Volatility

An Interactive Qualifying Project Report

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Abstract

Volatility is a crucial component the movement of the stock market. Although each transaction operates under relatively simple principals the movement as a whole is chaotic and unpredictable. The model presented simulates this movement using a simple methodology. In order to accurately simulate the movement of the stock market, the model uses a random number for each transaction. The culmination of all the random numbers used creates the familiar sporadic look of the stock market.

Introduction

Volatility is the relative rate in which a security fluctuates. The more volatile a security is the riskier it is as an investment. Risk is a crucial part of investing and the riskier something is the less attractive the security becomes. The stock market moves with what is known as Brownian motion. This movement can be modeled and simulated to produce accurate results and conclusions. Different risk factors affect the movement of a stock price in various ways. By taking the annualized daily change of a security one is able to quantify volatility. There are many different methods of estimating volatility however; some are more accurate than others. Volatility can be estimated using historical data and also using future call and put prices to infer implied volatility. Originally, volatility was estimated by simply taking the standard deviation of daily closes. Today it has transformed into using intra-day data to produce more efficient and precise results. The value obtained by estimating volatility is incorporated within the Black Scholes' model to quantify call and put options. The Black-Scholes' formula makes several assumptions including one concerning the consistency of volatility. Although it can be proven that volatility does not remain constant over time, as long as the time period remains small, this assumption holds true. The more efficient the technique used to estimate volatility, the more accurate the Black-Scholes' assumption of consistency becomes. This is because a fewer number of days are needed to obtain a significant level of accuracy. Formulas such as GARCH may be more efficient at estimating volatility because they do not rely on the assumption consistency. The history of valuating call and put options has evolved over the years and there is promise for more sophisticated methods in the future.

Model and Simulation

Introduction

The stock market is the open forum for publicly owned companies. It is important to understand the nature of the stock market in order to obtain the ability to arrive at conclusions with significant consequences. A model is a tool used to show the movement of the stock market. The movement of the stock market happens continuously and the inability to perfectly predict the future opens up the possibility for risk. The nature of movement in the stock market is unique and deceptively intricate. Therefore, understanding risk is crucial because quantifying risk gives investors the necessary information to protect themselves from unexpected losses. Articulating concepts concerning the movement of the stock market can be done through the use of a model. Once a model has been developed different risk factors can be investigated to understand the effects on the movement of the security.

Every publicly owned company is valued at a certain price. Shares of a company are known as stock and are available on the open market. Owning a stock is similar to owning anything else in that it has a supply, a demand and functional practicality with ownership. However, it is unique in that it can generate potential profits and losses that come from ownership. The price of a company is derived from the total amount of profits that will be generated over the lifetime of the company. Since this is largely predicated on future events, it is impossible to truly know the intrinsic value of the company's worth. Everyone interested in investing in a company has to evaluate how much the company is worth. A popular company might have a trade every fifteen seconds. Each trade is either higher or lower than the previous one and causes the stock price to shift up or down. If a broker believes that the stock price is under or overvalued they will trade the stock in order to make a potential profit. Since the intrinsic value of a stock is unknown, not all brokers can agree on the value of the stock. A plot of the price that every broker is willing to buy and sell a stock for, results in the formation of two normal distributions on each side of the intrinsic value of the stock. The right tail of the buyer's distribution would intersect with the left tail of the seller's distribution and trades would occur as seen in Figure 1.

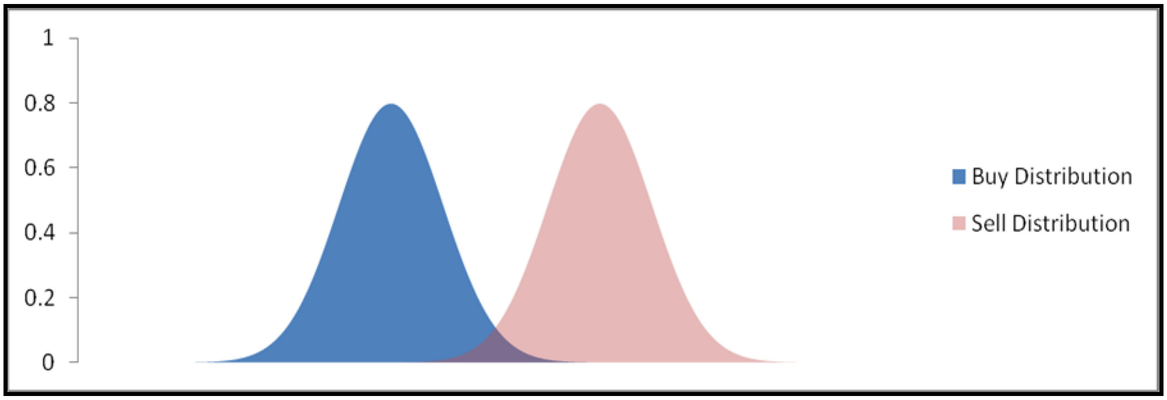

Figure 1: Normal Distribution of buy and seller stock value estimates
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Someone who is willing to buy the stock for more than the current stock price would negotiate a trade with someone who is willing to sell the stock and the price of the trade becomes the new stock price. A closer look at the values in which trades occur can be seen in Figure 2.

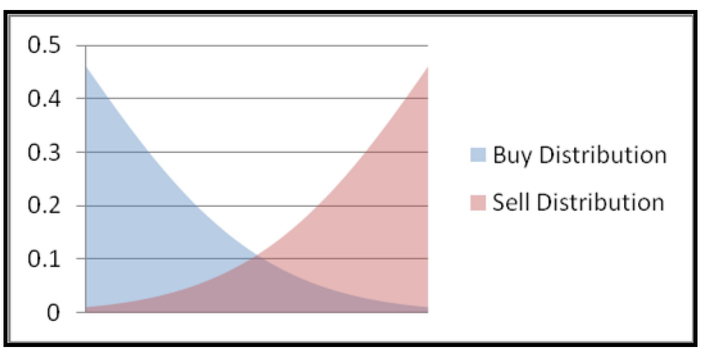

Figure 2: Overlap that occurs between buyer and seller trade values. The overlap of price estimation is the area in which trades occur.

It is assumed that the average broker evaluates the stock roughly at its intrinsic value. Because of this, the stock price will generally move towards the intrinsic value because there will be a greater demand on the side of the stock price that the intrinsic value is located.

Volatility and Risk Factors

One of the primary factors in the stock market is volatility. Volatility represents the standard deviation of returns over a specific period of time. This number is independent from the risk free rate of return that represents the change in price over a specific period of time if there is no risk. Volatility and risk are synonymous terms as they both pertain to the unpredictable movements of the stock market. Although the movement of the stock price may be the most important element of the stock market, risk plays a significant role. The movement of the stock price is important because it represents the changing value of the company. The change in price results in gains and losses for potential investors. However, the risk of the stock market is still significant because the more unpredictable a stock is, the less reassurance one has in investing in the stock.

Brokers are constantly trying to find stocks that minimize risk. A stock isn't risky because it might go down; a stock is risky because its movement is unpredictable. One could make a profit by correctly predicting a stock's fall or rise with equal precision. If nothing happens within a company, its stock price would stay the same. However, changes happen all the time within an organization. Changes become public and result in the change of the value of a stock.

Different factors of a company correspond to different levels of risk that change the stock's volatility. Risk factors can be quantified and incorporated into the model. Each of these risk factors affects the model differently and they all have a different impact on the volatility of the security. The effects of various levels of risk result in different movements of the value of the stock. Different movements can be contrasted and conclusions can be made.

When a change happens within a company a press conference is used as the medium in which this information becomes public. The purpose of information given at a press conference is to give investors a good idea concerning the intrinsic value of a company. The first risk factor to introduce is size. Size represents the average magnitude of intrinsic change resulting from the information given at a press conference. The first hypothesis tested by the model is the larger these changes are the more unpredictable a company will be.

In certain industries it may be harder for companies to explicitly quantify the changes that happen within their company. If the information mentioned in the conference is more obscure, then the larger the displacement of investor's evaluations will be. Obscurity is the second risk factor and represents the standard deviation of investor's evaluations. The second hypothesis that is addressed is that the more obscure the information is the more risky it is to invest in the stock.

Frequency is the last risk factor to take into account and it represents the total number of press conferences in a given time period. The higher the frequency the more changes in the intrinsic value of the stock. The last hypothesis investigated by the model is that the more changes the greater the unpredictability the stock becomes. Each time new information is given, the opportunity for unpredictability gets presented. The higher the rate of unpredictability within a given time, the higher the total unpredictability will be. When change happens within a company the stock price does not automatically shift to the value that the recent change suggests. If this was the case then stock prices would move smoothly from day to day and there would be little risk in investing.

The results of the model will prove the aforementioned hypotheses' to be supported or nullified. Based upon analyzing the risk factors size, obscurity and frequency within the model, justification of these three hypotheses will be obtained. After these hypotheses are addressed further analysis of the graphs of the risk factors versus volatility will allow for additional information about the effects of each risk factor to be obtained.

Model Methodology

The model simulates the actions of 200 brokers on eight companies that differ in size, frequency and obscurity with minute by minute data over the course of thirty trading days. Each broker evaluates the price of a stock whenever information concerning the company changes. The rate of change corresponds to the frequency number assigned to each company. The brokers form a buyer's distribution and seller's distribution that intersect over the intrinsic value of the stock.

In this model, the stock price for each company is initially announced at 100. The announcement is made from the company that suggests the intrinsic value should change. This means that the brokers in this simulation that are willing to buy or sell for under and over the intrinsic value, trade, and by the end of the trading period the stock price should move to close to the intrinsic value. Of course if by chance there are more traders that are willing to buy than sell then supply and demand will cause the final trade value to be a little over the intrinsic value. When the stock price is announced at the beginning of the trading period a buyer and a seller are randomly chosen and a trade is negotiated by averaging the buyers 'buy value' and the sellers 'sell value'. If there are more buyers than sellers the price will rise and vise versus. It is not an assumed that the price will abide by this generalization, but probability favors the side of demand. This should continue until the stock price reaches the new intrinsic value or a little higher or lower as previously stated. When a transaction is completed both brokers are eliminated from the playing field and 198 brokers remain. The process is repeated with the new stock price and continues until no one is willing to buy or sell the stock. In reality changes happen at a rate that the price would never become stagnant due to overlapped information, but in this model we will assume that once a change works itself out another change happens in succession.

Analysis and Conclusions

The model simulates eight companies stock movement over 30 days with minute by minute intervals. Each company has a different combination of the three risk factors; obscurity, size and frequency. Each company's movement is compared to the intrinsic data that the stock price is dependent on. The eight companies are then compared and the effects of each risk factor are shown. The two methods of quantifying risk and volatility are a standard deviation test and a sum of squares test. The standard deviation test finds the standard deviation of the stock movement over the thirty day period. A stock with a higher standard deviation is more risky and has a higher volatility. The sums of squares test takes the sum of the square of the difference between the stock price and the intrinsic value at every minute interval and averages them together. If a stock perfectly follows its intrinsic value then its risk solely lies in the standard deviation of the intrinsic value and not on the sporadic movement of the stock price. This will result in lower levels of risk and volatility.

Obscurity is the inability for a broker to access the value of a stock accurately. Mathematically obscurity is the standard deviation of the normal distribution of brokers' evaluation of the intrinsic value of the stock. A small obscurity means fewer buyers will think the value of the stock is more than the asking price and fewer sellers will think the opposite and less trades will occur.

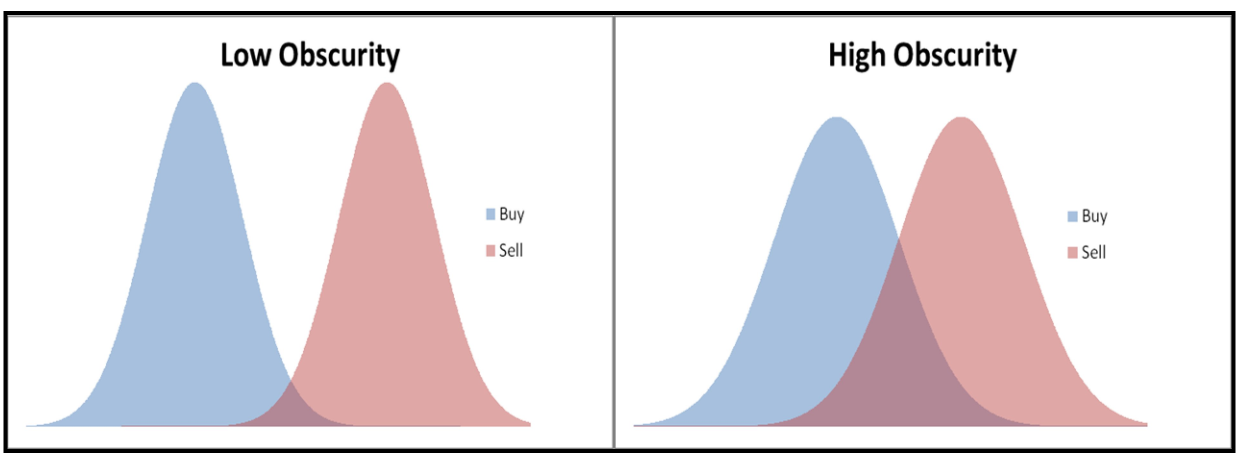

Figure 3: Comparison of obscurity distributions

Figure 3 demonstrates that a higher level of obscurity results in a larger intersection between the two distributions which suggests more trades will occur in each time period. The comparison of high and low obscurity, shown in Figure 4, suggests that the company with the higher level of obscurity contains more erratic movement and a higher sum of squares. However, the standard deviation of both levels of obscurity remains the same.

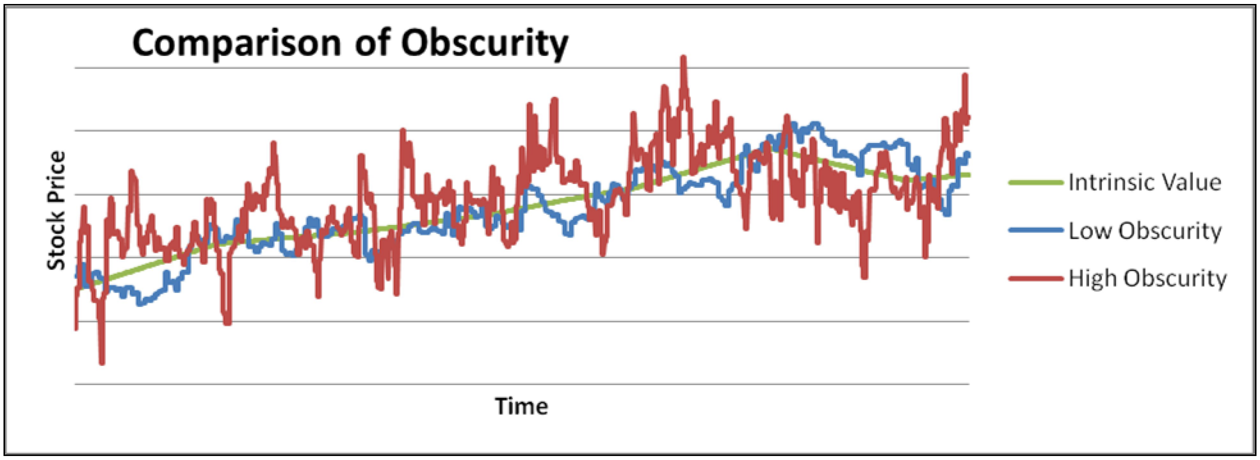

Figure 4: Comparison of obscurity levels to the intrinsic stock value

Size represents how high each change is in the company's value. Size has little impact the number of trades in a given period, but higher levels of size creates more unevenness in the initial number of willing buyers and sellers which fuels the demand in one direction and causes steeper movement.

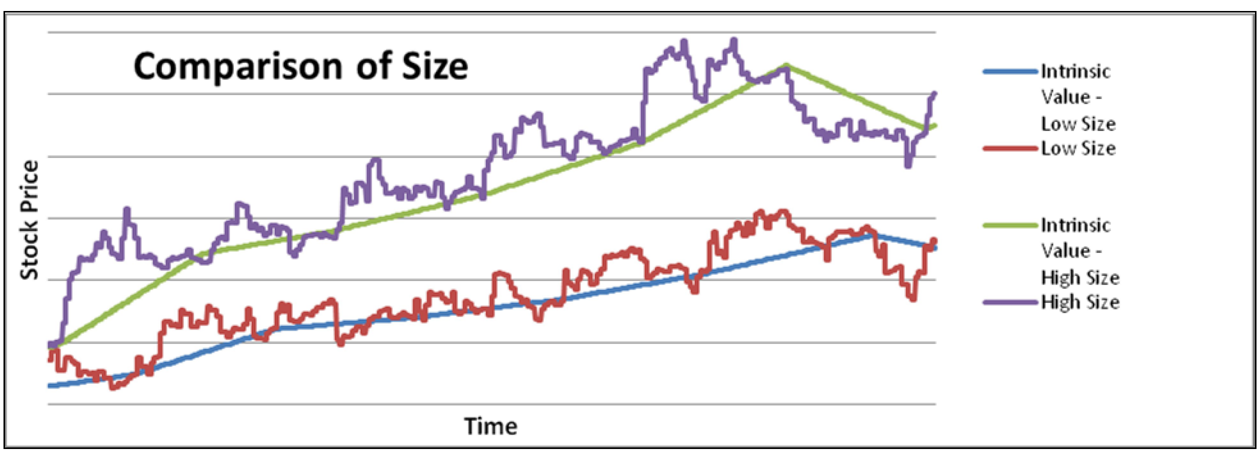

Figure 5: Comparison of size to the intrinsic stock value

Figure 5 shows two different levels of size. Although the sum of squares remains roughly constant, the standard deviation of the intrinsic value increases as size increases, resulting in higher levels of volatility.

Frequency represents the number of changes of intrinsic value in a given time period. Higher frequencies create more opportunities for trading to occur. Although each trading period will have roughly the same amount of trades, higher levels of frequency will cause for more trades in a given time period. The nature of the model causes the stock price to become close to the intrinsic value near the end of each trading period, but during a trade period the movement can be very sporadic. Higher levels of frequency mean that although more trades will occur there will be more instances where the stock price becomes very close to the intrinsic value.

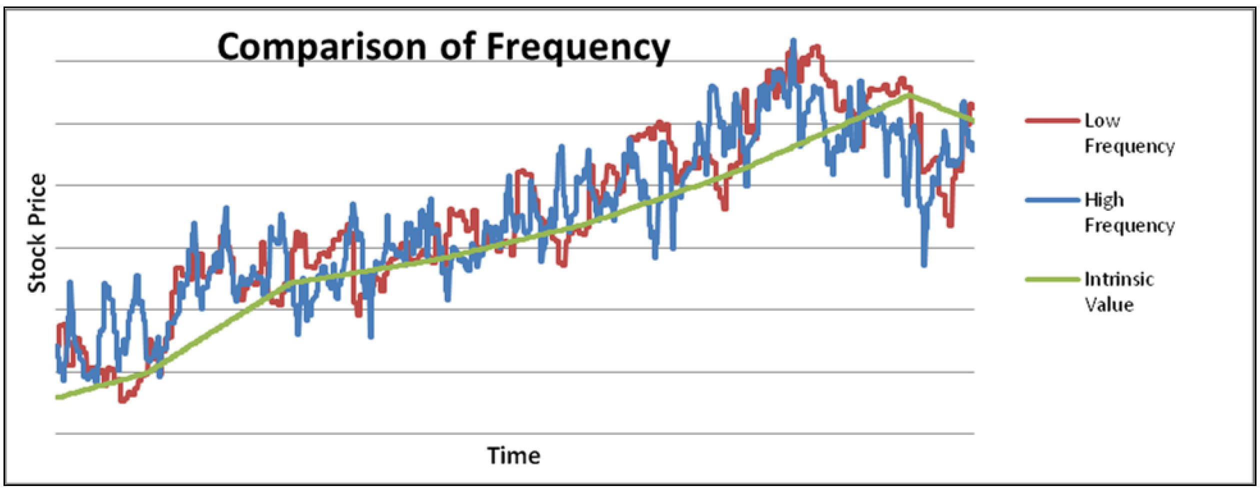

Figure 6: Comparison of frequency to the intrinsic stock value

Figure 6 shows two different levels of frequency. The standard stays relatively the same, but the sum of squares decreases due to the principals mentioned above. As seen in the graph, there are more instances of close proximities to the intrinsic value at higher frequencies.

An investor's goal is to determine which company is the least volatile. Although the individual effect of each risk factor can be examined, volatility depends on the interaction between the different factors. The standard deviation test shows that size and obscurity depend negatively on each other as size increases obscurity decreases and vice versa and changes in frequency does not change the standard deviation of the data. The Sums of Squares test shows that an increase in size or obscurity results in higher sums of squares and an increase in frequency results in a lower sums of squares. When size is low, an increase in obscurity results in a dramatic increase in sums of squares as demonstrated in Figure 4.

Figure 7 shows the ratio of each risk factor between its lower and higher level.

Avg Sqrs	Size 1	Size 2	Ratio
F 1 O 1	0.392652	1.144748	2.915427
F 1 O 2	1.251322	1.957501	1.564347
F 2 O 1	0.237952	0.413232	1.736616
F 2 O 2	0.95018	1.150115	1.210419
	Freq 1	Freq 2	Ratio
S 1 O 1	0.392652	0.237952	0.606014
S 1 O 2	1.251322	0.95018	0.759341
S 2 O 1	1.144748	0.413232	0.360981
S 2 O 2	1.957501	1.150115	0.587543
	Obs 1	Obs 2	Ratio
S 1 F 1	0.392652	1.251322	3.186848
S 1 F 2	0.237952	0.95018	3.993151
S 2 F 1	1.144748	1.957501	1.709984
S 2 F 2	0.413232	1.150115	2.783221

StdDev	Size 1	Size 2	Ratio
F 1 O 1	1.655274	3.152864	1.904739
F 1 O 2	1.849083	3.243493	1.754109
F 2 O 1	1.584117	3.066744	1.935932
F 2 O 2	1.795862	3.165166	1.762477
	Freq 1	Freq 2	Ratio
S 1 O 1	1.655274	1.584117	0.957012
S 1 O 2	1.849083	1.795862	0.971218
S 2 O 1	3.152864	3.066744	0.972685
S 2 O 2	3.243493	3.165166	0.975851
	Obs 1	Obs 2	Ratio
S 1 F 1	1.655274	1.849083	1.117086
S 1 F 2	1.584117	1.795862	1.133667
S 2 F 1	3.152864	3.243493	1.028745
S 2 F 2	3.066744	3.165166	1.032093

Background and Research

Estimating Volatility

The first volatility estimator used in the stock market took the standard deviation from close to close of historical data of a security. Since then Parkinson [1980], Garman and Klass [1980], Rogers and Satchell [1991], Alizahdeh, Brandt and Diebold [2001] and Yang and Zhang [2002] have all been created to replace the classical close to close estimator by incorporating more information which increases the efficiency of the estimator dramatically. An example of these new methods was demonstrated in 1997 when Anderson and Bollerslev used high frequency data at five minute intervals to very accurately predict volatility. (Brandt)

Although the current volatility estimators are more accurate, these estimators make inaccurate assumptions. These inaccurate conjectures include the assumption that Brownian motion is continuous, volatility is constant and drift is zero. Rogers and Satchell [1991] take time-varying drift into account, making it an innovative estimator. (Kinder) Another hidden factor concerning volatility is price jumps which occur when the open is considerably different than the close of the previous day. To adjust for the inconsistency of volatility estimators a short period of time should be used when analyzing historical data so the inconsistencies are relatively small. (iComp) For this to work, very efficient estimators need to be used or data must be taken as frequently as every five minutes as demonstrated by Anderson and Bollerslev in 1997. (Bali)

The motion of an asset with price S_t changes as follows:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

Where μ is the asset drift, σ is the volatility and Z_t is the Weiner process which follows a normal distribution. Many of the estimators look to estimate σ^2 including the following examples:

Classical - $\sigma^2 = \Sigma[(O_i+C_i)-1/n*\Sigma(O_i+C_i)]^2 / (n-1)$

Parkinson - $\sigma^2 = \Sigma(U_i-D_i)^2/[4n*\ln(2)]$

Garman-Klass - $\sigma^2 = \Sigma(U_i-D_i)^2-(.19/n)*\Sigma[(C_i*(U_i+D_i)-2(U_i*D_i)]-(.383/n)*\Sigma(C_i\hat{}2)$

Rogers & Satchell - $\sigma^2 = \Sigma[U_i(U_i-C_i)+D_i(D_i-C_i)]/n$

Where O - open, C – close, U – high, D – low and n - # of days (Brandt)

Parkinson, Garman-Klass and Rogers & Satchell are each five to eight times more efficient than the Classical estimator and are the current estimators in the constantly improving world of volatility estimation. The estimations produced by these equations get incorporated into other models such as the Black Scholes Model.

Black Scholes Model

A stock option is a contract between two parties where the writer of the contract sells the right to buy the underlying security at a specific price and time. Black Scholes and Merton developed a model to price stock options.

The movement of the stock market follows a lognormal distribution which means the natural log of the variable is distributed normally which uses the equation $d*\ln(S) = (\mu-\sigma^2/2)dt + \sigma SdZ$ where S is the stock price, μ is the risk free rate of return, σ is volatility and dZ is a normal distribution. (Hull)

The change of ln(S) over time has a normal distribution with a mean of $T*(\mu-\sigma^2/2)$ and a standard deviation of $\sigma*T^{1/2}$, where T is time. An example can help illustrate this:

If a stock is worth $40 today with a risk free rate of return of 16% and a volatility of 20% then we can use the above equations to find out where the stock will be in six months time.

The mean of the lognormal distribution would be, 3.759 with a standard deviation of .141. This would create a 95% confidence interval of ln(S)=3.759 +/- 1.96*.141. This is equivalent to saying S will range from 32.55 to 56.56 in six months time. (Hull)

The most important result given to us by the Black Scholes model is the method to evaluate call and put options. The stock market is generally thought of as a place to either buy or sell a stock. In recent history this process has become more complicated. Now, one can buy the right to buy or sell a stock at a predetermined date and amount. So the knowledge that a $40 stock with a risk free rate of 16% and volatility of 20% will be worth between 32.55 and 56.56 in six months is a useful piece of information. The equation for the price of call and put are as follows:

$$\text{Call} = S_o*N(d_1)-X^{e-rt} * N(d_2)$$

$$\text{Put} = X^{e-rt}* N(-d_2) - S_o * N(-d_1)$$

Where $d_1 = [\ln(S_o/X)+(\mu+\sigma^2/2)*T]/\sigma*T^{1/2}$ and $d_2 = d_1-\sigma*T^{1/2}$

And S_o is price X is strike price (The price in which the stock will be sold on the predetermined date), N() is a normal distribution. (Hull)

An example of an evaluation of a call and put price is illustrated below:

Suppose a stock is worth $42 with a risk free rate of 16% and a volatility of 20% and we wanted to know the call and put prices of a strike price of $40 in six months the equation would be as follows

$C = 42* N(.7693) – 38.049* N(.6278) = 4.76$

$P = 38.049* N(-6278)-42* N(-.7693) = .81$

If one owned a call option then he would need the stock to go up $2.76 to 44.76 to break even, and if one owned a put option he would need the stock to drop $2.81 to 39.19 to break even.

The key piece of information needed for this model to work is volatility. All of the other pieces of information are very easy to find out, some are even up to the negotiators, but volatility needs to be estimated, which is proven very difficult.

Because volatility isn't readily available and definitely not agreed upon, the Black Scholes equation is more of a model for individuals to price options and not the set price of the option on the open market. Prices for calls and puts change daily much like the stock's they depend on do and the information about them are just as public. If the call or put price, all the other information concerning the stock and the option conditions are known the volatility of the stock could be inferred. This is known as implied volatility and can be characterized in this example:

Suppose a stock is worth $21 and a risk free rate of return of 16% in three months with a $20 strike price the security has a call price of 1.875. We then estimate a value for volatility and change it until the equations above result in a call price of 1.875, in this case the implied volatility would be 23.5%. (Hull)

Estimating volatility is essential for quantifying risk and pricing call and put options with use of the Black Scholes Model. Although the risk factors used in the model and simulation presented within this paper are not included above in any of the estimating volatility techniques; those factors represent another way of arriving at the same conclusion. The statistics resulting from the presented model and simulation give insight into risk in the stock market and can be adapted to fit into the Black Scholes Model or into another that could be used to value options.

Conclusion

The Black Scholes model has revolutionized the way options are priced. It takes into account the illusive concept of volatility and requires the accurate estimation of historical volatility. Volatility represents the unpredictability of the stock market, and quantifying this concept gives investors the wherewithal needed to safely participate in the stock market. Different techniques have been used to estimate historical volatility. The classic technique involves taking the standard deviation of close to close values. Advancements in technology have resulted in the development of more powerful methods such as high frequency intraday techniques.

Volatility is simply a phenomenon resulting from the natural movement of the stock market. Risk factors cause different levels of volatility to exist, making a security either more or less desirable. By implementing a realistic methodology, a model can be created to accurately mimic the movement of the stock market. Different tests of risk can be used to determine how volatile a security is. In the model, three risk factors that all play a role in quantifying volatility are presented. The first factor is size, which represents the day to day movement of the value of the security. Second is frequency, which represents the amount of changes to a security in a given time. The last factor addressed is obscurity, which represents the inability of investors to agree upon the value of the security. These three risk factors were compared at different levels and two tests were completed to determine their effect on volatility. The first test took the standard deviation of the minute by minute data; this is a high frequency intraday volatility estimator. The second test took the average square of the difference between the stock price and the stock's intrinsic value. The second test gave a much more accurate portrayal of risk, but is not realistic because the intrinsic value of a company is unknown outside the confines of the model.

The model demonstrates that an increase in risk factors, size and obscurity, result in higher standard deviations and average squares which raises the amount of risk in the security. Increases in frequency show no change in standard deviation, however result in a dramatic decrease in average squares resulting in a lower volatility. Understanding these conclusions about the effects of different risk factors makes investing more manageable and secure.

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