Galaxy Angular Distribution in the Hubble Ultra Deep Field

A Major Qualifying Project Report:

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by

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Abstract

The Hubble Ultra Deep Field (HUDF) image surveyed a section of sky roughly 11 square arcminutes in area [1]. The image contains roughly 10,000 galaxies [11] spanning nearly 13 billion lightyears [17]. The information contained in this image has been the basis for numerous studies including redshift surveys, studies of galaxy composition, and research on galaxy distribution. In this study we present a survey of the angular sizes of the galaxies in the HUDF. The size distribution of the galaxies is compared to two mathematical models of galaxy density in the universe: the first, most basic model neglects both expansion and general relativity, while the second incorporates expansion but lacks general relativity. Both models suggest that the number density of galaxies depends on angular diameter to the negative fourth power. Future development of this model would involve the inclusion of general relativity in the model and refinements to account galaxy size distribution, the loss of faint galaxies, and other effects.

Acknowledgments

I would like to thank Professor Dick for all the work he has done in advising this project. I would also like to thank Professor Iannacchionne for his advice in the beginning stages of this project. And I would like to thank Lesley Drohan for her invaluable help in editing the project report.

Contents

1 Problem Statement

The Hubble Deep Field (HDF) and Ultra Deep Field (HUDF) experiments have yielded a diverse body of new information about the structure of the universe. Such studies range from redshift surveys to observation of these fields in infrared and radio wavelengths [8] [14] [9]. This study examined the number distribution of galaxy angular diameters. This data is compared to galaxy distribution models to determine if the expected distribution of galaxy angular diameters agrees with what is expected from theory.

2 Background

2.1 The Hubble Space Telescope

The Hubble Space Telescope (HST) is a large telescope, currently orbiting Earth. From this vantage point, the HST is far above the Earth's atmosphere, which allows it to obtain clearer, better resolution pictures of distant objects than ground based telescopes. The atmosphere distorts and absorbs light, making it difficult for ground based instruments to clearly visualize distant objects. It was designed with the intent of observing distant galaxies. It was launched in 1990 and is still operational. It is named for Edwin Hubble, an influential astronomer of the 1920's who developed a galaxy classification system that is still used today.

In 1995, a scientific team pointed the Hubble Space Telescope at a dark piece of sky in the northern hemisphere, containing very few stars [20]. Over the course of ten days, the HST took over 300 images of a carefully selected region in the constellation Ursa Major, at a variety of wavelengths [20]. The results were astonishing; the image, Appendix A, revealed a plethora of galaxies, instead of the darkness seen by the naked-eye.

The discovery sparked numerous research projects that deepened human understanding of galaxy formation and structure, and history of the early universe. It also introduced new types of deep space objects.

2.2 A Short History of the Early Universe

Data from Hubble Deep Field associated studies [10], the Sloan Digital Sky Survey (SDSS) [21] and Wilkinson Microwave Anisotropy Probe (WMAP) [6] help create a picture of the early universe, after the Big Bang, and how the universe evolved. The early universe is thought to have progressed as follows and as illustrated in figure 1:

- At 10^{-35}s inflation occurred [12]. The universe expanded at superluminal speeds.
- At 10^{-6} Baryons form [12].
- At 10^{-2}s the universe was in thermal equilibrium [23]. Electrons and positrons are spontaneously produced and then annihilated.
- At 3s basic elements such as Helium-3, Helium-4 and Lithium began to form [12].
- At 10^{3} years radiation was dominant in the universe [12], which continued to expand, thus stretching out the wavelengths of the radiation [23]. This became the cosmic microwave background currently measured to be about 3 K [12] [23].
- At 3×10^{5} years, nuclei to join with electrons to form stable atoms, this allows the universe became transparent to radiation [12] [23]. Thus matter became dominant over radiation [12] [23].

- At 3×10^8 years, galaxies were able to form [12] [23]. Atoms were able to clump together in clouds of gas [12]. From these clouds, stars and galaxies were born [12].
- At about 14×10^9 years, the current universe exists [12]. Heavier elements have been created though fusion in stars cores, and current galaxies have formed [23]. The universe continues to expand [23] [22] [15].

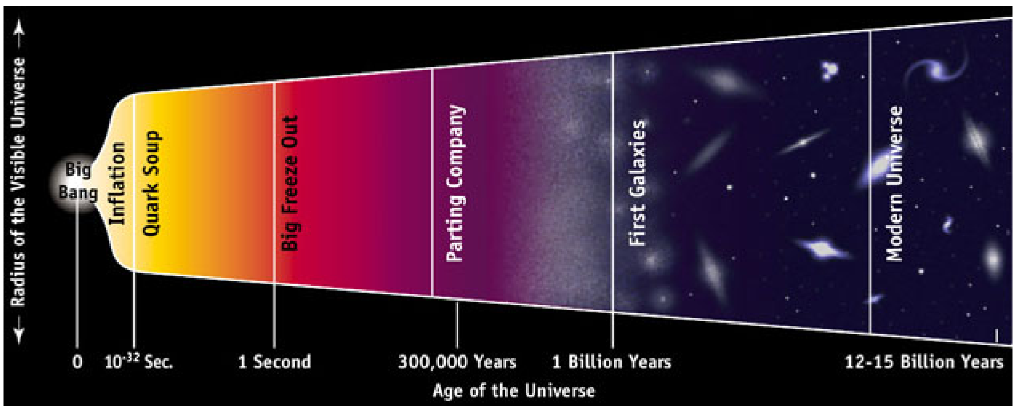

Figure 1: Visual representation of the evolution of the universe. The HST is capable of looking back to roughly one billion years after the Big Bang, when the first galaxies formed. [27].

In 2003, scientists aimed the HTS at an empty patch of sky, in the southern hemisphere [17]. The telescope produced a larger, higher resolution image, seen in Appendix B, of an area now known as the Hubble Ultra Deep Field (HUDF), looking back further than the HDF image [17].

While it is not currently possible to use a telescope to look back in time to the Big Bang, the HUDF image has provided one of the farthest looks back yet. This image contains galaxies as far away as 13 billion light years, meaning that the light we currently see from these galaxies originates from only 1 billion years after the beginning of the universe [17]. Like the HDF image, the HUDF image continues to be studied and contributes new information towards understanding the universe.

2.3 Galaxy Classification

Galaxies are easiest to study when classified by structure. Edwin Hubble developed the most commonly used galaxy classification system [15]. In this system, galaxies are divided into three major groups - ellipticals, spirals, and barred spirals - and further classified into subgroups based on several morphological features. Ellipticals are separated into numbered groups - E0, E1, etc - where the number designates the galaxy's eccentricity multiplied by ten and rounded to the nearest integer [15]. Spirals and barred spirals grouped by the size of their central bulge, the amount of dust present and how tightly their arms are wound about the central bulge [15]. Although the majority of galaxies can still be classified using this system [15], there are also galaxies, typically referred to as irregular galaxies, that do not fall neatly into the scheme. Irregular galaxies have a variety of structures including shapeless blobs and colliding galaxies [15].

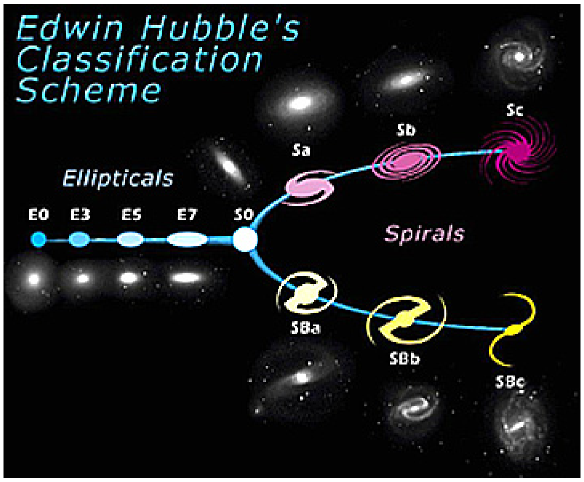

Figure 2: Hubble's Galaxy classification system [28].

Hubble placed the galaxies along a tuning fork shape because he assumed that galaxies started their lives as low eccentricity ellipticals and during the course of their lifetimes evolved into either a spiral or a barred spiral [15]. His classification system and evolutionary pathway are illustrated in figure 2. However, this is not actually the case. Galaxies do not evolve along the paths illustrated in Hubble's diagram, though it is possible for one galaxy to take on a wide variety of morphologies during its lifetime. A study involving the simulation of galaxy formation in a cold-dark-matter-dominated universe found that galaxies would take on different shapes as they acquired new matter in various ways [18]. Spiral structure appeared when a galaxy grew as a result of smooth accretion of gas. The study suggested a galaxy with spiral structure could evolve into either an elliptical or barred spiral, depending on the conditions. Ellipticals formed when two spirals of approximately equal size collided [18]. Collisions would lead to increased star formation in the galaxy, using up much of the galactic dust and resulting in a relatively dust free elliptical in which few new stars would form [18]. Barred spiral structure was seen when a spiral galaxy acquired new material from companion galaxies; the bars in the center resulted from tidal forces caused by the interactions between the galaxies [18]. Researchers suggest that these are not the sole methods through which these structural features can arise, but their work does suggest that galaxies evolve hierarchically [18]. Thus Hubble's notion of hierarchical galaxy evolution was correct, however the evolutionary paths a galaxy can take are more complex than illustrated in figure 2.

2.4 Galaxy Luminosity

Additional evidence for hierarchical formation of galaxies is suggested by a variety of galaxy luminosity studies. The galaxy luminosity function is defined as the number of galaxies in a certain luminosity range within a given volume of space. The galaxy luminosity function provides a method of statistically analyzing the distribution of galaxies in space [29]. The galaxy luminosity function can be used in conjunction with redshift data as done by [13] and [2] to examine how galactic structure changes over time. Another possible application of the galaxy luminosity function is the investigation the properties of a certain population of galaxies [29]. For example, using the luminosity function, ellipticals can be classified into three distinct populations - large, mid-sized, and dwarf ellipticals [29]. Analysis of the luminosities of large and midsize elliptical galaxies suggests that these two classes form in similar ways. Such an analysis also suggest that dwarf ellipticals formed through a different process [29].

In addition to the galaxy luminosity function, other statistical methods are being developed to study populations of galaxies and their evolution. S. Driver suggests two new distributions that may be useful in galaxy formation studies. Using these methods, S. Driver analyzed a sample of spirals and a sample of ellipticals [5]. From the "distinct, but overlapping" [5] distributions, it was concluded that these two morphologies arise from different mechanisms.

2.5 Applications of the HDF and HUDF Data

Several luminosity studies have used the galaxies contained within the HDF and HUDF. Other HST-image based research includes galaxy counts, redshift surveys - studies in which the properties of a population of galaxies as a certain redshift are statistically analyzed - and research into the kinematics of distant galaxies. Researchers often survey the section of sky surveyed by the HST a second time, using equipment such as radio or infrared ground based telescopes. They can survey larger sections of sky while using the HST results to draw conclusions about the data.

Many of the redshift surveys have been done in that manner. In one such study, S. Lilly et al. examined galaxy size, compared to galaxy luminosity of galaxies at redshifts between $z = 0$ and $z = 1$. They found that galaxies of different sizes are forming stars at different rates and thus evolving at different rates [9]. Smaller galaxies are evolving more than larger ones [9]. Another redshift survey conducted by M. Giavalisco examined a small sample of very redshifted galaxies with the intent of investigating galaxy properties and features in the early universe [7]. Another redshift survey examined the galaxy clustering at different redshifts [4]. The researchers found holes in the distribution of galaxies in the HDF [4].

2.6 Furthering the Hubble Ultra Deep Field Studies

Research from the HUDF and HDF continues to provide insight into the structure and evolution of the universe. Because the HUDF looks back in time, it allows us a way of visualizing what happened when the earliest galaxies formed. This not only presents new types of galactic objects to be understood, but also provides a way of studying the formation and evolution of galaxies throughout the evolution the universe. In this paper, we present a model of galaxy distribution compared to the number distribution of galaxies of different angular sizes. The flaws in the models are examined and possible improvements are suggested.

3 Methods

The angular diameters of the galaxies in the HUDF image, show in Appendix B, were measured using a computer program, presented in Appendix C, to determine the position of the end points of the galaxies' diameter. The galaxies' diameters were identified by eye and the program recorded the positions of the endpoints in a text file. Because the galaxies were identified by eye, only the diameter of the brighter, visible, star-rich portion of the galaxy was measured. In general the galaxies' halos were not visible in the HUDF image, and were not measured as part of the galaxies' angular diameters. Due to the size of the HUDF image, it was necessary to divide the image up into several smaller sections in order to analyze it.

The endpoint data was then transfered to an excel spread sheet where the galaxies' diameters, measured in number of pixels, were converted to radians. The diameters, in radians, were imported into matlab. Histogram plots of the data were created with a variety of bin numbers. In addition, kernel density plots were created, using matlab code from [3]. The plots were further analyzed based on two models, presented in section 4, of galaxy distribution in the universe, to find an average galaxy size.

4 Theory

4.1 Static Model

In this model, the expansion of the universe is neglected. The number of galaxies in the universe is assumed to remain constant over time, and thus because the size of the universe is also assumed to be constant, the galaxy density will be constant. In addition, all galaxies are assumed to have the same diameter D. The galaxy density is

$$\rho = \frac{N}{V} \tag{1}$$

where N is the number of galaxies and V is the volume of the universe.

In this experiment, the angular diameters of the galaxies, ϕ was measured. In Figure 4 d represents the distance between the observer and the galaxy being observed, and D represents the actual diameter of the galaxy.

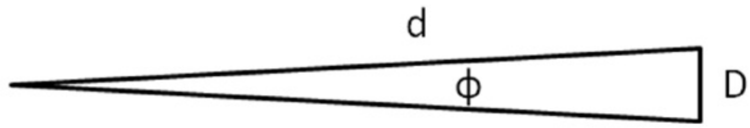

Figure 3: This figure shows the relationship between a galaxy's angular diameter, distance and actual diameter.

By using the small angle approximation, we find that the diameter is related to angular diameter and distance by

$$D = d\phi \tag{2}$$

and from this

$$d = \frac{D}{\phi}. \tag{3}$$

Now we must consider the space contained within the Hubble Ultra Deep Field image. The area covered in a portion of the image is

$$A = (r\theta)^2 \tag{4}$$

where r is the distance to that section of sky and θ is angular width of HUDF image. The volume element at distance r and thickness dr is

$$dV = Adr. \tag{5}$$

But r is just the distance d, to the galaxies being measured, that we worked with in earlier equations. So, dr can be determined in terms of $d\phi$ as follows.

$$r = \frac{D}{\phi} \tag{6}$$

$$\phi = \frac{D}{r} \tag{7}$$

$$\frac{d\phi}{dr} = \frac{-D}{r^2} \tag{8}$$

$$dr = \frac{-r^2}{D} d\phi \tag{9}$$

Substituting equation 9 into equation 5, yields a volume differential in terms of ϕ.

$$dV = \frac{-Ar^2}{D} d\phi \tag{10}$$

Substituting equation 4 into equation 10 gives

$$dV = \frac{-r^4\theta^2}{D} d\phi \tag{11}$$

But, since $r = \frac{D}{\phi}$, equation 10 becomes

$$dV = \frac{-\theta^2 D^3}{\phi^4} d\phi. \tag{12}$$

The galaxy density times the volume element gives the number of galaxies dn in that volume element,

$$dn = \rho \frac{D^3\theta^2}{\phi^4} d\phi. \tag{13}$$

Substituting equation 1 into equation 13, and dividing by $d\phi$ gives an equation for the number density as a function of angular diameter

$$\frac{dn}{d\phi} = \frac{ND^3\theta^2}{V\phi^4}. \tag{14}$$

So a non-expanding universe model suggests that the number of galaxies at a particular diameter is proportional to $\frac{1}{\phi^4}$.

4.2 Expansion, without General Relativity

This model takes the next step and accounts for the expansion of the universe, while not taking into account the effects of general relativity. As in the previous model, the number of galaxies in the universe is assumed to be constant over time, however, since the universe is assumed to expand at a constant rate, the galaxy density will decrease with time. As before, all galaxies are assumed to have the same diameter D.

First we must obtain an expression for the galaxy density as a function of time. We assume that the universe is expanding at the speed of light, c. Thus we find the radius of the universe to be

$$r(t) = ct \tag{15}$$

where t is time, measured from the beginning of the universe, such that $t = 0$ occurred at the Big Bang. Since the radius of the universe changes with time, the volume of the universe must also change with time such that

$$V(t) = \frac{4}{3}\pi(ct)^3 \tag{16}$$

From this we can obtain the galaxy density as a function of time, by simply dividing the number of galaxies in the universe, N, by $V(t)$.

$$\rho(t) = \frac{N}{\frac{4}{3}\pi(ct)^3}. \tag{17}$$

However, we would like to obtain an expression for galaxy density as a function of angular diameter, so it is necessary to find a way to express t in terms of the angular diameter, ϕ. A time variable t, which counts from the beginning of the universe has already been introduced. Now consider a second time variable, t', which counts time from the present, back to the Big Bang, such that $t' = 0$ occurs now and $T_0 = 13.7$ billion years is the time of the Big Bang [26]. t' can be represented in terms of t by

$$t' = T_0 - t. \tag{18}$$

In this experiment, the angular diameters of the galaxies, ϕ were measured. In Figure 4, d represents the distance between the observer and the galaxy being observed, and D represents the actual diameter of the galaxy.

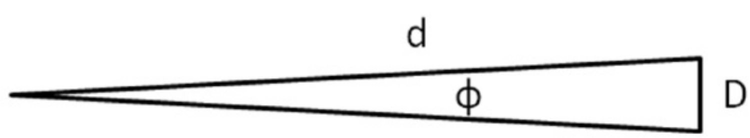

Figure 4: This figure shows the relationship between a galaxy's angular diameter, distance and actual diameter.

By using the small angle approximation, we find that the diameter is related to angular diameter and distance by

$$D = d\phi \tag{19}$$

and from this

$$d = \frac{D}{\phi}. \tag{20}$$

But since the universe is expanding the distance to galaxy can be represented by

$$d = ct'. \tag{21}$$

From this we obtain an expression for t'

$$t' = \frac{d}{c} \tag{22}$$

and by substituting equation 22 into equation 18 we find

$$t = \frac{D}{c\phi} - T_0 \tag{23}$$

We can now substitute equation 23 into equation 16 to obtain an equation for galaxy density as a function of angular diameter.

$$\rho(\phi) = \frac{N}{\frac{4}{3}\pi(c(\frac{D}{c\phi} - T_0))^3}. \tag{24}$$

As in the previous model, we must consider the space contained within the Hubble Ultra Deep Field image in order to obtain an expression for a differential volume of space,

$$dV = Adr. \tag{25}$$

But, as before, r is just the distance d, to the galaxies being measured, that we worked with in earlier equations. So, dr can be determined in terms of $d\phi$ in exactly the same way as in the non-expansion model,

$$dr = \frac{-r^2}{D} d\phi \tag{26}$$

Substituting equation 26 into equation 25, yields a volume differential in terms of ϕ.

$$dV = \frac{-Ar^2}{D} d\phi \tag{27}$$

Substituting equation 4 into equation 25 gives

$$dV = \frac{-r^4\theta^2}{D} d\phi \tag{28}$$

But, since $r = \frac{D}{\phi}$, equation 28 becomes

$$dV = \frac{-\theta^2 D^3}{\phi^4} d\phi. \tag{29}$$

The density of galaxies multiplied by the differential volume gives the number of galaxies in that volume,

$$\rho dV = dn(\phi). \tag{30}$$

From this, the number density of galaxies is found to be

$$\frac{dn}{d\phi} = \frac{-3N\theta^2 D^3}{4\pi\phi(D - \phi c T_0)^3} \tag{31}$$

and thus from this model we expect the angular size distribution of the galaxies to be proportional to $\frac{1}{\phi^4}$. Interestingly, this is a similar dependence to the constant density model.

5 Results and Discussion

5.1 Histograms

The 7,876 measured galaxies were plotted in several histograms. A histogram with 300 bins gave the best representation of the data. Figure 5 shows a histogram with 300 bins, which nicely displays the important features of the data set.

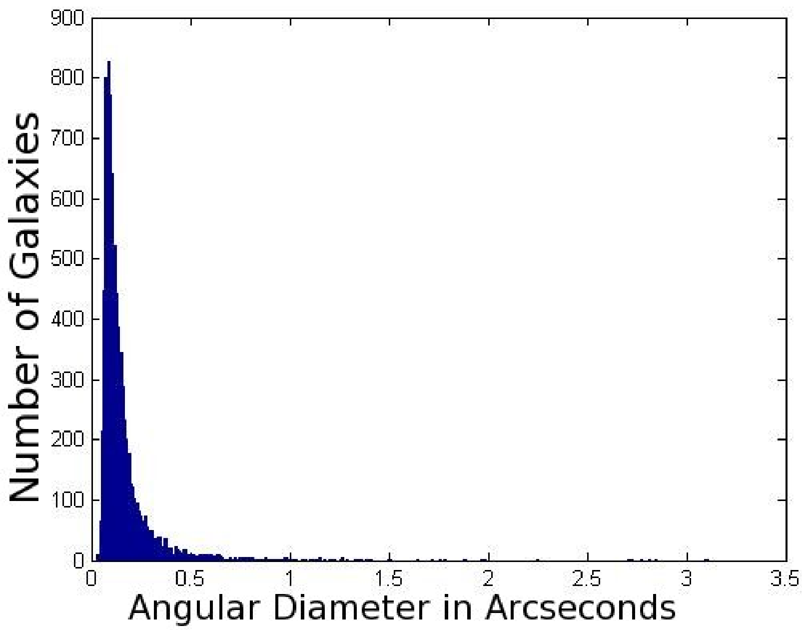

Figure 5: Histogram of galaxy angular size, using 300 bins.

The number of galaxies at a given angular diameter increases until a peak value is reached. After this point, the number of galaxies in a given angular diameter range falls off drastically. It is expected that, because, for smaller diameter galaxies, that are farther away, the HUDF window contains a larger volume space. This would suggest that the number of galaxies would continue to increase as the angular diameter decreased.

5.2 Kernel Density Plots

In addition to plotting histograms of galaxy angular size, kernel density plots of the data were constructed. Unlike histograms, which for a given data point, assign that data point to exactly one bin, kernel density plots account for some uncertainty in the position of a given data point [24].

For example, when measuring a galaxy's angular size, there is some uncertainty in the measurement. The kernel density plot can account for this by using a gaussian function to assign weights to a data point and those near it, thus allowing a data point to contribute to surrounding bins, rather than being placed into exactly one [24].

For these kernel density plots, matlab code was obtained from [3]. Figure 6 shows a kernel density plot of the galaxy data using the program's preset values were used for minimum and maximum, and 2^7 for σ, a quantity analogous to bin number when using histograms.

5.3 Curve Fits

Based on the models discussed in the theory section, curves proportional to $\frac{1}{x^4}$ were fit to the Kernel Density plots. Figure 7 shows a graph of the kernel density plot of the data (the blue line) and a plot of $y = \frac{1}{25x^4}$. This curve represented one of the best fits to the data, for a $\frac{1}{x^4}$ curve.

Curves proportional to $\frac{1}{x^3}$ were also fit to the graph, to examine other possible relations in the data. The best $\frac{1}{x^3}$ fit is shown in Figure 8. This plot shows a

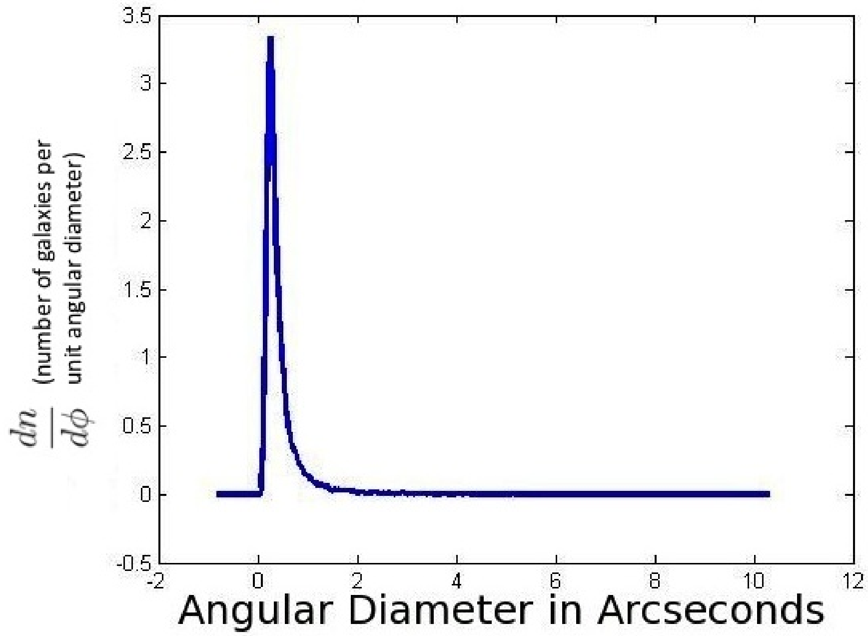

Figure 6: The kernel density plot of the galaxy angular size data.

graph of $y = \frac{1}{9.5x^3}$ plotted on top of the kernel density histogram. In this graph, the curve fit appears to fit the data better; the increase in the $\frac{1}{x^4}$ is more rapid than that of the data both at the bottom and the top of the curve.

5.4 Short-Comings of the Models

The $\frac{1}{x^4}$ curve fit does not really fit the data; it increases too rapidly, and falls off to quickly as angular sizes increase. In addition, this fit cannot account for the rapid drop off at small angular diameters. The $\frac{1}{x^3}$ curve fit fits the data much better than the $\frac{1}{x^4}$, although it also does not account for the rapid drop off seen at small angular diameters.

Both models depended on the assumption that all galaxies are the same, and thus from the models, it should have been possible to calculate this galaxy size.

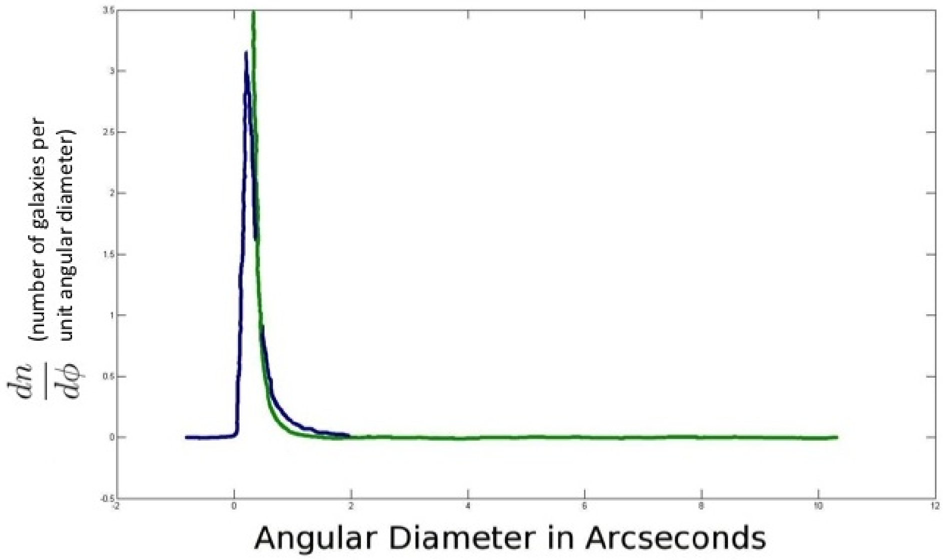

Figure 7: The kernel density plot, with a graph of $\frac{1}{25x^4}$ fit to it.

And this galaxy size should be approximately the average size of a galaxy. However, the models yielded very unreasonable values. Suggesting that the models need further refinement.

6 Future Work

6.1 Faintness and Dwarf Galaxies

As galaxies get farther and farther away, their angular diameters decrease and so does the amount of light received by telescopes, such as the HST, observing those galaxies. Dwarf galaxies at large distances may be so faint that they did not show up in the HUDF. It is also possible that during this study, many the angular diameters of such galaxies were not measured because it is difficult to visually separate very small faint galaxies from background noise in the HUDF.

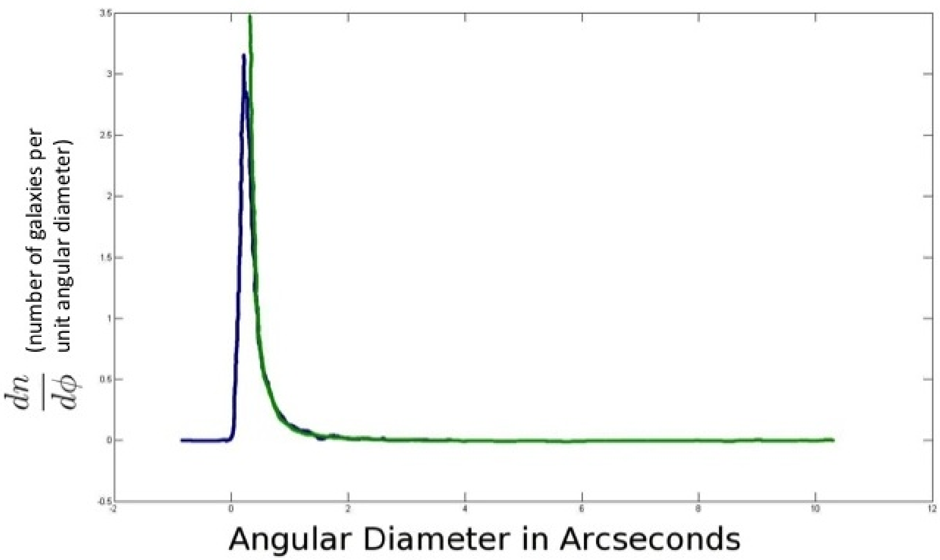

Figure 8: The kernel density plot, with a graph of $\frac{1}{9.5x^3}$ fit to it.

This may account for the rapid drop off in number of small angular diameter galaxies seen in the data. This source of error can be minimized by considering the maximum distance and magnitude that the Hubble Telescope can resolve galaxies. From this a reasonable cut off value for minimum angular diameter can be determined. In addition, the number of faint galaxies at large distances could be modeled and this could be incorporated in to the galaxy density model to give a more accurate model of number density of galaxies.

6.2 Distribution Function

In the data analysis, a log normal distribution of galaxy sizes is assumed. However, this may not be the case. Further refinement of the data, would benefit from an investigation of the distribution of galaxy sizes. A more accurate galaxy size distribution function could be factored into the kernel density function to produce more accurate plots of number density of angular size of galaxies.

6.3 Redshifts

The model could also be further refined through the use of redshift data. If the redshift of a given galaxy were to be measured in conjunction with the galaxy's angular diameter, this would provide a distance estimate to the galaxy. And from this, the size of the galaxy could be estimated. This would provide a way to construct a galaxy size distribution from the galaxy size estimations, as well as spatial distribution of galaxies from the redshift data. In addition, galaxy sizes at different redshifts could be examined to determine whether galaxies tend to be different sizes at different distances. From this it may be possible to draw some conclusions about galaxy evolution over time.

A The Hubble Deep Field

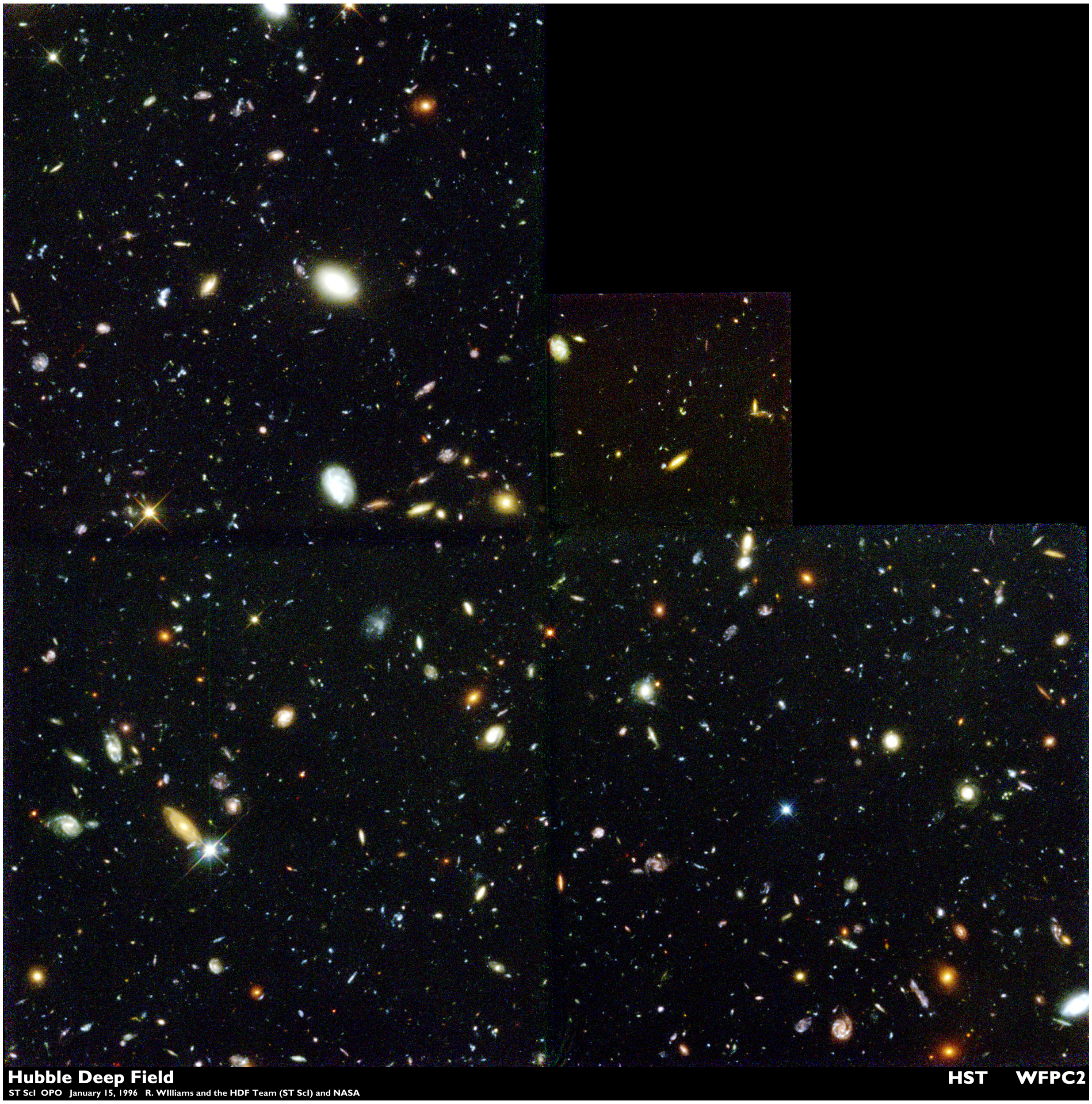

Figure 9: The composite image from the HST's 10 observation of the HDF region. [?].

B The Hubble Ultra Deep Field Image

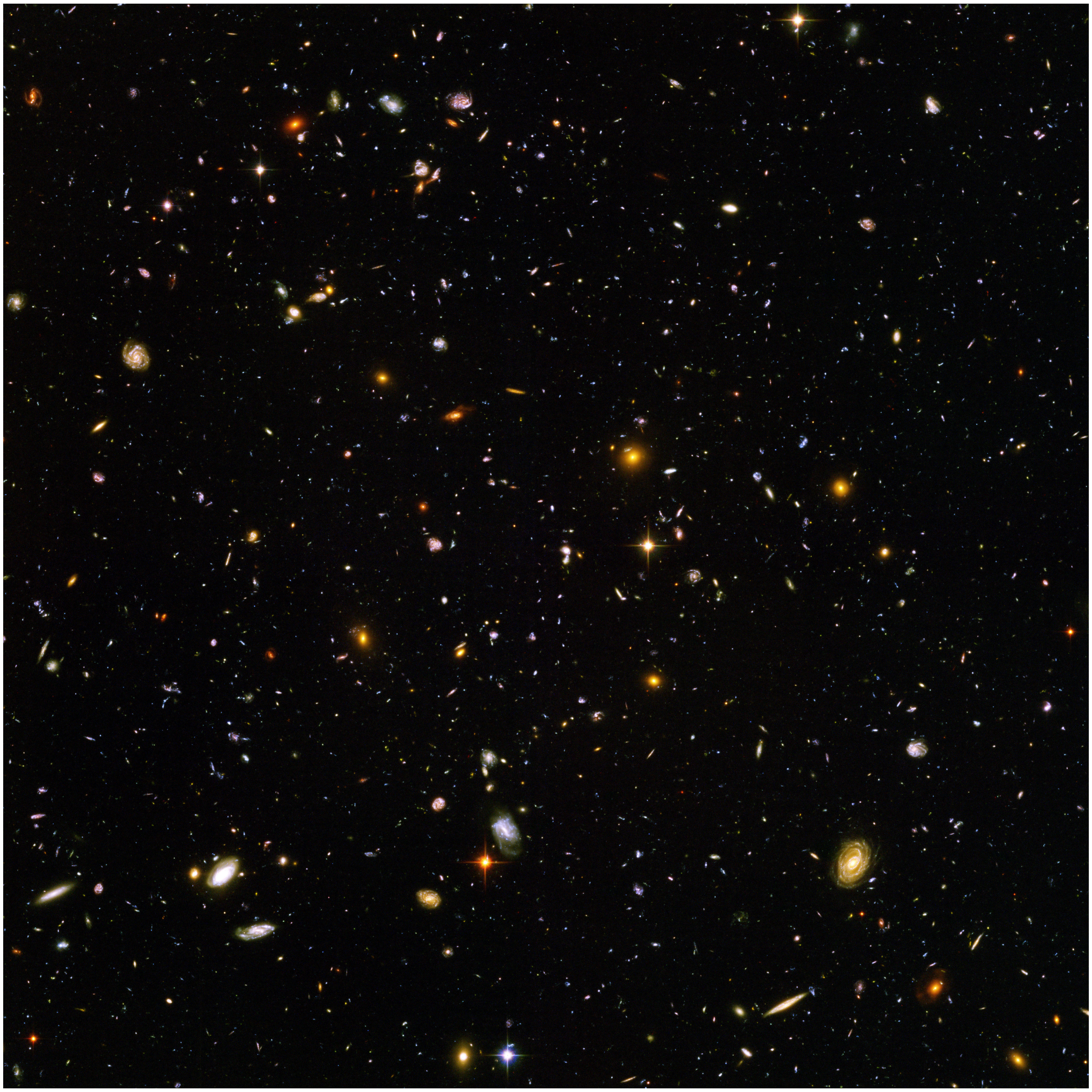

Figure 10: The Hubble Ultra Deep Field image, which is the source of the data in this study. [?].

C Computer Code

```
PrintWriter output;
color thecolor = color(0, 100, 200);
int [] pointarray = new int[4];
String spaces = ", ";
String spaces2 = " ";
int counter = 1;
void setup() {
size(200, 200);
// Create a new file in the sketch directory
output = createWriter("positions.txt");
}
void draw() { }
void mousePressed() {
set(mouseX, mouseY, thecolor); //attempting to color pixel on click. Success!
int Xplace = mouseX;
int Yplace = mouseY;
//prints co-ordinates to the screen
print("(");
print(Xplace);
print(", ");
print(Yplace);
print(") ");
//outputs the co-ordinate to a text file
/*output.print("(" + mouseX + ", " + mouseY + ") ");
```

```
counter = counter+1;
//print(counter);
if(counter%2 == 0){output.println();}*/
if(counter%2 == 1) {//add something to the first part of the array
pointarray[0] = Xplace;
pointarray[1] = Yplace; }
else {//adds something to the second part and prints whole array. Victory!
pointarray[2] = Xplace;
pointarray[3] = Yplace;
String joinedpoints = join(nf(pointarray, 0), " , ");
println(joinedpoints);
output.println(joinedpoints); }
counter = counter+1; }
//this is to print anything that has been stored to a text file
//and then stops the program
void keyPressed() {
output.flush(); // Writes the remaining data to the file
output.close(); // Finishes the file
exit(); // Stops the program }
```

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