Aerial Wind Turbine

A Major Qualifying Project Report

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Abstract:

Land based wind turbines are not used to their fullest potential due to the inconsistency of wind near the earth's surface. The goal was to determine if a structure could be designed and built to harness wind energy at high altitudes. Using a non-rigid airship, a design was created to lift wind turbines up to a desired height while still achieving a moderate power output.

Executive Summary:

The rising cost of oil is increasing the need to find alternative energy sources. One source is harnessing the power of wind which is less harmful to the environment. Commonly, wind turbines are fixed to the ground and can only reach heights of up to 125 meters. There are also issues with the consistency of the wind speeds and direction at these heights. Wind turbines installed at these heights do not produce as much power as they could due to the inconsistency of the winds.

The goal of this project was to determine a way to elevate the turbines up to heights of 300 meters using a lighter than air structure. At this altitude, the wind speeds are more constant and the direction of the wind does not vary. With these two factors significantly improved, the turbines operate at their maximum potential.

Many steps were involved to reach these goals. To start off, a preliminary design of the structure including the support beams, tether and turbines was created. The summation of the forces from weight and drag associated with the structure were found. These numbers were needed to determine the volume of hydrogen required to overcome the total weight. With the drag forces calculated, the strength of the tether combined with the angle formed between the tether and ground were determined. The turbines are aligned facing backwards so they do not interfere with the tethering system. Wings were added to provide additional lift to reduce the amount of hydrogen needed.

The future of aerial wind turbines is promising with the expectation that technology will reduce the weight and size of wind turbines while maintaining the same power capacity. The final design is a feasible solution to the goal of the project but would be better if the overall size of the structure could be smaller in the future.

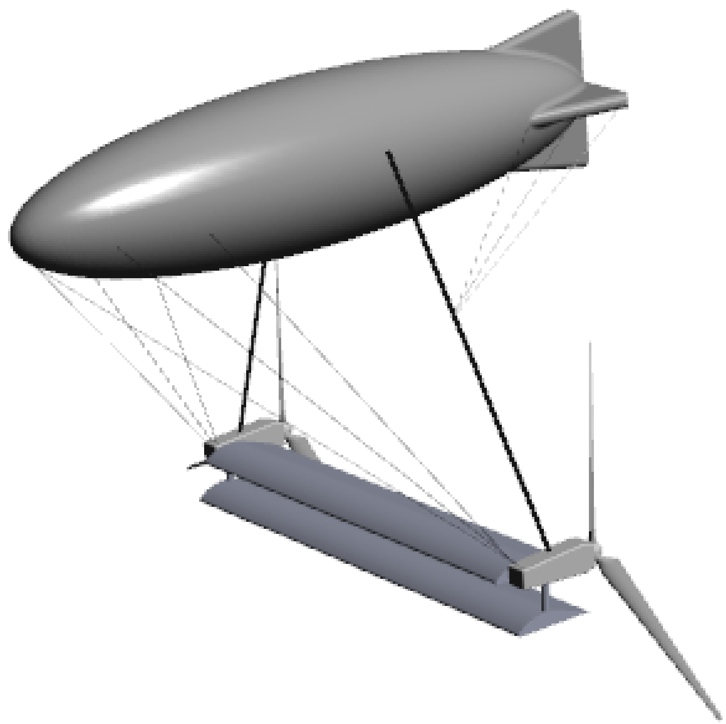

Figure 1: Isometric view of structure

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1.0 Introduction:

With the shortage of fossil fuels, alternative energy has been thrust into the national spotlight as a major necessity in order to keep up with the increasing energy demands of the world. One of these alternative energies is wind power. Although there has been an increasing interest in using turbines to harness the power of the wind, only a small percentage of those have focused on higher altitudes where the wind is more constant. The development and growth of land-based wind turbines has increased exponentially over the years as countries and businesses alike seek renewable energy sources. Whether the reason is funding or technical challenges, few companies are attempting to take on the winds at altitudes higher than 125 m [1].

Most designs for wind power have utilized a turbine anchored to the ground via a huge tower. Problems with these designs are that the intermittency of the wind causes most of these devices to be idle a majority of the time, with only 30 percent efficiency at most [2]. Cost is another major issue where a turbine with a 91 meter diameter that produces 2.5 megawatts costs about $14 million to design and build [2].

Reducing carbon dioxide emissions is a major contributing factor in alternative energy proposals. From the US average fuel mix, about 1.5 pounds of CO_2 are emitted for every kilowatt hour that is generated. Electricity consumption accounted for more than 2.3 billion tons of CO_2 in 2006. This accounted for 39.5 percent of the total emissions from human resources, according to the US Department of Energy [3]. Coal-fired plants alone released over

1.9 billion tons, which is one-third of the US total [3]. The US Department of Energy also projects that CO_2 emissions from power generation will increase 19 percent between 2007 and 2030. This is due to new or expanded coal plants [3]. A single 750-kW wind turbine produces roughly 2 million kilowatt hours of electricity annually [3]. With some simple calculations:

2 million kWh x 1.5 pounds CO_2 = 3 million lbs CO_2 = 1500 tons of CO_2 per year [3]

A forest absorbs about 3 tons of CO_2 per acre of trees per year [4]. Therefore, a single 750 kW turbine prevents the same CO_2 emitted each year as could be absorbed by 500 acres of forest [3].

The concept of this project is to use an existing wind turbine design and suspend it beneath a blimp. The blimp is the same shape as the Goodyear blimp but with larger dimensions. Two VestasV52's which have a capacity of 850 kW each were chosen as the wind turbines. The structure will be tethered to the ground with an operating height of 300m. The reasoning for this altitude is that the wind speeds are more steady there than at lower altitudes.

2.0 Background:

2.1 Wind Power:

Wind power first began around 5000 B.C. [5] with the use of sailboats. The sailors understood the use of aerodynamic lift, just not why it happened or how. But their experience and knowledge aided in the first versions of known windmills in Persia around 500-900 A.D. [5]. They were used for pumping water and grinding grain. The design evolved and was later used in Europe for similar purposes. Up until 1870 the blades were made out of wood [5]. The addition of steel blades to the design enabled them to be lighter and have more efficient shapes. These resulted in much higher speeds and a need to gear the windmills down to match the speed of the pump. In 1888, the first electricity generating windmill was built [5]. It produced 12 kilowatts with a 17 meter rotor [5]. Improvements to the design allowed for the production of up to 25 kW per windmill [5]. Jumping ahead, the United States government took an interest in wind energy after the “Arab oil crisis” of 1973 [5]. While this was not a large success, it does represent the beginning of the United States government realizing the weaknesses of oil based energy and looking towards more renewable energy sources. For the next twenty years much developmental research and testing was completed to solve many of the issues with windmill designs [5]. Current designs are far more efficient and able to produce energy in the magnitude of megawatts rather than kilowatts. However, as with most things, more can be done to increase the efficiency and power output of this renewable resource.

The modern design consists of a 1-3 Megawatt turbine [2] with 2 or more blades that have a rotor diameter between 70 and 100 meters [2]. Most turbine designs have three blades but it is said that the 2 blade design will become the norm for offshore wind farms [2]. The blades are shaped much like an airplane's wings in order to create lift from air moving over the blade [2]. Generally blades are made of a composite material structure. The best type is the wood laminates which prove to be the strongest and lightest.

Most modern wind turbine towers are conical tubular steel towers. They can range in height from 30 to 85 meters depending on the tower costs per meter, how much the wind locally varies with height above ground level, and the price the turbine owner gets for an additional kilowatt hour of electricity [5]. They are manufactured in sections of 20-30 meters [5] and are bolted together on site. The conical design is used to increase their strength while also saving materials at the same time.

When it comes to extracting power from the wind, there are some problems with finding a consistent wind. This greatly affects how efficient a wind turbine can be. From Betz's linear momentum theory, the maximum energy that can be extracted is 60 percent while most designs nowadays get around 30-40 percent [2]. A speed of 12 m/s is needed to get this maximum efficiency [2]. Any speeds higher than this will decrease efficiency and could potentially damage the turbine. Another way of increasing the efficiency of the turbine is to allow the rotor to change its rate of rotation as the wind speed changes. This is known as variable speed operation where the generators allow for variable rates of rotation while still producing alternating current electricity [2]. However, most designs are still of the fixed speed operation where all the components are much cheaper with a loss of 20 percent energy production [2].

There are numerous factors which affect the size and design of a wind turbine. Almost all the factors can be placed in two groups: time-dependent and frequency-dependent [2]. Forces from the wind, the unwanted vibrations of rotational frequency, and the fatigue effects of the constant rise and fall of the blades all influence the design process [2]. With power output goals in mind, the power capture is proportional to the swept rotor area. This will only increase to a certain range where the component size and machine cost outweighs the effectiveness. Conversely this “range” has greatly increased over the years as manufacturing and operational understanding is gained.

2.2 Wind Turbines:

The market for wind energy is expanding every day and many different companies are becoming involved or have been since the start. Companies all over the world are supplying wind energy to the residential market as well as building wind farms with hundreds of wind turbines for commercial use. A single wind turbine can output as little as 100 kW of power all the way up to 6 MW of power [3]. The amount of power produced is relative to the size and location of the turbines. The use of wind energy is increasing and potentially could power many areas of the world in the years to come.

There are several big names in the turbine business: Vestas (Denmark), GE (US), Gamesa (Spain), Enercon (Germany), Suzlon (India), Siemens (Germany), REpower (Germany). Each of these companies has developed a wide range of turbines for varying power levels and different locations with assorted wind conditions.

Vestas, a German company has wind turbines all across the world. They design 4 types of turbines that output power at a range from 850 kW to 3 MW. Their largest, 3 MW, turbine has a diameter of 90m with a weight of 41 tons which includes only the hub and rotor. Vestas has wind turbines in Europe, Denmark, the USA and China [6].

GE has been developing turbines for years now and has several models. Currently, GE has a 1.5 MW, 2.5 MW and a 3.6 MW turbine available to the market. The most advanced one is their 3.6 MW version that they have available for offshore installations. It has a rotor diameter of 111m and is rated at wind speeds of 14 m/s [7].

Gamesa is based in Spain and has turbines ranging from 850 kW to 2 MW. They have turbines designed for different wind speeds such as low, medium and high. The weights of their turbines rotor and blade combinations range from 10 tons up to 36 tons each [8].

Enercon, out of Germany, has the largest turbines on the market today. Their turbines are able to create power of over 6 MW each. These structures have a rotor diameter of 126 meters and are rated at 6 MW but have the potential to reach upwards of 7 MW. Enercon also has smaller scaled turbines ranging from 330 kW to 2 MW. They have close to 7,000 wind turbines in Germany and around 15,000 installed around the globe [9].

Suzlon, Siemens and REpower are all competing companies as well, with turbines starting at less than 1 MW. REpower was the leader for a while with their 5 MW turbine [10, 11, 12].

One must investigate the entire industry to get a grasp on what is going on in the world relating to wind turbine technology. There are wind turbines all over the world and some places are more advanced than others. According to the World Wind Energy Association [13], in 2008 the top five countries in terms of installed capacity are the US (25.17 GW), Germany (23.9 GW), Spain (16.74 GW), China (12.2 GW) and India (9.587 GW).

Wind energy is on the rise and should only increase with time. This environmentally friendly way to create electricity can power approximately 650 European households per every 1 MW turbine. Most turbines today are built to produce at least 2 MW [14].

Figure 1 compares some of the major wind turbine companies. The capacity is how much energy a single turbine can create. It ranges from 850 kW to 3.6 MW. The blade length is the span of a single blade and the hub height is the height of the tower which it is supported by. The total height is the distance from the ground to the tip of a vertical blade. The swept area is found by using the blade length as the radius and finding its circular area. This area is important for determining the total power output of the turbine. The area also affects how far each turbine must be placed away from one another. The last three columns show the different speeds of the spinning turbines and the ideal wind speed for the individual turbines.

The power output is related to the size of the turbine which also has a large impact on the weight of the product. The weight factor is important when choosing which type of turbine.

model	capacity	blade length	hub ht	total ht	area swept by blades	rpm range	max blade tip speed	Rated wind speed
GE 1.5s	1.5 MW	35.25 m	64.7m	99.95m	3904m^2	11.1-22.2	183 mph	12 m/s
GE 1.5sl	1.5 MW	38.5 m	80 m	118.5 m	4,657 m^2	10.1-20.4	184 mph	11.8m/s
GE 2.5xl	2.5 MW	50 m	100 m	150 m	7,854 m^2	NA	NA	12.5m/s
GE 3.6sl	3.6 MW	55.5 m	48.5 m	134 m	9,677 m^2	8.5-15.3	199 mph	14 m/s
Vestas V52	850 kW	26m	44 m	70 m	2,124m2	14-31.4	128 mph	16 m/s
Vestas V82	1.65 MW	41 m	70 m	111 m	5,281 m^2	12-14.4	138 mph	13m/s
Vestas V90	1.8 MW	45 m	80 m	125 m	6,362 m^2	8.8-14.9	157 mph	11 m/s
Vestas V100	2.75 MW	50 m	80 m	130 m	7,854 m^2	7.2-15.3	179 mph	15 m/s
Vestas V90	3.0 MW	45 m	80 m	125 m	6,362 m^2	9.0-19	200 mph	15 m/s
Gamesa G87	2.0 MW	43.5 m	78 m	121.5 m	5,945 m^2	9.0-19	194 mph	13.5 m/s
Siemens	1.3 MW	31 m	68 m	99 m	3,019 m^2	13-19	138 mph	14 m/s
Siemens	2.3 MW	41.2 m	80 m	121.2 m	5,333 m^2	11.0-17	164 mph	15 m/s
Suzlon 950	0.95 MW	32 m	65 m	97 m	3,217 m^2	13.9-20.8	156 mph	11 m/s
Suzlon S64	1.25 MW	32 m	73 m	105 m	3,217 m^2	13.9-20.8	156 mph	12 m/s
Suzlon S88	2.1 MW	44 m	80 m	124 m	6,082 m^2	NA	NA	14 m/s
Clipper Liberty	2.5 MW	44.5 m	80 m	124.5 m	6,221 m^2	9.7-15.5	163 mph	11.5 m/s
Repwer MM92	2.0 MW	46.25 m	100 m	146.25m	6,720 m^2	7.8-15	163 mph	11.2 m/s

Figure 2: Size specification of common industrial wind turbines [3]

2.3 Patent Search:

The present state of aerial wind turbine design is one in which designers have thought of many designs ranging from the simplest form of a kite/airfoil attached to a rope to the most radical proposals. However, there is a general picture that can be formed that encompasses most of the designs. Most of the designs have some sort of tethering system that anchors the structure to the ground or a platform in the water. All of them also have either a blimp or type of airfoil that keeps the whole system in the air. There next is a component that is used to harness the wind and convert it to electrical energy whether it is a turbine or a pulley design that turns a separate generator.

Although these ideas seem to be recent as of the last decade, patents were granted to engineers in the 70's for several aerial wind turbine designs. However, these designs consisted more of general ideas with multiple interpretations of the actual final product. Most of the patents found on this subject from the 70's and 80's had to do with the concepts of harnessing the wind and not all the logistics involving a final product.

US patent number 4,073,516 was one such example in which there are a lot of general ideas and little specifics. "Wind-Driven Power Plant" was issued a patent in June of 1975. Its design called for a power plant having a rotor assembly with at least one rotor connected to a generator. A gas-filled hollow body keeps the whole system in the air. The designer called for the whole system to be either anchored to the ground or be suspended in the air with a floating body. It would have some sort of means for aligning the rotors with the wind direction. There is also a possibility for the use of one cable to act as both the tethering system and the power transmitting wire. The designer suggests that the generated power be used while in the air through powering a transmitter or for energizing optical display devices. There is also at least one pair of coaxially supported counter rotating rotors that compensate for the spinning of the turbine. This causes rotors to always become aligned into the prevailing winds. Another key aspect of the design is a suspension body that is connected to a brace by a joint having three degrees of freedom (i.e. gimbal joint connection). This type of connector allows the suspended turbine to be readily changed to any pitch or attitude necessary for facing the wind. The balloon can be connected to the turbine via a net-like wrapping applied over the balloon. At the bottom there would be a connecting joint to which the rotor assembly attaches much like a gas balloon [15].

A much different design from the earlier decades would be US patent 4,491,739 entitled "Airship-Floated Wind Turbine." The goal for this patent was to create a wind turbine with a diameter reaching 1,000 feet with operational heights of several thousand feet. In this design, the airship holds up a large ring which supports the outer ends of the turbine blades that extend inward from the ring to the airship. A bearing assembly was created that allows the airship to rotate without twisting the tether line. The ring around the airship is basically a large "space frame" structure that holds all the turbine blades on the outer edges. This greatly reduces the weight of the whole structure [15].

A more conventional design is US patent 4,450,364 with the title of “Lighter than Air Wind Energy Conversion System Utilizing a Rotating Envelope.” This is a system where the main rotors spin independently of the gas-filled structures. It is self-orienting and includes aerodynamic damping of orientation motions. However, it still requires large heavy rotor blades. The rotor blades are designed to be rotated by the wind in a plane perpendicular to the longitudinal axis of the structure. There is also a non-rotating tail for orienting the structure while also providing some additional lift. Generators are positioned on the rotor blades in order to generate the electricity. The complete system is tethered to the ground by at least one tethering cable and one electrical cable. The designer also suggests that there be some type of mooring system on the cable with the ability to draw the cable in and tether it closer to the ground [15].

Many of the newer designs and patents have incorporated key aspects of earlier designs while taking them a step further by going into great detail about the tethering system or the actual design for the turbine. They are generally much more complete than the earlier patents.

One such design is patent 6,523,781 titled “Axial Load Linear Wind-Turbine.” This design can be used at any height from 300 feet to 3,000 feet. It is meant to capture the wind in the axial direction, perpendicular to the airfoil’s flight direction. Most of the expensive and heavy components are on the ground and only the airfoils are in the air. The system consists of three airfoil kites in tandem connected to a ground anchor. The kite operates at high speeds with the airfoil moving mostly perpendicular to the wind stream. Adjusting the length of the control lines in turn controls the airfoils direction and speed. The three airfoils are angled into the rising wind which causes the pulley and shaft to turn a generator on the ground. Since the generator is on the ground, there is no need for an electrical cable running to the apparatus that is in air. This greatly reduces one of the major problems in terms of receiving the energy from the turbines in the air. Once the airfoils reach the maximum height, their pitch becomes negative and they fall back to the starting position to start the cycle again [15].

Another present-day design for a wind turbine is US patent 7,129,596 entitled "Hovering Wind Turbine." This a fairly simple design in which the turbine is one that would be seen on a residential unit. The blades lie on the surface of an imaginary horizontal cylinder with their pitch angle changing as a result of the rotational angle. This allows the turbine to gather wind energy mainly in the upwind and downwind areas of the cylindrical path. It also uses a fraction of the gathered energy to create lift by deflecting air downwards, mostly in the upper and lower areas of the cylindrical path. The remainder of the wind energy is used to drive a pair of on board electrical generators. The anchoring tethers carry electrical current to a land or water site. The turbine is tied to a blimp that is purposely located in its wind shade and keeps it airborne during periods of light or no wind [15].

Generally most of the designs from the 70's and 80's have been similar to those used today. Almost all aspects of the designs are more comprehensive in comparison to the older designs in which some parts of the system are left the same. Presently the design and set up for a good working aerial wind turbine is out there. But the costs are just too high for a company to go through the trial and error of achieving a working design. Changes in the hardware and weight of the turbines are the most logical areas which need to be focused on.

2.4 Airships:

A prominent idea to deploy a wind turbine system in flight is to incorporate lighter-than-air structure. The main issue is the size of the airship. If the airship is much larger than any other airship ever made, then using lighter-than-air technology alone is a far-fetched pursuit. Other issues more quickly resolved are the type of medium used (helium, hydrogen, etc.) and the structure of the airship.

To create life, the gas inside the airship needs to be lighter than the surrounding air. Air can be lighter by heating it. Temperature is inversely proportional to density, meaning that the higher the temperature, the lighter the gas. However, hot air balloons require constant heat and a large volume of space, ruling out the possibility of using hot air. This leaves two possibilities: hydrogen and helium.

The main advantage of hydrogen is that it is readily available by splitting water molecules using electricity, sunlight, or radio waves. The electricity can be generated from the wind turbines automatically when needed, and the hydrogen can be sent upward by tubes inside or along the cables that attach the ground station to the aerial wind turbine. Plus, hydrogen is less dense than helium. However, hydrogen is explosive and requires fire protection measures for the airship. The two advantages of using the rare and expensive helium gas are that it is not flammable and does not leak as much as hydrogen. Though the airship will be flying high, susceptible to lightning strikes, pursuing a design that prevents lightning and leaks from igniting the airship would be a feasible concept.

There are three types of airships: rigid, semi-rigid, and non-rigid. Rigid airships such as the Hindenburg require an external shell to retain the shape of the airship. This requires a lot of heavy material that increases the weight of the unit. Semi-rigid airships have a frame around the envelope (the encasing of the medium) that is sometimes flexible. However, semi-rigid airships seem to be obsolete, minus the one and only Zeppilin NT (shown in Figure 2). Non-rigid airships rely on an over-pressure to retain the shape of the envelope and are the most popular type of airships today. In fact, there is very little difference between semi-rigid and non-rigid airships of today. It could save on material costs to use an airship that does not rely on a frame. It results in significant weight reduction and therefore makes sense to use a non-rigid airship if possible.

Figure 3: ZEPPELIN NT [16]

Lastly, the size of the airship portion of the aerial wind turbine must be within the range of already-designed airships. To use an already-made design would save time and ensure that the flight would be successful. Several airships have been created at a very large scale. The largest of them all was the Hindenburg, lifting over 112 metric tonnes, stretching 245 meters long, and filling a volume of 200,000 cubic meters with hydrogen. The Goodyear blimp, one of the largest airships of today, weighs 6 metric tonnes, and still generates enough lift for its passengers. Larger designs have been proposed up to 500,000 cubic meters, but never amassed the funding to get it off the ground.

3.0 Design Prototype:

In order to reach the altitudes needed, a blimp with 200,000 cubic feet of hydrogen was used. The weights of the turbines, the drag exerted on the swept area of a spinning rotor and the combined drag and weight of the tether were the main factors which lead to a blimp of such size. For the material used and the overall structure of the blimp, it was actually very similar to the Goodyear blimp that we see today. The blimp is a non-rigid airship without an internal supporting framework. The envelope is made out of a polyester composite fabric much like the fabric used for modern space suits. The higher pressure of the lifting gas inside the envelope and the strength of the envelope is what maintain the shape of the blimp.

Two Vestas 850 kilowatt turbines are attached under the blimp using steel supports. The supports are connected to the blimp using two catenary curtains and suspension cables inside the blimp, each located along the length of the airship. The curtains are made from folded fabric and are stitched into the envelope. The suspension cables then attach to the supports much like the gondola underneath the Goodyear blimp. Steel supports are attached between the two nacelles of the turbines to provide a more rigid structure. The turbines are set up to spin in opposing directions in order to counteract the forces involved with the spinning of the blades.

With an operating height of 300m, a material for the tether needed to have a high tensile strength and be a light as possible. Steel was first tried as the material for the tether but it was too heavy and not strong enough. After using calculations for the drag forces, the diameter of the steel tether came out to be 18 cm and this was considered to be too large. The selected tether is made out of a carbon epoxy composite with a tensile strength of 1100 MPa. By using the carbon composite, the diameter became 12cm which is more workable. This comes out to have a cross-sectional area of 256 cm^2.

The greatest challenge presented to the analysis was to calculate the amount of hydrogen that the airship must contain in order to stay afloat. All weights must be accounted for: the supports, the turbines, the envelope material, tethers, power lines, support cables, and possibly a wing. This analysis combined the sciences of fluid dynamics, stress analysis, heat transfer, electrical engineering, and elements of machine design to provide a basis for which a company could see that a design was possible, a proof of concept.

Further setup consisted of determining what safety factor to use, what lighter-than-air medium to fill the airship, and what proportions to assume for the airship envelope. Most safety factors tend to be around 2 or 2.5, but since this is a revolutionary idea, it made sense to go with a higher, more conservative safety factor of 3. The medium chosen was decided much earlier on to be hydrogen, due to its cost-effectiveness as well as the safety of the envelope being far from any type of spark. A proportion of 1:4 (height or depth by length) was the approximate ratio of many small blimp-like balloons that could serve as a test model for our design.

3.1 Preliminary Volume Calculations:

The gauge pressure in the Good Year Blimp was found to be around 0.7 psi. Both hydrogen and helium were taken into account in case a company taking this design into consideration favored the inert helium gas over hydrogen. This did not greatly affect the air density of the medium, whether it is Hydrogen or Helium.

Mass lifted is equal to the difference in density in air from Hydrogen times the volume (V_o). The mass of the structure was assumed by adding the mass of the turbines (W_{rotor}: 10 tonne; and W_{hub}: 22 tonne) times the safety factor. Initially, it made sense to go with the biggest turbines, but as the calculations were made, the size of the airship kept growing past the size of the largest airship ever made, the Hindenburg. Thus, a smaller turbines design, the 850kW Vestas wind turbine, was used in the calculations. A volume was estimated initially using the below equation.

$$2\cdot(W_{hub} + W_{rotor})\cdot SF = V_o\cdot(\rho_{air} - \rho_H)$$

$$W_{hub} = 22\,\text{tonne} \qquad W_{rotor} = 10\,\text{tonne} \qquad \rho_{air} = 1.225\times10^{-3}\,\frac{gm}{cm^3} \qquad \rho_H = 9.418\times10^{-5}\,\frac{gm}{cm^3}$$

$$V_o := \frac{2\cdot(W_{hub} + W_{rotor})\cdot SF}{\rho_{air} - \rho_H} = 1.698\times10^5\,m^3$$

The Hindenburg's volume is $2*10^5\,m^3$, so this renewable energy airship is still enormous, but not impossible. Keep in mind that this is still not the final calculation, but an estimate, thus a fair amount of analysis from this point on will be functions of the volume until the final volume is found. From the above calculation and the ratio of length to height, the equation of a spheroid can be used to find the height/width ($a_o(V_o)$) and the length ($b_o(V_o)$). The R_{ab} represents the 1:4 ratio (0.25) of width to length.

$$V = \frac{4}{3} \cdot \pi \cdot a^2 \cdot b \qquad R_{ab} = \frac{a}{b}$$

$$a_o(V_o) := \sqrt[3]{\frac{V_o \cdot 3 \cdot R_{ab}}{4 \cdot \pi}} \qquad a_o(V_o) = 19.654\ \mathrm{m} \qquad b_o(V_o) := \frac{a_o(V_o)}{R_{ab}} \qquad b_o(V_o) = 78.615\ \mathrm{m}$$

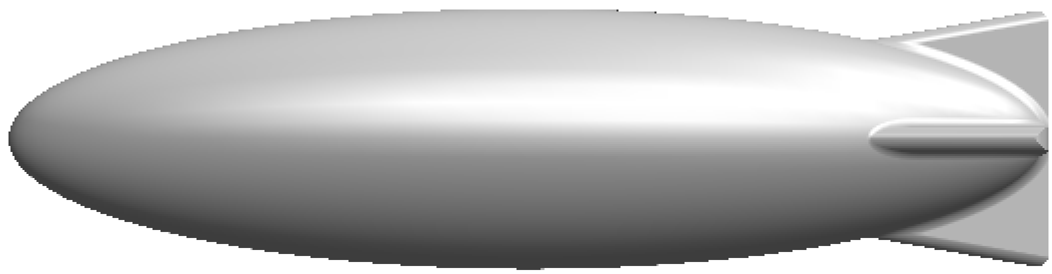


Figure 4: Envelope

3.2 Surface Area:

The envelope was assumed to have the same thickness and material as the Goodyear blimp. The surface area is calculated using the following formulas for prolate spheroids.

$$S = 2\cdot\pi\left(a^2 + \frac{a\cdot b\cdot \alpha\varepsilon}{\sin(\alpha\varepsilon)}\right) \quad \text{where...} \quad \alpha\varepsilon = \text{acos}\left(\frac{a}{b}\right)$$

$$\alpha\varepsilon(V_o) := \text{acos}\left(\frac{a_o(V_o)}{b_o(V_o)}\right) \qquad S_{fab}(V_o) := 2\cdot\pi\cdot\left(a_o(V_o)^2 + \frac{a_o(V_o)\cdot b_o(V_o)\cdot \alpha\varepsilon(V_o)}{\sin(\alpha\varepsilon(V_o))}\right)$$

$$S_{fab}(V_o) = 1.897 \times 10^4\, m^2$$

3.3 Hanging Supports:

The next important piece of the setup is the support system. The hanging supports consist of two rods extending from the centroid of the envelope (possibly using a netting or other canopy-type support to connect to the envelope) to the nacelles of the turbines. Because the width of the envelope is smaller than the sum of the radii of the blades plus a clearance (d_{clrnc} at 10m), this results in the two hanging supports being at obtuse angles from the envelope.

The support in the middle, under compression, needs to account for the blade radii (r_{bld} at 26m) plus the clearance. The height of the support is half the height of the envelope plus the radius of the blade plus the clearance minus half the height of the nacelle (w_{ncl} at 3m). The horizontal distance the support spans is half of the middle support length minus half of the envelope width. Using Pythagorean Theorem, the total length was calculated. Finally, the angle at which the supports hang is calculated.

$$d_{plt} := 2 \cdot r_{bld} + d_{clrnc} = 62.003\,m \qquad h_{sup}(V_o) := \frac{1}{2} \cdot a_o(V_o) + r_{bld} + d_{clrnc} - \frac{1}{2} \cdot w_{ncl}$$

$$w_{sup}(V_o) := \frac{d_{plt} - a_o(V_o)}{2} \qquad L_{sup}(V_o) := \sqrt{(h_{sup}(V_o))^2 + (w_{sup}(V_o))^2}$$

$$L_{sup}(V_o) = 49.612\,m \qquad \Theta := \operatorname{atan}\left(\frac{h_{sup}(V_o)}{w_{sup}(V_o)}\right) = 65.997 \cdot deg$$

Because this is facing the wind and will face a serious drag force at the end, the hanging supports are made from a hollow rectangular cross-section to ensure high moment of inertia in order to cut back on the bending stresses discussed further in the analysis called "Fatigue Load on Supports".

3.4 Power Lines:

Before getting an estimate of the different drag forces, the thickness of the power lines running up and down the tether must be considered. Because the resistance in a power line is relative to the current squared (I^2) and current directly proportional to the voltage, when the voltage increases, the resistance significantly decreases. However, when the voltage is too high, it may spark or lose charge through the corona effect (electron release into space when high charge is present on small point conductors). Thus, the range in voltage for high tension power lines was researched and found to be in the range of 3kV to 1MV. To be in that range, but cautious, 300kV was eventually chosen to be the voltage in the power lines.

A major limiting factor in the diameter of the power lines is the amount of heat transfer. The resistance is converted into heat, and on a sunny day, the heat from the power lines could melt the shielding right off. Using the equation of electrical power, the current in the power loss equation can be found from the voltage (E). Equations involving power are needed to combine electrical and heat transfer properties.

$$P_{gen} = I \cdot E \qquad P_{loss} = I^2 \cdot R = \frac{\Delta T \cdot k \cdot A_{surf}}{L} \qquad A_{surf} = \pi \cdot d \cdot L$$

Δ T is the change in temperature from a hot day (70°C) to the melting point of rubber (98°C). A_{surf} is the surface area of the wire. L is the length of the wire. "k" is the coefficient of heat transfer for copper, as copper has high conductivity, thus optimal for electrical power transfer. The power loss equation can be expanded knowing the R value. This is calculated from the below equation where ρ is the coefficient of resistivity.

$$R = \frac{\rho \cdot L}{A_{csec}} \qquad A_{csec} = \frac{\pi}{4} \cdot d^2$$

After combining the equations together, the calculated diameter is around 5.3 mm. The total power loss (using heat transfer) is roughly 178W, which is a small fraction of the overall power.

$$d_{p1} := SF \sqrt[3]{\frac{4}{\pi^2} \cdot \frac{I^2 \cdot \rho_{cua} \cdot L_{p1}}{k_{cu} \cdot (T_{max} - T_{hot})}} = 5.326 \cdot mm \qquad P_{loss} := (T_{max} - T_{hot}) \cdot k_{cu} \cdot \pi \cdot d_{p1} = 178.018\,W$$

3.5 Drag Force:

The "no slip" condition is a practical assumption that fluid velocity moving across a surface will approach zero as it nears the surface. Therefore, wind velocity is small at the ground and much larger the further away. Because of this, the aerial wind turbine must be able to undergo drag forces from high wind velocities.

Drag is a function of velocity squared, surface area, and fluid density. Therefore, the more area, the more drag. The most significant factor was not the surface area of the airship, nor the surface area of the tether or cables. The largest drag force came from the area of the turbine blades. From Betz' Law, it states that the maximum power that a turbine can take from the air is 16/27 (η_{max}), and further research estimates that the most efficient turbines in today's day only reach 80% of that value, resulting in an efficiency of 47.4% (η). Thus, spinning turbine blades only experience drag at a maximum of 47.4% from the drag on the swept area. Still, the area swept by the blades is the most significant surface area.

$$\eta_{max} := \frac{16}{27} = 0.593 \qquad \eta := 80\% \cdot \eta_{max} = 0.474$$

From there drag from the rotor is calculated, assuming wind velocity (v_{max}) to be 25 meters per second, the maximum wind velocity where the turbine can operate efficiently. Also drag on the envelope is calculated using a drag coefficient (C_d) of 0.025 [28] using an estimated cable radius times two plus the thickness of the power lines. The cable drag assumes an infinitely long cylinder with a drag coefficient (C_{dc})of 2.1 [29]. Also, a drag force from the wing is estimated from a previous iteration (to be around 17 kN) and used as a variable in the overall drag function.

$$F_{Dr} := \frac{1}{2} \rho_{air} \cdot A_{blades} \cdot v_{max}{}^2 \cdot \eta = 385.467 \cdot kN$$

$$F_{Db}(V_o) := \frac{1}{2} \cdot \rho_{air} \cdot C_d \cdot \frac{\pi}{4} \cdot v_{max}{}^2 \cdot \left(a_o(V_o)\right)^2 \qquad F_{Db}(V_o) = 3.52 \cdot kN$$

$$F_{dx}(V_o, F_{Dw}, r_{cble}) := 2 \cdot F_{Dr} + F_{Db}(V_o) + F_{Dc}(r_{cble}) + F_{Dw} \qquad F_{dx}(V_o, F_{Dw}, r_{cble}) = 846.882 \cdot kN$$

The calculations show that the drag on the two rotors accounts for almost all the horizontal force.

3.6 Area of Tether:

The tether has to be able to support the structure in heavy winds. This means that a tether that can support the drag forces as well as the upward forces of the wing must be attained. To do this, the drag force must be counteracted by another force equal to the drag force divided by the cosine of the angle of the tether projecting from the ground. Using the Pythagorean Theorem, the square root of the sum of those two forces is the overall force pulling on the tether.

$$F_{th}(V_o, F_{Dw}, r_{cble}) := \sqrt{1 + \frac{1}{\cos(\theta)^2}} \cdot F_{dx}(V_o, F_{Dw}, r_{cble})$$

Force divided by area is equal to the tensile stress. Using a safety factor of 3 and rearranging the equation, the radius of a steel cable was found to be 4.1 cm.

$$SF = \frac{S_{ut}}{\left(\frac{F}{\pi \cdot r_{cbl}^{2}}\right)} \qquad r_{cble} = \sqrt{\frac{F_{th}(V_o, F_{Dw}, r_{cble}) \cdot SF}{\pi \cdot S_{ut}}} \qquad r_{cbl}(V_o, F_{Dw}) = 4.1 \cdot cm$$

After going through the calculations and doing further research, it was concluded that a different material could be used to make the cable smaller and lighter. The material chosen was carbon epoxy, which has higher ultimate tensile values as well as a lower density.

$$r_{cble2} = \sqrt{\frac{F_{th}(V_o, F_{Dw}, r_{cble2}) \cdot SF}{\pi \cdot S_{ut.cac}}} \qquad r_{cbl2}(V_o, F_{Dw}) = 3.401 \cdot cm$$

3.7 Wing:

Lift needed to be provided beyond that of the airship part of the structure or else the angle of the tether from the ground would be much too small, resulting in the turbine blades interfering with the ground. The calculations assume a wing to be an elliptical shape with 4% height of width and thickness t_w of aluminum.

Lift force is a function of drag force, air density, wind velocity, the platform area (breadth divided by depth) and the lift coefficient. The lift coefficient is a function of the angle of attack (α) and the aspect ratio (AR) which is breadth divided by the depth of the wing.

Maximum effective angle of attack tends to be around 15 degrees from most graphs, and beyond that, the wind foil around a wing steadily declines.

$$F_L + F_D = \frac{1}{2}\cdot\rho\cdot v^2\cdot A_{plnfrm}\cdot C_L \qquad C_L = \alpha\cdot 2\cdot\pi\cdot C_{L1} \qquad C_{L1} = \frac{AR}{AR+2} \qquad AR = \frac{span^2}{A_{plnfrm}} = \frac{length}{depth}$$

Because of the complexity involving the aspect ratio, one dimension (breadth or depth) must be estimated first. While going through the first round of calculations, it became apparent that a large wingspan was needed. This complicated the placement of the wings. It would make sense to place the wing between the two nacelles, but the wing may be too deep, reaching into the area where the turbine blades will interfere. The calculations below demonstrate this complication.

$$L_w := d_{plt}$$

$$\tan(\theta)\cdot F_{dx}(V_o, F_{Dw}, r_{cble}) - g\cdot M_{lift} = \frac{1}{2}\cdot\rho_{air}\cdot v_{max}^2\cdot L_w\cdot D_w\cdot\cos(\alpha)\cdot\left(\frac{L_w}{L_w + 2\cdot D_w}\right)\cdot(2\cdot\pi\cdot\alpha)$$

The equation to solve for the wing depth is a function of wing drag and volume. Now that the equation is found, it can be plugged into a wing drag equation, then back into the depth equation. This results in a wing depth 3.9 times the length of the nacelle.

$$D_W(V_o, F_{Dw}) := \text{Find}(D_w) \qquad F_{Dw} = \frac{1}{2}\cdot\rho_{air}\cdot C_d\cdot v_{max}^2\cdot D_W(V_o, F_{Dw})\cdot L_w\cdot(0.04 + \sin(\alpha))$$

$$F_{dw} := \text{Find}(F_{Dw}) = 7.541\times 10^3\,\text{N}$$

$$D_W(V_o, F_{dw}) = 42.527\,m$$

$$D_W(V_o, F_{dw}) \cdot l_{ncl}^{-1} = 3.884$$

From this, the planform area is calculated at the maximum angle of attack. Also, the cross-sectional area of the wing is calculated.

$$A_{pf}(V_o) := D_W(V_o, F_{dw}) \cdot L_w \cdot \cos(\alpha) \qquad A_{pf}(V_o) = 2.547 \times 10^3\,m^2$$

$$A_w := 0.04 \cdot \frac{\pi}{2} \cdot \left[\left(D_W(V_o, F_{dw})\right)^2 - \left(D_W(V_o, F_{dw}) - t_w\right)^2 \right] = 0.027\,m^2$$

From the dimensions, it is now possible to calculate the lift coefficient and the lifting force provided by the wing, as well as the equivalent weight it lifts in metric tonnes.

$$C_{L0} := 2 \cdot \pi \cdot \alpha \qquad C_L := \left(\frac{L_w}{L_w + 2 D_W(V_o, F_{Dw})} \right) \cdot C_{L0} = 0.682$$

$$F_{lft} := \frac{1}{2} \cdot \rho_{air} \cdot v_{max}^2 \cdot A_{pf}(V_o) \cdot C_L = 6.653 \times 10^5\,N \qquad W_{lift} := \frac{F_{lft}}{g} = 67.837 \cdot tonne$$

The wing is much longer than the nacelle, so the design requires further attention. Perhaps the wings could be off the sides of the nacelle or spread out in between the supporting rods.

3.8 Horizontal Beam:

The horizontal beam connecting the two nacelles undergoes compressive stresses from the weight of the turbines hanging from the supports connected to the nacelles. Also, the beam undergoes torsion from the spinning turbine blades. The torque is a function of power over rotational speed.

$$P = \tau \cdot \omega \qquad \omega_{max} = 31.4\,\text{rpm} \qquad \tau := \frac{P_{gen}}{\omega_{max}} = 2.585 \times 10^5\,\text{J}$$

To determine the effects of the cross beam from torque, the moment of inertia is calculated from the geometry of the structure. The geometry of the horizontal beam structure must be designed in such a way that the compressive and bending stresses will not cause it to buckle. Therefore, it was decided to have a number of beams (n_{cbsup}) spaced a certain distance apart (x_{cb}) and held together by cross bars making a certain number of equally spaced sections (n_{cbsec}). For this design, it was chosen to have 3 hollow circular beams spaced 0.5 meters apart and connected in the middle. Values for the thickness (t_{cb} =~ 0.02 meters) of the tube-like beams and also the outside diameter (D_o =~ 0.15 meters) were guessed and re-calculate later on in this section.

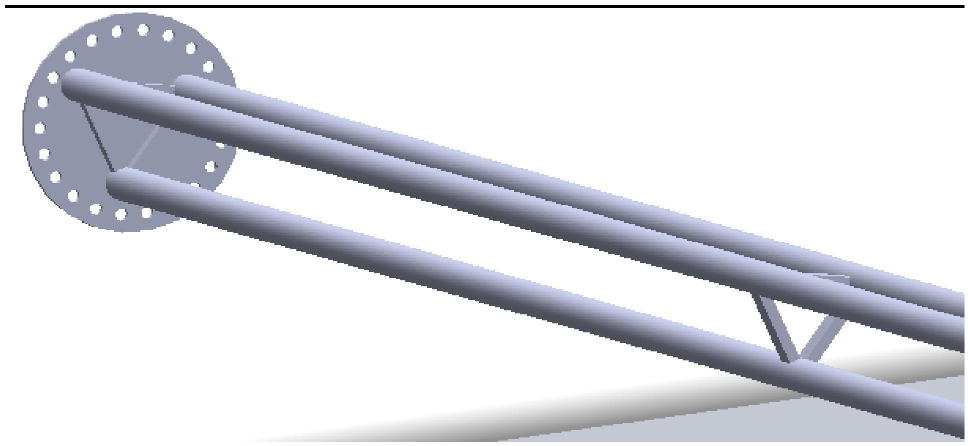


Figure 5: Horizontal Beam

$$n_{cbsec} := 2 \quad n_{cbsup} := 3 \quad I_{xx} = \frac{n \cdot \pi \cdot \left[D_o^{\,4} - (D_o - 2 \cdot t_{cb})^4\right]}{64} \quad I_{cb} = I_{xx} + n \cdot A \cdot x^2$$

$$x_{cb} = 0.5\,m \quad x_{cbt}(x_{cb}) := x_{cb} \cdot \sin(\theta_{nb}) \quad \phi_{nb} := \frac{2 \cdot \pi}{n_{cbsup}} = 120 \cdot deg \quad \theta_{nb} := \left|\phi_{nb} - 180deg\right| = 60 \cdot deg$$

$$I_{xxcb}(D_o, t_{cb}) := \frac{n_{cbsup} \cdot \pi \cdot \left[D_o^{\,4} - (D_o - 2 \cdot t_{cb})^4\right]}{64} \quad A_{cb1}(D_o, t_{cb}) := \frac{\pi}{4} \cdot \left[D_o^{\,2} - (D_o - 2 \cdot t_{cb})^2\right]$$

$$I_{cb}(D_o, t_{cb}, x_{cb}) := I_{xxcb}(D_o, t_{cb}) + n_{cbsup} \cdot A_{cb1}(D_o, t_{cb}) \cdot x_{cbt}(x_{cb})^2$$

The bending stress from torsion is equal to torque times outer-most edge from center divided by the area moment of inertia. This results in very high safety factors for steel and aluminum both for the estimated values, showing that the structure is quite robust in resisting bending stresses.

$$\sigma = \frac{M \cdot y}{I_{xx}} \quad \sigma_{cbm}(D_o, t_{cb}, x_{cb}) := \frac{\tau \cdot \left(x_{cbt}(x_{cb}) + \frac{D_o}{2} \right)}{I_{cb}(D_o, t_{cb}, x_{cb})} \quad \sigma_{cbm}(D_o, t_{cb}, x_{cb}) = 28.256 \cdot MPa$$

$$SF_{cbs} := \frac{S_{ut}}{\sigma_{cbm}(D_o, t_{cb}, x_{cb})} = 26.897 \qquad SF_{cba} := \frac{S_{ut.al}}{\sigma_{cbm}(D_o, t_{cb}, x_{cb})} = 16.103$$

The biggest concern overall was not the bending stresses, but the buckling. Long beams in compression will almost always longitudinally bend out of shape before the material snaps. The force that could cause buckling is a function of length, area moment of inertia, a constant (K_{st} = 0.5 for fixed-fixed connects), and the modulus of elasticity.

$$F = \frac{\pi^2 \cdot E \cdot I}{(K \cdot l)^2} \qquad K_{st} := 0.5 \qquad l_{cb} := \frac{1}{n_{cbsec}} \cdot d_{plt}$$

The compressive forces acting on the beam are functions of the angular geometry of the support beams. The compressive force from the torque of the wind turbines is equal to the torque over the length of the beam. The other compressive forces come from the tether at high winds and also from the weight of the hub and rotor. Knowing all of the geometry, it is possible to come up with the outer diameter.

$$Adj_{cb}(V_o) := \frac{d_{plt} - a_o(V_o)}{2} \quad Hyp_{cb}(V_o) := \sqrt{\left(Adj_{cb}(V_o)\right)^2 + \left(r_{bld} + d_{clrnc} + \frac{a_o(V_o)}{2}\right)^2}$$

$$F_{cb}(V_o, F_{dw}, r_{cble}) := 2\cos\left(\frac{d_{plt}}{2 \cdot L_{cbl}}\right) \cdot F_{lift}(V_o, F_{Dw}, r_{cble}) \dots$$
$$+ 2\cos\left(\frac{Adj_{cb}(V_o)}{Hyp_{cb}(V_o)}\right) \cdot (W_{hub} + W_{rotor}) \cdot g$$

$$F_{cb}(V_o, F_{dw}, r_{cble}) = 2.734 \times 10^6 N$$

$$F_{cb}(V_o, F_{dw}, r_{cble}) + F_T = \frac{\pi^2 \cdot E_{st} \cdot I_{cb}(D_o, t_{cb}, x_{cb})}{(K_{st} \cdot l_{cb})^2} \qquad D_{cb} := Find(D_o) = 0.03\,m$$

3.9 Support Cables/Rods:

Using the moment equilibrium, we calculate the x-directional force required to stabilize the nacelle. This assumes a parallel axis theorem for two cables located around connection. If F_{cuy} is negative, the support requires a rod, as it is in compression. Therefore, a value of F_{cuy} is estimated and the distance from the center of gravity of the nacelles that the supports hand from is solved for. For a static and stable structure, the moments (force times distance) about any point must equal zero. The moment is taken about the point where the hanging supports connect, and the function o_{cu} represents the distance from the point the cable connects to that point.

$$\Sigma M_{cb} = 0 \qquad o_r(o_{ncl}) := o_{cog} - o_{ncl} \qquad F_{cuy}(V_o, o_{ncl}) := \frac{g \cdot (W_{hub} + W_{rotor}) \cdot o_r(o_{ncl})}{o_{cu}(V_o, o_{ncl})}$$

$$\frac{F_{cuy}(V_o, o_{ncl})}{g} = 1000kg \qquad o_{sw} := Find(o_{ncl}) = 2.391\,m$$

The distance from the centroid of the blimp is calculated using a similar method. This assumes 3 cables pull from the blimp and are designed to constantly hold a force of 10 kN from being off-centered. This is again so that the cables stay in tension.

$$\Sigma M_{col} = 0 \qquad F_{sy}(V_o, \delta_{sup}) := \frac{\delta_{sup} \cdot V_o \cdot (\rho_{air} - \rho_H) \cdot g}{3 \cdot \delta_{sup} + \frac{9}{8} \cdot b_o(V_o)} \qquad F_{sy}(V_o, \delta_{sup}) = 10^4 N$$

$$\delta_{sw} := \text{Find}(\delta_{sup}) = 0.526 m$$

The point in time where the front cables would undergo the most force would be if there was a direct upward wind that tried to lift the envelope. Even if this were so, the cables only have to be one centimeter thick.

$$F_{Dbup}(V_o) := \frac{1}{2} \cdot \rho_{air} \cdot C_d \cdot \frac{\pi}{4} \cdot v_{max}^2 \cdot a_o(V_o) \cdot b_o(V_o)$$

$$F_{sw}(V_o) := \left(F_{sy}(V_o, \delta_{sw}) + F_{Dbup}(V_o) \cdot \frac{\delta_{sw} + b_o(V_o)}{\sin(\Theta) \cdot L_{sup}(V_o)} \right) \cdot \frac{1}{2} \qquad F_{sw}(V_o) = 1.853 \times 10^4 N$$

$$\frac{S_{ut}}{SF} = \frac{F}{A} \qquad A_{sw}(V_o, S_{ut}) := \frac{F_{sw}(V_o) \cdot SF}{S_{ut}} \qquad d_{sw}(V_o, S_{ut}) := \sqrt{\frac{4 A_{sw}(V_o, S_{ut})}{\pi}}$$

$$d_{sw}(V_o, S_{ut}) = 0.965 \cdot cm$$

Recoil Cables must be same clearance from blades to ensure blade does not interfere. The calculations for this are lengthy and thus are shown in Appendix A. The overall length of the support cables was 349.452 meters.

$$L_{swtot}(V_o, S_{ut}) = 349.452\,m$$

3.10 Maximum Fatigue Load on Supports:

Because the hanging beams undergo recoil when the wind stops suddenly from a gust of wind, the supports were designed with a larger moment area of inertia. The beams are rigid, hollow rectangular beams with the long direction facing the wind. R_{sup} is the fraction of length that is hollow. R_{sl} is the ratio of width to depth.

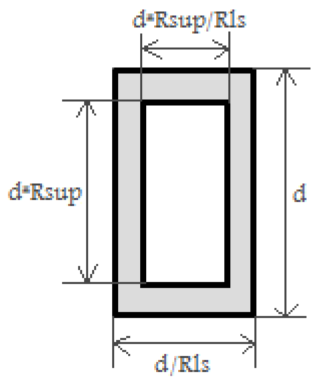


Figure 6: Cross section of hanging support

Both bending stresses and tensile stresses were taken into account. The following calculations were made to find the "d" in the above cross-sectional drawing labeled as the function d_{supi} for tensile stresses.

$$SF\cdot\sigma = \frac{F}{A} \qquad A = d\cdot\frac{d}{R_{ls}} - (d\cdot R_{sup})\cdot\left(\frac{d}{R_{sl}}\cdot R_{sup}\right)$$

$$F_{sup}(V_o) := V_o\cdot g\cdot(\rho_{air} + \rho_H) \qquad F_{sup}(V_o) = 2.197\times 10^{6}\,N$$

$$A_{sup}(V_o, S_{ut}) := \frac{F_{sup}(V_o)}{SF \cdot S_{ut}} \qquad d_{supi}(V_o, S_{ut}) := \sqrt{\frac{R_{ls} \cdot A_{sup}(V_o, S_{ut})}{\left(1 - R_{sup}^{2}\right)}} \qquad d_{supi}(V_o, S_{ut}) = 0.143 \cdot m$$

Bending stresses were calculated to find the maximum required "d" labeled as d_{sup} in the following calculations. Values for d_{sup} in both aluminum and high grade steel were determined.

$$SF \cdot \sigma = \frac{M \cdot y}{I_{xx}} \qquad M = F \cdot d \qquad I_{xx} = \frac{b \cdot h^3}{12}$$

$$L_{ff}(V_o) := L_{sup}(V_o) - \frac{H_b\left(V_o, \frac{1}{4}\right)}{\sin(\Theta)} \qquad L_{ff}(V_o) = 26.821\,m \qquad I_{sup}(d_{sup}) := \frac{d_{sup}^{4} \cdot \left(1 - R_{sup}^{4}\right)}{12 \cdot \sin(\Theta) \cdot R_{ls}}$$

$$F_{ffmax}(V_o, d_{sup}) := \frac{SF \cdot S_{syp.st} \cdot I_{sup}(d_{sup})}{L_{ff}(V_o) \cdot d_{sup}} \qquad F_{ffmax}(V_o, d_{sup}) = 4.224 \times 10^{4}\,N$$

$$F_{sw}(V_o) = F_{ffmax}(V_o, d_{sup}) \qquad d_{gd} := \max\left(d_{supi}(V_o, S_{ut}), \mathrm{Find}(d_{sup})\right) = 0.38\,m$$

$$F_{ffmaxa}(V_o, d_{sup}) := \frac{SF \cdot S_{syp.al} \cdot I_{sup}(d_{sup})}{L_{ff}(V_o) \cdot d_{sup}} \qquad F_{ffmaxa}(V_o, d_{sup}) = 2.322 \times 10^{4}\,N$$

$$F_{sw}(V_o) = F_{ffmaxa}(V_o, d_{sup}) \qquad d_{gda} := \max\left(d_{supi}(V_o, S_{ut.al}), \mathrm{Find}(d_{sup})\right) = 0.464\,m$$

Finally, the cross-sectional area was calculated where A_{gd} is for steel and A_{gda} for aluminum.

$$A = d \cdot \frac{d}{R_{ls}} - \left(d \cdot R_{sup}\right) \cdot \left(\frac{d}{R_{sl}} \cdot R_{sup}\right)$$

$$A_{gd} := \frac{d_{gd}^{\,2}}{R_{1s}} \cdot \left(1 - R_{sup}^{\,2}\right) = 67.646 \cdot cm^2 \qquad A_{gda} := \frac{d_{gda}^{\,2}}{R_{1s}} \cdot \left(1 - R_{sup}^{\,2}\right) = 100.805 \cdot cm^2$$

3.11 Calculating Mass:

Each material used carried its own mass, and this is calculated by multiplying volume and density. Two types of overall masses were determined: if only steel was used for cables and structures and if aluminum was used for structures and carbon epoxy was used for the tethers and cables. Most of the mass comes from the hub and rotors.

$$W_{cb} := n_{cbsup} \cdot \frac{\pi}{4} \cdot \left[D_{cb}^{\,2} - \left(D_{cb} - 2 \cdot t_{cb}\right)^2\right] \cdot d_{plt} \cdot \rho_{steel} \cdots + d_{cbar}^{\,2} \cdot l_{cbar} \cdot n_{cbsup} \cdot n_{cbsec} \cdot \rho_{steel} + 2 \cdot \frac{\pi}{4} \cdot D_{cbp}^{\,2} \cdot t_{cbp} \cdot \rho_{steel} = 1.217 \cdot tonne$$

$$W_{cables}(V_o) := L_{cbl} \cdot \left(r_{cbl}(V_o, F_{dw})\right)^2 \cdot \pi \cdot \rho_{steel} \quad W_{cables}(V_o) = 19.142 \cdot tonne$$

$$W_{cables2}(V_o) := L_{cbl} \cdot \left(r_{cbl2}(V_o, F_{dw})\right)^2 \cdot \pi \cdot \rho_{cac} \quad W_{cables2}(V_o) = 2.681 \cdot tonne$$

$$W_{pl} := 2 \cdot L_{cbl} \cdot \frac{\pi}{4} \cdot \left[d_{pl}^{\,2} \cdot \rho_{Cu} + \left[\left(d_{pl} + 2 \cdot t_r\right)^2 - d_{pl}^{\,2}\right] \cdot 2 \cdot \rho_{rub}\right] = 0.294 \cdot tonne$$

$$W_w(V_o) := A_w \cdot L_w \cdot \rho_{al} \quad W_w(V_o) = 4.473 \cdot tonne$$

$$W_{sup}(V_o) := 2A_{gd} \cdot L_{sup}(V_o) \cdot \rho_{steel} \quad W_{sup}(V_o) = 5.276 \cdot tonne$$

$$W_{supa}(V_o) := 2A_{gda} \cdot L_{sup}(V_o) \cdot \rho_{al} \quad W_{sup}(V_o) = 5.276 \cdot tonne$$

$$W_{sw}(V_o, S_{ut}, \rho_s) := 2L_{swtot}(V_o, S_{ut}) \cdot d_{sw}(V_o, S_{ut})^2 \cdot \frac{1}{4} \cdot \pi \cdot \rho_s \quad W_{sw}(V_o, S_{ut}, \rho_{steel}) = 0.402 \cdot \text{tonne}$$

$$W_{bmp}(V_o) := S_{fab}(V_o) \cdot \rho_{fbrc} \quad W_{bmp}(V_o) = 6.202 \cdot \text{tonne}$$

$$W_{all}(V_o) := W_{bmp}(V_o) + W_{cables}(V_o) \cdots + (W_{hub} + W_{rotor}) \cdot n_{blades} \cdots + W_{sup}(V_o) + W_w(V_o) + W_{p1} + W_{cb} + W_{sw}(V_o, S_{ut}, \rho_{steel}) + M_{lift}$$

$$W_{all}(V_o) = 102.006 \cdot \text{tonne}$$

$$W_{all2}(V_o) := W_{bmp}(V_o) + W_{cables2}(V_o) \cdots + (W_{hub} + W_{rotor}) \cdot n_{blades} \cdots + W_{supa}(V_o) + W_w(V_o) + W_{p1} + W_{cb} + W_{sw}(V_o, S_{ut.cac}, \rho_{cac}) + M_{lift}$$

$$W_{all2}(V_o) = 82.624 \cdot \text{tonne}$$

3.12 Solving for Volume:

Since someone may take the design forward and decide to use a helium-filled envelope, volume of helium needed is calculated. The below calculation is if using high strength steel structures and cables.

$$W_{all}(V_o) = V_o \cdot (\rho_{air} - \rho_{He}) \qquad V_{He} = 9.611 \times 10^4 \cdot m^3$$

$$a_o(V_{He}) = 17.901\,m \qquad b_o(V_{He}) = 71.604\,m$$

The below calculation is if using aluminum structures and carbon epoxy cables with a helium-filled envelope.

$$W_{all2}(V_o) = V_o \cdot (\rho_{air} - \rho_{He}) \qquad V_{He2} = 7.707 \times 10^4 \cdot m^3$$

$$a_o(V_{He2}) = 16.631\,m \qquad b_o(V_{He2}) = 66.522\,m$$

The below calculation is if using high strength steel structures and cables with a hydrogen-filled envelope.

$$W_{all}(V_o) = V_o \cdot (\rho_{air} - \rho_H) \qquad V_H = 8.797 \times 10^4 \cdot m^3$$

$$a_o(V_H) = 17.381\,m \qquad b_o(V_H) = 69.522\,m$$

The below calculation is if using aluminum structures and carbon epoxy cables with a hydrogen-filled envelope.

$$W_{all2}(V_o) = V_o \cdot (\rho_{air} - \rho_H) \qquad V_{H2} = 7.054 \times 10^4 \cdot m^3$$

$$a_o(V_{H2}) = 16.147\,m \qquad b_o(V_{H2}) = 64.589\,m$$

The largest blimp ever made so far was the Hindenburg. If all the calculations proved that this structure could be made smaller than the biggest blimp ever, this method of renewable energy generation will look more probable. Indeed, this structure uses an envelope much smaller than the Hindenburg.

$$TimesSmaller2 := \frac{2 \cdot 10^5 m^3}{V_{H2}} = 2.835$$

4.0 Conclusions/Future Work:

After researching this design, conclusions were made and ideas for future work came about. From the calculations, the major finding is that at the present time this project is feasible but will be challenging to complete. One reason for this conclusion is the size and weight of the turbines. The weight of the turbines and the drag forces cause the blimp to be massive in size. This project can be undertaken but in the future there are specific challenges that should be solved in order to have an upgraded prototype.

During our calculations for determining the forces on the structure we came up with some ideas for future development. Our first idea is to change the shape of the blimp. In the present design the blimp is the normal elliptical shape and we believe that if the blimp could be shaped as an airfoil, it could create its own lift. With our design, there needs to be a big wind added to create the extra lift needed. The placement of the wing is between the two turbines. We believe that if you could change the placement of the wing, attached to the sides of the blimp for example, it would create a pulling force up which might be more effective. We chose to use steel cables connecting the turbines to the blimp for our design. These cables are strong but they also weigh a lot. To lighten the load, it would be a better idea to use a different material for the cables. During our research we found a carbon-epoxy material that was both stronger and lighter than steel. The original design was conceived to be tethered at a 30 degree angle to the ground. After numerous calculations, it was determined that the optimal tethering angle should be approximately 60 degrees to minimize the force of the cable on the blimp.

Another aspect of the design that could be further looked into would be testing a model of this prototype to see how it might fair in extreme wind conditions. Calculating the forces on components of the structure will not give the full picture of how prototype might respond to stormy conditions. An example of this would be testing the torques caused by the spinning blades of the turbine. The blades will create a bending stress on the entire cross beam between the nacelles. These values can be obtained through experimentation to see if the beam fails and causes the whole structure to break. Also, there was a certain angle of the tethering that we figured out would be needed to keep the turbines aligned directly into the wind and a finished design would not be plausible without evaluating models. Testing a model to failure would also help to see where the weakest links of the structure are. If you can strengthen those weak links, then you can come up with a prototype that will have a much longer life span. The ability to see how the structure is affected by the different forces associated with swirling winds is of great importance in confirming what the calculations show.

Taking a turbine designed to sit atop a tower of no more than 100m and placing it hanging from a blimp upwards of 300m in the air could greatly affect the efficiency of the turbine itself. Designing a turbine specifically for high altitude use is crucial in obtaining the most efficient model for extracting power from high altitude winds. This could mean only changing some of the components to get a lighter overall weight but could also involve changing features in the turbine that constantly adjust the pitch of the blades. No companies are designing something like this now but if one were to look into high altitude structures a wind turbine specifically designed for the elevation difference is needed.

One major issue for keeping this structure in the air would involve replacing the hydrogen that leaks out of the balloon. Both hydrogen and helium leak out of the thin materials because they are very small molecules and find their way out of nearly any barrier. By determining the rate at which the gas seeps out, you could then come up with a method to replenish the blimp with new helium/hydrogen. A system that reels in the structure when a level sensor indicates low volume or pressure could be an answer to this problem. Another method could involve tubes in the tethering system that can feed the envelope when needed.

From the immense drag on the turbine blades, the structure will be blown so far backwards that it will be rendered useless. Thus, more lift must be generated in high winds. The solution is either more hydrogen or the addition of a wing to provide lift. From the calculations, the platform area (underside of the wing area) will be quite immense. However, the boundary layer around a wing's cross-section is generally quite thin in comparison to the dimensions of the cross-section. Therefore the wings can be layered between the supports much like a bi or tri plane. This will provide significant lift when the wind is strong, conserve space, and keep the bending stresses to a minimum.

Material selection is an additional aspect that should be looked into more. Whenever you have something this large in the air, it is important to reduce the weight wherever possible while also increasing its strength. Steel has very important fatigue properties. It has a fatigue limit in which it will theoretically never fail under certain repetitive load size. The high the grade of steel used, the less likely it is to fail and a lesser amount of material can be used to support the structure.

Works Cited

1) High Altitude Wind Power. Bryan Roberts. December 2008. Sky WindPower Corporation. November 19 2008 <http://www.skywindpower.com/ww/index.htm>.

2) Twidell, J., 2003, "Technology Fundamentals Wind Turbines," Renewable Energy World, pp. 102-111

3) Walker James February 5 2009, American Wind Energy Association. January 15 2009 <http://www.awea.org/>

4) Wackenmagel and Rees. 1996. "Our Ecological Footprint." New Society Publishers.1998

5) Dodge, D., 2006, "Illustrated History of Wind Power Development," Telos Net Web Development, Littleton, Colorado

6) Vestas. Feb 2009. Vestas Wind Systems December 2008 <http://www.vestas.com/en>

7) GE Energy, 2009 "1.5 MW Wind Turbine Technical Specifications." Fairfield, CT

8) Gamesa. January 2009. Gamesa Technology Corporation. November 10 2001 <http://gamescorp.com/en/gamesa>

9) Richard, G.R., Feb 2008, "Enercon E-126: The World's Largest Wind Turbine." Treehugger Discovery Communications. New York, NY.

10) RE Power. Feb 2009 RE Power Systems. November 2008. Hamburg, Germany. <http://www.repower.de/index>

11) Suzlon. Feb 2009. Suzlon Energy. November 15, 2008 <http://www.suzlon.com/Home.aspx>

12) Siemens. Feb 2009. Siemens Energy. November 2008 <http://www.powergeneration.siemens.com/home/>

13) World Wind Energy Report 2008. April 2009. World Wind Energy Association. <http://www.wwindea.org/home/images/stories/worldwindenergyreport2008_s.pdf>.

14) Wind Energy. 2007. European Renewable Energy Council. February 2009 <http://www.erec.org/renewableenergysources/wind-energy.html>.

15) Free Patents Online. January 2009. November 19 2008 <http://www.freepatentsonline.com/>.

16) "Zeppelin NT,"Wikipedia.com, April 5, 2009.

17) Goodyear Blimp. 2006 The Goodyear Tire and Rubber Company. January 2009. Akron, Ohio. <http://www.goodyearblimp.com>

18) Magenn Power Air Rotor System. Fred Ferguson. October 2008. Magenn Power. November 21 2008 <http://magenn.com/index.php>.

19) World Energy Outlook. February 2009. International Energy Agency. December 10 2008 <http://www.worldenergyoutlook.org/index.asp>.

20) LadderMill. Wubbo Ockels. September 2 2003. Delft University of Technology. December 2 2008 <http://www.ockels.nl/>.

21) Windfair. Martin Tschierschke. January 2009. December 12 2008 <http://windfair.net/welcome.html>.

22) Rosenbloom, E., 2009 "Size Specifications of Common Industrial Wind Turbines." Alternative Wind Energy Outlook. Vermont

23) Brain, M., 2009, "How Helium Balloons Work." How Stuff Works. Atlanta, GA.

24) Sussman, P., 2007 "Reach for the Sky: Could flying wind farms help beat global warming?" CNW. London, England.

25) Vahrenholt, F. 2005. "Germany Inaugurates 5 MW Wind Prototype." Renewable Energy Access. Hamburg, Germany.

26) Enercon. Feb 2009. Enercon Company. November 10 2008 <http://www.enercon.de/en_home.htm>

27) Wind Energy. 2009 Intelligent Energy. January 2009 <http://www.wind-energy-the-facts.org/>

28) Glenn Elert. "Aerodynamic Drag". April 2009. <http://hypertextbook.com/physics/matter/drag/>

29) "Drag Fluids". April 2009. <http://www.roymech.co.uk/Related/Fluids/Fluids_Drag.html>

Appendix A:

Calculations for Aerial Wind Turbine

Guess:

$V_o := 1.698 \times 10^5\ m^3$ $t := 2mm$ $t_w := 5mm$ $M_{lift} := 1tonne$ $SF := 3$

$n_{blades} := 2$ $d_{clrnc} := 10m$ $r_{cble} := 6.911\ cm$ $H_{turb} := 300m$ $\theta := 40deg$

$P_{gen} := 850kW$ $E := 300kV$ $t_r := 2mm$ $T_{hot} := 70\,°C$ $F_{Dw} := 1.707 \times 10^4\ N$

$r_{cble2} := 5cm$ $t_{cb} := 20mm$ $D_o := .15m$ $d_{cbar} := .05m$ $l_{cbar} := 0.5m$

$D_{cbp} := 1m$ $t_{cbp} := .02m$ $d_{bolt} := 20mm$ $l_{ncl} := 10.95m$ $d_{ncl} := 3.8m$

$o_{ncl} := 0.5m$ $o_{bld} := 1.05m$ $d_{bolt2} := 2.125in = 53.975 \cdot mm$ $d_{cfc} := 25\%$ $d_{end} := 1m$

$\delta_{sup} := 0m$ $w_{ncl} := 3m$ $v_{gusts} := 80\frac{m}{s}$ $d_f := \frac{1}{2}$ $\rho_s := 1.6\frac{gm}{cm^3}$

$R_{sup} := \frac{7}{8}$ $R_{ls} := 5$ $C_{L1} := 0.9$ $\alpha := 15deg$ $R_w := 3.5$

$d_{sup} := .5m$ $\omega_{max} := 31.4rpm$ $x_{cb} := 0.5m$ $C_d := 0.025$ $C_{dc} := 2.1$

$A_{blades} := 2124m^2$ $v_{max} := 25\frac{m}{s}$ $W_{hub} := 22tonne$ $W_{rotor} := 10tonne$

GE 1.5sl... W_{hub}: 50tonne W_{rotor}: 31tonne A_{blades}: 4657m^2 v_{max}: 20m/s
Bonus (Seimens) 1.3MW... W_{hub}: 50tonne W_{rotor}: 31tonne A_{blades}: 3019m^2 v_{max}: 25m/s
Vestas 850kW... W_{hub}: 22tonne W_{rotor}: 10tonne A_{blades}: 2124m^2 v_{max}: 25m/s

Given:

$\rho_{steel} := 7.86\frac{gm}{cm^3}$ $\rho_{air} := 1.225\frac{kg}{m^3}$ $L_{hub} := 13m$ $S_{ut} := 760MPa$ $S_{ut.al} := 455MPa$

$\rho_{He1} := 0.1786\frac{kg}{m^3}$ $\rho_{H1} := 0.0899\frac{kg}{m^3}$ $\rho_{fbrc} := 0.327\frac{kg}{m^2}$ $E_{st} := 190GPa$

$\rho_{Cu} := 8.96\frac{gm}{cm^3}$ $\rho_{rub} := 1.25\frac{gm}{cm^3}$ $S_{ut.cac} := 1100MPa$ $\rho_{cac} := 1.6\frac{gm}{cm^3}$

$\rho_{cua} := 1.96 \cdot 10^{-8}\Omega \cdot m$ $k_{cu} := 380\frac{W}{m \cdot K}$ $T_{max} := 98\,°C$ $\rho_{al} := 2.7\frac{gm}{cm^3}$ $E_{al} := 70GPa$

$cog := \frac{290 + 23}{526} = 0.595$ $p_{atm} := 14.7psi$ $p_{blimp} := 0.7psi$ $S_{us.bolt} := 187MPa$

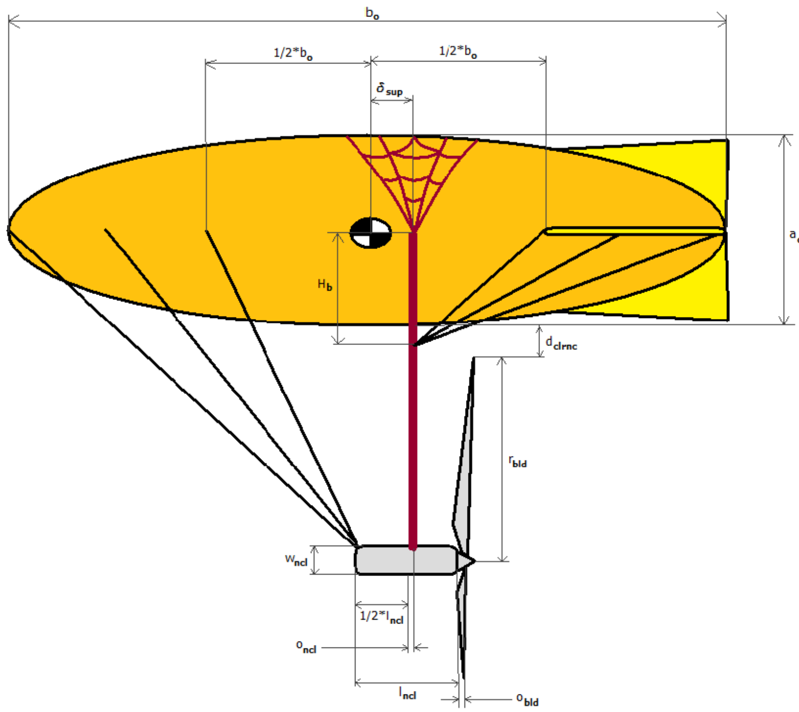
$S_{us.st} := 760 \cdot 0.75MPa = 570 \cdot MPa$ $S_{syp.st} := 690 \cdot 0.58MPa = 400.2 \cdot MPa$

$S_{us.al} := 455 \cdot 0.65MPa = 295.75 \cdot MPa$ $S_{syp.al} := 400 \cdot 0.55MPa = 220 \cdot MPa$

Material Stress Properties From...
http://www.roymech.co.uk/Useful_Tables/Matter/shear_tensile.htm
http://en.wikipedia.org/wiki/Yield_(engineering)

S_{ut} is using Steel, high strength alloy ASTM
$S_{ut.al}$ is using Aluminum 2014-T6

COG offset from...
http://www.roxburyny.com/windproject/pdf/DEIS/005%20Figure%201.4%20-%20Vestas%20V-90 %20Nacelle%20Overview.PDF



Accounting For Pressure (using Goodyear Blimp):

$$\rho_H := \rho_{H1} \cdot \frac{(\rho_{atm} + \rho_{blimp})}{\rho_{atm}} = 0.094 \frac{kg}{m^3} \qquad \rho_{He} := \rho_{He1} \cdot \frac{(\rho_{atm} + \rho_{blimp})}{\rho_{atm}} = 0.187 \frac{kg}{m^3}$$

Setting Proportions:

$$a := 50cm \qquad b := 2m \qquad R_{ab} := \frac{a}{b} = 0.25$$

New Dims:

$$a_o(V_o) := \sqrt[3]{\frac{V_o \cdot 3 \cdot R_{ab}}{4 \cdot \pi}} \qquad a_o(V_o) = 21.64\,m \qquad b_o(V_o) := \frac{a_o(V_o)}{R_{ab}} \qquad b_o(V_o) = 86.561\,m$$

Surface Area:

$$S = 2\cdot\pi\left(a^2 + \frac{a\cdot b\cdot\alpha\varepsilon}{\sin(\alpha\varepsilon)}\right) \qquad \text{where...} \qquad \alpha\varepsilon = \operatorname{acos}\left(\frac{a}{b}\right)$$

$$\alpha\varepsilon(V_o) := \operatorname{acos}\left(\frac{a_o(V_o)}{b_o(V_o)}\right) \qquad S_{fab}(V_o) := 2\cdot\pi\cdot\left(a_o(V_o)^2 + \frac{a_o(V_o)\cdot b_o(V_o)\cdot\alpha\varepsilon(V_o)}{\sin(\alpha\varepsilon(V_o))}\right)$$

$$S_{fab}(V_o) = 1.897\times 10^4\,m^2$$

Supports Lengths:

$$A = \pi r^2 \qquad r_{bld} := \sqrt{\frac{A_{blades}}{\pi}} = 26.002\,m$$

$$d_{plt} := n_{blades} r_{bld} + d_{clrnc} = 62.003\,m \qquad h_{sup}(V_o) := \frac{1}{2}\cdot a_o(V_o) + r_{bld} + d_{clrnc} - \frac{1}{2}\cdot w_{ncl}$$

$$w_{sup}(V_o) := \frac{d_{plt} - a_o(V_o)}{2} \qquad L_{sup}(V_o) := \sqrt{(h_{sup}(V_o))^2 + (w_{sup}(V_o))^2}$$

$$L_{sup}(V_o) = 49.612\,m \qquad \Theta := \operatorname{atan}\left(\frac{h_{sup}(V_o)}{w_{sup}(V_o)}\right) = 65.997\cdot deg$$

Power Lines:

$$L_{cbl} := \frac{H_{turb}}{\sin(\theta)} = 466.717\,m$$

$$P_{loss} = I^2\cdot R = \frac{\Delta T\cdot k\cdot A_{surf}}{L} \qquad A_{surf} = \pi\cdot d\cdot L \qquad R = \frac{\rho\cdot L}{A_{csec}} \qquad A_{csec} = \frac{\pi}{4}\cdot d^2$$

$$L_{pl} := 2\cdot L_{cbl} \qquad I := \frac{P_{gen}}{E} = 2.833\,A \qquad d_{pl} := SF\sqrt[3]{\frac{4}{\pi^2}\cdot\frac{I^2\cdot\rho_{cua}\cdot L_{pl}}{k_{cu}\cdot(T_{max} - T_{hot})}} = 5.326\,mm$$

$$P_{loss} := (T_{max} - T_{hot})\cdot k_{cu}\cdot\pi\cdot d_{pl} = 178.018\,W$$

Calculating Drag Force:

$$\eta_{max} := \frac{16}{27} = 0.593 \qquad \eta := 80\%\cdot\eta_{max} = 0.474 \qquad F_{Dw} = 17.07\cdot kN$$

$$F_{Dr} := \frac{1}{2}\rho_{air}\cdot A_{blades}\cdot v_{max}^2\cdot\eta = 385.467\cdot kN$$

$$F_{Db}(V_o) := \frac{1}{2}\cdot\rho_{air}\cdot C_d\cdot\frac{\pi}{4}\cdot v_{max}^2\cdot(a_o(V_o))^2 \qquad F_{Db}(V_o) = 3.52\cdot kN$$

$$F_{Dc}(r_{cble}) := \frac{1}{2}\cdot\rho_{air}\cdot C_{dc}\cdot v_{max}^{2}\cdot(2r_{cble} + d_{pl} + 2\cdot t_{r})\cdot H_{turb} \qquad F_{Dc}(r_{cble}) = 35.584\cdot kN$$

$$F_{dx}(V_o, F_{Dw}, r_{cble}) := 2\cdot F_{Dr} + F_{Db}(V_o) + F_{Dc}(r_{cble}) + F_{Dw}$$

$$F_{dx}(V_o, F_{Dw}, r_{cble}) = 827.107\cdot kN$$

Area of Tether:

$$SF = \frac{S_{ut}}{\left(\frac{F}{\pi\cdot r_{cbl}^{2}}\right)} \qquad F_{th} = r_{th}\cdot F_{dx} \qquad F_{lift} = \frac{F_{dx}}{\cos(\theta)} \qquad r_{th} = \sqrt{1 + \frac{1}{\cos(\theta)^{2}}}$$

$$F_{lift}(V_o, F_{Dw}, r_{cble}) := \frac{F_{dx}(V_o, F_{Dw}, r_{cble})}{\cos(\theta)}$$

$$F_{th}(V_o, F_{Dw}, r_{cble}) := \sqrt{1 + \frac{1}{\cos(\theta)^{2}}}\cdot F_{dx}(V_o, F_{Dw}, r_{cble})$$

Given $\quad r_{cble} = \sqrt{\frac{F_{th}(V_o, F_{Dw}, r_{cble})\cdot SF}{\pi\cdot S_{ut}}} \qquad r_{cbl}(V_o, F_{Dw}) := Find(r_{cble})$

$r_{cbl}(V_o, F_{Dw}) = 4.1\cdot cm \qquad r_{cbl}(V_o, F_{Dw})^{2}\cdot\pi = 52.808\cdot cm^{2}$

Given $\quad r_{cble2} = \sqrt{\frac{F_{th}(V_o, F_{Dw}, r_{cble2})\cdot SF}{\pi\cdot S_{ut.cac}}} \qquad r_{cbl2}(V_o, F_{Dw}) := Find(r_{cble2})$

$r_{cbl2}(V_o, F_{Dw}) = 3.401\cdot cm \qquad r_{cbl2}(V_o, F_{Dw})^{2}\cdot\pi = 36.334\cdot cm^{2}$

Wing:

Assumes wing to be an eliptical shape with 4% height of width and thickness t_w.
Maximum effective angle of attack tends to be around 15 degrees from most graphs.

$$F_L + F_D = \frac{1}{2}\cdot\rho\cdot v^{2}\cdot A_{plnfrm}\cdot C_L \qquad A_{plnfrm} = L_w\cdot D_w \qquad C_L = \alpha\cdot 2\cdot\pi\cdot C_{L1} \qquad C_{L1} = \frac{AR}{AR + 2}$$

$$AR = \frac{span^{2}}{A_{plnfrm}} = \frac{length}{depth} \qquad C_{L0} := 2\cdot\pi\cdot\alpha \qquad L_w := d_{plt} \qquad D_w := 40m$$

Given $\quad \tan(\theta)\cdot F_{dx}(V_o, F_{Dw}, r_{cble}) - g\cdot M_{lift} = \frac{1}{2}\cdot\rho_{air}\cdot v_{max}^{2}\cdot L_w\cdot D_w\cdot\cos(\alpha)\cdot\left(\frac{L_w}{L_w + 2\cdot D_w}\right)\cdot(2\cdot\pi\cdot\alpha)$

$D_W(V_o, F_{Dw}) := Find(D_w) \qquad D_W(V_o, F_{Dw}) = 43.74m$

Given $F_{Dw} = \frac{1}{2} \cdot \rho_{air} \cdot C_d \cdot v_{max}^2 \cdot D_W(V_o, F_{Dw}) \cdot L_w \cdot (0.04 + \sin(\alpha))$

$F_{dw} := \text{Find}(F_{Dw}) = 7.541 \times 10^3 N$ $\quad F_{dx}(V_o, F_{dw}, r_{cble}) = 817.578 \cdot kN$

$D_W(V_o, F_{dw}) = 42.527 m$ $\quad C_L := \left(\frac{L_w}{L_w + 2 D_W(V_o, F_{Dw})} \right) \cdot C_{L0} = 0.682$

$A_{pf}(V_o) := D_W(V_o, F_{dw}) \cdot L_w \cdot \cos(\alpha)$ $\quad A_{pf}(V_o) = 2.547 \times 10^3 m^2$

$A_w := 0.04 \cdot \frac{\pi}{2} \cdot \left[(D_W(V_o, F_{dw}))^2 - (D_W(V_o, F_{dw}) - t_w)^2 \right] = 0.027 m^2$

$F_{lft} := \frac{1}{2} \cdot \rho_{air} \cdot v_{max}^2 \cdot A_{pf}(V_o) \cdot C_L = 6.653 \times 10^5 N$ $\quad W_{lift} := \frac{F_{lft}}{g} = 67.837 \cdot tonne$

Times longer than nacelle...

$D_W(V_o, F_{dw}) \cdot l_{ncl}^{-1} = 3.884$

Horizontal Beam:

Calculating Torsion Stresses...

$\sigma = \frac{M \cdot y}{I_{xx}}$ $\quad P = \tau \cdot \omega$ $\quad I_{xx} = \frac{n \cdot \pi \cdot \left[D_o^4 - (D_o - 2 \cdot t_{cb})^4 \right]}{64}$ $\quad I_{cb} = I_{xx} + n \cdot A \cdot x^2$

$n_{cbsec} := 2$ $\quad n_{cbsup} := 3$ $\quad \tau := \frac{P_{gen}}{\omega_{max}} = 2.585 \times 10^5 J$ $\quad F_\tau := \frac{\tau}{d_{plt}} = 4.169 \times 10^3 \cdot N$

$\phi_{nb} := \frac{2 \cdot \pi}{n_{cbsup}} = 120 \cdot deg$ $\quad \theta_{nb} := \left| \phi_{nb} - 180deg \right| = 60 \cdot deg$

$x_{cbt}(x_{cb}) := x_{cb} \cdot \sin(\theta_{nb})$ $\quad I_{xxcb}(D_o, t_{cb}) := \frac{n_{cbsup} \cdot \pi \cdot \left[D_o^4 - (D_o - 2 \cdot t_{cb})^4 \right]}{64}$

$A_{cb1}(D_o, t_{cb}) := \frac{\pi}{4} \cdot \left[D_o^2 - (D_o - 2 \cdot t_{cb})^2 \right]$

$I_{cb}(D_o, t_{cb}, x_{cb}) := I_{xxcb}(D_o, t_{cb}) + n_{cbsup} \cdot A_{cb1}(D_o, t_{cb}) \cdot x_{cbt}(x_{cb})^2$

$\sigma_{cbm}(D_o, t_{cb}, x_{cb}) := \frac{\tau \cdot \left(x_{cbt}(x_{cb}) + \frac{D_o}{2} \right)}{I_{cb}(D_o, t_{cb}, x_{cb})}$ $\quad \sigma_{cbm}(D_o, t_{cb}, x_{cb}) = 28.256 \cdot MPa$

$SF_{cbs} := \frac{S_{ut}}{\sigma_{cbm}(D_o, t_{cb}, x_{cb})} = 26.897$ $\quad SF_{cba} := \frac{S_{ut.al}}{\sigma_{cbm}(D_o, t_{cb}, x_{cb})} = 16.103$

Checking Buckling Stresses...

$F = \frac{\pi^2 \cdot E \cdot I}{(K \cdot l)^2}$ $\quad K_{st} := 0.5$ $\quad l_{cb} := \frac{1}{n_{cbsec}} \cdot d_{plt}$ $\quad r_{cbl1} := r_{cbl}(V_o, F_{dw})$ $\quad t_{cb} = 0.02 m$

$$Adj_{cb}(V_o) := \frac{d_{plt} - a_o(V_o)}{2} \qquad Hyp_{cb}(V_o) := \sqrt{\left(Adj_{cb}(V_o)\right)^2 + \left(r_{bld} + d_{clrnc} + \frac{a_o(V_o)}{2}\right)^2}$$

$$F_{cb}(V_o, F_{dw}, r_{cble}) := 2\cos\left(\frac{d_{plt}}{2 \cdot L_{cbl}}\right) \cdot F_{lift}(V_o, F_{Dw}, r_{cble}) \cdots + 2\cos\left(\frac{Adj_{cb}(V_o)}{Hyp_{cb}(V_o)}\right) \cdot (W_{hub} + W_{rotor}) \cdot g$$

$$F_{cb}(V_o, F_{dw}, r_{cble}) = 2.734 \times 10^6 \, N$$

Given

$$F_{cb}(V_o, F_{dw}, r_{cble}) + F_T = \frac{\pi^2 \cdot E_{st} \cdot I_{cb}(D_o, t_{cb}, x_{cb})}{(K_{st} \cdot l_{cb})^2}$$

$$D_{cb} := Find(D_o) = 0.03 \, m$$

Finding Number of Bolts...

$$F_w := (W_{hub} + W_{rotor}) \cdot g = 3.138 \times 10^5 \, N \qquad F_{dx}(V_o, F_{dw}, r_{cble}) \qquad d_{bolt2} = 0.054 \, m$$

$$F_{shr}(V_o, F_{dw}, r_{cble}) := \sqrt{F_w^{\,2} + F_{dx}(V_o, F_{dw}, r_{cble})^2}$$

$$n_{bcb} := round\left(\frac{F_{shr}(V_o, F_{dw}, r_{cble}) \cdot SF}{S_{us.bolt} \cdot \frac{\pi}{4} \cdot d_{bolt2}^{\,2}}, 0\right) + 1 = 7$$

Support Cables/Rods:

Using a moment equilibrium, we calculate the x-directional force required to stabilize the nacelle. This assumes a parallel axis theorm for two cables located around connection. If F_{cuy} is negative, the support requires a rod, as it is in compression.

$$w_{sw}(V_o, o_{ncl}, d_f) := d_f \cdot b_o(V_o) + \delta_{sup} - o_{ncl} \qquad \phi(V_o, o_{ncl}) := atan\left(\frac{w_{sw}(V_o, o_{ncl}, d_f)}{h_{sup}(V_o)}\right)$$

$$\Sigma M_{cb} = 0 \qquad M_{cuy} = F_{cuy} \cdot d \qquad o_{cog} := cog \cdot (l_{ncl} + o_{bld} + d_{end}) - \frac{l_{ncl}}{2} = 2.261 \, m$$

$$\psi(o_{ncl}) := atan\left(\frac{\frac{1}{2} \cdot w_{ncl}}{\frac{1}{2} \cdot l_{ncl} + o_{ncl}}\right) \qquad \varepsilon_a(V_o, o_{ncl}) := \psi(o_{ncl}) - \phi(V_o, o_{ncl})$$

$$o_{cu}(V_o, o_{ncl}) := \sin(\varepsilon_a(V_o, o_{ncl})) \cdot \sqrt{\left(\frac{1}{2} \cdot w_{ncl}\right)^2 + \left(\frac{1}{2} \cdot l_{ncl} + o_{ncl}\right)^2} \qquad o_r(o_{ncl}) := o_{cog} - o_{ncl}$$

$$F_{cuy}(V_o, o_{ncl}) := \frac{g \cdot (W_{hub} + W_{rotor}) \cdot o_r(o_{ncl})}{o_{cu}(V_o, o_{ncl})}$$

Given $\frac{F_{cuy}(V_o, o_{ncl})}{g} = 1000kg$ $\quad o_{sw} := Find(o_{ncl}) = 2.391\,m$

$\Sigma M_{col} = 0$ $\quad F_{sy}(V_o, \delta_{sup}) := \frac{\delta_{sup} \cdot V_o \cdot (\rho_{air} - \rho_H) \cdot g}{3 \cdot \delta_{sup} + \frac{9}{8} \cdot b_o(V_o)}$ $\quad \phi_{sw}(V_o) := \phi(V_o, o_{sw})$

Given $F_{sy}(V_o, \delta_{sup}) = 10^4 N$ $\quad \delta_{sw} := Find(\delta_{sup}) = 0.526m$

Using drag at 90deg wind angle...

$$F_{Dbup}(V_o) := \frac{1}{2} \cdot \rho_{air} \cdot C_d \cdot \frac{\pi}{4} \cdot v_{max}^2 \cdot a_o(V_o) \cdot b_o(V_o)$$

$$F_{sw}(V_o) := \left(F_{sy}(V_o, \delta_{sw}) + F_{Dbup}(V_o) \cdot \frac{\delta_{sw} + b_o(V_o)}{\sin(\Theta) \cdot L_{sup}(V_o)} \right) \cdot \frac{1}{2} \qquad F_{sw}(V_o) = 1.853 \times 10^4 N$$

$\frac{S_{ut}}{SF} = \frac{F}{A}$ $\quad A_{sw}(V_o, S_{ut}) := \frac{F_{sw}(V_o) \cdot SF}{S_{ut}}$ $\quad d_{sw}(V_o, S_{ut}) := \sqrt{\frac{4 A_{sw}(V_o, S_{ut})}{\pi}}$

$d_{sw}(V_o, S_{ut}) = 0.965 \cdot cm$

Recoil Cables must be same clearance from blades to ensure blade does not interfere.

$\delta_{rcx}(V_o, d_f) := d_f \cdot b_o(V_o) - \delta_{sup} - \left(\frac{1}{2} \cdot l_{ncl} - o_{ncl} + o_{bld} \right)$ $\quad \delta_{rcy}(V_o) := \frac{1}{2} \cdot a_o(V_o)$

$$\psi_1(V_o, d_f) := atan\left(\frac{\delta_{rcy}(V_o)}{\delta_{rcx}(V_o, d_f)} \right)$$

$$H_b(V_o, d_f) := min\left[\left(\frac{1}{2} \cdot b_o(V_o) - \delta_{sup} \right) \cdot \tan(\psi_1(V_o, d_f)), \frac{1}{2} \cdot a_o(V_o) + d_{clrnc} \right]$$

$H_b(V_o, d_f) = 12.57\,m$

Z Distance...

$\frac{z^2}{\left(\frac{1}{2} \cdot a_o(V_o) \right)^2} + \frac{x^2}{\left(\frac{1}{2} \cdot b_o(V_o) \right)^2} = 1$ $\quad z_{sw}(V_o, d_f) := \frac{1}{2} \cdot \left(d_{plt} - a_o(V_o) \cdot \sqrt{1 - 2 \cdot d_f} \right)$

Length of Cables...

$$L_{sw1}(V_o, S_{ut}) := \sqrt{w_{sw}\left(V_o, o_{sw}, \frac{1}{2} \right)^2 + h_{sup}(V_o)^2 + z_{sw}\left(V_o, \frac{1}{2} \right)^2} \qquad L_{sw1}(V_o, S_{ut}) = 68.463\,m$$

$$L_{sw2}(V_o, S_{ut}) := \sqrt{w_{sw}\left(V_o, o_{sw}, \frac{3}{8}\right)^2 + h_{sup}(V_o)^2 + z_{sw}\left(V_o, \frac{3}{8}\right)^2} \qquad L_{sw2}(V_o, S_{ut}) = 60.11\,m$$

$$L_{sw3}(V_o, S_{ut}) := \sqrt{w_{sw}\left(V_o, o_{sw}, \frac{1}{4}\right)^2 + h_{sup}(V_o)^2 + z_{sw}\left(V_o, \frac{1}{4}\right)^2} \qquad L_{sw3}(V_o, S_{ut}) = 54.497\,m$$

$$L_{sb1}(V_o, S_{ut}) := \sqrt{H_b\left(V_o, \frac{1}{2}\right)^2 + h_{sup}(V_o)^2 + z_{sw}\left(V_o, \frac{1}{2}\right)^2} \qquad L_{sw1}(V_o, S_{ut}) = 68.463\,m$$

$$L_{sb2}(V_o, S_{ut}) := \sqrt{H_b\left(V_o, \frac{3}{8}\right)^2 + h_{sup}(V_o)^2 + z_{sw}\left(V_o, \frac{3}{8}\right)^2} \qquad L_{sw2}(V_o, S_{ut}) = 60.11\,m$$

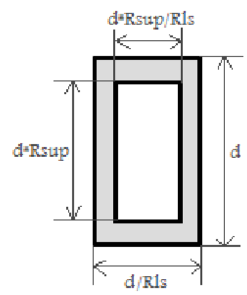
$$L_{sb3}(V_o, S_{ut}) := \sqrt{H_b\left(V_o, \frac{1}{4}\right)^2 + h_{sup}(V_o)^2 + z_{sw}\left(V_o, \frac{1}{4}\right)^2} \qquad L_{sw3}(V_o, S_{ut}) = 54.497\,m$$

$$L_{swtot}(V_o, S_{ut}) := L_{sw1}(V_o, S_{ut}) + L_{sw2}(V_o, S_{ut}) + L_{sw3}(V_o, S_{ut}) \ldots$$
$$+ L_{sb1}(V_o, S_{ut}) + L_{sb2}(V_o, S_{ut}) + L_{sb3}(V_o, S_{ut})$$

$$L_{swtot}(V_o, S_{ut}) = 349.452\,m$$

Maximum Fatigue Load on Supports:

R_{sup} is the fraction of length that is hollow.
R_{sl} is the ratio of width to depth.



$$SF \cdot \sigma = \frac{F}{A} \qquad A = d \cdot \frac{d}{R_{1s}} - (d \cdot R_{sup}) \cdot \left(\frac{d}{R_{sl}} \cdot R_{sup}\right)$$

$$F_{sup}(V_o) := V_o \cdot g \cdot (\rho_{air} + \rho_H) \qquad F_{sup}(V_o) = 2.197 \times 10^6\,N$$

$$A_{sup}(V_o, S_{ut}) := \frac{F_{sup}(V_o)}{SF \cdot S_{ut}} \qquad d_{supi}(V_o, S_{ut}) := \sqrt{\frac{R_{1s} \cdot A_{sup}(V_o, S_{ut})}{\left(1 - R_{sup}^{\ 2}\right)}} \qquad d_{supi}(V_o, S_{ut}) = 0.143 \cdot m$$

$$L_{ff}(V_o) := L_{sup}(V_o) - \frac{H_b\left(V_o, \frac{1}{4}\right)}{\sin(\Theta)} \qquad L_{ff}(V_o) = 26.821\,m$$

$$SF \cdot \sigma = \frac{M \cdot y}{I_{xx}} \qquad M = F \cdot d \qquad I_{xx} = \frac{b \cdot h^3}{12} \qquad I_{sup}(d_{sup}) := \frac{d_{sup}^{\ 4} \cdot \left(1 - R_{sup}^{\ 4}\right)}{12 \cdot \sin(\Theta) \cdot R_{1s}}$$

$$F_{ffmax}(V_o, d_{sup}) := \frac{SF \cdot S_{syp.st} \cdot I_{sup}(d_{sup})}{L_{ff}(V_o) \cdot d_{sup}} \qquad F_{ffmax}(V_o, d_{sup}) = 4.224 \times 10^4\,N$$

Given $\quad F_{sw}(V_o) = F_{ffmax}(V_o, d_{sup})$

$$d_{gd} := \max\left(d_{supi}\left(V_o, S_{ut}\right), \mathrm{Find}\left(d_{sup}\right)\right) = 0.38\,m$$

$$F_{ffmaxa}\left(V_o, d_{sup}\right) := \frac{SF \cdot S_{syp.al} \cdot I_{sup}\left(d_{sup}\right)}{L_{ff}\left(V_o\right) \cdot d_{sup}} \qquad F_{ffmaxa}\left(V_o, d_{sup}\right) = 2.322 \times 10^4\,N$$

Given $\quad F_{sw}\left(V_o\right) = F_{ffmaxa}\left(V_o, d_{sup}\right)$

$$d_{gda} := \max\left(d_{supi}\left(V_o, S_{ut.al}\right), \mathrm{Find}\left(d_{sup}\right)\right) = 0.464\,m$$

$$A_{gd} := \frac{d_{gd}^{\;2}}{R_{ls}} \cdot \left(1 - R_{sup}^{\;2}\right) = 67.646 \cdot cm^2 \qquad b_{gd} := d_{gd} \cdot R_{ls}^{\;-1} = 0.076\,m$$

$$A_{gda} := \frac{d_{gda}^{\;2}}{R_{ls}} \cdot \left(1 - R_{sup}^{\;2}\right) = 100.805 \cdot cm^2 \qquad b_{gda} := d_{gda} \cdot R_{ls}^{\;-1} = 0.093\,m$$

$$n_{bs} := \mathrm{round}\left(\frac{4A_{gd}}{\pi \cdot d_{bolt}^{\;2}}, 0\right) + 1 = 23$$

Calculating Weight:

$$W_{cb} := n_{cbsup} \frac{\pi}{4} \cdot \left[D_{cb}^{\;2} - \left(D_{cb} - 2 \cdot t_{cb}\right)^2\right] \cdot d_{plt} \cdot \rho_{steel} \cdots \qquad = 1.217 \cdot tonne$$
$$+ d_{cbar}^{\;2} \cdot l_{cbar} \cdot n_{cbsup} \cdot n_{cbsec} \cdot \rho_{steel} + 2 \cdot \frac{\pi}{4} \cdot D_{cbp}^{\;2} \cdot t_{cbp} \cdot \rho_{steel}$$

$$W_{cables}\left(V_o\right) := L_{cbl} \cdot \left(r_{cbl}\left(V_o, F_{dw}\right)\right)^2 \cdot \pi \cdot \rho_{steel} \qquad W_{cables}\left(V_o\right) = 19.142 \cdot tonne$$

$$W_{cables2}\left(V_o\right) := L_{cbl} \cdot \left(r_{cbl2}\left(V_o, F_{dw}\right)\right)^2 \cdot \pi \cdot \rho_{cac} \qquad W_{cables2}\left(V_o\right) = 2.681 \cdot tonne$$

$$W_{pl} := 2 \cdot L_{cbl} \cdot \frac{\pi}{4} \cdot \left[d_{pl}^{\;2} \cdot \rho_{Cu} + \left[\left(d_{pl} + 2 \cdot t_r\right)^2 - d_{pl}^{\;2}\right] \cdot 2 \cdot \rho_{rub}\right] = 0.294 \cdot tonne$$

$$W_w\left(V_o\right) := A_w \cdot L_w \cdot \rho_{al} \qquad W_w\left(V_o\right) = 4.473 \cdot tonne$$

$$W_{sup}\left(V_o\right) := 2A_{gd} \cdot L_{sup}\left(V_o\right) \cdot \rho_{steel} \qquad W_{sup}\left(V_o\right) = 5.276 \cdot tonne$$

$$W_{supa}\left(V_o\right) := 2A_{gda} \cdot L_{sup}\left(V_o\right) \cdot \rho_{al} \qquad W_{sup}\left(V_o\right) = 5.276 \cdot tonne$$

$$W_{sw}\left(V_o, S_{ut}, \rho_s\right) := 2\,L_{swtot}\left(V_o, S_{ut}\right) \cdot d_{sw}\left(V_o, S_{ut}\right)^2 \cdot \frac{1}{4} \cdot \pi \cdot \rho_s \qquad W_{sw}\left(V_o, S_{ut}, \rho_{steel}\right) = 0.402 \cdot tonne$$

$$W_{bmp}\left(V_o\right) := S_{fab}\left(V_o\right) \cdot \rho_{fbrc} \qquad W_{bmp}\left(V_o\right) = 6.202 \cdot tonne$$

$$W_{all}\left(V_o\right) := W_{bmp}\left(V_o\right) + W_{cables}\left(V_o\right) \cdots \qquad W_{all}\left(V_o\right) = 102.006 \cdot tonne$$
$$+ \left(W_{hub} + W_{rotor}\right) \cdot n_{blades} \cdots$$
$$+ W_{sup}\left(V_o\right) + W_w\left(V_o\right) + W_{pl} + W_{cb} + W_{sw}\left(V_o, S_{ut}, \rho_{steel}\right) + M_{lift}$$

$$W_{all2}(V_o) := W_{bmp}(V_o) + W_{cables2}(V_o) \cdots + (W_{hub} + W_{rotor})\cdot n_{blades} \cdots + W_{supa}(V_o) + W_w(V_o) + W_{p1} + W_{cb} + W_{sw}(V_o, S_{ut.cac}, \rho_{cac}) + M_{lift}$$

$W_{all2}(V_o) = 82.624\cdot tonne$

Solving for Volume for Helium:

Given

$W_{all}(V_o) = V_o\cdot(\rho_{air} - \rho_{He})$

$V_{He} := Find(V_o)$ $\quad V_{He} = 9.611 \times 10^4 \cdot m^3$

$a_o(V_{He}) = 17.901\, m$ $\quad b_o(V_{He}) = 71.604\, m$

Solving for Volume for Helium (Carbon Epoxy Composite Tether & Aluminum Supports):

Given

$W_{all2}(V_o) = V_o\cdot(\rho_{air} - \rho_{He})$

$V_{He2} := Find(V_o)$ $\quad V_{He2} = 7.707 \times 10^4 \cdot m^3$

$a_o(V_{He2}) = 16.631\, m$ $\quad b_o(V_{He2}) = 66.522\, m$

Solving for Volume for Hydrogen:

Given

$W_{all}(V_o) = V_o\cdot(\rho_{air} - \rho_H)$

$V_H := Find(V_o)$ $\quad V_H = 8.797 \times 10^4 \cdot m^3$

$a_o(V_H) = 17.381\, m$ $\quad b_o(V_H) = 69.522\, m$

Solving for Volume for Hydrogen (Carbon Epoxy Composite Tether & Aluminum Supports):

Given

$W_{all2}(V_o) = V_o\cdot(\rho_{air} - \rho_H)$

$V_{H2} := Find(V_o)$ $\quad V_{H2} = 7.054 \times 10^4 \cdot m^3$

$a_o(V_{H2}) = 16.147\, m$ $\quad b_o(V_{H2}) = 64.589\, m$

Times Smaller Than Biggest Blimp Ever:

$$TimesSmaller := \frac{2\cdot 10^5 m^3}{V_H} = 2.273$$

Times Smaller Than Biggest Blimp Ever (w/ Carbon Epoxy Composite):

$$\text{TimesSmaller2} := \frac{2 \cdot 10^5 m^3}{V_{H2}} = 2.835$$