



The Effects of Speed and Accuracy on Mathematical Fluency

Interactive Qualifying Project 2013

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Increasing studies have shown that the mathematical education of the United States, ranks far below the average of many other countries. This problem results from the early education and teaching of mathematics. One area in particular, math fluency, focuses on the retrieval of basic math facts from storage memory to working memory. This project targeted improvement of math fluency through interactive means. Specifically, a game to teach and improve math fluency was created through GameMaker. The game has not been tested and is only in the developmental stage but it has much promise into becoming useful in the success of math fluency in the future.

Table of Contents

1	Introduction.....	2
2	Background.....	5
2.1	Fluency.....	5
2.2	Types of Fluency.....	6
3	Materials and Methods.....	8
3.1	Introduction to Our Game.....	8
3.2	Our Game.....	9
4	Conclusion.....	12
5	Reference List.....	14
6	Appendix.....	16
6.1	Survey.....	16

1 Introduction

Educators and cognitive scientists both agree that math fluency is critical for students to attain higher-order math skills and abilities. Fluency means how fast a person can retrieve math facts to working memory from storage memory. Math fluency is determined by one's ability to quickly and accurately recall simple math facts or solutions. There are many ways to practice math fluency and attempt to increase one's fluency, the most common practice is constant timed tests administered by teachers in classrooms. At the federal Math Summit in 2003, the importance of mathematical fluency was brought to the forefront by Grover Whitehurst, the Director of the Institute for Educational Sciences. He discussed how cognitive psychologists have discovered that humans have maximum limits in the areas of attention and memory when solving problems. One way around these limits is to make parts of it routine and therefore not require energy to calculate and process. To do so one needs to over-learn math facts so that they become totally fluent, or seemingly automatic. (Math Fluency)

Being fluent in mathematics is no different from having basic fluency in any language. Retrieving basic math solutions for simple addition or subtraction needs to be as automatic as hello or goodbye. Once these basic mathematical skills are mastered the development of more difficult skills becomes easier. If the basic skill is not mastered then the development of these higher order skills such as multiple digit addition, long division and fractions will be severely impaired. There also have been studies that indicate lack of math fluency also causes students to participate less in class and have less confidence in the classroom. This will also negatively affect their learning and growth.

There is also medical research that provides evidence that math fluency training affects the way the mind solves mathematical questions. Recent research in cognitive science, using functional magnetic resonance imaging (fMRI), has revealed the actual shift in brain activation patterns as untrained math facts are learned. (Math Fluency) The fMRI results supported the theory that instruction and practice cause math fact processing to move from a quantitative area of the brain to one related to automatic retrieval. Many experts in the field suggest that this shift frees up space for the student to focus thinking capacity on different aspects of the problem besides the basic calculations associated with medium to high level problems.

Ivon Arroyo and James Royer are two leaders in the research and application of math fluency and mathematical cognitive studies. Their authorship of their study, *Using an Intelligent Tutor and Math Fluency Training to Improve Math Performance*, describes their study and assessment of using web tutoring and math fluency training to improve math achievement for 7th and 8th graders. Their study describes the impact of computer software such as the Wayang Tutoring software which is used in this study, and how it can improve strategic behavior and math fluency. Through using strategic tutoring software and fluency training, subjects were expected to free up memory and cognitive space that would usually be used for the task of setting the problem up and calculating small numbers. With this extra cognitive space the subjects could then focus on other areas of the problem and therefore, also the question with improved accuracy and speed. (Arroyo, Ivon, 2011) Their results did suggest that software interventions to help improve strategy and fluency strengthened the student's mathematical achievement level and can be proven by their improved test scores post-tutor. Their suggestion for

each was that mathematics fluency continues to impact the student's cognitive resources allowing them to have a larger capacity for more difficult problems. They also suggested that for the intelligent tutor they be enhanced with math fluency training, therefore students will be able to make calculations automatically with minimal memory load needed for the actual calculations. (Arroyo, Ivon, 2011)

Their end results in their study suggests a few interesting ideas. The first, that tutoring software is more effective at improving students' learning if part of the testing and work is directed to foundational skill activities, skills that the student is familiar with or have already mastered. This concept is proven by the following; students in the experimental group (Wayang/Fluency) had the highest achievement scores at posttest time, and they also concluded the test in the least amount of time.

Variable	Group			
	Wayang/Fluency	Wayang/No Fluency	NoWayang/Fluency	NoWaya/No Flu
Achievement	65.3	67.9	67.2	63.3
Pretest (%)	(21.7)	(21.2)	(22.0)	(21.3)
Achievement	78.0	67.2	61.7	66.3
Posttest (%)	(21.3)	(20.5)	(27.2)	(18.6)

Figure 1: The figure shows the posttest achievement score for Wayang/Fluency is the highest of the groups with Wayang/nofluency being the second highest posttest score. (Arroyo, Ivon, 2011)

This data is extremely valuable when making the case for the importance of fluency training. Wayang tutoring and other intelligent tutoring software as well as normal in class learning is important and can prove to be successful and helping students achieve high math scores. However, when coupled with fluency training and exercises, students excel even further and achieve higher scores.

2 Background

2.1 Fluency

Math fluency is defined as the “ability to recall basic math facts.” Basic math facts comprise of elementary math problems, including but not limited to single digit addition and subtraction. Many children are able to learn fluency in their first few years of schooling and thus it becomes part of the storage memory of a human.

The importance in becoming fluent in basic math skills is that it can lead to long term mathematical success as well as the ability to learn higher levels of mathematics. Multiplication in its basic form is large repetitions of addition and it is dependent on fluency at a basic level. The importance in developing fluency is also in the idea of being less dependent on outside sources (ie. calculators) for help in simple and standard mathematical functions. By enhancing fluency, one can develop more confidence and less dependence. This, again, creates a more intellectual and more educated body of citizens.

Math, however, is a subject that Americans have been struggling with recently. According to data released by the United States National Assessment of Educational Progress (NAEP), the U.S. only has 39% of fourth graders that are considered proficient or higher in Mathematics (Arroyo, Ivon 2011). The fact that less than half of the fourth graders scored proficient or higher is less than stellar, but what was much worse was that 18% of the fourth graders were considered below basic (Arroyo, Ivon 2011). The effects of this were seen when eighth graders were tested with the same test. For eighth graders, 34% scored proficient or above, while 27% of students scored below basic (Arroyo, Ivon 2011). This data shows that math is not being taught efficiently in both

elementary and middle schools. Without these basic skills, children will be unable to learn more complex subjects and subsequently will fall behind children who have learnt these skills. Another study shows that America is 14th in reading, 17th in science and 25th in math. (Sung-Jun, 2010) This study showed some improvement from the 2003 and 2006 tests but, still had countries such as Hong Kong and South Korea far in front of us scoring wise. (Sung-Jun, 2010) Fluency can be studied to help determine the best way to improve a student's learning and subsequent test scores.

Fluency studies have been performed to some degree throughout the country. Varying results have occurred but in general, these studies have shown large amounts of improvement between those learning fluency skills and those who are not. Another important aspect of fluency is its importance in providing new skills for students with disabilities. Some studies involving fluency implementation have shown a connection with higher learning in those with disabilities when using fluency programs. Experts, such as Ivon Arroyo and Michael Royer have contributed papers based on these studies and their developments will be laid out below.

2.2 Types of Fluency

There are many types of fluency designs that are done to help determine the best method for improving mathematical skills in children. Tests have been done to see if accuracy or speed is more important in fluency, but it is commonly believed that both are important for proper learning. (Ramos-Christian, 2008) A lot of studies have been done towards students with disabilities as well. These tests have been done to see whether or not children with disabilities can learn more using fluency methods than normally. A study done by Singer-Dudek et al. showed fluency was indeed helping

students with disabilities. (Singer-Dudek, 2005) It was done on four students with developmental disabilities and behavioral disorders. The ages of the students were not given unfortunately, but based off of the fact that simple addition and multiplication were used, it can be assumed that the students were most likely in either elementary or middle school. (Singer-Dudek, 2005) The study showed that children that had been a part of their two year study were able to maintain composite skills within one or two months. (Singer-Dudek, 2005) Scores of 80% were seen on the task at hand after two months if the child had been a part of the test, while children who had not been a part of the study had scores less than 80%. (Singer-Dudek, 2005) This is a rather good indication that fluency is important in learning and retention of information.

A slightly later study was done by Poncy et al. on a female child whose age was 10 years at the time. She had an IQ of 44 which is considered to be moderate mental retardation. (Poncy, 2007) The study was done using simple addition for two weeks, using the Cover, Copy, Control (CCC) method. (Poncy, 2007) Her accuracy using the said method increased during the two week test to 90%. (Poncy, 2007) It was also noted that her speed with respect to getting the answers correct increased slightly during this study. (Poncy, 2007) Most notable, however, was the fact that she increased the amount of correct problems by 100% by the end of the study. (Poncy, 2007) Overall, the (CCC) tends to work well in coordination with taped problems, especially for low cognitive functioning students. (Poncy, 2007) It is important to make sure that timing is kept track of during these tests though.

Another study that was done tested 36 kids in middle school, ages 12-15, using the Detect, Practice, Repair (DPR) method. (Axtell, 2009) The study was done on

children without disabilities and took approximately six weeks to complete. The test showed an improvement in accuracy, consistent with similar test completed. (Axtell, 2009) It is believed that this method should not be used on children who are in any school higher than middle school, except for rare circumstances. No testing has been done on children with disabilities using this method, but the next suggested testing according to the study, was going to be done using disabilities children. (Axtell, 2009) Because the method is more independent than other methods, it is also believed that this method will not work on very young students as well. The study also concluded that this test should be used for more complex mathematics rather than basic skills. (Axtell, 2009) In fact, it was suggested that the DPR method would not work with basic skills. This might be due to the fact that the DPR method has a more specific focus than general learning does. The DPR method is mainly used to practice a specific skill for improvement, rather than a general topic. (Axtell, 2009) It was also noted that this study was not done in a large group. It was in fact done in small groups. (Axtell, 2009) There was another method mentioned in this paper called the RTI method. (Axtell, 2009) This method is known as the Response to Stimuli method, and was done in coordination with the DPR method in this study.

3 Materials and Methods

3.1 Introduction to Our Game

The proposed idea for this project was to create a video game which increases math fluency in students with difficulty learning math. The proposed idea was based on the theories that exercise repetitions in a particular field can increase one's speed and ability to process in that particular field. The goal of this game was to increase the

overall abilities and speed of math calculations for children with deficiency in math and fluency.

One game which has been seen on the market is Math Blaster from Blaster Learning System. It is an educational video game in which missions are completed by solving math problems. The game gives “unlimited” amount of time to solve problems which does not contribute to fluency or speed training.

3.2 Our Game

The game created by this project is similar to the *Asteroids* video game by Atari, Inc. In this version of the game, the side bar displays an addition or subtraction equation without the answer. Four asteroids that scroll down have the correct answer as well as three distractor answers. The ship at the bottom of the screen shoots bullets/lasers/rockets that destroy all four answers when the correct answer is selected. When multiple correct answers are selected in a row, then the speed with which the asteroids fall increases. If an asteroid is selected incorrectly, then the rate of the asteroid line slows down. There are three “lives” or chances to select an incorrect answer. The level at which the speed is capped is after 50 correct answers have been made.

The fluency aspect of this game is incorporated in the increasing speed after correct answers. Fluency is increased when the subject is required to select the correct answer in a certain amount of time. Although, at first, the increasing speed can cause incorrect answers, eventually the students will adjust and become more fluent in selecting the correct answer. The time constraint in answer the question “causes” the students to improve in order to continue with the game.

The game consists of waves of asteroids falling from the top of the screen in line with each other. The numbers on the asteroids show the correct answer and three distractors. These numbers are chosen by the program after the equation is chosen. At the start of the game, the player is asked for the range of numbers to be used in the equations. They must input a low number and a high number. Two numbers in this range will be chosen as the operands. Then the code determines the correct answer and generates numbers close to that answer to be distractors. There is also a check that gets rid of any duplicate numbers chosen by the program and finds new ones, so as not to have a distractor that is the same as another distractor or the correct answer.

```
if (operator == "+")
{
    // determine which numbers will drop
    answer = operand1 + operand2

    if (operand1 - operand2 < 0 || answer == operand1 - operand2)
    {
        do
        {
            distractor1 = answer + (irandom(10)-5)
        }
        until (answer != distractor1 && distractor1 > 0)
    }
    else
        distractor1 = operand1 - operand2

    do
    {
        distractor2 = answer + (irandom(10)-5)
        distractor3 = answer + (irandom(10)-5)
    }
    until (distractor2 != distractor3
        && distractor1 != distractor3
        && distractor1 != distractor2
        && answer != distractor2
        && answer != distractor3
        && distractor2 >= 0
        && distractor3 >= 0)
}
```

Figure 2: Coding from the game is shown above. The code above shows how distractors are made using the GameMaker program.

The game was originally going to have four separate sections for addition, subtraction, multiplication, and division. We decided that we would limit it to just the first two, and then we came up with the idea of combining them into one section; the final game allows the player to choose whether to be challenged with addition, subtraction, or both. The type of problem does not affect the increase or decrease in speed at all, and the scoring is the same for both.

Players begin with a score of zero and with three lives. One point is given for every correct answer they hit, and there is no game-ending score: they must continue until they have run out of lives. A life is taken away every time a player fails to hit the correct answer before the asteroids reach the bottom of the screen, or when a player hits one of the incorrect answers. When the player has lost all three lives, the game is over, and the final score is displayed in a message. There is no way to lose points or to gain more lives.

The goal of this game is to make a significant increase to math fluency and to provide an entertaining way to implement it. The first step in this process is to provide a questionnaire or survey to all students that are to be participating in the study. The survey is intended to profile or create an overview of the level of interest and level of competence of each student. The survey requires students to rate their capability in math and to rate how much they are interested in pursuing mathematics. The survey is also planned to assess video game interest and how/what provides entertainment for the students. This is so that the game can be assessed as both an educational capsule and entertainment value. The second step in this process is to implement the “video game” by allowing each student to play it or “test” it out for a given amount of time. This

is to show an idea of how well the initial math skills can be rated. This can also be considered a baseline assessment of individual abilities. In a perfect setting, the next step would be to allow each student the opportunity to use the video game for an extended number of weeks or months to provide a timeline overview of whether the game can produce any improvements to the students' skills.

The hope of this project is to provide some sort of evidence that the game will provide a significant increase for each student to improve their skills and provide an entertaining way in which to achieve it. Math education through video games has been accomplished before but this project is intended to show results that fluency increases and that the entertainment value is enough provide a long lasting effect.

Expectations for the game are that it will be able to be used in classrooms as a way in which to enhance fluency. The educational value that this game can create is both as a fluency tool and an entertainment provider. The entertainment can make it more able for the students to continue playing the game and continue improving fluency.

4 Conclusion

Overall, the testing needs to be completed and continued. The inability to do testing this time around was because of there was not enough time to complete a prototype and to set up a classroom visit. The future projects related to this game may be able to continue with the testing and therefore assess the viability and feasibility of implementing this game into classroom settings. Once testing has been completed, results will be looked at and a newer version of the game would be created. It is a common belief by many of us that customization of certain aspects of the game such as

ship color, background, etc., might help students learn. These features would be available in a later version of the game if at all. Our goal, however, is to improve the way that students learn mathematics, starting with basic skills and working up to more complex skills in the long run.

5 Reference List

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6 Appendix

6.1 Survey

- 1) Are you a BOY or a GIRL? (circle one)
- 2) What are your two favorite subjects in school

- 3) Do you like competition? Having a winner and loser? (circle either YES or NO)
YES NO
- 4) Do you play video games? (Circle either YES or NO)
YES NO (if yes, how often) _____
- 5) What is your hardest subject? _____
- 6) How much time do you spend on homework in a day? (circle one)
0-15mins 30mins-1hour 1hour-2hours greater than 2 hours
- 7) Do you feel that you are above average in math for your age? (circle either YES or NO)
YES NO
- 8) On a scale of 1-10, how difficult is math for you. (1- wicked easy, 10- ridiculousness)
1 2 3 4 5 6 7 8 9 10
- 9) How fun was this game (1- worse than homework, 3- as fun as homework, 5-more fun than homework)
1 2 3 4 5
- 9) Do you feel that educational video games can improve your learning? (circle one)
YES NO
Please explain your answer
- 10) How hard was the game? (1-too easy, 3- just right, 5- too hard)
1 2 3 4 5

11) What changes do you suggest for this game?

Would you like multiplication and division levels?