PREDICTIVE LOSS RATIO MODELING WITH CREDIT SCORES, FOR INSURANCE PURPOSES

Major Qualifying Project
submitted to the faculty of Worcester Polytechnic Institute in partial fulfillment of the requirements for the Degree of Bachelor of Science in Actuarial Mathematics

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EXECUTIVE SUMMARY

Hanover Insurance Group is a publicly traded property and casualty insurance company that is based in Worcester, Massachusetts. They provide their customers with a wide range of insurance products for both personal and commercial business lines. In recent times, credit scores have gained widespread popularity within the insurance industry, especially in the underwriting and pricing, due to its powerful predictive value. Hanover Insurance commissioned a team of four students from Worcester Polytechnic Institute (WPI) to design a statistical model that incorporates the credit score of each policy to better predict the future level of risk associated with this policy.

The goal of this project was to create a loss ratio model that would improve the predictive ability of the current Hanover premium model through implementation of credit scores. This would enable Hanover to benefit from more informed underwriting and pricing techniques, greater competitive advantage in commercial insurance lines of business and most importantly, more robust underwriting profit. Steps included:

- ➢ Conducting weekly meetings with the project advisors to interpret and analyze weekly results.

- ➢ Utilizing statistical software such as Microsoft Excel, SAS and R to analyze and model data.

- ➢ Developing graphical charts of variables that are statistically significant to the model, in order to determine model accuracy.

- ➢ Recommendations for improving current underwriting and pricing techniques.

The methodology consists of 3 primary steps. Our first step was to analyze the current techniques that Hanover employs in pricing and underwriting. This was done in order to identify the risk factors considered and to develop a solution to uniquely address

their business issue. Our next step was to familiarize ourselves with the data set to detect data quality issues and prepare the data for modeling. It is vital that the data be cleaned before usage as our model's predictive ability depends primarily on the quality of the input data. We noted any outliers, missing data, and inconsistent or invalid data points and identified statistically significant variables to be used in the model. Once we completed our data preparation we proceeded with the final step, data modeling. We used a generalized linear model with Tweedie distribution to predict the loss ratios of the policies, and used trial and error methods to test model accuracy. Once we were satisfied with the model, we analyzed the results to develop appropriate risk categories to differentiate customers based on the level of risk indicated by their predicted loss ratio.

The purpose of this project was to improve Hanover's underwriting and pricing techniques through the implementation of credit scores. We believe that Hanover's usage of these credit scores combined with additional company specific data, will be a powerful tool in predicting incurred loss ratio of a policy. Incorporating these credit scores will ensure that Hanover's underwriting and pricing techniques are competitive and more advanced than similar companies within commercial insurance. Hanover's ability to better differentiate the risk types of their customers will ultimately improve their underwriting profit by ensuring that they do not underwrite policies with excessively high risk.

INTRODUCTION

The relatively recent integration of credit score information since the late 20th Century has significantly impacted the insurance industry; however its usage has resulted in some level of controversy (Wu & Guszcz, 2003). A credit score is a numeric value developed using statistical methods used to represent a customer's level of credit worthiness or ability to repay financial obligations. In insurance, credit scores have been used with additional variables (driver record, type of vehicle, location of vehicle, etc.) to establish relationships between individual responsibility and probability of a loss in the future. The underlying assumption here is that customers who are more responsible in managing personal finances will also be prudent in management of other aspects of their life. These traits would lead insurance underwriters to believe that customers with a respectable credit score will be low risk customers, hence making them desirable (Hartwig & Wilkinson, 2003, Wu & Guszczs, 2003). If an insurance company can better classify low and high risk customers from a pool of applicants, it can prevent them from taking on risky customers who will have more claims and cause the company to make more payouts in the future. Therefore more and more companies in the insurance industry have begun to harness the predictive value of credit information.

In spite of this widespread usage, there have been several concerns raised over the extent to which credit scores are an accurate predictive measure. Credit scores are generated using a combination of past credit behavior of a consumer; therefore there is an issue as to whether its usage can be used to conclusively represent their behavior in the future. Several studies have, however, indicated that a relationship does exist between credit scores and loss frequency. Furthermore, the inclusion of credit scores in combination with other variables increases the accuracy. Many consumers have challenged the link between credit scores and customer creditworthiness since a poor credit score can put them at a significant

disadvantage when seeking to purchase insurance. As a result of this the government has placed restrictions and regulations on the manner in which insurance companies can use credit information. Nonetheless, its continued usage attests to the fact that there is valuable information that can be gained by using credit information, which will ultimately serve to improve the financial position of a business (Hartwig & Wilkinson, 2003) (Wu & Guszcz, 2003) (Hanover Insurance Group, 2011).

The Hanover Insurance Group is a publicly traded property and casualty insurance provider that is looking to utilize consumer credit information to improve underwriting profit. Headquartered in Worcester, Massachusetts, Hanover focuses on providing comprehensive insurance products to consumers in personal and commercial business lines. Their core commercial business segments can be further broken down into commercial auto insurance, commercial property and liability insurance, worker's compensation insurance and other forms of specialized insurance packages (Hanover Insurance Group, 2011). Within the insurance industry, great emphasis is placed on developing techniques that allow for more accurate modeling and prediction of risk. As business trends have developed, there has been a noticeable shift to the inclusion of credit score data as a predictive measure of the likelihood of losses associated with a customer account. Innovative implementation of a credit score variable would allow Hanover to enhance their current pricing and underwriting mechanisms giving them a competitive advantage. This directly influences Hanover's long-term profitability and market control and as a result has a key business value to the firm. The business issue is therefore how the implementation of credit information can improve the predictive ability of the current Hanover premium model for the commercial lines of business.

This paper will outline the details surrounding the business issue facing the Hanover Insurance Group and the development of the solution. We will discuss the initial appearance of credit information in the insurance industry and how it has been used in predictive

modeling. Afterwards, we will indicate typical statistical models that have been used to conduct predictive analysis. From there, we will outline specific project goals and objectives and the approach that was taken to achieve them. Following this, we will display the results of the model, highlighting key points and trends and analyzing the significance that the model results have for the Hanover Insurance Group. At this stage, it will be necessary to understand the implications of implementing this model and additional future concerns that may affect the manner in which business is conducted at the Hanover Insurance Group.

BACKGROUND

Hanover Insurance Background

The Hanover Insurance Group is a property and casualty insurance company based in Worcester, Massachusetts. They have over four thousand employees and offer a wide range of products spanning personal and commercial lines of business. In addition, they are a holding company for a group of insurers, offering property and casualty products and services through a group of independent agents.

Within personal lines, Hanover offers home insurance, with four levels of coverage from basic to select premium. Basic coverage offers replacement benefits for most common causes of loss. In addition, it insures other structures such as sheds and garages, covers some personal belongings, handles liability claims, and offers medical payments to non-household members injured on the property. Each additional policy offers more coverage in addition to the basic coverage. Hanover also offers personal auto insurance, which offers various liability limits in addition to comprehensive and collision options, roadside assistance, transportation expense, medical payments and personal injury protection, and value-added endorsements. For personal insurance customers who already have home or auto insurance, Hanover offers personal add-ons. The add-on called "Toys" covers recreational vehicles, watercraft insurance, and account extras. Umbrella Coverage provides additional coverage on top of the original policy, the Identity Integrity Program protects against identity theft, and valuables insurance is also offered.

Hanover offers both small business insurance and midsize business insurance. For small business insurance, there is standard insurance that includes Business Owners Policy for businesses that fit specific criteria. They also offer Commercial Package Policy for the small businesses that do not fit the criteria for a Business Owners Policy. Hanover also offers workers' compensation, commercial auto, and commercial umbrella policies. In addition,

they have special coverage and policies for small technology businesses. Hanover offers similar policies for midsize businesses as they do for small businesses. In addition, they offer inland marine policies, policies for marinas, jewelers' block, and performance and surety bonds. They also offer special policies for schools, human services, chauffeured transportation, document management, moving and storage, religious institutions, specialty industrial, sports and fitness, and investment management.

Predictive Modeling

Predictive modeling is an analytical method used to create statistical models that predict future behavior or results. It is a form of data-mining that uses advanced statistical modeling techniques to forecast probabilities and trends (Mosley, 2005). For example, a company can use predictive modeling to identify insurance risks, which can lead to improved underwriting, pricing and marketing decisions. For many insurance companies, predictive modeling plays an important role in pricing techniques since it can help them differentiate low or high risk customers based on several characteristics and, most importantly, give them a price optimization strategy.

In recent times, predictive modeling has gained popularity and has been heavily utilized by property and casualty insurance companies in personal lines because it gives them a competitive advantage. Insurers believe that predictive modeling can significantly help their rating plans by identifying mispriced risks thus increasing profitability (Mosley, 2005). By identifying new rating variables or new relationships between existing variables, predictive modeling can find different ways to segment risks. One such method of segmenting risk is through the inclusion of a customer's credit history. Today, most companies have used credit score information in predictive analysis because they have identified a relationship between customer credit scores and likelihood of future loss. In addition, several companies have realized the accuracy of using multivariate analyses in

conjunction with credit information. Even for small to medium sized companies, a custom insurance score can greatly improve the results of current underwriting and pricing techniques. Therefore, the usage of credit scores as a predictor of losses in commercial lines will yield more accurate results.

There are three main stages involved in predictive modeling. Typically, identifying the issues a company wants the model to solve is the first stage. Collecting the appropriate data that would be needed to solve these issues is the second stage, and finally, developing the model that best fits the data and analyzing the results are the third stage (Mosley, 2005). The types of predictive modeling analysis methods that have received widespread attention are the Generalized Linear Modeling (GLM), Decision Tree Analysis and Multivariate Adaptive Regression Splines (MARS). A discussion of these modeling techniques will be done at the end of the section.

Credit score usage in Insurance

There are several risk factors that are used by insurance companies to evaluate the risk of customers in order to assign a proper individual premium to each. Competition continuously forces companies to find new and more accurate factors to add to their pricing strategies, thus ensuring more competitive pricing. In the 1980's through the early 1990's, insurance companies first began to formally use elements associated with credit such as bankruptcy history in premium pricing (Federal Trade Commission, 2007). Beginning in the 1990's, alternative credit scores were developed and made commercially available. As the years went on, new technology allowed for a wide range of differentiated scores to be developed using new modeling techniques (Oscherwitz & Reemts, 2011). In 1993, Fair Isaac Corporation developed the first credit-based insurance score. This and competing insurance scores were developed to reflect the predicted risk for an insurance loss based on a customer's credit report. Since then, a number of companies have developed their own

credit-based insurance scores for many different insurance markets (Federal Trade Commission, 2007). In addition, the wide availability of these scores and the increasing use of computers and database software allowed companies to better model this credit data to develop more practical applications of insurance scores. In fact, it was found that credit-based insurance scores were a reliable and successful predictor of risk. Experian, a major credit score company, found that credit scores are successfully indicative of loss 60-80% of the time (Krickus, 2011). Today the 15 largest personal auto insurers use credit scores for premium pricing (Federal Trade Commission, 2007).

Despite the wide acceptance of credit scores in the insurance industry, consumers have been reluctant to accept their use. In 2001 and 2002 a number of lawsuits were filed against GEICO and Safeco claiming they violated the Fair Credit Reporting Act, which "requires notice to any consumer who is subjected to adverse action … based in whole or in part on any consumer [credit] report" (Allen, Perry, Reynolds, & Long, 2008). The lawsuits suggested that insurance companies were required to inform customers whenever their credit history was used to negatively affect their premiums within personal auto insurance. Eventually the Supreme Court considered the case and the issue became public. The Supreme Court ruled "the insurance industry, under the Fair Credit Reporting Act, must notify customers that it is charging higher insurance rates only when the higher rate is based on a low credit score. Companies aren't in violation of the law, the high court said, if the consideration of a credit score, as one of several considerations, didn't alter the pricing" (Anderson, 2007). As long as the credit score was not the cause of a premium increase, insurance companies are not required to inform the customer their credit score was used. As a mere factor amongst many others, credit score does not have to be treated differently (Allen, Perry, Reynolds, & Long, 2008). Consumers were still not pleased with the practice, claiming it was unrelated to driving risk and possibly a discriminatory proxy for race. In 2007, the Federal Trade Commission made a report to congress detailing the effectiveness of

credit scores in insurance loss predictions. The study found that credit scores were a highly effective predictor of loss and that it was ineffective as a proxy for race (Federal Trade Commission, 2007). Even so, the general public is not fond of the practice, especially in harder financial times as their credit scores are often falling. In 2010 and 2011 the issue was again brought to the foreground in a number of news articles and new attempts to legally eliminate the practice (Lipka, 2011).

While the use of credit scores has permeated much of the personal line insurance business, commercial insurance has only recently introduced the idea. Most of the use of credit scores in commercial insurance appears on the underwriting side. Generally policies written in the middle market are at the discretion of the underwriter allowing for the use of credit scores subjectively. Companies are beginning to increase the use of credit in the pricing side in order to create a more uniform application of credit across the policies written (Walling III, 2011). However, there are some issues with the implementation of credit scores in commercial lines. There is a greater problem of thin file or no hit results as small businesses are less likely to have a commercial credit score available. This could be rectified by using the credit score of the small business owner, provided that the company has the right to do so (Krickus, 2011). Otherwise, the model will have to find an appropriate way to predict losses considering the lack of information of these no hit policies. There are also issues with relying too heavily on the credit data alone. Often the credit data will utilize other variables already considered in the model, so while it is an effective predictor of loss by itself, when incorporated into an existing model it may amplify its effects (Walling III, 2011). Modeling credit data should be done in conjunction with all the other factors to ensure that too much reliance is not placed on the credit scores, which might not give optimal results.

Government Regulatory Environment

The use of insurance scores within the commercial lines industry has been fairly recent; as such it is important to assess the role that government regulation will have in its usage. Some states have developed their own unique laws to govern the use of credit data in insurance. In Massachusetts these laws are detailed in the General Laws: Regulation of trade and certain enterprise and by Massachusetts instated regulations presided by the commissioner of insurance in Massachusetts. These laws state that credit information can be used for underwriting purposes. A section in the Massachusetts regulations, however, further states that credit information cannot be used for underwriting related to personal auto insurance. Due to the recent inclusion of credit scores in commercial insurance, there is very little literature available on this topic. A concern from the government could possibly be whether the insurance score inherently biases certain sectors of business more than others. In such a case this sector would be at a disadvantage and may be charged a higher premium because of limitations of the insurance scores. If the government feels as though the usage of insurance scores are unfair or pose undue restrictions on business customers they may feel the need to intervene. The relative uncertainty of the government regulations at this point may be an issue for concern in the future. Therefore, it will be necessary to monitor development within the industry and monitor how these could change if government were to impose more regulations (Massachusetts Government).

Multivariate Models

Generalized Linear Models

A generalized linear model (GLM) is a multivariate model which can be fit to datasets that follow probability distributions such as Poisson, Binomial and Multinomial distributions. Recently, there has been an increased interest in using GLMs in predictive

analysis. In particular, the incorporation of multivariate analysis is widely accepted as the results generally prove to be more conclusive and comprehensive than univariate results.

The purpose of a GLM is to quantify the relationship between several independent or predictor variables and a dependent variable. It can be seen as an extension of linear multiple regression for a single dependent variable. It is extremely useful in finding the best pricing method as it gives an overall view of how each independent variable relates to the price. Furthermore, GLM is more advantageous than univariate analysis as it allows the user to readily adjust for both exposure and response correlations that cause one-way analysis to fail. For example, when the underwriters do not want to change the internal constraints associated with their existing policies, they can force GLM to perform consistently by using offset techniques. Offset techniques allow users to adjust the input data and force the variables to be consistent with the desired values (Werner & Guven, 2007).

Generalized linear models can also remove unsystematic variation or the "noise" in the data. They are more robust and less susceptible to over-fitting than other predictive modeling techniques. Over-fitting occurs when the model incorporates patterns present in the sample data that are not present in the overall population. GLMs are not black box models because their calculations are relatively easy to follow and interpret. As such, the users are better able to understand how each of the predictors affects the final results of the predictive model. For our credit analysis project, GLM would be a suitable option to create our risk models. In addition to the advantages mentioned earlier, GLMs generally follow distributions such as Poisson and Gamma which are commonly used to model insurance data. This would allow for some level of consistency when being compared to other analyses (Werner & Guven, 2007).

Decision Tree Analysis

Besides Generalized Linear Modeling (GLM), other models such as Decision Tree Analysis are also popular models used in predictive modeling. Decision Tree Analysis is a type of predictive modeling that is used to separate a group of risks into homogenous groups based on an identified response variable. It is also an important tool for decision-making processes. Decision Tree Analyses are useful when there is a situation where a lot of decisions have to be made at each step and each decision could lead to determine the decision to be made in the next step. The user has to take into account how the choices and the outcomes of earlier events influence the events at later stages. In Decision Tree Analysis, analysis of each independent variable has to be done to determine which creates the largest degree of separation in the dependent variable. The whole data is split into different branches with two or more groups. Only then must each branch be analyzed to see which independent characteristic is most important in distinguishing the levels of the dependent variable for that branch (Mosley, 2005).

Multivariate Adaptive Regression Splines (MARS)

MARS is another type of regression analysis which also can be used to analyze the relationship between a response variable and independent variables. What MARS does is create a linear model based on the independent variables that you input into the model. Like other regression analyses, it shows the user how the value of the dependent variable changes when some of the independent variables vary while others are held fixed. Because of this nature, it can be used to model losses and insurance claims in the insurance industry. However, MARS differs from Generalized Linear Model and Decision Tree Analysis in that they are non-parametric statistical methods and they are useful in cases where the data do not have significant numerical interpretation but have a ranking. For example, MARS could be used by a company to assess whether customers have preference for high or low quality products. In such a case, MARS would be more suitable than a GLM due to the fact that a

customer's preference can be grouped in terms of categories; they can have high, medium or low preferences for the services or the products that they receive. If we are dealing with this type of data in which a ranking exists, we should use non-parametric methods like MARS.

Looking at the various types of models for multivariate analysis there are several benefits and drawbacks of using different models. While they are all a part of the same multivariate family, they are used in different ways depending on the type of the data being dealt with. For our data set, we have continuous variables in the form of the credit variables; however, in general MARS has been shown to be better at handling discrete data and as a result may not be suitable for our model. Regression Trees and MARS also require significantly large datasets in order to produce credible results, while with GLMs it is possible to generate concrete analysis given less data, and they are less susceptible to over fitting data. In addition, GLMs allow for the usage of numerous independent variables simultaneously and have a flexible model that can be fit to varied datasets. For these reasons, GLMs are widely used in the insurance industry and are effective in determining the impacts of different claim characteristics on the outcome. Therefore, based on our research of the relevant models we believe that a Generalized Linear Model will be the most suitable method for this project.

Model Distribution Type: Tweedie Distribution

The Tweedie distribution is a grouping of distributions that can consist of continuous distributions such as the Normal, Gamma, or the Pure Discrete Poisson. It can also be formed from the compound Poisson distributions which have positive mass at zero and are continuous elsewhere. The Tweedie distribution is useful in modeling data that is both discrete and continuous in which case it is called the compound Poisson process. It also has important parameters such as mean, variance, dispersion and power parameter p. Depending on this p-value, the Tweedie distribution can be modeled as Normal or Gamma,

or it can be modeled using the pure discrete Poisson distribution or the compound Poisson distribution (Shi, 2007).

If we were to incorporate the Tweedie distribution in our project, we would use compound Poisson distribution for several reasons. From our credit data, we are likely to incorporate claim frequency and the random number of claims filed which follows Poisson distribution. In addition, we will use random size of each claim which follows Gamma distribution. Therefore, it will be beneficial to use the compound Poisson distribution as it can handle both discrete and continuous distributions. Finally, since it is also a type of GLM, it can also take numerous variables as input. For our project, our concerns are how frequently a business account will report a claim and the severity or size of the claim. Therefore, using the insurance score data as well as additional risk factors as inputs we could create a compound Poisson distribution model to predict incurred losses, premium, and other future risks (Shi, 2007). The Tweedie distribution will also be appropriate as it is a form of GLM that has the ability to model both discrete and continuous distributions. Therefore, we believe that using a combination of the GLM and Tweedie models will offer more comprehensive predictive analysis results.

METHODOLOGY

In this section we outline the major steps that were taken to develop a predictive statistical model using credit information for Hanover's commercial lines of business. Hanover's commercial line of business is comprised of: Commercial Auto, Worker's Compensation, Business Owners Policy (BOP), and Commercial Package Policy (CPP). For our project we focused primarily on developing a model for the Commercial Auto and the Business Owners Policy (BOP) group. The statistical models incorporated the credit scores of a policy into current rating techniques (the process by which the company determines risk associated with each policy) and produced a single measurable summary, the incurred loss ratio. To develop this model there were three primary steps we took in order to ensure accurate and meaningful results. Our first step was to assess Hanover's current pricing and underwriting techniques to determine how to best incorporate credit score information into current calculations. Then we determined a base set of risk factors for each line of business that was used to predict the loss associated with a policy. Finally, we calculated an incurred loss ratio for each policy and then grouped policies based on their indicated level of risk determined by comparing their individual loss ratio to the overall loss ratio.

We decided to build our model design starting with the Commercial Auto line of business. We made this decision because Commercial Auto is a relatively simple policy with a single coverage. It would be the most straightforward to implement since the concepts behind the model design were relatively simpler to tackle than other business lines. Once we finished our work on Commercial Auto we moved onto the Business Owners Policy (BOP) group. Through our research and discussions with Hanover, we determined that BOP would most benefit from incorporating credit as the pricing techniques are the most automated which would allow for easier implementation of our model. Due to the limited underwriter involvement in BOP, any step to improve the accuracy of the pricing

calculations will have a greater effect on results than for lines that regularly incorporate underwriter judgment, such as Commercial Package Policy (CPP). With the time constraints on the project, we were able to complete these two lines of business, leaving Worker's Comp and CPP for future projects.

Before we began modeling the data, we first had to have a thorough understanding of the business issue. The goal of the project is to improve the predictive ability of the current Hanover premium model by implementing credit information. Using this information from Hanover's credit vendors, in addition to Hanover specific information ensured that current rating techniques used by Hanover are more informed and adequately account for varied risks. Our first step was, therefore, to analyze the current techniques that Hanover employs in pricing and underwriting to produce a solution that uniquely addresses their business needs. Through our discussion with Hanover representatives from each business department we identified the range of risk factors considered in that department and how they are measured and calibrated. We determined the stage in the current pricing model that would be most appropriate to input the credit variables and the most suitable form to output the results of our predictive analysis. Additionally, we determined the role that individual subjectivity plays in the calculation process and the extent to which an underwriter's experience affects the ultimate decision to underwrite a policy. This will be important for us in order to determine which aspects of the current system can be automated or which might need further individual attention. Ultimately, understanding their existing measurements of risk will allow us to ensure that the results of our credit model can easily be incorporated into current methods.

Once we were clear on the business issue, it was important to familiarize ourselves with the data set to detect data quality issues and prepare the data for modeling. For the Commercial Auto group, we were provided with a data set containing 165,142 policies and 35 fields with relevant information about each policy. The research at this stage can be

broken up into exploratory analysis and data preparation. During the exploratory analysis stage we noted any interesting relationships and factual details about the data and evaluated the data quality. We looked at the numeric and categorical data to test for validity, accuracy, reasonableness and completeness of data. This involved performing graphical representations of data variables such as histograms, bar graphs, scatterplots and box and whisker plots. We also conducted a range of statistical analysis on the data set in Excel and R statistical software to better understand the breakdown of each variable and their distributions. In addition, we observed relationships between various data fields and the loss ratio to identify whether any correlation existed between variables and loss ratio. Another major step was the identification of invalid, missing or inconsistent data. We observed data points that in their current form would not be suitable for use in the model (such as negative losses and rerated premiums). There were also several outliers and extreme values, mostly for incurred loss and loss ratios (we focused on loss ratio for extreme value cleaning since it considers loss relative to premium). More importantly we noted that 35.30% of credit data was missing (58,718 policies), this is vital as the credit variables are the main elements that we want to use in the model. It was important to understand this information early on as it assisted us in understanding what model results to expect. In addition, making certain that the data going in the model is of high quality will ensure that the model results are valid and meaningful.

In the data preparation stage we performed the specific data cleaning and adjustments in order to prepare the data for modeling. We began with 165,142 policies but based on our exploratory analysis we decided to delete policies with no credit information, missing rerated premium and that were manually written, reducing the data set to 95,173 polices. After these deletions, we adjusted the polices so that negative incurred losses were set to zero, incurred loss ratios were capped at the 95th percentile and rerated premiums were set to at least $500. Another aspect that had to be considered was correlation between

variables to be used in the model. Multi-collinearity is a situation where independent predictors have correlation with one or more other variables being used. This is a problem as it makes it hard to identify which of the variables has the greatest effect of the dependent variable being modeled. Based on our exploratory analysis, we found that the two credit variables used by Hanover were correlated. To correct this, an uncorrelated variable was created using factor analysis code in SAS software to combine these two credit variables into a new uncorrelated variable named Financial Stability, which was then assigned to each policy. Once we were confident that the quality of the data was at a superior level, we proceeded with the data modeling steps.

The first stage of the modeling process was to determine the response variable being modeled; we could either model pure premium or loss ratio of a policy. In determining whether to pursue a pure premium or loss ratio model, we looked at the business questions being asked and what we were hoping to accomplish with the model. Our goal is to improve the existing Hanover premium formulas with the inclusion of credit data as a variable. Hanover's existing premium formula is based on loss predicting models so in order to be consistent, a loss predicting model for credit scores is needed. Also, using the pure premium model would require that we have exposure information that was not provided in the data set given, as it is generally harder to attain. Therefore, our final choice was to use incurred loss ratio as the response variable.

Given that we chose the response variable, it was important to determine the type of model to be used. We chose to use the Generalized Linear Model for several reasons. GLMs are able to look at several variables at the same time and adjust for correlations that enhance the type of analysis that the predictive model can produce. They are also are helpful in reducing unsystematic noise from data, focusing more in statistical relationships that the modeler is attempting to test. Furthermore, GLMs can identify useful interactions and relationships between the rating variables, which can increase the predictive value of the

model. In comparison to other types of multiple regression analysis (such as Multivariate Analysis Regression Splines), GLMs allow for a transparent calculation process so the user is able to follow the steps and have a better understanding of the final result. However, we had to bear in mind a few significant assumptions of the GLM. First, the GLM assumes a probability distribution in the exponent family (normal, binomial, Poisson, multinomial, gamma, etc.). We chose to use a Tweedie distribution with p value between $1<p<2$ as a Tweedie distribution allows for a lump sum of probability at zero and continuous probability beyond that, perfectly suiting normal loss ratio distributions. Other primary assumptions of the GLM are that the relationship between dependent and independent variables that are linear in nature and the predictor variables are statistically independent.

The next stage was to identify which variables bear a predictive relationship with incurred loss ratio, focusing on the credit data variable. Correctly identifying explanatory variables is an important aspect of generalized linear modeling as results of the model are interpreted considering the impact and role of these predictors. Through trial and error we measured the impact that each variable had on the model accuracy and determined to use Policy Type, Business Type, Market Segment, Fleet Size and Financial Stability (combined credit variable) as input variables in the model. Before we moved on to modeling we had to separate the data into a build sample and a validation sample. When modeling we used a random 80% of the data (Build Sample) and saved 20% for testing of the model (Validation Sample). This was done to ensure that the data was not over-fit to the data set that was used to create the model, but rather would perform well on any data set provided. We separated the data by using the "randbetween" function in Excel to assign each policy a random number from 1-50000. After this we rearranged the policies in numerical order then chose the first 80% as the build sample. Afterwards, we tested the model on the validation sample which helped us to determine if the model works overall and was not over-fit to the build sample.

The loss ratios of the policies were fit to a Tweedie distribution using an iterative least squares method to fit data in R statistical software. An initial process determined the Tweedie distribution p value which was incorporated into the GLM model fitting process. A number of fits were created through a guided trial and error method using goodness of fit tests and variable significance tests to determine what changes to attempt. We used the Akaike Information Criteria (AIC) test to analyze the relative overall goodness of fit of each model iteration in comparison to the previous ones. The chi-squared test was used to determine the significance of each variable to the model fit and was used to determine which variables were to be kept in the next iteration and which were to be removed. Lift charts were used to assess the fit of each variable visually and determine the need for non-linear terms (lift charts will be further discussed in the following section). After the model was fit, we applied the proposed model to calculate predicted values for loss ratio on the validation sample of our data. We analyzed the fit on the validation sample for signs of over fitting in which the model is fit too tightly to the build sample, predicting build sample specific random occurrences in other samples. When we encountered errors and determined the possible cause of the error (such as over correlated input variables) we went through and changed the formula for the model and continued testing it on the 20% until the results on the test model seemed to be more accurate. Once we determined the fit on the test sample to be satisfactory, we refit the data on the full sample (100 percent of the policies) and made the necessary adjustments to the model formula until it produced solid results. After we completed the model adjusting, we presented our results to Hanover and got the feedback on the model. We finalized our model calculations based on the recommendations from Hanover to ensure that the final incurred loss ratios produced were valid and met their expectations from a business perspective.

ANALYSIS AND DISCUSSION

In this section we will display the results of our analysis through the use of lift charts. Lift charts serve primarily as a visual representation of the effectiveness of the model in predicting the loss ratio. The following section contains various lift charts demonstrating the particular effectiveness of the model measured against the variables in the data. Each chart breaks the data into groups based on the variable being measured and calculates the total loss ratio for each group. It does this for both the actual and the predicted loss ratios. By then charting each series of loss ratios, we are able to see the actual trend and the model's predicted trend of the loss ratio over the possible variable values. Total policy count and total premium for each group are included to allow for sample size and premium amount to be considered in analyzing the total loss ratios. In a lift chart, a noticeable trend indicates the predictive quality of a variable. An ideal model fit would have actual and predicted loss ratios that are equal to each other. For our project to determine the effectiveness of the model prediction, we made three lift charts for each of the eight variables present in our model: one showing the build sample, one showing the validation sample, and one showing the full sample. Included in this section are the eight full sample lift charts from the Commercial Auto group. The full display of all the graphs (Build, validation and full sample) for each Commercial Auto variable can be seen in the Appendix B and for each BOP variable in Appendix C.

1. Financial Stability Loss Ratio Relativities

Actual versus Predicted LR Relativities by Financial Stability

	1	2	3	4	5	6	7	8	9	10
Account count	9,839	10,648	9,931	10,334	10,278	9,837	9,372	9,733	9,169	6,032
Premium	$6,246.43	$6,239.23	$6,247.46	$6,246.28	$6,244.76	$6,241.51	$6,227.36	$6,260.65	$6,255.70	$6,245.82
Act LR Relativity	1.14	1.02	1.08	0.98	1.06	1.09	0.91	0.96	0.87	0.9
Pred LR Relativity	1.2	1.07	1.05	1	0.96	0.96	0.96	0.92	0.93	0.94
Lower Limit	-5.03	-1.31	-0.73	-0.29	0	0.26	0.49	0.72	0.93	1.2
Upper Limit	-1.31	-0.73	-0.29	0	0.26	0.49	0.72	0.93	1.2	3.17

This is a lift chart showing how financial stability affects the actual and predicted loss ratio relativities. It plots financial stability, which is a variable that takes into account both C and F Points. We grouped financial stability into ten categories for this chart, each with approximately the same premiums to make it easier to compare them. You can see that premium and account count are plotted as bar graphs next to each other and above them are two lines showing actual loss ratio relativity and predicted loss ratio relativity (bucket loss ratio divided by overall loss ratio) as affected by each bucket of financial stability. Since actual and predicted loss ratios relativities are close with only a few fluctuations both trending downwards, you can see that financial stability is an accurate predictor of loss ratio and thus is useful for our model. This downward trend indicates that policies with a low financial stability value tend to have higher loss ratios and vice versa; therefore there is business value that can be gained by looking at the financial stability value of a policy.

2. Policy Type Loss Ratio Relativities

Actual versus Predicted LR Relativities by Policy Type

	PP	LP	MH	Other
Account count	18,401	41,043	23,075	12,654
Premium	$5,464.66	$18,924.83	$24,593.18	$13,472.53
Act LR Relativity	1.03	0.98	0.98	1.06
Pred LR Relativity	1.02	0.98	0.94	1.14
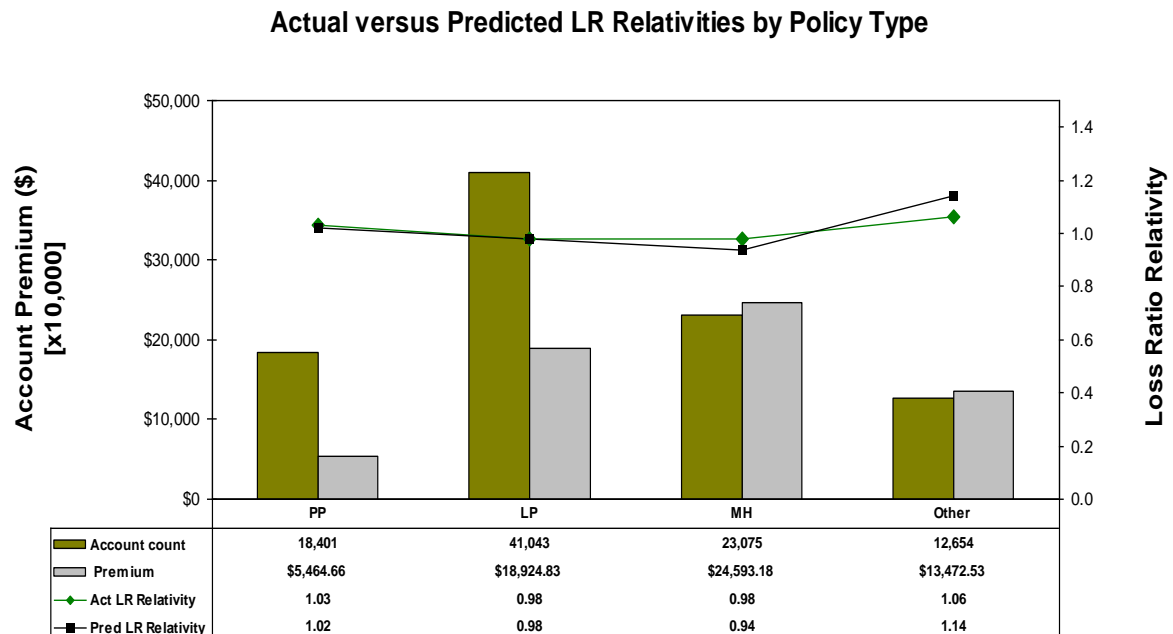
This lift chart shows the actual and predicted loss ratio relativities by policy type. We have four categories of policy type in the chart. PP represents policies that only have private passenger type vehicles, LP represents those that contain private passenger and light truck vehicles, and MH represents policies with private passenger, light truck, or medium/heavy vehicles with at least 25% of total vehicles being medium/heavy. "Other" represents all policy types not contained in the other three categories. The actual and predicted loss ratio relativities are closely matched with very minor fluctuation, so policy type is a valid predictor of loss ratio. The "Other" category has the highest relative loss ratio and is therefore riskier than the other categories, with PP being the second riskiest relative to the other three categories of policy type.

3. Business Type Loss Ratio Relativities

Actual versus Predicted LR Relativities by Business Type

	Corporation	Partnership	Sole Proprietorship	Other
Account count	73,435	5,859	11,274	4,605
Premium	$55,383.04	$2,846.00	$2,747.89	$1,478.26
Act LR Relativity	1.01	0.96	0.86	0.91
Pred LR Relativity	1.01	0.94	0.9	1.03
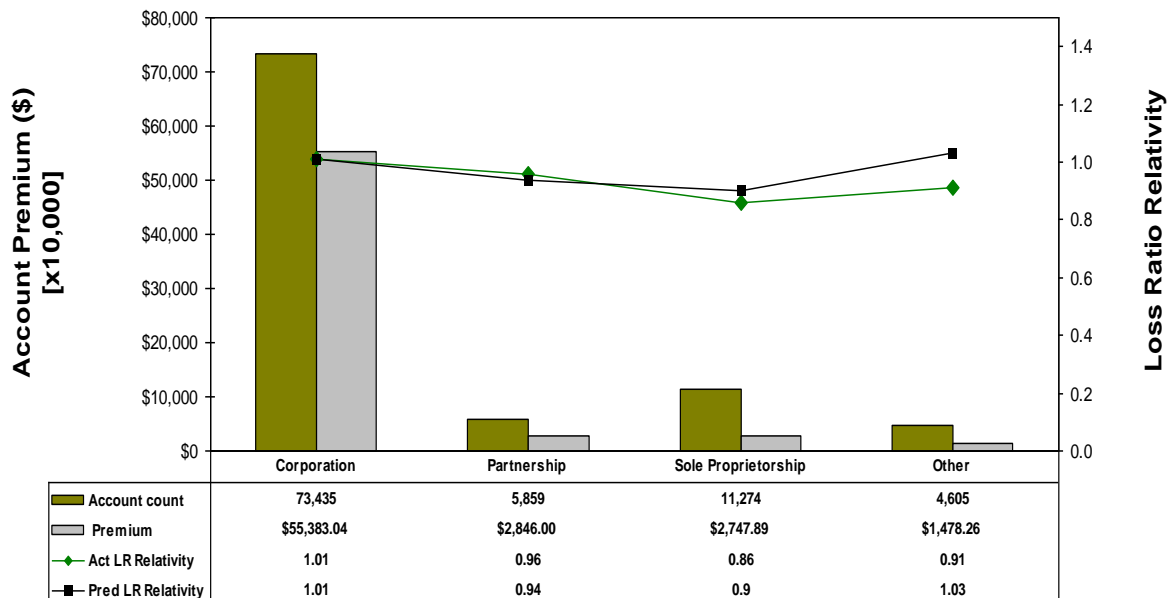
Here is a lift chart showing the actual versus predicted loss ratio relativities by business type. Business type indicates the ownership structure of the company, with the vast majority of both policies and premiums falling under the corporation category. The "Other" category has comparable predicted loss ratio relativity, but the actual is lower than for corporation. A possible reason for this significant difference in the actual and predicted loss ratio relativities is that there are fewer policies to model on. Because the actual and predicted loss ratio relativities are so close to each other, with them actually being identical for corporations, business type was shown to be a useful predictor of loss ratio.

4. Market Segment Loss Ratio Relativities

Actual versus Predicted LR Relativities by Market Segment

	SM	MM
Account count	67,476	27,697
Premium	$23,374.82	$39,080.38
Act LR Relativity	0.88	1.07
Pred LR Relativity	0.86	1.09
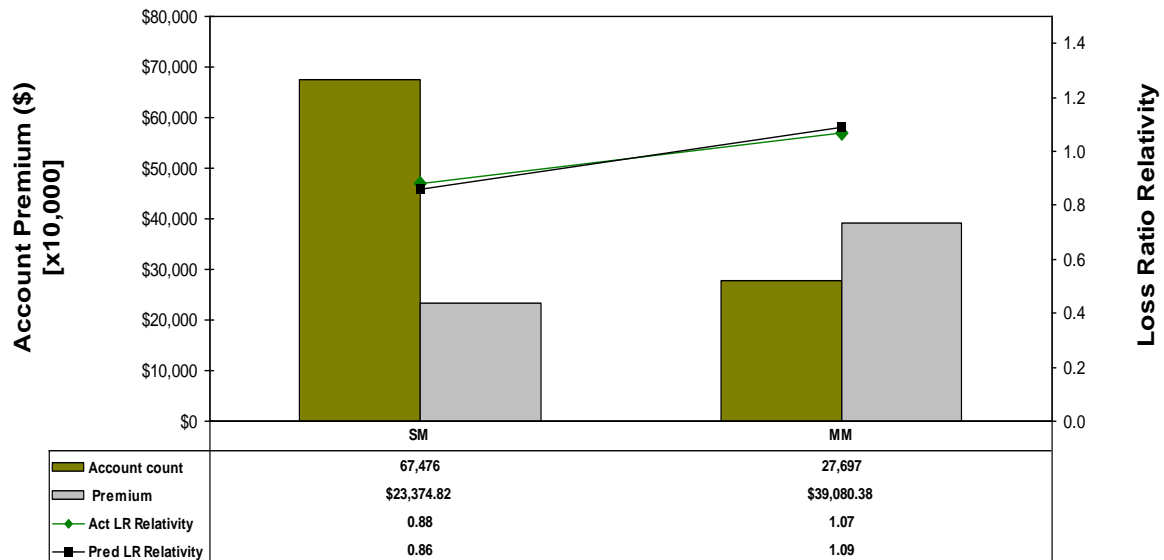
This lift chart shows the actual versus predicted loss ratio relativity by market segment, which is a Hanover-specific distinction based on the size and complexity of the specific account. There are two market segments: small commercial (SM) and middle market (MM). The predicted loss ratio relativity and actual loss ratio relativity were very closely matched and both increased from small market to middle market accounts. Thus, we determined that market segment is a good indicator of loss ratio and would be useful in our model.

5. Fleet Size Loss Ratio Relativities

Actual versus Predicted LR Relativities by Fleet Size

	0	1	2	3	4	5	6	7	8	9	≥ 10
Account count	1,485	26,157	17,626	11,900	8,547	6,007	4,313	3,224	2,517	2,000	11,397
Premium	$89.33	$3,658.74	$4,341.88	$4,273.81	$4,156.83	$3,929.52	$3,423.57	$2,919.19	$2,891.27	$2,605.45	$30,165.60
Act LR Relativity	0.36	0.67	0.84	1.01	1	0.93	0.93	0.93	0.95	0.87	1.1
Pred LR Relativity	0.28	0.65	0.81	0.91	0.95	0.93	0.97	1	1.01	1.05	1.1
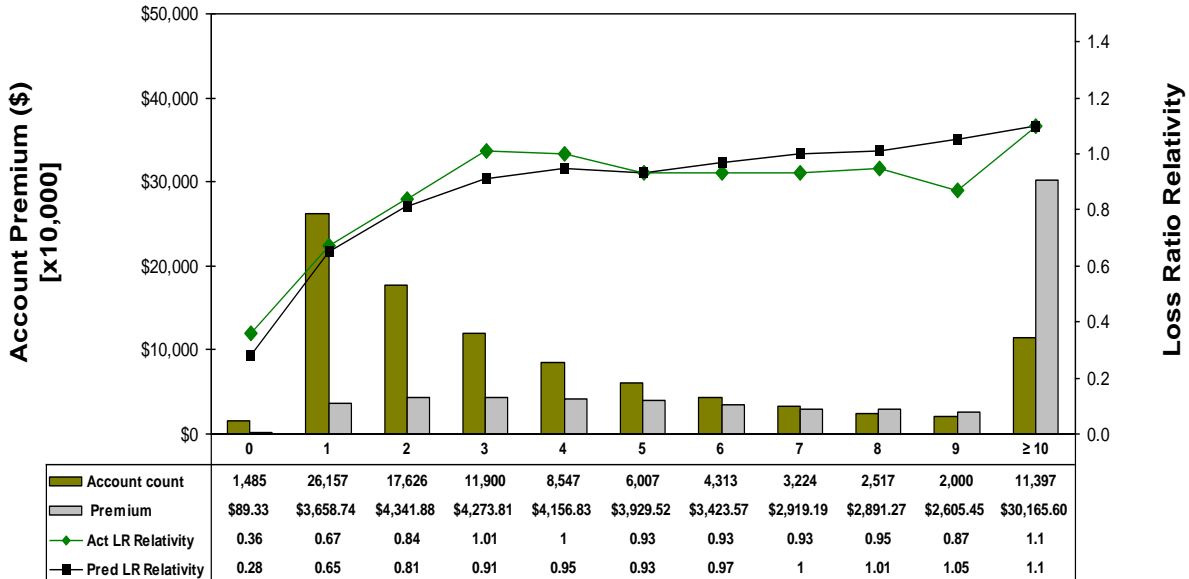
This lift chart shows the actual and predicted loss ratio relativities by fleet size. Fleet size is the number of powered motor vehicles on a policy, with a value of ten given to a policy with ten or more power units. A value of zero is given to a policy that may contain motor vehicles but which does not own any, such as a garage. When we first made a lift chart for fleet size, the actual loss ratio relativity had a non-linear plot, but the predicted loss ratio relativity was linear. We therefore had to try multiple things to get the predicted loss ratio to fit with the actual. Ultimately we used four variables with fleet size in our model. These were fleet size, (fleet size)^2, ln(fleet size + 1), and an interaction between fleet size and policy type. The combination of these predictors allowed the model to pick up on the non-linear aspects and become a much better predictor. In addition, there is some fluctuation between the actual and predicted loss ratios, in particular for fleet sizes between 0 and 3, the predicted loss ratio relativity underestimate the actual loss ratios, whereas from 6 to 9, the predicted relativities overestimate the actual loss ratio relativities.

6. Policy Effective Age Loss Ratio Relativities

Actual versus Predicted LR Relativities by Policy Effective Age

	1	2	3	4	5	6	7
Account count	5,575	18,403	16,830	16,694	16,118	15,416	6,137
Premium	$3,444.20	$12,007.22	$11,209.58	$11,209.22	$10,923.88	$10,065.32	$3,595.78
Act LR Relativity	0.12	0.7	1.13	1.13	1.2	1.1	1.19
Pred LR Relativity	0.14	0.77	1.12	1.11	1.16	1.1	1.12
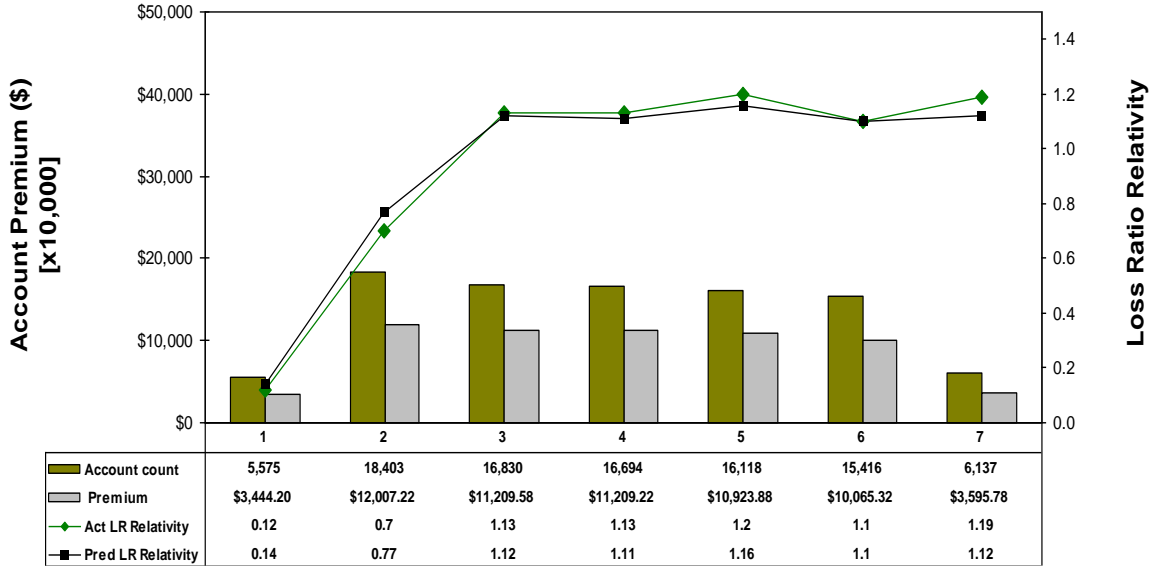
The window for claims is 36 months; therefore recent policies will have fewer claims resulting in lower loss ratios. This is reflected by looking into the Policy Effective Age. Policy Effective Age is a variable we created defined as the current year minus the Policy Effective Year. It is a better indicator than Policy Effective Year because it shows how old the policy is rather than what year it became effective and remains applicable as the current year changes. This results in a more predictive positive correlation between Policy Effective Age and the actual and predicted loss ratio relativities, as shown in the lift chart.

7. C-Points Loss Ratio Relativities

Actual versus Predicted LR Relativities by C-Points

	1	2	3	4	5	6	7	8	9	10
Account count	9,594	8,939	9,869	8,793	11,434	10,437	9,551	9,642	9,716	7,198
Premium	$6,213.14	$6,141.37	$6,376.25	$5,577.17	$6,475.39	$6,446.94	$6,075.53	$6,474.84	$6,294.70	$6,379.87
Act LR Relativity	1.1	1.12	1.24	0.9	0.97	0.97	0.95	0.97	0.88	0.9
Pred LR Relativity	1.2	1.09	1.04	1.03	0.99	0.95	0.96	0.92	0.93	0.91
Lower Limit	101	369	412	435	451	465	478	490	503	519
Upper Limit	369	412	435	451	465	478	490	503	519	655
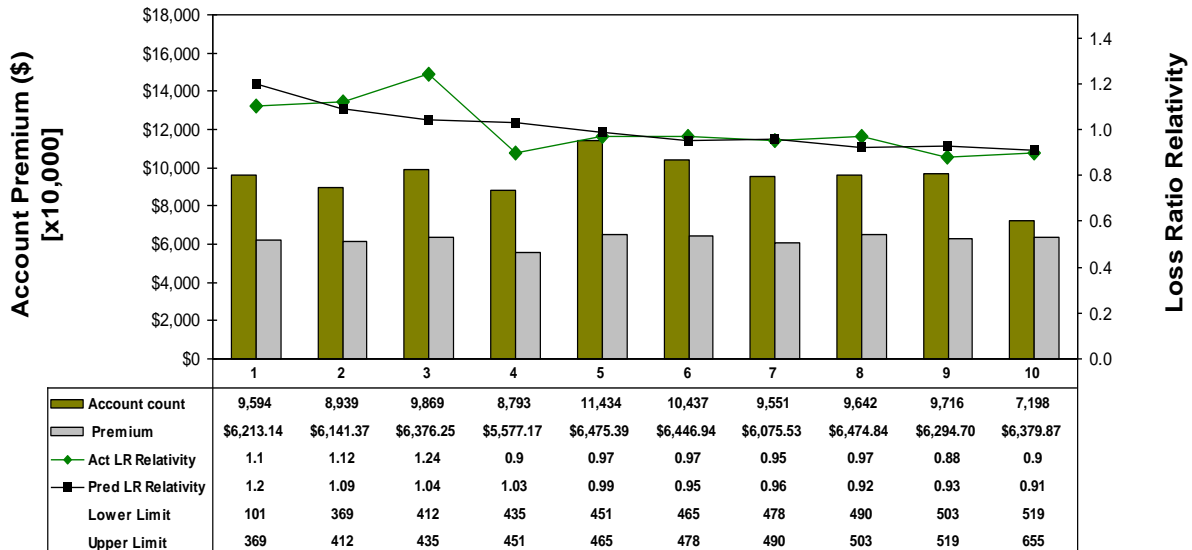
This lift chart shows how effective C-Points are in predicting Loss Ratio. The C-Point variable is an indicator of future severe delinquency. The scores range from 101-670, where a 101 represents the highest probability of severe delinquency, with the marginal odds of being good doubling for each 40 point increase. Thus, a score of 150 is twice as risky as a score of 190. You can see a downward trend in loss ratio as C-Points increase, which is to be expected since higher scores are given to less risky businesses. C-Points are a good predictor of loss ratio as shown in this chart because there is only a little difference between actual and predicted loss ratio relativities. There is some fluctuation as is to be expected from random sampling, but not enough to cause concern.

8. F-Points Loss Ratio Relativities

Actual versus Predicted LR Relativities by F-Points

	1	2	3	4	5	6	7	8	9	10
Account count	10,784	10,265	10,665	9,631	9,442	10,139	6,510	12,467	9,038	6,232
Premium	$6,026.29	$6,403.25	$6,188.18	$6,095.60	$6,058.55	$6,644.52	$4,436.31	$7,732.18	$6,298.98	$6,571.32
Act LR Relativity	1.11	1.04	1.02	1.04	1.06	1.07	0.99	0.89	0.89	0.93
Pred LR Relativity	1.15	1.09	1.02	1.03	0.97	0.94	0.97	0.92	0.96	0.97
Lower Limit	1,198	1,410	1,450	1,473	1,493	1,510	1,526	1,538	1,554	1,579
Upper Limit	1,410	1,450	1,473	1,493	1,510	1,526	1,538	1,554	1,579	1,827

This lift chart shows the predictability of F-Points. F-Points represent the risk of future financial stress. It is a score from 1,001-1,875, where 1,001 represents businesses with the highest chance of future financial stress. Similar to C-Points, the marginal odds of being good doubles with each 40 point increase. F-Points were a slightly better predictor of loss ratio, with less of a difference between the actual and predicted loss ratio relativities. The slight fluctuations we see are to be expected due to random sampling and appear to a lesser extent than those in the C-Points lift chart. In this case, a higher F-Score corresponds with a lower loss ratio, which is intuitive since a higher F-Score represents less of a chance of future financial stress.

CONCLUSIONS AND RECOMMENDATIONS

With the recent integration of credit score information, insurance companies within personal and commercial lines have changed rating methods to incorporate this information. This rapid transition has occurred because these insurance companies understand the value that credit scores have in better segmenting risk types of customers. For our project, we analyzed the current underwriting and rating techniques used by the Hanover group and determined how to best implement credit score information. We further developed a generalized linear model for their Commercial Auto and Business Owners Policy (BOP) groups to predict incurred loss ratio. Based on this incurred loss ratio, we grouped policies into different risk buckets and compared the actual incurred loss ratio to the predicted incurred loss ratio produced by the model. Looking at the results of this analysis we came to three main conclusions:

The credit variable, Financial Stability, when combined with additional company specific data, is a powerful predictor of future incurred loss ratio of a policy. Through our lift chart analysis, we were able to see that polices with lower financial stability scores have higher loss ratios and vice versa. Therefore this credit variable has predictive value which will enable Hanover to make more informed business decisions when underwriting.

Employing techniques that use credit information will allow for better differentiation of risk, ultimately improving underwriting profit. Due to the fact that the credit variable does have powerful predictive value, usage in the early stages of pricing and underwriting will enable Hanover to be more informed about the risk type of a customer on the outset. This will ensure that risky policies are priced an appropriately high premium and high risk policies that may have high losses in the future are rejected from the outset as they are just too risky to be underwritten.

Usage of credit scores will ensure that Hanover's underwriting and pricing techniques are competitive and more advanced. The usage of credit information in the insurance industry is still in its developing stages. Therefore, Hanover's ability to harness and strategically implement credit information can attract more desirable customers. If Hanover is able to price low risk customers a better rate than competitors because of credit information, they will attract more and more customers with low probability of future loss. This is the type of customer that is ideal for an insurance company and will enable them to maintain and increase profit while minimizing the underwriting of high risk policies.

In addition to these conclusions, we outlined several recommendations that could be used to improve the usability of our model in the future:

Our first recommendation is to use lift charts for financial stability scores to identify major tier groupings in the predicted loss ratio relativities. The loss ratios for each tier should be calculated by summing predicted losses for each policy and dividing by the sum of the rerated premiums. The predicted loss is therefore determined by multiplying the predicted loss ratio from our model output and multiplying that by the rerated premium. These should then be rearranged numerically based on this loss ratio relativity. This will help us to understand the average level of risk in each tier and that could indicate an ideal place where a break in risk tiers could be made. Based on this, we plan to identify range of loss ratios that seems to represent the average and assign that with a credit factor of one. From there we will rank other groupings in comparison to that average so that a tier with a credit factor above one would indicate a high risk policy and those below one would indicate low-good risk policies.

This assigned credit factor would be applied to pricing and would be a way of indicating the future level of risk of a policy holder indicated by their credit score information.

However, there will be an issue of dealing with policies with no credit information (no-hits), to handle this we suggest two recommendations. For one option, we would assign these policies a credit factor of 1 indicating that credit information should have no bearing on their premium as they do not have any information. The second possibility is to take the average loss ratios for all no-hits and determine an appropriate credit factor range it could fall into based on our data from our credit factors for the policies with credit information.

We also recommend that high-risk policies be flagged so that agents can determine whether they should be outright denied or if they should go to an underwriter for further investigation. In order to accomplish this flagging, a threshold for risky loss ratios needs to be determined to differentiate these customers based on our predictive analysis of their risk position. This step will primarily consist of analyzing and testing the model results against actual incurred losses. This will allow us to identify the general range and extent of customer losses which will inform any threshold set for extremely high risk polices. The business will also have to use business judgment to determine the appropriate steps that should be taken for these high risk policies, that is whether they should be underwritten or whether they need underwriter approval. In this way the model would allow for automation and ensure that those high risk polices are identified early on at which point the business has to determine the necessary actions suitable for that in order to underwrite that business customer. These recommendations will greatly improve the effectiveness of the model and ultimately ensure that the underwriting and pricing process is streamlined and able to produce concrete results in minimal time.

APPENDIX A: GLOSSARY

ANOVA (Analysis of Variance) — is a statistical analysis to measure the significance of each input variable to the overall model fit.

Data-mining — is the process of extracting predictive information from large databases.

Data capping — is the process by which extreme values in the data are modified or adjusted to smaller values in order to improve the data quality.

Commercial Line — refers to provision on insurance for businesses and corporations as opposed to individual consumers.

Credit score — is a numeric value developed by using statistical methods used to represent a customer's level of credit worthiness or ability to repay financial obligations.

Exposure — is the likelihood or probability that an entity will be in a situation that may have detrimental effects or negative influences (in other words risky situations).

Goodness of Fit — describes measures used to test how well a model is able to fit the data set to produce accurate results. This is generally measured by looking at the difference between actual and predicted values of the model, if there is a small fit then the model is a good fit.

Lift Chart — are graphs used to assess the performance and effectiveness of the statistical model developed. For out project, these charts serve primarily as a visual representation of the effectiveness of the model in predicting the loss ratio.

Loss Ratio — is calculated by dividing incurred loss by premium.

Multi-Collinearity — is a situation in which more than one variable are correlated with the other variables.

No-hits — are policies with no credit information provided, generally seen with new businesses that have little financial information.

Over-fitting- is a situation when the model incorporates patterns present in the sample data that are not present in the overall population. This means that the model is able to produce acceptable results for a given data set, but when the data set is changed results may not be reliable.

Premium— is the money charged by insurance companies for coverage.

Pricing- is the process of assigning a policy holder a premium or monetary charge. This premium is the cost the policy holder pays in exchange for coverage by the insurance company in the case of an occurrence of an event stipulated under their insurance agreement.

Property and Casualty insurance— is a type of insurance concerned with the protection against legal liabilities and losses caused from damages to persons or their properties.

Underwriting— is process by which an insurance company assesses the risk associated with a customer and determines where or not the company is willing to take on or insure that risk.

APPENDIX B: PROCESS OVERVIEW CHART
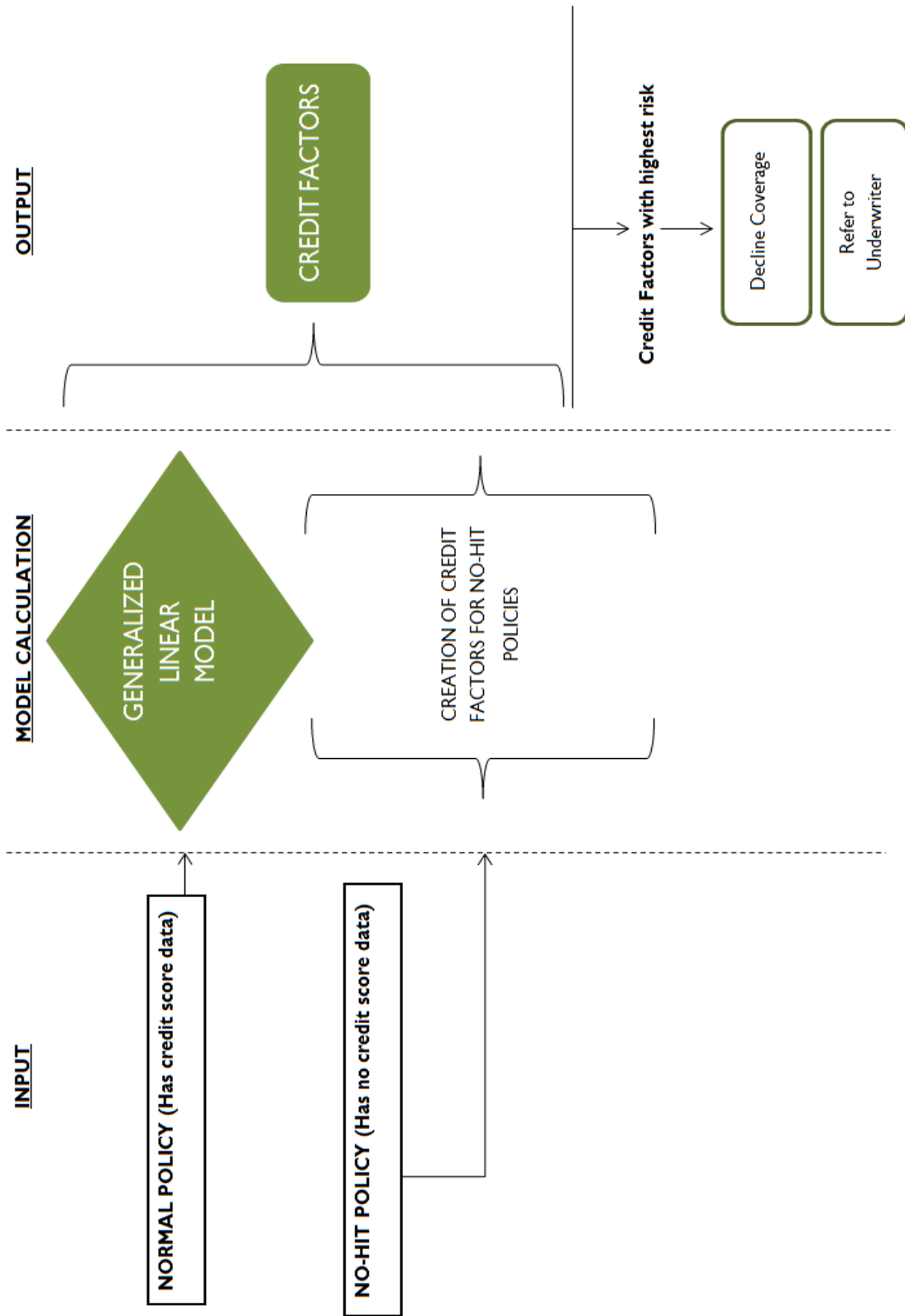
APPENDIX C: COMMERCIAL AUTO CHARTS

Financial Stability Lift Charts

Build Sample

Actual versus Predicted LR Relativities by Financial Stability

	1	2	3	4	5	6	7	8	9	10
Account count	7,880	8,518	7,993	8,348	8,287	7,900	7,452	7,710	7,322	4,727
Premium	$5,007.77	$5,009.78	$4,998.51	$5,027.92	$4,993.79	$5,014.25	$5,009.04	$5,007.46	$5,007.91	$5,010.24
Act LR Relativity	1.13	1.04	1.09	0.97	1.07	1.04	0.94	0.95	0.86	0.91
Pred LR Relativity	1.2	1.08	1.06	1	0.96	0.96	0.97	0.91	0.92	0.94
Lower Limit	-4.84	-1.3	-0.72	-0.29	0	0.27	0.5	0.72	0.94	1.21
Upper Limit	-1.3	-0.72	-0.29	0	0.27	0.5	0.72	0.94	1.21	3.17

Validation Sample

Actual versus Predicted LR Relativities by Financial Stability

	1	2	3	4	5	6	7	8	9	10
Account count	1,975.00	2,115.00	1,906.00	2,041.00	1,973.00	1,880.00	1,951.00	2,037.00	1,886.00	1,272.00
Premium	$1,231.17	$1,231.32	$1,248.05	$1,236.48	$1,237.40	$1,236.56	$1,235.29	$1,230.72	$1,235.52	$1,246.01
Act LR Relativity	1.16	0.94	1.02	1.09	0.94	1.18	0.95	0.96	0.95	0.83
Pred LR Relativity	1.17	1.08	1.04	1	0.98	0.98	0.94	0.93	0.94	0.95
Lower Limit	-4.58	-1.32	-0.73	-0.31	-0.03	0.22	0.45	0.69	0.91	1.18
Upper Limit	-1.32	-0.73	-0.31	-0.03	0.22	0.45	0.69	0.91	1.18	3.02

Policy Type Lift Charts

Build Sample
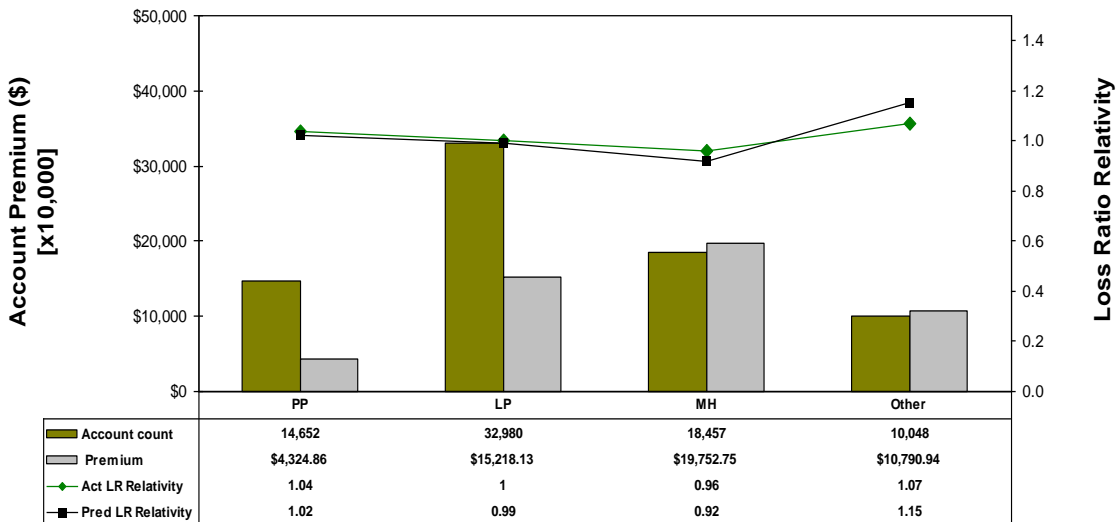
Actual versus Predicted LR Relativities by Policy Type

	PP	LP	MH	Other
Account count	14,652	32,980	18,457	10,048
Premium	$4,324.86	$15,218.13	$19,752.75	$10,790.94
Act LR Relativity	1.04	1	0.96	1.07
Pred LR Relativity	1.02	0.99	0.92	1.15

Validation Sample

Actual versus Predicted LR Relativities by Policy Type

	PP	LP	MH	Other
Account count	3,749	8,063	4,618	2,606
Premium	$1,139.80	$3,706.70	$4,840.42	$2,681.59
Act LR Relativity	1.02	0.89	1.06	1.03
Pred LR Relativity	1.03	1	0.93	1.11
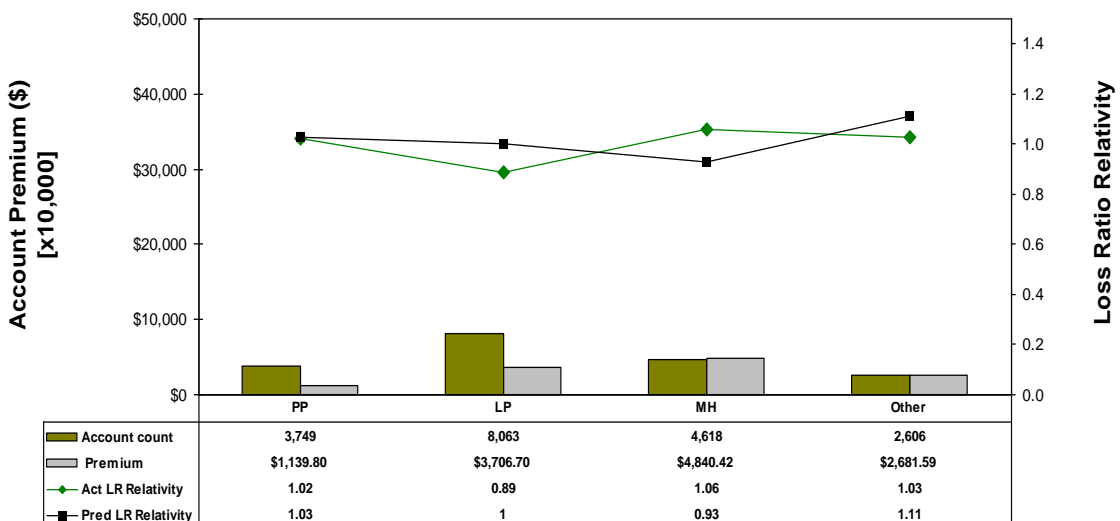
Business Type Lift Charts

Build Sample
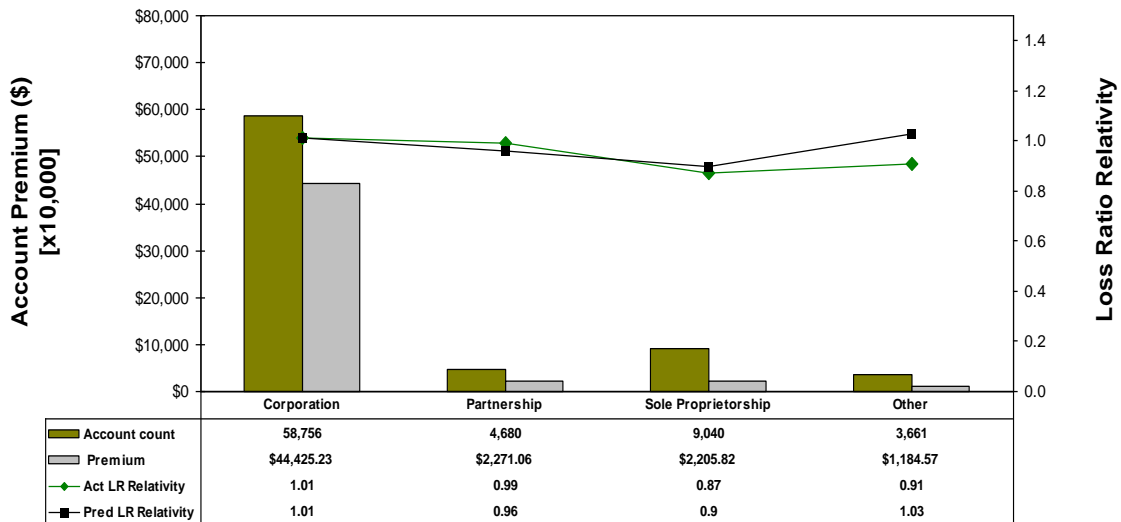
Actual versus Predicted LR Relativities by Business Type

	Corporation	Partnership	Sole Proprietorship	Other
Account count	58,756	4,680	9,040	3,661
Premium	$44,425.23	$2,271.06	$2,205.82	$1,184.57
Act LR Relativity	1.01	0.99	0.87	0.91
Pred LR Relativity	1.01	0.96	0.9	1.03

Validation Sample

Actual versus Predicted LR Relativities by Business Type

	Corporation	Partnership	Sole Proprietorship	Other
Account count	14,679	1,179	2,234	944
Premium	$10,957.81	$574.94	$542.07	$293.70
Act LR Relativity	1.02	0.83	0.85	0.91
Pred LR Relativity	1	0.97	0.93	1.1
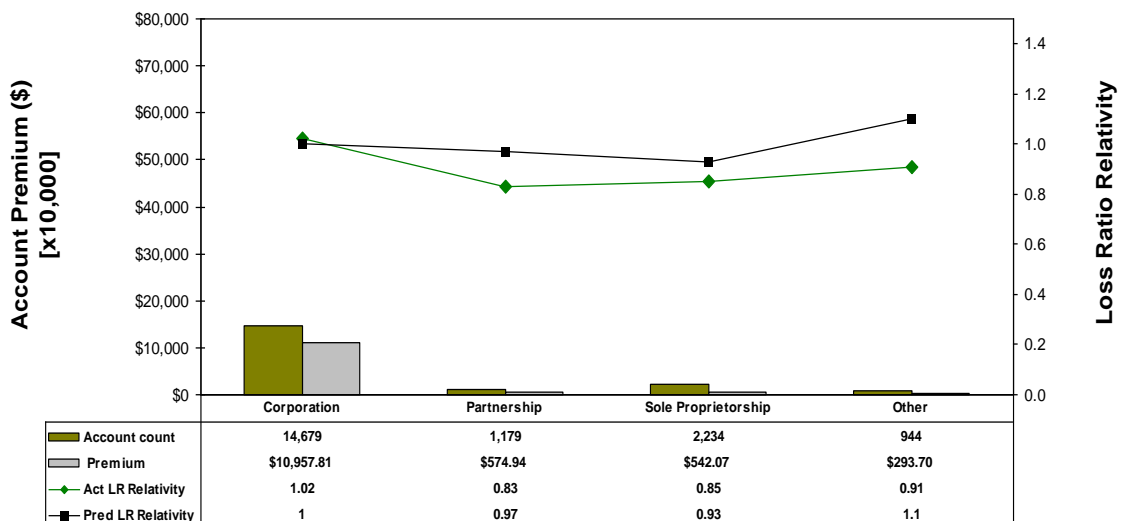
Market Segment Lift Charts

Build Sample
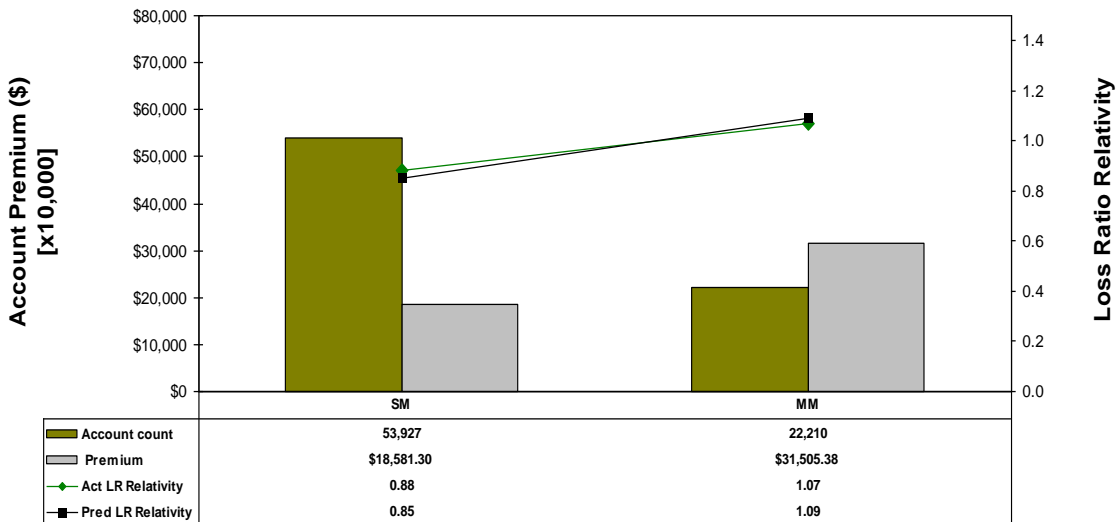
Actual versus Predicted LR Relativities by Market Segment

	SM	MM
Account count	53,927	22,210
Premium	$18,581.30	$31,505.38
Act LR Relativity	0.88	1.07
Pred LR Relativity	0.85	1.09

Validation Sample

Actual versus Predicted LR Relativities by Market Segment

	SM	MM
Account count	13,549	5,487
Premium	$4,793.52	$7,575.00
Act LR Relativity	0.87	1.08
Pred LR Relativity	0.86	1.09
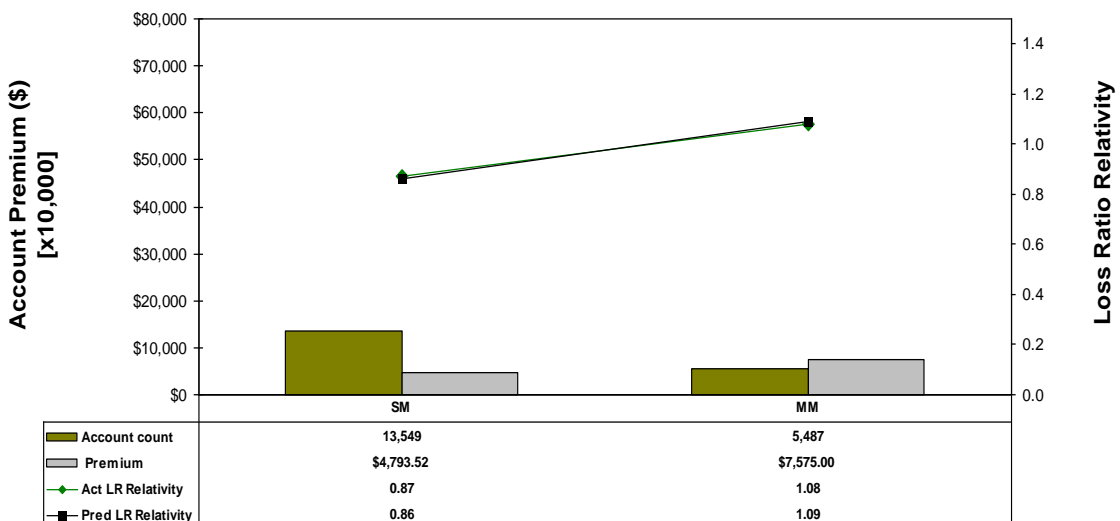
Fleet Size Lift Charts

Build Sample

Actual versus Predicted LR Relativities by Fleet Size

	0	1	2	3	4	5	6	7	8	9	≥ 10
Account count	1,163	20,948	14,106	9,515	6,805	4,810	3,484	2,540	1,995	1,623	9,148
Premium	$70.40	$2,928.44	$3,453.92	$3,432.91	$3,299.99	$3,157.91	$2,767.78	$2,293.97	$2,278.07	$2,105.29	$24,297.99
Act LR Relativity	0.35	0.68	0.84	1.04	0.98	0.93	0.92	0.91	0.98	0.83	1.1
Pred LR Relativity	0.27	0.65	0.81	0.9	0.95	0.93	0.97	1	1	1.04	1.1

Validation Sample

Actual versus Predicted LR Relativities by Fleet Size

	0	1	2	3	4	5	6	7	8	9	≥ 10
Account count	322	5,209	3,520	2,385	1,742	1,197	829	684	522	377	2,249
Premium	$18.93	$730.30	$887.97	$840.90	$856.84	$771.61	$655.79	$625.22	$613.19	$500.16	$5,867.62
Act LR Relativity	0.41	0.63	0.83	0.86	1.08	0.93	0.95	1	0.86	0.99	1.11
Pred LR Relativity	0.27	0.66	0.81	0.92	0.97	0.94	0.96	0.98	1.01	1.04	1.1

Policy Effective Age Lift Charts

Build Sample

Actual versus Predicted LR Relativities by Policy Effective Age

	1	2	3	4	5	6	7
Account count	4,451	14,666	13,487	13,385	12,904	12,360	4,884
Premium	$2,781.52	$9,605.59	$8,987.71	$8,968.34	$8,785.85	$8,109.44	$2,848.22
Act LR Relativity	0.13	0.71	1.16	1.12	1.15	1.12	1.2
Pred LR Relativity	0.15	0.77	1.13	1.1	1.14	1.1	1.16

Validation Sample

Actual versus Predicted LR Relativities by Policy Effective Age

	1	2	3	4	5	6	7
Account count	1,124	3,737	3,343	3,309	3,214	3,056	1,253
Premium	$662.68	$2,401.63	$2,221.87	$2,240.88	$2,138.03	$1,955.88	$747.56
Act LR Relativity	0.08	0.66	1.01	1.16	1.4	1.03	1.15
Pred LR Relativity	0.15	0.78	1.12	1.1	1.14	1.11	1.13

C-Points Lift Charts

Build Sample

Actual versus Predicted LR Relativities by C-Points

	1	2	3	4	5	6	7	8	9	10
Account count	7,581	7,471	7,600	7,936	9,000	7,637	8,133	7,175	7,783	5,821
Premium	$4,939.31	$5,043.44	$4,860.10	$5,034.92	$5,069.00	$4,738.45	$5,298.24	$4,852.65	$5,048.97	$5,201.58
Act LR Relativity	1.09	1.14	1.26	0.91	0.96	0.98	0.95	0.99	0.83	0.91
Pred LR Relativity	1.2	1.1	1.04	1.02	0.99	0.95	0.95	0.92	0.93	0.91
Lower Limit	101	368	413	435	452	466	478	491	503	519
Upper Limit	368	413	435	452	466	478	491	503	519	655

Validation Sample

Actual versus Predicted LR Relativities by C-Points

	1	2	3	4	5	6	7	8	9	10
Account count	1,944	1,669	1,774	1,987	2,456	2,140	1,846	1,793	1,970	1,455
Premium	$1,244.59	$1,258.69	$1,143.50	$1,239.97	$1,382.58	$1,250.61	$1,228.24	$1,285.17	$1,249.11	$1,278.49
Act LR Relativity	1.19	0.95	1.11	1.09	0.87	0.98	0.93	1.05	0.95	0.91
Pred LR Relativity	1.19	1.14	1.04	1	0.97	0.95	0.96	0.92	0.92	0.91
Lower Limit	101	368	410	433	451	466	479	491	503	519
Upper Limit	368	410	433	451	466	479	491	503	519	628

F-Points Lift Charts

Build Sample

Actual versus Predicted LR Relativities by F-Points

	1	2	3	4	5	6	7	8	9	10
Account count	8,889	7,949	8,538	7,669	7,569	8,234	4,998	10,016	7,562	4,713
Premium	$4,990.62	$4,963.43	$4,951.42	$4,866.05	$4,837.51	$5,434.85	$3,441.96	$6,199.23	$5,362.14	$5,039.48
Act LR Relativity	1.13	1	1.04	1.05	1.05	1.05	1.02	0.89	0.86	0.95
Pred LR Relativity	1.15	1.09	1.02	1.02	0.97	0.94	0.97	0.92	0.97	0.97
Lower Limit	1,210	1,411	1,450	1,473	1,493	1,510	1,527	1,538	1,554	1,580
Upper Limit	1,411	1,450	1,473	1,493	1,510	1,527	1,538	1,554	1,580	1,827

Validation Sample

Actual versus Predicted LR Relativities by F-Points

	1	2	3	4	5	6	7	8	9	10
Account count	2,165	1,997	2,108	2,030	1,873	1,845	1,572	2,420	1,788	1,238
Premium	$1,219.85	$1,209.76	$1,224.74	$1,287.48	$1,221.04	$1,136.68	$1,067.35	$1,509.81	$1,241.77	$1,250.05
Act LR Relativity	1.06	1.14	0.92	0.98	1.09	1.07	1.01	0.89	1.01	0.87
Pred LR Relativity	1.13	1.08	1.01	1.03	0.99	0.95	0.94	0.94	0.97	0.97
Lower Limit	1,198	1,410	1,449	1,472	1,493	1,510	1,524	1,538	1,553	1,578
Upper Limit	1,410	1,449	1,472	1,493	1,510	1,524	1,538	1,553	1,578	1,799

Financial Stability Lift Charts

Build Sample

Actual versus Predicted LR Relativities by Financial Stability

	1	2	3	4	5	6	7	8	9	10
Account count	7,880	8,518	7,993	8,348	8,287	7,900	7,452	7,710	7,322	4,727
Premium	$5,007.77	$5,009.78	$4,998.51	$5,027.92	$4,993.79	$5,014.25	$5,009.04	$5,007.46	$5,007.91	$5,010.24
Act LR Relativity	1.13	1.04	1.09	0.97	1.07	1.04	0.94	0.95	0.86	0.91
Pred LR Relativity	1.2	1.08	1.06	1	0.96	0.96	0.97	0.91	0.92	0.94
Lower Limit	-4.84	-1.3	-0.72	-0.29	0	0.27	0.5	0.72	0.94	1.21
Upper Limit	-1.3	-0.72	-0.29	0	0.27	0.5	0.72	0.94	1.21	3.17

Validation Sample

Actual versus Predicted LR Relativities by Financial Stability

	1	2	3	4	5	6	7	8	9	10
Account count	1,975.00	2,115.00	1,906.00	2,041.00	1,973.00	1,880.00	1,951.00	2,037.00	1,886.00	1,272.00
Premium	$1,231.17	$1,231.32	$1,248.05	$1,236.48	$1,237.40	$1,236.56	$1,235.29	$1,230.72	$1,235.52	$1,246.01
Act LR Relativity	1.16	0.94	1.02	1.09	0.94	1.18	0.95	0.96	0.95	0.83
Pred LR Relativity	1.17	1.08	1.04	1	0.98	0.98	0.94	0.93	0.94	0.95
Lower Limit	-4.58	-1.32	-0.73	-0.31	-0.03	0.22	0.45	0.69	0.91	1.18
Upper Limit	-1.32	-0.73	-0.31	-0.03	0.22	0.45	0.69	0.91	1.18	3.02

APPENDIX D: BUSINESS OWNER'S POLICY CHARTS

After we completed the work on Commercial Auto, we began the modeling process again with another line, Business Owners Policy (BOP). BOP focuses on insuring small businesses that fit a specific criteria. It was more complex to model than Commercial Auto due to having more predictive variables. BOP has more credit variables. In addition to C Points and F Points, there were liens, suits, judgements, and legal status. It also has different variables in general because it's pertaining to businesses and losses associated with buildings, contents, and liability rather than vehicles.

Due to BOP's complex nature, we ended up creating a model that took into account 22 different variables. These included C Points, F Points, Program Name, Region, Construction Type, Property Limit, Legal Status, Property Deductible, New/Renew, Financial Stability, Control Age, Effective Age, Limit Indicator, Location Count, and Protection Group. The next section depicts the full lift charts that were done for the variables.

Actual versus Predicted LR Relativities by CPoints

	1	2	3	4	5	6	7	8	9	10
Policy Count	17,556	16,741	17,219	19,979	20,581	19,384	18,181	18,528	16,197	14,514
Premium	$5,958	$6,225	$5,810	$6,217	$6,175	$5,990	$5,990	$6,263	$5,791	$6,530
Act LR Relativity	1.32	1.17	0.93	1.12	0.91	0.9	0.9	0.82	0.91	1.01
Pred LR Relativity	1.35	1.12	1.05	0.96	0.93	0.93	0.95	0.91	0.9	0.92
Lower Limit	101	369	411	433	450	462	473	484	497	511
Upper Limit	369	411	433	450	462	473	484	497	511	670

Actual versus Predicted LR Relativities by FPoints

	1	2	3	4	5	6	7	8	9	10
Policy Count	17,726	19,782	17,668	19,655	15,700	20,905	15,558	21,849	17,634	12,403
Premium	$5,773	$6,354	$6,122	$6,090	$5,598	$6,333	$5,462	$6,865	$6,232	$6,120
Act LR Relativity	1.19	1.11	1.04	1.12	1.13	0.88	0.87	0.88	0.84	0.97
Pred LR Relativity	1.32	1.11	1.08	0.99	0.99	0.95	0.89	0.88	0.85	0.97
Lower Limit	1,182	1,408	1,439	1,463	1,481	1,500	1,516	1,530	1,546	1,560
Upper Limit	1,408	1,439	1,463	1,481	1,500	1,516	1,530	1,546	1,560	1,783

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Actual versus Predicted LR Relativities by Financial Stability

	1	2	3	4	5	6	7	8	9	10
Policy Count	17,910	18,911	17,305	19,831	18,546	20,110	18,311	17,960	17,341	12,655
Premium	$6,095	$6,095	$6,094	$6,096	$6,091	$6,099	$6,083	$6,104	$6,095	$6,097
Act LR Relativity	1.31	1.1	1.03	1.07	1	0.94	0.82	0.88	0.83	1.01
Pred LR Relativity	1.36	1.09	1.07	1	0.94	0.93	0.92	0.88	0.87	0.95
Lower Limit	-5.21	-1.28	-0.8	-0.34	-0.04	0.23	0.46	0.7	0.94	1.18
Upper Limit	-1.28	-0.8	-0.34	-0.04	0.23	0.46	0.7	0.94	1.18	3.37

Actual versus Predicted LR Relativities by Property Limit

	1	2	3	4	5	6	7	8	9	10
Policy Count	42,109	34,623	24,257	19,365	15,513	13,229	10,651	8,303	6,582	4,248
Premium	$6,095	$6,095	$6,095	$6,095	$6,095	$6,094	$6,096	$6,092	$6,097	$6,096
Act LR Relativity	0.75	0.9	0.95	0.96	0.89	1	1.04	1.2	1.3	1.02
Pred LR Relativity	0.72	0.82	0.87	0.92	0.97	1.03	1.05	1.09	1.16	1.38
Lower Limit	100	44,557	140,932	264,596	413,750	599,245	856,752	1,234,223	1,890,036	3,391,567
Upper Limit	44,557	140,932	264,596	413,750	599,245	856,752	1,234,223	1,890,036	3,391,567	28,121,600

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Actual versus Predicted LR Relativities by Control Age

	1	2	3	4	5	6	7	8	9	10
Policy Count	22,916	17,397	22,536	20,933	16,183	19,064	17,346	15,737	14,938	11,830
Premium	$6,491	$4,713	$6,288	$6,751	$5,334	$6,415	$6,525	$6,049	$6,287	$6,097
Act LR Relativity	1.15	0.68	1.03	1.01	1.04	1.07	0.95	0.94	1.04	1.01
Pred LR Relativity	1.05	0.74	0.99	1.07	1.06	1.01	1.02	0.99	1	1.01
Lower Limit	0	1	7	12	17	21	26	31	37	48
Upper Limit	0	7	12	17	21	26	31	37	48	277

Actual versus Predicted LR Relativities by Effective Age

	1	2	3	4	5	6
Policy Count	41,550	29,348	27,484	27,442	26,584	26,472
Premium	$13,806	$9,838	$9,320	$9,698	$9,315	$8,973
Act LR Relativity	0.4	1.13	1.13	1.25	1.19	1.17
Pred LR Relativity	0.44	1.1	1.1	1.23	1.22	1.17
Year	2011	2010	2009	2008	2007	2006

Actual versus Predicted LR Relativities by Region

	West	ME	MA	NH	LA	Southeast	MI	Midwest	NJ	CT	IL	NY	GA	OK	TX
Policy Count	5,254	9,923	19,533	6,012	4,407	16,873	34,412	9,993	11,425	6,930	9,111	24,858	5,658	7,598	6,893
Premium	$1,507	$2,792	$7,428	$2,114	$1,744	$5,503	$8,943	$2,862	$5,589	$2,258	$3,165	$9,592	$1,867	$2,853	$2,732
Act LR Relativity	0.44	0.76	0.69	0.96	1.3	0.96	1.1	1.15	0.89	1	1.04	1.15	1.11	1.14	1.24
Pred LR Relativity	0.49	0.7	0.76	0.82	0.87	0.93	0.98	1.01	1.03	1.05	1.06	1.18	1.21	1.24	1.44

Actual versus Predicted LR Relativities by Market Segment

	Commercial Small	Commercial Middle
Policy Count	173,107	5,773
Premium	$51,645	$9,304
Act LR Relativity	0.97	1.16
Pred LR Relativity	0.94	1.34

LR Relativities by Protection Group

	"1-2"	"3"	"4-6"	"7-10"
Policy Count	31,360	43,410	94,535	9,575
Premium (x 10,000)	$11,841	$14,513	$31,700	$2,895
Act LR Relativity	1.09	0.99	0.99	0.76
Pred LR Relativity	1.09	1.08	0.96	0.71

LR Relativities by Book Transfer Code

	None	BTX	OBI
Policy Count	165,514	4,315	9,051
Premium (x 10,000)	$56,371	$1,090	$3,488
Act LR Relativity	1.05	0.47	0.43
Pred LR Relativity	1.04	0.45	0.46

LR Relativities by Construction Type

	Fire Resistive	Non-Combustible	Masonry and Non-Combustible	Joisted	Frame
Policy Count	8,398	13,611	41,397	61,364	54,110
Premium (x 10,000)	$2,995	$6,170	$16,470	$20,141	$15,174
Act LR Relativity	1.05	0.91	1.24	1.03	0.72
Pred LR Relativity	1.23	1.05	1.26	0.98	0.68

LR Relativities by Legal Status

	No Data Available	Proprietorship	Partnership	Corporation
Policy Count	48,492	36,314	13,197	80,877
Premium (x 10,000)	$13,209	$8,343	$4,497	$34,900
Act LR Relativity	0.96	0.95	1.05	1.02
Pred LR Relativity	0.87	0.94	1.05	1.05

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LR Relativities by Location Count2

	1	> 1
Policy Count	148,619	30,261
Premium (x 10,000)	$40,102	$20,847
Act LR Relativity	0.9	1.2
Pred LR Relativity	0.87	1.25
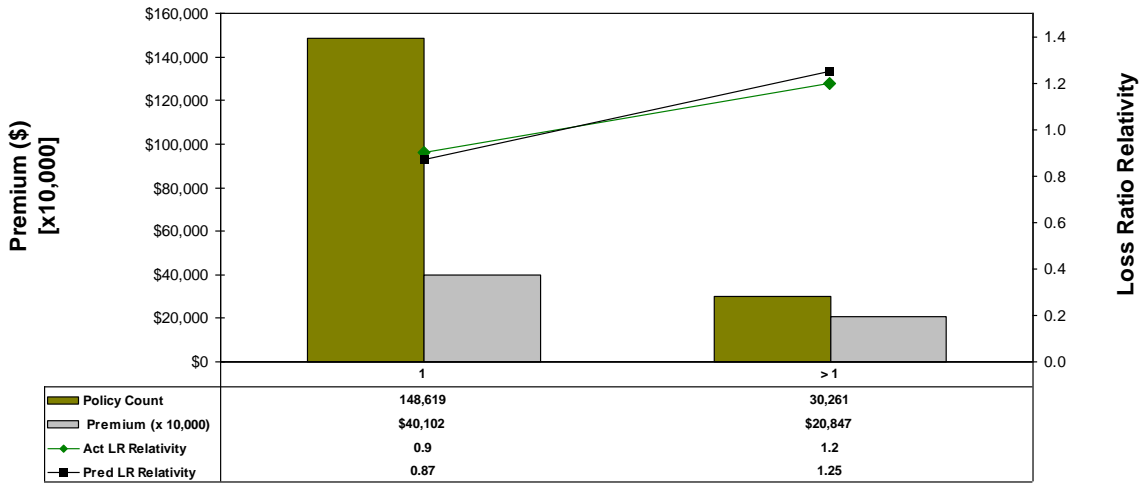
LR Relativities by New/Renew

	New	Renew
Policy Count	35,247	143,633
Premium (x 10,000)	$12,165	$48,784
Act LR Relativity	1.05	0.99
Pred LR Relativity	1.09	0.98
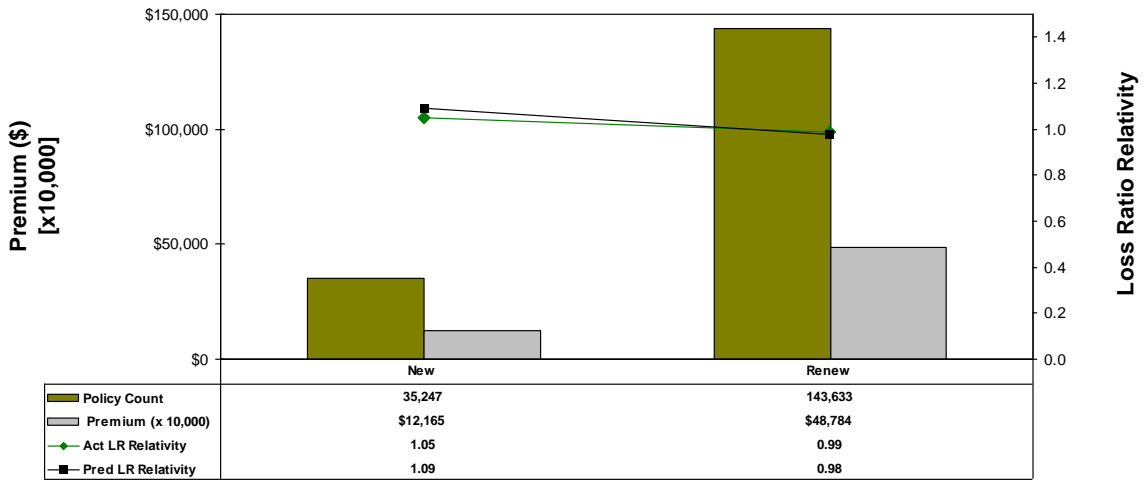
LR Relativities by Program Name

	Apartment & Condominium	Contractor	Manufacturer	Garage	Service	Office	Retail	Professional	Wholesale
Policy Count	3,038	13,288	12,628	3,793	24,670	61,849	32,366	14,706	12,542
Premium (x 10,000)	$2,765	$4,639	$6,059	$1,497	$7,675	$14,115	$14,475	$3,645	$6,079
Act LR Relativity	0.51	0.75	0.93	0.83	1.03	1.04	1.05	1.35	1.07
Pred LR Relativity	0.54	0.75	0.92	0.94	1	1.04	1.08	1.12	1.13
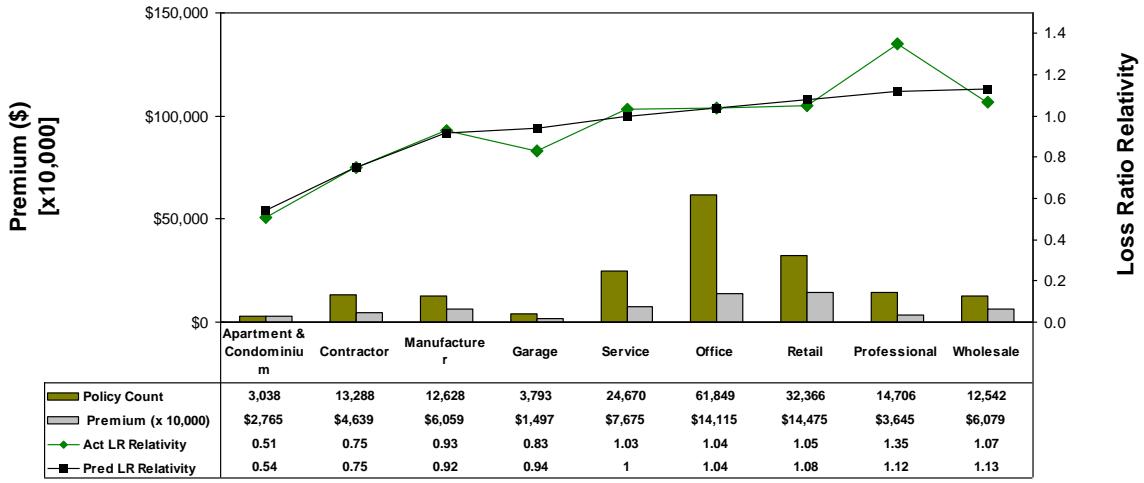
LR Relativities by Limit Indicator

	B	BC	C
Policy Count	22,084	53,111	103,685
Premium (x 10,000)	$12,202	$25,758	$22,989
Act LR Relativity	0.86	1.07	1
Pred LR Relativity	0.88	1.07	0.98
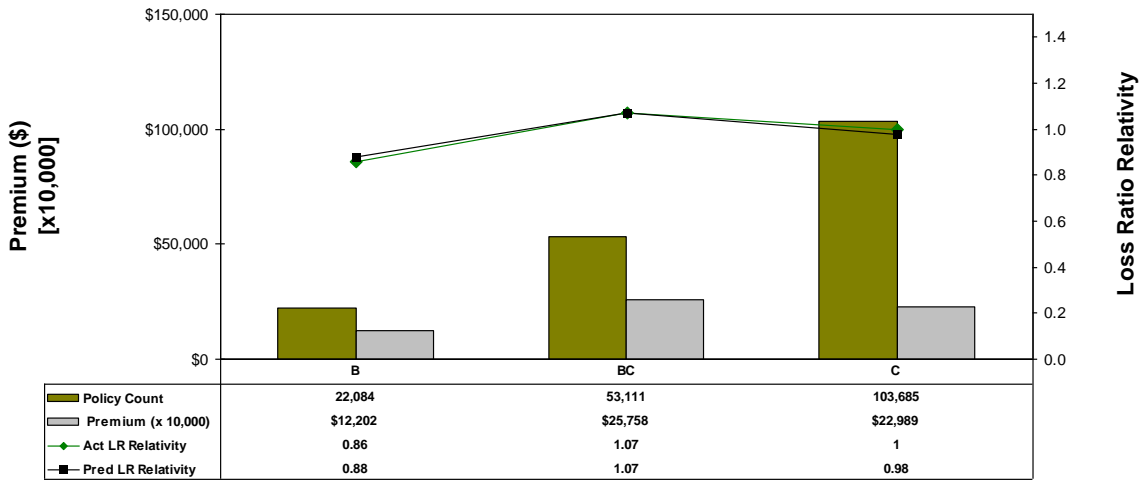
LR Relativities by Property Deductible2

	250	500	1,000	> 1,000
Policy Count	8,091	104,967	54,924	10,898
Premium (x 10,000)	$1,408	$24,126	$25,735	$9,679
Act LR Relativity	0.71	0.95	0.99	1.2
Pred LR Relativity	0.65	0.85	1.07	1.25

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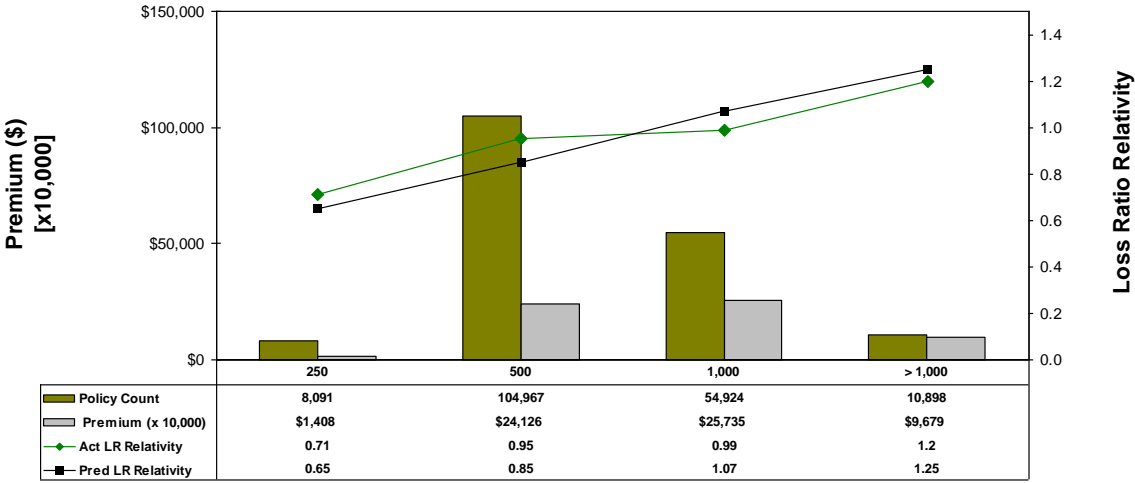
APPENDIX E: MODEL COEFFICIENTS

Commercial Auto

Variable	Coefficient	Chi-Squared	AIC
Intercept	-3.944		
Financial Stability	-0.106	1.746e-07	
Corp. Indicator	-0.015	0.0002085	
Policy Type 2 (LP)	-0.352	0.0002984	
Policy Type 3 (MM)	-1.358		
Policy Type 4 (MH)	-0.451		
Policy Type 5 (AO)	-0.573		
Market Segment (MM)	0.151	< 2.2e-16	

Variable	Coefficient	Chi-Squared	AIC
Policy Effective Age (2)	1.648	< 2.2e-16	
Policy Effective Age (3)	2.020		
Policy Effective Age (4)	2.005		
Policy Effective Age (5)	2.042		
Policy Effective Age (6)	2.003		
Policy Effective Age (7)	2.049		
Fleet Size	-0.273	< 2.2 e-16	
Fleet Size²	0.007	4.291e-05	
Ln(Fleet Size + 1)	1.185	< 2.2e-16	
Fleet Size : Policy Type 2	0.035	1.216e-09	
Fleet Size : Policy Type 3	0.129		
Fleet Size : Policy Type 4	0.029		
Fleet Size : Policy Type 5	0.137		126180.3

Business Owner's Policy

	Df	Deviance	Resid. Df	Resid. Dev	Pr(>Chi)	
NULL			143103	594478		
Financial_Stability	1	1275.0	143102	593203	2.290e-09	***
EffAge	5	10741.5	143097	582461	< 2.2e-16	***
Region	14	1843.6	143083	580618	3.244e-06	***
Control_Age	1	221.2	143082	580397	0.01281	*
Protection_Group	3	577.1	143079	579819	0.00105	**
Construction_Type	4	1343.8	143075	578476	1.332e-07	***
Property_Limit	1	599.7	143074	577876	4.165e-05	***
Legal_Status	3	285.8	143071	577590	0.04591	*
Location_Count2	1	1180.5	143070	576410	8.923e-09	***
Property_Deductible2	3	304.1	143067	576106	0.03646	*
ProgramLimit	24	4050.4	143043	572055	1.408e-13	***
NewTransfer	5	1416.6	143038	570639	1.737e-07	***
Property_Limit:LimitInd	2	275.3	143036	570363	0.02116	*

APPENDIX F: CODE

Commercial Auto

Data Preparation

LLdata←read.csv("data.csv",header=TRUE,sep=",")

LLdata←subset(LLdata,is.na(LLdata$Rerated_Prem)==FALSE)

LLdata←subset(LLdata,is.na(LLdata$CPOINTS_MODEL)==FALSE)

LLdata←subset(LLdata,LLdata$ManualPolicy=="Y")

LLdata←subset(LLdata,(LLdata$MarketSegment=="Other")==FALSE)

LLdata←subset(LLdata,(LLdata$LiabGrp==3)==FALSE)

LLdata$Rerated_Prem←ifelse(LLdata$Rerated_Prem<500,500,LLdata$Rerated_Prem)

LLdata$Incurred_Loss←ifelse(LLdata$Incurred_Loss<0,0,LLdata$Incurred_Loss)

LLdata$Loss_Ratio←LLdata$Incurred_Loss/LLdata$Rerated_Prem

Cap←quantile(LLdata$Loss_Ratio,0.99)

LLdata$Loss_Ratio←ifelse(LLdata$Loss_Ratio>Cap,Cap,LLdata$Loss_Ratio)

PolEffAge←2012-PoEffYear

CorpInd←ifelse(LLdata$Business_Type==A,1,0)

Fitting

model←tweedie.profile(Loss_Ratio~Financial_Stability+CorpInd+PolicyType+MarketSegment+PolEff
  Age+FleetSize+I(FleetSize^2)+I(log(FleetSize+1))+FleetSize:PolicyType,data=LLdata,verbose=2)

fit←glm(Loss_Ratio~Financial_Stability+CorpInd+PolicyType+MarketSegment+PolEffAge+
  FleetSize+I(FleetSize^2)+I(log(FleetSize + 1))+FleetSize:PolicyType, family = tweedie(link.power = 0,
  var.power = 1.636735), data = LLdata)

Lift Charts

Categorical Variables

```
sumPrem<-tapply(Rerated_Prem,MarketSegment,sum)

countPrem<-tapply(Rerated_Prem,MarketSegment,length)

sumLoss<-tapply(Loss_Ratio*Rerated_Prem,MarketSegment,sum)

sumPLoss<-tapply(fit22$fitted.values*Rerated_Prem,MarketSegment,sum)

LR<-sumLoss/sumPrem

PLR<-sumPLoss/sumPrem

cutoff<-data.frame(countPrem,sumPrem,sumLoss,sumPLoss,LR,PLR)

write.csv(cutoff," MarketSegment.csv")
```

Continuous Variables

```
VAR<-Financial_Stability

q <- wtd.quantile(VAR, weights=as.numeric(Rerated_Prem), probs=seq(0,1,.1), normwt=TRUE,
na.rm =TRUE)

a <- c(0,q[2],q[3],q[4],q[5],q[6],q[7],q[8],q[9],q[10],400)

quant<-rep(c("Z"),length(VAR))

for ( i in 2: (length(a))) { quant <- ifelse (Financial_Stability < a[i], ifelse (Financial_Stability >= a[i-
1], LETTERS[i-1],quant), quant)}

quant<-as.factor(quant)

sumPrem<-tapply(Rerated_Prem,quant,sum)

countPrem<-tapply(Rerated_Prem,quant,length)

sumLoss<-tapply(Loss_Ratio*Rerated_Prem,quant,sum)

sumPLoss<-tapply(fit22$fitted.values*Rerated_Prem,quant,sum)

LR<-sumLoss/sumPrem

PLR<-sumPLoss/sumPrem

lower_cut <- c(q[1],q[2],q[3],q[4],q[5],q[6],q[7],q[8],q[9],q[10])

upper_cut <- c(q[2],q[3],q[4],q[5],q[6],q[7],q[8],q[9],q[10],q[11])
```

```
cutoff <- data.frame(lower_cut, upper_cut, countPrem, sumPrem, sumLoss, sumPLoss, LR, PLR)

write.csv(cutoff, "Financial_Stability.csv")
```

Business Owners Policy

```
> model ← tweedie.profile(Loss_Ratio~Financial_Stability+EffAge+Region+Program_Name
    +Control_Age+Protection_Group+Construction_Type+Property_Limit+LimitInd
    +Legal_Status+Location_Count2+Property_Deductible2+NewRenew +Property_Limit:LimitInd
    +Program_Name:LimitInd +NewTransfer,data=LLdata,verbose=2)
```

Output:

ML Estimates: xi = 1.636735 with phi= 23.55729 giving L= -49755.23

```
> fit29←glm(Loss_Ratio~Financial_Stability+EffAge+Region+Program_Name
    +Control_Age+Protection_Group+Construction_Type+Property_Limit+LimitInd
    +Legal_Status+Location_Count2+Property_Deductible2+NewRenew+Property_Limit:LimitInd
    +Program_Name:LimitInd +NewTransfer,data=LLdata
    ,family=tweedie(link.power=0,var.power=1.636735))
```

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