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# Market and Credit Risk Models and Management Report

by

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## Abstract

This report is for MA575: Market and Credit Risk Models and Management, given by Professor Marcel Blais.

In this project, three different methods for estimating Value at Risk (VaR) and Expected Shortfall (ES) are used, examined, and compared to gain insightful information about the strength and weakness of each method.

In the first part of this project, a portfolio of underlying assets and vanilla options were formed in an Interactive Broker paper trading account. Value at Risk was calculated and updated weekly to measure the risk of the entire portfolio.

In the second part of this project, Value at Risk was calculated using semi-parametric model. Then the weekly losses of the stock portfolio and the daily losses of the entire portfolio were both fitted into ARMA(1,1)-GARCH(1,1), and the estimated parameters were used to find their conditional value at risks (CVaR) and the conditional expected shortfalls (CES).

Key Words: Portfolio Optimization; Value at Risk; Expected Shortfall; ARMA-GARCH Model; Risk Reduction

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## Contents

<b>1 Introduction</b>	<b>1</b>
<b>2 Background</b>	<b>3</b>
2.1 Basic Mathematical Finance Concepts	3
2.2 Basic Mathematical Formulas	3
<b>3 Securities Information</b>	<b>6</b>
<b>4 Computation</b>	<b>7</b>
4.1 Computational Process	7
4.2 Option Strategies	9
<b>5 Risk Analysis</b>	<b>15</b>
5.1 Value at Risk in Normal and t Distribution	15
5.2 Goodness of Fit	18
5.3 Value at Risk in Polynomial Tails	20
5.4 Value at Risk in GARCH Modeling	23
<b>6 Risk Reduction</b>	<b>31</b>
<b>7 Conclusion</b>	<b>34</b>
<b>References</b>	<b>35</b>
<b>A Appendix</b>	<b>36</b>
A.1 Matlab Code	36
A.2 Matlab Results	51

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## List of Charts

1	Each security's price tendency in one year	8
2	ADS's price tendency chart in 2011	9
3	ADS's price tendency chart in 2012	9
4	IBM's price tendency chart in 2011	10
5	IBM's price tendency chart in 2012	10
6	GS's price tendency chart in 2011	11
7	GS's price tendency chart in 2012	11
8	VXX's price tendency chart in 2011	12
9	VXX's price tendency chart in 2012	12
10	JPM's price tendency chart in 2011	13
11	JPM's price tendency chart in 2012	13
12	UPS's price tendency chart in 2011	13
13	UPS's price tendency chart in 2012	14
14	8 Stock's price tendency for week 1,2	16
15	7 Stock's price tendency for week 1, 2	16
16	8 Stock's price tendency week 2, 3	17
17	7 Stock's price tendency for week 2,3	18
18	VaR with alpha between 0.9 to 1	20
19	Expected shortfall	21
20	8 Stocks' performance in one week, April 5-April 12	21
21	7 Stocks' performance in one week, April 5-April 12	22
22	Conditional standard deviations and standardized residuals under t	24
23	$\mu$ over time under t	25

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24	conditional standard deviations and standardized residuals under Gaussian	26
25	$\mu$ over time under Gaussian	26
26	VaR, ES and actual loss under Gaussian Innovations	27
27	VaR, ES and actual loss percentages under Gaussian Innovations	28
28	VaR, ES and actual loss under t Innovations	28
29	VaR, ES and actual loss percentages under t Innovations	29
30	Comparison of ES and VaR under normal and t distribution	30
31	Average Actual Daily Loss of Each Stock	31
32	Daily Loss Movement of Each Stock	32

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## **Lists of Tables**

1	Expected daily return of each security	7
2	Each security's weight, shares and the money we need to invest	7
3	Details of each option strategy	14
4	VaR in each week	15
5	Chi-squared test for stock prices' weekly return under two distributions	19
6	Parameters under t innovation	23
7	Parameters under Gaussian innovation	25
8	Daily VaR and ES under normal and t distribution	29
9	New Status of Options	32
10	Comparison of VaR and ES before and after risk reduction	33

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# 1 Introduction

The project is a summary of the work I have completed in my course MA575: Market and Credit Risk Models and Management. The course helps me to measure and manage financial risk using the most important quantitative models with special emphasis on market and credit risk.

It starts with the introduction of metrics of risk such as volatility, value-at-risk and expected shortfall and with the fundamental quantitative techniques used in financial risk evaluation and management.

The next section is devoted to market risk, including volatility modeling, time series, non-normal heavy tailed phenomena, and multivariate notions of codependence such as copulas, correlations and tail-dependence. [1]

This project is based on the course lecture and the data collected through Yahoo Finance [2] website and Interactive Brokers [3] software.

Early attempts to measure risk, such as duration analysis, were somewhat primitive and of only limited applicability. Another traditional risk measure is volatility. The main problem with volatility, however, is that it does not care about the direction of an investment's movement. [4] "Since investors only want to reduce the risk of losing money and not gaining money, volatility is not a desirable risk measure. Value at Risk has been the most widely used risk measure because it focuses on the risk of losing money and can be applied to all kinds of risks." [5]

In the first part of this project, 15 stocks, ETNs or ETFs traded on the New York Stock Exchange were chosen. The weekly adjusted closing prices of stocks from 2011-01-03 to 2012-03-09 were converted into weekly log returns and then utilized to obtain the optimal weights for the stock portfolio. Option strategies were chosen for 6 component stocks in an attempt to reduce risks in those stock positions. The entire portfolio was formed using an Interactive Brokers paper trading account.

The loss distribution of the entire portfolio was estimated respectively under two assumptions, One is that risk-factor changes follow normal distribution, and the other one is that risk-factor changes follow t distribution with 4 degrees of freedom. The risk-factor



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changes in this project are comprised of stocks' log returns, change of risk-free rate and change of implied volatility. The weekly rate of return of the 1-year Treasury bill is used as the risk-free rate, and the weekly change of the VIX is used as change of volatility in this project. The historical weekly data for the risk-factor changes span from 2011-01-03 to 2012-03-09. Value at Risk was then calculated and updated weekly to measure the risk of our entire portfolio.

In the second part of this project, Value at Risk was calculated using semi-parametric model. "The characterization of semiparametric models as having a finite-dimensional parameter of interest (the "parametric component") and an infinite-dimensional nuisance parameter (the "nonparametric component") was given by Begun et al. (1983)." [6] Then the weekly stock portfolio losses were fitted into an ARMA(1,1)-GARCH(1,1) model, and the estimated parameters were used to find the conditional Value at Risk (CVaR) and conditional Expected Shortfall (CES). Finally, the daily losses of the entire portfolio were fitted into ARMA(1,1)-GARCH(1,1) and its CVaR and CES were estimated.

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## 2 Background

### 2.1 Basic Mathematical Finance Concepts

**Definition 1:** The loss operator maps risk-factor changes into losses,  $L_{[t]} : R^d \rightarrow R$  and is defined as  $L_{[t]}(\underline{X}) = -[f(t+1, \underline{Z}_t + \underline{X}) - f(t, \underline{Z}_t)]$  for  $\underline{X} \in R^d$ . [7]

**Definition 2:** Given a confidence level  $\alpha \in (0, 1)$ , the value-at-risk, or VaR of our portfolio is given by the smallest number  $l$  such that the probability that the loss  $L$  exceeds  $l$  is no larger than  $1 - \alpha$ . Formally,  $VaR_\alpha = \inf\{l \in R : P(L > l) \leq 1 - \alpha\} = \inf\{l \in R : F_L(l) \geq \alpha\}$ . [8]

### 2.2 Basic Mathematical Formulas

#### 1. Portfolio Loss

The loss of stocks:

$$-[\lambda_1 S_1 X_1^{\log} + \lambda_2 S_2 X_2^{\log} + \dots + \lambda_{15} S_{15} X_{15}^{\log}]$$

The loss of options:

$$-[C_t^{BS1} \Delta + C_s^{BS1} S_1 X_1^{\log} + C_r^{BS1} X_{16} + C_\sigma^{BS1} X_{17} + C_t^{BS2} \Delta + C_s^{BS2} S_2 X_2^{\log} + C_r^{BS2} X_{16} + C_\sigma^{BS2} X_{17}]$$

Combine the two equations:

$$\begin{aligned} & -[(C_t^{BS1} + C_t^{BS2}) \Delta + (C_r^{BS1} + C_r^{BS2}) X_{16} + (C_\sigma^{BS1} + C_\sigma^{BS2}) X_{17} \\ & + (C_s^{BS1} + \lambda_1) S_1 X_1^{\log} + (C_s^{BS2} + \lambda_2) S_2 X_2^{\log} + \lambda_3 S_3 X_3^{\log} + \dots + \lambda_{15} S_{15} X_{15}^{\log}] \end{aligned}$$

Then show it in matrix form:

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$$[(C_s^{BS1} + \lambda_1)S_1, (C_s^{BS2} + \lambda_2)S_2, \lambda_3 S_3, \dots, \lambda_{15} S_{15}, \Sigma C_r^{BS}, \Sigma C_\sigma^{BS}, \Sigma C_t^{BS}] * \begin{bmatrix} X_1^{\log} \\ X_2^{\log} \\ \vdots \\ X_{15}^{\log} \\ X_{16} \\ X_{17} \\ \Delta \end{bmatrix}$$

## 2. Value at Risk

Value at Risk is determined as follows:

$$\text{Suppose } F_L \sim N(\mu, \sigma^2), \text{ fix } \alpha \in (0,1)$$

$$\text{Then } VaR_\alpha = \mu + \sigma N^{-1}(\alpha)$$

$F_L$  is the cumulative loss distribution function and  $N^{-1}(\alpha)$  is the  $\alpha$ -quantile of the standard normal distribution.

Also suppose our loss  $L$  is such that  $\frac{L - \mu}{\sigma}$  has a standard t-distribution with  $\nu$  degree

freedom, denoted as  $L \sim t(\nu, \mu, \sigma^2)$ .

$$E(L) = \mu \quad \text{and} \quad Var(L) = \frac{\nu \sigma^2}{\nu - 2} \quad \text{for } \nu > 2$$

$VaR_\alpha = \mu + \sigma t_\nu^{-1}(\alpha)$ , where  $t_\nu$  is the cumulative distribution function of the standard t-distribution.

## 3. Expected Shortfall

ES for a Normal Loss Distribution

Suppose  $F_L \sim N(\mu, \sigma^2)$ ,  $F_L$  is the loss cdf.

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Let  $Z = \frac{L - \mu}{\sigma} \sim N(0,1)$ , then

$$ES_\alpha = E[L / L \geq VaR_\alpha] = E[\mu + \sigma Z / L \geq VaR_\alpha] = \mu + \sigma E[Z / Z \geq q_\alpha(Z)] = \mu + \sigma ES_\alpha(Z)$$

$$ES_\alpha(Z) = E[Z / Z \geq q_\alpha(Z)] = \frac{1}{1-\alpha} \int_{N^{-1}(\alpha)}^{\infty} l\phi(l)dl$$

$$\int l\phi(l)dl = \frac{1}{\sqrt{2\pi}} \int l e^{-\frac{l^2}{2}} dl = \frac{1}{\sqrt{2\pi}} \int (-du)e^u = -\frac{1}{\sqrt{2\pi}} e^u + C = -\frac{1}{\sqrt{2\pi}} e^{-\frac{l^2}{2}} + C$$

$$u = -\frac{l^2}{2} \quad \text{and} \quad du = -ldl$$

$$ES_\alpha(Z) = \frac{1}{1-\alpha} \left[ -\frac{1}{\sqrt{2\pi}} e^{-\frac{l^2}{2}} \right]_{N^{-1}(\alpha)}^{\infty} = \frac{1}{1-\alpha} [-\phi(l)]_{N^{-1}(\alpha)}^{\infty} = \frac{1}{1-\alpha} [-\lim_{l \rightarrow \infty} \phi(l) - (-\phi(N^{-1}(\alpha)))] = \frac{\phi[N^{-1}(\alpha)]}{1-\alpha}$$

So  $ES_\alpha = \mu + \sigma \frac{\phi[N^{-1}(\alpha)]}{1-\alpha}$  where  $\phi$  is the pdf of the standard normal distribution.

ES for t-Distribution

Suppose loss  $L$  is such that  $\tilde{L} = \frac{L - \mu}{\sigma}$  has a standard t distribution with  $\nu$  degrees of freedom.

As in the prior calculation,  $ES_\alpha = \mu + \sigma ES_\alpha(\tilde{L})$

Then  $ES_\alpha(\tilde{L}) = \frac{ES_\alpha - \mu}{\sigma} = \frac{g_\nu[t_\nu^{-1}(\alpha)]}{1-\alpha} \cdot \left( \frac{\nu + [t_\nu^{-1}(\alpha)]^2}{\nu - 1} \right)$ , where  $t_\nu$  is the cdf and  $g_\nu$  is the pdf of the standard t-distribution. [9]

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### **3 Securities Information**

#### **Stock:**

1. JP Morgan Chase & Co. Common St (JPM)
2. International Business Machines (IBM)
3. Goldman Sachs Group, Inc. (The) (GS)
4. Mastercard Incorporated Common (MA)
5. Visa Inc. (V)
6. Alliance Data Systems Corporati (ADS)
7. Walt Disney Company (The) Commo (DIS)
8. Exxon Mobil Corporation Common (XOM)
9. United Parcel Service, Inc. Com (UPS)
10. Home Depot, Inc. (The) Common S (HD)

#### **ETF:**

11. Vanguard MSCI Emerging Markets ETF (VWO)
12. iPath S&P 500 VIX Short-Term Futures ETN (VXX)
13. SPDR S&P Oil & Gas Explor & Pro (XOP)
14. Materials Select Sector SPDR (XLB)
15. iShares Russell 2000 (IWM)

## 4 Computation

### 4.1 Computational Process

First, we collect one year of historical price data for each security, from January 3, 2011 to March 9, 2012 from Yahoo Finance. [8] Then we calculate the expected daily return of each security using daily adjusted closing price.

The expected daily return of each security is illustrated below in Table 1:

<i>Security</i>	<i>Expected Daily Return</i>	<i>Security</i>	<i>Expected Daily Return</i>
<i>JPM</i>	0.000187776775545	<i>XOM</i>	0.000621438729831
<i>IBM</i>	0.001193004816320	<i>UPS</i>	0.000393152543798
<i>GS</i>	-0.000977257837232	<i>HD</i>	0.001277847300046
<i>MA</i>	0.002427719813423	<i>VWO</i>	-0.000068006827124
<i>V</i>	0.001917113620534	<i>VXX</i>	-0.000520637382104
<i>ADS</i>	0.001923727717883	<i>XLB</i>	0.000054368562369
<i>DIS</i>	0.000592716813379	<i>IWM</i>	0.000309975564376
<i>XOP</i>	0.000743118243926		

Table 1

From table 1 we can see that the expected daily returns of GS, VWO and VXX are negative, which means that we would lose money in the future if we bought these three securities.

The summation of each stock's weight value is 1:

$$\omega_1 + \omega_2 + \dots + \omega_{15} = 1$$

Then we use the portfolio optimization MATLAB code to calculate each security's weight, shares and the money we require to invest as follows:

<i>Security</i>	<i>Weight</i>	<i>Money</i>	<i>Shares</i>	<i>Security</i>	<i>Weight</i>	<i>Money</i>	<i>Shares</i>
<i>JPM</i>	0.1732	69280	1640	<i>UPS</i>	0.1571	62840	803

<i>IBM</i>	0.3901	156040	768	<i>HD</i>	0.2656	106240	2169
<i>GS</i>	-0.2394	-95760	-775	<i>VWO</i>	-0.1752	-70080	-1568
<i>MA</i>	0.1631	65240	155	<i>VXX</i>	0.1393	55720	2620
<i>V</i>	0.1281	51240	440	<i>XOP</i>	0.1366	54640	916
<i>ADS</i>	0.3323	132920	1075	<i>XLB</i>	-0.3128	-125120	-3383
<i>DIS</i>	0.0699	27960	639	<i>IWM</i>	-0.2896	-115840	-1400
<i>XOM</i>	0.0618	24720	285				

Table 2

Based the results we calculated, the weights for GS, VWO, XLB and IWM are all negative, which means the expecting trend for these three securities price is downward. So we possibly will lose money on these securities.

Using all the securities' historical price data, we construct a chart of their price movements as follows:

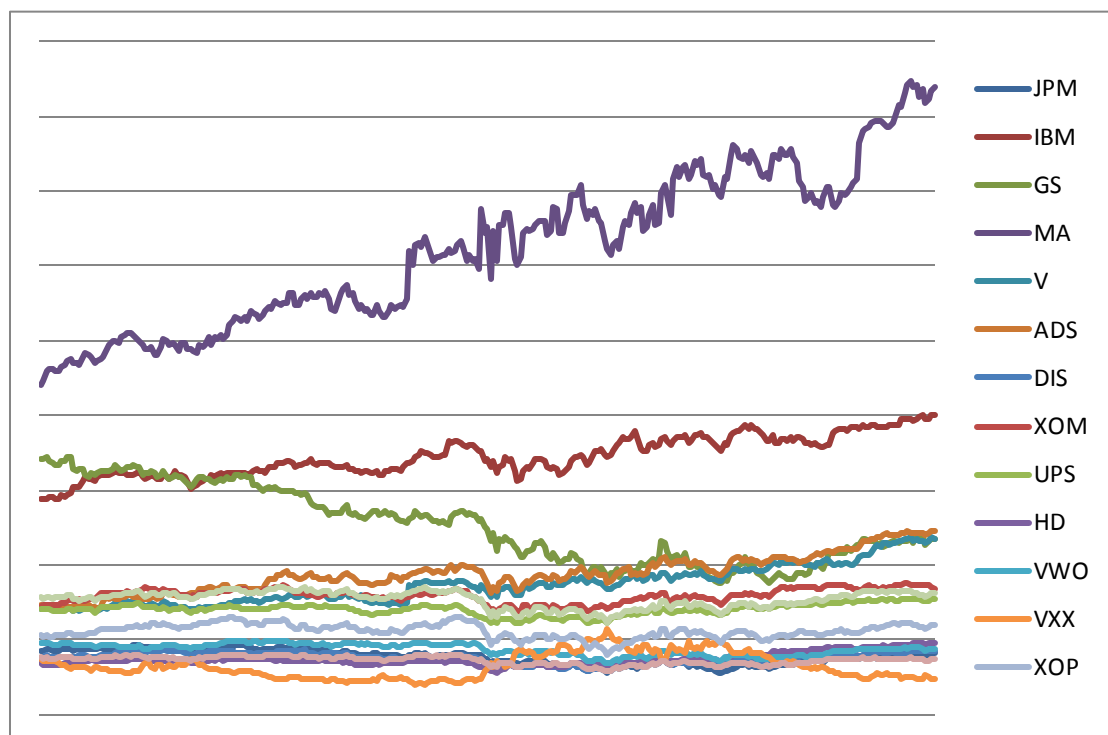


Chart 1

The weights we calculated are consistent with the tendency of securities' historical price data based on Chart 1 and Table 2. For example, GS's price continuously decreased in general

in the last year, so we expect that it will continue to decrease in 2012, and the weight of GS we calculate is negative, which is consistent with our expectation. For IBM, its stock price increased consistently in general in the last year, and the weight of IBM we calculate is also positive.

## 4.2 Option Strategies

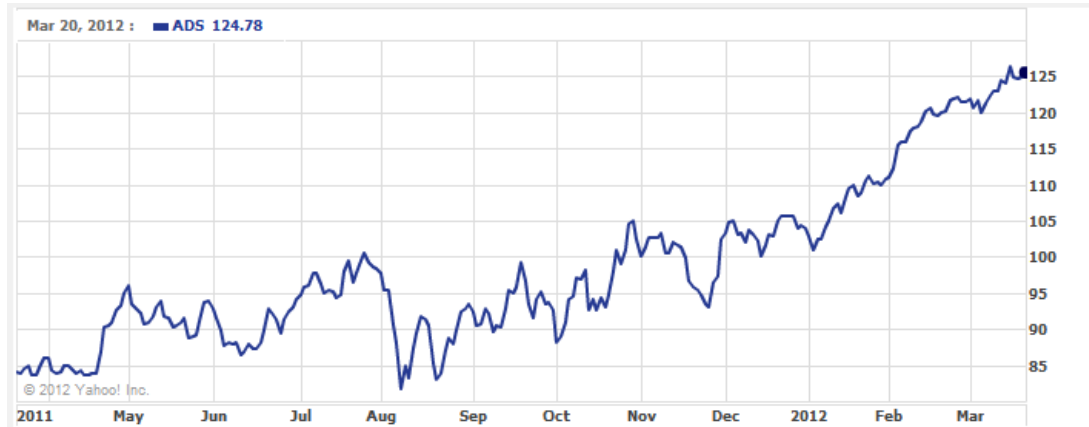


Chart 2

Strategy 1 on ADS: Bear Put Spread.

We buy 10 options with strike price  $K=125$ , expiration date June 15, 2012. We sell 10 options with strike price  $K=115$ , and the same expiration date of June 15.

From the chart 2 above, we can see that over the course of the year, the general trend for this stock's price is rising.



Chart 3

From Chart 3 above, we can see that in 2012, from January to March, the general trend of ADS stock's price is rising. The ADS's weight is 1075, which means we will lose money if



the ADS's price decreases. To reduce the risk of losing money, we plan a bear put spread strategy on ADS stock. We buy 10 put options with  $K=125$  and sell 10 put options with  $K=115$ , both with an expiry of June 15, 2012. So if the stock's price drops below \$125 but maintains above \$115, we can make a positive profit.

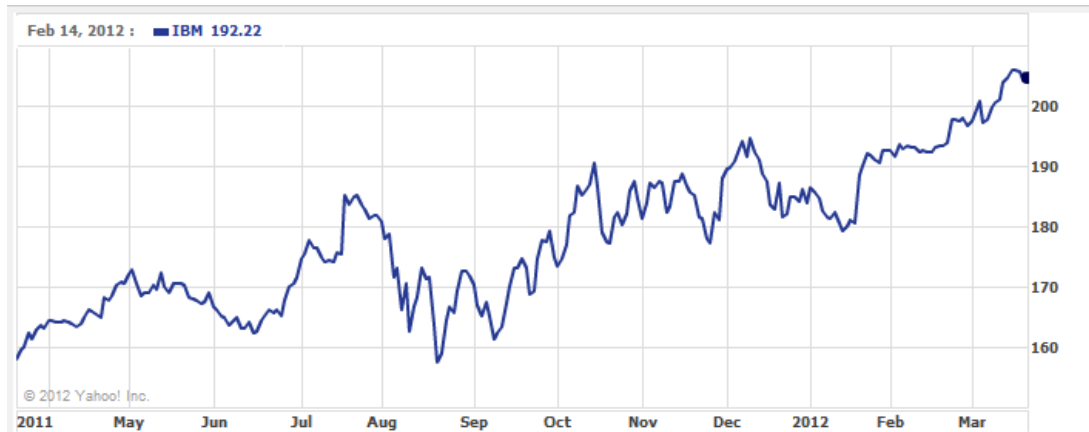


Chart 4

For IBM stock, we use a covered call strategy. We sell 7 options with strike price  $K=180$ , and expiration date July 12, 2012. From chart 4 above, we can see that in a year the general trend of IBM stock's price is rising.



Chart 5

From Chart 5 above, we can also see that from January to March, IBM stock's price is still rising. The weight of IBM is 768, which means we will lose money if the stock's price decreases. We sell 7 call options with strike price  $K=180$  and expiry date June 15, 2012. If the stock's price drops below \$180, we will make a profit using these options. Thus this helps to reduce the risk.

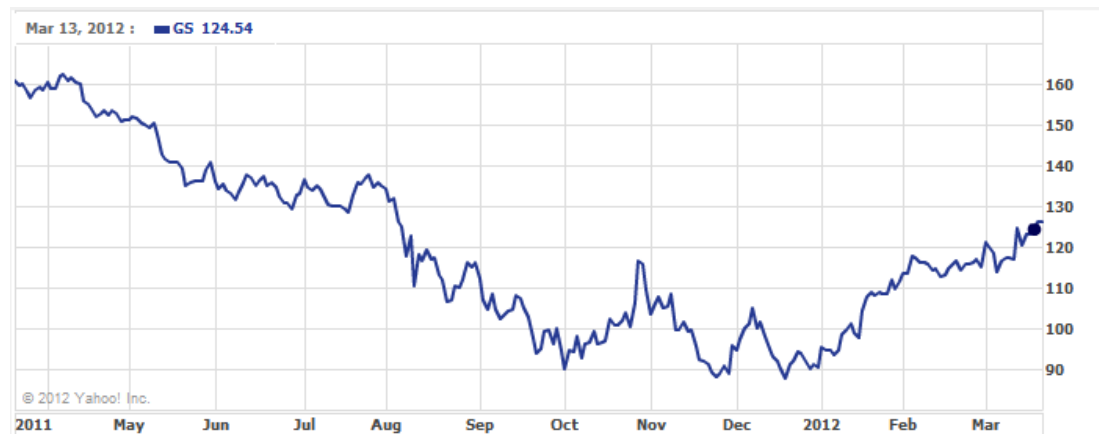


Chart 6

For GS stock, we use a bull call spread strategy. We buy 7 call options with strike price  $K=80$  and expiration date July 12, 2012, and we sell 7 call options with strike price  $K=140$  and expiration date July 12, 2012. From chart 6 above, we can see the general trend of GS stock price is decreasing. From the right part of chart 6, we can see this stock's price rises, so we can expect that in 2012, GS stock's price will continue to increase.



Chart 7

From chart 7, the increasing trend of GS's stock price is obvious. The weight we compute for GS is  $-775$ , which means we will earn money if GS's price decreases. From what we have known about this company, we expect the stock price will rise. So in order to reduce the risk, we have a bull call spread strategy on this stock.

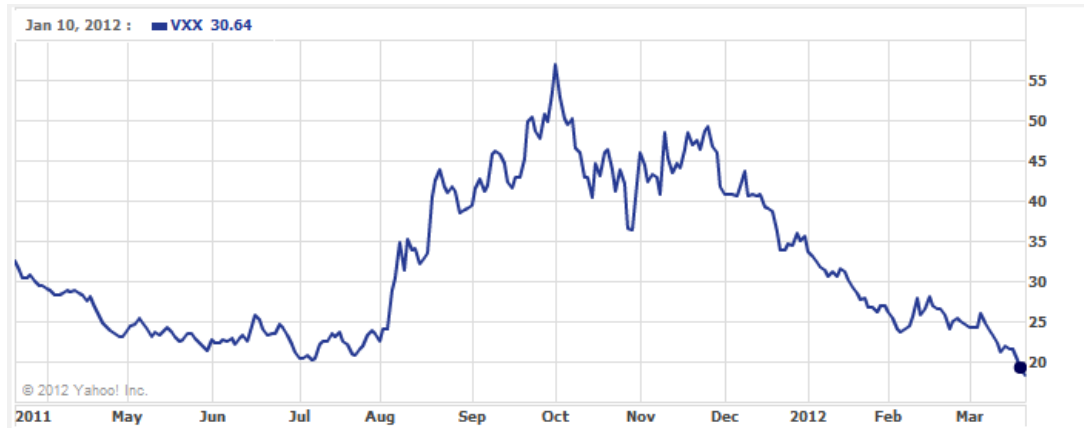


Chart 8



Chart 9

For VXX (iPath S&P 500 VIX Short Term Fu), we have a strangle strategy. We buy 25 call options with strike price  $K=22$  and expiration date June 12, 2012, and we buy 25 put options with strike price  $K=20$  and the same expiration. From the weight we compute for VXX, 2620 is positive, so we will lose money if the stock price decreases. From the charts above we can see that VXX's stock price fluctuates sharply. If the stock price drops below \$20, we can make a profit since we buy 25 put options with strike price  $K=20$ ; if the stock price rises above \$22, we can also make a profit using the call options we buy.

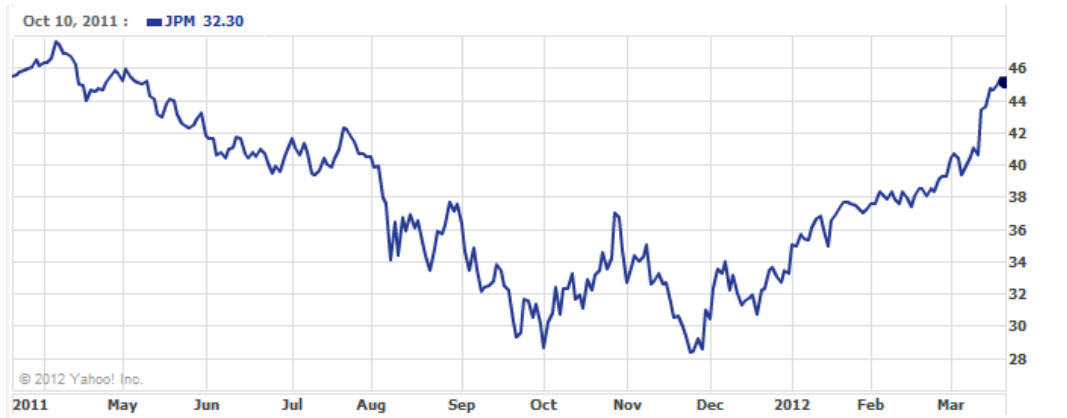


Chart 10



Chart 11

From the charts above, we can see JPM's stock price fluctuates sharply. In 2012 the stock price is generally increasing. We use a long straddle strategy on this stock. We buy 20 put options with strike price  $K=44$ , expiration date June 15, 2012, and buy 20 call options with strike price  $K=44$ , and the same maturity date.



Chart 12



Chart 13

As for the UPS stock, we have a short straddle strategy on this stock. We sell 8 call options and 8 put options with a strike price of 77.5 and maturity date July 15, 2012. The purpose of this position is to hedge the risk in our long position in the UPS stock.

So we get the results for option strategies as following:

<i>Security</i>	<i>Strategy</i>	<i>Details of Option</i>
<b>JPM</b>	Long Straddle	K=44, buy call, buy put, Ct=2.72,Pt=1.86
<b>IBM</b>	Covered Call	K=180, sell call, Ct=26.45, Pt=2.28
<b>GS</b>	Bull Call Spread	K=80, buy call, Ct=48.15, Pt=0.5 K=140, sell call, Ct=3.3, Pt=17.25
<b>ADS</b>	Bear Put Spread	K=125, buy put, Ct=5.7, Pt=5.3 K=115, sell put, Ct=11.6, Pt=1.9
<b>VXX</b>	Strangle	K=22, buy call, Ct=1.9, Pt=5.85 K=20, buy put, Ct=2.45, Pt=4.25
<b>UPS</b>	Short Straddle	K=77.5, sell put, sell call, Ct=4.35, Pt=1.89

Table 3

This completes the construction of our portfolio.

## 5 Risk Analysis

### 5.1 Value at Risk in the Normal and t Distributions

Each week we calculate the VaR and ES to update the data for analyzing the risk of our portfolio. We perform this analysis using two different distributions for our risk-factor changes: the normal distribution and the t-distribution.

For weekly losses, we update data:

<i>Date</i>	<i>Week 1</i>		<i>Week 2</i>		<i>Week 3</i>		<i>Week 4</i>		<i>Week 5</i>	
<i>Distribution</i>	Norm	T	Norm	T	Norm	T	Norm	T	Norm	T
<i>Mean</i>	-1995.3		-328.6799		-1669.5		-1613.3		-1647.6	
<i>Variance</i>	1.6028e+008		8.2438e+008		1.2065e+008		1.5956e+008		6.1238e+008	
<i>95%VaR</i>	3.77	3.42	9.38	8.59	3.26	2.98	3.83	3.49	7.81	7.13
<i>95%ES</i>	4.82	5.34	11.78	12.94	4.20	4.64	4.89	5.4	9.88	10.88
<i>99%VaR</i>	5.49	6.31	13.29	15.15	4.78	5.49	5.55	6.37	11.18	12.78
<i>99%ES</i>	6.35	8.95	15.24	21.13	5.52	7.78	6.41	9.00	12.86	17.94

Table 4

We can pick several weeks' data to analyze the risk of our portfolio.

For week 1 and week 2, comparing the two results based on the t distribution, we can see the expected value of our portfolio's loss has changed significantly. It changes from -1995.3 to -328.6799. Based on our observations on these days, each day our portfolio's realized loss is about \$2000.

In the mean time, the variance of our portfolio's loss ranges from 1.6028e+008 to 8.2438e+008. From the information we can expect that our portfolio's loss will fluctuate wildly. From our observation based on our IB paper trading account, we can see for some days the prices of some options rise quickly, and other options' prices fall significantly. That explains why the variance rises.

As for the 95% VAR under the t distribution, it rises from 1.6977e+004 to 4.2953e+004. Thus we can expect that since the prices of our options and stocks are likely to fluctuate wildly, the risk of our portfolio rises. The change between the two VAR values is significant,

which means our portfolio is very risky. 95% VAR under the t distribution in percentage rises from 0.0342 to 0.0859, which gives the same conclusion.

Further, in the light of the 95% ES under the t distribution (it rises from 26531 to 64697) and ES under t distribution on a percentage basis (it rises from 0.0534 to 0.1294), we can also conclude that the risk of our portfolio rises. The portfolio is riskier, and the profit is thus larger when it makes money.

We compare our 15 stocks' prices over a five day period, and the charts are displayed below.



Chart 14

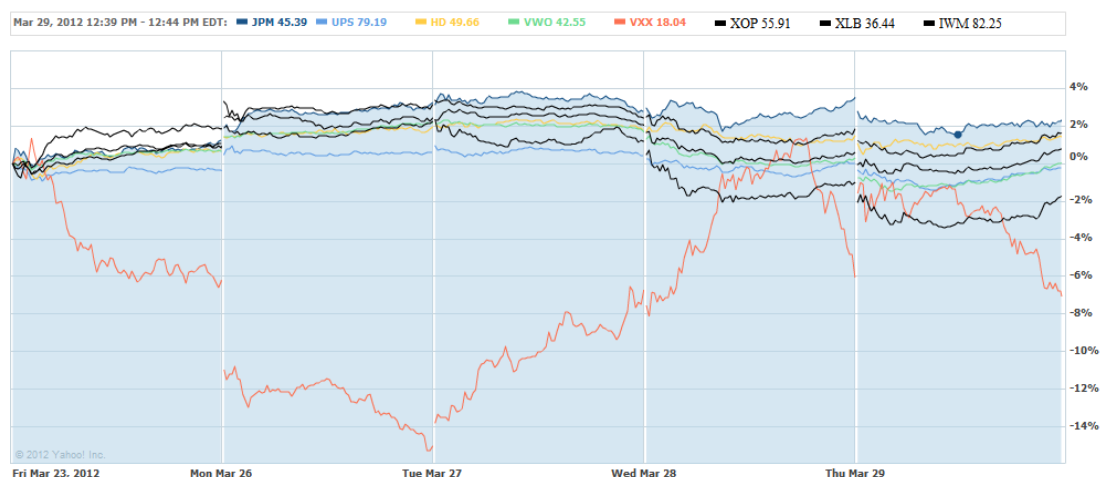


Chart 15

From the two charts above, we can see the prices of our stocks fluctuate wildly, that is consistent with the VAR and variance we compute. VXX is very special; it drops quite significantly on March 27. All other stocks' tendencies are very consistent, with a fluctuation

of about 5% over the course of the week.

For week 2 and week 3, comparing the two weeks' data based on the t distribution, we can see the expected value of our portfolio's loss has been changed slightly. It changes from -328.6799 to -1669.5, which means we can expect our portfolio to lose money in the future. With more risk comes more potential profit. Based on our observation over this week, each day our portfolio's profit is positive, like on April 4, the profit is \$4000.

In the mean time the variance of our portfolio's loss is from  $8.2438e+008$  to  $1.2065e+008$ . From this change, we can expect that our portfolio's loss will fluctuate more mildly than it did in the prior week. From our observation based on our IB paper trading account, we can see for some days, the prices of most stocks fluctuate mildly, only VXX's price changes significantly. That explains why the variance falls.

95% VAR under the t dist in percentage falls from 0.0859 to 0.0298. The prices of our options and stocks fluctuate wildly, and the risk of portfolio rises. The change between the two different 95% VAR values is not that significant, which means our portfolio is not that risky.

Further, in the light of 95% ES under the t distribution in percentage, which falls from 0.1294 to 0.0464, we can also expect that the risk of our portfolio falls. The portfolio is less risky, the profit is smaller when it makes money, and also the loss is smaller when it loses money.

We compare our 15 stocks' prices in a five day period, and the charts are displayed below:



Chart 16



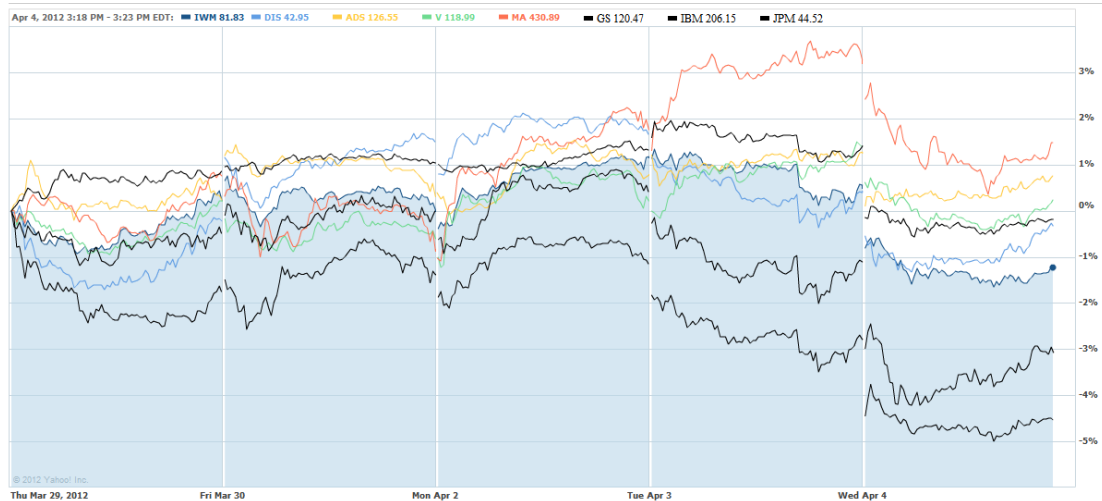


Chart 17

From the two charts, we can see the prices of our stocks fluctuate wildly. VXX is very special; it changes at a significant rate between March 29 and April 4. All other stocks' price movements are consistent, with a fluctuation of about 3%.

## 5.2 Goodness of Fit

To check the goodness of fit for the normal and t-distributions, we use a Chi-squared hypothesis test.

The Chi-squared formula is:  $\chi^2 = \sum \frac{(O - E)^2}{E}$

Distribution \ Stock	Normal		T	
	h	p	h	p
JPM	0	0.751386	0	0.768434
IBM	0	0.284991	0	0.277864
GS	0	0.415433	0	0.525641
MA	0	0.466590	0	0.543321
V	0	0.533544	0	0.460663
ADS	0	0.352118	0	0.331542

DIS	0	0.429882	0	0.362211
XOP	0	0.324518	0	0.266330
XOM	0	0.506732	0	0.410455
UPS	0	0.532294	0	0.432208
HD	0	0.294355	0	0.344587
IWM	0	0.387335	0	0.271233
XLB	0	0.603421	0	0.568832
VXX	1	0.015746	1	0.013247
VWO	0	0.342153	0	0.286673

Table 5

We use the stocks' weekly return to perform the Chi-square test. Under the confidence level  $\alpha = 0.05$ , if  $h=0$  we fail to reject the null hypothesis (which means that the weekly returns of this stock are from the population of this distribution); and if  $h=1$  we reject the null hypothesis. With bigger p-value comes a better goodness of fit.

For example, for JPM stock: under the normal distribution,  $h=0$  and the p-value is 0.751386, which means it is highly possible that the weekly return of the JPM stock price is from a normal distribution; under the t distribution,  $h=0$  and p-value is 0.768434, which means the probability of the weekly return of JPM stock price comes from a t distribution is 76.84%.

For our portfolio:

First, we test against normal distribution:

$h=0$ ,  $p=0.3655$

chi2stat: 3.1744

df: 3

edges: [-0.1220 -0.0474 -0.0225 0.0023 0.0272 0.0521 0.1267]

O: [6 9 15 9 7 8]

E: [8.9797 8.3931 10.5560 10.3329 7.8720 7.8663]

---

Since  $h=0$ , at the 5% significance level, we fail to reject the null hypothesis that the log returns of our stock portfolio come from a normal distribution. Thus the assumption that the log returns of stocks follow normal distribution is appropriate.

Second, we test against t distribution:

$h=0, p=0.2737$

chi2stat: 5.1352

df: 4

edges: [-2.4870 -0.9689 -0.4629 0.0431 0.5492 1.0552 2.5733]

O: [6 9 15 9 7 8]

E: [10.4612 7.5613 9.8505 9.5999 7.0539 9.4732]

Since  $h=0$ , at the 5% significance level, we fail to reject the null hypothesis that the log returns of our stock portfolio come from a t distribution with a degree of freedom of 4. Thus, the assumption that the log returns of stocks follow t distribution with 4 degrees of freedom is appropriate.

### 5.3 Value at Risk in Polynomial Tails

Assume that the tails of the loss distribution for the portfolio has a density  $f$  of the form  $f(y) = A|y|^{-(a+1)}$  for all  $y \leq c$  for some  $c < 0$  and  $A, a > 0$ .

We construct a historical times series of weekly returns for the portfolio to estimate the parameters  $a=1.820526458$  and  $A=0.000348587$ .

VaR with alpha between 0.9 and 1 is displayed below:

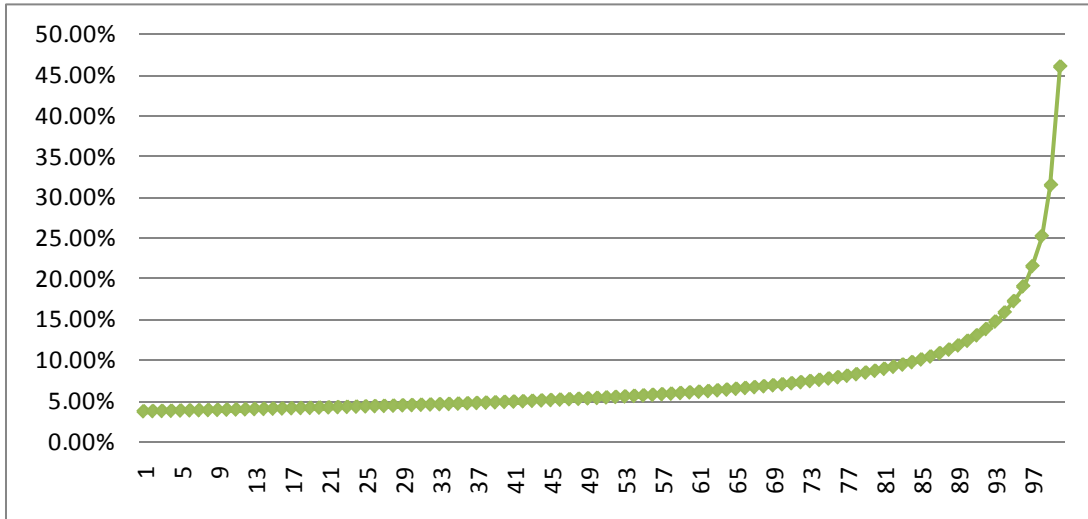


Chart 18

The expected shortfall is displayed below :

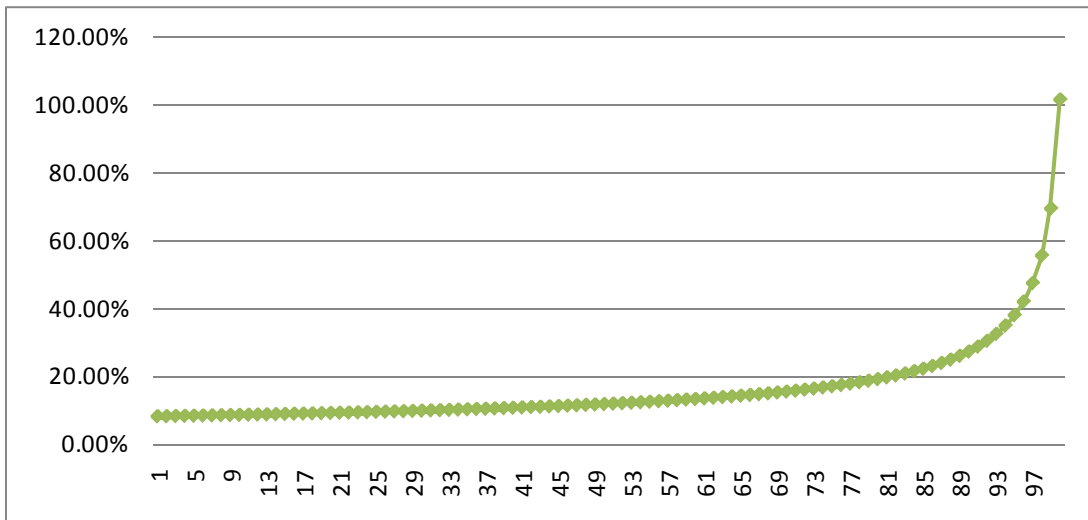


Chart 19

The charts of our stocks' performance in one week are given below in Charts 20 and 21:

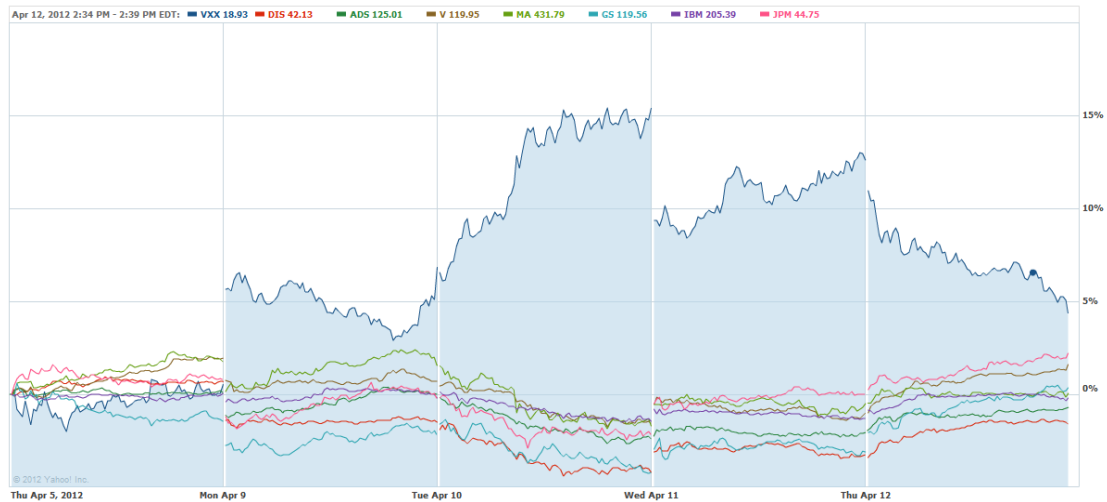


Chart 20

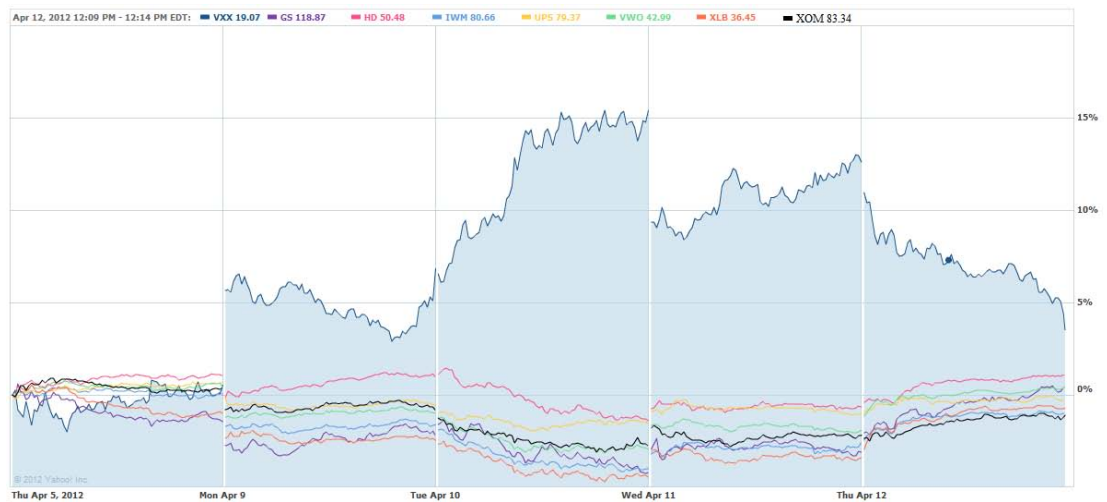


Chart 21

From the charts above, we can see that VXX is still very different than other stocks. It fluctuates quite significantly and in an opposite direction.

The data we calculated using the previous method for week 2 is as follows:

99% VAR in percentage: 13.29%

99% ES in percentage: 15.24%

And for week 3 we use the new method, the data is as follows:

99% VAR in percentage: 12.98%

99% ES in percentage: 28.79%

We can see that 99% VAR in percentage changes by 0.31% between 13.29% and

12.98%, and 99% ES in percentage changes by 13.54% between 15.24% and 28.79%. The 99% VAR in percentage is almost the same using the different methods, but the 99% ES in percentage changes significantly. So with the new method, the loss distribution will have a bigger tail.

#### 5.4 Value at Risk in GARCH Modeling

Assume that the weekly portfolio loss follows an ARMA(1,1)-GARCH(1,1) model of the form  $L_t = \mu_t + \sigma_t Z_t$ . [8]

Calculate  $VarR_\alpha$  and  $ES_\alpha$  for confidence levels of  $\alpha \geq 0.9$  for the conditional loss distribution in Matlab.

1. Under t innovations:

Mean: ARMAX(1,1,0); Variance: GARCH(1,1)

Conditional Probability Distribution: T

<i>Parameter</i>	<i>Value</i>	<i>Standard Error</i>	<i>T Statistic</i>
<i>C</i>	4343.9	2305.2	1.8844
<i>AR(1)</i>	-0.84929	0.11504	-7.3838
<i>MA(1)</i>	0.74191	0.14797	5.0139
<i>K</i>	8.9736e+007	0.00042775	209783741844.6989
<i>GARCH(1)</i>	0.61441	0.13276	4.6279
<i>ARCH(1)</i>	0.38559	0.2092	1.8431
<i>DoF</i>	3.1487	0.97521	3.2288

Table 6

The following are the plots of conditional standard deviations and standardized residuals.

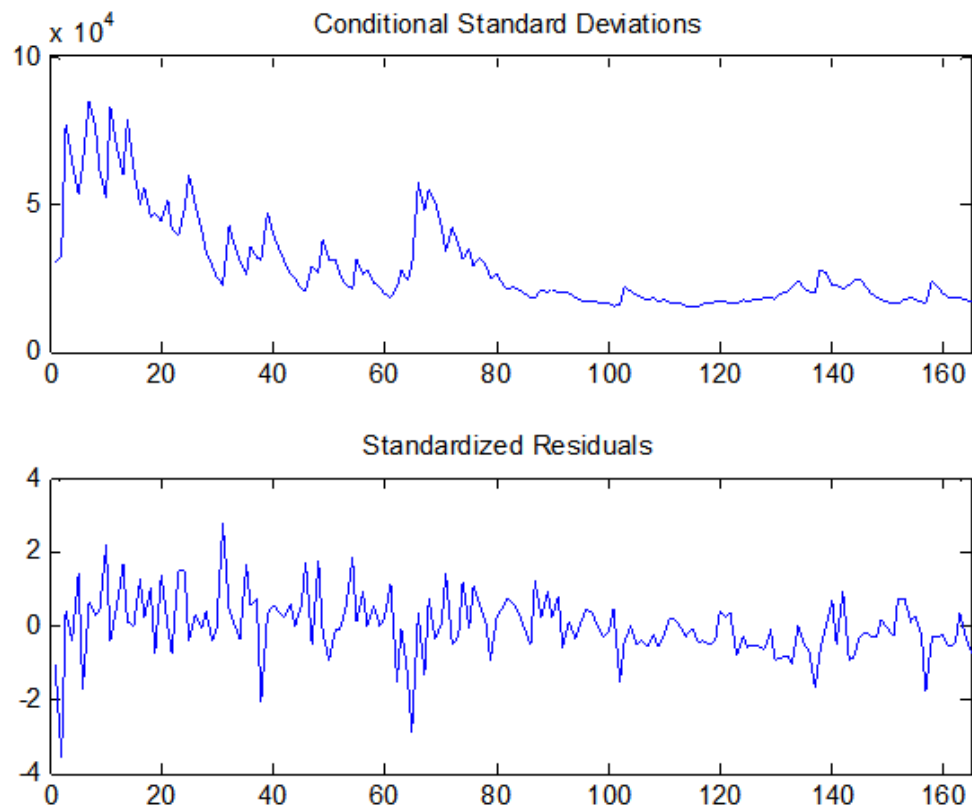


Chart 22

The following is the chart of the conditional expected value over time  $t$ .

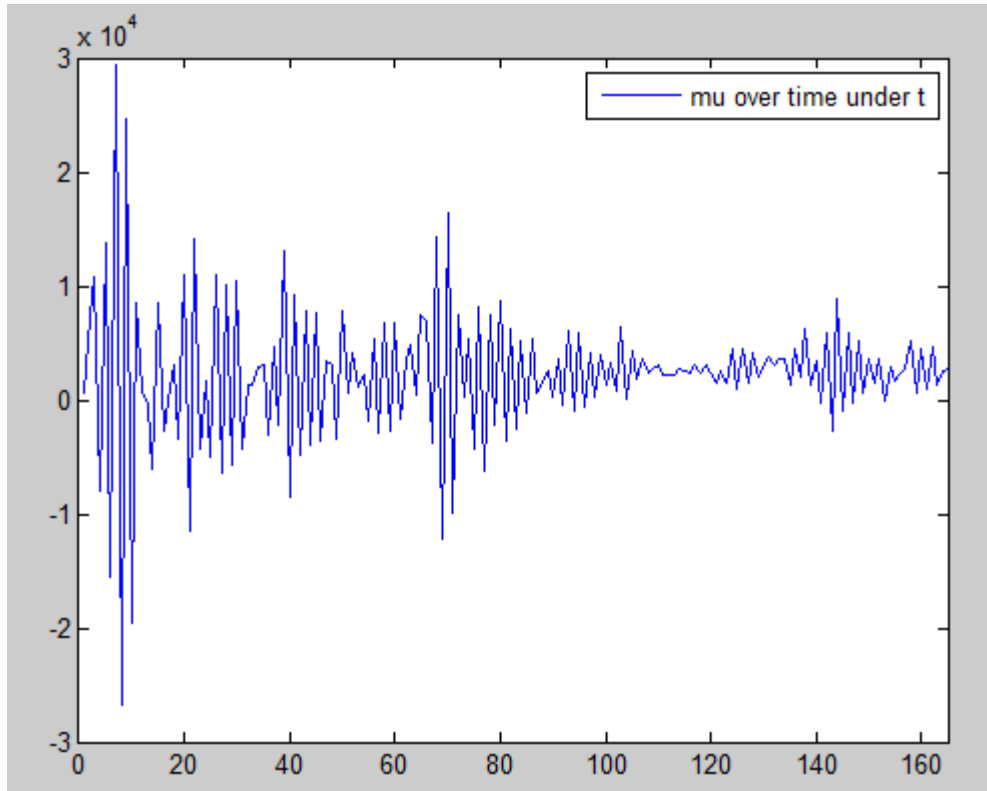


Chart 23

2. Under Gaussian innovations

Mean: ARMAX(1,1,0); Variance: GARCH (1,1)

Conditional Probability Distribution: Gaussian

Parameter	Value	Standard Error	T Statistic
C	4343.9	3118.1	1.3931
AR(1)	-0.79564	0.41444	-1.9198
MA(1)	0.76579	0.45805	1.6718
K	8.9736e+007	0.018637	4814832840.8266
GARCH(1)	0.46728	0.07623	6.1299
ARCH(1)	0.49553	0.1298	3.8177

Table 7

The following are the plots of conditional standard deviations and standardized residuals.



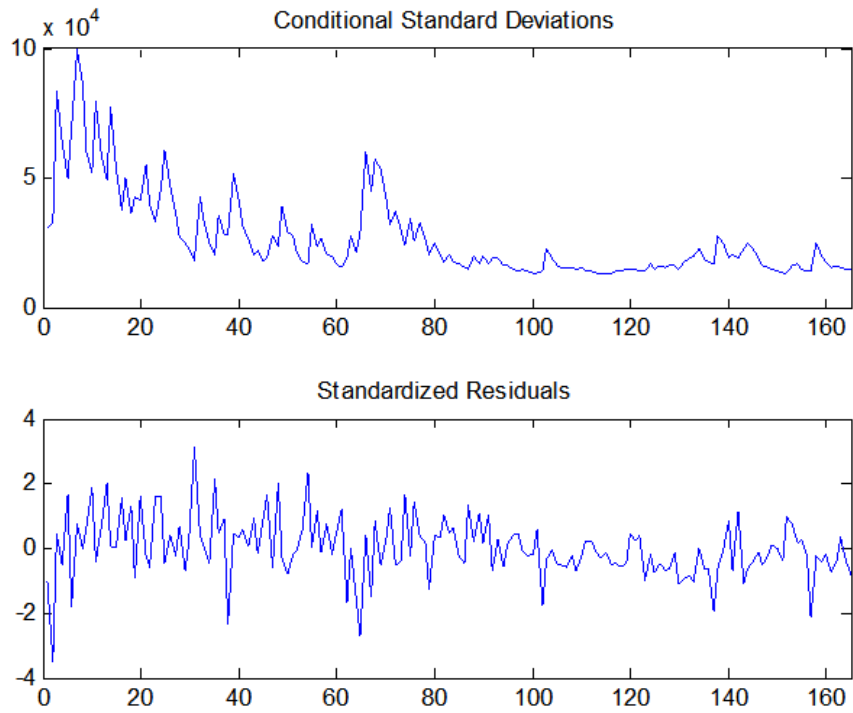


Chart 24

The following is the chart of conditional expected value over time.

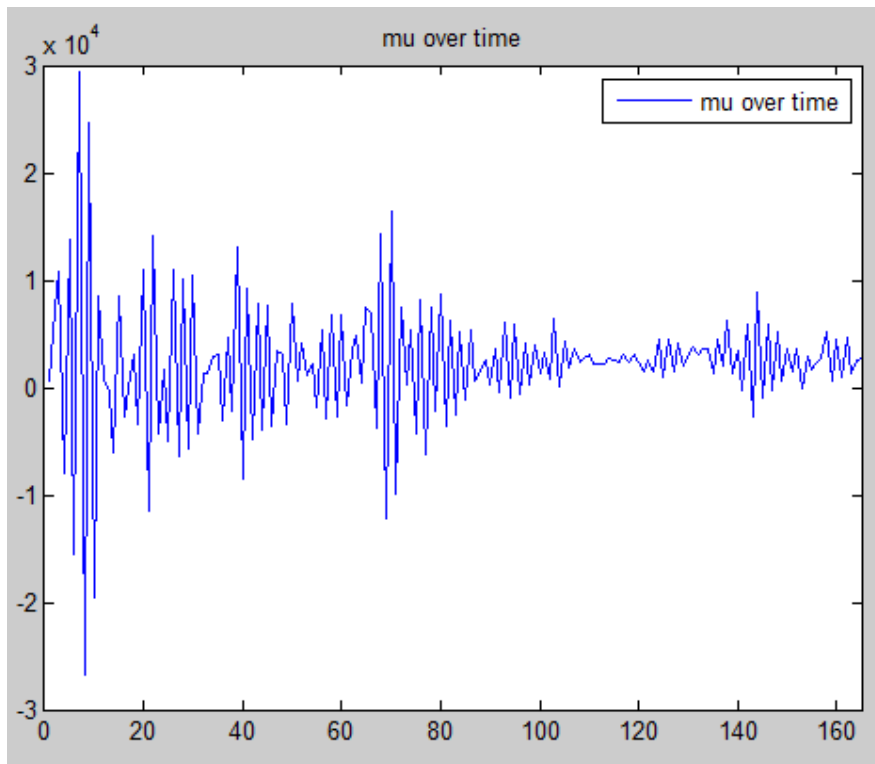


Chart 25

With both normal and student t innovations, the standardized residuals look like a white noise process with mean 0 and variance 1, which suggests that the estimated ARMA(1,1)-GARCH(1,1) is a good fit. Also, the graphs of expected value show that a high conditional mean value persists for a while and so does a low conditional mean value.

With t-student innovations the standard errors of the estimated parameters are clearly far less than those under Gaussian innovations. Thus GARCH model with t-student innovations is better than the GARCH model with Gaussian innovations.

In the plots we show that the VaR, ES and actual loss under the two different innovations:

Under Gaussian innovations:

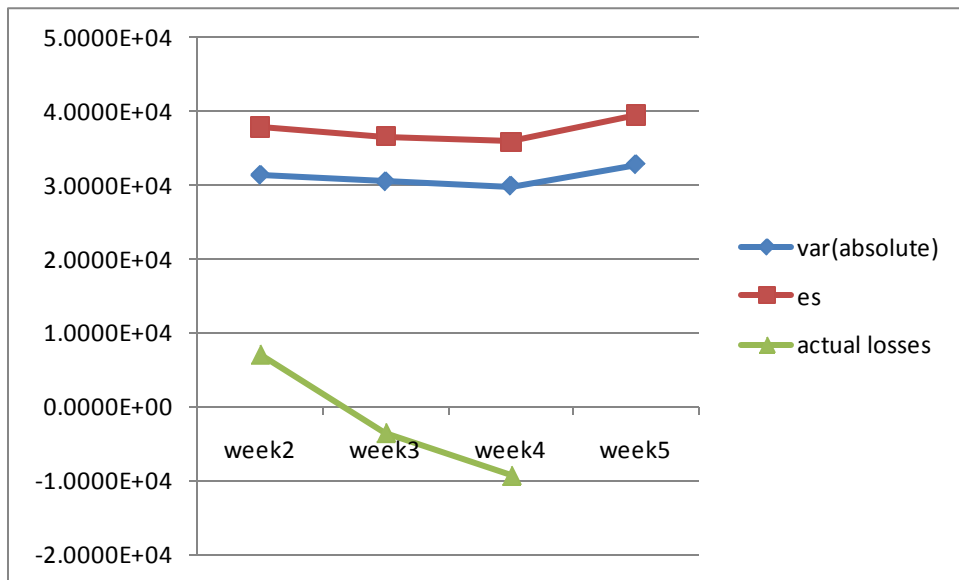


Chart 26

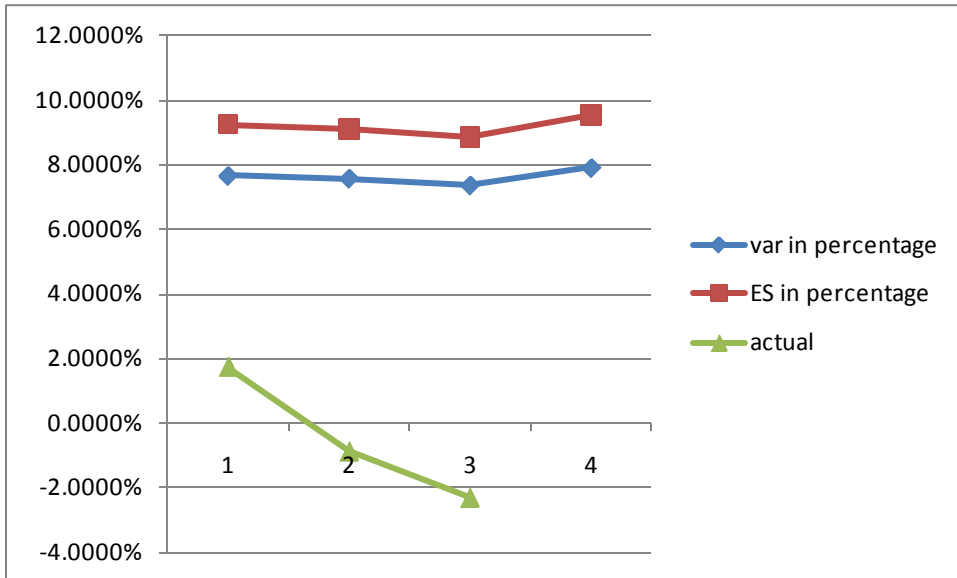


Chart 27

Under t-student innovations :

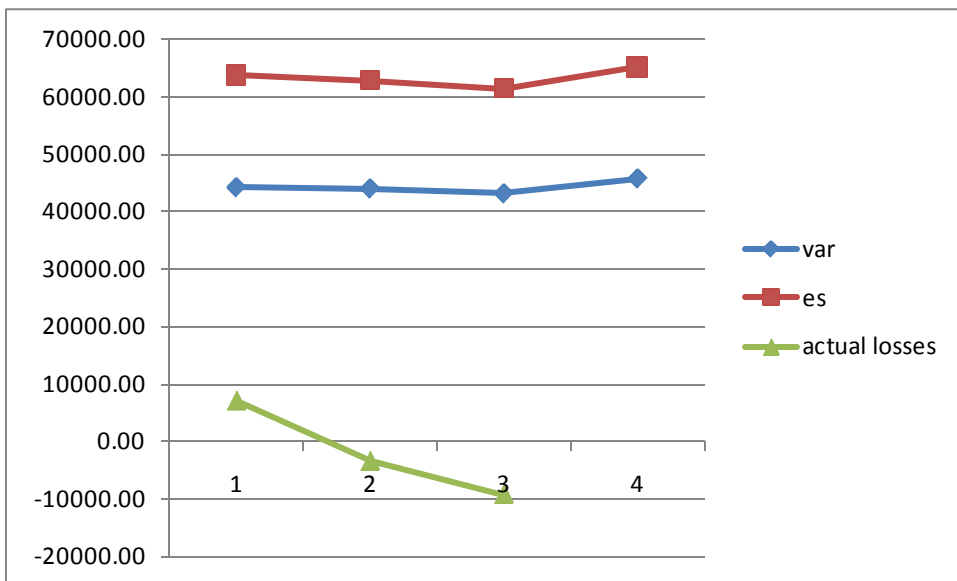


Chart 28

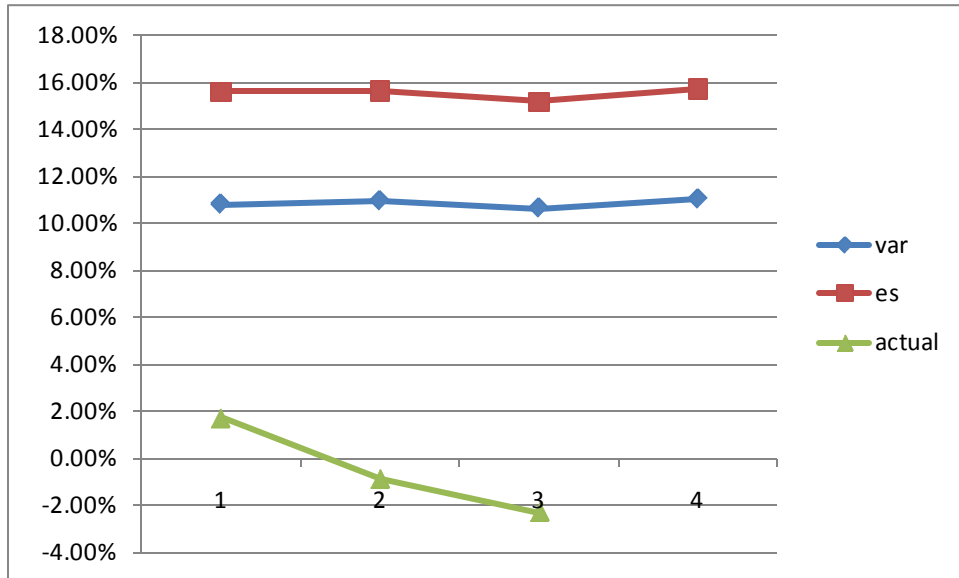


Chart 29

Under both innovations and under 95% confidence, the three weeks' actual losses are all below their corresponding VaR and ES.

We use a start date of April 2, 2012 and compute  $\sigma_t, \mu_t, VaR_\alpha, ES_\alpha$  for the conditional loss distribution of  $L$  on a rolling daily basis.

The results are as following:

<i>normal</i>	<i>VaR</i>	<i>ES</i>	<i>t dist</i>	<i>VaR</i>	<i>ES</i>
<b>4/2/2012</b>	1.54E+03	1.83E+03	<b>4/2/2012</b>	2.21E+03	3.00E+03
<b>4/3/2012</b>	1.38E+03	1.66E+03	<b>4/3/2012</b>	1.75E+03	2.50E+03
<b>4/4/2012</b>	2.46E+03	3.03E+03	<b>4/4/2012</b>	1.84E+03	3.79E+03
<b>4/5/2012</b>	2.30E+03	2.83E+03	<b>4/5/2012</b>	3.69E+03	5.02E+03
<b>4/9/2012</b>	6.26E+03	7.05E+03	<b>4/9/2012</b>	7.19E+03	9.23E+03
<b>4/10/2012</b>	2.42E+03	3.42E+03	<b>4/10/2012</b>	3.86E+03	6.54E+03
<b>4/11/2012</b>	3.82E+03	4.86E+03	<b>4/11/2012</b>	5.54E+03	8.90E+03

Table 8

Based on the results calculated, the data's tendency is as follows:

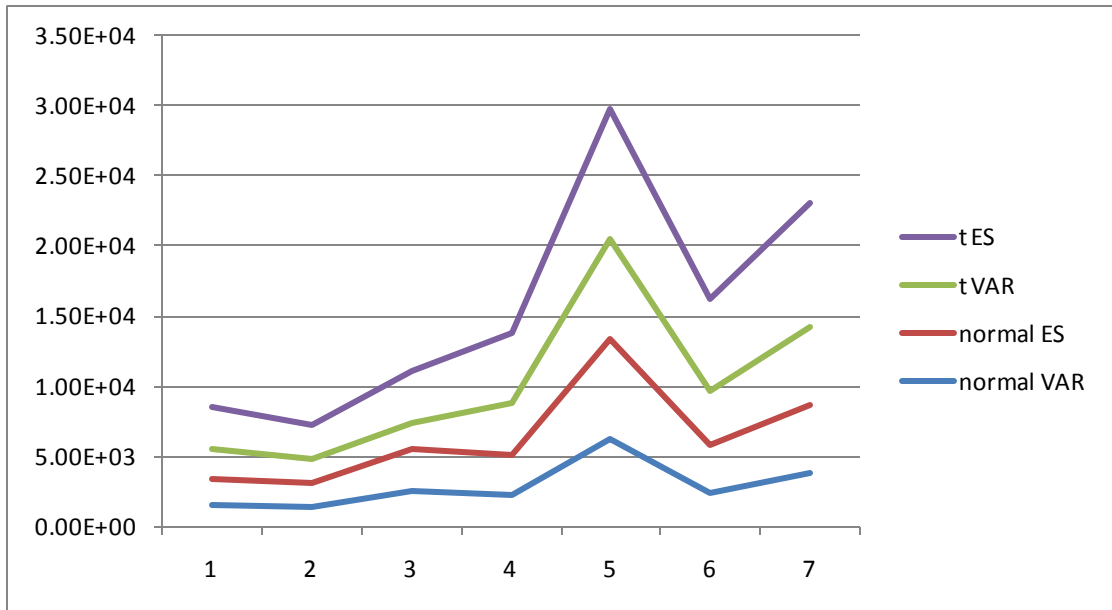


Chart 30

From the chart above we can see that under the two different distributions, ES and VaR do not change significantly, and the trend is the same. Under the normal distribution, the ES and VaR are smaller. The t distribution contains riskier factors, which lead to larger VaR and ES values.

## 6 Risk Reduction

From what we have discussed above and the portfolio's actual daily loss, we can see that our portfolio's risk is very high. In order to reduce the risk of our portfolio, we need to add some new positions into our portfolio.

We can reduce the portfolio risk by adding some new options.

First, we should look the average actual daily loss for each stock.

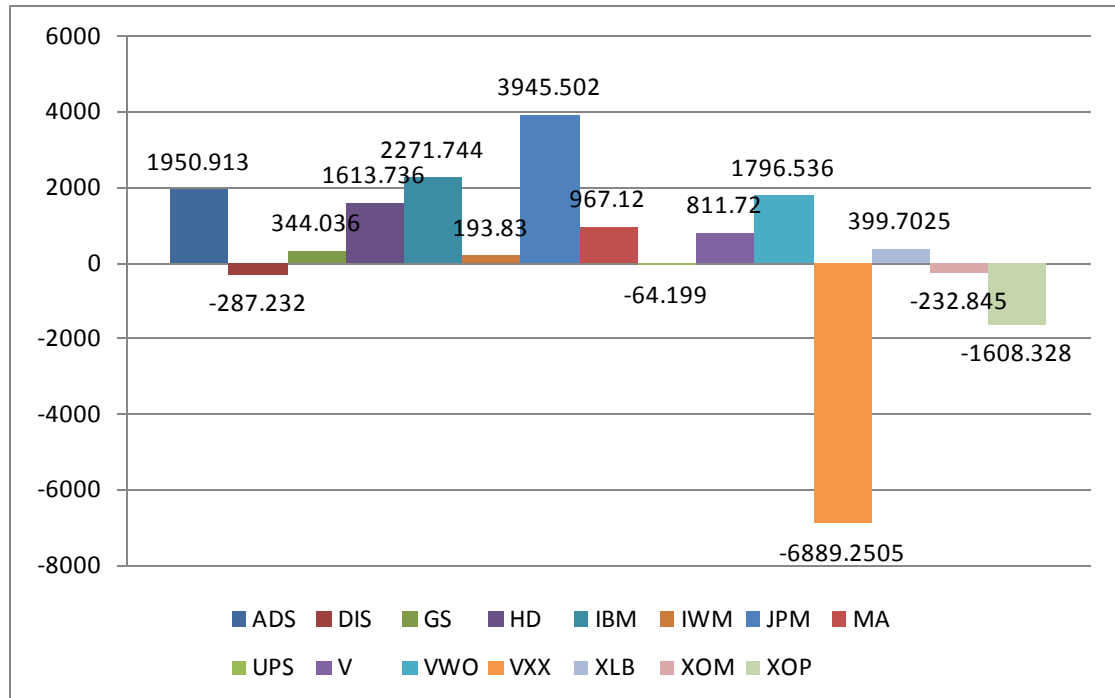


Chart 31

From Chart 31 we can see that VXX's daily loss is the biggest, and XOP's daily loss is the second biggest. The weights of VXX and XOP stocks are 0.1393 and 0.1366, respectively, and their stocks' prices decrease during the project period. Therefore we can buy put options or sell call options on these assets to reduce our portfolio's risk.

Then we should consider the daily loss movement of each stock.

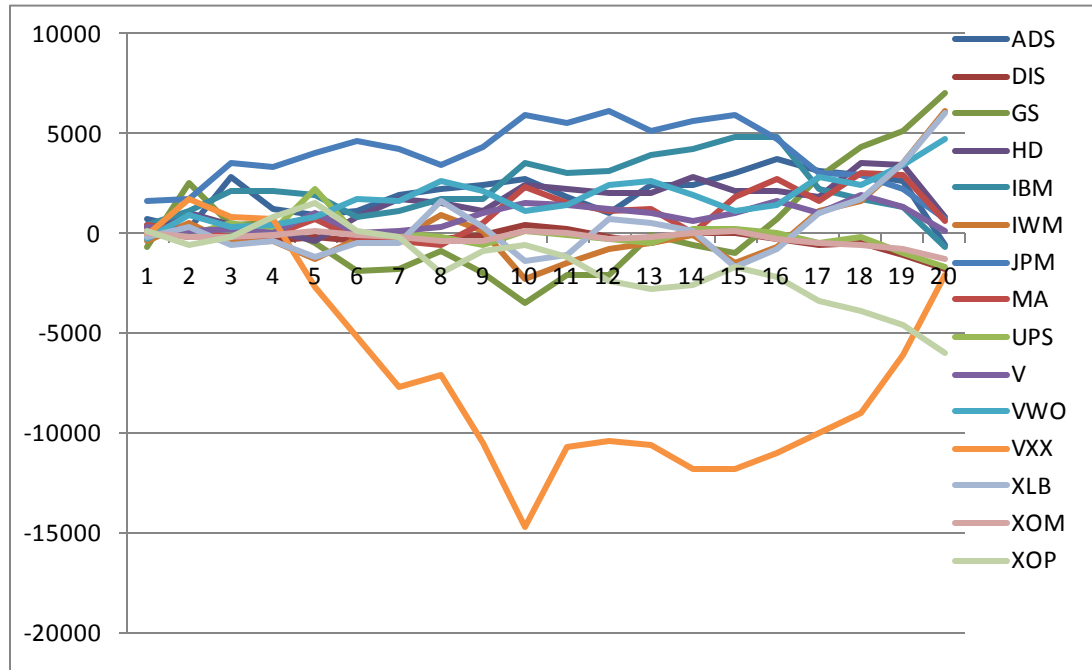


Chart 32

From Chart 32 we can see VXX stock price fluctuates most wildly. And in most days, we lose money on our VXX position. Considering the weight of VXX is positive, we consider buying a put option to reduce the risk and minimize the loss.

From the two different charts-Chart 31 and Chart 32, we arrive at the same conclusion. For VXX we buy another 25 put options with strike \$20 and expiration date June 12, 2012.

For the ADS and JPM stocks, their daily profits are our 2 best during the project period. Considering the weights of the two stocks are both positive, we can buy some more call option to increase the profit.

For ADS stock we can buy 10 call options with strike price  $K=\$115$  and expiration date June 15, 2012. For JPM stock we can buy 20 call options with strike price  $K=\$44$  and expiration date July 15, 2012.

The table below is the new status of our options:

<i>Security</i>	<i>Details of Option</i>
<b>JPM</b>	$K=\$44$ , buy 40 call; $K=\$44$ , buy 20 put
<b>IBM</b>	$K=\$180$ , sell 7 call
<b>GS</b>	$K=\$80$ , buy 7; $K=\$140$ , sell 7

<b>ADS</b>	K=\$125, buy 10 put; K=\$115, sell 10 put; K=\$115, buy 10 call
<b>VXX</b>	K=\$22, buy 25 call; K=\$20, buy 50 put
<b>UPS</b>	K=\$75, sell 8 call; K=\$82.5, sell 8 put

Table 9

After we add these new positions, we calculate the 95% VaR and 95% ES again to check whether the risk our portfolio is smaller than before.

Distribution	Before Risk Reduction		After Risk Reduction	
	Norm	T	Norm	T
Mean	-1647.6		-1044.7	
Variance	6.1238e+008		5.6045e+008	
95% VaR	7.81	7.13	7.46	7.05
95% ES	9.88	10.88	9.33	10.07
99% VaR	11.18	12.78	10.38	12.14
99% ES	12.86	17.94	12.32	17.24

Table 10

From Table 10, we can see that the VaR and ES in percentage are both smaller than they are before risk reduction, which means our strategy is working.

99% VaR under the normal distribution changes from 11.18% to 10.38%. It decreases by 0.8%, which means our portfolio's risk decreases under this risk measure. The mean of our portfolio's loss increases and the variance decreases. So some risk reduction is achieved.



---

## 7 Conclusion

Based on use of our Interactive Brokers paper trading account over the course period, our portfolio loses money almost every day. From what we have discussed above, the expected value of our portfolio's loss is consistent with the actual loss. Upon examination of all our stocks and options, it is clear that VXX's price movement is very different than that of other stocks. Our portfolio's risk may be smaller if we replace VXX with another asset. We compute a negative weight for GS stock, but based on what we have known about this company, we actually expect that its stock price will increase in 2012.

When VaR is estimated for a portfolio of assets rather than for a single asset, parametric estimation based on the assumption of multivariate normal or t-distributed returns is very convenient because the portfolio's return will have a univariate normal or t-distributed return. [10] From the results we have computed above we can see that VaR and ES values under a t distribution are larger than them under a normal distribution because the t distribution contains riskier factors because of its heavier tails. In our project results using the t distribution are closer to the actual data.

---

## References

- [1] Course syllabus of MA575: Market and Credit Risk Models and Management
- [2] Yahoo Finance, <http://finance.yahoo.com/>
- [3] Interactive Brokers, <http://www.interactivebrokers.com/>
- [4] David Harper, 2010, “An Introduction To Value at Risk (VAR)”
- [5] James L. Powell, “Estimation of Semiparametric Models”, Princeton University, *Definition of Semiparametric*, 2449
- [6] David Ruppert, 2010, “Statistics and Data Analysis for Financial Engineering”, First Edition, Springer, *Time Series Models: Basics*, 201-257.
- [7] David Ruppert, 2010, “Statistics and Data Analysis for Financial Engineering”, First Edition, Springer, *Risk Management*
- [8] David Ruppert, 2010, “Statistics and Data Analysis for Financial Engineering”, First Edition, Springer, *Portfolio Theory*
- [9] Tomas Bjork, 2009, “Arbitrage Theory in Continuous Time”, Third Edition, Oxford University Press, *Stochastic Integral*, 40-65
- [10] Rober A. Jarrow, 2002, “Modeling Fixed-Income Securities and Interest Rate Options”, Second Edition, Stanford University Press, *Trading Strategies, Arbitrage Opportunities, and Complete Markets*, 99-112

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## A Appendix

### A.1 Matlab Code

#### 1. T distribution

```
factors=xlsread('C:\data\JPM.xls','17factors','A2:R55');  
  
s=xlsread('C:\data\JPM.xls','shares','D2:D16');  
  
lamda=xlsread('C:\data\JPM.xls','shares','C2:C16');  
  
cdelta=xlsread('C:\data\JPM.xls','greeks','B10:B15');  
  
pdelta=xlsread('C:\data\JPM.xls','greeks','E10:E14');  
  
crho=xlsread('C:\data\JPM.xls','greeks','C10:C15');  
  
prho=xlsread('C:\data\JPM.xls','greeks','F10:F14');  
  
cvega=xlsread('C:\data\JPM.xls','greeks','D10:D15');  
  
pvega=xlsread('C:\data\JPM.xls','greeks','G10:G14');  
  
ctheta=xlsread('C:\data\JPM.xls','greeks','J10:J15');  
  
ptheta=xlsread('C:\data\JPM.xls','greeks','K10:K14');  
  
callshares=[1700 -800 800 -800 2500 -800];  
  
putshares=[1700 1100 -1100 2500 -800];  
  
DF=4;% degrees of freedom  
  
V0=500000;  
  
W(1,1)=s(1)*(lamda(1)+1700*cdelta(1)+1700*pdelta(1));  
  
W(2,1)=s(2)*(lamda(2)-800*cdelta(2));  
  
W(3,1)=s(3)*(-lamda(3)+800*cdelta(3)-800*cdelta(4));  
  
W(4,1)=s(4)*lamda(4);  
  
W(5,1)=s(5)*lamda(5);  
  
W(6,1)=s(6)*(lamda(6)+1100*pdelta(2)-1100*pdelta(3));
```

---

```

W(7,1)=s(7)*lamda(7);

W(8,1)=s(8)*lamda(8);

W(9,1)=s(9)*(lamda(9)-800*cde lta(6)-800*pdelta(5));

W(10,1)=s(10)*lamda(10);

W(11,1)=-s(11)*lamda(11);

W(12,1)=s(12)*(lamda(12)+2500*cde lta(5)+2500*pde lta(4));

W(13,1)=s(13)*lamda(13);

W(14,1)=-s(14)*lamda(14);

W(15,1)=-s(15)*lamda(15);

W(16,1)=callshares*crho+putshares*prho;

W(17,1)=callshares*cvega+putshares*pvega;

W(18,1)=callshares*ctheta+putshares*ptheta;

MU=mean(factors);% row vector need transpose

COVMATRIX=cov(factors);

TLossmu=-W'*MU';

TLossvar=((DF-2)/DF)*W'*COVMATRIX*W;% parameter for the combined t dist, not the
variance

Tvariance=W'*COVMATRIX*W;

disp('expected value for the combined t loss distribution of stocks and options is:');

disp(TLossmu);

disp('variance for the combined t loss distribution of stocks and options is:');

disp(Tvariance);

alpha1=0.95;

VaRalpha1=TLossmu+sqrt(TLossvar)*tinv(alpha1,DF);

ESalpha1=TLossmu+sqrt(TLossvar)*(1/(DF-1))*tpdf(tinv(alpha1,DF),DF)*(1/(1-alpha1))*(

```

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```

DF+(tinv(alpha1,DF))^2);

disp('95% VAR under t dist is:');

disp(VaRalpha1);

disp('95% VAR under t dist in percentage is:');

disp(VaRalpha1/V0);

disp('95% ES under t dist is:');

disp(ESalpha1);

disp('95% ES under t dist in percentage is:');

disp(ESalpha1/V0);

alpha2=0.99;

VaRalpha2=TLossmu+sqrt(TLossvar)*tinv(alpha2,DF);

ESalpha2=TLossmu+sqrt(TLossvar)*(1/(DF-1))*tpdf(tinv(alpha2,DF),DF)*(1/(1-alpha2))*
(DF+(tinv(alpha2,DF))^2);

disp('99% VAR under t dist is:');

disp(VaRalpha2);

disp('99% VAR under t dist in percentage is:');

disp(VaRalpha2/V0);

disp('99% ES under t dist is:');

disp(ESalpha2);

disp('99% ES under t dist in percentage is:');

disp(ESalpha2/V0);

```

## 2. VAR

```

function [callLinLoss,putLinLoss]=projectoption(delta,callPrice,putPrice,S,K,r,tau)

% logreturn=xlsread('C:\data\JPM.xls','St','A1:A54');

% logreturn=xlsread('C:\data\JPM.xls','St','B1:B54');

```

---

```

% logreturn=xlsread('C:\data\JPM.xls','St','C1:C54');

% logreturn=xlsread('C:\data\JPM.xls','St','D1:D54');

% logreturn=xlsread('C:\data\JPM.xls','St','E1:E54');

logreturn=xlsread('C:\data\JPM.xls','St','F1:F54');

twofactors=xlsread('C:\data\JPM.xls','twofactors','A1:B54');

X(:,1)=logreturn;

X(:,2)=twofactors(:,1);

X(:,3)=twofactors(:,2);

expect=mean(X);

covmatrix=cov(X);

Cvolatility=blsimpv(S,K,r,tau,callPrice,[],0,[],{'call'});

disp('call volatility is:');

disp(Cvolatility);

[ctheta,x]=blstheta(S,K,r,tau,Cvolatility);

[cdelta,x]=blsdelta(S,K,r,tau,Cvolatility);

[crho,x]=blsrho(S,K,r,tau,Cvolatility);

cvega=blsvega(S,K,r,tau,Cvolatility);

Pvolatility=blsimpv(S,K,r,tau,putPrice,[],0,[],{'put'});

disp('put volatility is:');

disp(Pvolatility);

[x,ptheta]=blstheta(S,K,r,tau,Pvolatility);

[x,pdelta]=blsdelta(S,K,r,tau,Pvolatility);

[x,prho]=blsrho(S,K,r,tau,Pvolatility);

pvega=blsvega(S,K,r,tau,Pvolatility);

```

---

```

Callvector=[cdelta,crho,cvega];

Putvector=[pdelta,prho,pvega];

disp('call option vector is:');

disp(Callvector);

disp('put option vector is:');

disp(Putvector);

end

factors=xlsread('C:\data\JPM.xls','17factors','A1:Q54');

s=xlsread('C:\data\JPM.xls','shares','B2:B16');

lamda=xlsread('C:\data\JPM.xls','shares','C2:C16');

cdelta=xlsread('C:\data\JPM.xls','greeks','B2:B7');

pdelta=xlsread('C:\data\JPM.xls','greeks','E2:E6');

crho=xlsread('C:\data\JPM.xls','greeks','C2:C7');

prho=xlsread('C:\data\JPM.xls','greeks','F2:F6');

cvega=xlsread('C:\data\JPM.xls','greeks','D2:D7');

pvega=xlsread('C:\data\JPM.xls','greeks','G2:G6');

V0=500000;

W(1,1)=s(1)*(lamda(1)+1700*cdelta(1)+1700*pdelta(1));

W(2,1)=s(2)*(lamda(2)-800*cdelta(2));

W(3,1)=s(3)*(-lamda(3)+800*cdelta(3)-800*pdelta(4));

W(4,1)=s(4)*lamda(4);

W(5,1)=s(5)*lamda(5);

W(6,1)=s(6)*(lamda(6)+1100*pdelta(2)-1100*pdelta(3));

W(7,1)=s(7)*lamda(7);

```

---

```

W(8,1)=s(8)*lamda(8);
W(9,1)=s(9)*(lamda(9)-800*cde lta(6)-800*pdelta(5));
W(10,1)=s(10)*lamda(10);
W(11,1)=-s(11)*lamda(11);
W(12,1)=s(12)*(lamda(12)+2500*cde lta(5)+2500*pde lta(4));
W(13,1)=s(13)*lamda(13);
W(14,1)=-s(14)*lamda(14);
W(15,1)=-s(15)*lamda(15);
W(16,1)=sum(crho)+sum(prho);
W(17,1)=sum(cvega)+sum(pvega);
MU=mean(factors);% row vector need transpose
COVMATRIX=cov(factors);
Lossmu=-W'*MU';
Lossvariance=W'*COVMATRIX*W;
disp('expected value for the combined loss distribution of stocks and options is:');
disp(Lossmu);
disp('variance for the combined loss distribution of stocks and options is:');
disp(Lossvariance);
alpha1=0.95;
VaRalpha1=Lossmu+sqrt(Lossvariance)*norminv(alpha1,0,1);
ESalpha1=Lossmu+sqrt(Lossvariance)*normpdf(norminv(alpha1,0,1),0,1)/(1-alpha1);
disp('95% VAR is:');
disp(VaRalpha1);
disp('95% VAR in percentage is:');

```



---

```

disp(VaRalpha1/V0);

disp('95% ES is:');

disp(ESalpha1);

disp('95% ES in percentage is:');

disp(ESalpha1/V0);

alpha2=0.99;

VaRalpha2=Lossmu+sqrt(Lossvariance)*norminv(alpha2,0,1);

ESalpha2=Lossmu+sqrt(Lossvariance)*normpdf(norminv(alpha2,0,1),0,1)/(1-alpha2);

disp('99% VAR is:');

disp(VaRalpha2);

disp('99% VAR in percentage is:');

disp(VaRalpha2/V0);

disp('99% ES is:');

disp(ESalpha2);

disp('99% ES in percentage is:');

disp(ESalpha2/V0);

```

### 3. Everyday T-Distribution Loss

```

dailyloss=xlsread('C:\Users\Jing\Desktop\JPM.xls','dailyloss','J4:J15');

spec=garchset('R',1,'M',1,'P',1,'Q',1,'DoF',4,'Dist','t');% under t innovations

spec=garchset(spec,'Display','off');

[coeff,error] = garchfit(spec,dailyloss);

garchdisp(coeff,error)

[res,sig,LogL] = garchinfer(coeff,dailyloss);

% t dist

```

---

```

alpha=0.95;

C=45.59;

AR(1)=-0.70463;

MA(1)=1;

K=7.4323e+005;

GARCH(1)=0.94799;

ARCH(1)=0;

l=length(dailyloss);

U=C+AR(1)*(dailyloss(l)-C)+MA(1)*res(l);

Sigma2=K+ARCH(1)*(res(l)^2)+GARCH(1)*(sig(l)^2);

ValAtRisk=U+sqrt(Sigma2)*tinv(alpha,4);

D=tinv(alpha,4);

EStdist=tpdf(D,4).*(4+(tinv(alpha,4)).^2)./(3*(1-alpha));

ES=U+sqrt(Sigma2)*EStdist;

disp('VAR');

disp(ValAtRisk);

disp('ES');

disp(ES);

```

#### 4. Every Normal Distribution Loss

```

% price=xlsread('C:\data\575project2\JPM.xls','sheet2','B2:P167');

% w=xlsread('C:\data\575project2\JPM.xls','sheet2','Q2:Q16');

% e=length(price);

% Pvalue1=price*w;

% Pvalue2=Pvalue1;

```

---

```

%Pvalue1(1)=[];

%Pvalue2(e)=[];

%Ploss=-(Pvalue1-Pvalue2);

% disp('portfolio loss is:');

% disp(Ploss);

dailyloss=xlsread('C:\Users\Jing\Desktop\JPM.xls','dailyloss','J4:J15');

spec=garchset('R',1,'M',1,'P',1,'Q',1);% under normal innovations

spec=garchset(spec,'Display','off');

[coeff,error] = garchfit(spec,dailyloss);

garchdisp(coeff,error)

[res,sig,LogL] = garchinfer(coeff,dailyloss);

```

## 5. Return

```

function[V]= HW3stocks2011(V0)

% read the friday adjusted close price from excel file

S=xlsread('C:\Users\xiaolei\Desktop\MA575\HW3stocks2011.xls','friday','B2:P58');

c=length(S);

V(1,1)=V0;

lambda=zeros(c,15);

lambda(1,:)=((V0/15)*ones(1,15))./S(c,:);

logreturn=zeros(c-1,1);

for i=1:(c-1)

V(i+1,1)=lambda(i,)*(S(c-i,:));

lambda(i+1,:)=((V(i+1,1)/15)*ones(1,15))./S(c-i,:);

logreturn(i,1)=log(V(i+1,1))-log(V(i,1));

```

---

```
end

V2011=zeros(c-4,1);

LOG2011=zeros(c-4-1,1);

for i=1:(c-4)

V2011(i,1)=V(i,1);

end

for i=1:(c-4-1)

LOG2011(i,1)=logreturn(i,1);

end

disp('lambda for 13 months is:');

disp(lambda);

disp('log return for 2011 is:');

disp(LOG2011);

figure(1);

% draw figure 1

title('graph 1');

plot(V2011,'r');

legend('portfolio value process for 2011');

figure(2);

% draw figure 2

title('graph 2');

plot(LOG2011,'g');

legend('log-return process for 2011');

hold off;
```

---

```

end

function[]=linearLossDistNormal(V0)

Lreturn=xlsread('C:\Users\xiaolei\Desktop\MA575\HW3stocks2011.xls','weekly
log-return','A2:O53');

U=(mean(Lreturn));

Sigma=cov(Lreturn);

W=(1/15)*ones(15,1);

V=HW3stocks2011(V0);

a=length(V);

expectloss=-V(a-4,1)*W'*U;

varianceloss=((V(a-4,1))^2)*W'*Sigma*W;

disp('expect value of loss distribution for January 2012 is:');

disp(expectloss);

disp('variance of loss distribution for January 2012 is:');

disp(varianceloss);

a1=[expectloss-2*sqrt(varianceloss);expectloss+2*sqrt(varianceloss)];

a2=[expectloss-sqrt(varianceloss);expectloss+sqrt(varianceloss)];

aleft1=expectloss-5*sqrt(varianceloss);

aright1=expectloss+5*sqrt(varianceloss);

step1=(aright1-aleft1)/1000;

aleft2=expectloss-100*sqrt(varianceloss);

aright2=expectloss+100*sqrt(varianceloss);

step2=(aright2-aleft2)/1000;

PDF=normpdf(aleft1:step1:aright1,expectloss,varianceloss);

CDF=normcdf(aleft2:step2:aright2,expectloss,varianceloss);

```

---

```

Loss=HW33Q(V0);

PDFa1=normpdf(a1,expectloss,varianceloss);

PDFa2=normpdf(a2,expectloss,varianceloss);

figure(1);

title('pdf plot');

plot(aleft1:step1:aright1,PDF,'g');

hold on;

y=0.000628;

plot(a1,PDFa1,'r*');

plot(a2,PDFa2,'ro');

plot(Loss,y,'b*');

legend('pdf','2 times of std.','1 time of std.','actual loss');

figure(2);

title('cdf plot');

plot(aleft2:step2:aright2,CDF,'r');

legend('cdf');

hold off;

end

function[Loss]=HW32012January(V0)

V=HW3stocks2011(V0);

V2012=zeros(5,1);

LOG2012=zeros(4,1);

Loss=zeros(4,1);

for i=1:5

```

---

```

V2012(i,1)=V(57-4+i-1,1);

end

for j=1:4

Loss(j,1)=- (V2012(j+1,1)-V2012(j,1));

LOG2012(j,1)=log(V2012(j+1,1))-log(V2012(j,1));

end

disp('portfolio value in Jan 2012 is:');

disp(V2012);

disp('log return in Jan 2012 is:');

disp(LOG2012);

disp('actual weekly loss in Jan 2012 is:');

disp(Loss);

figure(1);

title('Vt process in Jan2012');

plot(V2012);

legend('Vt process in Jan2012');

figure(2);

title('log return in Jan2012');

plot(LOG2012,'g');

legend('log return in Jan2012');

hold off;

end

```

## 6. Option Loss

```

function
[callLinLoss,putLinLoss]=optionLinLoss(delta,callPrice,putPrice,S,K,r,tau,riskFacChg)

```

---

```

Cvolatility=blsimpv(S,K,r,tau,callPrice,[],0,[],{'call'});
[ctheta,x]=blstheta(S,K,r,tau,Cvolatility);
[cdelta,x]=blsdelta(S,K,r,tau,Cvolatility);
[crho,x]=blsrho(S,K,r,tau,Cvolatility);
cvega=blsvega(S,K,r,tau,Cvolatility);
Pvolatility=blsimpv(S,K,r,tau,putPrice,[],0,[],{'put'});
[x,ptheta]=blstheta(S,K,r,tau,Pvolatility);
[x,pdelta]=blsdelta(S,K,r,tau,Pvolatility);
[x,prho]=blsrho(S,K,r,tau,Pvolatility);
pvega=blsvega(S,K,r,tau,Pvolatility);
callLinLoss=-(ctheta*delta+S*cdelta*riskFacChg(1,1)+crho*riskFacChg(2,1)+cvega*riskFac
Chg(3,1));
putLinLoss=-(ptheta*delta+S*pdelta*riskFacChg(1,1)+prho*riskFacChg(2,1)+pvega*riskFac
Chg(3,1));
end

```

## 7. Weight

```

function[weightmatrix] = weight(expReturns,CovMatrix)
E=size(expReturns);
d=E(1,1);
e=ones(d,1);
ngrid=500;
mup=linspace(min(expReturns),max(expReturns),ngrid);
rf=0.077885827846;
weightmatrix=zeros(d,ngrid);
Omega=inv(CovMatrix);

```



---

```

B=expReturns'*Omega*expReturns;

A=e'*Omega*expReturns;

C=e'*Omega*e;

D=B*C-A^2;

for i=1:ngrid

lamda1=(C*mup(i)-A)/D;

lamda2=(B-A*mup(i))/D;

weightmatrix(:,i)=Omega*(lamda1*expReturns+lamda2*e);

end

PortSigma=sqrt(diag(weightmatrix'*CovMatrix*weightmatrix));% standard deviation of the
portfolio

imin=find(PortSigma==min(PortSigma));% find the minimum variance portfolio

C=mup(imin);

disp('this is the return of the minimum variance portfolio');

disp(C);

Ieff=(mup>=mup(imin));

Sharperatio=(mup-rf)./PortSigma';%PortSigma is a vector,need transpose here

Itangency=find(Sharperatio==max(Sharperatio));

utangency=mup(Itangency);

lamda1=(C*utangency-A)/D;

lamda2=(B-A*utangency)/D;

weighttangency=Omega*(lamda1*expReturns+lamda2*e);

disp('weight of the tangency portfolio');

disp(weighttangency);

plot(PortSigma(Ieff),mup(Ieff),PortSigma(Itangency),mup(Itangency),'*',PortSigma(imin),mu

```

---

```
p(imin),'o',0,rf,'x');  
line([0 PortSigma(Itangency)],[rf,mup(Itangency)]);  
end
```

## **A.2 Matlab Results**

### **1. JPM**

```
projectoption(1/50,2.65,1.95,45,44,0.0017,0.248)
```

expect loss for call is: 0.8922

variance for call is: 1.1906e+003

expect loss for put is: 0.9063

variance for put is: 1.3075e+003

### **2. IBM**

```
projectoption(1/50,27.45,2.31,205.72,180,0.0017,0.336)
```

call volatility is: 0.2112

put volatility is: 0.2374

expect loss for call is: 1.6348

variance for call is: 9.0832e+003

expect loss for put is: 2.9024

variance for put is: 1.2890e+004

### **3. GS**

```
projectoption(1/50,45.95,0.51,124.3,80,0.0017,0.336)
```

call volatility is: 0.5936

put volatility is: 0.4550

expect loss for call is: 1.6530

variance for call is: 1.4313e+003

---

expect loss for put is: 0.5809

variance for put is: 537.0486

projectoption(1/50,2.74,18.5,124.3,140,0.0017,0.336)

call volatility is: 0.2729

put volatility is: 0.2790

expect loss for call is: 2.4827

variance for call is: 8.6520e+003

expect loss for put is: 1.9077

variance for put is: 9.5725e+003

#### **4. ADS**

projectoption(1/50,5.4,5.8,124.61,125,0.0017,0.248)

call volatility is: 0.2247

put volatility is: 0.2272

expect loss for call is: 2.0058

variance for call is: 9.9124e+003

expect loss for put is: 3.0345

variance for put is: 1.0540e+004

projectoption(1/50,11.6,2,124.61,115,0.0017,0.248)

call volatility is: 0.2300

put volatility is: 0.2331

expect loss for call is: 1.0997

variance for call is: 5.3812e+003

expect loss for put is: 2.1466

variance for put is: 5.9150e+003

---

## 5. VXX

projectoption(1/50,2.75,4.75,20.25,22,0.0017,0.248)

call volatility is: 0.8535

put volatility is: 0.9180

expect loss for call is: 0.6088

variance for call is: 302.9464

expect loss for put is: 0.4319

variance for put is: 241.1509

projectoption(1/50,3.25,3.4,20.25,20,0.0017,0.248)

call volatility is: 0.7853

put volatility is: 0.8897

expect loss for call is: 0.6004

variance for call is: 293.9302

expect loss for put is: 0.4272

variance for put is: 230.6454

## 6. UPS

projectoption(1/50,4.8,1.72,81.11,77.5,0.0017,0.336)

call volatility is: 0.1402

put volatility is: 0.1759

expect loss for call is: 1.4077

variance for call is: 3.8872e+003

expect loss for put is: 1.6753

variance for put is: 4.6431e+003

## 7. Risk free: 1 year treasury bill

---

Daily rate is 0.00043

weight of the tangency portfolio

0.1732

0.3901

-0.2394

0.1631

0.1281

0.3323

0.0699

0.0618

0.1571

0.2656

-0.1752

0.1393

0.1366

-0.3128

-0.2896