

HISTORICAL RISK ASSESSMENT OF A BALANCED PORTFOLIO
USING
VALUE-AT-RISK

by

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A Thesis

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APPROVED:

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Abstract

Calculation of the Value at Risk (VaR) measure, of a portfolio, can be done using Monte Carlo simulations of that portfolio's potential losses over a specified period of time. Regulators, such as the US Securities and Exchange Commission, and Exchanges, such as the New York Stock Exchange, establish regulatory capital requirements for firms. These regulations set the amount of capital that firms are required to have on hand to safeguard against market losses that can occur. VaR gives us this specific monetary value set by Regulators and Exchanges. The specific amount of capital on hand must satisfy that, for a given confidence level, a portfolio's losses over a certain period of time, will likely be no greater than the capital required a firm must have on hand.

The scenario used will be one of a Risk Manager position in which this manager inherited a portfolio that was set up for a client beginning in April 1992. The portfolio will have to meet certain parameters. The initial portfolio is worth \$61,543,328.00. The risk manager will be responsible for the calculation of the Value at Risk measure, at five percent, with a confidence level of 95% and 20 days out from each of the 24 business quarters, over a six year period, starting in 1992 and ending in 1996.

Acknowledgements

I am grateful and wish to thank my fellow students in the Financial Math Program for their fellowship, guidance and support. I am for ever indebted to Professor Vermes for his advisement and guidance. Professor Vermes was there every step of the way. He did not let me quit. His help throughout this project was priceless. His patients, and the time he spent critiquing me and reviewing my work, kept me in the right direction to complete this thesis. I will never forget Professor Vermes for his excellent support and will remember his lessons as I pursue my career in the Finance.

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1. Introduction

The subject of the thesis is to play the role of a financial advisor or asset manager whose client wants to make a long term investment in a portfolio of securities, diversified among all major asset classes and wishes to understand the inherent risks measured quantitatively and demonstrated based on historical data.

The task can be divided into three major components.

1. Portfolio Construction.

Select a set of securities which represent all major asset classes and for which long historical performance data are available. Assemble these securities into a portfolio that has a balanced exposure to all major risk factors:

- Stock market risk.
- Interest rate risk.
- Commodities risk.
- Foreign exchange risk.

2. Data Preparation.

Collect the historical performance data for the securities and the risk factors. Devise a methodology that allows a computerized detection of data errors. Make the necessary adjustments and error corrections that will result in a large clean data set.

3. Risk Assessment.

We needed to select a generally accepted quantitative measure of risk. For the current study we selected the Risk Metrics 95% Value at Risk measure, with a 20 day extrapolation horizon. With the Monte Carlo method, we implemented a computational methodology to calculate the risk measure for the large historical data set. This computation had to be performed in a way that is consistent with the real-time scenario, when data about the future is not available. The final step is to run the back testing and compare the actual performance with the predictions provided by the risk measure.

In the present study we consider the 13 year history window of 1990 – 2003. We perform the analysis and back testing on the first six years of the history, leaving the second half for other purposes. To achieve a sufficiently wide diversification, with relatively few securities, we compose the portfolio from mutual funds and large companies, like General Electric (GE) and British Petroleum (BP), which themselves are composed of a diversified set of distinct businesses. This way we are able to cover all major industry sectors and expose the portfolio to the major risk factors listed earlier with just 20 securities.

In the age of free information, widely available over the internet, the importance of the quality of the data is often overlooked. In finance, one single incorrectly adjusted stock split eight years ago can completely throw the results of an analysis off the mark.

It is the accuracy and cleanliness of the data that distinguishes a professionally usable dataset from a mere heap of useless numbers. Hence, in the present study a significant effort was devoted to the cleaning of the historical data downloaded from the internet (Yahoo Finance). The historical database was also designed and assembled to serve as a basis for further analysis by other subsequent students.

We use the Value-at-Risk (VaR) measure to assess the risk of the portfolio. The 95% VaR is the largest possible loss which will not be exceeded with a probability more than 5%. In other words, over the selected planning horizon, the portfolio value will remain above the 95% VaR threshold with a 95% probability. For the present study the planning horizon is 20 trading days, which roughly corresponds to one month out in the future.

VaR was first developed and introduced in 1995 as part of J.P. Morgan Risk Metrics. Since then, it has gained wide acceptance and is also used by regulators to set reserve requirements for the financial institutions, like banks and insurance companies. Its advantage as a measure of risk over the traditional metrics of (quadratic) variance is that VaR counts only losses as part of risk, while variance weights losses and gains equally. In statistical language, the 95% VaR is the 5% percentile of the probability distribution of the future portfolio values at the end of the planning horizon whereas it is related to the extreme values of the portfolio value distribution and is more difficult to evaluate or estimate than the variance.

We use Monte Carlo simulation to evaluate the Value-at-Risk. For the present Study, we assume that the daily returns of the securities are independent between days and have a correlated joint normal distribution. Under this assumption the value of a portfolio containing no options or other non-linear derivatives will have a log normal distribution. The VaR, at the 5th percentile, of this distribution could be calculated theoretically using a formula. Strictly speaking, under the above conditions the use of Monte Carlo simulations could be bypassed. Nevertheless, the objective of the present study is to develop and demonstrate a methodology that could equally well be used for non-normal (e.g. heavy tailed) return distribution and for portfolios that are hedged with the use of non-linear derivatives.

2. Construction of the Asset Portfolio

The composition of the portfolio is based on the following parameters:

- Relatively few (20ea) securities,
- Availability of long historical price data series (13 years),
- Diversification and coverage of all major asset classes and industry sectors,
- Exposure to all major risk factors,

To satisfy these requirements, we select mostly large mutual funds and companies which themselves are comprised of a diversified set of business (e.g. General Electric).

The major asset classes we wish to cover include

- Domestic U.S. Stocks
- International Stocks
- Bonds
- Commodities and Natural Resources

Within the different asset classes we attempt to diversify among all major industry groups or subclasses. Examples of domestic industry groups are Technology, Manufacturing, Utilities, or Financial sectors. Within the bond asset class we wish to diversify among instruments with short, intermediate and long maturities. In the commodities category we want to cover gold, industrial metals as well as oil and natural gas resources.

We seek exposure to the four major risk factors

- Stock Market Risk
- Interest Rate Risk
- Foreign Exchange Risk
- Commodities Risk

This will enable us to achieve risk diversification and to benefit from the sometimes negative correlation between these risk factors. To maximize risk exposure, we capped the allowable cash positions which are risk free. This is in accordance with the common requirement that portfolio managers do, in fact, invest the clients funds rather than keep cash. For the purpose of future hedging and risk management studies, historical price data for the following five risk factors were also collected and cleaned.

10Y TSY YLD (^TNX)

13WK TSY YLD NDX (^IRX)

30Y TSY YLD NDX (^TYX)

GBP to USD (GBPUSD=X), exchange pound to dollar

S&P 500 INDEX (^GSPC)

The twenty securities for the portfolio were selected to meet the above requirements .
 Their relative weight within the portfolio was set to satisfy the following numerical upper and lower bounds

- Cash \leq 5%
- U.S. Stocks \leq 50%
- Foreign Stocks \geq 25%
- Bonds \geq 20%

This resulted in the following portfolio composition

<u>SECURITY</u>	<u>TICKER</u>	<u>WEIGHT</u>
American Express NYSE	AXP	2.55%
BP PLC ADR NYSE	BP	4.04%
BT Group PLC ADR NYSE	BTY	7.69%
Dodge & Cox Stock	DODGX	4.02%
Duke Energy NYSE	DUK	1.36%
Fidelity Balanced	FBALX	10.57%
Fidelity Contrafund	FCNTX	8.42%
Franklin Federal Tax-Free Income A	FKTIX	2.08%
Fidelity Destiny II	FDETX	12.56%
Gabelli Growth	GABGX	4.63%
General Electric NYSE	GE	1.63%
General Motors NYSE	GM	2.37%
Harbor International Instl	HAINX	8.11%
Janus	JANSX	1.89%
Merrill Lynch Basic Value I	MABAX	1.85%
Neuberger Berman Partners Inv	NPRTX	2.07%
Scudder Growth & Income S	SCDGX	2.32%
Sequoia	SEQUX	2.73%
USAA Precious Metals and Minerals	USAGX	3.06%
Vanguard Wellesley Income	VWINX	16.05%

On the following pages we supply summaries of each of the securities included in the portfolio.

American Express Co (AXP)**American Express Co**

World Financial Center, 200 Vesey Street

New York, NY 10285

Phone: (212) 640-2000

Fax: (212) 619-9230

Email: ronald.stovall@aexp.comWeb Site: <http://www.americanexpress.com/>**DETAILS**

Index Membership:	<u>Dow Industrials</u> <u>S&P 500</u>
Sector:	<u>Financial</u>
Industry:	<u>Consumer Financial Services</u>
Employees (last reported count):	78,200

American Express Company is primarily engaged in the business of providing travel-related services, financial advisory services and international banking services worldwide. Through American Express Travel Related Services Company, Inc. (TRS), the Company offers travel-related products and services including charge cards, card member lending products, travelers checks and corporate and consumer travel services. Financial advisory services and products include financial planning and advice, investment advisory services and various products, including insurance and annuities, investment certificates and mutual funds. Through American Express Bank, the Company provides private, financial institution and corporate banking, as well as personal financial services and global trading.

BP PLC (BP)**BP PLC**

1 St. James's Square

London, EN SW1Y

Phone: (800) 638-5672

Fax: (312) 856-4883

Email: ir@bp.comWeb Site: <http://www.bp.com/>**DETAILS**

Index Membership:	N/A
Sector:	<u>Energy</u>
Industry:	<u>Oil & Gas - Integrated</u>
Employees (last reported count):	115,250

BP p.l.c. is an oil company with four main businesses: Exploration and Production; Gas, Power and Renewables; Refining and Marketing, and Chemicals. Exploration and Production includes oil and natural gas exploration and field development and production, together with pipeline transportation and natural gas processing. Gas, Power and Renewables activities include marketing and trading of natural gas, natural gas liquid, new market development, liquefied natural gas and solar and renewables. The activities of Refining and Marketing include oil supply and trading, as well as refining and marketing. Chemicals activities include petrochemicals manufacturing and marketing. In September 2003, BP merged its Russian assets into those of Tyumen Oil Co. (TNK), creating TNK-BP, a joint venture between the Company and the Alfa Group and Access-Renova (collectively, AAR). In addition, the Company plans to incorporate AAR's 50% interest in OAO Slavneft, a Russian oil company, into TNK-BP.

BT Group PLC (BTY)

BT Group PLC

BT Centre, 81 NewGate Street

London, EN EC1A

Phone: (212) 418-7787

Fax: (212) 418-7788

Email: investorrelations@bt.com

Web Site: <http://www.bt.com/>

DETAILS

Index Membership: N/A

Sector: [Services](#)

Industry: [Communications Services](#)

Employees (last reported count): 104,700

BT Group plc provides telecommunication services, principally in the United Kingdom, and essentially operates as a unitary business. The Company's main services and products are fixed voice and data calls, the provision of fixed exchange lines to homes and businesses, the provision of communication services to other operators, the provision of private services to businesses and the supply of telecommunication equipment for customers' premises. In the United Kingdom, BT serves over 20 million business and residential customers with more than 29 million exchange lines, and provides network services to other licensed operators. The Company consists principally of three lines of business: BT Retail, BT Wholesale and BT Global Services.

Dodge & Cox Stock (DODGX)

Dodge & Cox Stock

Dodge & Cox Funds One Sansome Street 35th Floor San Francisco, CA 94104
Phone: 800-621-3979

FUND SUMMARY

Dodge & Cox Stock Fund seeks long-term growth of principal and income; current income is a secondary consideration. In selecting investments, the fund invests in companies that appear to be temporarily undervalued by the stock market but have a favorable outlook for long-term growth. The fund intends to remain fully invested in equities with at least 80% of assets in common stocks. It may invest in preferred stocks and convertibles. The fund may invest up to 20% of assets in American Depository Receipts. Management seeks companies with financial strength and a sound economic background.

FUND OVERVIEW

Category:	Large Value
Fund Family:	Dodge & Cox
Net Assets:	32.57B
Year-to-Date Return:	5.99%
Yield:	1.33
Morningstar Rating:	★★★★★
Fund Inception Date:	04-Jan-65

OVERALL PORTFOLIO COMPOSITION

Stocks: 89.96%

Bonds: N/A

Preferred: N/A

Convertibles: N/A

Cash: 9.27%

TOP 10 HOLDINGS (23.65% OF TOTAL ASSETS)

Company Symbol % Assets YTD Return %

AT&T CORP T 2.88 -1.33

HEWLETT-PACKARD HPQ 2.70 -1.13

DOW CHEMICAL CO DOW 2.46 4.57

BANK ONE CORP ONE 2.39 18.40

SCHERING-PLOUGH SGP 2.33 3.58

HCA INC HCA 2.23 -0.98

SONY CORP SNE 2.23 18.08

News N/A 2.21 N/A

COMCAST CORP A CMCSA 2.20 -8.66

ELECTR DATA EDS 2.02 -21.40

Duke Energy Corp (DUK)

Duke Energy Corp
526 South Church Street
Charlotte, NC 28202
Phone: (704) 594-6200
Fax: (704) 382-4964
Email: investduk@duke-energy.com
Web Site: <http://www.duke-energy.com/>

DETAILS

Index Membership:	Dow Utilites S&P 500
Sector:	Utilities
Industry:	Electric Utilities
Employees (last reported count):	23,800

REUTERS ABRIDGED BUSINESS SUMMARY

Duke Energy Corporation, an integrated provider of energy services, offers physical delivery and management of both electricity and natural gas throughout the United States and abroad. It provides its services through seven business units: Franchised Electric, which generates, transmits, distributes and sells electricity; Natural Gas Transmission, which provides transportation and storage of natural gas; Field Services, which gathers, compresses, treats, processes, transports, trades, markets and stores natural gas; Duke Energy North America, which develops, operates and manages merchant power generation facilities; International Energy, which develops, operates and manages natural gas transportation and power generation facilities; Other Energy Services, which is engaged in diverse energy businesses, and Duke Ventures, which is composed of other diverse businesses, operating primarily through Crescent Resources, LLC and Duke Capital Partners, LLC.

Fidelity Balanced (FBALX)

Fidelity Balanced
82 Devonshire Street Mailzone Z1c Boston MA 02109
Phone: 800-343-3548

FUND OVERVIEW

Category:	Moderate Allocation
Fund Family:	Fidelity Group
Net Assets:	10.88B
Year-to-Date Return:	3.34%
Yield:	1.51
Morningstar Rating:	★★★★
Fund Inception Date:	06-Nov-86

FUND SUMMARY

Fidelity Balanced Fund seeks income and capital growth consistent with reasonable risk. The fund invests approximately 60% of assets in stocks and other equity securities and the remainder in bonds and other debt securities, including lower-quality debt securities, when its outlook is neutral. It invests at least 25% of total assets in fixed-income senior securities, including debt securities and preferred stock.

Holdings As of 31-Jan-04 Get Holdings for:

OVERALL PORTFOLIO COMPOSITION

Stocks: 65.63%
Bonds: 30.05%
Preferred: N/A
Convertibles: N/A
Cash: 3.36%

TOP 10 HOLDINGS (15.47% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return	%
FNMA N/A	3.61	N/A	
FNMA N/A	2.64	N/A	
CITIGROUP C	2.07	7.37	
FNMA N/A	1.59	N/A	
WEATHERFORD INTL WFT	1.06	16.75	
BANK OF AMERICA BAC	0.97	1.66	
US Treasury Note 5%	0.92	N/A	
AMER INTL GROUP AIG	0.89	7.74	
US Treasury Note 6%	0.89	N/A	
PRIDE INTL PDE	0.83	-8.48	

Fidelity Contrafund (FCNTX)

Fidelity Contrafund

Fidelity Investments Company 82 Devonshire Street Boston MA 02109
Phone: 800-343-3548

FUND OVERVIEW

Category:	Large Blend
Fund Family:	Fidelity Group
Net Assets:	37.68B
Year-to-Date Return:	3.75%
Yield:	0.10
Morningstar Rating:	★★★★★
Fund Inception Date:	17-May-67

FUND SUMMARY

Fidelity Contrafund seeks capital appreciation. The fund invests primarily in the common stock of companies believed to be undervalued. The types of companies in which the fund may invest include companies experiencing positive fundamental change such as a new management team or product launch or companies that are undervalued in relation to securities of other companies in the same industry.

OVERALL PORTFOLIO COMPOSITION

Stocks: 94.09%
Bonds: N/A
Preferred: N/A
Convertibles: N/A
Cash: 5.15%

TOP 10 HOLDINGS (20.42% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return %
3M COMPANY MMM	3.33	-3.28
BERKSHIRE CL A BRKa	2.92	N/A
AVON PRODS INC AVP	2.47	12.88
COLGATE PALMOLIV CL	2.09	10.61
LOCKHEED MARTIN LMT	1.93	-10.79
ENCANA CORP ECA	1.81	9.59
ZIMMER HLDGS ZMH	1.64	4.80
INTERACTIVECORP IACI	1.52	-6.78
RYANAIR HLDGS RYAAY	1.39	-32.51
NEWMONT MINING NEM	1.32	-3.96

Franklin Federal Tax-Free Income A (FKTIX)

Franklin Federal Tax-Free Income A
One Franklin Parkway San Mateo CA 94403-1906
Phone: 800-342-5236

FUND OVERVIEW

Category:	Muni National Long
Fund Family:	Franklin Templeton Investments
Net Assets:	6.59B
Year-to-Date Return:	2.33%
Yield:	4.60
Morningstar Rating:	★★★★
Fund Inception Date:	07-Oct-83

OVERALL PORTFOLIO COMPOSITION

Stocks: N/A
Bonds: 98.41%
Preferred: N/A
Convertibles: N/A
Cash: 1.59%

TOP 10 HOLDINGS (8.70% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return	%
Dallas Fort Worth Tex Intl Arpt	5.625%	N/A	1.22 N/A
Massachusetts St	5.25%	N/A	1.10 N/A
New York N Y	6.125%	N/A	0.98 N/A
North Carolina Eastn Mun Pwr Agy P	5.75%	N/A	0.92 N/A
Pope Cnty Ark Pollutn Ctl	6.3%	N/A	0.82 N/A
Puerto Rico Comwlth Hwy & Transn Auth	5%	N/A	0.81 N/A
New York N Y City Mun Wtr Fin Auth	5.5%	N/A	0.81 N/A
Massachusetts St Tpk Auth	5%	N/A	0.72 N/A
Golden St Tob Securitization	5.375%	N/A	0.67 N/A
California St	5%	N/A	0.65 N/A

Fidelity Destiny II (FDETX)

Fidelity Destiny II
82 Devonshire St Mailzone Z1c Boston MA 02109
Phone: 800-343-3548

FUND OVERVIEW

Category:	Large Blend
Fund Family:	Fidelity Group
Net Assets:	5.01B
Year-to-Date Return:	-0.37%
Yield:	0.73
Morningstar Rating:	★★★
Fund Inception Date:	30-Dec-85

OVERALL PORTFOLIO COMPOSITION

Stocks: 89.66%
Bonds: N/A
Preferred: N/A
Convertibles: N/A
Cash: 10.07%

TOP 10 HOLDINGS (43.22% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return %
MERCK & CO MRK	6.66	-3.60
JOHNSON&JOHNSON JNJ	6.52	-1.39
MICROSOFT CP MSFT	5.91	-8.91
PROCTER & GAMBLE PG	4.95	5.49
TYCO INTL TYC	4.82	8.16
SBC COMMS SBC	4.43	-4.78
VERIZON COMMS VZ	2.62	5.26
LOCKHEED MARTIN LMT	2.55	-10.79
3M COMPANY MMM	2.52	-3.28
NORTHROP GRUMMAN NOC	2.24	3.3

Gabelli Growth (GABGX)

Gabelli Growth

One Corporate Center Rye NY 10580-1434

Phone: 800-422-3554

FUND SUMMARY

Gabelli Growth Fund seeks capital appreciation; current income is a secondary consideration. The fund invests in a diversified portfolio of readily marketable common stocks and convertibles. It invests primarily in securities that management believes to be undervalued and to have favorable prospects for earnings growth. Other desired characteristics include above-average or expanding market shares, profit margins, and returns on equity. The fund may invest in foreign securities.

OVERALL PORTFOLIO COMPOSITION

Stocks: 99.57%

Bonds: N/A

Preferred: N/A

Convertibles: N/A

Cash: 0.42%

TOP 10 HOLDINGS (30.36% OF TOTAL ASSETS)

Company	Symbol	% Assets	YTD Return %
TIME WARNER INC	TWX	3.81	-6.28
ST STREET CP	STT	3.39	0.38
AMGEN	AMGN	3.29	-5.89
CITIGROUP	C	3.06	7.37
MEDTRONIC INC	MDT	3.04	-1.62
MICROSOFT CP	MSFT	2.98	-8.91
UNITEDHEALTH GP	UNH	2.93	10.81
TIFFANY & CO	TIF	2.71	-15.44
Merrill Lynch & Company	N/A	2.58	1.83
PFIZER INC	PFE	2.57	-0.35

General Electric Co (GE)

General Electric Co
3135 Easton Turnpike
Fairfield, CT 06828
Phone: (203) 373-2211
Fax: (203) 373-3131
Email: richard.wacker@corporate.ge.com
Web Site: <http://www.ge.com/>

DETAILS

Index Membership:	Dow Industrials S&P 500
Sector:	Conglomerates
Industry:	Conglomerates
Employees (last reported count):	305,000

REUTERS ABRIDGED BUSINESS SUMMARY

General Electric Company is a diversified industrial corporation engaged in developing, manufacturing and marketing a wide variety of products for the generation, transmission, distribution, control and utilization of electricity. Over the years, GE has developed or acquired new technologies and services that have considerably broadened the scope of its activities. The Company's business is divided into the following segments: Aircraft Engines, Commercial Finance, Consumer Finance, Consumer Products, Equipment Management, Insurance, NBC and Power Systems

General Motors Corp (GM)

General Motors Corp
300 Renaissance Center
Detroit, MI 48265
Phone: (313) 556-5000
Fax: (313) 556-5108
Web Site: <http://www.gm.com/>

DETAILS

Index Membership:	Dow Industrials S&P 500
Sector:	Consumer Cyclical
Industry:	Auto & Truck Manufacturers
Employees (last reported count):	326,000

REUTERS ABRIDGED BUSINESS SUMMARY

General Motors Corporation (GM) participates in the automotive industry through the activities of General Motors Automotive, which consists of four regions: GM North America (GMNA), GM Europe (GME), GM Latin America/Africa/Mid-East (GMLAAM) and GM Asia Pacific (GMAP). GMNA designs, manufactures and/or markets vehicles, primarily in North America under the following nameplates: Chevrolet, Pontiac, GMC, Oldsmobile, Buick, Cadillac, Saturn and Hummer. GME, GMLAAM and GMAP meet the demands of customers outside North America with vehicles under the following nameplates: Opel, Vauxhall, Holden, Saab, Buick, Chevrolet, GMC and Cadillac. GM's Financing and Insurance Operations primarily relate to General Motors Acceptance Corporation. It provides consumer vehicle financing, automotive dealership and other commercial financing, residential and commercial mortgage services, automobile service contracts, personal automobile insurance coverage and selected commercial insurance coverage.

Harbor International Instl (HAINX)

Harbor International Instl
One Seagate Toledo OH 43666
Phone: 800-422-1050

FUND OVERVIEW

Category:	Foreign Large Value
Fund Family:	Harbor
Net Assets:	6.19B
Year-to-Date Return:	3.86%
Yield:	1.30
Morningstar Rating:	★★★★
Fund Inception Date:	29-Dec-87

FUND SUMMARY

Harbor International Fund seeks long-term growth of capital. The fund primarily invests in equity securities issued by emerging market companies that have market capitalizations in excess of \$1 billion, typically from at least three countries. It focuses on companies located in Europe, the Pacific Basin and emerging industrialized countries whose economies and political regimes appear more stable.

OVERALL PORTFOLIO COMPOSITION

Stocks: 92.92%
Bonds: N/A
Preferred: N/A
Convertibles: N/A
Cash: 2.84%

TOP 10 HOLDINGS (23.40% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return	%
Bhp Billiton	N/A	3.76	N/A
Canon	N/A	2.41	N/A
Bp PLC	N/A	2.35	N/A
LM ERICS TEL ERICY	2.30	56.84	
China Petro & Chem	N/A	2.23	N/A
Nomura Hldgs	N/A	2.23	N/A
Continental	N/A	2.19	N/A
Richemont Cl A (Unit)	N/A	2.13	N/A
Abn-Amro Hldgs Nv	N/A	1.90	N/A
Novartis	N/A	1.90	N/A

Janus (JANSX)

Janus

100 Fillmore St 2nd Floor Denver CO 80206

Phone: 800-525-8983

FUND SUMMARY

Janus Fund seeks long-term growth of capital. The fund primarily invests in equity securities with growth potential. It generally invests in larger, more established companies. The fund may invest up to 35% of assets in high-yield bonds. It may also invest up to 15% of assets in illiquid investments and without limit in foreign equity and debt securities.

OVERALL PORTFOLIO COMPOSITION

Stocks: 98.51%

Bonds: N/A

Preferred: N/A

Convertibles: N/A

Cash: 1.05%

TOP 10 HOLDINGS (43.20% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return %
COMCAST A SPCL CMCSK	6.92	-36.08
MAXIM INTEGRTD MXIM	6.58	-5.12
LINEAR TECH LLTC	6.18	-11.67
TIME WARNER INC TWX	6.09	-6.28
CISCO SYSTEMS CSCO	4.05	-2.72
WALGREEN CO WAG	3.11	-9.32
UNIVISION COMM UVN	2.97	-16.83
CHARLES SCHWAB SCH	2.44	-1.83
MGIC INV CP MTG	2.43	12.86
UNITED PARCEL B UPS	2.43	-5.95

Merrill Lynch Basic Value I (MABAX)

Merrill Lynch Basic Value I
 P O Box 9011 Princeton NJ 08543
 Phone: 800-995-6526

FUND SUMMARY

Merrill Lynch Basic Value Fund seeks capital appreciation; income is a secondary consideration. The fund primarily invests in undervalued equity securities. It may invest in companies with below average price/earnings ratios but pay above average dividends. The fund primarily invests in companies with market capitalizations greater than \$5 billion but may also invest in small capitalization companies.

OVERALL PORTFOLIO COMPOSITION

Stocks: 94.74%
 Bonds: N/A
 Preferred: N/A
 Convertibles: N/A
 Cash: 5.03%

TOP 10 HOLDINGS (27.15% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return %
EXXON MOBIL XOM	4.13	2.06
WELLS FARGO & CO WFC	4.08	-3.01
CITIGROUP C	3.37	7.37
ROYAL DUTCH PETE RD	2.84	-9.18
BANK ONE CORP ONE	2.53	20.57
UNOCAL CORP DEL UCL	2.40	1.77
TIME WARNER INC TWX	2.04	-6.28
DEERE & CO DE	1.96	6.98
AMER INTL GROUP AIG	1.91	7.74
RAYTHEON CO RTN	1.89	4.33

Neuberger Berman Partners Inv (NPRTX)

Neuberger Berman Partners Inv

605 Third Ave 2nd Fl 605 Third Avenue 2nd Floor New York NY 10158-0006

Phone: 800-877-9700

FUND SUMMARY

Neuberger Berman Partners Fund seeks capital growth. The fund invests mainly in common stocks of mid- to large-capitalization companies. Management screens for a variety of characteristics in a company, including a strong market position relative to competitors, a high level of stock ownership among management, and a recent sharp decline in stock price that appears to be the result of a short-term market overreaction to negative news.

OVERALL PORTFOLIO COMPOSITION

Stocks: 98.41%

Bonds: N/A

Preferred: N/A

Convertibles: N/A

Cash: 1.59%

TOP 10 HOLDINGS (21.26% OF TOTAL ASSETS)

Company	Symbol	% Assets	YTD Return %
CITIGROUP	C	2.69	7.37
BERKSHIRE	CL B BRKb	2.66	10.52
JP MORGAN CHASE	JPM	2.11	15.27
LENNAR	CP CL A LEN	2.07	12.89
CENTEX CORP	CTX	2.03	11.69
AMER INTL GROUP	AIG	2.03	7.74
Merrill Lynch & Company	N/A	2.02	1.83
COUNTRYWIDE	FNCL CFC	1.93	26.75
NVR INC	NVR	1.89	-1.29
PFIZER INC	PFE	1.83	-0.35

Scudder Growth & Income S (SCDGX)

Scudder Growth & Income S
160 Federal St Boston MA 02110
Phone: 800-621-1048

FUND SUMMARY

Scudder Growth & Income Fund seeks long-term growth of capital, current income, and growth of income. The fund invests at least 65% of total assets in equities, mainly common stocks. Although the fund can invest in companies of any size and from any country, it invests primarily in large U.S. companies. The fund does not invest in securities issued by tobacco-producing companies. It may invest in dividend and non-dividend-paying stocks. Management uses bottom-up analysis, looking for companies with strong prospects for continued growth of capital and earnings.

OVERALL PORTFOLIO COMPOSITION

Stocks: 99.67%
Bonds: N/A
Preferred: N/A
Convertibles: N/A
Cash: 0.33%

TOP 10 HOLDINGS (31.40% OF TOTAL ASSETS)

Company Symbol	% Assets	YTD Return %
CITIGROUP C	3.80	7.37
MICROSOFT CP MSFT	3.57	-8.91
PFIZER INC PFE	3.49	-0.35
BANK OF AMERICA BAC	3.35	1.66
GENERAL ELEC CO GE	3.31	-0.88
EXXON MOBIL XOM	2.97	2.06
UNITED TECH CP UTX	2.91	-8.61
Morgan Stanley/Dean Witter	N/A	2.73 -0.56
JOHNSON&JOHNSON JNJ	2.70	-1.39
3M COMPANY MMM	2.57	-3.28

Sequoia (SEQUX)

Sequoia

767 Fifth Ave Suite 4701 New York NY 10153-4798

Phone: 800-686-6884

FUND SUMMARY

Sequoia Fund seeks growth of capital. The fund invests primarily in a limited number of common stocks that it believes have attractive long-term economic prospects relative to their market price. It may also invest up to 15% of assets in foreign securities.

The fund will usually invest cash reserves in U.S. Government securities.

OVERALL PORTFOLIO COMPOSITION

Stocks: 81.53%

Bonds: N/A

Preferred: N/A

Convertibles: N/A

Cash: 18.47%

TOP 10 HOLDINGS (79.16% OF TOTAL ASSETS)

Company	Symbol	% Assets	YTD Return	%
BERKSHIRE CL A	BRKa	33.16	N/A	
FIFTH THR BNCP	FITB	11.57	-5.77	
PROGRESS CORP OH	PGR	11.43	4.83	
TJX CO INC	TJX	7.35	11.55	
MOHAWK INDS	MHK	5.70	16.74	
FASTENAL CO	FAST	2.66	8.25	
EXPEDITORS	EXPD	2.43	4.51	
ETHAN ALLEN	ETH	1.88	-1.24	
WALGREEN CO	WAG	1.63	-9.32	
DOVER CORP	DOV	1.35	-2.09	

USAA Precious Metals and Minerals (USAGX)

USAA Precious Metals and Minerals
9800 Fredericksburg Road San Antonio TX 78288-0227
Phone: 800-382-8722

FUND SUMMARY

USAA Precious Metals and Minerals Fund seeks long-term capital appreciation and protection against inflation. Current income is a secondary objective. The fund normally invests at least 80% of assets in the equity securities of companies principally engaged in gold exploration, mining, or processing. The remainder of assets may be invested in the equity securities of other precious-metals and minerals concerns, and/or in U.S. government obligations having maturities of less than one year.

OVERALL PORTFOLIO COMPOSITION

Stocks: 90.4%
Bonds: N/A
Preferred: N/A
Convertibles: N/A
Cash: 4.59%

TOP 10 HOLDINGS (43.92% OF TOTAL ASSETS)

Company	Symbol	% Assets	YTD Return %
Newcrest Mng	N/A	5.87	N/A
Impala Platinum Hldgs	N/A	5.24	N/A
GLAMIS GOLD	GLG	5.06	5.20
NEWMONT MINING	NEM	4.89	-3.96
FRPRT-MCM GD	FCX	4.09	-6.75
BARRICK GOLD	ABX	4.06	4.71
ABER DIAMOND	ABER	3.91	-13.19
WHEATON RIV MIN	WHT	3.82	14.38
Anglo Amer Platinum	N/A	3.54	N/A
BUENAVENTURA	BVN	3.44	2.19

Vanguard Wellesley Income (VWINX)

Vanguard Wellesley Income

PO Box 2600 V26 Valley Forge PA 19482

Phone: 800-662-7447

FUND SUMMARY

Vanguard Wellesley Income Fund seeks current income consistent with reasonable risk. The fund normally invests at least 65% of assets in income-producing securities, including fixed-income securities and dividend-paying common stocks. Fixed-income securities usually account for approximately 60% of assets and may include government and corporate bonds, and preferred stocks. Bond holdings are usually rated no lower than BBB.

OVERALL PORTFOLIO COMPOSITION

Stocks: 38.71%
 Bonds: 60.19%
 Preferred: N/A
 Convertibles: N/A
 Cash: 1.10%

TOP 10 HOLDINGS (14.28% OF TOTAL ASSETS)

Company	Symbol	% Assets	YTD Return %
EXXON MOBIL	XOM	1.94	2.06
CHEVRONTEXACO	CVX	1.58	2.47
CITIGROUP	C	1.56	7.37
WEYERHAEUSER CO	WY	1.52	3.02
BANK OF AMERICA	BAC	1.47	1.66
SBC COMMS	SBC	1.34	-4.78
VERIZON COMMS	VZ	1.24	5.26
FPL GROUP INC	FPL	1.24	3.15
ROYAL DUTCH PETE	RD	1.20	-9.18
ST PAUL TRAVLRS	STA	1.19	2.19

3. Data Collection and Cleaning

Data sources:

Yahoo finance was the source for the historical data for the securities. The main reason for using Yahoo finance was due to the ease of downloading the historical prices. Other Data sources used were Wall Street Journal web site, Bloomberg.com, and Morning Star. The Wall Street Journal provided historical prices of the U.S. Dollar – British Pound conversion prices. Morning Star and Morningstar.com provide information for the bond funds and the analytical break down of the portfolio as seen below. The portfolio's asset class is seen in the figure 1 above.

Data Cleansing:

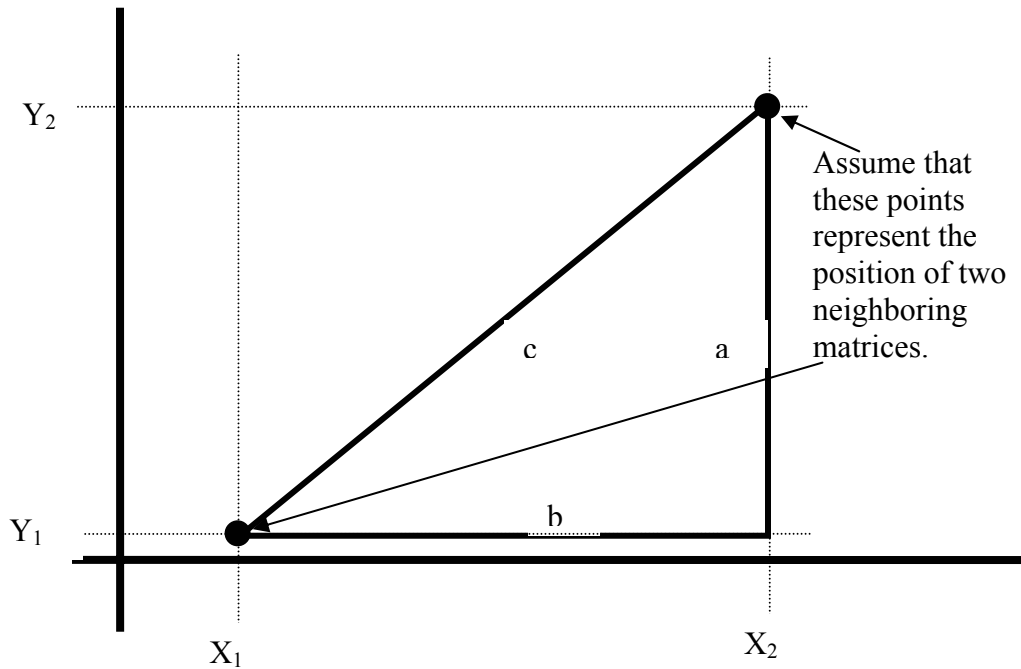
There were many problems with the data down loaded from Yahoo finance. The main problems had to do with the adjusted closing prices that accounted for dividends and stock splits. The adjusted closing price is specifically supposed to account for these things. However, it became apparent that it wasn't always done. Since there was a very large amount of data, and the data was suspect, it is necessary to come up with a quick and simple method of ensuring that the historical data was accurate. To accomplish this "quick" check of the data, the distance formula was used to see if any of the covariance matrices were out of step with the others. Since daily returns were being used, the covariance matrices should be pretty much close to each other. Most of the daily returns were less than 1%. So even a small mistake could make a daily return jump quiet significantly. The result would be that the matrix, where the mistake is, would be much farther from its neighbor, than the others.

Due to the central role covariance matrices play in risk assessment and management, we devised a computerized data screening method based on the estimated covariance matrices. We assume, that under normal circumstances correlations and covariance's between securities change gradually and in a continues fashion. Hence, a sudden jump or other discontinuity in the evolution of the estimated covariance matrices might be consequence of an indicator of some data error that requires closer inspection.

To implement this data verification process, we calculated and plotted the distances between neighboring covariance matrices as time evolved. We used the sum of squared element wise differences as a metric on the space of covariance matrices.

$$\|C(t+1) - C(t)\|^2 = \sum_{j=1}^{20} \sum_{i=1}^{20} [C_{ij}(t+1) - C_{ij}(t)]^2$$

Recall Pythagoras' theorem of right triangles where $c^2 = a^2 + b^2$. Using the figure below:

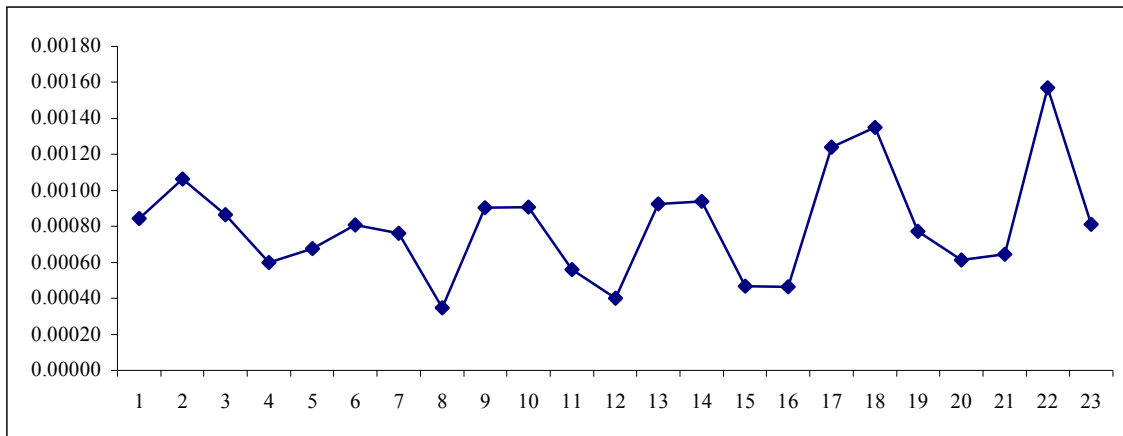


The distance between the points would be $(X_1 - X_2)^2 + (Y_1 - Y_2)^2$. Using this procedure the following results were obtained:

The table below shows the distance between the matrices along with the Standard Deviation, Average, Maximum result and the Minimum result.

	Distance			
1	0.000843	S.D.	0.000300	
2	0.001064	AVE	0.000805	
3	0.000866	MAX	0.001571	
4	0.000598	MIN	0.000346	
5	0.000675			
6	0.000808			
7	0.000760	0.001406	plus 2 S.D.s	
8	0.000346	0.000205	minus two S.D.s	
9	0.000903			
10	0.000908			
11	0.000559			
12	0.000400			
13	0.000926			
14	0.000938			
15	0.000468			
16	0.000465			
17	0.001239			
18	0.001349			
19	0.000771			
20	0.000614			
21	0.000644			
22	0.001571			
23	0.000810			

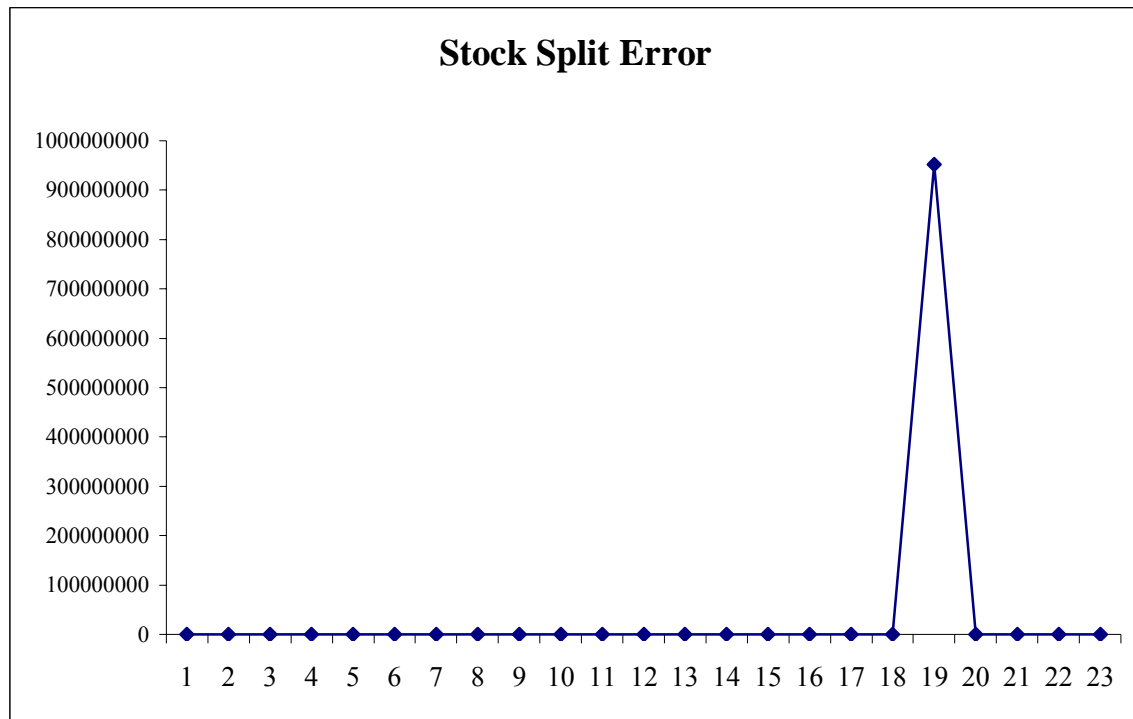
The graph is of the table above.



Just by looking at the above graph it's hard to tell if the data should be considered suspect. However, when we look at the table that gives us the standard deviation of the distances, we can see that all the points, with the exception of one point, are all within two standard deviations of the average. Having at least one out of twenty-three points out of two standard deviations is to be expected. We can also see from the graph that for the most part, the points are relatively in line with each other. So for this data at least, we can say that we don't have to account for any unusual jumps. So what would the data look like if there were in fact inaccurate returns? In other words, what indicators would we get if the data was suspect? By purposely corrupting the data, and then applying the same technique, we can compare the two results to see the differences.

test distance	original distance	AVE	41396753.183736000000
0.000810220018	0.000810220018	MAX	952114452.220719000000
0.001571223116	0.001571223116	MIN	0.000345665196
0.000644245800	0.000644245800	S.D.	198529484.088239000000
0.000614425893	0.000614425893		
0.000770979916	0.000770979916		
0.001349011167	0.001349011167		
0.001239366228	0.001239366228		
0.000464631505	0.000464631505		
0.000467768418	0.000467768418		
0.000937678384	0.000937678384		
0.000559089812	0.000926238029		
0.000400186703	0.000400186703		
0.000926238029	0.000559089812		
0.000467768418	0.000907837334		
0.000467768418	0.000902625480		
0.000902625480	0.000345665196		
0.000345665196	0.000760353891		
0.000760353891	0.000807804640		
952114452.220719000000	0.000675100408		
2698.467026138970	0.000598232902		
2731.472092968220	0.000865691846		
2713.312173621700	0.001064070029		
2727.740217640260	0.000843285536		

The table above shows what happens when a stock split occurs and then the returns are not adjusted. The error that was simulated was one of the stocks split 18 months before the end of the historical data. The split was 2:1. As you can see it quite dramatically shows up in the distance test. The graph is shown below.



You can see that there is quite a difference in the graphs also.

Other simulated data corruption test were performed also. By adding just 1% to a group of adjusted returns, the distance test picked up on the error. So it was concluded that this test was sufficient for ensuring that the data was accurate for our simulation runs.

4. Assessment of Portfolio Risk

1. The notion of Value at Risk

In the present study we will measure the risk of a portfolio by its 95% value at risk 20 days out in the future. As defined in J.P. Morgan's Risk Metric document, this is the largest amount that can be lost by the end of a 20 day period with a probability of no more than 5%. If L_{20} denotes the random variable, expressing the amount lost in 20 days, then $V_{95\%}$ is the largest value for which

$$P(L_{20} > V_{95\%}) \leq 0.05 \quad (1)$$

holds true. We will also use the closely related notion of the 95% VaR threshold $VaR_{95\%}$. The value of the portfolio will remain above the $VaR_{95\%}$ threshold at the end of the 20 day period with a probability of 95%. If P_k denotes the (random) portfolio's value at the end of the k-th day, then the definition of the $VaR_{95\%}$ threshold is

$$P(P_{20} > VaR_{95\%}) \leq 0.95. \quad (2)$$

In the language of probability theory, $VaR_{95\%}$ is the lower 5% percentile value of the probability distribution of the portfolio value 20 days out in the future. Assuming continuous probability distributions, the connection between the above quantities of the portfolio value, loss and risk are

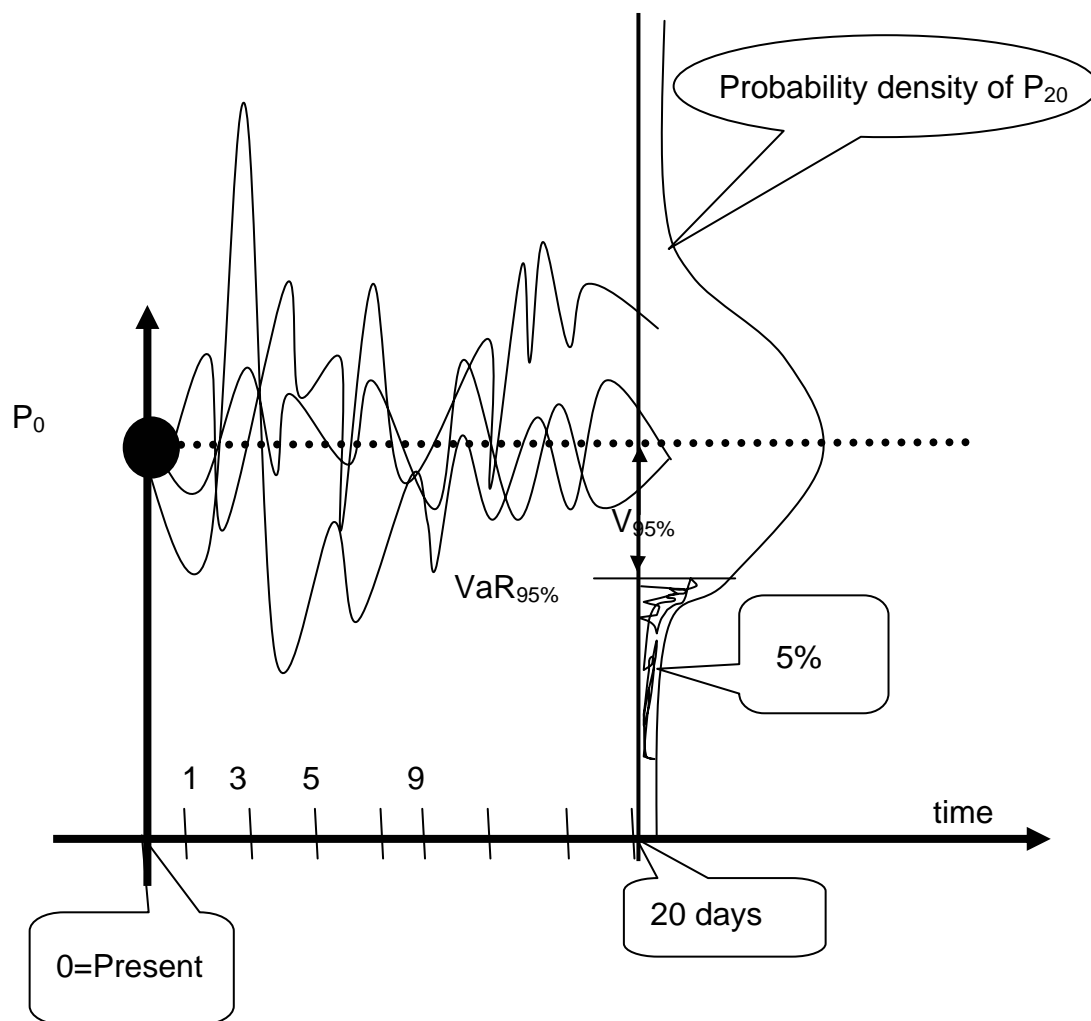
$$L_{20} = P_0 - P_{20} \quad (3)$$

and

$$VaR_{95\%} = P_0 - V_{95\%} \quad (4)$$

Here P_0 is the current known, i.e. nonrandom value of the portfolio. Unless the context requires a clear distinction, the subsequent text will refer the 95% VaR threshold or $VaR_{95\%}$ simply as "the VaR".

The following graph illustrates these ideas visually



2. Concept of VaR

We will evaluate the portfolio risk as measured by the VaR under certain assumptions. Our aim is to develop and test a methodology that could also be used in a more general framework under less restricting assumptions.

Our portfolio is composed of 20 securities. For the present thesis we will assume that the daily returns of the securities have a joint normal probability distribution and that they are independent on different days. As it is customary in risk assessment we will also assume that the mean of the return distributions is zero. As a consequence, the joint distribution of the 20 daily returns is characterized by its 20 x 20 covariance matrix.

The covariance matrix of the daily return is not known. We will estimate it from the historical security prices. Since the variance – covariance structure of the portfolio securities changes over time, we will re-estimate the covariance matrix from quarter to quarter.

If R_k denotes the return of a security on day k , then its price can be calculated according to the following relationship:

$$P_k = P_{k-1} + P_{k-1}R_k = P_{k-1}(1 + R_k) \quad (5)$$

Hence, the price 20 days out in the future will be:

$$P_{20} = P_0(1 + R_1)(1 + R_2)...(1 + R_{20}) = P_0 \prod_{k=1}^{20} (1 + R_k) \quad (6)$$

Assuming the daily returns to be independent and normally distributed with zero mean and known constant covariance, P_{20} will have a log-normal distribution.

Since the value of the portfolio is a linear combination of the prices of its constituent securities, the distribution of the portfolio value will also have a log-normal distribution. In this framework evaluation of the VaR threshold boils down to finding the 5% percentile of a log-normal distribution. This could be done by using the inverse of the cdf of the normal distribution or by numerically interpreting its density function.

In the present study we will not follow this inverse distribution function approach for the following two reasons.

a.) This cannot be generalized to non-normal daily return distributions. It is empirically known that security returns tend to have **fat-tailed**, i.e. non-normal distributions. There are no formulas available for the cdf of sums of fat-tailed distributions.

b. The approach cannot be generalized to portfolio containing securities whose prices depend in a non-linear way on some underlying risk factors or securities. One of the fundamental tools of risk management is the use of options, whose prices are not linear functions of the underlying indices, interest rates, exchange notes or securities.

Instead of the inverse cdf approach, we will use a Monte Carlo method to evaluate the VaR. We outline the steps that are needed to implement this approach.

1. We estimate the covariance matrix of the joint distribution of the 20 securities for one quarter of historical price data. This estimation will be repeated for each of the 24 quarters that we consider.
2. We simulate a 20 dimensional joint normally distributed vector, with a mean of zero and the covariance obtained in (1). This will represent one simulated daily return for each of the 20 securities.
3. We repeat step 2 twenty times independently to generate the daily returns corresponding to the 20 days of the planning horizon. (Recall that our aim is to evaluate the 95% VaR 20 days out in the future.)
4. We use formula (6) to combine the 20 simulated daily returns into the security price, 20 days out. We repeat this for every security. Then we combine the simulated security prices into the portfolio value. This gives one simulated sample of the portfolio value 20 days out in the future.
5. We repeat steps 1-4 2000 times to obtain 2000 simulated portfolio values 20 days out. The distribution of these 2000 simulated values will approximate the distribution of the future portfolio value.
6. We then sort the 2000 simulated portfolio values by size from smallest to largest and estimate the lower 5% percentile of the portfolio value distribution by the 100th smallest of the 2000 portfolio values. This will be the estimated $\text{VaR}_{95\%}$ threshold.

5. Mathematics needed to implement Value At Risk

Calculations

1. MONTE CARLO SIMULATIONS:

The Monte Carlo Method (MC) was used to find the value at risk measurement for this portfolio. This method is an accurate way to calculate the VaR measurement for all financial instruments linear and nonlinear. MC estimates the VaR by simulating random scenarios by using historical data and random variables to calculate the VaR at a specific confidence level.

Using the portfolio with twenty securities we want to calculate the VaR twenty business days from a specific date. Let today's date be December 29, 1995. Using the Monte Carlo Method you want to calculate the Value at Risk twenty business days from now based on the previous business quarter.

1. You start with a 20 x 20 covariance matrix that make up our portfolio.
2. You then generate a 20 dimensional independent standard normal random vector. These standard normal values will have a mean of "0" and a variance of 1.
3. You then must transform this 20 x 1 vector into another 20 x 1 vector which has the covariance matrix of your portfolio. In mathematical terms $N(0, \Sigma)$ if Σ is the covariance matrix. To do this you multiply the 20 x 1, $N(0,1)$ vector by $\sqrt{\Sigma}$ or $N(0, \Sigma) = \sqrt{\Sigma} * N(0,1)$. To get $\sqrt{\Sigma}$ you would use Cholesky decomposition technique that is explained in the next section.
4. The simulated next time interval increase/decrease portfolio security vector is can now be obtained by multiplying the 20 x 1 standard normal vector by the Cholesky matrix. This will give the new vector return ratio. In mathematical terms $N(0, \Sigma) = \sqrt{C}^T * N(0,1)$. This gives us our new return vector. Now we can apply this to the following formula to give our new price for each security:

$$P_0 * \frac{P_1 - P_0}{P_0} = (P_1 - P_0) + P_0 = P_1$$

$$P_0 \left(\frac{P_1 - P_0}{P_0} + 1 \right) = P_1$$

$P_0 =$ old price
 $P_1 =$ new price

$$N(0, \Sigma) = \frac{P_1 - P_0}{P_0}$$

$$\begin{pmatrix} S_0^1 \\ S_0^2 \\ S_0^3 \end{pmatrix} * \begin{pmatrix} x & b & k \\ y & c & l \\ z & d & m \end{pmatrix} + \begin{pmatrix} S_0^1 \\ S_0^2 \\ S_0^3 \end{pmatrix} = \begin{pmatrix} S_1^1 \\ S_1^2 \\ S_1^3 \end{pmatrix}$$

$$\begin{pmatrix} S_0^1 \\ S_0^2 \\ S_0^3 \end{pmatrix} = \text{old price return vector}$$

$$\begin{pmatrix} x & b & k \\ y & c & l \\ z & d & m \end{pmatrix} = N(0, \Sigma)$$

$$\begin{pmatrix} S_1^1 \\ S_1^2 \\ S_1^3 \end{pmatrix} = \text{new price return vector}$$

We keep using this algorithm to get to the desired future date.

To get this twenty business days from our current date, December 29, 1995,

we repeat this 20 times.

$$P_0 \rightarrow P_1 = P_0 * N_{\Sigma}^1 + P_0 \rightarrow P_2 = P_1 * N_{\Sigma}^2 + P_1 \dots$$

$$\dots \rightarrow P_{20} = P_{19} * N_{\Sigma}^{20} + P_{19}$$

P_{20} = Simulated prices of the securities of your portfolio 20 business days from the start date. This is one simulated scenario.

To keep the error as low as possible, you would run as many simulations needed to keep the Standard Error at an acceptable level. In this case you would run 2000 simulations.

5. For each simulated 20×1 security return vector that is 20 business days from the current date you would have to multiply the returns by the weights of the different securities that make up the portfolio. This gives you the simulated portfolio value.

2. CHOLESKY FACTORIZATION

Basic Matrix Algebra was the primary mathematical tool used to find the VaR values. In order to find the VaR values, you have to be able to take the square root of a matrix. This can be achieved by Cholesky factorization. A matrix that is positive definite can have its square found by Cholesky factorization.

A matrix X is said to be positive definite if $bXb' > 0$ for all row vectors $b \neq 0$. A positive definite matrix has a factorization of the form $x=gg'$ where g is a lower triangular matrix. For example, we can find the square root of the matrix below by applying Cholesky Factoring.

$$X = \begin{pmatrix} 8 & -3 & -10 \\ -3 & 14 & 5 \\ -10 & 5 & 18 \end{pmatrix}$$

The Matrix X now takes the form:

$$\begin{pmatrix} 8 & -3 & -10 \\ -3 & 14 & 5 \\ -10 & 5 & 18 \end{pmatrix} = \begin{pmatrix} g_{1,1} & 0 & 0 \\ g_{2,1} & g_{2,2} & 0 \\ g_{3,1} & g_{3,2} & g_{3,3} \end{pmatrix} * \begin{pmatrix} g_{1,1} & g_{2,1} & g_{3,1} \\ 0 & g_{2,2} & g_{3,2} \\ 0 & 0 & g_{3,3} \end{pmatrix}$$

Using basic matrix multiplication rules we can find the square root of x .

1. $g_{1,1}^2 = 8 \Rightarrow g_{1,1} = \sqrt{8} = 2.828$
2. $g_{1,1}g_{2,1} = -3 \Rightarrow g_{2,1} = \frac{-3}{g_{1,1}} = -1.061$
3. $g_{1,1}g_{3,1} = -10 \Rightarrow g_{3,1} = \frac{-10}{g_{1,1}} = -3.536$
4. $g_{2,1}^2 + g_{2,2}^2 = 14 \Rightarrow g_{2,2} = \sqrt{12.874} = 3.588$
5. $g_{2,1}g_{3,1} + g_{2,2}g_{3,2} = 5 \Rightarrow g_{3,2} = \frac{1.248}{g_{2,2}} = .3478$
6. $g_{3,1}^2 + g_{3,2}^2 + g_{3,3}^2 = 18 \Rightarrow g_{3,3} = \sqrt{5.376} = 2.319$

The Cholesky matrix is

$$G = \begin{pmatrix} 2.828 & 0 & 0 \\ -1.061 & 3.588 & 0 \\ -3.536 & 0.348 & 2.319 \end{pmatrix}$$

We can verify our result by using matrix multiplication. This can be done on an Excel worksheet.

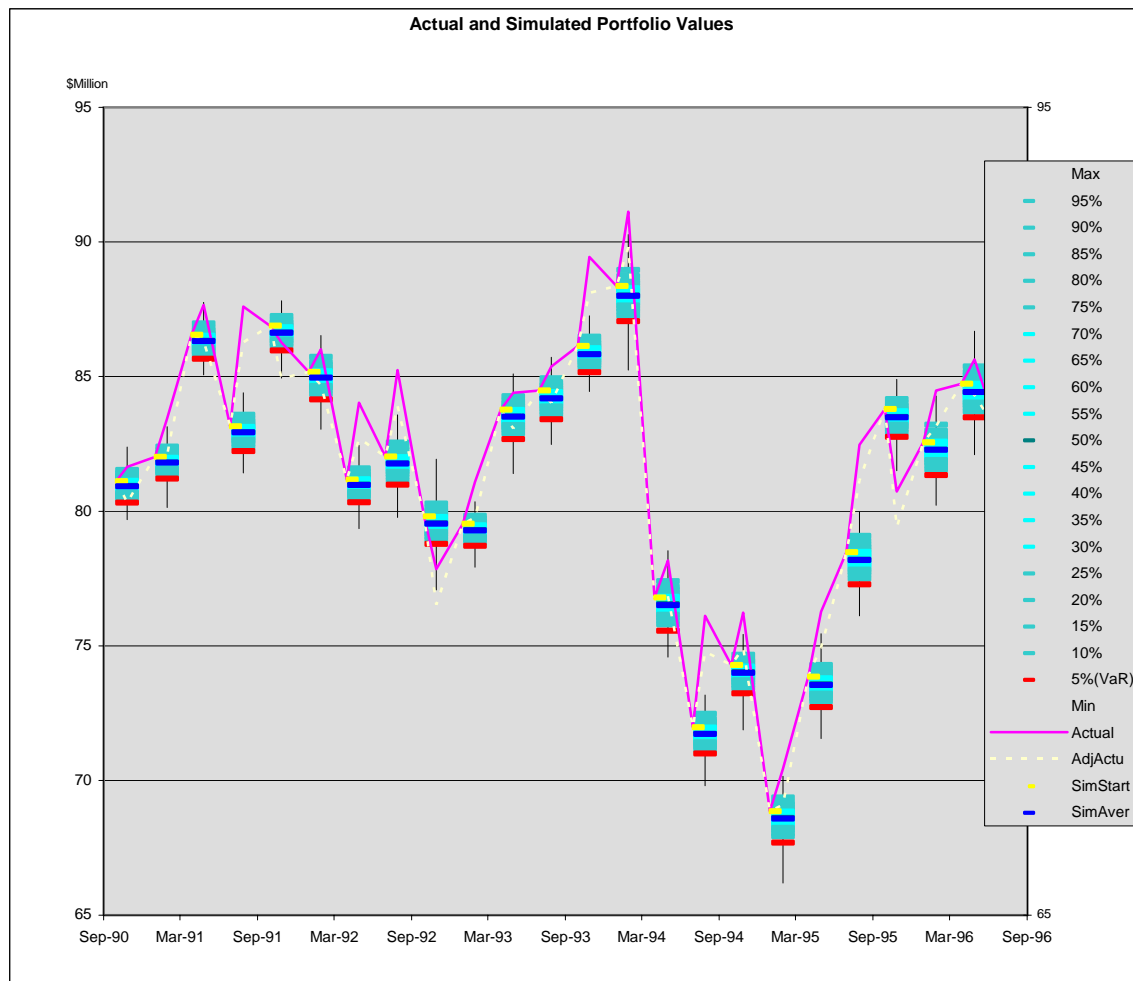
2.828	0.000	0.000	2.828	-1.061	-3.536	8	-3	-10
-1.061	3.588	0.000	0.000	3.588	0.348	-3	14	5
-3.536	0.348	2.319	0.000	0.000	2.319	-10	5	18

Another way to check to see if the result is correct is by multiplying the Cholesky Matrix by the Cholesky matrix transpose. In mathematical terms $g = (\sqrt{g}) * (\sqrt{g})^T$.

As you can see the end result by multiplying the upper Cholesky matrix and lower diagonal matrix gives us our original matrix.

6. Results

The following Box-Whisker graph shows the results for the VaR in all 24 quarters as well as the minimum portfolio simulated vales and maximum values and there relation to each other.



7. References

- 1.) Glyn A. Holden, 2003, “Value-at- Risk”, Academic Press
- 2.) Simon Benninga, 2001, “Financial Modeling”, MIT Press
- 3.) Chandan Sengupta, 2004, Financial Modeling, Wiley Finance
- 4.) Edwin J. Elton, Martin J. Gruber, Stephen J. Brown, William N. Goetzmann, “Modern Portfolio Theory and Investment Analysis”, 2003, Wiley & Sons
- 5.) Xitao Fan, Akos Felsevalyi, Stephen A. Sivo, Sean C. Keenan, “SAS for Monte Carlo Studies”, 2002, SAS Publishing

From the Internet, the following sites were used to collect data

www.yahoofinance.com This was the main source of the security prices and profiles.

www.morningstar.com

www.riskmetrics.com