**Incorporating survey weights Into**

 **Logistic Regression Models**

By

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**Abstract**

Incorporating survey weights into likelihood-based analysis is a controversial issue because the sampling weights are not simply equal to the reciprocal of selection probabilities but they are adjusted for various characteristics such as age, race, etc. Some adjustments are based on nonresponses as well. This adjustment is accomplished using a combination of probability calculations. When we build a logistic regression model to predict categorical outcomes with survey data, the sampling weights should be considered if the sampling design does not give each individual an equal chance of being selected in the sample. We rescale these weights to sum to an equivalent sample size because the variance is too small with the original weights. These new weights are called the adjusted weights. The old method is to apply quasi-likelihood maximization to make estimation with the adjusted weights. We develop a new method based on the correct likelihood for logistic regression to include the adjusted weights. In the new method, the adjusted weights are further used to adjust for both covariates and intercepts. We explore the differences and similarities between the quasi-likelihood and the correct likelihood methods. We use both binary logistic regression model and multinomial logistic regression model to estimate parameters and apply the methods to body mass index data from the Third National Health and Nutrition Examination Survey. The results show some similarities and differences between the old and new methods in parameter estimates, standard errors and statistical p-values.

**Keywords**: Sampling weights, Binary logistic regression, Multinomial logistic regression, Adjusted weights, Quasi-likelihood.

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**Contents**

[Chapter 1. The Old Method １](#_Toc354137265)

[1.1 Introduction １](#_Toc354137266)

[1.2 Sampling weights ３](#_Toc354137267)

[1.3 Adjusted weights and quasi-likelihood ４](#_Toc354137268)

[1.3.1 Probability weights ４](#_Toc354137269)

[1.3.2 Adjusted weights ５](#_Toc354137270)

[1.3.3 Generalized linear models ７](#_Toc354137271)

[1.3.4 Maximum likelihood ８](#_Toc354137272)

[1.3.5 Quasi-likelihood ９](#_Toc354137273)

[Chapter 2. The New Method １１](#_Toc354137274)

[2.1 Normalized distribution with sampling weights １１](#_Toc354137275)

[2.2.1 New view of sampling weights １１](#_Toc354137276)

[2.2.2 Summary old and new method １４](#_Toc354137277)

[Chapter 3. Illustrative Examples １６](#_Toc354137278)

[3.1 Binary logistic regression １７](#_Toc354137279)

[3.2 Multinomial logistic regressions ２６](#_Toc354137280)

[Chapter 4. Discussion ３５](#_Toc354137281)

[References ３６](#_Toc354137282)

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# Chapter 1. The Old Method

## 1.1 Introduction

 In recent years, logistic regression is applied extensively in numerous disciplines, such as medical or social sciences. In statistics, when the variables of interest have only two possible responses, we represent them as binary outcome. For example, in a study of obesity for adults, selected adults have a high (>30 kg/m2) body mass index (BMI) or do not have a high BMI (<30 kg/m2), with independent variables age, race and gender, the response variable Y is defined to have two possible outcomes: adults having a high BMI, not having a high BMI. Subsequently, we code them as 1 and 0, respectively. We can extend the binary logistic regression model to multinomial logistic regression model, in which the response variable has more than two levels. For example, in the study of obesity for adults, we can divide the BMI value into four different levels (underweight, normal, overweight and obese), then we build the multinomial logistic regression model with age, race and gender as covariates. We label the levels as 1, 2, 3 and 4, respectively, and define 1 as the reference category.

 When we consider estimating the regression coefficients using the survey data, the model does need include the sampling designs because the whole data is not available. The sampling weights should be considered if the sampling design does not give each individual an equal chance of being selected. Sampling weights can be thought as the number of observations represented by a unit in the population if they are scaled to sum to the population size. Gelman (2007) stated. “Sampling weight is a mess. It is not easy to estimate anything more complicated using weights than a simple mean or ratio, and standard errors are tricky even with simple weighted means. Contrary to what is assumed by many researchers, survey weights are not in general equal to the inverse of probabilities selection, but rather are constructed based on a combination of probability calculations and nonresponse adjustments.” Longford (1995), Graubard and Korn (1996), Korn and Graubard (2003), Pfeffermann et al. (1998) and others have discussed the use of sampling weights to rectify the bias problem in the context of two-level linear (or linear mixed) models, particularly random-intercept models. In this paper, we rescale the sampling weights to sum to an equivalent sample size because the variance is too small. These new weights are called the adjusted weights. The adjusted weights are incorporated into the logistic regression model to estimate the parameters.

 Traditionally, we use maximum likelihood methods to estimate and make inference about the parameters. However, the likelihood methods are efficient and attractive when the model follows the normal distribution assumption. In reality, not all the distributions are normal, such as a Poisson distribution, in which the variance is same as the mean. This means variance function is mostly determined by the mean function. The mean and variance parameters do not vary independently. In likelihood based analysis, it is standard to use quasi-likelihood method (QLM) to estimate the variance function from data directly without normal distributional assumption. In other words, the variance function and mean function vary independently. Therefore, QLM can be used to estimate parameters in the logistic regression model with adjusted weights. Grilli and Pratesi (2004) accomplished this by using SAS NLMIXED (Wolfinger, 1999) which implements maximum likelihood method for generalized linear mixed models using adaptive quadrature. In our model, we apply SURVEYLOGISTIC procedure in SAS software to analyze logistic regression with adjusted weights. This is the old method.

 We introduce a new method to analyze logistic regression model to include the adjusted weights. Under this, first, we give weights to logistic regression equation. Second, in order to keep the new function still to be a probability distribution function, we normalize the logistic regression equation with adjusted weights. Finally, we use adjusted weights to multiply the intercepts and covariates in the logistic regression model. Then, we use correct likelihood method (CLM) to estimate parameters in the logistic regression model with the adjusted covariates and intercepts. We achieve this by using PROC LOGISTIC procedure with link of CLOGIT (cumulative logit) in SAS software, this is the new method. The new method is normalized logistic regression with adjusted sampling weights, while the old method is un-normalized logistic regression with adjusted sampling weights. We use SURVEYLOGISTIC procedure in SAS software to analyze the old method, but we use LOGISTIC procedure in the SAS software to analyze new method. Nevertheless, both new method and old method incorporate adjusted weights. When both of them are used to analyze survey data, there are some similarities and differences.

 Later, we analyze body mass index data for adults from the Third National Health and Nutrition Examination Survey using QLM and CLM. BMI is a measure of human body shape based on an individual’s weight and height. This study is useful to diagnose overweight and obese adults. We construct model at the county level with age, race and gender as covariates. The BMI we study here has four levels, underweight, normal weight, overweight and obese, labeled as 1, 2, 3 and 4. First, we build the binary logistic regressions model, in which we compare underweight without underweight or normal weight without normal weight and so on. Thereafter, we build multinomial logistic regression model to analyze four levels of BMI at the same time; we use BMI = ‘1’ as the reference category which is in compared to the other three levels of BMI. We do this for each county. Observing the results produced from two models with two different methods, we find differences and similarities between traditional sampling weights methods and our new methods in terms of p-values, estimates etc.

 In Chapter 1, we review the old method which uses QLM to estimate the parameters with adjusted weights in the logistic regression model. In Chapter 2, we develop a new method that adjusts the covariates and intercepts in the model and use CLM to estimate the parameters. In Chapter 3, we illustrate both QLM and CLM by applying them to BMI survey data. We build both binary logistic regression and multinomial logistic regression models. The results show differences and similarities in the estimates, standard error, Wald chi-squared statistics and p-values using the two methods. This topic is of enormous correct contributor, and it is still an open topic for researchers to study in the future.

## 1.2 Sampling weights

 In order to reduce the cost, increase the speed of estimation, and ensure the accuracy and quality, we always select a subset of individuals from a population called a sample to make inference about the population characteristics. In general, a sample weight of an individual is the reciprocal of its probability of selection in the sample. If the th unit has probability to be included in the sample, then the weight would be ; see Kish (1965) and Cochran (1977).

We estimate population means, population totals or proportions from the survey data. If it is simple random sample (the probability of selection for each individual is equal), we can make descriptive inference of the population relying on the information in the survey data. However, not all the sample data is based simple random sample in reality, it can include other sample designs, such as systematic sampling, stratified sampling, probability proportional to size sampling, etc. In this case, sample weights compensate for some bias and rectify other departures between the sample and the reference population. With the inclusion of weights, the Horvitz-Thompson estimator of the finite population mean is given by .

 Pfeffermann (1993, 1996) discussed the role of sampling weights in modeling survey data. He developed methods to incorporate the weights in the analysis. The general conclusion of his study is:

1. The weights can be used to compensate non-ignorable sampling designs which have selection bias.
2. The weights can be used to rectify misspecifications of the model.

 When we consider estimating the regression coefficients, in the case of availability of the entire finite population data, it is easy to estimate regression parameters using least squares method. There is no bias, and estimators will be consistent. However, if we do not have the population data, the estimate would be inconsistent and bias without using the sampling weights. Pfeffermann (1993, 1996), Rubin (1976) and Little (1982) stated that not using the design probabilities will result in inconsistent estimators.

 The sample weights should be considered in general if the sample design does not give each individual an equal chance of being selected in the sample. Sampling weights correct some bias or other departures between the sample and the reference population (unequal probabilities of selection, non-response). Usually, the base weight of a sampled unit is the reciprocal of its probability of selection in the sample. For multi-stage designs, the base weights should be considered to reflect the probabilities of selection at each stage.

 Surveys often combine complex sampling designs where primary sampling units (PSUs)) are sampled in the first stage, sub-clusters in the second stage (SSUs) and so on. At each stage, the units at the corresponding level are often selected with unequal probabilities, typically leading to biased parameter estimates if standard multinomial modeling is used. Longford (1995), Graubard and Korn (1996), Korn and Graubard (2003), Pfeffermann et al. (1998) and others have discussed the use of sampling weights to rectify this problem in the context of two-level linear (or linear mixed) models, particularly random-intercept models.

## 1.3 Adjusted weights and quasi-likelihood

### 1.3.1 Probability weights

 When we consider estimating the regression coefficients in the survey data, we need to include sampling designs because the whole data is not available. The sample weights should be considered if sampling design does not give each individual an equal chance of being selected. Sampling weights can be thought of as the number of observations represented by a unit in the population if they are scaled to sum to the population size. Weights may vary for several reasons. Smaller selection probabilities may be assigned to the elements with high data collection costs. High selection probabilities may be assigned to the elements with larger variances. The estimator of total will be equal to , where is the overall probability that the th element is selected. We can define the sampling weight for the th element as .

### 1.3.2 Adjusted weights

 In the super population model, let denote the response variable for the th unit in the sample. Here, ’s are assumed to be independent random variables. Let us define the mean for th unit and variance , where is the sample size.

The mean and variance of the super population model are

The estimate of and variance of mean are

 Let us consider a “new” set of weights defined by , where is . We call as the adjusted weights and as an equivalent sample size (Potthoff, Woodbury and Manton 1992). The equivalent sample size is smaller than the population size. We rescale the sampling weights to sum to an equivalent sample size because the original variance is too small to include enough information. These new weights are called the adjusted weights.

We can rewrite the estimators using as

**Example of normal distribution with sampling weights**

 Let us consider as independent and identically distributed normal random variables from the population with mean equals and variance equals.

The density function of normal distribution is , for a random size.

 with the sampling weights it is In order to find the maximum likelihood estimators, let

=

When , it has:

 and

Define:

, where is a new adjusted weights

, where equivalent sample size ,

 The can be estimated by , the summations over is from 1 to (the sample size). The estimate of variance for mean can be expressed by , where . An approximate 95 % confidence interval for is.

 The procedures do not rely on conditioning on model elements such as covariates to adjust for design effects. Instead, we obtain estimators by rescaling sample weights to sum to the equivalent sample size. The equivalent sample size is smaller than the sample size. For some design, the equivalent sample size could be larger, but we restrict attention to simple random sampling. We rescale the sampling weights to sum to an equivalent sample size because the variance where respond weights is small.

### 1.3.3 Generalized linear models[[1]](#footnote-1)

 In statistics, when the outcome variables of interest only have two possible responses, we can represent them by the binary variables. For example, in a study of obesity for adults, if the selected adults have a high (>30kg/m2) or do not have a high BMI (<30kg/m2), with independent variables of age, race and gender, the response variable Y is defined to have two possible outcomes: adults have a high BMI, adults do not have a high BMI. We can label them as 1 and 0, respectively. We extend the binary logistic regression model to multinomial logistic regression model. The response variables in multinomial model have more than two levels. For example, in the study of obesity for adults, we divide the BMI value into four different levels, labeled as 1, 2, 3 and 4. In the multinomial logistic regression model, we use BMI = ’1’ as the reference category, in compare to the other three levels of BMI with the age, race and gender as covariates.

***Binary logistic regression model***

Let represent response variable, represent covariates, we get:

 ,

***Multiple logistic regression models***

 We can extend the simple logistic regression model easily to more than one predictor variable.

Let us define,

Then, we get,

So

***Multinomial logistic regression model***

 Sometimes, when the response variables have more than two levels, we still use logistic regression model. We divide the response into response categories, the variables will be . Then, let be the baseline, the logit for the th comparison is:

### 1.3.4 Maximum likelihood

Recall that, the joint probability function for binary logistic regression is:

Since and

Therefore, .

We are trying to find and to maximize the log-likelihood function:

.

Define:

 The model is . The estimator of is , where a diagonal matrix with th diagonal element .

### 1.3.5 Quasi-likelihood

We analyze the binary logistic regression with sampling weights.

The quasi likelihood is

Let =

Let

 We find estimators to maximize the quasi log-likelihood function: . The estimator of is .

 In general, we use maximum likelihood methods to make estimation and inference. For example, we always assume that the responses have normal distributions. The likelihood methods are efficient and attractive only when the models follow the distributional assumptions. In reality, not all of the distributions meet this assumption, such as binomial distribution or Poisson distribution for which the likelihood methods will not perform well. For example, we use logistic regression models to analyze binary data and use Poisson regression models to analyze count data. In a Poisson distribution, the variance is the same as the mean, and so the variance function is mostly determined by the mean function. However, if the data follow a normal distribution, their mean parameters and variance parameters do not connect with each other, which mean they can vary independently.

 In the old method, we use QLM to make estimation and inference, which is not a true likelihood. It does not need normal distributional assumptions. QLM estimates the variance function from the data directly without normal distributional assumption. Grilli and Pratesi (2004) accomplished this by using SAS NLMIXED (Wolfinger, 1999) which implements maximum likelihood estimation for generalized linear mixed models by using adaptive quadrature. The first moment is separate from the second moment . Therefore, they are uncorrelated to the normal distributional assumptions, but determined only by the first and second moments. There is the sandwich estimate of the covariance matrix of , , which could adjust the loss of efficiency.

 The SURVEYLOGISTIC procedure from SAS software provides us a quick way to analyze logistic regression for survey data. The WEIGHTS procedure incorporates adjusted weights we mentioned above. The PROC SURVEYLOGISTIC uses a Taylor expansion approximation and incorporates the sample design information. An adjustment is also used in the variance estimation to reduce the bias when the sample size is small.

# Chapter 2. The New Method

## 2.1 Normalized distribution with sampling weights

### 2.2.1 New view of sampling weights

 When we incorporate the weights into the *probability distribution function (pdf)*, in order to keep the new function still to be a *pdf*, we need to normalize it. For the discrete distribution, the new function becomes , and for the continuous distribution, the new probability distribution function becomes . We introduce the sampling weights in the probability distribution function, . The old method is to analyze using QLM. The new method is to normalize , then we use CLM to analyze We show some distributions and their normalization with sampling weights below. We compare their similarities and differences using the mean and variance.

**Example 1. Let with sampling weights**

The density function of normal distribution is:

 ,

Introducing the sampling weights we have:

Here,

 We see that the mean of normal distribution is , and the mean of normalized distribution with sampling weights is. There, the mean of normal distribution does not change after normalization. Similarly, the variance of normal distribution is , and the variance of normalized distribution with sampling weights is . The variance of the normal distribution changes after normalization.

**Example 2. Let with sampling weights**

The density function of Bernoulli distribution is:

Introducing the sampling weights we have:

Here,

 We see that the mean of Bernoulli distribution is . The mean of normalized Bernoulli distribution with sampling weights is . So that or or when .

**Example 3. Let with sampling weights**

The density function of Multinomial distribution is:

, where the unit is , otherwise 0.

Introducing the sampling weights we have:

Here, , where

 The mean of independent Bernoulli distributions is equal to without any sampling weights; it has changed to in the presence of the sampling weights.

**Example 4. Let with sampling weights**

Thedensityfunctionofbinarylogisticregressionis:

Introducing the sampling weights we have:

 Here, we see that the mean of binary logistic regression is . The mean of normalized binary logistic regression with sampling weights is . We use sampling weights to adjust the covariates and intercepts in the normalized binary logistic regression. When we estimate binary logistic regression coefficients, we multiply sampling weights with both covariates and intercepts to create new covariates and new intercepts, and use correct likelihood estimation methods. In the new method, we normalize the binary logistic regression with adjusted weights and use the correct likelihood to make estimation and inferences. Clearly, there are some differences between correct likelihood of normalized binary logistic regression with adjusted weights and quasi-likelihood of binary logistic regression with adjusted weights.

**Example 5. Let with sampling weights**

The density function of multinomial logistic regressions is:

Introducing the sampling weights we have:

 For multinomial logistic regression, we take one category as the reference category, then we compare others with it. We use sampling weights to adjust the covariates and intercepts in the normalized multinomial logistic regressions. When we estimate multinomial logistic regressions coefficients, we multiply both covariates and intercepts with sampling weights to create new covariates and new intercepts, and use correct likelihood estimation methods. In the new method, we normalize multinomial logistic regression with adjusted weights and use the correct likelihood to make estimation and inferences. Clearly, there are some differences between correct likelihood of normalized multinomial logistic regressions with adjusted weights and quasi-likelihood of multinomial logistic regressions with adjusted weights.

### 2.2.2 Summary old and new method

 In the new method, the sampling weights are the same as the old method; both of them are adjusted weights. In the old method, the intercepts are default as 1, and covariates are the regular covariates. However, the sampling weights are further used to adjust for both covariates and intercepts in the new method. We multiply adjusted weights with both intercepts and covariates; the intercepts are equal to adjusted weights in the new method. Also the covariates are equal to regular covariates multiply adjusted weights in the new method. We use quasi-likelihood method to estimate parameters in the old method, while we use the correct likelihood method to estimate parameters in the new method. The SAS procedure of SURVEYLOGISTIC is used by the old method, but LOGISTIC procedure is used in the new method. The details show on Table 2-1, differences between quasi-likelihood and correct likelihood below.

**Table 2-1 Differences between Quasi-likelihood and Correct Likelihood**

|  |  |  |
| --- | --- | --- |
| Methods | Quasi-likelihood | Correct Likelihood |
| Covariates | Regular Covariates | Adjusted Covariates |
| Intercepts | 1 | Adjusted Weights |
| SAS Procedure | SURVEYLOGISTIC | LOGISTIC |
| Likelihood | Not Normalized | Normalized |

# Chapter 3. Illustrative Examples

 We use BMI data from NHANES III (the Third National Health and Nutrition Examination Survey) to analyze the health condition of the U.S. population. We may diagnose underweight, overweight and obese adults. The variable we consider is body mass index (BMI: body weight in kilograms divided by the square of height), a measure of human body shape based on an individual’s weight and height. Our aim is to compare the old and new methods using BMI survey data.

 Overweight children often remain overweight in adulthood, and overweight in adulthood is a health risk (Wright et al. 2001). Numerous articles about obesity have been published recently. Using NHANES III data, Ogden et al. (2002) described national estimates of the prevalence and trends in overweight among U.S. children and adults. Based on a simple statistical analysis, they concluded that "the prevalence of overweight among children in the United States is continuing to increase, especially among Mexican-American and non-Hispanic black adults." (See also Flegal et al. 2005, 2007 for discussions of other aspects of the NHANES III data.)

 The Expert Committee on Clinical Guidelines for Overweight in Adults Prevention Services has published criteria for overweight to be integrated into routine screening of adults. BMI should be used routinely to screen for overweight and obesity in children and adults. Youths with a BMI at least the 95th percentile for age and gender should be considered overweight and referred for in depth medical follow-up to explore underlying diagnoses. Adults with a BMI in at least the 85th percentile (25 kg/m2) but below the 95th percentile (30 kg/m2) should be considered at risk of overweight and referred for a second-level screen.[[2]](#footnote-2)

 The BMI we study here has four levels, the first level is under 20 kg/m2 “underweight”, the second level is between 20 kg/ m2 and 25 kg/ m2 “normal”, the third level is between 25 kg/ m2 and 30 kg/ m2 “overweight”, the last level is over 30 kg/ m2 “obese”. For adults, the current value setting are as follows: a BMI of 18.5 kg/ m2 suggests the person is underweight while a number above 25 kg/ m2 may indicate the person is overweight; a person may have a BMI below 18.5 kg/ m2due to disease, a number above 30 kg/ m2suggests the person is obese.

 We constructed our models at the county level, with age (between 20 years old and 80 years old adults), race (white, non-white Hispanic, non-white Black and others), and gender (female and male) as covariates. The response variable is BMI with 4 levels in all counties. They are underweight, normal weight, overweight and obese. Data description below in Table 3-1shows some specific details about the number of observations. County 3 has a large amount of data compared to the other counties.

**Table 3-1 data description**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| County ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Sample Size | 194 | 221 | 1036 | 183 | 210 | 193 | 157 | 221 |

## 3.1 Binary logistic regression

 We build a binary logistic regression model to analyze the data. We label the four levels of BMI as 1, 2, 3 and 4 in which we compare underweight with no underweight or normal weight with not normal weight and so on. Specifically, in County 1 we take each variable as 1, and call the others 0, to do the binary regression. We call BMI equals to 1 as 1, and call BMI equals to 2 to 4 as 0 in County 1, do the first binary regression. Then we repeat the process, call BMI equals to 2 as 1, and call BMI equals to 1, 3 and 4 as 0 in County 1, do the second binary regression. We keep doing it until finishing the four binary regression analyses in County 1. We repeat it for each county. The following Table 1 to Table 8 show us the results of the binary regression. It shows the differences and similarities between the QLM and CLM.

 Here, we analyze eight counties one by one, basing on the two methods to compare their differences and similarities. We include p-values (Pr), estimates, Wald Chi-Square (WS) statistics and standard errors (SE). On the left side, it is the QLM. On the right side, it is the CLM. The variables age, race and gender are the independent variables for regression of BMI.

**Table 1. Comparing binary logistic regressions in County 1**

|  |
| --- |
| *BMI*  *Parameter*  *Quasi-likelihood Correct Likelihood* *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr****1 VS 2 3 4***  Intercept -4.7825 1.4828 10.4023 0.0013 -5.2799 1.1402 21.4412 <.0001 Age -0.00501 0.0326 0.0236 0.8779 0.0276 0.0183 2.2827 0.1308 Race 0.1989 0.3853 0.2663 0.6058 0.4322 0.3952 1.1962 0.2741 Gender 1.3892 0.7272 3.6495 0.0561 0.5252 0.5459 0.9257 0.336***2 VS 1 3 4***   Intercept -1.3805 1.0894 1.6058 0.2051 -0.694 0.7476 0.8617 0.3533 Age -0.00615 0.0128 0.2324 0.6297 -0.00488 0.00861 0.3216 0.5707 Race 0.3706 0.4241 0.7638 0.3821 0.4662 0.2646 3.1051 0.078 Gender 0.2656 0.5305 0.2507 0.6166 -0.2717 0.3116 0.7601 0.3833***3 VS 1 2 4***   Intercept 2.2644 1.0486 4.6633 0.0308 1.6912 0.7753 4.7584 0.0292 Age -0.0107 0.013 0.6808 0.4093 -0.00753 0.00846 0.7918 0.3735 Race -0.3342 0.4782 0.4885 0.4846 -0.9685 0.3829 6.397 0.0114\* Gender -1.2375 0.5255 5.5451 0.0185 -0.5547 0.3178 3.0464 0.0809\****4 VS 1 2 3***   Intercept -3.3464 1.2278 7.4284 0.0064 -2.5669 0.7686 11.1547 0.0008 Age 0.0233 0.0114 4.2263 0.0398 0.00385 0.00766 0.2523 0.6154\* Race -0.2363 0.3596 0.432 0.511 0.167 0.2739 0.3717 0.5421 Gender 0.8084 0.4734 2.9163 0.0877 0.7204 0.3403 4.4805 0.0343\* |

 In Table 1, comparing binary logistic regressions in County 1, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences[[3]](#footnote-3), when BMI equals to three compared to the others, the p-value of race is different; the old method is 0.4846 (>0.05), but the new is 0.0114 (<0.05). The p-value of gender is also different; the old method is 0.0185 (<0.05), but new method is 0.0809 (>0.05). When BMI equals to four compared to the others, the p-value of age is different; the old method is 0.0398 (<0.05), but the new is 0.6154 (>0.05). The p-value of gender is also different; the old method is 0.0877 (>0.05), but the new is 0.0343 (<0.05).

 Specific about similarities, when BMI equals to two compared to the others, the p-value of age is the same; the old method is 0.6297 (>0.05), and the new method is 0.5707 (>0.05). When BMI equals to three compared to the others, the p-value of age is the same; the old method is 0.4093 (>0.05), and the new method is 0.3735 (>0.05).

**Table 2. Comparing binary logistic regressions in County 2**

|  |
| --- |
| *BMI* *Parameter Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept -3.6242 1.2227 8.7858 0.003 -2.2043 0.3869 32.4673 <.0001 Age -0.007 0.0243 0.083 0.7733 8.34E-07 1.97E-06 0.1803 0.6711 Race 0.1044 0.4182 0.0624 0.8028 -0.00005 0.000141 0.1483 0.7002 Gender -0.0502 0.719 0.0049 0.9443 -0.00008 0.000181 0.2126 0.6447***2 VS 1 3 4***  Intercept -3.6443 0.9947 13.4225 0.0002 -1.445 0.1974 53.6029 <.0001 Age 0.0292 0.0138 4.4734 0.0344 4.30E-07 4.50E-07 0.9134 0.3392\* Race 0.1353 0.5491 0.0607 0.8054 -4.67E-06 0.000016 0.0846 0.7711 Gender 0.9622 0.6012 2.5612 0.1095 0.000011 0.000016 0.4718 0.4922***3 VS 1 2 4***  Intercept 4.144 1.1032 14.1092 0.0002 0.5573 0.1666 11.1844 0.0008 Age -0.0499 0.0149 11.1562 0.0008 -7.71E-07 4.70E-07 2.697 0.1005\* Race -0.0867 0.5397 0.0258 0.8723 0.000015 0.000022 0.4594 0.4979 Gender -1.096 0.5684 3.7174 0.0538 -3.41E-06 0.000018 0.0351 0.8513***4 VS 1 2 3***  Intercept -2.4016 0.933 6.6258 0.0101 -0.4886 0.1701 8.254 0.0041 Age 0.000726 0.0146 0.0025 0.9604 -4.24E-07 7.56E-07 0.3147 0.5748 Race 0.059 0.2759 0.0457 0.8308 -0.00003 0.000023 1.1596 0.2815 Gender 0.5568 0.5668 0.9648 0.326 8.93E-06 0.000023 0.1537 0.695 |

 In Table 2, comparing binary logistic regressions in County 2, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to two compared to the others, the p-value of age is different; the old method is 0.0344(<0.05), but the new method is 0.3392(>0.05). When BMI equals to three compared to the others, the p-value of age is different between these two methods, the old method is 0.0008(<0.05), but the new is 0.1005(>0.05).

 Specific about similarities, when BMI equals to three compared to the others, the p-value of race is the same, the old method is 0.8723 (>0.05), and the new method is 0.4979 (>0.05).

**Table 3. Comparing binary logistic regressions in County 3**

|  |
| --- |
| *BMI* *Parameter Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept -3.8232 1.9132 3.993 0.0457 -3.6586 0.8093 20.4371 <.0001 Age -0.0457 0.018 6.4528 0.0111 -0.0155 0.0128 1.4673 0.2258\* Race 0.3069 0.381 0.649 0.4205 0.4112 0.2188 3.5326 0.0602 Gender 1.4463 0.6971 4.3041 0.038 0.4589 0.3059 2.251 0.1335\****2 VS 1 3 4***  Intercept 0.1207 0.6112 0.039 0.8434 -0.0996 0.3226 0.0952 0.7576 Age -0.0222 0.0076 8.5075 0.0035 -0.0154 0.00436 12.3793 0.0004 Race 0.6976 0.174 16.0819 <.0001 0.3312 0.1116 8.8066 0.003 Gender -0.418 0.267 2.451 0.1174 -0.2572 0.1348 3.6398 0.0564***3 VS 1 2 4***  Intercept -0.3441 0.6064 0.3219 0.5705 0.1031 0.2991 0.1188 0.7303 Age 0.0162 0.00748 4.7099 0.03 0.00529 0.00354 2.2359 0.1348\* Race -0.4607 0.1851 6.1948 0.0128 -0.3237 0.1203 7.235 0.0071 Gender -0.2947 0.2698 1.193 0.2747 -0.2868 0.1295 4.9013 0.0268\****4 VS 1 2 3***  Intercept -1.9857 0.6881 8.3279 0.0039 -2.4033 0.3417 49.4699 <.0001 Age 0.0177 0.00716 6.1105 0.0134 0.0137 0.00366 13.9411 0.0002 Race -0.6825 0.1766 14.9299 0.0001 -0.1428 0.1241 1.3234 0.25\* Gender 0.5357 0.3083 3.0185 0.0823 0.5632 0.1474 14.595 0.0001\* |

 In Table 3, comparing binary logistic regressions in County 3, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to one compared to the others, the p-value of age is different; the old method is 0.011 (<0.05), but the new method is 0.2258 (>0.05). When BMI equals to three compared to the others, the p-value of age is different; the old method is 0.03 (<0.05), but the new is 0.1348 (>0.05). When BMI equals to four compared to the others, the p-value of race is different; the old method is 0.0001 (<0.05), but the new method is 0.25 (>0.05). The p-value of gender is also different; the old method is 0.0823 (>0.05), but the new method is 0.0001 (<0.05).

 Specific about similarities, when BMI equals to four compared to the others, the p-value of age is the same, the old method is 0.0134 (<0.05), and the new method is 0.0002 (<0.05). County 3 has a large amount of data; the table also shows differences and similarities between two methods. The methods can be used with large number of sample size.

**Table 4. Comparing binary logistic regressions in County 4**

|  |
| --- |
| *BMI Parameter*   *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept -4.3323 2.5322 2.9271 0.0871 -5.6719 1.5526 13.3451 0.0003 Age 0.00882 0.0227 0.1504 0.6982 0.00166 0.0148 0.0126 0.9107 Race 1.1816 0.5664 4.3523 0.037 0.8897 0.4768 3.4818 0.062\* Gender 0.2143 0.7482 0.082 0.7745 1.4934 0.5996 6.2028 0.0128\****2 VS 1 3 4***   Intercept 0.9177 1.4 0.4297 0.5121 0.3669 1.0264 0.1278 0.7208 Age -0.0372 0.0162 5.2701 0.0217 -0.0126 0.00859 2.1386 0.1436\* Race -0.9506 0.6019 2.4943 0.1143 -0.8192 0.6234 1.7267 0.1888 Gender 1.1778 0.6139 3.6803 0.0551 0.3398 0.3201 1.1266 0.2885***3 VS 1 2 4***   Intercept -0.8505 1.258 0.457 0.499 -0.4201 0.7753 0.2936 0.5879 Age 0.0192 0.0135 2.0186 0.1554 0.00523 0.00767 0.4659 0.4949 Race 0.2932 0.4637 0.3999 0.5272 0.3512 0.3634 0.9338 0.3339 Gender -0.9509 0.5504 2.9853 0.084 -0.5535 0.3177 3.035 0.0815***4 VS 1 2 3***   Intercept -1.1552 1.1641 0.9849 0.321 -0.5305 0.8608 0.3798 0.5377 Age 0.0306 0.0148 4.2832 0.0385 0.00935 0.00886 1.1145 0.2911\* Race -0.9859 0.5773 2.9161 0.0877 -0.6407 0.5573 1.3219 0.2502 Gender -0.7763 0.6834 1.2905 0.256 -0.5128 0.3914 1.7166 0.1901 |

 In Table 4, comparing binary logistic regressions in County 4, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to one compared to the others, p-value of gender is different; the old method is 0.7745 (>0.05), but the new method is 0.0128 (<0.05). The p-value of race is also different; the old method is 0.037 (<0.05), but the new method is 0.062 (>0.05). When BMI equals to two compared to the others, the p-value of age is different; the old method is 0.0217 (<0.05), but new method is 0.1436 (>0.05). When BMI equals to four compared to the others, p-value of age is different; the old method is 0.0385 (<0.05), but the new is 0.2911 (>0.05).

 Specific about similarities, when BMI equals to three compared to the others, the p-value of race is the same; the old method is 0.5272 (>0.05), and the new method is 0.3339 (>0.05).

**Table 5. Comparing binary logistic regressions in County 5**

|  |
| --- |
| *BMI Parameter*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept -5.0387 1.8591 7.3454 0.0067 -2.7062 0.3831 49.9097 <.0001 Age 0.0408 0.0225 3.3068 0.069 1.82E-06 1.40E-06 1.6886 0.1938 Race -0.617 0.864 0.51 0.4751 -0.00013 0.000083 2.4442 0.118 Gender 0.2627 1.1506 0.0521 0.8194 0.000013 0.000055 0.0554 0.8139***2 VS 1 3 4***  Intercept -0.5042 1.3112 0.1479 0.7006 -0.272 0.1726 2.4832 0.1151 Age 0.00673 0.0159 0.1784 0.6728 2.77E-07 5.52E-07 0.2511 0.6163 Race -0.4303 0.409 1.1067 0.2928 -0.00003 0.000035 0.8571 0.3546 Gender -0.0368 0.5652 0.0042 0.9481 5.18E-06 0.000016 0.1044 0.7467***3 VS 1 2 4***  Intercept 0.0807 1.3741 0.0035 0.9531 -1.0179 0.187 29.625 <.0001\* Age -0.00916 0.0164 0.3105 0.5774 -9.43E-08 5.59E-07 0.0285 0.8659 Race -0.2968 0.3888 0.5827 0.4452 8.07E-06 0.000028 0.082 0.7747 Gender -0.0341 0.6197 0.003 0.9562 2.00E-06 0.000014 0.0196 0.8885***4 VS 1 2 3***  Intercept -1.384 1.2914 1.1486 0.2838 -1.0946 0.1867 34.3546 <.0001\* Age -0.00567 0.0158 0.1287 0.7198 -4.62E-07 5.78E-07 0.6399 0.4238 Race 0.6961 0.3891 3.2012 0.0736 0.000037 0.000027 1.8579 0.1729 Gender 0.0259 0.6527 0.0016 0.9684 -7.34E-06 0.000015 0.2283 0.6328 |

 In Table 5, comparing binary logistic regressions in County 5, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to three compared to the others, the p-value of intercept is different; the old method is 0.9531(>0.05), but new method is <.0001(>0.05). When BMI equals to four compared to the others, the p-value of intercept is different between these two methods, the old method is 0.2838(>0.05), but the new is <.0001(<0.05).

 Specific about similarities, when BMI equals to one compared to the others, the p-value of gender is the same; the old method is 0.8184 (>0.05), and new method is 0.9139 (>0.05).

**Table 6. Comparing binary logistic regressions in County 6**

|  |
| --- |
| *BMI Parameter*   *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept -6.5774 2.3046 8.1456 0.0043 -3.2801 0.4306 58.0359 <.0001 Age -0.0234 0.0393 0.3554 0.5511 -2.15E-06 1.22E-06 3.0954 0.0785 Race 0.4738 0.6808 0.4842 0.4865 0.00002 0.000025 0.6332 0.4262 Gender 2.3553 1.1084 4.5154 0.0336 0.000048 0.000036 1.723 0.1893\****2 VS 1 3 4***  Intercept 0.7122 1.3181 0.2919 0.589 -0.7396 0.1798 16.9301 <.0001\* Age -0.00454 0.0164 0.0763 0.7824 2.43E-07 6.00E-07 0.1645 0.6851 Race 0.0842 0.352 0.0572 0.811 0.000016 0.000014 1.1465 0.2843 Gender -0.3891 0.5684 0.4687 0.4936 -2.44E-06 0.000017 0.021 0.8848***3 VS 1 2 4***  Intercept -0.0173 1.4904 0.0001 0.9908 -0.6117 0.1779 11.82 0.0006\* Age -0.00852 0.0195 0.1906 0.6624 -7.88E-08 6.20E-07 0.0161 0.8989 Race -0.1808 0.3676 0.2419 0.6228 -5.83E-06 0.000015 0.1605 0.6887 Gender -0.1767 0.5913 0.0893 0.765 -5.91E-07 0.000018 0.001 0.9745***4 VS 1 2 3***  Intercept -3.5746 1.2104 8.7224 0.0031 -0.6956 0.2189 10.095 0.0015 Age 0.0308 0.0138 5.0013 0.0253 1.62E-06 1.01E-06 2.5832 0.108\* Race -0.1765 0.3095 0.3253 0.5684 -0.00012 0.000053 5.2509 0.0219\* Gender 0.3671 0.752 0.2384 0.6254 4.85E-06 0.000037 0.0169 0.8964 |

 In Table 6, comparing binary logistic regressions in County 6, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to one compared to the others, p-value of gender between these two methods is different; the old method is 0.0336 (<0.05), but new method is 0.1893 (>0.05). When BMI equals to four compared to the others, the p-value of age is different; the old method is 0.0253 (<0.05), but new method is 0.108 (>0.05). The p-value of race is also different; the old method is 0.5684 (>0.05), but new method is 0.0219 (<0.05).

 Specific about similarities, when BMI equals to one compared to the others, the p-value of race is the same, the old method is 0.4865 (>0.05), and new method is 0.4262 (>0.05).

**Table 7. Comparing binary logistic regressions in County 7**

|  |
| --- |
| *BMI* *Parameter*   *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept -6.1528 2.3703 6.7383 0.0094 -3.5605 0.5726 38.6624 <.0001 Age 0.0149 0.0301 0.2449 0.6207 -5.65E-07 8.82E-07 0.4107 0.5216 Race 1.2626 0.4734 7.1121 0.0077 0.000031 0.000017 3.1575 0.0756\* Gender 0.5843 0.889 0.432 0.511 0.000025 0.000048 0.2652 0.6066***2 VS 1 3 4***  Intercept 2.1667 1.1765 3.3918 0.0655 -0.8439 0.2457 11.7924 0.0006\* Age -0.0312 0.0135 5.3105 0.0212 -9.18E-07 6.66E-07 1.9015 0.1679\* Race -0.869 0.3674 5.5958 0.018 -0.00002 0.000013 1.8502 0.1738\* Gender -0.0426 0.5065 0.0071 0.933 0.000059 0.000026 5.1325 0.0235\****3 VS 1 2 4***  Intercept -0.3762 1.1209 0.1127 0.7371 -0.314 0.2428 1.6731 0.1958 Age 0.0165 0.0122 1.8389 0.1751 1.12E-06 5.74E-07 3.8209 0.0506 Race 0.3185 0.3452 0.8511 0.3562 0.000023 0.000017 1.8201 0.1773 Gender -0.9763 0.5357 3.321 0.0684 -0.00008 0.00003 6.3796 0.0115\****4 VS 1 2 3***  Intercept -4.7367 1.242 14.5449 0.0001 -0.8827 0.3173 7.7384 0.0054 Age 0.0198 0.0101 3.8672 0.0492 3.42E-07 9.63E-07 0.1264 0.7222\* Race -0.1082 0.3907 0.0767 0.7818 -0.00008 0.000066 1.6044 0.2053 Gender 1.4592 0.5481 7.0864 0.0078 0.00003 0.000029 1.0151 0.3137\* |

 In Table 7, comparing binary logistic regression in County 7, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to one compared to the others, p-value of race is different; the old method is 0.0077 (<0.05), but new method is 0.0756 (>0.05). When BMI equals to two compared to the others, p-value of intercept, age, race and gender are different; the old method is 0.0212 (<0.05) for age, but new method is 0.1679 (>0.05). The old method is 0.018 (<0.05) for race, but new method is 0.1738 (>0.05). The old method is 0.933 (>0.05) for gender, but new method is 0.0235 (>0.05). When BMI equals to three compared to the others, p-value of gender is different; the old method is 0.0684 (>0.05), but new method is 0.0115 (<0.05). When BMI equals to four compared to the others, p-value of age is different; the old method is 0.0492 (<0.05), but new method is 0.7222 (>0.05). The p-value of gender is different; the old method is 0.0078 (<0.05), but new method is 0.3137 (>0.05).

 Specific about similarities, when BMI equals to one compared to the others, the p-value of gender is the same between these two methods, the old method is 0.511 (>0.05), and new method is 0.6066 (>0.05).

**Table 8. Comparing binary logistic regressions in County 8**

|  |
| --- |
| *BMI* *Parameter*   *Quasi-likelihood Correct Likelihood* *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| ***1 VS 2 3 4***  Intercept 9.1244 1.4653 38.7779 <.0001 -2.9621 0.3856 59.018 <.0001 Age 0.0318 0.0168 3.5731 0.0587 3.83E-06 1.69E-06 5.1783 0.0229\* Race -14.1071 0.7934 316.1864 <.0001 -0.00024 0.00009 6.9377 0.0084 Gender 0.4656 0.7123 0.4273 0.5133 0.000041 0.000034 1.4507 0.2284***2 VS 1 3 4***   Intercept 0.7124 1.1149 0.4083 0.5228 -1.06 0.2129 24.7996 <.0001\* Age -0.0307 0.00927 10.9774 0.0009 -1.48E-06 5.19E-07 8.0999 0.0044 Race 0.0907 0.304 0.0891 0.7654 0.000026 0.00002 1.7421 0.1869 Gender 0.2706 0.5367 0.2542 0.6141 0.00005 0.00002 6.5099 0.0107\****3 VS 1 2 4***   Intercept -0.4377 1.0475 0.1746 0.6761 -0.4234 0.1986 4.5463 0.033\* Age 0.0198 0.00834 5.6547 0.0174 1.45E-06 5.50E-07 6.9142 0.0086 Race -0.0453 0.292 0.0241 0.8767 -4.83E-06 0.000023 0.0453 0.8315 Gender -0.9195 0.5179 3.1531 0.0758 -0.00006 0.000025 6.6049 0.0102\****4 VS 1 2 3***   Intercept -2.7933 0.8848 9.9662 0.0016 -0.7759 0.2038 14.4974 0.0001 Age 0.00539 0.0087 0.3834 0.5358 -4.45E-07 6.24E-07 0.5073 0.4763 Race 0.136 0.2861 0.226 0.6345 -4.66E-06 0.000022 0.0469 0.8286 Gender 0.5988 0.4523 1.7526 0.1855 -8.45E-07 0.00002 0.0018 0.9665 |

 In Table 8, comparing binary logistic regression in County 8, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to one compared to the others, the p-value of age is different; the old method is 0.0587 (>0.05), but new is 0.0229 (<0.05). When BMI equals to two compared to the others, the p-value of gender is different; the old method is 0.6141 (>0.05), but the new is 0.0107 (<0.05). When BMI equals to three compared to the others, the p-value of gender is different; the old method is 0.0758 (>0.05), but new method is 0.0102 (<0.05).

 Specific about similarities, when BMI equals to one compared to the others, the p-value of intercept is the same between these two methods, the old method is <.0001 (<0.05), and new method is <.0001 (<0.05).

## 3.2 Multinomial logistic regressions

 We construct multinomial logistic regressions model to analyze the four levels of BMI together. We label the four levels of BMI as 1, 2, 3 and 4 where we compare underweight, normal weight, overweight and obese at the same time. Specifically, we construct this model for each county level, with age, race and gender as covariates. We use BMI =’1’ as the reference category and compare with the other three levels of BMI together in each county. It is the same to use BMI = ‘2’ as the reference category. Normal weights, overweight and obese are compared basing on the underweight. The optimization technique used by SAS here is Newton-Raphson. Among these eight counties, County 3 has a large amount of data.

 Here, we analyze data from the eight counties one by one, again we note that the two different methods to compare their differences and similarities. We include p-values, estimates, Wald Chi-Square statistics and standard errors. The outcomes are shown on Table 9-Table 16 multinomial logistic regressions with sampling weights. For the multinomial BMI variable, we put BMI equals one as the reference category, and BMI equals two to four compared to one. On the left side, it is the old quasi-likelihood method. On the right side, it is the new correct likelihood method. The variables of age, gender and race are the independent variables of the regression on BMI.

**Table 9. Multinomial logistic regression with sampling weights in County 1**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
|  Intercept 2 3.228 1.7589 3.3682 0.0665 4.1281 1.3295 9.6411 0.0019\*[[4]](#footnote-4)Intercept 3 5.4937 1.6219 11.4735 0.0007 5.7945 1.3002 19.8626 <.0001Intercept 4 1.6018 1.7957 0.7957 0.3724 2.6957 1.2941 4.3393 0.0372\*Age 2 -0.00043 0.0349 0.0002 0.9902 -0.029 0.0198 2.1466 0.1429Age 3 -0.00312 0.0349 0.008 0.9287 -0.0308 0.0196 2.4546 0.1172Age 4 0.0217 0.0335 0.4189 0.5175 -0.0226 0.0192 1.3886 0.2386Race 2 0.0337 0.4872 0.0048 0.9448 -0.1271 0.4434 0.0822 0.7744Race 3 -0.4104 0.5082 0.6522 0.4193 -1.1412 0.5284 4.6641 0.0308\*Race 4 -0.3767 0.4025 0.8759 0.3493 -0.2825 0.4437 0.4053 0.5244Gender 2 -1.1116 0.7753 2.0558 0.1516 -0.6755 0.5961 1.2841 0.2571Gender 3 -2.0661 0.8151 6.425 0.0113 -0.881 0.6011 2.148 0.1428\*Gender 4 -0.6529 0.8008 0.6646 0.4149 0.0381 0.6109 0.0039 0.9503 |

 In Table 9, multinomial logistic regression in County 1, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to three, the p-value of gender is different; the old method is 0.0113 (<0.05), but new method is 0.1428 (>0.05). The p-value of race is different; the old method is 0.4193 (>0.05), but new method is 0.0308 (<0.05).

 Specific about similarities, when BMI equals to two, the p-value of age is the same between these two methods, the old method is 0.9902 (>0.05), and new method is 0.1429 (>0.05). The p-value of gender is also the same; the old method is 0.1516 (>0.05), and new method is 0.2571 (>0.05). When BMI equals to four, the p-value of gender is the same between these two methods, the old method is 0.4149 (>0.05), and the new method is 0.9503 (>0.05).

**Table 10. Multinomial logistic regression with sampling weights in County 2**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 0.4913 1.415 0.1206 0.7284 0.6093 0.4275 2.0314 0.1541Intercept 3 4.6799 1.648 8.0644 0.0045 1.2921 0.4125 9.8117 0.0017Intercept 4 1.1201 1.3756 0.663 0.4155 1.3086 0.4017 10.612 0.0011\*Age 2 0.0262 0.0262 0.9981 0.3178 -0.018 0.0612 0.0863 0.7689Age 3 -0.00989 0.0264 0.1401 0.7082 -0.0254 0.0608 0.1749 0.6758Age 4 0.00866 0.0266 0.1061 0.7447 -0.0355 0.0636 0.3106 0.5773Race 2 -0.00337 0.6063 0 0.9956 1.7654 4.344 0.1652 0.6844Race 3 -0.2045 0.5511 0.1377 0.7106 2.0554 4.3314 0.2252 0.6351Race 4 -0.0458 0.4158 0.0121 0.9123 1.0972 4.3628 0.0633 0.8014Gender 2 0.7105 0.8421 0.7119 0.3988 2.6347 5.5549 0.225 0.6353Gender 3 -0.6597 0.7678 0.7381 0.3903 2.2189 5.5508 0.1598 0.6893Gender 4 0.5157 0.7846 0.432 0.511 2.7039 5.5672 0.2359 0.6272 |

 In Table 10, multinomial logistic regression in County 2, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to four, the p-value of intercept is different; the old method is 0.4155 (>0.05), but new method is 0.0011 (<0.05).

 Specific about similarities, when BMI equals to two, the p-value of age is the same; the old method is 0.3178 (>0.05), and new method is 0.7689 (>0.05). When BMI equals to three, the p-value of age is the same between these two methods, the old method is 0.7082 (>0.05), and new method is 0.6758 (>0.05). The p-value of gender is also the same; the old method is 0.3903 (>0.05), and new method is 0.6893 (>0.05). When BMI equals to four, the p-value of race is the same, the old method is 0.9123 (>0.05), and the new method is 0.8014 (>0.05). The p-value of gender is also the same, the old method is 0.511 (>0.05), and the new method is 0.6272 (>0.05).

**Table 11. Multinomial logistic regression with sampling weights in County 3**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood* *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 3.2099 1.8999 2.8543 0.0911 2.9793 0.8465 12.3869 0.0004\*Intercept 3 2.8722 1.9122 2.2561 0.1331 3.066 0.8356 13.4641 0.0002\*Intercept 4 1.6994 1.9353 0.771 0.3799 1.2916 0.8518 2.2991 0.1294Age 2 0.0305 0.0188 2.6359 0.1045 0.00442 0.0134 0.1094 0.7408Age 3 0.0566 0.0191 8.754 0.0031 0.0184 0.0132 1.9611 0.1614\*Age 4 0.0598 0.0188 10.1557 0.0014 0.0253 0.0132 3.6842 0.0549\*Race 2 0.0621 0.3871 0.0257 0.8726 -0.1847 0.2279 0.6567 0.4177Race 3 -0.6521 0.3951 2.7231 0.0989 -0.6105 0.2371 6.6303 0.01\*Race 4 -0.8994 0.3958 5.1633 0.0231 -0.5075 0.2387 4.5201 0.0335Gender 2 -1.578 0.7051 5.0085 0.0252 -0.6026 0.319 3.5688 0.0589\*Gender 3 -1.5233 0.7179 4.5021 0.0339 -0.6048 0.3175 3.6272 0.0568\*Gender 4 -0.9282 0.7355 1.5925 0.207 -0.00698 0.3269 0.0005 0.983 |

 In Table 11, multinomial logistic regression in County 3, we can see the differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to two, the p-value of gender is different; the old method is 0.0252 (<0.05), but new method is 0.0589 (>0.05). When BMI equals to three, the p-value of age is different; the old method is 0.0031 (<0.05), but new method is 0.1614 (>0.05). The p-value of race is also different; the old method is 0.0989 (>0.05), but new method is 0.01 (<0.05). The p-value of gender is also different; the old method is 0.0339 (<0.05), but new method is 0.0568 (>0.05). When BMI equals to four, the p-value of age is different; the old method is 0.0014 (<0.05), but new method is 0.0549 (>0.05).

 Specific about similarities, when BMI equals to two, the p-value of race is the same between these two methods, the old method is 0.8726 (>0.05), and new method is 0.4177 (>0.05). When BMI equals to four, the p-value of gender is the same; the old method is 0.207 (>0.05), and new method is 0.983 (>0.05).

 County 3 has a large amount of data; the table also shows differences and similarities between two methods. The methods can be used with large number of sample size.

**Table 12. Multinomial logistic regression with sampling weights in County 4**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 3.771 2.7228 1.9181 0.1661 5.0239 1.8422 7.4375 0.0064\*Intercept 3 2.7183 2.4862 1.1955 0.2742 4.4193 1.6013 7.6169 0.0058\*Intercept 4 2.6566 2.5379 1.0957 0.2952 4.4919 1.6806 7.1439 0.0075\*Age 2 -0.0277 0.0256 1.1755 0.2783 -0.00903 0.0164 0.3043 0.5812Age 3 0.00739 0.0237 0.0977 0.7547 0.00277 0.0158 0.0309 0.8605Age 4 0.0192 0.0253 0.5747 0.4484 0.00672 0.0168 0.1603 0.6889Race 2 -1.486 0.7604 3.8193 0.0507 -1.3424 0.7999 2.8162 0.0933Race 3 -0.6843 0.5885 1.3523 0.2449 -0.5245 0.4954 1.121 0.2897Race 4 -1.8194 0.7039 6.6805 0.0097 -1.299 0.6567 3.9126 0.0479Gender 2 0.4881 0.8637 0.3193 0.572 -1.1337 0.6478 3.0632 0.0801Gender 3 -0.863 0.7478 1.3321 0.2484 -1.6965 0.6373 7.0858 0.0078\*Gender 4 -0.858 0.9125 0.8842 0.347 -1.771 0.6889 6.6087 0.0101\* |

 In Table 12, multinomial logistic regression in County 4, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to three, the p-value of gender is different; the old method is 0.2484 (>0.05), but new method is 0.0078 (<0.05). When BMI equals to four, the p-value of gender is different; the old method is 0.347 (>0.05), but new method is 0.0101 (<0.05).

 Specific about similarities, when BMI equals to two, the p-value of age is the same between these two methods, the old method is 0.2783 (>0.05), the new method is 0.5812 (>0.05). The p-value of race is also the same; the old method is 0.0507 (>0.05), and new method is 0.0933 (>0.05). The p-value of gender is also the same; the old method is 0.572 (>0.05), and new method is 0.0801 (>0.05). When BMI equals to four, the p-value of age is the same, the old method is 0.4484 (>0.05), and the new method is 0.6889 (>0.05).

**Table 13. Multinomial logistic regression with sampling weights in County 5**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 4.0538 2.0896 3.7635 0.0524 1.9279 0.4043 22.7439 <.0001\*Intercept 3 4.438 2.1429 4.2891 0.0384 1.4305 0.417 11.7693 0.0006Intercept 4 3.499 2.0989 2.7791 0.0955 1.3756 0.4161 10.9301 0.0009\*Age 2 -0.0347 0.0257 1.8248 0.1767 -0.0631 0.0598 1.1122 0.2916Age 3 -0.0454 0.0255 3.1782 0.0746 -0.0735 0.0589 1.5587 0.2119Age 4 -0.0432 0.0251 2.9691 0.0849 -0.0843 0.059 2.0419 0.153Race 2 0.2897 0.9206 0.099 0.753 4.2678 3.5967 1.408 0.2354Race 3 0.393 0.9252 0.1804 0.671 5.4468 3.555 2.3475 0.1255Race 4 1.0199 0.932 1.1977 0.2738 6.2298 3.4899 3.1866 0.0742Gender 2 -0.2763 1.2197 0.0513 0.8208 -0.3885 2.2922 0.0287 0.8654Gender 3 -0.2758 1.2505 0.0487 0.8254 -0.5093 2.2962 0.0492 0.8245Gender 4 -0.2349 1.2725 0.0341 0.8535 -0.7475 2.3116 0.1046 0.7464 |

 In Table 13, multinomial logistic regression in County 5, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to two, the p-value of intercept is different; the old method is 0.0524 (>0.05), but new method is <.0001 (<0.05). When BMI equals to four, the p-value of intercept is different; the old method is 0.0955 (>0.05), but new method is 0.0009 (<0.05).

 Specific about similarities, when BMI equals to two, the p-value of age is the same between these two methods, the old method is 0.1767 (>0.05), and new method is 0.2916 (>0.05). The p-value of race is also the same, the old method is 0.753 (>0.05), and new method is 0.2354 (>0.05). The p-value of gender is also the same, the old method is 0.8208 (>0.05), and new method is 0.8654 (>0.05). When BMI equals to three, the p-value of gender is the same, the old method is 0.8254 (>0.05), and the new method is 0.8245 (>0.05). When BMI equals to four, the p-value of gender is the same, the old method is 0.8535 (>0.05), and the new method is 0.7464 (>0.05).

**Table 14. Multinomial logistic regression with sampling weights in County 6**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 6.1428 2.4443 6.3159 0.012 2.2056 0.4581 23.1847 <.0001Intercept 3 5.8796 2.6361 4.9748 0.0257 2.2845 0.4562 25.0755 <.0001Intercept 4 3.0271 2.5321 1.4292 0.2319 2.2874 0.4779 22.9062 <.0001\*Age 2 0.0196 0.0408 0.2314 0.6305 0.0649 0.0365 3.1566 0.0756Age 3 0.0158 0.0441 0.1291 0.7194 0.0607 0.0408 2.2185 0.1364Age 4 0.048 0.0415 1.3381 0.2474 0.1083 0.0474 5.2068 0.0225\*Race 2 -0.3954 0.7209 0.3008 0.5834 -0.4006 0.7649 0.2744 0.6004Race 3 -0.5681 0.7255 0.6132 0.4336 -0.7835 0.7992 0.9611 0.3269Race 4 -0.5839 0.7225 0.6531 0.419 -4.1063 1.7481 5.5178 0.0188\*Gender 2 -2.4364 1.1631 4.3875 0.0362 -1.3107 1.0766 1.4823 0.2234\*Gender 3 -2.3738 1.188 3.9926 0.0457 -1.3171 1.2416 1.1253 0.2888\*Gender 4 -1.9446 1.2948 2.2554 0.1331 -1.1265 1.5439 0.5324 0.4656 |

 In Table 14, multinomial logistic regression in County 6, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to four, the p-value of intercept is different; the old method is 0.2319 (>0.05), but the new method is <.0001 (<0.05). The p-value of age is also different; the old method is 0.2474 (>0.05), but new method is 0.0225 (<0.05). The p-value of race is also different, the old method is 0.419 (>0.05), but new method is 0.0188 (<0.05). When BMI equals to two, the p-value of gender is different; the old method is 0.0362 (<0.05), but new method is 0.2234 (>0.05).

 Specific about similarities, when BMI equals to two, the p-value of race is the same between these two methods, the old method is 0.5834 (>0.05), and new method is 0.6004 (>0.05). When BMI equals to three, the p-value of race is the same, the old method is 0.4336 (>0.05), and new method is 0.3269 (>0.05). When BMI equals to four, the p-value of gender is the same, the old method is 0.1331 (>0.05), and new method is 0.4656 (>0.05).

**Table 15. Multinomial logistic regression with sampling weights in County 7**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 6.6291 2.5896 6.5528 0.0105 2.3961 0.6109 15.3841 <.0001Intercept 3 5.0623 2.6042 3.7788 0.0519 2.7273 0.6086 20.0781 <.0001\*Intercept 4 1.5316 2.7238 0.3162 0.5739 2.3939 0.6483 13.6366 0.0002\*Age 2 -0.0351 0.0338 1.0743 0.3 -0.0009 0.0195 0.0021 0.9632Age 3 -0.006 0.0325 0.0342 0.8533 0.023 0.0181 1.625 0.2024Age 4 -0.00049 0.0326 0.0002 0.9879 0.0157 0.0252 0.3895 0.5326Race 2 -1.7112 0.5768 8.8007 0.5929 -0.8161 0.3916 4.3417 0.0372\*Race 3 -0.9374 0.5311 3.1151 0.0776 -0.2387 0.3813 0.3919 0.5313Race 4 -1.1985 0.5948 4.0609 0.4394 -2.0586 1.3086 2.4749 0.6157Gender 2 -0.5168 0.9666 0.2858 0.003 -0.2955 1.0155 0.0847 0.0771\*Gender 3 -1.1018 0.9655 1.3022 0.2538 -1.3902 1.0332 1.8103 0.1785Gender 4 -0.7337 0.9981 0.5404 0.0623 -0.1436 1.0708 0.018 0.2933 |

 In Table 15, multinomial logistic regression in County 7, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to three, the p-value of intercept is different; the old method is 0.0519 (>0.05), but new method is <.0001 (<0.05). When BMI equals to four, the p-value of intercept is different; the old method is 0.5739 (>0.05), but new method is 0.0002 (<0.05). When BMI equals to two, the p-value of race is different; the old method is 0.5929 (>0.05), but new method is 0.0372 (<0.05). The p-value of gender is also different; the old method is 0.003 (<0.05), but new method is 0.0771 (>0.05).

 Specific about similarities, when BMI equals to two, the p-value of age is the same between these two methods, the old method is 0.3 (>0.05), and new method is 0.9632 (>0.05). When BMI equals to four, the p-value of age is the same; the old method is 0.9879 (>0.05), and new method is 0.5326 (>0.05). When BMI equals to three, the p-value of gender is the same; the old method is 0.2538 (>0.05), and new method is 0.1785 (>0.05).

**Table 16. Multinomial logistic regression with sampling weights in County 8**

|  |
| --- |
| *Parameter BMI*  *Quasi-likelihood Correct Likelihood*  *(old method) (new method)*  *Estimate SE WS Pr Estimate SE WS Pr* |
| Intercept 2 -7.5622 1.6455 21.1216 <.0001 1.7104 0.4269 16.0551 <.0001Intercept 3 -8.2469 1.5597 27.9561 <.0001 2.1421 0.4242 25.4991 <.0001Intercept 4 -10.0191 1.4907 45.1745 <.0001 1.8857 0.4268 19.5192 <.0001Age 2 -0.0484 0.0192 6.3245 0.0119 -0.1052 0.0436 5.8333 0.0157Age 3 -0.0157 0.0182 0.7431 0.3887 -0.0597 0.0436 1.8727 0.1712Age 4 -0.0259 0.0189 1.8841 0.1699 -0.0916 0.0441 4.3193 0.0377\*Race 2 12.2064 0.7293 280.1113 <.0001 5.5784 2.0675 7.2799 0.007Race 3 12.1082 0.7534 258.3025 <.0001 5.1302 2.0826 6.0684 0.0138Race 4 12.2558 0.7451 270.5477 <.0001 5.195 2.0856 6.2048 0.0127Gender 2 -0.2444 0.8313 0.0865 0.7687 -0.2635 0.944 0.0779 0.7802Gender 3 -1.0373 0.807 1.6522 0.1987 -2.0626 1.0789 3.6546 0.0559Gender 4 0.0872 0.778 0.0126 0.9108 -0.9914 1.0251 0.9353 0.3335 |

 In Table 16, multinomial logistic regression in County 8, we can see differences and similarities between quasi-likelihood and correct likelihood methods. Specific about differences, when BMI equals to four, the p-value of age is different; the old method is 0.1699 (>0.05), but new method is 0.0377 (<0.05).

 Specific about similarities, when BMI equals to two, the p-value of intercept is the same between these two methods, the old method is <.0001 (<0.05), and new method is <.0001 (<0.05). When BMI equals to three, the p-value of intercept is the same, the old method is <.0001 (<0.05), and new method is <.0001 (<0.05). When BMI equals to four, the p-value of intercept is the same, the old method is <.0001 (<0.05), and new method is <.0001 (<0.05). When BMI equals to four, the p-value of gender is the same, the old method is 0.9108 (>0.05), and new method is 0.3335 (>0.05).

# Chapter 4. Discussion

 We use quasi-likelihood method as the old method for binary logistic regression model and multinomial logistic regression model. The maximum likelihood methods to make estimation and inference are no longer useful especially when the logistic regression fails to meet normal distribution assumption. As Pfeffermann et al (1998) pointed out maximum likelihood estimation will produce some bias.

 The contribution of this paper is to use the correct likelihood method as the new method for binary logistic regressions model and multinomial logistic regression model. We put weights in the *pdf*, and in order to keep the new function still a *pdf*, we should divide it by the integral or sum of distribution with weights (i.e., we accommodate the weights by normalization). In the new method, the weights are further used to adjust the covariates and intercepts. This process can be accomplished using the LOGISTIC procedure of SAS. The old method is the un-normalized distribution with sampling weights, but the new method is the normalized distribution with sampling weights. By comparing the results of data analysis of the two methods, we conclude that there are similarities and differences.

 The practical examples we used is to diagnose overweight and obesity for adults. The dependent variable is BMI with four levels, underweight, normal weight, overweight and obese. We analyze eight Counties and conclude there is significant different within counties. We build binary logistic regression models and multinomial logistic regression models to show the differences and similarities between un-normalized distribution with sampling weights and normalized distribution with sampling weights. We believe using the normalized distribution, the correct likelihood, is the right thing to do, although the use of survey weights is a controversial area Gelman (2007). It would be nice to compare our methods with the method of post stratification as described by Gelman (2007). One may want to post-stratify the survey weights to get approximately equal survey weights within strata.

#

# References

Agresti, A. (1990). *Categorical Data Analysis.* New York: John Wiley & Sons, Inc.

Anderson, R., & Bancroft, T. (1952). *Statistical Theory in Research.* New York: McGraw-Hill.

Archer, K. and Lemeshow, S.(2006). Goodness-of-fit test for a Logistic Regression Model Fitted using Survey Sample Data. *The Stata Journal*, 6, 97-105.

B., A. (n.d.). Performing Logistic Regression on Survey Data with the New Surveylogistic Procedure. *SAS Institute Inc., Cary, North Carolina, USA*, 258-27.

Balgobin Nandram and Jai Won Choi. (2010). A Bayesian Analysis of Body Mass Index Data From Small Domains Under Nonignorable Nonresponse and Selection. *Journal of American Statistical Association*, 105, 120-135.

R. F., Potthoff, Woodbury, M. A., and Manton, K. G. (1992). Equivalent Sample Size and Equivalent Degrees of Freedom Refinements for Inference using Survey Weights under Superpopulation Models. *Journal of the American Statistical Association*, 87, 383-396.

Gelman, Andrew. (2007) Struggles with Survey Weighting and Regression Modeling. *Statistical Science*, 22, 153-164.

Grilli, L.,and Pratesi, M. (2004). Weighted Estimation in Multinomial Ordinal and Binary Models in the Presence of Informative Sampling Designs. *Survey Methodology*, 30, 93-103.

J, C., Andersson, Verkuilen, J., and Peyton, B. L. (2010). Modeling Polytomous Item Responses using Simultaneously Estimated Multinomial Logistic Regression Model. *Journal of Educational and Behavioral Statistics*, 422.

Korn, Edward L and Graubard, and Barry I., Estimating Variance Components by Using Survey Data (2003). *Journal of the Royal Statistical Society, Series B* (Statistical Methodology), 65, 175-190.

Little, R. J. A., and Rubin, D. B. (2002). *Statistical Analysis With Missing Data*. New York: Wiley.

Longford, Nicholas T. (1995). Hierarchical Models and Social Sciences, *Journal of Educational and Behavioral Statistics*, 20, 205-209.

Michael, H., Kutner, C J., and Nachtsheim, J. N. (2004). *Applied Linear Regression Models.* McGraw-Hill/Irwin.

Morel, G. (1989). Logistic Regression Under Complex Survey Designs, *Survey Methodology*, 15, 203-223.

Nandram, B., and Choi, J. W. (2002), A Hierarchical Bayesian Nonresponse Model for Binary Data With Uncertainty About Ignorability, *Journal of the American Statistical Association*, 97, 381-388.

Nandram, B., and Choi, J. W. (2005), Hierarchical Bayesian Nonignorable Nonresponse Regression Models for Small Areas: An Application to the NHANES Data, *Survey Methodology*, 31, 73-84.

Pfeffermann, D. (1993). The Role of Sampling Weights When Modeling Survey Data. *International Statistical Review*, 61, 317-337.

Pfeffermann, D. (1996). The Use of Sampling Weights for Survey Data Analysis. *Statistical Methods for Medical Research*, 5, 239-261.

Pfeffermann, D., Skinner, C. J., Holmes, D. J., Goldstein, H., and Rasbash, J. (1998). Weighting for Unequal Selection Probabilities in Multinomial Models. *Journal of the Royal Statistical Society*, 60, 23-40.

Rittenhouse, C. D., Millspaugh, J. J., Andrew B, H., Michael W, S., Steven, L, Gitzen, R. A. (2008). Modeling Resource Selection using Polytomous Logistic Regression and Kernel Density Estimates. *Environmental and Ecological Statistics*, 15, 39-47.

Roberts, G., Rao, J. N., and Kumar, S. (1987). Logistic Regression Analysis of Sample Survey Data. *Biometrika*, 74, 1-12.

Rodriguez, G., and Goldman, N. (1995). An Assessment of Estimation Procedures for Multinomial Models with Binary Responses. *Journal of the Royal Statistical Society, A*, 158, 73-89.

Rodriguez, G., and Goldman, N. (2001). Improved Estimation Procedures for Multinomial Models with Binary Response: a Case-study. *Journal of the Royal Statistical Society, A*, 164, 339-355.

Skvondal, A., and Rabe-Hesketh, S. (2003). Multinomial Logistic Regression for Polytomous Data and Rankings. *Psychometrika*, 68, 267-287.

1. Details on Applied Linear Regression Models 4th, by Michael A. Kutner, Chris Nachtsheim and John Neter, published by McGraw-Hill Publishing Company. [↑](#footnote-ref-1)
2. Details in Himes and Dietz 1994 [↑](#footnote-ref-2)
3. Values with asterisk indicate differences, values without asterisk indicate similarities. [↑](#footnote-ref-3)
4. Values with asterisk indicate differences, values without asterisk indicate similarities. [↑](#footnote-ref-4)