

Developing an Algorithm for Optimal Call Center Utilization

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Sponsor:
SATMAP

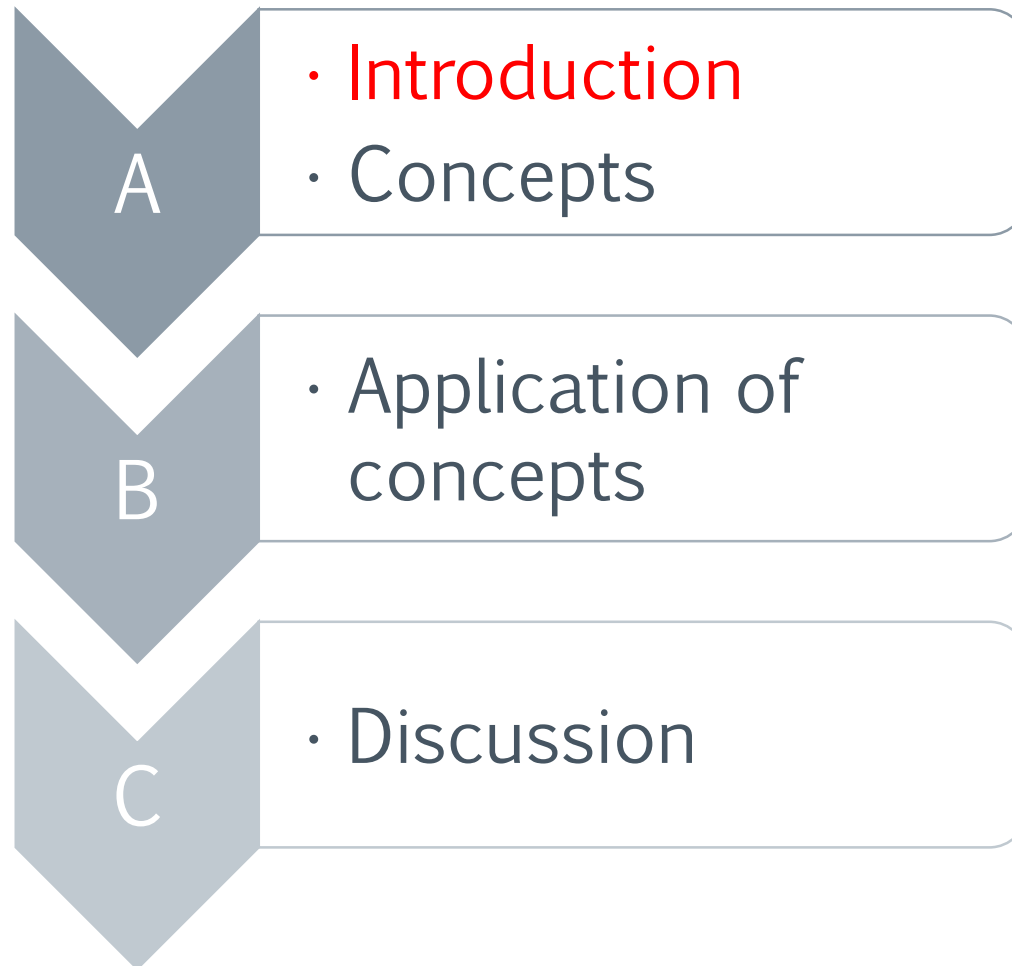
Introduction and Assumptions

- › SATMAP is a company that creates customer service strategies for calling centers.
- › Proprietary strategy maximizes revenue while maintaining agent utilization.
- › We consider that:
 - Each arriving call is concerned with only one issue that involves a certain skill
 - Agents are trained to answer calls that need different skills

The Problem

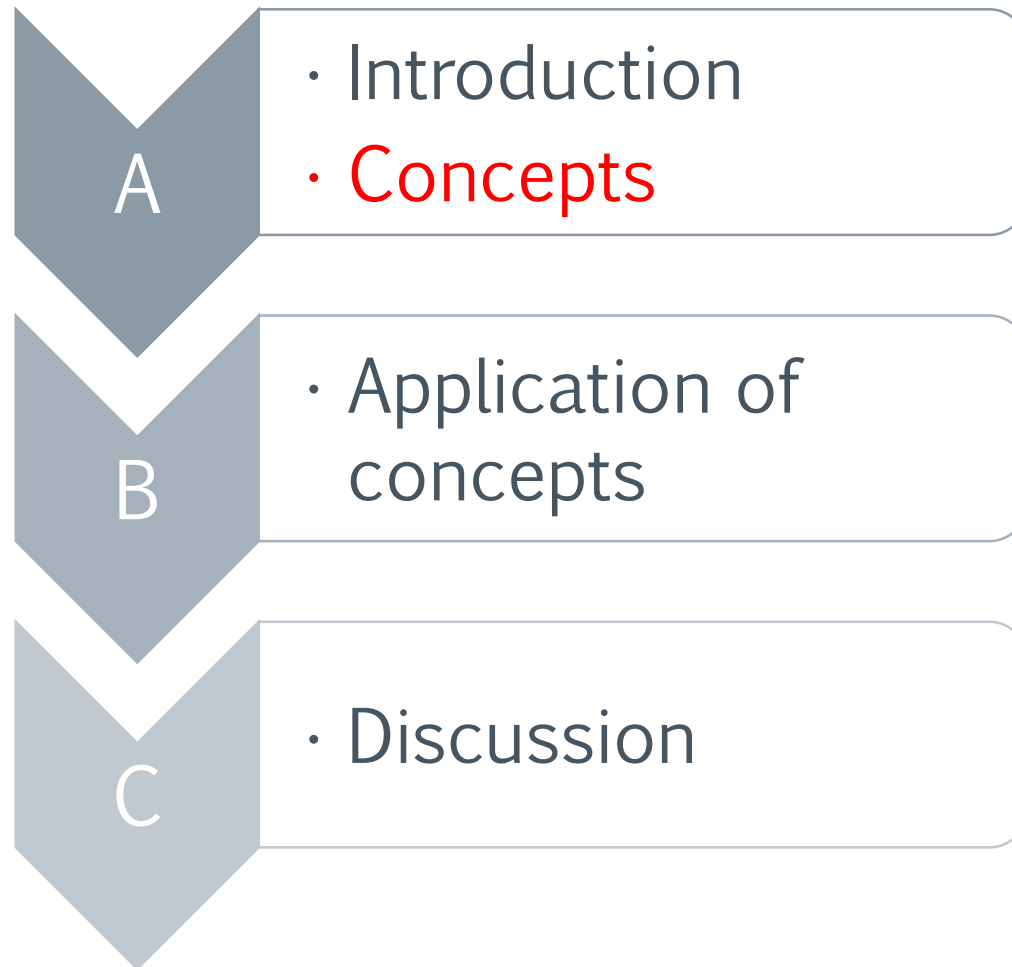
- › Given SATMAP's strategy, how much time should each agent spend on each skill in order to maximize revenue?
- › Factors of interest in the derivation of a model:
 - Comparative performance for each agent, and for each skill
 - Expected value of each skill per unit time
 - Target Utilization of each agent on each skill
- › Constraints to consider include maintenance of agent utilization across skills, and total representation of agents on each skill.

Linear Programming Method



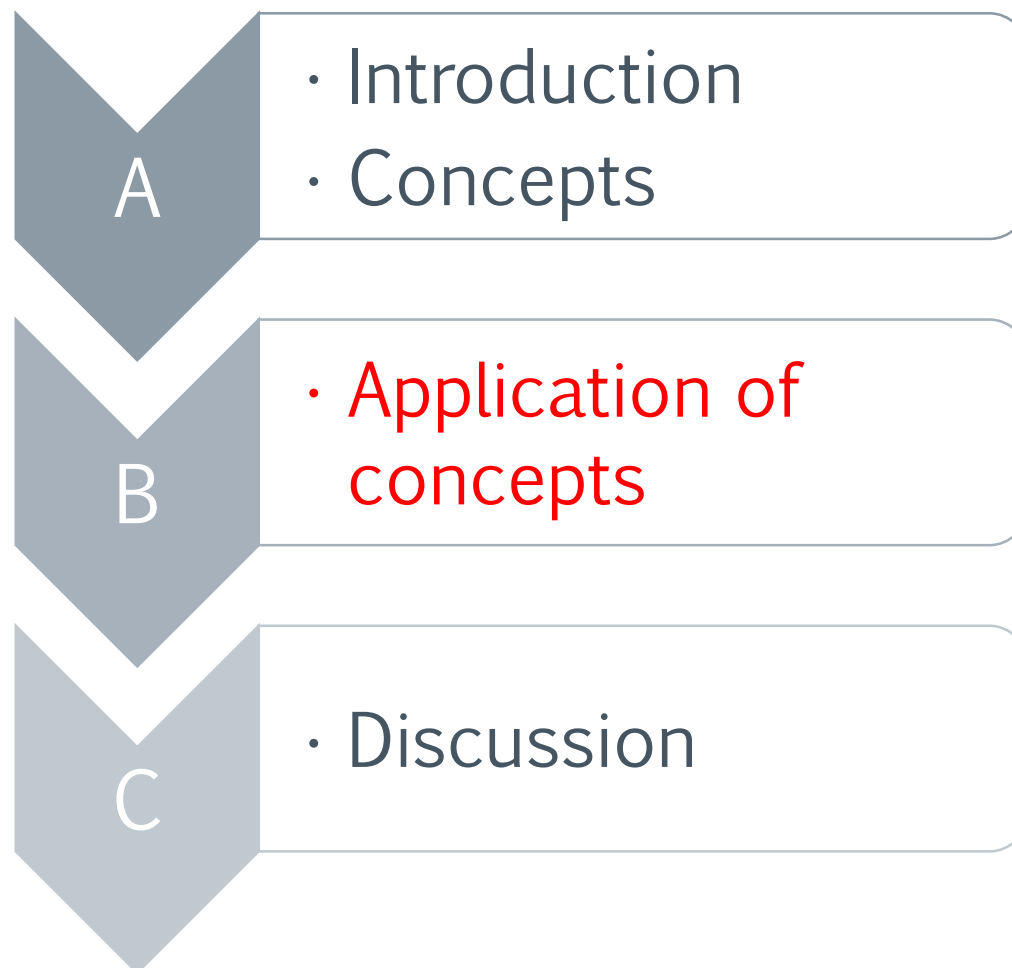
- › General linear programming function:
 - $\min c^T x$; such that $Ax \geq b, x \geq 0$
- › Let us assume that A has full row rank
- › Definition (Basic Feasible Point):
 - Let x be a feasible point with at most m nonzero components and let $B(x)$ be the set of indices i for which $x_i \neq 0$. Then define a new matrix B that has as columns the column of A , for all $i \in B(x)$. If the obtained matrix B is invertible, then we say that x is a basic feasible point.

Linear Programming Method



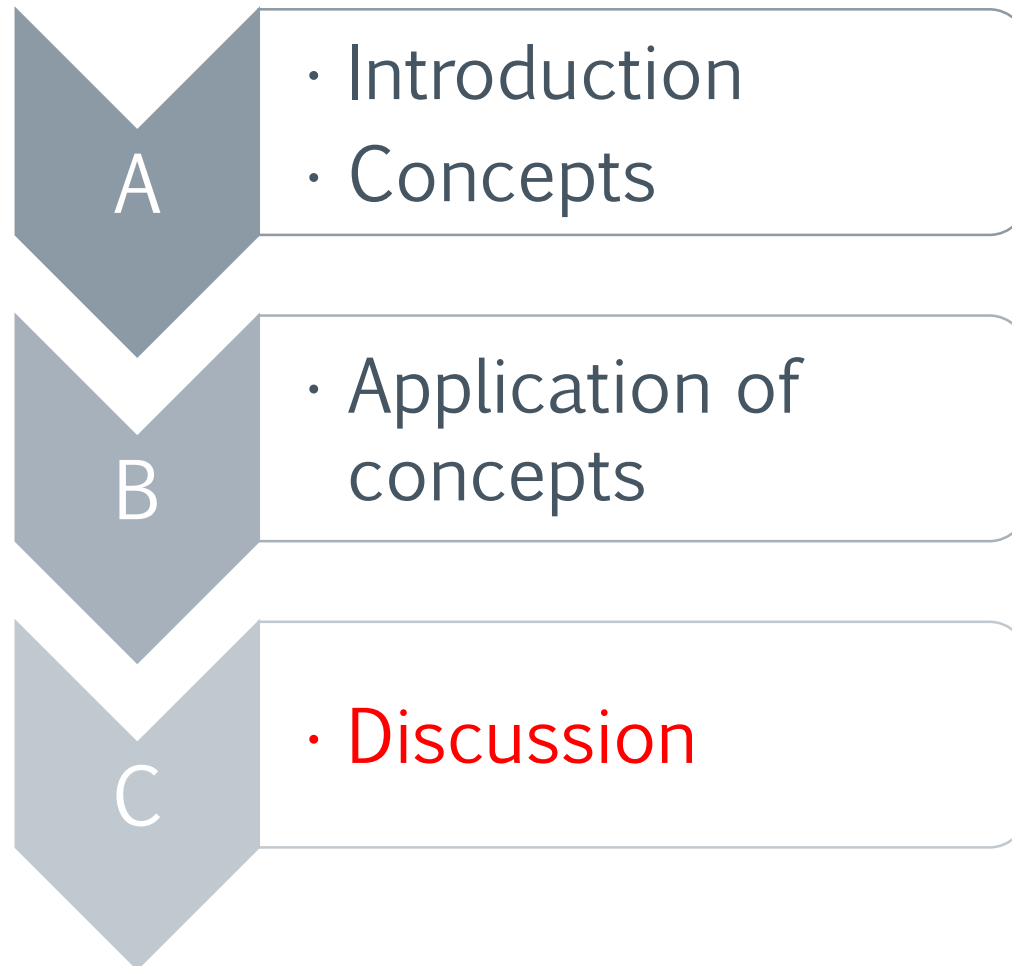
- › Theorem:
 - All basic feasible points are vertices of the feasible polytope $\{ x \mid Ax=b, x \geq 0 \}$.
- › Fundamental Theorem of Linear Programming:
 - If the system of constraints has a solution, then at least one such solution is a basic optimal point.

Linear Programming Method



- › If A has full row rank and if the system of constraints has a solution, then at least one such solution that optimizes the objective function is also the vertex of our polytope.

Linear Programming Method

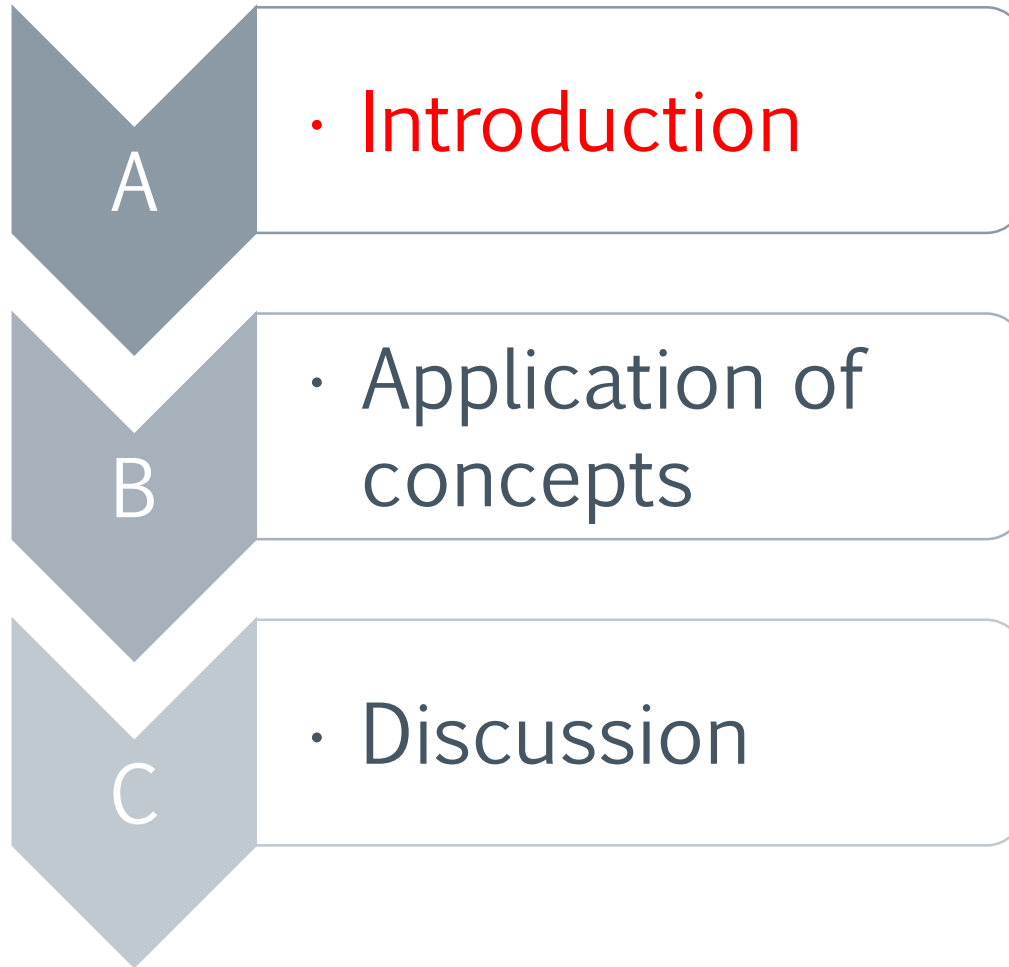


› Vertex solution to SatMap's LP is optimal in theory, but relaxation is desired

› Drawbacks:

- SATMAP would like to serve as many calls as possible
- Distributing an agent's time between multiple skills ensures more agent utilization across skills

Quadratic Programming Method

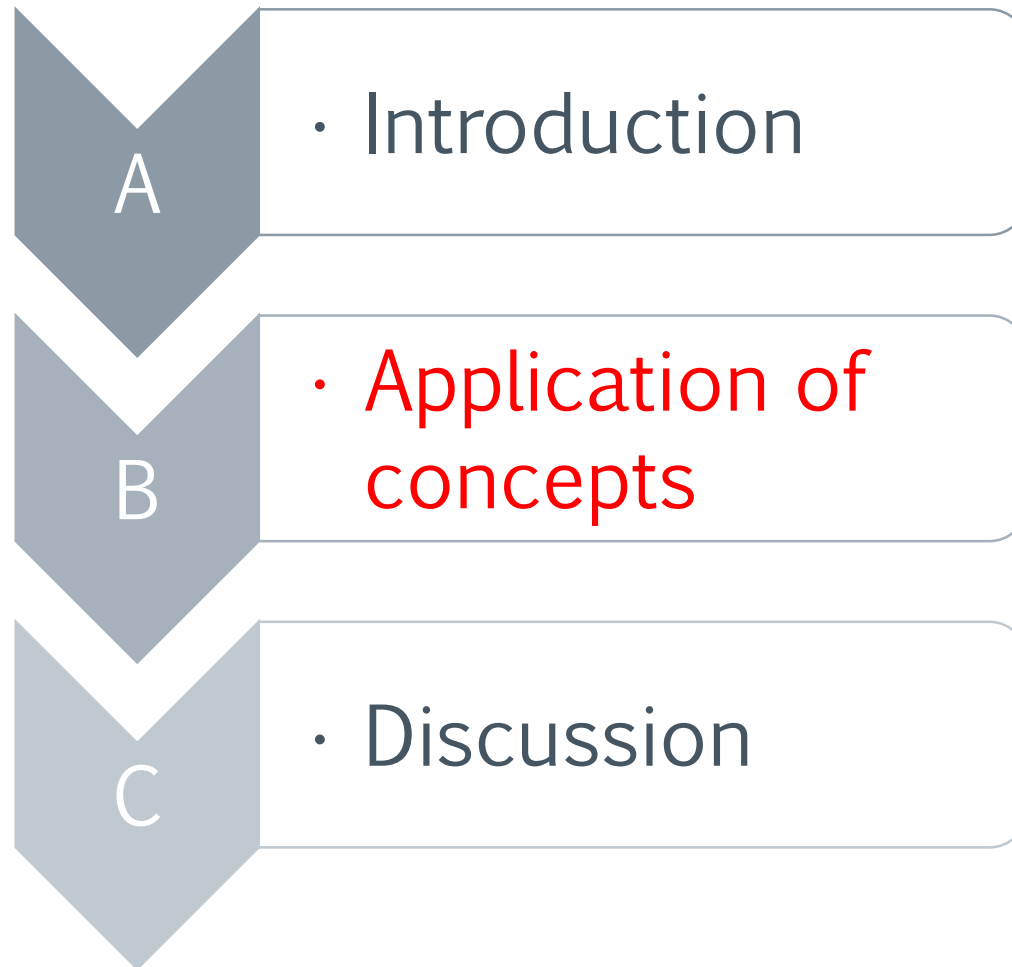


- › General Quadratic Programming function:

– Find $\min_x \frac{1}{2} x^T G x + x^T d$, subject to
 $a_i^T x = b_i$ for all $i \in E$ and
 $a_i^T x \geq b_i$ for all $i \in I$, where G is symmetric.

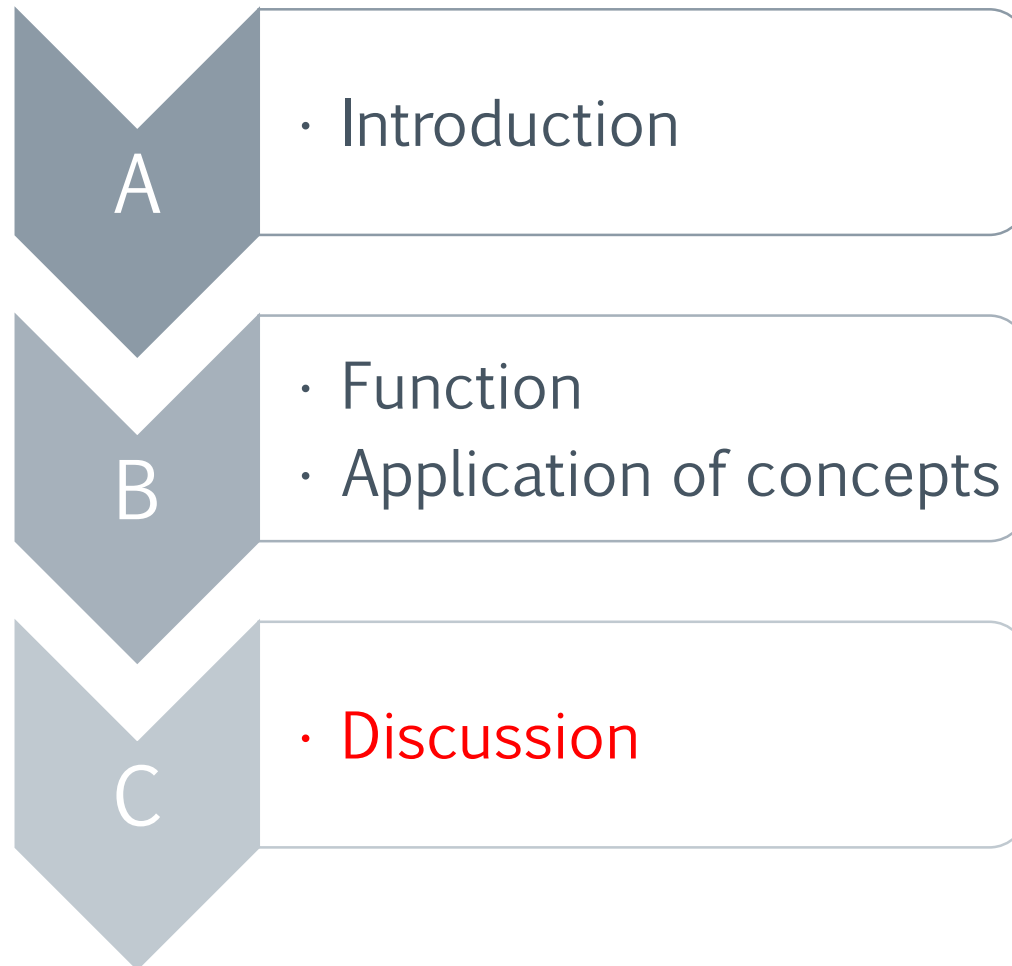
- › Generally, we denote by A the matrix that has as rows a_i^T for all $i \in E \cup I$

Quadratic Programming Method



- › Introduction of quadratic terms can allow for optimal solution away from vertices.
- › Original LP objective function maintained for linear terms, and quadratic terms attempt to emulate reality.

Quadratic Programming Method



- › Our devised G is indefinite
- › Pre-programmed functions can optimize a QP when G is positive semidefinite.
- › There are more complicated methods that can optimize a QP when G is negative definite and when G is indefinite, but they have not been implemented in simulation.

Lagrange Multipliers Method

A

- Introduction

B

- Application of concepts

C

- Discussion

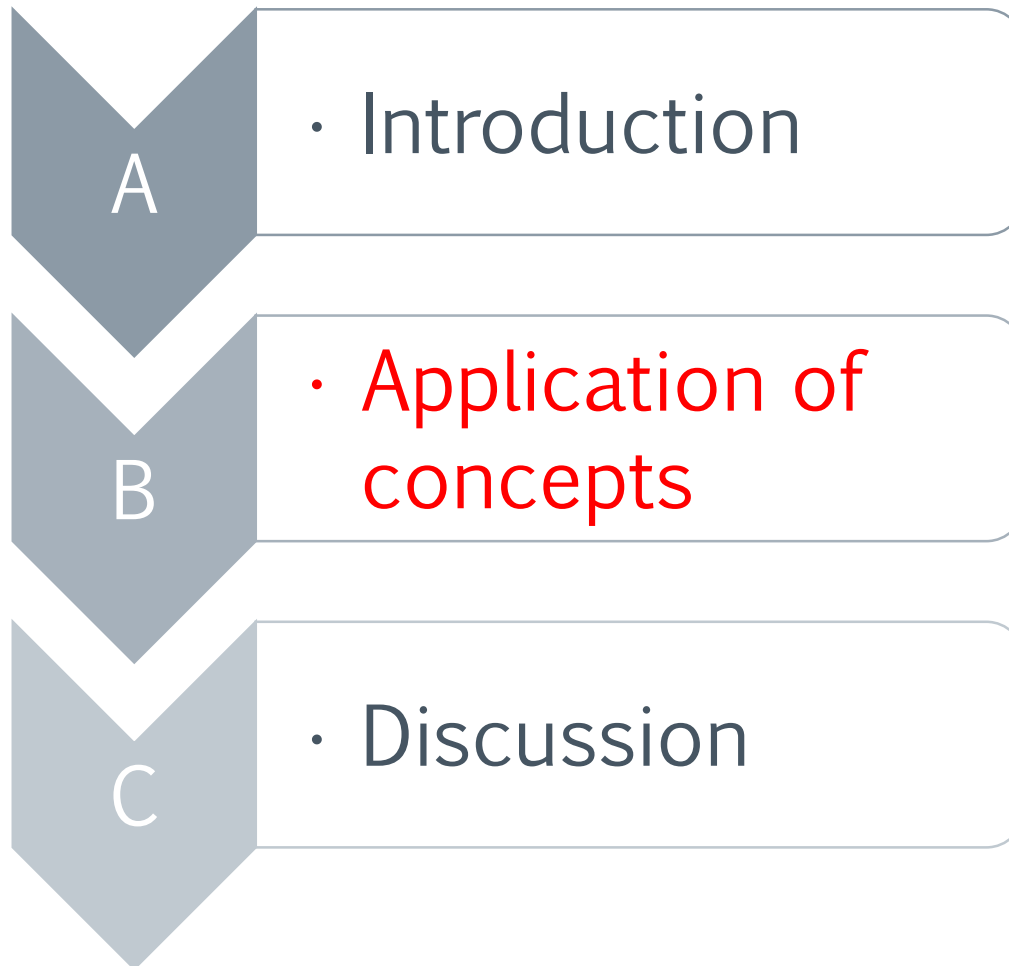
› $f(x_1, x_2, \dots, x_n)$ subject to
 $g_i(x_1, x_2, \dots, x_n) = c_i$, for $i = \overline{1, m}$

› $\mathcal{L}(\vec{x}, \vec{\lambda}) = f(x_1, x_2, \dots, x_n) -$
 $-\sum_{i=1}^n \lambda_i (g_i(x_1, x_2, \dots, x_n) - c_i)$

› Theorem:

– The 2 problems have the same extrema.

Lagrange Multipliers Method



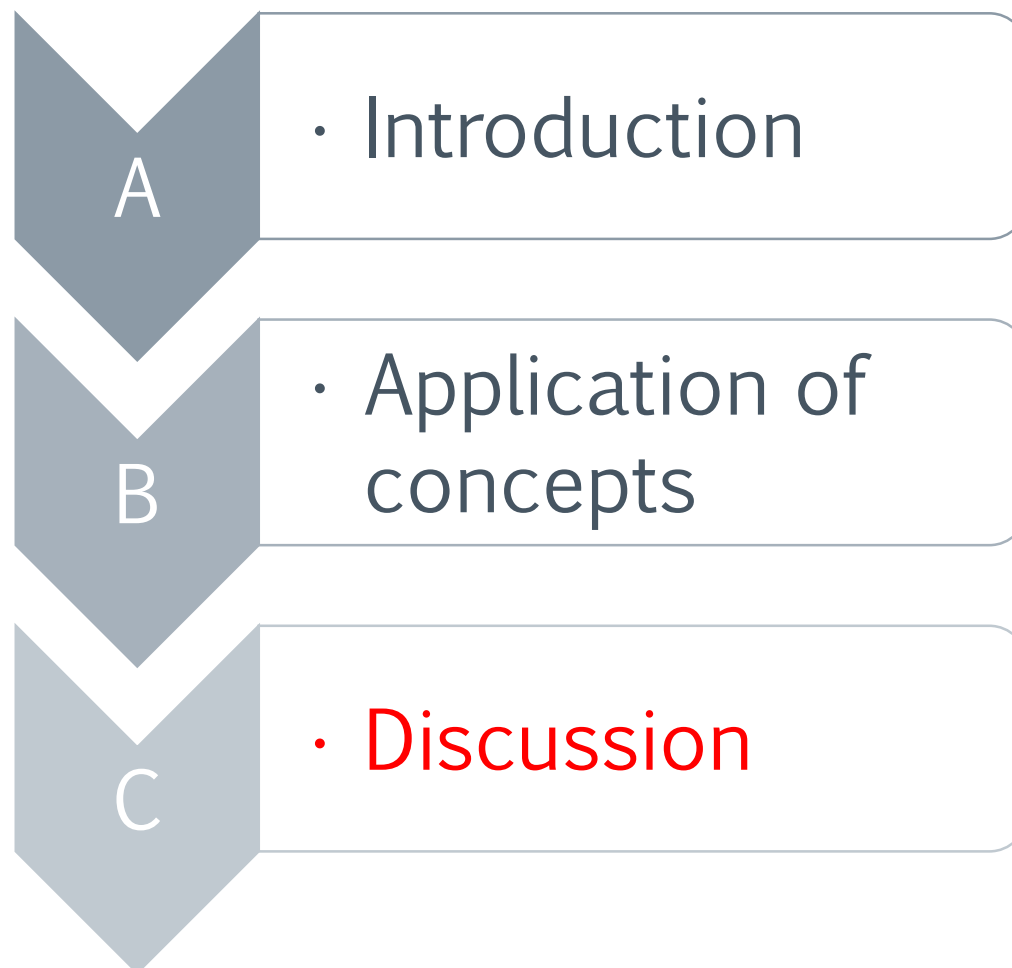
• Introduction

• Application of concepts

• Discussion

- › Since agent utilization on a skill requires positivity (an inequality), a work around was required.
 - Suppose we require $a \geq 0$.
 - Create new variable b
 - Create new constraint $a = b^2$
- › This ensures equal number of constraints and unknowns and allows an implied constraint on a .

Lagrange Multipliers Method



• Introduction

- › The matrix associated with the linear system previously mentioned is almost singular.

• Application of concepts

- › Computational error since multiple terms are significantly close to zero.

• Discussion

Revisiting Linear Programming

- › Due to various implantation issues which arose from Quadratic Programming and Lagrange Multipliers we revisited Linear Programming as a potential solution.
- › A new Linear Programming was devised originating from a different perspective on the problem and it allowed a solution which avoided the issues in the previous framework.

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Questions?

