Risk Analysis for Corporate Bond Portfolios

by

Qizhong Jiang   Yunfeng Zhao

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Professor Marcel Y. Blais, Project Advisor

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Professor Bogdan Vernescu, Head of Department

Contents

Abstract

This project focuses on risk analysis of corporate bond portfolios. We divide the total risk of the portfolio into three parts, which are market risk, credit risk and liquidity risk. The market risk component is quantified by value-at-risk (VaR) which is determined by change in yield to maturity of the bond portfolio. For the credit risk component, we calculate default probabilities and losses in the event of default and then compute credit VaR. Next, we define a factor called 'basis' which is the difference between the Credit Default Swap (CDS) spread and its corresponding corporate bond yield spread (z-spread or OAS[19]). We quantify the liquidity risk by using the basis. In addition we also introduce a Fama-French multi-factor model[16] to analyze the factor significance to the corporate bond portfolio.

Key words: market risk, credit risk, liquidity risk, value at risk, Fama-French multi-factor model

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1 Background

Issuing bonds to the public and to the investors is a main capital resource for a company. A corporate bond is a bond issued by a corporation. It is a bond that a corporation issues to raise money effectively in order to expand its business. All corporate bonds have a feature of maturity date falling at least a year after their issue date.

Generally speaking, a corporate bond is exposed to three main financial risks. The first one being market risk. Market risk is the risk derived by fluctuations of interest rates. We use the LIBOR (London Interbank Offered Rate)[9] as the risk-free interest rate. A corporate bond market price is determined by its yield to maturity. The yield to maturity can be seen as a required return for an investor. This required return is influenced by the risk-free interest rate. A slight fluctuation has an effect on the market price of the bond, and it can lead to a potential loss for the investor. Market risk is an important factor of concern.

Most corporate bonds will pay coupons to investors. The investors will also collect the principal of the bonds at maturity. If a corporation is not able to pay coupons or return the principal, then we say that this corporation has defaulted on its obligation. The second risk of corporate bonds is credit risk. Sovereign risk and counterparty risk are the two branches of credit risk. Credit risk can cause major turmoil in the financial world. For example, Russia defaulted on its foreign debts in 1998, which indirectly caused the Long Term Capital Management bankruptcy (1998)[21]. Another instance of this is the European Debt Crisis (2010)[41] which is still a critical issue today. Credit risk is an essential component of corporate bond risk.

Liquidity risk is the third main risk that a corporate bond investor faces. Sometimes liquidity risk can lead to a major loss or even bankruptcy. For instance, a phenomenon that was frequently observed during the liquidity crises was the flight to liquidity as investors exited illiquid investments and turned to secondary markets in pursuit of cash-like or easily saleable assets. Empirical evidence[11] points towards the widening of bid-ask spreads during periods of liquidity shortage among assets that are otherwise alike but differ in terms of their asset-market liquidity. An example of flight to liquidity occurred during the 1998 Russian financial crisis when the price of Treasury bonds rose sharply relative to the less-liquid debt instruments. This resulted in the widening of credit spreads and major losses at Long Term Capital Management and many other hedge funds. Liquidity risk is a key risk that cannot be ignored.

Many risk analysts say that risk management is an art rather than a science because we not only need to identify and assess risk but we also need to hedge and mitigate the risk. The remainder of this project report concentrates on quantitatively assessing the risk of corporate bond portfolios. Hedging and mitigating risk is left to future research.

2 Market Risk Analysis

In this section we quantify market risk by computing Value-at-Risk (VaR) which is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over a given time horizon exceeds the value (assuming normal markets and no trading in the portfolio) is the given probability level.[28] Before calculating VaR, we define invariants as shocks that steer the stochastic process of the risk drivers over a given time step $[t, t+1]$,

$$\boldsymbol{\epsilon}_{t,t+1} = \left(\epsilon_{1,[t,t+1]}, ... \epsilon_{Q,[t,t+1]} \right)^T$$

where Q is the number of the possible values.

The invariants satisfy the following two properties: a) they are identically and independently distributed (i.i.d) across different time steps; b) they become known at the end of the step, i.e. at time $t+1$.[30]

2.1 Invariant of Corporate Bond

First, we need to test whether the bond prices can act as market invariants. Let us say a corporate bond is issued today with maturity Apr-25-2023 and coupon rate 10%. It is clear that as the maturity date approaches, the bond price converges to the redemption value. If we buy the bond on Apr-24-2023 and the bond retires in the next second, then we will instantaneously get the principal back. In order to prevent arbitrage, the market price of the bond must be equal to the principal value. Therefore it is easy to understand that bond market price converges to its principal value as the maturity date approaches. Thus the market price of corporate bonds cannot be an invariant.

To find an invariant, we must formulate the problem in a time-homogenous framework by eliminating the redemption date.[31] Let P_t^T be the time t market price of a zero-coupon bond with time to maturity T and $P_{t+\tau}^T$ be the market price of another zero-coupon bond with the same time to maturity T at time $t+\tau$. The pricing formula for a zero-coupon bond is [39]:

$$P_t^T = Ne^{-Y_t T} \tag{1}$$

where Y_t is called yield to maturity at time t and N is its principal value. Then we define

$$R_{t,\tau} = \frac{P_{t+\tau}^T}{P_t^T} = \frac{Ne^{-Y_{t+\tau}T}}{Ne^{-Y_t T}} = e^{T(Y_t - Y_{t+\tau})}. \tag{2}$$

This implies that

$$Y_t - Y_{t+\tau} = \frac{1}{T} \ln(R_{t,\tau}). \tag{3}$$

To see if all these random variables are candidates for being invariants, we perform the two simple tests on the time series of the past realizations of these random variables.

First we split the series of $R_{t,\tau}$ in two halves and plot the histogram of each half. If all $R_{t,\tau}$ are identically distributed, the histogram from the first sample of the series should resemble the histogram from the second sample. In Figure 1 we see that this is the case.

The second test uses a scatter-plot of the time series of $R_{t,\tau}$ against the same time series lagged by one estimation interval. If each $R_{t,s}$ is independent of $R_{t+s,s}$ and they are identically distributed, then the scatter plot must resemble a circular cloud. For more on this subject, see e.g. Hamilton (1994)[18], Campbell, Lo, and MacKinlay (1997)[10], Lo and Mackinlay (2002)[29]. In Figure 1 we see that this is indeed the case.
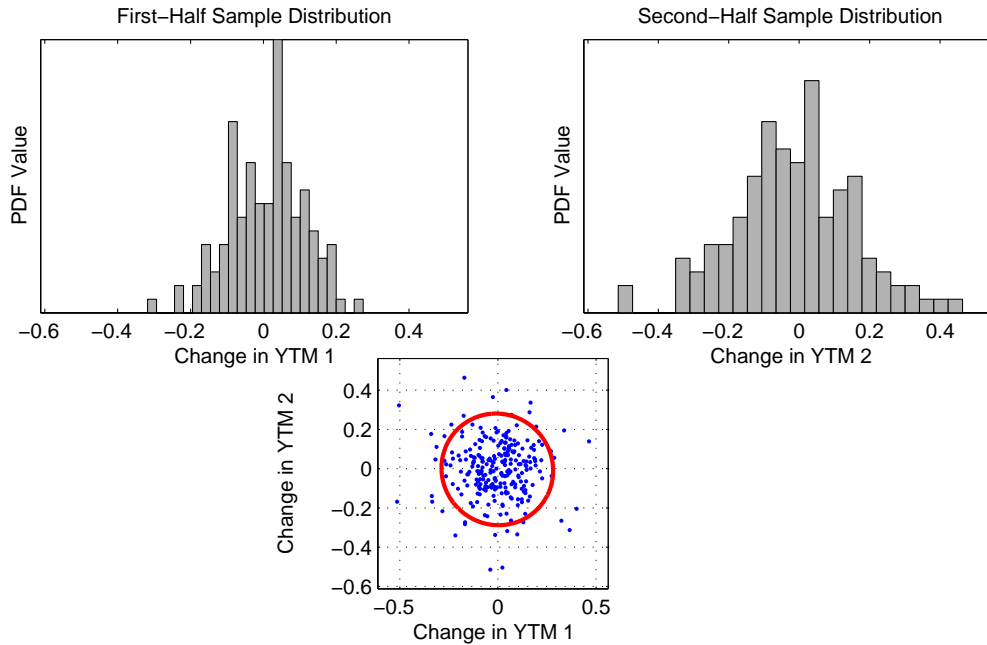
Figure 1: Invariants Test

Any linear function of independent identically distributed (i.i.d.) random variables is also i.i.d..[17] Therefore any linear function f of R defines new invariants. In addition, each coupon of a corporate bond can be viewed as a zero-coupon bond. Hence the change in yield to maturity $Y_t - Y_{t+\tau}$ can be the invariant for the corporate bond even for the fixed-income market.

2.2 Construction of Corporate Bond Portfolio

We select 30 corporate bonds issued by 10 different corporations from the corporate bond market and form an equally-weighted portfolio, with the weights of each of the 30 bonds being all 3.33%. Since the approach of the construction is simple we can concentrate on risk analysis, which is the main topic of the project. Additionally, in practice, portfolio managers do not usually form an optimal bond portfolio since there

are factors that need to be considered for bonds like default probabilities, recovery rates, embedded call features, and coupon rates.[40] Forming an optimal bond portfolio is challenging.

2.3 Computation of Market VaR of the Portfolio

There are three major approaches to calculating value-at-risk; the variance-covariance (delta-normal) method, historical simulation, and Monte Carlo simulation.

2.3.1 Variance-Covariance Approach

For the variance-covariance method, we need to assume risk variables (log-return in equity market and the change in yield to maturity in the fixed-income market) follow a specific distribution (usually the normal distribution). The two moments of the distribution, mean and variance, are calculated by the formulas:

$$\mu_p = \boldsymbol{W}^T \boldsymbol{\mu}_a$$
$$\sigma_p^2 = \boldsymbol{W}^T \boldsymbol{C} \boldsymbol{W}$$

where μ_p and σ_p^2 are the mean and variance of the portfolio respectively.
$\boldsymbol{W}$ is a vector of weights of each bond,
$\boldsymbol{C}$ is the covariance matrix of change in yield to maturity of different bonds in the time series,
$\boldsymbol{\mu}_a$ is a vector of the expectation of change in yield to maturity of each bond.
Then we can calculate VaR with confidence level 99% by the formula[27]:

$$\text{VaR}_{99\%} = \mu_p - \sigma_p \boldsymbol{N}^{-1}(99\%) \tag{4}$$

where $\boldsymbol{N}^{-1}(99\%)$ is 99% quantile of normal distribution $\boldsymbol{N}(\mu_p, \sigma_p^2)$.

The strength of the variance-covariance approach is that VaR is simple to compute once you have made an assumption about the distribution of risk factors and inserted the means, variances, and covariances of risk factors; however, this approach has three limitations. First, the normal distribution is not realistic in many cases. If risk factors are not normally distributed, then the formula for the computation of VaR will also be affected, otherwise it would understate the true VaR. The second limitation is input error. Even if the distribution assumption holds, the VaR can still be wrong if the variances and covariances that are used to estimate it are incorrect. The last limitation is non-stationary variables which occurs when technically the entire distribution can change across assets over time.

4

2.3.2 Historical Simulation Approach

Historical simulation is the simplest method of estimating the VaR for many portfolios. To run a historical simulation, we begin with time series data on each market risk factor just as we do in the variance-covariance approach; however, we do not use the data to estimate variances and covariances looking forward since the changes in the portfolio over time yield all the information we need to compute the VaR.

We form order statistics for the historical time series of risk factors and sort them in descending order. With 99% confidencel, the 99th percentile of the historical ordered risk factor data is the VaR$_{99\%}$.[3]

While this approach is popular and relatively easy to run, it has some weaknesses. For example, historical simulation is excessively dependent on past data. If some historical data is lost, then there may be a substantial error in the final result. Additionally, trends in the data are ignored in historical simulation. The approach assigns the same weight to every data point. In reality, yesterday's events weigh more on the present than the past year's events.

2.3.3 Monte Carlo Simulation Approach

The first two steps in a Monte Carlo simulation[4] are same as the variance-covariance approach where we identify the market risk factors and calculate the mean vector and covariance matrix including each asset. Next we will generate sample paths of the risk factors. For example, we suppose the risk factor follows multivariate normal distribution, which has the following probability density function [20]:

$$f_{\boldsymbol{x}}(x_1, ..., x_k) = \frac{1}{\sqrt{|(2\pi)^k \boldsymbol{C}|}} exp(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^T \boldsymbol{C}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})). \tag{5}$$

Once the distributions are specified, the simulation process can begin. In each run, the market risk variables take on different outcomes and the value of the portfolio reflects the outcomes. After a repeated series of runs, we will have a distribution of portfolio values that can be used to assess VaR. For example, assume we run a series of 1,000,000 simulations and derive corresponding values for the portfolio. These values can be ranked from highest to lowest, and the 99% VaR is the 100th lowest value.

Unlike the variance-covariance approach, Monte Carlo simulation is easily understood and can be applied to almost all kinds of financial securities and instruments; however, it usually requires a high time-cost in terms of computation because it depends upon the number of repeated runs.

Since the distribution of risk factors (change in yield to maturity) has a heavy fat-tail feature, we reject the variance-covariance approach. Additionally, due to the strong

data dependencies and trends in data, we also reject historical simulation. Although the multivariate normal distribution assumption is inapproriate for the Monte Carlo simulation, the central limit theorem (CLT)[2] and the law of large numbers[13] support that Monte Carlo simulation is more accurate than the other two methods in the computation of VaR for fixed-income securities.[8] By using the Monte Carlo simulation approach we derive the VaR of the market risk component from our dataset and estimate that $VaR_{99\%} = 0.48\%$. This implies that if we invest 1,000,000 USD in our portfolio today, then there is a 1% chance that we could lose more than 4,800 USD in the next month.

3 Credit Risk Analysis

Corporate bond investors are exposed to credit risk because the counterparty (corporation) may default on its obligations. Therefore in order to measure credit risk of corporate bond portfolios, we need to consider three important factors. They are default probabilities, losses given default, and exposures to each asset.

In this section we will discuss approaches of computing default risk. The Merton model[1] is an extension of the Black-Scholes option pricing model[6]. It estimates the value of the bondholders' and stockholders' claims, which are then used to estimate the implied probability of default. Moody's KMV[5] model improves the Merton model by relaxing some of the assumptions. These two methods are fequently used in analyzing the default probabilities of corporate bonds.

3.1 Merton Model

Before moving on to the Merton model, let us discuss the Black-Scholes option pricing formula first. The Black-Scholes model of the market for a specific stock makes the following explicit assumptions:

- There are no arbitrage opportunities.

- It is possible to borrow and lend cash at a known constant risk-free interest rate.

- Short selling is allowed.

- The stock price follows a geometric Brownian motion with constant drift and volatility.

- The underlying security does not pay a dividend.

- The financial market is frictionless.

- Fractional amounts of the underlying asset can be traded.

Based on these strong assumptions, we can deduce following Black-Scholes partial differential equation:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{6}$$

where f is a European style option price at time t. S is the underlying market price at time t. r is risk-free interest rate, and it is also the drift of the geometric Brownian motion under the risk-neutral measure. σ is the volatility of the underlying. The analytical solution is [37]:

$$c_0 = S_0 N(d_1) - Ke^{-rT}N(d_2)$$

$$p_0 = Ke^{-rT}N(-d_2) - S_0 N(-d1)$$

where:

$$d_1 = \frac{ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

c_0 is the price of the European call option with strike price K and maturity T, and p_0 is the price of the European put option with strike price K and maturity T.

For a corporation its asset value is equal to the sum of its liabilities and equity value. The Merton model assumes a corporation only issues one zero-coupon bond during its business cycle. Therefore the accounting formula: 'Asset = Liability + Equity' will be transformed into

$$A_T = B + S_T \tag{7}$$

where A_T is the corporation's asset value at maturity, S_T is the market capitalization of the corporation at maturity, and B is the face value of the zero-coupon bond. Similarly we assume the asset price follows a geometric Brownian motion. At maturity T, if $A_T \leq B$, then S_T will be equal to zero and the corporation faces default on its obligation. Therefore we can rewrite the formula as:

$$S_T = max(A_T - B, 0).$$

This implies the market capitalization of the corporation's equity value can be explained as a call option on the corporation's assets with strike price equal to the face value of the zero-coupon bond. Using the Black-Scholes model we can obtain the formula for S_0 under the Merton model,

$$S_0 = A_0 N(d_1) - Be^{rT}N(d_2) \tag{8}$$

where

$$d_1 = \frac{\ln(\frac{A_0 e^{rT}}{B})}{\sigma_A\sqrt{T}} + \frac{1}{2}\sigma_A\sqrt{T}$$

$$d_2 = d_1 - \sigma_A\sqrt{T},$$

σ_A is the volatility of the asset value, and r is the risk-free rate of interest, both of which are assumed to be constant.

Next we deal with the default probability. Define $B_0 = Be^{-rT}$ as the present value of the future payoff of the zero-coupon bond, and let $L = \frac{B_0}{A_0}$ be a measure of the leverage of the corporation[24]. Plug these two parameters back into the solution of the formula (8) to obtain

$$S_0 = A_0[N(d_1) - LN(d_2))] \tag{9}$$

where

$$d_1 = -\frac{\ln(L)}{\sigma_A\sqrt{T}} + \frac{1}{2}\sigma_A\sqrt{T}$$

$$d_2 = d_1 - \sigma_A\sqrt{T}.$$

Since the equity value is a function of the asset value we can apply Ito's lemma[25] to determine the instantaneous volatility of the equity from the asset volatility:

$$S_0\sigma_S = \frac{\partial S}{\partial A}A_0\sigma_A \tag{10}$$

where σ_S is the instantaneous volatility of the company's equity at time zero. From formula (9) we obtain

$$\frac{A_0}{S_0} = \frac{1}{N(d_1) - LN(d_2)}, \tag{11}$$

and from formula (8) we obtain

$$\frac{\partial S}{\partial A} = N(d_1). \tag{12}$$

Therefore equations (10), (11) and (12) lead to

$$\sigma_S = \frac{\sigma_A N(d_1)}{N(d_1) - LN(d_2)}. \tag{13}$$

Equations (9) and (13) allow A_0 and σ_A to be obtained from observed S_0, L, and T and the statistical estimate σ_S. The risk-neutral probability that the corporation will default at the maturity T is the probability that shareholders will not exercise their call option to buy the assets of the company for value B at the maturity time T. The probability is given by [34]

$$P = 1 - N(d_2). \tag{14}$$

The Merton model is simple to understand and easy to implement. It makes sense for non-mathematicians to use market data such as stock values and capital structure to predict the probability of default. While this is applicable, it also has some problems

due to its unrealistic assumptions. For example, if there is only one issue of equity and debt, and if the debt is in the form of a zero-coupon bond that matures at a given date, then default can only occur at the maturity date. The value of the firm is observable and follows a lognormal diffusion process (geometric Brownian motion). There is no negotiation between shareholders and debtholders, there is no need to adjust for liquidity, and the risk-free interest rate is constant.

3.2 Moody's KMV model

The KMV model is built on the Merton model and tries to improve some of its shortcomings. The most notable change is that it relaxes the assumption that all the debt mature at the same time. The KMV model divides the debt of a corporation into short-term liabilities and long-term liabilities which have different maturities. We combine these two values together to determine the default point. A practical rule for the determination of the default threshold[14] is:

$$B = SL + \frac{1}{2}LL \tag{15}$$

where SL stands for the short-term liabilities, LL refers to the long-term liabilities, and B is default threshold.

In the Merton model, we know there are three elements that determine the default probability of a firm: the value of company's assets, the uncertainty of the assets (i.e. volatility of the assets), and leverage.

There are six variables that determine the default probability of a firm over some horizon, from the current time until H (see Figure 2)
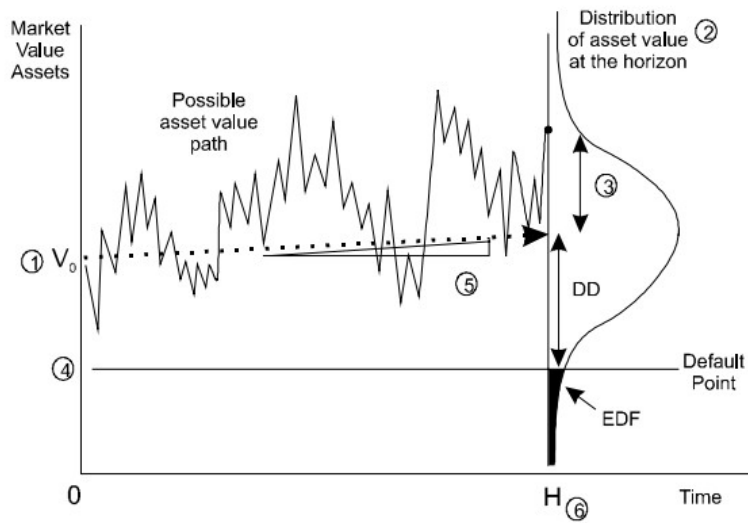

Figure 2: KMV model (Source: Crosbie and Bohn, 2004)

9

In the Figure 2 above,[12] ① is the current asset value. ② is the distribution of the asset value at time H. ③ is the volatility of the future assets value at time H. ④ is the level of the default point. ⑤ is the expected rate of growth in the asset value over the horizon. ⑥ is the length of the horizon, H.

If the value of the assets falls below the default point, then the firm defaults. Therefore the probability of default is the probability that the asset value will fall below the default point. This is depicted by the shaded area (Expected Default Frequency) below the default point in Figure 2.

There are essentially three steps in the computation of the default probability of a firm. First, estimate the asset value and the volatility from the formulas (9) and (13). We can observe market capitalization of the company which is the product of the stock price and the number of outstanding shares and then derive the σ_S by estimating the variance of the market value of the company. Second, KMV measures the distance-to-default as the number of standard deviations the asset value is away from default, and then uses empirical data to determine the corresponding default probability. The distance-to-default is calculated (DD) as

$$DD = \frac{E(A) - B}{\sigma_A} \tag{16}$$

where $E(A)$ refers to expected value of the asset and B is the default threshold.

From formula (16) we learn that DD is the number of standard deviations away from the default threshold B. Lastly after we obtain the DD we compare it to our historical database to find the corresponding DD in the historical default distribution. The KMV model needs a database that includes over 250,000 company-years of data and over 4,700 incidents of default or bankruptcy. From this data a lookup or frequency table can be generated which relates the likelihood of default to various levels of distance-to-default.[32]

For example, assume that we are interested in determining the default probability over the next year for a firm that is 7 standard deviations away from default. To determine this expected default frequency (EDF) value, we query the default history for the proportion of the firms. Seven standard deviations away from default that defaulted over the next year. The answer is about 5 basis points(bp), 0.05%, or an equivalent credit rating of AA according to Moody's.[38]

In order to estimate the default risk of the corporate bond portfolio under the KMV model, we require some fundamental data for the corporation like short-term and long-term liabilities and outstanding shares. The shortest period of financial statement issuing is quarterly and this project is focused on risk analysis in a 1-month investment

horizon. Hence we only illustrate the idea of the KMV model in the report rather than presenting the implementation process.

3.3 Calculation of Credit VaR of the Portfolio

The expected loss is equal to the product of the default probability, loss given default, and particular bond exposure,

$$L^t = W^T P_d^t LGD \tag{17}$$

where W is the weight of each asset in the portfolio, P_d^t is vector containing default probabilities of every asset at time t, LGD is a vector of loss given the default of each bond (we assume all bonds have a recovery rate of 30% over the bonds' life, which means loss given default is 70%). Therefore L^t is the vector of the expected loss driven by credit risk at time t.

Based on formulas (9), (13), and (14), we can generate a matrix of default probabilities which contains time series of default probabilities of each bond. Then by using formula (17), we can derive a time series of the expected loss of each bond.

In order to simplify the computation, we assume the default risks are linearly correlated. Then we use Monte Carlo simulation to generate the expected loss of the portfolio in one month. Next we select the 99th percentile of the simulation value to be the credit VaR with a 99% confidence level.[26] The result is VaR$_{99\%}$ = 4%. This means that if we invest 1,000,000 USD in our portfolio today, there is a 1% chance that we could lose more than 40,000 USD next month.

4 Liquidity Risk Analysis

4.1 Definition of Basis

Quantifying liquidity risk is the hardest part of risk analysis because it is difficult to find a parameter that represents the liquidity features of a corporate bond.

We have already discussed that a corporate bond is exposed to three main risks which are market risk, credit risk, and liquidity risk. In order to bear these three risks, an investor requires a specified yield from the corporate bond. We know that a major part of market risk is driven by the fluctuation of the interest rate. We could say that the bond yield spread (i.e. the difference between the bond yield and the risk-free interest) is the return we require to bear both the credit risk and the liquidity risk. In many financial markets there is an instrument called the credit default swap (CDS) which was invented by Blythe Masters from JP Morgan in 1994. "A credit default swap (CDS) is a financial swap agreement that the seller of the CDS will compensate the buyer in the

event of a loan default or other credit event. The buyer of the CDS makes a series of payments to the seller and, in exchange, receives a payoff if the loan defaults."[42] The CDS spread is defined by the payments on the face value of the protection, and these payments are made uptill a credit event occurs or uptill maturity.[35] The CDS spread represents the default risk of the specific corporate bond.[22] Therefore if we subtract the CDS spread from the bond yield spread, we conjecture that the remainder is the 'yield' we require to bear the liquidity risk.
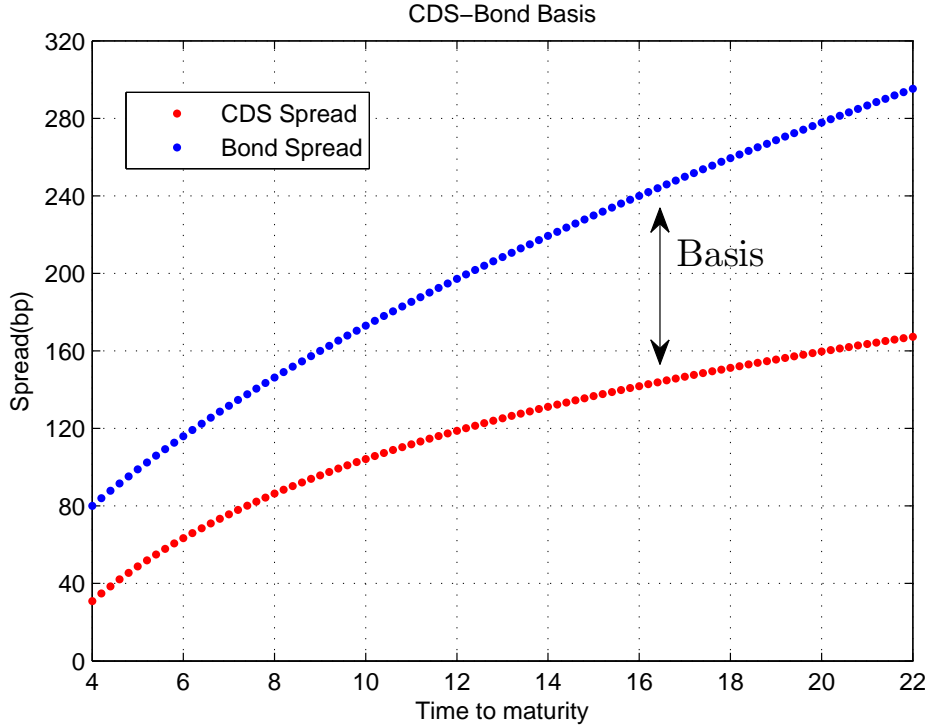
Figure 3: CDS spread-Bond yield Basis

The risk leads to a potential loss, so we define

$$b = SP_{CDS} - SP_{yield} \qquad (18)$$

where SP_{CDS} and SP_{yield} refer to the CDS spread and the bond's yield spread, respectively, and b is called the basis, which is used as a parameter to quantify the liquidity risk.

CDS spreads and corporate bond yield spreads are observable in the financial market, so the basis is easy to obtain; however, acquiring data for the CDS spreads is expensive, thus we are going to value the CDS spreads in the next step.

4.2 Valuation of Credit Default Swap Spread

A credit default swap (CDS) is the most popular form of protection against default. The goal of CDS valuation is to determine the value of the mid-market (i.e., average of bid and ask prices) CDS spread on the reference entity, which is the company upon which default protection is bought and sold. There are four steps to calculating the CDS spread[23]:

Step 1: Calculate the present value of expected payments. Payments are made at the CDS spread rate, s, and multiplied by the reference entity's probability of survival each year. The sum of the present value of each annual payment equals the present value of expected payments.

Step 2: Calculate the present value of the expected payoff in the event of default. To make this calculation, we first need to make an assumption about when defaults occur. It is common to assume that defaults occur halfway through a year. This is because almost all the corporate bonds pay coupons biannually. If the corporation cannot pay the coupon, then we can claim that it defaulted on its obligation. The assumptions make sense from a practical perspective. From these assumptions the annual default probability is multiplied by the reference entity's loss given default and discounted back to the present.

Step 3: Calculate the present value of the accrual payment in the event of default. Since payments are made in arrears, an accrual payment is required in the event of default to account for the time between the beginning of the year and the time when default actually occurs. If we assume that defaults occur halfway through the year, the accrual payment will be $\frac{1}{2}s$. The annual accrual payment is multiplied by the probability of default each year and discounted to the present.

Step 4: Calculate the CDS spread. Solve for s by equating the expected present value of total payments to the expected present value of payoff in the event of default.

From the above steps, we can derive the following formula:

$$s = \frac{PV(payoff)}{PV(payments)} \tag{19}$$

where PV_{payoff} means the present value of the expected payoff in the event of default and $PV_{payments}$ means the present value of the sum of expected payments made at the CDS spread rate, s, and accrual payments in the event of default.

It is also assumed that each CDS has the same maturity as the corresponding corporate bond and that the default probabilities and recovery rates remain constant. Based on these assumptions we can predict both the future payoff and the total ex-

pected payments (the sum of payments made at the CDS spread rate and accrual payment in the event of default).

We use this approach to approximate the CDS spread upon these 30 reference bonds. We have already estimated probability of default in each month, therefore we can generate a time series of CDS spreads for each reference bond.

4.3 Estimation of Liquidity VaR of the Portfolio

We have already defined a parameter called *basis* which is the difference between the CDS spread and its reference bond's yield spread. Usually the CDS spread is less than the bond yield spread. Thus the basis is a negative value. As we have already discussed above, the remainder of subtracting the CDS spread from the reference bond's yield spread is the investor's required return to bear the liquidity risk. So the following formula shows a potential loss of the portfolio because of the liquidity risk,

$$L_{liq} = \boldsymbol{W^T b} \tag{20}$$

where L_{liq} is the loss of the portfolio derived from liquidity risk, $\boldsymbol{W}$ is the vector of weights of each bond, and $\boldsymbol{b}$ is the vector of *basis* of each bond.

Formula (16) helps us estimate the basis of every bond at the beginning of each month. Thus we can obtain a time series for the basis. Next we estimate the mean and covariance of the estimated historical *basis* and apply the multivariate normal distribution assumption to run Monte Carlo simulations to calculate the liquidity VaR with 99% confidence one month later. The result is $\text{VaR}_{99\%} = 5.73\%$. This means that if we invest 1,000,000 USD in our portfolio today, there is a 1% chance that we could lose more than 57,300 USD next month.

5 Common Risk Factors in the Returns on Bonds

This section identifies five common risk factors in the returns on bonds; three factors of which come from the stock market. Stock returns have shared variation due to the stock-market factors and they are linked to bond returns through shared variation in the bond-market factors. Except for low-grade corporate bonds, the bond-market factors capture the common variation in bond returns.

When considering stock and bond returns, one famous approach is called the Fama-French common risk factors model[15]. In this report we are going to test our bond portfolio by using these common factors. Intuitively if we assume that the financial market is an integrated market (i.e. financial securities' prices among different locations or related goods follow similar patterns over a long period), we seek some common factors that affect both stocks' and bonds' average returns.

The list of empirical factors that we may consider contains securities' systematic risk β, issue size, leverage of stocks or bonds, book-to-market ratio, $TERM$, and DEF. The followings are these factors' definitions and considerations in our report.

5.1 Introduction to Fama-French Factors

First of all, let us consider some common factors that might be related to stock expected average returns.

The first one is the systematic risk β, which is derived from the Capital Asset Pricing Model (CAPM).[36] Under CAPM we have

$$E(R_i) = R_f + \beta_i(E(R_m - r)), \tag{21}$$

where R_m is the market expected return, and r is the risk-free return. The market risk β represents how risky one financial security is. If $\beta > 1$, we conclude that this security is more risky than the market and is less risky than the market otherwise. Based on Fama and French's research, the cross-section of average returns on U.S. common stocks shows little relation with this β. In this report we ignore this factor in our regression model.

Issue size, intuitively speaking, is related to one security's profitability. The bigger the issue size, the more liquid the security, and the less the bid-ask spread would be. In this sense investors prefer to invest in these bonds associated with large size issue, and large size issue could result in a negative relationship with average return. In the business world the small firms usually would get lower credit ratings, and they usually do not issue bonds as large as some big companies. The size effect would be more obvious during some specific periods. We use the issue size as the first common factor. The size is considered as following:

$$Size = SN \tag{22}$$

where S is stock market price and N is numbers of outstanding shares in the market.

The leverage factor is another important factor in this paper. Typically the higher the leverage of one security, the higher the expected return would be. The reason is quite simple, higher return would be associated with higher risk. For stocks the leverage is defined as[7]

$$Leverage = \frac{Earnings}{Price}. \tag{23}$$

Since the market is efficient under our assumption, the leverage factor is considered not only for stocks but also for bonds.

The book-to-market factor is naturally one important factor we should take into account. This risk factor is defined as

$$BM_{ratio} = \frac{BE}{ME} \tag{24}$$

where BE refers to the book value (the value at which an asset is carried on a balance sheet) of a corporation's equity, and ME is the market value of the equity.

Based on the definition of this risk factor, we expect that the higher book-to-market ratio, the higher the expected return will be, and vice versa. Given our assumptions, this risk not only affects the stock securities but also bond securities.

Now we form the first three Fama-French factors raised from the above considerations. First, we split the NYSE, Amex, and NASDAQ stocks into two groups (High and Low) using the median NYSE stock size. Second, break NYSE, Amex and NAS-DAQ stocks into three categories based on the breakpoints for the bottom 30% (Low), middle 40% (Medium), and top 30% (High) of the ranked values of book-to-market equity for NYSE stocks. Third, construct six portfolios from the intersection of the two ME and the three book-to-market equity groups. Fourth, set the SMB (small minus big) factor as the difference between simple average of the returns on the three small stock portfolios and the simple average of the returns on the big stock portfolios. Fifth, let the HML (high minus low) factor be the difference between simple average of the returns on the three high stock portfolios and the simple average of the returns on the small stock portfolios.

Finally, the last factor in our paper is the excess market return $R_m - r$. R_m is the return on the value-weighted portfolio of the stocks in the six $\frac{BE}{ME}$ portfolios plus the negative BE stocks excluded from the portfolios. r is the one-month Treasury bill rate.

In addition to all the above factors, we need to impose some term-structure bond market factors in our pool. Two different type indicators are natural choices. The first one is the term premium (TERM), the other one is from the default premium side.

The $TERM$ factor is the difference between the monthly long-term (more than 20y) government bond return and the one month Treasury bill rate measured at the end of the previous month, which is

$$TERM = R_{LTMG} - R_{bill} \tag{25}$$

where R_{LTMG} stands for the long-term government bond return and R_{bill} refers to the 1-month T-bill return.

In our case the 30-year Treasury constant maturity bond yield is chosen to be the

long-term government bond return. This risk factor should capture the risk premium that arises from an unexpected interest rate change. Especially for the bond market, the interest rate risk (duration risk) is one main risk source. We would use the sample data set to test this factor and try to quantify the term premium as accurately as possible.

The DEF factor is the other bond market variable in our regression model. As mentioned before, the default premium accounts for a significant portion of a bond's expected return. The U.S. Treasury bonds are typically considered to be risk-free investments, especially for short-term bonds; however, the corporate bonds (including investment-grade bonds) and long term government bonds can no longer be considered to be risk-free any more. Typically corporate bonds have a higher yield since it accounts for both default risk and liquidity risk. We define

$$DEF = R_{LTMP} - R_{LTMG} \tag{26}$$

where R_{LTMP} refers to the long-term market corporate bond portfolio return and R_{LTMG} is the long-term government bond portfolio return.

In this project the long-term government bond return is the 30-year U.S. Treasury bond's yield. The long-term corporate bonds' returns are from the composite portfolio on the corporate bond module of Ibbotson Associate.[33]

5.2   Cross-Sectional Regression

Once these factors are formed, we are ready to analyze the original Fama-French common factors model[15], which is

$$r = \beta_0 + \beta_1(R_m - r) + \beta_2 SMB + \beta_3 HML + \beta_4 TERM + \beta_5 DEF + \epsilon \tag{27}$$

where ϵ is the error term that follows normal distribution with mean zero and constant variance. In the above model the bonds' returns would be excess returns, so we subtracted from these returns the 1-month Treasury bill rate. The reason is his would result in a well-specified asset-pricing model that produces intercepts which are indistinguishable from 0. The estimated intercepts give us a simple return metric and it would give a relatively better test of how the combination of these factors affect the average returns.

6   Results

In Table 1 the second column R^2 tells us the explanatory power of our model. For example, $R^2 = 83.09\%$ shows that about 83.09% of the variation of the response variable average return is explained by this factor model. In Table 2 the $F - test$

statistic is large and we get quite a small $p-value$, thus that we conclude that this model is significant. Notice that about one half of the R^2 are about 50% or below. One main reason for this is that some historical bond yields are generated by sample data. Details are displayed in Table 1.

Table 1: R-square and Factor Coefficients

NASDSYMBOL	R-square	Intercept	Factor Coefficient				
			Mkt-RF	SMB	HML	TERM	DEF
ETFC3691819	83.09%	0.0379	0.0702	0.0011	-0.0276	1.8937	1.9238
FITB.HE	79.11%	0.5197	0.0281	-0.0675	-0.0249	0.4815	0.5364
BAC.HSL	74.57%	1.0969	0.0849	-0.0696	-0.0175	1.1731	0.5776
C.HFV	69.11%	0.4821	0.0221	-0.0217	-0.0471	0.4248	0.4471
BAC.GWX	62.84%	2.9003	0.0498	-0.1053	-0.0997	1.3004	1.7969
AIG3890850	62.29%	0.3779	-0.0335	-0.0197	0.0216	0.3538	0.7749
BAC.HAU	60.82%	0.7115	0.1255	-0.1576	-0.0246	1.455	0.8745
FITB.GZ	57.60%	0.9359	0.0512	-0.0913	-0.0163	0.5934	0.1084
BLK3858251	56.95%	0.008	-0.0216	0.0299	0.0427	0.563	0.9169
AVB3899398	55.87%	0.5844	0.0742	-0.1254	-0.0275	0.9411	1.0628
AMP.GF	53.58%	0.9828	-0.002	0.0074	-0.0113	0.4003	-0.169
AIG3857774	52.52%	0.528	0.0087	0.009	-0.0021	0.4034	0.4749
COF.HS	51.95%	3.3478	0.0382	-0.0949	0.0137	0.5536	0.8605
FITB.AB	51.49%	0.4719	0.0577	-0.1403	-0.0436	0.6017	0.7016
SCHW.AD	33.40%	0.3519	-0.0433	-0.065	0.0696	0.0774	1.2464
AMP.GF	31.43%	0.6556	0.0782	-0.1876	0.0825	0.5937	1.157
AVB3899398	27.69%	0.6069	-0.0596	-0.0079	0.0488	-0.0795	0.5995
SCHW.AD	27.69%	0.6069	-0.0596	-0.0079	0.0488	-0.0795	0.5995
BLK.GJ	27.67%	0.6069	-0.0596	-0.0079	0.0488	-0.0795	0.5995
COF.AC	27.67%	0.6069	0.0596	-0.0079	0.0488	-0.0795	0.5995
ETFC3719327	27.55%	1.7567	0.1372	-0.3198	0.0056	1.0342	-0.1713
AIG.JCZ	27.03%	0.5975	-0.0565	-0.0202	0.0463	-0.0796	0.6053
COF3833890	26.72%	1.5332	0.1895	-0.284	0.0008	1.3175	0.6383
AVB.GT	25.33%	0.6382	-0.0588	-0.0086	0.0552	-0.0754	0.5528
C.HFN	24.55%	2.4076	0.2585	-0.3727	-0.0969	1.5887	-0.1704
BLK.GJ	24.53%	0.6446	-0.0571	-0.0175	0.0685	-0.0835	0.6283
C.HEM	23.47%	2.3213	0.2895	-0.422	-0.0109	1.9269	0.4282
SCHW.AA	21.38%	0.5267	-0.0487	-0.0273	0.0624	0.0248	0.6105
ETFC3691819	20.56%	3.5671	0.0713	-0.1289	0.0557	0.7725	1.3802
AMP.GE	12.55%	1.3487	0.0208	-0.104	0.0029	0.3802	0.2767

Data from Feb-2-2006 to Feb-2-2013

We move on to the significance test of each factor. For most bonds' average returns, the $TERM$ and DEF factors are quite significant as the $p-value$ is less than 0.05. This indicates that the term premium significantly affects the bonds' returns. All the estimated parameters and their corresponding $p-values$ are listed in Table 2.

The results of VaR derived by Monte Carlo simulation are illustrated in Table 3. They

Table 2: p-value

NASDSYMBOL	Factor p-value					Model Test
	Mkt-RF	SMB	HML	TERM	DEF	F-test
ETFC3691819	0.0892	0.9901	0.0815	<0.0001	0.0004	0.0024
FITB.HE	0.0174	0.008	0.234	<0.0001	0.0006	<0.0001
BAC.HSL	0.0062	0.2868	0.7481	<0.0001	0.1416	<0.0001
C.HFV	0.1045	0.4564	0.0548	<0.0001	0.0118	<0.0001
BAC.GWX	0.308	0.316	0.426	<0.0001	0.0052	<0.0001
AIG3890850	0.0281	0.5437	0.4241	<0.0001	0.0001	<0.0001
BAC.HAU	0.0178	0.1617	0.7924	<0.0001	0.1948	<0.0001
FITB.GZ	0.0202	0.0528	0.6745	0.0001	0.6982	<0.0001
BLK3858251	0.3885	0.5791	0.3436	<0.0001	0.0056	<0.0001
AVB3899398	0.0622	0.1411	0.6968	<0.0001	0.0387	<0.0001
AMP.GF	0.8934	0.8209	0.6794	<0.0001	0.3897	<0.0001
AIG3857774	0.6379	0.8201	0.9495	<0.0001	0.0484	<0.0001
COF.HS	0.1464	0.0943	0.7704	<0.0001	0.0123	<0.0001
FITB.AB	0.0407	0.0211	0.3841	0.0001	0.0542	<0.0001
SCHW.AD	0.0189	0.0983	0.0351	0.1495	<0.0001	<0.0001
AMP.GF	0.0946	0.063	0.3235	<0.0001	0.0559	<0.0001
AVB3899398	0.0001	0.8021	0.0667	0.0688	0.0021	<0.0001
SCHW.AD	0.0001	0.8021	0.0667	0.0688	0.0021	<0.0001
BLK.GJ	0.0001	0.8201	0.0667	0.0688	0.0021	0.0004
COF.AC	0.0001	0.8021	0.0067	0.0688	0.0021	<0.0001
ETFC3719327	0.0592	0.0414	0.9657	<0.0001	0.8535	<0.0001
AIG.JCZ	0.0003	0.5297	0.0866	0.0728	0.0023	0.0001
COF3833890	0.0576	0.1832	0.9963	<0.0001	0.6161	0.0001
AVB.GT	0.0002	0.7898	0.0434	0.0912	0.0054	0.0003
C.HFN	0.0318	0.1563	0.6569	<0.0001	0.9133	0.0004
BLK.GJ	0.0006	0.6105	0.0188	0.0784	0.003	<0.0001
C.HEM	0.0638	0.2021	0.9686	<0.0001	0.8299	0.0006
SCHW.AA	0.0034	0.4336	0.034	0.6017	0.0043	0.0016
ETFC3691819	0.358	0.4393	0.6882	0.001	0.1689	<0.0001
AMP.GE	0.6446	0.2845	0.9713	0.0051	0.6457	0.0558

Data from Feb-2-2006 to Feb-2-2013

illustrate that the market risk is the lowest risk ($\text{VaR}_{99\%} = 0.48\%$) among these three components and that the liquidity risk is the largest ($\text{VaR}_{99\%} = 5.73\%$). Change in yield to maturity, default probability, and the difference between the CDS-Bond basis are three critical factors influencing the risk of the corporate bond portfolios. A bond portfolio manager needs to consider these three risk components at the same time. Separating the total risk into these three risk components helps the bond portfolio to manager manage the portfolio actively to meet his risk preference. For example, if a bond portfolio manager has a high risk preference to liquidity risk, he can select specific bonds with a larger CDS-Bond basis, but lower yield and higher credit rating to optimize his portfolio.

Table 3: $\text{VaR}_{99\%}$ of Different Components

Market VaR	Credit VaR	Liquidity VaR
0.48%	4.00%	5.73%

We assign a serial number to every bond and list them in the order of default probability in decreasing order. The results are illustrated in Table 4 and Figure 4. It is clear that the larger the CDS spread, the more likely the reference entity will default on its obligations. The more negative the basis, the more liquidity risk the bond bears. The corporate bond with the highest credit risk may suffer the highest liquidity risk because investors seldom want to buy a bond with high default risk. Our results show that the bond with the lowest credit risk may not have the lowest liquidity risk.

Table 4: Corporate Bond Portfolio

| Serial Number	NASDSYMBOL	Default Prob	CDS_spread	Basis(liquid)
1	C.HEM	18.54%	145.21	-2116.4
2	C.HFN	14.12%	104.93	-85.9
3	COF3833890	13.23%	97.33	-1200.9
4	ETFC3691819	12.29%	89.46	-182.4
5	BAC.GWX	11.30%	81.34	-569.6
6	ETFC3719327	9.59%	67.72	-1431.5
7	BAC.HAU	8.52%	59.49	-866.2
8	COF.HS	6.71%	45.94	-46.8
9	AVB3899398	6.22%	42.35	-46.8
10	AMP.GF	5.58%	37.73	-114.2
11	BAC.HSL	5.57%	37.71	-1067.7
12	AMP.GE	5.17%	34.86	-517.7
13	FITB.GZ	4.32%	28.85	-233.3
14	ETFC3691819	3.63%	24.06	-955.4
15	AIG3890850	3.39%	22.47	-46.8
16	FITB.AB	2.99%	19.70	-477.0
17	FITB.HE	2.13%	13.89	-181.4
18	AIG3857774	2.07%	13.48	-242.6
19	BLK.GJ	1.34%	8.71	-46.8
20	C.HFV	0.99%	6.37	-1364.3
21	SCHW.AD	0.64%	4.09	-46.8
22	AMP.GF	0.55%	3.55	-536.7
23	BLK3858251	0.50%	3.19	-12.4
24	SCHW.AA	0.50%	3.19	-34.7
25	AVB.GT	0.50%	3.19	-46.8
26	SCHW.AD	0.50%	3.19	-46.8
27	AIG.JCZ	0.50%	3.19	-243.6
28	BLK.GJ	0.50%	3.19	-267.9
29	AVB3899398	0.50%	3.19	-595.3
30	COF.AC	0.50%	3.19	-679.5
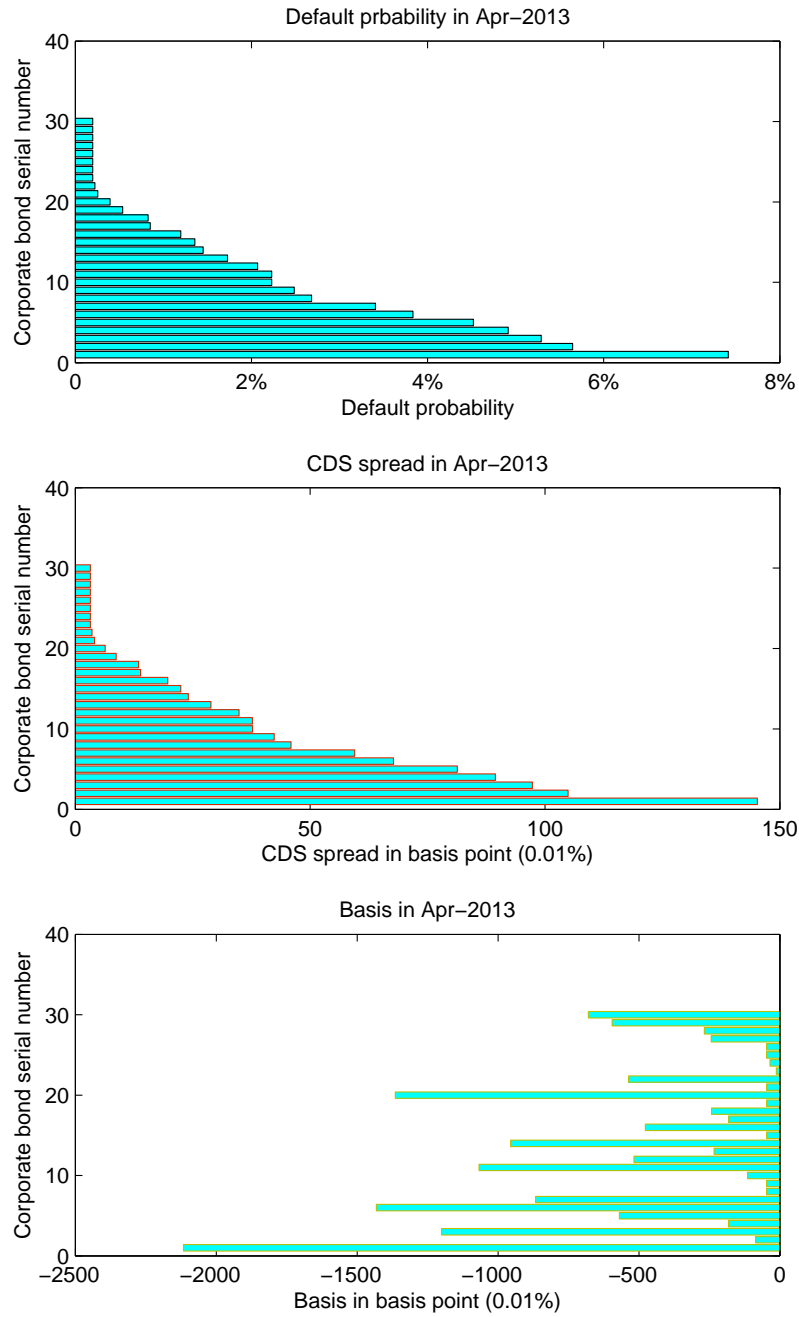
Figure 4: Estimated value in Apr-2013

7 Conclusion

In summary we quantify three different types of risk: market risk, credit risk and liquidity risk. The results are derived from the sample data using the Monte Carlo simulation method. The results are displayed in Table 3.

The first important risk in a bond portfolio is the market risk, which contains interest rate risk. If the interest rate changes, the value of the portfolio could have a significant fluctuation. According to duration and convexity approximation, which essentially tells us how much percentage change our bond portfolio value will experience, a bond portfolio manager can quantify its market risk.

Another risk that a bond portfolio manager faces is credit risk. The Merton model is used in this project to quantify credit risk. In the Merton model we consider the firm's equity as one European call option on the assets of the company with maturity T and a strike price B which is the face value of the debt.

The third risk is liquidity risk, which is quantified by the basis defined by the difference of the CDS spread and the bond yield spread. As we have discussed in the section 4.1, the CDS spread is considered as a protection against credit risk. Subtracting the CDS spread from the bond yield spread can capture the liquidity feature of the bond.

In addition, we use some common financial factors to capture the bond expected return. Basically the most important factors are related to size and the book-to-market equity ratio. These five factors provide a good description of the cross-section of the average returns, but they do not require that we have identified the true factors. As these factors capture the cross-section of average returns, they can be used to guide portfolio selection. The regression parameter estimates and the historical average premiums for the factors can be used to estimate the expected return of the portfolio. In practice for a specific firm's bond, we have to consider the sampling error seriously. Therefore we conclude that the $TERM$ and DEF bond market factors would significantly affect the bond returns at typical significance levels compared with the three other factors. Finally we check the model assumptions (i.e. error normality), which make our regression model results more reasonable.

8 Further Work

In the risk analysis portion of this report, Monte Carlo simulation is used to approximate the three risks. The major limitation of the Monte Carlo simulation approach is its model risk. Use of different distributions would result in quite different results. Minimization of the error in the approximation is the most important concern. In

addition the Merton model has some unrealistic assumptions. Relaxation of some of the assumptions is also a further research topic.

In our regression model there are some open questions. For example, for some cases with low R^2, some other common factors should be considered to improve the model's explanatory power. Also if we find new factors, do they improve the adjusted-R^2 and do they have a overfitting problem? All of these and other interesting questions are left to our future work.

9 Appendix

Matlab Code:

```matlab
clear;
clc;
%Calculate Market Risk VaR
r = xlsread('wellington.xls','Libor_rate','B2:B113')/100;%short
    -term libor rate
y = xlsread('wellington.xls','fake_data_yield','B2:AF113');%
    yield to maturity
delta_y = y(2:end,:)-y(1:end-1,:); %change in yield to maturity
     (risk factor)
Exp_deltaY = mean(delta_y);
C = cov(delta_y);
W = ones(1,30)*1/30;
Num = 10^4;
A = [];
%simulate change in YTM one month later
for i = 1:Num
    A(i,:) =  mvnrnd(Exp_deltaY, C);
end
FutureGain = A*W';
%Negative means loss
FutureGain_ordered = sort(FutureGain,'descend'); %order
    statistics
%0.99 confidence level VaR
j = Num*0.99-1;
VAR_market = FutureGain_ordered(j)
```

MarketVaR.m

```matlab
% Calculate Credit Risk VaR
for k = 1:length(y)
    PD(k,:) = 1-(1+r(k))./(1+y(k,:));
end
xlswrite('wellington.xls',PD,'default_probability','B2:AE113')
%Assume recover rate is 30% for all bonds
LGD = 0.7;
Loss = LGD*PD;
%Positive means loss here, so negative will mean gain
Exp_Loss = mean(Loss);
Gamma = cov(Loss);
Num = 10^4;
B = [];
%simulate expected loss one month later
for i = 1:Num
```

```
16      B(i,:) =  mvnrnd(Exp_Loss, Gamma);
17  end
18  FutureLoss = B*W';
19  FutureLoss_ordered = sort(FutureLoss,'ascend'); %order
        statistics
20  %0.99 confidence level VaR
21  jj = Num*0.99-1;
22  VAR_Credit = FutureLoss_ordered(jj)
```

CreditVaR.m

```
1   %Calculate CDS spread
2   %The largeste maturity of our bonds is Apr-15-2067
3   %Therefore in order to simplifying computation process we
        assume the
4   %short-term libor rate from May-2013 to Apr-2067 are constant
5   %r = 0.2641
6   %Default probability is also same from valuation date to the
        maturity
7
8   [~,zero_date] = xlsread('wellington.xls','Libor_rate','A2:A165'
        );%
9   zero_rate = xlsread('wellington.xls','Libor_rate','B2:B165')
        /100;%
10  [~,maturity] = xlsread('wellington.xls','portfolio','J2:J31');
11  pd = xlsread('wellington.xls','default_probability','Ae2:Ae165'
        );%
12  zero_data = [datenum(zero_date), zero_rate];
13  prob_data = [datenum(zero_date), pd(:,1)];
14  settle = zero_date(1:112);
15  for n = 2:112
16      spread(n-1) = cdsspread(zero_data,prob_data,settle(n),
            maturity(1));
17  end
18  xlswrite('Book1.xls',spread','Sheet1','Ae3:Ae113')%
```

cdsSpread.m

```
1   % Calculate Liquidity Risk VaR
2   % Basis = CDS spread - OAS(or Z-spread)
3   LIBOR = xlsread('wellington.xls','Libor_rate','B2:B113')*100;%
        in basis point
4   CDS = xlsread('wellington.xls','CDS_spread','B3:AE113');%in
        basis point
5   YIELD = xlsread('wellington.xls','fake_data_yield','B2:AE113')
        *10000;% in basis point
```

```matlab
6  SS = size(YIELD);
7  for NN = 1:length(LIBOR)
8      OAS(NN,:) = YIELD(NN,:) - LIBOR(NN)*ones(1,SS(1,2));
9  end
10
11 for N = 1:length(CDS)
12     BASIS(N,:) = CDS(N,:) - OAS(N,:);
13 end
14 liqui_loss = BASIS/10000;
15 %Negative means loss here
16 Exp_liqui = mean(liqui_loss);
17 Gamma_liqui = cov(liqui_loss);
18 Num = 10^4;
19 D = [];
20 %simulate the basis one month later
21 for i = 1:Num
22     D(i,:) =  mvnrnd(Exp_liqui, Gamma_liqui);
23 end
24 FutureLoss_liquid = D*W';
25 FutureLoss_liquidordered = sort(FutureLoss_liquid,'descend'); %
        order statistics
26 %0.99 confidence level VaR
27 kk = Num*0.99-1;
28 VAR_Liquid = FutureLoss_liquidordered(kk)
```

LiquidityVaR.m

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