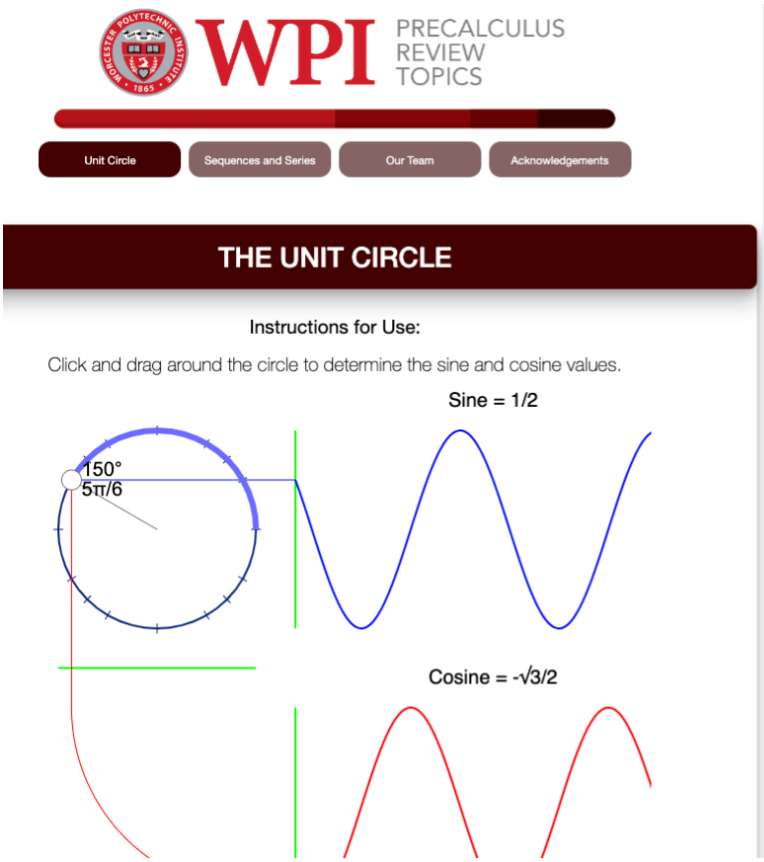
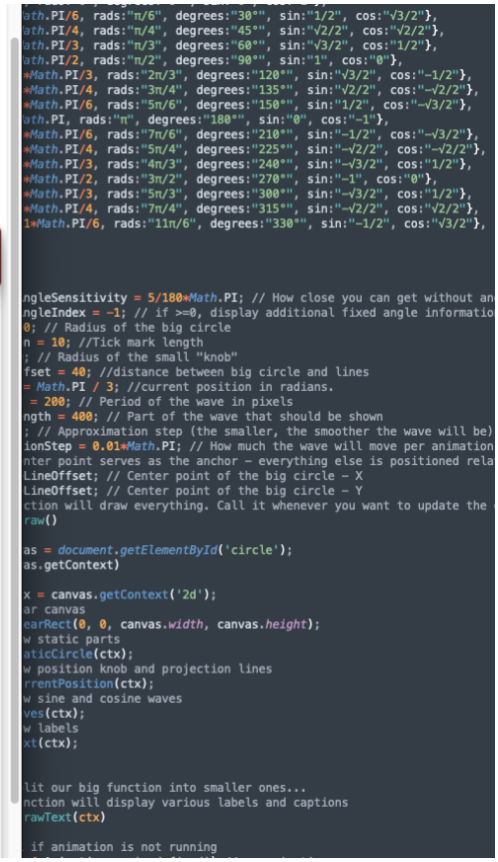
Worcester Polytechnic Institute

Development of an Educational Application for College-Level Mathematics





by

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An Educational

Application for College-Level Mathematics

Visualizing the Unit Circle and boosting Trigonometric Knowledge

An Interactive Qualifying Project
submitted to the Faculty of
WORCESTER POLYTECHNIC INSTITUTE
in partial fulfilment of the requirements for the
degree of Bachelor of Science

by
Joshua A. Galang
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Date:
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Report Submitted to:
Professors Brigitte Servatius and Joshua Cuneo
Worcester Polytechnic Institute

ABSTRACT

This project was developed with the goal of assisting and preparing college students for their required Calculus curriculum in a web application. The app was developed as a result of research to identify current gaps in students' learning experience vis-à-vis available resources. The newly-developed tool seeks to deepen the students' understanding of key mathematical skills required as a basis for Calculus content, as well as enhance students' experience by complementing in-classroom learning and textbook content with a readily-available and interactive interface.

ACKNOWLEDGEMENTS

We would like to thank our project advisors Professor Brigitte Servatius and Professor Joshua Cuneo, who provided insight and inspiration for the project.

We would also like to acknowledge the Calculus professors of WPI as well as all volunteer participating students in our surveys and questionnaires. These participants were surveyed and provided superb feedback in order to obtain the topical data needed to narrow down key topics of the Calculus curriculum.

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EXECUTIVE SUMMARY

Introduction

Calculus is a field with heavy reliance on the understanding of trigonometry, however, the current curriculum employed at WPI skips an introduction to trigonometry with the assumption that it was thoroughly covered in high school. The Calculus curriculum omits trigonometry altogether, with WPI professors observing issues with students' lack of understanding on the subject.

We first conducted a survey of professors currently teaching the Calculus curriculum in order to verify these claims, with responses confirming that Calculus students were not as prepared as they should be for the Calculus curriculum. We also posted a survey to the student population of WPI, learning that they too felt that the foundation of Calculus was lacking.

Another important portion of collecting background information involved research on available applications for Calculus. Using these existing sources provided an insight as to what was widely available and what was used on a variety of platforms, from mobile apps to web sites that are browser accessible. Many of these applications were based on a user-entered problem to solve and entering manual information or using the device's camera to take a picture of the problem with very few applications choosing to focus on the more conceptual perspective of solving problems.

Obtaining Data

We conducted our data collection using two surveys to current Calculus professors and as many students as possible, answering questions online voluntarily through email surveys and through Google Forms. Both surveys aided in our understanding of general Calculus topics and

solidified which topics to develop our application for. When surveying professors, it was through email correspondence and standardized question answering. Over the course of D-Term in 2018, we asked a set of questions involving what Calculus courses they taught as well as which Calculus skills that current students lacked from a topical standpoint. We found that among trigonometric operations, sequences and series were lacking among the students that were taught.

Surveying the student body required a larger survey, and to generate such we turned to Google Forms in order to create a better-equipped data collection service for innumerable responses, since we did not determine how many students would willingly participate in this survey. Questions on the survey included finding students' particular inclinations in technology, from the devices that they use - mobile or desktop platforms - to the software and websites they visit in order to help them better comprehend Calculus topics. Students reported that they were most familiar with using a desktop operating system as a majority, while they also used mobile devices such as smartphones to a lesser extent. Among the software and websites that were mentioned were ones that we had previously used to research similar applications.

From the online survey we determined general years which student was from, by major, as well as key Calculus topics which they found easy and difficult. Most students determined Derivatives and Integrals as fairly easy, while Sequences, Series, and Polar coordinates were what were deemed difficult, accounting for approximately 50% of students. A multitude of responses for Precalculus topics for trigonometry and its associated identities proved crucial in determining our topic for this application. Trigonometric functions were a keystone within the process of determining a basic Calculus topic to begin with and their presence holds prominence among existing applications, the correspondence and interviews with WPI Calculus professors,

and the surveyed student body. Students also reported that they were more visual learners, where visuals presented through images and videos could prove more effective in solidifying the foundation of Calculus.

Development

In creating the application, we wanted to develop a tool in addition to traditional methods to encourage students' thinking without giving them the answer directly. We also wanted to present Trigonometry in a more visual way as evidenced by our survey results. In order to do this, we settled on creating a Unit Circle, the foundation of trigonometry that produces the Sine and Cosine values.

Given previous experiences in programming interactive experiences using JavaScript, we determined that using such an application could be easily used between desktop and mobile systems with support on web browsers of both systems. We created a basic application that can be implemented on websites that displays an interactive Unit Circle. This Unit Circle visualizes the trigonometric functions Sine and Cosine, outlining their relationship as the X and Y axis of a marker that rotates about the circumference of the circle. This is represented in the form of two different curves which represent the graphs of both functions on a normal X-Y plane. In order to better represent the Cosine graph, the line is converted from a vertical standpoint to a horizontal one for familiarity. To supplement the circle, there are also values that correspond with the selected location on the circle's rim that display the angle of the circle from 0 to 360 in degrees as well as the corresponding values in radians. Displayed in corresponding color values are also the Sine and Cosine values at that particular point.

In setting up the HTML and JavaScript, the Unit Circle material composes its own section, and there have since been developed room for expansion and other acknowledgements. Other tabs can be opened for other applications such as sequences and series.

Final Recommendations

While we have met our basic project objectives, there is still room for improvement. In the future, different features will be employed in the Unit Circle that determine the Tangent values calculated from the Sine and Cosine values. Sine divided by Cosine is then shown and simplified and then the results are represented in an animated graph as well.

The next steps would involve adding more JavaScript functions and topics, such as Sequences and Series, detailed in both the professor surveys and the student surveys. There is always room for more features in the HTML and JavaScript file.

We believe that with further work on the application and more topics added, it will benefit WPI students as a complete course of preparation before formal Calculus courses begin. Having already conducted the base studies and surveying both current professors and students in order to determine new divisions of Precalculus topics to identify and add to the application, it would be evolutionary to add more from these topics to better benefit future students of WPI. In conjunction with WPI's established testing, the application can also provide supplementary study materials to better prepare incoming freshmen.

CHAPTER 1: INTRODUCTION

Calculus is a field that relies heavily on trigonometry, however, the current curriculum employed at WPI skips an introduction to trigonometry with the assumption that it was thoroughly covered in high school. Topics such as basic Sine, Cosine, and Tangent values are not immediately covered. The Calculus curriculum lacks this basic Trigonometry, save for a small review at the beginning of the course, with WPI professors observing issues with students' lack of understanding on the subject. Placement tests are an excellent way of determining which Calculus course a student should enter into, however, an evolution beyond the placement test could help better prepare students beforehand.

Therefore, we decided to conduct a study based on these claims in order to observe if students were truly lacking in Precalculus skill and create the beginnings of an application that covers fundamental Precalculus educational topics intended for use before a student arrives at WPI. This digital application is intended to provide a more visual side of Precalculus concepts featuring interactive graphics, helpful guides, and smooth animation. Hopefully with a better understanding of these concepts, students will be better prepared for the Calculus curriculum and improve their performance in these crucial set of courses.

The final goals of this IQP are to conduct studies with professors and students alike to survey Calculus topics that they found difficult, which Precalculus skills they felt they were lacking, and their learning styles, as well as to create a basic application featuring trigonometry in an interactive Unit Circle using the information obtained from these studies.

CHAPTER 2: BACKGROUND AND APPLICATION REVIEW

2.1 Existing Math Applications

Researching other similar smartphone applications, initial research provided a wide variety of software with varying methods and capabilities on both mobile and desktop systems. For the purpose of this project, we limited each application to their capability with topics from Calculus, namely in the forms of trigonometry and the integrals of the trigonometric functions. For this purpose, we sought ways to determine the integrals of $\sin(x)$, $\sin^2 x$, $\sin^3 x$, and $\sin^4 x$.

2.1.1 KHAN ACADEMY

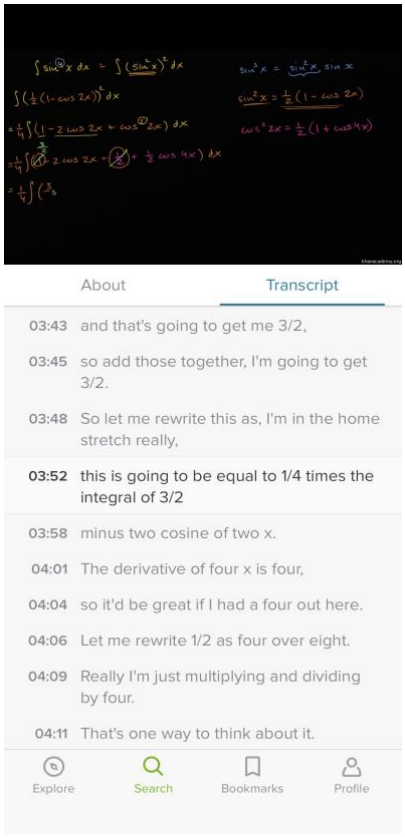

Figure 1: The Khan Academy application for iPhone

This free application is available for iOS and Android devices as well as accessible through the web browser. It takes a more topical approach to mathematics in dividing them according to subjects. Users navigate into the topic that best pertains to what they are learning, then watch instructional videos that show the reasoning behind problems and realistic application. A proven effective method through the use of tutorial videos, this is more of an education tool used as a primary form of education but it is difficult to access specific topics without completing previous modules. Users would also need to scrub through the video presentations in order to find the most relevant topics.

In terms of the sample problems, a relevant video was easily found but had to be watched completely in order to gain a complete understanding of Sine's integrals.

2.1.2 WOLFRAM ALPHA

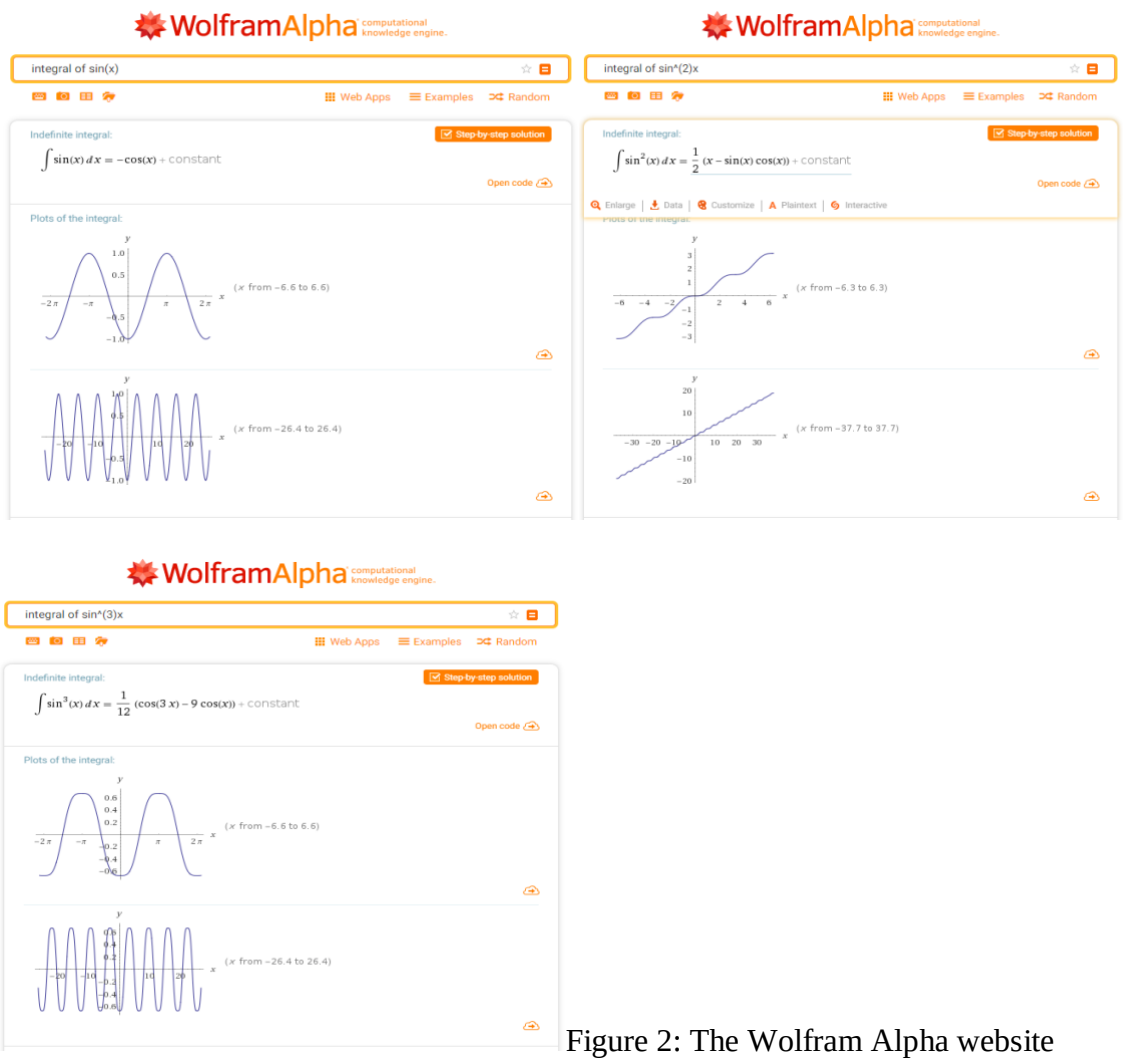

Figure 2: The Wolfram Alpha website

A paid app intended for iOS and Android mobile devices as well as an online website, Wolfram Alpha takes either a topical search or a problem search. When presented with a broad toolbar, users can input their search topic, question, or a simple function. Then are given readable methods for solving those types of problems, or, in most cases, the solutions directly. If a function is entered, it is plotted along with several other qualities, including integrals and derivatives.

Given the sample problems, the method was instantaneously calculated with the option of showing all steps required in order to reach the desired results along with graphs of the Sine waves both zoomed in and zoomed out.

2.1.3 SOCRATIC

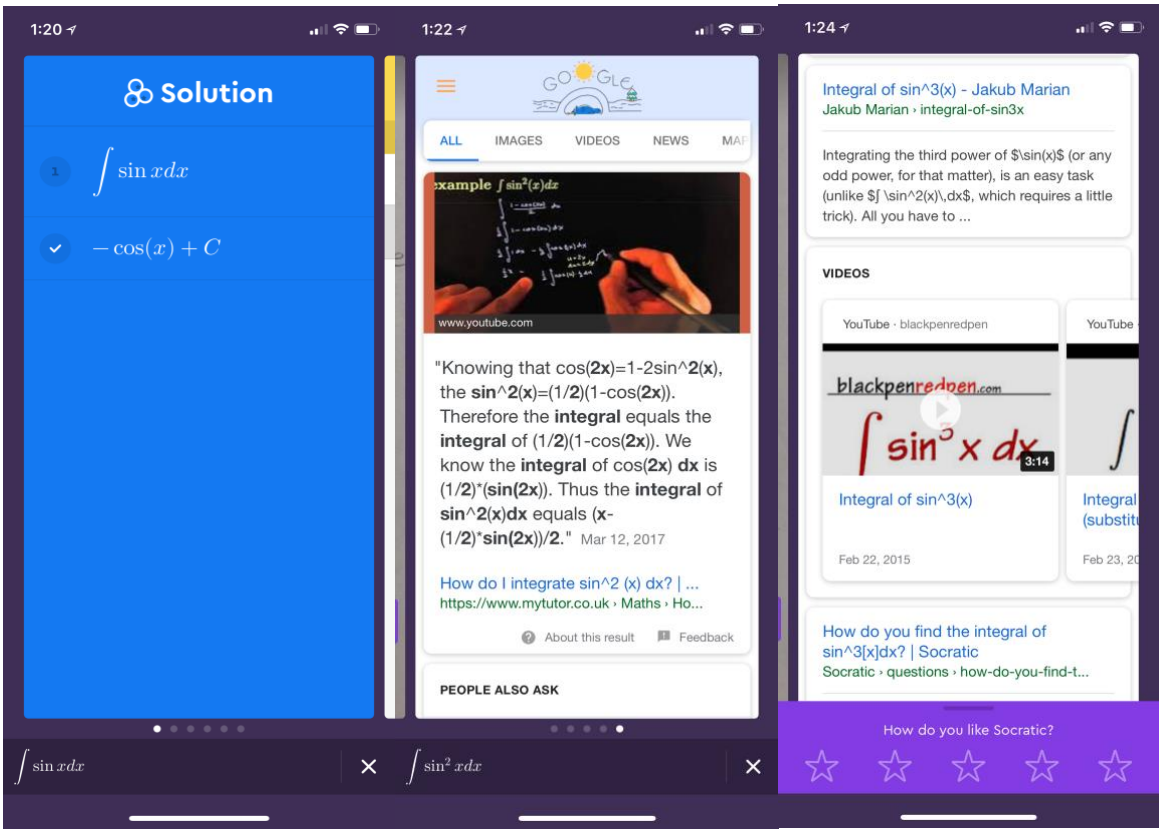

Figure 3: The Socratic application for iPhone.

This math application is free for both iOS and Android devices and is photography-based. Socratic can either solve a mathematical problem by taking pictures of a problem to solve or by having a user manually enter a problem. However, unlike previous sources, Socratic works as an index of resources that can help aid in the solution of the said problem. The user is then given sources of sites with input and solution, including Cymath, MathPapa, Wolfram Alpha, and a Google Search.

When given our Sine functions, Socratic returned the answer, but also provided alternative sources on slide able cards that linked to different web sites.

2.1.4 PHOTOMATH

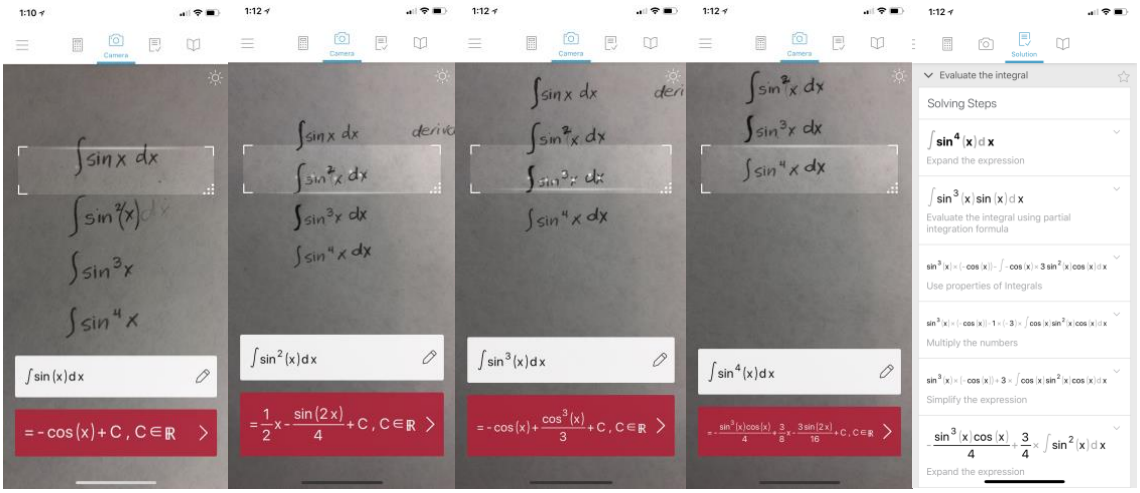

Figure 4: The Photomath application for iPhone

This application is free for both iOS and Android devices and is also photography-based. The user can either take photos of a problem to solve or manually enter one and return a solution back. The user is given step by step functions, including methods and reasons behind them.

After writing out each problem, it was easy to scan and obtain results. It was very useful to walk through the entire solution as the application not only went through the steps, it also showed what each step was called.

CHAPTER 3: APPLICATION RESEARCH AND DEVELOPMENT

The intent of this project was to perform research and begin development on a digital application as an auxiliary tool to help strengthen college students' foundations and understanding of Precalculus concepts. In order for the team to understand the current need for the app and define a subject to focus on, the current market and student needs were identified. This was the preliminary step to begin the development of the app. The following progression describes the methodology path leading up to the app's development:

1. Literature research was conducted to understand the evolution of the educational system, educational needs, learning resources available and how these shaped the learning experience for today's' students.
2. A variety of mobile apps were identified which are currently available in the market. A comparison of their interfaces, topics, functionalities and availability was conducted in order for the team to identify possible opportunities for development and gaps in the students' learning experience.
3. A survey was disseminated amongst the undergraduate student body in order to collect information on the students' learning experience with Calculus courses at WPI, their studying habits and preferences, subject areas of strength and weakness, as well as usage of technological devices and current market apps.
4. A questionnaire was sent out to Mathematics faculty in order to collect information on student areas of strength and weakness from the instructor's perspective.
5. Once a relevant amount of information was collected from the different sources listed above, development of the app took place. Based on the inputs collected, the team

would decide the subject focus, the desired device platform for the app, as well as the desired user interface and subsequent choice of programming methodology.

3.1 Professor Interviews and Correspondence

In D-Term of 2018, we interviewed all professors who were teaching some form of WPI's Calculus curriculum in order to get a better grasp of which topics students were lacking. Through our interviews, four professors brought trigonometry, basic algebra, and sequences and series to light as calculus concepts. After obtaining this information by contacting professors, our next step was to interview the student populace. As Calculus is a standard among all students at WPI, a wider audience and data collection method was needed in the form of an interactive survey, which we developed.

Table 1: Professor Email Survey Results, D-Term 2018

Professor	**Calculus Course**	**Topics**
Boris Iskra	Calculus I	Chain Rule, Product Rule, Derivative Applications
Michael Johnson	Calculus II	Trigonometry, Logarithms, Exponentials, Spherical and Polar Coordinates
Simone Cassani	Calculus III	Sequences and Series, Basic Algebra and Trigonometry
Zhongqiang Zhang	Calculus III	Basic Algebra, Trigonometry, Sequences and Series

Table 1: Data collected from WPI email questionnaire to all calculus professors D-18

It can be observed that within Calculus there are a variety of topics with which professors observed that students are lacking proficiency within. Among these topics, Trigonometry shows prominence above the rest, being mentioned not only by the team advisors, but also from a majority of the professors who have provided a response.

3.2 Student Surveys

After originally consulting with our faculty advisors, we asked professors either in-person or through email correspondence who were teaching portions of the Calculus curriculum in D-Term of 2018 if there were any topics that their students were lacking for those courses.

Surveying the WPI student body employed a completely different method through an online Google Forms survey. The questions we asked were as follows:

3.2.1 Introductory Questions

After obtaining individual consent, students were asked which devices they used the most and its operating system. This was asked to determine which type of application could be used by students, from either a smartphone app or an online website. Finding out the operating system helps with selecting which programming language and style could be used.

As students advance through their years at WPI they can determine how experienced they are in the Calculus curriculum or are currently completing Calculus, as well as whether or not Calculus plays an important role in the student's major. Students were asked their student year, their major and minor, if there is one, and the most recent Calculus course that they have completed, or if they have finished the curriculum.

Device Used and Operating System

67.5% of the students who were surveyed said that they use their mobile phones the most in their daily lives, while the remaining 32.5% use their laptops more often, with iOS and Android in the lead for operating systems.

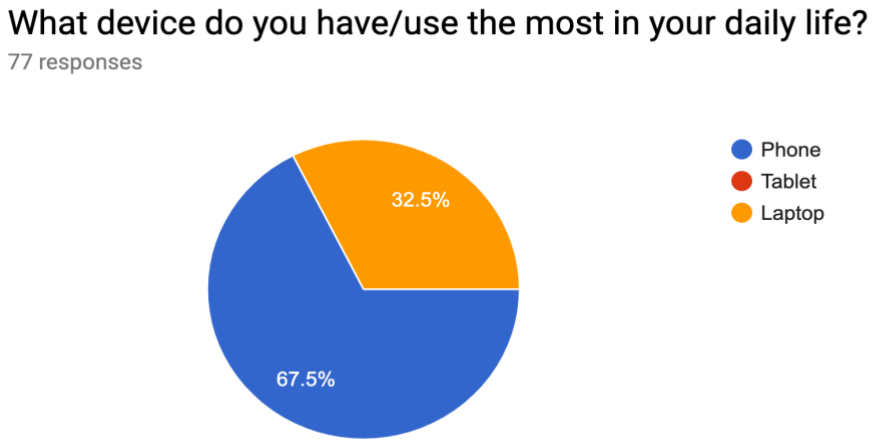

Figure 5: Survey Question – Devices Most Used in Daily Life

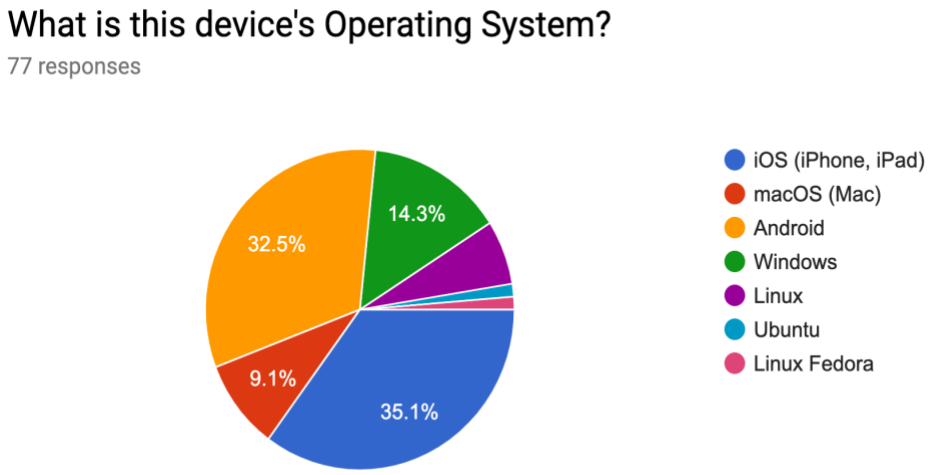

Figure 6: Survey Question – Devices Most Used Operating System

Student Year

A majority of the survey volunteers were Sophomores consisting of 31.2%, followed by 28.6% Freshmen, 23.4% Juniors and 16.9% seniors.

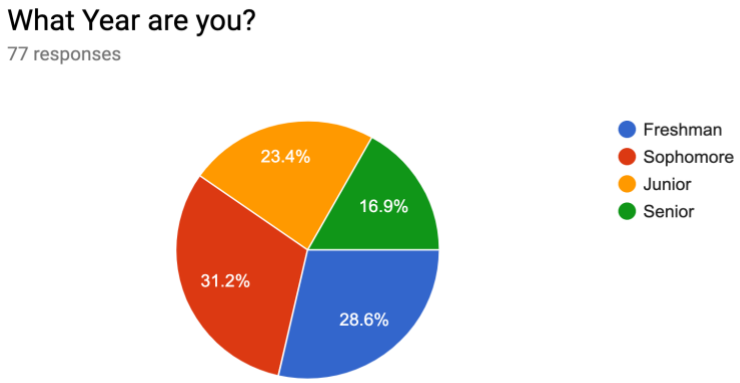

Figure 7: Survey Question – Student Year Distribution

Student Major

Due to an emailing mishap, a majority of the survey takers were Computer Science majors, consisting of a majority of the survey takers with the remainder being a combination of double majoring students or Majors and Minors.

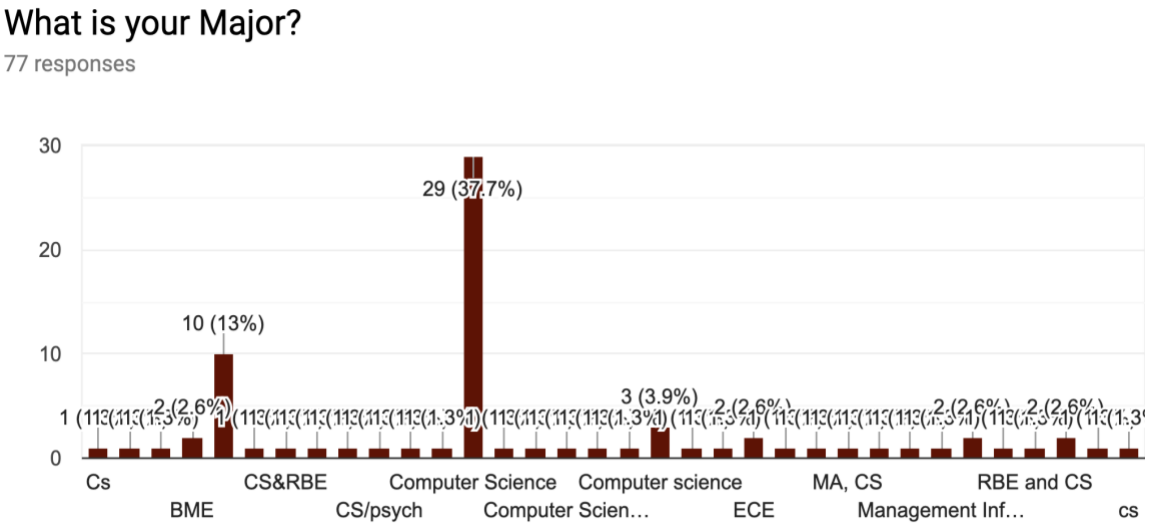

Figure 8: Survey Question – Student Major

Calculus Level completed and Topics

A majority of students have completed the Calculus curriculum with 86.8% of the survey subjects having completed Calculus and 9.2% currently taking the last calculus course, Calculus IV.

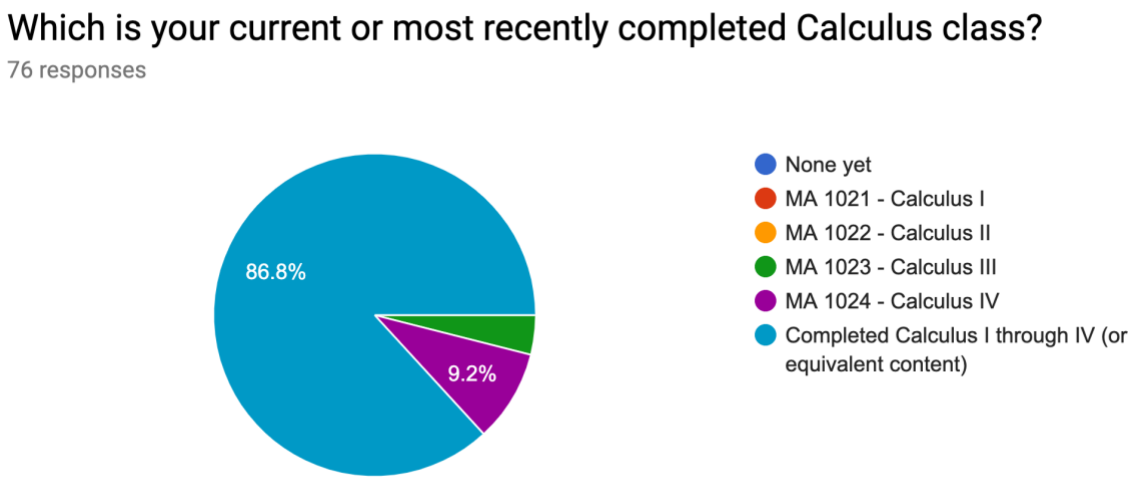

Figure 9: Survey Question – Student Calculus level completed

3.2.2 Current Calculus Student Questions

Free-response questions allowed students to describe which Calculus topics they found the easiest and the most difficult, as well as questions that inquire which Precalculus topics that students had encountered adversity against.

For students taking current Calculus courses, they were asked which websites and/or applications that they currently used to aid in the particular class, employing Free-Response questions for students to input whatever source they prefer. Students were also asked which platform, picking a variety of platforms. Choices for this were Smartphone, Tablet, Desktop, and a physical textbook. This question is meant to solidify that a majority of the students required a virtual method to better comprehend Calculus concepts.

Easy and Difficult Topics

Students were then asked which general Calculus topics were determined to be the easiest to comprehend and then the most difficult, with Series and Integrals being more difficult with derivatives being part of the easier side.

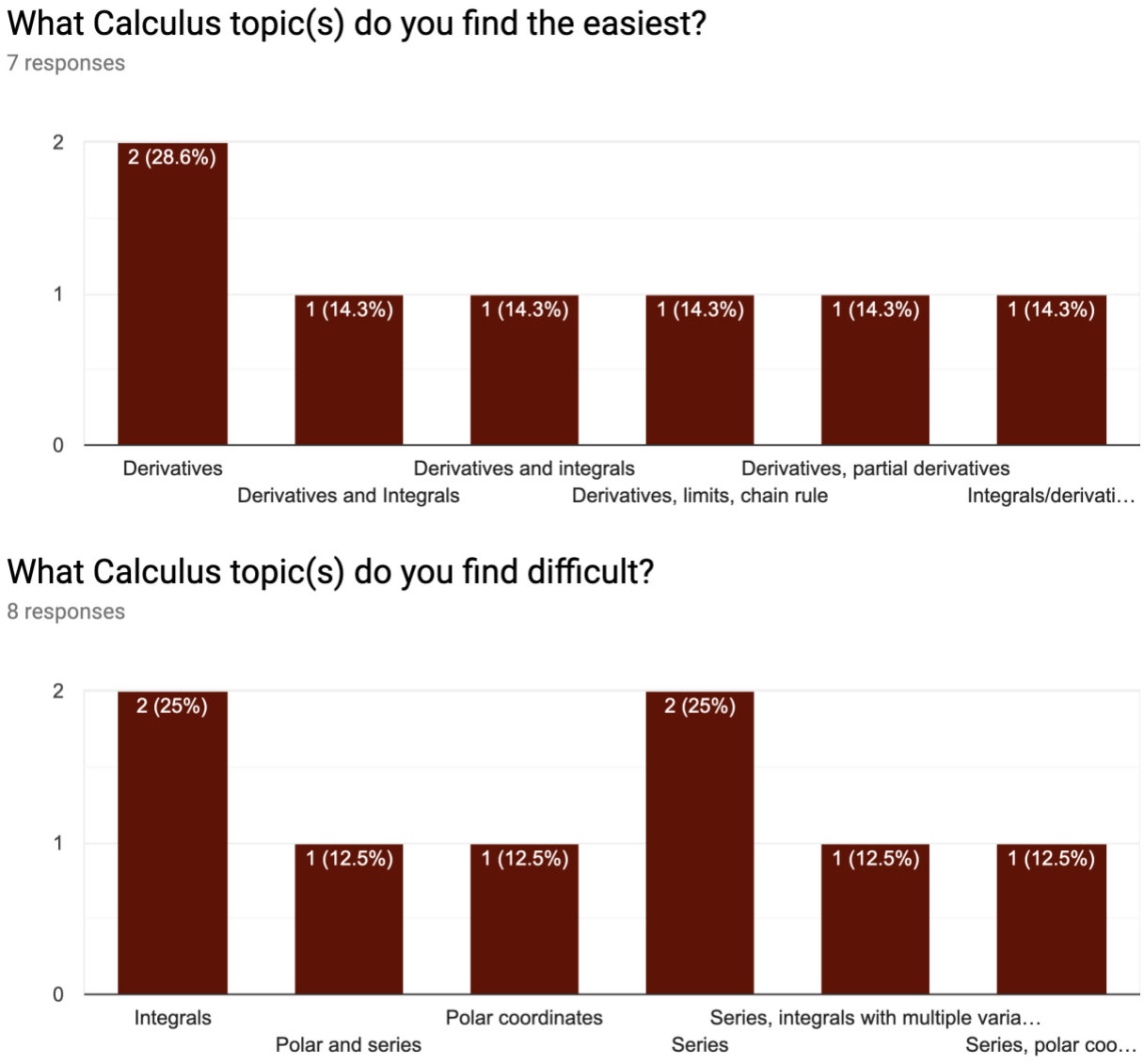

Figure 10: Survey Question – Current Student Easy and Difficult Topics

Precalculus Topics

Similar to the professor evaluation, students mentioned that their own Trigonometry skills were lacking prior to taking calculus courses.

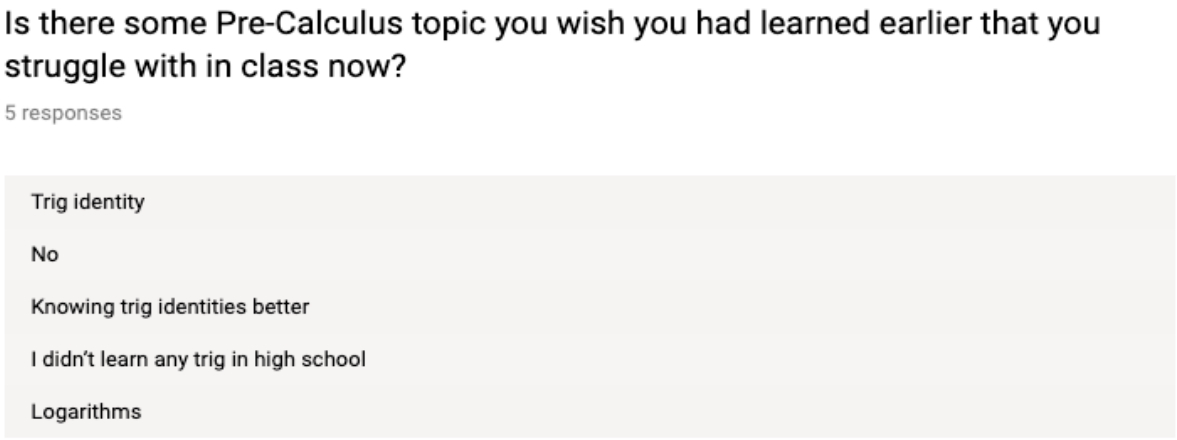

Figure 11: Survey Question – Current Student Unfamiliar Topics

Applications Used

A majority of students use applications and websites for aid in Calculus problems, particularly through Desmos for graphing, Khan Academy, and Wolfram Alpha.

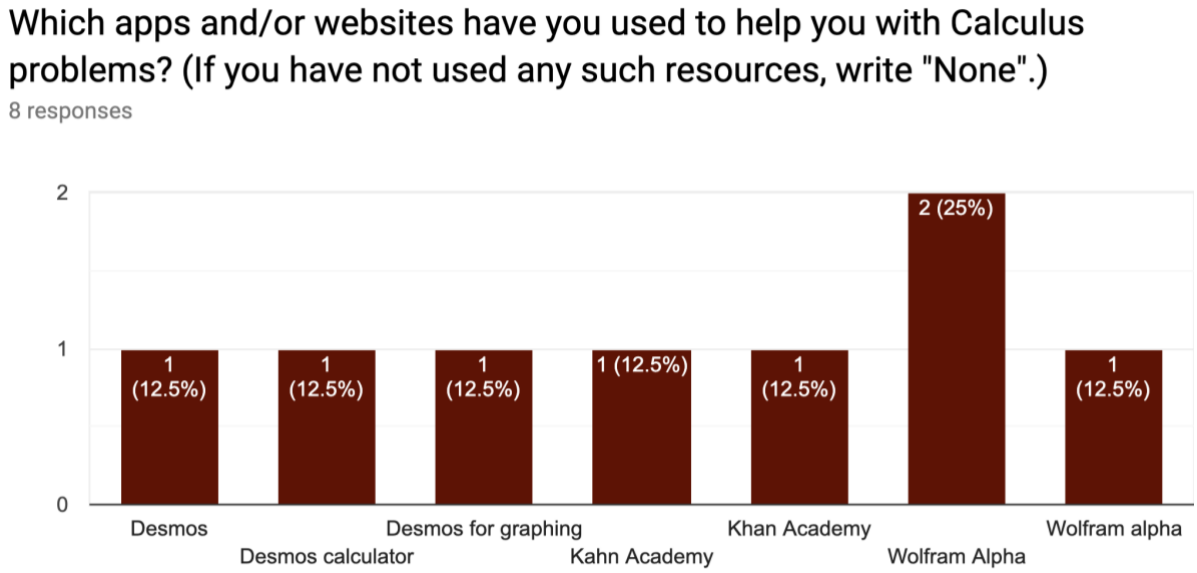

Figure 12: Survey Question – Current Student Online Resources

Precalculus Topics Familiarity

Students then would be asked which Precalculus topics they could best recall, ranging from Algebra, Trigonometry, and Geometry. While Algebra and Geometry were ranked highly, confidence faltered for trigonometry.

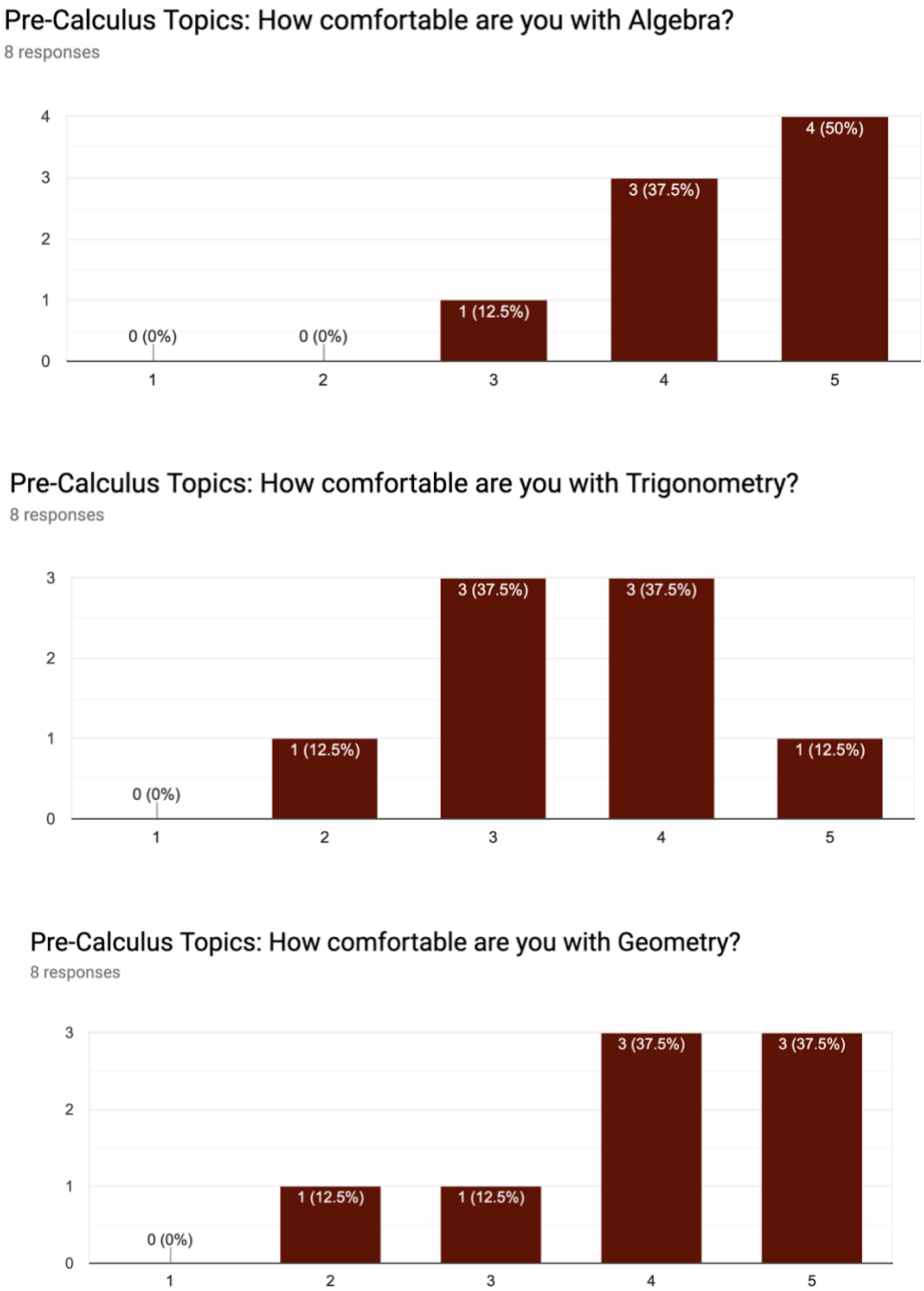

Figure 13: Survey Question – Precalculus Familiarity

3.2.3 Past Calculus Students Questions

This section was designed for students who have completed all four Calculus courses and therefore are better suited to answer questions pertaining to topics holistically. Students were asked which Calculus topics they remembered, what was easy and difficult, as well as which software they used to aid their studies.

Topics Remembered

A majority of students remembered Derivatives Integrals, Limits, Chain Rule, Vectors, Dot Product, and Cross Product. Student confidence waned, however, taking into account sequences and series.

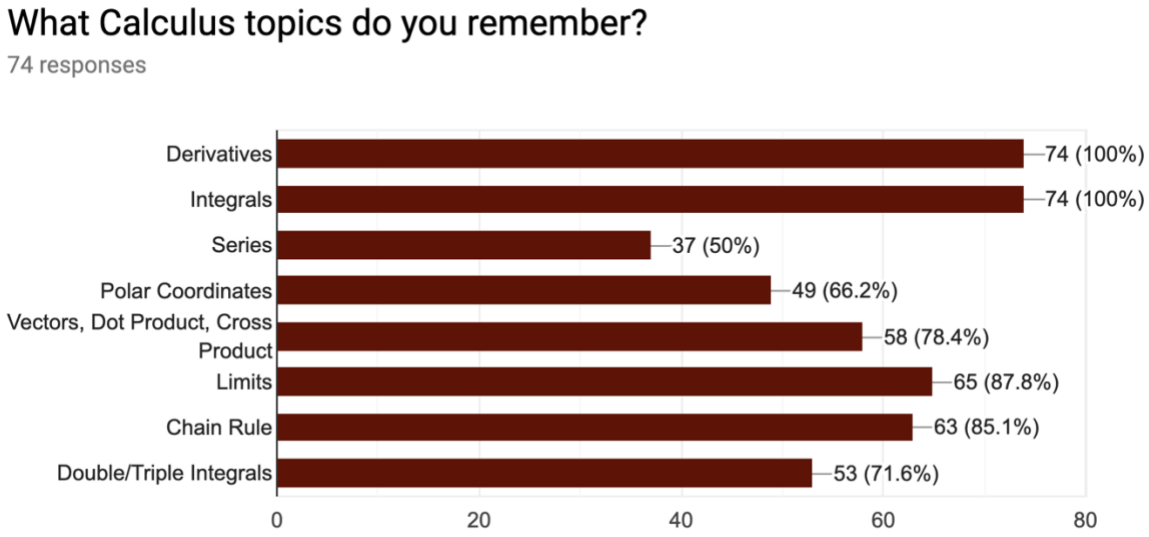

Figure 14: Survey Question – Past Students Topic Recollection

Easy and Difficult Topics

Students found basic Derivatives and Integrals easy, but deemed Series, Integrals, and Cross Products difficult.

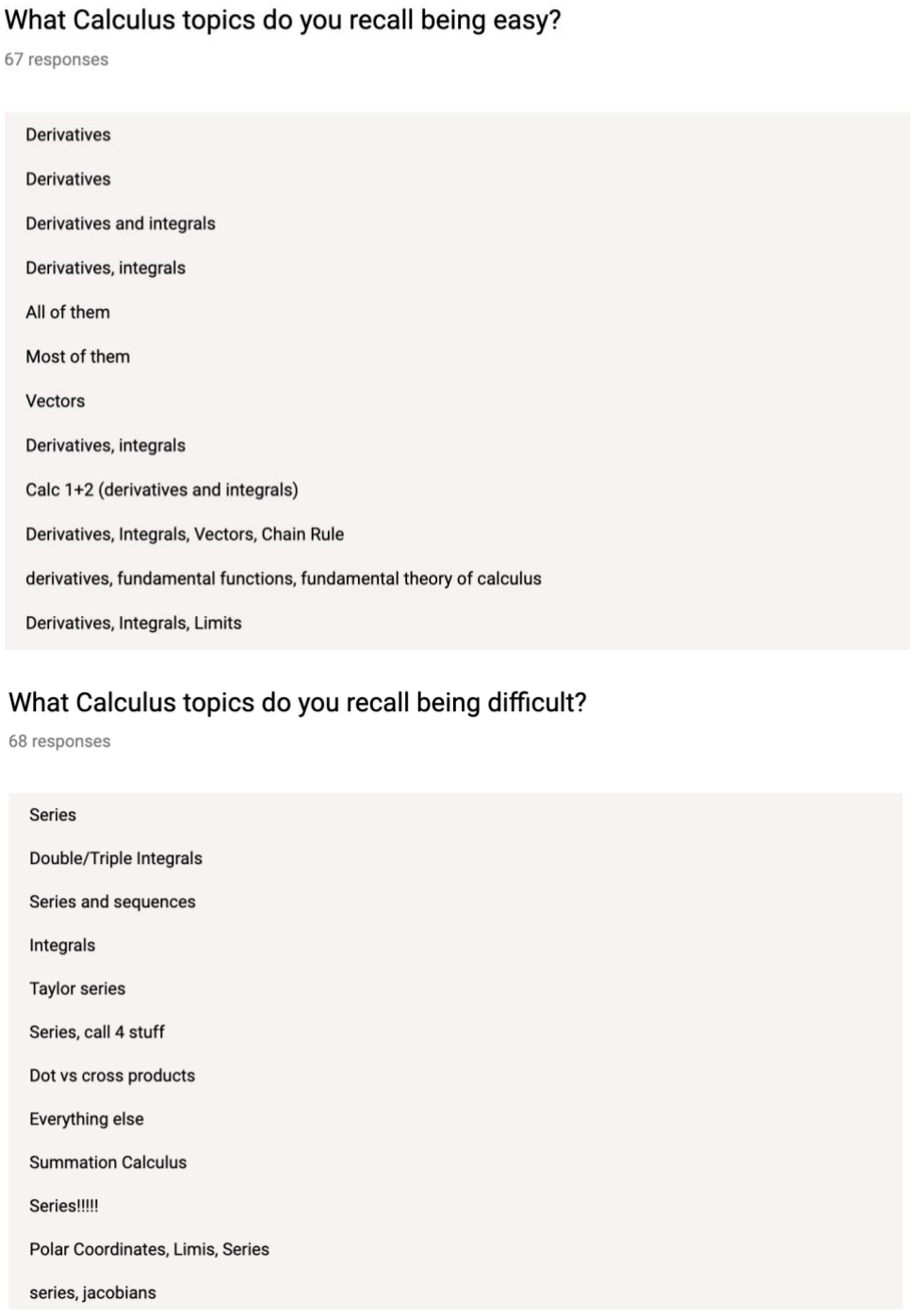

Figure 15: Survey Question – Past Student Topic Difficulty

3.2.4 Student Studying Techniques

This section is meant to conclude the survey summing up how students learn in terms of their preferred ways to learn and submit homework, which device is used for mathematical purposes, what they like in terms of learning through the methods they mentioned previously, and their preferred learning style – visual, auditory, linguistic, and kinaesthetic.

Homework Submission

A majority of students preferred electronic submissions due to its ease and quick grading ability. Virtual submission provides rapid feedback, and physical documents can be scanned as opposed to turning in at a specific location.

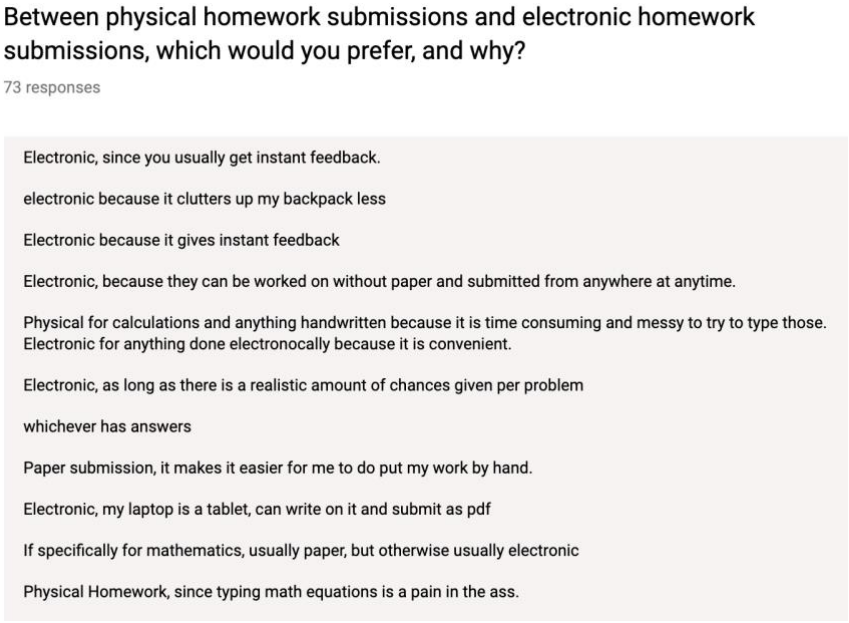

Figure 16: Survey Question – Preferred Homework Submission

Device used for Math

82.7% of students used their desktop for math purposes. This question was intended to clarify which device we would create the application for – with an emphasis on the desktop web browser.

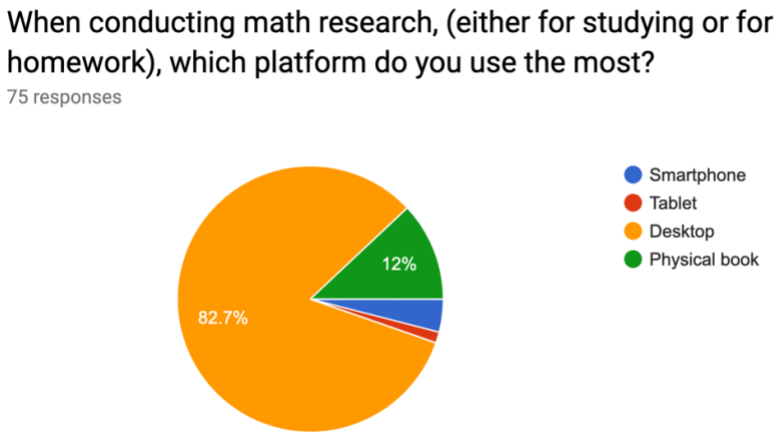

Figure 17: Survey Question – Preferred Device for Math

Learning Styles

Visual styles were preferred amongst the surveyed audience, consisting of 60.5% of responses with linguistic and kinesthetic making up 23.7% and 11.8% respectively.

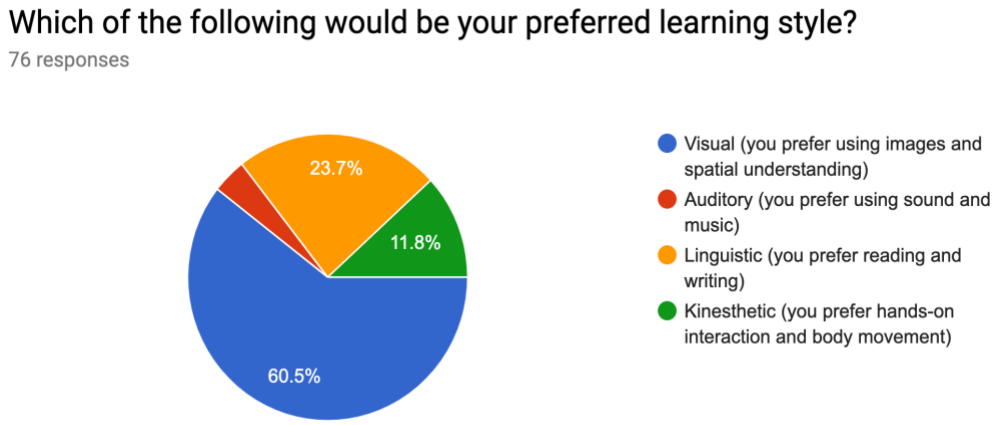

Figure 18: Survey Question – Preferred Learning Style

In conclusion, from both surveys, we affirmed that this application would cover the basics of trigonometry, a topic in which students did not feel as confidently prepared for, with possible additions for sequences and series. The application would then be web-based so as to be accessible through a browser, and it would use an interactive visual style so as to better teach students and aid them.

Application Development

After completing our research, we wanted to develop something that could be used as a tool in addition to traditional methods to encourage students' thinking without giving them the answer directly and to present Trigonometry in a more visual way as evidenced by our survey results. In order to do this, we settled on creating a Unit Circle, a visualization that essentially works as the core of trigonometry including the Sine and Cosine values. Sine and Cosine are associated with angles and degrees, particularly with finding the dimensions of corresponding triangles, where Sine is the opposite over hypotenuse and Cosine is adjacent over hypotenuse.

Given my previous experiences in programming interactive experiences using JavaScript, including data visualization and computer graphics, together with my team we determined that using such an application could be varied in terms of devices between desktop and mobile systems with web browser support. Therefore, this would be more accessible to the wide variety of students with an emphasis on those who use more desktop-oriented operating systems for math studies.

We created a basic application that can be implemented on websites that displays an interactive Unit Circle. This Unit Circle visualizes the trigonometric functions Sine and Cosine, outlining their relationship as the X and Y axis of a circular marker that rotates about the circumference of the placeholder circle. This is represented in the form of two different curves which represent the graphs of both functions on a normal X-Y plane. In order to better represent the Cosine graph, the line is converted from a vertical standpoint to a horizontal one for familiarity. To supplement the circle, there are also values that correspond with the selected location on the circle's rim that display the angle of the circle from 0 to 360 in degrees as well as

the corresponding values in radians. Displayed in corresponding color values are also the Sine and Cosine values at that particular point.

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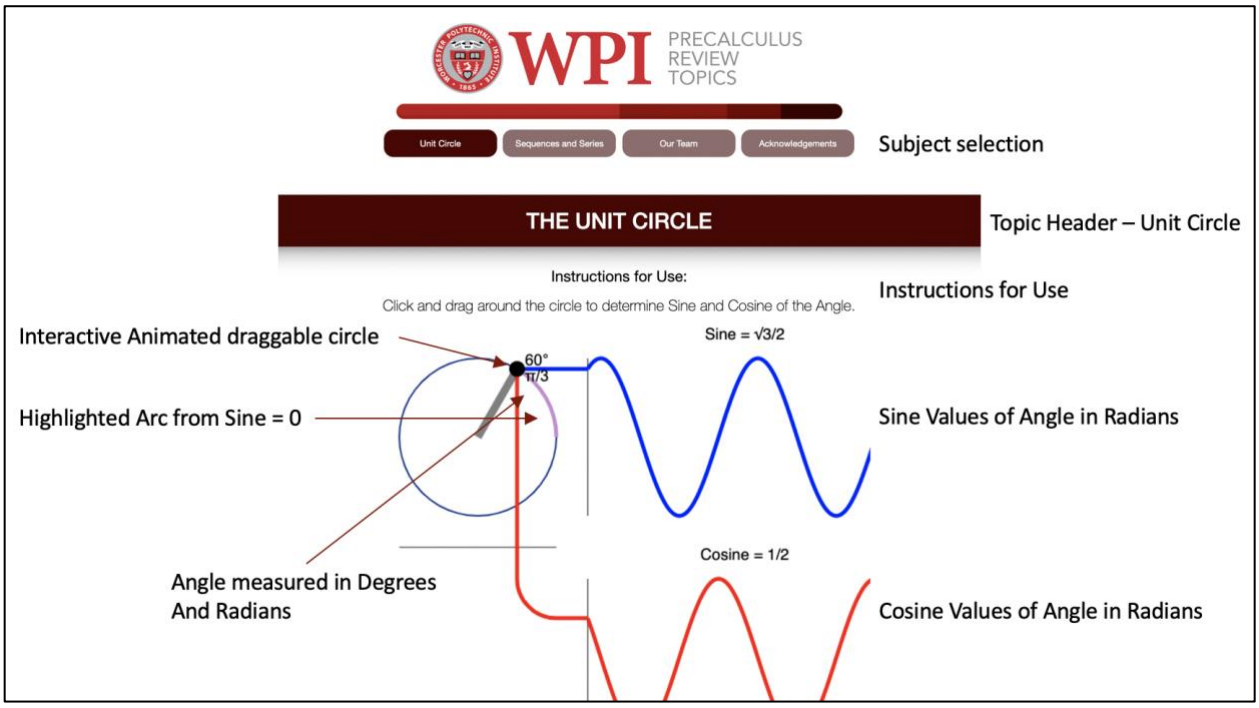

Figure 19: The Basic Application as it appears on Web Browsers.

Application Features

- Tab-based navigation provides a clear level of topics
- Usable on both Interactive colourful interface can be used on mobile and desktop alike
- Visualization of Sine and Cosine based on Unit Circle angles for users to associate the graphs with location on the circle
- Angle values in radians and degrees

CHAPTER 4: CONCLUSION

Creating an interactive Unit Circle was completed by the end of the project with room for more interactive features in separate canvases. In addition to the code, I designed all graphic aspects for the project and worked with Erika in order to develop the application. I also programmed the JavaScript. During the creation of this application I couldn't help but also observe that there is also more potential for this application, some of which require further discussion and research.

4.1 FUTURE FEATURES

1. More Topics

   Basic trigonometry is not the sole factor that professors found students lacking skills in. Therefore, more topics such as sequences and series and the basics of derivatives and integrals could better help prepare students for the Calculus Curriculum.

2. Adaptation into a Smartphone

   Another challenge would be creating an app specifically for smartphones, particularly for the two major operating systems iOS and Android.

3. More Visual Style and Animation

   Working with the IMGD department could yield a far better-looking and more easily navigable experience that also looks more professional and with smoother animation.

RESOURCES

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