# **Evaluation Bench for Portfolio Optimization**

A Major Qualifying Project Report

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by

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# Abstract

This project implements a MATLAB based evaluation workbench to assess and compare portfolio management strategies. Daily data for 170 stocks carefully selected to cover the market between Dec 24 1998 and March 31, 2008 forms the basis of the evaluation. Quarterly or yearly rebalancing is implemented. Three strategies are compared in the current implementation – uniform, top quartile and optimized, minimum variance portfolios.

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# Introduction

This project was intended to be a test bench for portfolio optimization – a system to which new strategies can be added such that their efficiency of maximizing value can be measured against other strategies.

The portfolio used in this project was chosen such that it was homogenous and large enough to represent the market and would have enough data history to enable analysis of the data over a period of nine years. The common market indeces, such as the Dow Jones Industrial Average or the S&P500 were not a good choice since in the past nine years they contained companies which went bankrupt during the bust of the internet bubble, other which got bought out or recent IPO's, like Google. These companies do not have long enough historical data to be used on a study that tests trading strategies implemented over the last nine years, since systematic investment in them is impossible.

The trading baseline strategies implemented were to uniformly distribute money over the entire portfolio at the end of each year or quarter or to invest in the top 40 companies of the past year or quarter. Other strategies would be compared to the results provided by these two initial strategies. In order to exemplify this, a third strategy was implemented - portfolio optimization to minimize risk subject to a minimum required return which was compared to the previous two.

# **1.** Portfolio Selection

### <u>Goals</u>

In order to attempt finding a diversified portfolio that would clearly exhibit the results of different optimization methods, a strategy needed to be devised to help with the choice of stocks composing the testing portfolio. The goal was to choose a portfolio that would satisfy the properties:

- large enough
- long enough history
- homogenous portfolio

The size of the portfolio was chosen such that it is **large enough** to represent the entire industry, but small enough to make repeated analysis on the data feasible. For example, if the portfolio included 50 stocks, then there would have existed the risk that they were not representing the industry well enough and then the portfolio would have not been uniformly distributed over the industry. In this case results would be skewed because parts of the market and their impact on a portfolio would be ignored. A very large portfolio, for example, one containing 2000 stocks could cover all areas of the industry, but would make computational analysis on the data associated with it take a long time every run, hindering research. This is why a portfolio of 170 stocks was chosen, picked such that it satisfies all the goals established.

The trading period chosen was from the beginning of 1999 to the end of the first quarter in 2008. For the data to be analyzed and in order to avoid large amounts of missing data giving inconclusive results, the companies in the portfolio were chosen such that they had **long enough historical data**, their first trading day needing to be before the end of 1998.

The portfolio was chosen in such a way that it is **homogenous** over market capitalization (its total stock market value), style (value, growth and blend) and market sectors such that the results would not be affected by investment in only one or a handful of areas of the market. This way the portfolio is affected only by market forces, and not by events that affected only small areas of the market.

### The Market Universe for the Present Study

The choice of stocks was made from the 2000 monitored by Morningstar, Inc. (www.morningstar.com). Morningstar is an investment research company whose financial data is often used by financial advisors and analysts. The way this company divides the stocks traded in the New York Stock Exchange is by placing each in one of nine categories, based on Value, Core and Growth, versus Small, Medium or Large market capitalization. Using this classification, the stocks were divided into the following nine categories:



Figure 1. Morningstar division table for the NYSE traded companies

These 2000 stocks constitute the "universe" out of which a data sample of stocks would be chosen to form the portfolio.

Restricting the portfolio to contain stocks which have enough historical data (going back to December 24'th, 1998) was a necessary condition. The reason this date was chosen has to do with the trading strategy. The year 1999 was considered a "buffer year", a period in which enough data is gathered in order to start making more informed investment decisions. This data is passed to a Matlab program written for this project, called the *Trader*. The trading period lasts 5 days, which means that the sum invested in the portfolio would be spread out evenly into five days such that to avoid a lucky or unlucky trading day that would affect the data. In order to start the new quarter/annual time periods from Jan 1<sup>st</sup> 1999, the last five trading days of 1998 were also included in the data. Having a long enough history assures that the analysis performed on the stocks is conclusive enough to affect trading. The downside to this is that it creates a bias – companies with new public offerings, bought out or bankrupt are not contained.

Stocks are also divided by market sectors, falling in one of the following categories: Basic Materials, Conglomerates, Consumer Goods, Financial, Healthcare, Industrial Goods, Services, Technology and Utilities. In each market sector there are subdivisions called industries.

C ---- 1 ----- 4 ---

Here is the breakdown of the market sectors:

#### **Basic Materials**

	Congiomerates
Agricultural Chemicals	
Aluminum	Conglomerates
Chemicals - Major Diversified	
Copper	Consumer Goods
Gold	
Independent Oil & Gas	Appliances
Industrial Metals & Minerals	Auto Manufacturers - Major
Major Integrated Oil & Gas	Auto Parts
Nonmetallic Mineral Mining	Beverages - Brewers
Oil & Gas Drilling & Exploration	Beverages - Soft Drinks
Oil & Gas Equipment & Services	Beverages - Wineries & Distillers
Oil & Gas Pipelines	Business Equipment
Oil & Gas Refining & Marketing	Cigarettes
Silver	Cleaning Products
Specialty Chemicals	Confectioners
Steel & Iron	Dairy Products
Synthetics	Electronic Equipment

Farm Products Food - Major Diversified Home Furnishings & Fixtures Housewares & Accessories Meat Products Office Supplies Packaging & Containers Paper & Paper Products Personal Products Photographic Equipment & Supplies Processed & Packaged Goods Recreational Goods, Other Recreational Vehicles Rubber & Plastics Sporting Goods Textile - Apparel Clothing Textile - Apparel Footwear &

#### Accessories

Tobacco Products, Other Toys & Games Trucks & Other Vehicles

#### **Financial**

Accident & Health Insurance Asset Management Closed-End Fund - Debt Closed-End Fund - Equity Closed-End Fund - Foreign Credit Services **Diversified Investments** Foreign Money Center Banks Foreign Regional Banks Insurance Brokers Investment Brokerage - National Investment Brokerage - Regional Life Insurance Money Center Banks Mortgage Investment Property & Casualty Insurance Property Management REIT - Diversified **REIT** - Healthcare Facilities REIT - Hotel/Motel REIT - Industrial **REIT** - Office **REIT - Residential** REIT - Retail Real Estate Development Regional - Mid-Atlantic Banks Regional - Midwest Banks Regional - Northeast Banks Regional - Pacific Banks Regional - Southeast Banks Regional - Southwest Banks Savings & Loans Surety & Title Insurance

#### <u>Healthcare</u>

Biotechnology Diagnostic Substances Drug Delivery Drug Manufacturers - Major Drug Manufacturers - Other Drug Related Products Drugs - Generic Health Care Plans Home Health Care Hospitals Long-Term Care Facilities Medical Appliances & Equipment Medical Instruments & Supplies Medical Laboratories & Research Medical Practitioners Specialized Health Services

#### Industrial Goods

Aerospace/Defense - Major Diversified Aerospace/Defense Products & Services Cement Diversified Machinery Farm & Construction Machinery General Building Materials General Contractors Heavy Construction Industrial Electrical Equipment Industrial Equipment & Components Lumber, Wood Production Machine Tools & Accessories Manufactured Housing Metal Fabrication Pollution & Treatment Controls **Residential Construction** Small Tools & Accessories Textile Industrial Waste Management

#### Services

Advertising Agencies Air Delivery & Freight Services Air Services, Other Apparel Stores Auto Dealerships Auto Parts Stores Auto Parts Wholesale Basic Materials Wholesale Broadcasting - Radio Broadcasting - TV Building Materials Wholesale **Business Services** CATV Systems Catalog & Mail Order Houses Computers Wholesale Consumer Services Department Stores Discount, Variety Stores Drug Stores Drugs Wholesale Education & Training Services Electronics Stores Electronics Wholesale Entertainment - Diversified Food Wholesale Gaming Activities General Entertainment Grocery Stores Home Furnishing Stores Home Improvement Stores Industrial Equipment Wholesale Jewelry Stores Lodging Major Airlines Management Services Marketing Services Medical Equipment Wholesale Movie Production, Theaters

Music & Video Stores Personal Services Publishing - Books Publishing - Newspapers Publishing - Periodicals Railroads Regional Airlines Rental & Leasing Services Research Services Resorts & Casinos Restaurants Security & Protection Services Shipping Specialty Eateries Specialty Retail, Other Sporting Activities Sporting Goods Stores Staffing & Outsourcing Services **Technical Services** Toy & Hobby Stores Trucking Wholesale, Other

#### **Technology**

Application Software Business Software & Services Communication Equipment Computer Based Systems Computer Peripherals Data Storage Devices Diversified Communication Services

Diversified Computer Systems Diversified Electronics Healthcare Information Services Information & Delivery Services Information Technology Services Internet Information Providers Internet Service Providers Internet Software & Services Long Distance Carriers Multimedia & Graphics Software Networking & Communication Devices Personal Computers Printed Circuit Boards Processing Systems & Products Scientific & Technical Instruments Security Software & Services Semiconductor - Broad Line Semiconductor - Integrated Circuits Semiconductor - Specialized Semiconductor Equipment & Materials Semiconductor- Memory Chips Technical & System Software Telecom Services - Domestic Telecom Services - Foreign Wireless Communications

### **Utilities**

Diversified Utilities Electric Utilities Foreign Utilities Gas Utilities Water Utilities

Subdivision into market sectors and the nine Morningstar categories provides two different ways to categorize stocks across the market. The market sector category divides the stocks according to the activity of the company, while the Morningstar style box presented earlier divides the 2000 stocks of the "universe" into nine categories, based on the market capitalization of the company and its style. Combining these two methods of classifying stocks, one can get a uniform portfolio across the market.

Provided with the list of stocks in each sector and industry and also with the list of stocks in each of the Morningstar categories, there was need for the information to be cross-referenced, along with information about the first trading day of the company. Having this data would facilitate uniformly picking out of each of the nine Morningstar

categories around three stocks per Sector pertaining of different Industries. The chosen stocks also needed to have enough history.

# Data gathering

By using a program written in Python for this project, the stocks in each of the nine categories developed by Morningstar have been cross-referenced with the industry and sector information, also adding for each the first trading date. What resulted were nine tables, each containing the stocks from each Morningstar category. Each stock had attached the industry and sector it was part of and its first trading day. The reason why the trading day was added was so that stocks without enough trading history not to be considered towards the portfolio. The first trading day was determined by using another program written for this program that would retrieve all trading history for a given set of stocks.

An average of three stocks was chosen from each industry per table, with the condition that they had historical trading data going back to the end of 1998. This resulted in a portfolio of 170 stocks chosen uniformly over style, market capital, market sectors and with enough historical data.

Once the portfolio was chosen, the tickers were put into a file. A program written in Python for this project created a .csv file with all the adjusted daily closing price of each stock starting from December 24'th, 1998. The data was taken from the finance.yahoo.com site.

This is the content of the portfolio, along with the Sector and first trading day:

				Morningstar
Symbol	Name	Sector	Industry	Category
ADBE	Adobe Systems Inc.	Application Software	Technology	MLGRT
AES	AES Corporation	Electric Utilities	Utilities	MLGRT
AET	Aetna Inc.	Health Care Plans	Healthcare	MLCRT
ATG	AGL Resources Inc.	Gas Utilities	Utilities	MMVLT
AA	Alcoa Inc.	Aluminum	<b>Basic Materials</b>	MLVLT
MO	Altria Group Inc.	Cigarettes	Consumer Goods	MLVLT
	AmerisourceBergen			
ABC	Corporation	Drugs Wholesale	Services	MMCRT
APA	Apache Corporation	Independent Oil & Gas	Basic Materials	MLVLT
AMAT	Applied Materials	Semiconductor Equip/Matrl	Technology	MLGRT
ACI	Arch Coal Inc.	Industrial Metals/Mineral	Basic Materials	MMGRT
ADSK	Autodesk Inc.	Technical/System Software	Technology	MMGRT
AN	AutoNation Inc.	Auto Dealerships	Services	MMVLT
BRL	Barr Pharmaceuticals Inc.	Drugs - Generic	Healthcare	MMCRT
BAX	Baxter International Inc.	Medical Instruments/Supls	Healthcare	MLGRT
BIO	Bio-Rad Laboratories Inc.	Scientific/Tech Instrmnts	Technology	MSCRT
		Networking & Commun		
BBOX	Black Box Corporation	Dvcs	Technology	MSVLT
BWA	BorgWarner Inc.	Auto Parts	Consumer Goods	MMVLT
BYD	Boyd Gaming Corporation	Resorts & Casinos	Services	MMCRT
BRC	Brady Corporation	Business Services	Services	MSCRT
BGG	Briggs & Stratton Corporation	Diversified Machinery	Industrial Goods	MSVLT
BRS	Bristow Group Inc.	Air Services Other	Services	MSVLT
BRKL	Brookline Bancorp Inc.	Savings & Loans	Financial	MSCRT
CBT	Cabot Corporation	Specialty Chemicals	Basic Materials	MMCRT
CDNS	Cadence Design Systems	Technical/System Software	Technology	MMCRT
CWT	California Water Service Group	Water Utilities	Utilities	MSVLT
CBM	Cambrex Corporation	Biotechnology	Healthcare	MSCRT
KMX	CarMax Inc.	Auto Dealerships	Services	MMGRT
		Farm/Construction		
CAE	Cascade Corporation	Machnry	Industrial Goods	MSCRT
	Central European Distribution	Deverence Winer /Distlere	Consumer Coode	MOODT
		Air Delivery/Ereight Succes	Consumer Goods	MAGRI
	CH Robinson Wondwide Inc.	Air Delivery/Freight Svcs	Services	
CHK	Chesapeake Energy Corp.	Independent Oil & Gas	Basic Materials	MLCRI
СВК	Corporation	Apparel Stores	Services	MSCRT
C		Money Center Banks	Financial	
0	Citizens Communications	Telecom		
CZN	Company	Services/Domestic	Technology	MMVLT
CLHB	Clean Harbors Inc.	Waste Management	Industrial Goods	MSGRT
CLX	Clorox Company	Cleaning Products	Consumer Goods	MMCRT
KO	Coca-Cola Company	Beverages-Soft Drinks	Consumer Goods	MLCRT
CL	Colgate-Palmolive Company	Personal Products	Consumer Goods	MLGRT
ED	Consolidated Edison Company	Diversified Utilities	Utilities	MLVLT
	Constellation Energy Group			
CEG	Inc.	Electric Utilities	Utilities	MLCRT
CMI	Cummins Inc.	Diversified Machinery	Industrial Goods	MMCRT

DECK	Deckers Outdoor Corporation	Textile-Apparel Footwr/Ac	Consumer Goods	MSGRT
		Farm/Construction		
DE	Deere & Company	Machnry	Industrial Goods	MLCRT
XRAY	DENTSPLY International Inc.	Medical Instruments/Supls	Healthcare	MMCRT
DO	Diamond Offshore Drilling Inc.	Oil & Gas Drilling/Explor	Basic Materials	MMGRT
DST	DST Systems Inc.	Information/Delivery Svcs	Technology	MMCRT
DD	DuPont EI de Nemours & Co	Agricultural Chemicals	Basic Materials	MLCRT
DY	Dycom Industries Inc.	Heavy Construction	Industrial Goods	MSCRT
SSP	E.W. Scripps Company	Publishing - Newspapers	Services	MMCRT
EXP	Eagle Materials Inc.	Cement	Industrial Goods	MSGRT
EMN	Eastman Chemical Company	Chemicals-Major Diversifd	Basic Materials	MMVLT
EK	Eastman Kodak Company	Photographic Equip/Supls	Consumer Goods	MMVLT
EV	Eaton Vance Corporation	Asset Management	Financial	MMGRT
EE	El Paso Electric Company	Electric Utilities	Utilities	MSCRT
EDS	Electronic Data Systems	Information Tachalay Syca	Tachnology	
EDS		Coo Utilition	Litilition	MMCDT
	Equitable Resources Inc.	BEIT Bosidential	Financial	
ELS	Equity Ellestyle Properties Inc.	REIT - Residential	Financial	MICDT
	Equity Residential	REIT - Residential	Consumer Coode	
	Estee Lauder Companies	Personal Products	Consumer Goods	
FASI	Fastenal Company	General Building Mathais	Industrial Goods	
		All Delivery/Freight Svcs	Services	
FBP	First BanCorp	Foreign Regional Banks	Financial Computer Coordo	MSCRI
F	Ford Motor Company	Auto Manufacturers/Major	Consumer Goods	
F51	Forest OII Corporation	Independent OII & Gas	Basic Materials	
FELE	Franklin Electric Co.		Thoustrial Goods	MSGRI
BEN	Franklin Resources	Asset Management	Financial	
GCI	Gannett Co. Inc.	Publishing - Newspapers	Services	
GPS	Gap Inc.	Apparel Stores	Services	
	Genentech Inc.	Biotechnology	Healthcare	MLGRI
GWR		Railroads	Services	MSGRI
GLF	GulfMark Offshore Inc.	Oil & Gas Equipment/Svcs	Basic Materials	MSCRI
FUL	H.B. Fuller Company	Specialty Chemicals	Basic Materials	MSCRI
HANS	Hansen Natural Corporation	Beverages-Soft Drinks	Consumer Goods	MMGRI
HHS	Harte-Hanks Inc.	Marketing Services	Services	MSVLT
HIG	Group Inc.	Prprty/Casualty Insurance	Financial	MLVLT
HLS	Healthsouth Corporation	Specialized Health Svcs	Healthcare	MSVLT
MLHR	Herman Miller Inc.	Business Equipment	Consumer Goods	MMGRT
HES	Hess Corporation	Oil & Gas Refining/Mrktng	Basic Materials	MLVLT
HPQ	Hewlett-Packard Company	Diversified Computer Sys	Technology	MLCRT
HOV	Hovnanian Enterprises Inc.	Residential Construction	Industrial Goods	MSVLT
IHP	IHOP Corp.	Restaurants	Services	MSCRT
ITW	Illinois Tool Works Inc.	Diversified Machinery	Industrial Goods	MLCRT
IR	Ingersoll-Rand Company Ltd.	Diversified Machinery	Industrial Goods	MLVLT
INTC	Intel Corporation	Semiconductor-Broad Line	Technology	MLCRT
	International Speedway		,,	
ISCA	Corporation	Sporting Activities	Services	MMVLT
10	ION Geophysical Corporation	Scientific/Tech Instrmnts	Technology	MSGRT

JKHY	Jack Henry & Associates	Business Software & Svcs	Technology	MSCRT
		Housewares &		
JAH	Jarden Corporation	Accessories	Consumer Goods	MMCRT
KG	King Pharmaceuticals Inc.	Drug Manufacturers/Other	Healthcare	MMVLT
LANC	Lancaster Colony Corporation	Food - Major Diversified	Consumer Goods	MSVLT
LEH	Lehman Brothers Holdings Inc.	Investmnt Brokerage-Natl	Financial	MLCRT
LXK	Lexmark International Inc.	Computer Peripherals	Technology	MMVLT
LECO	Lincoln Electric Holdings Inc.	Small Tools & Accessories	Industrial Goods	MMCRT
LNC	Lincoln National Corp.	Life Insurance	Financial	MLVLT
LMT	Lockheed Martin Corporation	Aerospace/Defense- Prd/Svc	Industrial Goods	MLCRT
LZ	Lubrizol Corporation	Specialty Chemicals	<b>Basic Materials</b>	MMVLT
MAC	Macerich Company	REIT - Retail	Financial	MMGRT
Μ	Macy's	Department Stores	Services	MLVLT
MAN	Manpower Inc.	Staffing/Outsourcing Svcs	Services	MMCRT
MKL	Markel Corporation	Prprty/Casualty Insurance	Financial	MMCRT
MAS	Masco Corporation	Lumber Wood Production	Industrial Goods	MMVLT
MTZ	MasTec Inc.	Heavy Construction	Industrial Goods	MSVLT
MRK	Merck & Co. Inc.	Drug Manufacturers/Major	Healthcare	MLCRT
	MFA Mortgage Investments		<u> </u>	
MFA	Inc.	Mortgage Investment	Financial	MSVLI
MGEE	MGE Energy Inc.	Diversified Utilities		MSVLI
MCHP	Microchip Technology Inc.	Semiconductor-Specialized	Technology	MMGRT
	Inc.	Drug Manufacturers/Other	Healthcare	MMCPT
MUD	Murphy Oil Corporation	Oil & Cas Refining/Mrktng	Basic Materials	MMCPT
MYCN	Murind Constinuing	Diagnostic Substances	Hooltheare	MSCRT
WITGIN	New Jersey Resources	Diagnostic Substances	Tiealiticale	WISGRI
NJR	Corporation	Gas Utilities	Utilities	MSVLT
NFX	Newfield Exploration Company	Independent Oil & Gas	Basic Materials	MMCRT
NEM	Newmont Mining	Gold	Basic Materials	MLGRT
NKE	Nike Inc.	Textile-Apparel Footwr/Ac	Consumer Goods	MLGRT
NUE	Nucor Corp.	Steel & Iron	Basic Materials	MLCRT
OGE	OGE Energy Corp	Electric Utilities	Utilities	MMVLT
ORI	Old Republic International	Surety/Title Insurance	Financial	MMVLT
DMTC	Parametric Technology	Tachnical/System Software	Technology	MECDT
		Pieteobaology	Healtheare	MACDT
	PDL DIOFIIalitia Inc.	Drug Related Products	Healthcare	
PKGU DI	Pelo Poloh Lauron Corporation	Toytile Apparel Clothing	Concurrer Goode	MMCPT
	Provoir Inc.	Suptration	Regio Metoriolo	
	Plaxall Inc.	Synthetics	basic materials	
	Precision Casipan's Corp.	Diversified Commun Svee	Toobhology	
	Proctor & Comble Company	Personal Products	Consumer Coode	
	Procier & Gamble Company	Personal Products	Consumer Goods	
			Concumer Coode	
	Roz Corporation		Einoncial	
REG	Regency Centers Corporation	KEII - Kelali		
ROD	Reliance Steel and Aluminum		Industrial Goods	
SLE	Sara Lee Corporation	Processed/Packaged	Consumer Goods	IVILVLI

		Goods		
SCG	Scana Corporation	Diversified Utilities	Utilities	MMVLT
	Sierra Pacific Resources			
SRP	Holding Company	Diversified Utilities	Utilities	MMCRT
SLGN	Silgan Holdings Inc.	Packaging & Containers	Consumer Goods	MSVLT
SPG	Simon Property Group Inc.	REIT - Retail	Financial	MLCRT
SFD	Smithfield Foods Inc.	Meat Products	Consumer Goods	MMVLT
BID	Sotheby's Holdings Inc.	Business Services	Services	MMGRT
SEH	Spartech Corporation	Rubber & Plastics	Consumer Goods	MSCRT
S	Sprint Nextel Corporation	Wireless Communications	Technology	MLCRT
JOE	St. Joe Corporation	Real Estate Development	Financial	MMCRT
SBUX	Starbucks Corporation	Specialty Eateries	Services	MLGRT
	Starwood Hotels & Resorts			
HOT	Worldwide Inc.	Lodging	Services	MLGRT
STT	State Street Corporation	Regional-Northeast Banks	Financial	MLCRT
SY	Sybase Inc.	Application Software	Technology	MMCRT
	TD Ameritrade Holding			
AMTD	Corporation	Investmnt Brokerage-Natl	Financial	MMGRT
TEV	Tana Orana di sa	Farm/Construction		MAODT
		Macnnry	Industrial Goods	
		Entertainment - Diversified	Services	MLCRI
I JX	TJX Companies	Department Stores	Services	MLCRI
TG	Tredegar Corporation	Aluminum	Basic Materials	MSCRT
TCI	Triumph Group Inc	Aerospace/Detense-	Industrial Goods	MSCRT
101	TrustCo Bank Corporation of	Flu/Svc	industrial Goods	MOUNT
TRST	NY	Regional-Northeast Banks	Financial	MSVLT
TYC	Tyco International Ltd.	Diversified Electronics	Technology	MLVLT
UNP	Union Pacific Corporation	Railroads	Services	MLCRT
UTX	United Technologies	Conglomerates	Conglomerates	MLCRT
UNH	UnitedHealth Group Inc.	Health Care Plans	Healthcare	MLGRT
UHS	Universal Health Services	Hospitals	Healthcare	MMCRT
UNM	Unum Group	Accident/Health Insurance	Financial	MMVLT
USG	USG Corporation	General Building Matrials	Industrial Goods	MMVLT
		Telecom		
VZ	Verizon Communications Inc.	Services/Domestic	Technology	MLVLT
VSAT	ViaSat Inc.	Communication Equipment	Technology	MSGRT
VPHM	ViroPharma Inc.	Biotechnology	Healthcare	MSGRT
VSH	Vishay Intertechnology	Diversified Electronics	Technology	MMVLT
BER	W.R. Berkley Corporation	Prprty/Casualty Insurance	Financial	MMVLT
WLT	Walter Industries Inc.	Industrial Metals/Mineral	Basic Materials	MSVLT
WMI	Waste Management Inc.	Waste Management	Industrial Goods	MLVLT
WY	Weyerhaeuser Company	Lumber Wood Production	Industrial Goods	MLVLT
WMB	Williams Companies Inc.	Oil & Gas Pipelines	Basic Materials	MLGRT
WTFC	Wintrust Financial Corporation	Regional-Midwest Banks	Financial	MSCRT
WWY	Wm. Wrigley Jr. Company	Confectioners	Consumer Goods	MLGRT
WYE	Wveth	Drug Manufacturers/Maior	Healthcare	MLCRT
XRX	Xerox Corporation	Business Equipment	Consumer Goods	MLCRT

# 2. Data Cleaning

When considering the history of stocks, some stocks had missing or wrong data for certain trading days. A method to fill in this missing data was required such that the missing or wrong data does not affect the trading or analysis.

To prevent errors in the data, rules about correct data were put in place – a spike over 20% in the price of a stock and then a return of over 20% in the span of 3 days was considered erroneous data and was deleted and replaced using the same method for missing data.

This method consists of replacing the missing data by a scaled new data that takes into account the historical variation. In order to keep the randomness of the data, a scalar value drawn from a normal distribution with mean 0 and standard deviation 1 is used.

			Stock i			
	46.9	100.31	15.27	50.09	110.64	Γ
	47.42	93.13	15.1	50.28	107.46	Γ
	47.81	90.4	15.04	49.37	105.89	Γ
	45.89	89.37	15.17	47.04	105.03	
	47.18	95.32	15.19	49.33	107.81	
	49.1	101.46	15.76	49,89	111.53	
- Point2	50.33	103.16	15.84	51.52	118.43	
	50.53	101.08	NaN	50.42	113.56	
	49.56	97.62	NaN	49.34	108.29	
	50.74	99.99	NaN	50.02	118.71	
- Point1	49.09	105.24	15.94	49.99	124.15	
	48.64	103.2	15.75	51.25	124.28	
	48.85	110.33	15.59	50.73	119.33	
	51.55	117.56	15.89	52.82	124.49	
	50.81	111.35	15.76	53.04	119.54	
	51.24	112.39	15.65	52.69	118.19	
	51.11	115.06	15.99	52.86	119.07	
	49.33	109.79	15.65	52.41	116.11	

**Example:** Imagine that for stock *i*, the data set looks like this:

Figure 2. Data Cleaning

The highlighted data is from stock *i*, which has data missing for 3 days. *Point 1* is defined as the last adjusted closing price before the missing data. *Point 2* is the fist data point after the missing data. To be remembered is that newer data comes before older data, which means that in the above example, the stock had an adjusted closing price of 15.94 and four days later recorded a 15.84 closing price. The below formula shows how the new data for the three missing points is calculated:

 $data(k,i) = point_1 + step * (k - (j - numbers_{missing})) + sqrt(volatility) * randn$ 

where  $numbers_{missing}$  represents the number of data points missing, in this case 3, and the formula for *step* is

$$step = \frac{point_2 - point_1}{numbers_{missing}}$$

The volatility is calculated from historical prices and then multiplied with a random number generated from a normal distribution with mean 0 and standard deviation 1. This way the randomness of the data is maintained, keeping the overall volatility and, because of the use of the *step*, keeping the overall trend.

Applying this overall process to any missing data or erroneous points, a new data set was generated, a complete data set with no missing points. This set is the one used for all the analysis.

Examples:

Consider the following data, where there is missing history for the first two stocks. This is only part of the data available for each stock. The volatility of Stock 1 before the missing data is of 1.2996 while the volatility of Stock 2 before the missing data is 12.5987.

Stock1	Stock2	Stock3	Stock4
41.77	43.04	100.21	50.34
42.52	44.22	101.33	50.46
41.76	42.09	99.3	48.23
41.52	42.54	100.18	47.89
NaN	43.25	100.04	48.33
41.83	NaN	101.23	48.78
41.66	NaN	102	49.63
42.15	44.23	100.78	49.14
41.65	43.09	100.05	48.04
40.81	42.3	100.71	47.59
40.78	42.2	101.55	48.77
39.7	42.82	100.05	47.35

After implementing the algorithm presented in this section, the following values resulted:

Stock1	Stock2	Stock3	Stock4
41.77	43.04	100.21	50.34
42.52	44.22	101.33	50.46
41.76	42.09	99.3	48.23
41.52	42.54	100.18	47.89
42.11	43.25	100.04	48.33
41.83	43.6	101.23	48.78
41.66	43.13	102	49.63
42.15	44.23	100.78	49.14
41.65	43.09	100.05	48.04
40.81	42.3	100.71	47.59
40.78	42.2	101.55	48.77
39.7	42.82	100.05	47.35

The new data keeps the volatility and the randomized nature of the rest of the data

# **3. Trading Strategy**

# <u>Goal</u>

The purpose of the project was to create a benchmark for trading strategies to be tested on. For exemplification, three trading strategies were chosen: uniformly distributing money over the entire portfolio at the end of every time period, investing in the top n companies(n = 40 was chosen for exemplification) of the portfolio and using Convex Optimization. A time period is considered either a quarter or a year.

# **Trading**

The system simulates the trading of a portfolio with the help of different portfolio optimization strategies. In order to evaluate the efficiency of the portfolio optimization strategy, the entire data is divided into time periods (quarter or year) and, at the end of each time period, by using the portfolio optimization strategy, a redistribution of the current value of the portfolio is done in accordance with the optimization.

The data used is from the current and previous time periods only, the rest of the data being considered unknown. At the end of each time period all the stocks in which money is invested are sold. The resulting value is reinvested by using the trading strategy of the trader's choice and by only using the "past" data (the data from the current period and the previous periods).

## The *m* day trading period

In order to avoid days where the market had results either above or below average, the decision has been made to invest the money in a period of several days. For the examples in this paper it was decided to give m a value of 5. This means that the sum that is to be invested in the portfolio is divided equally in 5 and then, on each day, invested in the portfolio according to the trading strategies.

### Initial Trading

The first trading period is common for all trading strategies. Starting from an initial investment, say \$1,000,000 and the portfolio created in the manner presented above, quarterly and annual investments were made. The same five day trading rule applies here. The initial trading period was chosen to be at the end of 1998, from December 24-th to December 30-st. This way the new quarter and year start from January 1999. The initial investment was divided uniformly in the five days and uniformly within the 170 stocks of the portfolio. This means that each day \$200,000 is invested uniformly over the portfolio (the same amount of money is invested in each company, buying as many stocks of each as the money allows).

## **Further Trading and Trading Strategies**

All the data is divided into quarterly and yearly partitions. This data is sent to the Trader.m file to be reinvested. At the end of each time partition a new five day time period starts. During this, the number of stocks invested in each unit is uniformly sold during a period of five days and, in each of these days, the daily resulted money from the sale is reinvested in the portfolio. This creates a 5 day sell-buy period. Using this data and the data at the end of the previous time period (quarter, year), the following tables were generated:

Yearly and quarterly

- Total Portfolio
- Total Distribution of Stocks
- Total Value per Stock
- Total Gain per Stock
- Total Percent Gain per Stock

What separates the different trading strategies is the way the money is reinvested in each of the five trading days.

### Reinvesting Uniformly Over All Stocks

This method implies uniformly investing each of the five trading day's profit uniformly across the portfolio. At the end of a trading period, on each day of the trading, 1/5 of the stocks is sold and, the total sum resulted from the sale is reinvested uniformly over the 170 stocks at the price of the day. This results in the following portfolio values at the end of each year and quarter:

Yearly Portfolio Value:

Date	Value
12/31/1999	1,284,760
12/29/2000	1,632,686
1/7/2002	2,047,591
1/7/2003	1,964,946
1/7/2004	2,867,571
1/6/2005	3,590,632
1/6/2006	4,375,438
1/9/2007	5,007,642
1/9/2008	5,182,133

Quarterly Portfolio Value:

Date	Value
4/5/1999	1,026,861.35
7/2/1999	1,202,972.41
10/1/1999	1,151,325.59
12/31/1999	1,249,955.89
3/31/2000	1,315,510.88
6/30/2000	1,389,819.57
9/29/2000	1,484,431.99
12/29/2000	1,584,192.95
4/2/2001	1,606,545.48
7/2/2001	1,813,016.45
10/5/2001	1,614,020.22
1/7/2002	1,902,091.99
4/9/2002	2,041,365.53
7/9/2002	1,858,754.26
10/7/2002	1,586,706.55
1/7/2003	1,821,251.24
4/8/2003	1,729,152.93
7/9/2003	2,271,561.69
10/7/2003	2,459,163.69
1/7/2004	2,798,396.71
4/7/2004	2,969,683.03
7/9/2004	2,988,789.06
10/7/2004	3,150,553.36
1/6/2005	3,487,500.34
4/8/2005	3,536,804.58
7/8/2005	3,766,200.33
10/6/2005	3,943,982.18
1/6/2006	4,208,310.96
4/7/2006	4,528,704.25
7/10/2006	4,357,910.71
10/6/2006	4,554,884.41
1/9/2007	4,873,460.53
4/11/2007	5,191,091.5
7/11/2007	5,490,931.8
10/9/2007	5,421,286.23
1/9/2008	4,962,929.55

# • Reinvesting Uniformly Over Top *n* Stocks

Another strategy is investing uniformly at the end of each time period in the top n stocks. The above strategy of uniformly distributing over all stocks is an application of this, with n = 170.

This strategy involves calculating the percent return at the end of the current time period (quarter or year) for each stock and singling out the top n stocks. The Trader will invest then uniformly in these n stocks following the same 5 day trading strategy. For exemplification of this strategy, n was chosen to be 40. The following portfolio values resulted at the end of every quarter and year:

Yearly Portfolio Value:

Date	Value
12/31/1999	1,284,760.23
12/29/2000	1,537,435.5
1/7/2002	1,708,320.76
1/7/2003	1,932,339.27
1/7/2004	2,715,143.56
1/6/2005	3,651,453.93
1/6/2006	4,982,715.92
1/9/2007	5,370,633.99
1/9/2008	5,776,921.03

Quarterly Portfolio Value:

Date	Value	
4/5/1999	1,026,861.35	
7/2/1999	1,180,977.1	
10/1/1999	1,164,115.36	
12/31/1999	1,502,352.24	
3/31/2000	1,682,604.51	
6/30/2000	1,764,676.51	
9/29/2000	2,024,598	
12/29/2000	2,040,107.08	
4/2/2001	2,007,561.01	
7/2/2001	2,338,822.37	

10/5/2001	1,884,074.7
1/7/2002	1,982,570.38
4/9/2002	2,242,480.34
7/9/2002	2,210,408.77
10/7/2002	1,949,542.77
1/7/2003	2,059,621.49
4/8/2003	1,967,801.63
7/9/2003	2,397,642.88
10/7/2003	2,763,322.83
1/7/2004	3,185,694.03
4/7/2004	3,345,854.13
7/9/2004	3,378,322.98
10/7/2004	3,551,027.48
1/6/2005	4,069,085.28
4/8/2005	4,199,760.16
7/8/2005	4,384,896.42
10/6/2005	4,852,948.23
1/6/2006	5,131,824.16
4/7/2006	5,869,688.46
7/10/2006	5,591,078.66
10/6/2006	5,438,122.43
1/9/2007	5,709,734.48
4/11/2007	6,142,800.26
7/11/2007	6,790,026.09
10/9/2007	7,020,910.88
1/9/2008	6,712,656.47

# • Markowitz Portfolio Optimization

A third trading strategy tested on the system was the Markowitz Portfolio Optimization. This method minimizes the return variance (associated with the risk of the portfolio) subject to achieving a minimum acceptable mean return. No short positions are accepted in this implementation, but this feature can be easily added.

At the end of each time period the program calculates the *matrix of daily returns* of each stock in the time period. This is calculated using the formula:

$$daily_{return} = \frac{price_{today} - price_{yesterday}}{price_{yesterday}}$$

The last row is discarded.

The vector  $\overline{p}$  is calculated, where  $\overline{p}$  is the vector of the mean of the daily returns for each stock. The next step is calculating the *matrix of daily centered returns* (P) by subtracting from the *matrix of daily returns* the respective average from the vector.

This means that for stock i, we will subtract from the entire i'th column of the *matrix of* daily returns the i'th element if  $\overline{p}$  ( $\overline{p}$ [i]).

The covariance of P, the *matrix of centered daily returns* can be calculated by either using the built in Matlab function *cov(Matrix)* or by using the formula:

$$\Sigma = \frac{1}{columnNumber(P)} * P^T \cdot P$$

The Trader uses quadratic optimization (the Matlab function quadprog()) to

Minimize  $x^T \Sigma x$ Subject to  $\overline{p}^T x \ge r_{\min}$  $1^T x = 1$  $lb \le x \le ub$ 

starting from x0, where x0 is the previous distribution.

The  $r_{min}$  above was chosen as 0.0001 and 0.0015. The lower boundary (lb) was chosen to be 0 and the upper boundary (ub) was taken to equal 0.025. The lower boundary was so chosen to prevent overinvestment in one asset. This way at least 40 stocks will be invested in.

The following portfolio values were obtained for the yearly and quarterly time period:

Date	Value	Value
Date	1 <sub>min</sub> =0.0001	1 <sub>min</sub> =0.0010
12/31/1999	1,284,760	1,284,760
12/29/2000	1,483,025	1,468,114
1/7/2002	1,986,954	1,939,264
1/7/2003	2,281,612	2,270,505
1/7/2004	2,755,897	2,983,702
1/6/2005	3,673,209	4,038,005
1/6/2006	4,366,913	4,990,428
1/9/2007	4,980,195	5,489,648
1/9/2008	5,043,998	5,646,559

Yearly Portfolio Value:

# Quarterly Portfolio Value:

	Value	Value
Date	r <sub>min</sub> =0.0001	r <sub>min</sub> =0.0015
4/5/1999	1,026,861	1,026,861
7/2/1999	1,170,161	1,182,884
10/1/1999	1,131,227	1,144,729
12/31/1999	1,155,527	1,276,501
3/31/2000	1,208,470	1,317,036
6/30/2000	1,246,045	1,361,044
9/29/2000	1,313,691	1,422,917
12/29/2000	1,397,930	1,512,026
4/2/2001	1,444,735	1,559,966
7/2/2001	1,591,195	1,725,698
10/5/2001	1,577,573	1,712,257
1/7/2002	1,706,924	1,828,181
4/9/2002	1,865,212	1,998,096
7/9/2002	1,835,137	1,972,306
10/7/2002	1,735,465	1,753,527
1/7/2003	1,865,153	1,836,435
4/8/2003	1,801,853	1,801,671
7/9/2003	2,124,413	2,052,560
10/7/2003	2,286,382	2,170,045
1/7/2004	2,539,765	2,408,587
4/7/2004	2,730,409	2,584,606
7/9/2004	2,804,207	2,633,172
10/7/2004	2,753,587	2,634,782
1/6/2005	2,976,855	2,855,994
4/8/2005	3,130,753	2,887,313

3,238,100	2,948,680
3,242,820	3,023,356
3,376,832	3,132,960
3,255,932	3,258,671
3,146,151	3,102,340
3,404,975	3,223,487
3,597,615	3,392,846
3,785,716	3,569,433
3,893,868	3,684,174
3,845,893	3,732,415
3,619,739	3,535,405
	3,238,100 3,242,820 3,376,832 3,255,932 3,146,151 3,404,975 3,597,615 3,785,716 3,893,868 3,845,893 3,619,739

## 4. Conclusions

In order to compare the different trading strategies, one must analyze what the value of the portfolio in time and its standard deviation (representing the risk of the portfolio) are for the different trading approaches. The comparison was done both yearly and quarterly. The Markowitz Optimization was taken with the minimum return of 0.0015.



## **Yearly Trading**

Figure 3. Yearly Portfolio Values

Since the data used spans only over nine years and trading is performed at the end of each year, there are only nine implementations for each strategy. Through the year the distribution of stocks in the portfolio remains unchanged. Figure 3 represents the yearly portfolio values of each stock. For this plot the value of the portfolio was calculated monthly. Both the Optimized and the Top40 strategies have similar returns at the end of the nine year period. On the other hand, the Optimized distribution of stocks generated a higher average portfolio value. The uniform distribution was the least successful strategy, generating overall the smallest portfolio value.

Figure 4 represents the standard deviation of each strategy when it was implemented yearly. It is a representation of the risk of each trading strategy. Since the Markowitz Optimization strategy is designed to minimize risk while achieving a minimum return, it was most likely to have the lowest overall standard deviation. As seen in the below figure, the optimized strategy had the lowest standard deviation, while the Top 40 strategy was overall the riskiest. The reason why the Top40 strategy is the riskiest is because of limited diversification. The other strategies distribute the money over more stocks, creating hedging.



### **Quarterly Trading**

The quarterly distribution involves 36 different instances of implementation of each strategy. On the other hand, since most of the trading decisions are made by taking into consideration the data from the current quarter and the previous one, there is less data to base the decision on.

Figure 5 represents the portfolio values when trading has been done at the end of each quarter. The top 40 strategy was throughout the 9 years the one with the highest returns. The Markowitz Optimization had the lowest returns.



Figure 5. Quaterly Portfolio Values

As seen in Figure 6, which represents the standard deviation of each strategy, the Markowitz Optimization has the lowest risk of all. The riskiest strategy remains the Top40 one.



Figure 6. Quarterly Standard deviation

### **Final Conclusions**

This system can be used to produce the data needed to compare trading strategies. The results reached by analyzing the data generated by the three implemented strategies are consistent with the fact that portfolios with lower risk generate lower return, while portfolios with higher risk generate higher return. Investing quarterly resulted in generating on average the highest returns since it allowed strategies to respond to the market and better correct themselves.

This project could be extended by adding other trading strategies. Better comparison tools for analyzing the differences between trading strategies could be added, Also, the algorithm could be tested on a larger set of stocks.

Annex 1 – Tables and Matlab code – The Data tables and the Matlab code is available and has been submitted separately.