What Goes Up Must Come Down

Development of a Modular Seesaw Safety Device

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A Major Qualifying Project

Worcester Polytechnic Institute

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Abstract

The goal of this project was to develop a device to improve the safety of playground seesaws. Seesaws can cause injury or discomfort due to the jarring impact experienced by the user when the seesaw hits the ground because of the other user dismounting the seesaw. To address this issue, we designed, built, and tested a modular shock-absorption system that applies a braking force to seesaw motion, thus limiting seesaw acceleration, and consequently, preventing large impact forces. The device attaches to a variety of seesaw geometries. Testing shows that the device successfully reduces the likelihood of user injury without impeding normal seesaw operation.

Acknowledgements

The team would like to thank our advisor Professor Cobb for his advice and consistent commitment, effort, and enthusiasm through all the phases of our project. We would also like to thank Barbara Furhman of the Mechanical Engineering department at Worcester Polytechnic Institute for being instrumental in obtaining the required components for the prototype. Without their contributions, this project would not have been possible.

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Introduction

For many years, the seesaw has served to entertain children at playgrounds. The seesaw, in its most basic form, consists of a linear structural member with a horizontally oriented pivot located at the midpoint of its length. During use, one child sits on each end of the beam. The children take turns pushing off the ground, which causes each end of the beam, and therefore each child, to move upward and downward in an alternating manner. This up and down motion of each child provides amusement for those involved.

For a seesaw to operate properly, both children must cooperate with each other. If this does not occur, one of the children may experience unpleasant results. One common occurrence of uncooperative behavior involves the child closer to the ground jumping off the seesaw while the other child is in an elevated position. This removes the load from one end of the beam, a load that was helping to keep one child in an elevated position. With this load removed, the child in the elevated position will accelerate rapidly downward. When this child reaches the ground, the impact will result in discomfort and possible injury.

To address this problem, our team intended to create a device that would prevent the elevated child from crashing to the ground if the other child dismounts the seesaw. This device had to be compatible with existing seesaws, and could not interfere with seesaw operation when both children remain seated on the device. If one child leaves the seat while the other child is in an elevated position, the device must prevent the rapid downward acceleration of the child who remains on the device. By preventing this acceleration, the device will prevent the child on the device from striking the ground at a high speed, thus minimizing the possibility of discomfort and/or injury.

Background

Before developing the device, the team conducted initial research to understand important topics, such as the current market and playground safety standards. This section details the findings from these studies.

Seesaw Overview

Seesaws, as shown in Figure 1 below, in their simplest form are rigid beams that pivot on a fulcrum, which acts as a hinge allowing either side to move up or down. This system is mechanically a lever and transfers energy accordingly.

Figure 1: Common playground seesaw.

The movement of a mass attached to either side of the seesaw - such as a rider - creates a torque about the seesaw's pivot axis at the fulcrum. This is why a small rider can easily lift a heavier rider on the other side.

Injury Statistics

Injuries as a result from playground equipment are commonplace in the United States. Almost 211,000 children in the U.S. treated annually for injuries sustained while at playground or using playground equipment (Tinsworth, 2001). The injuries treated are most commonly severe including severe fractures and even death. A study conducted by the US Consumer Product Safety Commission showed that injury cases occurred in children from as young as 1 month to 18 years of age with a mean age of 6.6 years and a deviation of 3.3 years. This study collected and analyzed data from 1996 until 2005 (Vollman, 2008). These injuries occurred to both males and females evenly, with males being 54.2% of the injury data collected. The age group of 5 to 12 years old accounted for 70% of the data. Knowing this the team can design the device to work most effectively for boys and girls ages 5 to 12 to prevent the most injuries possible.

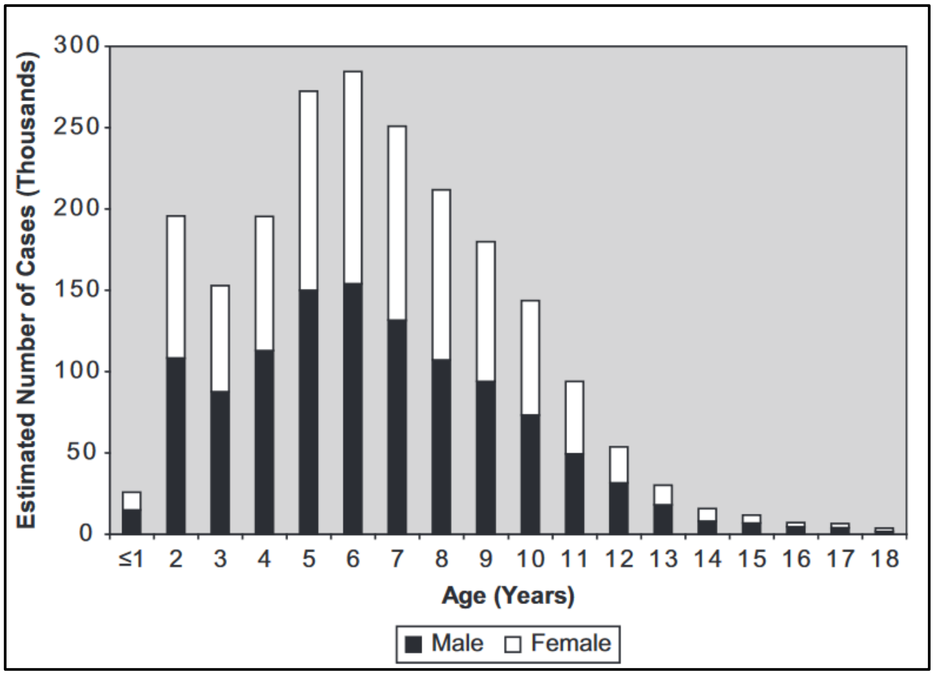
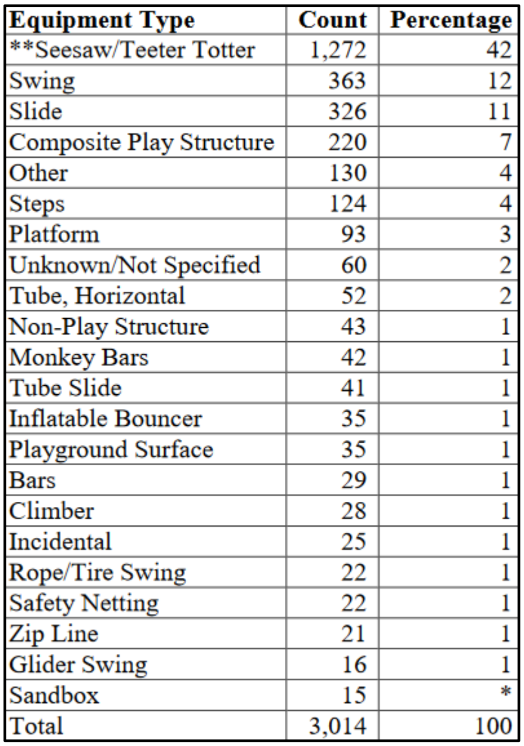


Figure 2: Playground injury statistics (USCSPC).

Falling injuries account for more than 75.1% of injury cases reviewed by hospitals in this study. This suggests that the current standards put forth by the U.S. Consumer Product Safety commission either require revision to make them more comprehensive and increase safety requirements or the playgrounds do not adhere to the standards and consist of unsafe and out of date equipment and layouts.

A similar study done for the years 2009 to 2014 yielded similar statistical data (U.S. Consumer Product Safety Commission, 2016). As shown in Table 1 below, this study also showed that 42% of the 3014 injuries reported to the CPSC and analyzed were a related to a seesaw or teeter-totter; however, this was due to a faulty device recalled in 2012. Without this device included, seesaw-related injuries fall to 33%.

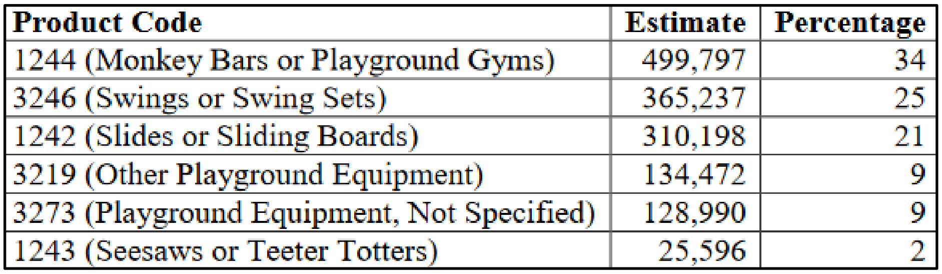
Table 1: Incidents associated with playground equipment by type of equipment, 2009-2014 (USCSPC).



Equipment Type	Count	Percentage
**Seesaw/Teeter Totter	1,272	42
Swing	363	12
Slide	326	11
Composite Play Structure	220	7
Other	130	4
Steps	124	4
Platform	93	3
Unknown/Not Specified	60	2
Tube, Horizontal	52	2
Non-Play Structure	43	1
Monkey Bars	42	1
Tube Slide	41	1
Inflatable Bouncer	35	1
Playground Surface	35	1
Bars	29	1
Climber	28	1
Incidental	25	1
Rope/Tire Swing	22	1
Safety Netting	22	1
Zip Line	21	1
Glider Swing	16	1
Sandbox	15	*
Total	3,014	100

As shown in the Table 2 below, the number of seesaw-related injuries rises to 25,596 from 2009-2014 when examining emergency department-treated injuries, which is about 2% of the total injuries. Seesaws only had one reported death investigation due to a head or neck injury.

Table 2: Estimated emergency department-treated injuries associated with playground equipment by product Code, 2009-2014 (USCPSC).



Product Code	Estimate	Percentage
1244 (Monkey Bars or Playground Gyms)	499,797	34
3246 (Swings or Swing Sets)	365,237	25
1242 (Slides or Sliding Boards)	310,198	21
3219 (Other Playground Equipment)	134,472	9
3273 (Playground Equipment, Not Specified)	128,990	9
1243 (Seesaws or Teeter Totters)	25,596	2

Safety Standards

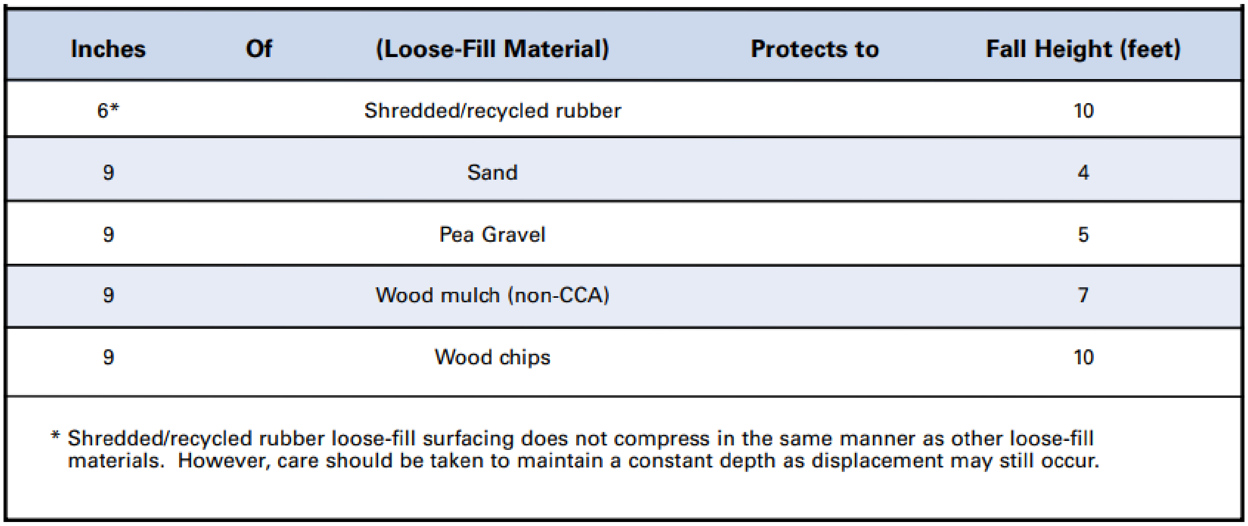
Consumers will use this device, therefore it must conform to several standards to ensure that it is safe to use. In particular, it must follow public playground standards set by the United States Consumer Product Safety Commission. Additionally, to prove the safety of the device, the Head Injury Criterion will be used, which is a common standard used in the automotive safety industry to determine the safety of the passengers during a crash. This section details these two standards.

Playground Safety Standards

The United States Consumer Product Safety Commission (CPSC) has documented standards regarding the design and construction of public playgrounds. This set of guidelines covers criteria from equipment materials to maintenance. The team will keep these standards in mind when designing any equipment that has an intended use in a playground.

The team must consider material selection in two aspects: surface material and equipment material, for two drastically different purposes. The surface that covers the ground controls the maximum fall height, as shown in Table 3. That is, the tallest structure on the playground - seesaw or otherwise - can only be as tall as the fall height below for the given ground material.

Table 3: Minimum compressed loose-fill surfacing depths (U.S. Consumer Product Safety Commission, 2015).



Inches	Of	(Loose-Fill Material)	Protects to	Fall Height (feet)
6*		Shredded/recycled rubber		10
9		Sand		4
9		Pea Gravel		5
9		Wood mulch (non-CCA)		7
9		Wood chips		10

* Shredded/recycled rubber loose-fill surfacing does not compress in the same manner as other loose-fill materials. However, care should be taken to maintain a constant depth as displacement may still occur.

Furthermore, the U. S. Consumer Product Safety Commission constrains the construction of playground equipment, ensuring that all fasteners should not be removable without the use of tools, and must be smooth as to not cause injury to the user. All fasteners should be corrosion resistant, and metal treated to prevent rust. This ensures that the components are safe and reliable (2015).

In addition, few guidelines exist for fulcrum seesaws. Besides the following: the fulcrum cannot present a crush hazard – if a child is able to crush a body part inside the point – during the operation of the seesaw, there should be no footrests, and the maximum attainable angle between the seats and the horizontal is 25° (U.S. Consumer Product Safety Commission, 2015). In addition, the CPSC recommends installing partial car tires under the seats to absorb the shock of impact, but it is not required (2015).

Injury Risk Standards

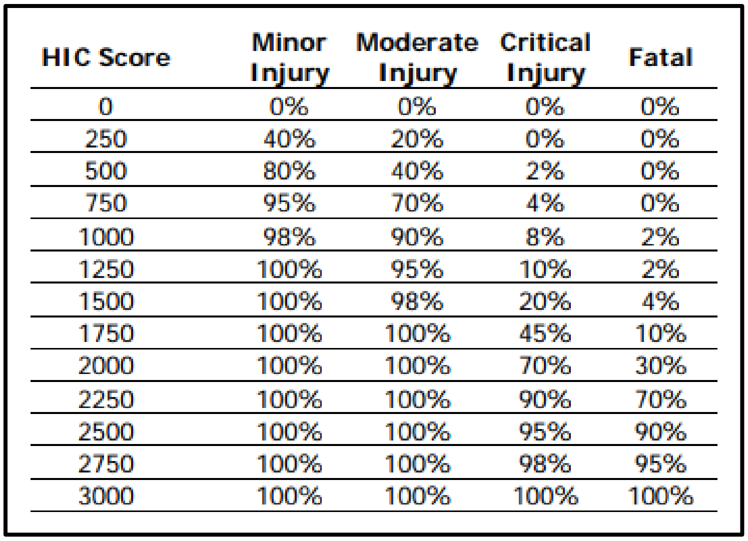
One industry that measures the likelihood of injury is automotive design. The United States National Highway Traffic Safety Administration (NHTSA) developed the Head Injury Criterion (HIC) to evaluate the risk of injury upon a crash. The test involves a frontal crash at thirty miles per hour and measures the acceleration of a crash test dummy's head. In order to calculate the HIC, the following equation is used:

$$HIC = \left\{\left[\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} a(t)dt\right]^{2.5}(t_2 - t_1)\right\}_{max}$$

Equation 1: The Head Injury Criterion (Henn, 1998).

The following chart relates the HIC score to the probability of injury, as shown in Table 4 (Canadian Association of Playground Practitioners). The American Society for Testing and Materials defines a minor injury as "a skull trauma without loss of consciousness; fracture of nose or teeth; superficial face injuries" and a moderate injury as "a skull trauma with or without dislocated skull fracture and brief loss of consciousness. Fracture of facial bones, without dislocation; deep wound(s)" (Canadian Association of Playground Practitioners).

Table 4: Probability of Head Injury Relative to HIC Score (Canadian Association of Playground Practitioners).



HIC Score	Minor Injury	Moderate Injury	Critical Injury	Fatal
0	0%	0%	0%	0%
250	40%	20%	0%	0%
500	80%	40%	2%	0%
750	95%	70%	4%	0%
1000	98%	90%	8%	2%
1250	100%	95%	10%	2%
1500	100%	98%	20%	4%
1750	100%	100%	45%	10%
2000	100%	100%	70%	30%
2250	100%	100%	90%	70%
2500	100%	100%	95%	90%
2750	100%	100%	98%	95%
3000	100%	100%	100%	100%

The Canadian Centre for Occupational Health and Safety (CCOHS) published a standard for children's playgrounds, which states that "the protective surfacing zone shall have a … HIC not exceeding 1000" (Canadian Centre for Occupational Health and Safety, 2014). While it refers to the design of surfacing material, it also serves as a benchmark for the design of a safety device. Because the equation is dependent on the change in time (t_2 and t_1), measured in seconds, and the acceleration, measured in *g*'s, a prediction of an HIC value can be calculated to design a safety device with a low injury risk.

Existing Devices and Practices

The team only found one design in searching for existing seesaw safety devices. This result was a patent filed in 1957 for a "Safety-Type Teeter Board." A sketch of this device is below in Figure 3.

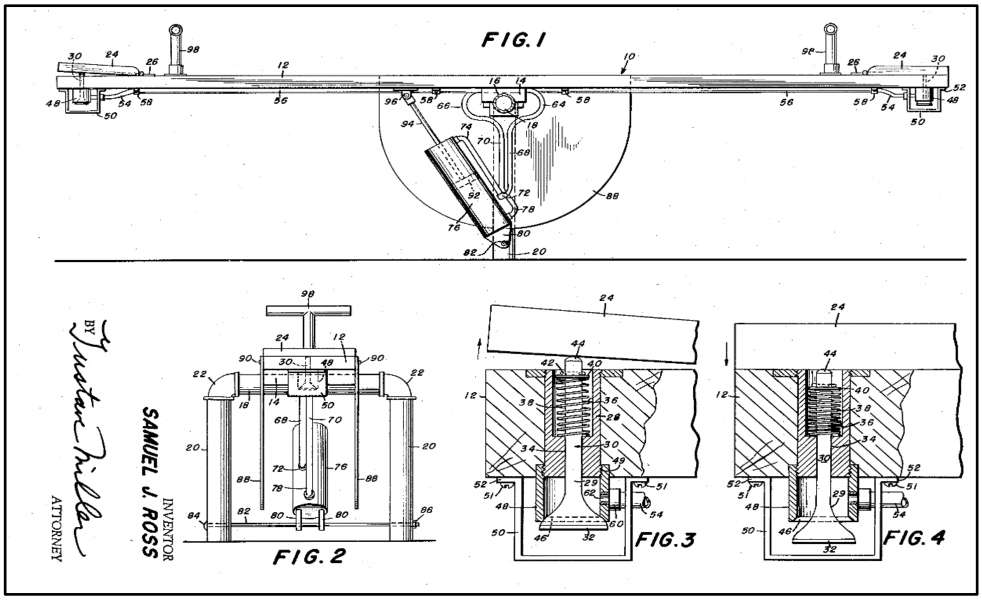


Figure 3: Sketch of "Safety-type Teeter Board" (United States Patent No. US2903263 A, 1957).

In the device, attached to the main board are hinged seat plates at the locations where users sit (Ross, 1957). Underneath each seat plate is a valve, which is spring-loaded in the closed position. In the closed position, the valve stems stick upward and push the seat plates upward about their hinges. The valves lead to air tubes, which run to opposite sides of a pneumatic cylinder. The cylinder body mounts to the base of the seesaw by a pin joint. The piston shaft of the cylinder attaches to the seesaw board by a pin joint at a distance offset from the lengthwise middle of the board. When both users of the seesaw sit on the seat plates, the plates move into a downward position. The seat plates push down on the valve stems, pushing the valves into the open position. This allows air to flow freely through the valves and air tubes, and into the cylinder. As the users move the seesaw, the piston shaft of the cylinder moves into and out of the cylinder, and the piston moves along the cylinder. Since air can flow freely into and out of both ends of the cylinder, minimal resistance to seesaw motion occurs.

If one user jumps off the seesaw, the seat plate that this user previously occupied lifts up. This allows the spring-loaded valve under the seat plate to move to the closed position. When one user jumps off, the end of the seesaw that the user previously occupied tends to swing upward. Due to the arrangement of the pneumatic cylinder and hoses, this motion moves the cylinder piston in a direction that tends to push air out of the valve under the previous user's seat. However, since the valve is closed, and the pressure in the system adds to the closing force in the valve, the air cannot escape. As a result, when the previous user's seat tries to swing upward, the cylinder piston compresses the air trapped on that side of the system by the valve. Since the force applied to the piston is not sufficient for significant compression, the piston, and thus the seesaw board and the remaining user, comes to a halt. This prevents the remaining user from crashing to the ground. Having no users sit on the seat plates prevents seesaw motion in the same manner.

Human Factors

The team will use anthropometric data for statistical data and analysis as presented by the National Center for Health Statistics (McDowell, Fryar, Ogden, & Flegal, 2008). The appendix titled anthropometric data contains tables for the weight, height, and appendage lengths based upon age and sex. Knowing that our target age range will correlate to a minimum supported weight specification, we can use the table to identify this value. Common playground seesaws have a target user base of children 5 to 12 years of age. Looking at the first table, we can take the age range and see the average mass of children in that age range and their percentiles. We also reference the height data to determine appropriate placement of potential device components to ensure that no obstruction of the rider occurs when seated. It also ensures any mechanism we want the rider to interface with will be within a reasonable distance for any rider. The team will use height data to calculate various body part or appendage lengths based on formulas derived from a dataset for height of children ages 5 to 12.

Design Strategy

When developing the device, the design process began by defining the design requirements that the device must satisfy to pass as functional. Following that, the team defined performance specifications, which further constrain the scope of the device and all preliminary concepts to be developed. Additionally, to ensure the completion of the project on time the team created a Gantt chart. This section details these steps of the design process.

Design Requirements

The first step of the design strategy for this device was to identify the objectives of the device: what we expected the safety device to perform, and at what performance level. The primary function of this safety device is to prevent injury to a user should the other user suddenly get off the seesaw. We determined that a mechanism designed to stop or slow the seesaw would be the most successful at achieving this main objective.

Safety of the device is of utmost importance, as its primary use is in a playground environment and used almost entirely around children. The team therefore must follow safety standards set forth by the American Society for Testing and Materials (ASTM). To provide the consumer a visually appealing and cost-effective product we will also be considering packaging and budget.

Design Standards

In order to propose a valid design, the team took into account certain design standards and considerations while moving along with design selection. These are design points that the team felt would be immediately limiting to development of the design and construction of the prototype. These eventually evolved to the design criteria that was used to narrow down to the final design within the selection matrix.

Performance Specifications

The team also developed performance specifications in order to define the preliminary concepts that serve as the basis for the final design. These specifications limit aspects of the design, such as the device weight and cost. Additionally, the team will use them to both assess the viability of the initial concepts and the final design to ensure that they perform as required. The exact specifications defined are in Table 5 below.

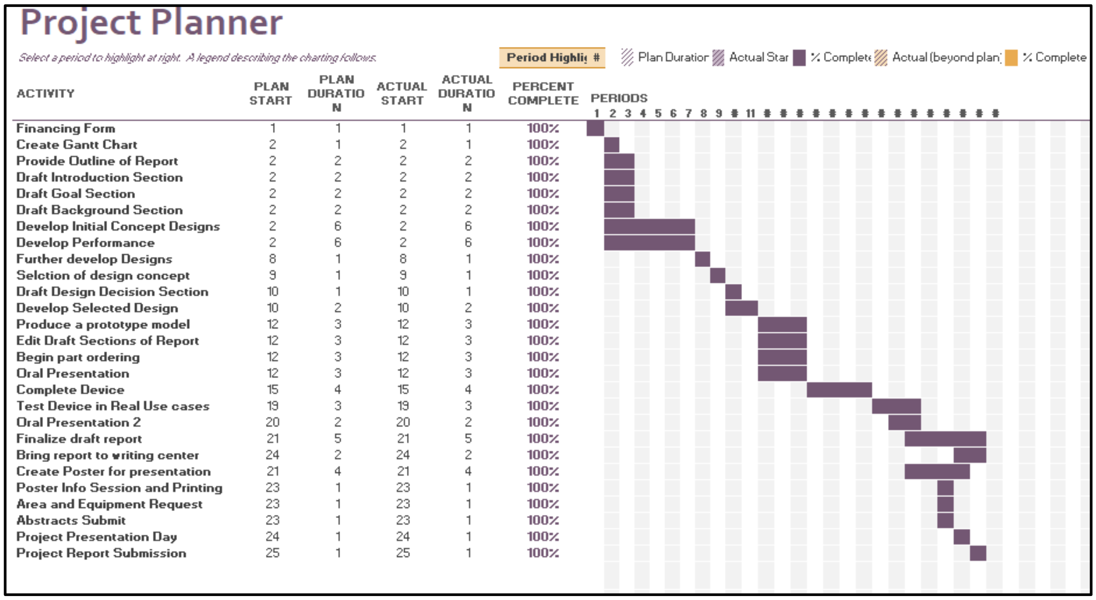
Table 5: Performance specifications for the seesaw safety device.

Specification	Criteria
Size of Container Used for Packaging	The size of the container is important to ensure that it can ship via standard FedEx and UPS ground trucks, not truck freight. According to FedEx packaging guidelines, a single package can be up to 150 lbs., up to 108” in length, and 165” in length and girth. Girth is defined as (2 * width) + (2 * height). Length is the longest side of the package or object.
Weight of Device	Device will be less than 50 kilograms in weight.
Tool Requirement	The device must assemble with simple hand tools. Such as wrenches, sockets and ratchets, screwdrivers, and Allen keys. Some tools can be included with the device packaging.
Ease of Assembly	No more than two individuals shall install the device. Minimal parts assembly is ideal.
User Age Limitation	Based upon the statistical data and safety standards researched we determined the age range this device should be designed for as 5 to 12 to limit total number of injuries.
User Weight Limitation	Based upon the anthropometric data the device must support a minimum user weight of 60 kilograms.
Positions of Controls Relative to User Torso/Arms/Legs	The control for the device must be within arms/legs reach of the user without full extension, and ideally be ambidextrous.
Weather Resistance	Device will be able to withstand varying weather conditions, and continue to be safe to use in varying weather conditions.
Temperature Resistance	Device will remain functional and withstand varying temperature conditions.
Seesaw Compatibility	The device must install on existing seesaw devices of varying geometries. The existing seesaw will require minimal modification to install the device.
Head Injury Criterion	The HIC should have an absolute maximum value of 250 to reduce acceleration/velocity to acceptable level.

ASTM Playground Standards	Meets ASTM F1847 standard.
Signage or Labeling	At a minimum, basic instructional labeling somewhere on device.
Electrical Components	No use of electrical components in device to eliminate point of device failure.
Cost	Cost of the device must be below $60, with a 15% markup.
Product Lifetime	Parts must be able to withstand 30 years of use or 1,000,000 cycles without failure.
Braking Operation	Must activate when one user leaves the seesaw. Must not interfere with normal seesaw operation with two users.
Safety Factor	Device will have a safety factor of at least 3 for one million cycles of use.

Timeline of Project Completion

To effectively manage and execute this project the team used the Gantt chart shown below in Figure 4. This created deadlines that enabled the efficient progress on the tasks outlined in the Gantt chart. It was regularly updated as each event was completed, noted by the categories of “actual start” and “actual duration,” to keep the project on track.



Project Planner

Select a period to highlight at right. A legend describing the charting follows.

ACTIVITY	PLAN START	PLAN DURATION	ACTUAL START	ACTUAL DURATION	PERCENT COMPLETE
Financing Form	1	1	1	1	100%
Create Gantt Chart	2	1	2	1	100%
Provide Outline of Report	2	2	2	2	100%
Draft Introduction Section	2	2	2	2	100%
Draft Goal Section	2	2	2	2	100%
Draft Background Section	2	2	2	2	100%
Develop Initial Concept Designs	2	6	2	6	100%
Develop Performance	2	6	2	6	100%
Further develop Designs	8	1	8	1	100%
Selction of design concept	9	1	9	1	100%
Draft Design Decision Section	10	1	10	1	100%
Develop Selected Design	10	2	10	2	100%
Produce a prototype model	12	3	12	3	100%
Edit Draft Sections of Report	12	3	12	3	100%
Begin part ordering	12	3	12	3	100%
Oral Presentation	12	3	12	3	100%
Complete Device	15	4	15	4	100%
Test Device in Real Use cases	19	3	19	3	100%
Oral Presentation 2	20	2	20	2	100%
Finalize draft report	21	5	21	5	100%
Bring report to writing center	24	2	24	2	100%
Create Poster for presentation	21	4	21	4	100%
Poster Info Session and Printing	23	1	23	1	100%
Area and Equipment Request	23	1	23	1	100%
Abstracts Submit	23	1	23	1	100%
Project Presentation Day	24	1	24	1	100%
Project Report Submission	25	1	25	1	100%

Figure 4: Gantt chart used to complete the project.

The team divided the project into four phases: research and initial concept development, prototype development, testing, and presentation. Each of these phases lasts approximately seven weeks. The team had to complete each phase before the next could begin, and was broken into smaller sub-phases, listed in the Gantt chart. This allowed for further assessment of the progress completed and created smaller goals and deadlines that are easier to manage.

Throughout this project, the overall objectives in the Gantt chart were met, although the timeline was adjusted several times to account for the design selection and analysis portion of the process taking longer than expected. However, this was acceptable in that the team agreed that a more thoroughly planned design often is more successful, even if it takes longer to develop.

Design Process

Once the team created the specifications and standards used to qualify a designs validity, the initial design process began. The team structured the design process to take our raw ideas and turn them into a final design concept. The design then progresses into system and design analysis to predetermine its capabilities before final construction and testing.

Conceptual Designs

In order to develop design concepts, we divided concepts into two groups. The first group included activation mechanisms for the device. The second group included braking mechanisms. The activation mechanism serves the purpose of activating the braking mechanism when one user jumps off the seesaw. The braking mechanism applies force to the seesaw beam to slow the descent of the remaining user on the seesaw.

The first activation mechanism concept, which we developed, is a manual switch. The remaining user activates this on a seesaw after the other user jumped off. The second activation mechanism concept is a dead man’s switch. This device would involve placing a switch on each handle of the seesaw for users to grab onto and pull. Users would have to pull the switch while using the seesaw. If one user jumps off, they would release the switch, causing the braking mechanism to activate. We created several sketches for potential dead man’s handle devices, as shown in Appendix E. Our third activation device mechanism was for a dead man’s seat. This device would involve a hinged seat plate with a spring underneath to push the seat in an upwards position. During regular use, the user’s weight would press the seat into the downward position. In this position, the braking mechanism deactivates, and the seesaw would function as normal. If one user jumped off the seesaw, the spring under the seat plate would push the seat plate into an upward position. In this position, the braking mechanism would activate. A sketch of the dead man’s seat concept is included in Appendix E.

The first braking mechanism concept, which we developed, is for a pneumatic or hydraulic piston to apply braking force to the seesaw beam. This design is the same as that of US Patent number 2903263A for a Safety-Type Teeter Board, as shown in Figure 5.

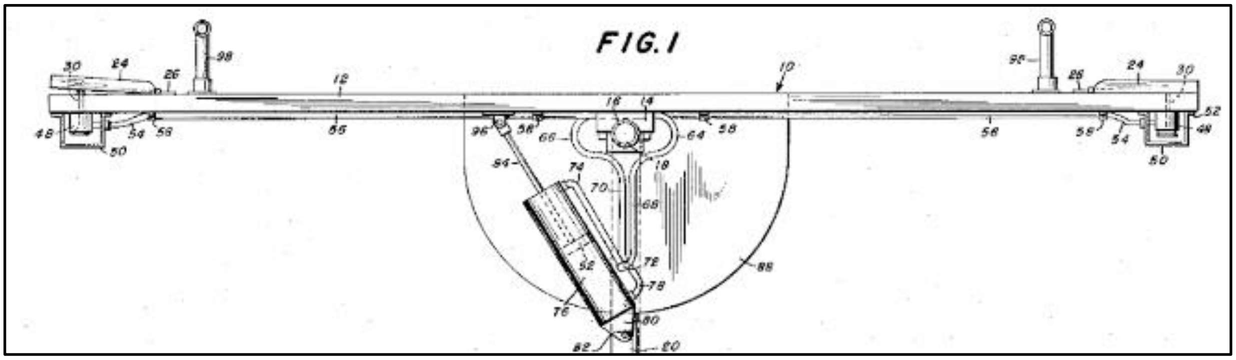


Figure 5: text (United States Patent No. US2903263 A, 1957).

The second braking mechanism concept is for a flywheel device. Such a device would be detached from the seesaw beam during normal operation. When the braking mechanism activates, the flywheel device connects to the seesaw beam through a clutch device. The large mass moment of inertia of the flywheel would slow down the acceleration of the seesaw beam as energy transfers to it. The third braking mechanism concept involved using springs to absorb energy from the seesaw beam, thus slowing the descent of the remaining user. The fourth braking mechanism concept involved a cable and brake. The fifth braking mechanism concept involved using magnetic repulsion to slow down the motion of the seesaw beam. The sixth braking mechanism concept involved the attachment of a disk brake to the seesaw beam using brackets. The brake caliper mounts to the seesaw frame using brackets as well. A sketch of this concept is in Appendix E. The seventh braking mechanism concept involved the use of a drum brake instead of a disk brake. The eighth braking mechanism concept involved the use of a fan system similar to that used in rowing machines. During normal operation, the fan disconnects from the seesaw beam. When the braking mechanism activates the fan connects to the seesaw beam through a clutch mechanism. As energy transfers to the fan wheel, it would spin and face air resistance. This concept operates similarly to the flywheel concept, except it uses air resistance as well as mass moment of inertia in order to slow the motion of the seesaw beam.

Design Selection

In order to select a design for further development, we paired each activation mechanism concept with each braking mechanism concept to create combined concepts. The team then analyzed these concepts using a decision matrix.

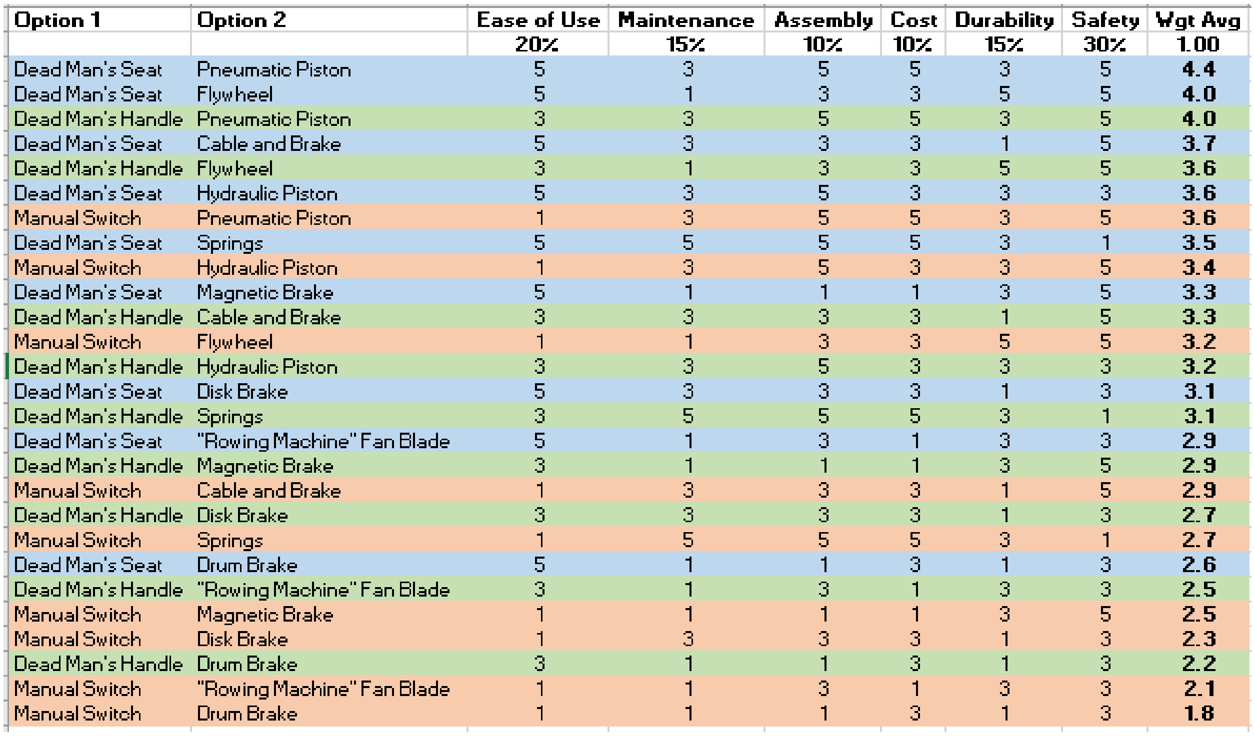
For the decision matrix, the team weighted a number of criteria based upon the importance of the factors. The criteria chosen, along with their respective weights are shown in the table below.

Table 6: Criteria for the decision matrix.

Criteria	Weight
Ease of Use	20%
Maintenance	15%
Assembly	10%
Cost	10%
Durability	15%
Safety	30%

This matrix also narrowed the list of the concepts from an initial 27 to 5, which the team further examined and evaluated. The design matrix is in Table 7 below.

Table 7: Design Selection Matrix



Option 1	Option 2	Ease of Use	Maintenance	Assembly	Cost	Durability	Safety	Wgt Avg
		20%	15%	10%	10%	15%	30%	1.00
Dead Man's Seat	Pneumatic Piston	5	3	5	5	3	5	**4.4**
Dead Man's Seat	Flywheel	5	1	3	3	5	5	**4.0**
Dead Man's Handle	Pneumatic Piston	3	3	5	5	3	5	**4.0**
Dead Man's Seat	Cable and Brake	5	3	3	3	1	5	**3.7**
Dead Man's Handle	Flywheel	3	1	3	3	5	5	**3.6**
Dead Man's Seat	Hydraulic Piston	5	3	5	3	3	3	**3.6**
Manual Switch	Pneumatic Piston	1	3	5	5	3	5	**3.6**
Dead Man's Seat	Springs	5	5	5	5	3	1	**3.5**
Manual Switch	Hydraulic Piston	1	3	5	3	3	5	**3.4**
Dead Man's Seat	Magnetic Brake	5	1	1	1	3	5	**3.3**
Dead Man's Handle	Cable and Brake	3	3	3	3	1	5	**3.3**
Manual Switch	Flywheel	1	1	3	3	5	5	**3.2**
Dead Man's Handle	Hydraulic Piston	3	3	5	3	3	3	**3.2**
Dead Man's Seat	Disk Brake	5	3	3	3	1	3	**3.1**
Dead Man's Handle	Springs	3	5	5	5	3	1	**3.1**
Dead Man's Seat	"Rowing Machine" Fan Blade	5	1	3	1	3	3	**2.9**
Dead Man's Handle	Magnetic Brake	3	1	1	1	3	5	**2.9**
Manual Switch	Cable and Brake	1	3	3	3	1	5	**2.9**
Dead Man's Handle	Disk Brake	3	3	3	3	1	3	**2.7**
Manual Switch	Springs	1	5	5	5	3	1	**2.7**
Dead Man's Seat	Drum Brake	5	1	1	3	1	3	**2.6**
Dead Man's Handle	"Rowing Machine" Fan Blade	3	1	3	1	3	3	**2.5**
Manual Switch	Magnetic Brake	1	1	1	1	3	5	**2.5**
Manual Switch	Disk Brake	1	3	3	3	1	3	**2.3**
Dead Man's Handle	Drum Brake	3	1	1	3	1	3	**2.2**
Manual Switch	"Rowing Machine" Fan Blade	1	1	3	1	3	3	**2.1**
Manual Switch	Drum Brake	1	1	1	3	1	3	**1.8**

The five concepts the team chose for further evaluation are:

- Dead man’s seat & pneumatic piston
- Dead man’s seat & flywheel
- Dead man’s handle & pneumatic piston
- Dead man’s seat & cable and brake
- Dead man’s handle & flywheel

Design Criteria

Explanation of each of the design criteria is as follows.

Ease of Use

1. Complicated design. The user is unable to understand the activation and purpose of the device intuitively. Difficult for a child to operate. May hinder seesaw operation.

3. Child can operate with some difficulty. Device is easy to understand but requires some simple instructional guidance.

5. Simple design. Child can operate freely with no explanation for operation. Does not require secondary person for assistance. Does not interfere with regular seesaw operation.

Maintenance

1. Requires use of uncommon tools for repair. Requires use of proprietary tools for repair. Requires maintenance at short intervals. Requires complicated repair procedures. Repairs require replacement of many components/large components. Repairs require skilled technician.
3. Repairs may require some uncommon tools. Maintenance is done at regular intervals but not extremely repetitive. Repairs may require instruction.
5. The assembler only needs common hand tools. Maintenance is required only if device is malfunctioning or damaged. Repairs are simple and intuitive.

Assembly

1. Lots of parts. Difficult to understand design and requires technician for installation. The device has many fasteners. Parts are difficult to assemble, may require awkward positioning and more than one assembler.
3. Minimal parts but more assembly is required. Single person to install with instruction.
5. Minimal parts. The factory pre-assembles a majority of the device into modules. Instructions are well thought out and easy to follow. Requires only single person to install.

Cost

1. More than recommended amount (~$50). Expensive replacement parts.
3. More than recommended amount or cheap and accessible replacement parts.
5. Minimized manufacturing costs. Cheap and accessible replacement parts.

Durability

1. Short life cycle. Easily damaged or rendered unusable. Parts require regular replacement. Cannot withstand expected environmental conditions.
3. Medium life cycle. Occasional replacement of parts. Meets minimums for standards and conditions.
5. High life cycle. Exceeds “x” standards. Made with tough and reliable materials. Exceeds requirements for environmental conditions.

Safety

1. Does not meet playground safety standard. Use of unsafe materials. Several aspects of design may present a hazard to the user. No warning, safety, or instructional labeling. Device does not function as intended. Head injury criterion greater than acceptable level.
3. Meets playground safe. Most materials are safe. Device functions as intended. HIC does not exceed acceptable level. Certain aspects of device may present hazard to the user. Labeling is minimal.
5. Device exceeds playground safety standards. Use of non-toxic child safe materials. Few aspects of the design present a hazard. Labeling is properly applied to the device, warning safety or instructional. Device functions as intended. Maximum value for head injury criterion is well below the accepted level.

Decision Matrix Rankings

Table 8: Dead Man's Seat & Pneumatic Piston Decision Matrix Rankings.

Criteria	Score	Reasoning
Ease of Use	5	This combination received a rating of ‘5’ for ease of use, in that the seat requires minimal instruction, the device is simple to use, and does not inhibit the user’s experience.
Maintenance	3	This combination received a rating of ‘3’ for maintenance. While maintenance is not frequently needed, the assembly is intricate and may be difficult to repair.
Assembly	5	This combination received a rating of ‘5’ for assembly, in that all of the components will be pre-assembled, making the implementation of the device straightforward and simple to perform.
Cost	5	This combination received a rating of ‘5’ for cost, in that the components used are relatively inexpensive, making both the initial cost and replacements affordable for the consumer.
Durability	3	This combination received a rating of ‘3’ for durability, in that the piston requires an airtight seal to perform effectively, and extreme weather conditions would likely affect this.
Safety	5	This combination received a rating of ‘5’ for safety. Non-toxic materials and no hazards, including pinch points, exist on the seesaw assembly.

Table 9: Dead Man's Seat & Flywheel Decision Matrix Rankings.

Criteria	Score	Reasoning
Ease of Use	5	This combination received a rating of '5' for ease of use, in that the seat requires minimal instruction, the device is simple to use, and does not inhibit the user's experience.
Maintenance	1	This combination received a rating of '1' for maintenance, in that parts are significantly complex to repair, and a trained technician may be required to maintain the device.
Assembly	3	This combination received a rating of '3' for assembly, in that while the device is modular and straightforward to assemble, it would require complex setup to work properly.
Cost	3	This combination received a rating of '3' for cost, in that the material cost of the flywheel may greatly raise the overall cost of the device.
Durability	5	This combination received a rating of '5' for durability, in that device will have a large life cycle and repairs should rarely be needed.
Safety	5	This combination received a rating of '5' for safety, in that non-toxic materials are used and no hazards, including pinch points, would be added to the seesaw assembly.

Table 10: Dead Man’s Handle & Pneumatic Piston Decision Matrix Rankings.

Criteria	Score	Reasoning
Ease of Use	3	This combination received a rating of ‘3’ for ease of use, in that the activation mechanism requires more complex user control, in that the user letting go of the handle during normal operation would engage the deceleration mechanism.
Maintenance	3	This combination received a rating of ‘3’ for maintenance, in that while maintenance is not frequently needed, the assembly is intricate and may be difficult to repair.
Assembly	5	This combination received a rating of ‘5’ for assembly, in that all of the components will be pre-assembled, making the implementation of the device straightforward and simple to perform.
Cost	5	This combination received a rating of ‘5’ for cost, in that the components used are relatively inexpensive, making both the initial cost and replacements affordable for the consumer.
Durability	3	This combination received a rating of ‘3’ for durability, in that the piston requires an airtight seal to perform effectively, and extreme weather conditions would likely affect this.
Safety	5	This combination received a rating of ‘5’ for safety, in that non-toxic materials are used and no hazards, including pinch points, would be added to the seesaw assembly.

Table 11: Dead Man’s Seat & Cable and Brake Decision Matrix Rankings.

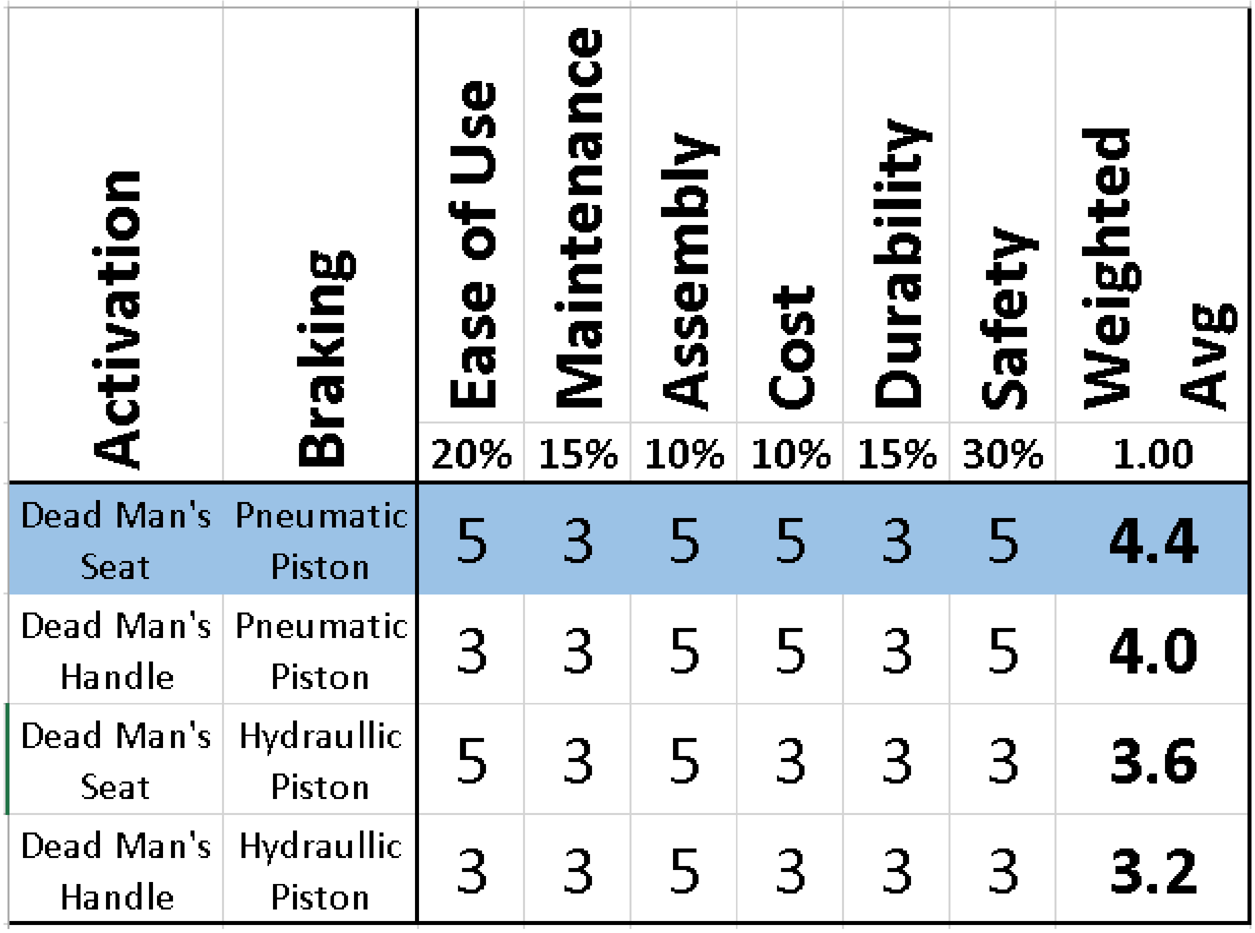
Criteria	Score	Reasoning
Ease of Use	5	This combination received a rating of ‘5’ for ease of use, in that the seat requires minimal instruction, the device is simple to use, and does not inhibit the user’s experience.
Maintenance	3	This combination received a rating of ‘3’ for maintenance, in that maintenance would be needed frequently due to the replacement of the brake pads and the cables after natural fatigue and wear.
Assembly	3	This combination received a rating of ‘3’ for assembly, in that the assembly of the device on the seesaw itself would be complex, specifically for the brake pad, which would need to be clamped onto the seesaw and calibrated.
Cost	3	This combination received a rating of ‘3’ for cost, in that while the initial cost may be low, the frequent maintenance would significantly increase the lifetime cost of the device.
Durability	1	This combination received a rating of ‘1’ for durability, in that the brake pads have a fairly short lifespan and would need frequent replacement, and the cables used would naturally wear and stretch from use and would need to be replaced as well.
Safety	5	This combination received a rating of ‘5’ for safety, in that non-toxic materials are used and no hazards, including pinch points, would be added to the seesaw assembly.

Table 12: Dead Man's Handle & Flywheel Decision Matrix Rankings.

Criteria	Score	Reasoning
Ease of Use	3	This combination received a rating of '3' for ease of use, in that the activation mechanism requires more complex user control, in that the user letting go of the handle during normal operation would engage the deceleration mechanism.
Maintenance	1	This combination received a rating of '1' for maintenance, in that parts are significantly complex to repair, and a trained technician may be required to maintain the device.
Assembly	3	This combination received a rating of '3' for assembly, in that while the device is modular and straightforward to assemble, it would require complex setup to work properly.
Cost	3	This combination received a rating of '3' for cost, in that the material cost of the flywheel may greatly raise the overall cost of the device.
Durability	5	This combination received a rating of '5' for durability, in that device will have a large life cycle and repairs should rarely be needed.
Safety	5	This combination received a rating of '5' for safety, in that non-toxic materials are used and no hazards, including pinch points, would be added to the seesaw assembly.

After completing the initial decision matrix, the team re-evaluated the top four designs in order to select a design for development. This resulted in the selection of the dead man's seat activation mechanism and pneumatic piston braking mechanism for development. The decision matrix is Table 13.

Table 13: Decision matrix of second round of concepts.



Activation	Braking	Ease of Use	Maintenance	Assembly	Cost	Durability	Safety	Weighted Avg
		20%	15%	10%	10%	15%	30%	1.00
Dead Man's Seat	Pneumatic Piston	5	3	5	5	3	5	**4.4**
Dead Man's Handle	Pneumatic Piston	3	3	5	5	3	5	**4.0**
Dead Man's Seat	Hydraullic Piston	5	3	5	3	3	3	**3.6**
Dead Man's Handle	Hydraullic Piston	3	3	5	3	3	3	**3.2**

Design Analysis

To ensure that the device will function properly and not fail under normal operating conditions, the team conducted several analyses. The team performed a system analysis to assess the forces within the seesaw assembly. This was used to determine the requirements for the pneumatic cylinders used within the device. These included system pressures and the angle and position of the pistons themselves relative to the seesaw arm. In addition, the team conducted a stress analysis on the critical points in the system to ensure that they do not fail under use. The results of these analyses are in this section below.

System Analysis

In designing the safety device, much of the functional analysis focused on the braking mechanism within the system. The primary concern of this analysis was to determine proper sizing and geometry of the pneumatic cylinders for the braking system in order to ensure that they would be able to provide sufficient stopping force without failure due to large applied loads. The team performed most of this analysis in a PTC Mathcad document located in Appendix F.

In this document, the team implemented an iterative design approach to create the geometry of the braking mechanism based upon the forces experienced within the system. Figure 6 and Figure 7 below, show the geometry of the seesaw and braking mechanism. Description of the variables used in these figures is in Appendix F.

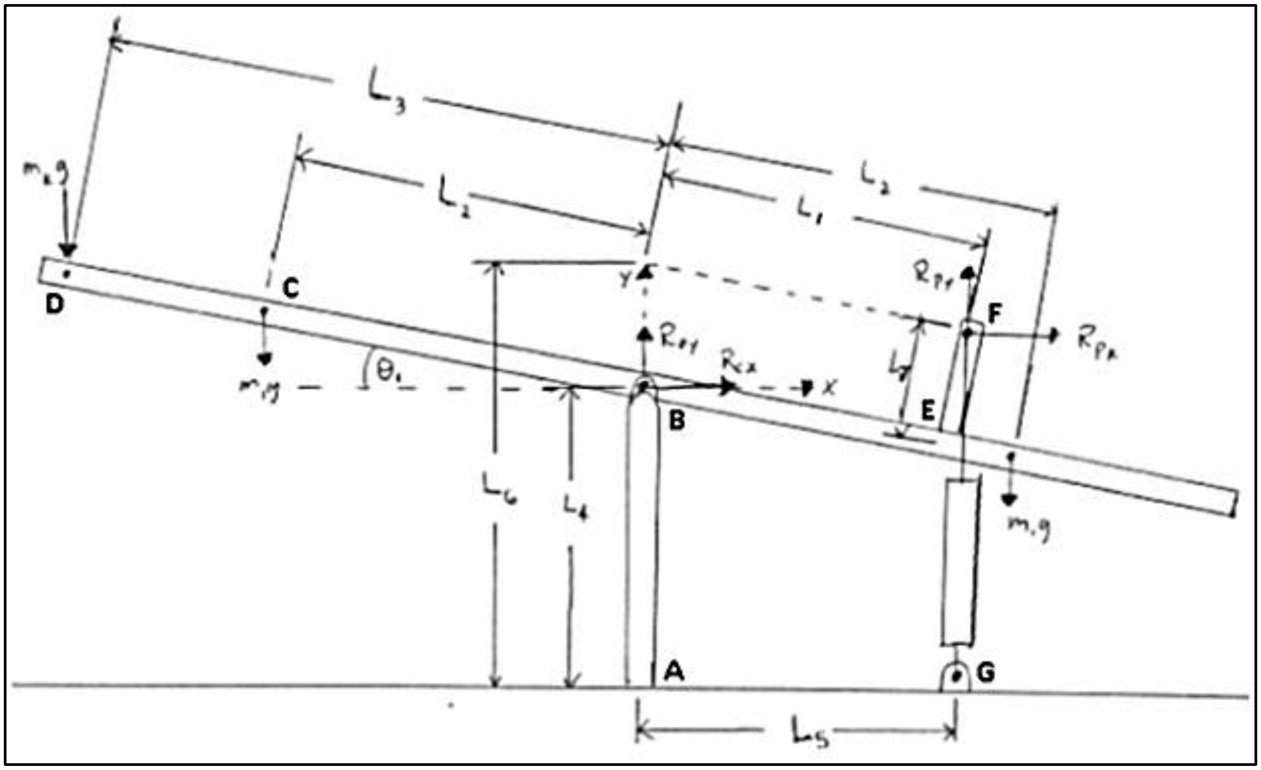


Figure 6: Geometry of the seesaw.

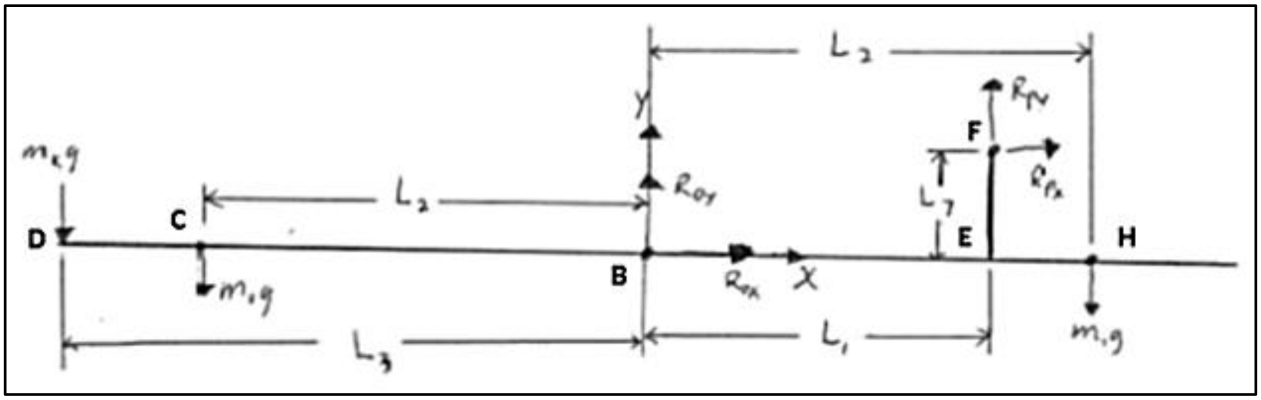


Figure 7: Forces on the seesaw arm.

Fixed values, based upon measurements from the seesaw prototype, were used for all lengths except for L_1 and L_7. It is important to note that seesaws of differing geometries will need different component sizing and placement. Based on these lengths and a maximum user mass of 60 kilograms, the value of L_1 was altered until sufficient results were obtained. For each value of L_1, the necessary piston stroke length was calculated to allow the seesaw to move from one end of the seesaw beam touching the ground to the other. This allowed for the selection of a suitable pneumatic cylinder, as the total retracted length was determined through the Mathcad calculations. With this value and the other geometry, the value for L_7 was calculated. Then, the force that the cylinders had to apply to the seesaw was calculated to achieve static equilibrium for the entire motion of the seesaw. The team created plots of the piston and pivot joint forces and shown below in Figure 8 and Figure 9. Additionally, the team created plots for the piston forces parallel and perpendicular to the seesaw beam, as shown in Figure 10 and Figure 11.

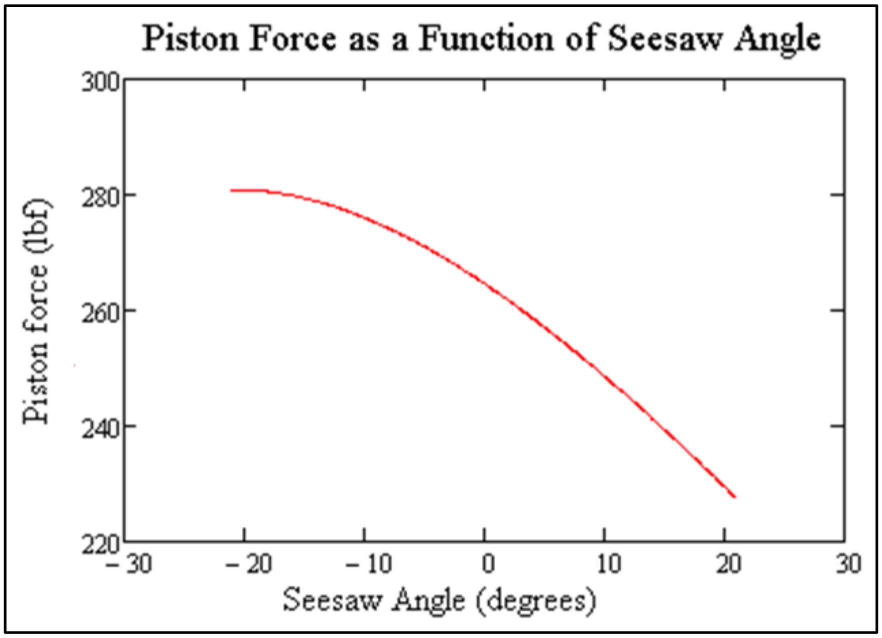


Figure 8: Piston Force as a Function of Seesaw Angle.

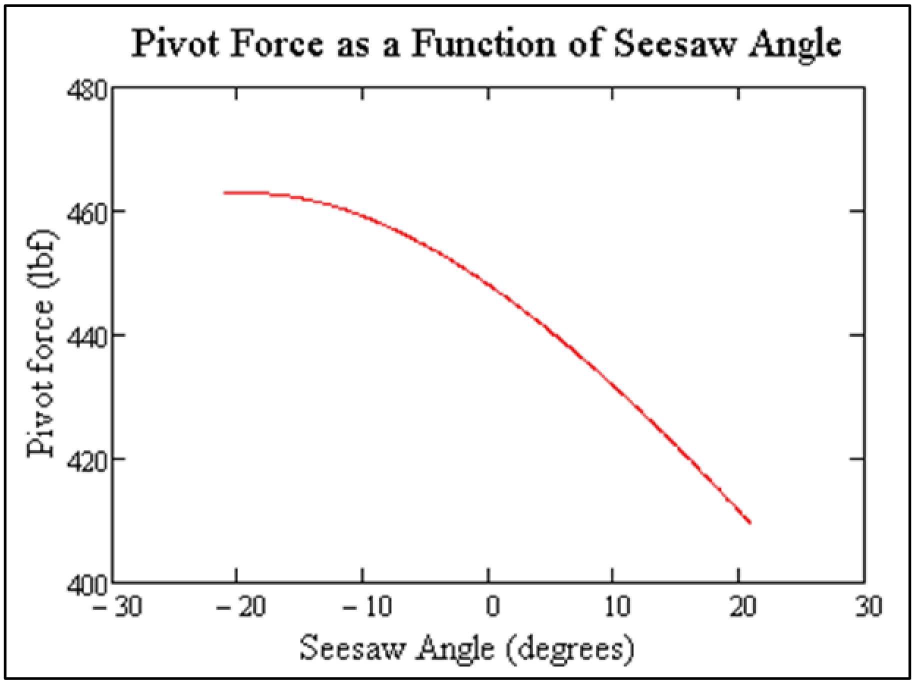


Figure 9: Force at the seesaw pivot pin as a function of seesaw angle.

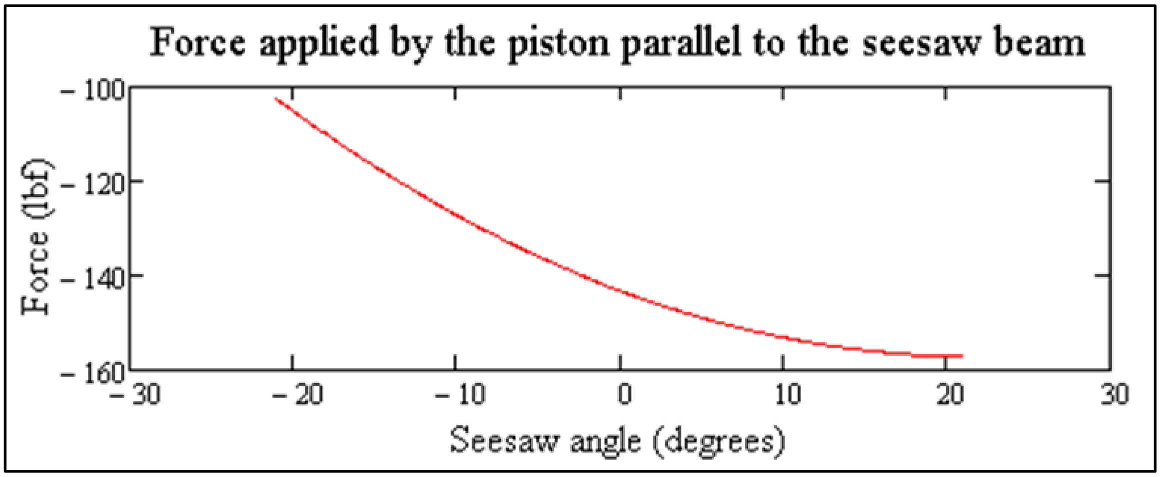


Figure 10: Force applied by the piston parallel to the seesaw beam.

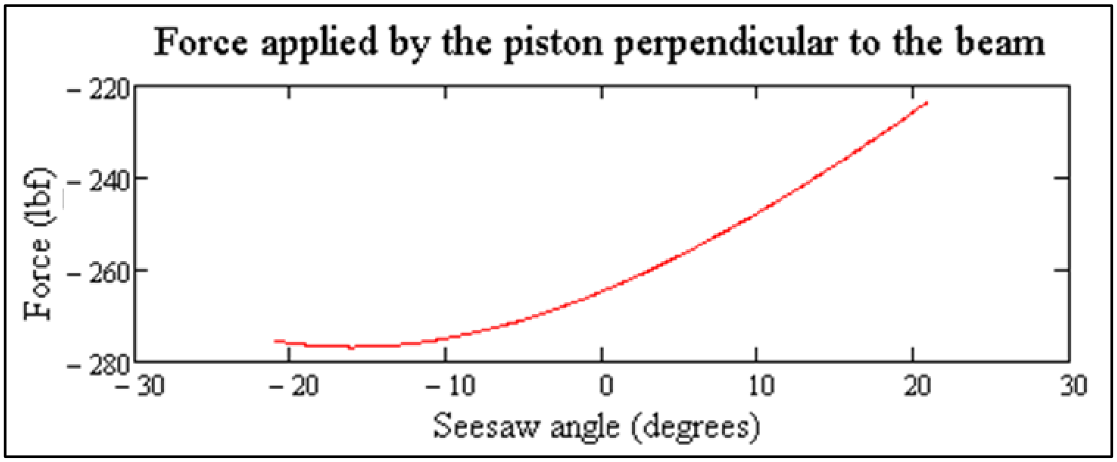


Figure 11: Force Applied by the Piston Perpendicular to the Seesaw Beam.

The objective of this analysis was to minimize the forces present in the plots above. This was done to prevent damage to the seesaw system - particularly, the pins in the braking mechanism - when the safety device activates. Additionally, the force for the piston should be minimized in order to minimize pressure within the piston and other pneumatic components, and to minimize the bore of the piston itself. The force parallel to the seesaw board should also be minimized to minimize the risk of breaking the board, and the force perpendicular to the board should be minimized in order to limit the possibility of slippage of the connection between the braking mechanism and the seesaw board. The value of L_1 was iterated several times until a reasonable value was found that resulted in safe forces in the system. This value and the other dimensional values are in the Variables section of the Mathcad document in Appendix F.

Following the calculation of these values, the piston pressure was determined at all angles the seesaw would reach during normal operation. This was based upon the piston force and the selected bore diameter, as shown in the figure below. This was iterated to obtain a maximum piston pressure of less than 100 pounds per square inch. This resulted in a piston bore size of 1.5 inches.

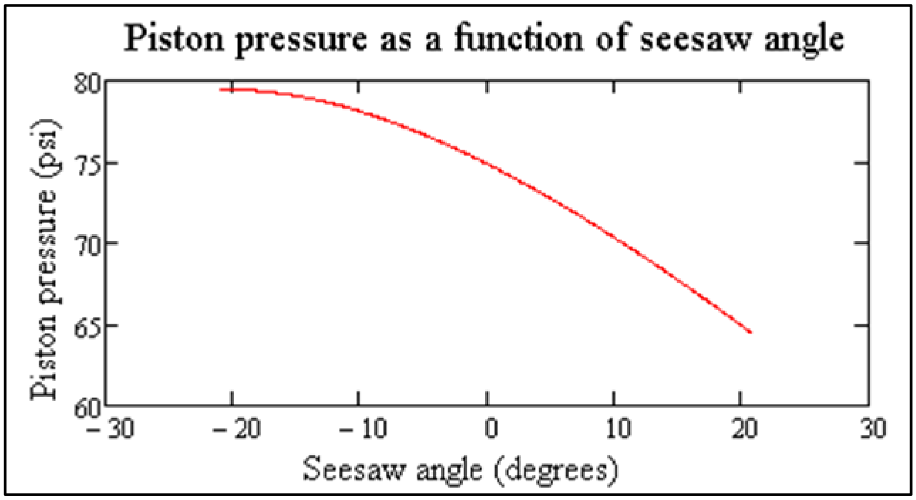


Figure 12: Piston pressure as a function of seesaw angle.

Based on the above analysis, we selected a piston with a stroke length of 28 inches and a 1.5 inch bore. The part of the braking mechanism attached to the seesaw beam needs to extend approximately 23.5 inches above the seesaw beam.

Stress Analysis

As shown in the figure below, the team identified two critical points in the system. Both of these points are located within the pins in the braking mechanism. Point A is located at the bottom two pins at the base of the seesaw, and Point B is located at the top two pins. Therefore, we conducted stress analyses for these two pins to ensure that they do not fail.

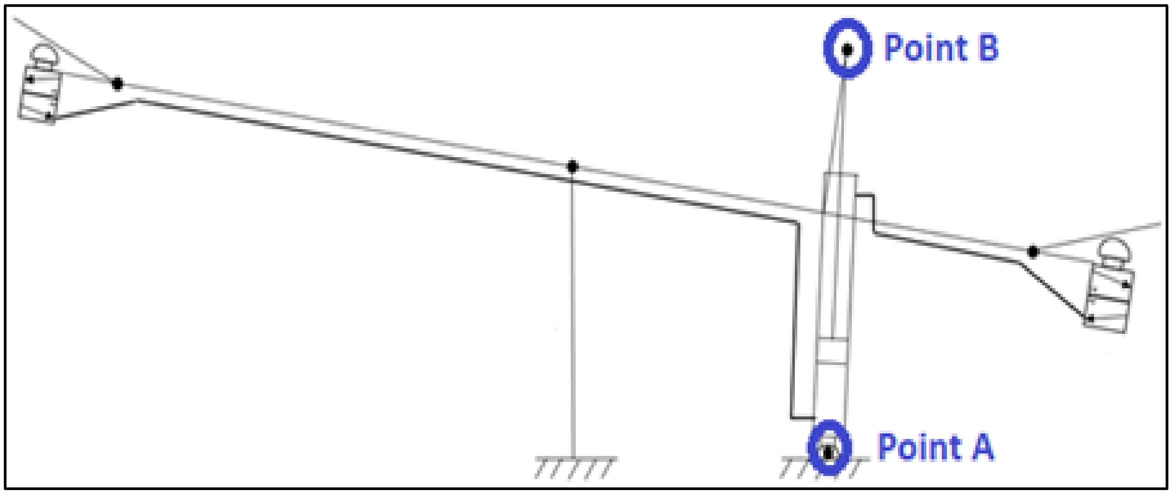


Figure 13: Location of critical points for the stress analysis.

To begin, the team created a free body diagram for each of the points to determine the locations and magnitudes of the forces involved. The diagrams for Points A and B are in Figure 14 and Figure 15, respectively.

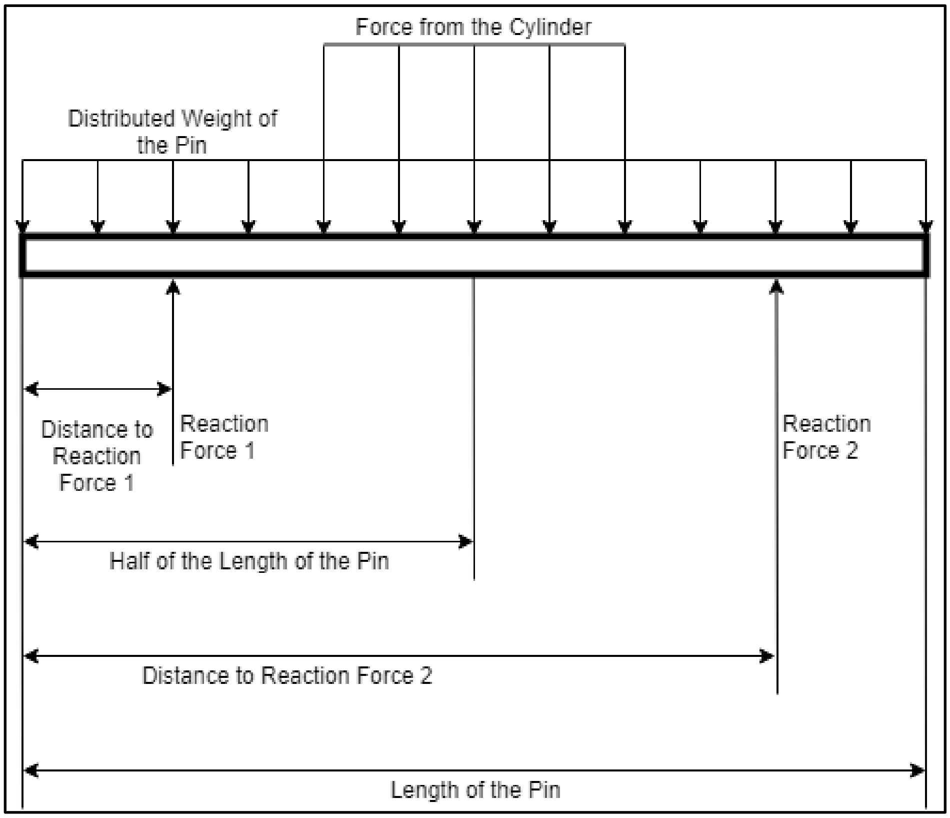


Figure 14: Free Body Diagram of Point A.

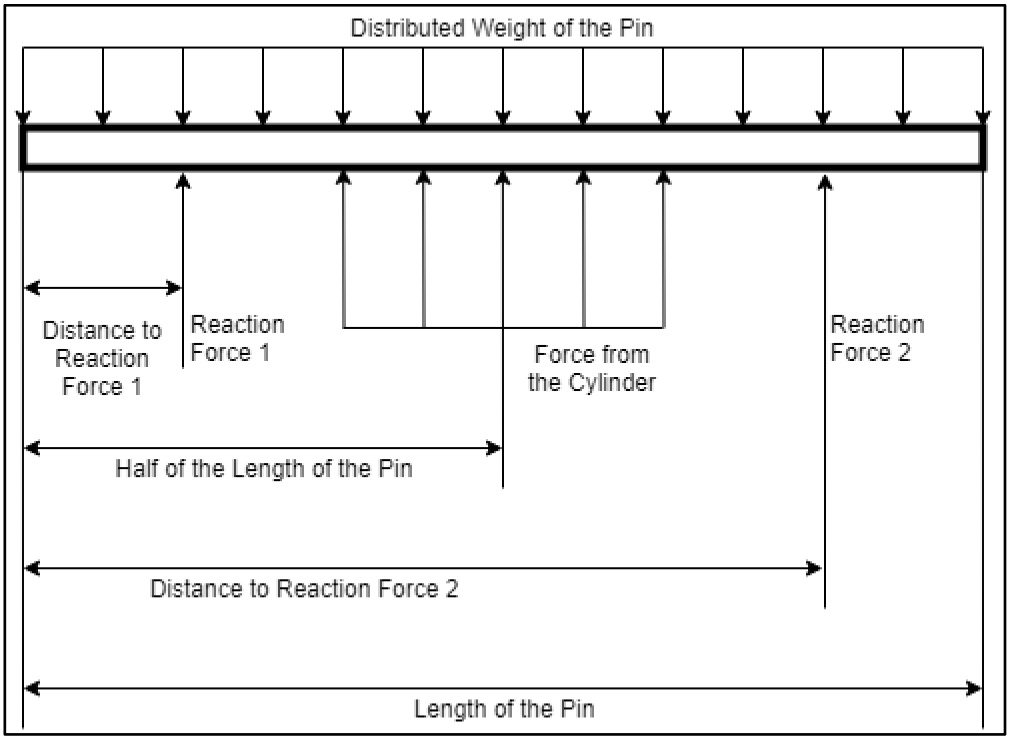


Figure 15: Free Body Diagram of Point B.

To determine the loading and moment functions for each pin, we used Mathcad by applying the method of singularity functions. As expected, the location of the highest moment is the midpoint of the pin. The details of this code are in Appendix F: Stress Analysis for Points A and B, respectively. Additionally, the key data and measurements used are in the table below.

Table 14: Key Data for Stress Analyses

Point	Material	Diameter	Length	Force
A	Galvanized Steel	6.35 mm	17.780 mm	1249.398 N
B	Galvanized Steel	6.35 mm	17.526 mm	1249.398 N

The results of these analyses are in the table below. The safety factor was calculated for one million cycles by defining a function based upon the number of cycles and several conditions, such as material and loading method. Additionally, the deflection at the midpoint was calculated to ensure that it does not interfere with the use and rotation of the pins.

Table 15: Results from Stress Analyses.

Point	Safety Factor (1,000,000 cycles)	Deflection at Midpoint
A	4.9	0.00098 mm
B	4.5	0.00025 mm

The safety factors for both Points A and B are well above the required value of 3 from the design parameters, as discussed in earlier sections, at 4.9 and 4.5, respectively. Additionally, the maximum deflection in each pin is well below 0.1% of the diameter of the pin, indicating that it will not obstruct the rotation of the pin. In conclusion, the device will withstand the forces it experiences for one million cycles.

Prototype Construction

Prototype assembly began with the construction of the prototype seesaw base, using a set of open sources plans found online (Kenny, 2018). We built this A-Frame base with basic construction lumber and a ten-foot galvanized steel pipe. All of which was easily purchased from the local big box hardware store. Using pipe clamps and a 2”x10” board, the axle attaches to the beam. The beam and axle assembly drop into the cutouts of the A-Frame base. The seesaw base was now complete and operational. A flange and pipe attaches to the sides to provide a mounting point for extra weights should the seesaw require it for stability during operation.

At this point in the construction, the team made the necessary modifications to install our braking device. The pneumatic piston braking assembly was attached to the seesaw beam at the previously calculated distance and then secured to a length of 2x4 beams connected to the seesaw A-Frame. This point is like grounding the cylinder in concrete or to an underground frame. The necessary adjustments were then made to fine tune the location and fit of the prototype braking module.

The attachment of the seating and activation mechanism came next. This assembly easily attaches using our custom adapter plate. At this time, a suitable handle was attached as well.

Finally, any necessary air fittings connect to the respectable locations and hosing runs along the system. For details regarding the connections and hosing runs see the figure below.

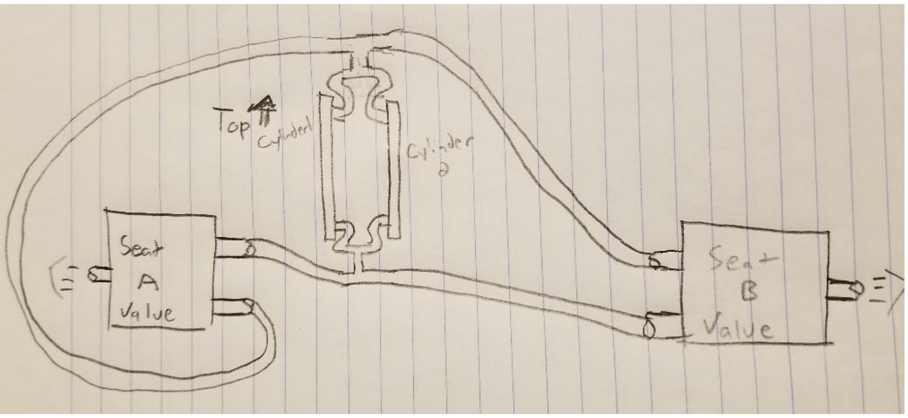


Figure 16: Hosing Diagram

Throughout the construction process, modularity was kept in mind. The team wanted the prototype to have ease of assembly and transportation. We accomplished this by making aspects of the prototype modular. The base, seesaw arm, pneumatic piston, braking module tower, and seat activation module are all individual assembly that are connected in one form or another. Takedown and setup is simple and takes only a few minutes.

Figure 17: View of Braking Mechanism

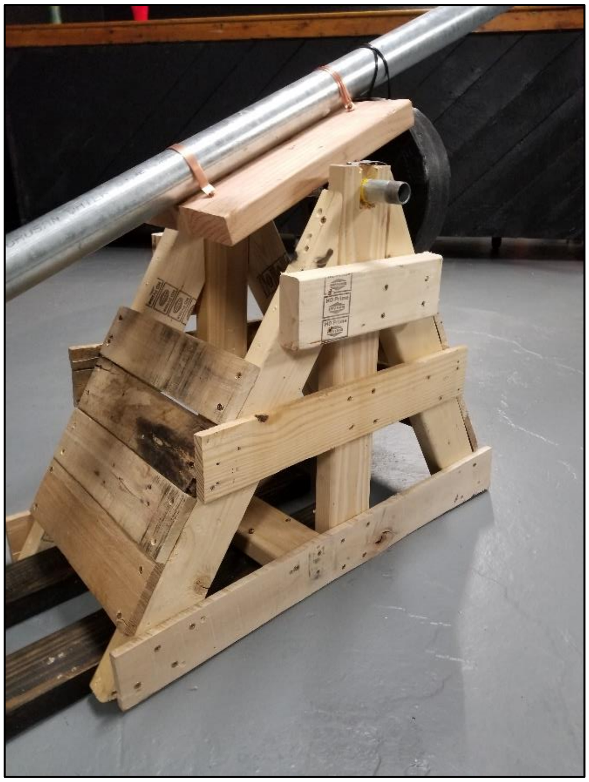


Figure 18: View of A-Frame Base

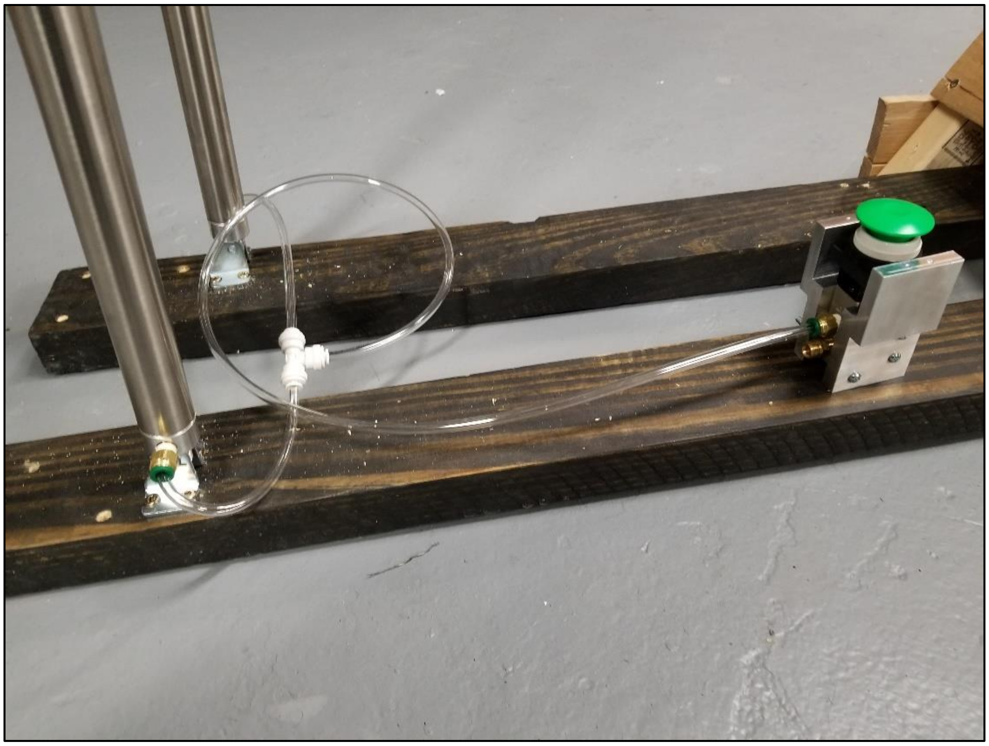


Figure 19: View of Valve and Fitting Testing

Prototype Testing

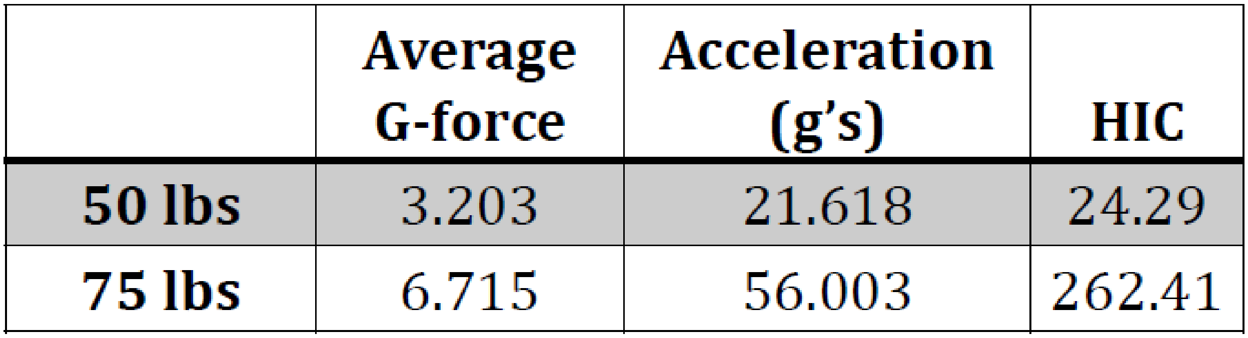
To properly test the device, the team revisited the performance specifications and developed a testing methodology that shows a pass or fail against those criteria.

Normal operation of the seesaw was tested cyclically to ensure that the seesaw did not fail or exhibit signs that the safety device was interfering with the seesaws standard up and down motion. This team operated the seesaw by hand repetitively until a predetermined count of runs occurred and the team was satisfied with the prototypes performance.

After we then tested the seesaws braking capabilities. Using a series of increasing weights, the seesaw was loaded with weight on both sides to simulate two children using the seesaw. We then removed one weight while the other caused that side of the seesaw to fall. The braking mechanism would then provide adequate braking force or not at this point depending on the weights used.

Accelerometer data was taken for various weights and multiple trials using a smartphone built in accelerometer via an app. This smartphone application takes data every millisecond for x, y, and z positions as well as g-force. Time of impact is also recorded and this data can be seen in Table 16: Results of Load Testing of our results. This data is used to determine the *g*'s experienced at the time of impact and is our main comparison showing whether the device is successful in braking the seesaw.

Table 16: Results of Load Testing.



	Average G-force	Acceleration (g's)	HIC
50 lbs	3.203	21.618	24.29
75 lbs	6.715	56.003	262.41

To ensure the viability of our pneumatic system, we performed a test prior to installation that guaranteed the function. The team assembled the pneumatic components separate from the safety device and pressure put on the cylinders to determine durability and reliability. The team wanted to ensure quality of the components before moving on with the construction as well as ensuring functionality of the system was exactly as the team had designed.

The team tested the size of container and weight together. We removed the device from the seesaw into its modular components and then placed into a box matching the packaging criteria of FedEx and UPS mentioned earlier in performance specifications. Standards for packaging filler is considered with this test. Once placed into the box it was then weighed to ensure it was under 50 kg.

We tested our devices tool requirement and ease of assembly simply by following those specifications in the construction of the prototype. Only tools which the end stakeholder would have readily available were used. The hardware is standard hardware readily available at a big box home improvement store, therefore ensuring no specialty tools are need during assembly of the prototype.

Discussion

When testing the seesaw device's ability to provide a braking force, the device performed as expected at weights below 50 pounds. The accelerometer data gathered showed that at a maximum weight of 50 pounds the g-forces experienced at the end of the seesaw where the user is located does not climb above 8.599, as shown in the graph below.

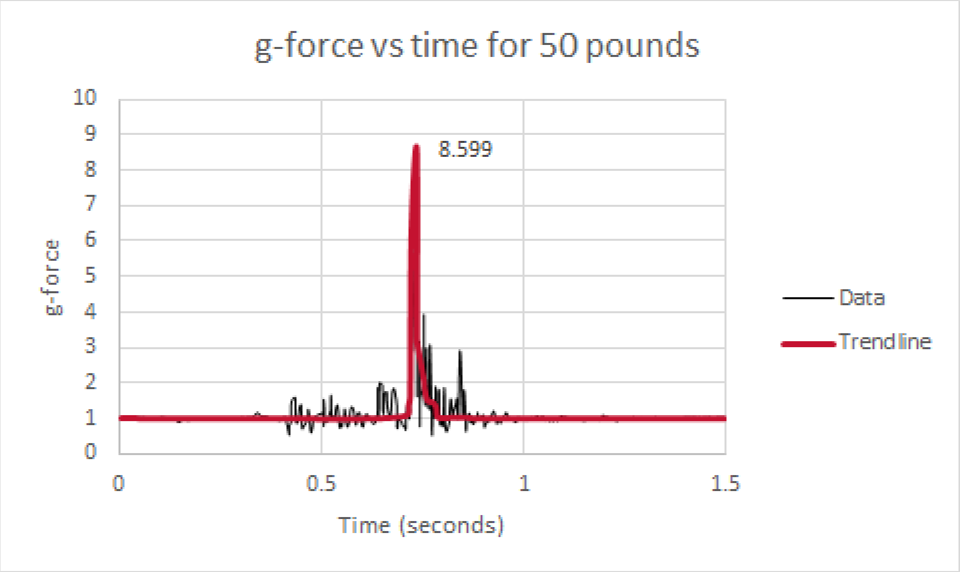


Figure 20: G-force vs Time graph of seesaw for load of 50 pounds.

Note that the time range of the impact is considerably small, less than half a second. The average g-force of 3.203 is calculated by averaging the g-force values within plus or minus 10 milliseconds of the maximum g-force value. Using this information an HIC value of 24.29 was calculated using Equation 1: The Head Injury Criterion (Henn, 1998).This is well below the HIC value of 150, which was determined to be the max acceptable value as noted earlier.

During this tests trials the tail of the seesaw seat never hit the ground and measuring the seat spacing showed that an average of 5 inches of space exists during and after the braking process. The team determined that this was a successful result.

When testing the seesaws ability to provide a braking force at weights above 50 pounds our device gave unfavorable results. Our previous analysis had shown this to be the case and we expected our device to do so. The accelerometer data gathered showed that at 75 pounds the maximum g-force to be 11.249. With an average value within plus or minus 10 milliseconds to be 6.715. This result more than doubled with only the addition of another 25 pounds of weight

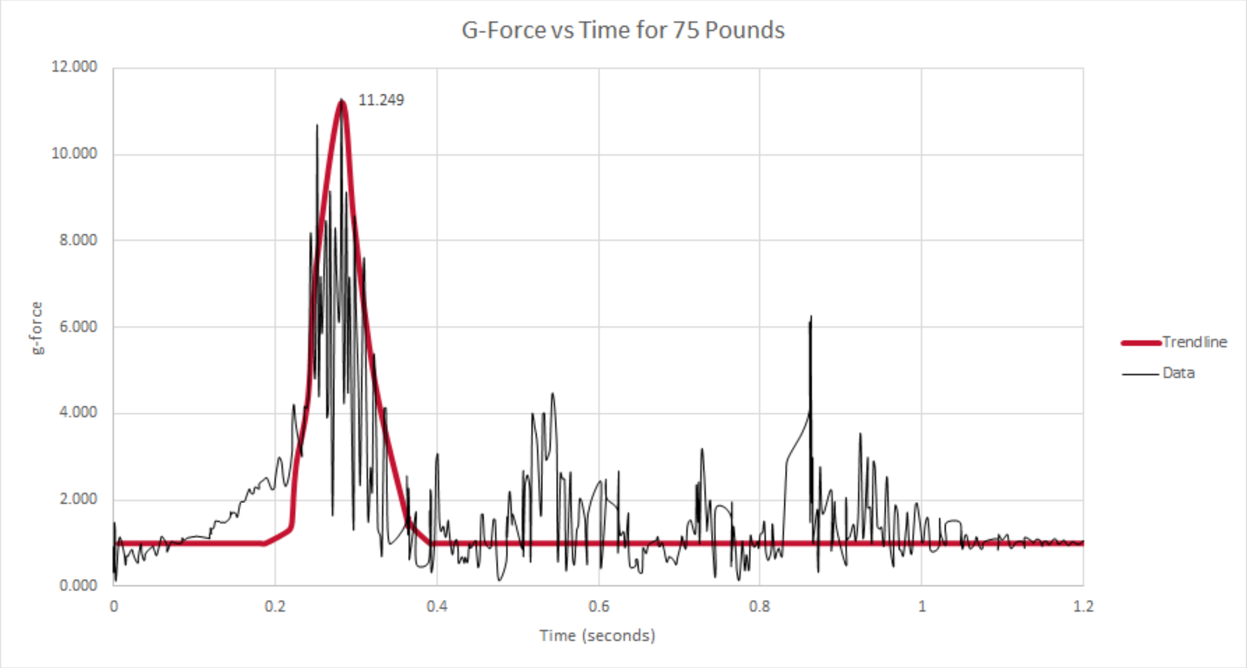


Figure 21: G-force vs Time graph of seesaw for load of 75 pounds.

The corresponding HIC value however increased tenfold to 262.41, which is well above the acceptable level of 150. At this high of an HIC head injury will occur and therefore this does not meet the requirements of our design.

This last result shows a roadblock we had with our design as a prototype. The device was limited due to the size of the cylinders chosen for the prototype. Reasonable braking after a certain user weight became unobtainable. If larger cylinders are chosen, this limitation goes away and our device can meet the minimum weight specification.

During testing of the pneumatic cylinders, we had hoses that blew out of some of the compression fittings. An alternate fitting was chosen for the locations in the pneumatic system to remedy this issue. No further issues occurred with the pneumatic system after this.

When testing a load heavier than 75 pounds, one of the piston rods had a critical failure. The piston arm succumbed to a bending moment. This was due to the clevis pin joint at the top of the cylinder being off the needed rotational axis. This caused the cylinder arm to lock up and then bend. We addressed this issue by tightening the fitting and ensuring that the eyebolt used in this joint could not rotate out of parallel axes. This brought up a valid concern that the team had not thought about beforehand. Even with larger cylinders what would the upper weight limitations of the device be? If for any reason an individual whose weight was greater than the max weight of the device got on the seesaw, what damage would result.

Conclusion & Recommendations

As discussed above, the goal of this project is to create a device that will improve the safety of playground seesaws by reducing the rapid downward acceleration experienced when one child dismounts the seesaw. The device was designed to support children from ages 5 to 12 and reduce the discomfort experienced to tolerable levels.

The findings above show that the device performs sufficiently as a “proof-of-concept” model. That is, it performs as expected, and serves as evidence that the concept has potential in becoming a fully functional model for commercial use. The prototype constructed was not intended to support high loads, as it was expected that stronger pneumatic cylinders would support this, rather than those used. For the lower load of 50 pounds, it slowed the child well below the comfortable HIC threshold of 150 to a value of approximately 24, which ensures that the child will not experience injury. However, to improve the model, we developed several recommendations.

While the prototype was built and performed successfully, there are several points of improvement that can be made to further the utility and performance of the device.

1. *Increase the diameter of the piston cylinders.* This will increase the pressure in the pneumatic cylinders, which will result in a greater braking force and the ability to support heavier loads during a braking operation.
2. *Development of devices for other models of seesaws.* We can reach a broader stakeholder audience by developing this prototype to function with other models of seesaws.
3. *Application of a visually appealing protective shroud over the braking mechanism.* This will serve two purposes. The first is for safety. It will provide an enclosure of the braking mechanism and prevent injury due to the normal movements of the various components of the system. Secondly, it is a good platform to dress up the seesaw to make it more visually appealing to children, i.e. disguise it as a dragon or other fantasy creature, and provides appeal to the playground as a whole.
4. *Develop the prototype into a more manufacturable design.* As the prototype stands, it serves the purpose of proving that the concept works. However, we can improve the design by making custom components rather than the store bought components that our budget allowed. By making a custom in house design the device can be manufactured easier and with stakeholder use in mind, i.e. ensuring that it can withstand environmental conditions.
5. *Reduction of device cost.* One of our functional requirements was to keep the device cost effective. We determined that a value of $60 with a 15% markup was acceptable based upon the existing cost of seesaws available on the market and our perception of what we believed a user would be willing to pay for the device. However, we were not

able to meet this as can be seen in our budget. Finding a way to reduce cost would greatly increase the value and potential popularity of this device.

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Appendices

Appendix A: Anthropometric Tables

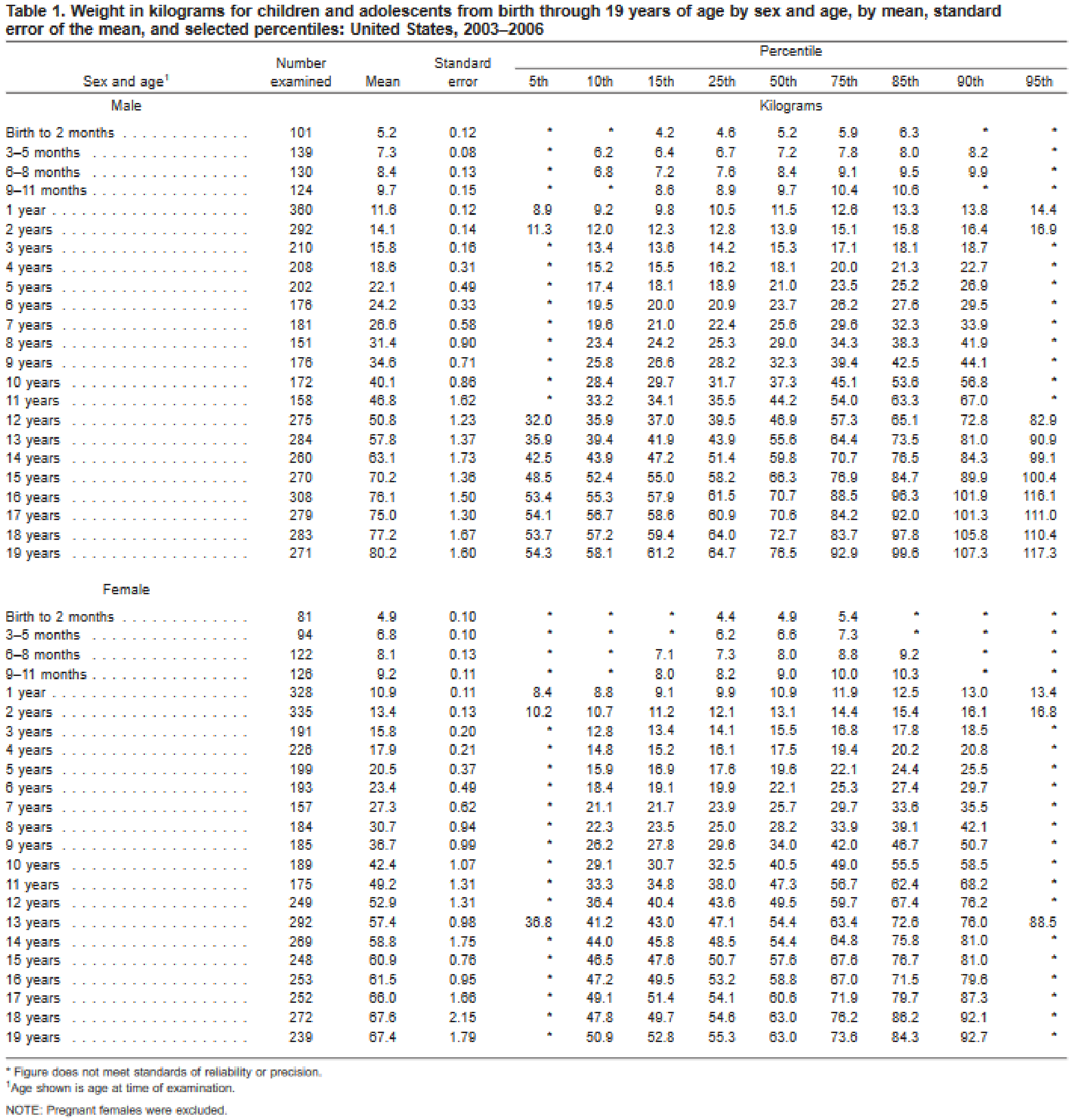


Table 1. Weight in kilograms for children and adolescents from birth through 19 years of age by sex and age, by mean, standard error of the mean, and selected percentiles: United States, 2003–2006

				Percentile								
Sex and age[1]	Number examined	Mean	Standard error	5th	10th	15th	25th	50th	75th	85th	90th	95th
Male								Kilograms				
Birth to 2 months	101	5.2	0.12	*	*	4.2	4.6	5.2	5.9	6.3	*	*
3–5 months	139	7.3	0.08	*	6.2	6.4	6.7	7.2	7.8	8.0	8.2	*
6–8 months	130	8.4	0.13	*	6.8	7.2	7.6	8.4	9.1	9.5	9.9	*
9–11 months	124	9.7	0.15	*	*	8.6	8.9	9.7	10.4	10.6	*	*
1 year	360	11.6	0.12	8.9	9.2	9.8	10.5	11.5	12.6	13.3	13.8	14.4
2 years	292	14.1	0.14	11.3	12.0	12.3	12.8	13.9	15.1	15.8	16.4	16.9
3 years	210	15.8	0.16	*	13.4	13.6	14.2	15.3	17.1	18.1	18.7	*
4 years	208	18.6	0.31	*	15.2	15.5	16.2	18.1	20.0	21.3	22.7	*
5 years	202	22.1	0.49	*	17.4	18.1	18.9	21.0	23.5	25.2	26.9	*
6 years	176	24.2	0.33	*	19.5	20.0	20.9	23.7	26.2	27.6	29.5	*
7 years	181	26.6	0.58	*	19.6	21.0	22.4	25.6	29.6	32.3	33.9	*
8 years	151	31.4	0.90	*	23.4	24.2	25.3	29.0	34.3	38.3	41.9	*
9 years	176	34.6	0.71	*	25.8	26.6	28.2	32.3	39.4	42.5	44.1	*
10 years	172	40.1	0.86	*	28.4	29.7	31.7	37.3	45.1	53.6	56.8	*
11 years	158	46.8	1.62	*	33.2	34.1	35.5	44.2	54.0	63.3	67.0	*
12 years	275	50.8	1.23	32.0	35.9	37.0	39.5	46.9	57.3	65.1	72.8	82.9
13 years	284	57.8	1.37	35.9	39.4	41.9	43.9	55.6	64.4	73.5	81.0	90.9
14 years	260	63.1	1.73	42.5	43.9	47.2	51.4	59.8	70.7	76.5	84.3	99.1
15 years	270	70.2	1.36	48.5	52.4	55.0	58.2	66.3	76.9	84.7	89.9	100.4
16 years	308	76.1	1.50	53.4	55.3	57.9	61.5	70.7	88.5	96.3	101.9	116.1
17 years	279	75.0	1.30	54.1	56.7	58.6	60.9	70.6	84.2	92.0	101.3	111.0
18 years	283	77.2	1.67	53.7	57.2	59.4	64.0	72.7	83.7	97.8	105.8	110.4
19 years	271	80.2	1.60	54.3	58.1	61.2	64.7	76.5	92.9	99.6	107.3	117.3
Female												
Birth to 2 months	81	4.9	0.10	*	*	*	4.4	4.9	5.4	*	*	*
3–5 months	94	6.8	0.10	*	*	*	6.2	6.6	7.3	*	*	*
6–8 months	122	8.1	0.13	*	*	7.1	7.3	8.0	8.8	9.2	*	*
9–11 months	126	9.2	0.11	*	*	8.0	8.2	9.0	10.0	10.3	*	*
1 year	328	10.9	0.11	8.4	8.8	9.1	9.9	10.9	11.9	12.5	13.0	13.4
2 years	335	13.4	0.13	10.2	10.7	11.2	12.1	13.1	14.4	15.4	16.1	16.8
3 years	191	15.8	0.20	*	12.8	13.4	14.1	15.5	16.8	17.8	18.5	*
4 years	226	17.9	0.21	*	14.8	15.2	16.1	17.5	19.4	20.2	20.8	*
5 years	199	20.5	0.37	*	15.9	16.9	17.6	19.6	22.1	24.4	25.5	*
6 years	193	23.4	0.49	*	18.4	19.1	19.9	22.1	25.3	27.4	29.7	*
7 years	157	27.3	0.62	*	21.1	21.7	23.9	25.7	29.7	33.6	35.5	*
8 years	184	30.7	0.94	*	22.3	23.5	25.0	28.2	33.9	39.1	42.1	*
9 years	185	36.7	0.99	*	26.2	27.8	29.6	34.0	42.0	46.7	50.7	*
10 years	189	42.4	1.07	*	29.1	30.7	32.5	40.5	49.0	55.5	58.5	*
11 years	175	49.2	1.31	*	33.3	34.8	38.0	47.3	56.7	62.4	68.2	*
12 years	249	52.9	1.31	*	36.4	40.4	43.6	49.5	59.7	67.4	76.2	*
13 years	292	57.4	0.98	36.8	41.2	43.0	47.1	54.4	63.4	72.6	76.0	88.5
14 years	269	58.8	1.75	*	44.0	45.8	48.5	54.4	64.8	75.8	81.0	*
15 years	248	60.9	0.76	*	46.5	47.6	50.7	57.6	67.6	76.7	81.0	*
16 years	253	61.5	0.95	*	47.2	49.5	53.2	58.8	67.0	71.5	79.6	*
17 years	252	66.0	1.66	*	49.1	51.4	54.1	60.6	71.9	79.7	87.3	*
18 years	272	67.6	2.15	*	47.8	49.7	54.6	63.0	76.2	86.2	92.1	*
19 years	239	67.4	1.79	*	50.9	52.8	55.3	63.0	73.6	84.3	92.7	*

* Figure does not meet standards of reliability or precision.

[1] Age shown is age at time of examination.

NOTE: Pregnant females were excluded.

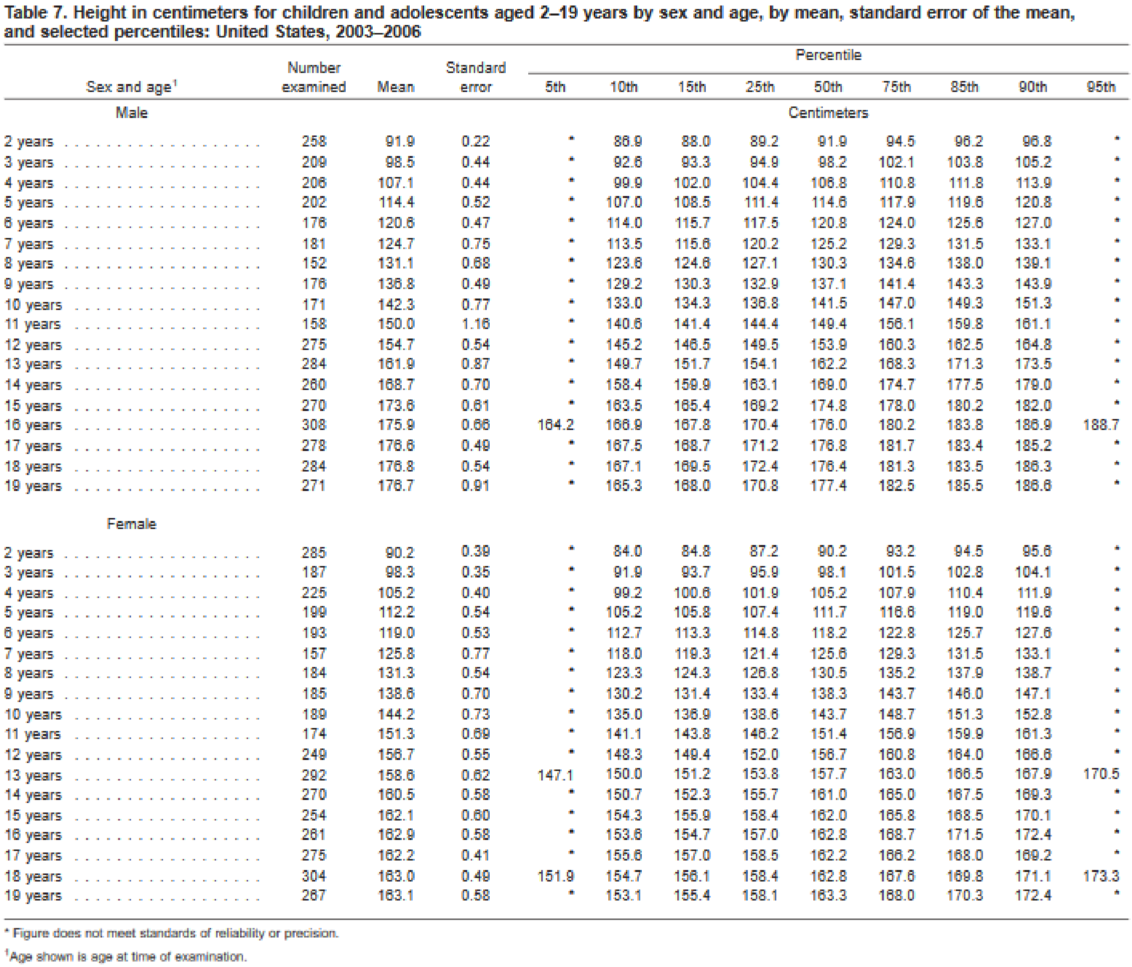


Table 7. Height in centimeters for children and adolescents aged 2–19 years by sex and age, by mean, standard error of the mean, and selected percentiles: United States, 2003–2006

Sex and age[1]	Number examined	Mean	Standard error	Percentile								
				5th	10th	15th	25th	50th	75th	85th	90th	95th
Male				Centimeters								
2 years	258	91.9	0.22	*	86.9	88.0	89.2	91.9	94.5	96.2	96.8	*
3 years	209	98.5	0.44	*	92.6	93.3	94.9	98.2	102.1	103.8	105.2	*
4 years	206	107.1	0.44	*	99.9	102.0	104.4	106.8	110.8	111.8	113.9	*
5 years	202	114.4	0.52	*	107.0	108.5	111.4	114.6	117.9	119.6	120.8	*
6 years	176	120.6	0.47	*	114.0	115.7	117.5	120.8	124.0	125.6	127.0	*
7 years	181	124.7	0.75	*	113.5	115.6	120.2	125.2	129.3	131.5	133.1	*
8 years	152	131.1	0.68	*	123.6	124.6	127.1	130.3	134.6	138.0	139.1	*
9 years	176	136.8	0.49	*	129.2	130.3	132.9	137.1	141.4	143.3	143.9	*
10 years	171	142.3	0.77	*	133.0	134.3	136.8	141.5	147.0	149.3	151.3	*
11 years	158	150.0	1.16	*	140.6	141.4	144.4	149.4	156.1	159.8	161.1	*
12 years	275	154.7	0.54	*	145.2	146.5	149.5	153.9	160.3	162.5	164.8	*
13 years	284	161.9	0.87	*	149.7	151.7	154.1	162.2	168.3	171.3	173.5	*
14 years	260	168.7	0.70	*	158.4	159.9	163.1	169.0	174.7	177.5	179.0	*
15 years	270	173.6	0.61	*	163.5	165.4	169.2	174.8	178.0	180.2	182.0	*
16 years	308	175.9	0.66	164.2	166.9	167.8	170.4	176.0	180.2	183.8	186.9	188.7
17 years	278	176.6	0.49	*	167.5	168.7	171.2	176.8	181.7	183.4	185.2	*
18 years	284	176.8	0.54	*	167.1	169.5	172.4	176.4	181.3	183.5	186.3	*
19 years	271	176.7	0.91	*	165.3	168.0	170.8	177.4	182.5	185.5	186.6	*
Female												
2 years	285	90.2	0.39	*	84.0	84.8	87.2	90.2	93.2	94.5	95.6	*
3 years	187	98.3	0.35	*	91.9	93.7	95.9	98.1	101.5	102.8	104.1	*
4 years	225	105.2	0.40	*	99.2	100.6	101.9	105.2	107.9	110.4	111.9	*
5 years	199	112.2	0.54	*	105.2	105.8	107.4	111.7	116.6	119.0	119.6	*
6 years	193	119.0	0.53	*	112.7	113.3	114.8	118.2	122.8	125.7	127.6	*
7 years	157	125.8	0.77	*	118.0	119.3	121.4	125.6	129.3	131.5	133.1	*
8 years	184	131.3	0.54	*	123.3	124.3	126.8	130.5	135.2	137.9	138.7	*
9 years	185	138.6	0.70	*	130.2	131.4	133.4	138.3	143.7	146.0	147.1	*
10 years	189	144.2	0.73	*	135.0	136.9	138.6	143.7	148.7	151.3	152.8	*
11 years	174	151.3	0.69	*	141.1	143.8	146.2	151.4	156.9	159.9	161.3	*
12 years	249	156.7	0.55	*	148.3	149.4	152.0	156.7	160.8	164.0	166.6	*
13 years	292	158.6	0.62	147.1	150.0	151.2	153.8	157.7	163.0	166.5	167.9	170.5
14 years	270	160.5	0.58	*	150.7	152.3	155.7	161.0	165.0	167.5	169.3	*
15 years	254	162.1	0.60	*	154.3	155.9	158.4	162.0	165.8	168.5	170.1	*
16 years	261	162.9	0.58	*	153.6	154.7	157.0	162.8	168.7	171.5	172.4	*
17 years	275	162.2	0.41	*	155.6	157.0	158.5	162.2	166.2	168.0	169.2	*
18 years	304	163.0	0.49	151.9	154.7	156.1	158.4	162.8	167.6	169.8	171.1	173.3
19 years	267	163.1	0.58	*	153.1	155.4	158.1	163.3	168.0	170.3	172.4	*

* Figure does not meet standards of reliability or precision.

[1] Age shown is age at time of examination.

Appendix B: Bill of Materials

Description	**Vendor**	**Item#**	**Quantity**
Pivot Bracket with Pin for 3/4" & 1-1/16" Bore Round Body Air Cylinder	McMaster-Carr	6498K72	2
Round Body Air Cylinder: Double-Acting, Universal Mount, 1-1/16" Bore, 28" Stroke	McMaster-Carr	6498K415	2
Rubber-Cushioned U-Bolt: Galvanized Steel, 3/8"-16 Thread Size, 2-3/8" ID	McMaster-Carr	30555T34	1
Rod Clevis with Pin for 1-1/16" Bore Size Round Body Air Cylinder	McMaster-Carr	6498K43	2
Corrosion-Resistant Fully Threaded Rod End Bolt: 1/4"-20 Shank Thread, 3-1/2" Shank Center Length	McMaster-Carr	2434K35	2
Pinless Surface-Mount Hinge with Holes: Polyolefin Plastic, 1-1/2" x 1/2" Door Leaf	McMaster-Carr	1637A71	1
Nylon Plug, 1/8" Stem OD for Brass Push-to-Connect Tube Fitting for Air	McMaster-Carr	51025K651	3
Brass Push-to-Connect Tube Fitting for Air: Straight Adapter, for 1/4" Tube OD x 1/8 NPTF Male	McMaster-Carr	51025K177	4
Brass Push-to-Connect Tube Fitting for Air: Straight Adapter, for 1/8" Tube OD x 1/8 NPTF Male	McMaster-Carr	51025K171	5
Compression Spring: Zinc-Plated, Tempered, Closed and Flat Ends, 2.5" Long, 1" OD	McMaster-Carr	9657K32	1
SMC TIUB07C-20 tubing, polyurethane, TIU POLYURETHANE TUBING	SMC Pneumatics	TIUB07C-20	1
NVM430-N01-30G VALVE, MECH L 1/8 NPT (GREEN)	SMC Pneumatics	NVM430-N01-30G	1
Seesaw Lumber	Home Depot		
Seesaw Arm - 10' 2" Dia Galvanized Steel Pipe	Home Depot		1
Custom Aluminum Brackets	Washburn Shops		2

Appendix C: Budget

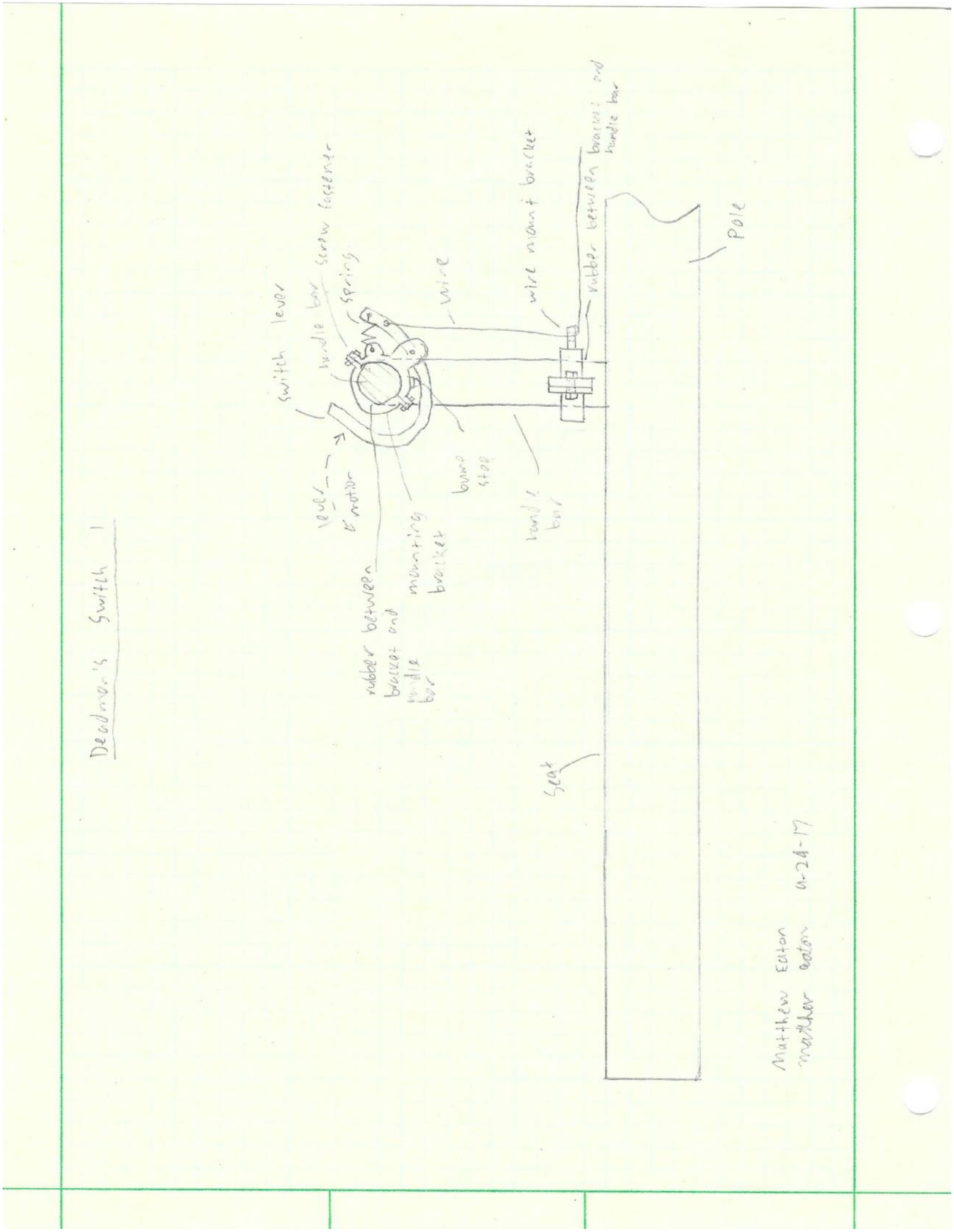
MQP Name:	***Seesaw Safety Device***
Advisor:	**Cobb**

ME Budget:	**750.00**
70%	**525.00**

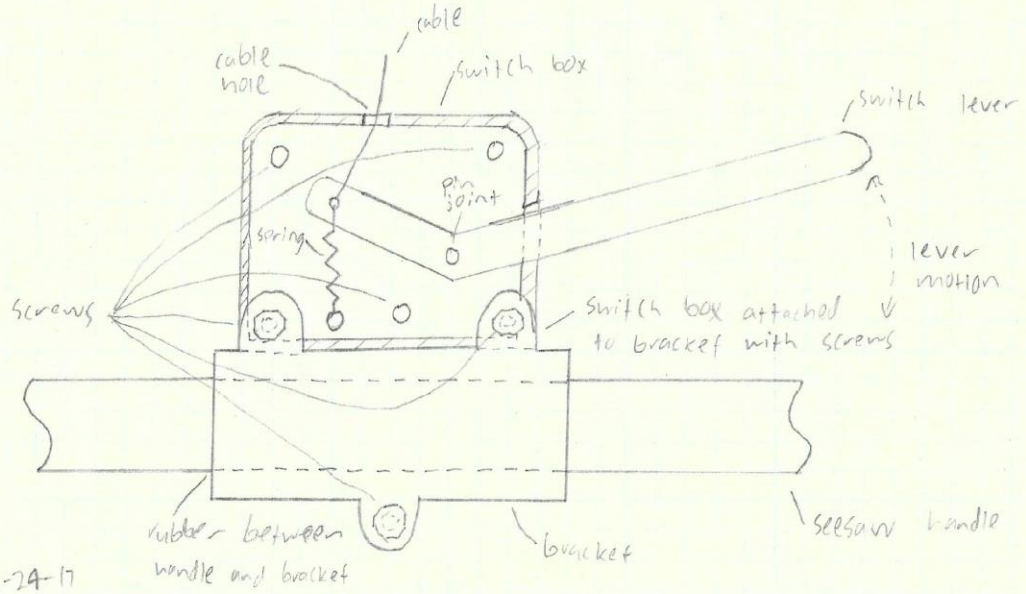
Students
Samuel Bickham ME
Matthew Garcia ME
Matthew Eaton ME

Date	**Name**	***Method***	***Materials***	***shipping***	***Expense***	***70% Balance***	***Full Budget Balance***
2/7/2018	**McMaster Carr**	***P0027963***	***253.77***	***7.70***	***261.47***	***263.53***	***488.53***
4/4/2018	**Matthew Garcia**	***exp report***	***211.44***	***n/a***	***211.44***	***52.09***	***277.09***
4/10/2018	**McMaster Carr**	***P0030731***	***91.42***	***8.14***	***99.56***	***-39.33***	***177.53***

Appendix D: Design Sketches

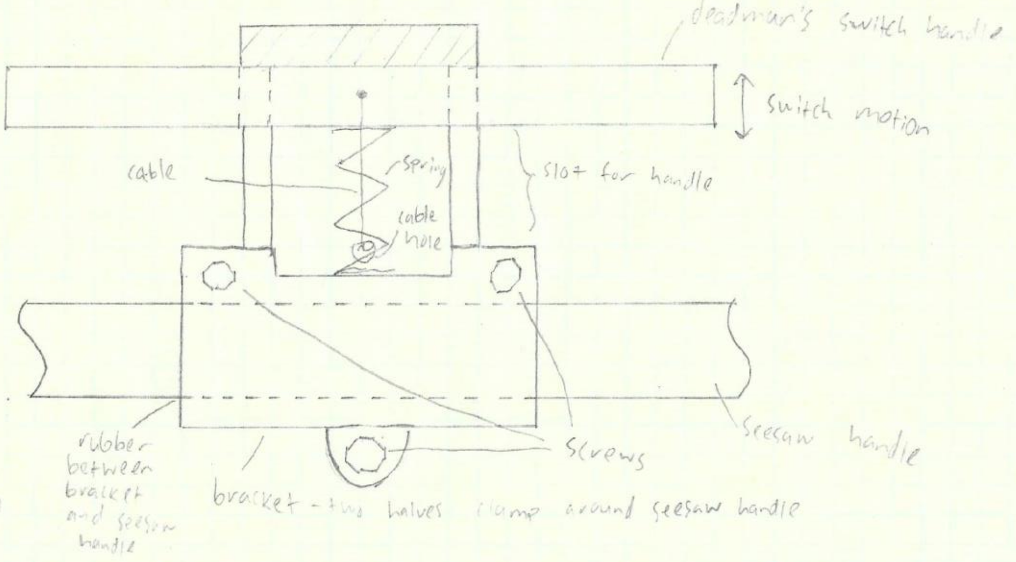


Deadman's switch 2



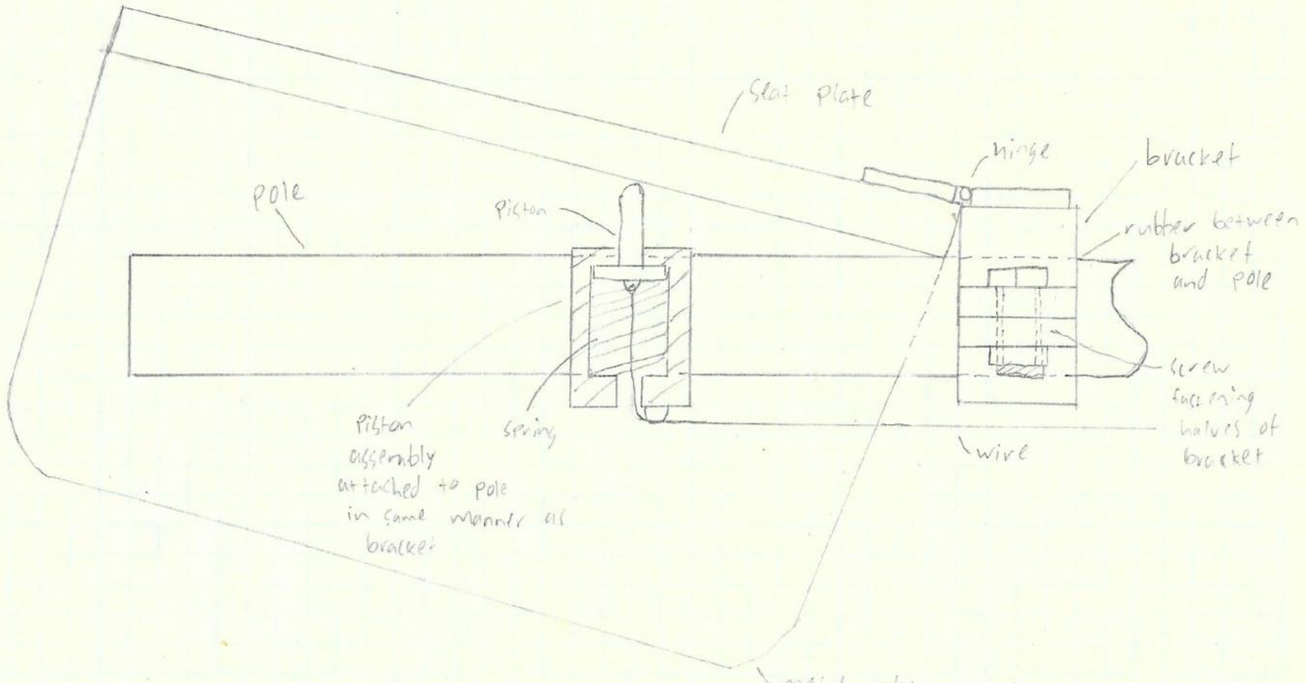
Matthew Eaton
Matthew Eaton 9-24-17

Deadman's switch 3



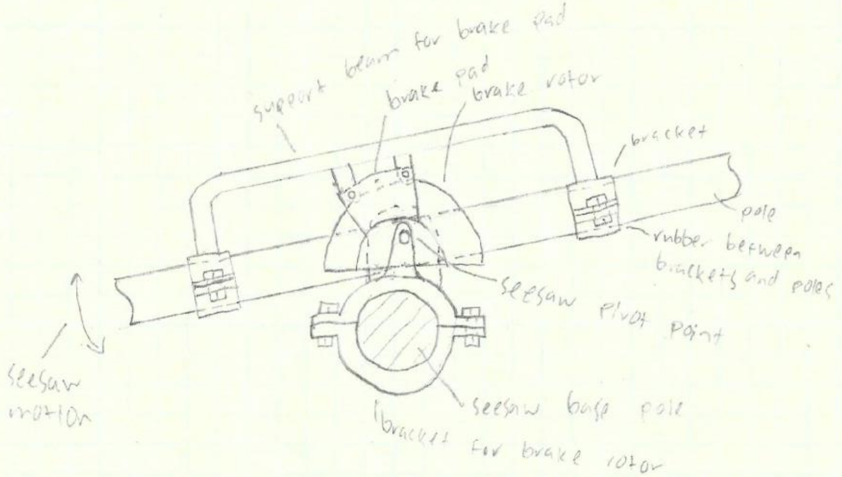
Matthew Eaton
Matthew Eaton 9-29-17

Dead Man's Seat



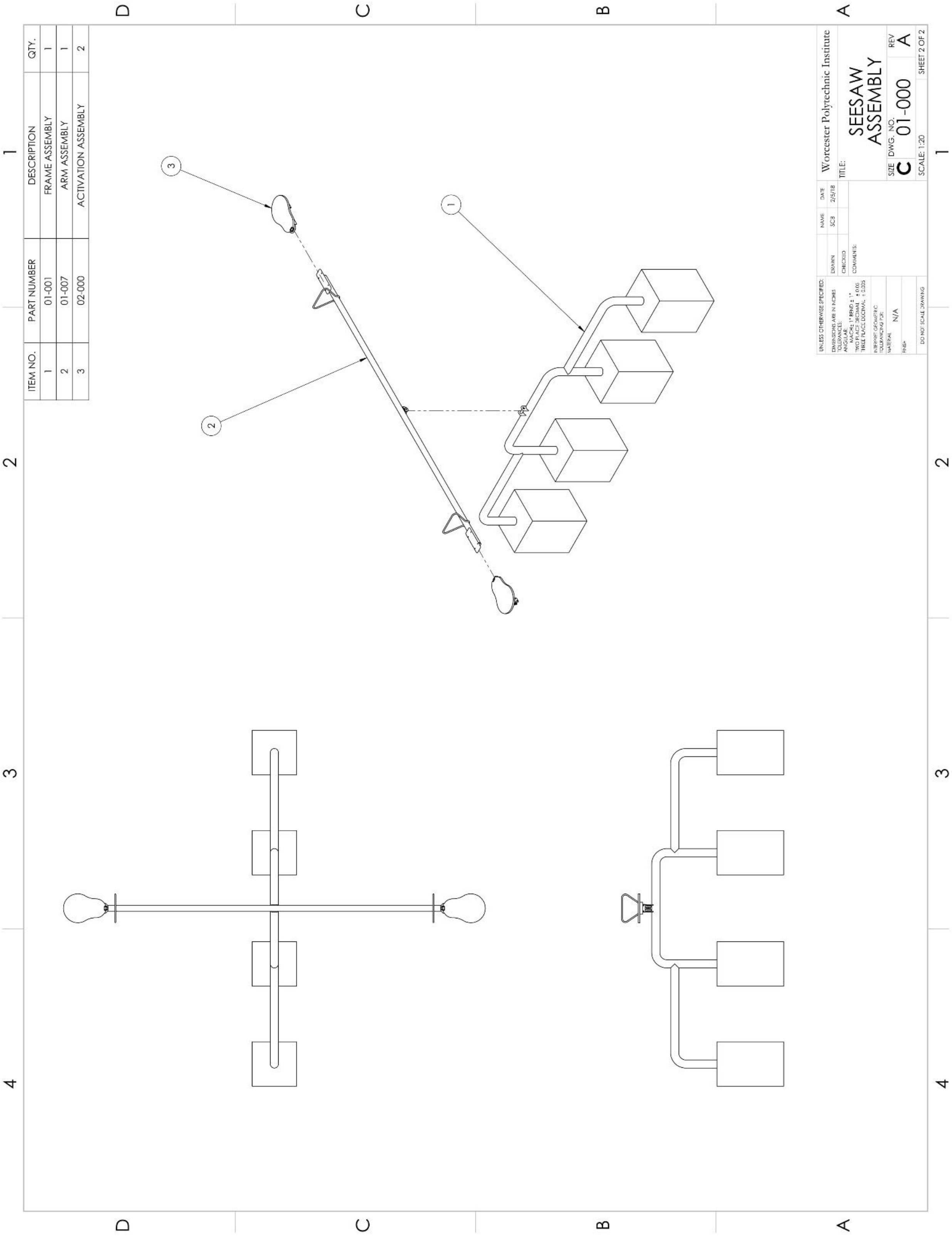
Matthew Eaton
Matthew Eaton 9-24-17

Disk Brake Stopping Device



Matthew Eaton
Matthew Eaton 9-25-17

Appendix E: Technical Drawings



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
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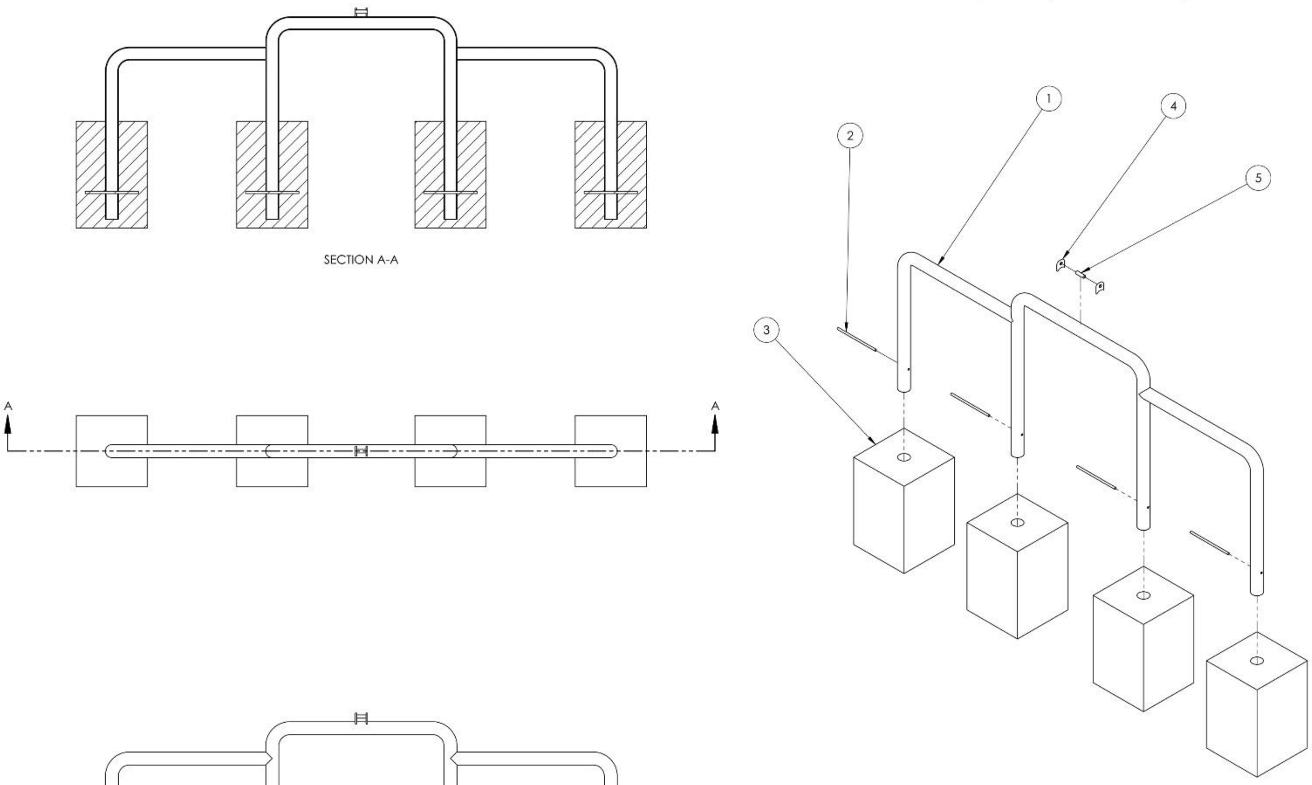
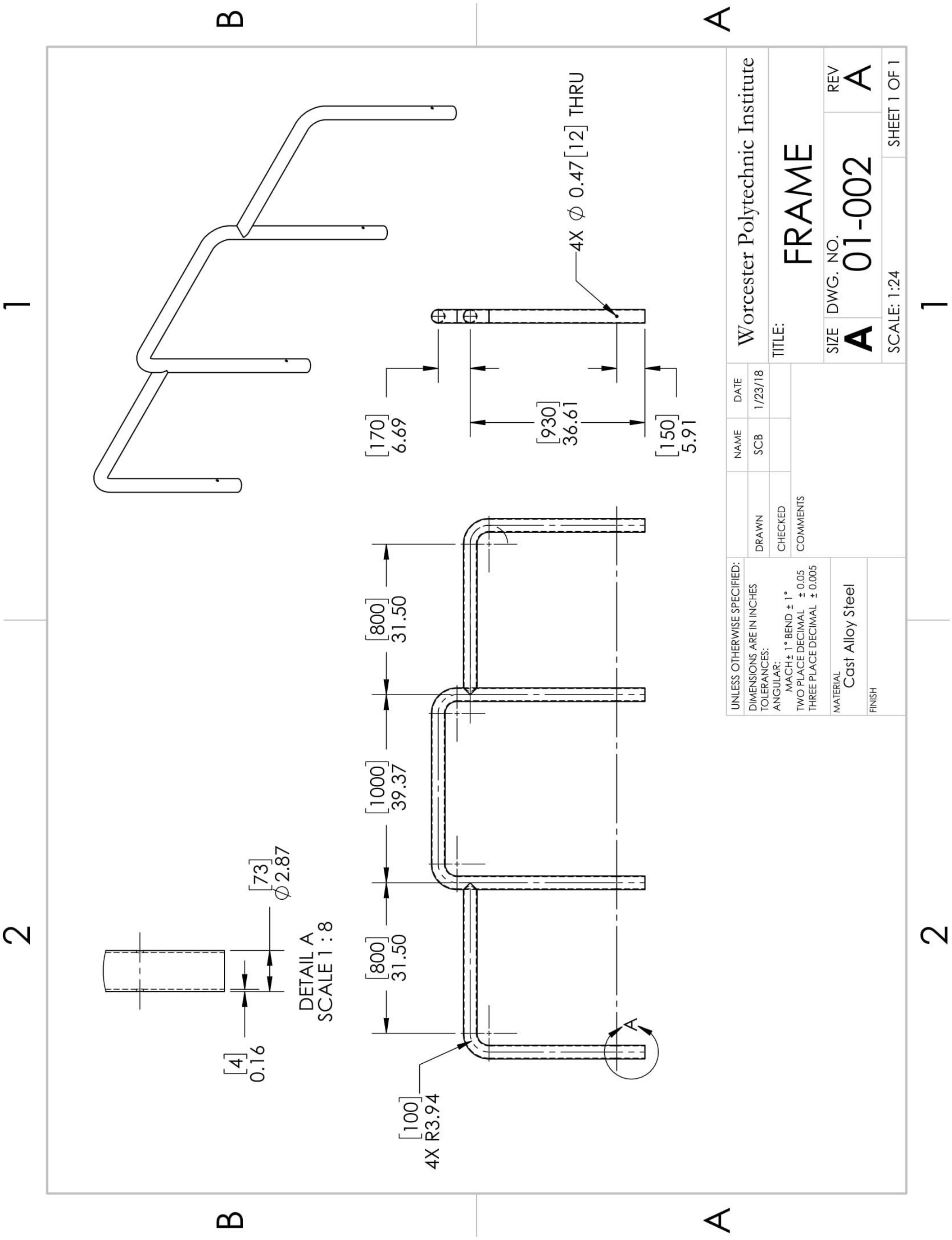
2	01-003	SUPPORT	2
3	01-004	AXLE	1
4	01-005	CONCRETE	4
5	01-006	BAR	4



UNLESS OTHERWISE SPECIFIED:		NAME	DATE	Worcester Polytechnic Institute
DIMENSIONS ARE IN INCHES TOLERANCES: ANGULAR: MACH ± 1° BEND ± 1° TWO PLACE DECIMAL ± 0.03 THREE PLACE DECIMAL ± 0.005	DRAWN	SCB	2/5/18	TITLE: FRAME ASSEMBLY
	CHECKED			
INTERPRET GEOMETRIC TOLERANCING PER:	COMMENTS:			
MATERIAL N/A				SIZE C DWG. NO. 01-001 REV A
FINISH				
DO NOT SCALE DRAWING				SCALE: 1:16 SHEET 2 OF 2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DETAIL A
SCALE 1 : 8
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
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FRAME
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DWG. NO.
01-002
REV
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1
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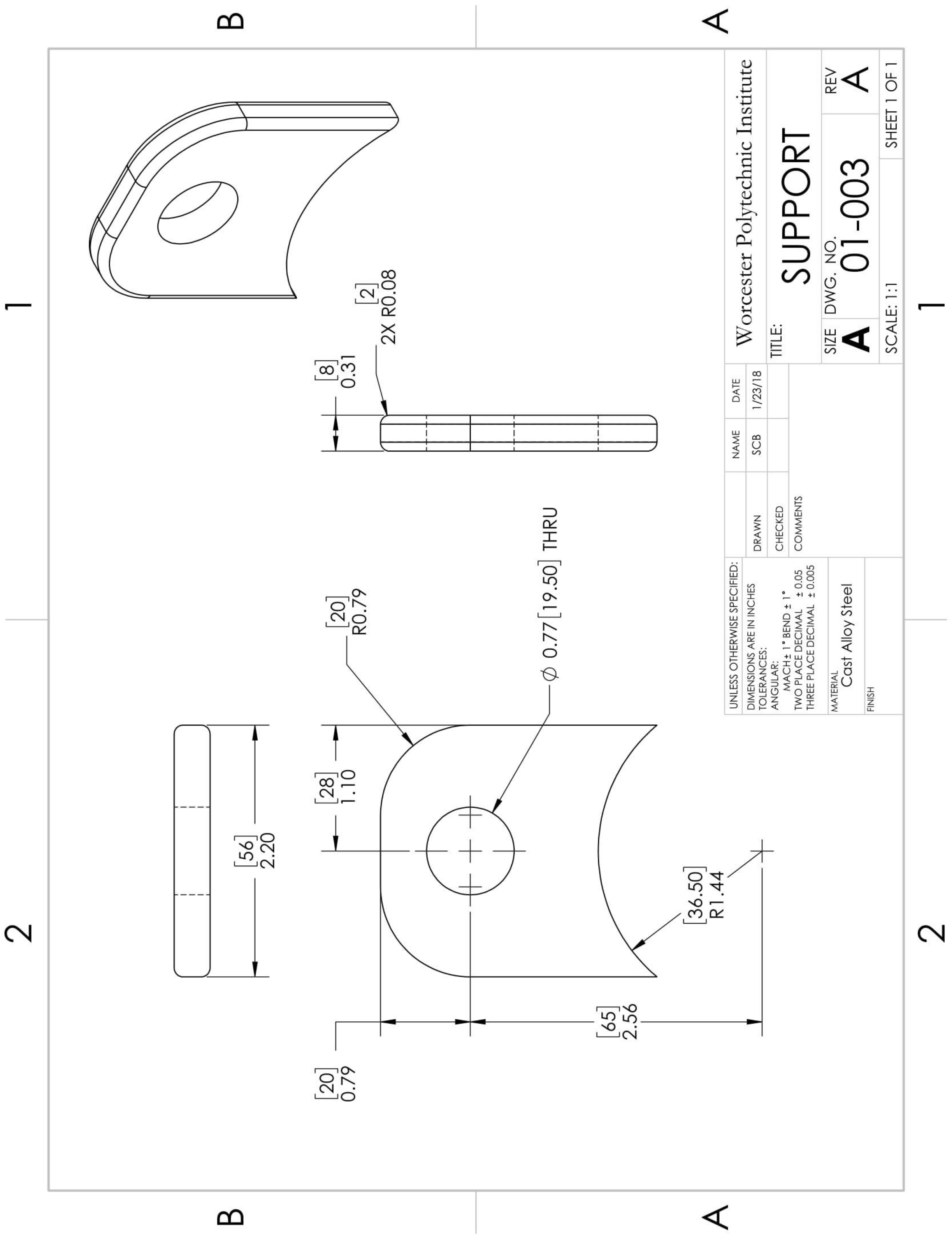
B



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⌀ 0.77 [19.50] THRU
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UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
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Cast Alloy Steel
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COMMENTS
Worcester Polytechnic Institute
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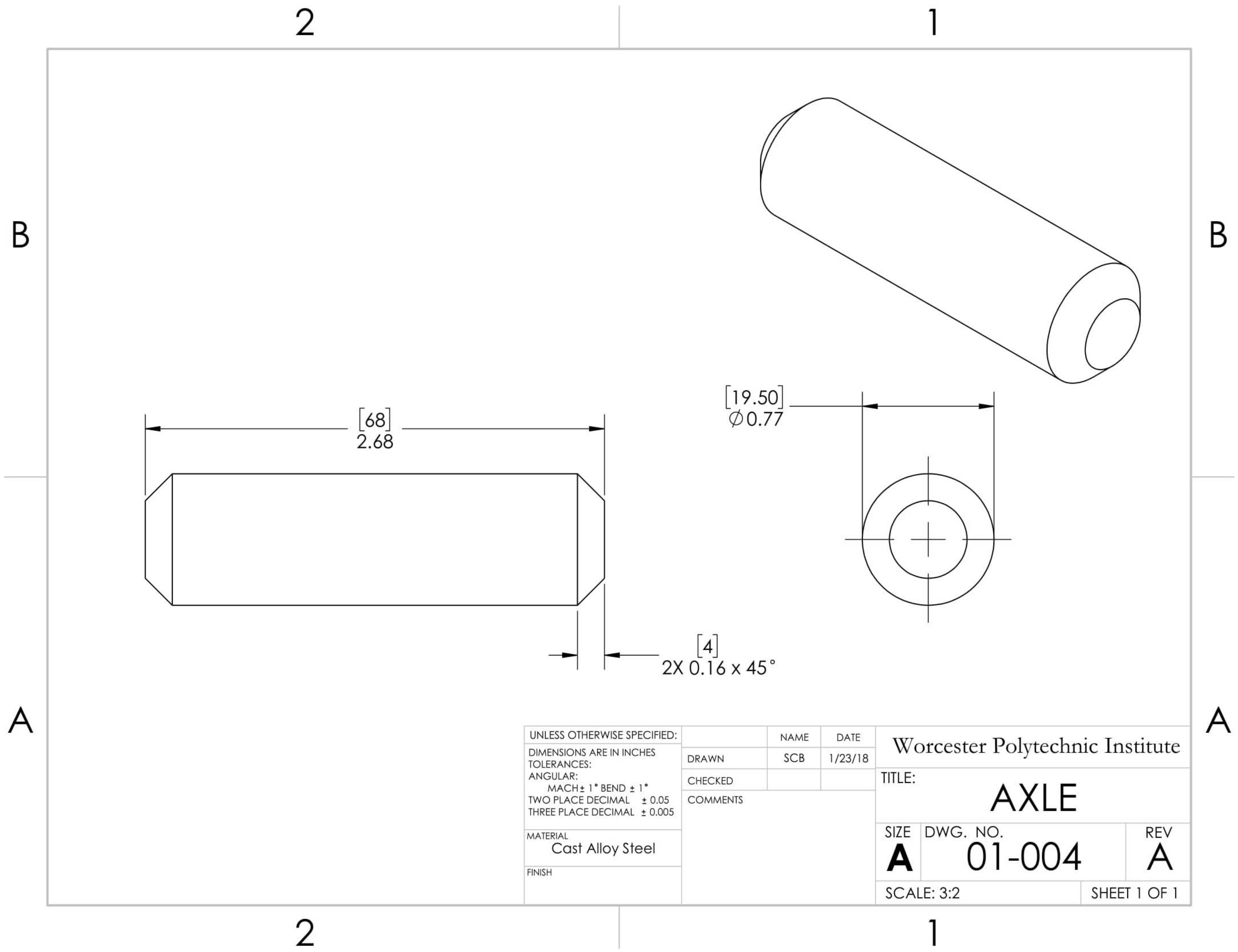
2
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DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
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DATE
DRAWN
SCB
1/23/18
CHECKED
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01-004
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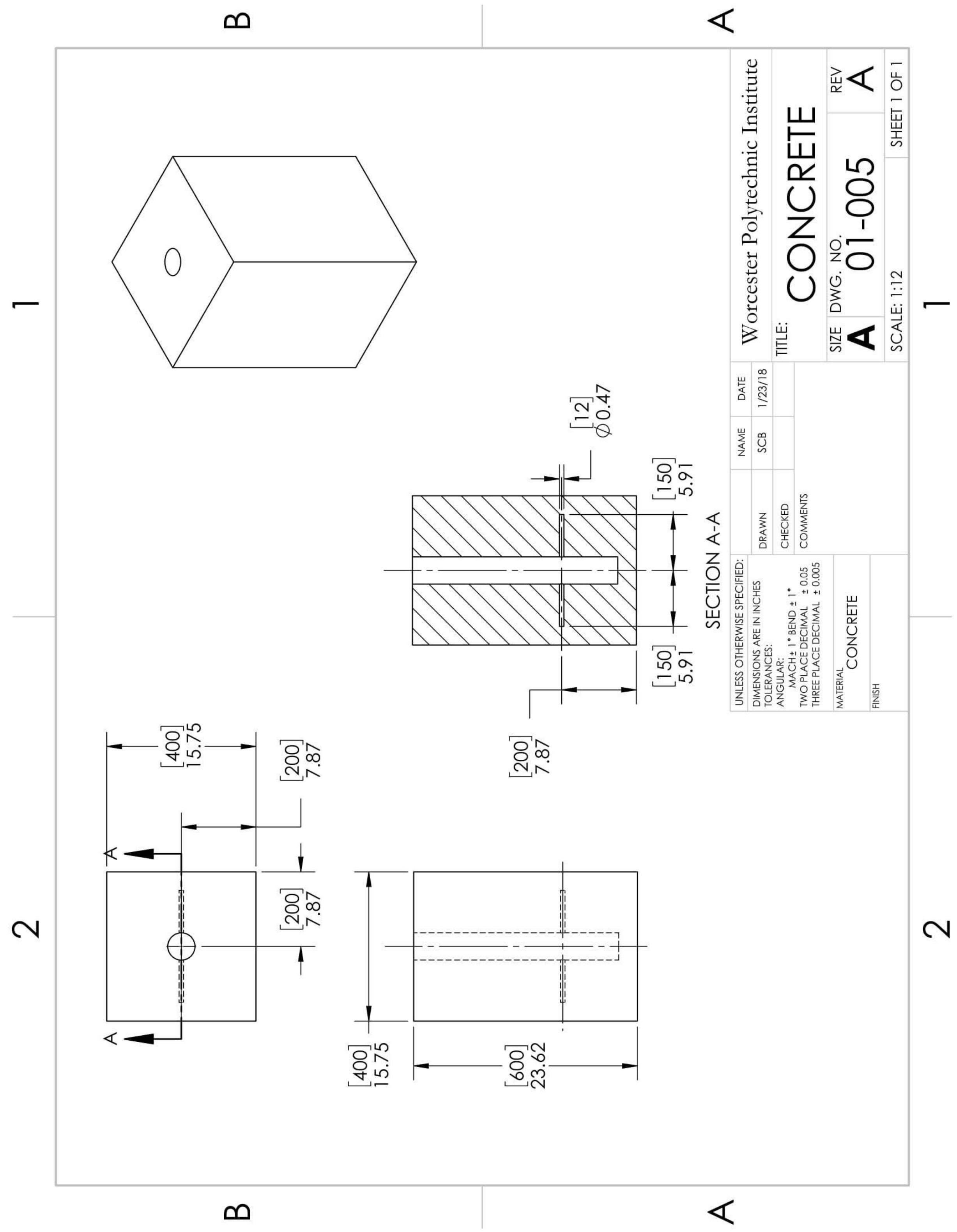
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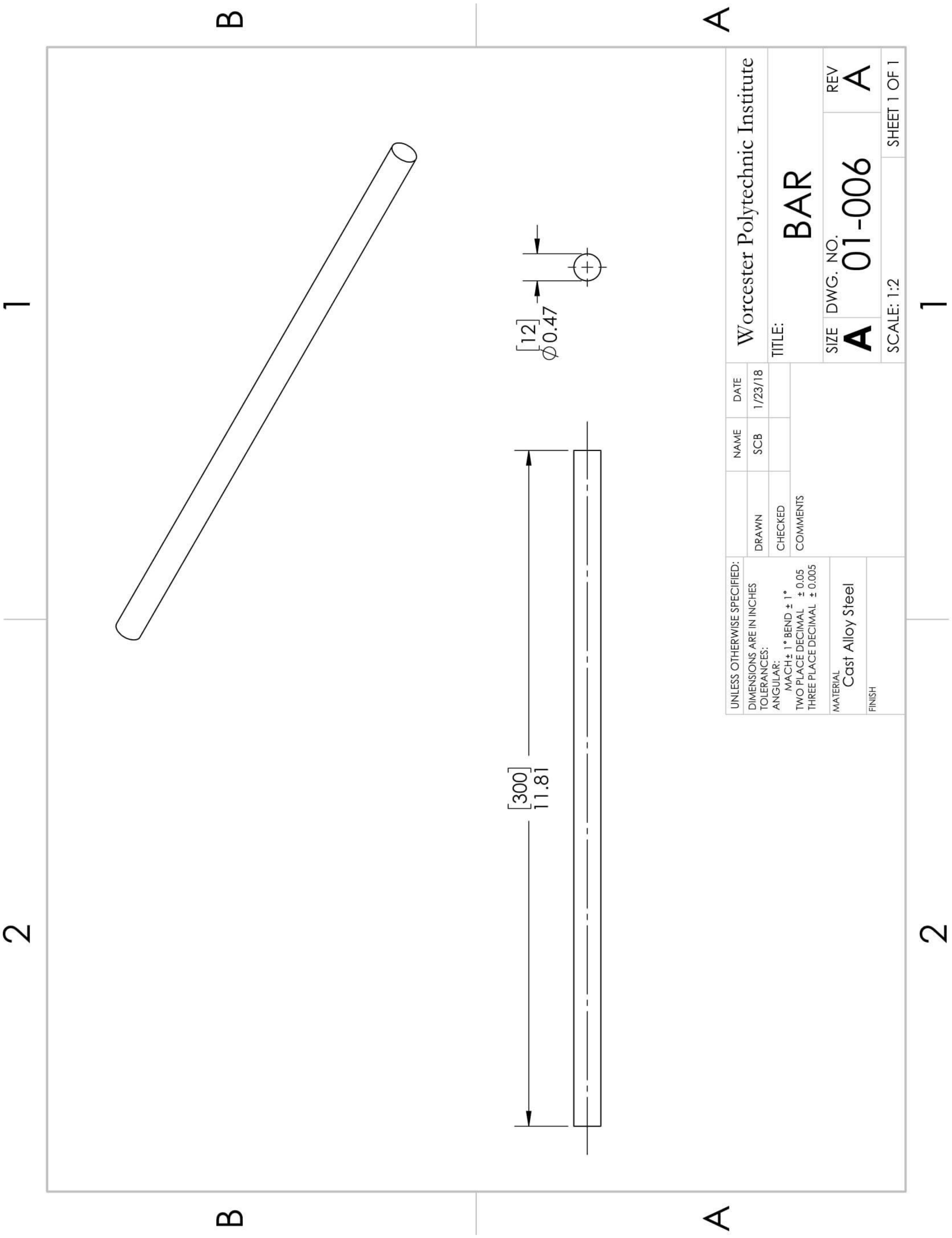
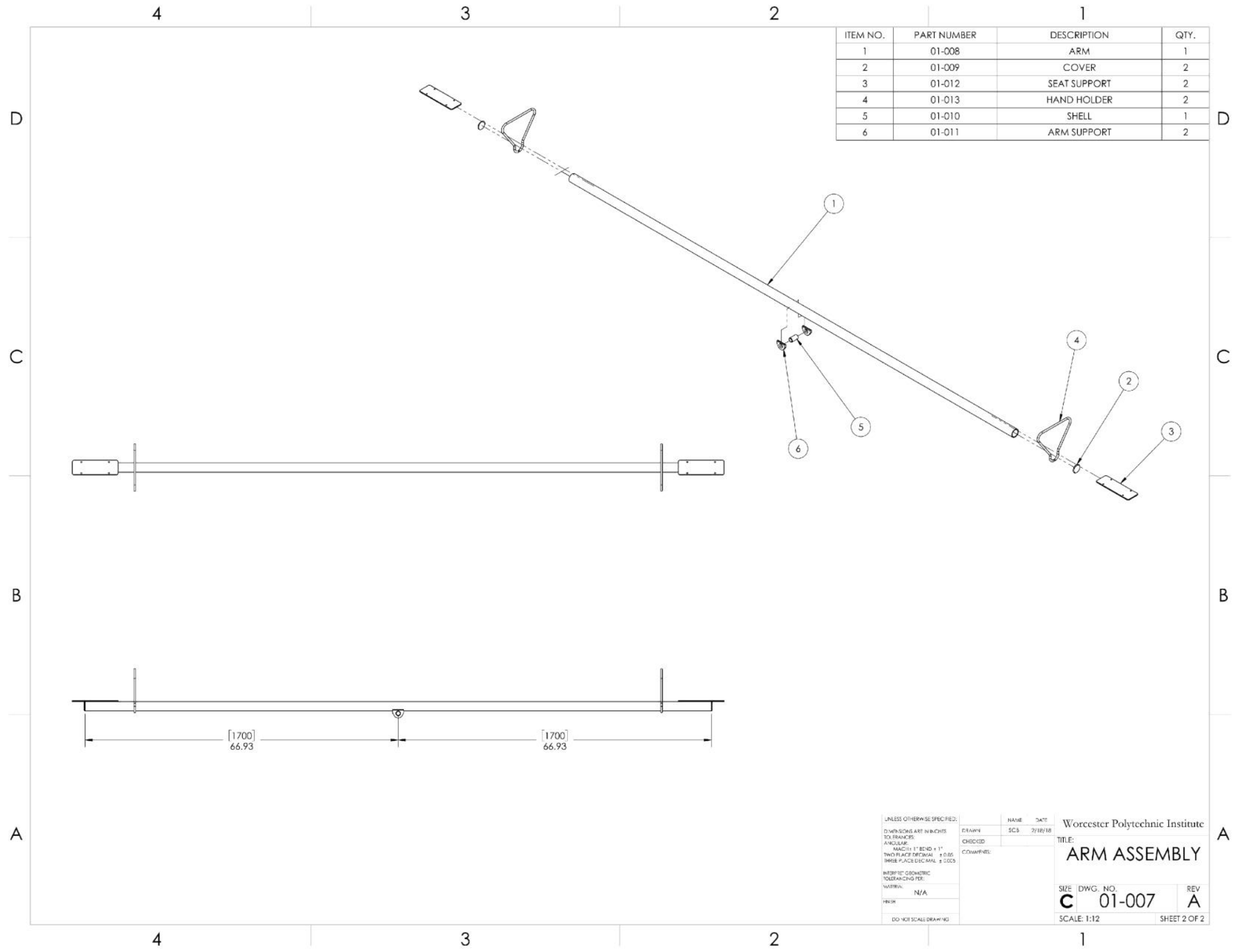
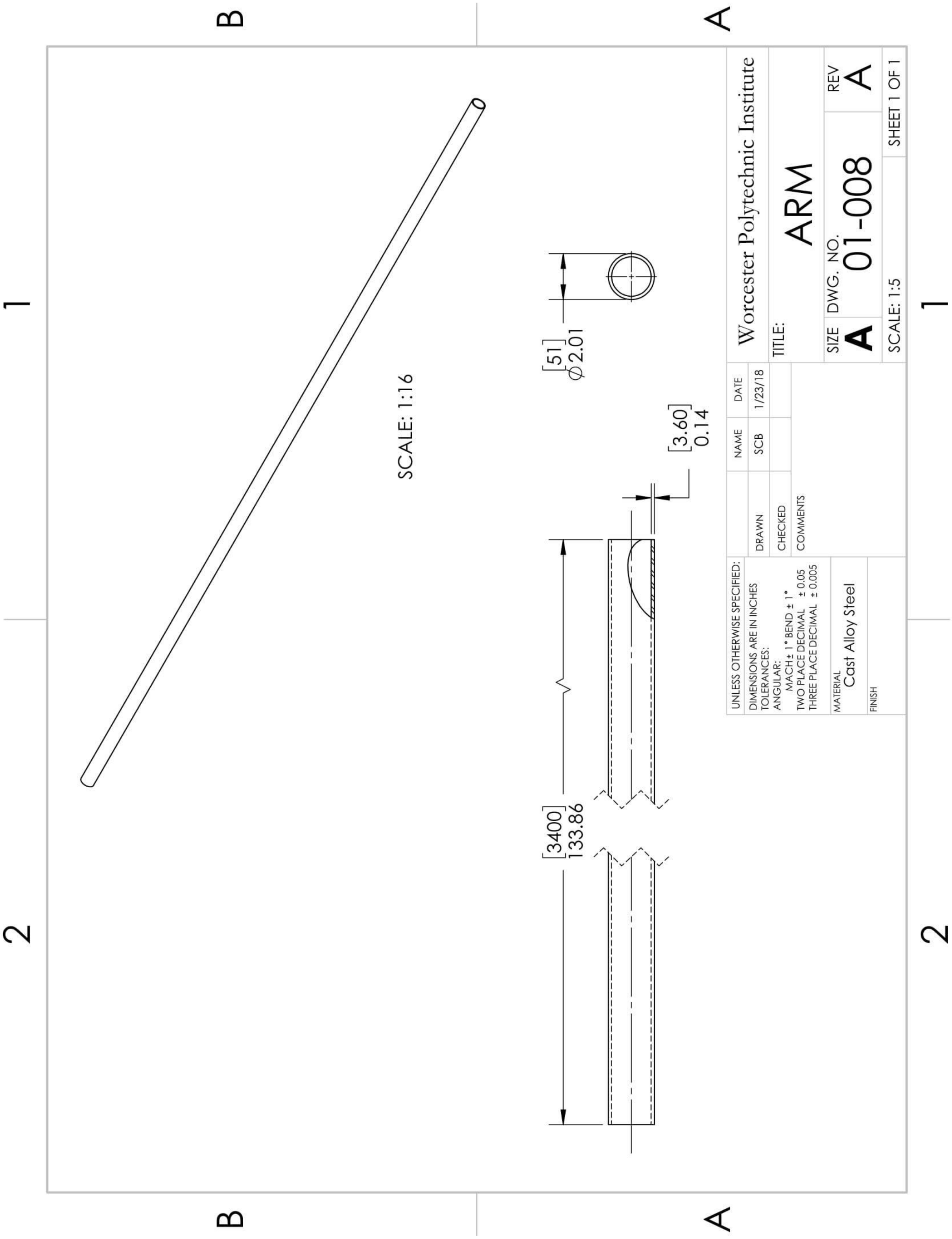
SECTION A-A
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH ± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
CONCRETE
FINISH
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
CONCRETE
SIZE
A
DWG. NO.
01-005
REV
A
SCALE: 1:12
SHEET 1 OF 1

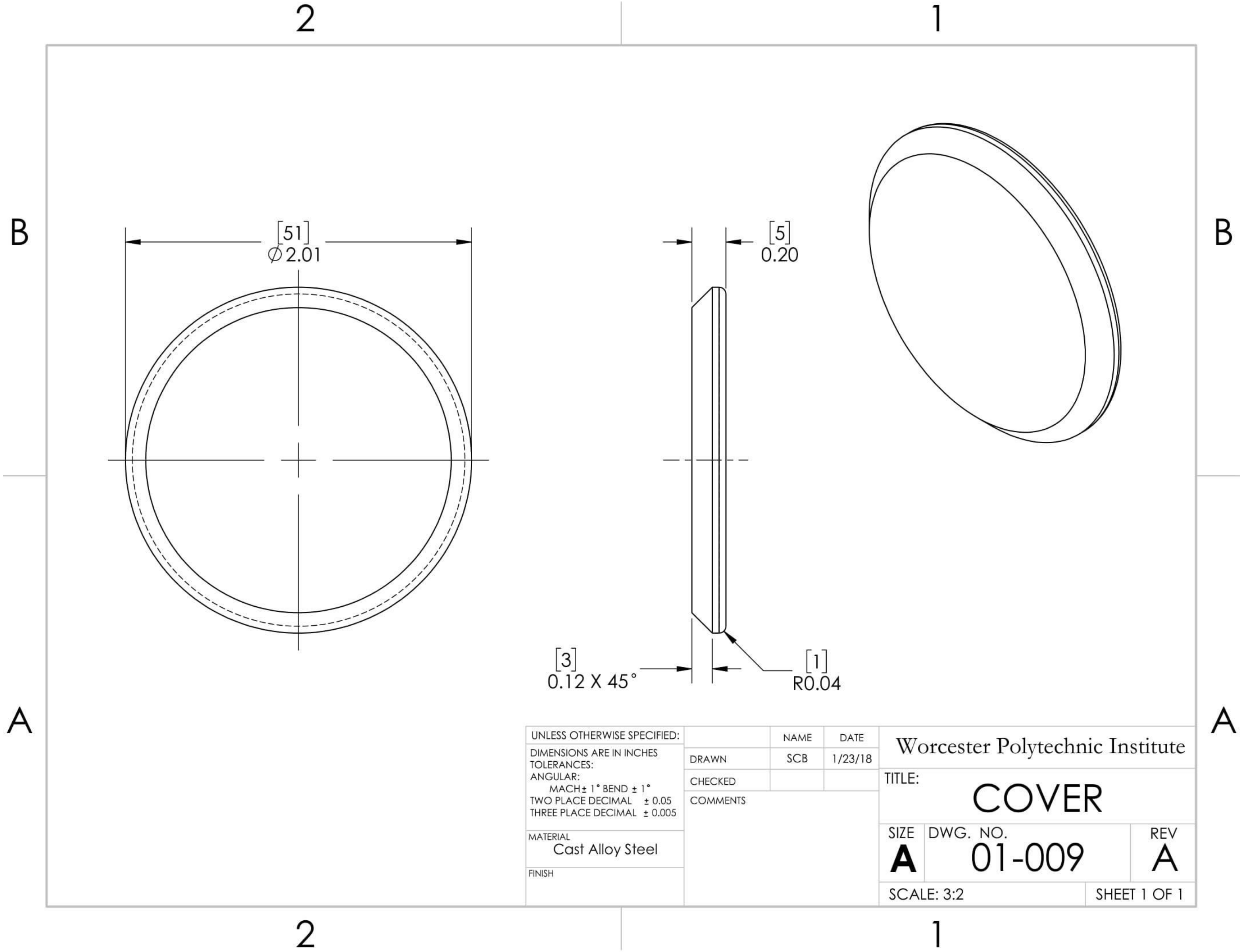

[12]
⌀0.47
[300]
11.81
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
BAR
SIZE
A
DWG. NO.
01-006
REV
A
SCALE: 1:2
SHEET 1 OF 1
A
B
1
2



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	01-008	ARM	1
2	01-009	COVER	2
3	01-012	SEAT SUPPORT	2
4	01-013	HAND HOLDER	2
5	01-010	SHELL	1
6	01-011	ARM SUPPORT	2



B
A
1
2
SCALE: 1:16
[51]
⌀2.01
[3.60]
0.14
[3400]
133.86
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH ± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
ARM
SIZE
A
DWG. NO.
01-008
REV
A
SCALE: 1:5
SHEET 1 OF 1

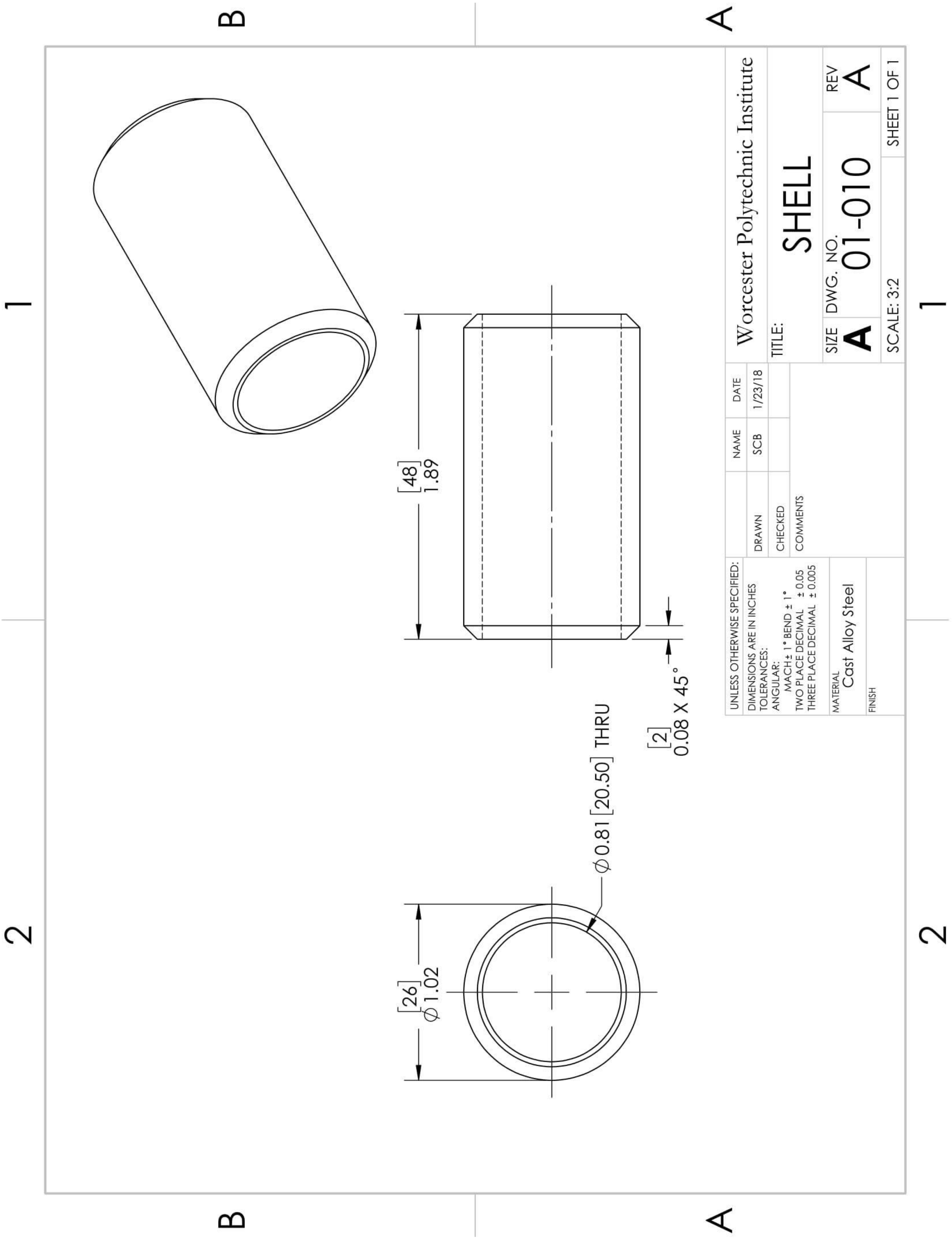


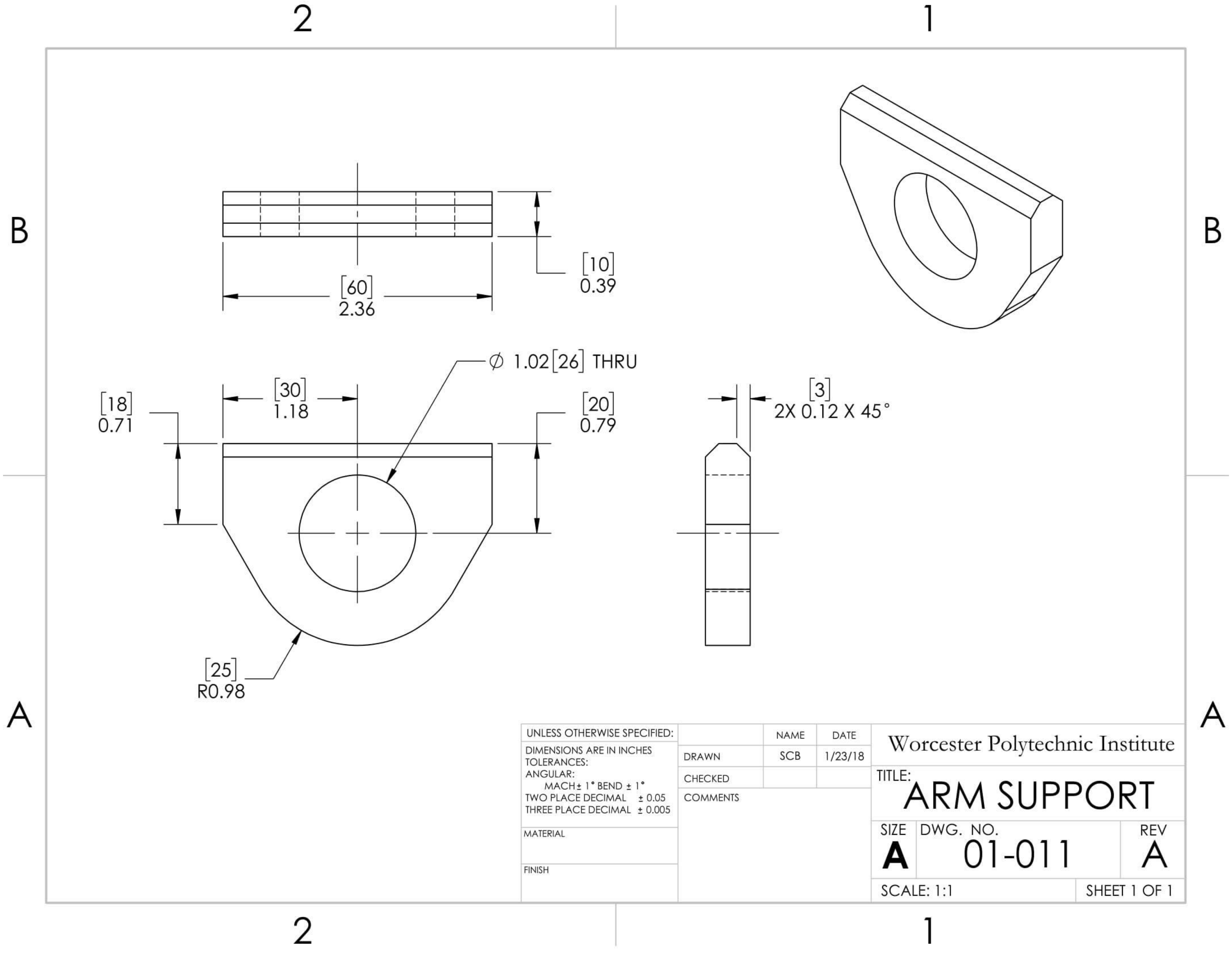
UNLESS OTHERWISE SPECIFIED:		NAME	DATE
DIMENSIONS ARE IN INCHES TOLERANCES: ANGULAR: MACH± 1° BEND ± 1° TWO PLACE DECIMAL ± 0.05 THREE PLACE DECIMAL ± 0.005	DRAWN	SCB	1/23/18
	CHECKED		
MATERIAL Cast Alloy Steel	COMMENTS		
FINISH			

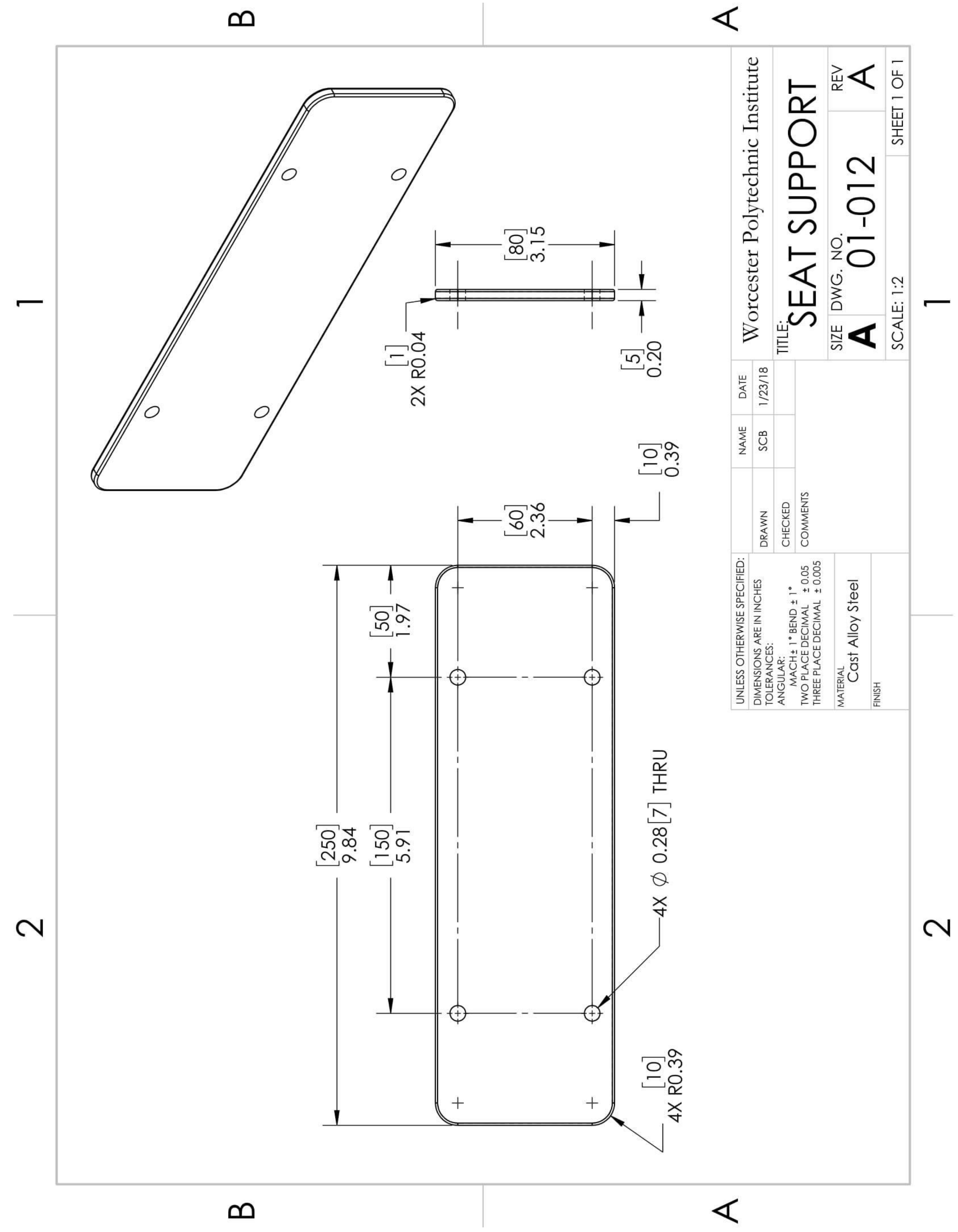
Worcester Polytechnic Institute

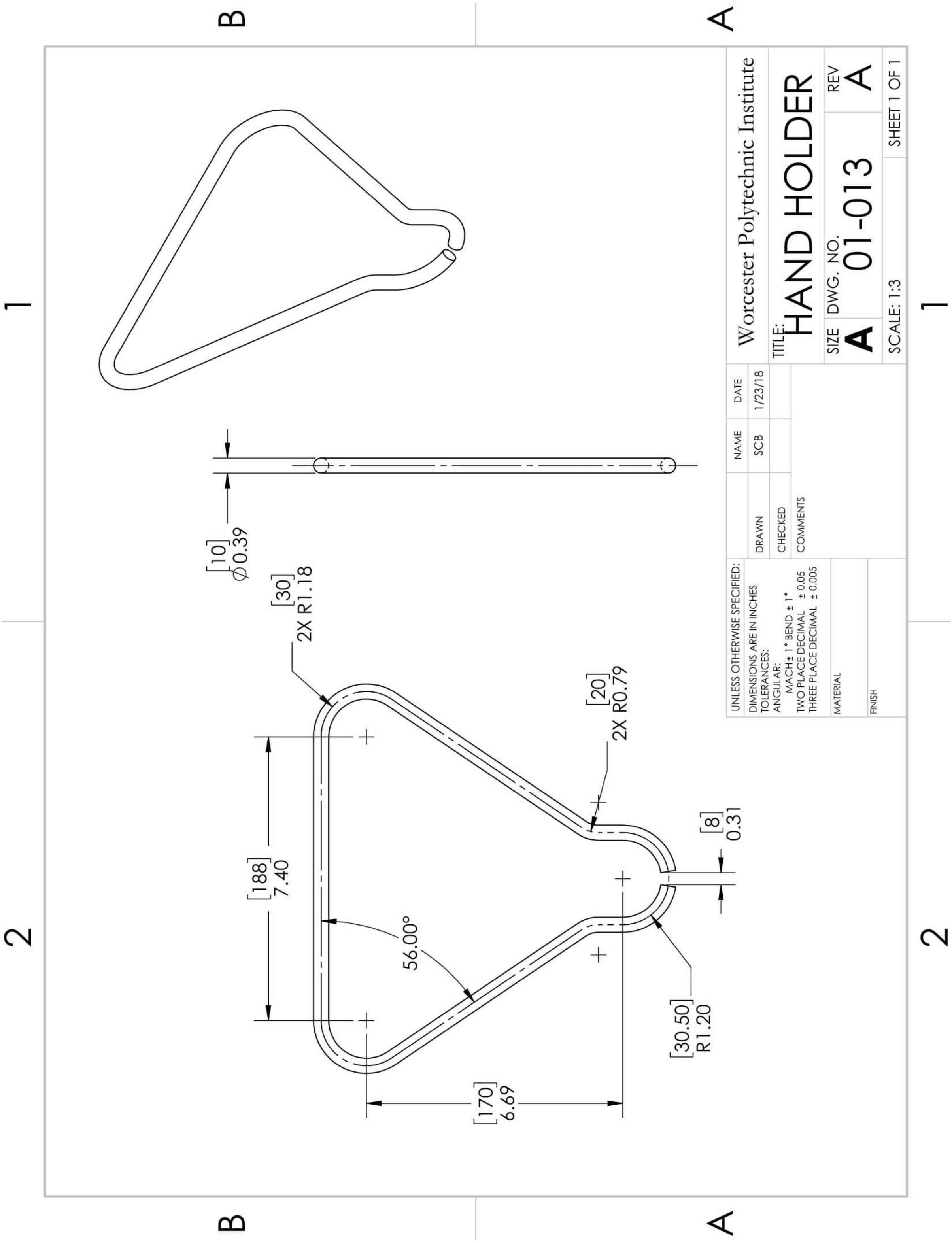
TITLE:
COVER

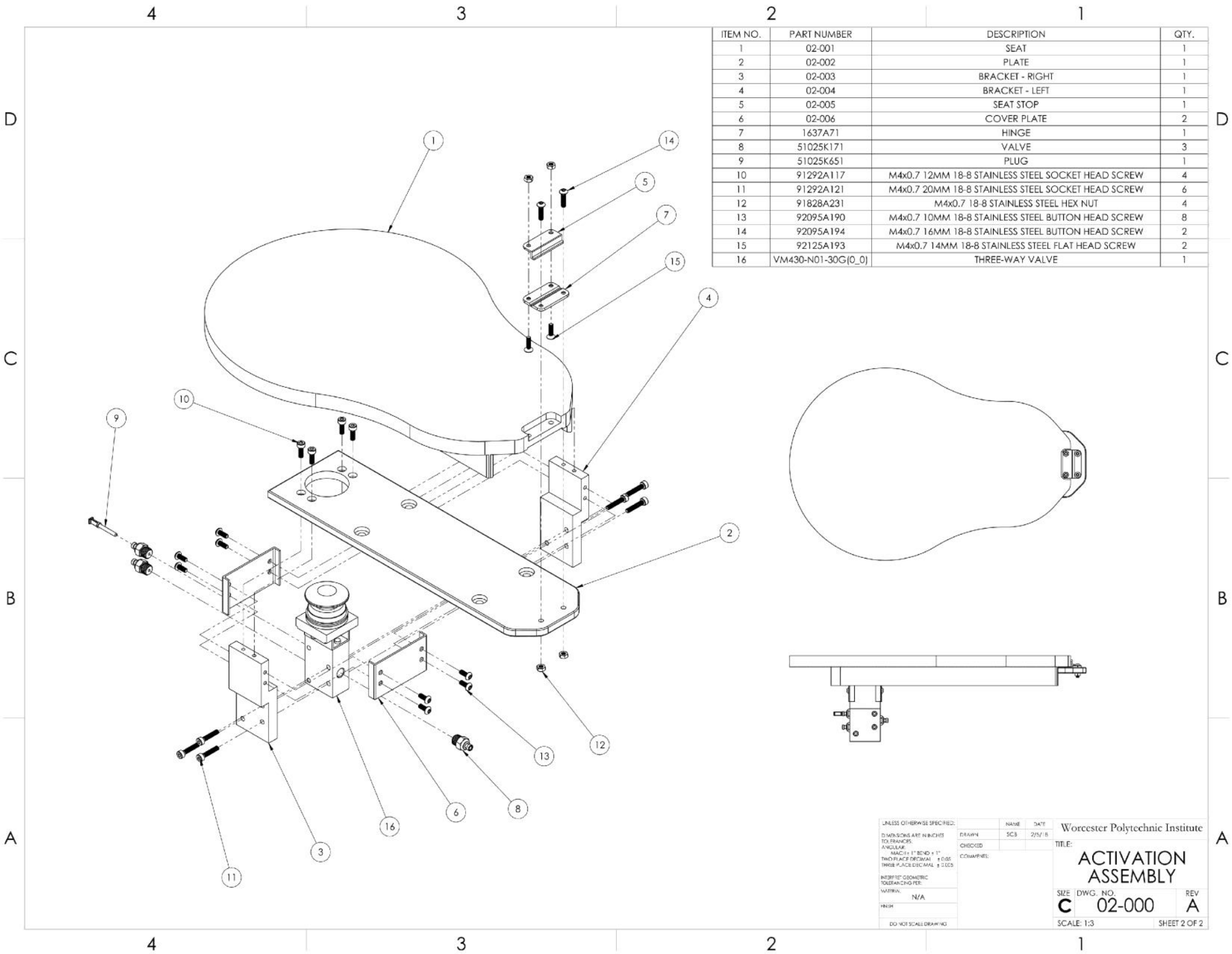
SIZE A
DWG. NO. 01-009
REV A
SCALE: 3:2
SHEET 1 OF 1


Worcester Polytechnic Institute
TITLE:
SHELL
SIZE A
DWG. NO. 01-010
REV A
SCALE: 3:2
SHEET 1 OF 1
NAME DATE
DRAWN SCB 1/23/18
CHECKED
COMMENTS
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH ± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
[48] 1.89
[2] 0.08 X 45°
⌀0.81 [20.50] THRU
[26] ⌀1.02
A
B
1
2

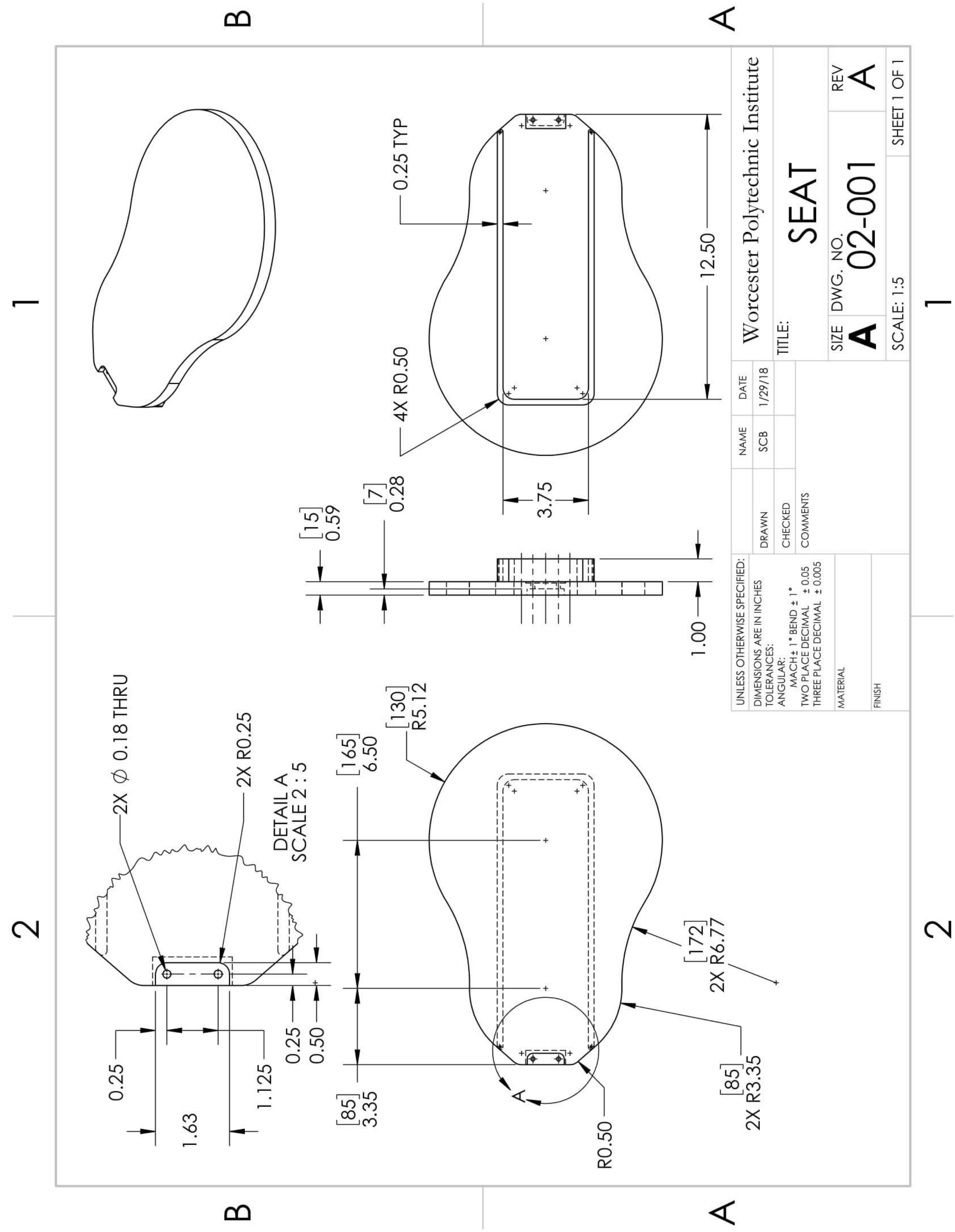

2
1
B
A
[60]
2.36
[10]
0.39
⌀ 1.02[26] THRU
[30]
1.18
[18]
0.71
[20]
0.79
[25]
R0.98
[3]
2X 0.12 X 45°
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
FINISH
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
ARM SUPPORT
SIZE
A
DWG. NO.
01-011
REV
A
SCALE: 1:1
SHEET 1 OF 1

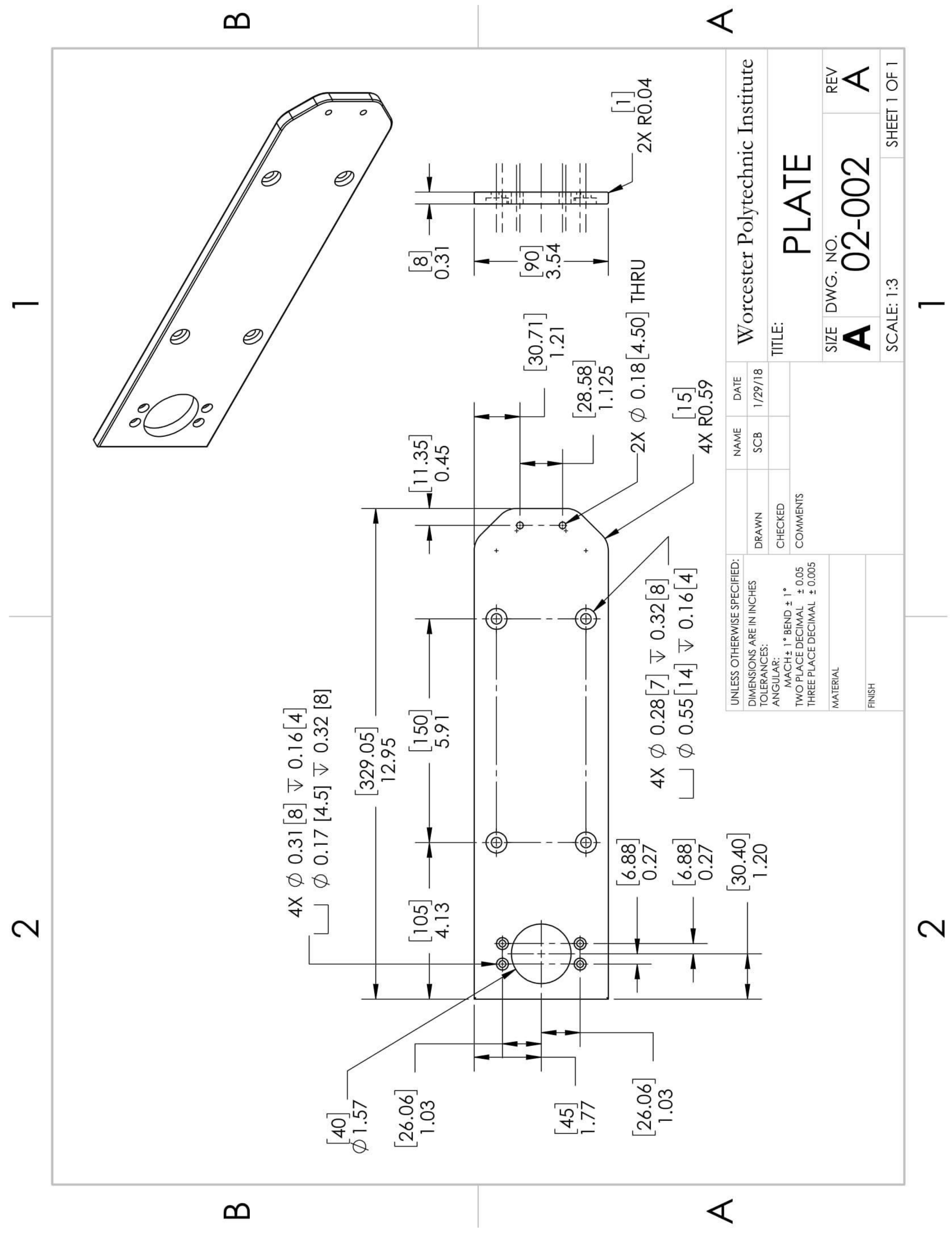

B
A
1
2
[80]
3.15
[1]
2X R0.04
[5]
0.20
[60]
2.36
[10]
0.39
[50]
1.97
[250]
9.84
[150]
5.91
4X ∅ 0.28[7] THRU
[10]
4X R0.39
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
MATERIAL
Cast Alloy Steel
FINISH
Worcester Polytechnic Institute
TITLE:
SEAT SUPPORT
SIZE
A
DWG. NO.
01-012
REV
A
SCALE: 1:2
SHEET 1 OF 1

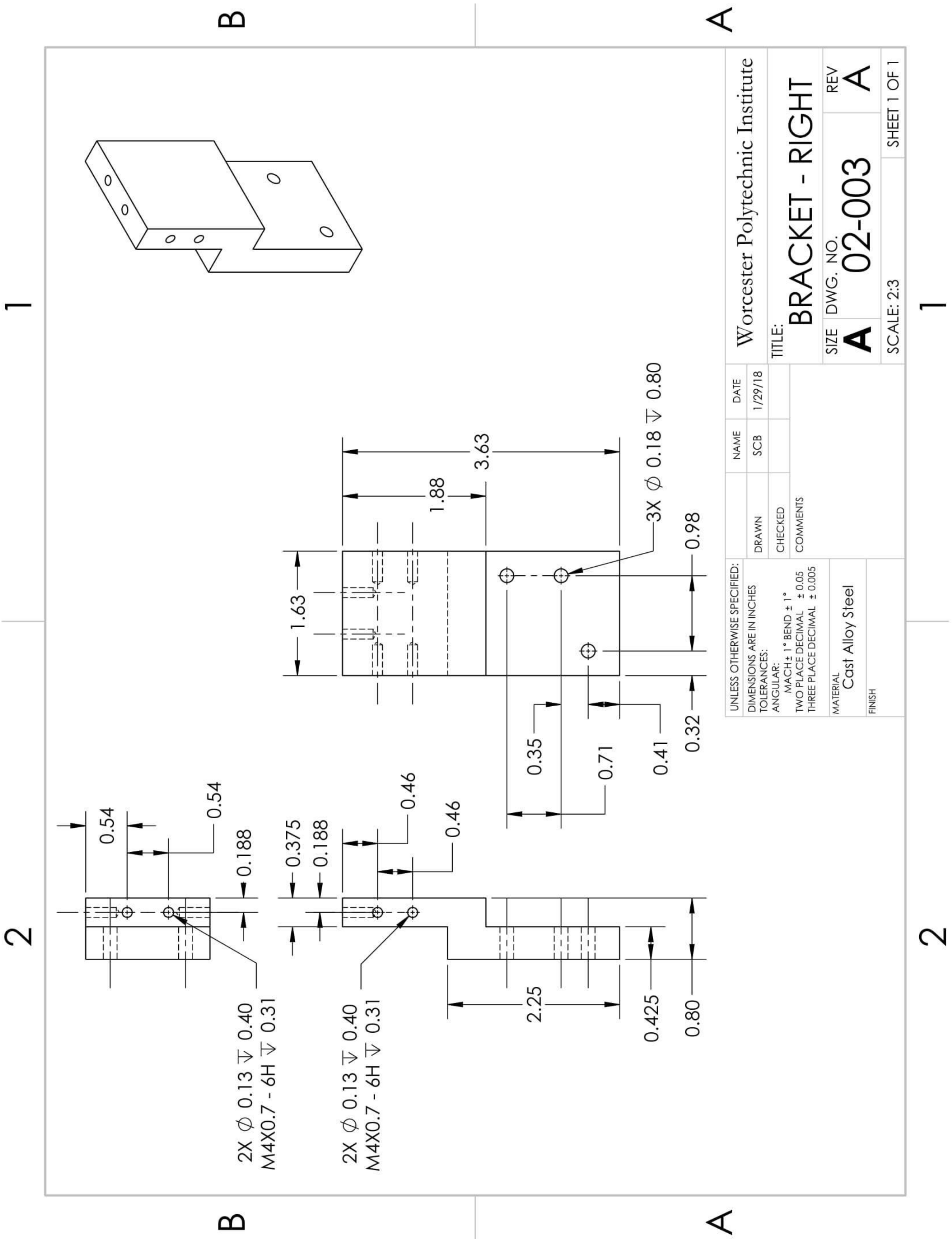

[10]
Ø0.39
[30]
2X R1.18
[20]
2X R0.79
[8]
0.31
[188]
7.40
56.00°
[30.50]
R1.20
[170]
6.69
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
NAME
DATE
DRAWN
SCB
1/23/18
CHECKED
COMMENTS
MATERIAL
FINISH
Worcester Polytechnic Institute
TITLE:
HAND HOLDER
SIZE
A
DWG. NO.
01-013
REV
A
SCALE: 1:3
SHEET 1 OF 1
A
B
1
2

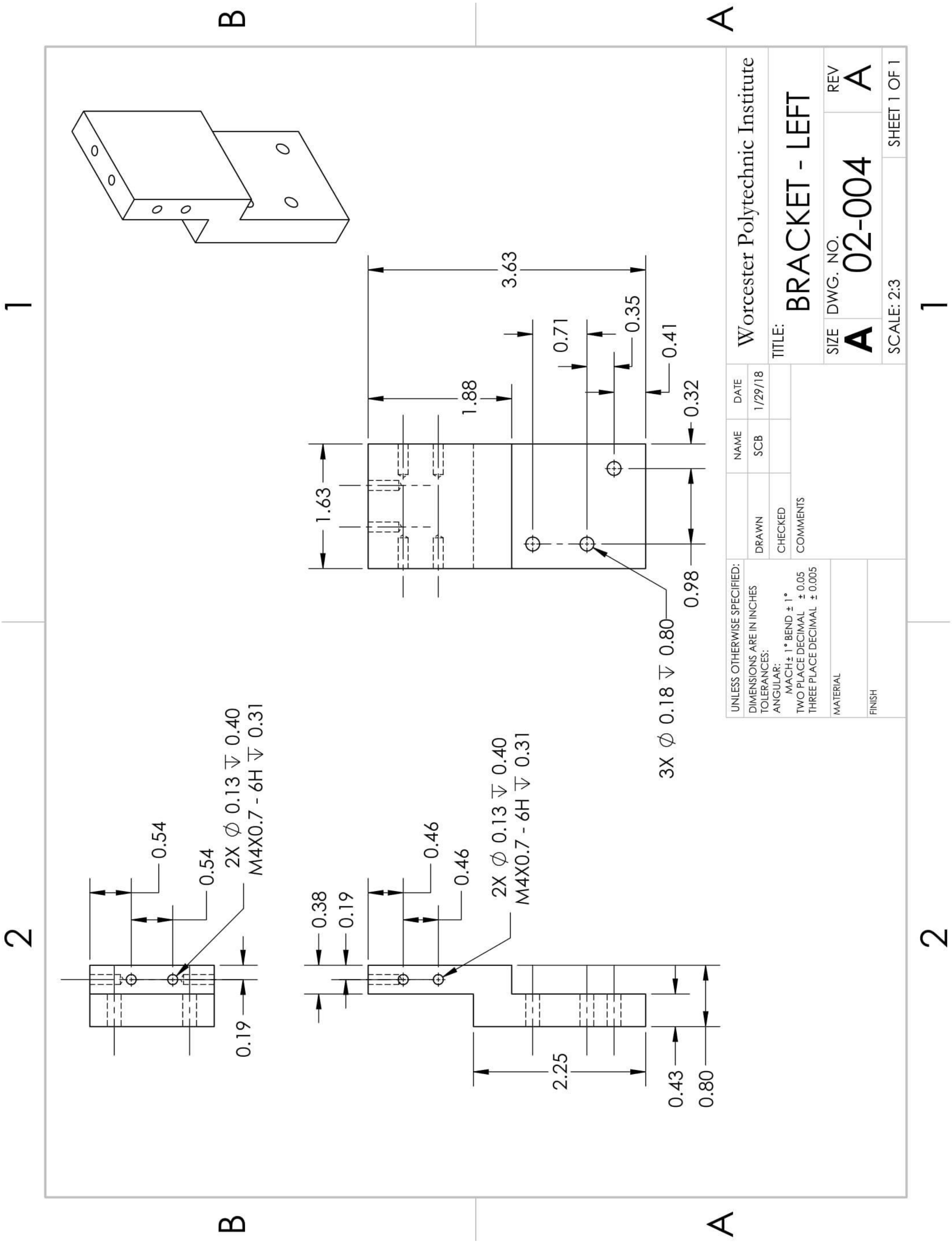


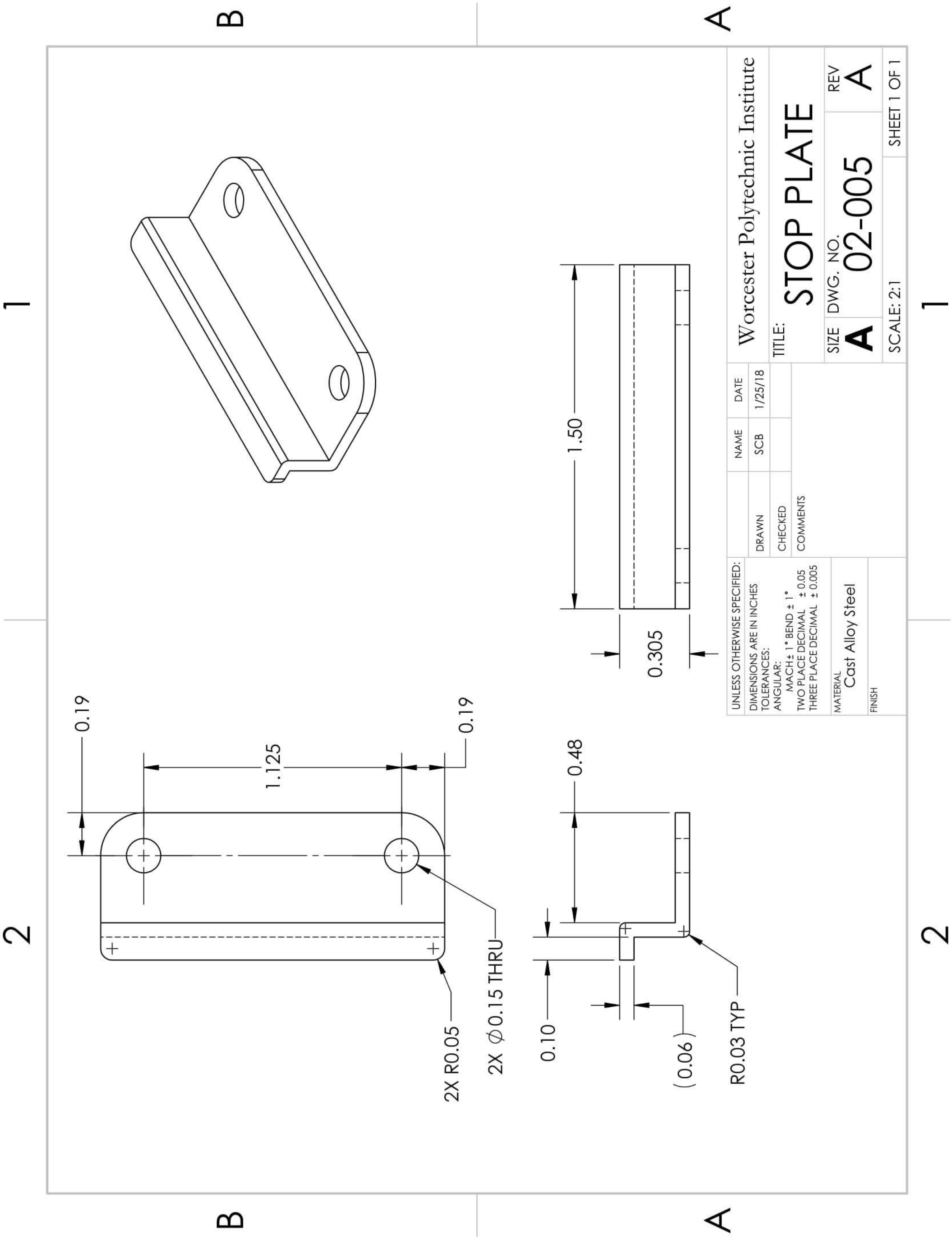
ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	02-001	SEAT	1
2	02-002	PLATE	1
3	02-003	BRACKET - RIGHT	1
4	02-004	BRACKET - LEFT	1
5	02-005	SEAT STOP	1
6	02-006	COVER PLATE	2
7	1637A71	HINGE	1
8	51025K171	VALVE	3
9	51025K651	PLUG	1
10	91292A117	M4x0.7 12MM 18-8 STAINLESS STEEL SOCKET HEAD SCREW	4
11	91292A121	M4x0.7 20MM 18-8 STAINLESS STEEL SOCKET HEAD SCREW	6
12	91828A231	M4x0.7 18-8 STAINLESS STEEL HEX NUT	4
13	92095A190	M4x0.7 10MM 18-8 STAINLESS STEEL BUTTON HEAD SCREW	8
14	92095A194	M4x0.7 16MM 18-8 STAINLESS STEEL BUTTON HEAD SCREW	2
15	92125A193	M4x0.7 14MM 18-8 STAINLESS STEEL FLAT HEAD SCREW	2
16	VM430-N01-30G(0_0)	THREE-WAY VALVE	1

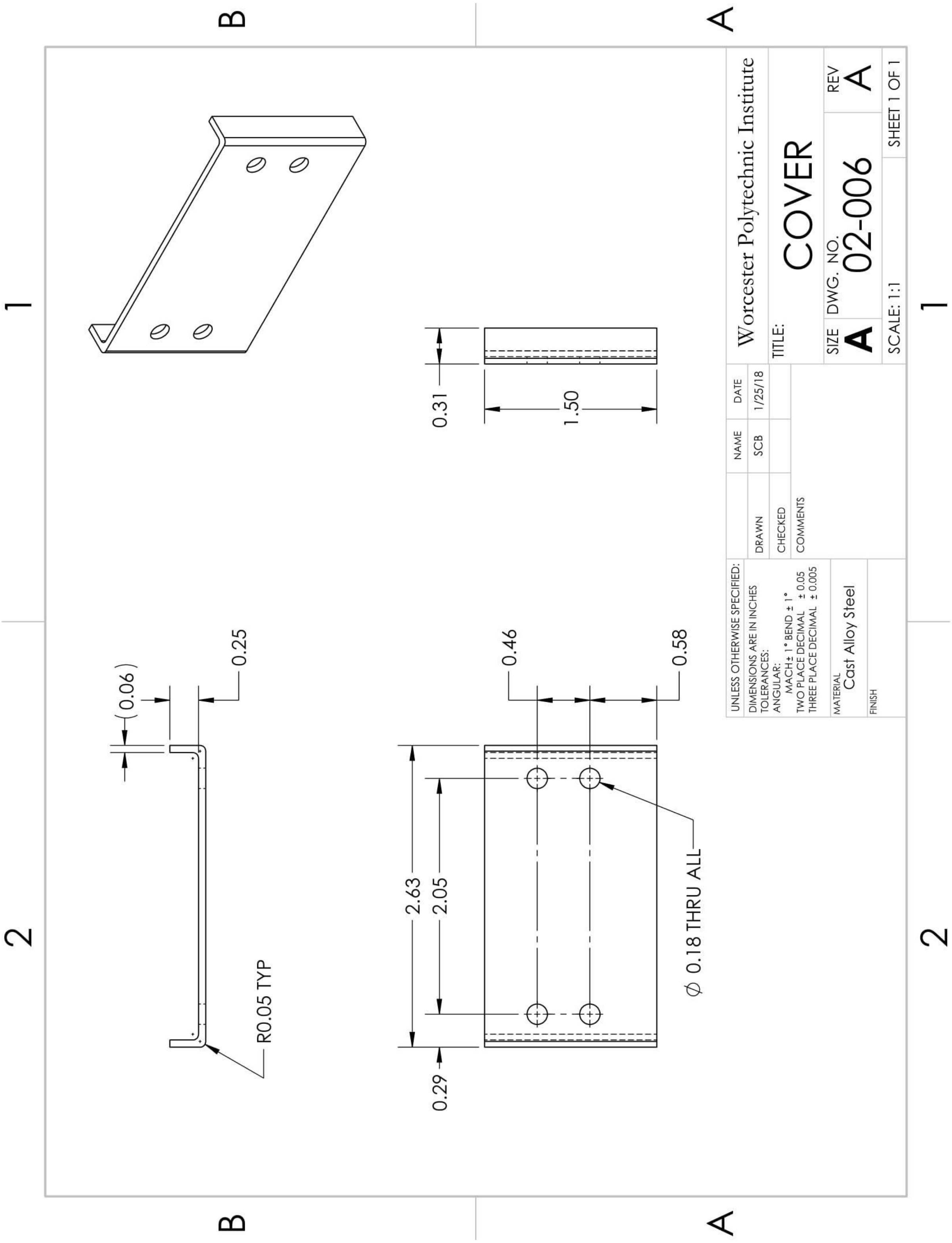

0.25 TYP
4X R0.50
12.50
3.75
[15]
0.59
[7]
0.28
1.00
2X ⌀ 0.18 THRU
2X R0.25
DETAIL A
SCALE 2 : 5
0.25
1.63
1.125
0.25
0.50
[165]
6.50
[130]
R5.12
[85]
3.35
[172]
2X R6.77
[85]
2X R3.35
R0.50
A
1
2
A
B
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH ± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
FINISH
NAME
DATE
DRAWN
SCB
1/29/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
SEAT
SIZE
A
DWG. NO.
02-001
REV
A
SCALE: 1:5
SHEET 1 OF 1


B
1
2
A
4X ⌀ 0.31 [8] ⩡ 0.16 [4]
⌴ ⌀ 0.17 [4.5] ⩡ 0.32 [8]
[329.05]
12.95
[150]
5.91
[105]
4.13
[11.35]
0.45
[30.71]
1.21
[28.58]
1.125
2X ⌀ 0.18 [4.50] THRU
[15]
4X R0.59
4X ⌀ 0.28 [7] ⩡ 0.32 [8]
⌴ ⌀ 0.55 [14] ⩡ 0.16 [4]
[6.88]
0.27
[6.88]
0.27
[30.40]
1.20
[40]
⌀1.57
[26.06]
1.03
[45]
1.77
[26.06]
1.03
[8]
0.31
[90]
3.54
[1]
2X R0.04
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
FINISH
NAME
DATE
DRAWN
SCB
1/29/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
PLATE
SIZE
A
DWG. NO.
02-002
REV
A
SCALE: 1:3
SHEET 1 OF 1


Worcester Polytechnic Institute
TITLE:
BRACKET - RIGHT
SIZE A
DWG. NO. 02-003
REV A
SCALE: 2:3
SHEET 1 OF 1
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
NAME
DATE
DRAWN
SCB
1/29/18
CHECKED
COMMENTS
3.63
1.88
1.63
3X ∅ 0.18 ↧ 0.80
0.98
0.32
0.41
0.71
0.35
0.54
0.54
0.188
0.375
0.188
0.46
0.46
2X ∅ 0.13 ↧ 0.40
M4X0.7 - 6H ↧ 0.31
2X ∅ 0.13 ↧ 0.40
M4X0.7 - 6H ↧ 0.31
2.25
0.425
0.80
A
B
1
2


B
A
1
2
Worcester Polytechnic Institute
TITLE:
BRACKET - LEFT
SIZE
A
DWG. NO.
02-004
REV
A
SCALE: 2:3
SHEET 1 OF 1
NAME
DATE
DRAWN
SCB
1/29/18
CHECKED
COMMENTS
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
FINISH
3.63
1.88
0.71
0.35
0.41
0.32
1.63
0.98
3X ⌀ 0.18 ↧ 0.80
0.54
0.54
2X ⌀ 0.13 ↧ 0.40
M4X0.7 - 6H ↧ 0.31
0.19
0.38
0.19
0.46
0.46
2X ⌀ 0.13 ↧ 0.40
M4X0.7 - 6H ↧ 0.31
2.25
0.43
0.80


B
1
2
A
0.19
1.125
0.19
2X R0.05
2X ⌀0.15 THRU
0.48
0.10
(0.06)
R0.03 TYP
1.50
0.305
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH ± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
NAME
DATE
DRAWN
SCB
1/25/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
STOP PLATE
SIZE
A
DWG. NO.
02-005
REV
A
SCALE: 2:1
SHEET 1 OF 1



B
1
2
A
0.31
1.50
(0.06)
0.25
0.46
0.58
2.63
2.05
0.29
R0.05 TYP
⌀ 0.18 THRU ALL
UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN INCHES
TOLERANCES:
ANGULAR:
MACH± 1° BEND ± 1°
TWO PLACE DECIMAL ± 0.05
THREE PLACE DECIMAL ± 0.005
MATERIAL
Cast Alloy Steel
FINISH
NAME
DATE
DRAWN
SCB
1/25/18
CHECKED
COMMENTS
Worcester Polytechnic Institute
TITLE:
COVER
SIZE
A
DWG. NO.
02-006
REV
A
SCALE: 1:1
SHEET 1 OF 1

Appendix F: Mathcad

System Analysis

Variable Labels

L_1 = distance from pivot pin to piston-beam attachment

L_2 = distance from pivot pin to center of mass of half of the seesaw beam

L_3 = distance from pivot pin to application point of user weight on seesaw beam

L_4 = distance from pivot pin to ground

L_5 = distance from frame to piston-ground mount

L_p = piston length (pin joint to pin joint)

L_s = piston stroke length

A,B = triangle side lengths (see included figure)

R_{ox} = x-direction reaction force at the pivot point

R_{oy} = y-direction reaction force at the pivot point

R_{px} = x-direction reaction force at the piston-beam attachment

R_{py} = y-direction reaction force at the piston-beam attachment

R_p = magnitude of the reaction force at the piston-beam attachment

θ_1 = seesaw beam angle measured counter-clockwise from +x-axis

θ_{max} = maximum seesaw angle possible before beam strikes ground

d = piston head diameter

A = piston face area

P = piston pressure

m_k = mass of the seesaw user

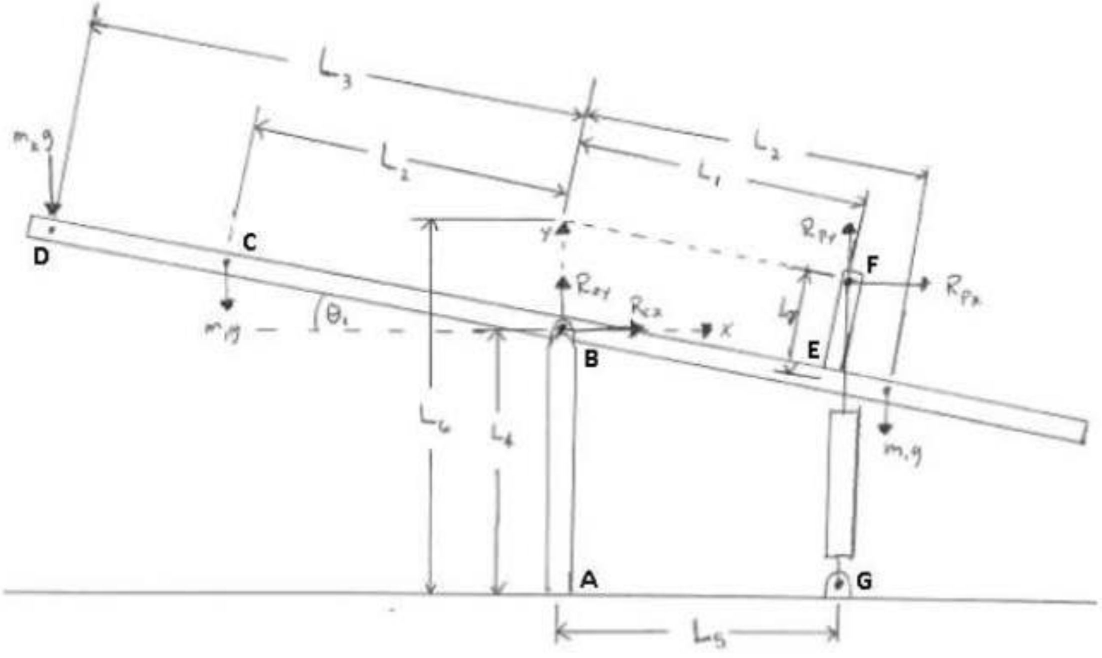
g = gravitational constant

m_1 = mass of half of the seesaw beam

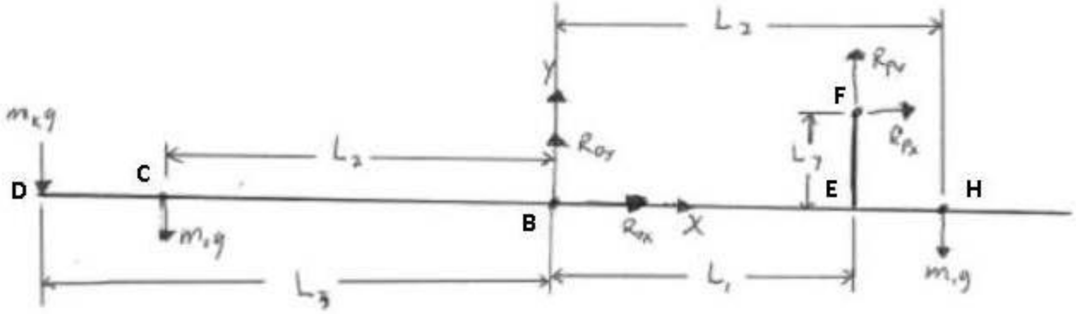
$F_{parallel}$ = force applied from the piston to the beam, parallel to the beam length

$F_{perpendicular}$ = force applied from the piston to the beam, perpendicular to the beam length

Forces on Seesaw Board



Seesaw setup.



Forces on seesaw board.

Variables:

Select L1, Lp, and d values for optimization. Select other values based on seesaw geometry and user weight.

$m_k := 60\text{kg}$ user mass

$g := 9.81 \frac{m}{s^2}$ gravitational constant

$L_3 := 1.7m$ distance from seesaw pivot to user

$L_2 := 1.2m$ distance from seesaw pivot to center of mass of seesaw half

$L_4 := .613m$ height of seesaw pivot above ground

$m_1 := 11.6kg$ mass of half of the seesaw

$L_1 := .85m$ distance along seesaw board length from seesaw pivot to pneumatic piston attachment

$d := 1.5in$ Pneumatic piston bore. Note that there will be 2 pistons.

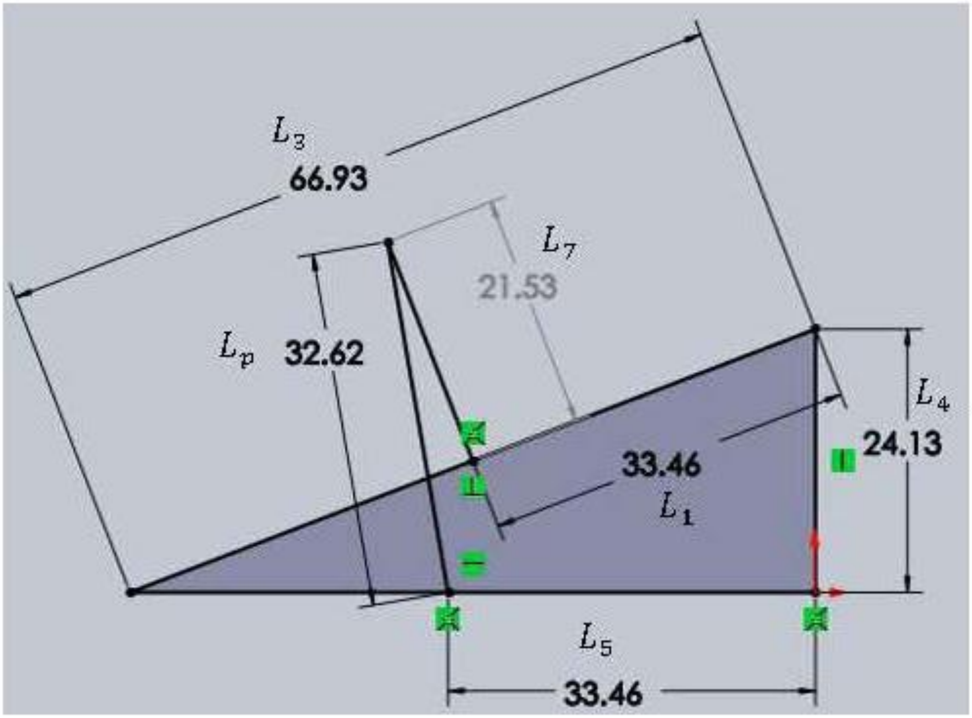
$L_5 := L_1$ distance from seesaw frame to pneumatic piston attachment to ground

$\theta_{max} := \operatorname{asin}\left(\frac{L_4}{L_3}\right) = 21.136 \cdot deg$ maximum seesaw angle possible before beam strikes ground

$L_s := 2 \cdot L_1 \cdot \sin\left(\theta_{max}\right) = 24.134 \cdot in$ Piston stroke length necessary to move through full range of motion. Select piston based on this length.

$L_p := 32.62in$ retracted length of selected piston

$L_7 := 21.53in$ Distance above seesaw beam of pneumatic piston attachment point. L7 is determined using existing seesaw geometry as shown in the image below from SolidWorks.



Seesaw geometry for L7.

Plots:

$\theta_1 := -21\text{deg}, -20.9\text{deg} .. 21\text{deg}$

Equations:

$$R_{py}(\theta_1) := \frac{m_k \cdot g \cdot L_3 \cdot \cos(\theta_1)}{\frac{\left(L_1 \cdot \cos(\theta_1) - L_7 \cdot \cos(90\text{deg} - \theta_1) - L_5\right)}{L_4 + L_1 \cdot \sin(\theta_1) + L_7 \cdot \sin(90\text{deg} - \theta_1)} \cdot \left(L_1 \cdot \sin(\theta_1) + L_7 \cdot \sin(90\text{deg} - \theta_1)\right) - L_1 \cdot \cos(\theta_1)}$$

$$R_{px}(\theta_1) := \frac{R_{py}(\theta_1) \cdot \left(L_1 \cdot \cos(\theta_1) - L_7 \cdot \cos(90\text{deg} - \theta_1) - L_5\right)}{L_4 + L_1 \cdot \sin(\theta_1) + L_7 \cdot \sin(90\text{deg} - \theta_1)}$$

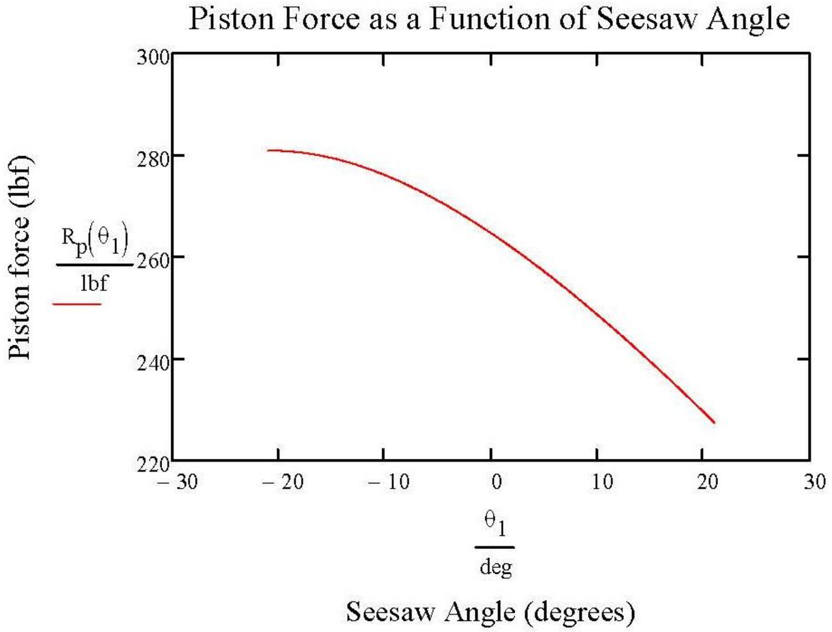
$$R_{oy}(\theta_1) := m_k \cdot g + 2 \cdot m_1 \cdot g - R_{py}(\theta_1)$$

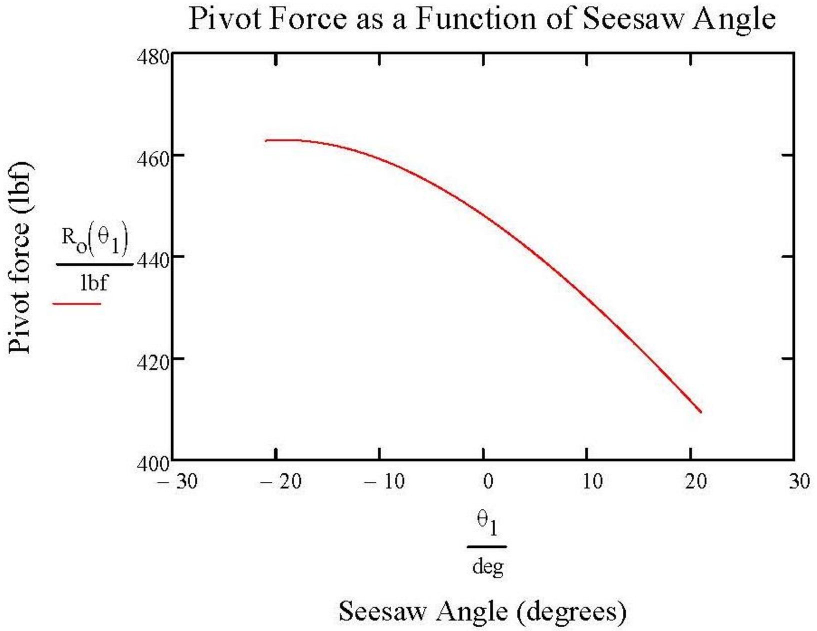
$$R_{ox}(\theta_1) := R_{px}(\theta_1)$$

$$R_o(\theta_1) := \sqrt{R_{oy}(\theta_1)^2 + R_{ox}(\theta_1)^2}$$

$$R_p(\theta_1) := \sqrt{R_{px}(\theta_1)^2 + R_{py}(\theta_1)^2}$$

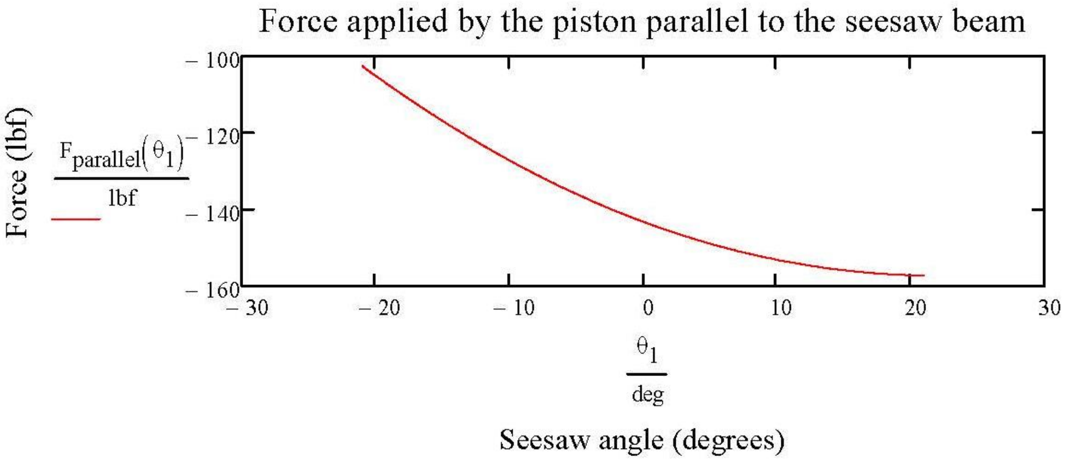
Plots:





$$P_{O2}(\theta_1) := \sqrt{\left(L_1 \cdot \cos(\theta_1) - L_7 \cdot \cos(90deg - \theta_1)\right)^2 + \left(L_1 \cdot \sin(\theta_1) + L_7 \cdot \sin(90deg - \theta_1)\right)^2}$$

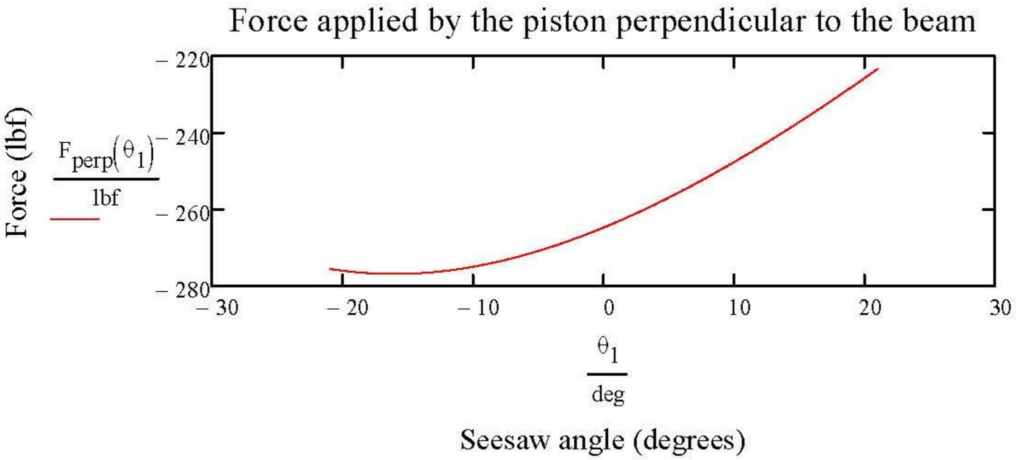
$$F_{parallel}(\theta_1) := R_{px}(\theta_1)\cdot\frac{\left(L_1\cdot\cos(\theta_1) - L_7\cdot\cos(90deg - \theta_1)\right)}{P_{O2}(\theta_1)} \ldots$$
$$+ R_{py}(\theta_1)\cdot\frac{\left(L_1\cdot\sin(\theta_1) + L_7\cdot\sin(90deg - \theta_1)\right)}{P_{O2}(\theta_1)}$$



$F_{parallel}(21deg) = -157.233\cdot lbf$

$F_{parallel}(-21deg) = -102.597\cdot lbf$

$$F_{perp}(\theta_1) := R_{px}(\theta_1)\cdot\cos(\theta_1 + 90deg) + R_{py}(\theta_1)\cdot\sin(\theta_1 + 90deg)$$

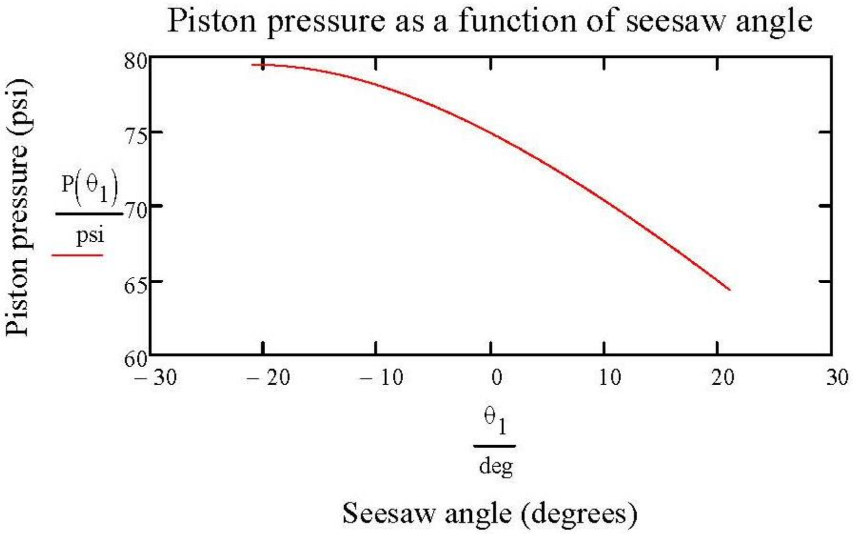


$F_{perp}(0deg) = -264.645\cdot lbf$

Piston Pressure:

$$A := \frac{2\pi \cdot d^2}{4} = 2.28 \times 10^{-3}\, m^2$$

$$P(\theta_1) := \frac{R_p(\theta_1)}{A}$$



Conclusion:

As a result of this analysis, we determined that the pistons should act on the seesaw beam at a distance of 0.85 meters from the seesaw pivot. Pistons should have a stroke length of 28 inches and a bore of 1.5 inches. The upper portion of the braking mechanism which is attached to the seesaw beam should extend 21.5 inches above the seesaw beam.

Seesaw Beam Forces Equation Derivations

Sum of the forces in the x-direction

$$\sum F_x = 0$$

$0 = R_{ox} + R_{px}$ 1

$R_{px} = -R_{ox}$ 2

Sum of the forces in the y-direction

$$\sum F_y = 0$$

$0 = -m_k g - 2m_1 g + R_{py} + R_{oy}$ 3

$R_{oy} = m_k g + 2m_1 g - R_{py}$ 4

Sum of the moments about the pivot pin

$$\sum M_o = 0$$

$0 = m_k g L_3 \cos(\theta_1) + m_1 g L_2 \cos(\theta_1) - R_{px}(L_1 \sin(\theta_1) + L_7 \sin(90° - \theta_1)) - R_{py} L_1 \cos(\theta_1) - m_1 g L_2 \cos(\theta_1)$

$0 = m_k g L_3 \cos(\theta_1) - R_{px}(L_1 \sin(\theta_1) + L_7 \sin(90° - \theta_1)) - R_{py} L_1 \cos(\theta_1)$ 5

Position and force vectors in the direction of the piston force

$$\overrightarrow{P_{12}} = [L_1 \cos(\theta_1) - L_7 \cos(90° - \theta_1) - L_5]\,\hat{\imath} + [L_4 + L_1 \sin(\theta_1) + L_7 \sin(90° - \theta_1)]\,\hat{\jmath}$$

$$\left|\overrightarrow{P_{12}}\right| = \sqrt{(L_1 \cos(\theta_1) - L_7 \cos(90° - \theta_1) - L_5)^2 + [L_4 + L_1 \sin(\theta_1) + L_7 \sin(90° - \theta_1)]^2}$$

$$\widehat{P_{12}} = \frac{[L_1 \cos(\theta_1) - L_7 \cos(90° - \theta_1) - L_5]}{\left|\overrightarrow{P_{12}}\right|}\,\hat{\imath} + \frac{[L_4 + L_1 \sin(\theta_1) + L_7 \sin(90° - \theta_1)]}{\left|\overrightarrow{P_{12}}\right|}\,\hat{\jmath}$$

$$\widehat{P_{12}} = \widehat{R_p}$$

$$\overrightarrow{R_p} = R_{px}\hat{\imath} + R_{py}\hat{\jmath}$$

$$\left|\overrightarrow{R_p}\right| = \sqrt{R_{px}^2 + R_{py}^2}$$

$$\widehat{R_p} = \frac{R_{px}}{\left|\overrightarrow{R_p}\right|}\,\hat{\imath} + \frac{R_{py}}{\left|\overrightarrow{R_p}\right|}\,\hat{\jmath}$$

$$\frac{R_{px}}{\left|\overrightarrow{R_p}\right|} = \frac{[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]}{\left|\overrightarrow{P_{12}}\right|}$$

$$\left|\overrightarrow{R_p}\right| = \frac{R_{px}\left|\overrightarrow{P_{12}}\right|}{[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]}$$

$$\frac{R_{py}}{\left|\overrightarrow{R_p}\right|} = \frac{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]}{\left|\overrightarrow{P_{12}}\right|}$$

$$\left|\overrightarrow{R_p}\right| = \frac{R_{py}\left|\overrightarrow{P_{12}}\right|}{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]}$$

$$\frac{R_{px}\left|\overrightarrow{P_{12}}\right|}{[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]} = \frac{R_{py}\left|\overrightarrow{P_{12}}\right|}{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]}$$

$$R_{px} = \frac{R_{py}[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]}{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]} \tag{6}$$

Combining sum of the moments and piston position/force vectors

$$0 = m_k g L_3\cos(\theta_1) - R_{px}(L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)) - R_{py}L_1\cos(\theta_1) \tag{5}$$

$$0 = m_k g L_3\cos(\theta_1) - \frac{R_{py}[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]}{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]}(L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)) - R_{py}L_1\cos(\theta_1)$$

$$0 = m_k g L_3\cos(\theta_1) - R_{py}\left[\frac{[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]}{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]}(L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)) - L_1\cos(\theta_1)\right]$$

$$R_{py} = \frac{m_k g L_3\cos(\theta_1)}{\left[\frac{[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1) - L_5]}{[L_4 + L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)]}(L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)) - L_1\cos(\theta_1)\right]} \tag{7}$$

Position vector from seesaw pivot pin to piston attachment on seesaw board

$$\overrightarrow{P_{O2}} = [L_1\cos(\theta_1) - L_7\cos(90° - \theta_1)]\,\hat{\imath} + (L_1\sin(\theta_1) + L_7\sin(90° - \theta_1))\hat{\jmath}$$

$$\left|\overrightarrow{P_{O2}}\right| = \sqrt{[L_1\cos(\theta_1) - L_7\cos(90° - \theta_1)]^2 + (L_1\sin(\theta_1) + L_7\sin(90° - \theta_1))^2}$$

$$\widehat{P_{O2}} = \frac{L_1\cos(\theta_1) - L_7\cos(90° - \theta_1)}{\left|\overrightarrow{P_{O2}}\right|}\hat{\imath} + \frac{L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)}{\left|\overrightarrow{P_{O2}}\right|}\hat{\jmath}$$

Force applied by the piston to the board, parallel to the board direction

$$F_{parallel} = \overrightarrow{R_p} \cdot \widehat{P_{O2}}$$

$$F_{parallel} = R_{px}\frac{L_1\cos(\theta_1) - L_7\cos(90° - \theta_1)}{\left|\overrightarrow{P_{O2}}\right|} + R_{py}\frac{L_1\sin(\theta_1) + L_7\sin(90° - \theta_1)}{\left|\overrightarrow{P_{O2}}\right|} \tag{8}$$

Force applied by the piston to the board, perpendicular to the board direction

$$\hat{n} = \cos(\theta_1 + 90°)\,\hat{\imath} + \sin(\theta_1 + 90°)\hat{\jmath}$$

$$F_{perpendicular} = \overrightarrow{R_p} \cdot \hat{n}$$

$$F_{perpendicular} = R_{px}\cos(\theta_1 + 90°) + R_{py}\sin(\theta_1 + 90°) \qquad 8$$

Piston pressure

$$P = \frac{F}{A}$$

$$P = \frac{R_p}{A}$$

$$A = \frac{\pi d^2}{4}$$

Stress Analysis – Point A

$L := 17.78\text{mm}$ $\quad D := 6.35\text{mm}$ $\quad \gamma := 7870\dfrac{\text{kg}}{\text{m}^3}$ $\quad F := 280.876\text{lbf} = 1.249\cdot\text{kN}$

$a := \left(1.27 + \dfrac{3.048}{2}\right)\text{mm} = 2.794\cdot\text{mm}$ $\quad b := L - a = 14.986\cdot\text{mm}$

$f := 12.7\text{mm}$ $\quad f_1 := \dfrac{L}{2} - \dfrac{f}{2} = 2.54\cdot\text{mm}$ $\quad f_2 := \dfrac{L}{2} + \dfrac{f}{2} = 15.24\cdot\text{mm}$

$w := \left(\dfrac{\pi}{4}\right)\cdot D^2\cdot\gamma\cdot g = 2.444\cdot\dfrac{\text{N}}{\text{m}}$ $\quad W := w\cdot L = 0.043\,\text{N}$

$S_y := 203943242.6\text{Pa} = 203.943\cdot\text{MPa}$ $\quad S_{ut} := 356900674.5\text{Pa} = 356.901\cdot\text{MPa}$

Defining the singularity functions:

$x := 0\text{m}, 0.005L .. L$ $\quad S(x,z) := \left|\begin{array}{l}\text{return } 1 \ \text{ if } x > z\\ 0\end{array}\right.$

At x = R_1, M = 0

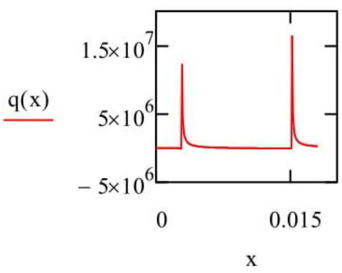
$$R_2 := \frac{1}{(b-a)}\cdot\left[F\cdot\left(\frac{L}{2} - a\right) + W\cdot\left(\frac{L}{2} - a\right)\right] = 624.721\,\text{N}$$

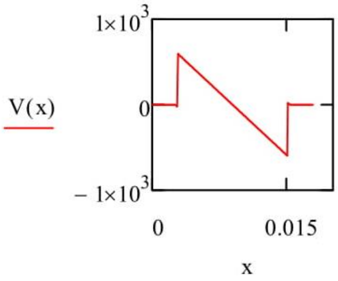
$$R_1 := F + W - R_2 = 624.721\,\text{N}$$

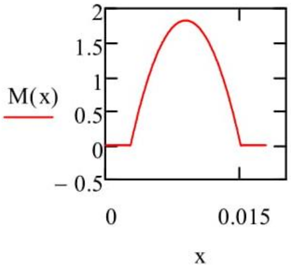
$$q(x) := R_1\cdot S(x,a)\cdot(x-a)^{-1} + R_2\cdot S(x,b)\cdot(x-b)^{-1} - w\cdot S(x,0)\cdot(x-0)^0 - \frac{F}{f}\cdot S(x,f_1)\cdot(x-f_1)^0 \ldots$$
$$+\frac{F}{f}\cdot S(x,f_2)\cdot(x-f_2)^0$$

$$V(x) := R_1\cdot S(x,a)\cdot(x-a)^0 + R_2\cdot S(x,b)\cdot(x-b)^0 - w\cdot S(x,0)\cdot(x-0)^1 - \frac{F}{f}\cdot S(x,f_1)\cdot(x-f_1)^1 \ldots$$
$$+\frac{F}{f}\cdot S(x,f_2)\cdot(x-f_2)^1$$

$$M(x) := R_1\cdot S(x,a)\cdot(x-a)^1 + R_2\cdot S(x,b)\cdot(x-b)^1 - \frac{w}{2}S(x,0)\cdot(x-0)^2 - \frac{F}{2\cdot f}\cdot S(x,f_1)\cdot(x-f_1)^2 \ldots$$
$$+\frac{F}{2f}\cdot S(x,f_2)\cdot(x-f_2)^2$$







Locating the critical point and the calculation of the resulting moment:

Based upon the location of the maximum value of M(x), the critical point of the shaft is located at *L/2*, the midpoint of the shaft.

$$M\left(\frac{L}{2}\right) = 1.825 \cdot N \cdot m$$

Calculation of principal stresses at each of the axial points at the cross-section of L/2:

A is located at the positive y intersection; B is located at the negative x intersection;
C is located at the negative y intersection; D is located at the positive x intersection.

D:

$$\sigma_{Dxx} := \frac{0}{\left(\frac{\pi}{4}\right) \cdot D^2} = 0 \cdot Pa$$

$$\tau_{Dxy1} := \frac{4}{3} \cdot \frac{\left|V\left(\frac{L}{2}\right)\right|}{\frac{\pi}{4} \cdot D^2} = 4.786 \times 10^{-9} \, Pa$$

$$\tau_{Dxy2} := \frac{0 \cdot \frac{D}{2}}{\frac{\pi}{32} \cdot D^4} = 0 \cdot Pa$$

$$\tau_{DxyT} := -\tau_{Dxy1} + \tau_{Dxy2} = -4.786 \times 10^{-9} \, Pa$$

$$\sigma_{1D} := \frac{\sigma_{Dxx}}{2} + \sqrt{\left(\frac{\sigma_{Dxx}}{2}\right)^2 + \tau_{DxyT}^{\;2}}$$

A:

$$\sigma_{Axx1} := \frac{\left|M\left(\frac{L}{2}\right)\right| \cdot \frac{D}{2}}{\left(\frac{\pi}{32}\right) \cdot D^4} = 3.63 \times 10^{7} \, Pa$$

$$\sigma_{Axx2} := \frac{0}{\frac{\pi}{4} \cdot D^2} = 0 \cdot Pa$$

$$\tau_{Axz} := \frac{0 \cdot \frac{D}{2}}{\frac{\pi}{32} \cdot D^4} = 0 \cdot Pa$$

$$\sigma_{AxxT} := -\sigma_{Axx1} + \sigma_{Axx2} = -3.63 \times 10^{7} \, Pa$$

$$\sigma_{1A} := \frac{\sigma_{AxxT}}{2} + \sqrt{\left(\frac{\sigma_{AxxT}}{2}\right)^2 + \tau_{Axz}^{\;2}}$$

$$\sigma_{1D} = 4.786 \times 10^{-9}\,\mathrm{Pa}$$

$$\sigma_{3D} := \frac{\sigma_{Dxx}}{2} - \sqrt{\left(\frac{\sigma_{Dxx}}{2}\right)^2 + \tau_{DxyT}^{\;2}}$$

$$\sigma_{3D} = -4.786 \times 10^{-9}\,\mathrm{Pa}$$

$$\sigma_{vmD} := \sqrt{\sigma_{1D}^{\;2} + \sigma_{3D}^{\;2} - \sigma_{1D}\cdot\sigma_{3D}}$$

$$\sigma_{vmD} = 8.29 \times 10^{-15}\cdot\mathrm{MPa}$$

B:

$$\sigma_{Bxx} := \frac{0}{\left(\frac{\pi}{4}\right)\cdot D^2} = 0\cdot\mathrm{Pa}$$

$$\tau_{Bxy1} := \frac{4}{3}\cdot\frac{\left|V\left(\frac{L}{2}\right)\right|}{\frac{\pi}{4}\cdot D^2} = 4.786 \times 10^{-9}\,\mathrm{Pa}$$

$$\tau_{Bxy2} := \frac{0\cdot\frac{D}{2}}{\frac{\pi}{32}\cdot D^4} = 0\cdot\mathrm{Pa}$$

$$\tau_{BxyT} := \tau_{Bxy1} + \tau_{Bxy2} = 4.786 \times 10^{-9}\,\mathrm{Pa}$$

$$\sigma_{1B} := \frac{\sigma_{Bxx}}{2} + \sqrt{\left(\frac{\sigma_{Bxx}}{2}\right)^2 + \tau_{BxyT}^{\;2}}$$

$$\sigma_{1B} = 4.786 \times 10^{-9}\,\mathrm{Pa}$$

$$\sigma_{3B} := \frac{\sigma_{Bxx}}{2} - \sqrt{\left(\frac{\sigma_{Bxx}}{2}\right)^2 + \tau_{BxyT}^{\;2}}$$

$$\sigma_{3B} = -4.786 \times 10^{-9}\,\mathrm{Pa}$$

$$\sigma_{vmB} := \sqrt{\sigma_{1B}^{\;2} + \sigma_{3B}^{\;2} - \sigma_{1B}\cdot\sigma_{3B}}$$

$$\sigma_{1A} = 0\,\mathrm{Pa}$$

$$\sigma_{3A} := \frac{\sigma_{AxxT}}{2} - \sqrt{\left(\frac{\sigma_{AxxT}}{2}\right)^2 + \tau_{Axz}^{\;2}}$$

$$\sigma_{3A} = -3.63 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{vmA} := \sqrt{\sigma_{1A}^{\;2} + \sigma_{3A}^{\;2} - \sigma_{1A}\cdot\sigma_{3A}}$$

$$\sigma_{vmA} = 36.296\cdot\mathrm{MPa}$$

C:

$$\sigma_{Cxx1} := \frac{\left|M\left(\frac{L}{2}\right)\right|\cdot\frac{D}{2}}{\left(\frac{\pi}{32}\right)\cdot D^4} = 3.63 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{Cxx2} := \frac{0}{\frac{\pi}{4}\cdot D^2} = 0\cdot\mathrm{Pa}$$

$$\tau_{Cxz} := \frac{0\cdot\frac{D}{2}}{\frac{\pi}{32}\cdot D^4} = 0\cdot\mathrm{Pa}$$

$$\sigma_{CxxT} := \sigma_{Cxx1} + \sigma_{Cxx2} = 3.63 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{1C} := \frac{\sigma_{CxxT}}{2} + \sqrt{\left(\frac{\sigma_{CxxT}}{2}\right)^2 + \tau_{Cxz}^{\;2}}$$

$$\sigma_{1C} = 3.63 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{3C} := \frac{\sigma_{CxxT}}{2} - \sqrt{\left(\frac{\sigma_{CxxT}}{2}\right)^2 + \tau_{Cxz}^{\;2}}$$

$$\sigma_{3C} = 0\,\mathrm{Pa}$$

$$\sigma_{vmC} := \sqrt{\sigma_{1C}^{\;2} + \sigma_{3C}^{\;2} - \sigma_{1C}\cdot\sigma_{3C}}$$

$\sigma_{vmB} = 8.29 \times 10^{-15} \cdot MPa$

$\sigma_{vmC} = 36.296 \cdot MPa$

At cross section *L/2*, points A and C are the most important for evaluation. This is due to the fact that they have the higher von Mises stress values.

$\sigma_{vmA} = 36.296 \cdot MPa$

$\sigma_{vmB} = 8.29 \times 10^{-15} \cdot MPa$

$\sigma_{vmC} = 36.296 \cdot MPa$

$\sigma_{vmD} = 8.29 \times 10^{-15} \cdot MPa$

Calculation of safety factors at section *L/2* using the Distortion Energy Theory:

$$N_B := \frac{S_y}{\sigma_{vmB}} = 2.46 \times 10^{16}$$

$$N_A := \frac{S_y}{\sigma_{vmA}} = 5.619$$

$$N_D := \frac{S_y}{\sigma_{vmD}} = 2.46 \times 10^{16}$$

$$N_C := \frac{S_y}{\sigma_{vmC}} = 5.619$$

Calculation of fatigue strength:

$\text{load} := \text{"bending"} \qquad \text{surface} := \text{"machined"} \qquad T := 72$

$$S_e := \left|\begin{array}{l} \text{return } \left(0.5 \cdot S_{ut}\right) \text{ if } S_{ut} \le 200\text{ksi} \\ (100\text{ksi}) \text{ otherwise} \end{array}\right. = 178.45 \cdot \text{MPa} \qquad R := 0.9999 \qquad d := D$$

$d_{equiv} := d = 0.25 \cdot \text{in}$

$$C_{size} := \left|\begin{array}{ll} \text{return } 1 \text{ if } d \le 0.3\text{in} & \\ \text{return } 0.869 \cdot \left(\dfrac{d_{equiv}}{\text{in}}\right)^{-0.097} & \text{if } 0.3\text{in} < d \le 10\text{in} \\ \text{return } 1.869 \cdot \left(\dfrac{d_{equiv}}{\text{in}}\right)^{-0.097} & \text{if } 8\text{mm} < d \le 250\text{mm} \\ 0.6 \text{ otherwise} & \end{array}\right. = 1$$

$$C_{load} := \left|\begin{array}{l} \text{return } 1 \text{ if load} = \text{"bending"} \\ \text{return } 1 \text{ if load} = \text{"torsion"} \\ \text{return } 0.7 \text{ if load} = \text{"axial"} \end{array}\right. = 1$$

$$A := \left|\begin{array}{l} \text{return } 1.34 \text{ if surface} = \text{"ground"} \\ \text{return } 2.70 \text{ if surface} = \text{"machined"} \\ \text{return } 2.70 \text{ if surface} = \text{"cold\_rolled"} \\ \text{return } 14.4 \text{ if surface} = \text{"hot\_rolled"} \\ \text{return } 39.9 \text{ if surface} = \text{"forged"} \end{array}\right. = 2.7$$

$$b := \left|\begin{array}{l} \text{return } (-0.085) \text{ if surface} = \text{"ground"} \\ \text{return } (-0.265) \text{ if surface} = \text{"machined"} \\ \text{return } (-0.265) \text{ if surface} = \text{"cold\_rolled"} \\ \text{return } (-0.718) \text{ if surface} = \text{"hot\_rolled"} \\ \text{return } (-0.995) \text{ if surface} = \text{"forged"} \end{array}\right. = -0.265$$

$$C_{temp} := \left| \begin{array}{l} \text{return } 1 \text{ if } T \le 840 \\ [1 - 0.0032 \cdot (T - 840)] \text{ otherwise} \end{array} \right. = 1$$

$$C_{reliab} := \left| \begin{array}{l} \text{return } 1.000 \text{ if } R = 0.50 \\ \text{return } 0.897 \text{ if } R = 0.90 \\ \text{return } 0.814 \text{ if } R = 0.99 \\ \text{return } 0.753 \text{ if } R = 0.999 \\ \text{return } 0.702 \text{ if } R = 0.9999 \\ \text{return } 0.659 \text{ if } R = 0.99999 \end{array} \right. = 0.702 \qquad C_{surf} := A \cdot \left(\frac{S_{ut}}{ksi} \right)^{b} = 0.949$$

$$S_{e1} := C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S_{e} = 1.189 \times 10^{8} \cdot Pa$$

$$S_{m} := \left| \begin{array}{l} \text{return } (0.75 \cdot S_{ut}) \text{ if load} = \text{"axial"} \\ (0.9 \cdot S_{ut}) \text{ otherwise} \end{array} \right. = 3.212 \times 10^{8} \cdot Pa$$

Calculation of function pertaining to fatigue strength:

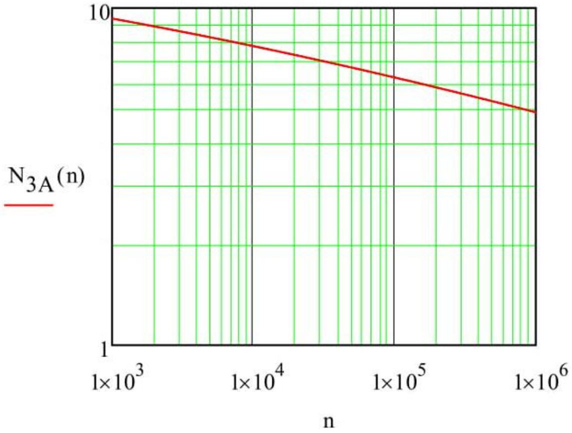
$$S_{f} = a \cdot N^{b} \qquad z := -3.000$$

$$b := \frac{1}{z} \cdot \log\left(\frac{S_{m}}{S_{e1}} \right) = -0.144 \qquad a := \frac{S_{m}}{\left(10^{3}\right)^{b}} = 8.681 \times 10^{8} \cdot Pa$$

$$S_{f}(n) := \left| \begin{array}{l} \text{return } a \cdot n^{b} \text{ if } n < 10^{6} \\ S_{e1} \text{ otherwise} \end{array} \right.$$

Calculation of Final Safety Factor:

$$N_{3A}(n) := \frac{S_{f}(n) \cdot S_{ut}}{\frac{\sigma_{vmA}}{2} \cdot S_{ut} + \frac{\sigma_{vmA}}{2} \cdot S_{f}(n)}$$



$\text{infinL} := 10^6$

$\text{finL} := 10^5$

$N_{3A}(\text{infinL}) = 4.913$

$N_{3A}(\text{finL}) = 6.232$

Deflection at point *L/2*:

$l := f_2 - f_1 = 0.013\,\text{m}$

$E := 200\text{GPa}$

$I := \frac{1}{12} \cdot l \cdot D^3 = 2.71 \times 10^{-10}\,\text{m}^4$

$\delta := \frac{F \cdot l^3}{48 \cdot E \cdot I} = 9.838 \times 10^{-7}\,\text{m}$

$\delta = 9.838 \times 10^{-4} \cdot \text{mm}$

Stress Analysis – Point B

$L := 17.526\text{mm}$ $D := 6.35\text{mm}$ $\gamma := 7870\,\frac{\text{kg}}{\text{m}^3}$ $F := 280.876\text{lbf} = 1.249\cdot\text{kN}$

$a := \left(2.413 + \frac{3.175}{2}\right)\text{mm} = 4\cdot\text{mm}$ $b := L - a = 13.525\cdot\text{mm}$

$f := 6.35\text{mm}$ $f_1 := \frac{L}{2} - \frac{f}{2} = 5.588\cdot\text{mm}$ $f_2 := \frac{L}{2} + \frac{f}{2} = 11.938\cdot\text{mm}$

$w := \left(\frac{\pi}{4}\right)\cdot D^2\cdot\gamma\cdot g = 2.444\cdot\frac{\text{N}}{\text{m}}$ $W := w\cdot L = 0.043\,\text{N}$

$S_y := 203943242.6\text{Pa} = 203.943\cdot\text{MPa}$ $S_{ut} := 356900674.5\text{Pa} = 356.901\cdot\text{MPa}$

Defining the singularity functions:

$x := 0\text{m}, 0.005L\,..\,L$ $S(x,z) := \left|\begin{array}{l}\text{return } 1 \text{ if } x > z\\ 0\end{array}\right.$

At x = R_1, M = 0

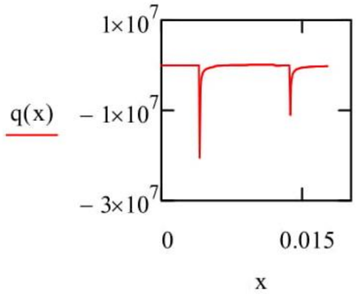
$$R_2 := \frac{-1}{(b-a)}\cdot\left[F\cdot\left(\frac{L}{2} - a\right) + W\cdot\left(\frac{L}{2} - a\right)\right] = -624.721\,\text{N}$$

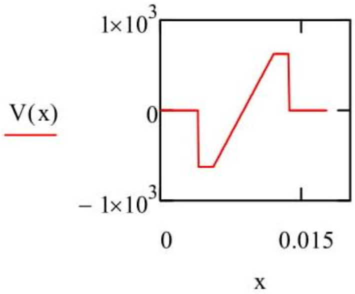
$$R_1 := -F - W - R_2 = -624.721\,\text{N}$$

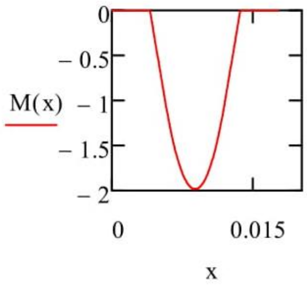
$$q(x) := R_1\cdot S(x,a)\cdot(x-a)^{-1} + R_2\cdot S(x,b)\cdot(x-b)^{-1} - w\cdot S(x,0)\cdot(x-0)^0 + \frac{F}{f}\cdot S(x,f_1)\cdot(x-f_1)^0 \;... \\ + \frac{-F}{f}\cdot S(x,f_2)\cdot(x-f_2)^0$$

$$V(x) := R_1\cdot S(x,a)\cdot(x-a)^0 + R_2\cdot S(x,b)\cdot(x-b)^0 - w\cdot S(x,0)\cdot(x-0)^1 + \frac{F}{f}\cdot S(x,f_1)\cdot(x-f_1)^1 \;... \\ + \frac{-F}{f}\cdot S(x,f_2)\cdot(x-f_2)^1$$

$$M(x) := R_1\cdot S(x,a)\cdot(x-a)^1 + R_2\cdot S(x,b)\cdot(x-b)^1 - \frac{w}{2}\cdot S(x,0)\cdot(x-0)^2 + \frac{F}{2\cdot f}\cdot S(x,f_1)\cdot(x-f_1)^2 \;... \\ + \frac{-F}{2\cdot f}\cdot S(x,f_2)\cdot(x-f_2)^2$$







Locating the critical point and the calculation of the resulting moment:

Based upon the location of the maximum value of M(x), the critical point of the shaft is located at *L/2*, the midpoint of the shaft.

$$M\left(\frac{L}{2}\right) = -1.984 \cdot N \cdot m$$

Calculation of principal stresses at each of the axial points at the cross-section of L/2:

A is located at the positive y intersection; B is located at the negative x intersection;
C is located at the negative y intersection; D is located at the positive x intersection.

D:

$$\sigma_{Dxx} := \frac{0}{\left(\frac{\pi}{4}\right) \cdot D^2} = 0 \cdot Pa$$

$$\tau_{Dxy1} := \frac{4}{3} \cdot \frac{\left|V\left(\frac{L}{2}\right)\right|}{\frac{\pi}{4} \cdot D^2} = 1.804 \times 10^3 \, Pa$$

$$\tau_{Dxy2} := \frac{0 \cdot \frac{D}{2}}{\frac{\pi}{32} \cdot D^4} = 0 \cdot Pa$$

$$\tau_{DxyT} := -\tau_{Dxy1} + \tau_{Dxy2} = -1.804 \times 10^3 \, Pa$$

$$\sigma_{1D} := \frac{\sigma_{Dxx}}{2} + \sqrt{\left(\frac{\sigma_{Dxx}}{2}\right)^2 + \tau_{DxyT}^{\,2}}$$

A:

$$\sigma_{Axx1} := \frac{\left|M\left(\frac{L}{2}\right)\right| \cdot \frac{D}{2}}{\left(\frac{\pi}{32}\right) \cdot D^4} = 3.946 \times 10^7 \, Pa$$

$$\sigma_{Axx2} := \frac{0}{\frac{\pi}{4} \cdot D^2} = 0 \cdot Pa$$

$$\tau_{Axz} := \frac{0 \cdot \frac{D}{2}}{\frac{\pi}{32} \cdot D^4} = 0 \cdot Pa$$

$$\sigma_{AxxT} := -\sigma_{Axx1} + \sigma_{Axx2} = -3.946 \times 10^7 \, Pa$$

$$\sigma_{1A} := \frac{\sigma_{AxxT}}{2} + \sqrt{\left(\frac{\sigma_{AxxT}}{2}\right)^2 + \tau_{Axz}^{\,2}}$$

$$\sigma_{1D} = 1.804 \times 10^{3}\,\mathrm{Pa}$$

$$\sigma_{3D} := \frac{\sigma_{Dxx}}{2} - \sqrt{\left(\frac{\sigma_{Dxx}}{2}\right)^{2} + \tau_{DxyT}{}^{2}}$$

$$\sigma_{3D} = -1.804 \times 10^{3}\,\mathrm{Pa}$$

$$\sigma_{vmD} := \sqrt{\sigma_{1D}{}^{2} + \sigma_{3D}{}^{2} - \sigma_{1D}\cdot\sigma_{3D}}$$

$$\sigma_{vmD} = 3.124 \times 10^{-3}\cdot\mathrm{MPa}$$

$$\sigma_{1A} = 0\,\mathrm{Pa}$$

$$\sigma_{3A} := \frac{\sigma_{AxxT}}{2} - \sqrt{\left(\frac{\sigma_{AxxT}}{2}\right)^{2} + \tau_{Axz}{}^{2}}$$

$$\sigma_{3A} = -3.946 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{vmA} := \sqrt{\sigma_{1A}{}^{2} + \sigma_{3A}{}^{2} - \sigma_{1A}\cdot\sigma_{3A}}$$

$$\sigma_{vmA} = 39.455\cdot\mathrm{MPa}$$

B:

$$\sigma_{Bxx} := \frac{0}{\left(\frac{\pi}{4}\right)\cdot D^{2}} = 0\cdot\mathrm{Pa}$$

$$\tau_{Bxy1} := \frac{4}{3}\cdot\frac{\left|V\left(\frac{L}{2}\right)\right|}{\frac{\pi}{4}\cdot D^{2}} = 1.804 \times 10^{3}\,\mathrm{Pa}$$

$$\tau_{Bxy2} := \frac{0\cdot\frac{D}{2}}{\frac{\pi}{32}\cdot D^{4}} = 0\cdot\mathrm{Pa}$$

$$\tau_{BxyT} := \tau_{Bxy1} + \tau_{Bxy2} = 1.804 \times 10^{3}\,\mathrm{Pa}$$

$$\sigma_{1B} := \frac{\sigma_{Bxx}}{2} + \sqrt{\left(\frac{\sigma_{Bxx}}{2}\right)^{2} + \tau_{BxyT}{}^{2}}$$

$$\sigma_{1B} = 1.804 \times 10^{3}\,\mathrm{Pa}$$

$$\sigma_{3B} := \frac{\sigma_{Bxx}}{2} - \sqrt{\left(\frac{\sigma_{Bxx}}{2}\right)^{2} + \tau_{BxyT}{}^{2}}$$

$$\sigma_{3B} = -1.804 \times 10^{3}\,\mathrm{Pa}$$

$$\sigma_{vmB} := \sqrt{\sigma_{1B}{}^{2} + \sigma_{3B}{}^{2} - \sigma_{1B}\cdot\sigma_{3B}}$$

C:

$$\sigma_{Cxx1} := \frac{\left|M\left(\frac{L}{2}\right)\right|\cdot\frac{D}{2}}{\left(\frac{\pi}{32}\right)\cdot D^{4}} = 3.946 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{Cxx2} := \frac{0}{\frac{\pi}{4}\cdot D^{2}} = 0\cdot\mathrm{Pa}$$

$$\tau_{Cxz} := \frac{0\cdot\frac{D}{2}}{\frac{\pi}{32}\cdot D^{4}} = 0\cdot\mathrm{Pa}$$

$$\sigma_{CxxT} := \sigma_{Cxx1} + \sigma_{Cxx2} = 3.946 \times 10^{7}\,\mathrm{P}$$

$$\sigma_{1C} := \frac{\sigma_{CxxT}}{2} + \sqrt{\left(\frac{\sigma_{CxxT}}{2}\right)^{2} + \tau_{Cxz}{}^{2}}$$

$$\sigma_{1C} = 3.946 \times 10^{7}\,\mathrm{Pa}$$

$$\sigma_{3C} := \frac{\sigma_{CxxT}}{2} - \sqrt{\left(\frac{\sigma_{CxxT}}{2}\right)^{2} + \tau_{Cxz}{}^{2}}$$

$$\sigma_{3C} = 0\,\mathrm{Pa}$$

$$\sigma_{vmC} := \sqrt{\sigma_{1C}{}^{2} + \sigma_{3C}{}^{2} - \sigma_{1C}\cdot\sigma_{3C}}$$

Calculation of fatigue strength:

$$\text{load} := \text{"bending"} \qquad \text{surface} := \text{"machined"} \qquad T := 72$$

$$S_e := \left| \begin{array}{l} \text{return } \left(0.5 \cdot S_{ut}\right) \text{ if } S_{ut} \le 200\text{ksi} \\ (100\text{ksi}) \text{ otherwise} \end{array} \right. = 178.45 \cdot \text{MPa} \qquad R := 0.9999 \qquad d := D$$

$$d_{equiv} := d = 0.25 \cdot \text{in}$$

$$C_{size} := \left| \begin{array}{l} \text{return } 1 \text{ if } d \le 0.3\text{in} \\ \text{return } 0.869 \cdot \left(\dfrac{d_{equiv}}{\text{in}}\right)^{-0.097} \text{ if } 0.3\text{in} < d \le 10\text{in} \\ \text{return } 1.869 \cdot \left(\dfrac{d_{equiv}}{\text{in}}\right)^{-0.097} \text{ if } 8\text{mm} < d \le 250\text{mm} \\ 0.6 \text{ otherwise} \end{array} \right. = 1$$

$$C_{load} := \left| \begin{array}{l} \text{return } 1 \text{ if load} = \text{"bending"} \\ \text{return } 1 \text{ if load} = \text{"torsion"} \\ \text{return } 0.7 \text{ if load} = \text{"axial"} \end{array} \right. = 1$$

$$A := \left| \begin{array}{l} \text{return } 1.34 \text{ if surface} = \text{"ground"} \\ \text{return } 2.70 \text{ if surface} = \text{"machined"} \\ \text{return } 2.70 \text{ if surface} = \text{"cold\_rolled"} \\ \text{return } 14.4 \text{ if surface} = \text{"hot\_rolled"} \\ \text{return } 39.9 \text{ if surface} = \text{"forged"} \end{array} \right. = 2.7$$

$$b := \left| \begin{array}{l} \text{return } (-0.085) \text{ if surface} = \text{"ground"} \\ \text{return } (-0.265) \text{ if surface} = \text{"machined"} \\ \text{return } (-0.265) \text{ if surface} = \text{"cold\_rolled"} \\ \text{return } (-0.718) \text{ if surface} = \text{"hot\_rolled"} \\ \text{return } (-0.995) \text{ if surface} = \text{"forged"} \end{array} \right. = -0.265$$

$$C_{temp} := \left| \begin{array}{l} \text{return } 1 \ \text{ if } T \le 840 \\ [1 - 0.0032 \cdot (T - 840)] \ \text{ otherwise} \end{array} \right. = 1$$

$$C_{reliab} := \left| \begin{array}{l} \text{return } 1.000 \ \text{ if } R = 0.50 \\ \text{return } 0.897 \ \text{ if } R = 0.90 \\ \text{return } 0.814 \ \text{ if } R = 0.99 \\ \text{return } 0.753 \ \text{ if } R = 0.999 \\ \text{return } 0.702 \ \text{ if } R = 0.9999 \\ \text{return } 0.659 \ \text{ if } R = 0.99999 \end{array} \right. = 0.702 \qquad C_{surf} := A \cdot \left(\frac{S_{ut}}{ksi} \right)^{b} = 0.949$$

$$S_{e1} := C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S_e = 1.189 \times 10^{8} \cdot Pa$$

$$S_m := \left| \begin{array}{l} \text{return } \left(0.75 \cdot S_{ut}\right) \ \text{ if load} = \text{"axial"} \\ \left(0.9 \cdot S_{ut}\right) \ \text{ otherwise} \end{array} \right. = 3.212 \times 10^{8} \cdot Pa$$

Calculation of function pertaining to fatigue strength:

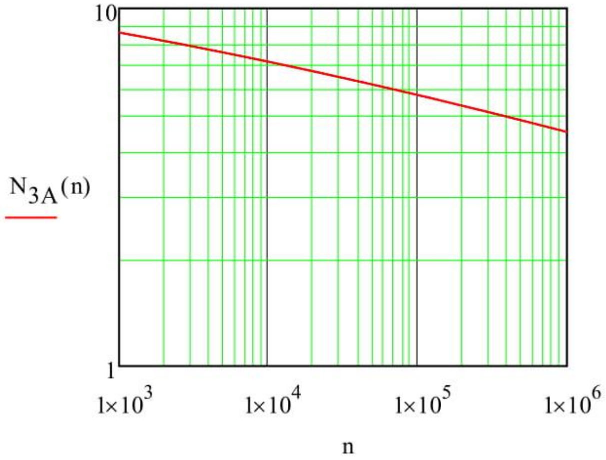
$$S_f = a \cdot N^{b} \qquad z := -3.000$$

$$b := \frac{1}{z} \cdot \log\left(\frac{S_m}{S_{e1}} \right) = -0.144 \qquad a := \frac{S_m}{\left(10^{3}\right)^{b}} = 8.681 \times 10^{8} \cdot Pa$$

$$S_f(n) := \left| \begin{array}{l} \text{return } a \cdot n^{b} \ \text{ if } n < 10^{6} \\ S_{e1} \ \text{ otherwise} \end{array} \right.$$

Calculation of Final Safety Factor:

$$N_{3A}(n) := \frac{S_f(n) \cdot S_{ut}}{\frac{\sigma_{vmA}}{2} \cdot S_{ut} + \frac{\sigma_{vmA}}{2} \cdot S_f(n)}$$



$infinL := 10^6$

$N_{3A}(infinL) = 4.52$

$finL := 10^5$

$N_{3A}(finL) = 5.733$

Deflection at point *L/2*:

$$l := f_2 - f_1 = 6.35 \times 10^{-3}\,m$$

$$E := 200GPa$$

$$I := \frac{1}{12} \cdot l \cdot D^3 = 1.355 \times 10^{-10}\,m^4$$

$$\delta := \frac{F \cdot l^3}{48 \cdot E \cdot I} = 2.459 \times 10^{-7}\,m$$

$$\delta = 2.459 \times 10^{-4} \cdot mm$$

$$\sigma_{vmB} = 3.124 \times 10^{-3} \cdot MPa$$

$$\sigma_{vmC} = 39.455 \cdot MPa$$

At cross section *L/2*, points A and C are the most important for evaluation. This is due to the fact that they have the higher von Mises stress values.

$$\sigma_{vmA} = 39.455 \cdot MPa$$

$$\sigma_{vmB} = 3.124 \times 10^{-3} \cdot MPa$$

$$\sigma_{vmC} = 39.455 \cdot MPa$$

$$\sigma_{vmD} = 3.124 \times 10^{-3} \cdot MPa$$

Calculation of safety factors at section *L/2* using the Distortion Energy Theory:

$$N_B := \frac{S_y}{\sigma_{vmB}} = 6.529 \times 10^4$$

$$N_A := \frac{S_y}{\sigma_{vmA}} = 5.169$$

$$N_D := \frac{S_y}{\sigma_{vmD}} = 6.529 \times 10^4$$

$$N_C := \frac{S_y}{\sigma_{vmC}} = 5.169$$