A Major Qualifying Project Submitted to the Faculty of Worcester Polytechnic Institute in partial fulfillment of the requirements for the Degree in Bachelor of Science in Mechanical Engineering By

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Abstract

The goal of this MQP was to design a tabletop demonstration device to be used to aid students in understanding combined bending and torsional stresses. The device is a cantilever beam capable of being twisted as well as bent. The geometry of the beam was selected to balance easily measured amplitudes of strain with longevity of the device. We included the addition of strain gauges on the critical section of the main shaft of the beam to allow theoretical values of stress to be compared to measured values of stress.

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Introduction:

The device designed is intended to demonstrate combined torsional and bending stresses to be used to further students understanding at WPI. In order to complete a design concentration with a Mechanical Engineering degree at WPI students are required to take the Design of Machine Elements course. In this introductory course students spend the 7 week term examining stress and fatigue in various machine elements. According to the WPI course catalog "machine elements are studied and methods of selection and design are related to the associated hardware" (WPI Course Catalog). It is important for students to understand the stresses and forces that are applied to various potential failures. From analyzing these potential failures students learn how to account for such failures in their design and selecting materials in order to acquire a satisfactory safety factor.

While learning throughout this seven week course students are also required to design various components for certain devices and structures to put into practice what they are learning. One device that has been designed by students in past Design of Machine Elements courses has been the main rotor shaft for a Wind Turbine. Students underwent research to understand the function and design of the rotor shaft and what specific forces and torques they needed to design for. From their students determined singularity functions and calculated the stresses acting at critical sections on the shaft to determine if it would withstand the forces applied. As the course went on students learned about stress concentrations and made stepped shafts with a radius of curvature and calculated the stresses acting at that section. They then used their desired material and those stresses to determine the safety factor of the rotor shaft.

A problem that students tend to face in this class is understanding the concept of combined stresses and the affects they have on a system. Designing for such systems can be difficult when there isn't a fundamental understanding which could be helped through the use of visual aid. The purpose of this MQP is to design a tabletop demonstration device to demonstrate

combined stresses in a system. This would be used in future design classes as a teaching aid to further students' understanding of combined stresses.

Design Specifications

- The beam must be able to withstand bending and torsional stresses with a safety factor over 1 (preferably 2).
- The device must be able to withstand the max load desired without flipping over or breaking.
- The connection between the beam and back plate must be able to hold without causing damage to the beam or having the beam slip from the grip.
- The connection between the beam and the arm must be able to hold without causing damage to the beam or arm or by having the grip fail.
- Must be able to measure stresses, both bending and torsional, acting on the shaft.
- The device must be easily transported.
- The beam must deflect enough to be visually distinct but not cause any plastic deformation to the shaft.
- The device must be able to be accurately measured with the use of strain gages based on the theoretical values calculated for the device.
- The device must be able to withstand 200,000 cycles of a maximum load of 45 lbf applied without failing.
- The device must be able to adjust the amount of force applied between 0 and 45 lbf.
- The hanging weight must be able to adjust the amount of distance it applies the force across the length of the shaft.

Design Section

Design of Shaft

When designing the beam for our demonstration device we needed to determine some important variables and characteristics to take into account. We determined that the length of the beam would be 0.5 meters with a 0.75 inch diameter. We then assumed that we would hang a weight on the end of the arm we designed for our device that weighs 45lbs. This would generate approximately 200N of force as well as 100N*m of torque from the weight both bending and twisting the main beam. We then needed to determine a material for our beam. When looking through materials we wanted something that would be lightweight but also have a high strength value to be able to handle the amount of force and torque being applied. After some researching we settled on a 2024 Heat Treated Aluminum. Using its specific gravity we were then able to calculate the weight of the beam and thus begin determining our singularity functions.

When creating the singularity functions we had to determine the forces acting on the beam. We analyzed the beam having a reaction force, R1, at the base of the back plate, the weight of the beam being evenly distributed across its entire length and then the applied force acting at the end of the beam. From here we created the Force, Shear and Moment singularity functions as shown in the figure below.

$$\begin{aligned} q(x) &:= R_1 \cdot S(x, 0m) \cdot (x - 0)^{-1} - w_1 \cdot S(x, 0m) \cdot (x - 0)^0 + w_1 \cdot S(x, L) \cdot (x - 0)^0 - F \cdot S(x, L) \cdot (x - L)^{-1} \\ V(x) &:= R_1 \cdot S(x, 0m) \cdot (x - 0)^0 - w_1 \cdot S(x, 0m) \cdot (x - 0)^1 + w_1 \cdot S(x, L) \cdot (x - 0)^1 - F \cdot S(x, L) \cdot (x - L)^0 \\ M(x) &:= R_1 \cdot S(x, 0m) \cdot (x - 0)^1 - \frac{w_1}{2} \cdot S(x, 0m) \cdot (x - 0)^2 + \frac{w_1}{2} \cdot S(x, L) \cdot (x - 0)^2 - F \cdot S(x, L) \cdot (x - L)^1 \end{aligned}$$

Figure 01: Singularity Functions

We then had to solve for the integration constants and calculate the value of the reaction force at the back plate solving the moment function for when x equals the length of the beam.

This solution derived that the reaction force equaled 1.934 N. After solving for these variables we were able to create the final versions of the singularity functions. From there we created a graph that contained both the shear and moment diagrams to determine where the critical section was across the length of the beam. Based on the graph we determined the critical section to be located approximately 0.25 meters from the base of the back plate. We also determined the four critical points: "A" located at the top of the beam, "B" located on the left side of the beam, "C" located on the bottom of the beam and "D" located on the right side of the beam.

From here we calculated the stresses acting at each critical point on the beam. At critical point A there was the bending stress and the torsional stress acting on the beam. After calculating both of those stresses we then converted them into the von mises stress values for that specific critical point. The total von mises stress was calculated to be 1.276*10^8 Pascals. The final calculations are shown in the figure below and the full calculations can be found in the Appendix at the end of this document.

$$\sigma_{\rm vma} := \sqrt{\sigma_{1a}^2 + \sigma_{3a}^2 - \sigma_{1a} \cdot \sigma_{3a}} = 1.276 \times 10^8 \, {\rm Pa}$$

Figure 02: Calculations of Von-Mises Stresses at critical point A.

Next we calculated the stresses acting at point B on the critical section. We determined a transverse shear and a torsional shear were both at this specific point. We calculated both these stresses and added them together because they are both acting on the same axis and are both in the same direction. We then calculated the von mises stresses at this point. The calculations for the von mises stresses at point C were very similar to that at point A except both forces were in the opposite directions of the forces at A. These calculations ultimately lead to the same value that was calculated at point A. Then for the calculations at point D we determined again both a transverse and a torsional stress present but they were acting in the opposite directions. When calculating this though we still came up with the same value as in point B for the von mises stresses. We then did a preliminary safety factor calculation dividing the yield strength of the material by the total von-mises stress calculations that we completed at each point to figure out

how well we were doing at this point of the design process. This showed us that our current safety factor was approximately 2.3 which was a satisfactory value but we needed to account for different loading and our desired lifetime of the device. The calculations of this initial safety factor can be found in the figure below.

Max Shear Stress Theory:

$$N_{sfamss} := \frac{.5 \cdot S_y}{\tau_{xza}} = 1.968$$
$$N_{sfcmss} := \frac{.5 \cdot S_y}{\tau_{xzc}} = 1.968$$

Figure 03: Preliminary Safety Factor

To ensure a more accurate safety factor we took into account a cyclic loading scenario and had to calculate for the endurance strength value. This was determined to be half the ultimate tensile strength which meant the endurance strength is equal to 2.205*10^8 Pascals. We then calculated the various correction factors that would help us acquire a more accurate safety factor for our design analysis. We determined our size, load, and temperature factors to all be 1 meaning they didn't alter our beams safety factor. However, we didn't use a surface correctional factor because we could not find a table of constants to determine the values needed to calculate such a factor. With basic bending and torsional loading our correction factor for loading is 1. For the size correction factor, with the beam diameter being over 0.3inches the correction factor is equal to 0.894 after the calculations. The temperature correction factor is 1 since we are not working at extreme temperatures. Lastly we want a high reliability so we decided to go with a reliability of 99.99% which bad our correction factor 0.702. After multiplying these factors to our endurance strength Se, our new corrected endurance strength value was calculated at 1.548*10^8 Pascals.

We then created a log mean diagram for the device and graphed it. We used 10³ cycles and 10⁶ cycles for the range of our linear slope. To determine our necessary strength Sf we needed to determine what number of cycles to design for. An estimation was made that each student in a course would perform 10 cycles on the device each term for the course of 10 years.

Based on a class size of approximately 30 students we rounded the number of cycles to 200,000 for the life of our device. This lead to a value of 1.928*10^8 Pascals for Sf at 200,000 cycles.



Figure 04: Log Mean Diagram and Sf value.

We determined this would be a cyclic loading condition for our device and designed for such. This is because the loads we expect to design for are a full load of a certain weight We used a the current Force and Torque values we designed for as our maximum values and then used 0 for the minimums of both. From here we calculated the alternating and mean values for both Force and Torque which were 100N and 50J respectively. We then used these alternating and mean values to recalculate the alternating and mean von-mises stresses at each critical point on the critical section of the beam. The calculations of these values at critical point A can be seen below.

$$\sigma_{aalt'} := \sqrt{\sigma_{1aalt}^{2} + \sigma_{3aalt}^{2} - \sigma_{1aalt'}\sigma_{3aalt}} = 6.38 \times 10^{7} \text{ Pa}$$
$$\sigma_{am'} := \sqrt{\sigma_{1am}^{2} + \sigma_{3am}^{2} - \sigma_{1am'}\sigma_{3am}} = 6.38 \times 10^{7} \text{ Pa}$$

Figure 05: Alternating and Mean Von-Mises stresses

Lastly with each point we used the alternating and mean von-mises stresses to calculate the safety factors for each critical point. With a finite life value for 200,000 cycles, the von-mises stresses, and the ultimate tensile strengths we determined safety factors of approximately 2 at each point. We felt this was a very reasonable safety factor for our final design and would work

within our design parameters. The calculation of the safety factor can be seen below for point A with the full calculations available in the appendix at the end of this document.

$$S_{nfin} := S_{f}(2 \cdot 10^{5}) = 1.928 \times 10^{8} Pa$$
$$N_{sffina} := \frac{S_{nfin} \cdot S_{ut}}{S_{ut} \cdot \sigma_{aalt'} + S_{nfin} \cdot \sigma_{am'}} = 2.102$$

Figure 06: Safety Factor Calculations

Design of Device

Through calculations of the stresses, bending deflection and twist from torsion; we were able to define the material, diameter, and length of cantilevered circular beam. The solid circular rod would be made of aluminum, and would need to deflect and twist enough to offer an easily observable deformation, in both bending and torsion under combined loading. Initially the device was a pretty complicated system of parts that would require careful and skillful machining for it to work correctly.



Figure 07: The first design which was overly complex.

This first solid model we devised, but it required precision machining of the shaft, having flat sides to keep it from rotation when the torsion is applied. Further complicating the logistics of making the parts, the post in which it mated would have to be machined to a matching tolerance, having the same shape as the shaft to constrain it. This design also had a series of ball

joint, bearings and other overly complicated elements; The realization of the complexity led to a deeper discussion of its intended use, and how to choose different methods that would simplify the device.

With further focus on what the true core requirements of this device were; we designed a very simplistic mechanism assembly, from which our final design would grow, as we honed the parameters, fixturing methods of the circular beam, and moment arm; each parameter we refined, allowed us to strip the design down to it's core elements and configure it for our precise needs, then build up from there to meet our space, weight, durability, safety and aesthetic needs.

Also considering the ease of storage, transportation and use; We wanted the input load to be created by a system that was both variable and easily transportable. For this, we decided to use a 5 gallon water bottle as the weight. A varied amount of water could be added to the bottle, for a range of roughly 5 to 195 Newtons. Since our full bottle would be just under 200 N, and our goal was 100 Nm force, our beam would want to be about 0.5 meter from the cantilevered end, to the hanging weight.



Figure 08: First Simplified Design. The design was easier to make but still lacked a way to apply a torque to the device.

These are two versions of the simplified device, where the rod would be pinned into one block, which could then be bolted or otherwise mounted to the base, constraining the torque and bending with a pinned element. One concern that arose from examining this design was the stresses caused by drilling and pinning the elements together, along with the requirement of the parts to have a sliding fit to allow them to be assembled before they are pinned. The sliding fit meant that the aluminum rod could move inside the plate, creating a weakness in the design; The

rod could be deformed by repeated movement inside the plate. We were able to take this "failure" and turn it into the solution we used in our final device. One of us had experience with shrink-fits at work; this method of heating and/or cooling parts to aid in assembly or disassembly is relatively simple in execution. However, it requires careful calculating to find a balance of torque-holding capability and building excess internal stresses.



Figure 09: First rendition of our final design. The addition of the arm allows a torque to be applied. This is the first design that incorporated the interference fit at the shaft interfaces.

An interference fit is where the two parts are in a state of compression and tension due to the inside part having a larger diameter than the hole in the outer part at their normal operating temperature. This state can be achieved via pressing the parts together under great force, or via shrink-fit; where heating or cooling of the parts causes expansion or contraction of the interfering elements, to the point that they no longer interfere, and can be slid together without force. As they return to room temperature, the outer part squeezes the inner part, creating a joint that has no play, and can hold a torque without any pinning elements. When designed with only as much

interference as is required to hold the torque required, the mating has a much lower concentration factor than any pinning method.



Figure 10: Diagrams showing the slide fit of the shaft into the device.

The rod shown here is 0.75" round stock, that measured 0.7501"(19.05mm) with exceptional roundness, so we used that dimension as our baseline to machine the Rod Plate, after calculating the required dimensions and interference to hold the torque we chose as its max. Given our max intended applied force of 200N, on the end of the 0.5m moment arm, we know the torque applied on the shaft and the connections to the arm and base, is 100Nm or less.

Torque =
$$T = r x F$$
;

Given the relatively small angular displacement, we chose to assume the force is applied perpendicular to the arm, simplifying the equation to:

$$T = F * L = 200N * 0.5m = 100Nm$$

We needed to include a factor of safety, and chose 2.5 for these joints, meaning the minimum sustainable torque that can be held by the joint must be 250Nm.

For these calculations, there are a number of calculators available online; We used one created by Alex Slocum(MIT), then checked the results with another from Amesweb.info, then sanity checked a few calculations by hand to confirm we could trust and understood the outputs.

Using the excel spreadsheet from Alex Slocum as our primary calculator, we were able to vary numerous parameters, and save the results and equations for each together. That interface can be found at the end of the report in Appendix A. with our final project inputs shown.

Shaft Results (For Max. Diametric	cal Interference)		
Radial displacement of outer surface	Uouter	-0.008	mm 👻
Radial press fit stress at outer diameter	σ _{r,pressure}	-87.3	
Circumferential press-fit stress at outer diameter	0 _{e,pressure}	-87.3	
Axial stress from applied axial force	σz	0.0	
Shear stress from applied torque at outer diameter	T	184.2	MPa 🔻
Max radial centrifugal stress	σ _{r,centrifugal}	0.0	
Max circumferential centrifugal stress	0 _{e,centrifugal}	0.0	
Max Von Mises stress	omises	330.7	
Factor of safety against yielding of shaft *	fos _{y,s}	0.98	l

Shall be larger than relevant design factor. Green color means safe, red color means not safe according to input arameters.



Selection of Unit System for Graphs: MPa - mm 🔻

Figure 11: AMESweb Calculation of the various stress parameters

Building the Device

Design to Part

Having calculated the parameters that allow us to meet our goals, the device design is updated to account dimensions, fits, and bending clearances. The following engineering drawings were created in a format prefered by machinists, using ordinate dimensions with a common origin point at which all points can be referenced during machining, with physical measurement. The updated design used 10-series 80/20 extrusion as the base frame of the assembly, on which the other parts would sit; allowing simplified assembly and weight savings over solid square stock.





Figure 12: Engineering Drawings of the arm and backplate, using ordinate dimensioning.

Redesigns During Physical Build

We started these calculations using the 2024-T351 Aluminum round bar stock, ¹/₂" 1018-Steel plate for the torque arm, and a block of 1018 for our cantilever base. The 2024 proved somewhat difficult to get, so we ordered 6061-T651 Aluminum which has very similar properties but is readily available. After hours of machining and working with the 1018 base block, we felt the weight of these pieces would be inhibitive to our goal of the whole device being easy to carry and set up. Additionally we found the great amount of machining time required for our original 1-piece base was inefficient, where a 2-piece base assembly made from 6061 Aluminum would allow for faster, simplified machining, assembly, and future maintenance. Through the process we had numerous redesigns; and we had to learn from the data that we received and readjust.

The result was this, our final design iteration; upsized aluminum extrusion to offer a stiffer and slightly heavier frame to counteract the weight applied on the machine, reducing it's likelihood of flipping off the table. The updated aluminum 2-piece cantilever base assembly

sitting atop the extrusion frame, paired with a 1018 steel plate at the cantilevered end of the bar, were our torque-holding shrink-fit elements. The steel plate was easy to heat and manipulate onto the bar, then providing a strong plane on which the torque arm could be mounted.



Figure 13: Final Design Concept of the Tabletop Demonstration Device





Figures 14 and 15: Engineering Drawings of the two backplate pieces and a front view of the devices arm used to apply the torque and force.

The ¹/₂ meter intended torque arm was more than we needed to visually represent the torsion, so we changed to a shorter arm. The point on the arm on which the weight is applied, can be adjusted with a thumb screw, allowing fine tuning of the moment arm length, or setting an "unknown" length parameter, for which students can solve for. The Plate Base and Rod Plate were both made on the CNC Haas Mini Mills, after creating CAM programs in Esprit, and outputting the NC code into the machine to machine the parts.

After creating the device and testing, we found there were a few features that the project should include in order to be more complete. A safety feature was designed and built from stock we had available, taking advantage of the 80/20 extruded aluminum of the base frame. Some alignment shims, and a tensioning handle were modified and added to a piece of extrusion to act as a sliding outrigger. This piece allows easy extension past the back side of the table, where the adjustable clamping end can be adjusted to hold the back of the device from both lifting off the table, and sliding off the front edge.

A thumb screw allows easy adjustment of the table clamp, for tables up to about 8" thick, while the sliding arm can accommodate tables about 42" deep.

Making clearance for the sliding arm, bracketry, and adding safety; we added pliable rubber feet, allowing the device to safely sit on the table without moving around.

The reality of heating, then simultaneously aligning and sliding the ends onto the aluminum bar was more difficult than expected, leading to a loosening of the joint over the following weeks of being transported and tested; please see our recommendations section for our thoughts on how this could be made better in the future.

Strain Gages

What is a Strain Gage



Figure 16: Diagrams of the parts and workings of a standard strain gage.

The basic concept of a strain gage, is the gage is a thin circuit that has a single "wire" that zig-zags back and forth in a configuration on the thin film, and has connections for both ends of the circuit. This circuit has a given resistance across its leads, which changes as the gage is deformed. The change in resistance is caused by deformation of the circuit, when stretched or compressed along the length of the circuit. The "strain direction" as mentioned in the above figure, is the one in which the circuit is designed to stretch the wire of the circuit; the layout of the small circuit "wire" is thin in the strain direction. This configuration allows the circuit to elongate perpendicular to the strain direction, like pulling on a light spring, without changing the internal resistance of the wire. Whereas along the strain direction, an elongation of the circuit is translated directly into stretching of the wires, changing the resistance of the circuit. Thus, when we drive an excitation voltage through this circuit, and an appropriate balancing circuit; we can output a voltage that changes linearly with the tension or compression under which the gage is

subjected. When securely attached to a surface, the gage will read the strain at the surface of that object, and allow us to measure the deformation of the object.

Choosing Strain Gage Sensor(s) and Configurations

For our application, we needed to monitor the combined stresses of bending and torsion when a force is exerted on the end of the torque arm. For this we initially considered the Delta Rosette specialty strain gages. Here multiple strain gages are built into one gage, and oriented at one of many different configurations. The one we decided on was the 45° Delta Rosette.



Solving for ε_x , ε_y and γ_{xy} gives:

 $\epsilon_{\chi} = \epsilon_{a} \qquad \epsilon_{y} = \epsilon_{c} \qquad \gamma_{\chi y} = 2\epsilon_{b} - \left(\epsilon_{a} + \epsilon_{c}\right)$

Figure 17: Diagrams and equations for the degree Delta Rosette.

These rosette configurations allowed for differing types of readings, where the gages required calculation of the output voltages to gain a useful reading. With a Labview type program, this would be uncomplicated for the final user to read the output, but there were a number of downfalls to this configuration.

The first was it required the use of three (3) separate amplifier circuits, which would be calibrated and then the outputs calculated in LabView or by hand if reading the output voltages on a voltmeter. Second, the calculations added a step to the process where a student could become disconnected and not be able to have a natural correlation of the force exerted on the rod, to the reaction of the gages. Third is the aforementioned high cost of the custom gages; though the group of three gages could be replicated using three individual ones, this added an additional

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complexity to the precision orientation of these gages and created an avenue for loss of precision and output linearity.

A less complicated method would be creating two separate circuits, using 4 simple strain gages, the MMF006835 "Student gages" by Micro-Measurements. Two would be mounted along the rod,

Additionally, the gages are meant to be used by teachers and students to further the learning; So using these odd gages would be less applicable to what the students have available outside of this device, so we decided to use the standard gage that is used in the ME3901 course: Engineering Experimentation. This meant a student could become more intimately familiar with setup, calibration and output extrapolation of these gages through the use of our device; then apply the same gages and equipment that are readily available, to their own projects in a more efficient manner.

Calculation of Bending Stress (2-Gage Method)



Figure 18: 2-Gage Bending Stress Measurement

Two strain gages bonded at the contrasting positions of the front and rear surfaces of a beam are equal in their absolute values and the mark of (+) or (-) will come reverse. If the strain gages are bonded on a beam in such a manner that they may be neighboring ones each other, their bending strain will become double and the strains caused by the force to the axial direction may be negated. In this case, calculated to: $\sigma = \varepsilon 0^* E/2$

Calculation of Torsional Stress (2-Gage Method)



Figure 19: 2-Gage Torsional Stress Measurement



In the axle catching the torsional moment "M ω " like a figure, the shearing stress " τ " becomes greatest at the axis surface; the value is :

$\tau_{max} {=} M\omega/Zp$

Where, Zp: Polar modulus of section The surface shearing strain " γ " is :

$\gamma = \tau_{max}/G = M\omega/G*Zp$

The indicated strain " ϵo " becomes the value of the surface shearing strain " $\gamma = 2 \epsilon o$ " when it's measured by 2-Gage Method, and the shearing stress " τ " be calculated using a next formula. $\tau_{max} = G \epsilon o'$

> $\varepsilon o' = \gamma = 2 \varepsilon o$ $M \omega = G Z p \varepsilon o'$

Figure 20: Bending-Torsion Arm Diagram with Strain Gage Placement

Installation and Testing of Strain Gages

For accurate measurements with the strain gages; they need to be accurately placed, securely attached, with little to no strain from the wire leads. The device was placed on a level table, top-dead-center was located by marking the point in contact with the level placed on the bar, and a line was etched into the bar. This line represents the absolute top of the bar, and the center plane on which the first bending strain gage would be aligned. As in figure 21, the two gages would be oriented with the strain direction along the length of the beam, and because it is a circular beam; both the top and bottom would need to be centered and aligned for a linear and accurate measurement.



Figure 21: 2-Gage Bending Stress Measurement

Similarly, the torsion gages needed to be accurately installed, but their orientation was to be centered 90° from the bending gages, and once again that line would be centered on the gage. Orienting these was made somewhat easier by the fact that the gages have small indicator lines on them which allow you to align the gages at 45° and 90° increments. The two of these gages would be mounted as shown in figure 22, at 45° and 135° from the length of the beam, which is 90° from each other.



Figure 22: 2-Gage Torsional Stress Measurement

The orientation of these gages would allow for each one to be in tension while the other is in compression; this is critical because we are inducing bending on the beam while reading the torsion. If we did not orient the gages this way, the bending would affect the torsion reading, and we could not use it as an accurate measure of torsion. The gage aligned with the twist of the beam gives the torsional output, which is slightly inaccurate due to the bending of the beam also causing deformation of the gage. The second gage is perpendicular to the twist, meaning it is not affected by the torsion, but is still sensitive to the bending. These gages are then wired through the wheatstone bridge, such that the anomaly caused by the bending cancels between the two gages, and we are left with an accurate and linear torsion output.



Figure 23: Strain Gage Amplifiers

Figure 24: Strain Gages installed on beam

The laboratory amplifiers in figure 23, were used in testing, and measuring of out strain gage circuits. They have an output power circuit, and adjustable wheatstone bridge circuit inside, allowing simplified connection, calibration, and use of the bending-torsion device.

They were connected to the amplifiers, then a calibration procedure done in order to set the circuit to output 0 volts at rest on both circuits. From here, a known load was added, in order

to dial in the gain factor, and set our 10 volt available output to match our maximum measured weight to be added.

Per our previous calculations and designs, this would be scaled as a 100Nm bending force acting at the static end of a 0.5 meter cantilevered beam, and adjustable length torque arm at the cantilevered end. This means we have our 200N force applied at a variable distance along the torque arm. We decided on a moment arm of 30cm, For the initial testing, we placed a 6.66N weight 30cm along the torque arm, and since this was different from the previously calculated torque, we had to calculate the max torque with the simple torque equation.

T = F * L = 200N * 0.300m = 60Nm

This means that for bending;	200N @ 0.5m = 100Nm = 10 volt output.
Resolution would therefore be:	10v/100Nm = 0.1v/Nm

The value could also be calculated for a direct correlation to the weight added, by calculating Volts/Newton, which would be: 10 volts/200 Newton = 0.05 v/N

For Torsion, we have: 10v/60Nm = 0.167v/Nm

The range and resolution can be adjusted for any maximum load or moment arm length, or it can be tuned to have a stable output correlation to the weight applied. Because of this adjustability, it is useful for training how adjusting the circuit can affect the output data, for good and bad.

After our circuits were adjusted, we applied varying known weights to the device, to check the linearity of the output, and it was generally accurate within about 1%, with a few fluctuations outside that range that we believe were caused by, and corresponded to, a slight drift in the amplifier's output voltage, or calibration/balance circuit that we had noticed.

A user can set the device up to output a voltage, to be read directly from a volt meter for simple usage and data tracking, however, another more powerful way to make use of this device is via LabView, with a custom designed interface that can take the input voltages from the bending and torsion circuits, and do anything you'd like- the limits are your programming ability in LabView. We created an interface that could read either of the inputs, and output them on a graph, calculate the weight, deflection, and unit conversions, but it was flawed and unable to control both circuits simultaneously. We ran out of time to fix the program, and instead focussed

on other aspects of the device and project that would be more useful for future users, or MQP groups that wish to improve upon it. Please see figure 26 for an idea of how the labview program we created was laid out, and intended to work.



Figure 25: Bending/Torsion with 195 Newton mass installed

Trille Laft, Veren Project, Operate Tools Window, Help The Laft, Veren Project, Operate Tools Window, Help The Laft, Veren III, ISpt Application Fort i v too too too too too too too too too t	- Seach
Torsion and Bending Reado	out
Al Chennetic Ves and Vescer	Al Channels: You and Veder 2 Ded/fail
Shaft Length(m) 3 5 3 7 23 7 23 7 10 0.031500 Weight[kg] 0.686000 Weight[kg] 0.686000 Viroue 80 10 3 9 10 5 Torque Expected 0.0 -0.0	-0.016377
Measured Torque[N*m]	Measured Bending 0

Figure 26: LabView Interface: Torsion and Bending Readout

Conclusion

Over the course of the three terms for this project we were successful in completing the task we sought out to complete and learned a lot about the engineering design process because of it. We learned the amount of work and time that can go into designing and re-designing alone which took up a huge majority of our time on this project. The various different iteration of the device consistently had improvements that needed to be made in order to successfully complete the task desired. Even once we had our "final design" we still made changes during the build process and added on new additions to add new features for safety or ease of use such as the table gripper that prevents the device from flipping. A lot of thought went into the design of this device, a lot more than we anticipated, so it was a lesson learned thinking this would have been relatively easy to create.

However, we were able to finalize and build the complete device and it functions rather well with the desire we had intended. The beam is able to maintain its structural integrity when facing the maximum load designed for and can be easily seen visually bending. The strain gages applied to the device to measure bending and torsion stresses are also easily able to be set up to be hooked up to a LabView program to measure the exact stresses applied on the shaft. There are still improvements that could be made to the device that we realized through the completion of this project but with the time we had available we weren't able to fix these issues. We do list all of the suggestions that we would like to make to improve this device in the recommendations section of this paper going into detail of what changes should be made and why. Overall, this device has been very successful and we are confident it will be a great teaching aid to be used in future design classes.

Recommendations

Even after successfully completing our desired task and building this tabletop demonstration device we still found some potential improvements that could be made in the future for anyone who wants to design a device such as this. We hope to use this section to allow others to learn from mistakes and choices we have made that probably weren't the most optimal in completing the task to help them in future designs and builds.

USING STEEL

One of the top recommendations we had was making the back plate, the main shaft, and the connection to the arm all out of steel. We did a shrink fit for our beam into the wall and the arm and this was done by heating up the back plate and arm slot to a high temperature so it would expand. Then we would force slide the shaft into the openings and as the pieces cooled off they would shrink back down and have a strong, high friction grip that could withstand the torques applied.

However, the fact we used aluminum for the shaft and back plate made it very difficult to obtain a successful shrink fit. This is because the aluminum would cool of significantly fast to the point where the shaft would fail to slide in properly. This lead to the fit being redone multiple times until it was finally fitted properly. Even so, the last ½" of the fit had to be forced in after it was cooled which lead to the degradation of the fit. After only two weeks of use and transporting the device the bar started to become loose on the back plate. For these reasons we would suggest using steel for those three components, but maintaining the aluminum for the rest of the design. The steel stays heated longer and will be much easier to get a successful shrink fit and will maintain its integrity longer and better than the aluminum will.

However, steel is much heavier compared to the same volume of aluminum so make sure only these three components are steel. It will still allow your device to be easily transported and also will be easier to machine. The first attempts at making the backplate out of steel from hand took almost five hours of cutting and was barely half complete. If time and equipment permit

though steel would be a nice option still. Also we suggest if the shaft were to also be made out of steel design for a smaller diameter cross section. With the steel being much stronger than aluminum the steel would need less material for the same safety factor with its higher yield and ultimate tensile strength but still be able to have the visually distinct bend for students and users of the device to see. If after all of this you still desire to maintain the aluminum shaft for any reason then we would suggest have a closer tolerance for machining the fits. This could help overcome some of the issues we saw when building and fitting our shaft for the shrink fit. Even setting up a jig to allow for a quicker and more accurate slide fit could help with the short time response needed to counteract the fast cooling tendencies of the aluminum material.

SIMPLIFY YOUR DESIGN

Another big piece of advice we have for others who wish to design such a device is to make sure you don't overdesign and over complicate your components. From the beginning our original designs were very complex in how they would be built and were very large and bulky. There was also talk of some complicated fits into the wall which involved some very complex machining that neither of us really had the ability to do. This can be seen in what was our first design which involved a large system to house the beam involving many pieces and a complex key fit for the shaft into the back plate. These can be seen in Figure 07.

The device overall was way to complex for us to be able to build or design and included many components that may not have gotten the job done properly. Additionally we still didn't have a way to apply a torque to the shaft but only a bending force which was applied from a threaded rod that fit through the end of the shaft. This made not only the build and design more complex, but the analysis more complex as well since we would have to design for a pin that could withstand the force applied and could potentially be a component that failed.

From here when we moved onto our simpler versions and eventually our final designs we were much more successful at being able to analyze and build. If time, materials, and equipment permitted there could potentially be some design choices that could be more effective than what we were able to accomplish with our design and could in turn make the device much more successful at completing its job. However, if you do not have access to the equipment and

materials necessary, and if you have a short time frame then making sure your design is simple and effective is the best way to succeed well.

STRAIN GAGE PROGRAM

The program was not completed in a functional manner, as such it should be recreated with a usable interface from which users can gather and log data. The creation of a robust base program, would allow students to further build on these inputs and outputs to form their own interfaces that would meet their own specific needs.

VIBRATION AND RESONANCE FREQUENCY TESTING

We noted that during the devices use, some buildings were transmitting a vibration into the device, causing an oscillation of the arm- we think it would be used for vibration and resonant frequency learning, in addition to the bending and torsion.

HEIGHT ADJUSTABLE FEET

Most tables on which the device would be used, will have a flat top- but we would suggest adding height adjustable feet to allow it's use on any surface. Additionally, if the table were not level, the feet could allow for leveling of the system, allowing greater accuracy to some measurements.

Appendix

Appendix A: Tables and Diagrams for the Design of the Device













Shaft Results (For Max. Diametri	cal Interference)		
Radial displacement of outer surface	Uouter	-0.008	mm 👻
Radial press fit stress at outer diameter	σ _{r,pressure}	-87.3	
Circumferential press-fit stress at outer diameter	σ _{e,pressure}	-87.3	
Axial stress from applied axial force	σz	0.0	
Shear stress from applied torque at outer diameter	Т	184.2	MPa 🔻
Max radial centrifugal stress	σ _{r,centrifugal}	0.0	
Max circumferential centrifugal stress	$\sigma_{\theta, centrifugal}$	0.0	
Max Von Mises stress	σ _{mises}	330.7	
Factor of safety against yielding of shaft *	fos _{y,s}	0.98	

Shall be larger than relevant design factor. Green color means safe, red color means not safe according to input arameters.













2-Gage Method for bending stress measuring

2-Gage Method for torsional stress measuring

INPUT PARAMETERS			
Parameter/Definition	Symbol	Value	Unit
Working Conditions			
Torque to be transmitted	Т	250	N*m ▼
Axial force to be transmitted	F	0	N 👻
Coefficient of friction	Ч	1.05	
Operation temperature	T _o	20	°C ▼
Rotation speed	w	0	rpm
Engagement length	L	12.7	mm 🔻
Hub Parameters			
Hub outer diameter	D _{ho}	50.8	
Hub inner diameter	D _{hi}	18.999	
Inner diameter upper deviation	$\Delta_{h,up}$	0.005	
Inner diameter lower deviation	$\Delta_{h,low}$	0	
Stress concentration factor	Kt	1	
Modulus of elasticity	E _h	68.9	GPa 🔻
Yield strength	Sy	324	MPa 💌
Poisson's ratio	Vh	.33	
Coefficient of thermal expansion	α _h	0.0000232	m/m °C ▼
Density	Ρh	2.78	g/cm^3 ▼
Shaft Parameters			
Shaft outer diameter	D _{so}	19.05	mm 🔻
Shaft inner diameter	D _{si}	0	
Outer diameter upper deviation	∆ _{s,up}	0.005	mm 🔻
Outer diameter lower deviation	$\Delta_{\rm s, low}$	0	
Stress concentration factor	Kt	1	
Modulus of elasticity	E,	68.9	GPa 🔻
Yield strength	Sy	324	MPa 🔻
Poisson's ratio	Vs	.33	
Coefficient of thermal expansion	α,	0.0000232	m/m °C ▼
Density	ρs	2.78	g/cm^3 ▼
Shrink Fit Design			
Desired clearance to be able to make assembly (clearance obtained between shaft and hub after thermal process)	Δ_{d}	0.005	mm 🔻
Standard reference temperature(reference temperature for geometrical product specification and verification defined by ISO and ANSI)	Ta	20	°C ▼
Design Factors			-11
Design factor against sliding *	nd _s	2	
Design factor against yielding (Hub) *	nd _{y,h}	1.5	
Design factor against yielding (Shaft) *	nd _{y.s}	.95	

RESULTS				
Parameter	Symbol	Va	lue	Unit
Dimensional Parameters				
Max. shaft outer diameter	d _{so_max}	19.	055	
Min. shaft outer diameter	d _{so_min}	19.	050	
Max. hub inner diameter	d _{hi_max}	19.	004	
Min. hub inner diameter	d _{hi_min}	18.	999	mm 👻
Limit values for diametrical interference (minimum and maximum diametrical interference values before assembly. Not include rotation, thermal, poisson's effects)	delta	Max 0.056	Min 0.046	
Limit values for diametrical interference (minimum and maximum diametrical interference values during operation. Includes rotation, thermal, poisson's effects)	delta	0.056	0.046	
Interference Parameters				
Minimum required interface pressure (interface pressure required to transmit torque and force)	rpi	33	8.1	MPa 👻
Limit values for resultant interface pressure due to diametrical interference (pressure values for minimum and maximum interference condition)	Р	87.3	71.7	MPa ▼
Factor of safety against sliding [= P _{min} / rpi]*	fose	2	17	
Assembly Parameters For Shrink Fit (For Max. Dian	netrical Inter	ference	÷)	
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft	T _{r,s}	ference -18	a) 0.4	
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub	T _{r,h}	ference -18 31	2) 30.4 7.1	°F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I	T _{r,s} T _{r,h}	ference -18 31	*) 0.4 7.1	°F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference	T _{r,s} T _{r,h}	ference -18 31 Max	0.4 7.1 Min	●F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range	Tr,s Tr,h Fit	ference -18 31 Max 69460	60.4 7.1 Min 57056	°F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter	T _{r,s} T _{r,h} Fit Fpf	ference -18 31 Max 69460	e) 60.4 7.1 Min 57056	°F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface	Tr,s Tr,h Fit Fpf ference)	ference -18 31 Max 69460 0.0	2) 60.4 7.1 Min 57056 020	°F ▼ N ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter	Tr,s Tr,h Fit Fpf ference) U _{inner} σ _{r,pressure}	ference -18 31 Max 69460 0.0	2) 0.4 7.1 Min 57056 020 7.3	°F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter Circumferential press-fit stress at inner diameter	Tr,s Tr,h Fit Fpf ference) U _{inner} σ _{e,pressure}	ference -18 31 Max 69460 0.0 -8	 a) a) a) b) c) c) d) <	°F ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter Circumferential press-fit stress at inner diameter Axial stress from applied axial force	Tr,s Tr,h Fit Fpf ference) U _{inner} σ _{e,pressure} σ _z	ference -18 31 Max 69460 0.0 -8 11 0	 a) a) a) b) c) <	oF ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter Circumferential press-fit stress at inner diameter Axial stress from applied axial force Shear stress from applied torque at inner diameter	Tr,s Tr,h Fit Gr,pressure $\sigma_{e,pressure}$ σ_z T	ference -18 31 Max 69460 0.0 -8 11 0 3	20.4 7.1 57056 020 7.3 5.7 .0 .7	°F ▼ N ▼ mm ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter Circumferential press-fit stress at inner diameter Axial stress from applied axial force Shear stress from applied torque at inner diameter Max radial centrifugal stress	Tr,s Tr,h Fit Fpf ference) U_{inner} $\sigma_{r,pressure}$ σ_{z} T $\sigma_{r,centrifugal}$	ference -18 31 69460 0.0 -8 11 0 3 0	2) 0.4 7.1 7.1 57056 020 7.3 5.7 .0 .0 .7	°F ▼ N ▼ MPa ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter Circumferential press-fit stress at inner diameter Axial stress from applied axial force Shear stress from applied torque at inner diameter Max radial centrifugal stress Max circumferential centrifugal stress	Genetrical Inter Tr,s Tr,h Fit Fpf ference) Uinner $\sigma_{r,pressure}$ $\sigma_{e,pressure}$ σ_z T $\sigma_{r,centrifugal}$ $\sigma_{e,centrifugal}$	ference -18 31 69460 0.0 -8 11 0 3 0 0) 0.4 7.1 7.1 57056 020 7.3 5.7 .0 .0 .7 .0 .0 .0	°F ▼ N ▼ MPa ▼
Assembly Parameters For Shrink Fit (For Max. Dian Required temperature of shaft for assembly if cooling shaft Required temperature of hub for assembly if heating hub Assembly Parameters For Press I Assembly force range to press fit for calculated interference range Hub Results (For Max. Diametrical Inter Radial displacement of inner surface Radial press fit stress at inner diameter Circumferential press-fit stress at inner diameter Axial stress from applied axial force Shear stress from applied torque at inner diameter Max radial centrifugal stress Max circumferential centrifugal stress Max Von Mises stress	Genetrical Inter Tr,s Tr,h Fit Fpf ference) Uinner $\sigma_{r,pressure}$ $\sigma_{e,pressure}$ σ_{z} T $\sigma_{r,centrifugal}$ $\sigma_{e,centrifugal}$ $\sigma_{e,centrifugal}$ σ_{mises}	ference -18 31 69460 0.0 -8 11 0 3 0 3 0 0	 a) a) a) a) b) c) <	°F ▼ N ▼ MPa ▼

Shaft Results (For Max. Diametr	ical Interference)		
Radial displacement of outer surface	Uouter	-0.008	mm 👻
Radial press fit stress at outer diameter	σ _{r,pressure}	-87.3	
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Max radial centrifugal stress	σ _{r,centrifugal}	0.0	
Max circumferential centrifugal stress	$\sigma_{\theta,centrifugal}$	0.0	
Max Von Mises stress	omises	330.7	
Factor of safety against yielding of shaft *	fos _{y,s}	0.98	

Shall be larger than relevant design factor. Green color means safe, red color means not safe according to input arameters.

Last modified 4/16/07 b	by Alex Slocu	m, with thanks to Clea	Joint_interference_fit.xls By Alex Slocum Kue'en Yang, Stephen Jaman, Richard Blakelock, Alexandra Nelson, Kurt McMurtrie are and pressure in shrink-fit bodies r numbers in BOLD, Results in RED		
Loads			terférence narameters		Fonations Ref
Torque to be transmitted (N-mm)	torque	250,000	Interference interface pressure to cause vielding in inner body (N/mm2) Pobmax	191402.66	
Axial force to be transmitted (N)	force	0	Interference interface pressure to cause yielding in outer body (N/mm2) Pibmax	229102.60	
Coefficient of friction	nu	1.05	Maximum interface pressure (interference fit causes yielding) Pimax	191402.66	
Operating temperature (°C)	ot	20	Minimum required interface pressure (N/mm ²) rPI	32.97	
Rotation speed (rpm)	npm	0	Differential Poisson radial interference due to axial force (mm) ddp	0.00E+00	1
Outer body input parameteres			Differential thermal radial expansion (mm) ted	0.00E+00	2
Outside diameter (mm)	poqo	50.800	Outer body rotating inner diameter radial displacement (mm) robd	0.00E+00	3
Inside diameter (mm)	obido	19.025	Inner body rotating outer diameter radial displacement (mm) nbd	0.00E+00	-7
Plus tolerance (nun)	obptol	0	Total additional diametrical interference amount to be added (to ibod) (mm) addi	0.00E+00	
Minus tolerance (nun)	obmtol	0	nterference fit calculations (assumes addi has been added to ibod)		
Outer body stress concentration factor	obsef	-	Maximum diametrical interference (nm) mardelta	2.50E-02	(See diagrams)
Modulus of elasticity (N/mm ²)	obe	6.89E+04	Maximum resulting interface pressure (N'mm ²) maxip	3.89E+01	5
Yield strength (N/mm ²)	obsy	324000	Minimum diametrical interference (mm) mindelta	2.50E-02	(See diagrams)
Poisson's ratio	obn	0.33	Minimum resulting interface pressure (N'nm ²) minip	3.89E+01	5
Coefficient of thermal expansion $(1/^{\circ}C)$	obcte	2.32E-05	Minimum safety margin (nin obtained pressure/required pressure) msmp	1.18	
Density (g/cm ³)	obtho	2.78	Maximum sustainable torque (N-mm) maxt	294,965.72	
Inner body input parameters			Minimum sustainable torque (N-mm) mint	2.95E+05	
Engagement length (mm)	1	12.700	Maximum torque limited by interference fit causing yielding yieldt	1.45E+09	
Outside diameter (mm)	bodi	19.050	Minimum safety margin (min obtained torque/required torque) msmt	1.18	1
Inside diameter (mm)	ibidi	0	buter body material stresses at maximum interface pressure		
OD if to compensate for rotational expansion (nm)		19.050	Radial displacement of inner surface (nm)	8.90E-03	6
Plus tolerance (mm)	ibptol	0	Radial press-fit stress at ID (Nmm ²) obsr	-3.89E+01	7
Minus tolerance (mm)	ibmtol	0	Circumferential press fit stress at ID (N'mm ²) obsc	5.16E+01	8
Inner body stress concentration factor	ibsef	1	Axial stress from applied axial Force (N'mm ²) obs2	0.00E+00	6
Modulus of elasticity (N'mm ²)	ibe	6.89E+04	Shear stress from applied Torque (N'mm ²) obtau	3.71E+00	10
Yield strength, obsy (N/mm ⁴)	ibsy	324000	Max radial centrifugal stress (Ninm2) obrc	0.00E+00	
Poisson's ratio	ibn	0.33	Max circumferential centrifugal stress (Nimm ⁴) obcc	0.00E+00	11
Coefficient of themal expansion $(1^{\circ}C)$	ibcte	2.32E-05	Max Von Mises stress (N/mm ²) obvm	7.89E+01	12
Density (g/cm ³)	ibrho	2.78	Resulting safety factor (yield stress) (scf ⁴⁴ Von Mises stress)	4106.572	
Shrink-fit design			mer body material stresses at maximum interface pressure		
Desired assembly clearance at deltaT (mm)	ddt	0.1	Radial displacement of outer surface (mm)	-3.60E-03	13
Required differential temperature if heating outer body (°C)	robdt	2.83E+02	Radial press-fit stress (Nmm ²) ibsr	-3.89E+01	14
Required differential temperature if cooling inner body (°C)	nbdt	2.83E+02	thsc	-3.89E+01	15
Press-fit design			Axial stress from applied axial Force (Nmm ²) ibsz	0.00E+00	16
Maximum assembly force to press fit (N)	Fpfmax	3.10E+04	Shear stress from applied Torque (Nimm ⁴) ibtau	1.84E+02	17
Minimum assembly force to press fit (N)	Fpfmin	3.10E+04	Max radial centrifigal stress (N'mm ²) ibro	0.00E+00	
			Max circumferential centrifugal stress at ID (Nimm ²) thee	0.00E+00	18
			Von Mises stress at ID (N'mm ²) thym	3.21E+02	19
Outer body heating temp in F 5	541 76483		Resulting safety factor (Vield stress) (scf*Von Mises stress)	1008.215	
May Sustainable Tarante 3	10 07220 NOC				00

Appendix B: Mathcad Calculations for the Design of the Shaft

Cantilevar Beam Calculations:

First we determined the values of some specific characteristics of our cantilever beam and the forces applied. We used a length of half a meter, a 3/4 inch diamter, a downward force of 200N which is equivilent to that of a 45lb plate weight, and then a Torque of 100N*m which is the equivalent to that same weight applying a torque at a distance of half a meter. We then based on material properties for ALUMINUM 2024 HEAT TREATED chose the specific gravity and calculated the weight of the beam.

$$L_{m} := .5m \qquad \gamma := .1 \frac{1bf}{in^{3}}$$

$$d := .75in$$

$$E_{m} := 200N \qquad A_{m} := \pi \cdot \left(\frac{d}{2}\right)^{2} = 2.85 \times 10^{-4} \text{ m}^{2}$$

$$I_{m} := 100N \cdot m \qquad w_{1} := \gamma \cdot A = 7.737 \cdot \frac{N}{m}$$

Based on the device and how the weight will be applied we were able to determine the singularity functions for the force, shear and moment diagrams and solving for the integration constants which both came out to 0.

$$S(x,z) := if(x \ge z,1,0)$$

x := 0,0.001·L..L

$$q(x) = R1 < x - 0 >^{-1} - w1 < x - 0 >^{-0} + w1 < x - L >^{-0} - F < x - L >^{-1}$$

$$V(x) = R1 < x - 0 >^{-0} - w1 < x - 0 >^{-1} + w1 < x - L >^{-1} - F < x - L >^{-0} + C1$$

$$M(x) = R1 < x - 0 >^{-1} - (w1)/2 < x - 0 >^{-2} + (w1)/2 < x - L >^{-2} - F < x - L >^{-1} + C1x + C2$$

$$V(0-) = 0$$

$$M(0-) = 0$$

C1 = 0
C2 = 0

$$R_1 := \frac{\frac{w_1}{2} \cdot (L - 0)^2}{L} = 1.934 \cdot N$$

 $\begin{aligned} & \mathsf{SINGULARITY} \ \mathsf{FUNCTIONS}: \\ & \mathsf{q}(x) := \mathsf{R}_1 \cdot \mathsf{S}(x, 0\mathsf{m}) \cdot (x-0)^{-1} - \mathsf{w}_1 \cdot \mathsf{S}(x, 0\mathsf{m}) \cdot (x-0)^0 + \mathsf{w}_1 \cdot \mathsf{S}(x, L) \cdot (x-0)^0 - \mathsf{F} \cdot \mathsf{S}(x, L) \cdot (x-L)^{-1} \end{aligned}$

$$\underbrace{V}_{W}(x) := R_1 \cdot S(x, 0m) \cdot (x - 0)^0 - w_1 \cdot S(x, 0m) \cdot (x - 0)^1 + w_1 \cdot S(x, L) \cdot (x - 0)^1 - F \cdot S(x, L) \cdot (x - L)^0$$

$$\underbrace{M(x)}_{W} := R_1 \cdot S(x, 0m) \cdot (x - 0)^1 - \frac{w_1}{2} \cdot S(x, 0m) \cdot (x - 0)^2 + \frac{w_1}{2} \cdot S(x, L) \cdot (x - 0)^2 - F \cdot S(x, L) \cdot (x - L)^1$$

We then graphed the shear and moment diagrams and compared the values across the length of the beam to determine the critical section.

CROSS SECTION OF BEAM

The critical points will be the top (A) the middle left side (B) the bottom (C) and the middle right (D).

From here we calculated the inertia and polar moments of iniertia at each of the critical points and calculated the stresses acting on each point based on stress cube diagrams. We then used these individual stresses to calculate the overall von-mises stress acting at each critical point. V(0m) = 1.934 N V(1m) = -198.066 N M(.37m) = 0.186 J M(.38m) = 0.176 J M(.385m) = 0.171 J $M_{max} := M(.25m) = 0.242 J$

CRITICAL POINT A

$$I := \frac{\pi \cdot d^{4}}{64} = 6.465 \times 10^{-9} \text{ m}^{4}$$

$$J := \frac{\pi \cdot d^{4}}{32} = 1.293 \times 10^{-8} \text{ m}^{4}$$

$$\sigma_{xxa} := \frac{M(0.25m) \cdot \frac{d}{2}}{I} = 3.562 \times 10^{5} \text{ Pa}$$

$$\tau_{xza} := \frac{T \cdot \frac{d}{2}}{J} = 7.367 \times 10^{7} \text{ Pa}$$

von mises stresses

$$\sigma_{1a} := \frac{\sigma_{xxa}}{2} + \sqrt{\left(\frac{\sigma_{xxa}}{2}\right)^2 + \tau_{xza}^2} = 7.385 \times 10^7 \text{ Pa}$$
$$\sigma_{3a} := \frac{\sigma_{xxa}}{2} - \sqrt{\left(\frac{\sigma_{xxa}}{2}\right)^2 + \tau_{xza}^2} = -7.349 \times 10^7 \text{ Pa}$$
$$\sigma_{2a} := 0$$

$$\sigma_{\rm vma} := \sqrt{\sigma_{1a}^2 + \sigma_{3a}^2 - \sigma_{1a} \cdot \sigma_{3a}} = 1.276 \times 10^8 \, {\rm Pa}$$

CRITICAL POINT B

$$\tau_{xyb1} := \frac{4}{3} \cdot \frac{V(0.25m)}{A} = 0 Pa$$

 $\tau_{xyb2} := \frac{T \cdot \frac{d}{2}}{J} = 7.367 \times 10^7 Pa$

 $\tau_{xybtotal} \coloneqq \tau_{xyb1} + \tau_{xyb2} = 7.367 \times 10^7 \text{ Pa}$

von mises stresses

$$\sigma_{1b} := \sqrt{\tau_{xybtotal}^2} = 7.367 \times 10^7 \text{ Pa}$$

 $\sigma_{3b} := -\sqrt{\tau_{xybtotal}^2} = -7.367 \times 10^7 \text{ Pa}$
 $\sigma_{vmb} := \sqrt{\sigma_{1b}^2 + \sigma_{3b}^2 - \sigma_{1b} \cdot \sigma_{3b}} = 1.276 \times 10^8 \text{ Pa}$

CRITICAL POINT C

$$\sigma_{\text{xxc}} \coloneqq \frac{-M(0.25\text{m}) \cdot \frac{d}{2}}{I} = -3.562 \times 10^5 \text{ Pa}$$

$$\tau_{\text{xzc}} \coloneqq \frac{\mathbf{T} \cdot \frac{\mathbf{d}}{2}}{J} = 7.367 \times 10^7 \, \text{Pa}$$

von mises stresses

$$\sigma_{1c} := \frac{\sigma_{xxc}}{2} + \sqrt{\left(\frac{\sigma_{xxc}}{2}\right)^2 + \tau_{xzc}^2} = 7.349 \times 10^7 \text{ Pa}$$

$$\sigma_{3c} := \frac{\sigma_{xxc}}{2} - \sqrt{\left(\frac{\sigma_{xxc}}{2}\right)^2 + \tau_{xzc}^2} = -7.385 \times 10^7 \text{ Pa}$$

$$\sigma_{2c} := 0$$

$$\sigma_{vmc} := \sqrt{\sigma_{1c}^2 + \sigma_{3c}^2 - \sigma_{1c} \cdot \sigma_{3c}} = 1.276 \times 10^8 \text{ Pa}$$

CRITICAL POINT D

$$\tau_{xyd1} := \frac{4}{3} \cdot \frac{V(0.25m)}{A} = 0 Pa$$

$$\tau_{xyd2} := \frac{T \cdot \frac{d}{2}}{J} = 7.367 \times 10^7 Pa$$

$$\tau_{xydtota1} := \tau_{xyd1} - \tau_{xyd2} = -7.367 \times 10^7 Pa$$

von mises stresses $\sigma_{1d} := \sqrt{\tau_{xydtotal}^2} = 7.367 \times 10^7 \text{ Pa}$ $\sigma_{3d} := -\sqrt{\tau_{xydtotal}^2} = -7.367 \times 10^7 \text{ Pa}$ $\sigma_{vmd} := \sqrt{\sigma_{1d}^2 + \sigma_{3d}^2 - \sigma_{1d} \cdot \sigma_{3d}} = 1.276 \times 10^8 \text{ Pa}$

After calculating these stresses we then used the material properties of our beam to calculate the safety factors based on the Max Shear Stress theory with the yield strength.

MATERIAL 2024 HEAT TREATED WROUGHT ALLUMINUM ALLOY

$$S_{ut} \coloneqq 441 \text{MPa} \qquad S_{utc} \coloneqq 441 \qquad S_{ut} = 4.41 \times 10^8 \text{Pa}$$
$$N_{sfa} \coloneqq \frac{S_y}{\sigma_{vma}} = 2.273 \qquad N_{sfc} \coloneqq \frac{S_y}{\sigma_{vmc}} = 2.273$$

Max Shear Stress Theory:

S. := 290MPa

$$N_{sfamss} := \frac{.5 \cdot S_y}{\tau_{xza}} = 1.968$$
$$N_{sfcmss} := \frac{.5 \cdot S_y}{\tau_{xzc}} = 1.968$$

To ensure a more accurate safety factor taking into account cyclic loading properties we decided to solve for the safety factors with the inclusion of calculating equivalent Strength value. We used correction factors for the size of the beam, the load, the temperature and the surface of the beam. Since there is no radius of curvature then there is no Kt, Kts, Kf, or Kfs values

$$S_{e^{i}} := .5 \cdot S_{ut} = 2.205 \times 10^{8} Pa$$

 $C_{size} := 0.869 \cdot .75^{-0.097} = 0.894$
 $C_{load} := 1$
 $C_{temp} := 1$

Creliab := .702

 $S_e := C_{size} \cdot C_{load} \cdot C_{temp} \cdot C_{reliab} \cdot S_{e'} = 1.383 \times 10^8 Pa$

for 99.99% reliability

$$S_m := .9 \cdot S_{ut} = 3.969 \times 10^8 Pa$$

We then created the log mean diagram for the device and graphed it accordingly using 10^A3 cycles and 10^A6 cycles as the range for our linear slope. We estimated a value of 200,000 cycles over the course of 10 years. This took into account the average size of the class, each student performing 10 cycles of force each term of classes over the course of 10 years.

$$\begin{split} N_{c} &:= 1000, 5000...10^{9} \\ & \underbrace{S(N_{c}, z) := if(N_{c} \ge z, 1, 0)}_{N_{1} := 10^{3}} \\ & N_{2} := 10^{6} \\ & z := log(N_{1}) - log(N_{2}) = -3 \\ & b_{sf} := \frac{1}{z} \cdot log\left(\frac{S_{m}}{S_{e}}\right) = -0.153 \\ & a_{sf} := \frac{S_{m}}{10^{3 \cdot b_{sf}}} = 1.139 \times 10^{9} Pa \\ & S_{f}(N_{c}) := \left[S(N_{c}, 0) \cdot \left(a_{sf} \cdot N_{c}^{\ b_{sf}}\right) - S\left(N_{c}, 10^{6}\right) \cdot \left(a_{sf} \cdot N_{c}^{\ b_{sf}}\right) + S\left(N_{c}, 10^{6}\right) \cdot S_{e}\right] \end{split}$$

$$S_f(2.10^5) = 1.768 \times 10^8 Pa$$

Considering this would be a cyclic loading, we used the maximum and minimum values of the force and torsional values to calculate the mean and alternating Force and Torque values as shown below. This would help better simulate the idea of someone loading the 45lbf wegiht and then removing the load entirely each time.

$$F_{max} := F = 200 N$$

$$F_{min} := 0 = 0$$

$$F_{alt} := \frac{F_{max} - F_{min}}{2} = 100 N$$

$$F_{m} := \frac{F_{max} + F_{min}}{2} = 100 N$$

$$T_{max} := T = 100 J$$

$$T_{min} := 0$$

$$T_{alt} := \frac{T_{max} - T_{min}}{2} = 50 J$$

$$T_{m} := \frac{T_{max} + T_{min}}{2} = 50 J$$

F

From here we used these alternating and mean values to recalculate the vonmises stresses at each critical point on the critical section of the beam. From there we then calculated the safety factors at each critical point with the new von-mises stresses, the ultimate tensile strength, and the finite strength of 200,000 cycles.

POINT A $\tau_{xzaalt} := \frac{T_{alt} \cdot \frac{d}{2}}{J} = 3.683 \times 10^7 \text{ Pa}$ $\tau_{xzam} := \frac{T_{m} \cdot \frac{d}{2}}{J} = 3.683 \times 10^7 \text{ Pa}$ $\sigma_{1aalt} := \frac{\sigma_{xxa}}{2} + \sqrt{\left(\frac{\sigma_{xxa}}{2}\right)^2 + \tau_{xzaalt}^2} = 3.701 \times 10^7 \text{ Pa}$ $\sigma_{1am} := \frac{\sigma_{xxa}}{2} + \sqrt{\left(\frac{\sigma_{xxa}}{2}\right)^2 + \tau_{xzam}^2} = 3.701 \times 10^7 \text{ Pa}$

$$\sigma_{\text{3aalt}} \coloneqq \frac{\sigma_{\text{xxa}}}{2} - \sqrt{\left(\frac{\sigma_{\text{xxa}}}{2}\right)^2 + \tau_{\text{xzaalt}}^2} = -3.666 \times 10^7 \text{ Pa}$$

$$\sigma_{3am} \coloneqq \frac{\sigma_{xxa}}{2} - \sqrt{\left(\frac{\sigma_{xxa}}{2}\right)^2 + \tau_{xzam}^2} = -3.666 \times 10^7 \text{ Pa}$$

$$\sigma_{\text{aalt}} := \sqrt{\sigma_{1\text{aalt}}^2 + \sigma_{3\text{aalt}}^2 - \sigma_{1\text{aalt}} \cdot \sigma_{3\text{aalt}}} = 6.38 \times 10^7 \, \text{Pa}$$

$$\sigma_{\rm am'} \coloneqq \sqrt{\sigma_{\rm 1am}^2 + \sigma_{\rm 3am}^2 - \sigma_{\rm 1am} \cdot \sigma_{\rm 3am}} = 6.38 \times 10^7 \, \rm Pa$$

$$S_{nfin} := S_f(2.10^5) = 1.768 \times 10^8 Pa$$

 $N_{sffina} := \frac{S_{nfin} \cdot S_{ut}}{S_{ut} \cdot \sigma_{aalt'} + S_{nfin} \cdot \sigma_{am'}} = 1.978$ The safety factor calculated for critical point A.

POINT B

$$J_{xybbls} = \frac{4}{3} \cdot \frac{V(0.25m)}{A} = 0 Pa$$

$$\tau_{xyb2alt} := \frac{T_{alt} \cdot \frac{d}{2}}{J} = 3.683 \times 10^7 Pa$$

$$\tau_{xyb2m} := \frac{T_{m} \cdot \frac{d}{2}}{J} = 3.683 \times 10^7 Pa$$

$$\tau_{xybtotalam} := \tau_{xyb1} + \tau_{xyb2alt} = 3.683 \times 10^7 Pa$$

$$von \text{ mises stresses}$$

$$\sigma_{1bam} := \sqrt{\tau_{xybtotalam}}^2 = 3.683 \times 10^7 Pa$$

$$\sigma_{3bam} := -\sqrt{\tau_{xybtotalam}}^2 = -3.683 \times 10^7 Pa$$

$$\sigma_{vmbam} := \sqrt{\sigma_{1bam}}^2 + \sigma_{3bam}^2 - \sigma_{1bam} \cdot \sigma_{3bam} = 6.38 \times 10^7 Pa$$
The safety factor calculated for critical point B.

$$N_{sffinb} := \frac{S_{nfin} \cdot S_{ut}}{S_{ut} \cdot \sigma_{vmbam} + S_{nfin} \cdot \sigma_{vmbam}} = 1.978$$

POINT C

$$\sigma_{\text{maxev}} = \frac{-M(0.25\text{m}) \cdot \frac{d}{2}}{I} = -3.562 \times 10^5 \text{ Pa}$$
$$\tau_{\text{maxem}} := \frac{T_{\text{m}} \cdot \frac{d}{2}}{J} = 3.683 \times 10^7 \text{ Pa}$$

von mises stresses

$$\sigma_{1\text{cam}} \coloneqq \frac{\sigma_{\text{xxc}}}{2} + \sqrt{\left(\frac{\sigma_{\text{xxc}}}{2}\right)^2 + \tau_{\text{xzcam}}^2} = 3.666 \times 10^7 \text{ Pa}$$
$$\sigma_{3\text{cam}} \coloneqq \frac{\sigma_{\text{xxc}}}{2} - \sqrt{\left(\frac{\sigma_{\text{xxc}}}{2}\right)^2 + \tau_{\text{xzcam}}^2} = -3.701 \times 10^7 \text{ Pa}$$
$$\sigma_{\text{vmcam}} \coloneqq \sqrt{\sigma_{1\text{cam}}^2 + \sigma_{3\text{cam}}^2 - \sigma_{1\text{cam}} \cdot \sigma_{3\text{cam}}} = 6.38 \times 10^7 \text{ Pa}$$

 $N_{sffinc} := \frac{S_{nfin} \cdot S_{ut}}{S_{ut} \cdot \sigma_{vmcam} + S_{nfin} \cdot \sigma_{vmcam}} = 1.978$ The safety factor calculated for critical point C.

POINT D

$$T_{xyd2am} := \frac{4}{3} \cdot \frac{V(0.25m)}{A} = 0 \text{ Pa}$$

$$\tau_{xyd2am} := \frac{T_m \cdot \frac{d}{2}}{J} = 3.683 \times 10^7 \text{ Pa}$$

$$\tau_{xydtotalam} := \tau_{xyd1} - \tau_{xyd2am} = -3.683 \times 10^7 \text{ Pa}$$

von mises stresses

$$\sigma_{1 \text{dam}} := \sqrt{\tau_{xy \text{dtotalam}}^2} = 3.683 \times 10^7 \text{ Pa}$$

 $\sigma_{3 \text{dam}} := -\sqrt{\tau_{xy \text{dtotalam}}^2} = -3.683 \times 10^7 \text{ Pa}$
 $\sigma_{vm \text{dam}} := \sqrt{\sigma_{1 \text{dam}}^2 + \sigma_{3 \text{dam}}^2 - \sigma_{1 \text{dam}} \cdot \sigma_{3 \text{dam}}} = 6.38 \times 10^7 \text{ Pa}$

 $N_{sffind} := \frac{S_{nfin} \cdot S_{ut}}{S_{ut} \cdot \sigma_{vmdam} + S_{nfin} \cdot \sigma_{vmdam}} = 1.978$ The safety factor calculated for critical point D.

With a beam 0.5m in length and 3/4 of an inch in diamter when there is 45lbf (200N) of force and 100N*m of torque applied the Von Mises stresses at critical points A(top), B(left), C(bottom) and D(right) are $6.38*10^{\circ}7$ pascals. This leads to a safety factor of 2.102 for the beam.

Appendix C: Photos of the final device

The final device with the addition of the hanging weight and the adjustable gripper arm.

A view of the adjustable gripper holding onto the back of a table

Close up of the strain gages attached to measure both bending and torsional stresses.

The device under the effect of the maximum weight designed for (45lbf).

Side view of the device under the effect of the maximum load designed for (45lbf).

The devices' strain gages hooked up and being measured by a volt meter.