recommended for Calculus I) and still be recommended for a Jump Start module in one subject area.

Figure 5 shows the score distribution for each subject area of the Calculus I Placement Test. A total of 489 students completed this test. Note that 43 students took either the Calculus II or Calculus III Placement Test without taking the Calculus I Test; these individuals were placed in the "No Score" category in the Figure 5. For those that completed the test, the same general pattern is evident for each subject area: the distribution is negatively skewed, with a perfect score of 4 in a category as the most common score. More interesting than the general pattern are differences among the four subject areas. It appears that students were most successful with the Algebra questions, with fully 67.1% of students obtaining a perfect score of

4. Students were least successful in completing the Geometry and Trigonometry questions. Geometry was the subject for which the largest percentage of students (20.3% of those taking the test) received a Jump Start recommendation. With less than half (42.1%) of students earning a score of 4 in Trigonometry, this subject exhibited the smallest percentage of perfect scores. This might be explained by the difficulty of Question 7 on the Calculus I Placement Test, which involved the Law of Sines and displayed a low average score (See 4.1. *Internal Consistency*)



**Figure 5: Distribution of Scores on Individual Subject Areas of the Calculus I Placement Test**

Given the apparent differences in performance among the subject areas, it would of course be useful to quantify the statistical significance these differences. To that end, a two-sample  $t$ -test was performed for every possible two-subject combination of the four subjects. The null hypothesis  $H_0$  was that differences in the distribution of scores for the two different subjects were due to chance. The  $p$ -values for those tests are given in Table 1 below.

**Table 1:  $p$ -values for two-sample  $t$ -tests, examining differences between individual sections of the Calculus I Placement Tests**

	<b>Algebra</b>	<b>Trigonometry</b>	<b>Geometry</b>	<b>Functions</b>
<b>Algebra</b>		$5.2817 \times 10^{-30}$	$6.0709 \times 10^{-22}$	$1.4757 \times 10^{-02}$
<b>Trigonometry</b>			$2.1717 \times 10^{-01}$	$3.4478 \times 10^{-21}$
<b>Geometry</b>				$2.1955 \times 10^{-17}$
<b>Functions</b>				

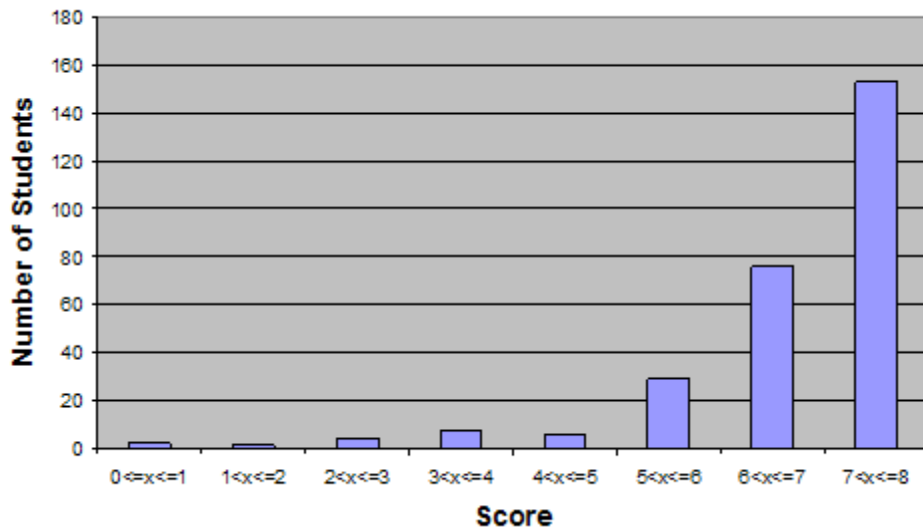
Examination of the results of hypothesis testing validates the supposition that performance on Geometry and Trigonometry was significantly different from that of Algebra and Functions. All four of those subject combinations (Geometry-Algebra, Geometry-Functions, Trigonometry-Algebra, and Trigonometry-Functions) had  $p$ -values small enough to reject the null hypothesis at any commonly used level of significance. There are certainly no grounds for rejecting the null hypothesis regarding Geometry and Trigonometry, for which  $p = 0.217$ . Finally, for Algebra and Functions  $p = 0.0148$ , which fails to be significant at the 1% level. These results confirm in a more rigorous manner the conclusion that performance on the Geometry and Trigonometry sections of the test was lower than that of Algebra and Functions to a significant extent.

It is important to note that although the aforementioned hypothesis testing has demonstrated that score differences in particular subject areas are not random, it does not demonstrate or suggest a cause. The most tempting conclusion is that the Placement Test is accurately measuring a real deficiency in student preparation in the areas of trigonometry and geometry. Nevertheless, it is also possible that differences arose as a result of other factors, such as the difficulty and complexity of questions in different sections of the Placement Test.

As a result of subdividing the Calculus I Placement Test in the aforementioned manner, some students could receive contradictory recommendations. In some cases, students received passing scores for all three placement tests – suggesting that they should begin in Calculus III –

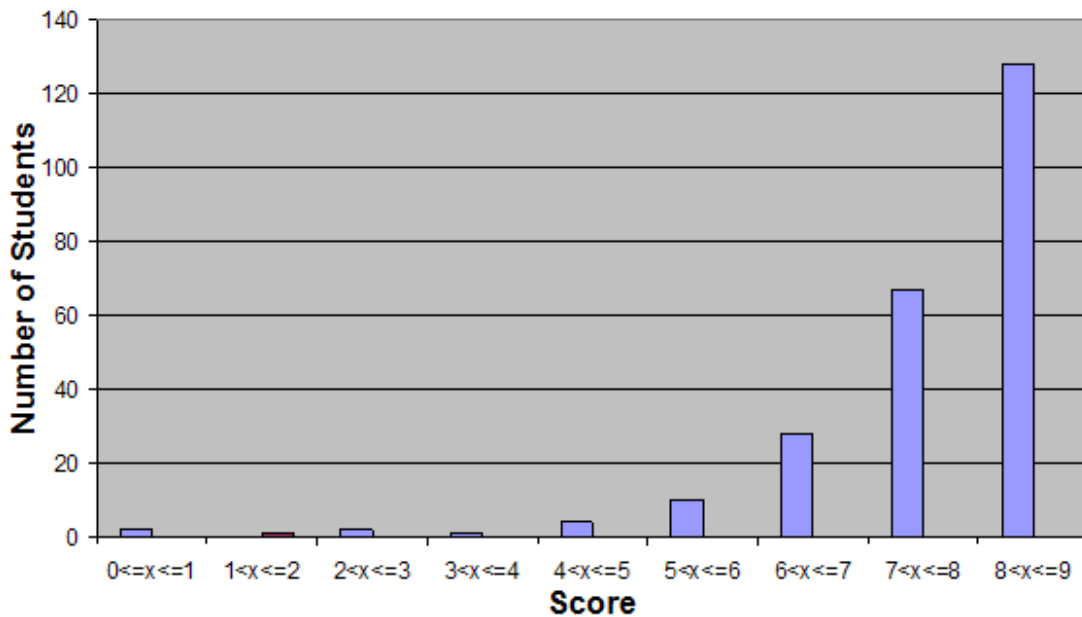
but scored 2 or lower on Algebra, Geometry, Trigonometry, or Functions – suggesting that remedial work in some area of pre-calculus was necessary. The specific course recommendation that a student in this situation received could vary, and in some cases was the result of personal consultation with an advisor at WPI. This should not be regarded as a failing of the Placement Tests. Instead, it underscores the fact that the goal of an Internet-based system should not be to completely replace personal academic advising, but rather to complement, streamline, and objectify the advising process. Those students who score very high or very low on particular placement tests probably have little need for extensive personal advising. On the other hand, the Internet-based test can identify “borderline” cases and direct them to an advising system that will make a personalized decision. Indeed, such students probably received contradictory results because of gaps in their knowledge of precalculus fundamentals, and the Placement Tests can help to identify and to remedy those gaps in understanding.

Figure 6 shows the distribution of scores for the students who took and completed the Calculus II Placement Test. 278 students completed the Calculus II Placement Test. Like the Calculus I Test, the distribution for the set of Calculus II Placement Test scores is also left-skewed, with the mode of the data being a perfect score of 8. The number of students receiving passing scores was high, with 251 students (90.2%) receiving scores of 6 or higher.



**Figure 6: Distribution of scores on Calculus II Placement Tests**

Figure 7 shows the distribution for the Calculus III Placement Test Scores. 242 students completed this test, and out of those 242 students 227 students were recommended to start Calculus III. This could imply that the test was too easy, but we have to take into consideration the students who did not take the test. Why didn't they take the test?



**Figure 7: Distribution of scores on the Calculus III Placement Test**

Although completion of these Placement Tests is highly recommended, they are not mandatory. One possible reason that high scores were so prevalent is that students only attempted the tests that they felt comfortable taking. There were students who attempted the Calculus II and Calculus III tests but did not complete them. There were also several students who did not attempt the Calculus II or Calculus III Placement Tests after passing the Calculus I Test. This might suggest that students took the tests they felt they could pass and for which the material was familiar, which is a perfectly reasonable expectation for an optional placement test.

### 3.2. Performance in A-Term Mathematics Courses

Before considering the relationship between the Calculus Placement Tests and the final grades of in the first mathematics courses at WPI, we consider the distributions of course grades independently.

In examining student performance in the various math courses, it is important to mention that WPI freshmen were not evenly distributed among the possible math courses. Figure 8 below shows the number of students enrolled in the different math courses. The most common courses, MA1020-MA1024 and MA101X, were given their own categories, and all other courses were combined into the category “Other.”

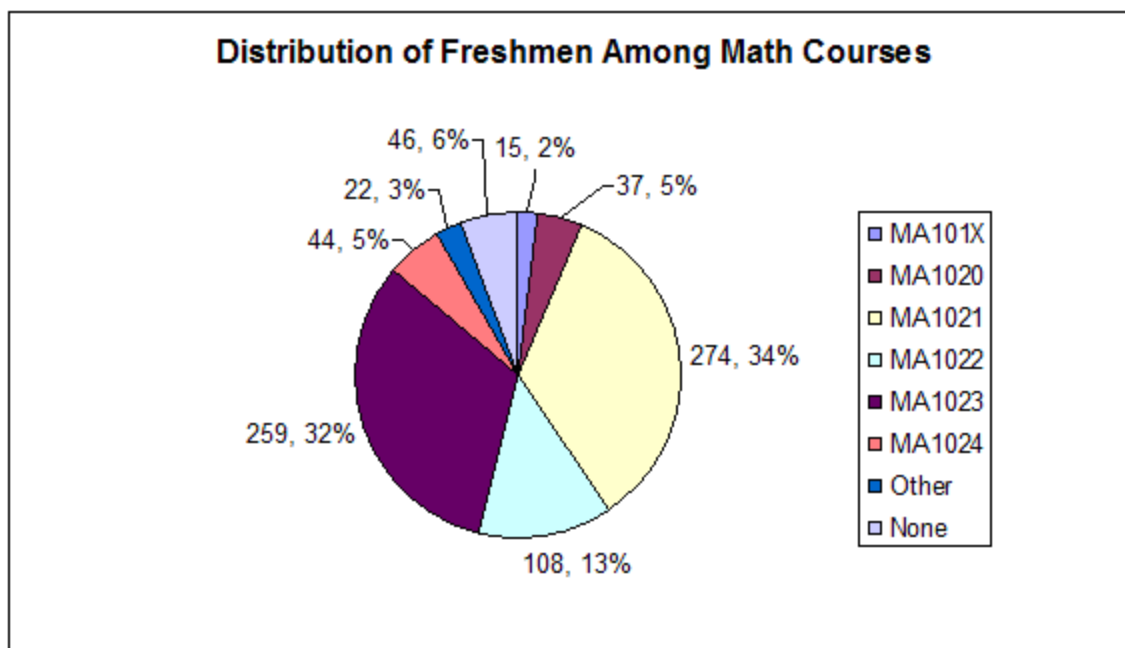


Figure 8: Distribution of freshmen among mathematics courses



Figure 8 shows that the most common first math courses were Calculus I (MA 1021) and Calculus III (MA1023), with 274 (34%) and 259 (32%) students respectively. Together, therefore, students in Calculus I and III account for two-thirds of the freshman class. The next most common course was Calculus II, with 108 students (13%). Comparatively fewer students took Calculus IV, Calculus I with Preliminary Topics (MA1020), and Calculus for Management and Biology Majors (MA 101X). Only 46 freshmen (6%) did not take any math course in A term.

The next point to consider is the distribution of grades for different math courses: how commonly did students receive an A in a particular course? An NR? Note that MA1020 extends over A and B terms so there are no grades available at the end of A term. MA 101X is only offered in B term. For all other freshman math courses, the grade distribution is shown in Figure 9 below. Roughly one-third of these students earned an A, one-third earned a B, one-quarter earned a C, and remaining students (about 11%) either failed the course (NR) or received an Incomplete (I).

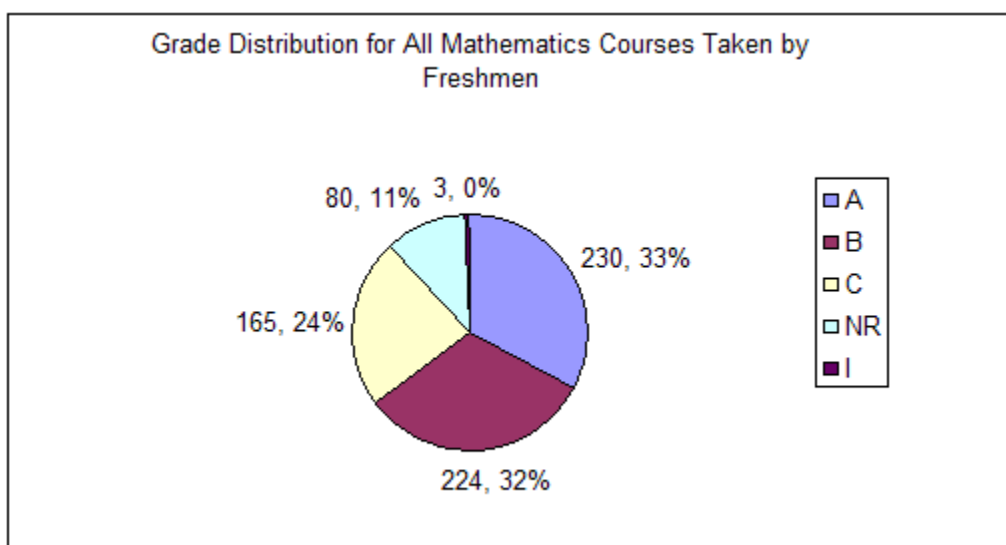


Figure 9: Grade distribution for all mathematics courses taken by freshmen

Figure 10 shows the grade distributions for individual math courses, with that numbers listed in Table 2. The graph clearly emphasizes the differences in total course size, with Calculus I and Calculus III clearly being the largest. Calculus III had the highest number of A's. Figure 11 and Table 3 show the same grade distributions, but expressed as percentages of the total course population.

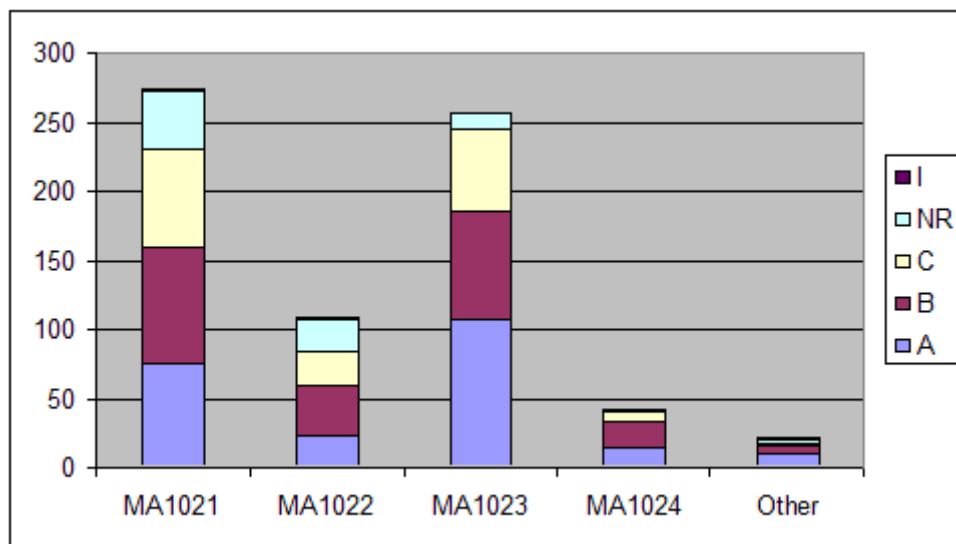


Figure 10: Final grade distribution by math course

Table 2: Final grade distribution by math course

	MA1021	MA1022	MA1023	MA1024	Other	Total
<b>A</b>	76	23	107	14	10	230
<b>B</b>	84	37	78	19	6	224
<b>C</b>	71	24	60	8	2	165
<b>NR</b>	42	23	11	1	3	80
<b>I</b>	1	1	0	0	1	3
<b>Total</b>	274	108	256	42	22	702

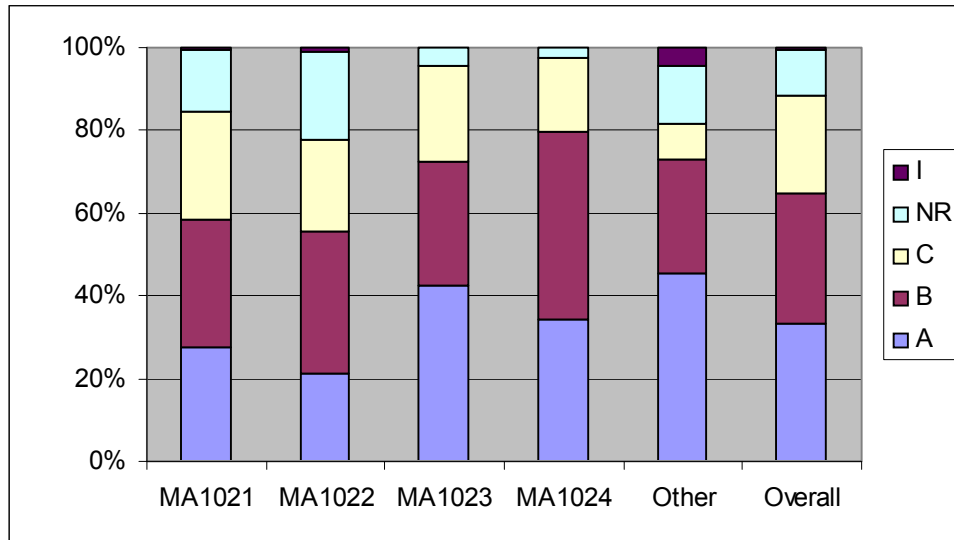


Figure 11: Final grade distribution by math course (percentages)

Table 3: Final grade distribution by math course (percentages)

	MA1021	MA1022	MA1023	MA1024	Other	Overall
<b>A</b>	27.7 %	21.3 %	41.8 %	33.3 %	45.5 %	33.1 %
<b>B</b>	30.7 %	34.3 %	30.5 %	45.2 %	27.3 %	31.8 %
<b>C</b>	25.9 %	22.2 %	23.4 %	19.0 %	9.1 %	23.3 %
<b>NR</b>	15.3 %	21.3 %	4.3 %	2.4 %	13.6 %	11.3 %
<b>I</b>	4.0 %	9.0 %	0.0 %	0.0 %	4.5 %	0.4 %
<b>Total</b>	100 %	100 %	100 %	100 %	100 %	100 %

Among the four basic calculus courses (that is, excluding the “Other” category), Calculus III had the highest proportion of A’s (41.8%). In Calculus II, the proportion of A’s was about half as large. In addition, the proportion of students with passing grades (A, B, or C) appeared to be significantly larger in MA 1023 (95.7%) or MA 1024 (97.5%) than in MA 1021 (84.3%) or MA 1022 (77.8%). In general, student performance in MA 1021 and MA 1022 appears to be

poorer than performance in the other classes. Note that the column marked “Overall” shows the grade distribution for all students in all classes in the graph.

It is interesting to determine whether these and other apparent conclusions are indeed statistically significant. More precisely, it should be determined whether the proportion of students receiving a particular grade in a given class is significantly different from the proportion of students in the entire group who received that grade. This determination may be made via a statistical hypothesis test. The particular test used was a one-tailed, one-proportion  $z$  test. Consider the example of comparing the proportion of students receiving A’s in MA 1021 with the proportion receiving A’s in all math classes. If  $p$  represents the proportion of A’s in the entire population and  $\hat{p}$  represents the proportion of A’s in MA 1021, then the null and alternative hypotheses may be stated as follows:

$$\begin{aligned}H_0 &: p \geq \hat{p} \\H_a &: p < \hat{p}\end{aligned}$$

Note that if  $p$  was observed to be greater than  $\hat{p}$ , then the above hypotheses were used, and if  $p$  was less than  $\hat{p}$ , the opposite hypotheses (i.e.  $H_0 : p \leq \hat{p}$ ) would be used.

The test statistic for a one-proportion  $z$ -test is:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Table 4 below shows the  $p$ -values (now referring to “probability,” not “proportion”) associated with the above hypothesis test for each grade distribution. Values that are significant at the 5% level are highlighted in yellow. For MA 1021, MA 1022, the proportions of A’s were significantly lower, and the proportion of NR’s significantly higher, than in the overall distribution. Conversely, MA 1023 showed more A’s and fewer NR’s than the overall group.

Also, MA 1024 showed significantly more B grades and fewer NR's. The results of hypothesis testing largely confirm the intuitive assertions made about the grade distributions: overall student performance in Calculus I and Calculus II was generally lower than in Calculus III or IV. (The final significant  $p$ -value shows that since 1 of 22 students taking "Other" courses received an "Incomplete" designation, there are significantly more grades of I in the "Other" category. While statistically significant, this result has no practical significance.)

**Table 4:  $p$ -values for two-tailed hypothesis test**

	<b>MA1021</b>	<b>MA1022</b>	<b>MA1023</b>	<b>MA1024</b>	<b>Other</b>
<b>A</b>	0.0297	0.0046	0.0007	0.4456	0.1090
<b>B</b>	0.3391	0.2935	0.2786	0.0290	0.3233
<b>C</b>	0.1569	0.3920	0.4741	0.2148	0.0571
<b>NR</b>	0.0180	0.0005	0.0002	0.0322	0.3656
<b>I</b>	0.4399	0.2113	0.1481	0.3361	0.0015

Analysis of grades of WPI freshmen in their first math courses has yielded a number of useful insights. The number of students taking Calculus II or IV was much smaller than those taking Calculus I or Calculus III. Among courses for which A term grades exist (that is, excluding MA 1020), roughly 89% of students received passing grades. When individual courses are considered, performance in Calculus III and IV appeared to be superior to that of Calculus I or II. These insights into the general course choices and overall performance of students should prove very useful in analyzing the Calculus Placement Tests. Hopefully, a thorough analysis of the Tests will demonstrate their influence on course choice and performance.

### 3.2.1. Calculus II Final Grades

As explained in Section 2.1. *Format and Content of the Calculus Placement Tests*, the current Calculus II course is a study of integration and the applications of integration techniques. The Calculus II placement exam's purpose is to determine if a student has an appropriate understanding the key concepts of Calculus I, differentiation. The placement exam itself covers topics that a student should've learned in Calculus I in order to be ready to enroll in Calculus II. This exam is only one placement test out of three. In this section we are examining the Calculus II final grades for A-Term 2007. This class contained a mix of students, some who took the placement exams and some who did not. Of the students who took the placement exams and passed, some chose to follow the recommendation given and some did not. The differences between these groups in their performance in Calculus II can be an indicator of the placement tests' effectiveness.

Figure 12 summarized the Calculus II Placement test scores. Note that grades were only given if the student completed the entire test.

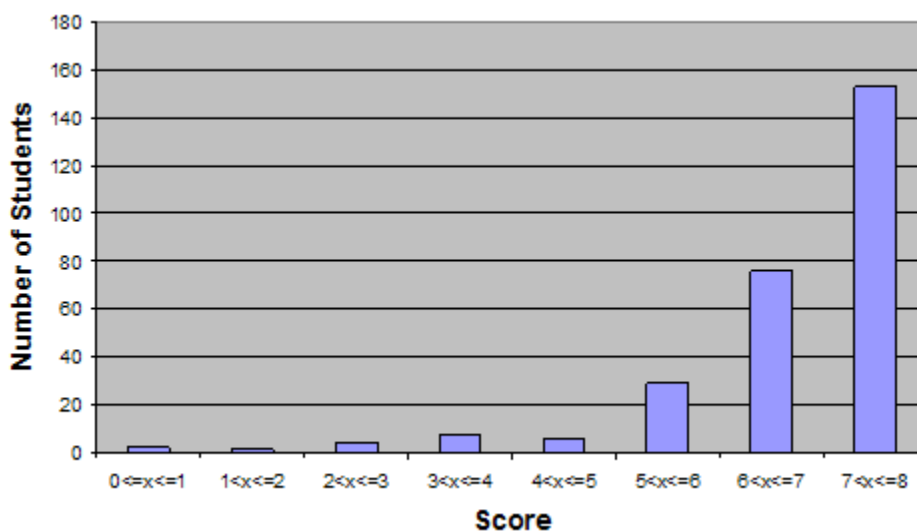
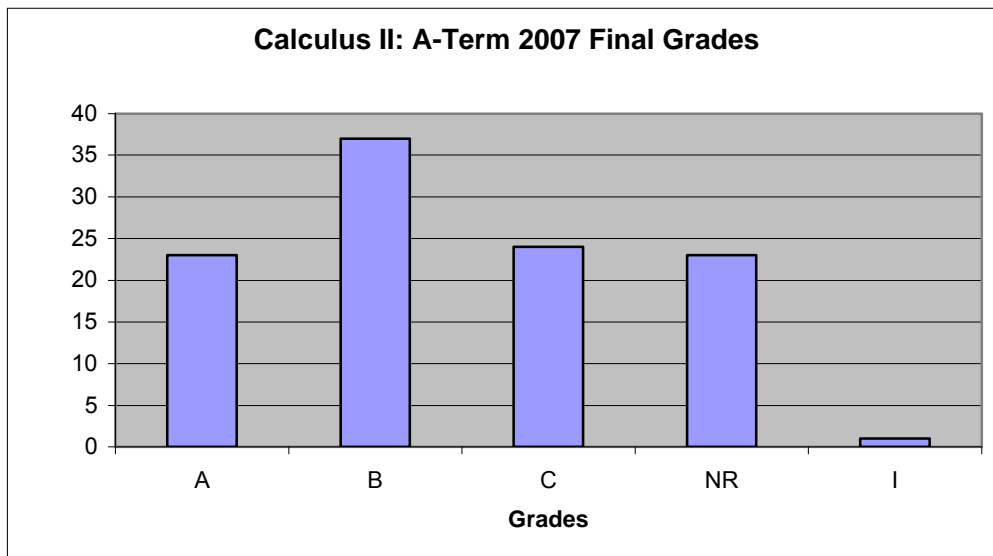


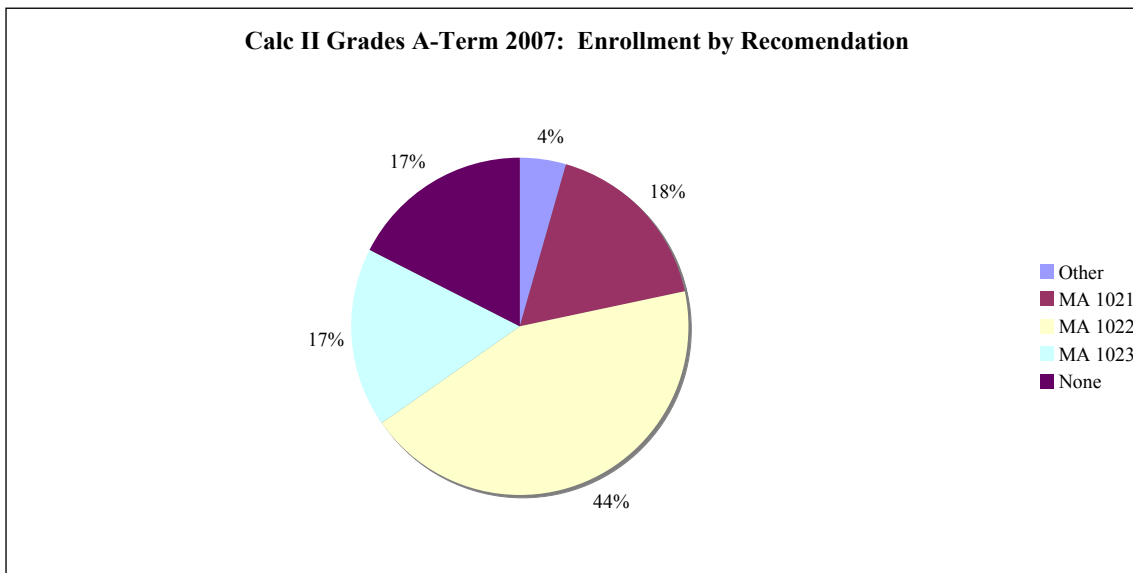
Figure 12: Distribution of Calculus II Placement Scores

Figure 13 shows the grade distribution for the students who took Calculus II A-Term 2007 at WPI. There are just as many A's as there are NR's in this distribution. There are also a substantial number of C's. The greatest proportion of students received B's.



**Figure 13: Distribution of Calculus II Final Grades**

Figure 14 shows the enrollment of the A-Term 2007 section of Calculus II broken down by the students' placement testing recommendation. The largest proportion, 44% of students enrolled in the course, did take the placement test and had received a recommendation to enroll in Calculus II. Many students (39%) enrolled in Calculus II after having received a recommendation to start in another course. 17% of students who enrolled and completed the course either did not complete the placement test. Considering that the placement tests are not mandatory, this indicates that many students do take advantage of their availability and have confidence in their results.

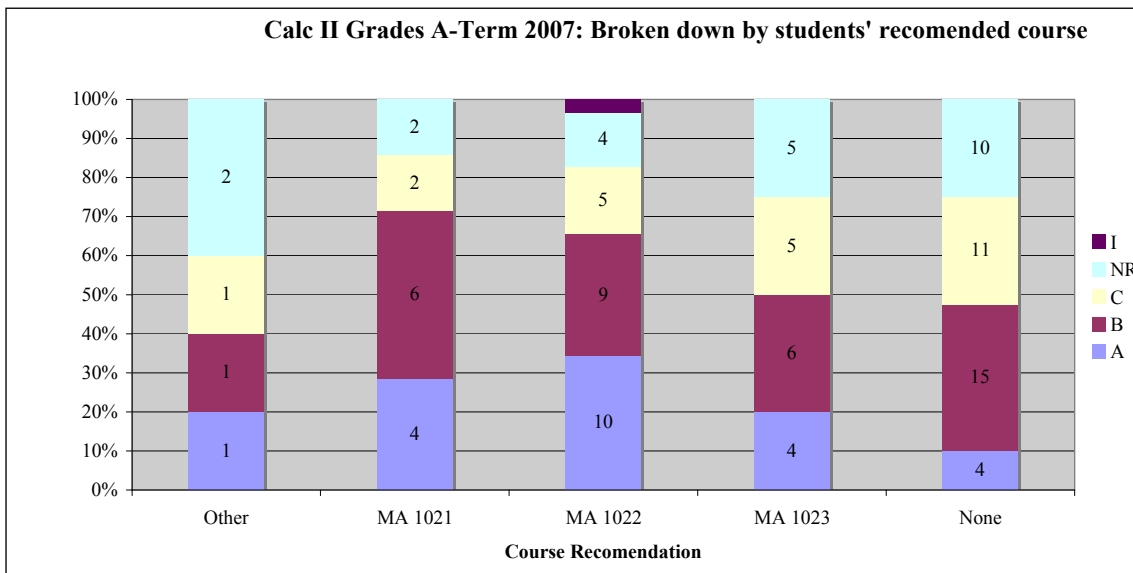


**Figure 14: Distribution of recommendation given to students enrolled in MA1022 A-Term 2007**

Figure 15 shows the grade distributions for students in Calculus II in A-Term 2007, broken down by the students' Placement Test recommendation. Students were broken down by the recommendation they received during placement testing. Students who received a recommendation for Calculus II are represented by the center column labeled MA1022 and received the greatest percentage of A's and a smaller percentage of NR's than students who did not receive a recommendation for Calculus II. The group labeled "Other" on the graph represents the students who received the following recommendations: MA101X, MA1024, and borderline. The group labeled "None" represents students who received no recommendation because they did not complete any of the Placement Tests.

The students who did not complete the placement tests had a distribution that was skewed toward the lower end, with fewer A's, a majority of B's and a more C's and NR's.





**Figure 15: Calculus II Final Grades, Grouped by Students' Original Course Recommendation**

Figure 16 shows the Calculus II grades broken down by various groups of interest. For the students who took at least one test and received a recommendation; some chose to take the course that was recommended (represented in Figure 16 by the “Took Recommendation” column), some chose to take a harder class than which they were recommended (represented in Figure 16 by the “Went Up” column), and some chose to take a class easier than the course they were recommended for (represented in Figure 16 by the “Went Down” column). The students in the “No Recommendation” column in Figure 16 represent the students who did not receive a course recommendation because they chose not to complete any of the Placement Tests. The column labeled “Whole Group” represents the grade distribution of the entire class for comparison with the other groups.

It is valuable to know if students who followed their recommendation did better than those who did not follow the recommendation. The students who did not take the placement test were also compared to the other groups.

In this distribution the students who received the greatest percentage of A's in the course were the students who were recommended to start in Calculus II. This same group also received the fewest NR's. The students who did not complete the placement testing and entered Calculus II show a lower percentage of A's and a higher percentage of NR's than the other groups of students.

The placement test serves as a tool to provide students with information about what they are getting into when they chose a class. It can help students assess their own readiness for a particular math course. It could be argued that a student who took the tests and gave it a fair amount of effort could make a better informed decision on what class to enroll in than the students who chose not to take the placement exams.

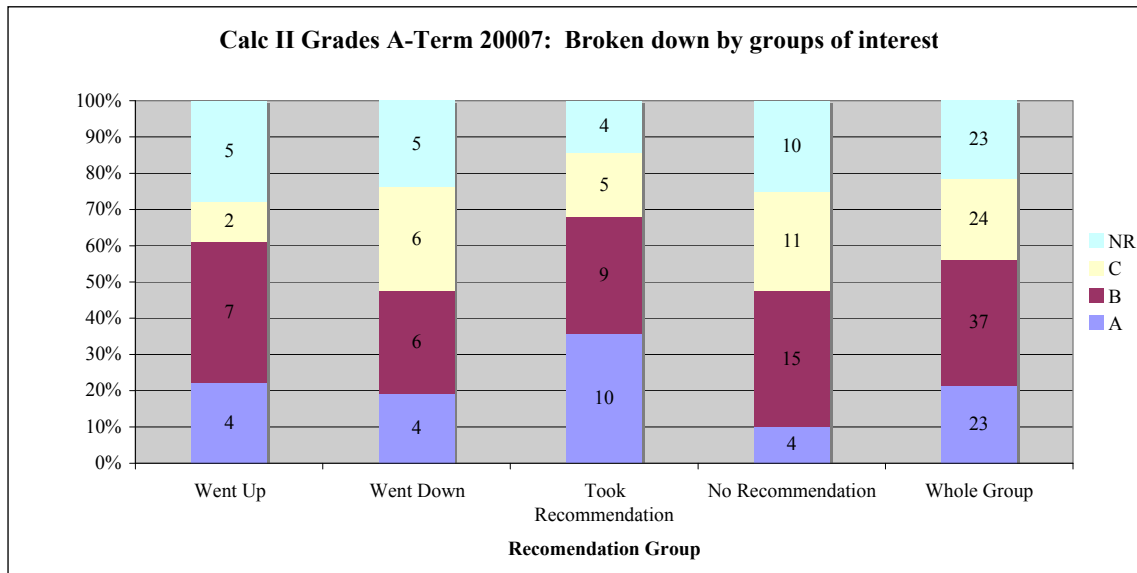


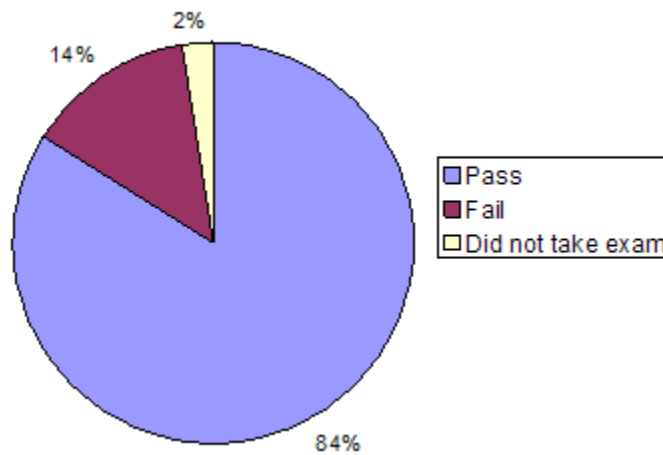
Figure 16: Calculus II Final Grades, Grouped by Students' Decisions Regarding their Recommendation

### 3.2.2. Student Scores on Calculus I Final Exam

At the conclusion of Calculus I (MA 1021) at WPI, which studies differential calculus, students take a final exam with two distinct parts. The first part, the Basic Skills Test, consists of seven questions on basic course concepts which are graded either completely correct or completely incorrect. At least five correct answers are required to pass this portion of the test. The second part, the General Test, is a more typical final exam with more complicated problems for which partial credit is available. The General Test contains seven problems of varying point values, and the maximum possible score is 100 points.

Student scores on each question of the Calculus I final exam given in A term of 2007 were obtained. A total of 321 students were enrolled in Calculus I during the term, and 313 of those students took the final exam. What follows is an analysis of student performance on the Basic Skills Test and the General Test, and a discussion of the relationship between these two scores.

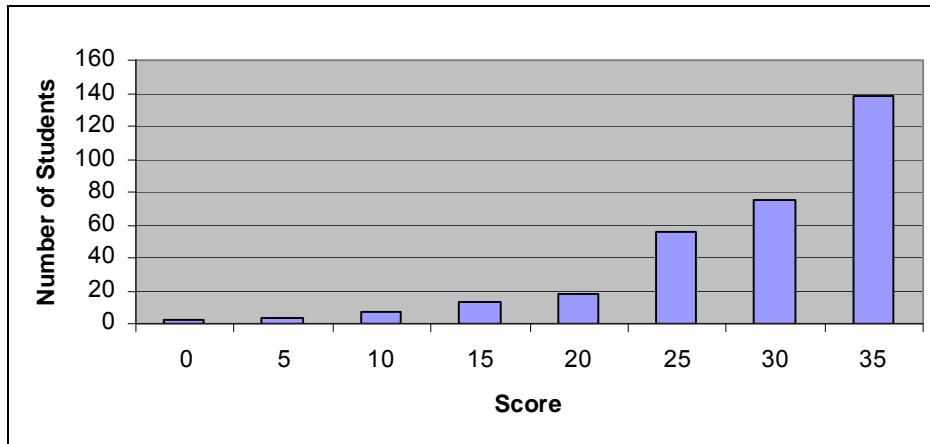
In analyzing student performance on the Basic Skills test, the most important consideration is whether or not a student passed the exam. Five correct answers, out of seven questions, are required to pass the exam. The chart shown in Figure 17 shows the proportion of students that passed, failed, or did not take the Basic Skills Exam. By far the majority of students, 84%, passed the Basic Skills Test. A total of 16% of students failed this portion of the final exam, with 2% doing so because they did not take the exam.



**Figure 17: Performance on the basic skills exam**

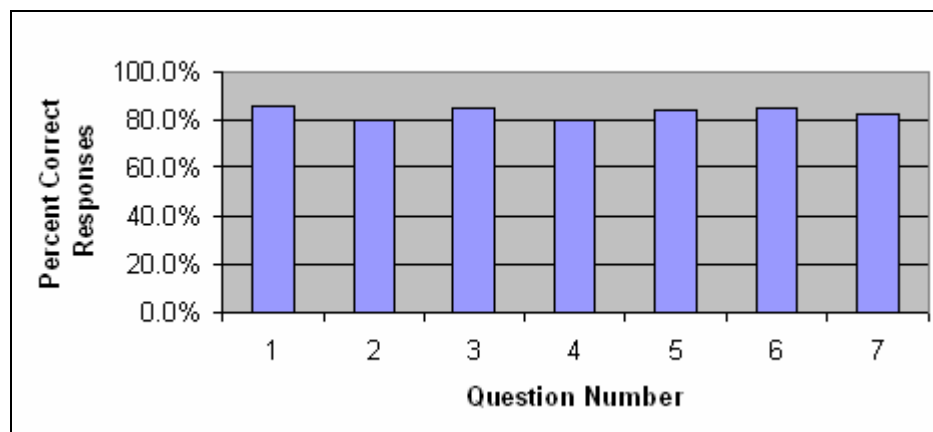
From this point forward, students who did not take the exam will not be considered in the analysis, since this would distort the apparent performance of students who did take the exam and whose performance we wish to analyze.

More detailed information about the Basic Skills Tests can be obtained by computing the score distribution. Since each of the seven problems was worth five points, and students can earn scores of either 5 or 0 on each problem, only seven different test scores are possible. The score distribution is shown in Figure 18. The most common score was a perfect score of 35, with 138 students earning this score. As the score decreases, the number of students earning that score is consistently less. Scores less than 25 – that is, failing grades – were fairly uncommon, with less than 20 students earning each of the possible failing scores.



**Figure 18: Basic Skills Score Distribution**

It is interesting to consider whether any one question on the Basic Skills Test was significantly more difficult, or more confusing to students, than the others. This is especially relevant for this iteration of the Basic Skills Test, since an error in the writing of Question 6 unintentionally caused an implicitly-defined curve in the problem simply to be a horizontal line. The graph in Figure 19 below shows the percentage of students who answered each question of the Basic Skills Exam correctly.

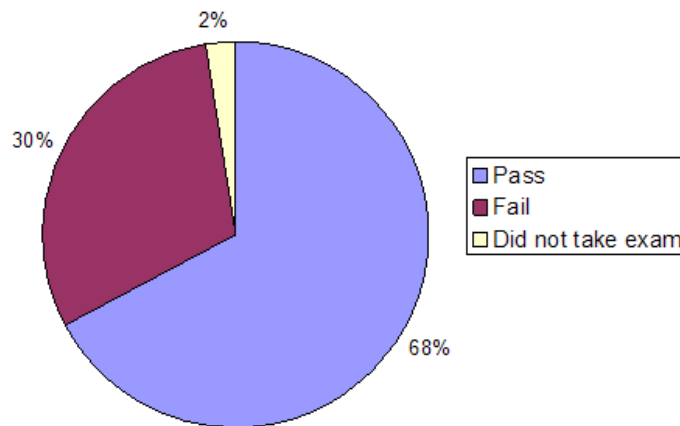


**Figure 19: Percent correct responses on each Basic Skills Test question**

In general, the percentage of correct responses was quite uniform, with typically just over 80% of students answering a particular question correctly. Any confusion about the

aforementioned Question 6 did not manifest itself in the data, with 85.3% of students answering correctly. When all seven questions are considered as a whole, 83% of all answers were correct.

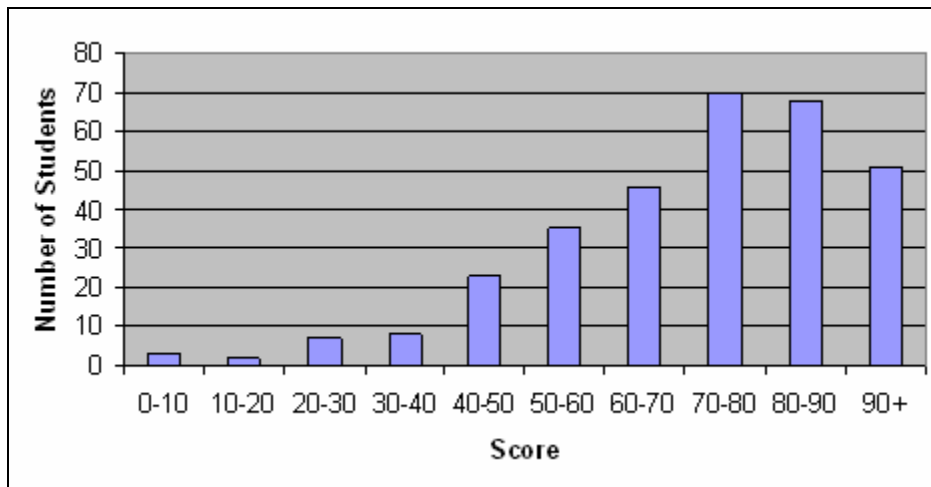
Shifting attention to the General Test, a basic measurement of student performance overall is how many students achieved a “passing” grade on the test. The concept of “passing” an exam is somewhat arbitrary, since the final exam is not the sole factor determining a passing or failing grade for the course; however, it is valuable to determine how many students scored higher than a certain threshold. If a score of 65% is required to pass the exam, then Figure 20 shows how many students passed the exam. Despite a lower passing threshold (65% for General Test versus 71% for Basic Skills Test), fewer students passed the General Test (68%) than the Basic Skills Test. This is to be expected, because of the higher level of complexity of the General Test.



**Figure 20: Performance on the General Test**

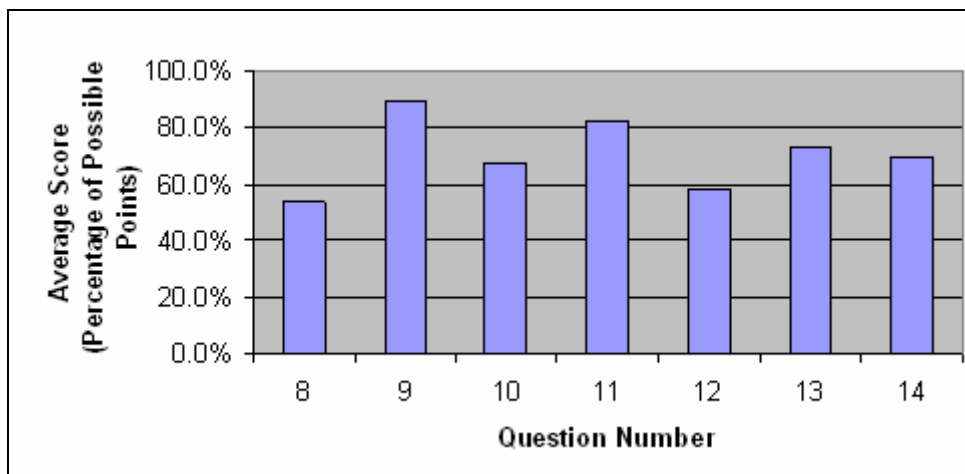
Examination of the score distribution of General Test scores, shown in Figure 21, reveals a very different distribution from that of the Basic Skills Test. For the Basic Skills Test, the most common score was the full score of 35, with significantly fewer students earning the other

possible scores. For the General Test, the most common scores were in the range 70-80, with smaller (but still significant) numbers of students scoring 80 or higher.



**Figure 21: General Test score distribution**

Every Basic Skills Test question was answered correctly by an approximately equal number of students (about 82%). A similar analysis of the General Test indicated that some problems were much more likely to be answered correctly than others. It should be reiterated that the Basic Skills questions are graded on an “all-or-nothing” basis, whereas partial credit is available on the General Test. Figure 22 shows the average score on each of the seven General Test problems, expressed as a percentage of possible points on the problem. A considerable amount of variation in average score was observed. Average scores ranged from 53.4% correct for Question 8 to 89.5% correct for Question 9. This suggests that the difficulty of the General Test was less uniform than the difficulty of the Basic Skills Test.



**Figure 22: Average score on General Test questions**

The preceding analysis has outlined a number of differences between the Basic Skills Test and the General Test. In summary, the Basic Skills Test exhibited a relatively high passing rate with a large percentage of nearly-perfect scores. Scores for the General Test were much more widely distributed, with an apparently higher level of difficulty overall. These differences are precisely what should be expected given the different purposes of the tests. The Basic Skills Test determines whether a student has learned the core mathematical operations on which differential calculus is based. Such a test should have a fairly high passing rate. The General Test, on the other hand, tests application of basic concepts in longer, more complex problems to determine a student's broader grasp of calculus. This test therefore would have more widely distributed scores.

It is also of interest whether success on the Basic Skills Test is correlated with success on the General Test. Conceptually, this addresses whether mastery of basic concepts predicts mastery of the application of those concepts. This question is explored in Figure 23, which plots a student's General Test score against their Basic Skills Test score. A moderately strong positive correlation of  $r = 0.668$  was observed. This is a satisfyingly high correlation, considering that



there are only eight possible values for a Basic Skills score, versus one hundred possible General Test scores. Success on the Basic Skills exam was indeed a moderate predictor of General Test performance.

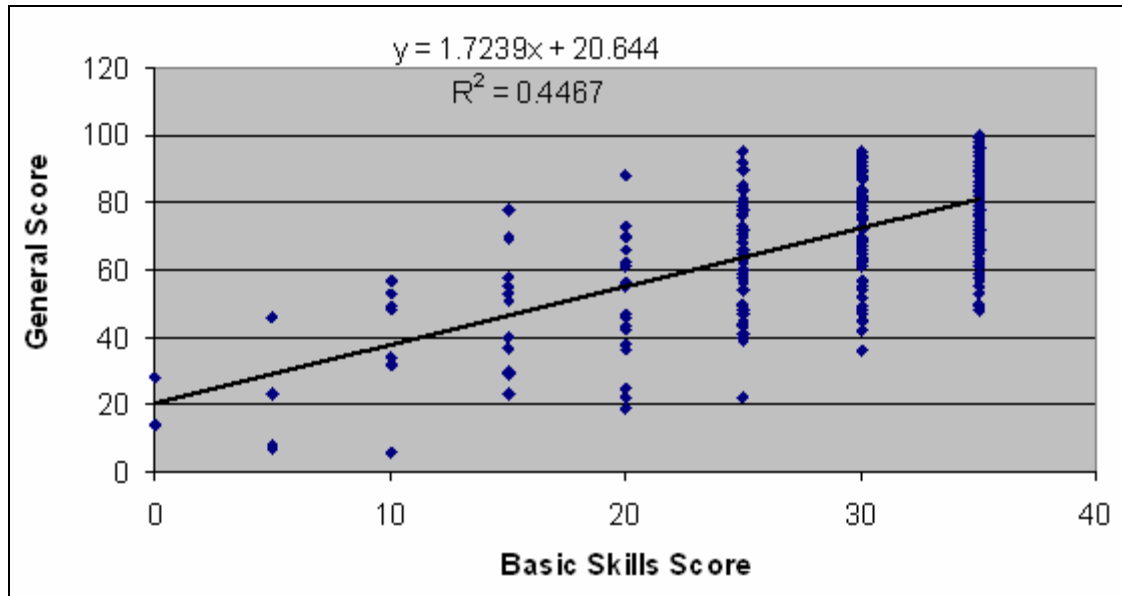


Figure 23: General Test Score versus Basic Skills Test Score

### 3.3. Performance in B-Term Mathematics Courses

The information extracted from the A-Term grades is valuable in the trends that it can show us. The data we examined from the placement tests were from the summer before A-Term '07. The placement tests are in place to help students determine which math class they would be most appropriate to start in. The range of the placement tests effectiveness is most predominant in the students' first math course. However, one interesting thing to examine is how well these students did after they completed their first math class. This data can serve as an examination of whether the students are moving successfully along the math courses track based upon what they were recommended. It can also serve as a control since the effect of the placement test doesn't necessarily affect the students grades in their second math course as their grade in their previously taken course would. For example, if a student received an A in Calculus I, they would probably do better in a Calculus II course than a student who received a C in Calculus I. Their grade is much less likely to have been determined by their performance on the placement tests.

Figure 24 and Figure 25 show the grade distribution for each of the math courses. Figure 24 shows the number of students receiving each grade whereas Figure 25 shows the same data by percentage of students. Looking at these graphs we can notice a few things. The percentages of the students in each class receiving a final grade of a B are relatively close to one another. The students in MA101X received more A's by percentage than any other course, though the most A's were received in MA1024. The percentages of students failing these math courses in B-Term are low, less than 20% in all cases.

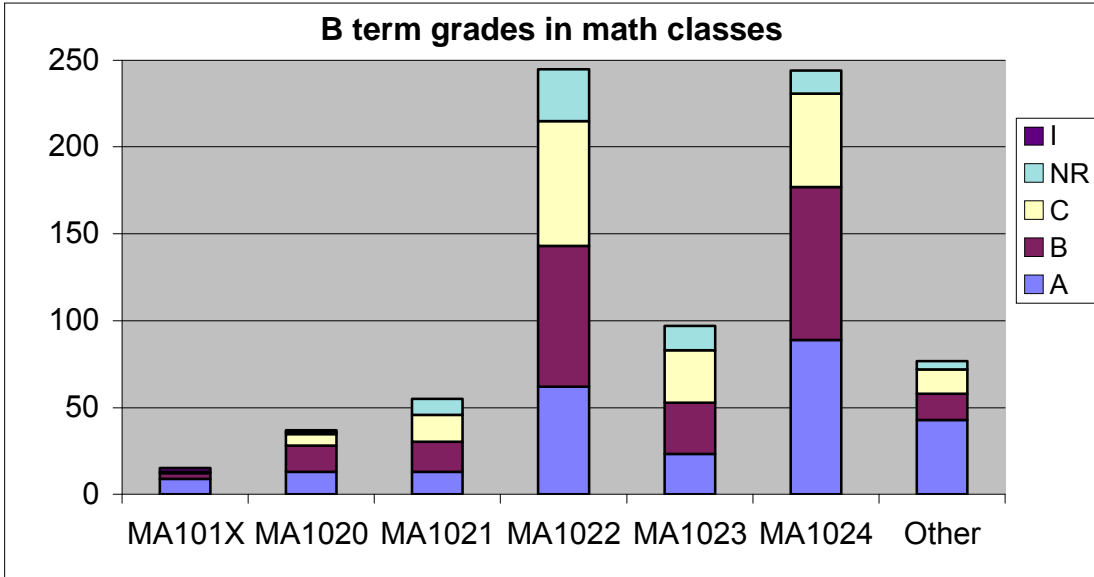


Figure 24: B-Term Grades in Math Courses, year 2007

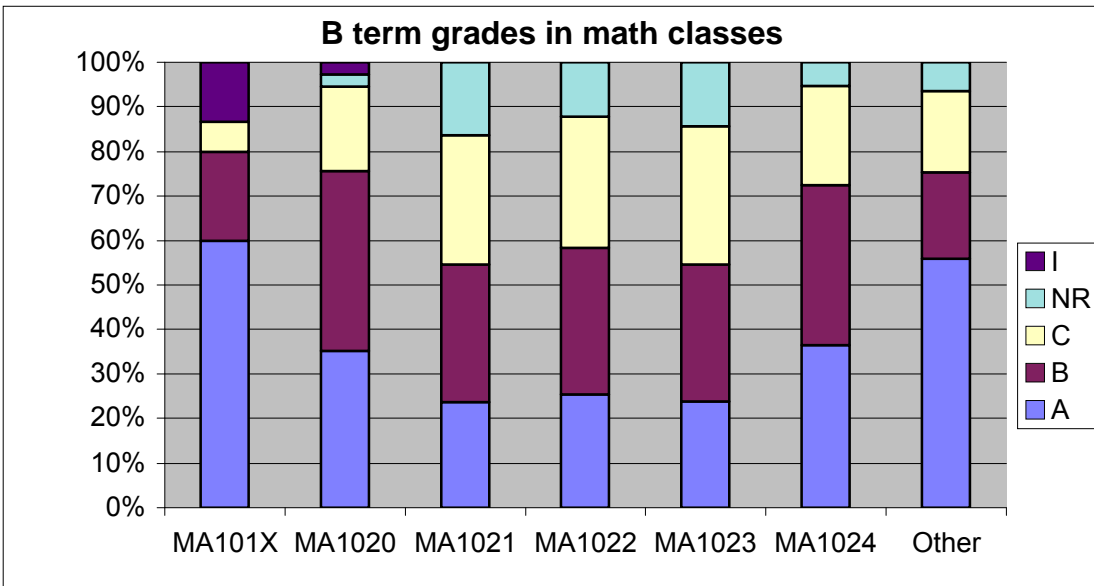
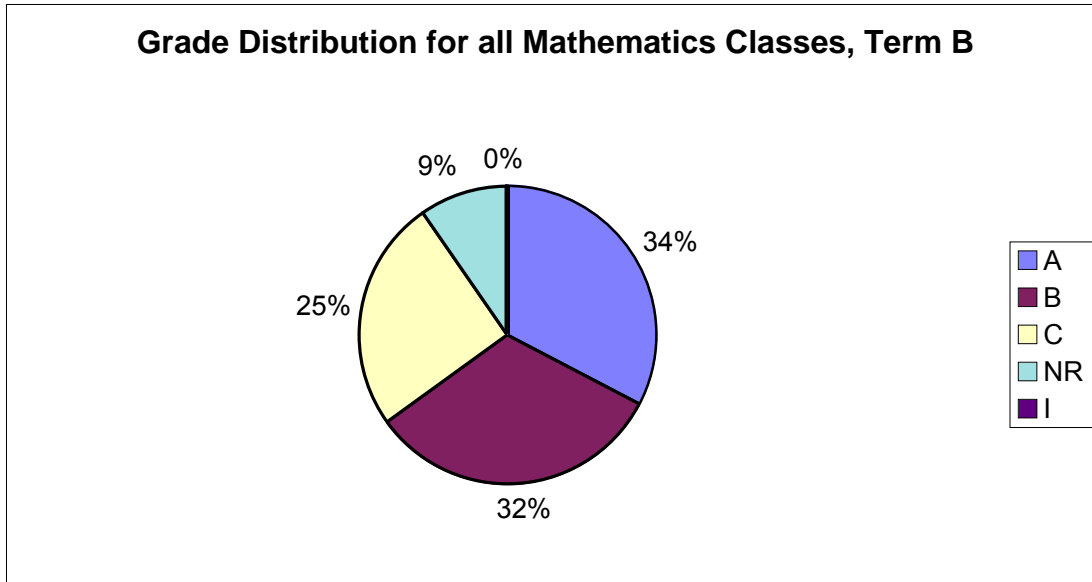


Figure 25: B-Term Grades in Math Courses by percentage, year 2007

Figure 26 shows the grade distribution for all math classes in B-Term. Encouragingly, less than 10 percent of students received an NR in all of the math classes and the largest percentage of students (34%) received an A in their math course for B-Term.



**Figure 26: Grade Distributions for all math courses, Term B 2007**

The distribution of freshmen among the math courses is shown in Figure 27. The largest chunks of students participated in Calculus II and Calculus IV B-Term. The students in the Calculus II group for B-Term are perhaps the most interesting to examine. They represent a large chunk of the freshmen population who, this past summer, had the option to participate in the online Placement Testing program. Within this chunk of students are the students who took the placement test and took Calculus I A-Term, and moved on to Calculus II for B-Term. While the placement test has less of an effect in the subsequent terms after it was taken, it is interesting to examine how well the students we examined for A-Term did in B-Term.

### Distribution of Freshmen Among Math Classes, Term B

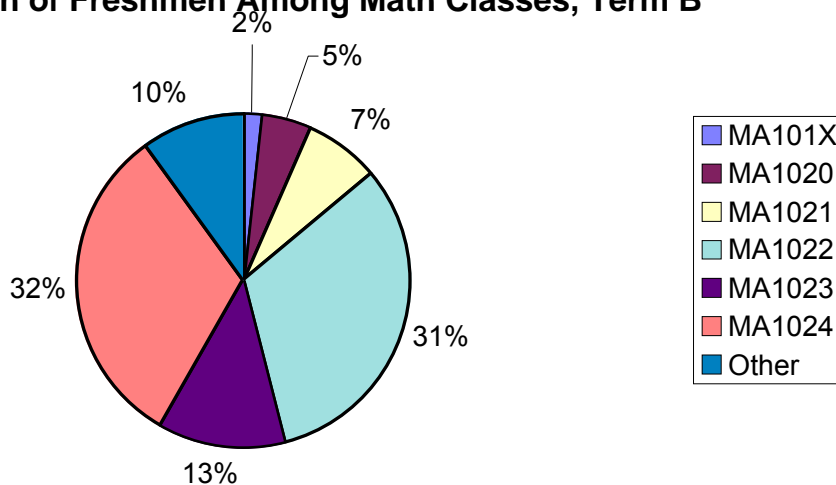


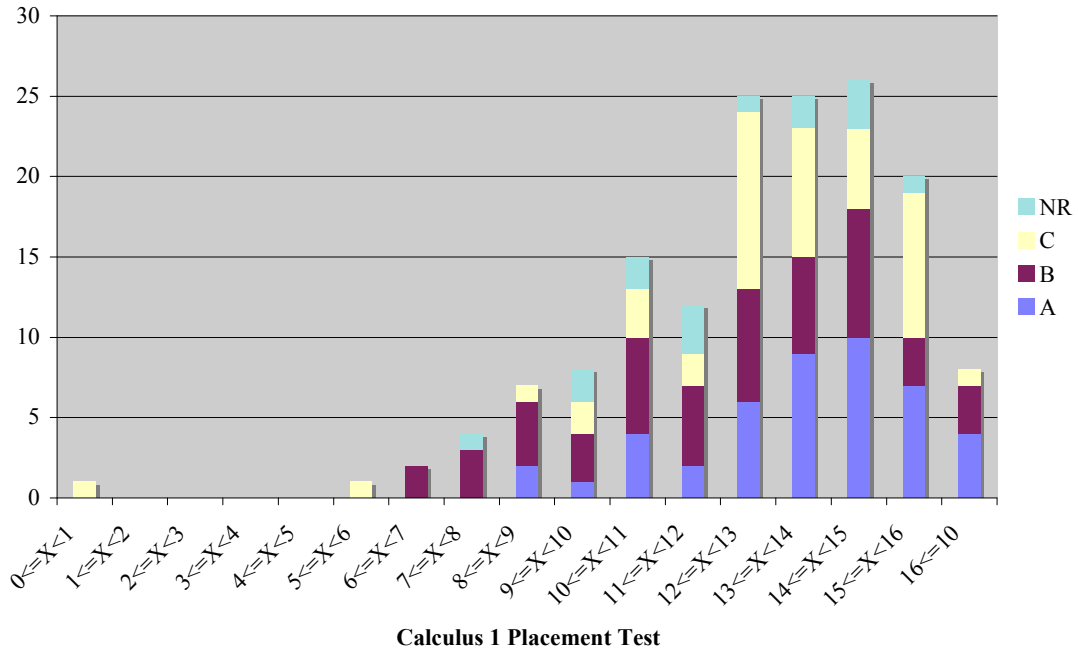
Figure 27: Distribution of Freshmen Among all Math Courses, Term B 2007

Figure 28 shows the B-Term 2007 Calculus II final grades for students who took the Calculus I placement exam broken down by the scores they received on that exam. One thing one can immediately notice is that the graph is left skewed. Figure 29 displays the same data except on a percentage basis. Very few students in this class did poorly on the Calculus Placement exam. The cutoff for the placement exam was a score of 8 as long as the student performed wasn't deficient in one of the areas examined on the placement exam (i.e. trigonometry, algebra, etc). Most of the students in Calculus II this term would have received a recommendation for Calculus I A-Term granted they didn't take any more placement tests. Students below the cutoff didn't perform as well in Calculus II as did the students who did well on the Calculus I placement exam.

Though, what this says about the placement test itself is not much. A better determinant of the placement tests effectiveness would be the grades these students received A-Term. Though, this examination can help take out factors that a student would have to deal with their

first term in college. Students who did poorly in the course due to the fact that they are adjusting to college life will have had time to adjust to the environment of WPI.

**MA1022 B08 Final Grades by Calc I Placement Scores**



**Figure 28: B-Term Calculus 2 Grades Grouped by Calculus I Placement Test Score**

### MA1022 B08 Final Grades by Calc I Placement Scores

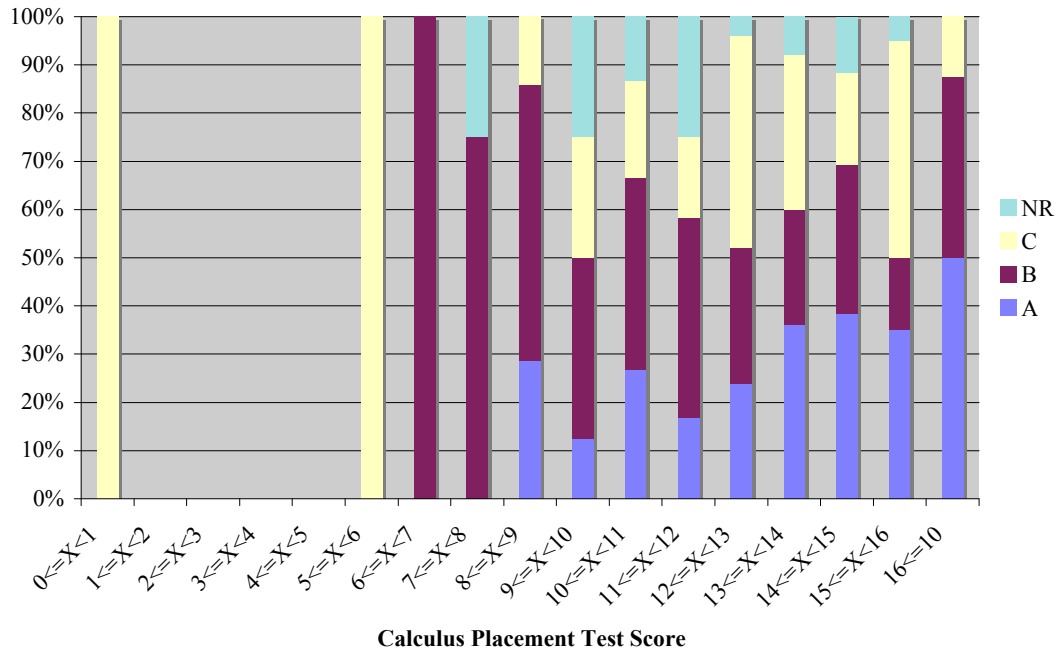


Figure 29: B-Term Calculus 2 Grades Grouped by Calculus I Placement Test Score (Percent)

## 4. Results: Analysis of Placement Test Structure and Design

### 4.1. Internal Consistency

One desirable property of a placement test, or indeed of any scientific measurement, is *reliability*. The reliability of a placement test may be defined as extent to which the test yields consistent, reproducible results. A reliable test displays very little random error, making differences in results mostly systematic; that is, score differences represent true differences among students, not idiosyncrasies of the test itself.

Although reliability may be measured using several different methods, some of these methods are better suited to this study than others. For example, two measures of reliability that are poorly suited to this study are *test-retest* reliability and *alternate forms* reliability. Test-retest reliability measures the correlation between two separate administrations of identical tests, and alternate forms reliability measures the correlation between two different forms of the same test. Using either of these methods in this study would have required the Calculus Placement Tests to be administered twice; furthermore, there was only one form of each of the Placement Tests.

A much more convenient, and much more widely used, indicator of reliability is *internal consistency*. This measurement is “internal” in the sense that it measures whether a test is consistent with itself rather than being consistent with separate administrations of similar tests. If the items of a placement test are well-correlated with each other, then the test is said to have an acceptable degree of internal consistency. Perhaps the simplest method of measuring the internal consistency of a test is to determine its *split-half* reliability. Split-half reliability is the correlation of scores on one half of the test with scores on the other half. However, difficulty arises in determining how to split the test in two. Of course, the halves may be chosen randomly,



but there are certainly several different ways to divide a test in half, and each such division of the test would yield a different numerical value for split-half reliability. A better measurement of internal consistency would address this problem.

Perhaps the most widely used indicator of internal consistency is *Cronbach's alpha*, which represents the mean of *all* possible split-half reliability values. Alpha is defined as follows:

$$\alpha = \frac{k}{k+1} \left( 1 - \frac{\sum_{i=1}^k \sigma_i^2}{\sum_{i=1}^k \sigma_i^2 + 2 \sum_{i>j}^k \sum_{j=1}^k \sigma_{ij}} \right)$$

where  $k$  represents the number of items in the test,  $\sigma_i^2$  represents the variance in score for item  $i$ , and  $\sigma_{ij}$  represents the *covariance* between item  $i$  and item  $j$  (Peter, 9). Note that when the covariance is divided by the product of the standard deviations for the two items, Person's correlation coefficient is the result. Another formula for alpha involving average correlation coefficients is sometimes used, but this formula only applies when the differences in variance among test items either are negligible or have been standardized.

Possible values of alpha range from 0 to 1, with a higher value being associated with a greater degree of internal consistency. Although judging what value of alpha constitutes an acceptable degree of internal consistency is subjective, it is generally considered that values higher than 0.6 are acceptable. It should be cautioned that very high values of alpha (0.95, for example) suggest that test items are so well-correlated that they are probably redundant. Finally, note that alpha will be close to zero for large  $k$ ; qualitatively, when there are more items the random error of each item is more likely to be canceled by the random error of other items.

Cronbach's alpha was computed for each of the three Calculus Placement Tests to serve as a measure of their internal consistency. Because this indicator measures the internal consistency of a test as a whole, only students who completed every question on the test (rather than attempting some problems without finishing) were considered. The results of these computations are shown in Table 5.

**Table 5: Internal Consistency - Cronbach's Alpha**

<b>Placement Test</b>	<b>Number of Items</b>	<b><math>\alpha</math></b>
Calculus I Placement	16	0.84
Calculus II Placement	8	0.65
Calculus III Placement	9	0.60

The Calculus I Placement Test, with  $\alpha = 0.84$ , displayed the highest degree of internal consistency. This indicates that in terms of reliability, the Calculus I Placement Test is very well-designed. Note also that the Calculus I Placement Test had approximately twice as many items as the other Placement Tests and that a larger number of items tends to increase the value of alpha. The Calculus II Placement Test ( $\alpha = 0.65$ ) and Calculus III Placement Test ( $\alpha = 0.60$ ) also displayed acceptably high levels of internal consistency, although alpha took lower values for these tests than for the Calculus I Placement Test. This discrepancy may be explained to some extent by the smaller number of items compared to the Calculus I Test. Overall, then, all three of the Calculus Placement Tests displayed an acceptable degree of internal consistency.

Using Cronbach's alpha to analyze the reliability of the Calculus Placement Tests has yielded the encouraging result that all three tests displayed a sufficient degree of internal consistency. A natural extension of this analysis would be to examine the contribution of individual test questions to this overall measure of reliability. For example, one abnormally difficult question that was poorly correlated with the other items of a test could detrimentally

affect the internal consistency of the entire test. Even a very easy item would have a detrimental effect on internal consistency, since that item would also be poorly correlated with the overall test. By calculating the *item-total* correlation, or the correlation of scores on a particular question with scores on the test as a whole, it could be determined whether a particular item was of an improper level of difficulty or otherwise unrelated to the bulk of test material.

The item-total correlation of each item on each of the three Calculus Placement Tests was calculated. The results for the Calculus I Placement Test are shown in Table 6, along with the average and standard deviation of scores on each question (the maximum possible score on each question was 1).

**Table 6: Statistics for Individual Questions on the Calculus I Placement Test**

<b>Question</b>	<b>Average Score</b>	<b>Standard Deviation</b>	<b>Item-total Correlation</b>
1	0.98	0.13	0.38
2	0.80	0.40	0.55
3	0.93	0.25	0.53
4	0.86	0.31	0.58
5	0.95	0.17	0.40
6	0.84	0.30	0.68
7	0.76	0.43	0.47
8	0.54	0.50	0.58
9	0.73	0.42	0.63
10	0.61	0.46	0.63
11	0.92	0.26	0.59
12	0.88	0.32	0.57
13	0.93	0.21	0.63
14	0.91	0.29	0.58
15	0.70	0.45	0.63
16	0.96	0.19	0.52

Table 6 provides a wealth of information regarding the individual questions on the Calculus I Placement Test. Highlighted in yellow are questions whose item-total correlation was less than 0.5, perhaps indicating that these questions are either more or less difficult than normal and are adversely affecting the internal consistency of the test. The choice of 0.5 as the acceptable lower

limit of the item-total correlation was arbitrary. However, it did appear that for all three placement tests, the item-total correlation for most items was higher than 0.5, suggesting that items with lower correlation coefficients might require particular attention. There were three such questions on the Calculus I Placement Test: Questions 1, 5, and 7.

Question 1 of the Calculus I Placement Test is reproduced in Figure 30. This question displayed the lowest item-total correlation (0.38) of all questions on the Calculus I Placement Test. With a nearly-perfect average score of 0.98 and a relatively small standard deviation of 0.13, nearly every student answered this question correctly. This analysis indicates that Question 1 was very easy relative to other questions on the test, and perhaps that this question is too easy to contribute meaningful information to the Calculus I Placement Test as a whole.

**Figure 30: Question 1, Calculus I Placement Test**

### Calculus 1 Placement: Problem 1

This set is visible to students.

(1 pt) Find  $m$  and  $b$  such that the equation  $y = mx + b$  describes the equation of the straight line through the points  $(9, 1)$  and  $(5, 2)$

$m =$  ,  $b =$

**Note:** *You can earn partial credit on this problem.*

While an item-total correlation analysis does indeed suggest that the level of difficulty of Question 1 was abnormally low, there is another more qualitative consideration that was also made when designing the Placement Tests. Since the Placement Tests were optional, every effort was made to encourage students to complete them. Indeed, one such effort was to make the first question of each test relatively simple in order to build student's confidence and to encourage them to continue the test. In a sense, the item-total correlation of the first question

was sacrificed as part of this appeal to the “softer” side of educational testing. With this in mind, a low item-total correlation is acceptable for the first question.

Question 5 of the Calculus Placement Test exhibited an item-total correlation of 0.40, which was almost as low as that of Question 1 (0.38). The results for Question 5 were also similar to those of Question 1 in that the average score was high (0.95) and the standard deviation was relatively low (0.17). It appears, then, that Question 5 was also of a lower difficulty than most of the other questions on the Calculus I Placement Test. Question 5, which is reproduced in Figure 31, tested knowledge of the Pythagorean Theorem and of basic definitions of the sine and cosine functions. More meaningful information about students’ knowledge of trigonometry might be obtained by increasing the difficulty of Question 5 to lower the average score and, hopefully, to raise the item-total correlation.

Figure 31: Question 5, Calculus I Placement Test

### Calculus 1 Placement: Problem 5

This set is visible to students.

(1 pt) If a right triangle has a hypotenuse of length 13 and one leg of length 5 and  $\theta$  is the angle between that leg and the hypotenuse, please

a) Determine the length of the other leg.

b) Determine  $\cos(\theta)$

c) Determine  $\sin(\theta)$

**Note:** *You can earn partial credit on this problem.*

Unlike Questions 1 or 5, Question 7 may have exhibited a low item-total correlation because it was more difficult than other questions. Question 7, which involved the Law of Sines, is shown in Figure 32. With an average score of 0.76, this question was answered correctly less often, although this is certainly not the lowest average score (The average score for Question 8 was 0.54.). The item-total correlation of 0.47 was low, but not abnormally so, as item-total

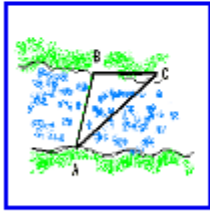
correlations of 0.52 and 0.53 were also observed. The standard deviation of 0.43 was relatively high, but this could possibly be explained by the lack of a possibility for partial credit on this question; scores were either 0 or 1. In short, while this question did appear to be one of the more difficult items on the Calculus I Placement Test, there is insufficient evidence to recommend immediate revision.

Figure 32: Question 7, Calculus I Placement Test

### Calculus 1 Placement: Problem 7

This set is visible to students.

(1 pt) To find the distance  $AB$  across a river, a distance  $BC = 235$  is laid off on one side of the river. It is found that  $B = 109^\circ$  and  $C = 18^\circ$ . Find  $AB$ .  
See the picture below. Click on the picture to see it more clearly.



AB =

A similar item-total correlation analysis was also performed for the Calculus II Placement Test, the results of which are shown in Table 7. Two questions, Questions 1 and 5, showed item-total correlations less than 0.5 and are highlighted in yellow. The text of Questions 1 and 5 are given in Figure 33 and Figure 34. With its extremely high average score (0.997) and very low standard deviation (0.06), Question 1 was a very easy question relative to the rest of the test. As described previously, the first question of each test was intentionally made less difficult, and this simple differentiation problem is no exception. Question 5 was less strikingly simple than Question 1; its average score was high (0.93), its standard deviation was typical (0.25), and its item-total correlation was moderately low. As shown in Figure 34, Question 5 involved

computing the equation of a tangent line to a fourth-order polynomial. Although it is not entirely obvious that this question requires revision, its item-total correlation might be improved by using a more complicated function than a polynomial for which differentiation would be more difficult.

**Table 7: Statistics for Individual Questions on the Calculus II Placement Test**

Question	Average Score	Standard Deviation	Item-total Correlation
1	0.997	0.06	0.34
2	0.91	0.29	0.57
3	0.91	0.25	0.66
4	0.83	0.37	0.61
5	0.93	0.25	0.47
6	0.74	0.44	0.61
7	0.97	0.14	0.56
8	0.87	0.33	0.64

**Figure 33: Question 1, Calculus II Placement Test**

### Calculus 2 Placement: Problem 1

This set is visible to students.

(1 pt) If  $f(x) = 4x^2 - 2x - 3$ , find  $f'(x)$ .

Figure 34: Question 5, Calculus II Placement Test

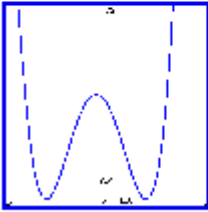
## Calculus 2 Placement: Problem 5

This set is visible to students.

(1 pt)

For the function graphed below,  $f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36$ , find the equation of the line tangent to the curve at  $x = 1$ .

**Note:** Click on the picture to enlarge the image.



$y =$

Finally, an item-total correlation analysis was also performed for the Calculus III Placement Test. The results of this analysis are shown in Table 8. Of the nine questions on this placement test, Questions 1 and 7 had low item-total correlation values. Question 1 is shown in Figure 35, and Question 7 in Figure 36. In fact, the item-total correlation was undefined for Question 1 because *every* student who completed the Calculus III Placement Test answered Question 1 correctly. This is also evident by the values of the average score (1.00) and standard deviation (0.00) for Question 1. Question 1 involved computation of the antiderivative of a quadratic function and represents another instance of a simple first question to build students' confidence as they begin this optional test. Question 7 exhibited a low item-total correlation (0.35) along with a high average score and low standard deviation, suggesting that this question was not sufficiently difficult. As shown in Figure 36, Question 7 involved integration of a simple linear function with use of initial conditions. Increasing the complexity of the function to



be integrated might raise the difficulty of this question such that it contributes more meaningfully to the overall test score.

**Table 8: Statistics for Individual Questions on the Calculus II Placement Test**

Question	Average Score	Standard Deviation	Item-total Correlation
1	1.00	0.00	Undefined
2	0.96	0.19	0.53
3	0.97	0.16	0.59
4	0.93	0.26	0.52
5	0.75	0.43	0.51
6	0.75	0.43	0.64
7	0.96	0.15	0.35
8	0.88	0.32	0.62
9	0.91	0.29	0.54

**Figure 35: Question 1, Calculus III Placement Test**

## Calculus 3 Placement: Problem 1

This set is visible to students.

(1 pt)

Find the antiderivative:

$$\int (7x^2 - 7x + 8)dx = \boxed{\phantom{000000}} + C$$

Figure 36: Question 7, Calculus III Placement Test

## Calculus 3 Placement: Problem 7

This set is visible to students.

(1 pt) Given

$$f''(x) = 4x + 4$$

and  $f'(-1) = 0$  and  $f(-1) = 0$ .

Find  $f'(x) =$

and find  $f(3) =$

**Note:** *You can earn partial credit on this problem.*

Each of the Calculus Placement Tests has been shown to possess an acceptable degree of internal consistency, as measured by Cronbach's alpha. The Calculus I Placement Test exhibited the highest value of alpha, possibly due to the fact that this test had roughly twice as many items as the other two Placement Tests. Analysis of the item-total correlation of each question revealed questions that were much easier or more difficult than others. Although the first question of each Placement Test was intentionally simple, other questions with low item-total correlations could be revised in an attempt to improve the internal consistency of the entire test.

## 4.2. Analysis of Placement Test Completion Time

A unique feature of the Calculus Placement Tests at Worcester Polytechnic Institute is their extremely open-ended format. The tests were available to students for a period of slightly longer than one month. Students could begin a test at any time, there was no time limit once the test was begun, and students could leave the testing website and resume at a later time. Students could work the test problems while logged into the testing system, or they could download a hardcopy of the test questions (an activity which is not logged) to complete with pencil and paper and enter their answers electronically later.

The variety of student approaches to the Calculus Placement Tests would explain the total time spent on the tests. A student who completed a test in one sitting, without interruptions, might have spent roughly one hour. For a student who completed the test in multiple sittings, many hours or even days might elapse before the test was finished. On the other hand, a student who downloaded a hardcopy of the test, completed the questions on paper, and entered answers electronically in quick succession would spend a very short time completing the computerized portion of the test (where time measurements can be made).

There is certainly the possibility that a student's score would depend on, or at least be correlated with, the manner in which they completed the test. Perhaps students who complete the test in multiple sittings over several days are able to obtain outside assistance with the test questions, making their score higher than if they had completed the test in one sitting. On the other hand, perhaps students who are very knowledgeable concerning the subject of a test complete it more quickly and earn higher scores than those who spend more time. The decision of Worcester Polytechnic Institute to offer a very open-ended placement testing system was

based on the implicit assumption that the value of the test for placement does not depend on the manner in which it is taken. The analysis that follows serves to test that assumption. If the score on the placement test was independent of the time spent taking it, then the current format of the placement tests would largely be justified. If it were discovered that success on the test depended significantly on the time spent completing it, then a restriction of the currently open-ended format might be in order.

Whenever a student submits an answer to the WeBWorK Internet-based homework/testing system, the date and time of submission are recorded, along with the student's username and the specific problem being answered. By using this information to calculate the total elapsed time between a student's first and last answer submissions on a given placement test, a good approximation of the total time spent taking the test could be obtained. This measurement is an approximation because there is no information about the time spent working on the test before any answers were submitted. This inherent error was assumed to be insignificant, both because the first question of each test typically was simple to solve, and because the error is probably relatively constant among students. This provides another reason for making the first question on the placement test relatively easy (other reasons include maintaining student confidence as they begin the test).

Information about the time of WeBWorK answer submissions (and other data not relevant to this discussion) is stored in a database and is downloadable. After downloading the database information to a spreadsheet file, irrelevant information was removed to leave the following data: the year, month, day, and time of an answer submission; the student username; and the particular placement test and problem being answered. Because this log contained *every* answer ever submitted to the Calculus Placement Tests, the student usernames in the log were

synchronized with the list of current WPI freshmen who had taken the tests (As is usual, student names were replaced with numbers to preserve student privacy). This removed entries resulting from the development, testing, and piloting of the Calculus Placement Tests. In addition, several students began a placement test without finishing it, and these students were removed from consideration as well, since test completion time was of interest. (Answering the final question of a placement test served no purpose but to indicate that the student had completed the test, so there was no uncertainty about whether a student was finished.)

To determine the total time spent on a test, it would of course be necessary to compare the time of first answer submission with that of the last answer. To compare times quantitatively, the WeBWorK time format of year, month, day, and 24-hour time was converted to a single number representing the number of seconds elapsed since midnight on May 1, 2007. For example, the log file might contain a time entry with the information “Year: 2007, Month: 6, Day: 8, Time: 0.5000.” This corresponds to a time of 12:00 p.m. on June 8, 2007, which is 38 days and 12 hours after midnight on May 1, 2007. In seconds, the time elapsed since May 1 for this example is 3326400 seconds. Formatting times in this manner allowed for easy numerical comparison of otherwise intractably-formatted times. The placement tests were not available to incoming first-year students until after that date, so representing times in this manner gave positive values that were easily compared by subtraction.

A histogram of the time spent completing the Calculus Placement Tests is shown in Figure 37. All three of the placement tests are grouped together for this chart. To improve the readability of the Figure, only one endpoint of each bin is displayed on the horizontal axis; note that the bin marked “2” in the Figure represents times greater than 1 day and less than or equal to 2 days. A large majority of students completed their chosen placement tests in one day or less,

with 715 of 1016 student-test combinations being completed in that time. There were some instances, however, where the time between a student's first and last answers exceeded 20 days. While Figure 37 indicates that there was indeed a great deal of variability in the time spent completing a placement test, much information about the distribution is hidden in the very large group of students completing a placement test in one day or less.

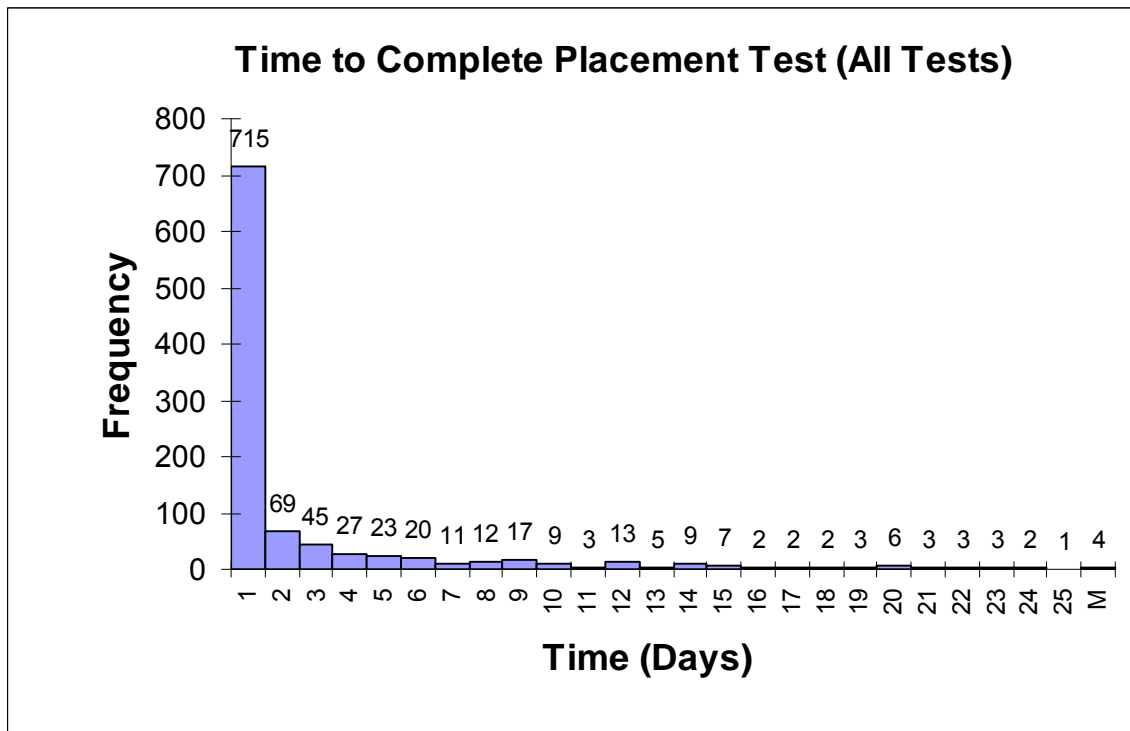


Figure 37: Time to complete Calculus Placement Tests (All Three Tests)

The foregoing discussion suggests that much more information could be gleaned from the data by plotting time on a logarithmic scale, rather than a linear scale. To that end, Figure 38 shows the same histogram with a logarithmic (base 10) time axis. It is immediately clear that such a representation is far more illustrative. A qualitative examination of this distribution suggests that it is bimodal, with frequency maxima at approximately -1.31 (roughly one hour)

and a lower local maximum at approximately 0.50 (slightly more than three days). It therefore appears that while many students completed the test in one sitting that was about one hour long on average, a significant number of students distributed their efforts over several days. Students taking longer than 20 days to complete a placement exam appear at the far right of the histogram. Interestingly, there were also students whose completion times were exceedingly brief – the two leftmost bins indicate that seven students fully completed a test in less than 5.6 minutes ( $10^{-2.41}$  days). To achieve such a completion time would be difficult even for experienced mathematicians, and thus it was concluded that these students most likely downloaded a hardcopy of the test questions, worked the problems after logging out of the test system, and later returned to enter their answers in quick succession, completing the test in a very short time from the computer’s perspective.

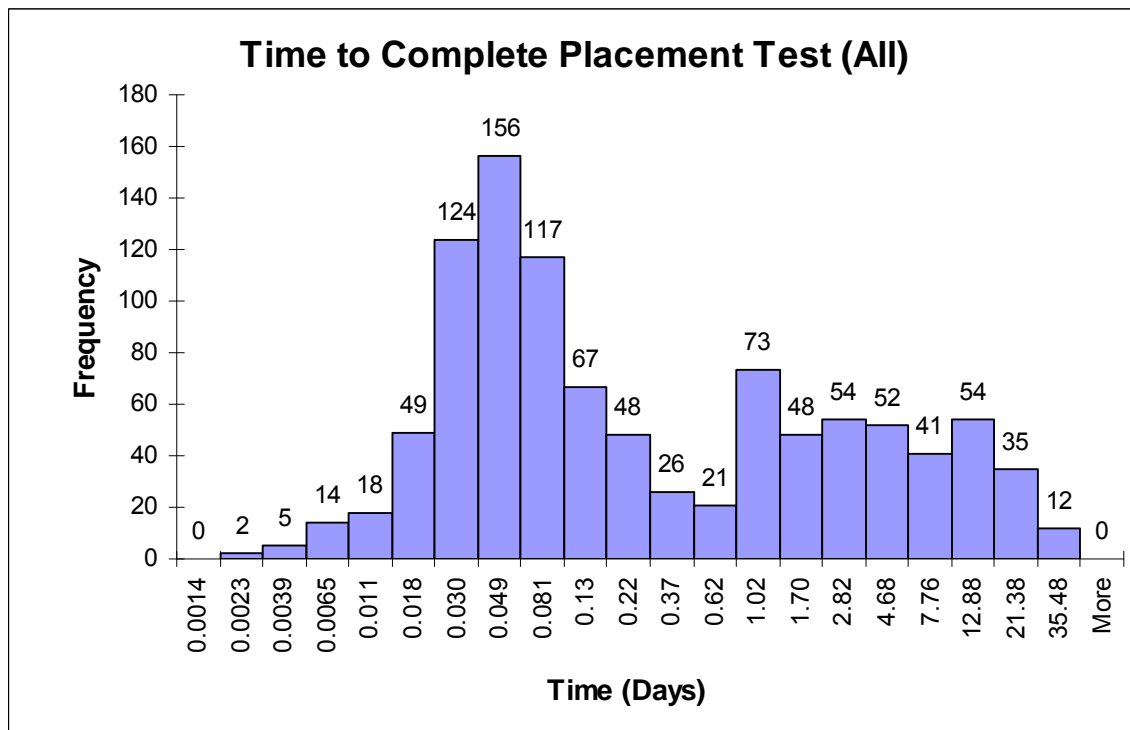


Figure 38: Time to complete placement tests (Logarithmic scale)

Examination of the histogram in Figure 38 therefore demonstrates clearly that the population of students took advantage of the open-ended test format to take the Calculus Placement Tests in a manner most comfortable and convenient for them. The histogram in Figure 38 is color-coded in Figure 39 to serve as a very approximate guide to student approaches to the placement tests (the very qualitative nature of the figure is stressed).

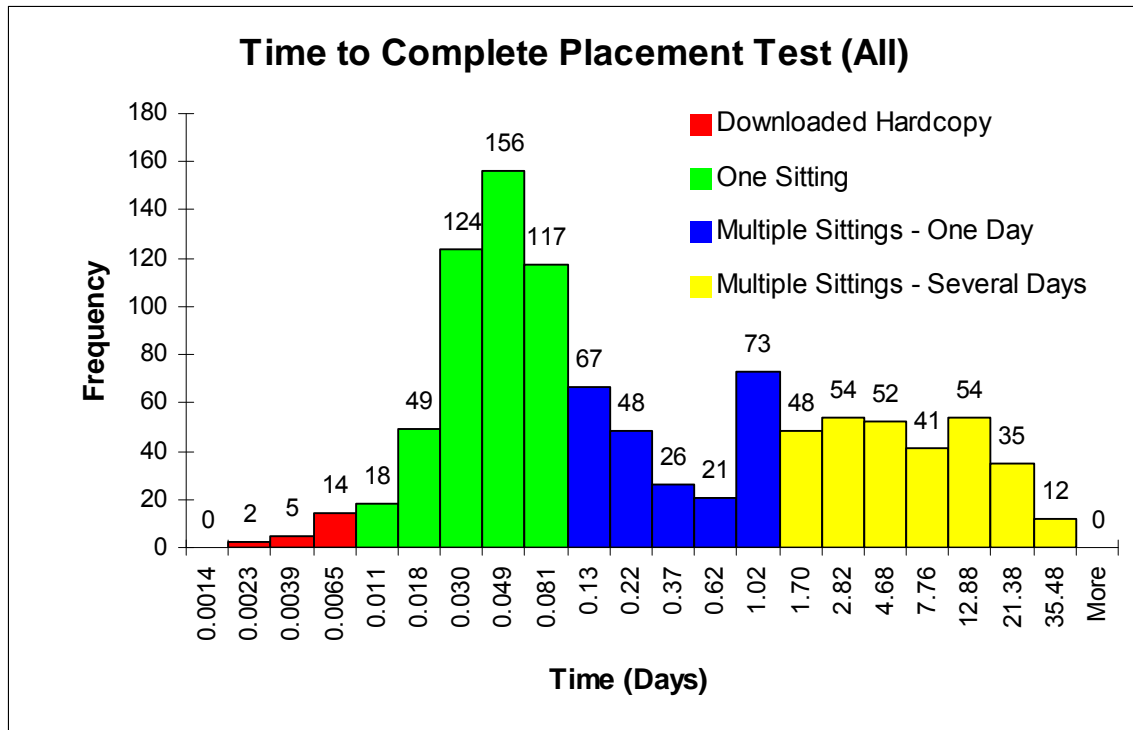


Figure 39: General completion patterns for placement test

The analysis of completion times has thus far considered all three placement tests as a single data set. It is therefore important to consider whether the time-based trends that hold for all three tests generally also hold for the three different tests when considered separately. The histogram of the logarithm of completion time is given for the Calculus I, Calculus II, and Calculus III Placement Tests in Figure 40, Figure 41, and Figure 42, respectively. Clearly, the overall completion time distribution is closely mirrored in the distributions for each test



individually. There are of course slight variations from test to test; for example, it appears that a greater proportion of students spent more than one day on the Calculus I Placement Test than on the other two tests. Despite these differences, it is clear that regardless of the test in question, many students completed it within one day, and fewer but not insignificant numbers of students spent a longer time.

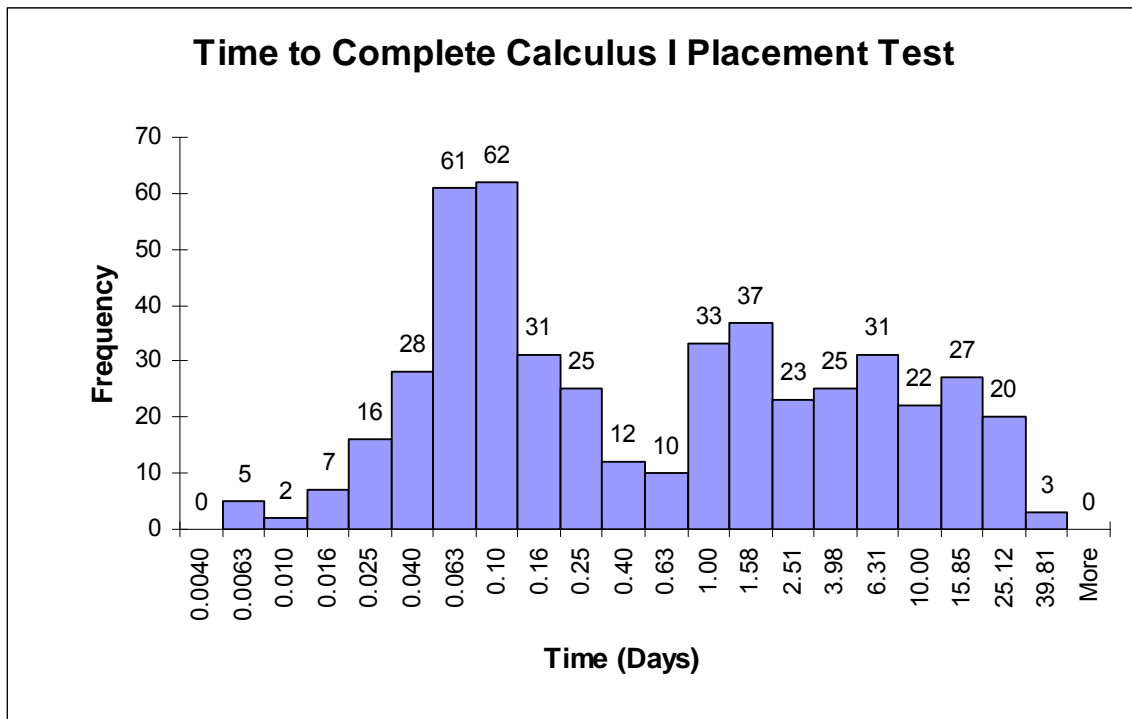


Figure 40: Time to complete Calculus I Placement Test

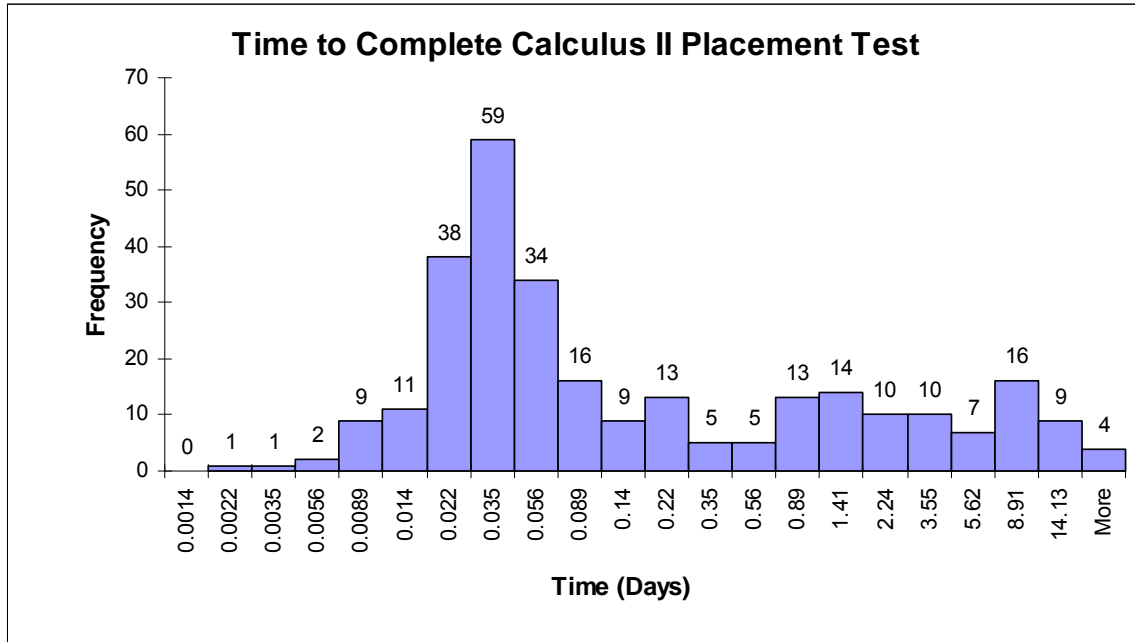


Figure 41: Time to complete Calculus II Placement Test

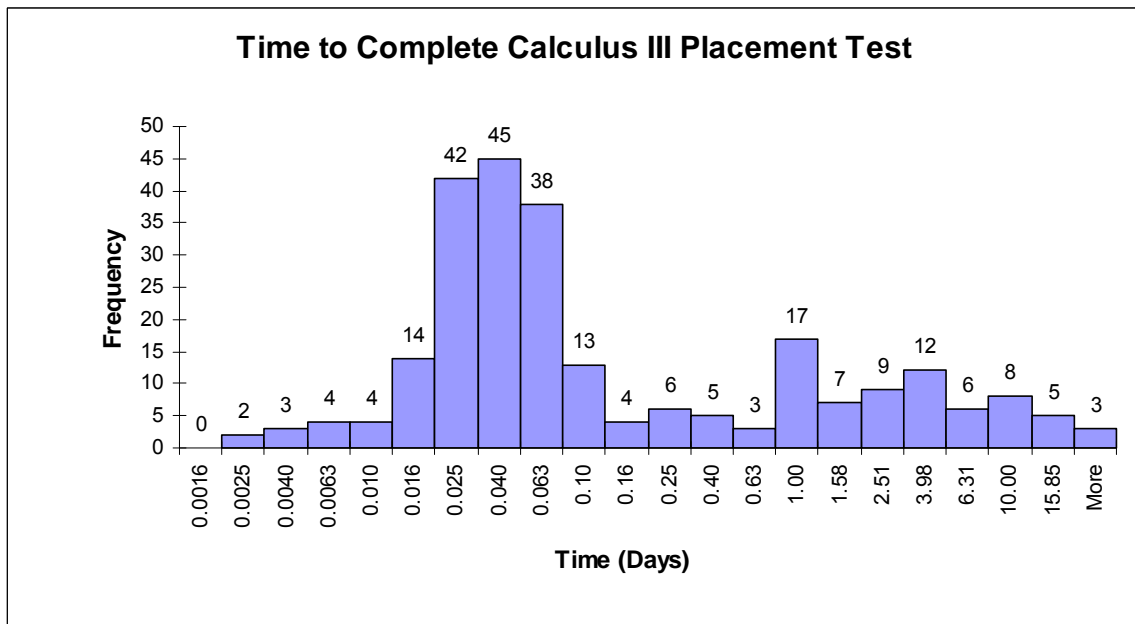
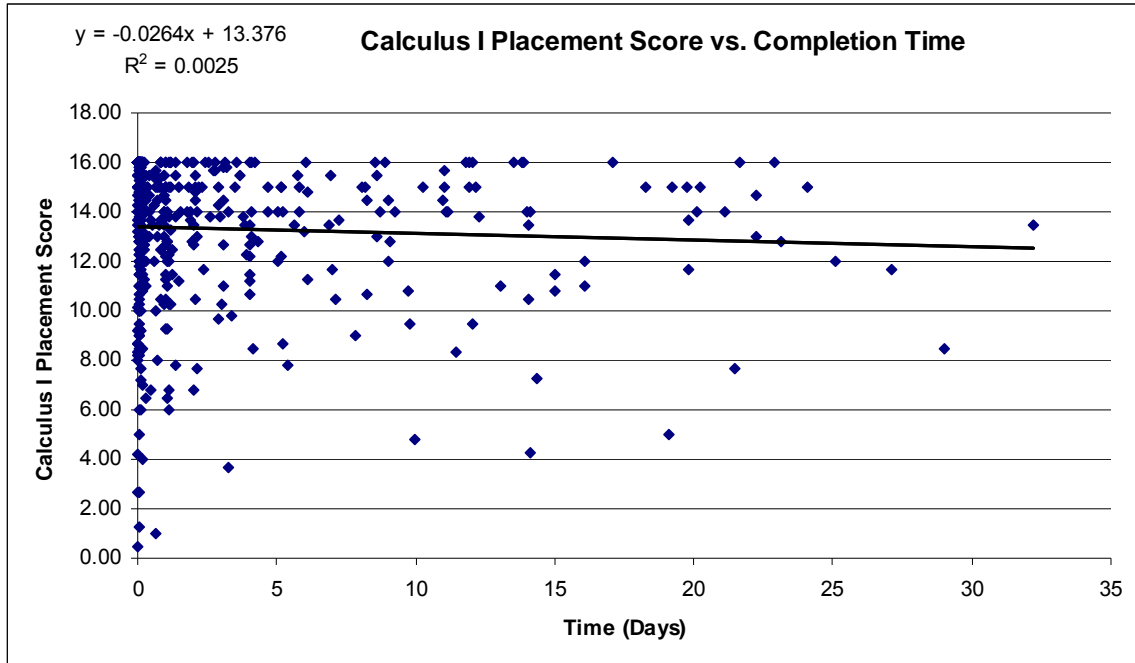


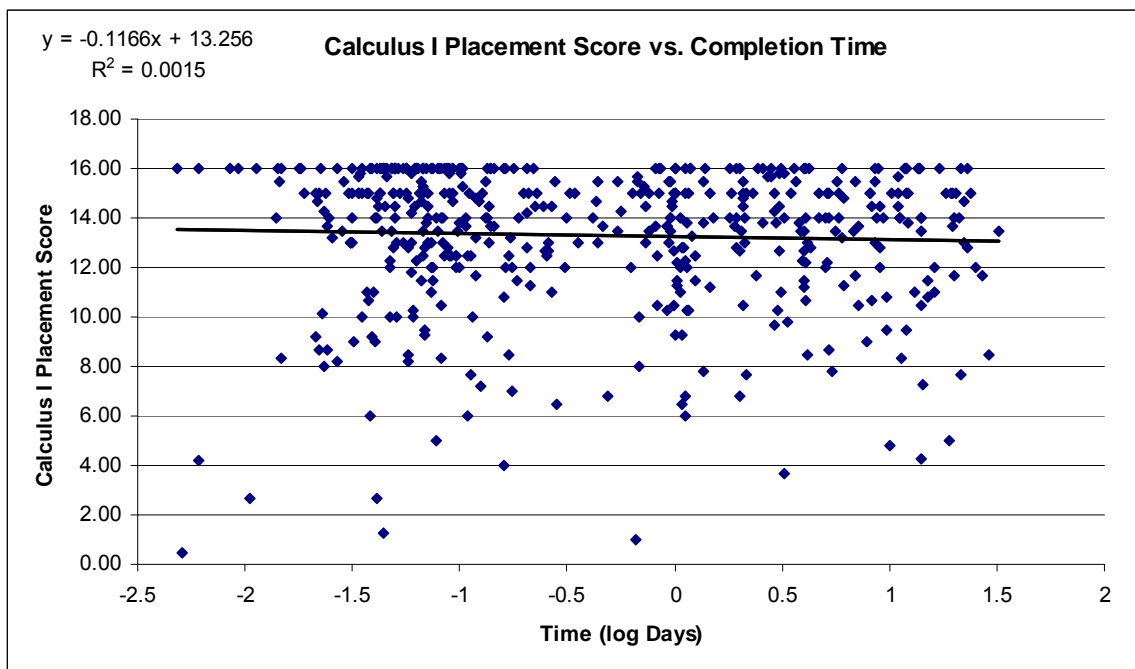
Figure 42: Time to complete Calculus III Placement Test

From the preceding analysis it is clear that completion time varied widely, and that completion time is an indication of more general test-taking behavior (downloading a hardcopy, completing the test in one sitting, etc.). For these reasons it was of interest whether completion time is in any way related to the score earned on the placement test. The most basic relationship that could be observed would be simple correlation: Did students with good placement test scores work more quickly, or vice versa? If completion time and test score are correlated, then there is either a causal relationship between the two variables or an additional factor that explains both.

A plot of Calculus I Placement Test scores versus completion time is shown in Figure 43. Qualitatively, it is obvious that there is almost zero correlation between the two variables; indeed,  $r = -0.05$ ,  $r^2 = 0.0025$ , and the variables cannot be said to be correlated. The clustering of values near low completion times suggests a logarithmic time scale as a way to glean more useful information from this plot. The plot of Figure 43 is therefore reproduced in Figure 44 with a logarithmic time scale. Although this strategy resolves the closely grouped data values at low completion times, the correlation of test score with the logarithm of time is even lower than the correlation with time itself, with  $r = -0.039$  and  $r^2 = 0.0015$ . Clearly, there is no correlation, in the sense of Pearson's correlation coefficient, between Calculus I Placement Test scores and completion time.



**Figure 43: Calculus I Placement Test score vs. Completion Time**



**Figure 44: Calculus I Placement Test Score vs. Completion Time (Logarithmic Scale)**

Since scores on the Calculus I Placement Test were not correlated with completion time, and considering the similarity of the completion time histograms for the Calculus I, II, and III tests, it would be reasonable to expect that scores on the Calculus II and Calculus III Placement Tests also were not correlated with completion time. This was indeed the case. The relevant plots of score versus the logarithm of completion time are shown in Figure 45 and Figure 46 for Calculus II and Calculus III Placement, respectively.

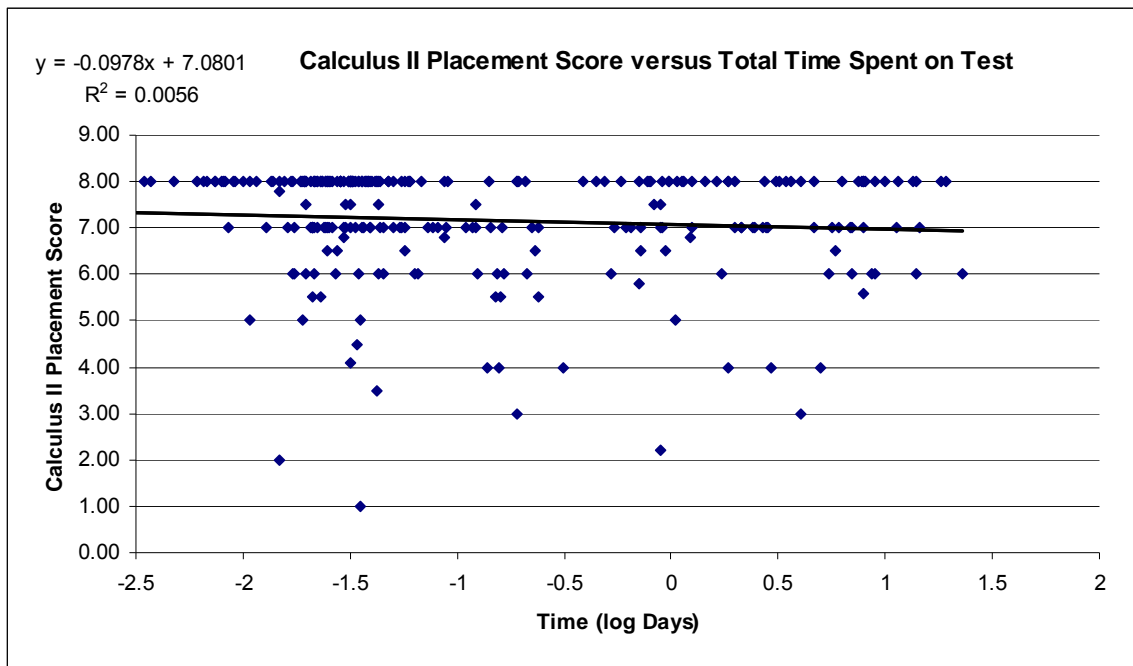
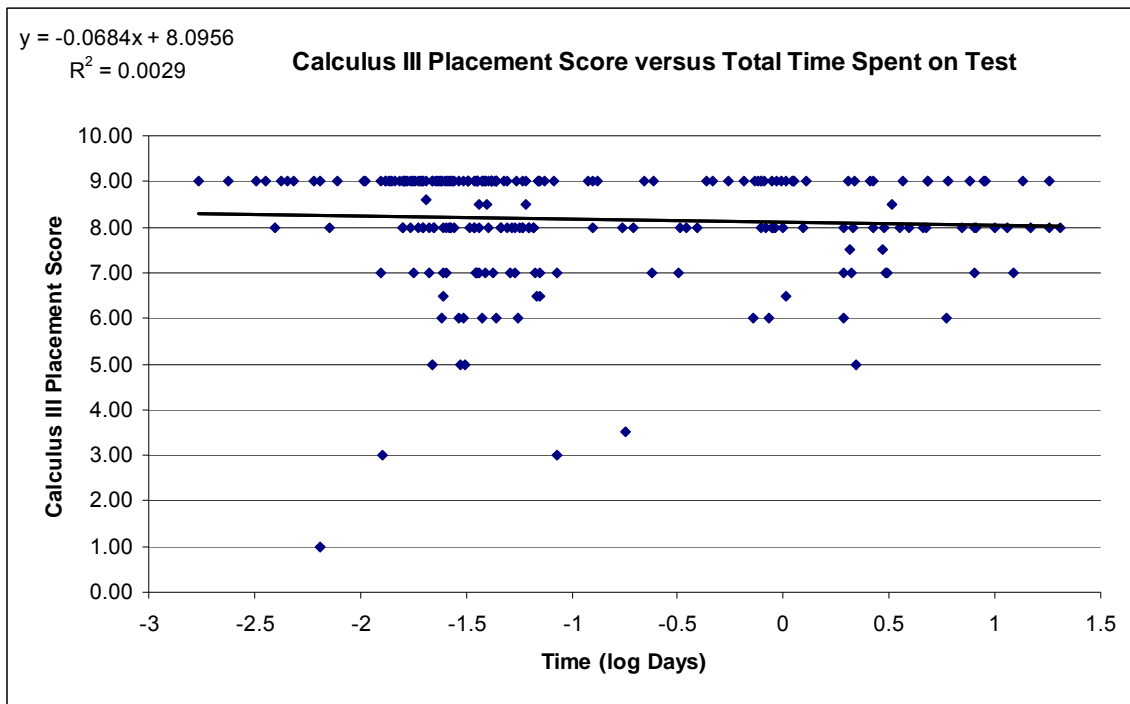


Figure 45: Calculus II Placement Test Score vs. Completion Time (Logarithmic Scale)



**Figure 46: Calculus III Placement Test Score vs. Completion Time (Logarithmic Scale)**

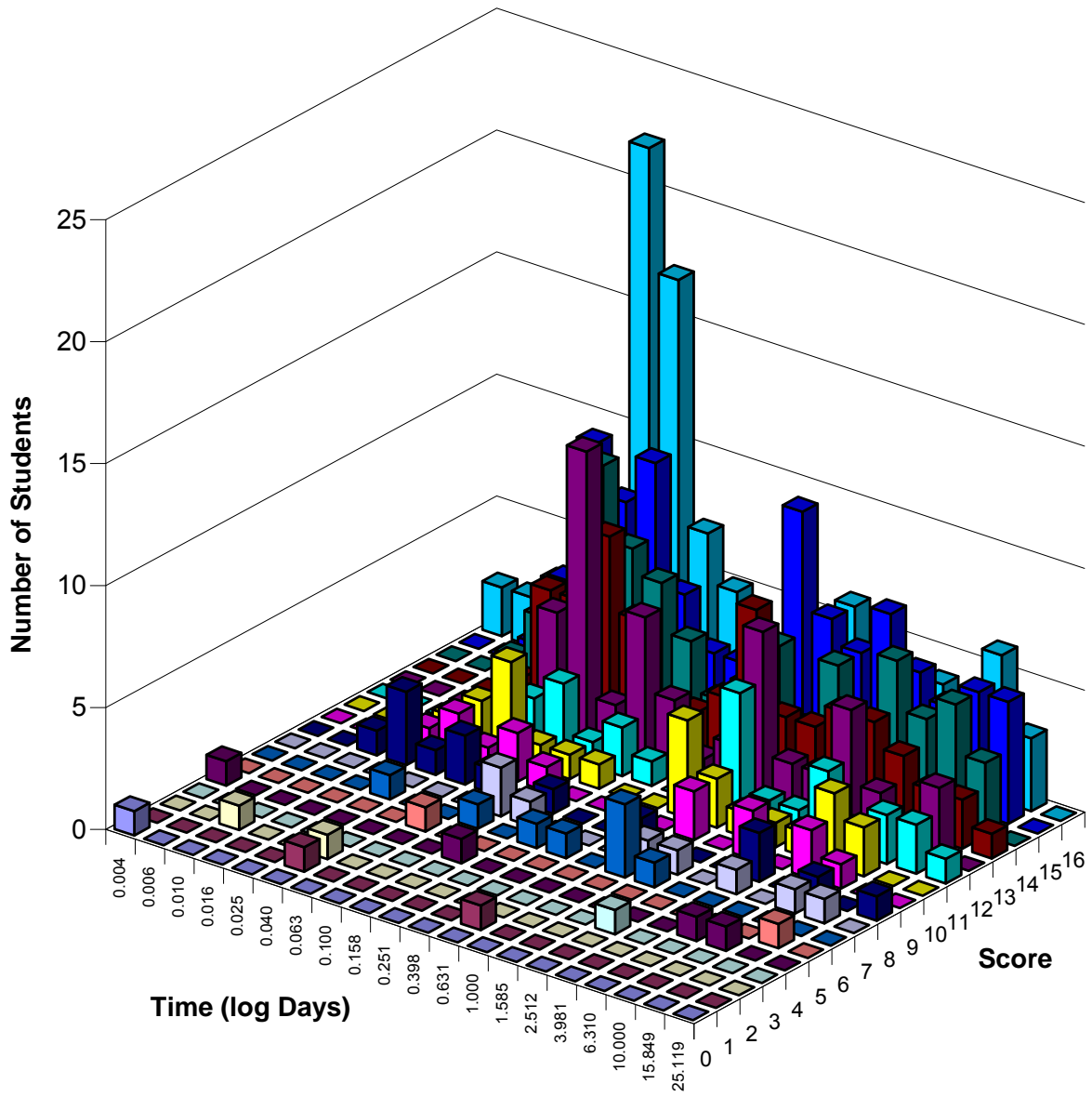
Clearly, a simple correlation between placement test score and completion time cannot be inferred from the data. However, an argument may be made that to expect such a correlation would be unreasonable, given the complexity of the data set and the numerous confounding variables. For example, there are a number of reasons related to one's personal schedule of activities – and not to one's mathematics ability – that could cause a longer or shorter completion time of an open-ended placement test. This analysis does not consider the specific mathematics background of students or their level of comfort with a computer system. It is impossible to know whether students completed the tests entirely on their own or if they received help from another individual. Given these confounding factors, it may indeed be unreasonable to expect test scores to be well-correlated with completion time. Perhaps it is possible that a relationship

that is “weaker” than a simple correlation, but still significant, is at work. Rather than comparing a continuum of time measurements with a continuum (or nearly so) of test scores, it was considered whether grouping these sets of data into discrete bins would shed light on an important pattern.

In performing such an analysis with the Calculus I Placement Tests, the range of values for the logarithm of completion time was divided into 20 equally-sized intervals. The range of placement test scores, which ranged from 0 to 16, was similarly divided into 17 intervals. This created  $20 \cdot 17 = 340$  possible time-score combinations, a large number but certainly a simplification compared to the continuous axes in the scatter plots of Figures Figure 44, Figure 45, and Figure 46. A chart showing the frequency of each time-score combination is given in Figure 47. The most obvious features of Figure 47 are some general trends: scores of 13 and above were far more common than lower scores, and the highest frequencies are found at high test scores and completion times of less than one day. Of course, precise mathematical justification must accompany any qualitative notions. It was necessary to determine, for a given test score, whether the distribution of completion times differed from those of other score values. Simply put, it was necessary to determine if test score and completion time were independent.

To test for the independence of test score and completion time, a chi-square test was performed. The expected distribution was defined as the distribution of completion times over all possible scores; that is, the distribution in Figure 40. The distribution of completion times for a single score bin was tested against this expected overall distribution, yielding the  $p$ -values shown in Table 9.

Figure 47: Calculus I Placement Test Scores vs. Completion Time (Discrete Bins)





**Table 9: Chi-Square Analysis of Calculus I Placement Test Scores vs. Completion Time**

<b>Score Bin</b>	<b><i>n</i></b>	<b><i>p</i></b>
0	1	2.114 x 10 <sup>-16</sup>
1	2	0.9707
2	2	0.0104
3	1	0.5144
4	4	0.0027
5	2	0.7949
6	8	0.1906
7	8	0.8106
8	14	0.0205
9	14	0.8092
10	24	0.8678
11	26	0.6331
12	52	0.1628
13	59	0.8540
14	70	0.9335
15	93	0.8722
16	98	0.0665

The chi-square analysis for the Calculus I Placement Test is summarized in Table 9. Note that the score bin labeled “0” in fact represents scores greater than or equal to 0 and less than 1. If the time distribution for a single score bin differed significantly from the overall completion time distribution at the 90% level, that *p*-value was highlighted in Table 9. The vast majority of observed time differences were not significant and in many cases were associated with very high *p*-values. Of the five time bins where a chi-square test indicated a significant difference, three of these bins represented less than 5 individuals each, in which case the assumptions of the chi-square test are not valid. Only two score bins, namely bins 8 and 16, have both  $n \geq 5$  and significant *p* values. This is fairly convincing evidence that for the Calculus I Placement Test, completion time was independent of test score.

A similar analysis was performed for the Calculus II and Calculus III Placement Tests, with similar results. Graphs and tables analogous to those generated for the Calculus I Test are given in Figure 48 and Table 10 for Calculus II, and Figure 49 and Table 11 for Calculus III.

Figure 48: Calculus II Placement Test Scores vs. Completion Time (Discrete Bins)

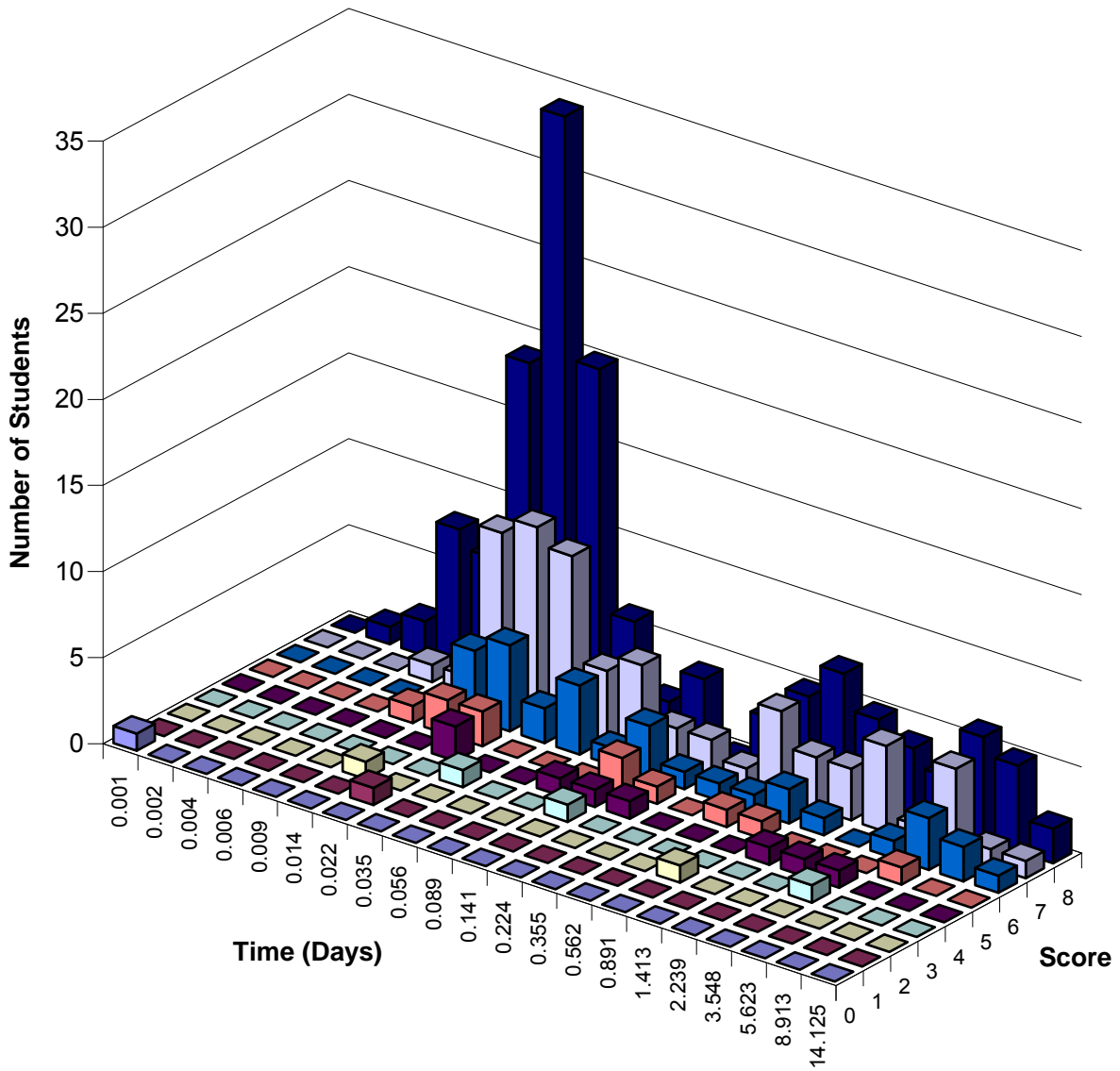
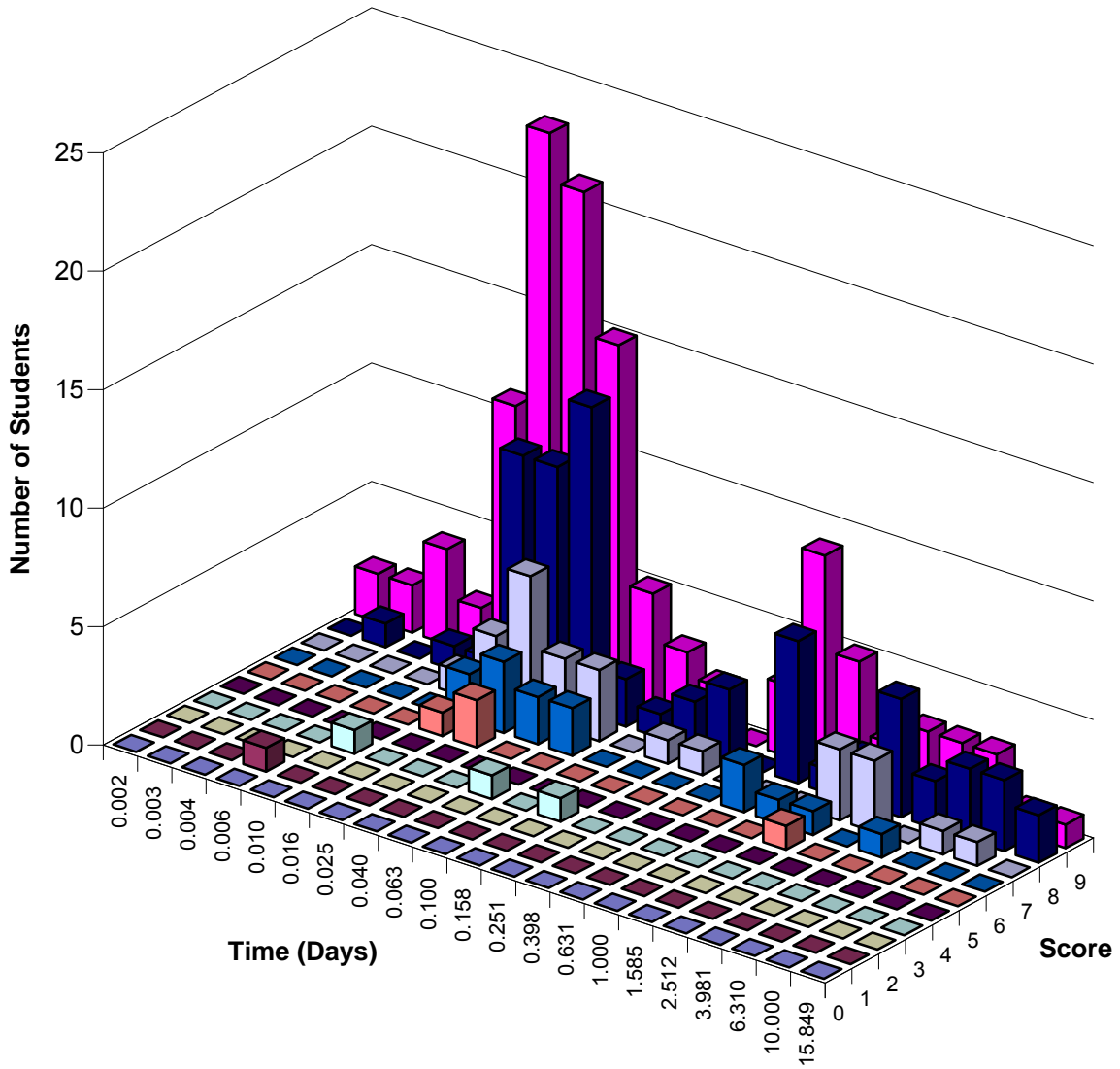


Table 10: Chi-Square Analysis of Calculus II Placement Test Scores vs. Completion Time

Score Bin	$n$	$P$
0	1	$6.462 \times 10^{-46}$
1	1	1.0000
2	2	0.9325
3	3	0.4785
4	8	0.5009
5	11	0.8138
6	32	0.8645
7	71	0.9017
8	143	0.8562

**Figure 49: Calculus III Placement Test Scores vs. Completion Time (Discrete Bins)**



**Table 11: Chi-Square Analysis of Calculus III Placement Test Scores vs. Completion Time**

Score Bin	<i>n</i>	<i>P</i>
1	1	$1.446 \times 10^{-5}$
3	3	0.3525
5	4	0.9735
6	14	0.9744
7	26	0.6736
8	68	0.7046
9	120	0.7796

Note: Among those completing the test, no individuals obtained scores in bins 0, 2, or 4, and these score ranges are omitted from the Table.

The chi-square analyses for the Calculus II and Calculus III Placement Tests are even more convincing than the fairly conclusive results of the Calculus I Placement Test. For both Calculus II and Calculus III, in no case was a significant difference in completion times observed for a score bin containing 5 or more individuals. Said another way, the chi-square test never showed that a significant difference in completion time for any particular cross-section of scores. This point is illustrated quite well visually in Figure 48 and Figure 49. For a given score bin (i.e. a set of similarly-colored bars), the shape of the distribution of completion times visually resembles the distribution shape for other score bins. For the students earning a particular score, the distribution of their completion times is nearly identical to the completion times of the group as a whole. This is strong evidence that the score earned on the Calculus Placement Tests was independent of completion time.

The Calculus Placement Tests are offered in a flexible, Internet-based format that allows students to employ a variety of test-taking strategies. Students may download a hardcopy of the Internet-based questions, complete the test in one sitting, or log out of the test and complete it hours or days later. This flexibility represents a great convenience for students, and is also convenient for the university – it avoids the logistical difficulties of a single, limited-time administration of the test. To ensure the validity of such a test, a student's score should not depend on the manner in which the test is taken; that is, one's score should be independent of completion time. An analysis based on the correlation coefficient and the chi-square test of independence indicates strongly that scores were indeed independent of completion time. From this standpoint, Worcester Polytechnic Institute is justified in offering the Calculus Placement Test electronically over a long period of time for student convenience.

## **5. Results: Correlation Analysis for the Placement Tests**

### **5.1. Using Calculus I Placement Tests to Predict Calculus I Performance**

In creating placement tests for the WPI calculus sequence, an attempt was made to coordinate the placement test with the prerequisite material of the corresponding course. By taking the series of placement tests, students would learn which calculus topics are covered in a particular course, and they could test their proficiency in those subject areas. It may be, then, that a student's performance on the Calculus Placement Tests would to some extent directly predict his or her success in the chosen math course. The goal of this section is to determine the extent to which success on the Calculus I Placement Test predicted success in Calculus I (MA 1021), the course for which the most data was available regarding student performance.

For Calculus I (MA 1021), a great deal of information about student performance was available. The scores obtained on the Calculus I Placement Test are of course known. Regarding course performance, the final grades for the course were available for each student. In addition, scores on the MA 1021 final examination were provided for this study. The final examination for this course is identical for all students taking the course during a given term, regardless of professor or course meeting time, adding a degree of uniformity to the examination results. Furthermore, the examination is divided into two distinct sections, termed the Basic Skills Test and the General Test. As mentioned previously, the Basic Skills Test consists of seven questions on essential concepts that are graded either completely correct or completely incorrect. The General Test contains more complicated problems for which partial credit is available.

Scores on the MA 1021 final examination are the most quantitative information about student performance that this study possesses, increasing the number of analytical techniques that may be used. Specifically, linear regression analysis becomes possible when both sets of data (Calculus I Placement Test scores and MA 1021 final examination scores) are quantitative rather than categorical (i.e. grades of “A,” “B,” etc.). It would be interesting to determine whether scores on the Calculus I Placement Test were to any extent correlated with the scores on the MA 1021 final examination.

As a first step in this determination, scores on the MA 1021 General Test were plotted against scores on the Calculus I Placement Test, as shown in Figure 50 below. Students were included in this analysis only if they in fact completed both tests; that is, if their scores on both were nonzero. Fitting a least squares linear regression equation to this set of data reveals a negligible correlation, with  $R^2 = 1.39\%$ . Overall, then, the Calculus Placement Tests are not useful in directly predicting a student’s score on a particular examination.

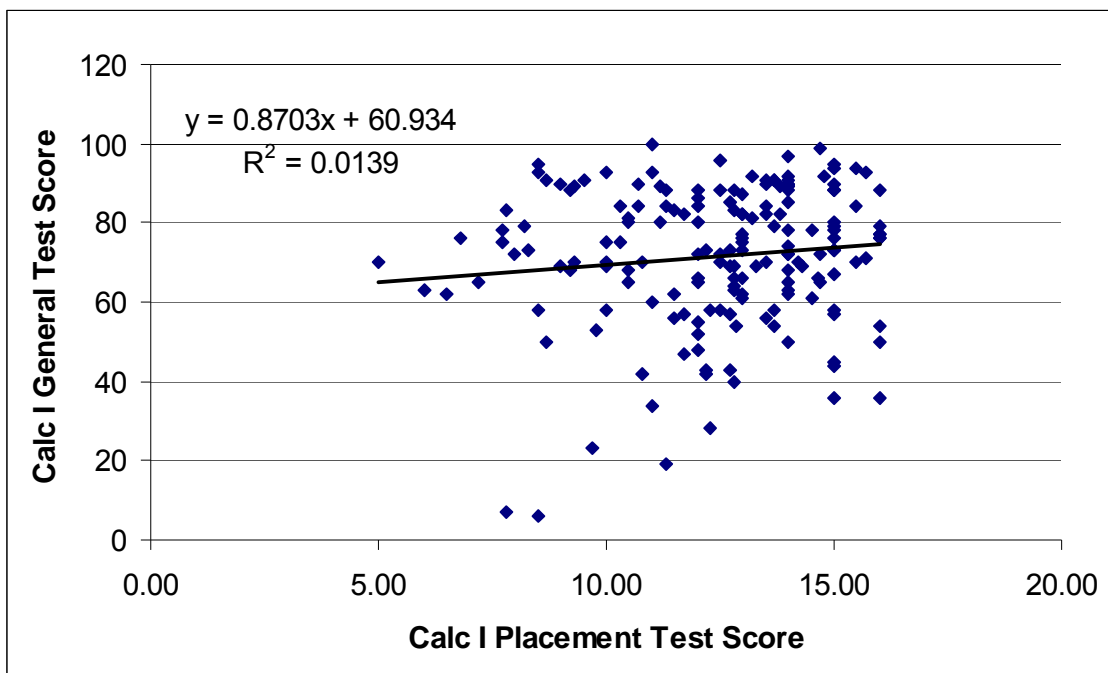
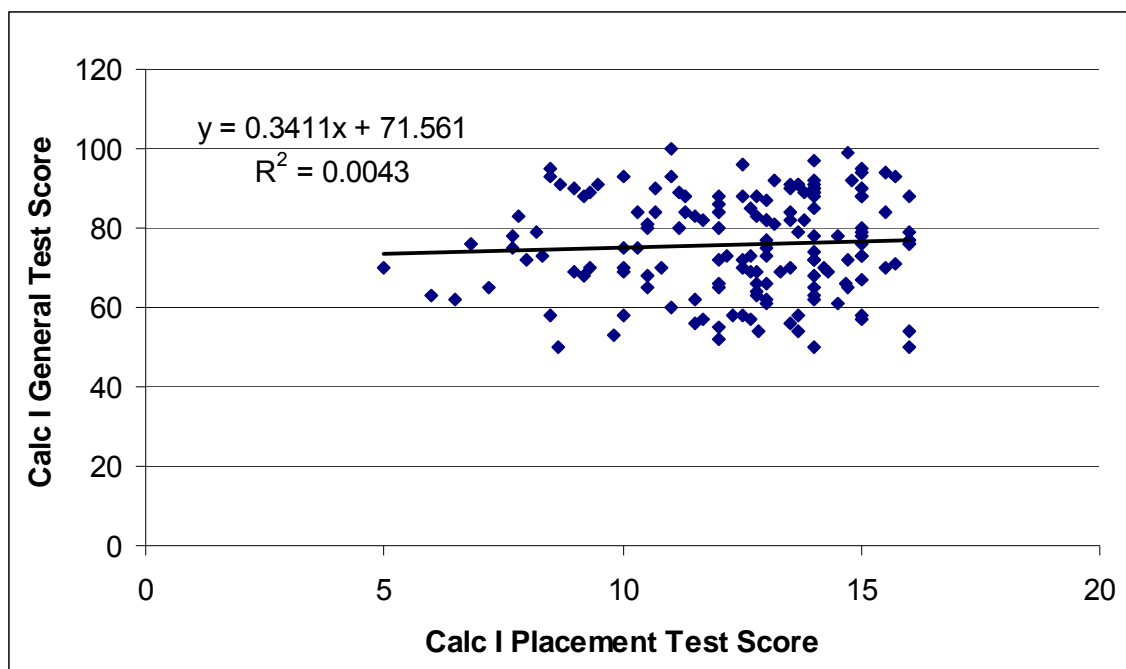


Figure 50: Calculus I General Test vs. Calculus I Placement Test Scores

A lack of correlation between placement test scores and examination scores in the corresponding course might suggest a failure of the placement test system currently in place; however, given the vast number of confounding variables there is no reason to expect that such a direct prediction could indeed be made. Students completed the Calculus I Placement Tests in the comfort of their homes, had no time limit for completion of the tests, could avail themselves of help from other students or calculators, and had three attempts at each problem with feedback after each attempt. In contrast, during a traditional examination there is a time limit, no outside assistance is available, and only one attempt at each question is allowed. Furthermore, the results of a single exam would be affected by other factors – fatigue, degree of preparedness, personal factors, and schoolwork from other classes – whose effects hopefully would be diluted over the course of an entire academic term.

It could be argued that the relatively small number of very low scores on the General Test was disproportionately influential in the regression analysis of Figure 50. To that end, students the lowest 10% of General Test scores were excluded from the graph shown in Figure 51.

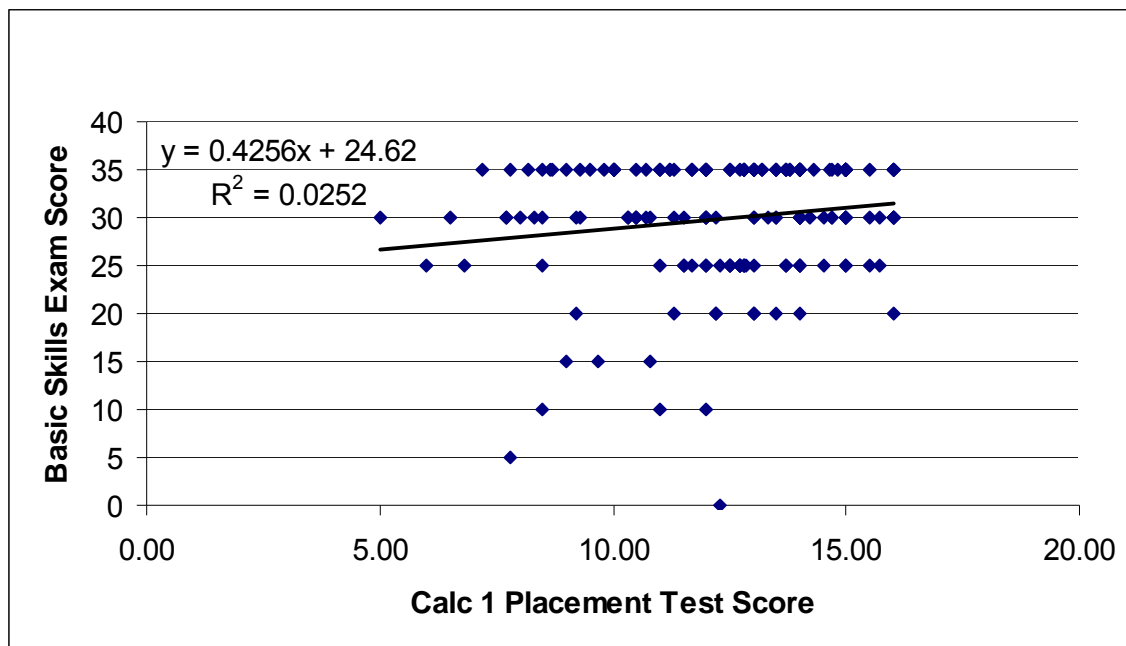


**Figure 51: Calculus I General Test Scores vs. Calculus I Placement Test Scores (Lowest 10% Trimmed)**

The coefficient of determination is still quite small, with  $R^2 = 0.43\%$ . The slope of the regression line did change appreciably after trimming, from 0.87 to 0.34; however, with such a weak correlation it is difficult to attribute much significance to the slope of the line. It is clearly seen that with or without the lowest General Test scores, there is essentially no correlation between scores on the Calculus I Placement Test and the MA 1021 General Test.

Despite the foregoing, for the sake of completeness the extent to which the Calculus I Placement Test predicted success on the Basic Skills Test was also examined. Figure 52 below shows a plot of Basic Skills score versus Calculus I Placement score. A similarly low correlation, with  $R^2 = 2.52\%$ , was observed, demonstrating that the Placement Tests also cannot directly predict Basic Skills Test scores. This should be even less surprising than the similar result with the General Test, since Basic Skills questions were graded on an all-or-nothing basis, with only eight discrete scores possible for the entire test.



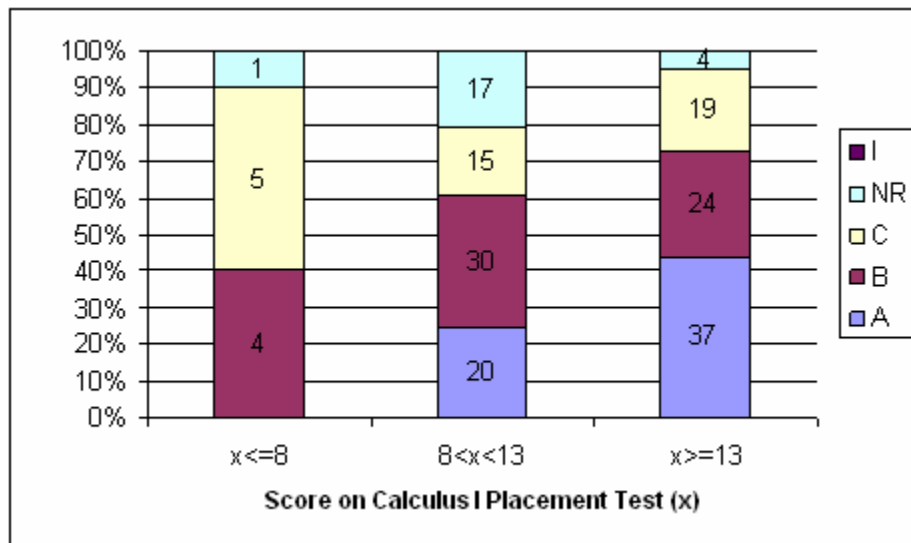


**Figure 52: Calc I Placement Score vs. Basic Skills Score**

It is quite clear that the Calculus I Placement Test is not a useful predictor of specific scores on either component of the MA 1021 final exam. It was discussed, however, that such predictive power should not be expected, given the myriad confounding variables associated with a student's performance on one particular examination. What should be expected is that a placement test can determine whether a student is prepared, in a broad sense, for the course in question. Therefore, while placement test scores cannot predict performance on a single test, they should have some bearing on a student's success in the course as a whole. The analysis that follows will attempt to determine the relationship of Calculus I Placement Test scores to the final course grade in MA 1021.

Numerical scores on the Calculus Placement Test were grouped into three broad categories. A score of 8 or lower (the maximum score possible was 16) lead to a recommendation for MA 1020, Calculus I with Preliminary Topics. A score of 13 or greater indicated that a student should probably choose MA 1021, and students with scores between 8

and 13 were classified as “Borderline” and asked to consider additional factors in making a course choice. Of course, the recommendation that a particular student received was not solely determined by their score – consultation with an advisor at WPI was also common – but these score cutoffs provide a good starting point for analysis. To that end, Figure 53 shows the final grades received by students in MA 1021 who also took the Calculus I Placement Test, with students grouped by their Placement Test score.



**Figure 53: Calculus I Final Grades, Grouped by Scores on the Calculus I Placement Test**

It does appear that higher Placement Test scores do indeed appear to suggest higher success in MA 1021. Among the 10 students in MA 1021 who had Calculus I Placement Test scores of 8 or lower, no students received A’s, and 6 received a C or NR. Students in this group probably took MA 1021 contrary to a recommendation to take MA 1021. Students with “borderline” scores of 8 to 13 achieved a slightly better distribution of scores, although almost 20% of this group did not pass the course. The best grades were for those of students scoring 13 or above on the Placement Test. On the whole, then, it appears that at the level of grade distributions, higher Placement Test scores imply generally better grades in the resulting mathematics course. This is

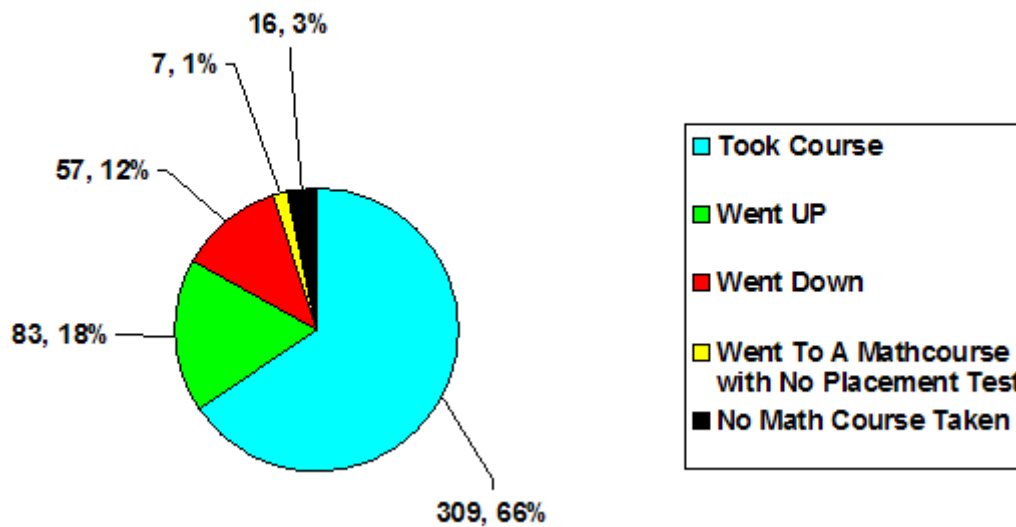
one indication of validity for the Calculus I Placement Test; it does seem to measure readiness for Calculus I.

As mentioned previously, three possible recommendations could arise from the Calculus I Placement Test – MA 1020, “Borderline,” or MA 1021 – with the respective “cutoff” scores being 8 and 13. In addition, these cutoff scores were chosen arbitrarily in the first implementation of the Calculus Placement Tests. Since one goal of this study is to suggest improvements to the Placement Tests, a discussion of the appropriateness of these particular cutoffs would be relevant. This topic is considered in depth in the Section 5.3. *Placement Test Cutoff Scores*.

## 5.2. Comparing Placement Recommendations with Course Choices

Students who took the Calculus Placement Exams were given a recommendation based upon their scores. To determine how effective a placement test is, we need to assess how seriously students take the recommendations. Did students take the course recommended to them? Rather, did they pick a different course? If they did choose courses outside of their recommendation, how did they come to this decision?

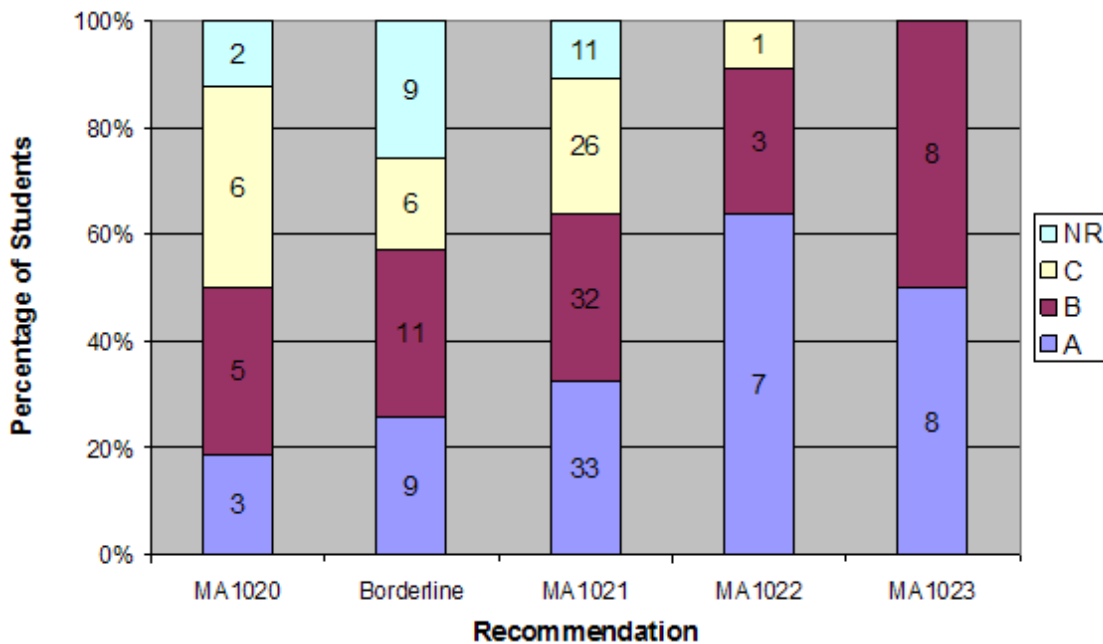
Figure 54 illustrates what students did with the recommendation they received. Courses were ranked in the following manner; MA1020, MA1021, MA1022, MA1023, and MA1024 (Pre-calculus, Calculus I, Calculus II, Calculus III and Calculus 4 respectively). Any other math course was placed in a separate category as a course without a placement test. As illustrated by the graph, 66% of students did follow the recommendation they received. What was interesting was that 18% of students chose to take a course that was ranked above their recommended course, and 12% chose a course at a lower level than the one they recommended. More students chose to go up a level, then down a level. These could be the group of students with high aspirations. How well did these students do? Did moving up a class have any effect on their grade? This data could possibly show that the test was too easy. If students can move up a recommendation and succeed just as well as students who did follow the recommendation, the test could be too easy.



**Figure 54: Where students went with their recommendation**

Figure 55 shows the student body that completed Calculus 1 (MA1021) A-Term of 2007. The students are broken down by the recommendation they received from the calculus placement. The sample of students in this graph includes only those who took and completed the Calculus Placement Exam and attended MA1021. Another important question focuses on how well the students did who didn't take the placement exams. The numbers on the graph indicates the number of students in each group. So, reading the graph we could see that 32 students who were recommended MA1021 and took the course received a B for a grade.

Looking at the graph we can see that the number of people who received an NR (Not Recorded) seems to decrease (by percentage). Students who were recommended for MA1022 and MA1023 all received passing scores and by percentage and more A's (percentage) than the other groups. This trend seems to be expected. The more background knowledge a student has the better they will do in a given course.



**Figure 55: MA 1021 final grades, grouped by original course recommendation**

Figure 56 shows where students went with their recommendations. The bars represent a class and the different sections within the bars represent percentage of students who followed their recommendation they received in a particular way. Included also is the number of students who took the classes who did not take a placement test at all, the students in this category make a notable fraction of the students in each class. The graph shows the proportions of students in each class who chose to take their recommendation and follow it, ignore their recommendation and go up, or ignore their recommendation and go down.

The classes are ranked as follows MA1020, MA1021, MA1022, MA1023, and MA1024 (Pre-Calculus, Calculus 1, Calculus 2, Calculus 3, and Calculus 4 respectively). If a student was recommended for MA1020, but took MA1021 instead, they went up a course. If a student was recommended for MA1022, but took MA1020 instead, they went down a course.

Note that Calculus 4 does not have a placement test, but it was interesting to see the number of students who decided to take it.

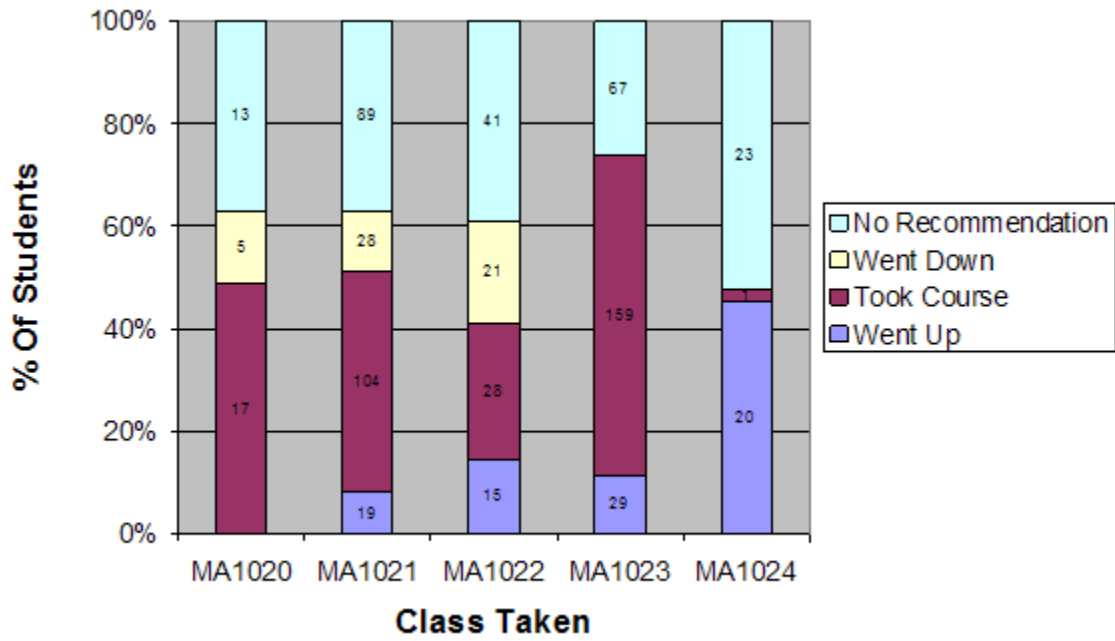


Figure 56: Course populations, grouped by original recommendation

### 5.3. Placement Test Cutoff Scores

Even if the content of a placement test has been carefully selected, there are several other factors that are required for a successful test. One of the most important elements of a placement test besides the content itself is a consistent and reasonable method for determining the course recommendation that results from a given test score. Often, a score threshold or cutoff is implemented such that students scoring higher than the cutoff are recommended for a more advanced course, with students scoring lower being recommended for a less advanced course. These cutoff scores must be chosen very carefully, as poorly chosen cutoffs can have a drastic, detrimental impact on enrollment patterns for the courses in question, even for an otherwise well-designed placement test. This analysis serves to examine the appropriateness of the cutoff scores that were used on the Calculus Placement Tests and to recommend changes where necessary.

Recall that each of the three Calculus Placement Tests had at least one cutoff score that led to a suggested recommendation. For the Calculus I Placement Test, a score of 8 or below (out of a possible score of 16), led a student to be recommended for Calculus I with Preliminary Topics (MA 1020). Students with scores of 13 or above received a recommendation for Calculus I (MA 1021). A score between 8 and 13 gave a student a “Borderline” recommendation, and these students were advised to consult with an academic advisor to determine their first mathematics course. For the Calculus II and Calculus III Placement Tests, there was no perceived need for a “Borderline” classification and only a single cutoff score was used. For the Calculus II Placement Test, a score greater than or equal to 6 of 8 possible points led a student to be recommended for Calculus II (MA 1022), with scores below 6 resulting in a recommendation



for Calculus I (assuming the Calculus I Placement Test was passed). For the Calculus III Placement Test, students were recommended to enroll in Calculus III (MA 1023) if they earned a score of 7 or greater out of a possible 9 points. Table 12 below summarizes the cutoff scheme for these three tests.

**Table 12: Cutoff Scores for the Calculus Placement Tests**

	<b>Failing</b>	<b>Borderline</b>	<b>Passing</b>
<b>Calculus I Placement</b> (16 points possible)	8 or Below	Between 8 and 13	13 or Above
<b>Calculus II Placement</b> (8 points possible)	Below 6	–	6 or Above
<b>Calculus III Placement</b> (9 points possible)	Below 7	–	7 or Above

It is important to stress that because the Calculus Placement Tests were optional, and because the choice of a mathematics course was ultimately left to the student, these cutoff scores are by no means the sole determinant of a student's courses. Indeed, in some cases the Calculus Placement Test resulted in a somewhat contradictory recommendation. For example, 24 students passed the Calculus III Placement Test while also being classified as "Borderline" by the Calculus I Placement Test. While this group was certainly a small subset of the 525 entering freshmen who took the Placement Tests, it illustrates that the tests, as currently implemented, need not produce a definitive recommendation. Also, it is important throughout this discussion to stress that any course recommendation from the Placement Tests could be changed after personal consultation with WPI faculty or with the Office of Academic Advising. The goal of the Placement Tests was not to impose unilateral course recommendations on students, but rather to facilitate course choices by communicating the content and goals of the calculus sequence. Still, the recommendation arising from the Placement Tests was an important starting point in the

decision-making process for most students, and it is of critical importance to optimize the accuracy of that recommendation.

Because the Calculus II and Calculus III Placement Tests used only one cutoff score, these tests are fundamentally simpler to discuss and will be addressed first. The insights obtained from a discussion of these tests are then applicable to the more complicated case of the Calculus I Placement Test, which had two different cutoff scores. Note that in the actual administration of the Calculus Placement Tests, several students did indeed change their recommendation after consultation with an advisor. Such changes, however, are very unpredictable and cannot be analyzed mathematically to any meaningful level. For the purposes of this discussion, then, any statement about a student's course recommendation refers to the recommendation they *would have* received had their recommendation been determined solely by the numerical values of their placement test scores. Furthermore, although most students took more than one placement test, for this analysis tests are considered one at a time.

Table 13 below shows the results of the Calculus II Placement Test in terms of the recommendations received. Approximately 90% percent of students taking this test were recommended for Calculus II, with only about 10% of students failing to meet the cutoff score. Solely in light of the information in Table 13, it is unclear whether there is cause for concern in such a high passing rate. This was an optional test, so students who chose to complete it were very likely to have substantial prior experience with differential calculus, making them likely to earn a passing score of 6 or higher. However, the high pass rate calls attention to the possibility that the requirements for passing this test were too lenient.

**Table 13: Results of the Calculus II Placement Test**

<b>Recommendation</b>	<b>Number of Students</b>	<b>Percentage</b>
Calculus II ("Passing")	249	90.2%
Calculus I ("Failing")	27	9.8%
Total	276	100.0 %

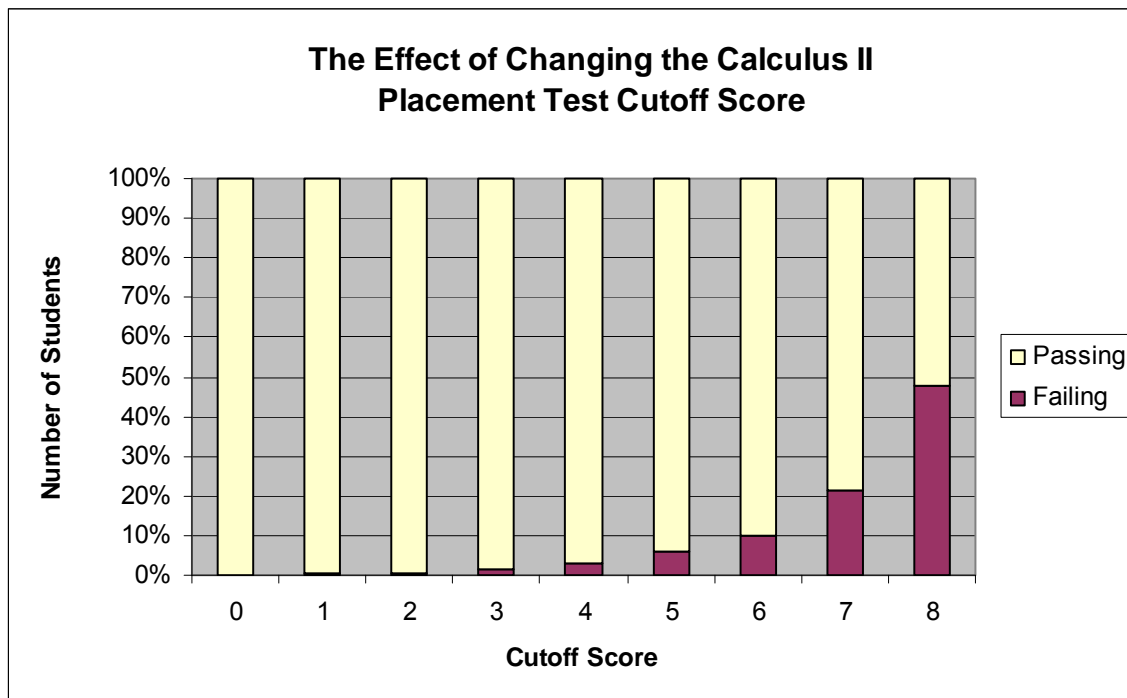
To determine whether the cutoff score for the Calculus II Placement Test was appropriate, consider the effect of a change in the cutoff score on the recommendations of the 276 students who completed this test. Table 14 shows the percentages of students who would have passed or failed the placement test if the cutoff score had been a particular value. Information for the true cutoff score (6) is highlighted in blue. The same information is also presented graphically in Figure 57.

Consider the effect of increasing or decreasing the cutoff score by one point. Increasing the cutoff score from 6 to 7 would have more than doubled the percentage of students who failed the test, from 9.8% to 21.4%. Decreasing the cutoff score to 5 would have roughly halved the number of students failing the test, with the failure percentage decreasing from 9.8% to 5.8%. It is clear that cutoff scores above 7 (which would require a perfect score to pass) and below 5 are unreasonable in terms of the number of students who would fail the test under such a scenario – either far too many or too few. The question of the appropriateness of the Calculus II Placement Test cutoff score can therefore be posed as follows: Would increasing or decreasing the cutoff score by one unit be likely to result in more appropriate recommendations for most students? To determine whether a recommendation was appropriate, some measure of student performance

beyond the placement tests is needed, and grades in the first mathematics course are a natural choice.

**Table 14: The Effect of Changing the Calculus II Placement Test Cutoff Score**

Cutoff Score	Failing Students	Passing Students	Failing %	Passing %
0	0	276	0.0%	100.0%
1	1	275	0.4%	99.6%
2	2	274	0.7%	99.3%
3	4	272	1.4%	98.6%
4	8	268	2.9%	97.1%
5	16	260	5.8%	94.2%
6	27	249	9.8%	90.2%
7	59	217	21.4%	78.6%
8	132	144	47.8%	52.2%



**Figure 57: The Effect of Changing the Calculus II Placement Test Cutoff Score**

Surely, if the cutoff score for the Calculus II Placement Test had been changed, then the number of students taking the course also would have changed. Raising the cutoff score would decrease enrollment in Calculus II, but it would also, in theory, improve the grade distribution of

those taking the course, since they were ostensibly better-prepared. Conversely, decreasing the cutoff score would allow enrollment in Calculus II to be increased, but this might also worsen the overall grade distribution by admitting students who were not as well-prepared for the course. Fortunately, the data available for this study allow some quantitative statements to be made about the effect of the placement test cutoff score on performance in the ensuing course.

Consider the population of students who completed Calculus II in Term A. Most of these students did indeed achieve passing scores on the Calculus II Placement Test, but they need not have passed this test to enroll in Calculus II. Other students did not complete the placement tests at all. The graph in Figure 58 shows the grade distribution of students taking Calculus II in Term A, broken down by the recommendation they would have received as a result of their Calculus II Placement Test Results. For reference, the grade distribution of all students taking the course – whether or not those students completed a placement test – is provided in the rightmost column. Figure 59 displays the same information in terms of percentages. The most striking feature of these graphs is that only *one* student who received a failing score on the Calculus II Placement Test subsequently completed the Calculus II course. This is a strong affirmation of the influence of the Placement Tests, and it also provides even more incentive to set cutoff scores properly. Additionally, students who took the Calculus II Placement Test and received a “MA 1022” recommendation appeared to perform somewhat better in Calculus II than the general population, with roughly 10% more A’s and fewer failing grades (See Figure 59).

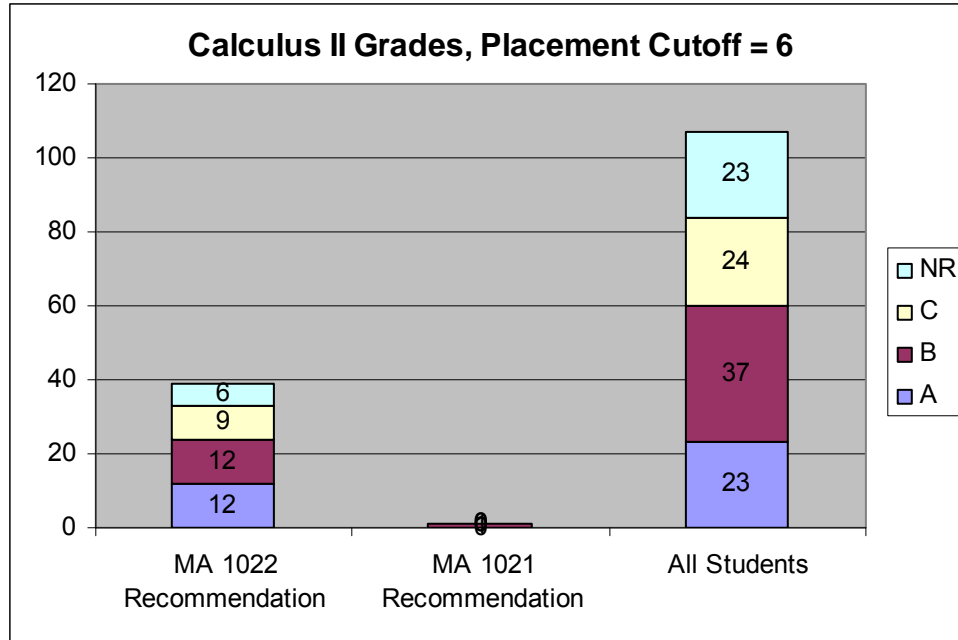


Figure 58: Distribution of Calculus II Grades by Recommendation, Original Cutoff Score

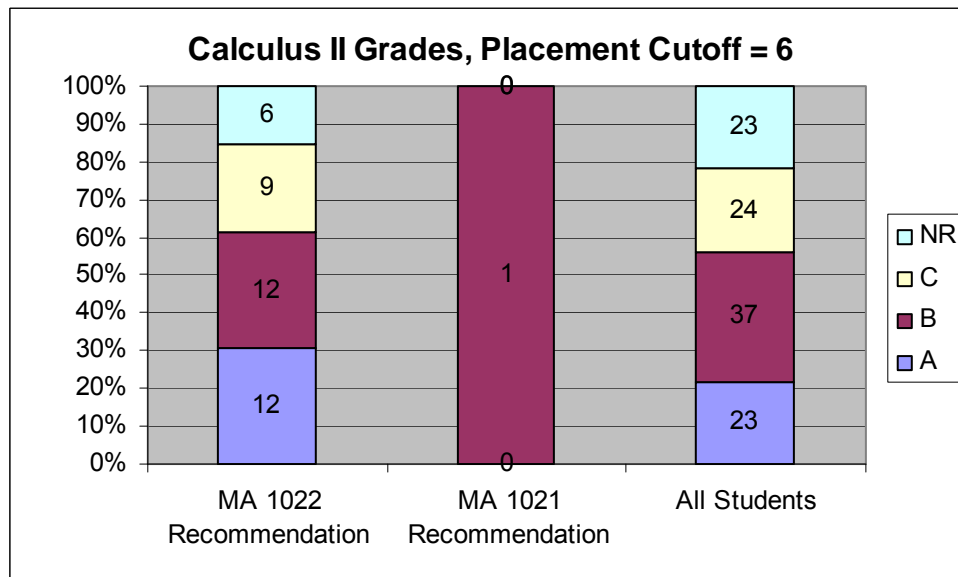


Figure 59: Distribution of Calculus II Grades by Recommendation, Original Cutoff Score

By studying the group of students who both completed the Calculus II Placement Test *and* completed Calculus II, the hypothetical effect of a change in the cutoff score can be considered. For example, suppose that the same group of students completed the Calculus II course, but that the cutoff score had been raised to 7 of 8 points. If this were the case, then more

students would have failed the Calculus II Placement Test and would have overridden this recommendation to take Calculus II. This situation is obviously quite hypothetical but represents the most quantitative prediction that can be made regarding a change in cutoff scores.

Figure 60 and Figure 61 show the result of this hypothetical increase of the cutoff score from 6 to 7. The principal effect is that, of the students who did indeed take Calculus II, eight more students in the class would have been recommended to take Calculus I instead. Did these students, who under this stricter placement scheme probably would not have taken Calculus II, actually perform poorly in Calculus II? Comparison of Figure 61 with Figure 59 shows that students who would have received an “MA 1022 Recommendation” had virtually identical grade distributions even after increasing the cutoff score. This means that making the passing requirement more stringent would have done nothing to improve the grade distribution of the course, in effect decreasing enrollment without a commensurate improvement in overall performance. Taken with the previous observation that increasing the cutoff score to 7 would more than double the Calculus II Placement Test failure rate (see Table 14), there appears to be no justification for increasing the Calculus II Placement Test cutoff score.

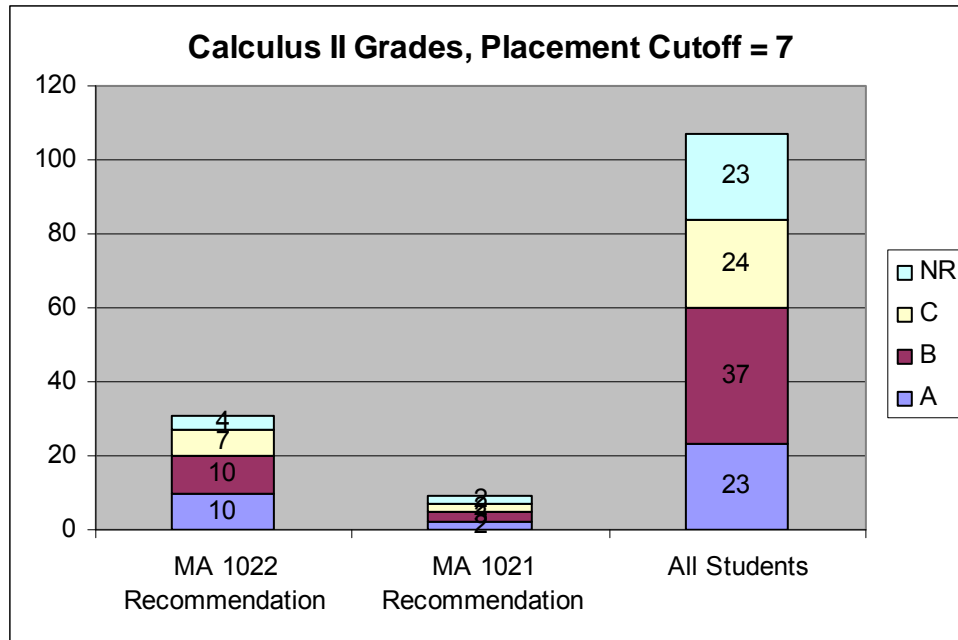


Figure 60: Distribution of Calculus II Grades by Recommendation, Cutoff Score of 7

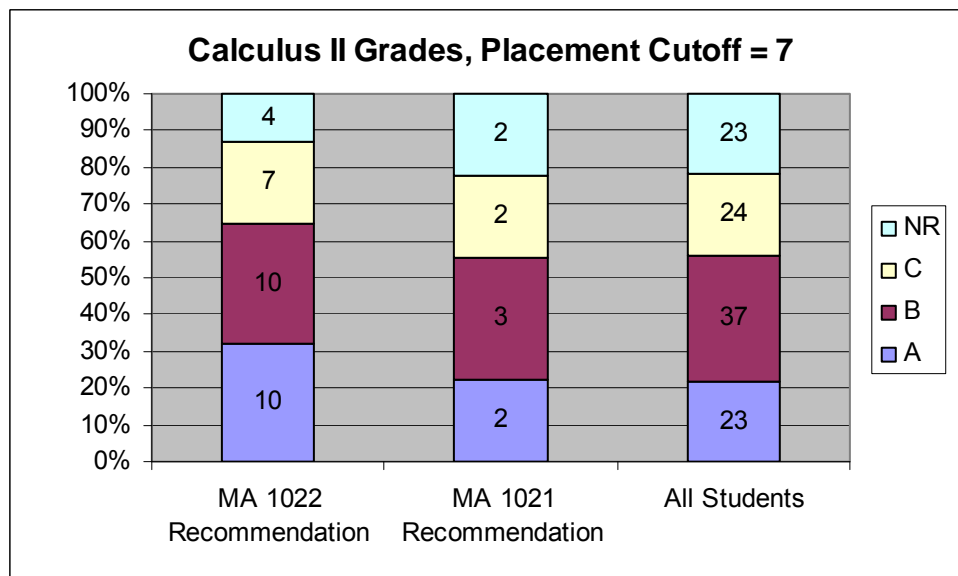


Figure 61: Distribution of Calculus II Grades by Recommendation, Cutoff Score of 7

In principle, a similar hypothetical situation could be constructed to represent lowering the cutoff score from 6 to 5. Recall, however, that only *one* student who took Calculus II received a failing score on the Calculus II Placement Test. With regard to the Calculus II grade



distribution as seen in Figure 58 through Figure 61, then, lowering the cutoff score would only change the recommendation of that one student. Of course, with a lower cutoff score many more students who did *not* take Calculus II in Term A might have been convinced to do so now that their placement scores were in the passing range, but since they did not actually take Calculus II in Term A, no statement can be made here about their Calculus II grade. This illustrates the principal shortcoming of this approach.

Still, it was observed earlier (see Table 14) that decreasing the cutoff from 6 to 5 would lower the placement test failure rate from 9.8% to 5.8%. Since it was also observed that students who complete the Calculus II Placement Test are, through self-selection, generally quite competent in differential calculus, there seems to be little justification for further lowering the passing score when the vast majority of students pass the test already. This collection of evidence suggests that there is no justification for changing the cutoff score for the Calculus II Placement Test, which should remain at 6 of 8 possible points.

Similar arguments may be used to examine the cutoff score of the Calculus III Placement Test, which like the Calculus II Test had one cutoff score above which students would be recommended to enroll in Calculus III. Table 15 shows the distribution of passing and failing scores for the Calculus III Placement Test. Interestingly, the distribution is almost identical to that of the Calculus II Placement Test, with approximately 90% of students achieving a passing score and 10% failing to achieve that score. The foregoing discussion concluded that there was insufficient evidence for a change in the Calculus II cutoff score, so that observation of a strikingly similar passing rate for the Calculus III Placement Test is at least a preliminary indicator that there is also no reason to change the Calculus III cutoff score. Further analysis will serve to confirm this indication.

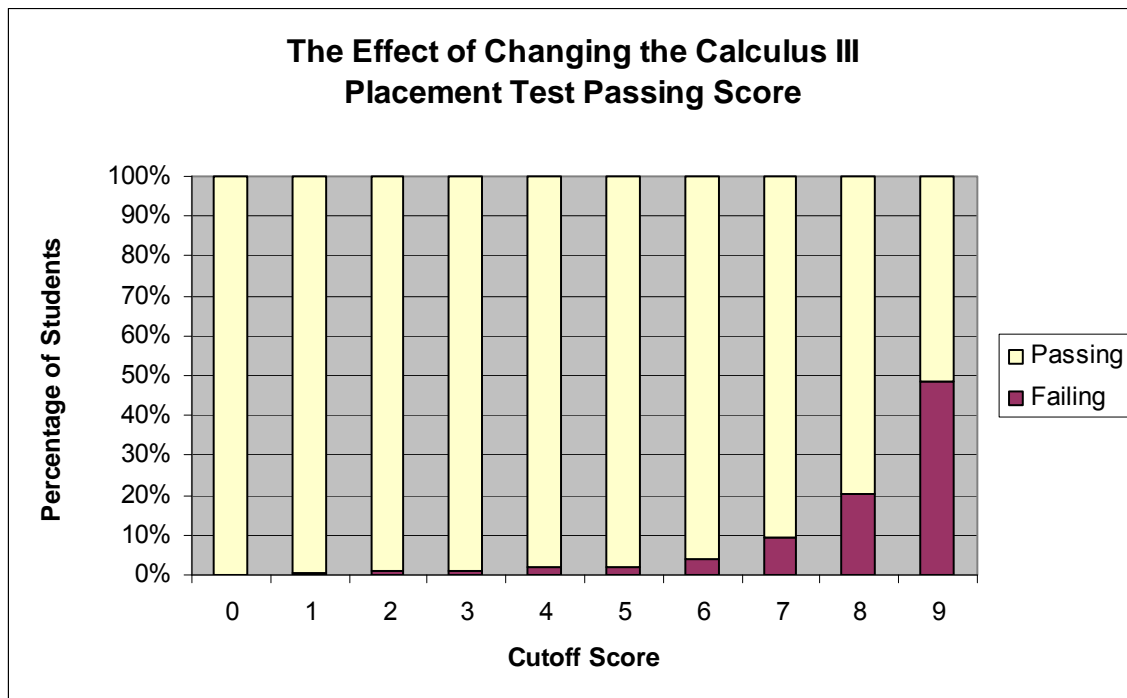
**Table 15: Results of the Calculus III Placement Test**

<b>Recommendation</b>	<b>Number of Students</b>	<b>Percentage</b>
Calculus III ("Passing")	217	90.4 %
Calculus II ("Failing")	23	9.6 %
Total	240	100.0 %

As before, consider the effect of changing the Calculus III Placement Test cutoff score on the course recommendations of the 240 students who completed the test. Table 16 shows the number of students who would have passed (Calculus III recommendation) or failed (Calculus II recommendation) the placement test for a given cutoff score. The same information is provided graphically in Figure 62. The true cutoff score of 7 is highlighted in blue and shows the approximately 90% passing rate mentioned previously. Increasing the cutoff score from 7 to 8 would more than double the percentage of failing students to 20.4%. Considering that students who have completed this placement test did so of their own volition, they probably have substantial previous knowledge of integral calculus, and there is probably little reason to make passing requirements strict enough to deny Calculus III recommendations to 1 in 5 of these students. On the other hand, decreasing the cutoff score to 6 would result in a failure rate of only 3.8%, and there appears to be little justification for lowering standards for an already strong group of students. The remaining evidence to be examined for the Calculus III Placement Test links performance in the first mathematics course to performance on the placement test.

**Table 16: The Effect of Changing the Calculus III Placement Test Cutoff Score**

Cutoff Score	Failing Students	Passing Students	Failing %	Passing %
0	0	240	0.0%	100.0%
1	1	239	0.4%	99.6%
2	2	238	0.8%	99.2%
3	2	238	0.8%	99.2%
4	5	235	2.1%	97.9%
5	5	235	2.1%	97.9%
6	9	231	3.8%	96.3%
7	23	217	9.6%	90.4%
8	49	191	20.4%	79.6%
9	117	123	48.8%	51.3%



**Figure 62: The Effect of Changing the Calculus III Placement Test Cutoff Score**

To examine the hypothetical effect of the Calculus III cutoff score on the distribution of grades in Calculus III, consider the group of students that both completed the Calculus III Placement Test and enrolled in Calculus III. Figure 63 and Figure 64 show the distribution of final grades for these students, who are divided according to the recommendation they would have received based on their Calculus III Placement Test score. A column showing the grades of

all students who took Calculus III, regardless of whether they completed a placement test, is given in the rightmost column for reference. Note that only 7 students overrode a recommendation not to enroll in Calculus III and took the course nonetheless. Also, 158 of the 259 students who completed the Calculus III course took the Calculus III Placement Test. These observations indicate many students not only took the placement tests but followed their course recommendation. Encouragingly, Figure 64 indicates that students who received and abided by a recommendation for Calculus III received almost exactly the same distribution of grades as the general population, showing that the placement test confirmed that these students were as well-prepared to take Calculus III as any other student who did so.

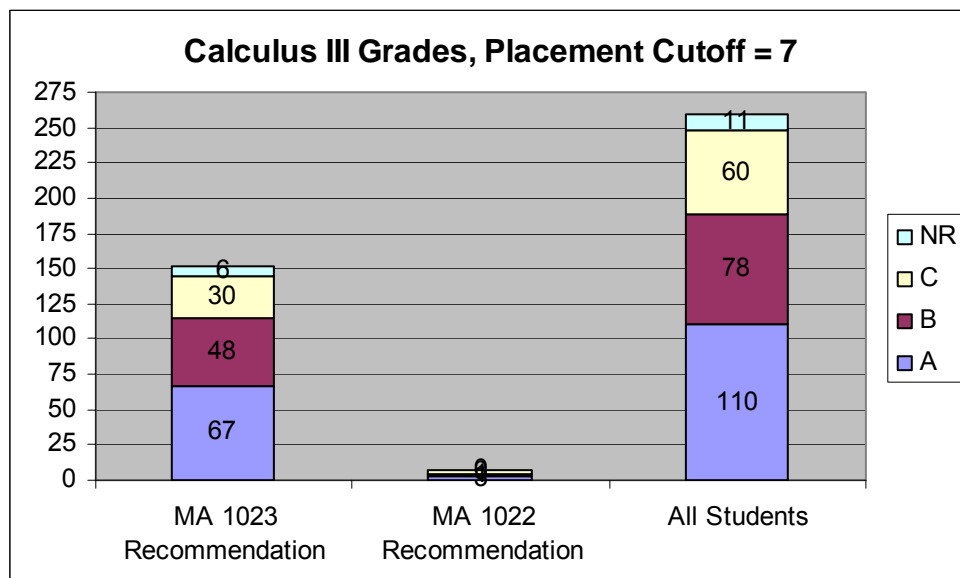
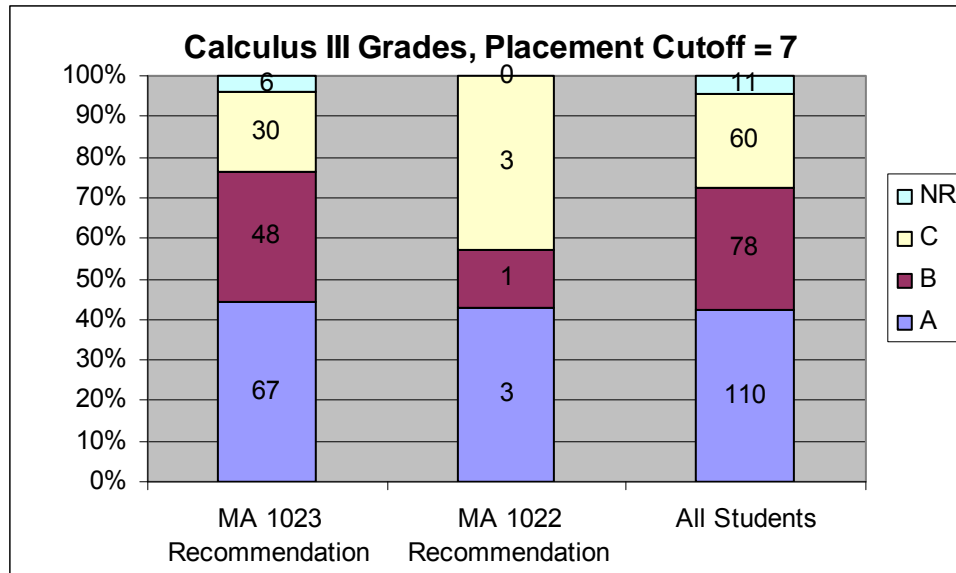


Figure 63: Distribution of Calculus III Grades by Recommendation, Original Cutoff Score



**Figure 64: Distribution of Calculus III Grades by Recommendation, Original Cutoff Score**

Now the effect of a hypothetical increase in the Calculus III Placement Test cutoff score from 7 to 8 can be considered. Figure 64 and Figure 65 show the redistribution of recommendations based on a cutoff score of 8. Such a change would have caused 15 students who took Calculus III to receive a recommendation for Calculus II rather than for Calculus III. Although Figure 65 shows that these students who would have been recommended for Calculus II generally received somewhat lower grades than the general course population, the small number of students in this group is not a convincing source of evidence to change the cutoff score. Indeed, comparison of Figure 63 with Figure 65 also shows that the grade distribution for students receiving an “MA 1023 Recommendation” is virtually unchanged despite an increased cutoff score. It appears, then, that increasing the cutoff score by one point would decrease course enrollment – recall from Table 16 that the placement test failure rate would double to ~20% -- without providing convincing evidence that grade distributions would improve. Taken together, these arguments indicate that, like the Calculus II Placement Test, there is insufficient evidence to warrant changing the Calculus III Placement Test cutoff score.

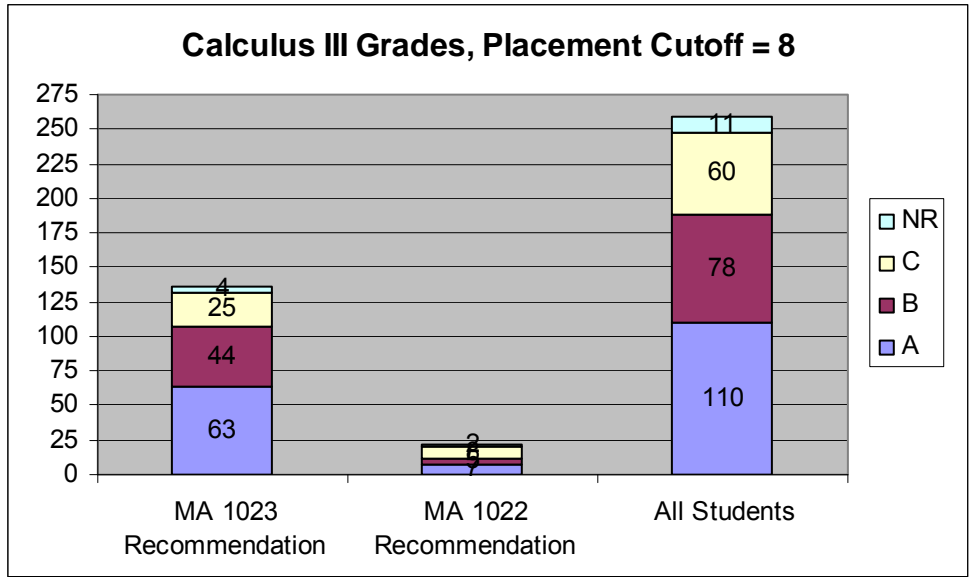


Figure 65: Distribution of Calculus II Grades by Recommendation, Cutoff Score of 8

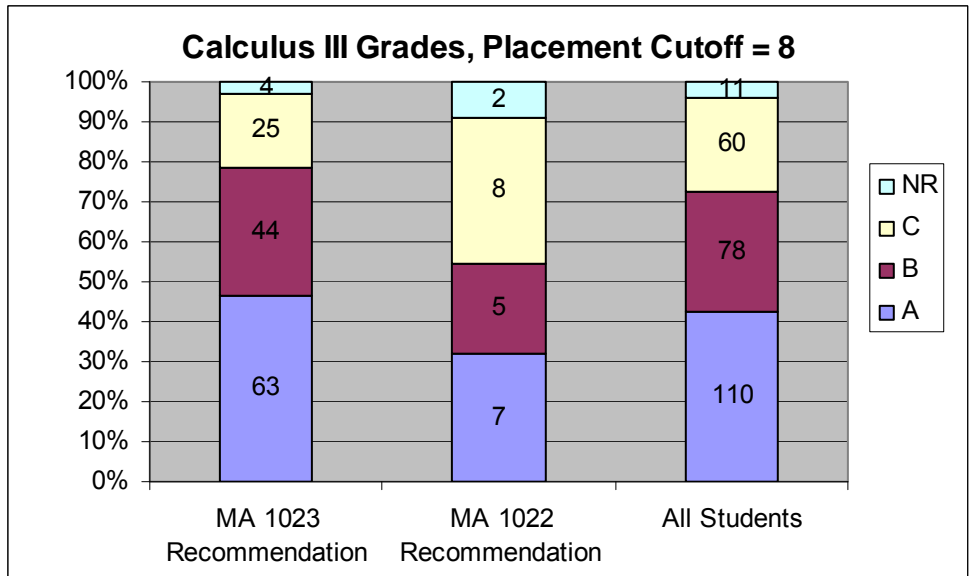


Figure 66: Distribution of Calculus II Grades by Recommendation, Cutoff Score of 8

It is acknowledged that for Calculus II and Calculus III, the methods of “predicting” the effect of a change in cutoff score rely on the changed recommendations of very few students. (Recall that increasing the Calculus III cutoff by one point would have changed the recommendation of only 15 students who took Calculus III.) For this reason, this line of

evidence is less helpful than, for example, the observation that increasing the placement test cutoff score would double the number of students failing the test. It will be seen, however, that a “predictive” argument is very useful in analyzing the two cutoff scores of the Calculus I Placement Test. For this reason, this type of argument was introduced for the simpler cases of Calculus II and Calculus III despite being less useful for these tests.

The preceding discussion has established that the placement test results for the Calculus II Placement Test and the Calculus III Placement Tests were very similar in terms of score distributions, failure rates, and the expected effect of a change in the cutoff score. The Calculus I Placement Test, on the other hand, is unique among the set of three tests in that there were two different cutoff scores, making decisions about cutoff changes correspondingly less clear-cut. Despite the increased complexity of this problem, the methods developed for analyzing the Calculus II and Calculus III cutoff scores can be effectively extended to the Calculus I Placement Test.

The results of the Calculus I Placement Test, using the original cutoff scores, are given in Table 17. Note that the number of students who took the Calculus I Placement Test (483) far exceeds the number of students who completed either the Calculus II (276) or Calculus III (240) Placement Tests. Of these students, a majority (67.1%) met or exceeded the upper cutoff score of 13 and received a recommendation for Calculus I. Only 6.4 % of students did not meet the lower cutoff score of 8, and these students were recommended for MA 1020, Calculus I with Preliminary Topics. Students whose scores were between these two cutoffs were classified as “Borderline.” This category applied to roughly one-fourth (26.5%) of students taking the placement test.

**Table 17: Results of the Calculus I Placement Test**

<b>Recommendation</b>	<b>Number of Students</b>	<b>Percentage</b>
MA 1020 ("Failing")	31	6.4 %
"Borderline"	128	26.5 %
Calculus I ("Passing")	324	67.1 %
Total	483	100.0 %

As before, we may now consider the effect of changes in cutoff scores on the distribution of the different recommendations. A change in the lower cutoff score would affect the number of MA 1020 recommendations relative to Borderline recommendations, and changing the upper cutoff score would affect the number of Calculus I recommendations relative to Borderline recommendations. So that changes in either cutoff score can be considered simultaneously, the effect of such changes on the number of Borderline recommendations will be considered. Table 18 shows the effect of changing either or both cutoff scores on the number of students classified as Borderline. The entry corresponding to the current scheme, with an upper cutoff score of 13 and a lower cutoff of 8, is highlighted in yellow. Note that scenarios that would have resulted in a "lower" cutoff that was higher than the "upper" cutoff simply are not considered in the Table.



**Table 18: The Effect of Different Calculus I Placement Test Cutoff Scores on the Percentage of Students Classified as Borderline**

		Upper Cutoff Score															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Lower Cutoff Score	0	0.2%	0.6%	1.0%	1.2%	2.1%	2.5%	4.1%	5.8%	8.7%	11.6%	16.6%	21.9%	32.7%	45.3%	59.8%	79.3%
	1		0.2%	0.6%	0.8%	1.7%	2.1%	3.7%	5.4%	8.3%	11.2%	16.1%	21.5%	32.3%	44.9%	59.4%	78.9%
	2			0.4%	0.6%	1.4%	1.9%	3.5%	5.2%	8.1%	11.0%	15.9%	21.3%	32.1%	44.7%	59.2%	78.7%
	3				0.2%	1.0%	1.4%	3.1%	4.8%	7.7%	10.6%	15.5%	20.9%	31.7%	44.3%	58.8%	78.3%
	4					0.6%	1.0%	2.7%	4.3%	7.2%	10.1%	15.1%	20.5%	31.3%	43.9%	58.4%	77.8%
	5						0.0%	1.7%	3.3%	6.2%	9.1%	14.1%	19.5%	30.2%	42.9%	57.3%	76.8%
	6							1.0%	2.7%	5.6%	8.5%	13.5%	18.8%	29.6%	42.2%	56.7%	76.2%
	7								1.4%	4.3%	7.2%	12.2%	17.6%	28.4%	41.0%	55.5%	74.9%
	8									2.5%	5.4%	10.4%	15.7%	26.5%	39.1%	53.6%	73.1%
	9										2.3%	7.2%	12.6%	23.4%	36.0%	50.5%	70.0%
	10											3.7%	9.1%	19.9%	32.5%	47.0%	66.5%
	11												3.7%	14.5%	27.1%	41.6%	61.1%
	12													7.5%	20.1%	34.6%	54.0%
	13														8.3%	22.8%	42.2%
	14															6.8%	26.3%
	15																6.6%
	16																

Table 18 is an exhaustive listing of the possible cutoff score scenarios that can be used to determine what changes in the current scheme might be practical. For example, note that the percentage of Borderline students is fairly insensitive to changes in the lower cutoff score. (Observe the entries in Table 18 above or below the yellow-colored cell.) A one-point increase in the lower cutoff from 8 to 9 only would have decreased the number of Borderline students by 3.1%. A one-point decrease in this cutoff from 8 to 7 would only reclassify another 1.9% of students as Borderline. Further lowering this cutoff to 6 would only reclassify another 1.2% of students. Since changes in the lower cutoff score would have a very small effect on the recommendations that students received, and especially because this was only the first time this particular placement test was used, there is little support for a change in the lower cutoff score, which should remain at 8.

On the other hand, small changes in the upper cutoff score would affect the number of Borderline students much more significantly. (Observe the entries in Table 18 to the left or right of the yellow-colored cell.) Increasing the upper cutoff from 13 to 14 would increase the number of Borderline students by 12.6%, from 26.5% to 39.1%. Decreasing this cutoff from 13 to 12 would lower the number of Borderline students by 10.6%. A small change in this cutoff score would have a significant effect on the recommendations students received, so further investigation is warranted to determine whether those changes would be beneficial.

Consider the group of students who both took Calculus I in Term A and completed the Calculus I Placement Test. Information regarding the performance of these students in Calculus I is given in Figure 67 and Figure 68, with the grade distribution of all students in the course provided in the rightmost column as usual. Interestingly, Figure 67 indicates that the group of students who completed the Calculus I Placement Test and subsequently completed Calculus

contained almost equal numbers of Borderline students (83) and students actually recommended for Calculus I (85). Only a small number of students (13) enrolled in Calculus I despite receiving a score qualifying for a MA 1020 recommendation. Figure 68 indicates rather clearly that students receiving an “MA 1021” recommendation generally received higher grades than Borderline students, who in turn performed better than students recommended for MA 1020. This is most evident by noticing the decreasing percentage of “A” grades moving from left to right in the Figure. This trend is further evidence of the placement accuracy of the Calculus I test as well as an indication of the need for a Borderline class to differentiate the wide range of performance levels observed in this group.

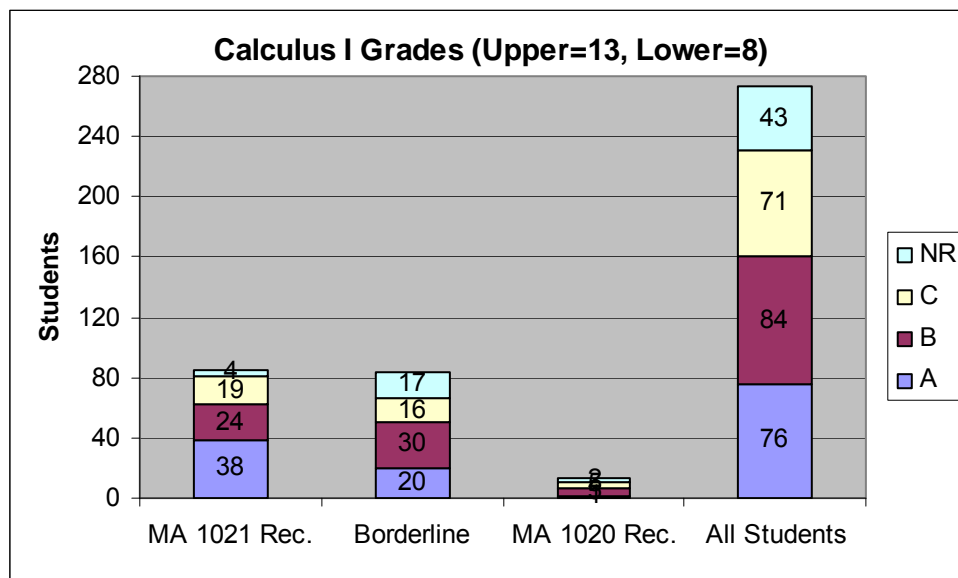
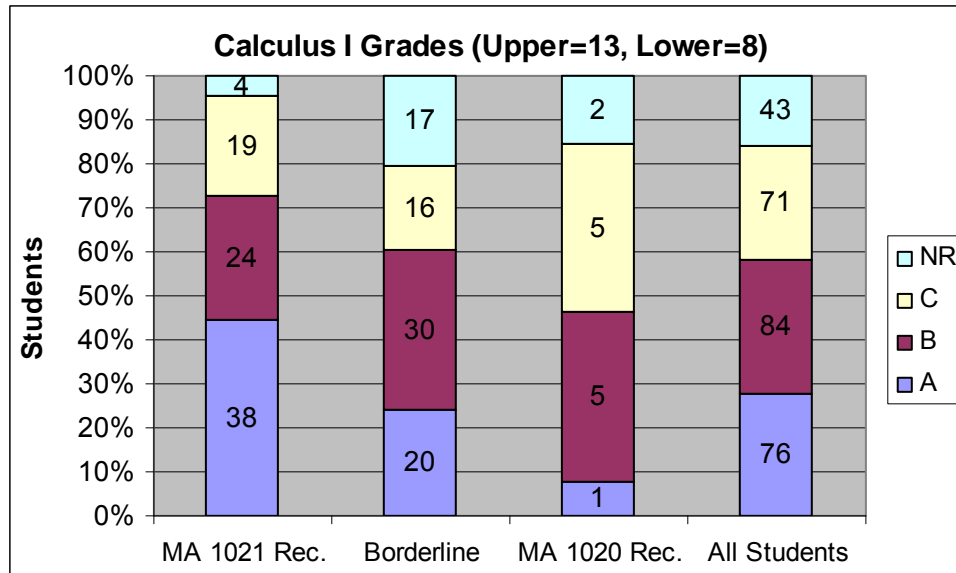


Figure 67: Distribution of Calculus I Grades by Recommendation, Original Cutoff Score



**Figure 68: Distribution of Calculus I Grades by Recommendation, Original Cutoff Score**

Recall that changes in the upper cutoff score were deemed to be most worthy of consideration for the Calculus I Placement Test. Suppose that the upper cutoff score had been increased from 13 to 14. Figure 69 and Figure 70 show the hypothetical effect of this change on the recommendations and grade distributions of the Calculus I course population. Note that by increasing the upper cutoff score, in effect increasing the score range that is considered to be Borderline, Calculus I would contain far more students who received Borderline recommendations (110) than recommendations for Calculus I itself (58). This essentially undermines the function of a Borderline classification, which implies that one's readiness for Calculus I is questionable. Making a Calculus I recommendation more difficult to obtain might be justified, however, if marked improvement in course grades were observed for students meeting this more difficult requirement. Comparison of Figure 68 with Figure 70 shows that despite the drastic increase in the number of Borderline students, the grade distributions shown in the Figures change insignificantly. As a result, there is no evidence for increasing the upper cutoff score of the Calculus I Placement Test.

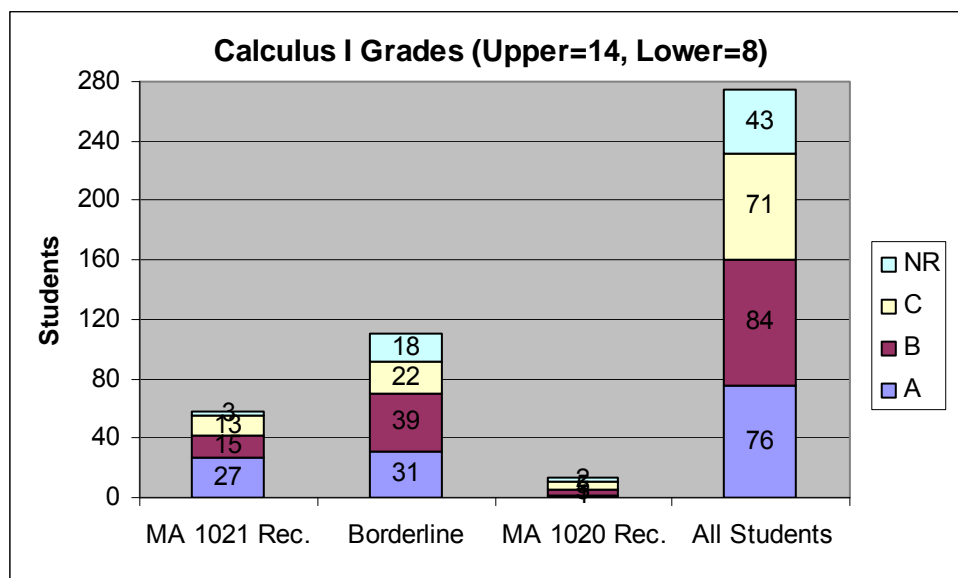


Figure 69: Distribution of Calculus I Grades by Recommendation, Upper Cutoff = 14

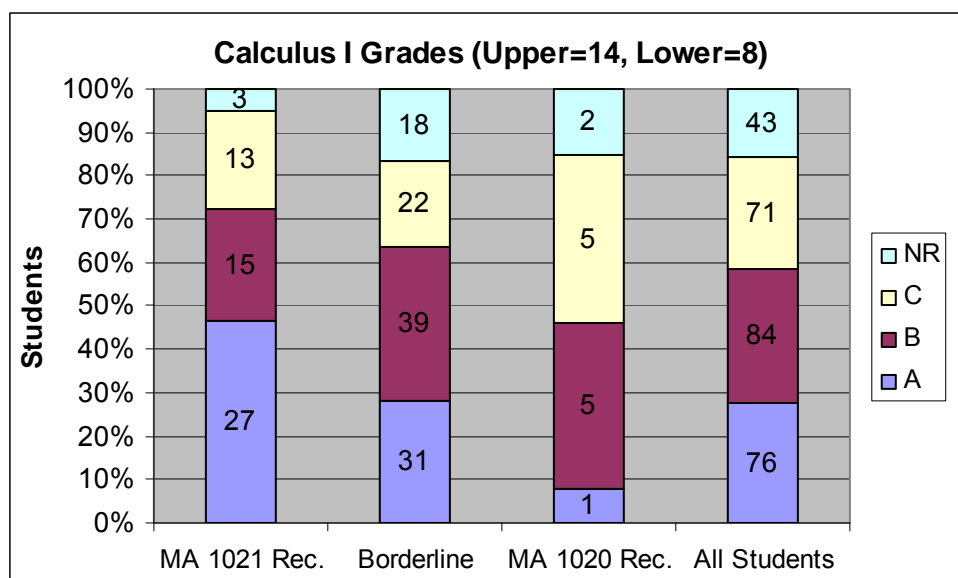


Figure 70: Distribution of Calculus I Grades by Recommendation, Upper Cutoff = 14

The remaining case to consider is the effect of decreasing the upper cutoff of the Calculus I Placement Test from 13 to 12. The hypothetical effect of such a change is presented in Figure 71 and Figure 72. In Figure 71 it is seen that narrowing the range of scores considered to be Borderline would have increased the number of students receiving Calculus I recommendations

(now 120 students) relative to Borderline recommendations (48 students). This would imply that more students were expected to perform well enough in Calculus I to warrant a recommendation for the course. The hypothetical grade distributions in Figure 72 indicate that this would not, in fact, be the case. Indeed, the difference in grades between the MA 1021 and Borderline recommendations that was so clearly observed in Figure 68 is much less pronounced in Figure 72. With an upper cutoff score of 12, students recommended for Calculus I would have earned a roughly similar number of “A” grades (35.8%) as Borderline students (31.3%). The combined percentage of “B” and “C” grades would also be strikingly similar for Calculus I recommendations (53.3%) and Borderline recommendations (52.1%). In short, lowering the upper cutoff score to 12 would destroy the very meaningful difference between students recommended for Calculus I and Borderline students, and the potential loss of this layer of the significance indicates that such a change is not warranted. In terms of Calculus I course performance, Borderline students were, indeed, “borderline,” earning better grades than those with MA 1020 recommendations but poorer grades than students recommended for Calculus I.

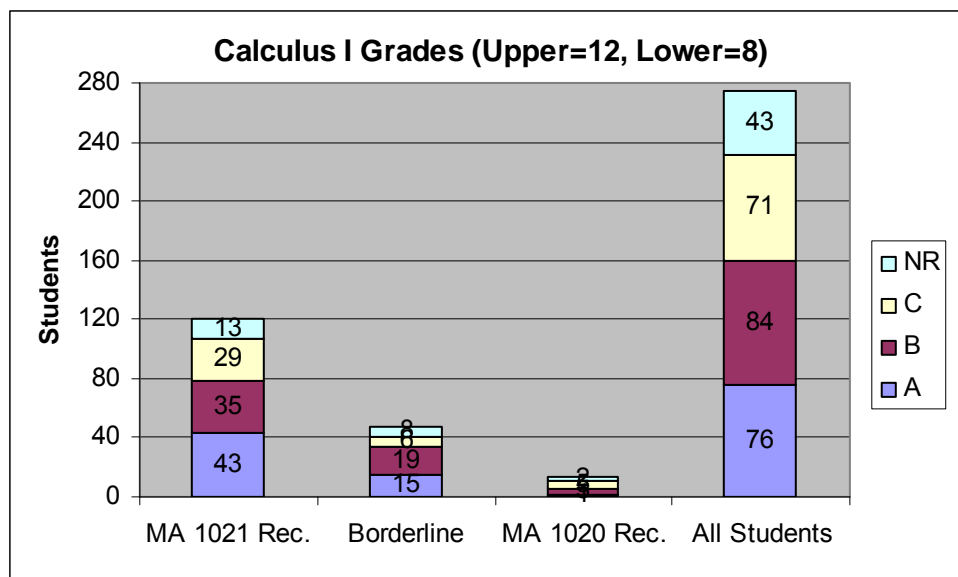
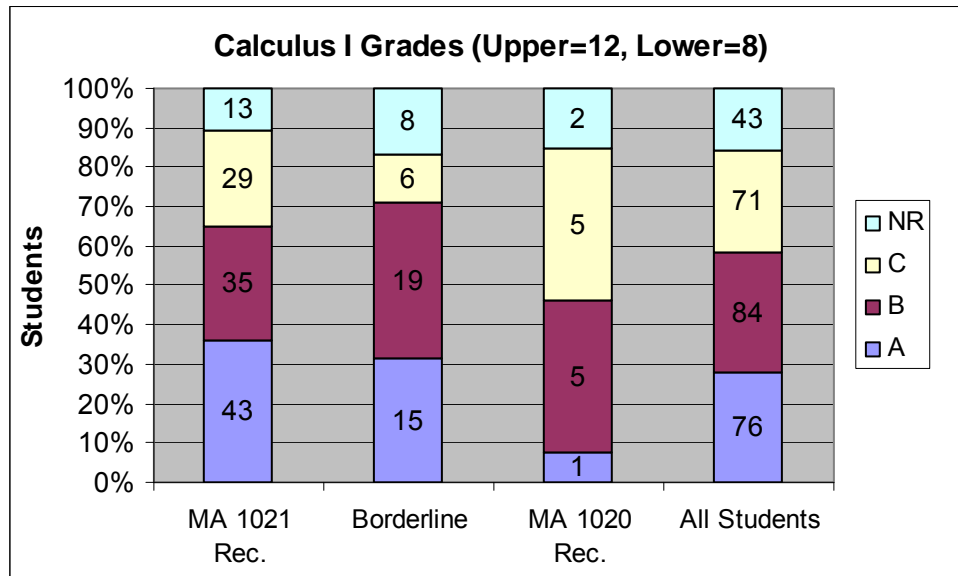


Figure 71: Distribution of Calculus I Grades by Recommendation, Upper Cutoff = 12



**Figure 72: Distribution of Calculus I Grades by Recommendation, Upper Cutoff = 12**

## 6. Conclusions and Recommendations

A thorough study of the content, results, and predictive power of the Calculus Placement Tests at Worcester Polytechnic Institute has been completed in order to evaluate the success of this new placement system and, when necessary, to make recommendations for its improvement. Several lines of evidence have converged on two overarching conclusions. It is clear that the Calculus Placement Tests were very successful in their first year of use. The Placement Tests effectively communicated the content of mathematics courses and made appropriate course recommendations based on placement scores. Obviously, since these Placement Tests have been in use for only one year and further study is highly recommended to verify that the trends observed in this first year. This study has clearly indicated that there is no aspect of the Placement Tests that must be changed immediately; rather, the recommendations made herein serve primarily to identify areas that particularly warrant attention in future studies.

The new Calculus Placement Tests brought two important improvements over the previous Calculus Readiness Test. First, the WeBWorK system provided short-answer questions (in place of old multiple-choice questions) with automatic grading so that students see immediately when an answer was correct (or not). Second, the new system had separate tests for Calculus I, Calculus II, and Calculus III. This gave WPI detailed information to help place students in the best first class and it also gave students a better understanding of introductory calculus at WPI.

Statistical analysis has shown that the Calculus Placement Tests possess internal consistency and are also able to provide good placement for first year students. Each of the Calculus Placement Tests displayed, as a whole, a satisfactorily high degree of internal consistency. In



addition, analysis of item-total correlations revealed individual test items that may have been much easier or more difficult than average. In particular, the first question of each test was almost invariably answered correctly. However, an argument can be made for the retention of a simpler question at the start of these optional tests to encourage completion of the test. On the other hand, certain items were more difficult than average. Those test items that exhibited either a very high or very low average score combined with a lower than normal item-total correlation should be given particular attention in future administrations and studies of the Placement Tests.

Scores on the Calculus Placement Test were not strongly correlated with the numerical data that was available for final examination scores in mathematics courses. The placement test is not a perfect predictor for grades in the corresponding course. However, considering the many confounding variables influencing the early university experience, it would be unreasonable to expect to observe a strong correlation in such a specific situation. Nevertheless, it was consistently observed that students who earned higher scores on a given Placement Test were, in general, more likely to receive a high grade in the corresponding mathematics course. This is an important result and the principal predictive accomplishment of the Placement Tests.

Another important result involved the group of students in the Calculus II course. It is not possible to receive Advanced Placement credit for Calculus I only. As a result, students who took Calculus II either skipped Calculus I without AP credit, or received credit for Calculus II and took the course anyway. It was found that compared to the other calculus courses, students in Calculus II generally earned lower grades. Furthermore, among the students in Calculus II, those who were actually recommended for Calculus II by the Placement Tests outperformed students with different recommendations. This indicates that whether a course decision is based

on Advanced Placement credit or the Placement Tests, results are best when the recommendation is followed.

Although the Calculus Placement Tests were optional, a majority (66%) of incoming first-year students completed one or more of the tests. Within the group of students completing a test and therefore receiving a course recommendation, a majority followed that recommendation in choosing a first university mathematics course.

The open-ended format of the Calculus Placement Tests allowed students essentially unlimited time to complete the tests. For each of the Placement Tests the completion time – and by extension the manner in which the test was taken – had little, if any, discernible effect on eventual performance. As a result, continuation of this open-ended format is recommended, as it appears to mitigate the problematic logistics of a timed placement test without confounding the results.

To determine course recommendations, each of the placement tests had either one or two cutoff scores that specified the range of placement test scores associated with a particular course recommendation. The current set of cutoff values and several possible alternative cutoff scores was carefully evaluated. The suitability of a particular cutoff score was based on its ability to consistently discriminate between high- and low- performing students. Analysis of cutoff scores for each of the Calculus Placement Tests suggested that the existing cutoffs were indeed appropriate, and it is thus recommended that the current scheme of cutoff scores should remain unchanged.

The Calculus Placement Tests at Worcester Polytechnic Institute were delivered using an Internet-based format that allowed for flexibility in logistics and accuracy in mathematical content, producing a very successful placement system. The Placement Tests displayed an

encouraging degree of predictive power and as such were able to inform the course choices of first-year students in a beneficial manner. There is no evidence to suggest that any significant revisions of the Placement Tests are necessary before their second year of use, and this study has identified aspects of the tests that should be important targets of future work. In the coming years, the Internet-based Calculus Placement Tests are expected to become a crucially important component of first-year academic advising in mathematics at Worcester Polytechnic Institute.

## 7. Future Work

While we have reasonably concluded that, at least for now, the Calculus Placement Test does not require immediate revision, there are different avenues and approaches that have yet to be explored. Here we pose some questions that another interested group may choose to investigate.

- Is there anything atypical about our data? Or do the distributions we found remain consistent in future years?
- Are there certain trends specific to certain math courses?

Only recently have the Calculus Placement Tests become available online. The data used in this study only encompasses one year of testing and coursework. It is important to consider the possibility that this year may have been atypical. There is also the concern of differences between groups of students. In this study, it was found that the data for Calculus II was different than that of the other math courses. This may or may not remain true for future students in Calculus II. It is probably important that students who start in Calculus II at WPI are “unusual.” Students can receive AP credit only for both Calculus I and Calculus II or for Calculus I, II, and III. It is not possible to have AP credit for Calculus I only. Students who choose to start in Calculus II are either trying to get ahead (without AP credit) or have AP credit but hope for an easy first math class at WPI.

- Did students follow the directions? Use a calculator? Acquire outside help?
- How influential did the students find the recommendations provided by the test?
- What are potential flaws in the online system? Did students have any particular difficulty that may have been overlooked?

- Which questions on the test did students have questions about? Were any questions unclear?

There is also an interest in collecting more information from the students who complete the Placement Tests. A simple survey at the end of the placement tests could collect additional information from the students while the test is still fresh in their memory. Something to explore is *how* the students took the tests. For example, did they use calculators? Did they ask their family for help? Another area of exploration is how seriously did the students take the recommendation provided by the tests?

- Is the format of the test appropriate?
- Can any improvements be made to the format of the test?
- Can the test be made more effective?

This question was brought up because the placement tests are broken up into sections for their respective courses. A student could complete any of tests they wished and potentially could receive several recommendations (one from each test they took). Did this confuse students? Or more simply, is there a better way to do it? This study has found that there is no justification for immediate changes to the tests, but that is not to say that they could not be made more effective. Indeed, the possibilities to continue research on this project are innumerable.

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## 9. Appendix

### 9.1. Outline of Goals

### 9.2. Calculus Placement Test Hardcopies

### 9.3. Calculus Readiness Test Hardcopy

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The following is an outline our project's goals, ideas for methods, and so on.

- **Accuracy of Placement: Predictive Validity** – Did students succeed in the courses that the Placement Test recommended for them?
  - Placement Test scores are available
  - Obtain quantitative indicator of success in classes
    - Final exam scores for MA 1021, MA 1022 available
    - Data will be available for MA 1020 also
    - Final grades are available for all mathematics courses
    - MA 1024 outside the scope of the existing Placement Tests
  - Determine how well Placement Test performance predicts class performance
    - Find a relationship between final exam score and Placement Test score
    - Compare pass rates for students who did take tests with those who did not
    - Compare pass rates for students who did follow Placement Test recommendation with those who did not
    - Compare student performance with that of past years

- Probably difficult to obtain data
  - What Placement Test systems were in place in past years?
    - Consider students who dropped the class separately
- Other factors: What else, besides the placement tests, might explain variability in grades?
- Confidentiality
  - Need to be blind to student names during analysis
  - Encode names using randomly-assigned integers
- **Content Validity** – Do the Tests contain all the appropriate material? Are any questions unhelpful?
  - Are there important topics not addressed by the Tests?
  - Are any questions irrelevant?
  - Ask professors to list topics that the Tests should address
  - Determine the item-total correlation of individual questions
    - Why do certain questions have low item-total correlations? Are they too easy? Too difficult? Not relevant?
  - Determine overall measurements of reliability
    - Cronbach's alpha
- **Appropriateness of Internet-Based System**
  - Search literature on placement testing
  - Compare WPI's WeBWork-based test with the placement tests of other schools
  - Assess advantages/disadvantages of current format
  - Because of Internet-based format, students may complete tests in a variety of ways

- Download hardcopy and enter answers later
  - Complete test(s) in one sitting
  - Complete test(s) in multiple sittings, possibly over many days
- Did the time to complete the tests affect the resulting score?
- **Influence of Tests** – Did students follow the recommendation generated by the Placement Tests? How seriously did students take the Placement Tests?
  - Recommendations generated by Placement Tests are known
  - Determine the first math course each student took
  - Determine proportion of students who followed recommendation
    - Group by course
  - Compare course choices of students who did and did not take the tests
    - Did tests affect student choice?
  - Study characteristics of students who did not take the tests. Is AP credit a reason?
- **Student Attitude** – What is their opinion of the difficulty, usefulness, etc. of the Tests?
  - Possible addition of a brief questionnaire to the revised Placement Tests.

The Calculus 1 Placement Test will close on 01/01/2008 at 09:00pm EST.

The purpose of this set of questions is to determine whether you are ready for Calculus I (MA1021), or if you should instead start in Calculus I with Preliminary Topics (MA1020).

When you hit "Submit Answers," WeBWorK will tell you whether or not you got the right answer. In most problems, you have **3 attempts** to enter your solution.

If you have trouble with a particular problem, use the "Email Instructors" button at the bottom of the problem to send email to WPI. This gives us a link to your specific problem and we can see the solutions you have already tried.

Most of these problems are set up so that you can answer all of the questions without a calculator. For example, if the answer for a problem is  $\sqrt{2}$ , you should enter "sqrt(2)" or " $2 \wedge (1/2)$ " instead of 1.414213562. If a problem does require a decimal answer, include at least 4 correct decimal places.

**1. (1 pt) setCalculus.1.Placement/problem1.pg**

Find  $m$  and  $b$  such that the equation  $y = mx + b$  describes the equation of the straight line through the points  $(8, 8)$  and  $(4, 9)$   
 $m = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$

**2. (1 pt) setCalculus.1.Placement/problem7.pg**

Simplify  $\frac{\sqrt{(x^5)^3}}{x}$ . Enter the **exponent** of  $x$  in the blank.  
 \_\_\_\_\_

**3. (1 pt) setCalculus.2.Placement/problem12.pg**

Find the points of intersection of the functions  $f(x) = 2x + 1$  and  $g(x) = -x^2 + 13x - 17$ .

Enter your answers as standard Cartesian coordinates, in the form  $(x, y)$  (with no spaces in your answer).

The leftmost point is: \_\_\_\_\_

The rightmost point is: \_\_\_\_\_

**4. (1 pt) setPreCalculus.Placement/problem1.pg**

Solve the following system of equations. If there are no solutions, type "No Solution" for both  $x$  and  $y$ . If there are infinitely many solutions, type " $x$ " for  $x$ , and an expression in terms of  $x$  for  $y$ .

$$-1x + 1y = 8$$

$$1x - 3y = 2$$

$x = \underline{\hspace{2cm}}$ .

$y = \underline{\hspace{2cm}}$ .

**5. (1 pt) setCalculus.1.Placement/problem2.pg**

If a right triangle has a hypotenuse of length 13 and one leg of length 5 and  $\theta$  is the angle between that leg and the hypotenuse, please

a) Determine the length of the other leg. \_\_\_\_\_

b) Determine  $\cos(\theta)$  \_\_\_\_\_

c) Determine  $\sin(\theta)$  \_\_\_\_\_

**6. (1 pt) setCalculus.1.Placement/problem6B.pg**

Write each expression in terms of sines and/or cosines and then simplify.

$$\frac{1}{\sin^2(x)} - \frac{1}{\tan^2(x)} = \underline{\hspace{2cm}}$$

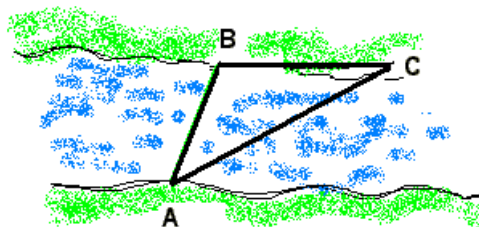
$$\frac{\cos(x)}{\sec(x)} + \cos^2(x) = \underline{\hspace{2cm}}$$

$$\frac{\cot(x)}{\csc(x)} = \underline{\hspace{2cm}}$$

**7. (1 pt) rochesterLibrary/setTrig09Laws/p6.pg**

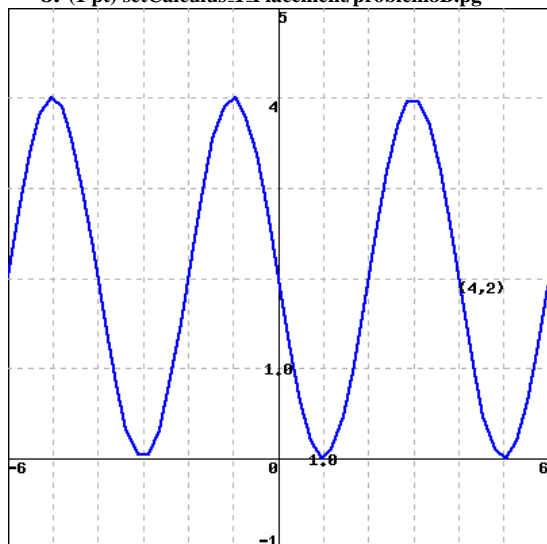
To find the distance AB across a river, a distance  $BC = 290$  is laid off on one side of the river. It is found that  $B = 105^\circ$  and  $C = 15^\circ$ . Find AB.

See the picture below. Click on the picture to see it more clearly.



AB = \_\_\_\_\_

8. (1 pt) setCalculus.1.Placement/problem8B.pg



To get a better look at the graph, you can click on it.

Write an equation of the form  $f(x) = A \sin[B(x - C)] + D$  whose graph is the sine wave shown above. The curve goes through the points  $(0, 2)$  and  $(4, 2)$ .

If needed, you can enter  $\pi=3.1416\dots$  as 'pi' in your answer.

$f(x) =$  \_\_\_\_\_

9. (1 pt) setPreCalculus.Placement/problem6.pg

Given the following equation of a circle, find the coordinates of the center and its radius.

$$x^2 + 2x + y^2 + 8y = 0$$

Center: \_\_\_\_\_ (Enter the center as standard Cartesian coordinates, in the form  $(x, y)$ .)

Radius: \_\_\_\_\_

10. (1 pt) setCalculus.1.Placement/problem21.pg

Find an equation for the ellipse whose graph goes through the points  $(16, 0)$  and  $(8, 6)$ .

If your answer is  $\frac{x^2}{A} + \frac{y^2}{B} = 1$ , then

$A =$  \_\_\_\_\_ and  $B =$  \_\_\_\_\_.

11. (1 pt) setCalculus.1.Placement/problem22.pg

The parabola given by the equation  $y = -x^2 + 16x - 12$  has its vertex at  $(h, k)$  for:

$h =$  \_\_\_\_\_

and

$k =$  \_\_\_\_\_

12. (1 pt) setPreCalculus.Placement/problem8.pg

A hemisphere has a radius of 3 feet. If a cylinder that is 3 feet tall has the same volume as the hemisphere, what is the radius of the cylinder?

**Note:** The following formulas may be useful:

$$\text{Volume of a Cylinder} = \pi r^2 h$$

$$\text{Volume of a Sphere} = \frac{4}{3} \pi r^3$$

Radius = \_\_\_\_\_ feet.

13. (1 pt) setCalculus.1.Placement/problem3.pg

If  $f$  is a function defined by  $f(x) = x^2 + 3$ ,

a) Find  $f(y)$ .

$f(y) =$  \_\_\_\_\_

b) Find  $f(t + 1)$ , and expand the result.

$f(t + 1) =$  \_\_\_\_\_  $t^2 +$  \_\_\_\_\_  $t +$  \_\_\_\_\_

c) Find  $f(f(2))$ .

$f(f(2)) =$  \_\_\_\_\_

d) True or False: The function  $f$  has an inverse on the interval  $-\infty < x < \infty$ . \_\_\_\_\_

14. (1 pt) setPreCalculus.Placement/problem2.pg

Find the solution of the exponential equation

$$5e^x - 12 = 20$$

in terms of logarithms, or correct to four decimal places.

$x =$  \_\_\_\_\_

15. (1 pt) setCalculus.1.Placement/problem20.pg

Let  $f(x) = \sqrt{12 - x}$  and  $g(x) = x^2 - x$ .

Then the domain of  $f \circ g$  is equal to  $[a, b]$  for

$a =$  \_\_\_\_\_

and

$b =$  \_\_\_\_\_

16. (1 pt) setPreCalculus.Placement/problem4.pg

Solve the equation  $|2x + 3| = 13$ .

The solutions are  $x_1 =$  \_\_\_\_\_ and  $x_2 =$  \_\_\_\_\_

where  $x_1 \leq x_2$ .

17. (1 pt) setCalculus.1.Placement/newfinalproblem.pg

This is the final problem for the Calculus 1 Placement Test. Please complete this problem only after you have finished Problems 1-16. Once you click the button below and submit your answer, we will evaluate your performance. Your score will not be officially recorded - these scores are for your benefit only. We will only use the results to recommend (via e-mail) what we think would be an appropriate first math class for you. Our recommendation will tell you whether you should continue on to take the other placement tests, or we might suggest that you participate in one of our online Jump Start programs in Algebra, Trigonometry, Functions, or Geometry.

If you are finished with the Calculus 1 Placement Test, answer this question now. If you are not finished, then complete the rest of the test, and submit an answer to this problem when you want your exam to be evaluated.

- A. I have completed the Calculus 1 Placement Test. Evaluate my score and send me your recommendations.

Within 48 hours of answering this question, you will receive an e-mail containing information about your score and our recommendations. **This message will be sent to your WPI e-mail address.** You may check your WPI e-mail address by following the link below:

**WPI Exchange E-mail Server**

Under "User Name:", enter the text "student\username" (without the quotation marks), replacing USERNAME with your WPI username. This is the same username that you use to log in to WeBWorK.

Under "Password:", enter your WPI password.

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The Calculus 2 Placement Test will close on 03/15/2008 at 10:02am EDT.

The purpose of this set of questions is to determine whether you are ready for Calculus II (MA 1022), or if you should instead start in Calculus I (MA 1021). At WPI, Calculus II is all about integration and assumes that you already have knowledge of derivatives, graphing and limits.

When you hit "Submit Answers," WeBWorK will tell you whether or not you got the right answer. In most problems, you have **3 attempts** to submit the correct solution.

If you have trouble with a particular problem, use the "Email Instructors" button at the bottom of the problem to send email to WPI. This gives us a link to your specific problem and we can see the solutions you have already tried.

Most of these problems are set up so that you can answer all of the questions without a calculator. For example, if the answer for a problem is  $\sqrt{2}$ , you should enter "sqrt(2)" or " $2 \wedge (1/2)$ " instead of 1.414213562. If a problem does require a decimal answer, include at least 4 correct decimal places.

1. (1 pt) setCalculus.3.Placement/problem2.pg

If  $f(x) = 7x^2 - 4x - 1$ , find  $f'(x)$ .

2. (1 pt) setCalculus.2.Placement/problem8.pg

Find  $f'(x)$  if  $f(x) = 4 \sin(4x) \cos(x)$ .

$f'(x) =$  \_\_\_\_\_

3. (1 pt) setCalculus.2.Placement/problem9.pg

$f(x) = 2 \ln(x^2 + 5x + 5)$

Find  $f'(x)$ :

$f'(x) =$  \_\_\_\_\_

Find  $f''(x)$ :

$f''(x) =$  \_\_\_\_\_

4. (1 pt) setCalculus.2.Placement/problem7.pg

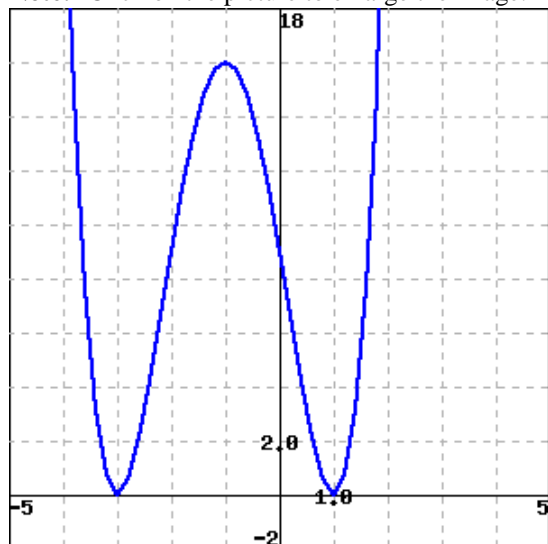
If  $y = (1 - u)^4$  and  $u = e^{5x}$ , find  $\frac{dy}{dx}$ .

$\frac{dy}{dx} =$  \_\_\_\_\_

5. (1 pt) setCalculus.2.Placement/problem14.pg

For the function graphed below,  $f(x) = x^4 + 4x^3 - 2x^2 - 12x + 9$ , find the equation of the line tangent to the curve at  $x = 0$ .

**Note:** Click on the picture to enlarge the image.



$y =$  \_\_\_\_\_

6. (1 pt) setCalculus.2.Placement/problem11.pg

The lengths of the sides of an equilateral triangle are increasing at 3 inches per second. How fast is the area increasing at the time when the sides have length 9 inches?

$\frac{dA}{dt} =$  \_\_\_\_\_

7. (1 pt) setCalculus.3.Placement/problem18.pg

The function

$$f(x) = 2x^3 + 6x^2 - 210x + 5$$

is decreasing on the interval ( \_\_\_\_\_, \_\_\_\_\_ ).

It is increasing on the interval (  $-\infty$ , \_\_\_\_\_ ) and the interval ( \_\_\_\_\_,  $\infty$  ).

The function has a local maximum at  $x =$  \_\_\_\_\_.

8. (1 pt) setCalculus.2.Placement/problem15.pg

Let  $f(x)$  be the function  $2x^2 - 9x + 6$ . Then

$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} =$  \_\_\_\_\_

9. (1 pt) setCalculus.2.Placement/final\_problem.pg

This is the final problem for the Calculus 2 Placement Test. Please complete this problem only after you have finished Problems 1-8.

If you are not finished, complete the rest of the test, and then come back to this problem.

- A. I have completed the Calculus 2 Placement Test.







## CALCULUS READINESS TEST I

**This test consists of 25 questions. Each question is followed by five suggested answers, labeled (A) through (E). Select the one best answer to each problem and indicate the answer on the ANSWER SHEET in the box provided.**

A calculator is not required for any question on this test.

**→ DO NOT WRITE IN THIS BOOKLET ←**

**Scratch paper is provided**

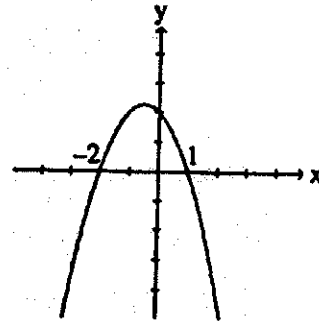
*You have 30 minutes to complete the exam.*

**PART I**  
(In all graphing problems, assume the usual coordinate system.)

---

1. If  $f$  is a function whose graph is the parabola shown to the right, then  $f(x) < 0$  whenever

- (A)  $x < 0$
- (B)  $x < 1$
- (C)  $x > -2$
- (D)  $-2 < x < 1$
- (E)  $x < -2$  or  $x > 1$



- 
2. If money in a bank triples every 11 years, then by what factor does it increase over a 22 year period?

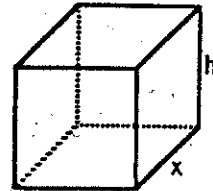
- (A) 2
- (B) 6
- (C) 8
- (D) 9
- (E) 11

- 
3. The  $x$ -coordinate of the point of intersection of the graphs  $-2x + y = -18$  and  $x + y = -9$  is

- (A) -9
- (B) -6
- (C) -3
- (D) 0
- (E) 3

- 
4. The box shown to the right has a square base and a closed top. Express its surface area in terms of  $x$  and  $h$ .

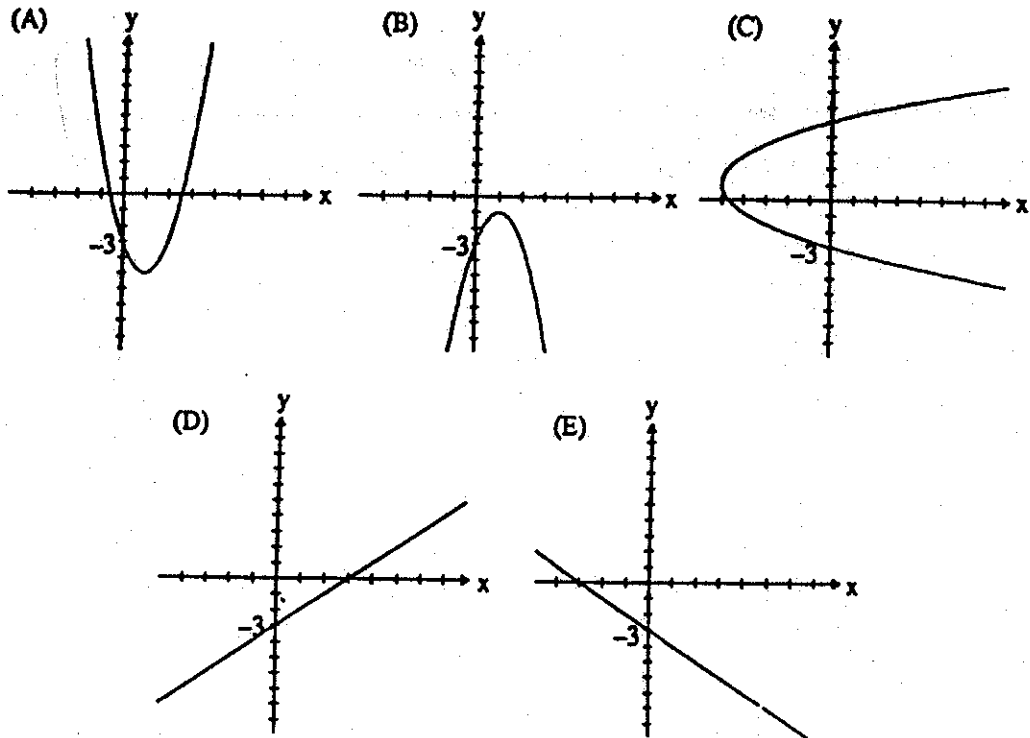
- (A)  $x^2h$
- (B)  $4x + h$
- (C)  $8x + 4h$
- (D)  $x^2 + 4xh$
- (E)  $2x^2 + 4xh$



- 
5. If  $\log_4(x + 3) = 2$ , then  $x =$

- (A) 5
- (B) 8
- (C) 11
- (D) 13
- (E)  $\frac{2}{\log_4 2} - 3$

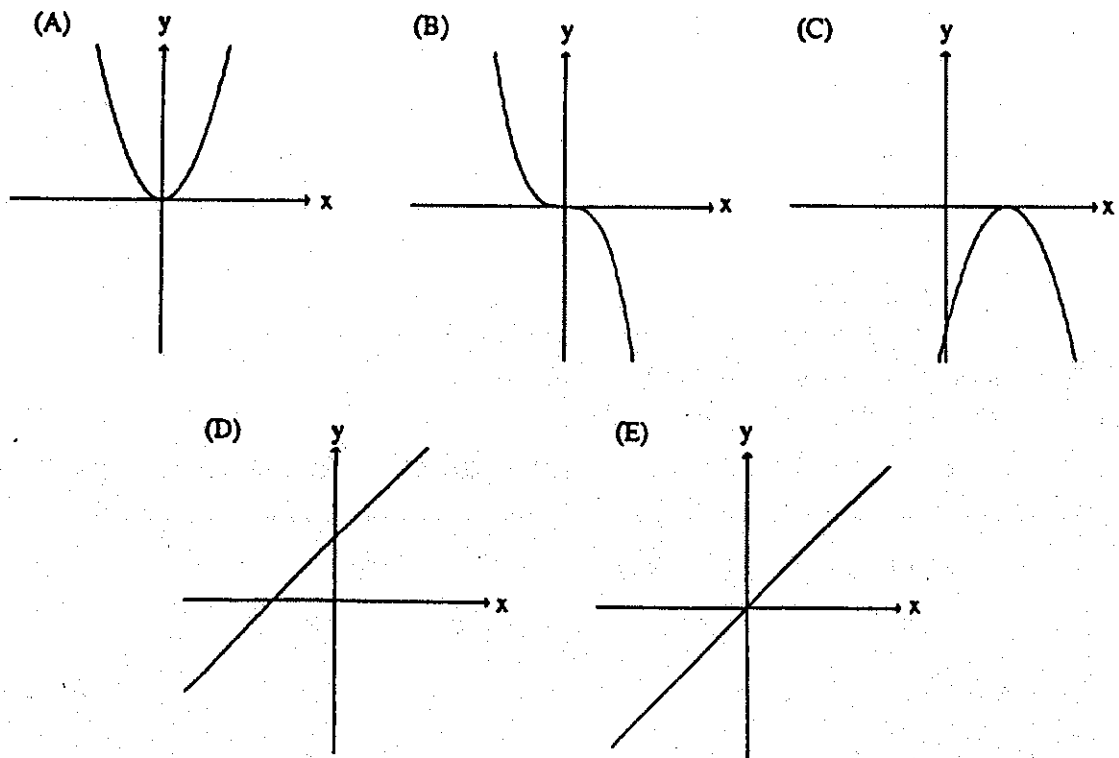
6. Which of the following best represents the graph of  $y = 2x^2 - 4x - 3$ ?



7.  $(8)^{1/3}(81)^{-1/4} =$

- (A) -6      (B)  $(648)^{-1/12}$       (C)  $\frac{2}{3}$       (D)  $\frac{3}{2}$       (E) 6

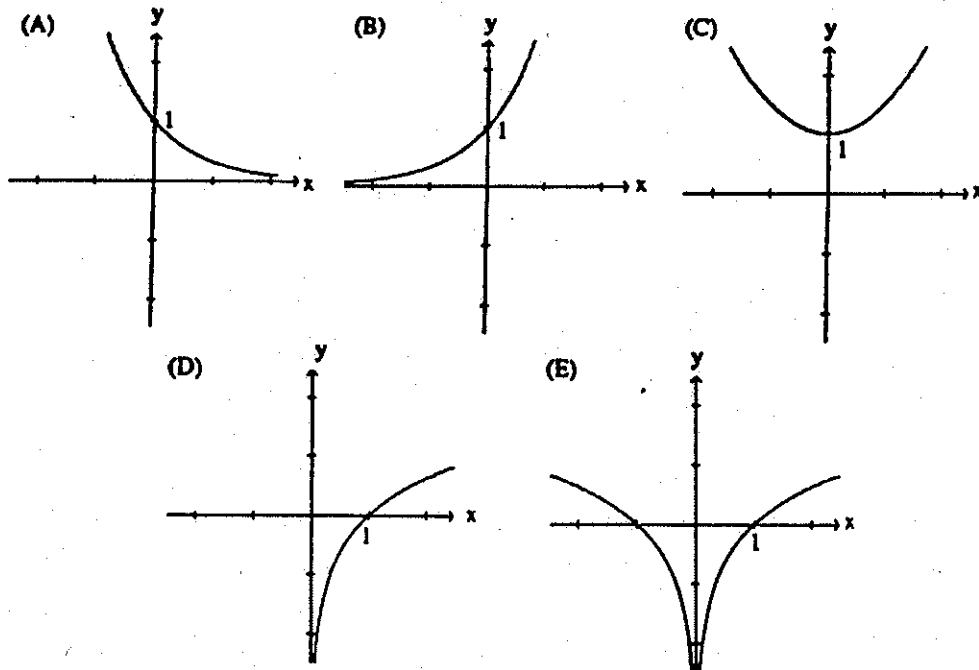
8. Definition: A function  $f$  is even if  $f(-x) = f(x)$  for each  $x$  in the domain of  $f$ .  
Which of the functions whose graphs are shown below is even?



9. If  $\frac{(4x+1)(x-2)}{x+1} = 0$ , then  $x =$

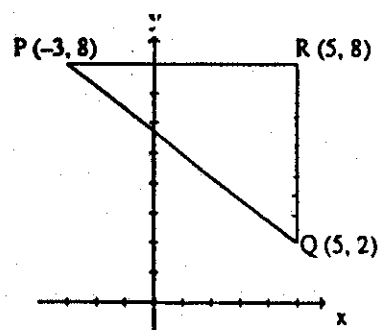
- (A)  $-2$  or  $\frac{1}{4}$     (B)  $-1$  or  $2$     (C)  $-\frac{1}{4}$  or  $2$     (D)  $-2, \frac{1}{4}$  or  $1$     (E)  $-1, -\frac{1}{4}$  or  $2$

10. Which of the following best represents the graph of  $f(x) = 5^x$ ?



11. In the figure shown to the right, what is the distance between the points P and Q?

- (A) 8  
(B) 10  
(C) 12  
(D) 14  
(E) 16



12. Definition: The symbol " $\approx$ " means "is approximately equal to."

Given that  $2^{12} \approx 4000$ , then  $2^{24} \approx$

- (A) 8000    (B) 160,000    (C) 1,600,000    (D) 16,000,000    (E)  $(4000)^2$

13. The quantity  $a - b$  is a factor of how many of the following:

$a^2 - b^2$      $a^2 + b^2$      $a^3 - b^3$      $a^3 + b^3$

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

14. If  $f(x) = \frac{5x+3}{x+5}$ , then  $f(a+2) =$

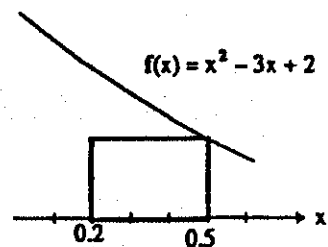
- (A)  $\frac{13}{7}$       (B)  $\frac{5a+13}{a+7}$       (C)  $\frac{5a+5}{a+7}$       (D)  $\frac{5a+3}{a+5}$       (E)  $\frac{5a+3}{a+7}$

15. In a standard coordinate system, the graph of the equation  $y = -7x + 4$  is

- (A) a line rising to the right.      (B) a line falling to the right.      (C) a vertical line.  
(D) a horizontal line.      (E) not a line.

16. In the figure shown to the right, the area of the rectangle is

- (A) .0225      (B) .1      (C) .225      (D) .3      (E) .75



The figure is not drawn to scale.

17. If the sides of a rectangle with length  $x$  and width  $y$  are each made 5 times longer, then the area of the rectangle is increased by

- (A)  $xy$       (B)  $5xy$       (C)  $24xy$       (D)  $25xy$       (E)  $x^5y^5$

18. The inequality  $|x - 3| \leq 4$  is equivalent to

- (A)  $x \leq -1$       (B)  $x \geq 7$       (C)  $-7 \leq x \leq 1$       (D)  $-1 \leq x \leq 7$       (E)  $-7 \leq x \leq 7$

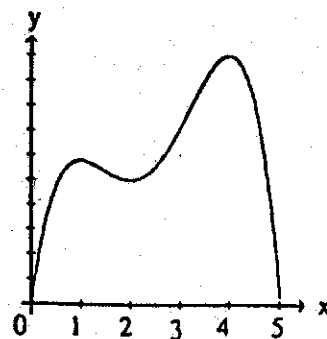
19. The length of a certain rectangle is 6 meters more than twice its width. What is the width of the rectangle if the perimeter of the rectangle is 96 meters?

- (A) 6 m.      (B) 12 m.      (C) 14 m.      (D) 18 m.      (E) 30 m.

20. Definition: A function  $f$  has a maximum value at the number  $c$  if  $f(c) \geq f(x)$  for every  $x$  in the domain of  $f$ .

If the domain of the function, whose graph is shown to the right, is  $[0, 5]$ , at which number does the function have a maximum value?

- (A) 0  
(B) 1  
(C) 2  
(D) 4  
(E) 5



(Assume radian measure in all trigonometric expressions.)

21. If  $f(x) = \cos 3x$ , then  $f\left(\frac{\pi}{6}\right) =$

- (A) 0      (B)  $\frac{1}{2}$       (C)  $\frac{1}{\sqrt{2}}$       (D)  $\frac{\sqrt{3}}{2}$       (E) 1

22.  $\sin^2 \theta \cot \theta \sec \theta =$

- (A)  $\sin \theta$       (B)  $\cos \theta$       (C)  $\cot \theta$       (D)  $\csc \theta$       (E)  $\sec \theta$

23. For which of the following values of  $x$  is  $\csc x$  not defined?

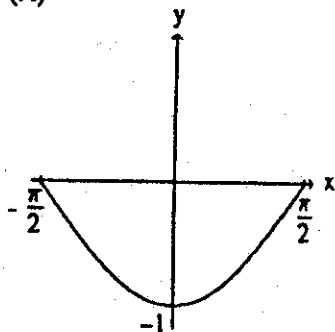
- (A)  $-\frac{\pi}{2}$       (B)  $-\frac{\pi}{3}$       (C)  $\frac{\pi}{6}$       (D)  $\frac{\pi}{4}$       (E)  $\pi$

24.  $\cos^2 \theta - 1 =$

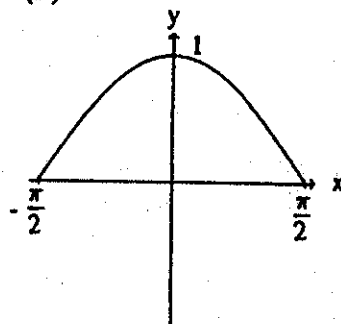
- (A)  $\sin \theta$       (B)  $\cos 2\theta$       (C)  $\sin^2 \theta$       (D)  $-\sin^2 \theta$       (E)  $\sec^2 \theta$

25. Which of the following best represents the graph of  $y = -\cos x$  for  $x$  between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$ ?

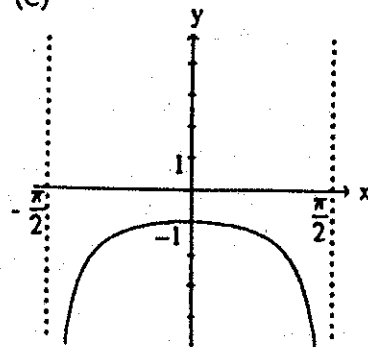
(A)



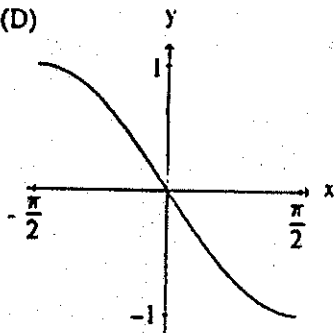
(B)



(C)



(D)



(E)

