Improving U.S. Foster Care Service Allocation

A Major Qualifying Project submitted to the faculty of

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Abstract

The goal of this MQP is to improve the allocation of services for the United States foster care system. The current foster care system is strained, which is in part due to the opioid epidemic that has impacted the nation as a whole. Our team has investigated the use of predictive and prescriptive analytics to improve the service allocation of the foster care system. Through careful estimation of the effects of various factors (including allocated services) on time in the system for each foster child, along with our proposed mechanism to reallocate such services, our results demonstrate that there is significant potential to improve the foster care system as a whole, and should be carefully considered for future implementation.

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Authorship

This report was equally written by Benjamin Diefendorf, Riley Doherty, Ryan Tropeano, and Kameron Yagoobi, with the exception of the IE Reflection in Chapter 6, which was written by Benjamin Diefendorf, Riley Doherty, and Kameron Yagoobi as part of the Industrial Engineering degree requirements of the Foisie Business School. While sections of this paper were drafted individually, the final report was revised and edited in a combined effort.

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Chapter 1: Introduction

The impact of the opioid epidemic on the foster care system is a unique problem that must first be examined holistically. It is important to gain background knowledge on the opioid epidemic itself, as well as the current state of the foster care system and its associated services, before being able to examine the impact the epidemic has on the foster care system.

1.1 National Opioid Epidemic

The United States has been experiencing an opioid epidemic for a number of years (Center for Disease Control, 2017). As of March 2018, there are an estimated 115 opioid deaths daily in the United States. Nationally the use of all types of opioids (refer to Table 1 below), has increased since pharmaceutical companies pushed for the use of prescription opioids for chronic pain in the late 1990s. At that time, it was thought that these drugs were not addictive. Today we know that opioids are addictive. Research shows that among patients who use opioid prescription medications for more than 30 days and who did not previously use opioids, 47% used opioid-prescribed medications for three years or longer. (National Institute on Drug Abuse, 2018). Figure 1 shows the consistent increase in deaths from prescription opioids since 1999, as well as the more recent drastic increase in deaths from illicit opioids, or non-prescription drugs, such as heroin, beginning in 2010. This rise of deaths from illicit opioid use coincides with the most recent opioid epidemic.

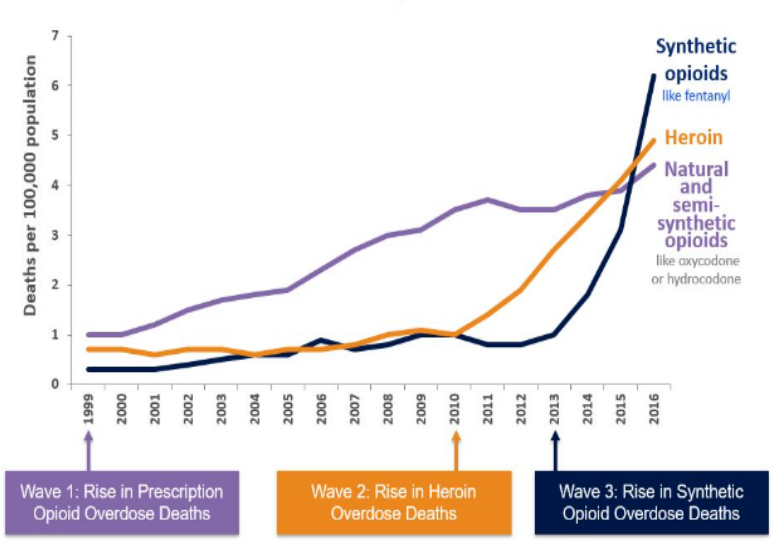

Figure 1 - Rise in Deaths by Opioid Type (National Center for Health Statistics, 2018)

Type of Opioid	Description
Natural Opiates	Are alkaloids, nitrogen-containing base chemical compounds that occur in plants such as the opium poppy
Semi-Synthetic/manmade opioids	Are created in labs from natural opiates. Semi-synthetic opioids include hydromorphone, hydrocodone, and oxycodone (the prescription drug OxyContin), as well as heroin, which is made from morphine.
Fully synthetic/manmade opioids	Are completely man made, including fentanyl, pethidine, levorphanol, methadone, tramadol, and dextropropoxyphene.

Table 1 - Types of Opioids (Bellum, 2014)

The growing availability of prescription opioids as well as illicitly obtained drugs such as heroin is proving to be a public health crisis in the United States. The Centers for Disease Control and Prevention estimates that the total 'economic burden' of prescription opioid addiction alone in the United States is $78.5 billion a year (National Institute on Drug Abuse, 2018), including the costs of healthcare, lost productivity, addiction treatment, and criminal justice involvement.

The opioid epidemic has affected certain areas of the country more than others, such as the Northeast, West, and Midwest of the U.S. (HIDTA, 2017). However, the metric of suspected opioid overdoses per 10,000 visits to the emergency department shows increases in all regions of the United States, from the third quarter of 2016 to the third quarter of 2017. The largest increase has been in the Midwest (69.7%), followed by the West (40.3%), Northeast (21.3%), Southwest (20.2%), and Southeast (14.0%). Some states within each region also had troublesome variations over the same period of time; within the Northeast significant increases occurred in Delaware (105.5%), Pennsylvania (80.6%), and Maine (34.0%). In the Southeast, increases were seen in North Carolina (31.3%). In the West, an increase in Nevada (17.9%) was reported. All the states

in the Midwest reported significant increases, including Wisconsin (108.6%), Illinois (65.5%), Indiana (35.1%), Ohio (27.7%), and Missouri (21.4%). Looking at the demographics within the same report and metric of suspected overdoses due to opioids, shows that no one age range or gender is dominating the amount of apparent opioid overdoses across all classes of opioids. Substantial increases occurred across the board among males (30.2%), females (24.0%), and persons aged 25-34 years (30.7%), 35-54 years (36.3%), and those 55 years or older (31.9%) (Vivolo-Kantor et al., 2018). With this information, it is easy to suggest that no matter the region in the United States, that someone lives in or the demographics of the individual, there is a substantial impact on abusers and their families.

1.2 Current State of Foster Care System

In 2018, there were upwards of 430,000 children in the United States foster care system. These children spend an average of two years in state care, and six percent have been in the system for more than six years (Children's Rights, 2018). Foster care becomes necessary when children do not receive adequate care in their own homes. Such determinations are made by members of the child welfare profession. The most common reason for why children would be removed from their home and place into foster care are abuse and/or neglect.

The purpose of the foster care system is to provide a temporary arrangement where adults can provide care to a child or children whose birth/legal parents are not able to do so. Typically, the goal of the foster care system is reunification with the children's family of origin, but this can be changed to adoption or other developmentally appropriate placements or goals if it is deemed to be in the child's best interest. In situations where reunification is feasible, the child welfare system focuses on matching social, therapeutic, and educational services to the identified needs of the children, parents/caregivers, and family unit (Children's Rights, 2018).

Resources for these children are limited, as it is extremely difficult to find and satisfy the complex needs for each one of these children and their families on a case by case basis. States struggle to allocate their resources effectively and are plagued by low funding as well as increases of children in the foster care system (Kohomban, et al., 2018).

The annual foster care system costs to states are substantial. In 2011, the total costs of maintaining children in foster care were upwards of nine billion dollars (Zill, 2011). The total of maintenance costs and administrative costs per child per year was $25,782 in 2011 as well. With

more and more children entering the foster care system as a result of the opioid epidemic, these costs can only be expected to rise (Kohomban, et al., 2018).

1.3 Effects of Opioid Epidemic on Foster Care System

The opioid epidemic has had a significant impact on the foster care system, because the foster care system was already strained prior to the epidemic. A federal study determined that a 10% rise in overdose deaths correlated to a 4.4% increase in the number of children entering foster care (Ghertner et al., 2018). In 2016, the number of children entering the foster care system due to parental drug abuse reached 92,000, accounting for 34% of the entries into the foster care system (Administration for Children and Families, 2017). This is not the first time that a drug epidemic has caused an increase in demand for the foster care system. The crack cocaine epidemic in the 1980s caused the number of children in the foster care system to reach a high in 1987 (Sengupta, 1999), and due to minimal government intervention at this time, the effects of the crack cocaine epidemic on the foster care system persisted until 2005 when many of the children aged out of the system at the age of 18 (Kohomban, et al., 2018).

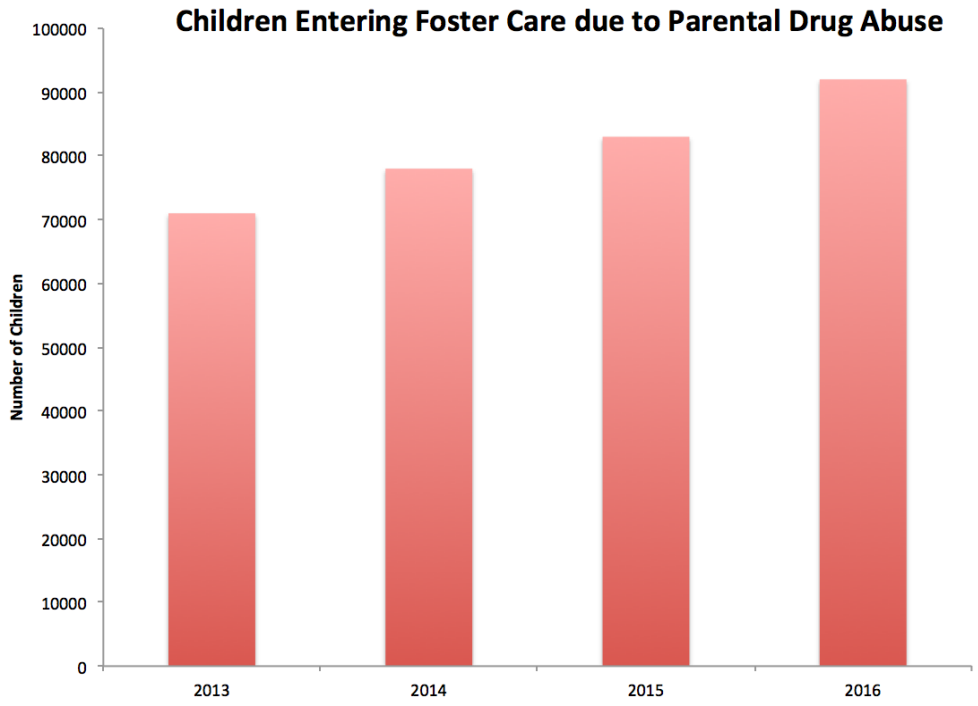

Figure 2- Nationwide Increase in Number of Drug-Related Foster Care Entries (Sedensky, 2017)

Figure 2 shows the increase in proportion of children entering the foster care system due to parental drug abuse from 2013-2016. The increased number of children entering the foster care system is not the only issue created by the opioid epidemic. The long-lasting impacts and the multi-generational nature of the opioid epidemic have decreased the number of children exiting the foster care system through parental reunification and kinship care. Research shows that opioid abuse cases result in parents who are less compliant with court orders and more likely to relapse, resulting in children's placements being longer and a lower chance of parental reunification. When parental reunification is not possible, the foster care system often looks to place children in kinship care, or relative foster care, due to the lack of traditional foster homes. Kinship care has seen a 10% increase in placement between 2006 to 2016 (Child Welfare Information Gateway, 2017), with grandparents or relatives often caring for the child, but the opioid epidemic is threatening the growth in kinship care. The opioid epidemic often affects multiple generations within a family, and case workers have found it difficult to find family members to care for children since many extended family members may also struggle with opioid addiction (Abramo, 2018). With the number of children entering the system increasing, and the number of children exiting the system decreasing, resources within the system have been strained and unable to adequately meet the needs of the children and parents entering the foster care system.

The increasing number of children entering the foster care system has resulted in increased costs for its maintenance. In 2016, the estimated cost of child and family assistance services due to the opioid epidemic alone was $6.1 billion (Rhyan, 2017), which exceeds the entire federal foster care budget of $5.0 billion for 2016 (U.S. Department of Health & Human Services, 2015). With the increasing costs of the opioid crisis, it is critical for the foster care system to use its resources in an efficient manner.

1.4 Providing Services

Programs and services are often offered to parents in an effort to either prevent their child from entering the foster care system, or to reunify with their child after an out-of-home placement. In 2012, 45 states reported that approximately 3.2 million children and families received prevention services (U.S Department of Health and Human Services, 2017). As of 2015, three in five children in the foster care system returned to their parents or other family members (Child Welfare Information Gateway, 2017). Emily Palmer, a *Boston Globe* Spotlight reporter who is

investigating the effect of the opioid crisis on the foster care system, made clear that services that are currently being offered to these parents are generally neither sufficient, nor helpful, because the services do not address all of the problems that the family is facing (Palmer, 2018). Often, these services have limitations that prevent many families from receiving the full breadth of services that are truly needed.

This is supported by research. Parents suffering from opioid addiction tend to have more complex cases that require additional services (Choi & Ryan, 2007). Oftentimes parents suffering from opioid addiction are given access to substance abuse services but are not given access to services to treat the underlying causes of the substance abuse. The National Institute of Drug Abuse identified many factors that can lead to substance abuse, including a history of trauma, limited social skills, and community poverty (National Institute of Drug Abuse, 2018). Without providing services that also attempt to resolve these problems, substance abuse treatment is less likely to be effective. Therefore, when focusing on parents facing substance abuse problems in the foster care system, it is important that effective services are provided that aim to resolve more of the underlying causes of substance abuse as well as receiving substance abuse treatment.

The child welfare system currently offers services that cover a wide range of problems. The most common services, along with a short description, can be found in Table 2 below (National Child Abuse and Neglect Data System, 2016).

Service	Description
Adoption Services	Services provided to assist in preparing a child for adoption
Case Management Services	Close monitoring of the situation by a caseworker to ensure that proper services are being provided and completed
Counseling Services	Counseling provided to the family or individual to improve family function
Education and Training Services	Services provided to the family to improve knowledge or daily living skills
Employment Services	Services provided to assist in securing employment
Family Preservation Services	Services that are typically provided to families that are preparing for reunification to ensure that they receive all necessary services in an appropriate manner
Foster Care Services	Services that include any form of 24-hour substitute care for a child whom the state has responsibility for care
Home Based Services	Services that are provided within their own home in the hopes of preventing the child from entering the foster care system or to aid with reunification
Housing Services	Services to help a family find safe, permanent housing
Mental Health Services	Services to assist in overcoming mental health problems
Substance Abuse Services	Services to help solve substance abuse problems

Table 2 - Common Services Provided

There are a number of issues that prevent families from receiving appropriate services. First, the problems need to be identified for each case by a child welfare caseworker, and the most effective services to address these problems need to be determined. If a problem is not identified by the caseworker, then the family will not receive services for this problem. Once the problems are identified, there is often a lack of resources. Because the opioid crisis typically affects certain regions more strongly than others, the availability of resources within these regions is often limited due to a very high demand for the services.

With the opioid crisis increasing the number of families interacting with the foster care system, it is extremely important for the foster care system to find a way to allocate available services economically to ensure that the right services are provided to families in need in the most effective manner possible.

Chapter 2: Background

Prior to developing a project, we had to conduct a literature review in the field of operations research, with a focus on the foster care system. Then, shortcomings of the current research could be identified, and consolidated to create our project scope.

2.1 Literature Review of Operations Research in Foster Care

There has been extensive research conducted to determine the effects of the opioid crisis on the foster care system. In 2018, the United States Department of Health and Human Services conducted a nationwide study to examine the relationship between opioid-related deaths and foster care entry. The results were significant and determined that a 10% increase in opioid deaths corresponded to a 4.4% increase in children who entered the foster care system. The study also found that substance abuse cases were deemed to be more complex by child welfare workers that were involved in the case than non-substance abuse cases. This complexity is expected to be a result of the large number of problems that were found to also be prevalent in families experiencing substance abuse, such as domestic violence and mental health concerns (Radel, 2018).

These co-occurring problems described in the 2018 federal study are very similar to Choi and Ryan's 2007 study of matching services for mothers involved with the child welfare system living with addiction in the child welfare system. Choi and Ryan focused on a set of substance

abusing mothers in the Illinois area and attempted to match these mothers with the most beneficial services to increase the likelihood of reunification. Similar to the 2018 federal study, Choi and Ryan found that domestic violence and mental health problems were common co-occurring problems, but also determined that inadequate housing, lack of employment opportunities, lack of education, and the need for family counseling were common co-occurring problems amongst these mothers. In many cases, the mothers suffering from these co-occurring problems did not receive treatment for all of the issues, resulting in less than optimum treatment and low rates of reunification. By matching the services based on needs, the study was able to achieve a statistically significant increase in reunification rates (Choi & Ryan, 2007). Many other studies have looked to expand on Choi and Ryan's findings. Grant (2011) used a similar approach to Choi and Ryan's matching method but studied the families for a longer period of time, examining each family for three years. Grant's findings support those of Choi and Ryan, concluding that mothers who receive comprehensive services such as education and training services, employment services, housing services, and mental health services in addition to substance abuse services, were 60% more likely to be caring for their own child at the end of the three-year period (Grant, 2011).

Other studies have been conducted in an attempt to predict reunification rates and reason for exit of the foster care system. Lloyd, Akin, and Brook examined the likelihood of various permanency options for children that entered the foster care system due to parental drug abuse. Using competing risk analysis, the study found that parental drug abuse negatively affected reunification rates for children aged 0-3 but did not have any effect on children over the age of 3 (Lloyd, 2017). This is inconsistent with the findings of Akin et. al. who found that age is significant for all age groups, and that type of drug use is a more significant predictor, with methamphetamine use being a greater barrier to reunification than other illicit drug use (Akin, 2015).

Van Santen sought to build upon these studies by factoring in exit types other than reunification to predict length of stay in foster care. Using a multinomial model, Van Santen found that gender of the child is not a significant predictor of length of stay. One of the few significant predictors that Van Santen discovered was child age at entry, with younger children expected to remain in the system for a longer length of time (Van Santen, 2015).

Russell and Macgill explored factors that most efficiently predict foster care entry rates and how long maltreated children typically stay in foster care. Through the use of predictive analytics, such as classification and regression trees, 104 potential explanatory variables and their

combinations were measured. The amount of child welfare expenditures from state to state was found to be the single most efficient predictor of length of time in care (Russell and Macgill, 2015). That finding suggests that providing services to families will decrease the time in the foster care system as services are one of the highest expenditures for any child welfare agency.

Brook and McDonald reported on the impact of parental substance abuse on the stability of family reunifications from foster care. Their data was restricted to the Oklahoma Division of Children and Family Services during the time period of 1999 through 2003. They isolated all the cases that led to reentry of a child into foster care. As expected, of the four groupings of Alcohol use, Drug use, both Alcohol and other Drug use (AOD), or neither AOD: the group with the highest likelihood of causing reentry amongst parents is the grouping that struggles with both alcohol and drug use (Brook and MacDonald, 2009).

2.2 Shortcomings from Literature Review

The literature shows that co-occurring problems are common amongst parents suffering from opioid addiction, and Choi and Ryan (2007) take a further step in attempting to match services to address these issues. The limitation of these studies is that they do not examine the foster care system as an entire system, rather studying these effects on a case by case basis. With a growing number of children entering the foster care system, it becomes even more important to provide services in an optimal manner since these services are scarce resources. In many cases the need for a service can be determined by the case worker, but often times these services cannot be provided since the finite amount of services are already being consumed by other cases.

Several studies have attempted to predict length of stay in the foster care system but have only examined parental and child characteristics, disregarding services that are provided to these families. The objective of the services is to provide the family with resources that increase the likelihood of reunification. By not including services provided, these studies are missing important predictive factors.

2.3 Project Overview

Building upon the findings from prior research on co-occurring problems in the foster care system, we chose to examine the issue on a system-level rather than an individual-level. By taking

a more holistic approach, we could incorporate other constraints such as the finite amount of certain services that can be provided.

To do so we had to first develop a predictive model that could predict the length of stay in the foster care system as a function of various factors. Using prior research, we were able to incorporate certain characteristics that had been found to be significant in other studies and we also incorporated services that are provided to the family in an attempt to develop a more accurate model. The results of the predictive model were then used in conjunction with an optimization model to effectively allocate services.

By developing an optimization model, we were able to determine the optimal allocation of services for each case, ensuring that each case is only receiving the necessary resources and not consuming excess resources that could be used to assist other cases. The objective of our optimization model was to reduce the overall number of days that are spent in the foster care system. While there are certain fixed costs that exist for each child that enters the system, the main cost of the foster care system is related to the time spent in the system. By decreasing the overall time in the system, we were able to provide insights into how to allocate resources to decrease the overall costs of the foster care system, while also improving reunification rates by providing effective combinations of services through the use of interaction terms in our model.

Chapter 3: Methods

This project was broken down into three different stages. In the initial stage we formatted the data and divided the dataset in to groups for testing. We then developed a predictive model to predict expected days in care for each child and finally developed an optimization model to minimize the total expected days in care as predicted by the predictive model.

3.1 General Data Information

For this research we used the National Child Abuse and Neglect Data System (NCANDS) and the Adoption and Foster Care Analysis and Reporting System (AFCARS) datasets for the years 2010 to 2015. These datasets are managed by Cornell and contain data from all 50 states. The NCANDS dataset contains data for all neglect or abuse cases regardless of whether or not the

child enters the foster care system, while the AFCARS dataset is further limited to only children that enter the foster care system. We were able to combine the two datasets using key child identifiers to access information about each case from the initial abuse or neglect visit throughout the duration of the child's stay in foster care. To join and clean the datasets we used Microsoft Access. The SQL query used for the joining and initial cleaning can be found in Appendix D of this report. The query joins the two datasets on the unique child ID and also converts all categorical variables to multiple binary variables so that the categorical variables can be used in the regression analysis. The script also removes all data entries with a null value for total days in foster care as well as removes all entries for children that have not yet exited the foster care system. This combined dataset provided us with information about the problems that each household faced, as well as the services that the household received while the child was in foster care.

All cases within the datasets of AFCARS and NCANDS are voluntarily submitted by US states. With that said, not every foster care case in the United States is reported, leading to limitations within the datasets of AFCARS and NCANDS. Specifically, to this project, it is very likely that there were foster care cases that involved parental or legal guardian substance abuse that were not included in the datasets. There were a number of cases that had incomplete data, leading our team to omit those cases as they may have created misleading results. Other cases that were omitted were those that had children that were not discharged from the foster care system, as our goal was to capture cases for which we had the "whole picture", thus giving us our final discharged only dataset, and identifying the service(s) that assisted in that child exiting the foster care system. One potentially useful, but absent, piece of information, was the number of times that a child had entered the system. Knowing this would have allowed the predictive model to better forecast the child's number of days in care. Even so, after taking into account all limitations to the final dataset, there were roughly 147,000 children and 60 data points for each case for the years 2010-2015.

This final discharged only dataset was then divided into three different datasets. The years 2010-2014 were randomly divided to develop a Lasso training dataset and a Linear training dataset. Data from the year 2015 was not included in either training dataset and instead the 16,310 records from 2015 were used as a testing dataset to test both the regression model and the optimization model. More information on the datasets used for the analysis can be found in Table 3 below.

Dataset	Years	Number of Entries
Training (Lasso)	2010-2014	61,964
Training (Linear)	2010-2014	69,418
Testing (Regression and Optimization)	2015	16,310

Table 3 – Dataset Descriptions

3.2 Regression Analysis

The purpose of the regression analysis was to develop a predictive model to estimate the total days a child will spend in the foster care system based on certain risk factors, services the child receives, as well as other general information about the child. Because this research is focused on the effects of services depending on the problems that are present, for each case we determined that it was necessary to include interaction terms in our regression model to determine the effect of services provided together or services provided in response to specific problems the child is facing. After adding all possible interaction terms up to the second degree, our dataset contained approximately 1,700 possible features for the predictive model. Due to the large number of features in the dataset, linear regression was not the ideal method due to high multicollinearity and potential overfitting. To avoid these problems, we adopted the Least Absolute Shrinkage and Selection Operator (Lasso) regression technique. The Lasso method is useful for feature selection, as it will determine which features are the most significant predictors and remove all other features as well as remove variables that could lead to multicollinearity problems. Unfortunately, Lasso does not allow for inference testing to determine the significance of each individual variable. Therefore, we adopted a two-phase approach. The first phase used the Lasso method to eliminate a majority of the insignificant features, and the subsequent phase created a linear regression model using the remaining features.

The two-phased method we used for the regression analysis suggests dividing the original dataset into three separate groups. The first two groups, the Lasso training and the linear training groups, each contained half of the children, randomly selected, that were discharged from the foster care system between 2010 and 2014. This random division was made prior to removing any data points that contained null values in the required fields for the analysis. There were more required

variables for the Lasso regression analysis since no variables had been dropped yet from the model. Therefore, when null values were removed the total number of records for the Lasso training dataset was slightly less than the number of records for the linear training dataset. The final group, the testing group, contained all children that were discharged in 2015. The testing group was used to determine the accuracy of the developed predictive model. The model was found to have a margin of error of approximately 20% using the testing dataset. The final regression results are reported in Table 4 in Section 3.2.1 below, and the corresponding regression coefficient values can be found in Appendix D of this report.

3.2.1 Regression Results

The final results of our regression model can be seen in Table 4 below.

Number of Observations	69,418
Degrees of Freedom	53
R-Squared	0.707
Adjusted R-Squared	0.706

Table 4: Regression Results

The results show that our model had more predictors that originally desired with 53, but all predictors included in the final model were significant to the 0.005 significance level. The inference testing results can be found in Appendix D. Our model also had a relatively high R-Squared as well as Adjusted R-Squared values. This shows that our model was fit to the data fairly accurately but could suggest that our model may be overfitting the data. To determine whether or not our model was overfit, we then tested the final regression model with data that was not included in the training data.

3.2.2 Testing Results

The testing results were conducted using data from 2015 from the final discharged only dataset. This data was not included in the regression analysis. The results using the testing data can be found in Table 5 below.

Actual Days in Care	Predicted Days in Care	Margin of Error
6,711,901	8,072,969	20.27%

Table 5: Regression Testing Results

Table 5 shows that after testing our model with testing data, our model is likely to over predict the total days in care. Our final model has a fairly high margin of error at 20.27% and ideally would be closer to the 10% range. This could potentially have been caused by the limited data available for each case or potentially overfitting with the final regression model.

3.3 Optimization Model

The predictive model was constructed to estimate the total time spent in care for each child. This predictive model could then be used to drive an optimization model that aims to minimize the total number of days that children spend in care based on the allocation of services. Using the data available from 2015, we determined the total number of units of each service that was used. We then used these counts of services as the capacities. Using these capacities, we reallocated services to different cases in order to minimize the total expected number of days in care.

Because the objective function estimates the overall time in the system per child, the objective coefficients were determined by the regression coefficients from our final regression model. The model also needed to use data from our final discharged only dataset to have case data such as child information or problems the child was facing.

Our optimization model features two main sets of decision variables: whether a child receives a specific *child* level service (x), and whether a child receives a specific *family* level service (y). Due to our objective function being a linear regression with interaction terms that include products of (binary) service provision indicators, and because such nonlinearities can cause difficulties with optimization solvers, it became necessary to linearize these interaction product terms using auxiliary variables and constraints. Although the auxiliary variables are present within the model, they are dependent on the decision variables x and y. This can be seen in the constraint sets that are present within the model for each auxiliary variable.

There are three primary constraint sets in the optimization model. The first two primary constraint sets are child service resource limit and family service resource limit constraints. The foster care system has a finite amount of services that can be provided. Since data about the available resources was not available, the resource constraints for each service was determined by the count of cases that received the service within our testing dataset. By using this approach to define the service limits, any improvements within the model can be attributed to the reallocation of services since the same amount of services are being used. The final primary constraint was to ensure that every child within the same household received the benefits of a family level service being provided to the caretakers.

The algebraic model can be found below, and the Python script used to find the optimal solution can be found in Appendix C of this report.

3.3.1 Algebraic Model:

Since this is a problem that has data that is constantly changing, it was extremely important to develop an algebraic optimization model that can be applied to any set of data. Below is the algebraic notation detailing the sets (Table 6), parameters (Table 7), variables, objective function, and constraints in mathematical form.

Set	Definition
$\boldsymbol{D}$	The set of all case variables available in the dataset indexed by d
$\boldsymbol{T}$	The set of all child services that are present in the model indexed by t
$\boldsymbol{F}$	The set of all family services that are present in the model indexed by f
$\boldsymbol{H}$	The set of all households in the model indexed by h
$\boldsymbol{C_h}$	The set of all children in household h, $\forall\ h\ \in H$
$\boldsymbol{C}$	The set of all children in the model indexed by c, $C = \bigcup_{h \in H} C_h$

Table 6 – Sets and Definitions

Parameter	Definition
r_t	The value of the coefficient from the regression model for a specific child service. Parameter r is also defined for other sets including r_f, r_d, and all interaction terms
l_t	The limit of a child service t
l_f	The limit of family service f
m_{cd}	The value of case variable d for child c
$m_{cd\bar{d}}$	The value of case the interaction term for variable d and variable $\bar{d}$ for child c

Table 7 – Parameter Definitions

Decision Variables:

$$x_{ct} = = \begin{cases} 1 & if\ child\ c\ receives\ child\ service\ t \\ 0 & otherwise, child\ c\ does\ not\ receive\ child\ service\ t \end{cases}$$

$$y_{cf} = = \begin{cases} 1 & if\ child\ c\ receives\ family\ service\ f \\ 0 & otherwise, child\ c\ does\ not\ receive\ family\ service\ f \end{cases}$$

Auxiliary Variables:

The auxiliary variables defined below are included to introduce the interaction terms in the model. Since the interaction terms are the products of two binary variables, new auxiliary variables need to be introduced to the model to ensure linearity remains. In order to linearize these variables, each auxiliary variable has a constraint set that can be found in the constraint section of the model. The definition of each auxiliary variable can be found in Table 8 below.

Auxiliary Varible	Definition	Constraint Equations	Set
$i_{ct\bar{t}}$	The interaction term for two child services (Equation 1)	Equations 9-11	
j_{ctf}	The interaction term for a child service and a family service (Equation 2)	Equations 12-14	
$k_{cf\bar{f}}$	The interaction term for two family services (Equation 3)	Equations 15-17	
v_{ctd}	The interaction term for a child service and case data (Equation 4)	N/A	
h_{cfd}	The interaction term for a family service and case data (Equation 5)	N/A	

Table 8: Auxiliary Variable Definitions

$$i_{ct\bar{t}} = x_{ct} * x_{c\bar{t}} \; for \; c \in C, t, \bar{t} \in T \; where \; t < \bar{t} \quad (1)$$

$$j_{ctf} = x_{ct} * y_{cf} \; for \; c \in C, t \in T, f \in F \quad (2)$$

$$k_{cf\bar{f}} = y_{cf} * y_{c\bar{f}} \; for \; c \in C, f, \bar{f} \in F \; where \; f < \bar{f} \quad (3)$$

$$v_{ctd} = m_{cd} * x_{ct} \; for \; c \in C, t \in T, d \in D \quad (4)$$

$$h_{cfd} = m_{cd} * y_{cf} \; for \; c \in C, f \in F, d \in D \quad (5)$$

Objective Function:

$$Min \sum_{c=1}^{|C|}\left(\sum_{d=1}^{|D|} r_d m_{cd} + \sum_{t=1}^{|T|} r_t x_{ct} + \sum_{f=1}^{|F|} r_f y_{cf} + \sum_{t=1}^{|T|-1}\sum_{\bar{t}=t+1}^{|T|} r_{t\bar{t}} i_{ct\bar{t}} + \sum_{t=1}^{|T|}\sum_{d=1}^{|D|} r_{td} v_{ctd} + \sum_{t=1}^{|T|}\sum_{f=1}^{|F|} r_{tf} j_{ctf} + \sum_{f=1}^{|F|-1}\sum_{\bar{f}=f+1}^{|F|} r_{f\bar{f}} k_{cf\bar{f}} + \sum_{f=1}^{|F|}\sum_{d=1}^{|D|} r_{fd} h_{cfd} + \sum_{d=1}^{|D|}\sum_{\bar{d}=1}^{|D|} r_{d\bar{d}} m_{cd\bar{d}}\right)$$

Subject To

$$\sum_{h=1}^{|H|}\sum_{c=1}^{|C_h|} x_{ct} \leq l_t, \forall\, t \in T \quad (6)$$

$$\sum_{h=1}^{|H|}\sum_{c=1}^{|C_h|} y_{cf} \leq l_f, \forall\, f \in F \quad (7)$$

$$y_{cf} = y_{\bar{c}f}, \forall\, f \in F, \forall\, c, \bar{c} \in C_h \text{ where } c \neq \bar{c} \quad (8)$$

$$i_{ct\bar{t}} \leq x_{ct}, \forall\, c, t, \bar{t} \text{ where } t < \bar{t} \quad (9)$$

$$i_{ct\bar{t}} \leq x_{c\bar{t}}, \forall\, c, t, \bar{t} \text{ where } t < \bar{t} \quad (10)$$

$$i_{ct\bar{t}} \geq x_{ct} + x_{c\bar{t}} - 1, \forall\, c, t, \bar{t} \text{ where } t < \bar{t} \quad (11)$$

$$j_{ctf} \leq x_{ct}, \forall\, c, t, f \quad (12)$$

$$j_{ctf} \leq y_{cf}, \forall\, c, t, f \quad (13)$$

$$j_{ctf} \geq x_{ct} + y_{cf} - 1, \forall\, c, t, f \quad (14)$$

$$k_{cf\bar{f}} \leq y_{cf}, \forall\, c, f, \bar{f} \text{ where } f < \bar{f} \quad (15)$$

$$k_{cf\bar{f}} \leq y_{c\bar{f}}, \forall\, c, f, \bar{f} \text{ where } f < \bar{f} \quad (16)$$

$$k_{cf\bar{f}} \geq y_{cf} + y_{c\bar{f}} - 1, \forall\, c, f, \bar{f} \text{ where } f < \bar{f} \quad (17)$$

$x_{ct} \in \{0,1\}$, $y_{cf} \in \{0,1\}$, $i_{ct\bar{t}} \in \{0,1\}$, $j_{ctf} \in \{0,1\}$, $k_{cf\bar{f}} \in \{0,1\}$

The child service resource limit and family service resource limit constraints in equations (6) and (7) limit the amount of each service that can be provided. The two constraint sets ensure that the total number of service units allocated is within the service capacities that are available. The household service constraint in equation (8) ensures that children in each household receive the benefit of their caretaker receiving a family level service. Equations (9) through (17) are the auxiliary variable constraints. The auxiliary variables represent the product of two binary variables, and these three constraints for each variable ensure the auxiliary variable is implicitly represented as the product of the two binary variables.

Chapter 4: Results

We tested the effectiveness of the optimization model using the testing dataset from the final only discharged dataset. The predicted numbers of days in care for the 2015 data set used was 8,072,969 days. This number was calculated using the developed predictive model, and so is subject to inherent variability. The output of our regression analysis provided confidence intervals for each regression coefficient. Using this information, we developed two estimates of the impact of our analytical modeling: a conservative scenario, and a most likely scenario.

The conservative estimate was developed by using the worst-case scenario for each individual regression coefficient within the optimization model. There is data available that states the average cost per child per day in foster care is approximately $70 (National Council for Adoption, 2011), and in 2016 it was reported that there were 273,000 children that entered the U.S. foster care system, and of those cases 37% suffered from parental substance abuse (Child Welfare Information Gateway, 2018). Using these statistics, the average savings per year can be calculated by determining the average savings per child and multiplying this by the number of children that enter the foster care system due to parental substance abuse on an annual basis, as well as the total number of children that enter the foster care system on an annual basis, or alternatively, if the same process is applicable for the entire foster care system, the total yearly number of children that enter the foster care system.

	Conservative Estimate	Most Likely Case
Total Days in Care	6,327,684	5,913,547
Actual Days in Care	6,411,901	6,411,901
Total Days in Care Saved	84,217	498,347
Average Days in Care Saved	5.2	31
Average Savings per Child ($)	$361	$2,138
Total Annual Savings (Substance Abuse Cases)	$36,589,536	$216,518,233
Total Annual Savings (Entire Foster Care)	$98,890,367	$585,185,233

Table 9: Summary Results and Impacts of Predictive and Prescriptive Analytics Study

These values are highly dependent on the predictive model that was developed. Unfortunately, the data was fairly coarse, and the total expected savings have a fairly wide range. As expressed in Chapter 6, the results show that due to the high cost per day per child in care, as well as the high number of children entering the foster care system each year, any decrease in time in care will result in significant savings for the foster care system.

Chapter 5: Future Ideas

Although we faced limitations in our project, the results of our study clearly demonstrated that there might be better and more efficient ways to use the resources and provision of services within the foster care system. Thus, we found it important to consider potential future ideas which could serve useful as a basis for continuing the project in the future. It is important to note that these are just a few of the potential areas for future improvement.

The first idea we identified was a more specialized implementation of the ideas outlined in this study. Future groups could partner with local organizations, or even at the state level, and utilize specialized data to make more specific regression analyses and predictive models. Similarly, another potential area of future improvement would to consider different objective functions. Our objective function of minimizing total days in foster care was unique to the data that we received, so it would be beneficial to see what other unique objective functions could be utilized to improve the foster care system. Additionally, future work could focus on improving the

predictive modeling. Our results have a relatively high margin of error, and it would prove beneficial to reduce this, as the results would then be more reliable. Our final idea would be to conduct a stress analysis of the foster care system. By looking at the foster care system as a whole, it could be possible to analyze how the system reacts to stress - and identifying what the system does well, and what the system needs to improve. This would allow for improvements in the foster care system that would prove useful as future stressors occur.

Chapter 6: Study Limitations

In our study, there were several limitations that were important to be acknowledged. First, not every foster care case is reported within the United States, thus affecting the data that is made available through the NCANDS and AFCARS. Also, within the NCANDS dataset there is no indication whether a service is given multiple times to a family or a child. The only indication is whether a service has ever been received. Due to the coarseness of this data, information on the quantity and volume of services received is lacking. While such information very likely affects the length of stay in the foster care system, the unavailability of this information necessarily led to less effective predictive modeling.

Furthermore, The NCANDS dataset did not indicate how many times a child had entered the foster care system, which consequently effects the accuracy of the predictive models' forecasting capability for each predictor. Additionally, The NCANDS dataset had substantial missing data and only gave us partial information on the case, which led us to choose to omit these from final consideration. It is unclear how these omissions would have affected the final predictive modeling. Our predictive model is based on data reported at the national level; thus, it is unclear how the predictors and the forecasting capability of the model would fit regional or state level data.

Chapter 7: IE Reflection

Our group chose to design our own project and develop our own research question. Without a sponsor it was difficult at times to determine the proper objectives for the project, but with the help of our advisors as well as other resources we were able to overcome many of these challenges and develop a project of significance, and one that we hope to see continued in the future.

7.1: Design of Project Scope

Without an official sponsor it was fairly difficult to develop the proper scope for our project. At the start of the project we initially were planning to focus on a project related to the current opioid epidemic, but as we continued with the project it shifted more towards a project focused on the foster care system that had connected to the opioid epidemic. We were able to leverage a very useful outside resource in Emily Palmer, a reporter from the Boston Globe, to gain valuable insight into how the opioid epidemic has affected the foster care system and the lack of services that are available within the foster care system.

After speaking with Emily Palmer, the goal of our project began to become clear. We were able to recognize that the issues that Emily Palmer discussed with us were very similar to resource allocation optimization problems. To develop an effective optimization model, we needed to first gain a strong understanding of the foster care system. Once we had a strong understanding of the foster care system, we were able to design our approach for the problem, but quickly ran into our greatest challenge of accessing quality data. Because the data related to the foster care system is fairly sensitive data, there were many necessary, additional steps to ensure the data would be secure. This resulted in a large portion of the project where we were not able to access the data but needed to continue with other areas of the project to ensure we were still able to meet the project deadline.

Once we began working with the data and developing our predictive and optimization models it was difficult to determine which factors could be deemed important. To overcome this challenge and avoid arbitrarily selecting predictors, we designed a quantitative process to select the most important predictors.

7.2: Constraints and Limitations in the Design

The major constraint in the design of this project was the data that was available for use. There were multiple other factors that we wished to consider in the design of our project but at the moment this data is either not collected or not easily accessible for a large number of children. Throughout the project we needed to ensure that the design of our project was feasible with the data that we were able to access.

Another challenge that we faced throughout the project was determining the best software for our needs. At the beginning of the project no members of the group had any experience with Python, and limited experience in other scripting languages. This project, due to the large amount of data that needed to be used, quickly became very dependent on scripting in Python. Members of our group needed to learn Python at an accelerated pace in order to deliver our final product prior to the deadline.

Finally, we needed to consider the constraints of the foster care system. There were multiple recommendations or ideas that our group considered, such as reallocating resources from one service to another service, but some of these ideas were not plausible given the current foster care system. Our goal was to design a deliverable that could potentially have a meaningful impact on the foster care system, and to do this we determined that it would most likely be adopted if the foster care system did not need to make major changes to adopt the tool.

7.3: Life-Long Learning

This project was a tremendous learning experience that has inspired us to find different areas that our technical and analytical skills can be applied. Although many large corporations are using predictive modeling or optimization modeling similar to the work that we conducted in this project, many nonprofit or social service agencies do not have the technical know-how, or the funding to pursue projects such as these. This project has shown us that our education has put each of us in a position to make a meaningful impact in other areas through the use of the skills we have learned as Industrial Engineering students.

We also learned the importance of continuing to develop new skills while working on this project. Each member of our group had a very strong technical skillset that we learned through a wide range of classes at WPI, but we quickly found that the most effective tool for this project, Python, was not something that any of us had used before. Therefore, we quickly learned the importance of continuously adding new skills to your skillset in order to be prepared for as many projects as possible.

7.4: Interdisciplinary Aspects

This project required a wide range of skills. Without an objective provided by a sponsor, we initially relied on our problem-solving skills to analyze the foster care system and identify what

problem would be most beneficial to address. After identifying the problem we wished to address, we relied on a wide range of technical skills to complete the project. Our courses in database management were necessary to properly combine the two datasets that we needed to work with. We then needed to leverage statistical skills that were learned in various statistics and econometrics courses to analyze the data and conduct a proper regression analysis to develop a predictive model. Finally, both scripting and optimization courses provided the skills we needed to develop an optimization model to develop our final model. Economic analysis and business skills were then needed to analyze these results and provide meaningful context for the numbers that were gained from our optimization model.

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Appendix A: Glossary Terms

1. **Abandonment:** When the parent's identity or whereabouts are unknown, the child has been left by the parent in circumstances in which the child suffers serious harm, or the parent has failed to maintain contact with the child or to provide reasonable support for a specified period of time.

2. **Adoption and Foster Care Analysis and Reporting System (AFCARS):** Collects case-level information from state and tribal title IV-E agencies on all children in foster care and those who have been adopted with title IV-E agency involvement.

3. **Adoption:** Is the social, emotional, and legal process in which children who will not be raised by their birth parents become full and permanent legal members of another family while maintaining genetic and psychological connections to their birth family.

4. **Constraint:** Is an inequality or equality defining limitations on decisions. Constraints arise from a variety of sources such as limited resources, contractual obligations, or physical laws.

5. **Decision Variables:** Describe the quantities that the decision makers would like to determine. They are the unknowns of a mathematical programming model. Typically, we will determine their optimum values with an optimization method.

6. **Emotional Abuse:** Injury to the psychological capacity or emotional stability of the child as evidenced by an observable or substantial change in behavior, emotional response, or cognition and injury as evidenced by anxiety, depression, withdrawal, or aggressive behavior.

7. **Foster Care:** Is a temporary service provided by States for children who cannot live with their families. Children in foster care may live with relatives or with unrelated foster

parents. Foster care can also refer to placement settings such as group homes, residential care facilities, emergency shelters, and supervised independent living.

8. **Kinship Care:** Refers to the care of children by relatives or, in some jurisdictions, close family friends (often referred to as fictive kin).

9. **National Child Abuse and Neglect Data System (NCANDS):** Is a voluntary data collection system that gathers information from all 50 states, the District of Columbia, and Puerto Rico about reports of child abuse and neglect. NCANDS was established in response to the Child Abuse Prevention and Treatment Act of 1988. The data are used to examine trends in child abuse and neglect across the country.

10. **Neglect:** The failure of a parent or other person with responsibility for the child to provide needed food, clothing, shelter, medical care, or supervision to the degree that the child's health, safety, and well-being are threatened with harm.

11. **Objective Function:** Is the mathematical function one wants to maximize or minimize, subject to certain constraints.

12. **Opioid:** Are a class of drugs that include the illegal drug heroin, synthetic opioids such as fentanyl, and pain relievers available legally by prescription, such as oxycodone (OxyContin®), hydrocodone (Vicodin®), codeine, morphine, and many others.

13. **Optimization:** Is finding a best element from some set of alternatives – according to some criteria for evaluation.

14. **Physical Abuse:** Any non-accidental physical injury to the child" and can include striking, kicking, burning, or biting the child, or any action that results in a physical impairment of the child.

15. **Predictive Modeling:** Is the process of using known results to create, process and validate a model that can be used to forecast future outcomes. It is a tool used in predictive analytics, a data mining technique that attempts to answer the question "what might possibly happen in the future?

16. **Regression Model:** Is used to investigate the relationship between two or more variables and estimate one variable based on the others.

17. **Reunification:** Is a process of reconnecting children in foster care (or substitute care) with their families.

Appendix B: Predictor Definitions

1. **Abandonment Removal:** As a condition associated with a child's removal from home and contact with the foster care system, the child has been left alone or with others; caretaker did not return or make whereabouts known.

2. **Adoption Services:** Services or activities provided to assist in bringing about the adoption of a child.

3. **Caretaker Alcohol Abuse:** The principal caretaker(s)' compulsive use of alcohol that is not of a temporary nature.

4. **Caretaker Drug Abuse:** The principal caretaker(s)' compulsive use of drugs that is not of a temporary nature.

5. **Caretaker Emotional Problems:** A condition exhibiting one or more of the following characteristics over a long period of time and to a marked degree: an inability to build or maintain satisfactory interpersonal relationships; inappropriate types of behavior or feelings under normal circumstances; a general pervasive mood of unhappiness or depression; or a tendency to develop physical symptoms or fears associated with personal problems. The term includes persons who are schizophrenic or autistic. The term does not include persons who are socially maladjusted, unless it is determined that they are also seriously emotionally disturbed. This condition must be clinically diagnosed. The diagnosis is based on the Diagnostic and Statistical Manual of Mental Disorders (the most recent edition of DSM).

6. **Caretaker Learning Disability:** A disorder in one or more of the principal caretaker(s)'s basic psychological processes involved in understanding or using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell or to use mathematical calculations. The term includes conditions such as perceptual disability, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.

7. **Caretaker Medical Problems:** A medical condition other than mental retardation, visual or hearing impairment, physical disability, or being emotionally disturbed, that significantly affects the functioning or development of the primary caretaker(s) and their ability to provide a suitable child care environment.

8. **Caretaker Mental Retardation:** Significantly sub average general cognitive and motor functioning existing concurrently with deficits in adaptive behavior that adversely affect socialization and learning. This condition must be clinically diagnosed.

9. **Caretaker Physically Disabled:** A physical condition that adversely affects the caretaker(s)' day to day motor functioning, such as cerebral palsy, spina bifida, multiple sclerosis, orthopedic impairments, and other physical disabilities.

10. **Caretaker Visual or Hearing Impairment Problems:** A clinically diagnosed handicapping condition of the principal caretaker(s) related to a visual impairment or permanent or fluctuating hearing or speech impairment that may significantly affect functioning or development.

11. **Case Management Services:** Services or activities for the arrangement, coordination, and monitoring of services to meet the needs of children and their families.

12. **Child Age:** Age, calculated in years, as of the date of the report of alleged child maltreatment.

13. **Child Alcohol Abuse Removal:** As a condition associated with a child's removal from home and contact with the foster care system, the child's compulsive use of or need for alcohol. This element should include infants addicted at birth.

14. **Child Alcohol Abuse:**A compulsive use of or need for alcohol by the child. This element should include infants addicted at birth, or who are victims of Fetal Alcohol Syndrome, or who may suffer other disabilities due to the use of alcohol during pregnancy.

15. **Child Behavior Removal:** As a condition associated with a child's removal from home and contact with the foster care system, child's behavior in the school and/or community that adversely affects socialization, learning, growth and moral development. These may include adjudicated or non-adjudicated child behavior problems. This would include the child's running away from home or other placement.

16. **Child Behavioral Problem:** Behavior in the school and/or community that adversely affects socialization, learning, growth, and moral development. These may include adjudicated or non-adjudicated child behavior problems. This would include the child's running away from home or a placement.

17. **Child Disability Removal:** As a condition associated with a child's removal from home and contact with the foster care system, a clinical diagnosis by a qualified professional of one or more of the following: mental retardation; emotional disturbance; specific learning disability; hearing, speech or sight impairment; physical disability; or other clinically diagnosed handicap. Include only if the disability(ies) was at least one of the factors which led to the child's removal.

18. **Child Drug Abuse Removal:** As a condition associated with a child's removal from home and contact with the foster care system, the child's compulsive use of or need for narcotics. This element should include infants addicted at birth.

19. **Child Drug Abuse:** The compulsive use of or need for narcotics by the child. This element should include infants addicted at birth.

20. **Child Emotional Problem:** A condition exhibiting one or more of the following characteristics over a long period of time and to a marked degree: an inability to build or

maintain satisfactory interpersonal relationships; inappropriate types of behavior or feelings under normal circumstances; a general pervasive mood of unhappiness or depression; or a tendency to develop physical symptoms or fears associated with personal problems. The term includes persons who are schizophrenic or autistic. The term does not include persons who are socially maladjusted, unless it is determined that they are also seriously emotionally disturbed. This condition must be clinically diagnosed. The diagnosis is based on the Diagnostic and Statistical Manual of Mental Disorders (the most recent edition of DSM).

21. **Child ID:** A unique identification assigned to each child. This identification is not the State child identification but is an encrypted identification assigned by the State for the purposes of the NCANDS data collection.

22. **Child Learning Disability:** A disorder in one or more of the child's basic psychological processes involved in understanding or using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell or to use mathematical calculations. The term includes conditions such as perceptual disability, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.

23. **Child Medical Problem:** A medical condition other than mental retardation, visual or hearing impairment, physical disability, or being emotionally disturbed, that significantly affects the functioning or development of the child or requires special medical care such as chronic illnesses. Included are children diagnosed as HIV positive or with AIDS.

24. **Child Mental Retardation:** Significantly subaverage general cognitive and motor functioning existing concurrently with deficits in adaptive behavior manifested during the developmental period that adversely affect a child's/youth's socialization and learning. This condition must be clinically diagnosed.

25. **Child Sex:**The gender of the child at the time of the report.

26. **Child Visual or Hearing Impairment Problems:** A clinically diagnosed handicapping condition of the child related to a visual impairment or permanent or fluctuating hearing or speech impairment that may significantly affect functioning or development

27. **Counseling Services:** Services or activities that apply the therapeutic processes to personal, family, situational or occupational problems to bring about a positive resolution of the problem or improved individual or family functioning or circumstances.

28. **Court Appointed Representative:** A person required to be appointed by the court to represent a child in a neglect or abuse proceeding. May be an attorney or a court appointed special advocate (or both) and is often referred to as a guardian ad litem. Makes recommendations to the court concerning the best interests of the child.

29. **Court Ordered Removal:** The court has issued an order which is the basis of the child's removal.

30. **Daycare Services:** Services or activities provided in a setting that meets applicable standards of State and local law, in a center or in a home, for a portion of a 24-hour day.

31. **Domestic Violence:** Incidents of inter-spousal physical or emotional abuse perpetrated by one of the spouses or parent figures upon the other spouse or parent figure in the child victim's home environment.

32. **Education Services:** Services provided to the victim and/or the family to improve knowledge or daily living skills and to enhance cultural opportunities.

33. **Employment Services:** Services or activities provided to assist individuals in securing employment or acquiring of learning skills that promote opportunities for employment.

34. **Ever Adopted:** Has the child ever been legally adopted?

35. **Family Planning Services:** Educational, comprehensive medical or social services or activities which enable individuals, including minors, to determine freely the number and spacing of their children and to select the means by which this may be achieved.

36. **Family Preservation Services:** Family preservation services typically are services designed to help families alleviate crises that might lead to out-of-home placement of children; maintain the safety of children in their own homes; support families preparing to reunify or adopt; and assist families in obtaining services and other supports necessary to address their multiple needs in a culturally sensitive manner. (If a child cannot be protected from harm without placement or the family does not have adequate strengths on which to build, family preservation services are not appropriate.

37. **Family Support Services:** Family support services are primarily community-based preventative activities designed to alleviate stress and promote parental competencies and behaviors that will increase the ability of families to successfully nurture their children; enable families to use other resources and opportunities available in the community; and create supportive networks to enhance child-rearing abilities of parents and help compensate for the increased social isolation and vulnerability of families.

38. **Financial Problem:** A risk factor related to the family's inability to provide sufficient financial resources to meet minimum needs.

39. **Foster Care Services:** Services or activities associated with 24-hour substitute care for all children placed away from their parents or guardians and for whom the State agency has placement and care responsibility.

40. **Health and Home Health Services:** Services to attain and maintain a favorable condition of health.

41. **Home Based Services:** In-home services or activities provided to individuals or families to assist with household or personal care activities that improve or maintain adequate

family well-being. Includes homemaker services, chore services, home maintenance services and household management services.

42. **Housing Services:** Services or activities designed to assist individuals or families in locating, obtaining or retaining suitable housing.

43. **Inadequate Housing Removal:** As a condition associated with a child's removal from home and contact with the foster care system, housing facilities were substandard, overcrowded, unsafe or otherwise inadequate resulting in their not being appropriate for the parents and child to reside together. Also includes homelessness.

44. **Inadequate Housing:** A risk factor related to substandard, overcrowded, unsafe, or otherwise inadequate housing conditions, including homelessness.

45. **Informational and Referral Services:** Services or activities designed to provide information about services provided by public and private service providers and a brief assessment of client needs (but not a diagnosis and evaluation) to facilitate appropriate referral to these community resources.
46. **Legal Services:** Services or activities provided by a lawyer, or other person(s) under the supervision of a lawyer, to assist individuals in seeking or obtaining legal help in civil matters such as housing, divorce, child support, guardianship, paternity and legal separation.

47. **Mental Health Services:** Services to overcome issues involving emotional disturbance or maladaptive behavior adversely affecting socialization, learning, or development. Usually provided by public or private mental health agencies and includes residential services (inpatient hospitalization, residential treatment, and supported independent living) and nonresidential services (partial day treatment, outpatient services, home-based services, emergency services, intensive case management and assessment).

48. **Neglect Removal:** As a condition associated with a child's removal from home and contact with the foster care system, alleged or substantiated negligent treatment or maltreatment, including failure to provide adequate food, clothing, shelter or care.

49. **Number of Placements in Current Episode:** The number of places the child has lived, including the current setting, during the current removal episode. Do not include trial home visits as a placement setting.

50. **Parent Alcohol Abuse Removal:** As a condition associated with a child's removal from home and contact with the foster care system, the principal caretaker's compulsive use of alcohol that is not of a temporary nature.

51. **Parent Coping Removal:** As a condition associated with a child's removal from home and contact with the foster care system, physical or emotional illness or disabling condition adversely affecting the caretaker's ability to care for the child.

52. **Parent Drug Abuse Removal:** As a condition associated with a child's removal from home and contact with the foster care system, the principal caretaker's compulsive use of drugs that is not of a temporary nature.

53. **Parents Died Removal:** As a condition associated with a child's removal from home and contact with the foster care system, family stress or inability to care for child due to death of a parent or caretaker.

54. **Parents Jail Removal:** As a condition associated with a child's removal from home and contact with the foster care system, temporary or permanent placement of a parent or caretaker in jail that adversely affects care for the child.

55. **Physical Abuse Removal:** As a condition associated with a child's removal from home and contact with the foster care system, alleged or substantiated physical abuse, injury or maltreatment of the child by a person responsible for the child's welfare.

56. **Post Investigation Services:** The child protective services agency, social services agency, and/or the child welfare agency provides or arranges post investigation services for the child/family as a result of needs discovered during the course of the investigation. If services were being provided prior to or as a result of the report of alleged child maltreatment, the continuation of the service provisions after the disposition of the investigation would constitute post investigation services. Post investigation services are delivered within the first 90 days after the disposition of the report and would include: Family Preservation, Family Support, Foster Care and other services listed in the NCANDS record layout.

57. **Pregnancy and Parenting Services:** Services or activities for married or unmarried adolescent parents and their families to assist them in coping with social, emotional, and economic problems related to pregnancy and in planning for the future.

58. **Public Assistance:** Any one or combination of the following welfare or social services programs: AFDC, General Assistance, Medicaid, SSI, Food Stamps, etc.

59. **Relinquishment Removal:** As a condition associated with a child's removal from home and contact with the foster care system, parent(s), in writing, assigned the physical and legal custody of the child to the agency for the purpose of having the child adopted.

60. **Removal Adoption:** Child discharged due to being adopted.

61. **Removal Emancipation:** Child discharged due to turning 18 years old, making them legal adults.

62. **Removal Guardianship:** Child is discharged due to change in guardianship.

63. **Removal Kinship:** Child discharged due to being placed with a close relative or close family friend.

64. **Removal Reunification:** Child discharged due to being reunified with his/her parent or guardian.

65. **Removal Runaway:** The child has run away from the foster care setting.

66. **Removal Transfer:** Child is discharged to do transitioning to another foster care organization.

67. **Respite Services:** Services involving temporary care of the child(ren) to provide relief to the caretaker. May involve care of the children outside of their own home for a brief period of time, such as overnight or for a weekend. Not considered by the State to be foster care or another placement.

68. **Sex Abuse Removal:** As a condition associated with a child's removal from home and contact with the foster care system, alleged or substantiated sexual abuse or exploitation of a child by a person who is responsible for the child's welfare.

69. **Special Services Disabled:** Services for persons with developmental or physical disabilities, or persons with visual or auditory, impairments, or services or activities to maximize the potential of persons with disabilities, help alleviate the effects of physical, mental or emotional disabilities, and to enable these persons to live in the least restrictive environment possible.

70. **Special Services Juvenile Delinquent:** Services or activities for youth (and their families) who are, or who may become, involved with the juvenile justice system.

71. **State:** The U.S. Postal Service two-character abbreviation for the State or Territory submitting the Child File.

72. **Substance Abuse Services**: Services or activities designed to deter, reduce, or eliminate substance abuse or chemical dependency.

73. **Total Days in Care:** Can only be computed for kids who have 1 or 2 removals (TotRem). If TotalRem = 1, CareerLOS = LatRemLOS If TotalRem = 2, CareerLOS = LatRemLOS + PreviousLOS

74. **Total Number of Removals:** The number of times the child was removed from home, including the current removal.

75. **Transitional Living Services:** Services and activities designed to help older youth in foster care or homeless youth make the transition to independent living.

76. **Transportation Services:** Services or activities that provide or arrange for travel, including travel costs of individuals, to access services, or obtain medical care or employment.

77. **Voluntary Removal:** An official voluntary placement agreement has been executed between the caretaker and the agency. The placement remains voluntary even if a subsequent court order is issued to continue the child in foster care.

78. **Year:** The submission year is the Federal Fiscal Year 12-month period. The Federal Fiscal Year is from October 1 through September 30. All records in the data submission must have report disposition dates that fall within the Federal Fiscal Year. The report date may have occurred in a previous year. All records must have the same year in this field.

Appendix C: Optimization Python Script

```
from gurobipy import *
import pandas as pd

# Read in the CSV files as panda dataframes
optimization_df = pd.read_csv('TestDataset.csv', header=0)
child_services_df = pd.read_csv('ChildServices.csv', header=0)
family_services_df = pd.read_csv('FamilyServices.csv', header=0)
child_child_services_df = pd.read_csv('ChildChildServices.csv', header=0)
child_family_services_df = pd.read_csv('ChildFamilyServices.csv', header=0)
family_family_services_df = pd.read_csv('FamilyFamilyServices.csv', header=0)
service_limits_df = pd.read_csv('ServiceLimits.csv', header=0)
regression_coefficients_df = pd.read_csv('RegressionCoefficients.csv', header=0)
case_data_df = pd.read_csv('CaseData.csv', header=0)

# Create Gurobi optimization model named foster
mod = Model('foster')

# Define the empty lists for x and y, x is the list of child services and y is list of family services, u is the list of child service - child service interactions,
# v is the list of child service - family service interactions, and w is the list of family service - family service interactions
x = []
y = []
u = []
v = []
w = []
z = []
```

```
casedata_child_services = []
casedata_family_services = []
casedata_child_services_interactions = []
casedata_family_services_interactions = []

# Convert the dataframes to a list
included_variables = regression_coefficients_df.columns.values.tolist()
child_services = child_services_df.columns.values.tolist()
family_services = family_services_df.columns.values.tolist()
child_child_interactions = child_child_services_df.columns.values.tolist()
child_family_interactions = child_family_services_df.columns.values.tolist()
family_family_interactions = family_family_services_df.columns.values.tolist()
case_data = case_data_df.columns.values.tolist()

# Create a list of all child IDs
children = optimization_df['Child ID'].tolist()

# Create a new list that contains all possible interaction terms
all_interactions = child_child_interactions + child_family_interactions + family_family_interactions

# Change the index of the optimization dataframe to be Child ID
optimization_df.set_index('Child ID', inplace = True)

# Iterate through the list of all variables that are included and append any child service that is included to list x
for variable in included_variables:
        for service in child_services:
                if service in variable:
                        x.append(service)
```

```
# Iterate through the list of all variables that are included and append any family service that is included to list y
for variable in included_variables:
        for service in family_services:
                if service in variable:
                        y.append(service)

# Iterate through the list of all variables that are included and append any child child interaction that is included to list u
for variable in included_variables:
        for interaction in child_child_interactions:
                if interaction in variable:
                        interaction_terms = interaction.split(' x ')
                        u.append(interaction_terms)

# Iterate through the list of all variables that are included and append any child family interaction that is included to list v
for variable in included_variables:
        for interaction in child_family_interactions:
                if interaction in variable:
                        interaction_terms = interaction.split(' x ')
                        v.append(interaction_terms)

# Iterate through the list of all variables that are included and append any family family interaction that is included to list w
for variable in included_variables:
        for interaction in family_family_interactions:
                if interaction in variable:
                        interaction_terms = interaction.split(' x ')
                        w.append(interaction_terms)
```

```
# Iterate through the list of all included variables and append any child child interactions that is included to list z
split_z = []
for term1 in case_data:
    for term2 in case_data:
        if term1 + " x " + term2 in included_variables:
            split_z = [term1, term2]
            z.append(split_z)

# Convert the lists to a dictionary and back to a list to remove any duplicate values
x = list(dict.fromkeys(x))
y = list(dict.fromkeys(y))

# Finding all variables that are included in the model that are an interaction between case data and a child service and reordering
for variable in included_variables:
    for service in x:
        if service in variable and ' x ' in variable and variable not in all_interactions:
            individual_terms = variable.split(' x ')
            individual_terms.insert(0, service)
            individual_terms = list(dict.fromkeys(individual_terms))
            casedata_child_services_interactions.append(individual_terms)
            casedata_child_services.append(variable)

# Finding all variables that are included in the model that are an interaction between case data and a family service and reordering
for variable in included_variables:
    for service in y:
        if service in variable and ' x ' in variable and variable not in all_interactions:
            individual_terms = variable.split(' x ')
            individual_terms.insert(0, service)
```

```
                individual_terms = list(dict.fromkeys(individual_terms))
                casedata_family_services_interactions.append(individual_terms)
                casedata_family_services.append(variable)

# DECISION VARIABLES

# Create decision variables s to determine if a child receives a child level service
s = {}
for child in children:
        for service in x:
                s[child, service] = mod.addVar(vtype = GRB.BINARY,
name='s('+str(child)+')('+str(service)+')')


# Create decision variables f to determine if a child receives a family level service

f = {}
for child in children:
        for service in y:
                f[child, service] = mod.addVar(vtype = GRB.BINARY,
name='f('+str(child)+')('+str(service)+')')

# Create auxiliary variable p to assign obj value for individual child service terms
p = {}
for child in children:
        for variable in included_variables:
                if variable in x:
                        p[child, variable] = mod.addVar(obj =
regression_coefficients_df[variable][0], vtype = GRB.BINARY, name =
'p('+str(child)+')('+str(variable)+')')
```

```
# Create auxiliary variable q to assign obj value for individual family service terms

q = {}
for child in children:
        for variable in included_variables:
                if variable in y:
                        q[child, variable] = mod.addVar(obj = regression_coefficients_df[variable][0], vtype = GRB.BINARY, name = 'q('+str(child)+')('+str(variable)+')')

# Create auxiliary variables m to determine if a child receives a child service for certain case data

obj_value = 0
m = {}
for child in children:
        for service in x:
                for interaction in casedata_child_services_interactions:
                        if service == interaction[0]:
                                if interaction[0] + ' x ' + interaction[1] in included_variables:
                                        obj_value = regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0] * optimization_df[interaction[1]][child]
                                        m[child, interaction[0], interaction[1]] = mod.addVar(obj = obj_value, vtype = GRB.BINARY, name='m('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')
                                else:
                                        obj_value = regression_coefficients_df[interaction[1] + ' x ' + interaction[0]][0] * optimization_df[interaction[1]][child]
                                        m[child, interaction[0], interaction[1]] = mod.addVar(obj = obj_value, vtype = GRB.BINARY, name='m('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')
```

```
# Create auxiliary variable n to determine if a child receives a family service for certain case data
obj_fam_value = 0
n = {}
for child in children:
    for service in y:
        for interaction in casedata_family_services_interactions:
            if service == interaction[0]:
                if interaction[0] + ' x ' + interaction[1] in included_variables:
                    obj_fam_value = regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0] * optimization_df[interaction[1]][child]
                    n[child, interaction[0], interaction[1]] = mod.addVar(obj = obj_fam_value, vtype = GRB.BINARY, name='n('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')
                else:
                    obj_fam_value = regression_coefficients_df[interaction[1] + ' x ' + interaction[0]][0] * optimization_df[interaction[1]][child]
                    n[child, interaction[0], interaction[1]] = mod.addVar(obj = obj_fam_value, vtype = GRB.BINARY, name='n('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')


# Create auxiliary variables i for all child service - child service interactions

i = {}
for child in children:
    for interaction in u:
        for service in x:
            if service == interaction[0]:
                i[child, interaction[0], interaction[1]] = mod.addVar(obj = regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0], vtype = GRB.BINARY, name='i('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')
```

```
# Create auxiliary variables j for all child service - family service interactions

j = {}
for child in children:
        for interaction in v:
                for service in x:
                        if service == interaction[0]:
                                j[child, interaction[0], interaction[1]] = mod.addVar(obj = regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0], vtype = GRB.BINARY, name='j('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')
                for service in y:
                        if service == interaction[0]:
                                j[child, interaction[0], interaction[1]] = mod.addVar(obj = regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0], vtype = GRB.BINARY, name='j('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')

# Create auxiliary variables k for all family service- family service interactions

k = {}
for child in children:
        for interaction in w:
                for service in y:
                        if service == interaction[0]:
                                k[child, interaction[0], interaction[1]] = mod.addVar(obj = regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0], vtype = GRB.BINARY, name='k('+str(child)+')('+str(interaction[0])+')('+str(interaction[1])+')')

# Create variable for the constant term and determine objective value of constant term
constant_value = 0
for child in children:
```

```
        for data in case_data:
                for interaction in z:
                        if data == interaction[0]:
                                constant_value = constant_value + (regression_coefficients_df[interaction[0] + ' x ' + interaction[1]][0] * optimization_df[interaction[0]][child] * optimization_df[interaction[1]][child])
for child in children:
        for variable in included_variables:
                if variable in case_data:
                        constant_value = constant_value + (regression_coefficients_df[variable][0] * optimization_df[variable][child])

test_list = [1]
g = {}
for items in test_list:
        g[items] = mod.addVar(obj = constant_value, vtype = GRB.BINARY, name = 'Constant Term')

mod.update()

# CONSTRAINTS

# Constant term must be equal to 1
for items in test_list:
        mod.addConstr(g[items] == 1)

# Create child service limit constraints

for service in x:
        mod.addConstr(quicksum([s[child, service] for child in children]) <= service_limits_df[service][0])
```

```
# Create family service limit constraints
for service in y:
    mod.addConstr(quicksum([f[child, service] for child in children]) <= service_limits_df[service][0])

# Create constraint for auxiliary variable m, the value of m must be equal to the value of s for the same child and service

for child in children:
    for interaction in casedata_child_services_interactions:
        for service in x:
            if service == interaction[0]:
                mod.addConstr(m[child, interaction[0], interaction[1]] == s[child, interaction[0]])

# Create constraint for auxiliary variable n, the value of n must be equal to the value of s for the same child and service

for child in children:
    for interaction in casedata_family_services_interactions:
        for service in y:
            if service == interaction[0]:
                mod.addConstr(n[child, interaction[0], interaction[1]] == f[child, interaction[0]])

# Create constraint for auxiliary variable p, the value of p must be equal to the value of s for the same child and service

for child in children:
    for variable in included_variables:
```

```
            if variable in x:
                mod.addConstr(p[child, variable] == s[child, variable])

# Create constraint for auxiliary variable q, the value of must be equal to the value of q for the same child and family service

for child in children:
    for variable in included_variables:
        if variable in y:
            mod.addConstr(q[child, variable] == f[child, variable])

# Create constraint set for child service - child service auxiliary variable i

for child in children:
    for interaction in u:
        for service in x:
            if service == interaction[0]:
                mod.addConstr(i[child, interaction[0], interaction[1]] <= s[child, interaction[0]])
                mod.addConstr(i[child, interaction[0], interaction[1]] <= s[child, interaction[1]])
                mod.addConstr(i[child, interaction[0], interaction[1]] >= s[child, interaction[0]] + s[child, interaction[1]] - 1)

# Create constraint set for child service - family service auxiliary variable j

for child in children:
    for interaction in v:
        for service in x:
            if service == interaction[0]:
```

```
                                mod.addConstr(j[child, interaction[0], interaction[1]] <= s[child,
interaction[0]])
                                mod.addConstr(j[child, interaction[0], interaction[1]] <= f[child,
interaction[1]])
                                mod.addConstr(j[child, interaction[0], interaction[1]] >= s[child,
interaction[0]] + f[child, interaction[1]] - 1)
                for service in y:
                        if service == interaction[0]:
                                mod.addConstr(j[child, interaction[0], interaction[1]] <= f[child,
interaction[0]])
                                mod.addConstr(j[child, interaction[0], interaction[1]] <= s[child,
interaction[1]])
                                mod.addConstr(j[child, interaction[0], interaction[1]] >= f[child,
interaction[0]] + s[child, interaction[1]] - 1)

# Create constraint set for family service - family service auxiliary variable k

for child in children:
        for interaction in w:
                for service in y:
                        if service == interaction[0]:
                                mod.addConstr(k[child, interaction[0], interaction[1]] <= f[child,
interaction[0]])
                                mod.addConstr(k[child, interaction[0], interaction[1]] <= f[child,
interaction[1]])
                                mod.addConstr(k[child, interaction[0], interaction[1]] >= f[child,
interaction[0]] + f[child, interaction[1]] - 1)

# Create constraint that children in the same family must receive the same family services

for child1 in children:
```

```
        for child2 in children:
                for service in y:
                        if optimization_df['Report ID'][child1] == optimization_df['Report ID'][child2]:
                                mod.addConstr(f[child1, service] == f[child2, service])



mod.update()



mod.write('foster.lp')

# Solve

mod.optimize()

# Write Solutions

mod.write('foster.sol')

for child in children:
        for service in x:
                if s[child, service].X == 1:
                        print("Child " + str(child) + " receives " + service)
        for service in y:
                if f[child, service].X == 1:
                        print("Child " + str(child) + " receives " + service)
```

Appendix D: SQL Script

```
SELECT NCANDS.ID AS [Child ID],
        NCANDS.chage AS [Child Age],
        NCANDS.subyr AS [Entry Year],
        AFCARS.STATE AS State,
        NCANDS.strptid AS [Report ID],
        SWITCH(
            NCANDS.chsex=1,0,
            NCANDS.chsex=2,1) AS [Child Sex],
        NCANDS.cdalc AS [Child Alcohol],
        NCANDS.cddrug AS [Child Drug],
        NCANDS.cdrtrd AS [Child Mental Retardation],
        NCANDS.cdemotnl AS [Child Emotional Problem],
        NCANDS.cdvisual AS [Child Visual Problems],
        NCANDS.cdlearn AS [Child Learning Disability],
        NCANDS.cdbehav AS [Child Behavioral Problem],
        NCANDS.cdmedicl AS [Child Medical Problem],
        NCANDS.fcalc AS [Caretaker Alcohol Abuse],
        NCANDS.fcdrug AS [Caretaker Drug Abuse],
        NCANDS.fcrtrd AS [Caretaker Mental Retardation],
        NCANDS.fcemotnl AS [Caretaker Emotional Problems],
        NCANDS.fcvisual AS [Caretaker Visual Problems],
        NCANDS.fclearn AS [Caretaker Learning Disability],
        NCANDS.fcphys AS [Caretaker Physically Disabled],
        NCANDS.fcmedicl AS [Caretaker Medical Problems],
        NCANDS.fcviol AS [Domestic Violence],
        NCANDS.fchouse AS [Inadequate Housing],
        NCANDS.fcmoney AS [Financial Problem],
        NCANDS.fcpublic AS [Public Assistance],
        NCANDS.postserv AS [Post Investigation Services],
```

NCANDS.famsup AS [Family Support Services],
NCANDS.fampres AS [Family Preservation Services],
NCANDS.fostercr AS [Foster Care Services],
NCANDS.cochrep AS [Court Appointed Representative],
NCANDS.adopt AS [Adoption Services],
NCANDS.casemang AS [Case Management Services],
NCANDS.counsel AS [Counseling Services],
NCANDS.daycare AS [Daycare Services],
NCANDS.educatn AS [Education Services],
NCANDS.employ AS [Employment Services],
NCANDS.famplan AS [Family Planning Services],
NCANDS.health AS [Health and Home Health Services],
NCANDS.homebase AS [Home Based Services],
NCANDS.housing AS [Housing Services],
NCANDS.transliv AS [Transitional Living Services],
NCANDS.inforef AS [Informational and Referral Services],
NCANDS.legal AS [Legal Services],
NCANDS.menthlth AS [Mental Health Services],
NCANDS.pregpar AS [Pregnancy and Parenting Services],
NCANDS.respite AS [Respite Services],
NCANDS.ssdisabl AS [Special Services Disabled],
NCANDS.ssdelinq AS [Special Services Juvenile Delinquent],
NCANDS.subabuse AS [Substance Abuse Services],
NCANDS.transprt AS [Transportation Services],
AFCARS.EVERADPT AS [Ever Adopted],
AFCARS.TOTALREM AS [Total Removals],
AFCARS.PHYABUSE AS [Physical Abuse Removal],
AFCARS.SEXABUSE AS [Sexual Abuse Removal],
AFCARS.NEGLECT AS [Neglect Removal],
AFCARS.AAPARENT AS [Alcohol Abuse Removal],
AFCARS.DAPARENT AS [Drug Abuse Removal],

```
AFCARS.DACHILD AS [Drug Abuse Child],
AFCARS.PRTSDIED AS [Parents Died],
AFCARS.PRTSJAIL AS [Parents in Jail],
AFCARS.RELINQSH AS [Rights Relinquished],
AFCARS.HOUSING AS [Inadequate Housing Removal],
SWITCH(
        AFCARS.DISREASN=0,1,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [No Discharge],
SWITCH(
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=1,1,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge-Reunification],
SWITCH(
        AFCARS.DISREASN=0,0,
```

```
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,1,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge-Kinship],
SWITCH(
        AFCARS.DISREASN=0,0,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,1,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge - Adoption],
SWITCH(
        AFCARS.DISREASN=0,0,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,1,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
```

```
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge - Emancipation],
SWITCH(
        AFCARS.DISREASN=0,0,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,1,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge - Guardianship],
SWITCH(
        AFCARS.DISREASN=0,0,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,1,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge - Transfer],
SWITCH(
        AFCARS.DISREASN=0,0,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
```

```
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,1,
        AFCARS.DISREASN=8,0,
        AFCARS.DISREASN is null,0
        ) AS [Discharge - Runaway],
    SWITCH(
        AFCARS.DISREASN=0,0,
        AFCARS.DISREASN=1,0,
        AFCARS.DISREASN=2,0,
        AFCARS.DISREASN=3,0,
        AFCARS.DISREASN=4,0,
        AFCARS.DISREASN=5,0,
        AFCARS.DISREASN=6,0,
        AFCARS.DISREASN=7,0,
        AFCARS.DISREASN=8,1,
        AFCARS.DISREASN is null,0
        ) AS [Discharge - Death],
    AFCARS.LIFELOS AS [Total Days in Care]
FROM AFCARS, NCANDS
WHERE AFCARS.StFCID=NCANDS.StFCID
AND AFCARS.LIFELOS is not null
AND ReportIDQuery.[No Discharge]=0
ORDER BY NCANDS.ID;
```

Appendix E: Regression Coefficients

Variable	coef	std err	t	P>|t|	[0.025	0.975]
Child Age	1.69	0.28	6.13	0.00	1.15	2.23
Public Assistance	32.15	2.85	11.28	0.00	26.56	37.73
Daycare Services	24.67	1.46	16.94	0.00	21.82	27.53
Ever Adopted	150.75	5.85	25.78	0.00	139.29	162.21
Total Removals	145.07	1.84	78.85	0.00	141.46	148.67
Physical Abuse Removal	71.05	5.34	13.31	0.00	60.59	81.52
Drug Abuse Child	33.11	6.73	4.92	0.00	19.92	46.29
Inadequate Housing Removal	39.82	5.91	6.74	0.00	28.24	51.39
Child Age x Child Emotional Problem	7.88	0.67	11.69	0.00	6.56	9.20
Child Age x Child Learning Disability	5.63	1.08	5.21	0.00	3.51	7.75
Child Age x Caretaker Drug Abuse	1.69	0.28	6.13	0.00	1.15	2.23
Child Age x Caretaker Learning Disability	6.78	0.81	8.39	0.00	5.19	8.36
Child Age x Domestic Violence	-1.42	0.30	-4.78	0.00	-2.00	-0.84
Child Age x Financial Problem	-1.77	0.37	-4.82	0.00	-2.48	-1.05
Child Age x Post Investigation Services	-6.12	0.63	-9.80	0.00	-7.35	-4.90
Child Age x Foster Care Services	-4.90	0.47	-10.55	0.00	-5.81	-3.99
Child Age x Health and Home Health Services	-3.58	0.75	-4.76	0.00	-5.05	-2.11
Child Age x Housing Services	-4.02	0.91	-4.44	0.00	-5.80	-2.25
Child Age x Transitional Living Services	4.46	1.27	3.51	0.00	1.97	6.96
Child Age x Informational and Referral Services	2.96	0.47	6.28	0.00	2.03	3.88
Child Age x Substance Abuse Services	-1.89	0.45	-4.20	0.00	-2.77	-1.01

Child Age x Total Removals	4.78	0.56	8.60	0.00	3.69	5.87
Child Age x Sexual Abuse Removal	7.60	0.88	8.66	0.00	5.88	9.31
Child Age x Alcohol Abuse Removal	5.48	0.55	10.00	0.00	4.41	6.55
Child Age x Drug Abuse Removal	1.69	0.28	6.13	0.00	1.15	2.23
Child Drug x Total Removals	-21.35	4.03	-5.29	0.00	-29.26	-13.45
Caretaker Alcohol Abuse x Public Assistance	52.81	6.07	8.71	0.00	40.92	64.70
Caretaker Alcohol Abuse x Family Preservation Services	-44.79	7.06	-6.34	0.00	-58.63	-30.94
Caretaker Alcohol Abuse x Home Based Services	51.18	10.14	5.05	0.00	31.30	71.05
Caretaker Drug Abuse x Public Assistance	32.15	2.85	11.28	0.00	26.56	37.73
Caretaker Drug Abuse x Foster Care Services	36.98	12.66	2.92	0.00	12.16	61.81
Caretaker Drug Abuse x Daycare Services	24.67	1.46	16.94	0.00	21.82	27.53
Caretaker Drug Abuse x Ever Adopted	150.75	5.85	25.78	0.00	139.29	162.21
Caretaker Drug Abuse x Total Removals	145.07	1.84	78.85	0.00	141.46	148.67
Caretaker Drug Abuse x Physical Abuse Removal	71.05	5.34	13.31	0.00	60.59	81.52
Caretaker Drug Abuse x Inadequate Housing Removal	39.82	5.91	6.74	0.00	28.24	51.39
Inadequate Housing x Public Assistance	-37.74	5.93	-6.36	0.00	-49.37	-26.12
Inadequate Housing x Foster Care Services	31.60	3.87	8.17	0.00	24.01	39.18
Financial Problem x Family Support Services	41.18	6.75	6.10	0.00	27.95	54.40
Financial Problem x Foster Care Services	21.50	3.96	5.43	0.00	13.74	29.27
Public Assistance x Foster Care Services	-47.17	6.29	-7.50	0.00	-59.50	-34.83

Public Assistance x Health and Home Health Services	65.59	12.12	5.41	0.00	41.84	89.34
Public Assistance x Alcohol Abuse Removal	-57.44	9.03	-6.36	0.00	-75.14	-39.73
Post Investigation Services x Foster Care Services	128.13	10.99	11.66	0.00	106.59	149.67
Post Investigation Services x Court Appointed Representative	44.72	8.88	5.04	0.00	27.31	62.14
Post Investigation Services x Physical Abuse Removal	-88.10	13.81	-6.38	0.00	-115.16	-61.04
Family Support Services x Case Management Services	38.08	6.57	5.80	0.00	25.20	50.95
Family Support Services x Substance Abuse Services	-48.22	7.39	-6.52	0.00	-62.71	-33.74
Family Support Services x Total Removals	-27.96	4.46	-6.27	0.00	-36.70	-19.22
Family Preservation Services x Foster Care Services	-33.91	5.00	-6.78	0.00	-43.72	-24.11
Family Preservation Services x Case Management Services	99.55	5.08	19.62	0.00	89.60	109.50
Foster Care Services x Case Management Services	-33.68	3.43	-9.82	0.00	-40.40	-26.95
Foster Care Services x Total Removals	-23.59	6.56	-3.60	0.00	-36.44	-10.73
Foster Care Services x Physical Abuse Removal	-40.88	10.28	-3.98	0.00	-61.04	-20.72
Court Appointed Representative x Total Removals	-34.91	7.47	-4.67	0.00	-49.56	-20.27
Adoption Services x Substance Abuse Services	112.49	7.89	14.25	0.00	97.02	127.96
Counseling Services x Daycare Services	-37.72	7.22	-5.22	0.00	-51.87	-23.57
Daycare Services x Drug Abuse Removal	24.67	1.46	16.94	0.00	21.82	27.53

Substance Abuse Services x Total Removals	-55.86	3.72	-15.00	0.00	-63.16	-48.56
Transportation Services x Total Removals	26.66	4.17	6.39	0.00	18.48	34.83
Total Removals x Neglect Removal	24.57	2.32	10.58	0.00	20.02	29.13
Total Removals x Inadequate Housing Removal	-41.92	10.17	-4.12	0.00	-61.85	-22.00