Developing an Algorithm for Optimal Call Center Utilization

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Sponsor:
SATMAP

Introduction and Assumptions

- SATMAP is a company that creates customer service strategies for calling centers.
- Proprietary strategy maximizes revenue while maintaining agent utilization.
- We consider that:
  - Each arriving call is concerned with only one issue that involves a certain skill
  - Agents are trained to answer calls that need different skills

The Problem

- Given SATMAP's strategy, how much time should each agent spend on each skill in order to maximize revenue?
- Factors of interest in the derivation of a model:
  - Comparative performance for each agent, and for each skill
  - Expected value of each skill per unit time
  - Target Utilization of each agent on each skill
- Constraints to consider include maintenance of agent utilization across skills, and total representation of agents on each skill.

Linear Programming Method

A
- Introduction
- Concepts

B
- Application of concepts

C
- Discussion

› General linear programming function:
  - $\min c^T x;\ \ such\ that\ \ Ax \geq b, x \geq 0$

› Let us assume that A has full row rank

› Definition (Basic Feasible Point):
  - Let x be a feasible point with at most m nonzero components and let B(x) be the set of indices i for which $x_i \neq 0$. Then define a new matrix B that has as columns the column of A, for all $i \in B(x)$. If the obtained matrix B is invertible, then we say that x is a basic feasible point.

Linear Programming Method

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› Theorem:
  - All basic feasible points are vertices of the feasible polytope $\{ x \mid Ax=b, x \geq 0\}$.

› Fundamental Theorem of Linear Programming:
  - If the system of constraints has a solution, then at least one such solution is a basic optimal point.

Linear Programming Method

A
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› If A has full row rank and if the system of constraints has a solution, then at least one such solution that optimizes the objective function is also the vertex of our polytope.

Linear Programming Method

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› Vertex solution to SatMap's LP is optimal in theory, but relaxation is desired

› Drawbacks:
  - SATMAP would like to serve as many calls as possible
  - Distributing an agent's time between multiple skills ensures more agent utilization across skills

Quadratic Programming Method

A
- Introduction

B
- Application of concepts

C
- Discussion

- General Quadratic Programming function:
  - *Find* $\min_{x} \frac{1}{2} x^T G x + x^T d$, *subject to* $a_i^T x = b_i \; for \; all \; i \in E$ *and* $a_i^T x \geq b_i \; for \; all \; i \in I$, *where G is symmetric.*

- Generally, we denote by A the matrix that has as rows $a_i^T \; for \; all \; i \in E \cup I$

Quadratic Programming Method

A
- Introduction

B
- Application of concepts

C
- Discussion

› Introduction of quadratic terms can allow for optimal solution away from vertices.

› Original LP objective function maintained for linear terms, and quadratic terms attempt to emulate reality.

Quadratic Programming Method

A
- Introduction

B
- Function
- Application of concepts

C
- Discussion

- Our devised G is indefinite
- Pre-programmed functions can optimize a QP when G is positive semidefinite.
- There are more complicated methods that can optimize a QP when G is negative definite and when G is indefinite, but they have not been implemented in simulation.

Lagrange Multipliers Method

A
- Introduction

B
- Application of concepts

C
- Discussion

- $f(x_1, x_2, \dots, x_n)$ subject to $g_i(x_1, x_2, \dots, x_n) = c_i\,,\quad for\ i = \overline{1, m}$
- $\mathcal{L}(\vec{x}, \vec{\lambda}) = f(x_1, x_2, \dots, x_n) - - \sum_{i=1}^{n} \lambda_i (g_i(x_1, x_2, \dots, x_n) - c_i)$
- Theorem:
  - The 2 problems have the same extrema.

Lagrange Multipliers Method

- A · Introduction
- B · Application of concepts
- C · Discussion

- Since agent utilization on a skill requires positivity (an inequality), a work around was required.
  - Suppose we require $a \geq 0$.
  - Create new variable b
  - Create new constraint $a = b^2$
- This ensures equal number of constraints and unknowns and allows an implied constraint on a.

Lagrange Multipliers Method

- A
  - Introduction
- B
  - Application of concepts
- C
  - Discussion

› The matrix associated with the linear system previously mentioned is almost singular.

› Computational error since multiple terms are significantly close to zero.

Revisiting Linear Programming

- Due to various implantation issues which arose from Quadratic Programming and Lagrange Multipliers we revisited Linear Programming as a potential solution.
- A new Linear Programming was devised originating from a different perspective on the problem and it allowed a solution which avoided the issues in the previous framework.

Questions?