

# Managed Account Plans

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A Major Qualifying Project submitted to the faculty of

Worcester Polytechnic Institute

In partial fulfillment of the requirements for the Degree of Bachelor of Science

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This report represents the work of four WPI undergraduate students submitted to the faculty as evidence of completion of a degree requirement. WPI routinely publishes these reports on its web site without editorial or peer review.

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## **Abstract**

Due to recent economic changes, Aon Hewitt has begun research into an alternative pension plan called a Managed Account Plan (MAP). To provide Aon Hewitt with information about the risks and benefits of the MAPs to employers, this MQP simulated a population and its additional liabilities at different guaranteed rates. The final model compares the risk of a MAP against a traditional Cash Balance (CB) plan. The analysis has shown that MAPs have potential benefits and warrant further research.

## **Acknowledgements**

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## Executive Summary

Aon Hewitt is continually pursuing research into cutting edge developments and human resource solutions. One of these new developments is the Managed Account Plan (MAP), a variant of more traditional Cash Balance Plans (CB). This plan gives Aon Hewitt the chance to offer new pension plans to companies that will reduce the companies' risk no matter the state of the economy. The objective of this project was to develop a Microsoft Excel program that would compare the company's liabilities at different points of time dependent on many different factors such as what guarantee is given, the population of the company and the current economic state.

The first step was to create a model that would show the liability to a company for one person at one point in time. This application was referred to as "The Straw Man." This approach made it possible to analyze how each plan worked for one person, at a certain age and a certain year of service. This ensured that the application of both the MAP and the CB plan were working properly before the model was expanded to an entire population.

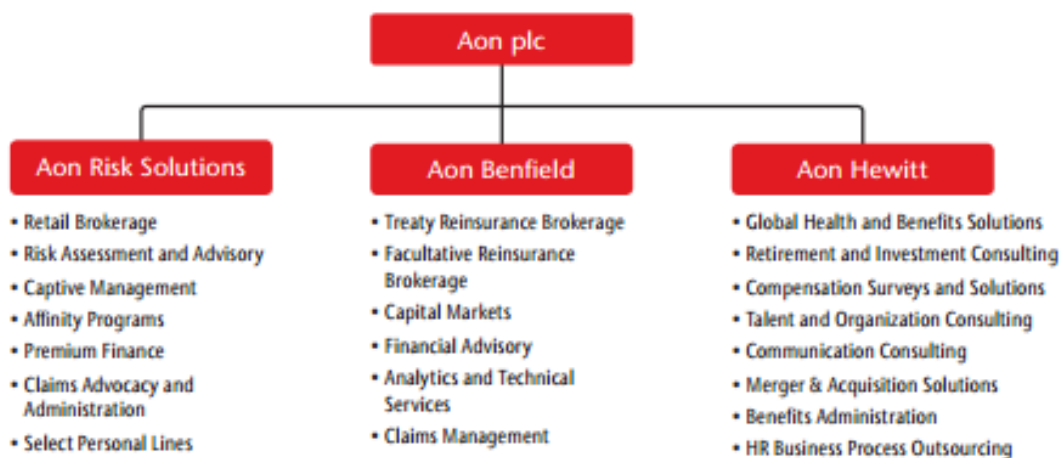
Once The Straw Man was completed, it was applied to a population which included people entering and leaving the population as time progressed (but which remained essentially stationary). This population was created using information provided by Aon Hewitt. The model made it possible for the program to run through each age with each possible plan contract year and year of service. To do this, macros were written in Microsoft Visual Basic for Applications.

A Monte Carlo Simulation was used to simulate future returns on a variety of possible investments. The forms of investments, Equity, Bonds and Emerging Markets, were simulated using Microsoft Visual Basic for Applications. These investment results were used to calculate the projected "actual" value of an employee's account would be worth. This amount could then be compared to what the company had guaranteed would be in the account.

The model makes it possible for users to analyze the risks of the MAPs and the CB plans but does not provide a guarantee for the outcome of either plan. This model can be used to show the different possibilities of each plan given different circumstances. Initial analysis of this data has shown promising results, and our team recommends expansion of the model and further research in order to provide more accurate information to companies considering a MAP.

## Background

In 2010, The Aon Corporation acquired Hewitt Associates and became Aon Hewitt. Aon had originally been comprised of Aon Risk Solutions, Aon Benfield and Aon Hewitt. The roles of these divisions can be found in Figure 1. Aon Hewitt now provides global health and benefit solutions, retirement and investment consulting, benefit administration and other tasks. The merger nearly tripled Aon's consulting business and Aon Hewitt is now a global leader providing risk management services, insurance and reinsurance brokerage, and human resource consulting and outsourcing with more than 62,000 employees worldwide. These employees are stationed in over 500 offices in over 120 countries. Aon Hewitt's primary goal is "to be the world's most responsive, client-focused insurance and consulting services company" (About Aon, 2013). Aon is currently the number one global reinsurance broker, global manager of captive insurance companies, and global employee benefits consultant.



**Figure 1: Aon Organizational Chart**

Pension plans were originally created to reward employees for loyalty, to provide employees with some economic security during employment and after retirement, and to recruit and retain employees. These plans qualify the employer for tax deductions and the plan assets accumulate tax free. However, the employer's tax deduction cannot exceed the maximum deductible contribution and the employer's contribution is required to be at least as large as the minimum required contribution. Pension plans have Federal Regulators and the Pension Benefit Guaranty Corporation (PBGC) that verifies if the plan is complying with laws and regulations, if future changes in laws and regulations would help to promote "public policy", if pension plans



are inadequately funded, and whether the sale or purchase of a business impacts the funded status of the plan.

A Defined Benefit (DB) pension plan guarantees the employer a lifetime benefit initiated at retirement for employees that meet the necessary requirements. With this plan, the risk lies with the employer rather than the employee. A Defined Contribution (DC) pension plan guarantees an annual contribution from the employer to the employee's account. The success of the plan is dependent on the investment performance of the plan assets. In a Defined Contribution pension plan, the risk mostly lies with the employee rather than the employer. Two common forms of Defined Contribution plans include 401(k) and profit sharing. With the Defined Contribution option, employees need to make their own investment decisions. Cash Balance (CB) plans are Defined Benefit plans which operate more like a Defined Contribution plan, with the retirement benefit defined in terms of a notional account. In these plans, the employer takes the risk for the investments and the account is available to be paid upon retirement or termination.

There have been changes to the legislature involving Defined Benefit and Cash Balance plans. There had been numerous lawsuits against employers that were transitioning from Defined Benefit plans to Cash Balance plans. These lawsuits resulted in amendments to the *Age Discrimination in Employment Act* (1967, amended in 2006 by the Pension Protection Act, PPA). This act stated that employers may not cease benefit accrual, or reduce benefit accrual, because of age but that service maximums are acceptable. It was later stated that Cash Balance plans were not discriminatory if the accrued benefits of any older employee were more than the accrued benefits for any younger employee that has all of the same attributes of the older employee except for age. This change clarified that in certain situations, Cash Balance plans are not age discriminatory.

Managed Account Plans (MAP's) are an innovative new form of pension plans that are intended to share the risk between the employer and the employee while also giving more protection to people's retirement funds. The current issue is that Defined Contribution plans have cost certainty but they also have benefit volatility. The MAP's pay credits are similar to the typical cash balance plans' and the floor is the sum of the pay credits. The interest involved with MAPs is linked to the total return on equity and bonds. The risk is spread between the employee and the employer since the employee's long-term risk is reduced from the credited interest and

the employer's short-term risk is reduced by the account value fluctuations. This results in the employees having less pressure to make the correct financial decisions and improved efficiency.

## Assumptions

Aon Hewitt splits the employee information into active and inactive data. Active data consists of date of birth, date of hire, benefit service, pay history, hourly history and sex. The inactive data should also contain date of birth, sex, payment form, benefit amount, beneficiary start date and beneficiary information.

There were assumptions that needed to be made to properly simulate the developments of a single person, and eventually a population. Assumptions were made for the death, termination rates, salary, and retirement age. Aon Hewitt provided information on termination given the person's age and experience and the average salary amounts dependent on the age of the employee. To correctly simulate a population, data from one of Aon Hewitt's clients was used (anonymously, with no identifying data provided). This data gave information on when employees leave the company due to retirement or any other reason besides death. Death was not incorporated into the model due to insignificance (relatively low mortality rates in the working population, particularly when compared with other, more significant assumptions). Information about the expected mean and the standard deviation of the investments for various asset categories was also provided by Aon Hewitt. The mean (denoted by  $\mu$ ) signifies the expected return and the standard deviation (denoted by  $\sigma$ ) is the risk associated with the asset.

## Lognormal Distribution

The Lognormal Distribution was used in this project to approximate the Large Capital U.S. Equity and the Long Duration Bonds. The Lognormal Distribution is a continuous probability function of a random variable where the natural log of the function follows a Normal Distribution (a more appropriate name for the Lognormal Distribution might be "Expnormal", since that is more descriptive of how the distribution works). The Lognormal Distribution is always non-zero and follows the formula:

$$S_1 = S_0 * (\exp^r)$$

$S_1$  is the stock price after one period and  $S_0$  is the stock price at the current time. The rate of return, "r," is Normal and the future price is Lognormal. The stock price is continuously compounding the current stock price at a rate of "r." The lognormal distribution is unimodal and

is positively right skewed. The function is increasing on  $(0, m)$  and decreasing on  $(m, \infty)$  where  $m$  is the mode. The mode,  $m$ , is equal to  $\exp(\mu - \sigma^2)$ . In this application, the desired result is that the mode occurs in the first few years of the plan. This would result in a severe decrease in the risk as time goes on. The Lognormal distribution is considered to be more accurate in estimating stock prices compared to the normal distribution.

## Investments

Both MAPs and Cash Balance plans rely upon the return on investments to build and maintain employee account balances. These investments can be a number of different combinations, but most often, investing in both bonds and equity is a common strategy of the companies offering retirement plans. The bond investments are a less risky option compared to the equity investments due to the market volatility affecting the equity risk. For this reason there is often a mix of multiple forms of investments, and the overall return on investments will depend upon the various asset classes, their returns, and any interaction (correlation) between the classes

The first step is to understand the average return of each investment type. The Monte Carlo Framework is the appropriate approach to achieving stock prices, or in this case any investment returns, that simulate the expected average of those investments. After formulating the mean investment returns from each investment type, the next step involves how different types of investments relate to one another.

When considering the returns on multiple investment types, users must be conscious of the fact that investment returns can demonstrate a high level of correlation. It then becomes necessary to randomize the potential returns while considering their correlation. The relationship between correlation, variance and covariance is shown below:

$$\text{Corr}(X_1, X_2) = \frac{\text{Cov}(X_1, X_2)}{\sqrt{\text{Var}(X_1)\text{Var}(X_2)}}$$

The above formula considers the correlation between two investment types.

The covariance of each pair can be solved for, given the correlation and variance of each pair of investment types. The covariance of each combination results in a covariance matrix of each possible pairing of investment types. There are three properties of a covariance matrix:

1. The matrix is symmetric.

2. The diagonal of the matrix has entries greater than or equal to zero.
3. The matrix is positive semi-definite so that  $\chi^T \Sigma \chi \geq 0$  for all  $\chi \in R^n$  where  $\Sigma$  is the covariance matrix of  $\chi$ .

Since the covariance matrix will be a positive semi-definite matrix by definition, the Cholesky decomposition of a symmetric positive-definite matrix can then be utilized. This final step in the Monte Carlo Simulation will allow for the investment mixture to be accurately applied with proper consideration for the correlation between different types of investments (Haugh).

## Methodology

### Creation of The Straw Man

In beginning to design a model to compare the financial benefits and risks between a Managed Account Plan (MAP) and a Cash Balance (CB) plan we first considered one employee individually. The model was created in Microsoft Excel using Visual Basic for Applications since this is the software Aon Hewitt has readily available. This became known as “The Straw Man.” This straw man model represented a single employee with certain characteristics. These characteristics include age and service for that employee, employee salary information, investment projections and termination information. Based on these inputs, the employee’s overall retirement benefit at the time of departure was determined in both a MAP and CB scenario. The difference between the sum of the pay credits, aggregated at a percentage input by the user, and the notional account of the employee was considered in both scenarios. This difference represents the additional liability the company could anticipate.

The original straw man was designed to act as a means of understanding the difference between a MAP and a CB plan. Ultimately there would be a need to convert to a full size population, incorporating termination probabilities and the possibility of having multiple investment types contributing to the same employee account. An entirely new model was designed to begin to incorporate these additional variables. The same theory and approaches applied to the creation of the straw man were carried over to the development of the final model.

### Developing the Model

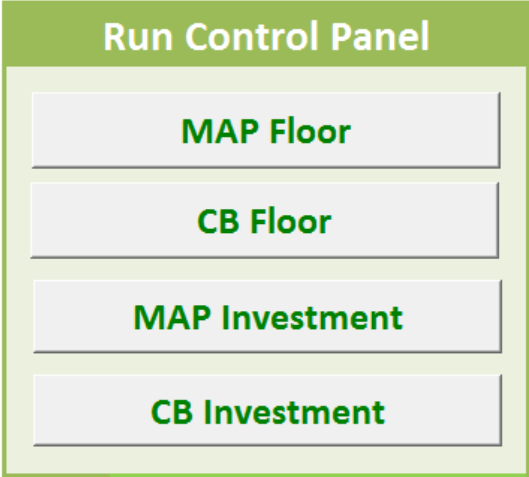
The model was designed to display the differences between four separate pension plan approaches. The four options include a MAP considering the floor risk, a CB plan considering the floor risk, a MAP considering investment risk, and a CB plan considering investment risk. The four options are held in four separate workbooks due to their large size. The difference between the floor risk and investment risk options involves the client guaranteeing a minimum return to each employee at their time of departure from the company. The company would be required to make up any gap between the guarantee and the actual account balance that particular employee has accrued over their time of employment.

The model requires five workbooks. These workbooks include “Control.xlsm,” “MAPFloor.xlsm,” “MAPInvest.xlsm,” “CBFloor.xlsm,” and “CBInvest.xlsm.” The user only

needs to be manually working in the “Control” workbook to enter inputs as well as observe outputs. For best results, having only the workbook that corresponds to the macro the user is currently running is ideal. At any point in time, the user should only have the “Control” workbook and the workbook corresponding to the desired macro open.

**The “Control” Workbook**

The “Control” workbook receives inputs from the user in the “Inputs” worksheet. These inputs are explained in the following section. The “Control” worksheet has two panels that enable the macros in the models and a third panel of Clear Outputs for convenience. The Run Control Panel displays four buttons for each of the types of pension plan scenarios that can be run for comparison. A snapshot of this section of the “Control” worksheet is shown below in Figure 2:



**Figure 2: Run Control Panel**

The Investment Control Panel displays two buttons, one to clear the investments and one to refresh the investment make-up. It is important to note that the “Clear Investments” button only clears the investments in the “Control” workbook. The investments in the sub-workbooks will be automatically updated when the model macros are run. A snapshot of this section of the “Control” worksheet is shown below in Figure 3:



Figure 3: Investment Control Panel

The third panel of the “Control” worksheet named “Clear Outputs” is explained in a later section entitled Ease of Use.

## Model Inputs

The model requires the user to input the following material into the “Inputs” worksheet in the “Control” workbook. The inputs on the “Inputs” worksheet can be thought of in three sections. A snapshot of the first section is shown below in Figure 4.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Inputs														
2			Nominal Account			Investment Account			Salary Scale	Termination Rates by Age and Service					
3	Year	Age	Equity	Bonds	Risky	Equity	Bonds	Risky	% Increase	0	1	2	3	4	5+
4	0	25	50%	50%	0%	70%	30%	0%	7.9%	0.2270	0.2270	0.1920	0.1620	0.1370	0.1080
5	1	26	49%	51%	0%	69%	31%	0%	7.8%	0.2220	0.2220	0.1880	0.1590	0.1340	0.1050
6	2	27	48%	52%	0%	68%	32%	0%	7.7%	0.2170	0.2170	0.1840	0.1550	0.1310	0.1030
7	3	28	47%	53%	0%	67%	33%	0%	7.4%	0.2160	0.2160	0.1830	0.1550	0.1310	0.1020
8	4	29	46%	54%	0%	66%	34%	0%	7.0%	0.2140	0.2140	0.1820	0.1550	0.1310	0.1020
9	5	30	45%	55%	0%	65%	35%	0%	6.5%	0.2120	0.2120	0.1810	0.1540	0.1310	0.1010
10	6	31	44%	56%	0%	64%	36%	0%	6.0%	0.2110	0.2110	0.1800	0.1540	0.1310	0.1010
11	7	32	43%	57%	0%	63%	37%	0%	5.6%	0.2090	0.2090	0.1790	0.1530	0.1310	0.1000
12	8	33	42%	58%	0%	62%	38%	0%	5.3%	0.2020	0.2020	0.1720	0.1470	0.1250	0.0970
13	9	34	41%	59%	0%	61%	39%	0%	5.0%	0.1950	0.1950	0.1650	0.1400	0.1190	0.0940
14	10	35	40%	60%	0%	60%	40%	0%	4.8%	0.1880	0.1880	0.1590	0.1340	0.1130	0.0910
15	11	36	39%	61%	0%	59%	41%	0%	4.6%	0.1820	0.1820	0.1530	0.1280	0.1080	0.0890
16	12	37	38%	62%	0%	58%	42%	0%	4.4%	0.1750	0.1750	0.1470	0.1230	0.1030	0.0860
17	13	38	37%	63%	0%	57%	43%	0%	4.2%	0.1780	0.1780	0.1470	0.1210	0.1000	0.0830
18	14	39	36%	64%	0%	56%	44%	0%	4.1%	0.1810	0.1810	0.1470	0.1200	0.0980	0.0800
19	15	40	35%	65%	0%	55%	45%	0%	4.0%	0.1830	0.1830	0.1480	0.1190	0.0960	0.0760
20	16	41	34%	66%	0%	54%	46%	0%	3.9%	0.1860	0.1860	0.1480	0.1180	0.0940	0.0730
21	17	42	33%	67%	0%	53%	47%	0%	3.8%	0.1890	0.1890	0.1490	0.1170	0.0920	0.0710
22	18	43	32%	68%	0%	52%	48%	0%	3.7%	0.1860	0.1860	0.1450	0.1140	0.0890	0.0670
23	19	44	31%	69%	0%	51%	49%	0%	3.7%	0.1830	0.1830	0.1420	0.1110	0.0860	0.0650
24	20	45	30%	70%	0%	50%	50%	0%	3.6%	0.1800	0.1800	0.1390	0.1070	0.0830	0.0620
25	21	46	29%	71%	0%	49%	51%	0%	3.6%	0.1780	0.1780	0.1360	0.1040	0.0800	0.0590
26	22	47	28%	72%	0%	48%	52%	0%	3.5%	0.1750	0.1750	0.1330	0.1020	0.0770	0.0560
27	23	48	27%	73%	0%	47%	53%	0%	3.4%	0.1750	0.1750	0.1340	0.1020	0.0780	0.0560
28	24	49	26%	74%	0%	46%	54%	0%	3.4%	0.1760	0.1760	0.1340	0.1030	0.0790	0.0560
29	25	50	25%	75%	0%	45%	55%	0%	3.3%	0.1760	0.1760	0.1350	0.1030	0.0790	0.0550
30	26	51	24%	76%	0%	44%	56%	0%	3.3%	0.1760	0.1760	0.1350	0.1040	0.0800	0.0550
31	27	52	23%	77%	0%	43%	57%	0%	3.2%	0.1770	0.1770	0.1360	0.1040	0.0800	0.0550
32	28	53	22%	78%	0%	42%	58%	0%	3.2%	0.1770	0.1770	0.1360	0.1050	0.0810	0.0550

Figure 4: Inputs Panel 1

The inputs required in this section include:

1. The mixture of how employees are expected to invest throughout their career. This mixture will be displayed as percentages of each investment type and will sum to a total of 100% for each year. In the model, the percentage of investment in bonds was increased and the percentage of investment in equity was decreased as an employee ages. Different account make-ups are typically used between the Nominal Account (Figure 4, blue) and the Investment Account (Figure 4, purple).
2. The salary scale percent increase over time as an employee ages (Figure 4, green).
3. The current termination rates by age and service (Figure 4, orange).

The second section of inputs on the “Inputs” worksheet requires the following information. A snapshot of the second section is shown below in Figure 5.

Starting Salary	50000		
Pay Credit %	5.00%		
Chosen Output Discount Rate	2.00%		
Guaranteed % Notional Acct	1.00%		
	Annualized Mean	Annualized Sigma	
Large Cap U.S. Equity	7.50%	19.00%	
Long Duration Bonds – Credit	4.10%	14.50%	
Emerging Markets	9.80%	31.50%	
Starting Age	25		
Service Year	0		
Current Plan Year	0		
Correlation Matrix	Large Cap U.S. Equity	Long Duration Bonds – Credit	Emerging Markets
Large Cap U.S. Equity	1.0000	0.1100	0.6200
Long Duration Bonds – Credit	0.1100	1.0000	0.0700
Emerging Markets	0.6200	0.0700	1.0000

**Figure 5: Inputs Panel 2**

1. The Starting Salary of a 25 year old employee with zero years of service.
2. The Pay Credit % that will apply to an employee’s salary determining their pay credits.
3. The Chosen Output Discount Rate will allow the user to add their desired level of risk. In the outputs this will be an outputted array of additional liabilities.
4. The Guaranteed % Notional Acct.
5. The types of investment returns the user would like to see applied to the employees’ nominal and investment accounts. We used the large cap U.S. equity, long duration credit bonds, and made room for the option of an additional “risky asset” as well.



6. The Mean and Sigma of these types of investment returns chosen by the user (previously referred to in number 5 above).
7. The Starting Age, Service Year, and Current Plan Year. Note that these should begin as 25, 0, and 0 respectively before any of the macros are run.
8. The correlation percentages, entered as decimals, between the different types of investment types.

The third and final section of the “Inputs” worksheet in the “Control” workbook is the population percentages. A snapshot of the population percentages is shown below in Figure 6:

V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL
Population Percentages																
Age/Service	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
25	0.00477															
26	0.005	0.00371														
27	0.005	0.00392	0.00303													
28	0.005	0.00392	0.0032	0.00256												
29	0.005	0.00393	0.00321	0.0027	0.00222											
30	0.005	0.00394	0.00322	0.00271	0.00235	0.002										
31	0.005	0.00395	0.00323	0.00272	0.00236	0.00211	0.00169									
32	0.005	0.00396	0.00324	0.00274	0.00237	0.00212	0.00179	0.00143								
33	0.005	0.00399	0.00327	0.00276	0.00239	0.00214	0.00181	0.00153	0.00122							
34	0.005	0.00403	0.00333	0.00282	0.00243	0.00217	0.00184	0.00156	0.00131	0.00105						
35	0.005	0.00406	0.00339	0.00289	0.0025	0.00221	0.00188	0.00159	0.00135	0.00114	0.00091					
36	0.005	0.00409	0.00344	0.00295	0.00257	0.00228	0.00193	0.00164	0.00139	0.00118	0.00099	0.00079				
37	0.005	0.00413	0.00349	0.00302	0.00265	0.00235	0.002	0.00169	0.00144	0.00122	0.00103	0.00087	0.0007			
38	0.005	0.00411	0.00352	0.00307	0.00271	0.00243	0.00207	0.00175	0.00149	0.00126	0.00107	0.00091	0.00076	0.00061		
39	0.005	0.0041	0.00351	0.0031	0.00277	0.0025	0.00214	0.00182	0.00154	0.00131	0.00111	0.00094	0.0008	0.00067	0.00054	
40	0.00482	0.00409	0.00349	0.00309	0.0028	0.00256	0.0022	0.00188	0.0016	0.00136	0.00115	0.00098	0.00083	0.0007	0.00059	0.00047
41	0.00423	0.00392	0.00348	0.00308	0.0028	0.00259	0.00225	0.00194	0.00166	0.00141	0.0012	0.00102	0.00086	0.00073	0.00062	0.00052
42	0.00401	0.00343	0.00334	0.00307	0.00279	0.0026	0.00229	0.00199	0.00171	0.00147	0.00125	0.00106	0.0009	0.00076	0.00065	0.00055
43	0.00354	0.00326	0.00293	0.00296	0.0028	0.00261	0.0023	0.00203	0.00176	0.00152	0.0013	0.00111	0.00094	0.0008	0.00068	0.00057
44	0.00331	0.00289	0.0028	0.00261	0.0027	0.00262	0.00232	0.00205	0.0018	0.00157	0.00135	0.00115	0.00098	0.00083	0.00071	0.0006
45	0.00298	0.00271	0.00249	0.0025	0.00239	0.00254	0.00234	0.00207	0.00183	0.00161	0.0014	0.00121	0.00103	0.00088	0.00075	0.00063
46	0.00266	0.00245	0.00234	0.00223	0.0023	0.00225	0.00227	0.00209	0.00185	0.00164	0.00144	0.00125	0.00108	0.00092	0.00079	0.00067
47	0.00236	0.00219	0.00212	0.00211	0.00206	0.00217	0.00202	0.00204	0.00188	0.00167	0.00147	0.0013	0.00113	0.00097	0.00083	0.00071
48	0.00232	0.00194	0.0019	0.0019	0.00194	0.00194	0.00195	0.00181	0.00183	0.00169	0.0015	0.00132	0.00116	0.00101	0.00087	0.00075
49	0.00229	0.00191	0.00168	0.0017	0.00175	0.00183	0.00174	0.00175	0.00163	0.00164	0.00152	0.00134	0.00119	0.00104	0.00091	0.00078
50	0.00211	0.00189	0.00165	0.00151	0.00157	0.00166	0.00164	0.00156	0.00157	0.00146	0.00147	0.00136	0.0012	0.00106	0.00094	0.00081
51	0.00204	0.00174	0.00163	0.00148	0.00139	0.00148	0.00149	0.00147	0.0014	0.0014	0.00131	0.00132	0.00122	0.00108	0.00095	0.00084
52	0.002	0.00168	0.0015	0.00146	0.00136	0.00131	0.00133	0.00133	0.00132	0.00126	0.00126	0.00117	0.00118	0.00109	0.00097	0.00085
53	0.00191	0.00165	0.00145	0.00134	0.00134	0.00129	0.00118	0.00119	0.00119	0.00118	0.00112	0.00113	0.00105	0.00106	0.00098	0.00086
54	0.00175	0.00157	0.00142	0.0013	0.00124	0.00127	0.00115	0.00105	0.00106	0.00106	0.00106	0.001	0.00101	0.00094	0.00095	0.00087
55	0.00273	0.00144	0.00136	0.00132	0.00121	0.00115	0.00118	0.00107	0.00098	0.00099	0.00099	0.00098	0.00093	0.00094	0.00087	0.00088
56	0.00254	0.00224	0.00124	0.00126	0.00123	0.00112	0.00107	0.0011	0.001	0.00091	0.00092	0.00092	0.00091	0.00087	0.00087	0.00081
57	0.00234	0.00209	0.00193	0.00116	0.00117	0.00114	0.00104	0.00099	0.00102	0.00093	0.00084	0.00085	0.00086	0.00085	0.00081	0.00081
58	0.00215	0.00192	0.0018	0.0018	0.00107	0.00109	0.00106	0.00097	0.00092	0.00095	0.00086	0.00079	0.00079	0.0008	0.00079	0.00075
59	0.00195	0.00177	0.00166	0.00167	0.00167	0.001	0.00102	0.00099	0.0009	0.00086	0.00088	0.0008	0.00073	0.00074	0.00074	0.00073
60	0.00224	0.00161	0.00152	0.00153	0.00154	0.00154	0.00092	0.00093	0.00091	0.00083	0.00079	0.00081	0.00074	0.00067	0.00068	0.00068
61	0.00196	0.00184	0.00138	0.0014	0.0014	0.00141	0.00142	0.00085	0.00086	0.00084	0.00076	0.00073	0.00075	0.00068	0.00062	0.00063
62	0.00248	0.00162	0.00158	0.00125	0.00126	0.00126	0.00127	0.00076	0.00077	0.00075	0.00069	0.00065	0.00067	0.00061	0.00056	
63	0.002	0.00204	0.00139	0.00143	0.00112	0.00114	0.00114	0.00115	0.00115	0.00069	0.0007	0.00068	0.00062	0.00059	0.00061	0.00055
64	0.00328	0.00165	0.00176	0.00116	0.00118	0.00093	0.00094	0.00094	0.00095	0.00095	0.00057	0.00058	0.00056	0.00051	0.00049	0.0005
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 6: Inputs Panel 3

This population is based on the client’s current employee make-up. The entries should reflect the percentage of the company’s population that corresponds to each age and number of

years of service with the company. For a more in depth view of how we designed the population percentages for our utilization of the model please refer to Appendix C.

### Creation of “Investments” Macro

To create scenarios of possible investment returns over time, a macro in Microsoft Visual Basic for Application was created. This macro is run using the “Refresh Investments” button on the “Control” worksheet in the “Control” workbook. For convenience, there is also a “Clear Investments” button located on the “Control” worksheet in the “Control” workbook as well. This should be used before refreshing the investments. A snapshot of the two buttons can be seen below in Figure 7:



Figure 7: Investment Control Panel

The macro uses the data from the “Inputs” tab in the “Control” workbook. When the “Refresh Investments” macro is run, the macro first creates a covariance matrix based on the sigmas of each investment type inputted by the user. This covariance matrix utilizes the percentages of correlation from the correlation matrix inputted by the user on the “Inputs” worksheet of the “Control” workbook. These percentages represent the correlation between each pair of investment types. The sigmas and correlations are used to solve for the covariance of each pair of investments. The macro then calculates the Cholesky decomposition of the covariance matrix. For a more in depth view of these calculations please refer to Appendix B.

The macro then creates six arrays of randomly generated numbers between zero and one, three for monthly returns and three for annual returns. The dimensions of the monthly arrays are

1000 rows by 960 columns. The 1000 rows represent the 1000 trials that we had previously decided would be a reasonable sample size for the project's purposes. The 960 columns represent twelve months a year for 80 years. The dimensions of the annual arrays are 1000 rows by 80 columns representing 100 trials and 80 years of returns. We projected our results out 80 years to give the plans time to mature to a stationary state.

The six arrays of randomly generated numbers between zero and one are then multiplied by the previously determined Cholesky decomposition of the covariance matrix. Since the Managed Account Plans and Cash Balance plans operate differently, the MAP requires monthly investment returns while the CB requires yearly investment returns. The final part of the macro takes the created monthly investment returns and outputs them for the MAP Floor and MAP Invest macros. However, the monthly investment returns for each year are multiplied together to achieve yearly investment returns for the CB Floor and CB Invest macros. The monthly and yearly investments for each of the three types of investments output the six arrays to the "Investments" worksheet in the "Control" workbook. When each of the four model macros run, they utilize the appropriate investment arrays.

### **Creation of "MAPFloorRisk" Worksheet**

The first macro can be run using the first button on the control panel, labeled "MAP Floor." When run, this macro copies the "Inputs" worksheet the user has manually entered their data into in the "Control" workbook and copies it to the "Inputs" worksheet in the "MAPFloor" workbook. To understand how the macro runs and outputs, we first must understand how the "MAPFloorRisk" worksheet works within the "MAPFloor" workbook.

The MAP is designed to monitor monthly pay credits and sum them over the course of an employee's career with the client company. The pay credits are aggregated using the prior month's sum of pay credits scaled using one twelfth of the "Monthly Guarantee % Notional Acct" input. This raises the sum of the pay credits according to the desired percentage inputted by the user. The monthly pay credit from the previous month is then added to the scaled sum of pay credits from the previous month. It is important to note that the monthly pay credits are calculated using a percentage of the salary of an employee, given their age. The percentage of an employee's salary that makes up their monthly pay credit is one of the inputs from the user.

The "MAP Floor Risk" macro begins by producing 1000 trials of notional account balances over the course of 960 months. This is the array highlighted in red in the

“MAPFloorRisk” worksheet. Depending on which month in the lifetime of the plan that particular employee begins working, the array begins to fill at different months of the plan. This array will fill in as many months of an individual’s account balance as necessary. Depending on what age the individual is, the array will fill in account balances through the end of the year in which the individual is 65 years old. The array also begins filling in notional account balances depending on the year of the plan. For example, if a company has had the MAP implemented for 10 years, you will see the notional account balances begin at the 120<sup>th</sup> month of the array. The notional account continuously accumulates based on the pay credit percentage that was inputted by the user.

The second array in the “MAPFloorRisk” worksheet is simply a placeholder array for consistency. In the “MAPInvest” workbook this array will be populated.

An additional array is created in the “MAPFloorRisk” worksheet based on the monthly overhang. This array is located under the Notional Account Balance array in the “MAPFloorRisk” worksheet and is highlighted in green. This array simply shows the difference between the sum of the pay credits and the notional account balance. However, if the sum of the pay credits is less than the notional account balance the array displays a zero for that entry in the overhang array. This is because when the sum of the pay credits exceeds the balance of the notional account, the company does not realize any additional liabilities.

The fourth array in the “MAPFloorRisk” worksheet produces the expected additional liabilities. This array applies the monthly termination probability at any given time to the monthly overhang, from the previously explained array. The monthly termination probabilities are determined using the termination rates by age and service, which are inputted by the user. The rates we are currently using in the model needed adjustments before we could apply them. Because our termination inputs were based on ages 25 through 65 and 0 to 5+ years of service corresponding to each age, we had to adjust the termination rates in the “Calculated” worksheet. The “Lookup Termination Rates,” column C in the “Calculated” worksheet, retrieves the termination probabilities based on the employee’s age and years of service. As you move down the column the termination probabilities correspond to an additional year of service and the employee becoming one year older. Column D in the “Calculated” worksheet, labeled “Termination Probabilities,” takes the lookup termination rates and conditions them to consider the probability of not terminating before a given age and then actually terminating at that

particular age. These termination probabilities are divided by twelve to span over the twelve months in a year and applied as the “Monthly Termination Probability” in the “MAPFloorRisk” worksheet.

## Creation of “MAPMonthlyFloor” Macro

Now that we understand the “MAPFloorRisk” worksheet, we can approach how the macro creates the outputs in the “Output” worksheet in the “Control” workbook. The macro works through each possible age, 25 to 65, and possible years of service, 0 to 40, for 1000 trials. The macro stores the data while it runs and aggregates based on years of service to output five arrays in the “Output” worksheet. These are labeled, in order, “Overhang,” “PV at Input %,” “PV at 5%,” “PV at 0%,” and “80 Plan Years.” We will walk through each one individually.

The “Overhang” array first sums the overhang outputs from the “MAPFloorRisk” worksheet by trial. We are then left with the total amount of overhang per trial. These are then multiplied by the corresponding percentage of the employee population that is of the same age and years of service. All 1000 trials are run through for each age and service combination. The output is an array in the “Output” worksheet with dimensions 1000 by 40. This represents the overhang of all 1000 trials and aggregated by each of the 40 years of service.

The “PV at Input %” array first stores the net present value of each additional liability determined in the “MAPFloorRisk” worksheet. This is done using the NPV function in Microsoft Visual Basic in Excel. The percentage is based on the “Chosen Output Discount Rate” that is inputted by the user. This percentage is then put through the NOMINAL function in excel. This returns the nominal annual interest rate, given the effective rate and the number of compounding periods per year, in this case 12, for 12 months per year. This NPV array is then multiplied by the corresponding percentage of the employee population that is of the same age and years of service. All 1000 trials are run through for each age and service combination. The output is an array in the “Output” worksheet in the “MapFloor” workbook with dimensions 1000 by 40. This represents the present value of the additional liabilities at the inputted percentage of all 1000 trials and aggregated by each of the 40 years of service.

The “PV at 5%” array first stores the net present value of each additional liability determined in the “MAPFloorRisk” worksheet. This is done using the same NPV function as before. This is then multiplied by the corresponding percentage of the employee population that is of the same age and years of service. All 1000 trials are run through for each age and service

combination. The output is an array in the “Output” worksheet with dimensions 1000 by 40. This represents the present value of the additional liabilities at 5% of all 1000 trials and aggregated by each of the 40 years of service.

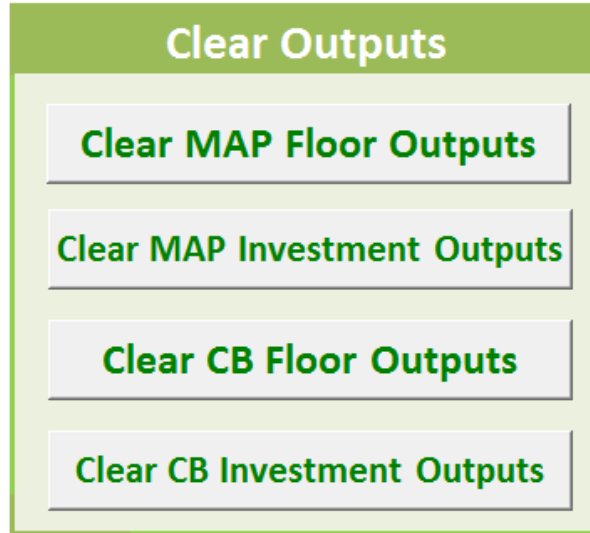
The “PV at 0%” array first sums the additional liability outputs from the “MAPFloorRisk” worksheet by trial. This is then multiplied by the corresponding percentage of the employee population that is of the same age and years of service. All 1000 trials are run through for each age and service combination. The output is an array in the “Output” worksheet with dimensions 1000 by 40. This represents the present value of the additional liabilities at 0% of all 1000 trials and aggregated by each of the 40 years of service.

The fifth and final array in the “Output” worksheet has different dimensions than the others. This array outputs 1000 trials by 960 months of service in the “MapFloor” workbook. This was created to display the additional liabilities summed by the year in which they occurred. The aggregated additional liabilities by year are displayed in the “Control” workbook in the “MAPFloorOutputs” worksheet. This allows for a view of how a company would experience a MAP once the plan has had time to mature. We implemented the Managed Account Plan for 0 through 80 years.

All of these arrays are then used to populate the outputs tab in the “Control” workbook, the same workbook where the user applied the inputs.

## **Ease of Use**

For the benefit of the user, we incorporated separate buttons in the “Control” workbook to allow for quick data refreshing and clearing. A snapshot of the third panel of buttons in the “Control” worksheet in the “Control” workbook is shown below in Figure 8:



**Figure 8: Clear Outputs**

All four buttons shown above clear out all five arrays in the “Outputs” worksheet in the respective sub-workbook.

### **CB Floor Macro**

The “CBFloor” workbook operates similarly to the “MAPFloor” workbook. The differences first arise in the “CBFloorRisk” worksheet. There is no tracking of the sum of the pay credits for the Cash Balance plans. Also, the pay credits are accrued yearly instead of monthly. The termination probabilities are also yearly instead of monthly. These differences arise because of the monthly and yearly difference between the Managed Account Plan and the Cash Balance respectively. The notional account of the employee in a MAP aggregates the pay credits monthly because there is a monthly guarantee. In the CB, however, the employee only has a guarantee of their nominal account through the end of the previous year.

The differences in the “CBFloor” workbook when compared to the “MAPFloor” workbook first bring us to the “CBFloorRisk” worksheet in the “CBFloor” workbook. The first array present in the worksheet is the Guarantee Account, the previous balance of the account scaled by either the three chosen investment types, scaled by the inputted percentage make-up, or the return on bonds alone. The larger of the two options is chosen and used to scale the previous account balance and the yearly pay credit is also added to the new balance of the Guarantee Account.

The second array in this worksheet is the Nominal Account which removes the bond guarantee from the first array. This is identical to the Guarantee Account array except that the larger of the two investment options does not exist. The maximum between the three chosen investment types, scaled by the inputted percentage make-up of each investment, or the return on bonds is not applied because there is no guarantee of the bond.

The next array is a placeholder in the “CBFloorRisk” worksheet for consistency throughout the multiple sub-workbooks. This array is utilized later in the CB investment macro. The next two arrays represent Overhang and Additional Liability respectively in the same way as the “MAPFloor.”

The “CBYearlyFloor” macro then utilizes the “CBFloorRisk” worksheet in the “CBFloor” workbook and runs 1000 trials for each combination of ages 25 through 65 and years of service 0 through 40. The same five arrays are outputted in the “Output” worksheet in the “CBFloor” workbook as in the “Output” worksheet in the “MAPFloor” workbook.

It is important to note that the “80 Plan Year” array is outputted to the sub-workbook in dimensions 1000 by 80 representing the additional liabilities of 1000 trials aggregated by year of accrument. This differs from the “80 Plan Year” array outputted to the “MAPFloor” workbook because the “MAPFloor” sub-workbook displays 1000 trials of additional liabilities for each of 960 months. It is not until the outputs are transferred to the “Control” workbook that the “MAPFloor” outputs are converted to yearly sums.

All sub-workbook outputs are then transferred to the “CBFloorOutput” of the “Control” workbook, the same workbook where the user applied the inputs.

## **MAP Investment Macro**

The MAP Investment macro functions similarly to the MAP Floor macro. The difference lies in the distribution of investment percentages amongst the three chosen types of investments. Looking at the differences in the “MAPInvestmentRisk” worksheet of the “MAPInvest” workbook, the arrays differ in the following ways:

The notional account is aggregated is the same way as the MAP Floor. The second array is only populated in the “MAPInvest” workbook, however. This array aggregates the Investment Account using the same method as the Notional Account. The difference between the first and second array is present in the scaling of the investment returns. The user is able to input two different investment structures, one for the Notional Account and one for the Investment



Account. For the purposes of our model we designed the Notional Account to have a less risky make-up of investments and the Investment Account has a more risky make-up of investments. A less risky make-up would involve a smaller percentage of investments dedicated to equity and the remainder of the investment make-up to come from the return on bonds. It is worth noting that for our current model we do not include the third “Risky” asset as any percentage of our investment make-up, but the functionality does exist within the model.

The next array, “Overhang,” takes the larger of two considerations. The first consideration is the difference between the sum of the pay credits at any given point in time and the corresponding notional account. The second consideration is the difference between the sum of the pay credits and the investment account. Whichever difference is larger, that is what the company will be responsible for making up to the employee. The final array, identically to the MAP Floor, is the Additional Liability to the company. This takes the Overhang array and applies the probability of termination in that particular month of employment.

It is important to understand that the Overhang array represents what the company would have to pay out to the employees based on the asset returns for that point in time in the market. The Additional Liability array also considers the probability the company will actually have to pay out at that time since the pay out only occurs when the employee actually terminates.

The MAP Investment macro then utilizes the “MAPInvestmentRisk” worksheet and runs 1000 trials for each combination of ages 25 through 65 and years of service 0 through 40. The same five arrays are outputted in the “Output” worksheet in the “MAPInvest” workbook as in the “Output” worksheet in the “MAPFloor” and “CBFloor” workbooks, previously explained.

These outputs are then transferred to the “MAPInvestOutput” worksheet of the “Control” workbook, the same workbook where the user applied the inputs.

## **CB Investment Macro**

The CB Investment macro functions similarly to the CB Floor and MAP Investment macro. The “CBInvestmentRisk” worksheet in the “CBInvest” workbook has the same arrays that function in the same way as the “CBFloorRisk,” previously explained. The difference is that in place of the “Placeholder” array that existed in the “CBFloor” workbook, there is an array that uses the investment make-up percentages inputted by the user. Much like the MAP Investment macro, the Overhang considers the larger difference, either the sum of the pay credits and the guarantee account or the sum of the pay credits and the nominal account. Recall that the

difference between the guarantee account and the nominal account is that the guarantee account ensures that at least the bond return will be matched in the employee's account. The nominal account does not carry the same bond guarantee. The Additional Liability array once again simply applies the termination probabilities to those overhang values.

The CB Investment macro then utilizes the "CBInvestmentRisk" worksheet and runs 1000 trials for each combination of ages 25 through 65 and years of service 0 through 40. The same five arrays are outputted in the "Output" worksheet in the "CBInvest" workbook as in the "Output" worksheet in the "MAPFloor," "CBFloor," and "MAPInvest" workbooks, previously explained.

These outputs are then transferred to the "CBInvestOutput" worksheet of the "Control" workbook, the same workbook where the user applied the inputs.

### **"Control" Workbook Outputs**

The model outputs six arrays in each of the sub-workbooks as previously explained. These outputs are then transferred back over to the "Control" workbook in each of the sub-workbooks' respective "Outputs" worksheets and transformed into percentages. The first four arrays are the present values at different interest rates by years of service divided by the expected pay credits for employees with those years of service at the same interest rates. In the array labeled "80 Plan Years", these percentages are the additional liabilities at each plan year compared to the total amount of retirement benefits the company can expect to pay out at that plan year for the entire population. The final array, labeled "PV at 5%/PV at 0%" divides the present values at those interest rates for each possible service year.

To complete these calculations, the "Denominators" worksheet in the "Control" workbook was designed to automatically create denominators from the Inputs tab. The number labeled "Expected Pay Credit" on the Denominators tab divides the array representing additional liabilities for 80 years ("80 Plan Years"). This is calculated by multiplying the pay credits at each age based on the salary scale by the percent of the total population at each age then summing these numbers. This reflects the amount of money the company expects to pay out at each plan year, making it a good comparison to the additional liabilities that the model projects.

The second set of denominators are comparisons to the present value of liabilities for each possible service year. So, the denominator for these arrays must be a present value of expected pay credits for each service year. The worksheet scales the total pay credits for each

combination of age and service by the probabilities of that individual remaining with the company until the year of that pay credit. These scaled sums of pay credits are then present valued at zero, five, and the input percentage to correspond to the three respective additional liability arrays produced in the “Outputs” worksheets in the sub-workbooks.

The present valued sums of pay credits are then scaled by the percentage of the population with the corresponding age and number of years of service. Finally, to complete the denominator calculations, the scaled and present valued sums of pay credits are aggregated based on service year. These are used as the denominators to divide into the additional liability outputs from each sub-workbook based on discount rate. The five percentage arrays can be found in the four worksheets in the in the “Control” workbook labeled “MAPFloorOutput,” “CBFloorOutput,” “MAPInvestOutput,” and “CBInvestOutput.”

The “Graphs” worksheet in the “Control” workbook is designed to visualize the differences between the four plans. The graphs labeled “CB Floor-80 Years,” “MAP Floor-80 Years,” “MAP Invest-80 Years” and “CB Invest-80 Years” display the continuation of the model over 80 plan years. This was created in order to analyze how the plans would change after their initialization in companies. The additional liabilities summed by plan year are compared to pay credits at that year. Because of limitations within Microsoft Excel, and for the purpose of achieving effective visual outputs, only the first fifty trials are displayed in each graph. These first fifty trials are random, however, and allow for an accurate portrayal of the 1000 trials overall.

Another output for consideration by the user on this worksheet is the Zero Top Up Probability chart, found on the “Charts” worksheet. Based on plan year, this chart displays the percentage of the 1000 trials that did not require any form of top up payments. This means there was no additional liability or overhang for that percentage of the trials.

Graphs of the by service summarizations from the “Charts” worksheet and the by plan year chart are also portrayed on the “Graphs” worksheet. The “Charts” worksheet in the “Control” workbook is designed to allow the user to compare the means, medians, 90<sup>th</sup> percentiles, and the largest 10% of the 1000 trials based on service year. These same characteristics are shown for all four plans.

The Graphs labeled “PV at 5%/PV at 0% -by Service Year” are summarizations of the arrays on the “Inputs” worksheet which divide the present values by service years at 5% by the

present values by service years at 0%. These numbers are intended to give a comparison of the time that liabilities occur, since liabilities that occur sooner are less affected by the 5% discounting and will be closer to the present value at 0%. Therefore, results closer to one imply shorter liability streams over the careers of the employees at the first year of the MAP or CB plan.

Also, there are automatically created arrays which summarize the same “PV at 5%/PV at 0%” for the entire population. This is accomplished by weighting the “by service year” arrays by the percent of the population at each service year. All 1000 trials of these are summarized in the graphs labeled “PV at 5%/PV at 0%-Population Histogram.” Finally, an average of all 1000 trials which give a single-number comparison for all four models is available on the Charts worksheet.

## Conclusions

There is a great deal of information that can be extracted from the model and the information that has been provided in the output sheets in the “Control” workbook. This information demonstrates that the Managed Account Plans have many added benefits as a pension plan as a whole while also having comparable benefits to the Cash Balance plans.

It is interesting to note that, in general, the “Investment” models have higher risk compared to the corresponding “Floor” models. Since the “Investment” model calculates the future liabilities for a company that is attempting to generate higher returns than the employee accounts, it is reasonable that this risky choice generates more liabilities. Of course, there may be benefits to this choice in the form of higher returns on company investments.

In reference to the MAPs, liability for a newly implemented plan increases as the employee service year increases. This is beneficial to the company since as a percentage, there are fewer employees with a higher account balance and with more years of service. As percentage of payroll, these people with higher liabilities with respect to the present value of their pay credits have less of an impact on the company. This can be seen in the following chart that summarizes the tail value at risk of the 90<sup>th</sup> percentile, the 90<sup>th</sup> percentile, the mean and the median of the MAP Floor and Investment. This is shown by graphing the present value of the liabilities of the company and dividing that by the total present value of the expected pay credits for the entire company. This is plotted against the service year of the employee entering the plan at its first year (plan year 0).

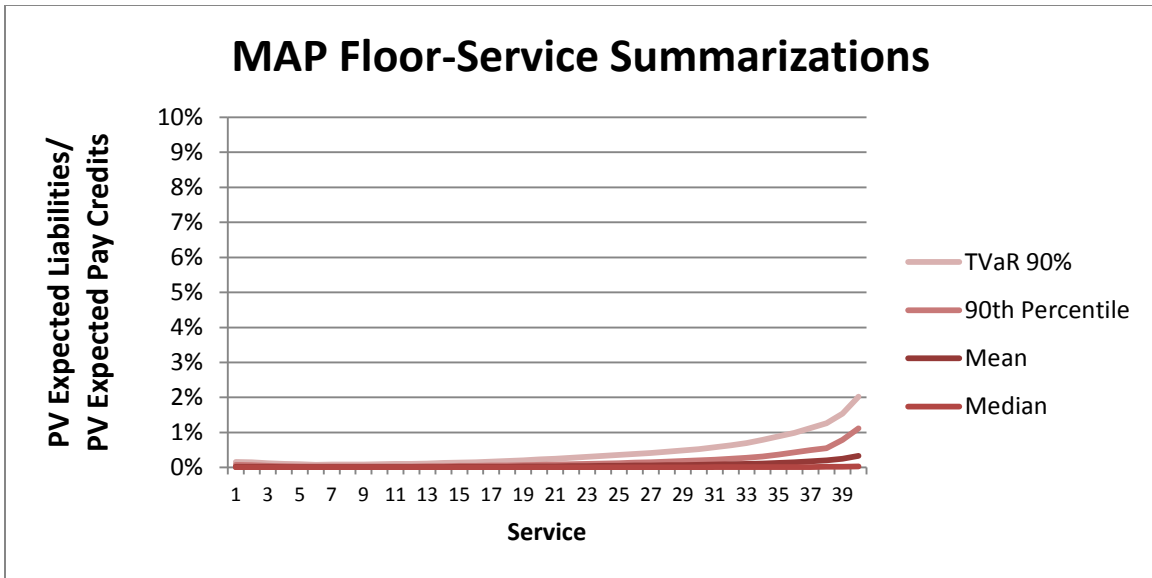


Figure 9: Map Floor- Service Summarizations

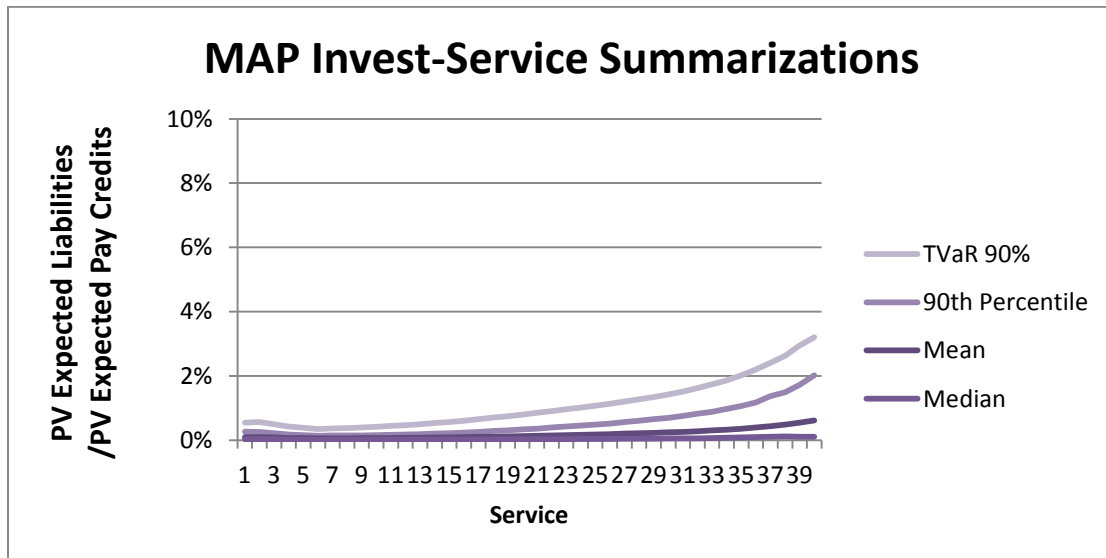


Figure 10: MAP Investments- Service Summarizations

There is a significant difference between the trends of the MAPs and the CBs. The Cash Balance plans have a higher liability when the employees' years of service are lower. This can be shown in the same way as the previous graphs as seen below in the "CB Floor-Service Summarizations" and the "CB Invest-Service Summarizations". Both of the following graphs show that as the service of the employees increases in a newly-implemented CB plan, the liability as a percentage and present value of the total liability decreases.

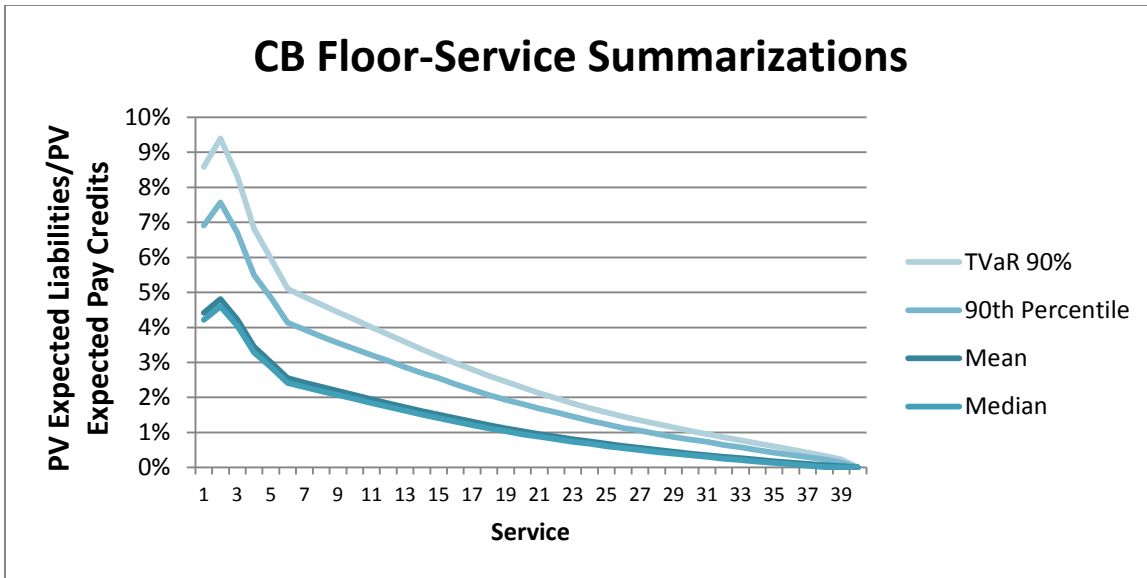


Figure 11: CB Floor- Service Summarizations

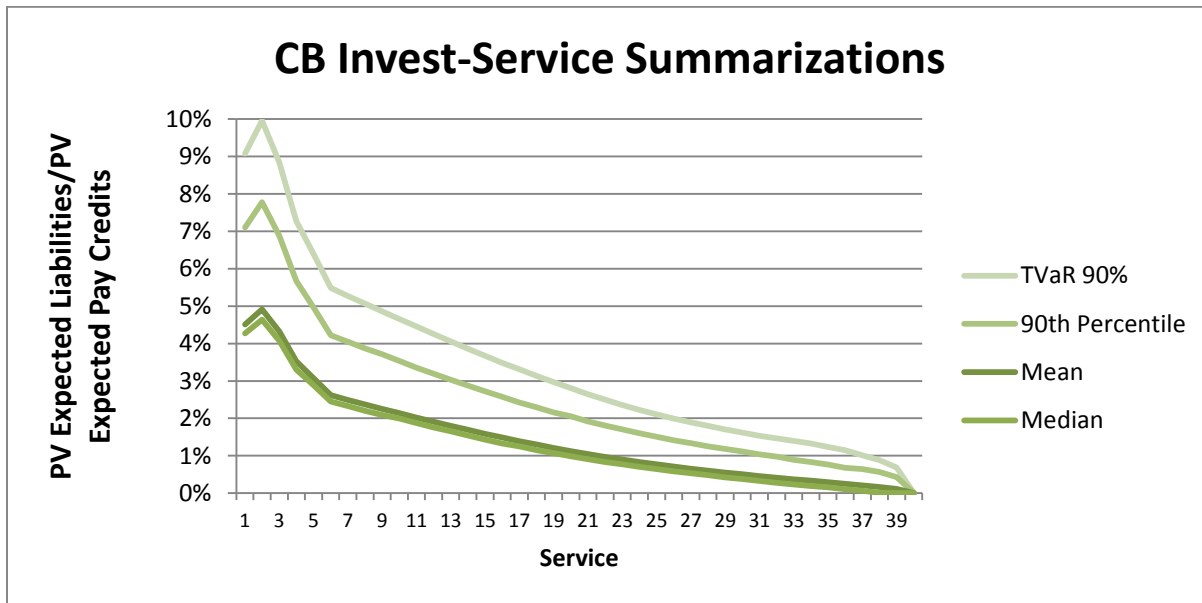


Figure 12: CB Investments- Service Summarizations

Overall, in the comparison between the two forms of pension plans, the percentages of the liability are less with the MAPs and the higher percentages (approximately 2%) have less weight than the higher percentages in the Cash Balance plans (approximately 9.5%). The weight is affected by how many employees of that service is currently employed and how large the account balance is. However, differences in inputs between the MAP and CB models must be taken into consideration when making direct comparisons between the graphs.

The models also produce a similar output to the Present Value of Expected Liabilities over the Present Value of Pay Credits, using the Present Value of Overhang. These outputs

would represent the same stream of liabilities, without consideration of the probability of termination at the time of the liability. This could be a valuable summarization for a company that was interested in the total possible liability at each year in order to plan reserves for their plan guarantee.

The eighty year graphs show the projected additional liability divided by the expected pay credit at that year. The following graphs show a random fifty trials from the original thousand. These outputs would be of interest to a company since these outputs can be scaled to show a company how much more they may have to pay in additional liabilities compared to what they originally expected to pay as a retirement benefit.

Assume that a trial shows a ten percent additional liability over expected pay credit in a given year. This means that a company that is already paying one million a year in pay credits would have paid one hundred thousand in additional liability in said year.

In general the MAPs show a decreasing trend in the additional liability while the CB plans show an increasing trend over eighty plan years.

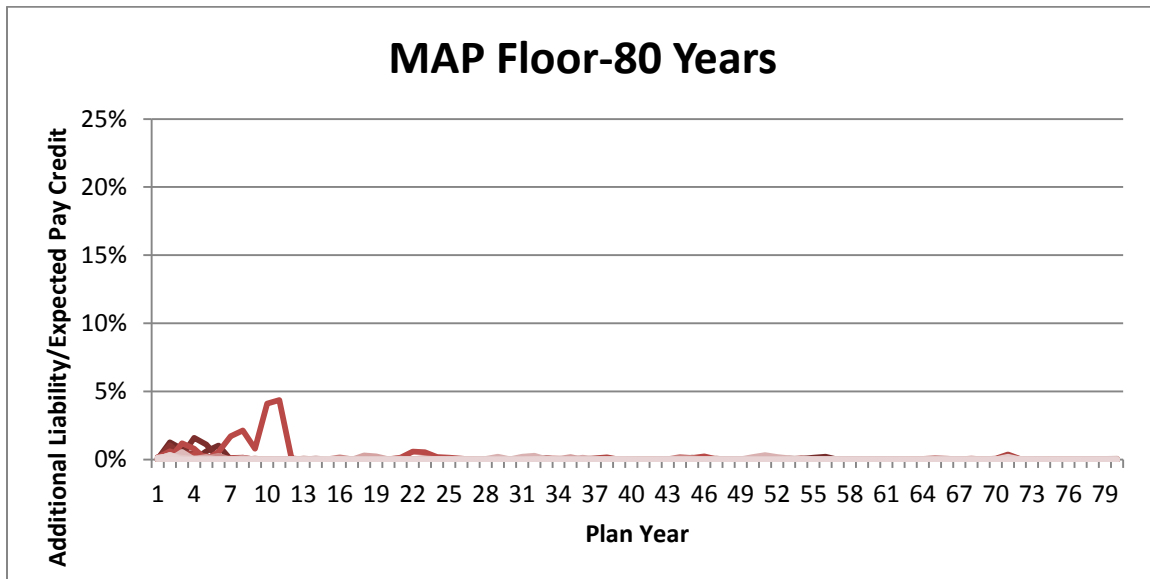


Figure 13: MAP Floor-80 Years



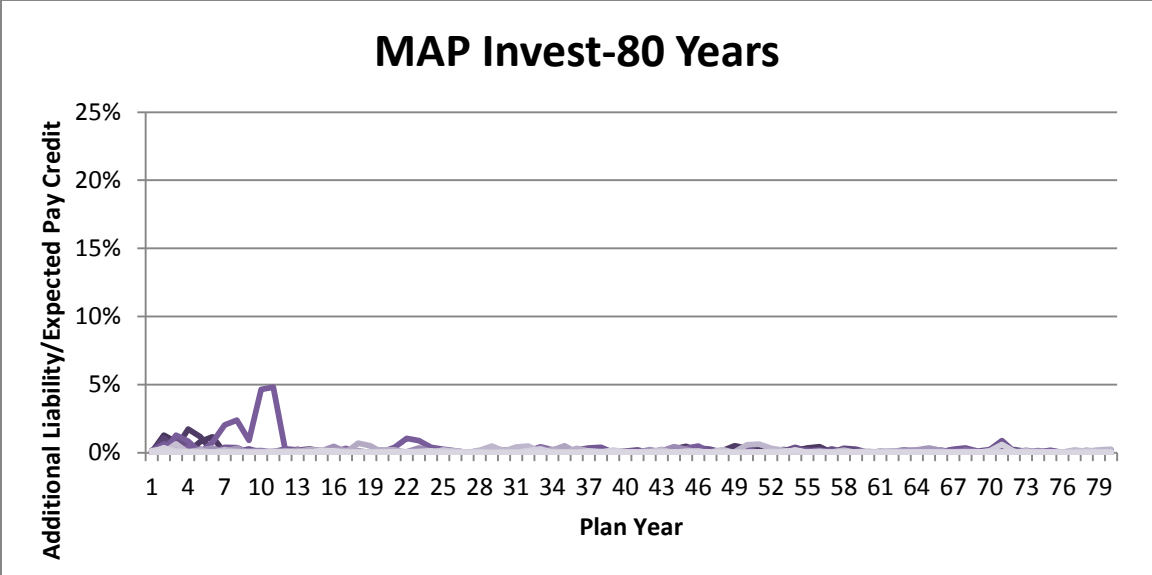


Figure 14: MAP Investments-80 Years

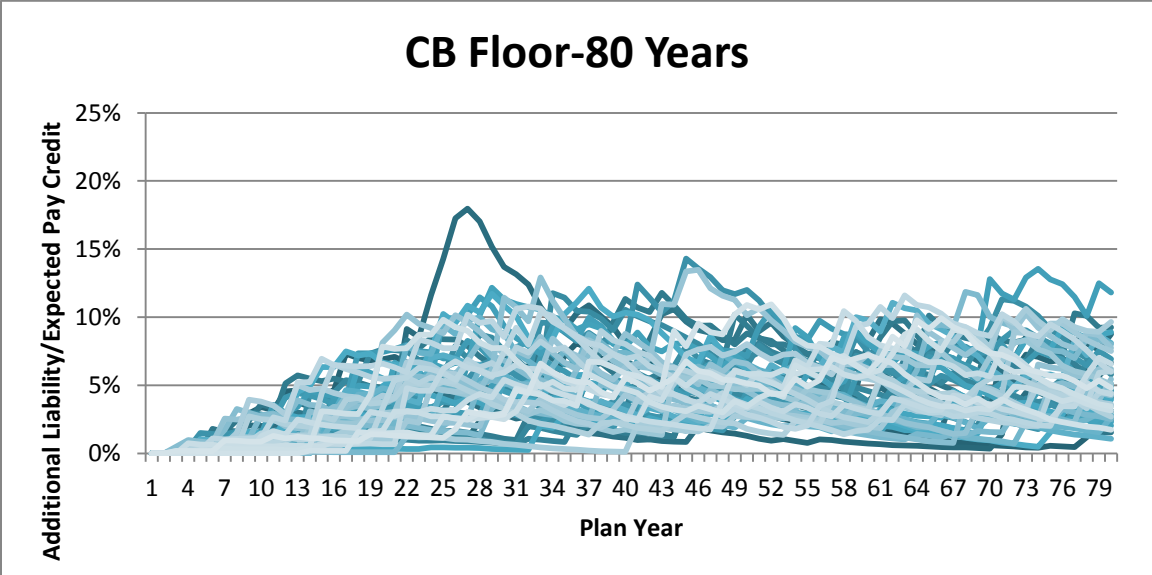


Figure 15: CB Floor-80 Years

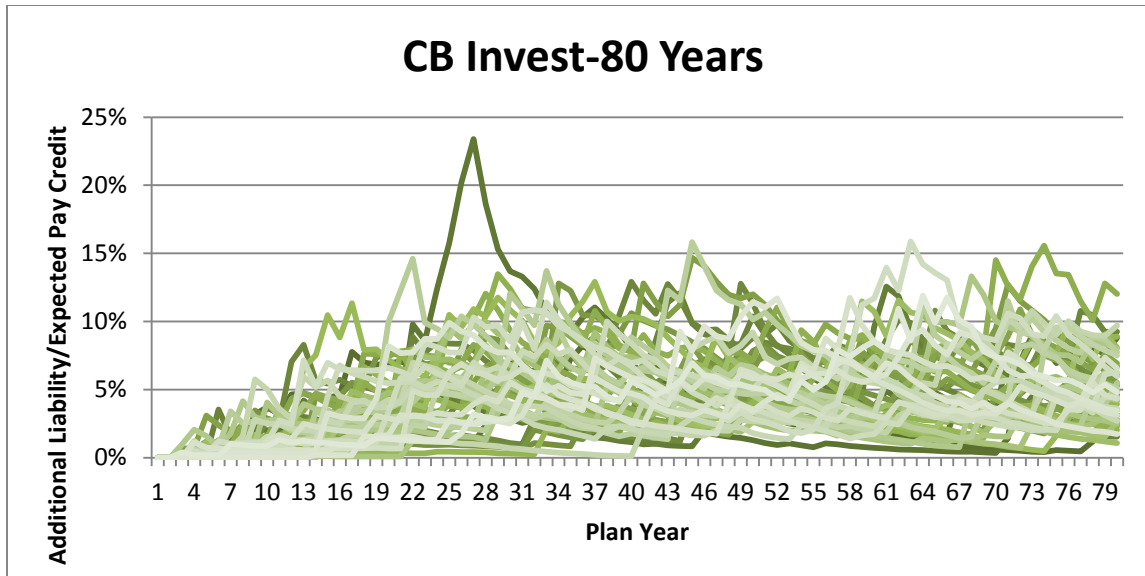


Figure 16: CB Investments-80 Years

It had been discussed in our meetings with Aon Hewitt that it would be beneficial to see a measure of the liabilities that allowed for a comparison of the timing when the liabilities occur. This can be seen in the diagrams listed below. Results closer to one reflect an earlier stream of liabilities since the numerator (present value at 5%) has been less effected by discounting when compared to the denominator (present value at 0%). The MAP values for all service years are close to one while the CB values for all service years show later liabilities for employees with less service.

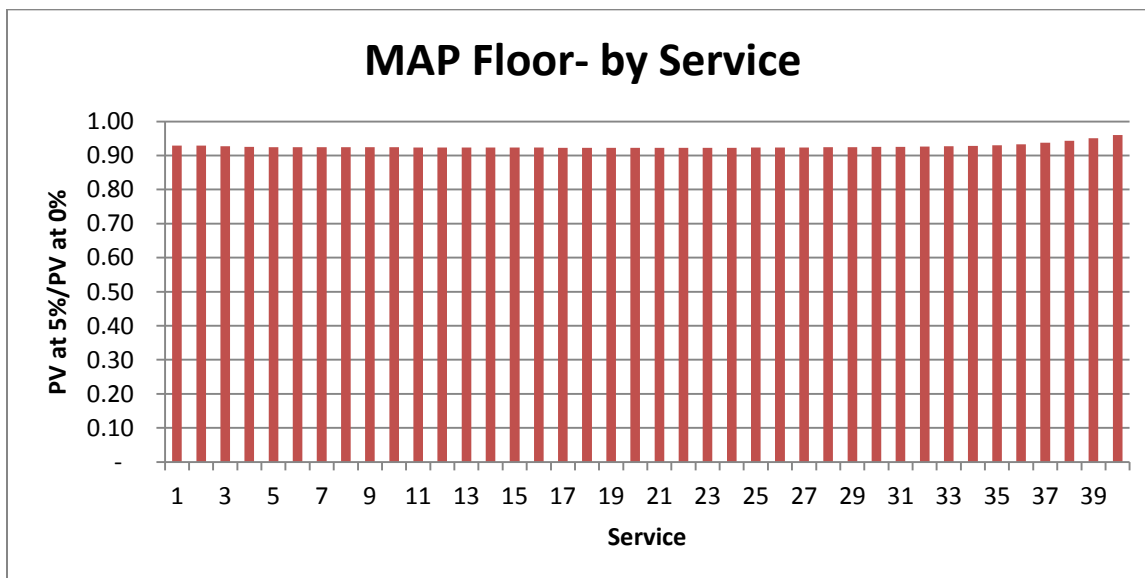


Figure 17: MAP Floor-Service PV at 5%/PV at 0%

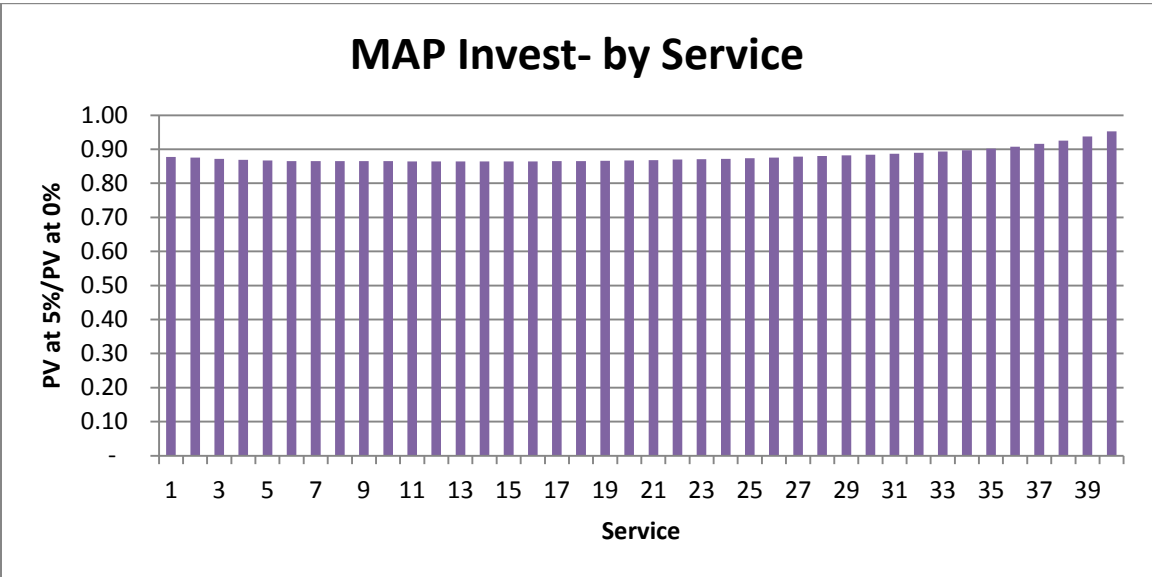


Figure 18: MAP Investments-Service PV at 5%/PV at 0%

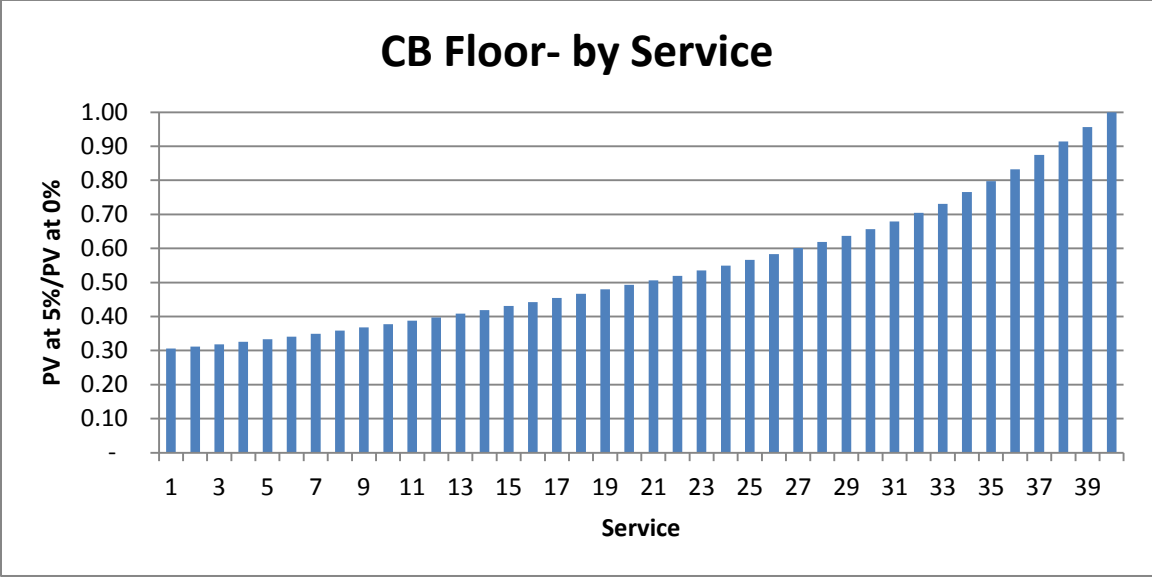


Figure 19: CB Floor-Service PV at 5%/PV at 0%

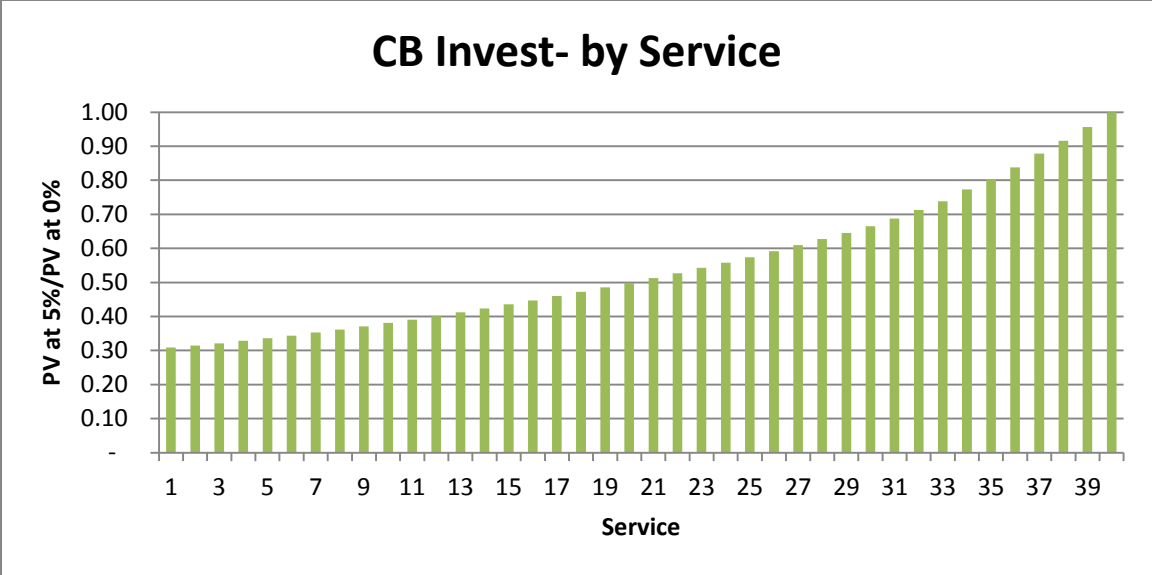


Figure 20: CB Investments-Service PV at 5%/PV at 0%

Using the by service year information above, we were able to consolidate the data into one population summary, using the weights of the population for each service year. As before, these numbers express when the additional liability is expected to occur, and values closer to one represent liabilities occurring closer to time 0. The MAP in general has shorter expected liability streams compared to the CB, and the Floor models have shorter liability streams compared to the corresponding Investment models.

Service Year	Population
MAP Floor	0.9257
CB Floor	0.3620
MAP Invest	0.8697
CB Invest	0.3655

Figure 21: Population PV at 5%/PV at 0%

As previously stated, there are many different interesting summarizations of this data. We have created a model that is flexible enough for many different outputs to be created and for the information to be analyzed in different ways. These summaries are only a small sample of what can be analyzed with the model and we believe this model can provide answers to many different questions Aon Hewitt may have pertaining to Managed Account Plans.

## Recommendations

### Next Steps

The next portion of the project will involve making some alterations to the current model and manipulations of the outputs. The future changes to the model will be in the interest of marketing the product to future clients. Many of the upcoming outputs will be manipulations of the current outputs and available data within the model.

The first consideration towards designing the model for professional use will involve ensuring that the new Managed Account Plans meet all legislative standards. Particular attention must be paid to the Age Discrimination in Employment Act (and its various amendments). There must be no discrimination from one employee's account to the next based on age. It must also be demonstrated that the MAPs are designed to enhance the benefits of the employees instead of simply benefitting the employer. Highlighting the benefits to the individual will aid in legislative approval of this new retirement plan design.

The next consideration will be how to most effectively market the idea to Aon's clients. Allowing for ease of client specific outputs will be a crucial functionality of the model. For example, a tool designed to take the current company profile and automatically project the population make-up into the future could be very helpful to the client's understanding of the MAP. This could also involve the functionality of scaling the population up or down as the company sees fit for its own future. Ensuring that the client can easily manipulate their own company information to be inputted into the model will aid in the comparison between the MAPs and CB plans.

Another point of interest will be to add the functionality of having a designated vested period, perhaps of three years. This functionality would allow for the company to only apply the guarantee benefits of the Managed Account Plan to those employees who have been with the company for a minimum period of time. This would allow the company to add incentive to remaining with the company and security against some of the early, typically most volatile years of the Managed Account Plan.

Designing multiple options for the employer to be able to hedge the risk presented by the Managed Account Plan will aid in the selling of this new, unprecedented retirement plan offering. One potential option would be the purchasing of swaps to make up for any additional

liabilities they may potentially pay out to their employees who terminate. Combined with the comparison outputs of the model, having additional ways of defraying the potential risk will encourage the benefits of the MAP.

Since not all additional liabilities can realistically be avoided, designing options for paying out the employees' benefits at the time of their retirement will encourage companies to adopt the MAP. For example, designing plans that offer an annuity to the employee upon time of retirement would be one option. Ideally these options and suggestions would aid in the presentation of the MAP to legislature.

For the use of Aon Hewitt in future, we will also have a finalized and user-friendly version of the original straw man. The straw man will be used for Aon's clients who may want to view the idea of a MAP on a smaller, individual employee scale. The current straw man needs investment type correlation considerations, other minor adjustments, documentation and outputs designed to meet the needs of Aon and their clients.

## **Recommendations Beyond this Project**

Moving forward with the use of the model and the future additions to the project, we have made the following recommendations to enhance the current model.

The first recommendation is to take a closer look into the correlation matrix inputted by the user and how it is applied. The current application utilizes the Monte Carlo Framework and the Cholesky decomposition matrix to determine the covariance between multiple investment type pairings. Further research should be considered to ensure proper utilization and application of the investment correlations.

Another suggestion is to add functionality to the model that allows for a scaling of the pay credit percentage based on an employee's age and service. Currently the model applies one percentage no matter what age or number of years of service an employee represents. For a company with information regarding the fluctuation in pay credit percentage of salary over time, this could add to the accuracy of the model.

Designing a way to "shock the system" is an additional functionality a client may find interesting. For example, if the market was to experience a severe spike, either positive or negative, how that would affect the outcomes of the plan, and how those outcomes of a MAP compare to a CB plan under the same market conditions should be considered. This is a reality for companies to consider especially in recent decades with unpredictable peaks and lulls.

Understanding the best and worst case scenarios and having plans for those times could ensure the company's confidence in the MAP.

Currently the model only compares a MAP to a Cash Balance plan. There could be interesting considerations when comparing other types of pension plans or 401K plans.

While designing the model, consideration was put towards designing the MAP to produce yearly outputs as appose to the current monthly outputs. We determined too much information about the fluctuation of the employee's account was lost by not including the aggregation by month. Therefore, the model currently compares the MAP accrued monthly and the CB plan accrued yearly. Additional consideration should be given to how the MAP would operate yearly and if there is a way to avoid losing accuracy when operating the MAP this way.

The final suggestion is to add functionality to the model that allows for the employee to have some percentage of the decision about what kinds of investments their account utilizes. As time goes on, the employee would ideally have less and less say because there is a higher account balance at stake for the company. The company, and most likely the employee, would both be better off having the company decide which investment types will be used in the latter years of the account. This functionality could add to the positive aspects of a MAP when presenting the plan to legislature. The more consideration to the employee's benefit, the more likely the MAP will be approved for industry use.

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## Appendices

### Appendix A: Documentation

#### Purpose of Model

This model estimates the guarantee and investment risks of a population enrolled in two retirement plans, Managed Account Plan (MAP) and Cash Balance plan (CB). The model creates lognormal stock returns using the Monte Carlo Method, and estimates the amount of money that the company might need to add to a participant's account based on the guarantee if the participant's employment is terminated. For a MAP, the guarantee is a 0% overall return on investments, so the company will need to make up any deficit between the sum of pay credits, up to the time the employee leaves, and the actual investments. For a CB, the guarantee is a minimum investment return at each year, so the company would need to make up the difference between the account balance with this guarantee and without it. The guarantee is a fixed percentage in this model, but in practice may be linked to a bond or index.

#### Assumptions

##### Lognormal

This model assumes lognormal stock returns, which uses historical information about a stock or index to estimate a mean return and the standard deviation of those returns. The following formula takes the mean ( $\mu$ ) standard deviation ( $\sigma$ ) and correlation as inputs to estimate 1000 trials of 960 monthly returns.

$$e^{\left(\alpha - \frac{\sigma^2}{2}\right)T + \sqrt{T}Z}$$

The Monte Carlo method, while widely accepted as a valid method of creating random lognormal returns, has some limitations. It is more accurate for short term returns. It may not have a heavy enough tail to accurately represent the returns. However, it is still the most commonly used equation for this application since it is simple to code and not calculation-intensive.

*See Appendix B for more information on the lognormal assumption and the investment returns.*

### ***Termination at End of Years***

The CB model assumes all terminations take place at the end of the year, using the turnover assumptions. This means that even if an employee has a deficit in their account during the year, the guarantee is applied as if they had left at the end of the previous year when that deficit might not exist. This method was chosen in order to reflect the real limitations of the CB plan, where the guarantee is only applied once a year.

The MAP model assumes that employee terminations occur at the end of the month. The probability of an employee termination in the year is distributed equally across the months of that year. Since a MAP guarantee can create an additional liability for the company at any point in time (when an employee with an account deficit leaves the company) the monthly assumption was chosen as a more accurate representation.

### ***Constant Pay Credit %***

The model only has the capability to accept a fixed percentage for calculating pay credits. This may not reflect the real situation of companies that use a scaled pay credit percentage based on service, salary or age.

### ***Stable Population***

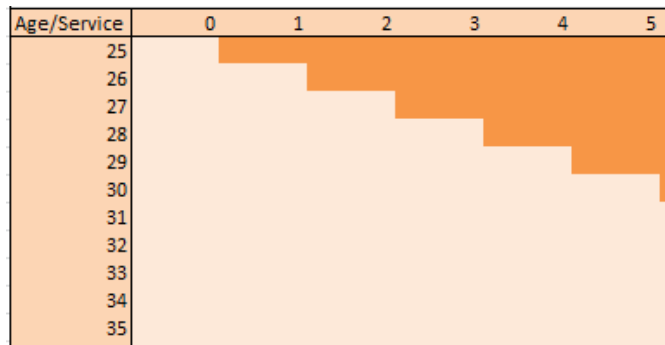
The current population model creates a population that does not change, given a hiring profile. Of course, this is not realistic since the number of employees in a company and their ages will vary over time.

### **Inputs**

*Inputs should be changed ONLY on the Control Panel workbook. Running the model will automatically copy these inputs into all of the necessary workbooks. This functionality is intended to keep the inputs consistent between runs.*

	Termination Rates by Age and Service					
Age	0	1	2	3	4	5+
25						
26						
27						
28						
29						
30						
31						
32						
33						
34						
35						

**Termination Rates** are the yearly probabilities of a person with a given age and service leaving in the next year. These should be probabilities, but do not need to add to one since each probability is dependent on still being employed at the beginning of the year.



The **Hiring Profile and Population Percentages** are input in the lighter colored portion of this matrix, with zeros in the dark orange cells. The entire matrix should sum to one. See Appendix C for information on how to create a population profile that will work for this.

	Salary Scale
Age	% Increase
25	
26	
27	
28	
29	
30	
31	
32	
33	
34	
35	

A **Salary Scale** grows salaries by the given percentage each year, so older employees make relatively more than younger ones.

Age	Nominal Account			Investment Account		
	Equity	Bonds	Risky	Equity	Bonds	Risky
25						
26						
27						
28						
29						
30						
31						
32						
33						
34						
35						
36						

The **Investment Mixes** determine how much of the notional and investment accounts are invested in each of the three assets (generally labeled here as Equity, Bonds and Risky but could be any three investments). The models that estimate ‘floor’ risk only use the Nominal Account weights, and ‘investment risk’ uses both Nominal and Investment Account. The Investment Account should be more risky than the Nominal Account. If they are the same, there will be no difference between the ‘floor’ and ‘investment’ risk when the models are run. In a realistic situation, as the employee gets older the investments should become less risky. However, as long as the account weights sum to one for each year (Nominal Equity+Nominal Bonds+Nominal Risky=1) the model will run without a problem.

Starting Salary	
Pay Credit %	
Chosen Output Discount Rate	
Guaranteed % Notional Acct	

The **Starting Salary** is used as the salary for a 25 year old and is subject to the salary scale. Since all of the outputs are ratios, this number could theoretically be anything. However, if there is a chance that the outputs will be presented without creating ratios, it may be best to keep this at a reasonable level.

The model uses a flat **Pay Credit Percentage** to calculate the percent of an employee’s salary that is paid into their account as a pay credit each year. This input creates the pay credits for the CB and MAP with the same percentage. *Make sure this is entered as a percentage, such as .02.*

The model has the option of using a **Chosen Output Discount Rate**, which will adjust the discount rate used to present value some of the outputs (these will be labeled as ‘Input Rate’).

This rate should be entered as an annual effective rate, and where necessary the equivalent monthly nominal rate will be automatically created. *Make sure this is entered as a percentage, such as .02.*

The **Guaranteed % Notional Account** modifies the guarantee in the MAP models. This rate is the annual interest rate that the guaranteed account grows by. When this is zero the employee is guaranteed that their account will never fall below the sum of the pay credits that have been paid into it. *Make sure this is entered as a percentage, such as .02.*

	Annualized Mean	Annualized Sigma
Large Cap U.S. Equity		
Long Duration Bonds – Credit		
Emerging Markets		

The **Annualized Mean and Annualized Sigma** are created from historical returns of different investments. These should be entered as *annualized* because the model automatically creates the monthly inputs when they are needed.

Correlation Matrix	Large Cap U.S. Equity	Long Duration Bonds – Credit	Emerging Markets
Large Cap U.S. Equity	1.0000		
Long Duration Bonds – Credit		1.0000	
Emerging Markets			1.0000

**Investment Correlations** should also be calculated from historical returns. The diagonal should always be 1’s, and the other cells should all be filled in (so there will be two instances of each correlation in this matrix) It is also important to fill this with only correlations, because the *covariance* matrix is calculated in the model using these numbers.

## Calculations

*See Appendix B for information on how investments are calculated using the “Refresh Investments” macro.*

## Straw Man

At the center of every calculation is “The Straw Man” model, which projects 1000 trials of additional liabilities for an employee with a given age and service at a particular plan year. On the “Inputs” worksheet of the “Control” workbook, there are inputs for Starting Age, Service Year and Plan Year that are changed by the macro to loop through all possible employees.

When the starting age or service is changed, this refreshes the “Calculated” worksheet in each of the sub-workbooks. This refresh pulls the inputted decrements from the “Inputs” worksheet and uses them to calculate termination probabilities at each year. The worksheet also uses the input salary scale and starting salary to project the salaries and calculate expected pay credits for this profile.

These age, service and plan year inputs also move the projected liabilities to the appropriate areas in the 80-year (CB) or 960-month (MAP) matrix on the ‘Risk’ worksheet in each of the sub-workbooks. Therefore, inputting age 35, service 5, and plan year zero would activate the formulas in the first 30 or 360 columns and use the salary, pay credits and termination from the ‘Calculated’ worksheet for a 35 year old with 5 years of service in the first column, a 36 year old with 6 years in the second, etc. Similarly, age 40, service 0 and plan year 10 would move the first column of liabilities to the tenth (or 120<sup>th</sup>) column of the array.

### ***Account Balances***

The model uses the projected investments to create account balances, which are used to identify the times when the guarantee would force the company to add money to an employee’s account and the amounts of those payments. These formulas (at time t) are:

$$\text{MAP Guarantee}(t) = \text{MAP Guarantee}(t-1) * (1 + \text{MAP Guarantee } \%) + \text{Pay Credit}(t)$$

$$\text{MAP Notional}(t) = \text{MAP Notional}(t-1) * (\text{Notional Account Return}(t)) + \text{Pay Credit}(t)$$

$$\text{MAP Investment}(t) = \text{MAP Investment}(t-1) * (\text{Investment Account Return}(t)) + \text{Pay Credit}(t)$$

$$\text{CB Guarantee}(t) = \text{CB Guarantee}(t-1) * \text{Max}(\text{Nominal Account Return}(t), \text{Bond Return}(t)) + \text{Pay Credit}(t)$$

$$\text{CB Nominal}(t) = \text{CB Guarantee}(t-1) * (\text{Nominal Account Return}(t)) + \text{Pay Credit}(t)$$

$$\text{CB Investment}(t) = \text{CB Guarantee}(t-1) * (\text{Investment Account Return}(t)) + \text{Pay Credit}(t)$$

### ***Population Projection***

The workbook takes 1000 trials of investment projections for 80 years (monthly or yearly) and applies formulas to those 1000 by 960 or 80 arrays. Then, for any realistic combination of age, service and plan entrance year (on the “Inputs” worksheet in the sub-workbooks) these formulas will create a stream of liabilities representing that person.

Generally, the macro calculates additional liabilities in two steps. First, we consider a “cohort” population which represents the initial implementation of the plan (whether CB or MAP). These calculations happen at plan year 0, where not only are the new employees’ (those

hired in plan year 0) liabilities estimated, but those who are new to the plan and not the company. At this point we consider all possible combinations of age and service at plan year 0. These liabilities are scaled by the population percentages and summarized by service year.

The second step is a continuation of the plan, which adds employees with 0 years of service at each plan year. For each plan year (1 to 80) the model calculates the future liabilities for all possible new employees (ages 25 to 65) and scales those liabilities by the ‘hiring profile’, given by the zero service column of the population percentages. These liabilities are summed by the year in which they occur.

### *Overhang and Additional Liabilities*

For the MAP, an overhang is considered the positive difference between the sum of pay credits (with optional credited interest) up to that point and the account balance (investment or notional). For the CB plan, the overhang is the positive difference between the accounts with or without a guaranteed return. The formula used to create these is:

MAP Additional Liabilities=MAX(0, Sum of Pay Credits-Account Balance)

CB Additional Liabilities=MAX(0, Account with Guarantee-Account without Guarantee)

The investment models also take into consideration positive differences between the investment account and the Sum of Pay Credits or Account with Guarantee.

In order to convert these overhangs into additional liabilities, they are scaled by the probabilities that the employee with that liability leaves at that time. So, if an employee had a \$100 overhang in a given month or year and had a .1 probability of leaving in that year, the additional liability would be \$10.

### **Outputs**

The information created in a model run is summarized in two ways, by service year and plan year. This creates four 1000x40 matrices and one 1000x80 or 1000x960 matrix for each model run.

The service summarizations take the projected additional liabilities and overhangs for every combination of age and service at plan year zero and present values them at different interest rates. The projected liabilities for a 30 year old with 4 years of service would be present valued at 0%, 5% and the input rate. Those present values would be scaled by the percentage of



the population made up by 30 year olds with 4 years of service and added to the running total of liabilities created by employees with 4 years of service. The output created is a 1000x40 matrix, where each column represents 1000 trials of present valued additional liabilities or overhangs created by the employees with that number of years of service.

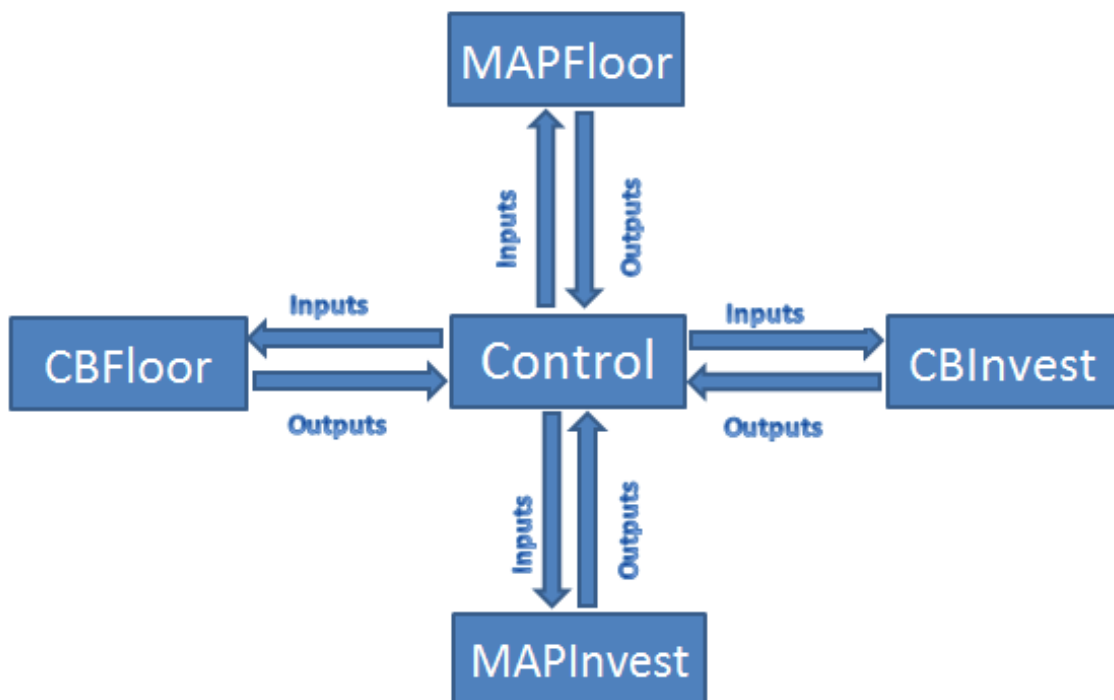
The plan year summarizations simply add the 1000x80 or 1000x960 matrices created on the 'Risk' worksheet of each of the sub-workbooks for every combination of age and service, weighted by the percentage of the population for that age and service.

### How to Run the Model

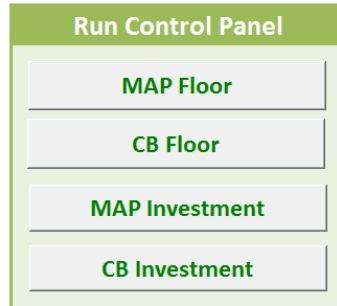
*Note: The model runs more efficiently when all of the workbooks are set to manual calculations.*

The model consists of five interconnected workbooks, one of which is the "Control" workbook. This workbook controls the other four, so it must be open at all times when the model is in use and should be the only workbook that the user interacts with. However, it is best to keep all five workbooks in the same folder to avoid problems with links.

The "Control" workbook is connected to the other workbooks as described in this diagram:



Within the “Control” workbook is the “Control” worksheet, which has ten buttons that run different macros, and a table of timestamps. The first set of buttons is the Run Control Panel area:



When the user selects one of these buttons, they must be sure to have the corresponding sub-workbook open. For example, to run MAP Floor, the user would need to have “Control.xlsx” and “MAPFloor.xlsx” open and named as such. Then, the following will automatically happen when the corresponding button is clicked.

1. Beginning timestamp.
2. Copy Inputs from Control to sub-workbook.
3. Copy Investments from Control to sub-workbook.
4. Run the macro as described in the Calculations section.
5. Output to the “Outputs” worksheet of the sub-workbook.
6. Refresh links to corresponding “Outputs” worksheet in the “Control” workbook.
7. Ending timestamp.

The next set of buttons is the Investment Control Panel area:



When the user selects the Refresh Investments button, they only need to have the “Control.xlsx” workbook open. Pressing this button will create six new matrices of random,

correlated investments in the “Investments” worksheet in the “Control” workbook using the method described in Appendix B.

The 1000x960 arrays are used in the MAP sub-workbooks, and the 1000x80 arrays are used in the CB sub-workbooks. It is important to remember that these are not automatically copied to the sub-workbooks when the Refresh Investments button is pressed, but are copied when the Run Control Panel buttons are activated by the user.

The Clear Investments button clears the arrays in the Investments worksheet of the “Control” workbook but does not affect the sub-workbooks. It is best to clear the investment arrays using this button before refreshing the investments.

*If a direct comparison between models is desired, the user should not refresh investments in between model runs. That way, the models are all run with the same set of investments.*

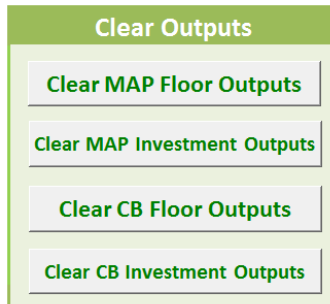
For the user’s convenience, the “Control” workbook includes a table of timestamps:

	Start	Finish
MAP Floor		
CB Floor		
MAP Investment		
CB Investment		

Last Investment Refresh	
-------------------------	--

These timestamps are created at the beginning and end of the most recent model runs, and at the end of the most recent investment refresh. The timestamps are intended to help the user keep track of which models have been run with the most recent investments.



The Clear Outputs area contains buttons which clear the arrays in the “Outputs” worksheet of the corresponding sub-workbook, given that the correct workbook is open. This will affect the linked cells in the “Output” worksheet in the “Control” workbook, since the cells they link to will all be cleared. It is best to clear the outputs before running each model.

## Appendix B: Investments

The Monte Carlo method is used to project randomized investment returns with an inputted mean and sigma, and a given correlation matrix. For more information about creating correlated stock returns, see the paper *The Monte Carlo Framework, Examples from Finance and Generating Correlated Random Variables* by Martin Haugh from Columbia University. The steps used to create these returns using the Investments macro are:

- 1. Convert annualized mean and sigma to monthly.** The “Inputs” worksheet in the “Control” workbook uses a yearly mean and sigma. These must be converted to monthly to create monthly returns. The following formulas are used:

$$\sigma_{monthly} = \frac{\sigma_{yearly}}{\sqrt{12}}$$

$$\alpha_{monthly} = \frac{\alpha_{yearly}}{12}$$

- 2. Calculate Covariance Matrix.** The covariance matrix is computed using this formula for all combinations of investments, using the inputted sigma and correlations.

$$Cov_{i,j} = \sigma_i \sigma_j \rho_{ij}$$

- 3. Calculate Cholesky decomposition of covariance matrix.** In order to apply the covariances to the projected returns and create correlated returns, the Cholesky decomposition of the covariance matrix must be created. The formulas are found in this table.

$Chol_{AA}$ $= \sqrt{Cov_{AA}}$	0	0
$Chol_{AB}$ $= \frac{Cov_{AB}}{Chol_{AA}}$	$Chol_{BB} = \sqrt{Cov_{BB} - Chol_{AB}^2}$	0
$Chol_{AC} = \frac{Cov_{AC}}{Chol_{AA}}$	$Chol_{BC}$ $= \frac{Cov_{BC} - (Chol_{AB}Chol_{AC})}{Chol_{BB}}$	$Chol_{CC}$ $= \sqrt{Cov_{CC} - (Chol_{AC}^2 - Chol_{AB}^2)}$

- 4. For 1000 investment trials, do the following and save in three 1000x960 matrices:**

- a. **Generate random standard normal numbers.** This is accomplished by generating uniform random numbers from 0 to 1 (representing probabilities) and taking the inverse of the standard normal distribution of those random numbers.
- b. **Multiply matrix of random standard normal numbers by Cholesky-decomposed covariances.** This will create a 960x3 matrix of correlated normal numbers.
- c. **Use inverse normal matrix in the Monte Carol equation.** This equation generates monthly (when T=1/12) returns for 960 months of each of the three investment types, where Z is an entry in the matrix of correlated normal numbers.

$$e^{\left(\alpha - \frac{\sigma^2}{2}\right)T + \sqrt{T}Z}$$

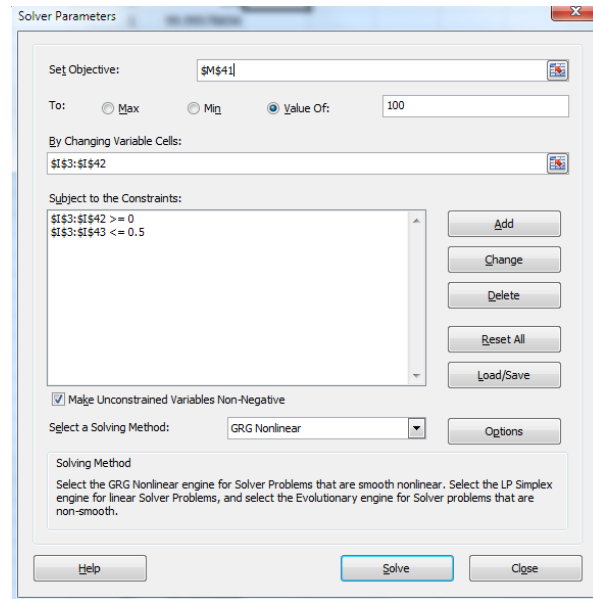
- 5. **Create yearly investments.** In order to create comparable investments for use in the yearly macros, yearly investments are created by multiplying the first 12 months of returns to create the first year, etc.

## Appendix C: Population Projection

In order to create a population using provided decrements, it was necessary to create a hiring profile that would maintain a stationary (level) population by hiring new people to replace those terminating.

The model accepts yearly termination probabilities for all ages, and service years 0 to 5+. These can be used to create a hiring profile by the following process:

1. Fill a triangle matrix of all possible age/service combinations with 1s (i.e. there are no 25 year olds with 20 years of service, so those should be 0). This is an initial population, but of course does not have the correct ratios.
2. Create 39 additional matrices, each time applying the correct decrements to that initial population of 1s and moving the numbers as the population ages (so a 25 year old with 0 years of service is a 26 year old with one year of service in the next array). The population will get smaller as the decrements are applied and no new employees are hired.
3. In a separate location, create a hiring profile and use links to automatically “hire” that profile of employees in the first column of each matrix. Initially, this can be filled with 1s since it will be changed later on. The last hiring age (in this case, 65 year olds) should be set to 0 in order for Solver to find a solution.
4. It will be useful to create a table which tracks the total population by summing the entire matrix at each year.
5. Use the Solver tool in Excel to modify the hiring profile. The objective of the solver tool should be to set the last matrix population to the “Value Of: 100.” The “Variable Cells” should be the hiring profile, except the final year which is always zero. Additional constraints may be necessary in order to find a reasonable solution, one that does not have any large jumps from one age to the next and hires more young employees than old ones.



6. Check that the solution created a population that is stationary by checking that future years still have a population sum of 100.
7. Divide the entire matrix at year 40 by 100 in order to find the population percentages, which will be used as an input.

## Appendix D: Marketing the Managed Account Plan

### Introduction

The Managed Account Plan (MAP) and Cash Balance (CB) Plan comparative model has a wide range of functionality that allows the user to cater the comparisons directly to Aon Hewitt's client specific criteria. This is going to be an important strategy point for selling the new MAP pension plan to prospective and current Aon Hewitt clients. Concurrently, the MAP must be approved by legislature as a pension plan design with the employee's best interests in mind. Laws and regulations are designed to mind the interests of the individual employee while Aon Hewitt is not only concerned for the benefits of the individual employees but also the benefit to their clients, the employers, as well. To better understand the obstacles in legislature approval exploration into the history of pension plan acceptance and adoption was conducted.

The National Conference of State Legislatures maintains legislation summaries for each year that include information concerning state efforts to address pension issues. The most current summary available was for 2011 (Snell, 2012). Over the past five years state legislatures have taken steps towards increasing the amount employees are required to contribute to their own employee accounts. A shift from defined benefit to defined contribution or "hybrid" plans has also become more prevalent. In 2011, Rhode Island "transferred to a hybrid plan that will consist of a reduced defined benefit plan and an individual account for each member" (Snell, 2012). The risk associated with a defined benefit plan lies in guaranteeing a benefit at the time of retirement for an undetermined amount of time, until that employee passes away. A defined contribution plan, in opposition, is very deterministic as the employer contributes a certain amount to the employee throughout their career with the company. Therefore, the risk lies with the employee who, what the time of retirement, must ration their benefits at that time for "the rest of their lives." A hybrid is an ideal situation to transfer the risk equivalently onto both the employee and the employer. The most recent changes in some state legislatures provide hope for the implementation of a hybrid plan such as the MAP.

Another shift towards less responsibility having been placed onto the employer was the change in requirements of the employee towards their own benefit contributions. According to the 2011 state legislation summaries, "Sixteen legislatures increased age and service requirements for normal retirement for state employees... to or closer to 65, and increase(d) the minimum amount of service credit a person must have for any alternative earlier age of



retirement” (Snell, 2012). Additionally, vesting period application was often increased from five or six years to eight to ten years. All of these changes suggest that less risk on the employer is a direction state legislatures are comfortable moving towards going forward.

Another contributor to law and regulation standards is the Pension Benefit Guaranty Corporation (PBGC). The goal of PBGC is to “protect the retirement incomes of more than 40 million American workers in more than 26,000 private-sector defined benefit pension plans” (Pension Benefit Guaranty Corporation, 2013). To manage the “continuation and maintenance of private-sector defined benefit pension plans (and) provide timely and uninterrupted payment of pension benefits, and keep pension insurance premiums at a minimum,” the Employee Retirement Income Security Act of 1974 created the PBGC (Pension Benefit Guaranty Corporation, 2013). However, with a shift to a hybrid between a defined benefit and a defined contribution, the U.S. Government Agency, the creator of PBGC, could potentially have an issue with a move towards MAPs.

Especially since 2008, the tough economic situation has placed a lot of pressure on companies to supply adequate benefits to their employees but to also be wise and diligent about their investments. At the end of 2008 there was a “\$1 trillion” gap between the actual funds set aside for employee retirement benefits and what was actually promised (Keegan, 2010). This deficit results in company stability issues. Some other results from the current economic downturn include:

- From 2000 to 2008 the United States went from having more than half of the states have fully funded pension plans to only six states in 2006 and only four by 2008.
- Eight states fell into the category of having less than a third of their total pension liability funded. This left more than two thirds unfunded in these states.
- Forty out of fifty states were determined to need improvement considering most had available less than 10 percent of the funds they actually promised to their residents.

With so many effects from the state of the economy, it is imperative that state legislatures are not finding themselves with too much debt to cover the costs of the pension plan promises made to employees (Keegan, 2010). This is just as important as the risk associated with companies who make pension plan promises to their employees and take on the risks of investing those

reserves in areas such as bonds or equity. A downturn in the economy could greatly affect those returns and lead to large problems for the company.

The process of reforming currently accepted pension plan designs can be a long and tedious road to adoption. The state legislature is first and foremost concerned with the benefit to the individual employees of a company rather than the employer themselves (Gilliland, 2013). Aon Hewitt's clients being the employers themselves, they hold more weight on how to exploit the benefit to the employer. There then must be a very unique balance in marketing the MAP. There must be a simultaneous consideration for both the benefit to the employee as well as the benefit to the employer. A look into the adoption of the 401(k), a widely used retirement plan by many corporations, showed a multi-step process with many road blocks along the way.

In order to estimate the approach as to how to market the MAP to legislature, a look into how the 401(k) was adopted was conducted. As an answer to moving away from the defined benefit retirement plan, the 401(k) was designed to be a "cash or deferred arrangement under which a covered employee can elect to have a portion of his or her compensation contributed to a qualified retirement plan...invested in a wide variety of investment vehicles such as stocks, bonds, guaranteed investment contract..." (Employee Benefit Research Institute, 2005). The defined benefit plans were proving to be too generous to the employee and would result in major gaps between the money owed to the employees by the employer and the actual reserves they had available. To avoid the inability to meet the employee benefits a new plan was designed, the 401(k). The 401(k) plan was first approved in 1978 through a provision in The Revenue Act of 1978. The next major milestones were the adoption of large name corporations such as Hughes Aircraft Company in 1978 and Johnson & Johnson in 1979. By 1982 other large name companies such as PepsiCo, Honeywell, and JC Penny had all began 401(k) proposals in the plan to switch over their retirement benefit options. Between 1980 and 1992 the 401(k) underwent many regulation changes and adjustments to better meet the needs of the employee. Amongst these changes were regulations against discrimination and certain sanctions such as on the use of employee salary reductions as a source of retirement plan contributions (Employee Benefit Research Institute, 2005).

## Materials and Methods

With so many avenues to pursue when considering the marketing and plan for review and explanation of the new MAP, the following approaches were decided on as those with the most potential to see the MAP through to regulatory approval.

When looking into the current model as it was given to Aon Hewitt by the team, there was a variety of outputs to be pulled from all of the data and model manipulation. Using different realistic inputs into the model, some of which were suggested by Aon and other's we attributed to economic norms such as starting salary, we were able to view multiple real life scenarios of how a Cash Balance Plan would stand up to a Managed Account Plan given the same company's criteria. Depending on the population make-up of a company or how they choose to invest their employee's accounts made a difference as to how the liability would result for the company in the end. Two graphs such as Figure 22 and Figure 23 below would be a great way of explaining the risk associated with a Cash Balance Plan versus a Managed Account Plan over the lifetime of the plans themselves.

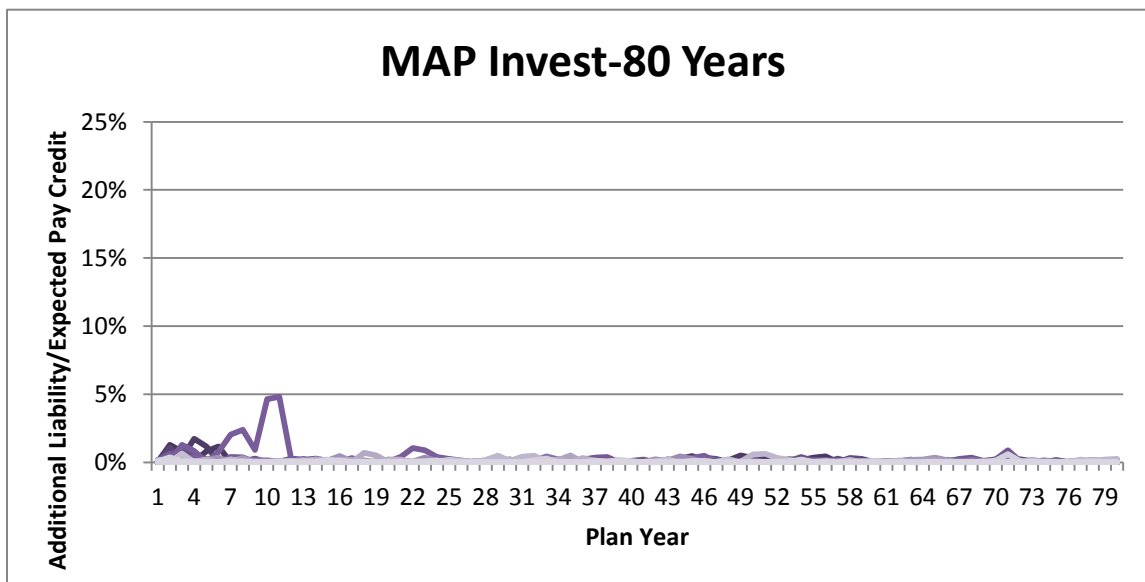


Figure 22: MAP Invest- 80 Years

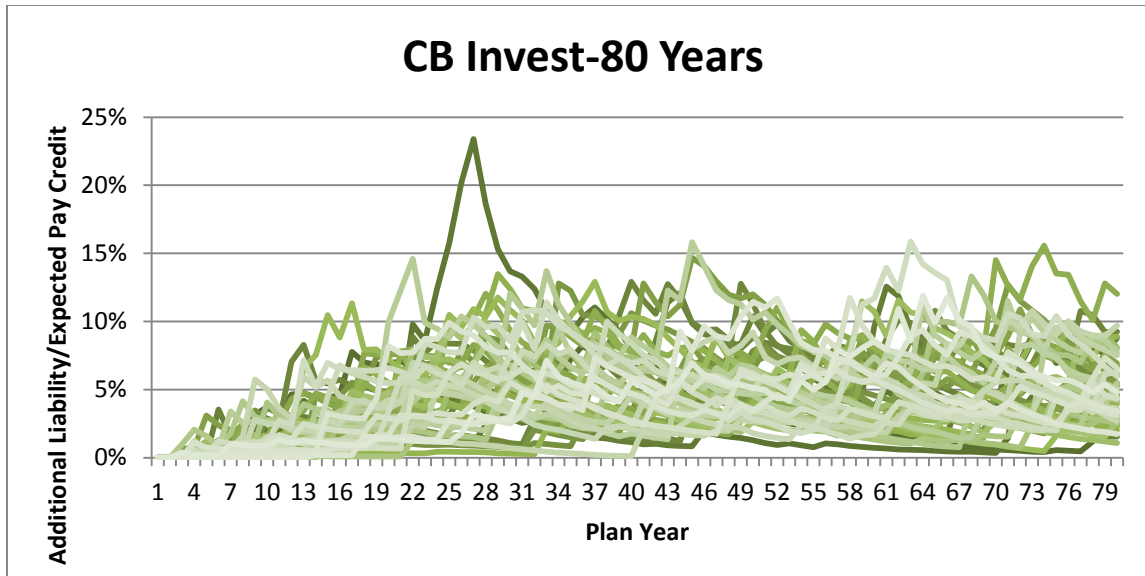


Figure 23: CB Invest- 80 Years

Note that both plans have identical inputs resulting in Figure 22 and Figure 23. To better understand the denominators used to acquire the percentage scales of the graph please refer to the model documentation in Appendix A. These graphs can quickly and easily show and employer the immediate difference between the Cash Balance Plan and the MAP. As previously discussed, overall the Cash Balance plan shows a greater amount of risk in the long term of the plan’s use, but in the short term of the plan’s existence, the MAP shows more additional liability.

The model has many outputs that will be beneficial depending on the client Aon Hewitt is presenting to. For example, a client with employees that on the whole tend to stay with the company longer will be very intrigued by the difference in additional liability in the long-term consideration of both the MAP and Cash Balance Plan.

Using the straw man was another helpful tool in approaching the marketing of the MAP. Because the model is very intricate and provides more information and outputs than any one company is likely to ever need, finding a way to simplify the numbers and combine the “big ideas” of the outputs into a more simplistic visual can be very beneficial. Figure 24 below shows

the straw man of a CB Plan and a Managed Account Plan at the same time.

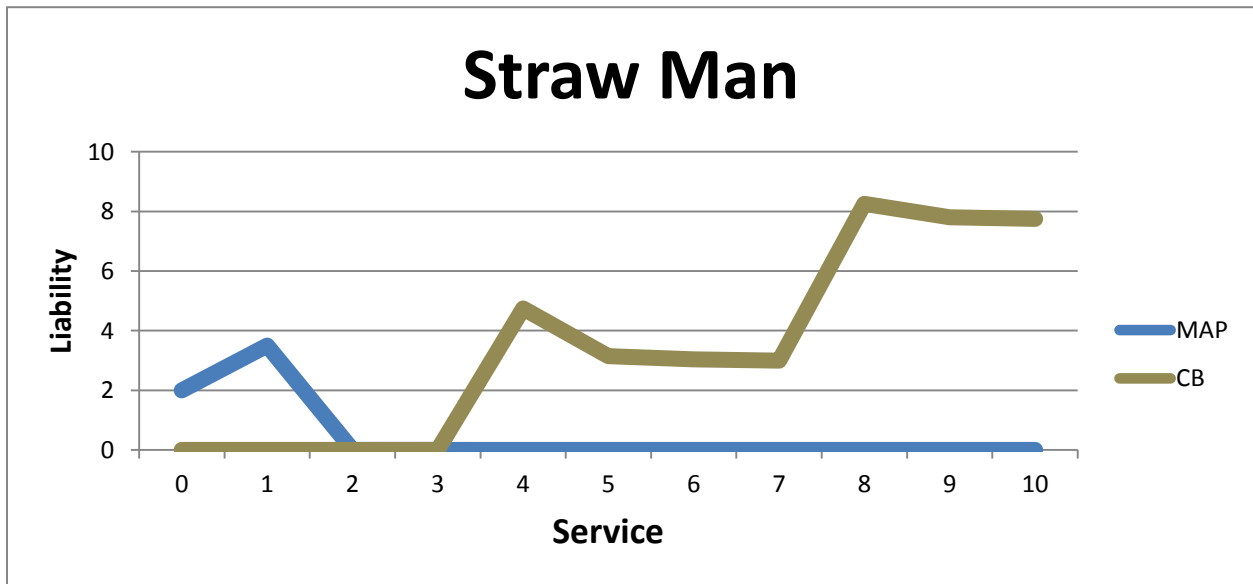


Figure 24: Straw Man CB MAP Comparison

With the correct “average” inputs of any one employee from one of Aon Hewitt’s clients, this straw man can essentially sum up all of the major points that make the MAP a less risky choice to the employee and employer.

## Results

There are a number of suggested approaches to use to make the Managed Account Plan a more desirable, employee friendly product when presented to legislature. The best approach in describing the new MAPs will be to advertise the plan as a “hybrid” between a defined benefit plan and a defined contribution plan. Using the term hybrid will combine already accepted jargon with the comfort of two current plans the audience is already accustomed to.

Designing tools to clean-up the real-world data of Aon Hewitt’s clients is also very important. Having a filter to accept the current records of the client, employer, and output clean data that is easily accepted by the current model will be very beneficial to both the user of the model as well as Aon Hewitt’s clients. The employers will then be able to see direct comparisons between the Cash Balance Plans and the Managed Account Plans using their own data. The more personalized the outputs can be to the client, the more relatable the results will be.

Another suggestion in moving forward with the marketing of the MAP is to consider conducting data analysis on the company's retention rates of their employees. This would provide feedback as to how long the employees are staying with the company and more importantly how long they are accruing retirement benefits through the company. Considering in the short term the MAP holds more liability than the Cash Balance Plan and that the Cash Balance Plan holds more liability in the long-term of a plan's lifetime, an argument can be made in favor of the long term benefits of having a MAP over a CB Plan.

When Aon Hewitt presents this new type of retirement plan they want to relate to their client as much as possible. Instilling confidence in the client that they are Aon Hewitt's number one priority and that they are looking out for the best interests of a company that they know inside and out, they will reassure their client that what they are presenting must be in the best interests of their company. Use as much data from the actual company as possible will prove to be very beneficial. If the client is able to see direct comparisons of their company projected using both the old and new types of plans, they will better understand how the outputs of the model relate to their company specifically.

## Discussion

In order to market the new Managed Account Plan, Aon Hewitt's clients have to understand the benefit to their company as well as to their employees. There are two major obstacles Aon Hewitt will have to overcome to encourage their clients to adopt this new plan. The first is why they should be switching their retirement plan offerings at all. For some of Aon Hewitt's clients there may be a need or want to convert to a plan with less risk in the latter years of the plan and the accrual of retirement benefits by the employee. However, for those who are not looking for a change to their current retirement plan options, Aon Hewitt must approach this offering with an open-mind and exploit the benefits to each company individually. Making the new MAP relatable to each of Aon Hewitt's clients will allow each of them to feel as though they are the most important, a key to sales and marketing. Also, convincing someone that change is good and necessary can be one of the largest hurdles to overcome.

Many times, even when the need for change is recognized, there is still resistance towards any new approach to an old problem. People do not like change; it is that simple. The unwillingness to explore the unknown is surpassed only by the unwillingness to formulate and

infiltrate the change themselves. It is Aon Hewitt's job to make this new Managed Account Plan look more desirable than the Cash Balance Plan or adoption will not be an easy feat.

One of the keys to conveying the benefits of the MAP will be getting to the correct target market. It may be true that not all of Aon Hewitt's clients would in fact benefit from a change from Cash Balance Plan to Managed Account Plan. Those who have loyal employees who tend to stay with company past the first five to ten years make for terrific candidates for the MAP. Our results have shown that after the first five to ten years most liability tapers off in the Managed Account Plan. This is in opposition with the slow but steady rise in additional liability seen in the Cash Balance Plan as the plan's lifetime matures. This is a great way to convey the benefits of the MAP to a client.

## Recommendations

After consideration for the way that the 401(k) was adopted as well as the typical norms for marketing and attaining approval of a new type of retirement plan, we would make the following recommendations. Many of these recommendations deal with working on the model once again and adding certain functionality that would either allow for an easier view of the outputs the model provides or more functionality to the model itself.

Aon Hewitt must be conscious of who their audience is at all times and who else that audience will be concerned with. Making a proposal to regulators or legislature would require a specific consideration for the benefit to the employees. Regulators will be concerned that the individual employees are not getting a sour deal from their employer. At the same time, the regulators will also need to be sure the companies are not setting themselves up for a large deficit with the risk associated with the new plan. Therefore a balance of consideration of the employer and likewise the "employer's consideration for their employees" must be conveyed. Marketing is very much about the point of view. Knowing who your target market or audience is can make or break a sale or proposal for approval.

Adding the functionality of having a designated vesting period, perhaps of three years, could be a useful tool to relating to the client. Many companies require a certain amount of time with the company for the employee to receive benefits beyond just their employee account balances. This functionality would allow for the company to only apply the guarantee benefits of the Managed Account Plan to those employees who have been with the company for a minimum period of time. This would allow the company to add incentive to remaining with the company

and security against some of the early, typically most volatile years of the Managed Account Plan.

Designing multiple options for the employer to be able to hedge the risk presented by the Managed Account Plan will aid in the selling of this new, unprecedented retirement plan offering. One potential option would be the purchasing of swaps to make up for any additional liabilities they may potentially pay out to their employees who terminate. Combined with the comparison outputs of the model, having additional ways of defraying the potential risk will encourage the benefits of the MAP. This should be further explored and potential “Risk Hedging Options” would be a positive way to ease the minds of any clients who are still hesitant of the MAP risk even after considering the model’s outputs.

Since not all additional liabilities can realistically be avoided, designing options for paying out the employees’ benefits at the time of their retirement will encourage companies to adopt the MAP. For example, designing plans that offer an annuity to the employee upon time of retirement would be one option. Ideally these options and suggestions would aid in the presentation of the MAP to legislature.

Utilization of the original straw man model, where only one person is reflected in comparison between their risks with a Cash Balance Plan versus a Managed Account Plan, can help to simplify the model for clients. The straw man could be used for Aon’s clients who may want to view the idea of a MAP on a smaller, individual employee scale. The current straw man needs investment type correlation considerations, other minor adjustments, documentation and outputs designed to meet the needs of Aon and their clients.

Moving forward with the use of the model and the future additions to the project, we have made the following recommendations to enhance the current model.

The first recommendation is to take a closer look into the correlation matrix inputted by the user and how it is applied. The current application utilizes the Monte Carlo Framework and the Cholesky decomposition matrix to determine the covariance between multiple investment type pairings. Further research should be considered to ensure proper utilization and application of the investment correlations.

Another suggestion is to add functionality to the model that allows for a scaling of the pay credit percentage based on an employee’s age and service. Currently the model applies one percentage no matter what age or number of years of service an employee represents. For a



company with information regarding the fluctuation in pay credit percentage of salary over time, this could add to the accuracy of the model.

Designing a way to “shock the system” is an additional functionality a client may find interesting. For example, if the market was to experience a severe spike, either positive or negative, how that would affect the outcomes of the plan, and how those outcomes of a MAP compare to a CB plan under the same market conditions should be considered. This is a reality for companies to consider especially in recent decades with unpredictable peaks and lulls. Understanding the best and worst case scenarios and having plans for those times could ensure the company’s confidence in the MAP.

Currently the model only compares a MAP to a Cash Balance plan. There could be interesting considerations when comparing other types of pension plans or 401K plans.

While designing the model, consideration was put towards designing the MAP to produce yearly outputs as appose to the current monthly outputs. We determined too much information about the fluctuation of the employee’s account was lost by not including the aggregation by month. Therefore, the model currently compares the MAP accrued monthly and the CB plan accrued yearly. Additional consideration should be given to how the MAP would operate yearly and if there is a way to avoid losing accuracy when operating the MAP this way.

The final suggestion is to add functionality to the model that allows for the employee to have some percentage of the decision about what kinds of investments their account utilizes. As time goes on, the employee would ideally have less and less say because there is a higher account balance at stake for the company. The company, and most likely the employee, would both be better off having the companies decide which investment types will be used in the latter years of the account. This functionality could add to the positive aspects of a MAP when presenting the plan to legislature. The more consideration to the employee’s benefit, the more likely the MAP will be approved for industry use.

## Appendix E: VBA Code

### Model Macros

#### *Model Macros-MAP Floor*

```
Sub MAPMonthlyFloor()  
  
Dim InputArray(1 To 1000, 1 To 80) As Double 'Input Array  
Dim FiveArray(1 To 1000, 1 To 80) As Double 'Five Percent Array  
Dim OverhangArray(1 To 1000, 1 To 80) As Double 'Overhang Array  
Dim ZeroArray(1 To 1000, 1 To 80) As Double 'Zero Percent Array  
Dim SaveArray(1 To 1000, 1 To 960) As Double 'Array for 80 years  
Dim age As Double  
Dim service As Double  
Dim C As Double  
Dim d As Double  
Dim FiveNPV As Double  
Dim InputNPV As Double  
Dim Overhang As Double  
Dim PlanYear As Double  
Dim ZeroNPV As Double  
Dim s As Double  
Dim ScalarLoop As Double  
Dim ScalarLoop2 As Double  
Dim NewAge As Double  
Dim t As Double  
  
'Activates the correct workbook  
Windows("Control.xlsm").Activate  
  
'Creates a timestamp for the start  
Sheets("Control").Cells(3, 22).Value = Now  
  
'Activates the correct workbook  
Windows("MAPFloor.xlsm").Activate  
  
'Plan Year 0  
Sheets("Inputs").Range("R16") = 0  
  
'Sets the service year to the count of the loop  
For service = 0 To 39 Step 1  
    Sheets("Inputs").Range("R15") = service  
    'Creates the age for a reduction of time in the loop  
    C = 25 + service  
  
    For age = C To 65 Step 1  
        'Sets the age as the loop  
        Sheets("Inputs").Range("R14") = age  
  
        Calculate  
        'Selects the correct sheet
```

```

Sheets("MAPFloorRisk").Select

    'Loops through the 1000 trials for the 80 year array
    For s = 1 To 1000 Step 1

        'Adds current liability to past to get an array for 80
        years at plan year 0.
        For ScalarLoop = 1 To 960 Step 1
            SaveArray(s, ScalarLoop) = SaveArray(s,
            ScalarLoop) + (Sheets("MAPFloorRisk").Cells(s + 3010, ScalarLoop +
            1).Value * Sheets("Inputs").Cells(age - 22, 23).Value)
            Next ScalarLoop

        Next s

        'Loops for the 1000 trials for there zero, five, and
        inputted guarentee. Also for the overhang
        For d = 1 To 1000 Step 1
            'Calculates the net present value with a zero percent
            guarentee
            ZeroNPV = WorksheetFunction.Sum(Range(Cells(d + 3010,
            2), Cells(d + 3010, 481))) 'NPV at 0%
            'Calculates the net present value with the inputted
            percent guarentee
            InputNPV =
            WorksheetFunction.NPV(Sheets("Calculated").Cells(2, 18).Value / 12,
            Range(Cells(d + 3010, 2), Cells(d + 3010, 481))) 'NPV at Input %
            'Calculates the net present value with a five percent
            guarentee
            FiveNPV = WorksheetFunction.NPV(0.05 / 12,
            Range(Cells(d + 3010, 2), Cells(d + 3010, 481))) 'NPV at 5%
            'Calculates the net present value of the overhang
            Overhang = WorksheetFunction.Sum(Range(Cells(d + 2009,
            2), Cells(d + 2009, 481))) 'Overhang

            'Accumulates the NPV's of the above arrays
            ZeroArray(d, service + 1) = (ZeroNPV *
            Sheets("Inputs").Cells(age - 22, 23 + service).Value) + ZeroArray(d,
            service + 1) 'For Zero Percent
            InputArray(d, service + 1) = (InputNPV *
            Sheets("Inputs").Cells(age - 22, 23 + service).Value) + InputArray(d,
            service + 1) 'For Inputted Percent
            FiveArray(d, service + 1) = (FiveNPV *
            Sheets("Inputs").Cells(age - 22, 23 + service).Value) + FiveArray(d,
            service + 1) 'For Five Percent
            OverhangArray(d, service + 1) = (Overhang *
            Sheets("Inputs").Cells(age - 22, 23 + service).Value) +
            OverhangArray(d, service + 1) 'For the Overhang

        Next d

    Next age
Next service

```

```

'Sets the Service to Zero
Sheets("Inputs").Range("R15") = 0

'Loops Service Years 1 to 80
For PlanYear = 1 To 79 Step 1
    'Sets the cell to the plan year of the loop
    Sheets("Inputs").Range("R16") = PlanYear

        'Loops through the ages
        For NewAge = 25 To 65 Step 1
            'Sets the cell to the current age
            Sheets("Inputs").Range("R14") = NewAge
            Calculate
            'Selects the correct sheet
            Sheets("MAPFloorRisk").Select

                'Loops for the 1000 trials of the additional liability
for the 80 years
                For t = 1 To 1000 Step 1
                    For ScalarLoop2 = 1 To 960 Step 1
                        SaveArray(t, ScalarLoop2) = SaveArray(t,
ScalarLoop2) + (Sheets("MAPFloorRisk").Cells(t + 3010, ScalarLoop2 +
1).Value * Sheets("Inputs").Cells(NewAge - 22, 23).Value)
                    Next ScalarLoop2
                Next t
            Next NewAge
        Next PlanYear

'Selects the correct worksheet
Sheets("Output").Select
'Outputs all of the calculated arrays
Range("A2:AN1001") = OverhangArray 'Overhang
Range("A1004:AN2003") = InputArray 'Input%
Range("A2006:AN3005") = FiveArray 'Five %
Range("A3008:AN4007") = ZeroArray 'Zero%
Range("A4010:AJX5009") = SaveArray '80 plan years (additional
liability)

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Outputs the timestamp for the completion of the macro
Sheets("Control").Cells(3, 23).Value = Now

End Sub

```

### ***Model Macros-CB Floor***

```
Sub CBYearlyFloor()
```

```

Dim InputArray(1 To 1000, 1 To 80) As Double 'Input Array
Dim FiveArray(1 To 1000, 1 To 80) As Double 'Five Percent Array
Dim OverhangArray(1 To 1000, 1 To 80) As Double 'Overhang Array
Dim ZeroArray(1 To 1000, 1 To 80) As Double 'Zero Percent Array
Dim SaveArray(1 To 1000, 1 To 960) As Double 'Array for 80 years
Dim age As Double
Dim service As Double
Dim C As Double 'Added to service for loop
Dim d As Double 'Loop
Dim FiveNPV As Double
Dim InputNPV As Double
Dim Overhang As Double
Dim PlanYear As Double
Dim ZeroNPV As Double
Dim s As Double 'Loop
Dim ScalarLoop As Double
Dim ScalarLoop2 As Double
Dim NewAge As Double
Dim t As Double 'Loop

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Creates a timestamp for the start
Sheets("Control").Cells(4, 22).Value = Now

'Activates the correct workbook
Windows("CBFloor.xlsm").Activate

'Plan Year 0
Sheets("CBInputs").Range("R16") = 0

'Loop for service
For service = 0 To 39 Step 1
    'Sets the service year to the count of the loop
    Sheets("CBInputs").Range("R15") = service
    'Creates the age for a reduction of time in the loop
    C = 25 + service

    'Loop for age
    For age = C To 65 Step 1
        'Sets the age as the loop
        Sheets("CBInputs").Range("R14") = age

        Calculate
        'Selects the correct sheet
        Sheets("CBFloorRisk").Select

        'Loops through the 1000 trials for the 80 year array
        For s = 1 To 1000 Step 1

```

```

        'Adds current liability to past to get an array for 80
years at plan year 0.
        For ScalarLoop = 1 To 80 Step 1
            SaveArray(s, ScalarLoop) = SaveArray(s,
ScalarLoop) + (Sheets("CBFloorRisk").Cells(s + 4009, ScalarLoop +
1).Value * Sheets("CBInputs").Cells(age - 22, 23).Value)
            Next ScalarLoop

        Next s

        'Loops for the 1000 trials for the zero, five, and
inputted guarentee. Also for the overhang
        For d = 1 To 1000 Step 1
            'Calculates the net present value with a zero percent
guarentee
            ZeroNPV = WorksheetFunction.Sum(Range(Cells(d + 4009,
2), Cells(d + 4009, 81))) 'NPV at 0%
            'Calculates the net present value with the inputted
percent guarentee
            InputNPV =
WorksheetFunction.NPV(Sheets("CBInputs").Cells(5, 18).Value,
Range(Cells(d + 4009, 2), Cells(d + 4009, 81))) 'NPV at Input %
            'Calculates the net present value with a five percent
guarentee
            FiveNPV = WorksheetFunction.NPV(0.05, Range(Cells(d +
4009, 2), Cells(d + 4009, 81))) 'NPV at 5%
            'Calculates the net present value of the overhang
            Overhang = WorksheetFunction.Sum(Range(Cells(d + 3008,
2), Cells(d + 3008, 81))) 'Overhang

            'Accumulates the NPV's of the above arrays
            ZeroArray(d, service + 1) = (ZeroNPV *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) + ZeroArray(d,
service + 1) 'For Zero Percent
            InputArray(d, service + 1) = (InputNPV *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) +
InputArray(d, service + 1) 'For the Inputted Percent
            FiveArray(d, service + 1) = (FiveNPV *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) + FiveArray(d,
service + 1) 'For Five Percent
            OverhangArray(d, service + 1) = (Overhang *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) +
OverhangArray(d, service + 1) 'For the Overhang

        Next d
    Next age
Next service

'Sets the Service to Zero
Sheets("CBInputs").Range("R15") = 0

```

```

'Loops Service Years 1 to 80
For PlanYear = 1 To 80 Step 1
    'Sets the cell to the plan year of the loop
    Sheets("CBInputs").Range("R16") = PlanYear

        'Loops through the ages
        For NewAge = 25 To 65 Step 1
            'Sets the cell to the current age
            Sheets("CBInputs").Range("R14") = NewAge
            Calculate
            'Selects the correct sheet
            Sheets("CBFloorRisk").Select

                'Loops for the 1000 trials of the additional liability
for the 80 years
                For t = 1 To 1000 Step 1
                    For ScalarLoop2 = 1 To 80 Step 1
                        SaveArray(t, ScalarLoop2) = SaveArray(t,
ScalarLoop2) + (Sheets("CBFloorRisk").Cells(t + 4009, ScalarLoop2 +
1).Value * Sheets("CBInputs").Cells(NewAge - 22, 23).Value)
                    Next ScalarLoop2
                Next t
            Next NewAge
        Next PlanYear

'Selects the correct worksheet
Sheets("Output").Select
'Outputs all of the calculated arrays
Range("A2:AN1001") = OverhangArray 'Overhang
Range("A1004:AN2003") = InputArray 'Input%
Range("A2006:AN3005") = FiveArray 'Five %
Range("A3008:AN4007") = ZeroArray 'Zero%
Range("A4010:CB5009") = SaveArray '80 plan years (additional
liability)

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Outputs the timestamp for the completion of the macro
Sheets("Control").Cells(4, 23).Value = Now

End Sub

```

### ***Model Macros-MAP Investment***

```

Sub MAPMonthlyInvest()
Dim InputArray(1 To 1000, 1 To 80) As Double 'Input Array
Dim FiveArray(1 To 1000, 1 To 80) As Double 'Five Percent Array
Dim OverhangArray(1 To 1000, 1 To 80) As Double 'Overhang Array
Dim ZeroArray(1 To 1000, 1 To 80) As Double 'Zero Percent Array
Dim SaveArray(1 To 1000, 1 To 960) As Double 'Array for 80 years
Dim age As Double

```

```

Dim service As Double
Dim C As Double
Dim d As Double
Dim FiveNPV As Double
Dim InputNPV As Double
Dim Overhang As Double
Dim PlanYear As Double
Dim ZeroNPV As Double
Dim s As Double
Dim ScalarLoop As Double
Dim ScalarLoop2 As Double
Dim NewAge As Double
Dim t As Double

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Creates a timestamp for the start
Sheets("Control").Cells(5, 22).Value = Now

'Activates the correct workbook
Windows("MAPInvest.xlsm").Activate

'Plan Year 0
Sheets("Inputs").Range("R16") = 0

'Loop for service
For service = 0 To 39 Step 1
    'Sets the service year to the count of the loop
    Sheets("Inputs").Range("R15") = service
    'Creates the age for a reduction of time in the loop
    C = 25 + service

    'Loop for age
    For age = C To 65 Step 1
        'Sets the age as the loop
        Sheets("Inputs").Range("R14") = age

        Calculate
        'Selects the correct sheet
        Sheets("MAPInvestmentRisk").Select

        'Loops through the 1000 trials for the 80 year array
        For s = 1 To 1000 Step 1

            'Adds current liability to past to get an array for 80
            years at plan year 0.
            For ScalarLoop = 1 To 960 Step 1
                SaveArray(s, ScalarLoop) = SaveArray(s,
                ScalarLoop) + (Sheets("MAPInvestmentRisk").Cells(s + 3010, ScalarLoop
                + 1).Value * Sheets("Inputs").Cells(age - 22, 23).Value)
            Next ScalarLoop
        Next s
    Next age
Next service

```



```

Next s

'Loops for the 1000 trials for the zero, five, and
inputted guarentee. Also for the overhang
For d = 1 To 1000 Step 1
    'Calculates the net present value with a zero percent
guarentee
    ZeroNPV = WorksheetFunction.Sum(Range(Cells(d + 3010,
2), Cells(d + 3010, 481))) 'NPV at 0%
    'Calculates the net present value with the inputted
percent guarentee
    InputNPV =
WorksheetFunction.NPV(Sheets("Calculated").Cells(2, 18).Value / 12,
Range(Cells(d + 3010, 2), Cells(d + 3010, 481))) 'NPV at Input %
    'Calculates the net present value with a five percent
guarentee
    FiveNPV = WorksheetFunction.NPV(0.05 / 12,
Range(Cells(d + 3010, 2), Cells(d + 3010, 481))) 'NPV at 5%
    'Calculates the net present value of the overhang
    Overhang = WorksheetFunction.Sum(Range(Cells(d + 2009,
2), Cells(d + 2009, 481))) 'Overhang

    'Accumulates the NPV's of the above arrays
    ZeroArray(d, service + 1) = (ZeroNPV *
Sheets("Inputs").Cells(age - 22, 23 + service).Value) + ZeroArray(d,
service + 1) 'For Zero Percent
    InputArray(d, service + 1) = (InputNPV *
Sheets("Inputs").Cells(age - 22, 23 + service).Value) + InputArray(d,
service + 1) 'For Inputted Percent
    FiveArray(d, service + 1) = (FiveNPV *
Sheets("Inputs").Cells(age - 22, 23 + service).Value) + FiveArray(d,
service + 1) 'For Five Percent
    OverhangArray(d, service + 1) = (Overhang *
Sheets("Inputs").Cells(age - 22, 23 + service).Value) +
OverhangArray(d, service + 1) 'For the Overhang

Next d
Next age
Next service

'Sets the Service to Zero
Sheets("Inputs").Range("R15") = 0

'Loops Service Years 1 to 80
For PlanYear = 1 To 79 Step 1
    'Sets the cell to the plan year of the loop
    Sheets("Inputs").Range("R16") = PlanYear

    'Loops through the ages
    For NewAge = 25 To 65 Step 1
        'Sets the cell to the current age

```

```

    Sheets("Inputs").Range("R14") = NewAge
    Calculate
    'Selects the correct sheet
    Sheets("MAPInvestmentRisk").Select

        'Loops for the 1000 trials of the additional liability
for the 80 years
        For t = 1 To 1000 Step 1
            For ScalarLoop2 = 1 To 960 Step 1
                SaveArray(t, ScalarLoop2) = SaveArray(t,
ScalarLoop2) + (Sheets("MAPInvestmentRisk").Cells(t + 3010,
ScalarLoop2 + 1).Value * Sheets("Inputs").Cells(NewAge - 22,
23).Value)
            Next ScalarLoop2
        Next t
    Next NewAge
Next PlanYear

'Selects the correct worksheet
Sheets("Output").Select
'Outputs all of the calculated arrays
Range("A2:AN1001") = OverhangArray 'Overhang
Range("A1004:AN2003") = InputArray 'Input%
Range("A2006:AN3005") = FiveArray 'Five %
Range("A3008:AN4007") = ZeroArray 'Zero%
Range("A4010:AJX5009") = SaveArray '80 plan years (additional
liability)

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Outputs the timestamp for the completion of the macro
Sheets("Control").Cells(5, 23).Value = Now

End Sub

```

### ***Model Macros-CB Investment***

```

Sub CBYearlyInvestment()

Dim InputArray(1 To 1000, 1 To 80) As Double 'Input Array
Dim FiveArray(1 To 1000, 1 To 80) As Double 'Five Percent Array
Dim OverhangArray(1 To 1000, 1 To 80) As Double 'Overhang Array
Dim ZeroArray(1 To 1000, 1 To 80) As Double 'Zero Percent Array
Dim SaveArray(1 To 1000, 1 To 960) As Double 'Array for 80 years
Dim age As Double
Dim service As Double
Dim C As Double 'added to service
Dim d As Double 'Loop
Dim FiveNPV As Double
Dim InputNPV As Double

```

```

Dim Overhang As Double
Dim PlanYear As Double
Dim ZeroNPV As Double
Dim s As Double 'Loop
Dim ScalarLoop As Double
Dim ScalarLoop2 As Double
Dim NewAge As Double
Dim t As Double 'Loop

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Creates a timestamp for the start
Sheets("Control").Cells(6, 22).Value = Now

'Activates the correct workbook
Windows("CBInvest.xlsm").Activate

'Plan Year 0
Sheets("CBInputs").Range("R16") = 0

'Loop for service
For service = 0 To 39 Step 1
    'Sets the service year to the count of the loop
    Sheets("CBInputs").Range("R15") = service
    'Creates the age for a reduction of time in the loop
    C = 25 + service

    'Loop for age
    For age = C To 65 Step 1
        'Sets the age as the loop
        Sheets("CBInputs").Range("R14") = age

        Calculate
        'Selects the correct sheet
        Sheets("CBInvestRisk").Select

        'Loops through the 1000 trials for the 80 year array
        For s = 1 To 1000 Step 1

            'Adds current liability to past to get an array for 80
            years at plan year 0.
            For ScalarLoop = 1 To 80 Step 1
                SaveArray(s, ScalarLoop) = SaveArray(s,
                ScalarLoop) + (Sheets("CBInvestRisk").Cells(s + 4010, ScalarLoop +
                1).Value * Sheets("CBInputs").Cells(age - 22, 23).Value)
            Next ScalarLoop

        Next s

        'Loops for the 1000 trials for the zero, five, and
        inputted guarentee. Also for the overhang
        For d = 1 To 1000 Step 1

```

```

        'Calculates the net present value with a zero percent
guarentee
        ZeroNPV = WorksheetFunction.Sum(Range(Cells(d + 4010,
2), Cells(d + 4010, 81))) 'NPV at 0%
        'Calculates the net present value with the inputted
percent guarentee
        InputNPV =
WorksheetFunction.NPV(Sheets("CBInputs").Cells(5, 18).Value,
Range(Cells(d + 4010, 2), Cells(d + 4010, 81))) 'NPV at Input %
        'Calculates the net present value with a five percent
guarentee
        FiveNPV = WorksheetFunction.NPV(0.05, Range(Cells(d +
4010, 2), Cells(d + 4010, 81))) 'NPV at 5%
        'Calculates the net present value of the overhang
        Overhang = WorksheetFunction.Sum(Range(Cells(d + 3008,
2), Cells(d + 3008, 81))) 'Overhang

        'Accumulates the NPV's of the above arrays
        ZeroArray(d, service + 1) = (ZeroNPV *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) + ZeroArray(d,
service + 1) 'For Zero Percent
        InputArray(d, service + 1) = (InputNPV *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) +
InputArray(d, service + 1) 'For the Inputted Percent
        FiveArray(d, service + 1) = (FiveNPV *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) + FiveArray(d,
service + 1) 'For Five Percent
        OverhangArray(d, service + 1) = (Overhang *
Sheets("CBInputs").Cells(age - 22, 23 + service).Value) +
OverhangArray(d, service + 1) 'For the Overhang
        Next d
    Next age
Next service

'Sets the Service to Zero
Sheets("CBInputs").Range("R15") = 0

'Loops Service Years 1 to 80
For PlanYear = 1 To 80 Step 1
    'Sets the cell to the plan year of the loop
    Sheets("CBInputs").Range("R16") = PlanYear

    'Loops through the ages
    For NewAge = 25 To 65 Step 1
        'Sets the cell to the current age
        Sheets("CBInputs").Range("R14") = NewAge
        Calculate
        'Selects the correct sheet
        Sheets("CBInvestRisk").Select

        'Loops for the 1000 trials of the additional liability
for the 80 years

```

```

        For t = 1 To 1000 Step 1
            For ScalarLoop2 = 1 To 80 Step 1
                SaveArray(t, ScalarLoop2) = SaveArray(t,
ScalarLoop2) + (Sheets("CBInvestRisk").Cells(t + 4010, ScalarLoop2 +
1).Value * Sheets("CBInputs").Cells(NewAge - 22, 23).Value)
            Next ScalarLoop2
        Next t
    Next NewAge
Next PlanYear

'Selects the correct worksheet
Sheets("Output").Select
'Outputs all of the calculated arrays
Range("A2:AN1001") = OverhangArray 'Overhang
Range("A1004:AN2003") = InputArray 'Input'
Range("A2006:AN3005") = FiveArray 'Five
Range("A3008:AN4007") = ZeroArray 'Zero'
Range("A4010:CB5009") = SaveArray '80 plan years (additional
liability)

'Activates the correct workbook
Windows("Control.xlsm").Activate

'Outputs the timestamp for the completion of the macro
Sheets("Control").Cells(6, 23).Value = Now

End Sub

```

## Paste Inputs

### *Paste Inputs- MAPFloor*

```

Sub PasteMAPFloorInputs()

    'Activates the correct workbook
    Windows("Control.xlsm").Activate
    'Selects the correct worksheet
    Worksheets("Inputs").Activate
    'Selects the start cell
    Range("A1").Select
    'Selects the range
    Range("A1:BK44").Select
    Range("BK44").Activate
    'Copies the selected range
    Selection.Copy
    Windows("MAPFloor.xlsm").Activate
    Worksheets("Inputs").Activate
    Range("A1").Select
    'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
    ActiveSheet.Paste

```

End Sub

### ***Paste Inputs- CBFloor***

```
Sub PasteCBFloorInputs()  
  
    'Activates the correct workbook  
    Windows("Control.xlsm").Activate  
    'Selects the correct worksheet  
    Worksheets("Inputs").Activate  
    'Selects the start cell  
    Range("A1").Select  
    'Selects the range  
    Range("A1:BK44").Select  
    Range("BK44").Activate  
    'Copies the selected range  
    Selection.Copy  
    Windows("CBFloor.xlsm").Activate  
    Worksheets("CBInputs").Activate  
    Range("A1").Select  
    'Pastes to the new worksheet in the following workbook, worksheet,  
and cell.  
    ActiveSheet.Paste  
  
End Sub
```

### ***Paste Inputs- MAPInvest***

```
Sub PasteMAPInvestInputs()  
    'Activates the correct Workbook  
    Windows("Control.xlsm").Activate  
    'Selects the correct worksheet  
    Worksheets("Inputs").Activate  
    'Selects the start cell  
    Range("A1").Select  
    'Selects the range  
    Range("A1:BK44").Select  
    Range("BK44").Activate  
    'Copies the selected range  
    Selection.Copy  
    Windows("MAPInvest.xlsm").Activate  
    Worksheets("Inputs").Activate  
    Range("A1").Select  
    'Pastes to the new worksheet in the following workbook, worksheet,  
and cell.  
    ActiveSheet.Paste  
  
End Sub
```

## ***Paste Inputs- CBInvest***

```
Sub PasteCBInvestInputs ()

    'Selects the correct workbook
    Windows("Control.xlsm").Activate
    'Activates the correct worksheet
    Worksheets("Inputs").Activate
    'Selects the start cell
    Range("A1").Select
    'Selects the range
    Range("A1:BK44").Select
    Range("BK44").Activate
    'Copies the selected range
    Selection.Copy
    Windows("CBInvest.xlsm").Activate
    Worksheets("CBInputs").Activate
    Range("A1").Select
    'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
    ActiveSheet.Paste

End Sub
```

## **Paste Investments**

### ***Paste Investments- MAP Floor***

```
Sub PasteMAPFloorInvest ()

    'Equity
    'Selects the correct workbook
    Windows("Control.xlsm").Activate
    'Activates the correct worksheet
    Worksheets("Investments").Activate
    'Selects the start cell
    Range("A2").Select
    'Selects the range
    Range("A2:AJX1001").Select
    Range("AJX1001").Activate
    'Copies the selected range
    Selection.Copy
    Windows("MAPFloor.xlsm").Activate
    Worksheets("Calculated").Activate
    Range("u2").Select
    'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
    ActiveSheet.Paste

    'Bonds
    'Selects the correct workbook
```

```

Windows("Control.xlsm").Activate
'Activates the correct worksheet
Worksheets("Investments").Activate
'Selects the start cell
Range("A1004").Select
'Selects the range
Range("A1004:AJX2003").Select
Range("AJX2003").Activate
'Copies the selected range
Selection.Copy
Windows("MAPFloor.xlsm").Activate
Worksheets("Calculated").Activate
Range("AKT2").Select
'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
ActiveSheet.Paste

'Risky
'Selects the correct workbook
Windows("Control.xlsm").Activate
'Activates the correct worksheet
Worksheets("Investments").Activate
'Selects the start cell
Range("A2006").Select
'Selects the range
Range("A2006:AJX3005").Select
Range("AJX3005").Activate
'Copies the selected range
Selection.Copy
Windows("MAPFloor.xlsm").Activate
Worksheets("Calculated").Activate
Range("BVS2").Select
'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
ActiveSheet.Paste

End Sub

```

### ***Paste Investments- CB Floor***

```

Sub PasteCBFloorInvest()

'Equity
'Selects the correct workbook
Windows("Control.xlsm").Activate
'Activates the correct worksheet
Worksheets("Investments").Activate
'Selects the start cell
Range("A3008").Select
'Selects the range
Range("A3008:CB4007").Select

```



```

Range("CB4007").Activate
'Copies the selected range
Selection.Copy
Windows("CBFloor.xlsm").Activate
Worksheets("Calculated").Activate
Range("u2").Select
'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
ActiveSheet.Paste

'Bonds
'Selects the correct workbook
Windows("Control.xlsm").Activate
'Activates the correct worksheet
Worksheets("Investments").Activate
'Selects the start cell
Range("A4010").Select
'Selects the range
Range("A4010:CB5009").Select
Range("CB5009").Activate
'Copies the selected range
Selection.Copy
Windows("CBFloor.xlsm").Activate
Worksheets("Calculated").Activate
Range("CY2").Select
'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
ActiveSheet.Paste

'Risky
'Selects the correct workbook
Windows("Control.xlsm").Activate
'Activates the correct worksheet
Worksheets("Investments").Activate
'Selects the start cell
Range("A5012").Select
'Selects the range
Range("A5012:CB6011").Select
Range("CB6011").Activate
'Copies the selected range
Selection.Copy
Windows("CBFloor.xlsm").Activate
Worksheets("Calculated").Activate
Range("GC2").Select
'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
ActiveSheet.Paste
End Sub

```

### ***Paste Investments- MAP Investments***

```
Sub PasteMAPInvInvest()  
  
    'Equity  
    'Selects the correct workbook  
    Windows("Control.xlsm").Activate  
    'Activates the correct worksheet  
    Worksheets("Investments").Activate  
    'Selects the start cell  
    Range("A2").Select  
    'Selects the range  
    Range("A2:AJX1001").Select  
    Range("AJX1001").Activate  
    'Copies the selected range  
    Selection.Copy  
    Windows("MAPInvest.xlsm").Activate  
    Worksheets("Calculated").Activate  
    Range("u2").Select  
    'Pastes to the new worksheet in the following workbook, worksheet,  
and cell.  
    ActiveSheet.Paste  
  
    'Bonds  
    'Selects the correct workbook  
    Windows("Control.xlsm").Activate  
    'Activates the correct worksheet  
    Worksheets("Investments").Activate  
    'Selects the start cell  
    Range("A1004").Select  
    'Selects the range  
    Range("A1004:AJX2003").Select  
    Range("AJX2003").Activate  
    'Copies the selected range  
    Selection.Copy  
    Windows("MAPInvest.xlsm").Activate  
    Worksheets("Calculated").Activate  
    Range("AKT2").Select  
    'Pastes to the new worksheet in the following workbook, worksheet,  
and cell.  
    ActiveSheet.Paste  
  
    'Risky  
    'Selects the correct workbook  
    Windows("Control.xlsm").Activate  
    'Activates the correct worksheet  
    Worksheets("Investments").Activate  
    'Selects the start cell  
    Range("A2006").Select  
    'Selects the range  
    Range("A2006:AJX3005").Select  
    Range("AJX3005").Activate
```

```

    'Copies the selected range
    Selection.Copy
    Windows("MAPInvest.xlsm").Activate
    Worksheets("Calculated").Activate
    Range("BVS2").Select
    'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
    ActiveSheet.Paste
End Sub

```

### ***Paste Investments- CB Investments***

```

Sub PasteCBInvInvest()
    'Equity
    'Selects the correct workbook
    Windows("Control.xlsm").Activate
    'Activates the correct worksheet
    Worksheets("Investments").Activate
    'Selects the start cell
    Range("A3008").Select
    'Selects the range
    Range("A3008:CB4007").Select
    Range("CB4007").Activate
    'Copies the selected range
    Selection.Copy
    Windows("CBInvest.xlsm").Activate
    Worksheets("Calculated").Activate
    Range("u2").Select
    'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
    ActiveSheet.Paste

    'Bonds
    'Selects the correct workbook
    Windows("Control.xlsm").Activate
    'Activates the correct worksheet
    Worksheets("Investments").Activate
    'Selects the start cell
    Range("A4010").Select
    'Selects the range
    Range("A4010:CB5009").Select
    Range("CB5009").Activate
    'Copies the selected range
    Selection.Copy
    Windows("CBInvest.xlsm").Activate
    Worksheets("Calculated").Activate
    Range("CY2").Select
    'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
    ActiveSheet.Paste

    'Risky

```

```

'Selects the correct workbook
Windows("Control.xlsm").Activate
'Activates the correct worksheet
Worksheets("Investments").Activate
'Selects the start cell
Range("A5012").Select
'Selects the range
Range("A5012:CB6011").Select
Range("CB6011").Activate
'Copies the selected range
Selection.Copy
Windows("CBInvest.xlsm").Activate
Worksheets("Calculated").Activate
Range("GC2").Select
'Pastes to the new worksheet in the following workbook, worksheet,
and cell.
ActiveSheet.Paste
End Sub

```

## **Run Macros**

### ***Run Macros-MAP Floor***

```

Sub RunMAPFloor()

Call PasteMAPFloorInputs
Call PasteMAPFloorInvest
Call MAPMonthlyFloor

End Sub

```

### ***Run Macros-CB Floor***

```

Sub RunCBFloor()

Call PasteCBFloorInputs
Call PasteCBFloorInvest
Call CBYearlyFloor

End Sub

```

### ***Run Macros-MAP Investments***

```

Sub RunMAPInvest()

Call PasteMAPInvestInputs
Call PasteMAPInvInvest
Call MAPMonthlyInvest

End Sub

```

### ***Run Macros-CB Investments***

```
Sub RunCBInvest ()

Call PasteCBInvestInputs
Call PasteCBInvInvest
Call CBYearlyInvestment

End Sub
```

## **Investments**

### ***Calculate Investments***

```
Sub Investments ()

Dim Returns(1 To 3, 1 To 960) As Double
Dim MonthlyA(1 To 1000, 1 To 960) As Double
Dim MonthlyB(1 To 1000, 1 To 960) As Double
Dim MonthlyC(1 To 1000, 1 To 960) As Double
Dim YearlyA(1 To 1000, 1 To 80) As Double
Dim YearlyB(1 To 1000, 1 To 80) As Double
Dim YearlyC(1 To 1000, 1 To 80) As Double
Dim SigmaA As Double
Dim SigmaB As Double
Dim SigmaC As Double
Dim AlphaA As Double
Dim AlphaB As Double
Dim AlphaC As Double
Dim Cov(0 To 2, 0 To 2) As Double
Dim h As Double
Dim i As Double
Dim j As Double
Dim e As Double
Dim Chol(0 To 2, 0 To 2) As Double
Dim V As Variant
e = 2.71828182845905 'set e

SigmaA = Sheets("Inputs").Range("S10").Value / (12 ^ 0.5)
SigmaB = Sheets("Inputs").Range("S11").Value / (12 ^ 0.5)
SigmaC = Sheets("Inputs").Range("S12").Value / (12 ^ 0.5)

AlphaA = Sheets("Inputs").Range("R10").Value / 12
AlphaB = Sheets("Inputs").Range("R11").Value / 12
AlphaC = Sheets("Inputs").Range("R12").Value / 12

'calculate covariance matrix
Cov(0, 0) = SigmaA ^ 2
Cov(0, 1) = SigmaA * SigmaB * Sheets("Inputs").Range("R24").Value
Cov(1, 0) = SigmaA * SigmaB * Sheets("Inputs").Range("R24").Value
Cov(1, 1) = SigmaB ^ 2
Cov(0, 2) = SigmaA * SigmaC * Sheets("Inputs").Range("R25").Value
```

```

Cov(2, 0) = SigmaA * SigmaC * Sheets("Inputs").Range("R25").Value
Cov(2, 2) = SigmaC ^ 2
Cov(1, 2) = SigmaB * SigmaC * Sheets("Inputs").Range("S25").Value
Cov(2, 1) = SigmaB * SigmaC * Sheets("Inputs").Range("S25").Value

'Calculates Cholesky decomposition of covariance matrix
Chol(0, 0) = Cov(0, 0) ^ 0.5
Chol(0, 1) = 0
Chol(0, 2) = 0
Chol(1, 0) = Cov(1, 0) / Chol(0, 0)
Chol(1, 1) = (Cov(1, 1) - (Chol(1, 0) ^ 2)) ^ 0.5
Chol(1, 2) = 0
Chol(2, 0) = Cov(2, 0) / Chol(0, 0)
Chol(2, 1) = (Cov(2, 1) - (Chol(2, 0) * Chol(1, 0))) / Chol(1, 1)
Chol(2, 2) = (Cov(2, 2) - (Chol(2, 0) ^ 2 - Chol(1, 0) ^ 2)) ^ 0.5

For h = 1 To 1000 Step 1 'loop for 1000 trials
    For i = 1 To 3 Step 1 'loop for 3 investments
        For j = 1 To 960 Step 1 'loop for 960 months
            Returns(i, j) =
Application.WorksheetFunction.Norm_S_Inv(Application.WorksheetFunction
.RandBetween(1, 999999) / 1000000) 'create random numbers from 0 to 1
        Next j
    Next i

    V = Application.WorksheetFunction.MMult(Chol, Returns) 'multiply
matrix of random numbers by Chol

    For j = 1 To 960 Step 1
        MonthlyA(h, j) = e ^ (AlphaA - (((SigmaA ^ 2) / 2) * (1 / 12))
+ (((1 / 12) ^ 0.5) * V(1, j)))
        MonthlyB(h, j) = e ^ (AlphaB - (((SigmaB ^ 2) / 2) * (1 / 12))
+ (((1 / 12) ^ 0.5) * V(2, j)))
        MonthlyC(h, j) = e ^ (AlphaC - (((SigmaC ^ 2) / 2) * (1 / 12))
+ (((1 / 12) ^ 0.5) * V(3, j)))
    Next j
Next h

'Creates Yearly investments
For h = 1 To 1000 Step 1
    For j = 1 To 80
        YearlyA(h, j) =
Application.WorksheetFunction.Product(MonthlyA(h, (j - 1) * 12 + 1),
MonthlyA(h, (j - 1) * 12 + 2), MonthlyA(h, (j - 1) * 12 + 3),
MonthlyA(h, (j - 1) * 12 + 4), MonthlyA(h, (j - 1) * 12 + 5),
MonthlyA(h, (j - 1) * 12 + 6), MonthlyA(h, (j - 1) * 12 + 7),
MonthlyA(h, (j - 1) * 12 + 8), MonthlyA(h, (j - 1) * 12 + 9),
MonthlyA(h, (j - 1) * 12 + 10), MonthlyA(h, (j - 1) * 12 + 11),
MonthlyA(h, (j - 1) * 12 + 12))
        YearlyB(h, j) =
Application.WorksheetFunction.Product(MonthlyB(h, (j - 1) * 12 + 1),
MonthlyB(h, (j - 1) * 12 + 2), MonthlyB(h, (j - 1) * 12 + 3),

```

```

MonthlyB(h, (j - 1) * 12 + 4), MonthlyB(h, (j - 1) * 12 + 5),
MonthlyB(h, (j - 1) * 12 + 6), MonthlyB(h, (j - 1) * 12 + 7),
MonthlyB(h, (j - 1) * 12 + 8), MonthlyB(h, (j - 1) * 12 + 9),
MonthlyB(h, (j - 1) * 12 + 10), MonthlyB(h, (j - 1) * 12 + 11),
MonthlyB(h, (j - 1) * 12 + 12))
    YearlyC(h, j) =
Application.WorksheetFunction.Product(MonthlyC(h, (j - 1) * 12 + 1),
MonthlyC(h, (j - 1) * 12 + 2), MonthlyC(h, (j - 1) * 12 + 3),
MonthlyC(h, (j - 1) * 12 + 4), MonthlyC(h, (j - 1) * 12 + 5),
MonthlyC(h, (j - 1) * 12 + 6), MonthlyC(h, (j - 1) * 12 + 7),
MonthlyC(h, (j - 1) * 12 + 8), MonthlyC(h, (j - 1) * 12 + 9),
MonthlyC(h, (j - 1) * 12 + 10), MonthlyC(h, (j - 1) * 12 + 11),
MonthlyC(h, (j - 1) * 12 + 12))
    Next j
Next h

```

```

Sheets("Investments").Range("A2:AJX1001") = MonthlyA
Sheets("Investments").Range("A1004:AJX2003") = MonthlyB
Sheets("Investments").Range("A2006:AJX3005") = MonthlyC

```

```

Sheets("Investments").Range("A3008:CB4007") = YearlyA
Sheets("Investments").Range("A4010:CB5009") = YearlyB
Sheets("Investments").Range("A5012:CB6011") = YearlyC

```

```

Sheets("Control").Cells(8, 23).Value = Now
End Sub

```

### **Clear Investments**

```

Sub ClearInvestments()

Sheets("Investments").Select
Range("A2:AJX1001") = ""
Range("A1004:AJX2003") = ""
Range("A2006:AJX3005") = ""
Range("A3008:CB4007") = ""
Range("A4010:CB5009") = ""
Range("A5012:CB6011") = ""

End Sub

```

### **Clear Outputs**

#### **Clear Outputs- MAP Floor**

```

Sub ClearMAPFloorOutputs()

'Clears MAP Floor Outputs
Windows("MAPFloor.xlsm").Activate
Sheets("Output").Select
Range("A2:AN1001") = "" 'Overhang

```

```
Range("A1004:AN2003") = "" 'Input'  
Range("A2006:AN3005") = "" 'Five'  
Range("A3008:AN4007") = "" 'Zero'  
Range("A4010:CB5009") = "" '80 years'
```

End Sub

### ***Clear Outputs- MAP Investments***

```
Sub ClearMAPInvestOutputs()  
'Clears MAP Investment Outputs  
Windows("MAPInvest.xlsm").Activate  
Sheets("Output").Select  
Range("A2:AN1001") = "" 'Overhang  
Range("A1004:AN2003") = "" 'Input'  
Range("A2006:AN3005") = "" 'Five'  
Range("A3008:AN4007") = "" 'Zero'  
Range("A4010:CB5009") = "" '80 years'
```

End Sub

### ***Clear Outputs- CB Floor***

```
Sub ClearCBFloorOutputs()  
'Clears CB Floor Outputs  
Windows("CBFloor.xlsm").Activate  
Sheets("Output").Select  
Range("A2:AN1001") = "" 'Overhang  
Range("A1004:AN2003") = "" 'Input'  
Range("A2006:AN3005") = "" 'Five'  
Range("A3008:AN4007") = "" 'Zero'  
Range("A4010:CB5009") = "" '80 years'
```

End Sub

### ***Clear Outputs- CB Investments***

```
Sub ClearCBInvestOutputs()  
'Clears CB Investment Outputs  
Windows("CBInvest.xlsm").Activate  
Sheets("Output").Select  
Range("A2:AN1001") = "" 'Overhang  
Range("A1004:AN2003") = "" 'Input'  
Range("A2006:AN3005") = "" 'Five'  
Range("A3008:AN4007") = "" 'Zero'  
Range("A4010:CB5009") = "" '80 years'
```

End Sub