Analyzing the Effects of Trade Compression on Risk Propagation in Over-the-Counter Derivative Markets

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Abstract

Following the 2008 financial crisis, a new and mostly unstudied technique has become a central tenet of today's financial markets: portfolio trade compression. Trade compression is a service offered by third party vendors that lowers a bank's gross notional exposures, while keeping net exposures the same. However, the effects of compression on systemic risk are unknown. In order to test the effectiveness of trade compression in risk mitigation, we compare the loss after default in markets with a variety of structures.

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1 Background

In financial markets, participants can take additional risks by writing Over-the-Counter (OTC) derivatives, which come in the form of swaps, forwards, futures, and options. These contracts increase profit or loss by betting on change in an underlying asset. Many experts agree that it was the use of OTC derivatives that lead to the financial crisis of 2007-2008; large institutions wrote these OTC derivative contracts to bet against mortgage defaults. The most notable of these institutions was Lehman Brothers, who leveraged their assets 44:1 trading credit default swaps. However, the risk associated with these contracts was not well understood; these institutions thought default was highly unlikely, and thus looked at the OTC derivative market as virtually risk free. Unfortunately, this was not the case. In the first quarters of 2008, many people started to default on mortgage payments, causing Lehman, and many others, to lose out on their positions. Lehman contacted other lenders, such as Bank of America and the London based Barclays, looking for a buyout, but no offer was made. Due to Lehman's large level of leverage, and the lack of a buyout, they did not have the physical capital to pay their losses. As a result, Lehman defaulted, creating huge losses for institutions that held contracts with Lehman. AIG was one such institution. The lack of payout from Lehman to AIG would have caused the bankruptcy of AIG. Due to a bailout, this did not happen, but had AIG not been bailed out, losses would have even further propagated throughout the system, causing more institutions to default.

In response to the financial crisis that followed, the US passed the Dodd-Frank act that mandated the clearing of certain OTC derivatives, alongside many other regulations. This means that institutions trading OTC derivatives now have to go through a central counterparty (CCP), which keeps various default safety funds to protect against the kind of leveraging that lead to the Lehman brothers default. International policy changed as well. For example, the EU passed EMIR mandating the clearing of various classes of OTC derivatives.

In addition to clearing, banks started applying other risk management prac-

tices in the form of portfolio trade compression, a key tool in handling the fallout from the financial crisis. Due to the use of portfolio compression, Lehman trade positions were considerably smaller than their gross totals: while cleaning up trades in October 2008, after the default, CLS Group, a third party middleman for interbank transactions, processed $5.2 billion in net settlements, corresponding to $72 billion notional amount (London Clearing House, 2012). In addition, trade compression has gained traction since the financial crisis: TriOptima and LCH.Clearnet Limited (LCH.Clearnet) compressed out $110 trillion in total notional volume in EUR, JPY, GBP and USD interest rate swaps... using TriOptima's triReduce since 2008 (TriOptima, 2017). However, while it is clear that trade compression can significantly reduce market exposure levels, the effect of trade compression on systemic risk is unclear.

Theoretically, trade compression looks to eliminate chains of trades in a network.

(a) Before Compression   (b) After Compression
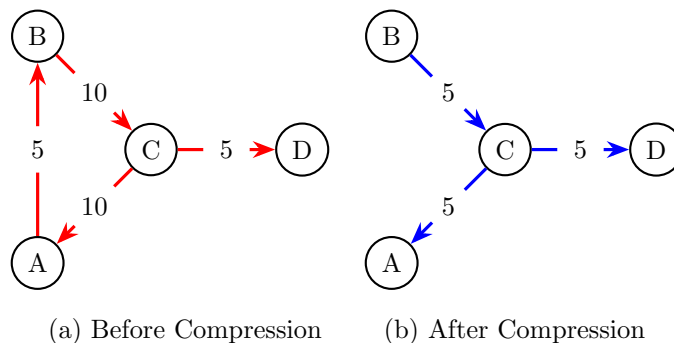
Figure 1: A graphical example of compression (from: D'Errico, Roukny)

Ideally, compression would eliminate all such cycles from a network. However, the complexity of markets often prevents this (D'Errico, 2017). Therefore, firms offering compression services often use a conservative approach to compression, wherein trades are only removed to a given extent.

Currently, the main provider of trade compression is TriOptima, with over 260 clients globally (TriOptima, 2017). LCH, SwapClear and CLS all have deals

with TriOptima to use their service on cleared and settled trades respectively. TriOptima's compression service, triReduce, uses a hybrid of conservative and nonconservative compression, cycling through dealer and client trades and compresses trades based on their own constraints, as well as constraints set by customers detailing the exposures they are open to taking on (TriOptima, 2017). According to TriOptima, triReduce has greatly reduced counterparty exposure. Figure 2 shows the exposure levels (z-axis) between two counterparties (x-axis and y-axis intersection) before (left) and after (right) applying compression.

SMOOTHING OUT BILATERAL COUNTERPARTY RISK EXPOSURES

Counterparty risk **before** triReduce cycle   Counterparty risk **after** triReduce cycle

■ -2.5--1.5 ■ -1.5--0.5 ■ -0.5-0.5 ■ 0.5-1.5 ■ 1.5-2.5   **Net Notional € Billion**
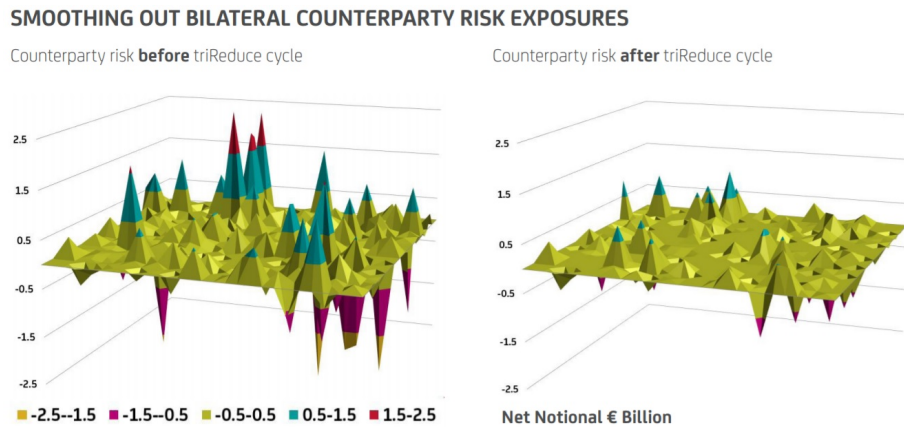
Figure 2: Claimed reductions in counterparty risk exposures after the use of triReduce on uncleared trades [Source: TriOptima (2017)]

While figure 2 demonstrates a reduction in gross exposures. It is unclear if systemic risk is lowered; a reduction in exposure may not correlate to a decrease in risk of default. It is the goal of this paper to provide a framework for understanding default risk in trade compressed OTC derivative markets.

2 Market Structures

In order to test trade compression, we must model various asset classes of OTC derivatives. First, we model the bilateral case; to this, we can apply each

form of compression, and then central clearing. In a discussion with Roukny, he stressed that "the interaction between central clearing and compression is not well understood", so the models presented in this paper are a simplified case where compression is only applied to markets prior to clearing (personal communication, Oct 30, 2017).

To define the base structure for the markets presented in this paper, we consider the weighted adjacency matrix of counterparty exposures E. In this matrix, a given counterparty i has exposures given by the corresponding row i, where an expected inflow of capital is a positive position, whereas an outflow is negative. Note then, that this adjacency matrix will be skew-symmetric, as can be seen in the example matrix given in figure 3.

	CP1	CP2	CP3	CP4	CP5	CP6	CP7	CP8	CP9	CP10
CP1		-70	115	30	65	55	20	-5	-15	-45
CP2	70		-60	70	85	65	-55		40	-10
CP3	-115	60		50	110	-80	35	80	20	100
CP4	-30	-70	-50		-155	5	30	-95	-50	130
CP5	-65	-85	-110	155		-45	30	-65	30	-30
CP6	-55	-65	80	-5	45		-15	-20	70	30
CP7	-20	55	-35	-30	-30	15		-25	-35	-115
CP8	5		-80	95	65	20	25		-45	40
CP9	15	-40	-20	50	-30	-70	35	45		65
CP10	45	10	-100	-130	30	-30	115	-40	-65	

Figure 3: The weighted adjacency matrix for a given market with 10 counterparties

This adjacency matrix can then be represented as directed market graph, where directions denote the flow of capital, i.e. an arrow from counterparty i to counterparty j denotes a trade where i is expected to pay j an amount equal to the weight of the edge between the two parties. Figure 4 is an example of the directed graph defined by the matrix in figure 3.
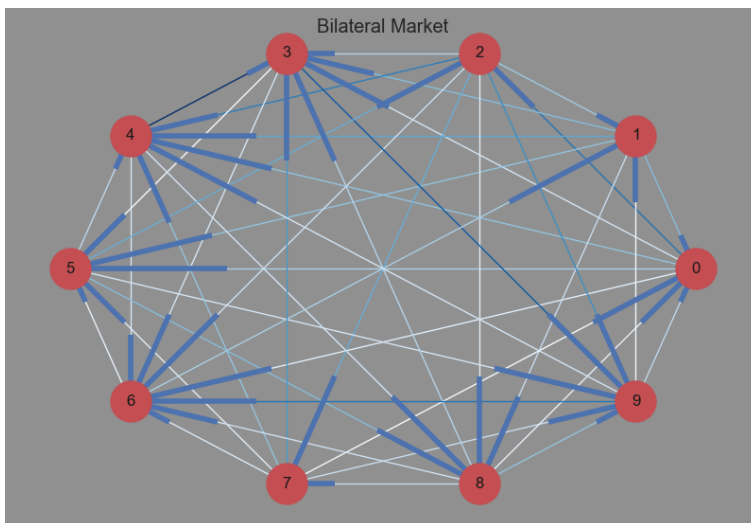
Figure 4: The graph representation of the given adjacency matrix (edge darkness indicates weight)

Note that as each row in the adjacency matrix corresponds to a different counterparty's positions, the net assets a_i and net liabilities l_i for a given institution i are simply the sum of row i and column i respectively. Thus we define for counterparty i the assets a_i and liabilities l_i as

$$a_i = \sum_{j=1}^{n} e_{ij} \qquad l_i = \sum_{j=1}^{n} e_{ji}$$

where e_{ij} is row i, column j of the matrix E.

In other words, net assets are row sums, and net liabilities are column sums. Note that, as the matrix E is skew symmetric, we have $a_i = -l_i$ for each counterparty i. However, in real world markets, there is often very little data available to the public, and the only consistently available data are gross notional amounts for each counterparty. Thus we must impose further structure to calculate a, l, and counterparty exposures. This structure is the basis of our bilateral market model.

With the bilateral model, we calculate net positions given gross notional for each counterparty. This is accomplished using methodology outlined by Gandy

and Veraart: first, we sample net notional from a normal distribution, then we find an initial feasible network, finally, we use Gibbs sampling to converge to our target distribution (Gandy, 2016). To this bilateral market, we can apply central clearing. For central clearing, we define two models: a market with a single central clearing party, and a market with multiple central clearing parties. The following sections outline the exact methodologies used.

2.1 Bilateral Market Model

As we often only have gross notional amounts for the counterparties in a given market, we will model the entire market from this data alone. While net notional is equivalent to net assets a_i as defined above, gross notional is equivalent to gross assets, i.e. for counterparty i, the gross notional g_i is

$$g_i = \sum_{j=1}^{n} |e_{ij}|.$$

While we do not have data on each counterparties individual assets and liabilities, (i.e. each entry in a given row of matrix E), the ratio of net to gross notional amounts is known. For a given counterparty i, we define the assets e_i^+ and liabilities e_i^- values as

$$e_i^+ = \frac{g_i + a_i}{2} \qquad e_i^- = \frac{g_i - a_i}{2}.$$

Note that e_i^+ and e_i^- are vectors containing the positive and negative values of e_i, respectively. Thus $e_i = e_i^+ - e_i^-$. However, to compute this, we need to estimate a_i, as the net asset data is not available. To do this, we must define an initial feasible network. First, we define an Erdös-Rényi graph, using assets and liabilities to constrain the market. We then apply the Edmonds-Karp max flow algorithm to this graph. Now we have our initial feasible network. Finally, we use Gibbs sampling, a type of Monte Carlo Markov Chain, on the initial network to build a chain of networks that converge to the assumed distribution of our market.

2.2   Single Central Clearing Party Model

CCPs are simply middle men in a financial market. Thus, modeling a network with a single CCP is just a restructuring of the adjacency matrix for the market. Figure 5 demonstrates the case of adding a single CCP to the market in figure 4.
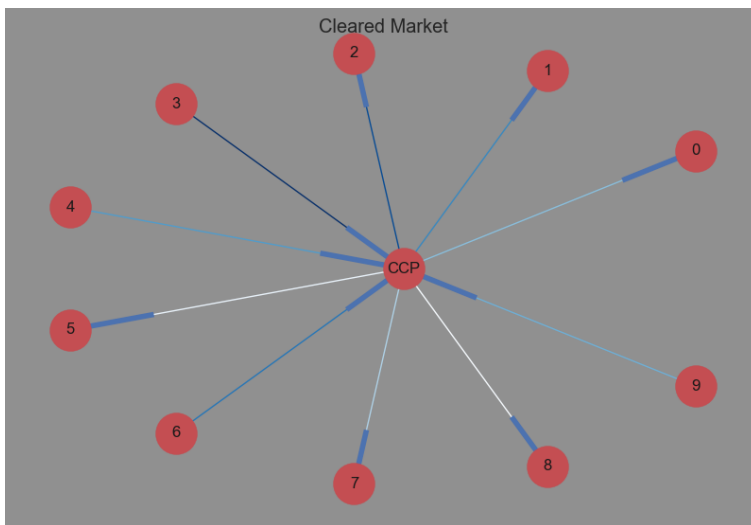

Figure 5: A centrally cleared version of the market in figure 4

To restructure an arbitrary market, we do the following: First, we define an $(n+1) \times (n+1)$ matrix where n is the number of counterparties in the original market. Now, for each entry i in the first row of our new matrix, we look at the sum of column $i-1$ in the original matrix, with the first entry being zero. Similarly, for the first column, we look at the sum of row $i-1$ in the original matrix. All other entries of the new matrix are zero.

Notice that the restructured structured market is simply the net trades in or out of a given counterparty $i-1$. Thus the exposures for each counterparty remain the same in both markets.

2.3　Multiple Central Clearing Party Market

In real world markets, there are often more than one CCP for a single type of derivative asset class. Thus, we will look into adding multiple CCPs to a market with and without the various types of compression.

Similar to the single CCP case, the multi-CCP case is a restructuring of the original market. In this case, we create an $(n+c) \times (n+c)$ matrix, where c is the number of CCPs in the market. Now we populate the entries in the first c rows. To get the first c entries in column i, we sum the entries of column $(i-c)$ in the original matrix, with the first c entries in the new matrix being zero. Note that this sum is equal to the exposure between i and a single CCP. In this case, we take this exposure, and scale it by the proportion i is exposed to each CCP, i.e. we distribute the net exposure of i among each CCP. Each entry is then the given proportion of the total sum for column $i-c$. We do likewise for the rows. The first c entries of a given row i, are the sum of row $(i-c)$ in the original matrix. Then a normal distribution is sampled, and the c entries are populated with the given proportion of the row sum. The rest of the matrix is populated with zeros.

3　Trade Compression Overview

There are many competing models of trade compression, this paper is based on the model proposed by D'Errico and Roukny. Here we define a market as a graph G(N,E), where N is the set of counterparties, and E is the set of trades, and compression is an operation $c : G \to G^*$ where $G^* = (N, E^*) := c(N, E)$ satisfies

$$a_i^* = a_i \quad and \quad g_i^* \leq g_i \quad \forall i \in N$$

(note that at least one inequality must be strict). Thus compression keeps net positions, or assets, constant, and reconfigures edges such that the gross position is decreased for at least one counterparty. In order to optimally apply compression to a market, we will further define the market itself; in a market, a

counterparty is defined as a dealer if they are both buying and selling, otherwise they are defined as a customer. A market can be partitioned into two subsets, G^d and G^c where $G^d = (N, E^d)$, $G^c = (N, E^c)$, $E^d \cap E^c = \emptyset$ and $E^d \cup E^c = E$.

In order to analyze the efficiency of various methods of compression, we compare the decrease in the value of positive trades. Note that this decrease is bounded by the net positions for each counterparty. The difference between the value of positive trades and net value for each counterparty is defined as the excess. Thus, for a given market G, the excess $\Delta(G)$ is defined as

$$\Delta(G) = \frac{\sum_{i \in N} \sum_{j \in N} |e_{ij}| - \sum_{j=1}^{n} |e_{ij}|}{2}.$$

Trade compression always reduces excess in a market (D'Errico, 2017).

We now define four types of compression that can be applied to the above system. The methods are differentiated by a_{ij} and b_{ij}, the upper and lower bounds, respectively, for trade volume between institutions i and j.

Definition 1. We define the following types of loop compression algorithms for use on a market:

- Conservative: $a_{ij} = 0 \quad and \quad b_{ij} = e_{ij}$

- Nonconservative: $a_{ij} = 0 \quad and \quad b_{ij} = \infty$

- Hybrid: $a_{ij} = 0 \quad and \quad b_{ij} = e_{ij} \quad \forall i, j \in E^C$
  $a_{ij} = 0 \quad and \quad b_{ij} = \infty \quad \forall i, j \in E^D$

- Bilateral: $a_{ij} = b_{ij} = max\{e_{ij} - e_{ji}, 0\}$

As non-conservative compression has no upper bound on edge weights, we can always find a solution that results in no excess (D'Errico, 2017), thus the network is maximally compressed. Conversely, conservative compression is bound by the initial trade amount, and thus cannot always reduce all excess. Hybrid compression combines the above two methods by being conservative on customers and non-conservative on dealers. Finally, bilateral compression looks at each bilateral trade and conservatively compresses the loop between the two

counterparties. Therefore, the efficiency of each operation, defined by the reduction in excess, is as follows:

$$\Delta(G)^{bilateral} \geq \Delta(G)^{conservative} \geq \Delta(G)^{hybrid} \geq \Delta(G)^{non-conservative}.$$

While it would be optimal to apply nonconservative compression, the complexity of real world markets often prevents this. Thus the standard for compression services is a more conservative approach.

Note that nonconservative compression contains conservative as a subset, and thus it is possible for both methods to result in the same compressed market.

4   Modeling Overview

In this section, we will outline the exact models for compression and risk analysis in the aforementioned market structures. For our trade compression algorithms, we will implement non-conservative, hybrid, and conservative compression. For conservative and hybrid compression, we will be using the network simplex method, as outlined in D'Errico and Roukny. The network simplex is simply a minimum-flow algorithm. In this case, we define node positions and trade bounds to constrain the network, then we apply the network simplex to find the minimum flow that allows for our network to be feasible. Non-conservative compression will use L_1 matrix minimization as an equivalent algorithm to network compression.

To measure risk levels in each market structure, we will apply the interbank contagion model proposed in Eisenberg and Noe. As this model simulates default at a single counterparty, the model will be applied to each counterparty in the market, and the average risk will be calculated over all cases. We simulate a trigger at each node individually with a shock. For simplicity, the shock will completely wipe out the triggering counterparty. As the relation between clearing and compression is not well understood, any CCPs in the market will be ignored in the triggering step, and will be regarded as unable to default.

To check if a bank defaults, we look to their reserve levels to see if there

is sufficient capital to avoid a default. Reserve levels are taken from Federal Reserve data on the top 25 U.S. banks. To calculate reserves, we take the consolidated assets, normalize to find the ratio of assets for each bank, and then multiply by the total level of net assets for all U.S. banks.

In the CCP case, we assume "cover two" default model, where each CCP holds enough in reserve to cover the larger of either the largest exposure, or the sum of the second two largest exposures. In addition, we add a buffer to account for surplus reserve levels.

4.1 Compression Models

According to Roukny, conservative compression is the most commonly implemented form of compression, with most real world algorithms based on conservative models (personal communication, Oct 30, 2017). Conservative compression looks to minimize excess while using current trade levels as an upper bound for changes on the network. Thus, we can reformulate conservative compression as finding the minimum cost flow in the network. Thus an optimal solution to conservative compression can be found using the network simplex algorithm (Appendix B.2). In the conservative case, we define node demand to be the net position for each counterparty, then we use trade levels to bound the maximum flow an edge con support, thus the problem is equivalent to conservative compression. For details on how network simplex, and how provides an optimal solution to conservative compression, we refer to D'Errico and Roukny, Appendix E.2.

In addition to conservative compression, the network simplex method can be applied to hybrid compression as well. As hybrid compression is conservative over customers and nonconservative over dealers, we bound the flow to and from customers by the trade level, while giving no bound to inter-dealer flow, i.e. the maximal flow, or trade, between two dealers is unconstrained.

Non-conservative compression, in contrast to conservative and hybrid compression, is boundless over the entire market. Thus, there exist many algorithms

for non-conservative compression. In this paper, we propose the use of L_1 matrix minimization as a form of non-conservative compression.

Definition 2. *L_1 minimization is a constrained optimization problem that minimizes the sum*

$$\sum_{i=1}^{n}\sum_{j=1}^{n}|e_{ij}| = \sum_{i=1}^{n}\sum_{j=1}^{n}e_{ij}^{+} + e_{ij}^{-}$$

where $E^{+} - E^{-} = E$, subject to

$$\sum_{j=1}^{n}(e_{ij}^{+} - e_{ij}^{-}) = a_i, \quad e_{ij} = -e_{ji} \quad \forall i,j \in N.$$

Note that E is skew-symmetric, so we have

$$a_i = \sum_{j=1}^{n}e_{ij} = \sum_{j=1}^{n}e_{ij} = -l_i.$$

In order to determine if a given matrix is L_1 minimal, we define an the following optimality check.

Lemma 1. *If $e_{ij} = 0$, or $sgn(e_{ij}) = sgn(a_i)$ and $sgn(e_{ij}) = sgn(l_j)$ $\forall i,j \in N$ then E is L_1 minimal.*

Proof. See Appendix A.2 □

Now that we have defined an optimality check for L_1 minimization, we will prove that any matrix can be compressed to be L_1 minimal.

Theorem 2. *Any market can be compressed to be L_1 minimal.*

In addition to always finding a feasible solution, we will now prove that the solution to L_1 minimization is a form of non-conservative compression.

Corollary. *After applying L_1 minimization to a market, every participant in the market becomes a customer.*

Proof. If every institution has either positive or negative trades, they are either only buying or only selling. By definition, this makes every institution a customer. □

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Corollary. *Minimizing the L_1 norm of the adjacency matrix E of a market eliminates all excess in the underlying market*

Proof. Any market where all participants are customers has 0 excess (D'Errico, 2017). Thus, as L_1 minimization results in a market with only customers, it eliminates all excess. $\square$

Thus we use L_1 minimization as it is equivalent to nonconservative compression and a feasible solution can always be found.

4.2   Risk Propagation Model

To get an accurate measure of the effects of default on a given financial market, we will simulate a default of each counterparty in the market, and take the average result of all cases. The following is a model for market value lost after the default of an arbitrary counterparty:

Given a financial market $G = (N, E)$, we define the vector r as the 'capital reserves' of each counterparty in the network (this reflects the counterparty's absorption capacity). Now we simulate the default of an arbitrary i in N:

First, define the set Γ to be the set of all counterparties that have defaulted, note that initially, $\Gamma^1 = \{i\}$. The default is triggered by wiping out all assets of i. This means the reserves of i are depleted, and all incoming trades are needed to pay off additional debts. Thus each j adjacent to i receives 0 on any assets from i. We now define the updated matrix of trades E^1 as follows:

$$E^1 = \begin{cases} 0 & e_{ij} > 0 \\ e_{ij} & e_{ij} \leq 0 \end{cases} \qquad \forall j \in N, \ i \text{ fixed.}$$

Thus E^1 now represents the total amount traded after taking into account the default of i. We also define r^1 as:

$$r^1 = \begin{cases} r(i) & i \notin \Gamma^1 \\ 0 & i \in \Gamma^1 \end{cases}.$$

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Now, to determine if this causes j to default, we calculate p_j^1, the net position of j:

$$p_j^1 = r_j^1 + \sum_{k \in N} e_{kj}^1.$$

Thus j defaults if $p_j^1 < 0$. Now we define Γ^2 as $\Gamma^2 = \Gamma^1 \bigcup \{j \in N | p_j^1 < 0\}$. We now define E^2 for all j in Γ^2:

$$E^2 = \begin{cases} \frac{l_j^1 + p_j^1}{l_j^1} e_{jk}^1 & e_{jk}^1 > 0, \ k \in \Gamma^2 \\ e_{jk}^1 & \text{otherwise} \end{cases}.$$

Note that if a trade exists between two defaulted counterparties, say j to k, k will collect on the trade, despite the default of j, to mitigate losses. We also define

$$r^2 = \begin{cases} r^1(i) & i \notin \Gamma^2 \\ 0 & i \in \Gamma^2 \end{cases}.$$

Now look at all counterparties k adjacent to all new j in Γ^3. Calculate each p_k^3 and we define $\Gamma^3 = \Gamma^2 \bigcup \{k \in N | p_k^2 < 0\}$. Now calculate E^3 similar to E^2. Continue the above until no new counterparties default, we will call this step $*$. At this point either all counterparties have defaulted, or the parties remaining are resistant to default.

Now we calculate the loss of value due to the default of the triggering counterparty i:

$$VL_i = \sum_{j \in N} (r_j - r_j^*) + \sum_{i,j} (e_{ij} - e_{ij}^*).$$

This takes into account the loss to capital reserves, as well as value lost on assets traded. After repeat the above algorithm for all i. Now we compute the average value lost as follows:

$$L = \frac{1}{N} \sum_{i=1}^{N} VL_i.$$

5   Results

The data used in the base bilateral market model are the gross notional amounts from the fourth quarter of 2016 for the top 25 commercial banks, savings associations, and trust companies in the United States (Comptroller, 2017). This data contains the gross notional for three asset classes: Interest Rate Swaps, Foreign Exchange, and Credit Derivatives. In addition, the surplus reverse for the CCP case were calculated using EU stress tests data; the average CCP held a 13% surplus on required capital, thus each CCP's reserve levels have a multiplier of 1.13 (ESMA, 2018).

Using the aforementioned gross notional data with the bilateral market model, we create a chain of 10,000 bilateral trading networks for each asset class. For the Gibbs sampling, we want the output to start when the data has already converged, thus we define a burn-in period of 1,000,000 samples. Furthermore, we do not want successive markets to be correlated, so we thin the data by sampling between each market. The thinning step used was 10,000 samples, to ensure little correlation between subsequent markets in the chain.

For each market in the aforementioned chain, we apply conservative, hybrid, and non-conservative compression algorithms, resulting in three additional chains, one for each compression method. Then, for each market in each chain, we restructure the market to account for all CCPs in the given asset class, resulting in a four additional chains. Thus, for each of the three asset class used, we are left with eight chains: four bilateral, and four cleared. We apply the risk propagation model to each network in all 24 chains. Table 1 shows the average result for each chain.

Table 1: Value lost calculated from risk propagation model in each market. Average of results from 10,000 market chain (Loss in millions $)

Compression Method	IRS		FX		Credit	
	Bilateral	Cleared	Bilateral	Cleared	Bilateral	Cleared
Base Market	3,441.92	1,638.32	1,365.75	496.49	675.77	278.57
Conservative	3,282.71	1,638.32	1,222.33	496.49	614.13	278.57
Hybrid	1,719.35	1,638.32	592.06	496.49	360.48	278.57
Non-conservative	1,687.42	1,638.32	588.31	496.49	341.85	278.57

The graphs in figures 6 through 11 show the risk results for each of the 10,000 networks in a given chain. The averages shown in table 1 were calculated using the data in figures 6 through 11.
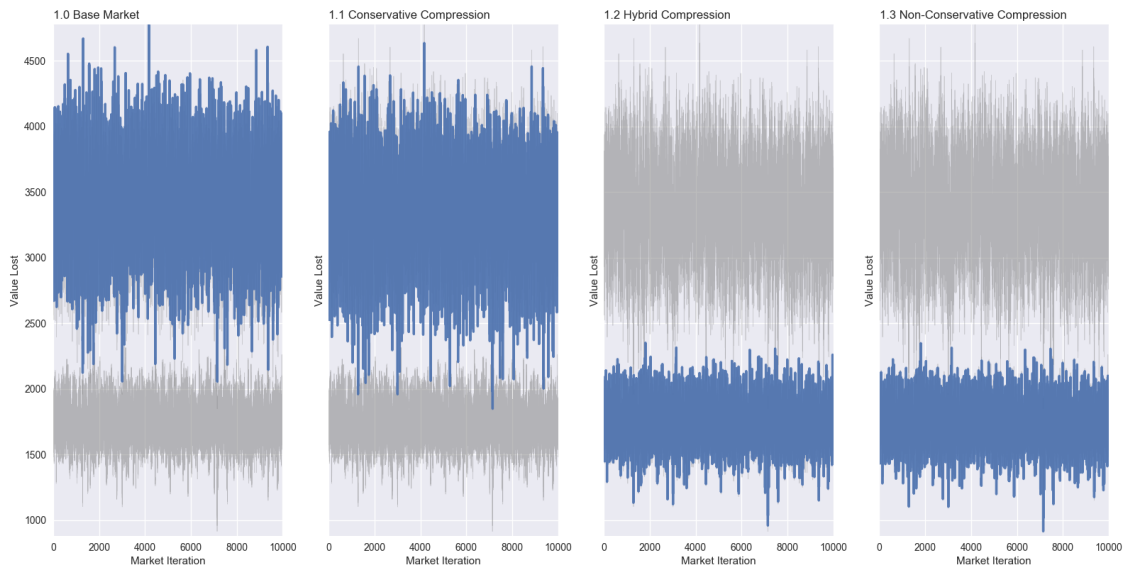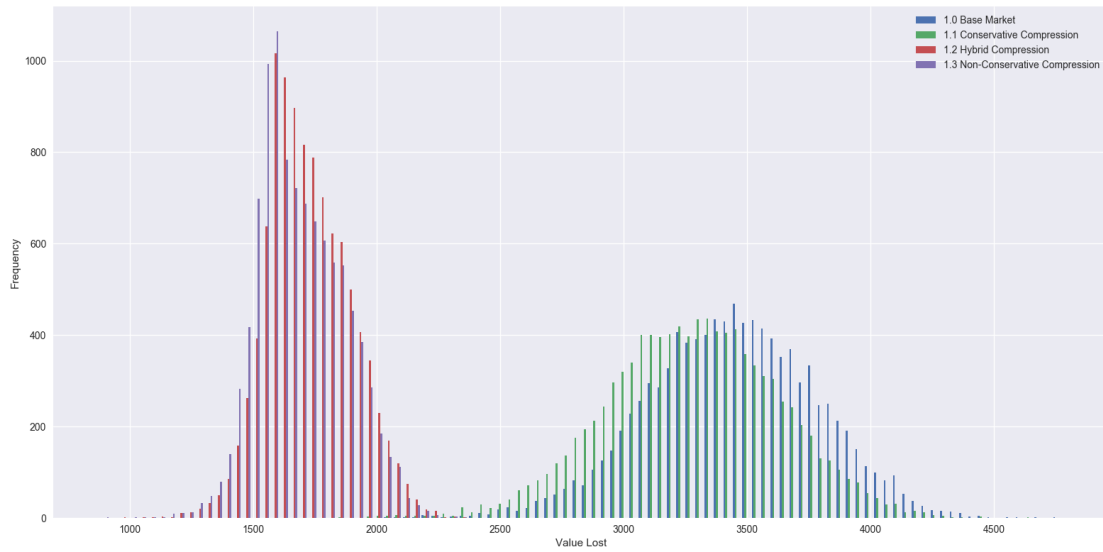
Figure 6: Bilateral IRS market loss after triggering default (title market in blue, other markets in gray), and the distribution of losses
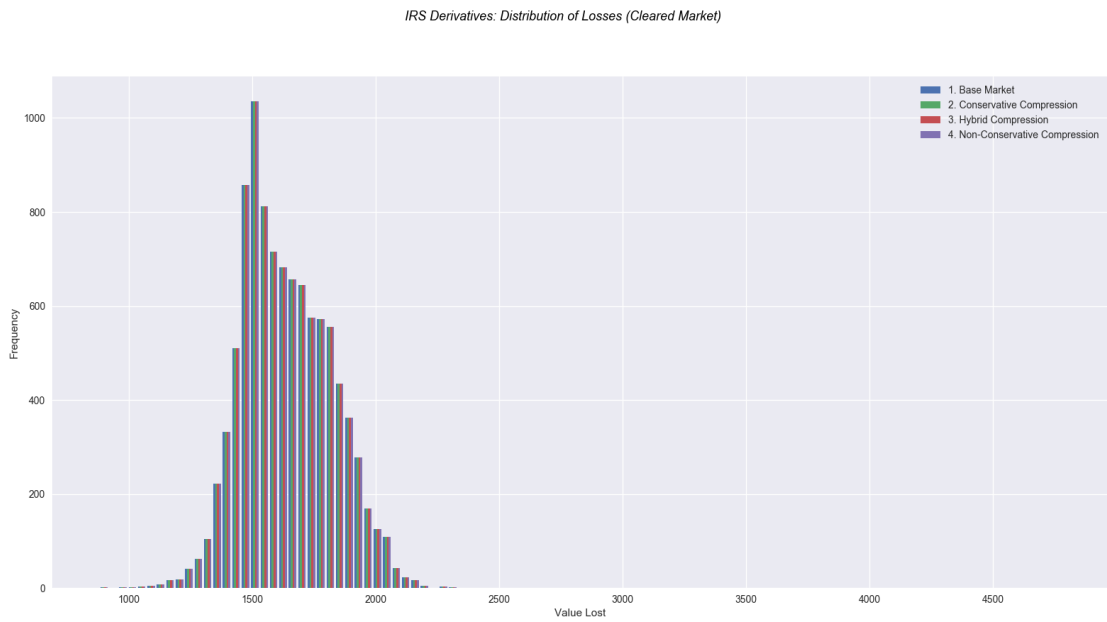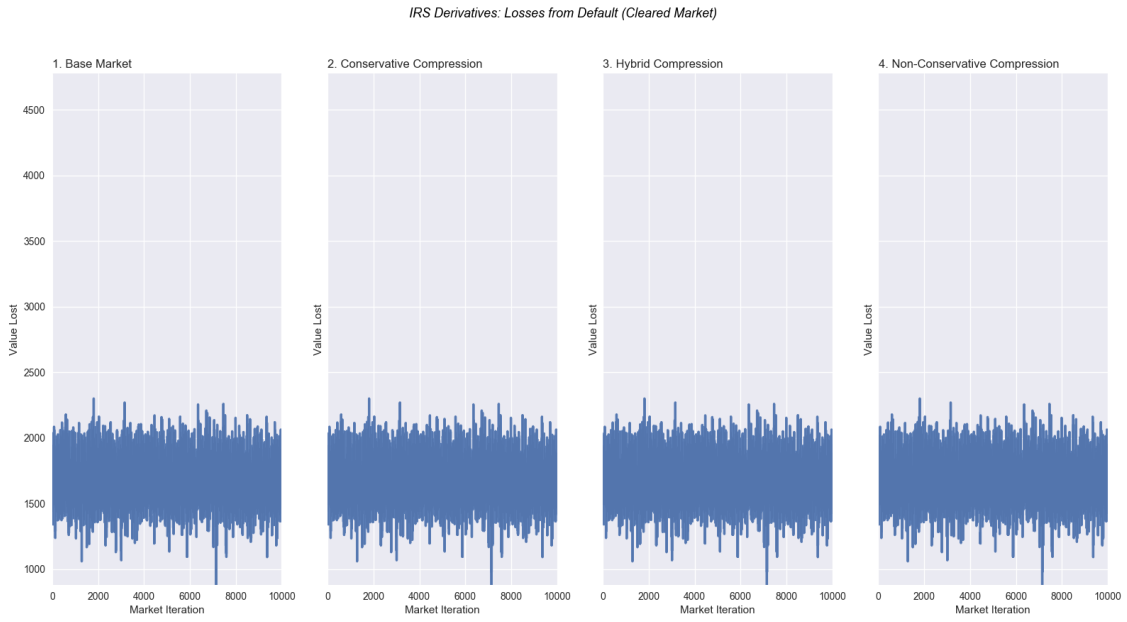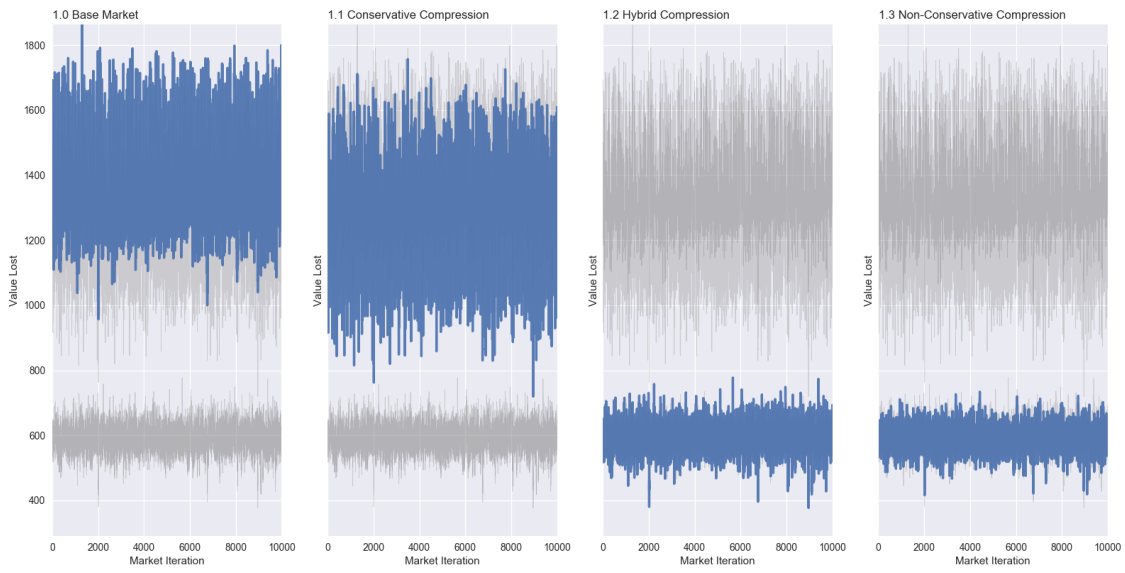
Figure 7: Centrally cleared IRS market Loss after triggering default (title market in blue, other markets in gray), and the distribution of results
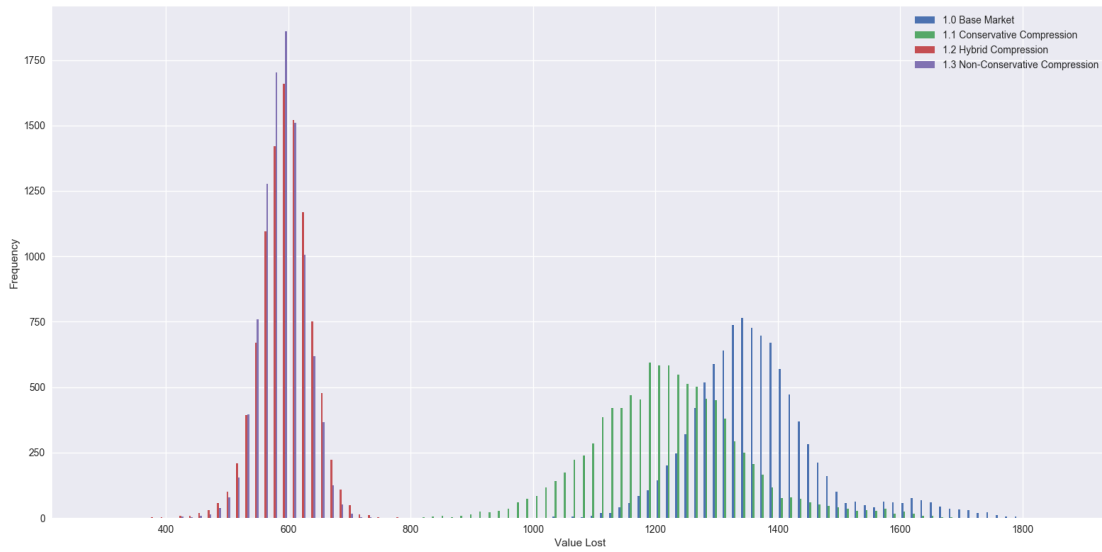
Figure 8: Bilateral Forex market loss after triggering default (title market in blue, other markets in gray), and the distribution of losses

Figure 9: Centrally cleared Forex market Loss after triggering default (title market in blue, other markets in gray), and the distribution of losses
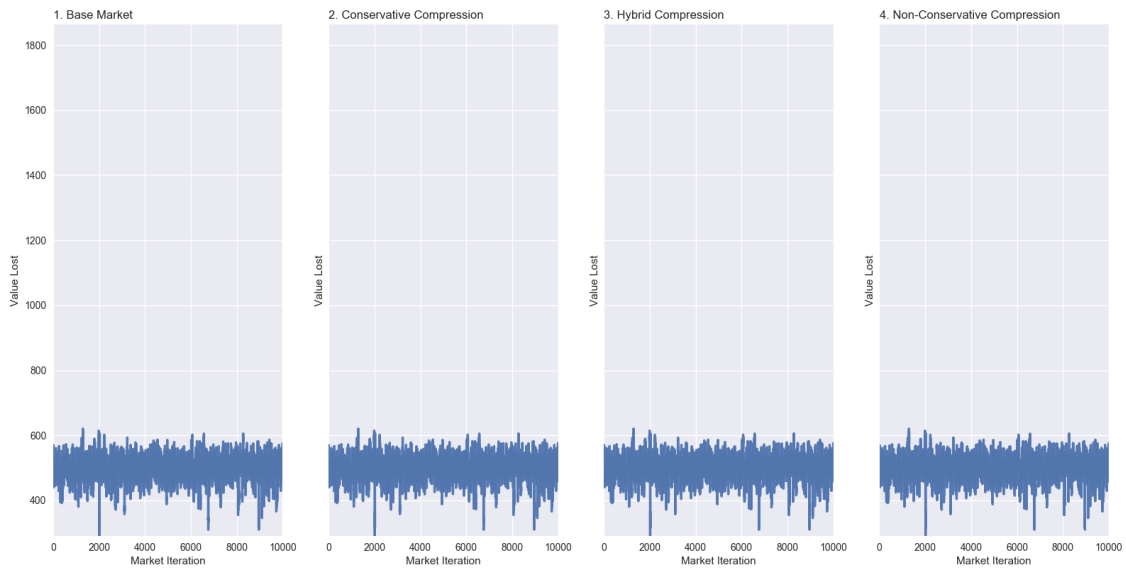
Figure 10: Bilateral Credit market loss after triggering default (title market in blue, other markets in gray), and the distribution of losses
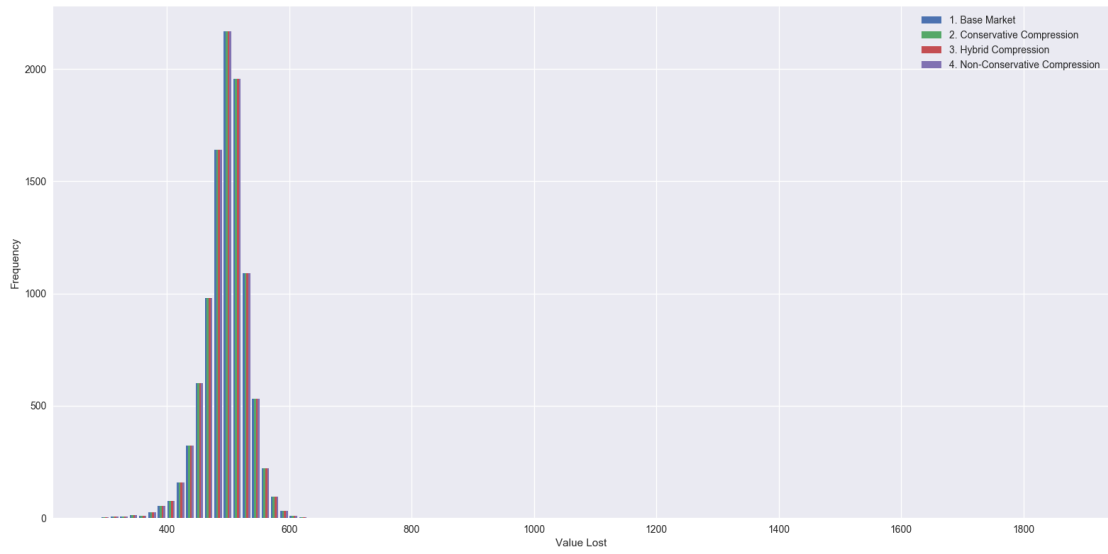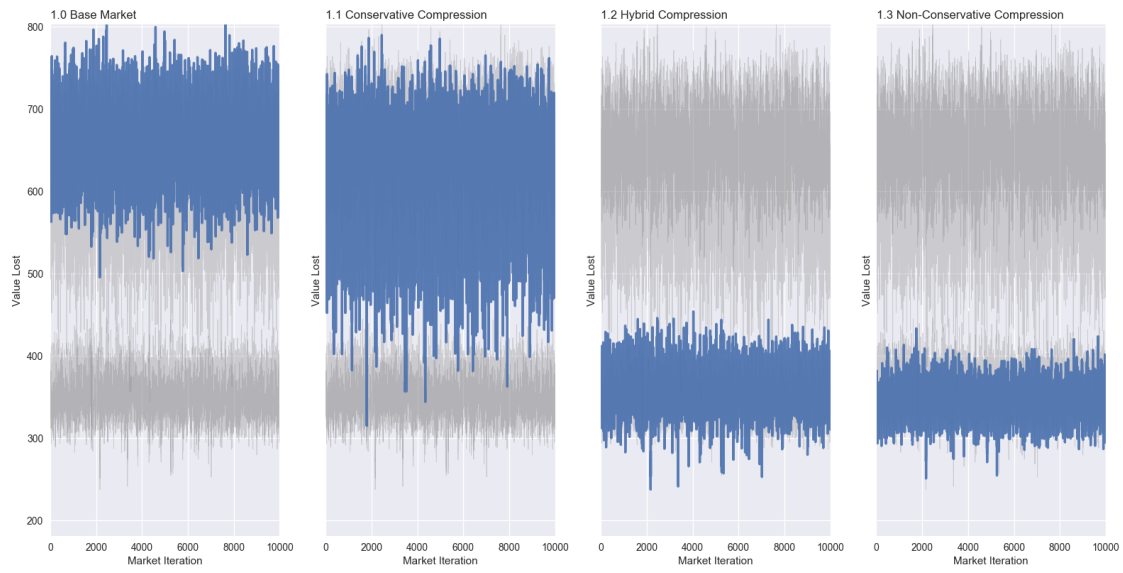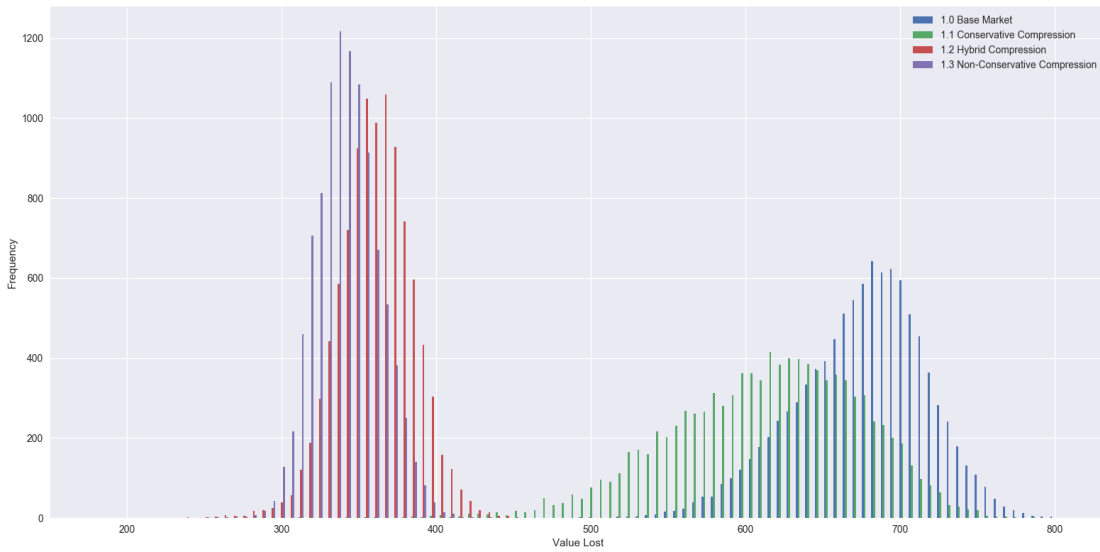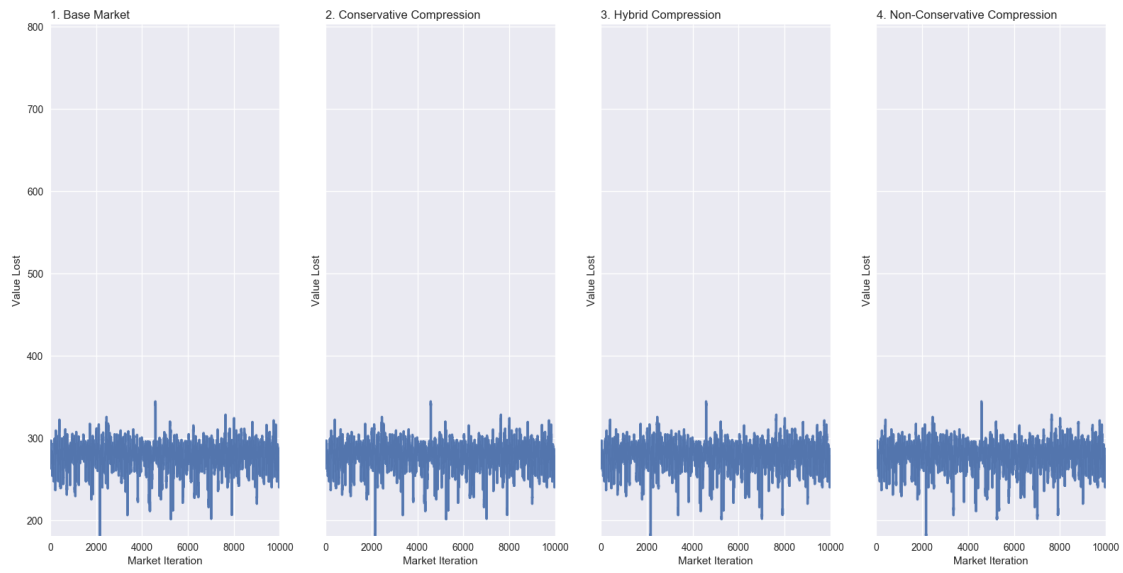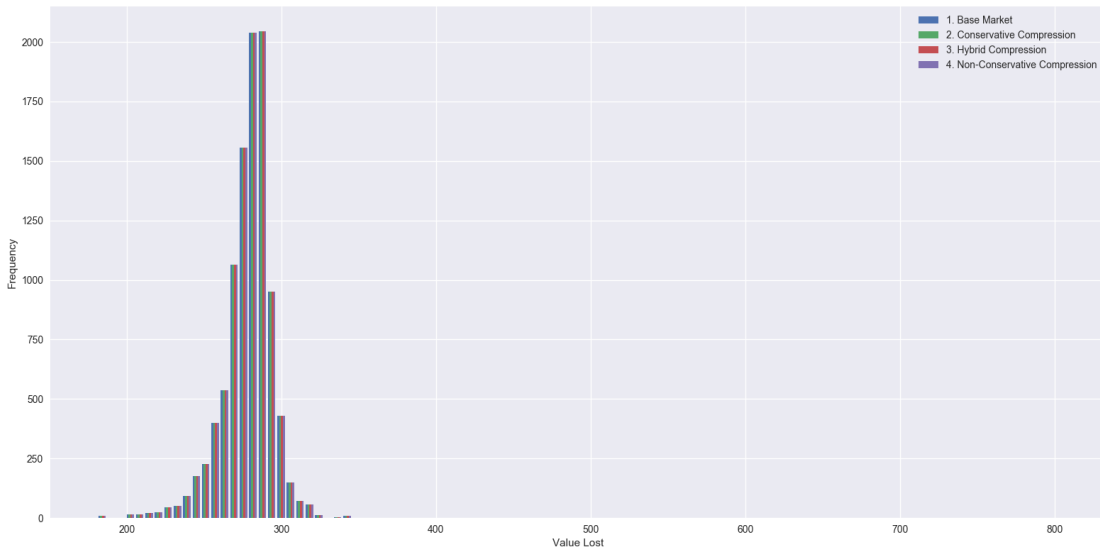
Figure 11: Centrally cleared Credit market Loss after triggering default (title market in blue, other markets in gray), and the distribution of losses

6   Conclusions

For each asset class, compression had no effect on the loss in the cleared market. Thus, it is apparent that compression does not make a difference in cleared markets. A possible explanation for this could be that central clearing results in a restructuring of the market based on net positions. As compression does not change net positions, it is obvious that compression will not change the structure of a market after clearing. In addition, the model used did not take into account cross-asset class netting, and the data is a "snapshot" of the market where time to maturity is disregarded. The effects of either of these cases on results is unknown.

In all three asset classes, we see similar levels of reduction in loss for bilateral, compression, and clearing. Furthermore, between the different types of compression, we again see similar levels of reduction for conservative, hybrid, and non-conservative methods. Table 3 shows these differences in relation to the level of loss seen in the bilateral market. Note that central clearing is generalized to a single case for each asset class, as compression did not effect the value lost.

Table 2: The difference in average value lost compared to the base (bilateral) market

Market Structure	Reduction from Base		
	IRS	FX	Credit
Base	0.0%	0.0%	0.0%
Conservative Compresssion	4.6%	10.5%	9.1%
Hybrid Compression	50.0%	56.6%	46.6%
Non-conservative Compression	52.4%	56.9%	49.4%
Central Clearing	52.4%	63.6%	58.8%

The reason that conservative compression resulted in only a marginal reduction is due to the fact that conservative compression only allows for the elimination of closed chains of intermediation, which are not always present, thus conservative compression is not always possible. Non-conservative com-

pression, on the other hand, is always possible, and reduces gross positions by as much as possible, thus, we see more of a difference between non-conservative and the bilateral market.

Interestingly, hybrid compression offered a reduction similar to non-conservative. There are several possible explanations for this. First, it is possible that the simulated markets did not have many customers, and thus hybrid compression mostly dealt with non-conservative compression over many dealers. Another possibility is dealer exposures are more important than client exposures, and thus the non-conservative compression over dealers was key to preventing default. The data do not strongly support either case, as most markets average a 50/50 split between dealers and customer.

As all forms of compression reduced value lost compared to the base market, with non-conservative resulting in a reduction comparable to clearing, we conclude that a reduction in exposure may correlate with a reduction in default risk. Additionally, as cleared markets had equal loss over all forms of compression, it would seem that compression is not necessary in those markets. However, the interaction between compression and clearing is not well understood. Real world markets are far more complex than the models used, and the addition of model complexity in the form of cross-asset netting or time to maturity might have effects on value lost, but this is not in the scope of this project.

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A Proofs

A.1 Lemma 1

Proof. Let E be an L_1 minimal matrix such that row r has values $e_{ra} > 0$ and $e_{rb} < 0$. Then

$$|E|_1 = \sum_{i,j \in N} |e_{ij}| \geq |e_{ra}| + |e_{rb}|$$

but for row r, $a_r = e_{ra} + e_{rb} < |e_{ra}| + |e_{rb}|$. Thus we have

$$|E|_1 > \sum_{i \in N} |a_i|$$

However, a_i is the lower bound for each row i, so $|E|_1$ is greater than the minimum bound, meaning E is not L_1 minimal, and we have a contradiction. Thus, if E is L_1 minimal, then $e_{ij} = 0$, or $sgn(e_{ij}) = sgn(a_i)$ and $sgn(e_{ij}) = sgn(g_j)$ $\forall i, j \in N$. $\qquad\square$

A.2 Theorem 2

Proof. Let E be the adjacency matrix associated with a given market, and a the vector of assets. If $a = \vec{0}$, then E can be redefined as a matrix of 0s and we are done. If $a \neq \vec{0}$, then there exists an a_i in a such that $a_i \neq 0$. We know that

$$\sum_{i \in N} a_i = 0$$

Thus there exists some $a_j \leq -a_i$. Now we partition the vector a into $a^+ = \{a_i \in a | a_i < 0\}$, and $a^- = \{a_i \in a | a_i > 0\}$. Similarly we partition l into $l^+ = \{l_i \in l | l_i < 0\}$, and $l^- = \{l_i \in l | l_i > 0\}$. Note that $a_i^+ = -l_i^-$ and $a_i^- = -l_i^+$ for all i in N.

Let $r \in a^+$ be given, there exist $c \in a^-$ such that $a_c \leq -a_r$. Now we construct an optimal. skew-symmetric matrix E^* by populating row r of E^* with values such that E^* is L_1 minimal. Clearly $r \neq c$, and $sgn(e_{rc}) = 1 = sgn(l_c)$, so we set $e_{rc}^* \leq a_r$, and as E^* is skew symmetric, $e_{cr}^* \leq -a_r = l_r$. If $a_r \leq l_c$ then we set $e_{rc}^* = a_r$ and the row is optimal. Else, we set $e_{rc}^* = l_r$, and as

we know there exists j such that $a_j \leq -a_i$ we find other entries in l to populate the rest of the row similarly. Now let another row in a^+ be given. This row can be populated similarly, with the exception that now l_c is bounded by $l_c - a_r$. Continue this for all rows in l^+. As we have not used negative values yet, the same operation can work for all rows in a^-. Finally, for any rows not in a^+ or a^-, the value of a_i here is clearly 0, and thus the row can be populated with 0. By definition, our constructed E^* contains only values that pass the optimality check in lemma 1. As E^* was defined using an arbitrary matrix E, it is always possible to compress a network to be L_1 minimal. $\qquad\square$

B   Pseudocode for Algorithms

B.1   Network Simplex

Algorithm 1: Network Simplex for trade Compression

   Input   : Original market $G = (N, E)$, set of risk tolerances
   Output: G^* such that x' is minimized

1 begin
2   start with an initial tree structure $E^T = (T, L, U)$;
3   compute total notional x' , reduced cost and node potentials;
4   **while** *there exists some arc* $\notin E^T$ *that violates optimality conditions* **do**
5   choose an edge (i, j) that violates conditions;
6   add (i, j) to E' and select the leaving edge (k, l);
7   update E', x' and node potentials;
8   **end**
9 end

B.2   Non-conservative Compression (L_1 Minimization)

Algorithm 2: A deterministic conservative compression algorithm

---
   Input   : Original market $G = (N, E)$
   Output: G^* such that $\Delta(G^*) \leq \Delta(G)$ and $E^* \subset E$

1 begin
2   Calculate assets $a_i = \sum_{j=1}^n e_{ij}$, and liabilities $l_i = -a_i$;
3   **for** *each row in E* **do**
4   **for** *each entry j in row_i* **do**
5   **if** $sgn(l_j) = sgn(a_i)$ *and* $sgn(l_j) = 1$ **then**
6 $E_{ij}^* = min(l_j, [a_i - \sum_{j=1}^n e_{ij}^*])$
7   **end**
8   **else if** $sgn(l_j) = sgn(a_i)$ *and* $sgn(l_j) = -1$ **then**
9 $E_{ij}^* = max(l_j, [a_i - \sum_{j=1}^n e_{ij}^*])$
10   **end**
11   **end**
12   **end**
13 end

B.3   Risk Propagation

Algorithm 3: Algorithm for applying and analyzing a shock to a given market

---

    Input   : Original market $G = (N, E)$, reserve vector r
    Output: The average loss of value L

1 **begin**

2   **for** $i \in N$ **do**

3   Shock i;

4   Add i to Γ;

5   Update e_{ij} $\forall j$ and c_i;

6   **while** *New i are added to Γ* **do**

7   **for** *j adjacent to $i \in \Gamma$* **do**

8   Calculate p_j;

9   **if** $p_j < 0$ **then**

10   Add j to Γ

11   **end**

12   **end**

13   Update E, r $\forall i \in \Gamma$;

14   **end**

15 $VL_i = \sum_{i \in \Gamma}(r_i - r_i^*) + \sum_{i,j}(e_{ij} - e_{ij}^*)$;

16 $\Gamma = \emptyset$;

17   **end**

18 $L = \frac{1}{N} \sum_{i=1}^{N} VL_i$

19 **end**