

**Student Teaching in Mathematics at  
Tahanto Regional Middle and High School**

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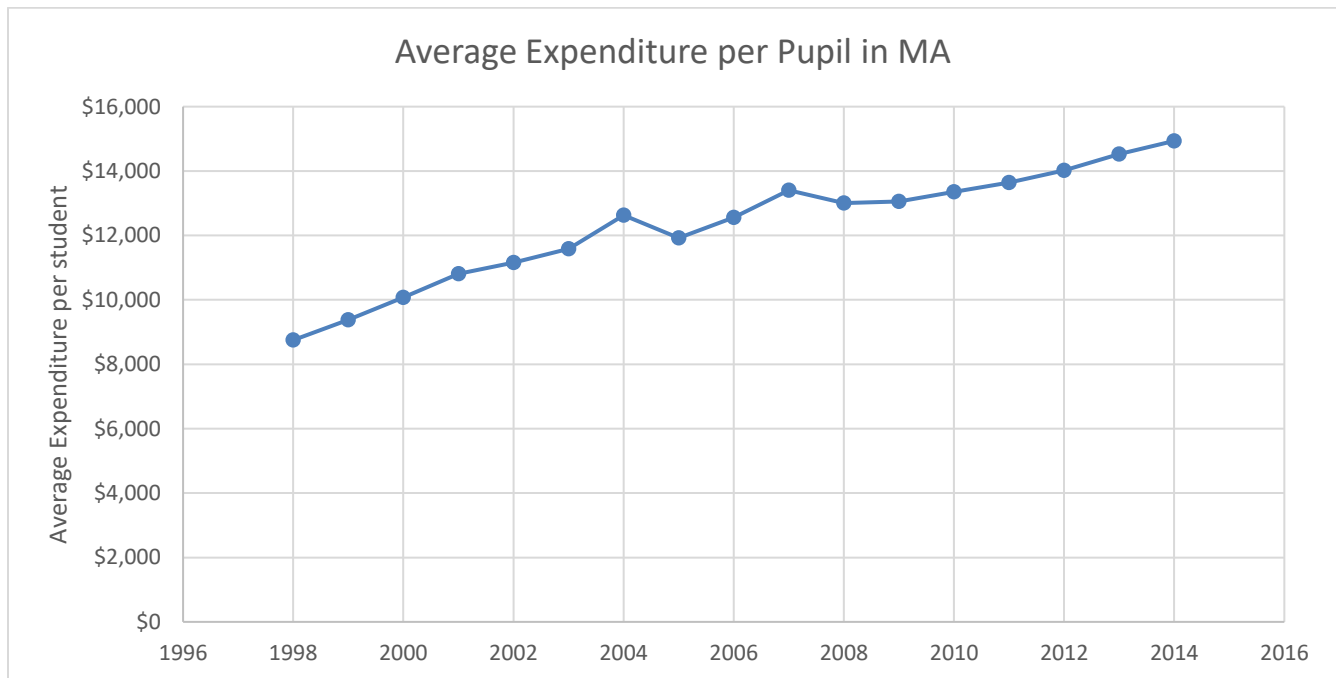
**Abstract**

Through the WPI STEM Education Center, I completed the practicum requirement for an Initial Teaching License in Massachusetts. At Tahanto Regional Middle and High School, I taught three calculus classes under my supervising practitioner, Francene Gleason, and a special needs math class under the guidance of Mrs. Derderian. This program licenses me to teach professionally, and the educational best practices, personal development, and professional habits I learned during this IQP will empower me to share my knowledge in any setting.

## Chapter 1 – Background

Twenty years ago, the State of Massachusetts passed the Massachusetts Educational Reform Act of 1993 (MERA) in response to equality suits such as *McDuffy vs. Robertson*, a financial squeeze in many parts of Massachusetts, and generally mediocre student performance relative to other industrialized nations (*Building on Reform*). The act made significant alterations to the funding and accountability structures within Massachusetts’s existing framework of local school systems.

MERA was designed to increase and equalize expenditure per student across the state (*Building on Reform*). In 1989, the average expenditure per pupil in Massachusetts public schools was \$6,237 (\$10,544 adjusted to 2008 dollars). By 2000, it was \$9,375 (\$11,416 adjusted to 2008 dollars) . This past year (2014-2015) it was \$14,936. Because of MERA, approximately \$2 billion have flown into Massachusetts Public schools in the form of reform aid. The goal of MERA funding was to bring all schools up to a foundational level of about \$5,500 per pupil in 1993 (WPI source). Although funding still varies widely, with some districts spending twice as much per student as others, all the districts on the DOE websites have comfortably exceeded the \$5,500 benchmark, and most spend double that amount per student today.



Made with data from [http://profiles.doe.mass.edu/state\\_report/ppx.aspx](http://profiles.doe.mass.edu/state_report/ppx.aspx) and [https://nces.ed.gov/programs/digest/d10/tables/dt10\\_194.asp](https://nces.ed.gov/programs/digest/d10/tables/dt10_194.asp)

MERA also dramatically increased oversight and accountability for local schools throughout the state of Massachusetts. The Act requires each school to have a school council, spells out the role of school committees, and increases the authority of school principals. (WPI source). It also introduced a statewide curriculum framework called Massachusetts Standards for Learning, which listed specific



content all students must learn, to serve as a minimum requirement for teachers in all core subjects. Most famously, MERA instituted the Massachusetts Comprehensive Assessment System (MCAS), a series of tests all students must take in 4<sup>th</sup>, 8<sup>th</sup>, and 10<sup>th</sup> grade. Funding is tied to the results of these high-stakes tests, and underperformance, especially chronic underperformance, can result in the state asserting its authority over a local school. MERA requires all students within Massachusetts to pass the 10<sup>th</sup> grade MCAS exam in order to earn a high school diploma, including all special needs students. Students who fail the 10<sup>th</sup> grade MCAS may retake the portions failed until they pass all portions of the exam or until they age out.

MERA instituted a verity of other requirements as well. Students are required to spend a fixed number of hours learning core subjects in all districts (900 hours in elementary schools and 990 in middle and high schools). This impacts student schedules and attendance policies. All new certifications for teachers require a passing score on two tests: one on subject content and another on English communication and literacy skills. MERA also instituted charter schools, chartering 22 in 1993, 8 more in 1998, and an additional 3 in 2017 (mass live).

Massachusetts generally performs well compared to the international community. The Trends in International Mathematics and Science Study (TIMMS) samples students across the country and around the globe, testing them in science and mathematics. Massachusetts students are above average in both subjects regardless of sex. Black students, Hispanic students, and students from schools where >75% of the population is eligible for reduced lunch perform only average compared to the international community. 5 countries, all in Southeast Asia, outperform Massachusetts students on the TIMMS: Korea, Singapore, China, Hong Kong, and Japan.

I completed my student teaching in Boylston, MA at Tahanto Regional Middle and High School, a relatively small 6-12 school about 10 miles Northeast of Worcester MA. The Middle School and High School are separate wings of one building, sharing a gymnasium, a cafeteria, and bussing. The 6<sup>th</sup> grade joined the school when the new building was opened in 2013. Other than shared lunch periods, the middle schoolers and high schoolers are kept separate, although one of my special needs students, despite being an 8<sup>th</sup> grader, is taught in the high school in order to keep his environment age-appropriate. There are 542 high school students and roughly the same number of middle school students. The school is 99% white, which means there are less than a dozen non-white students. 53% of the students are female, and 12% of students are eligible for free or reduced lunch. We spend less than average per student, but our scores are above average and we have very few behavior issues in our school.

Year	Expenditure per student (MA)	Expenditure per student (Boylston MA)
2005-2006	\$11,210	\$10,038
2009-2010	\$13,055	\$11,420
2014-2015	\$14,936	\$13,226

Our Mascot is the stag and we seek to foster DEER virtues: Determination, Education, Enrichment, and Responsibility. Our mission and vision are “to support and to challenge students in achieving personal and academic excellence in a safe, collaborative, and student-centered environment” and to “is to create a tradition of developing responsible and reflective citizens who are college/career ready and life-long learners”. Less than a dozen students dropout per year, less than half a dozen fail the 10<sup>th</sup> grade MCAS most years, and the vast majority of students go to college, although a few join the military or attend vocational schools before entering the workforce.



*Tahanto Regional Middle and High School Mascot*

I saw students from all across the academic spectrum at TRMHS. I taught four periods per day: two sections of Honors Calculus, one section of Applications of Calculus, and one section of Special Needs Math. Tahanto offers five levels of Calculus: College Preparation Calculus, Honors Calculus, Advanced Placement Calculus AB, Advanced Placement Calculus BC, and Applications of Calculus. My Honors Calculus sections are my largest classes, with 17 students in the 4<sup>th</sup> period and 13 students in the 5<sup>th</sup> period, a mix Juniors and Seniors. Applications of Calculus is a new course this year for the three seniors who finished the AP track a year early. We explore advanced topics such as series, polar equations, equations of multiple variables, applications in engineering, and whatever the boys are interested in. My most challenging class is Special Needs Math, a pullout session for four students with moderate disabilities that prevent them from learning in a standard classroom environment. I have two tenth graders, a ninth grader, and an eighth grader. They are learning algebra, largely based on Massachusetts Curriculum Framework for Mathematics, with a special focus on money math and topics that appear heavily on the MCAS exam. Of necessity, and as required by their IEPs, their class involves both new topics from the High School standards, such as modeling with linear equations, and significant review of Middle school standards such as basic operations with fractions.

My three calculus classes have completed all Massachusetts standards, and the material I teach them is primarily guided by the need to prepare them for college-level math courses. For the Honors Calculus class I am guided by *Calculus: graphical, numerical, algebraic* (3<sup>rd</sup> edition, Finney), Francine Gleason’s years of expertise, my personal knowledge of the demands of college math courses, and to a lesser extent the interests of the students. For Applications of Calculus the opposite is true; I am primarily guided by student interest and my knowledge of the underlying topics required for more complex topics, and to a lesser extent Francine’s experience and preferences, various textbooks, and MIT open courseware class *18.02SC Multivariable Calculus Fall 2010*. Overall, I have a great deal of flexibility in my curriculum because my students are either well ahead of Massachusetts standards, or require a great deal of individually tailored instruction.

## Works Cited:

Chester, Mitchell D. "Building on 20 Years of Massachusetts Education Reform." Massachusetts Board of Elementary and Secondary Education. Web. 17 May 2017.  
<<http://www.doe.mass.edu/commissioner/BuildingOnReform.pdf>>.

"Digest of Education Statistics, 2010." National Center for Education Statistics. Web. 17 May 2017.  
<[https://nces.ed.gov/programs/digest/d10/tables/dt10\\_194.asp](https://nces.ed.gov/programs/digest/d10/tables/dt10_194.asp)>.

Francis W. Parker Charter Essential School's Regional Teachers Center. "Overview of the Massachusetts Education Reform Act of 1993." Worcester Polytechnic Institute. Web. 17 May 2017.  
<[https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&ved=0ahUKEwiPi7bA5\\_fTAhXCiFQKHV72AW0QFgg7MAI&url=https%3A%2F%2Fusers.wpi.edu%2F~goulet%2Fteacher\\_prep%2FOverview%2520of%2520the%2520Massachusetts%2520Education%2520Reform%2520Act%2520of%25201993.doc&usq=AFQjCNFYDPPm8tE8mJtosFQvO6DKqix6Gw&sig2=18P9R9TmAc5xz2sKvqRMTQ&cad=rja](https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&ved=0ahUKEwiPi7bA5_fTAhXCiFQKHV72AW0QFgg7MAI&url=https%3A%2F%2Fusers.wpi.edu%2F~goulet%2Fteacher_prep%2FOverview%2520of%2520the%2520Massachusetts%2520Education%2520Reform%2520Act%2520of%25201993.doc&usq=AFQjCNFYDPPm8tE8mJtosFQvO6DKqix6Gw&sig2=18P9R9TmAc5xz2sKvqRMTQ&cad=rja)>.

"Massachusetts Comprehensive Assessment System - TIMSS." *Massachusetts State Seal*. Massachusetts Department of Elementary and Secondary Education. Web. 17 May 2017.  
<<http://www.doe.mass.edu/mcas/natl-intl/timss/>>.

"Massachusetts Curriculum Frameworks." *Massachusetts State Seal*. Massachusetts Department of Elementary and Secondary Education, 21 Feb. 2017. Web. 17 May 2017.  
<<http://www.doe.mass.edu/frameworks/current.html>>.

"Massachusetts Curriculum Frameworks." *Mass.gov*. Massachusetts Department of Elementary and Secondary Education. Web. 17 May 2017. <<http://www.doe.mass.edu/frameworks/current.html>>.

"Massachusetts School and District Profiles." *Total Expenditure Per Pupil, All Funds, By Function - Boylston (00390000)*. Massachusetts Board of Elementary and Secondary Education. Web. 17 May 2017.  
<<http://profiles.doe.mass.edu/profiles/finance.aspx?orgcode=00390000&orgtypecode=5&leftNavID=501&fycode=2015>>.

Provasnik, Stephen. "TIMSS 2011 Results for Massachusetts." *TIMSS Mathematics Results for Massachusetts*. National Center for Education Statistics. Web. 17 May 2017.  
<[https://nces.ed.gov/timss/pdf/results11\\_Massachusetts\\_Math.pdf](https://nces.ed.gov/timss/pdf/results11_Massachusetts_Math.pdf)>.

"2014-15 Per Pupil Expenditures Statewide Report." Massachusetts Board of Elementary and Secondary Education. Web. 17 May 2017. <[http://profiles.doe.mass.edu/state\\_report/ppx.aspx](http://profiles.doe.mass.edu/state_report/ppx.aspx)>.

"Welcome." Tahanto Regional Middle and High School. Web. 17 May 2017.  
<<http://tahanto.bbrsd.org/pages/TahantoRegionalMiddleHighSch+>>.

Williams, Michelle. "Three New Charter Schools Approved to Open in Massachusetts, Serving Students from Agawam to Wareham." *Masslive.com*. 28 Feb. 2017. Web. 17 May 2017.  
<[http://www.masslive.com/news/index.ssf/2017/02/three\\_new\\_charter\\_schools\\_appr.html](http://www.masslive.com/news/index.ssf/2017/02/three_new_charter_schools_appr.html)>.

## Chapter 2 – Elements of CAP: Well-Structured Lessons (1.A.4)

The Six Essential Elements of CAP:

Standard	Element	Proficient Descriptor*
1: Curriculum, Planning & Assessment	1.A.4: Well-Structured Lessons	Develops well-structured lessons with challenging, measurable objectives and appropriate student engagement strategies, pacing, sequence, activities, materials, resources, technologies, and grouping.
	1.B.2: Adjustments to Practice	Organizes and analyzes results from a variety of assessments to determine progress toward intended outcomes and uses these findings to adjust practice and identify and/or implement appropriate differentiated interventions and enhancements for students.
2: Teaching All Students	2.A.3: Meeting Diverse Needs	Uses appropriate practices, including tiered instruction and scaffolds, to accommodate differences in learning styles, needs, interests, and levels of readiness, including those of students with disabilities and English language learners.
	2.B.1: Safe Learning Environment	Uses rituals, routines, and appropriate responses that create and maintain a safe physical and intellectual environment where students take academic risks and most behaviors that interfere with learning are prevented.
	2.D.2: High Expectations	Effectively models and reinforces ways that students can master challenging material through effective effort, rather than having to depend on innate ability.
4: Professional Culture	4.A.1: Reflective Practice	Regularly reflects on the effectiveness of lessons, units, and interactions with students, both individually and with colleagues, and uses insights gained to improve practice and student learning.

The Candidate Assessment of Performance system (CAP) consists of three standards, which are further broken down into 6 elements. Standard 1 is Curriculum, Planning, and Assessment, which contains the element Well-structured Lessons (1.A.4). An educator demonstrates proficiency in well-structured lessons when he or she “develops well-structured lessons with challenging, measurable objectives and appropriate student engagement strategies, pacing, sequence, activities, materials, resources, technologies, and grouping”.

Well-structured lessons has always been my strongest element. My disposition and my engineering education naturally lend themselves to goal-oriented lessons and carefully selected methods. Whenever I begin an endeavor, I first answer the question “What are you trying to do?”, then let everything else flow from that goal. My lesson planning begins with the class curriculum and objectives, including state math standards, IEP and student goals, and the overarching mission of preparing students both for future classes and life beyond TRMHS’s walls. From those goals flow lesson objectives and topics, including enrichment topics. Then those goals are hammered into measurable objectives. Finally I select, modify, or design methods and materials for instruction based on the lesson goals and measurable objectives. After each lesson I evaluate how class went to determine if the lesson was completed, or if I need to insert another learning activity tomorrow to meet the lesson goals.

Well-structured lessons are important for several reasons. The first is so obvious to an engineering mind it hardly bears mentioning; if you don’t know your goal you’re unlikely to reach it. Teachers tell their students “Shoot for the moon. Even if you miss you’ll land among the stars”. Challenging goals push the students to learn more, and to learn deeper. Goals provide meaning and direction to lessons for both students and teachers. Goals motivate, guide, and accelerate student learning. Measurable objectives provide reliable feedback for teachers and students. They ensure that the class is going somewhere significant, not just running around like a chicken with its head cut off.

Without a well-structured lesson, some students truly will run around like a bunch of headless chickens. Well-structured lessons keep students engaged and occupied in their learning activities, while poorly

structured lessons force students to wait or allow them to wander. Parents and teachers warn “The devil makes work for idle hands.” Well-structured lessons are not primarily for maintaining behavior in the classroom, but they serve as an excellent preventative measure.

I used the same template for all my lesson plans, but the lesson planning process looked different in each of my four classes. In Applications of Calculus, I began with broader goals of college preparedness and life skills because those students had fulfilled all the state and school math requirements. I selected topics and projects of interest, then built in math objectives that were appropriate for the students and built units around those objectives. Each unit involved a few days of direct instructions and packet work for practice and formative assessment, then usually a small project, then review, then a test, then a review of the test. The students were self-motivated, well-behaved, and focused, so lesson openers and closers were more freewheeling, and pacing was rapid, about a topic per day. My Small Group Math for Special Education Students (SPED) class was very much the opposite. They lacked the learning behaviors necessary to learn readily in a classroom, so openers for them were urgently needed. Most days began with specific instructions on the materials required (pencil, paper, calculator), the instruction to sit down, a brief description of the lesson objective, and an short assessment of the student’s retention of the previous lesson. A well-structured lesson, with well-suited materials, was essential to establishing and maintaining momentum in that class. My lesson plans brought an element of order to the chaos of a special needs room, and they kept us on track to learn material. I also taught two sections of honors calculus, so I used the same lesson plans for both sections. I experimented with a variety of teaching methods in my lesson plans. In the beginning I focused on finding a method of instruction that worked for me, and on ending my lessons with a closer instead of getting hit by the bell. Later in my student teaching I focused on beginning with stronger openers, usually mini-worksheets, definitions, or calculations that would be needed for the main lesson.

My lesson objectives always centered on math I thought the students would need later for future lessons, future classes, or future life. They were always challenging for the students, although sometimes early in my student teaching I didn’t get to them on the first day of a lesson. I also refused to avoid topics students had missed in earlier years, preferring to reteach them concepts such as arithmetic with fractions even though those objectives seemed at once too easy (because they were from lower grades) and too hard (because the students had struggled with them for years). The appropriateness of life-skills math to an upper level math course is an interesting question, but I tried to incorporate it without compromising the other material, and I was usually successful.

I managed both my pacing and my assessment primarily with formative assessments, asking students questions and surveying students’ work several times per class period to evaluate student comprehension, fluency, and interest. On summative assessments I made one question to evaluate each aspect of each objective. For example, if I wanted students to be able take indefinite integrals with the power rule, I would include  $x^3$  as a general test, then  $x$ ,  $1$ , and  $x^{-3}$  to evaluate their ability to work with unusual exponents, then  $x^{\frac{1}{2}}$ , and  $\sqrt[3]{x}$  to assess their skill with fractional exponents. I prefer a battery of small questions to a single large one that contains subparts because it enables students to demonstrate

partial understanding, shows me what confuses them, and puts less emotional pressure on the students, which was important as several of my students had anxiety, academic confidence, or perseverance problems which interfered with their ability to attempt difficult problems.

Over the course of my student teaching I tweaked how I used the lesson planning template and improved my openers and closers, but lesson planning was my strongest element going in so it changed less than other elements did. Nearly every lesson had a lesson plan (although several plans covered multiple days of instruction, especially for projects), and my mentor teacher called my lesson plans “nearly perfect”. Several of the lesson plans I created and used for each class are attached in the appendix.

**Works Cited:**

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017.  
<<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

### **Chapter 3 – Elements of CAP: Adjustment to Practice (1.B.2)**

The second of the six elements in the Candidate Assessment of Performance system (CAP) is Adjustment to Practice (1.B.2), which is part of Standard 1, Curriculum, Planning, and Assessment. A teacher who “organizes and analyzes results from a variety of assessments to determine progress toward intended outcomes and uses these findings to adjust practice and identify/implement appropriate differentiated interventions and enhancements for students” demonstrates proficiency in adjustment to practice.

Adjustment to Practice was the element I struggled most with, but it was also the element I grew the most in. There are two halves to adjustment to practice: ascertaining where your students are, and meeting them there. The first is accomplished through instinct, experience, and formative assessment. The second is accomplished through tailoring instruction to the students with flexibility, creativity, and patience. I struggled with both, but all my teachers were impressed at how much I improved over my student teaching experience. The same disposition that motivated detailed and well-designed lesson plans also wanted to stick to those plans as much as possible, especially whenever I was nervous or unsettled (especially early in my student teaching but also whenever students were acting up or I was being observed or when students didn't understand the lesson). I was willing to change my plans between lessons, but adapting to the flow of teaching live was a struggle, and learning to read my students was a challenge as well, especially whenever they refused to answer questions such as “What's wrong?”, or “What don't you understand?” or “What's getting in your way?”. Through experimenting with different methods and a great deal of support from my mentor teacher, the special needs teacher, and my practicum teacher, I found a methods of formative assessment that worked well, such as thumb checks, coming around while students were working to examine their work, hand-in homework, exit tickets, and creating a cooperative learning environment where students both asked questions and answered them.

Adjustment to Practice is an art form essential to promoting learning and maintaining student engagement in a classroom. Teaching without it is like driving in a city with the windows covered using just the speedometer and a stopwatch. Any miscalculation will cause a crash. Students require constant tracking to assess where they are in the material, and whether they are engaged. Then teachers must be constantly adjusting their teaching to the students. Otherwise their pupils are either confused and bored or impatient and bored, and bored students are hard to teach effectively. Basically, adjustment to practice is reacting to the dynamics of a classroom as they occur; it is teaching live.

The input data for adjustment to practice comes from formative assessment of the students. This can be done informally (by observing students' behavior and expressions), formally (by quizzing or testing), or semi-formally (by asking students for questions and inviting them to assess their own comprehension levels). I used all three methods but relied most heavily on semi-formal methods such as thumb checks, asking for questions, inviting students to explain words and methods to the class, and having students demonstrate their understanding instead of verbalizing it.

These assessment methods took different forms with different student populations. The students in my applications of calculus class were bright but reserved, so I working independent examples into the lessons early on in each topic so they could demonstrate their mastery or confusion rather than verbalizing it. They were able to explain their confusion whenever they got stuck, but some days they were as chatty as rocks and I had to work around their silence. My SPED students, on the other hand, were often very talkative but lacked the advocacy skills to call my attention to their confusion, and would often goof off, refuse to engaged, or simply stare whenever they got stuck. I learned to use Dalfanzo as a translator for them. I always knew what he was thinking because it was whatever was coming out of his mouth at the time, and he could pinpoint the aspect of a lesson that confused or intimidated the group if I asked. I also used a battery of quizzing when teaching them, going around in a circle and asking them what to do next in a problem, or why a method worked, which both let me assess their learning and kept them engaged in the lesson (although it also annoyed them sometimes). In my honors calculus classes I used some quizzing and some independent examples, but I also did a lot of hand-raising and thumb-check questions, which let me pinpoint struggling students faster.

In all my classes, I put a great deal of effort into creating and maintaining an environment where questions and mistakes were welcome. I never scolded or belittled students for being wrong, and I forbid the students from doing so. Above all, I lead by example, admitting and changing my mistakes without showing frustration or shame (even at those moments when I wanted to melt into the floor). Students seemed very comfortable asking questions or trying answers in my class, but if I went too long without noticing they were lost they were resistant to renegading. One metaphor I often used with my honors calculus students was “Are you on the bus now?”. They would say things like “Back up the bus” or “I just got hit by the bus”. When multiple attempts to explained the same idea failed they’d say “Nope, I just got run over by the bus again” and I’d try different methods, or letting the students explain to one another, until we were “all on the bus”.

Once I located where my students were in the spectrum from totally lost to total mastery, I had to move them forward. In the beginning I had a one-size-fits-all concept of how to present material, sticking closely to my lesson plans whenever I could, and falling back on illustrative examples on the white board whenever I needed to slow down or explain an idea I hadn’t expected to cover. As I grew more comfortable, I grew more flexible, and I developed a larger arsenal of interventions, such as letting the students support each other’s learning, providing multiple definitions for the same word and letting the students pick one, drawing pictures, giving metaphors, and providing checklists for multistep methods. I learned to leave a spare board for side notes instead of planning to use the entire span, to make extra materials so I could adjust my pace, and to color-code. I also developed the patience to wait for students, and trained them to tell me when I needed to wait for them, and to let me know when they were done copying notes or examples. It was a learning experience for all parties.

I had great growth in adjustment to practice, and the flexibility and resilience skills in this element will likely be the most useful in any profession, whether I go on to work with special needs students, or as a mainstream teacher, or as an engineer.



**Works Cited:**

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017.  
<<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

**Chapter 4 – Elements of CAP: Meeting Diverse Needs (2.A.3)**

Meeting Diverse Needs (2.A.3) is the third elements of Canidate Assesment of Performance (CAP), under standard 2, Teaching All Students. An educator demonstrates profienency in meeting diverse needs by using “appropriate practices, including tiered insturction and scaffolds, to accomidate differences in learning styles, needs, interests, and levels of readiness, including those of students with diabilities and English language learners.”

Meeting Diverse Needs was my favorite element because I love working with special needs students, and at Tahanto Regional Middle and High School (TRMHS) I had the opportunity to teach a special education (SPED) pullout math class. I also had to accommodate less severe diverse needs in my other classes. In all four classes, I used Gardner’s theory of multiple intelligences to provide all my students with variety and a chance to learn with their strengths.

Meeting Diverse Needs holds a special place in public education because it offers students from all walks of life a fighting chance for success. Providing an equitable education to all students was the core motivator of the Massachusetts Educational Reform Act (MERA). Equal education for students with mental, emotional, and physical disabilities is a moral imperative because it success in school opens doors for the students after the graduate (if they graduate, in some cases). It’s also the law. I have friends who are special needs adults and watching their struggles and successes motivates me to meet the diverse needs in my classroom.

I describe the accommodations, modifications, and methods I employed in my SPED algebra class in great detail in chapter 9, but I’d like to highlight a few of the ways I meet their diverse needs here as well. To accommodate varying levels of readiness, I inserted lessons middle school topics such as understanding what fractions are, graphing with on the Cartesian plane, and arithmetic with integers. I also provided students with checklists and visual organizers for following along in mini-lectures to accommodate poor note-taking skills. I taught them with specially designed lessons, shortened to accommodate brief attentions spans, with simple instructions to accommodate trouble with task completion. I did a lot of scaffolding with the SPED class, doing several simple problems, then several problems with one distracting feature (such as a fraction or a new presentation), then more applied questions once students understood the process. Sometimes I didn’t even start with doing a problem to completion, teaching just part of the process on the first day, then the entire process on the second day. I made my lessons very visual and tactile, color-coding equations, making flip charts, playing games, and using cutouts and fake money. I made almost all their materials myself so I could accommodate their reading levels, incorporate their names and hobbies, maintain consistency in instructions, and keep only one problem per page to avoid overwhelming them. Some of these are in the appendix, and others are posted online at [teacherspayteachers.com](http://teacherspayteachers.com). Those students require English language support and in some cases speech therapy, not because they were learning English as a second language but because they were still developing first language skills. I provided definitions for key vocabulary words and point to them whenever they came up in the lesson so students could use them as a reference. I managed my vocabulary in the materials I made, providing context clues for difficult words and using shorter

sentences and simpler sentence structures. I also repeated the instructions regularly and succinctly for the students. They required and were provided with personalized explanations of classroom expectations such as hand raising, keeping their hands to themselves, getting and returning materials, keeping cellphones away, not speaking to students in other classes, and focusing on the lesson. Individual prompts were given when necessary, which meant several times a class and sometimes multiple times a minute. The student's emotion needs were accommodated through encouragement and praise when behavior was good, and short stark corrections when behavior was poor. Because they could be resistant, and would suck me into an argument if I let them, I had to disengage and walk away after each correction, even if I just went to shuffle papers and then came back. This move was partly about escaping and partly about giving them space to recollect themselves, unbend their pride, and refocus on their work, and it was very effective.

Teaching students with needs as extensive as theirs was at once tiring and rewarding. The aids, the special needs teacher, and my mentor teacher were all impressed that I could keep them focused and learning as much as I did despite their distractibility and behaviors. The special needs teacher's letter of recommendation is in the appendix, as are some examples of student work.

Applications of Calculus meets diverse needs in a completely different way than SPED algebra. Those students needed special instruction because they were high performing and highly motivated and had exhausted all the mainstream math courses at TRMHS. The math they were learning was challenging, beyond most adults and some teachers, but their behavior was so good they were by far my easiest class to teach. Meeting Diverse Needs takes a different angle with them, but they still needed deliberate grouping separate from typical students, specialized materials, and instruction that focused on their interests (sports, roller coasters, bullet-resistant glass, and other high-adrenaline topics).

I also had to meet diverse needs within the mainstream environment in my honors calculus classes. Some of the special needs in that class were relatively easy to meet, such as waiving homework for a while when a student had a concussion, discussing the hopes and pitfalls of a career in mental health, and my family's experience with the field, with a student who'd struggled with depression, or providing makeup quizzes for students who were out sick. Helping these students was a vital part of personally connecting with them, and it was a privilege to have the power to do so as their teacher. Some students required more effort to help than others. One of my students was nearsighted and didn't wear his glasses, so I gave him preferential seating, wrote bigger on the board, used high-contrast text on my presentation, and read key points and small text aloud. (This was especially important with integral bounds, which are always small). A few of my students had anxiety problems, which we coped with in a variety of ways depending on the severity. I was more soft-spoken with them, gave them warning before cold-calling on them, tried to take advantage whenever their hands were raised, seated them beside calmer students or by themselves, and insisted the other students treat them with respect whether or not they were in class at the time. One of the students had severe anxiety and was out for extended periods of time. Towards the beginning of my student teaching I worked one-on-one with him a great deal, and he was able to learn the material and pass his midterm exams despite missing over a month of school. I took him to a calm and quiet space, usually the library conference room, instead of tutoring him in the

mainstream classroom. As was my habit when working with students with emotional challenges, I started each day by just chatting for a minute or two, asking how his day or weekend was going, just trying to connect, help him relax, and gauge his mood. I ended each lesson by asking what his plans were, discussing when I would see him again, and negotiating how much work he would do in the interim, rather than dictatorially assigning work and hoping he got it done, because that just wasn't feasible with him. I encouraged him to get sleep and exercise to help with his anxiety, but he never really seemed to take that advice. We worked together for hours in the weeks before midterms, making a big push to get him through so he could learn the material and get credit for the first semester. He stayed for a little while after midterms, and we switched him into the smaller section of the class and sat him at his own table instead of at a desk so he could relax more. After the midterms the class did a project on optimization, and I partnered with him to limit the social anxiety and support him on the material. He turned in the project the day it was due, but never came to school again. After that I sent materials home to him, including notes, PowerPoints, YouTube videos, homework, classwork, and topic lists, as well as encouragement and promises of individual accommodations if he returned. The principal's office and guidance office took over managing attendance, put him on a student contract, tried to get medical documentation, and so on, but it's unlikely he'll graduate, which is a shame. He's bright and he can learn the content if he just comes to school, but supporting a chronically absent student is difficult, and it's impossible if they won't do the work on their own.

Supporting students with diverse needs is crucial to the role of a teacher, and it's the main reason I want to go into teaching. Supreme Court Justice Sonia Sotomayor warned "Until we get equality in education, we won't have an equal society," because the quality of students' education will ripple out into their professional opportunities and quality of life. But it's also tiring, requiring not just extra preparation time but also extra maturity, energy, and patience to deal with disruptive behavior. I grew a lot in terms of learning to manage behavior and meet diverse needs, but I also got a sense that the challenge is ever-expanding, requiring both personal and professional growth. There's a mural on the wall here at TRMHS claiming "If it doesn't challenge you, it won't change you" and I think that's true for students and teachers alike, especially in the element of meeting diverse needs.

**Works Cited:**

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017.

<<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

Sotomayor, Sonia. "Sonia Sotomayor Quotes." BrainyQuote. Xplore. Web. 17 May 2017.

<<https://www.brainyquote.com/quotes/quotes/s/soniasotom454926.html>>.

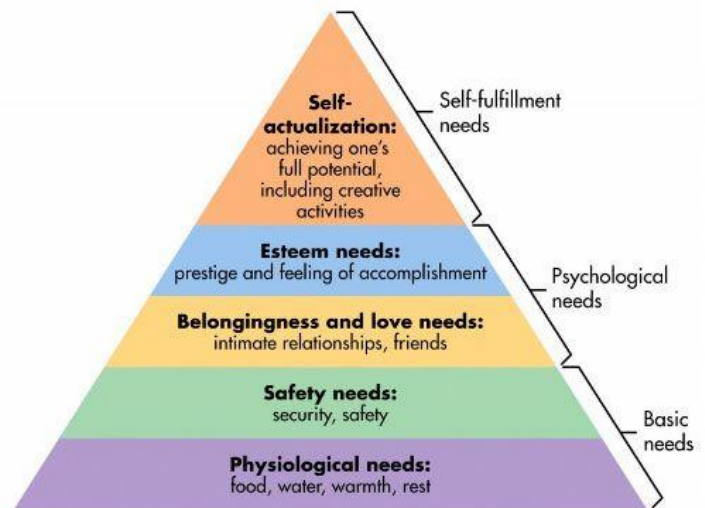
## Chapter 5 – Elements of CAP: Safe Learning Environment (2.B.1)

The fourth element of The Candidate Assessment of Performance system (CAP) is maintaining a Safe Learning Environment (2.B.1), which is part of the second standard, Teaching All Students. An educator has proficiently created a safe learning environment when he or she “uses rituals, routines, and appropriate responses that create and maintain a safe physical and intellectual environment where students take academic risks and most behaviors that interfere with learning are prevented.”

There are many levels to maintaining a safe learning environment, just as there are many levels to student misbehavior, so Maslow's Hierarchy of Needs is a clear way to define a safe learning environment. At the base of the triangle are the needs that must be met first, physiological needs, such as food, water, shelter, and rest, without which learning is nearly impossible. Although there were occasions when teachers crowdsourced money for groceries for families in crisis and so on, basic provision was rarely a problem at Tahanto Regional Middle and High School

(TRMHS). I often had to encourage my students to get enough sleep though, especially students who worked part-time, played sports, or struggled with anxiety problems. For this reason, I never assigned homework on weekends, and I always gave multiple days of warning about exams. Next came safety needs, which were also rarely a problem at TRMHS, where fights are rare, the building is new, and the students are well-behaved. Meeting the student’s safety needs in my classroom meant participating in fire drills, learning the school safety and evacuation policies, and coping with snow days whenever it wasn’t safe to drive. I was also able to incorporate information on fire safety and vehicle safety into my word problems, which engaged the students because they thought fire was cool. After the basic needs of safety and physiology are met, students are motivated by belongingness and love needs, such as being part of a community, being respected, being cared about, being noticed, and generally being cool. These needs have a powerful pull on teenagers and the bulk of my effort establishing a safe environment was centered on meeting these needs. According to Maslow’s theory, only if these previous needs are met will students be motivated to learn, because learning is an esteem need or a self-actualization need, based on a student’s innate desire for accomplishment and achieving their fullest potential.

Although Maslow’s theory is not all-encompassing, it provides a rationale for why a physically, intellectually, and emotionally safe environment is a prerequisite for learning. Students who are hungry, tired, or worried cannot focus on their schoolwork. Students who are worried about being teased, manipulated, or bullied will not be motivated to learn, and they won’t be comfortable taking the risks necessary to learn effectively, such as asking questions, attempting difficult problems, and interacting with the teacher in front of the class. A teacher creates and maintains a safe learning environment



through preventative measures such as clear class expectations, personal connections with students, and constant presence in the classroom, as well as corrective measures when hazards or misbehaviors occur. The implementation of these elements varied across my student populations.

Maintaining a safe learning environment in Applications of Calculus was fairly easy because of the students' maturity and respectfulness. When I performed demonstrations with boiling water, I had to keep the temperature low to protect their physical safety, and I used a coffee machine instead of a hot plate. (For instructions on boiling water at lower temperatures, see the lesson on linearization in the appendix). Also, we used scissors instead of utility knives when making cardboard contour plots. Other than that, maintaining a safe learning environment meant making the students comfortable asking questions, admitting they didn't understand things, and learning from their mistakes. I always encouraged the boys to ask questions, planned in time for them to do so, and encouraged them to attempt difficult problems and persist in understanding upper-level concepts. I believe in leading by example, admitting to my own mathematical and preparation errors without fireworks whenever they come up, so that students see mistakes as part of learning and part of life, nothing to be overly self-conscious about, just something to be fixed.

I used the same methodology in my special education (SPED) class but the implementation was very different. I admitted my mistakes, demonstrating that they are a big deal, and the students were comfortable recovering from errors, just like the neurotypical students. Other aspects of maintaining a safe learning environment were harder with the SPED class though. Telling my applications of calculus students not to call each other stupid was a one-time correction. With the SPED class, I discussed or corrected inappropriate behavior most days, especially in the beginning and whenever the material got challenging. I never let them get away with calling each other stupid, autistic, retarded, dumb, useless, or hopeless, but social expectations and impulse control were difficult for them. I also never allowed them to call themselves names, which was vastly more common. They got distracted or demoralized easily, but most of the time a brief redirection ("Inez, try this problem now") got them back on track. We tried to minimize distractions by quieting down the environment, but SPED classrooms are noisy places. Next year, TRMHS won't put two classes SPED in the same space during the same period. We tried bringing the class to the library but it made things worse because the change in environment was more distracting than the noisy but familiar environment. I supported my students with encouragement and praise whenever they got a concept correct or maintained good behavior. This made the intellectual environment safer for the students, and they were all willing to ask questions, to complain when they didn't understand a concept, or even to occasionally come up to the board and explain their work to the class. I consider my ability to maintain order in that class, to keep the students willing to take academic risks despite the inherent challenges they face to be evidence of my proficiency in maintaining a safe learning environment in that class.

I struggled to maintain a safe and productive learning environment with my Honor Calculus students, but I generally had success. I forbid name calling and belittling other students, even if the students being teased were ok with it. Some of the students in that class had severe anxiety and were usually absent, and the students were surprised that I expected them to treat absent students with the same respect as

present students—including no name-calling. I believe this promoted an emotionally safe environment both for the student being belittled and for bystanders who were later absent themselves. I explained these principles to the students and they were bemused but complied. The students were comfortable asking questions or explaining what they didn't understand, often asking myself or other students for clarification during the lesson. We used a ritual that one of the students came up with to identify who needed more support in a low-pressure way. I'd say, "Are we on the bus?" and they'd say "Yup, I'm on the bus" if they were tracking the lesson. If they were struggling but managing they'd say things like "I'm chasing after the bus" or "I'm hanging onto the bus" or something like that. If they were totally lost students would say "I just got hit by the bus" and I'd say "ok, lets back up the bus" and go over the material again. Sometimes that didn't work and they'd tell me the bus ran over them while backing up, but usually I could get them back on track. In Honors Calculus, another behavior that interfered with student learning was "checking out", or a refusal to pay attention to the lesson or do work. Often simple teacher proximity fixed this, but sometimes I had to give individual instructions. If students were really resistant I'd leave them be for a few minutes, then bring them back in when we started a new problem or a new aspect of the lesson. I also used Do Nows, individual problems, and cold call to prevent students from checking out. I believe I am proficient at maintaining a safe learning environment because most behaviors that interfere with learning were prevented, although special needs students were still special cases.

In all my classes, I endeavored to explain the rationale for my expectations. Some rules were formalities that helped me be responsible for the students, such as requiring them to sign out. Many students had babysat or been camp councilors themselves and they all understood the sentiment "you really hate to lose a kid." Some of my expectations were more involved, such as not taunting a boy about to undergo knee surgery, and I had to explain how that behavior might make someone else feel. I originally assumed high school juniors and seniors were familiar with such principles, but I was willing to explain them for two reasons. First, I felt the students had to be considerate of one another so we could maintain a safe learning environment for vulnerable students and it was only fair to explain what I meant by that. Second, I felt I should take advantage of teachable moments to explain principles of empathy and citizenship to the students. Teachers say, "Teaching a child not to step on a caterpillar is as important to the child as it is to the caterpillar." I did my level best both to require and model good citizenship, and I know the students will need those skills both in the college classrooms and in non-academic settings.

As my student teaching progresses I'm learning to be more efficient at maintaining a safe learning environment. I improved at stating my expectations explicitly, and my corrections became clearer, more effective, and significantly shorter. Because of my experience tutoring, I began student teaching with the vital habit of accepting my own mistakes and leading others to recover from theirs comfortably. I think I'm skillful at inspiring students to take academic risks but still have room to grow at managing and eliminating behavioral issues.

## Works Cited:

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017.  
<<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

McLeod, Saul. "Maslow's Hierarchy of Needs." *Simply Psychology*. 16 Sept. 2016. Web. 17 May 2017.  
<<https://www.simplypsychology.org/maslow.html>>.

Miller, Bradley. *Google+*. Victoria Bubis. Web. 17 May 2017.  
<<https://plus.google.com/101118065394149099178>>.



## Chapter 6 – Elements of CAP: High Expectations (2.D.2)

The second to last element of The Candidate Assessment of Performance system (CAP) is called High Expectations (2.D.2), which is also part of the second standard, Teaching All Students. Proficiency is shown when an educator “effectively models and reinforces ways that students can master challenging material through effective effort, rather than having to depend on innate ability.”

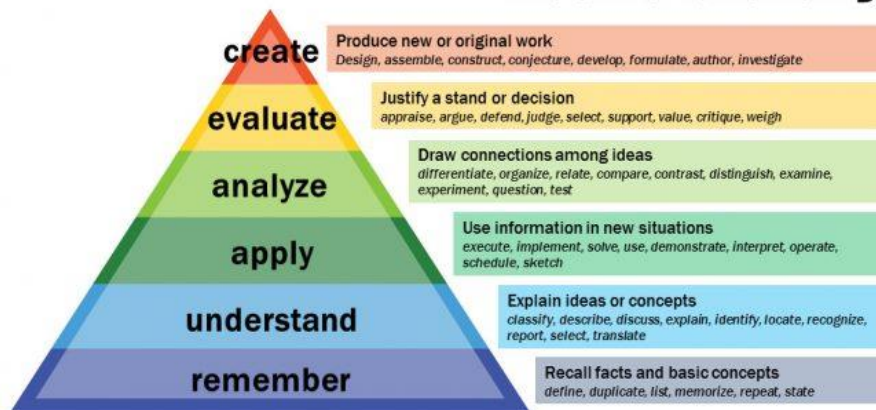
An educator should set high expectations for student learning, comprehension, and behavior, and keep them high for all students. High expectations are important because people will often rise or sink to the expectations around them, especially when they are school-age. High expectations for student behavior are key to maintain a safe and effective learning environment. Maintaining high expectations, even for high needs students, can be difficult or feel harsh, but it is essential to providing them with a quality education.

One high expectation I had for all my students was deep engagement with and thorough understanding of the material. I wanted students to understand the logic underlying the mathematics they were learning, not just perform monkey-math. Bloom’s taxonomy is useful framework for evaluating the depth of understanding. The

lower levels of Bloom’s taxonomy, remember and understand, are not unworthy goals of instruction but they are insufficient. I began each lesson with vocabulary and steps to remember, but I never left it there. I asked student to apply those concepts to familiar and novel problems, and analyze the answers they found for connections to previous topics, regardless of the level of the students. Whenever the time and topic allowed, I had students evaluate the methods they were learning, and create projects or presentations on the material they were learning in class.

In my special needs small group pullout class, I asked a lot of low-level questions on arithmetic and vocabulary to give the students practice and keep them focused on the class. Nonetheless, I still required them to apply the methods I taught first to familiar problems, and then to novel problems. We discussed the relevance of math to daily life, and used visual organizers to make connections and comparisons between the concepts learned in class, and to evaluate their usefulness. I had them compare the methods for solving equations of two variables, and select their favorite method, and they enjoyed doing word problems about themselves so we did that when we had the opportunity. The special needs students struggled with reading, answering open-ended questions, and planning projects, but I always tried to ask them upper-level questions, and I learned to give them more wait time as they tried to answer. Over the course of my student teaching, the students became more comfortable attempting open-ended questions

### Bloom’s Taxonomy



and they got better at connecting ideas across lessons or topics. It was a real pleasure to watch those students rise to meet high expectation for deep learning in algebra. I also maintained high expectations for their classroom behavior, as discussed in the previous chapter, which kept my expectations for learning algebra possible.

In my other small group class, Applications of Calculus, I spent very little time on behavior, vocabulary, and arithmetic because the students had such a strong educational background. The concepts they were learning were harder to understand but I took the time to explain why the methods worked, not just how they worked. I connected new formulas and methods to previous topics, deriving geometry formulas that 5<sup>th</sup> graders memorize and tying three-dimensional math to previously learned two-dimensional math. The students created projects, including cardboard contour plots and continuous rollercoasters, to deepen their understanding. We covered advanced topics, including Taylor series, calculus of multiple variables, and approximation with lines and planes. I had high expectations for the content the students would learn, including material from MIT's open courseware and topics from my background in mechanical engineering. The students rose to meet high expectations in both scope of learning and breadth of learning.

Honors Calculus was a more traditional classroom setting, but the principle of setting high expectations was very similar with them. I maintained the expectation that students would remain engaged in the lesson every day, and other behavioral expectations, as discussed in the previous chapter. I kept my expectations high regardless of the behavior of the student, the attendance record of the student, or how talented the students said they were. I provided review material for students who missed the lesson, whether they were out for a month or just in the bathroom, so they had the background to participate in class, and I made sure to check their comprehension when they rejoined the class. In Honors Calculus, I asked a fairly equal amount of questions from each level of Bloom's taxonomy, asking them to remember vocabulary and basic rules, apply those rules to familiar and unfamiliar problems, and explain how the rules were dependent on the fundamental theorem of calculus. The students also created presentations on optimization and designed posters on the basic rules of integration. I was a gentle grader but I always included some novel questions on assessments so I knew if students could apply the methods learned in class, and the students usually performed well on those questions, although the students who struggled needed personal encouragement to attempt them.

In all my classes, I encouraged the students to try problems they struggled with. On tests, I promised to give partial credit for partial answers, and for some students that was enough. Some students needed individual encouragement, or just needed me to tell them to try the problems, one problem at a time. Some students needed to take a break, then come back to the test. I discussed with my classes the value of hard work, stressed the importance of studying, and explained that success in college isn't about talent, it's about diligence. I explained to the students that just because a student is slow doesn't mean the student can't understand the material, or will never grasp the concept fully, it just means it takes them longer. We discussed methods of studying, such as learning vocabulary by adding words to the list each time you memorize the whole list, and the difference between cramming and healthy study skills. I made a point of this my first day by memorizing all my students names each period, then explaining

how I learned to memorize despite struggling with it as a child. I expected students to memorize key facts as well as apply key ideas, even if they struggled with mathematics, keeping my expectations high for all my students, and I was impressed that they generally rose to the challenge. Throughout my practicum, I had high expectations for my students, and over the course of the practicum I learned to communicate that to my students so they had high expectations for themselves as well.

**Works Cited:**

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017. <<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

Mcdaniel, Rhett. "Bloom's Taxonomy." *Vanderbilt University*. Vanderbilt University, 10 June 1970. Web. 17 May 2017. <<https://cft.vanderbilt.edu/guides-sub-pages/blooms-taxonomy/>>.

## Chapter 7 – Elements of CAP: Reflective Practice (4.A.1)

The final element of Canadite Assessment of Performance (CAP) is Reflective Practice, which falls under the fourth standard, Professional Culture. An educator who “regularly reflects on the effectiveness of lessons, units, and interactions with students, both individually and with colleagues, and uses insights gained to improve practice and student learning” demonstrates reflective practice proficiently.

Reflective practice involves not merely looking back on individual learning activities to evaluate their effectiveness, but also reflecting on class dynamics, overall pacing, and interpersonal interactions. My personality predisposes me to reflect daily, evaluating how well things went and why. After each lesson, I determined what I would change if I did it again, which was particularly helpful for Honors Calculus because I taught each lesson twice and could incorporate the lessons of hindsight. At the beginning of student teaching I was very uncomfortable reflecting with others, especially about the more personal aspects of teaching, such as interacting with resistant students or developing a teaching persona. Over the course of my practicum I became more comfortable both with teaching and with taking feedback, and I did it more often and more thoroughly. I also learned to suspend my evaluation of feedback until I heard all of it, and to explain how I was going to incorporate or tweak the suggestions I received instead of saying “I don’t think that will work”, thinking about it for a day or so, and then trying something along those lines, as was my habit when I first started student teaching. Receiving feedback and making people feel heard is an art form I will continue to work on, but I became proficient at it over the course of my practicum.

Individual reflection is still my preferred aspect of reflective practice, and it has always been my strongest. I journaled weekly during my practicum, setting weekly goals for student learning, new methods of instruction, or interactions with struggling students. At the close of each lesson, I compared my delivery of the lesson with the lesson plan and modified the lesson plan to reflect what I would do if I were to teach the lesson the following year. At the end of each day, after all the students went home, I considered the day as a whole, looking for occasions when I had grown, or needed to grow, as a teacher, a student, and a person, such as giving more wait time or not taking snarky comments as personally.

I also learned to reflect with other teachers at Tahanto Regional Middle and High School (TRMHS). I regularly discussed my lesson plans with my mentor teacher, Mrs. Gleason, selecting methods of instruction and modifying materials for the students. I also discussed student behavioral issues with Mrs. Gleason, as well as with the special needs teacher, Mrs. D., to learn better methods of interacting with the students, gauge the effectiveness of lessons, and get feedback on the overall atmosphere of my class. They were both able to give me ideas for helping my students I never would have considered, as well as a benchmark for how my class compared to a typical class for students of a given level. They also helped me estimate how much time learning activities would take, which is difficult for a new teacher to gauge. I also discussed my students with other staff at TRMHS, including teachers who had them in other classes, aids who worked with them one-on-one, and guidance counselors. This gave me a more rounded view of my students, exposed me to methods for supporting struggling students that other professionals had developed, and provided a support network for evaluating and coping with

misbehaviors. I also discussed my students and my teaching methods with Katie and Shari, who I met with at least once a week throughout my practicum, getting feedback on everything from class management to project-based learning to school policies. There was a small village of professionals supporting throughout my teaching practicum and I would not have learned nearly as much, or had as much success as I did, without their help.

I also received feedback from my students. Some of this came in the form of survey responses, both to the short form survey discussed in chapter 9, and to exit tickets I asked for on specific topics such as riddles, learning styles, or teaching methods. Most of the feedback came in the form of student interactions during and after class. I learned to consider student preferences on topics and teaching methods, weighing student preference with educational effectiveness. I also learned to take the less helpful and less specific comments less personally, again with the help of teachers at TRMHS and WPI, especially Mrs. Gleason and Mrs. D.

Reflective practice is important for educators because it enables continuous lifelong improvement. It allows educators to adapt to changes in culture, curriculum, or demographics. It allows teachers to improve their weaker areas as well as build on their strengths, becoming better over time rather than stagnating like an athlete plateauing. I told my students to be lifelong learners, because they will never stop growing and maturing as long as their alive, so they should stay adventurous and stay curious. Reflective practice allows me to take my own advice, to keep growing intellectually, professionally and personally.

I never met at teacher who loves to be observed, or to hear criticism on her teaching, but I have grown more comfortable being observed and taking feedback. I incorporate feedback into my lessons where I can, and I'm will to try new methods and phrases when their suggested to me by colleagues and mentors, or even sometimes students. I'm also skillful at reflecting independently, both at the end of each lesson and at the close of each day, week, and unit, which allows me to improve continuously and never stop learning, just as I encourage my students to do.

### **Works Cited:**

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017.  
<<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

## Chapter 8 – My WPI Education

I will graduate from Worcester Polytechnic Institute (WPI) with a Bachelors of Science in Mechanical Engineering (ME) in May of 2018. I have taken the entire calculus series, Ordinary Differential Equations, Matrices and Linear Algebra I, Applied Statistics I, and Probability for Applications, which thoroughly exceeds the material taught in high school calculus classes. Most of this math was required for my major, but the probability course I took merely for its usefulness across all fields.

Term	Subject	Course	Title	Attribute	Credits	Grade	Source
201501	MA	1021	CALCULUS I		3.000	L	T
201501	MA	1022	CALCULUS II		3.000	A	H
201501	MA	1023	CALCULUS III		3.000	B	H
201502	MA	1024	CALCULUS IV		3.000	A	H
201601	MA	2051	ORDINARY DIFFER. EQUATIONS		3.000	A	H
201501	CH	1010	MOLECULARITY		3.000	A	H
201501	CH	1020	FORCES AND BONDING		3.000	B	H
201501	PH	1110	GENERAL PHYSICS-MECHANICS		3.000	L	T
201501	BB	1000	BIOLOGY ELECTIVE		3.000	L	T
201601	MA	2071	MATRICES&LINEAR ALGEBRA I		3.000	A	H
201502	MA	2611	APPLIED STATISTICS I		3.000	A	H
201602	MA	2621	PROBABILITY FOR APPLICATIONS		3.000	A	H

In addition to the seven math classes and a more than a dozen engineering courses, I have also taken two courses in Chemistry and have AP credit in Biology and Calculus-based Physics. WPI also requires humanity courses, and in my five English classes I have read and written on classic literature, and performed in a play. This background enables me to speak coherently to subjects beyond my specialty when asked. As one of my students marveled “You know physics?”. Yes, I’m an engineering student, and I understand math, and the sciences, and even history and grammar, at the high school level. I took Kinematics of Mechanisms and Design of Machine Elements and Stress Analysis and Thermodynamics and Introduction to Fluids and Introduction to Electrical and Computer Engineering and more, and I applied Newtons laws and conservation of mass and energy until I dreamed them. I know physics.

Because I have studied engineering at WPI, I understand how mathematics are applied. This empowers me to defend my curriculum to students who don’t see the use for imaginary numbers (they’re used in electrical engineering to evaluate alternating current). It also enables me to enrich and engage my students with the history and application abstract mathematics. Some branches of mathematics were developed as curiosities, but most were due to the technical limitations of the time or needed for new advances in science. For example, before calculators, logarithms were used to multiply long strings of numbers accurately. Calculus was invented for the study of accelerations. Students have told me they find this information cool, and if math is cool last period on a Friday I must be doing something right.

Understanding the potential use for upper level mathematics isn't just motivation for my students. It motivates me to teach them, to make sure they understand ideas that will be foundations for other learning. Engineering is merely the application of math, science, and technology to solve real world problems inside an economic environment. Math is the language of science and engineering. It's a foundation required to understand the world around us. If you don't understand exponents, really understand them, you may know  $E = mc^2$  is that thing Einstein said, but you don't understand how it turns an x-ray machine into an atomic bomb. If you don't understand place values, the distinction between 7.5 billion people on the planet and 8.5 million people in New York City is lost on you. WPI requires us to take humanities courses on the grounds that if you haven't understood great poetry, or listened to great music, or traced human history, you're missing out on a huge body of human knowledge. Shakespeare is the calculus of the humanities world, complex, creative, and beautiful. In our scientific world, algebra is like reading; without real fluency, everything is just a jumble of symbols that look familiar. I want to open that world for my students so they can understand that some of the greatest discoveries known to man are equations.  $F_{net} = ma$  is why airplanes stay up, but they can only stay aloft when going forward because of Bernoulli's equation  $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ . I think that's fascinating. Aside from comprehending the world, WPI has shown me another use for mathematics.

Mastery of mathematics opens up a host of educational and economic opportunities. Even if you don't get to claim the next billion dollar patent, an engineer will rarely be unemployed even in the harshest of economies. Although the field is dominated by white men, there is real opportunity for women and non-whites who are willing to stick out. Mathematics can be a great equalizer for students who struggle with English, or lack social skills. You don't have to be socially inept to be a nerd, but if you struggle with social skills or are confused by American culture, engineering is a great place to earn your living.

No, I don't have a degree in Education. I've taken five courses in Education (Sheltered English Immersion, Teaching Methods in Mathematics and Science, Introduction to Psychological Science, The Psychology of Education, and Cross-cultural Psychology) but there's still more to learn, and if I become a teacher I'll read a lot more about the field, and probably get a Masters Degree in Education. Still, WPI has given me a wide base of knowledge in technical subjects and a solid education in mathematics, and this is fine education for a new educator to begin her teaching career with.

### **Works Cited:**

"Unofficial Transcript." *Bannerweb*. Worcester Polytechnic Institute. Web. 17 May 2017.  
<[https://bannerweb.wpi.edu/pls/prod/bwskotrn.P\\_ViewTran](https://bannerweb.wpi.edu/pls/prod/bwskotrn.P_ViewTran)>.

## Chapter 9 – My Classes

I taught four classes at Tahanto Regional Middle and High School (TRMS): Applications of Calculus (Apps.), Small Group Math for Special Education Students (SPED), and two sections of Honors Calculus. Each had a unique blend of students, and each was a challenge and a joy in its own way.

Despite the advanced mathematics, Applications of Calculus was easy to teach. The class is new this year, created to meet the unique needs of three boys who have exhausted the calculus options at TRMHS. All three students had excellent behavior (aside from never doing homework because this is an optional enrichment class for all three). The class was quite democratic and relaxed. I didn't work from a set curriculum with them. Instead I asked what they want to learn, then collected and taught all the topics required to master their topic of interest, then presented the topic itself. The math was fairly advanced and sometimes pushed the limits of my own knowledge of calculus, sending me to textbooks and WPI professors. I taught Series and Polynomials (including the derivation of  $e$ ), Calculus with Polar Equations (Including the derivation of geometry formulas), and Multivariable Calculus (Derivatives and Integrals in 3-D). We started most topics with group readings, then proofs, then scaffolded example problems, and finally independent problems. After the foundations were laid, students did projects, including making cardboard models of contour plots and designing a roller coaster with piecewise functions. Each topic ended with a review day, an in-class test, a review of the test, and a retest as needed. I asked them regularly (about once a week) what they think of the subtopics covered and methods of instructions, and incorporated their requests into future lesson plans, adding more proofs and more projects to the curriculum. All three students are bright, quiet, and college bound this coming fall. These students were a splendid example of the dark secret of student teaching: the most difficult part of teaching math is not the math.

If Applications of Calculus was my most peaceful class, Small Group Math for Special Education students, or SPED math, was my most chaotic. The SPED students are learning algebra, largely based on Massachusetts Curriculum Framework for Mathematics, with a special focus on money math (which they have expressed an interest in) and topics that appear heavily on the MCAS exam, which we are concerned about them passing. Our curriculum is a revolving mix of new topics from the High School standards, such as modeling with linear equations, and significant review of Middle school standards such as basic operations with fractions, regularly returning to reassess and reinforce previous topics to support retention. The class had only four students: Zack, Inez, Dalfanzo, and Sarah. Two are in tenth grade, one is in ninth grade, and one is in eighth grade but taught in the high school so that his environment will be age appropriate even though he once was held back a year. They are all on IEPs and have moderate disabilities that prevent them from learning in a standard classroom environment. They have significant struggles with reading level, work perseverance, attention span, processing speed, and appropriate classroom behavior. If pushed too hard, they may shut down entirely, refusing to work or once even to speak for a few minutes or even until the end of the period. I learned to give them a break when they needed it, and to teach fractions in spurts instead of long hauls. I gave a series of one-question worksheets instead of packets because the students got intimidated or distracted by multiple problems, and because Inez, Sarah, and Zack struggle to plan out work on a page, running out of space



and getting disorganized. Two of them required prompts every minute or every 5-10 minutes to stay on task while working independently, and three required individual reminders to re-engage with the lesson during lectures. I gave regular individual prompts for attention, and constructive succinct feedback on appropriate classroom behavior and the reasons for such expectations. These corrections ranged from daily standards like “You need to get a pencil and a calculator”, “Please sit down”, and “You need to your cell phone away” to more unique corrects such as “Dalfanzo, you need to answer me in English because you don’t speak Spanish and we are not having math class in gibberish” (Dalfanzo was very excited to pick his own pseudonym for this paper and wants to take Spanish next year, but currently speaks not a word of the language). The special needs/math teacher responsible for the class complimented me on my ability to keep them engaged on topics that challenge them (especially fractions and solving equations of two variables) but teaching learning behaviors, and teaching despite poor behavior, was a challenge with them. We didn’t have any nightmare problems (violence, bullying, leaving the classroom, vulgar comments, etc.) but keeping the class moving forward was a challenge, requiring me to expand my battery of teaching methods and heavily modify my lesson plans. We color-code our equations with colored pencils and white board markers (this idea was Inez’s and it’s brilliant and effective). With them I usually taught mini-lessons instead of full lessons, and I used flip charts or cut and paste notes instead of traditional notetaking strategies. To accommodate low reading levels, I always defined vocabulary in simple terms on the board, and I rarely used reading for instruction. To compensate for poor arithmetic skills, students have modifications allowing four-function calculators in class, on tests, and even on the MCAS exam. I frequently had them evaluate fractions on their calculators and I honored their request to keep fractions out of the first few examples we do with any new topic, feathering them in once students are proficient. I taught and tested fractions separately, which the scientist in me considered a form of changing only one variable at a time in an experiment. As we progressed through a unit, I frequently had to insert a lesson on underlying skills, such as adding integers. This need for review is part of why the students are pulled out of standard classes, and naturally creates breaks within difficult topics, which is emotionally valuable for these students.

The classroom is set up so that students and teachers have constant support, with two or three small groups lead by teachers and aids going on at the same time in a large room with its own computers, printer, kitchen, and bathroom. This allows any of the special needs students (not just my four) to step out with a teacher at any time to discuss an issue before it explodes, and it allows teachers to back each other up, like players on a baseball field.

All the SPED students were kind souls who genuinely want to learn, especially when they see the relevance of the material to their future independence. Dalfanzo, however, is resistant to just about everything, and I have struggled greatly to connect with him and get his cooperation. My first SPED class was simply filling in on a Friday when their main teacher had to leave early, playing a fractions review game (steal the old man’s pack) with a standard deck of cards. Most of the students were willing to try the game with me, but Dalfanzo refused to sit with the group. I have since learned that fractions are difficult for all of them, and everyone struggles with change. Changing teachers was a struggle for them, and changing seats was just overload. Nonetheless, it was an inauspicious start.

I tried a wide variety of methods with Dalfanzo, but I'll focus on the ones that worked. In a SPED class, not everything can be a fight, so I learned to pick my battles, focusing on eliminating behaviors that were distracting to other students, such as poking Zack or criticizing Sarah. Encouraging him to be persistent, and giving him a taste of success was effective, as was telling him that if he finished the worksheet he could be released (whether he went on his phone, or just sat there, being released was a reward that really motivated him, and since he's faster than the other students it wasn't too much of a time drain). I had to learn to read the subtle difference between Dalfanzo can't and Dalfanzo won't, interconnected as they were. He also had to realize that being stubborn wouldn't get him out of the work. I would work with him, and I would wait for him, and I would not give up, even if that meant scaffolding problem by problem. Early on, working with a packet on percentages, I went through problem by problem in a table, drawing arrows and having him fill in the numbers until he just took the table and filled out the whole thing so he could be released. Dalfanzo took that kind of patience to get him to perform, but he was capable when he put his mind to it. I also learned to diffuse power struggles whenever possible, because he lacks the self-control to back down smoothly. One thing that worked well was giving him an option like "You can do questions 3, then question 10, or you can do question 10 now".

I also learned to take advantage of his strengths. Dalfanzo has guts, whereas some of the students struggled with their self-esteem. Dalfanzo was almost always willing to put his answers on the board and explain his reasoning. He became my barometer for the class, articulating when they were confused and what exactly the point of confusion was. Class was quiet whenever he was out, and although presenting the lesson was easier without having to handle him, I missed him because I always knew what he was thinking. The others weren't as articulate or introspective, no matter how long I waited. As tiring as he was, objectively I did well with him. He learned the material, stayed engaged, and only shut down once throughout my 15 weeks. I even got him to work during MCAS week and I never had to send him out of the room. He's young, and hopefully he'll mature over the summer, but he's learned some math with me, and he's practiced the vital life skill of coping with change.

Dealing with a change in teachers was also an issue in my more traditional classes, two sections of Honors Calculus. There were thirty of them, mostly seniors, with a thirteen juniors mixed in. As an engineering student, I found the calculus quite manageable but gaining the student's cooperation and respect was a challenge, especially for the first period I took over, which my mentor teacher actually took back for a week when I was first starting to teach. After experimenting with demonstrations, video mini-lessons, project-based learning, instruction from the white board with traditional note taking, learning games, and teaching from PowerPoint slides with flip chart notes, I found a balance that worked well. I generally used PowerPoint-based instruction with flip chart notes, and held review games or mini-projects about once a week. These students were less adept at math than the students in Applications of Calculus, AP Calculus BC, and AP Calculus AB. They required more scaffolding, more example problems, and more practice than I had originally anticipated, but I learned to include that in my lesson planning, and to make up examples on the fly. I taught optimization with differentiation, Riemann sums, the Fundamental Theorem of Calculus, rules for integration, indefinite integrals, and

definite integrals. They have serious cases of senioritis, so towards the end of the year they did more project-based work.

I did not have any English Language Learners in any of my classes because the school only has about a dozen of them, and none happened to be in my mentor teacher's classes.

Besides the special education class, I had a few mainstreamed students that needed individual accommodations. One student was out for the majority of my student teaching due to severe anxiety, so whenever he was available during the beginning of my student teaching, I worked with him one-on-one to get him caught up on all the material he missed during the first half of the year so he could pass his midterms, which he did. Although he was originally apprehensive, we worked together well, and we covered many weeks of material in the first few weeks he was in. Then he was out for weeks, in for a span of days, and has generally been out ever since, although he emailed in his project the day it was due. His main barrier to graduation is not grades, or even content, but absences, and many people, including myself, have impressed on him the importance of attendance. In addition to one-on-one work, preferential seating, careful partnering, and personal encouragement whenever he was in school, I also supported him by emailing him assignments, PowerPoint presentations, YouTube videos, and notes from other students. The principal is handling his student contract, and he is seeing a counselor both in school and out of school. It's uncertain if he will graduate, but he has been provided a great deal of assistance. I sent letters home and one of them did get him to come back to school for a few days, but additional letters didn't have the same effect. Parent communication was spotty with his family. It was difficult for me, my mentor teacher, or the school counselor to extract responses from him and his parents. I learned that family communication, like everything else in teaching, is a two-way street, requiring effort on both sides. Certainly, he and his parents were made well aware of the consequences of his absences, and I don't know if he'll lose credit, but every effort was made. It's hard to teach a student who doesn't come to school.

Another very challenging student in my Honors Calculus class was as resistant as Dalfanzo, but she was neurotypical and never accepted the change in teachers. She was resistant to me personally because she had struggled with all previous math teachers except my mentor teacher, and now I was taking that teacher away from her. She behaved for me much as she had for previous teachers, refusing to listen, frequently putting her head down on the desk, and occasionally defying direct instructions.

I learned to wander through the desks as the students worked without tripping, as just walking past her sometimes got her to start her work, and sometimes a personal invitation to work, or take notes, was effective. If her head went down, I'd ask her to pick it up, and I learned correction was more effective when it was brief, included her name, and was followed by immediately returning to what I had been doing before. In the beginning my mentor teacher often sat next to her while I taught, ensuring she took notes and being an emotional support. Eventually I found teaching methods that worked well for the class and as frustration went down, her behavior improved somewhat. Partnering her with a close friend who's also a solid student insured that she learned the material by asking him frequent questions, and as I encourage students to collaborate during lecture, asking and answering questions freely, this worked

well, although getting her (or really that whole class) quiet remained a problem throughout my student teaching experience. With her, as with that class at large, I had to learn to pick my battles. I was willing to wait a minute or more for students to work on an integral, but if no one wanted to tell me how their vacation was after April break, I let it go. Connecting with that class was difficult. Although I shared knowledge from my engineering courses and stories and skills from my quirky life (I am a gold award girl scout, a second degree blackbelt, and a clipart artist), they were much less receptive than other students, sometimes refusing to engage with me at all, then answering my mentor teacher with enthusiasm when she asked the very same question.

So, I settled on being honest. When they were unreasonable, I said so. I told them what I was going to do, and then I did it. I promised to check homework frequently and did so, ensuring all my students, including the most resistant girl, did their homework most nights. I also used homework as my main formative assessment for her, compensating for her unwillingness to speak in class. She was willing to participate in games and learning activities so I incorporated those regularly into my instruction. I never got her to stop being resistant, and I certainly never became her friend, but she did learn the material. She learned against her will, but she did learn, and she got steadily better, though Fridays and lectures were always tough.

By the end of my student teaching, most of the students were learning quite well. I gave my students in honors calculus class a quiz on definite integrals. It covered five properties of integrals (Order of Integration, Zero, Constant Multiple, Sum and Difference, and Additivity), evaluating integrals as signed area between the curve and the x-axis, evaluating definite integrals using a graphing calculator, and calculating average value using integrals. My mentor teacher and I discussed the students expected performance, which decided should have been about a third A's, a third bridging A's and B's, and a third B's. The students performed almost exactly as expected, with a few low outliers. Students who refused to pay attention when I was teaching performed poorly on the quiz, but otherwise student performance generally matched their performance before I took over teaching. Many students struggled with the question on average value so I went over that question again.

I attended a staff development day at TRMHS on teaching with games, which focused on iCivics and integrating civic involvement into other subjects. I learned that integrating games into a curriculum requires careful consideration of the learning objectives, but can be very rewarding because students may play the games at home. I learned about civic engagement in general, and decided to integrate current events and global concerns into my mainstream math classes. I found my students were interested in word problems that involved civics issues like population size, global warming, and child mortality rates, which I would not have considered before attending the staff development day. They said it made them sad, but it also made them interested and informed. My students have big hearts, and I'm glad I took the chance to expose them to some of the joys and sorrows of being aware of global issues.

Most of my training and feedback was far less structured. Often my mentor teacher had specific advice on my materials, and she provided pacing and order of instruction, and served as the eyes behind my

head whenever I wasn't facing the class. She suggested methods I wouldn't have thought students liked, such as 3-2-1 worksheet races and flip charts. The special needs teacher also had a lot of advice on my materials and teaching methods, and always complimented me on finding and using methods that worked. (She asked to write a letter of recommendation, which is in the appendix). Both told me to do more review, which I incorporated into my lesson planning, and to take things less personally.

I administered the CAP student feedback survey in short form, grades 6-12, to all my students, including my special needs students. The CAP survey was never designed or validated for special needs students, and their responses were largely incoherent, irrelevant, or obvious, but their comments on teaching methods were instructive. Based on the survey results, I tried to involve the students in their learning more. At the request of the students, I tried to incorporate the students in their learning more, having them come up to the board even though I was afraid they would shut down if they made an error and I had to correct them in front of the class. I was impressed. They were excited to come up to the board, tried problems with me that they wouldn't attempt on their worksheets, and never even seemed upset to be corrected. I'd made it clear throughout my teaching that mistakes are part of life; I make them, students make them, everybody makes them from time to time. I've always believed in teaching by example, and if the only thing my students take away from my tenure at Tahanto Regional Middle and High School is not to be ashamed of mistakes, just to fix them and carry on, I've taught them something of great worth.

The results of the CAP were more cohesive in my mainstream classes. I averaged near b (agree, but not strongly) in each of the standards. The survey affirmed my habit of frequent comprehension checks while providing direct instruction (thumb checks, put that in your own words, brief practice problems, and answering each other's questions), which I used for formative assessment, for engaging students who are drifting, for variety, for alternative perspectives which the students might connect to better, for improving retention, and for deepening understanding. The students felt my lessons were well planned, a praise I have heard from day one from my mentor teachers and practicum staff, as the engineering mind is well-trained to begin with the end in mind, and I'm a skillful and detail-oriented planner. For teaching all students I got a low b, primarily because of issues managing behavior in my classroom, which interferes with teaching at times. Students also cross-wired a negative tone when correcting work (which I don't use) with a negative tone when correcting behavior (which I use when necessary). This is one of the disadvantages of student assessment, but in other cases the it was useful. In response to lower scores (averaging a low b, or slightly agree, in II.A and II.D), I included more graded projects into the lessons, especially for review, and provided more optional enrichment topics, which the quieter students, easily neglected as they don't cause problems, were receptive to. Students generally felt heard and respected in my class, and confirmed they received personalized attention and concern with even b's in II.B and II.C. I made word problems to appeal to student interests (primarily animals, but also driving, and makeup), and make the time to ask them about their lives beyond my class, which some appreciated. The responses were polarized, suggesting that students who disliked one aspect of the class (me) rated my class poorly across the board rather than homing in on what specifically bothered them. Individual

comments such as “Ms. Cole is a great teacher, and she makes cool clipart” (I made them a specialized piece of clipart for the trigonometric integrals) or

A few students commented that I treat them childishly, requesting “to be taught and treated the age we are, not elementary school age”. One student said “While I understood content most of the time, connecting and relating to students is crucial. Understanding that we may not be motivated rather than forcing us to be motivated would be considerate. We are high schoolers so we know what is expected of us. This should be expressed but accepted when we aren’t motivated. Lastly, childish methods aren’t very effective when we are juniors and seniors.” I always tried to treat my students maturely but some students acted really immaturely. No matter how you say it, “Don’t play fight with the safety scissors” will sound like a correction that belongs in an elementary school because it is a correction that belongs in an elementary school. If I seemed exasperated when it took several corrections, and then several individual corrections as well, to get the class quiet, it’s because I was. The survey confirmed that I needed to work on treating my students maturely and maintaining appropriate classroom behavior.

Many students and staff expressed that I grew dramatically better over the course of my student teaching. One student explained on the survey “I think the teaching in the beginning was hard to understand by I find the way she’s teaching a lot easier to understand now”. My practicum teacher told me she’s “never seen anyone improve so much over the course of the practicum.” Overall student teaching was a challenging but enriching experience which will be valuable whether or not I become a professional teacher.

**Works Cited:**

none

## Conclusion

After seventeen weeks of student teaching, I have concluded teaching is a lot like shepherding because you spend some of your time keeping everyone moving in the right direction, but most of your energy goes into taking care of strays. The example a teacher sets and the material they teach should contribute to the success of every student they work with, but the largest impact is felt by a small subset of the students, the ones you have to put a lot of energy into helping, the strays. These are the students who miss class, who struggle to understand the material, or who have behavioral and emotional needs that you have to work around or work through. In some of my classes, you could take all but four students out of the room and it wouldn't be any easier to teach them. Nonetheless, that energy isn't wasted, because a great teacher impacts those students' chances of success more than they impact the class as a whole. They will benefit from the influence of their teacher's advice and example. Their self-esteem and aspirations will both be higher because of the high expectations, extra encouragement, and safe learning environment a good teacher provides. The right teacher opens doors for them, including things other students take for granted such as graduation, workplace readiness, and wise career planning. All students are impacted by their teachers, but it's the students that need the most help that a good teacher impacts the most. A great school system will benefit all its students, but it's most important for the most needy students, who need the support, resources, and healthy environment that a quality school system provides.

Over the course of my student teaching, I experienced the teamwork and commitment required to teach all students. My mentor teacher is the head of the math department at Tahanto Regional Middle and High School (TRMHS), and teachers were constantly popping into her room asking for feedback on their lesson plans, new instructional methods for topics their students were struggling with, guidance on grading, and materials she has developed over the decades. Within each subject, teachers coordinated tests, divided up content across grade levels, wrestled through the nuances of content, and shared suggestions for how to explain specific concepts. Sometimes teachers coordinated projects and topics across disciplines, such as exploring the plays of Shakespeare while students were studying Elizabethan history. However, when teachers collaborated with teachers of other subjects, they generally discussed the behavior of specific students. Hearing how difficult students behaved in other classes provided insight for pinpointing the source of the problem (frustration, boredom, disrespect, seniorities, emotional problems, trouble reading, poor attention span, medical problems, etc.). Often more experienced teachers could share methods that worked well with the student, such as alternative notes, a five minute break, or project-based work, which was extraordinarily helpful to me as a student teacher.

Collaboration is very much part of the school culture at TRMHS, and I grew more in the collaborative aspect of reflective practice than I did in any other aspect of teaching. I expect to go into industry when I finish my engineering degree at Worcester Polytechnic Institute, and the progress I made in collaborating with coworkers and receiving feedback will serve me well in my career. Were I to go directly into teaching after graduation, my biggest goal would be to reflect more effectively with mentors, and to more clearly communicate with them. Often I struggled to get feedback on specific areas that I was struggling in, and although I often incorporated the suggestions of my mentors, I often forgot

to tell them I had done so, so they felt unheard. If I went directly into teaching, I would want to make those mentoring exchanges both more fruitful and more pleasant for all involved. Getting teaching advice is like getting parenting advice. At its best, collaboration is empowering, comforting, practical, and inspiring. If handled poorly, it becomes stressful even when it's helpful and accurate. I underestimated just how collaborative the teaching profession is, and how dramatically that environment permeates the school experience.

Sportswriter Red Smith once wrote "Fighters fight. And teachers, they teach," underscoring how teaching is more than a profession, it's a calling, a way of life, and a personality. When I spoke to a teacher about current politics, or breeding guppies, or the possibility of a technologically advanced civilization predating our own before the axis of Earth shifted, I learned volumes, because there's something about teachers that's inherently didactic. All my mentors and most of my students agreed I have the heart of a teacher, and I'll find a way to incorporate teaching into my lifestyle, whether that's as a professional teacher or a math tutor or a Sunday school teacher or a special needs mentor or something I haven't even tried yet. The best practices, differentiated methods, and tricks of the trade student teaching has taught me will be put to good use, and I'm grateful for the experience.

#### **Works Cited:**

"Guidelines for the Candidate Assessment of Performance." Massachusetts Department of Elementary and Secondary Education, June 2016. Web. 17 May 2017.  
<<http://www.doe.mass.edu/edprep/cap/guidelines.pdf>>.

Gladwell, Malcolm. *Tipping Point*. Little, Brown, 2014. Print.



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# APPLICATIONS OF CALCULUS

## Lesson Plan Title: Linear Approximation

**Teacher's Name: Priscilla Cole      Subject/Course: Applications of Calculus**  
**Unit: Taylor Polynomials              Grade Level: 12**

### Overview of and Motivation for Lesson:

This lays a foundation for learning Taylor Polynomials, which the students are interested in. This topic will help them with calculus and higher math courses they plan to take in college. Also, linearization is a useful skill and the boys need to practice their derivatives and fractions.

Stage 1-Desired Results	
<p><b>Standard(s):</b></p> <ul style="list-style-type: none"> <li>These students have completed Mass. Standards.</li> </ul>	
<p><b>Understanding(s):</b> <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>Linearization allows us to extrapolate and interpolate</li> <li>Tangent linearization is a first order Taylor Polynomial</li> <li>Secant linearization is used with tables.</li> </ul>	<p><b>Essential Question(s):</b></p> <p>Why do we use linearization?            How do we linearize a function?            How do we find a value not shown on a table?</p>
<p><b>Content Objectives:</b></p> <p><b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>Linearization allows us to extrapolate and interpolate</li> <li>Tangent linearization is a first order Taylor Polynomial</li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>Use linearization to find approximations of given functions</li> <li>Use linearization to interpolate and extrapolate from a table of values</li> </ul>	<p><b>Language Objectives:</b></p> <p>ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Perform the kind of linearization requested</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Determine what form of linearization is requested from a complex word problem.</li> </ul>
<p><b>Key Vocabulary</b>  <b>Tangent, Secant, Linear, Linearization, Taylor Polynomial, 1<sup>st</sup> order</b></p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Completing worksheet problems, ultimately without assistance</li> <li>Interpolating from tables</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>Assisting others</li> <li>Asking insightful questions</li> <li>Connecting concepts with science and life experiences.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Are the answers correct?</li> <li>Is the work shown correct?</li> </ul>	

## Stage 3- Learning Plan

### Learning Activities:

Do Now/Bell Ringer/Opener: Read linearization notes (should be a review).

#### Learning Activity 1:

Linearization from equations (packet based). I have an extra sheet with 3 more problems if the students struggle.

#### Learning Activity 2:

I will explain secant interpolation and we will interpolate using the steam tables to find the pressure needed to boil water at room temperature, and the temperature needed to boil water at atmospheric pressure. Students will work in a group of 3.

#### Closing

We will demonstrate that the boiling point of water changes with pressure using hot water from the coffee machine and a syringe. The class is small enough for everyone to take a turn.

### Multiple Intelligences Addressed:

- |  |  |  |  |
|--|--|--|--|
| <input checked="" type="checkbox"/> Linguistic | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input checked="" type="checkbox"/> Bodily-kinesthetic |
| <input type="checkbox"/> Spatial               | <input checked="" type="checkbox"/> Interpersonal        | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic                  |

### Student Grouping

- Whole Class     Small Group     Pairs     Individual

The whole class is a small group of 3 (4 with me).

### Instructional Delivery Methods

- |  |                                  |   |
|--|----------------------------------|---|
| <input checked="" type="checkbox"/> Teacher Modeling/Demonstration | <input type="checkbox"/> Lecture | <input type="checkbox"/> Discussion                 |
| <input checked="" type="checkbox"/> Cooperative Learning           | <input type="checkbox"/> Centers | <input checked="" type="checkbox"/> Problem Solving |
| <input type="checkbox"/> Independent Projects                      |                                  |   |

### Accommodations

All students are seated near the teacher.

### Modifications

Window is opened, even though it's January, because students are hot after gym.

### Homework/Extension Activities:

Because this is an optional math class, and they have completed their requirements, this class has no homework

### Materials and Equipment Needed:

- Linearization packet <https://www.teacherspayteachers.com/>
- Linearization extra worksheet <https://www.teacherspayteachers.com/>
- Steam tables [http://www.engineeringtoolbox.com/saturated-steam-properties-d\\_101.html](http://www.engineeringtoolbox.com/saturated-steam-properties-d_101.html)
- Coffeemaker
- Syringe
- Mug of water

<u>Absolute pressure</u> (kPa, kN/m <sup>2</sup> )	<u>Temperature</u> (°C)	<u>Specific Volume</u> (m <sup>3</sup> /kg)	<u>Density</u> - ρ - (kg/m <sup>3</sup> )	Specific Enthalpy of			Specific Entropy of Steam - s - (kJ/kgK)
				Liquid - h <sub>l</sub> - (kJ/kg)	Evaporation - h <sub>e</sub> - (kJ/kg)	Steam - h <sub>s</sub> - (kJ/kg)	
0.8	3.8	160	0.00626	15.8	2493	2509	9.058
2.0	17.5	67.0	0.0149	73.5	2460	2534	8.725
5.0	32.9	28.2	0.0354	137.8	2424	2562	8.396
10.0	45.8	14.7	0.0682	191.8	2393	2585	8.151
20.0	60.1	7.65	0.131	251.5	2358	2610	7.909
28	67.5	5.58	0.179	282.7	2340	2623	7.793
35	72.7	4.53	0.221	304.3	2327	2632	7.717
45	78.7	3.58	0.279	329.6	2312	2642	7.631
55	83.7	2.96	0.338	350.6	2299	2650	7.562
65	88.0	2.53	0.395	368.6	2288	2657	7.506
75	91.8	2.22	0.450	384.5	2279	2663	7.457
85	95.2	1.97	0.507	398.6	2270	2668	7.415
95	98.2	1.78	0.563	411.5	2262	2673	7.377
100	99.6	1.69	0.590	417.5	2258	2675	7.360
<b>101.33<sup>1)</sup></b>	<b>100</b>	<b>1.67</b>	<b>0.598</b>	<b>419.1</b>	<b>2257</b>	<b>2676</b>	<b>7.355</b>
110	102.3	1.55	0.646	428.8	2251	2680	7.328
130	107.1	1.33	0.755	449.2	2238	2687	7.271
150	111.4	1.16	0.863	467.1	2226	2698	7.223
170	115.2	1.03	0.970	483.2	2216	2699	7.181
190	118.6	0.929	1.08	497.8	2206	2704	7.144
220	123.3	0.810	1.23	517.6	2193	2711	7.095
260	128.7	0.693	1.44	540.9	2177	2718	7.039
280	131.2	0.646	1.55	551.4	2170	2722	7.014
320	135.8	0.570	1.75	570.9	2157	2728	6.969
360	139.9	0.510	1.96	588.5	2144	2733	6.930
400	143.1	0.462	2.16	604.7	2133	2738	6.894
440	147.1	0.423	2.36	619.6	2122	2742	6.862
480	150.3	0.389	2.57	633.5	2112	2746	6.833
500	151.8	0.375	2.67	640.1	2107	2748	6.819
550	155.5	0.342	2.92	655.8	2096	2752	6.787
600	158.8	0.315	3.175	670.4	2085	2756	6.758
650	162.0	0.292	3.425	684.1	2075	2759	6.730
700	165.0	0.273	3.66	697.1	2065	2762	6.705
750	167.8	0.255	3.915	709.3	2056	2765	6.682

<u>Absolute pressure</u> (kPa, kN/m <sup>2</sup> )	<u>Temperature</u> (°C)	<u>Specific Volume</u> (m <sup>3</sup> /kg)	<u>Density</u> - ρ - (kg/m <sup>3</sup> )	Specific Enthalpy of			Specific Entropy of Steam - s - (kJ/kgK)
				Liquid - h <sub>l</sub> - (kJ/kg)	Evaporation - h <sub>e</sub> - (kJ/kg)	Steam - h <sub>s</sub> - (kJ/kg)	
800	170.4	0.240	4.16	720.9	2047	2768	6.660
850	172.9	0.229	4.41	732.0	2038	2770	6.639
900	175.4	0.215	4.65	742.6	2030	2772	6.619
950	177.7	0.204	4.90	752.8	2021	2774	6.601
1000	179.9	0.194	5.15	762.6	2014	2776	6.583
1050	182.0	0.186	5.39	772	2006	2778	6.566
1150	186.0	0.170	5.89	790	1991	2781	6.534
1250	189.8	0.157	6.38	807	1977	2784	6.505
1300	191.6	0.151	6.62	815	1971	2785	6.491
1500	198.3	0.132	7.59	845	1945	2790	6.441
1600	201.4	0.124	8.03	859	1933	2792	6.418
1800	207.1	0.110	9.07	885	1910	2795	6.375
2000	212.4	0.0995	10.01	909	1889	2797	6.337
2100	214.9	0.0945	10.54	920	1878	2798	6.319
2300	219.6	0.0868	11.52	942	1858	2800	6.285
2400	221.8	0.0832	12.02	952	1849	2800	6.269
2600	226.0	0.0769	13.01	972	1830	2801	6.239
2700	228.1	0.0740	13.52	981	1821	2802	6.224
2900	232.0	0.0689	14.52	1000	1803	2802	6.197
3000	233.8	0.0666	15.00	1008	1794	2802	6.184
3200	237.4	0.0624	16.02	1025	1779	2802	6.158
3400	240.9	0.0587	17.04	1042	1760	2802	6.134
3600	244.2	0.0554	18.06	1058	1744	2802	6.112
3800	247.3	0.0524	19.08	1073	1728	2801	6.090
4000	250.3	0.0497	20.09	1087	1713	2800	6.069

## Lesson Plan Title: Polar Area

**Teacher's Name:** Priscilla Cole      **Subject/Course:** Applications of Calculus  
**Unit:** Polar                              **Grade Level:** 12

### Overview of and Motivation for Lesson:

The students are interested in learning double derivatives. This topic will help them with calculus and higher math courses they plan to take in college. Also, the boys have largely forgotten polar, so I will be reteaching that.

Stage 1-Desired Results	
<p><b>Standard(s):</b></p> <ul style="list-style-type: none"> <li>These students have completed Mass. Standards.</li> </ul>	
<p><b>Understanding(s):</b>  <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>Polar is an alternative to the Cartesian plane</li> <li><math>X=r\cos\theta</math> , <math>y=r\sin\theta</math></li> <li><math>R=\sqrt{(x^2+y^2)}</math> and <math>\tan\theta=y/x</math></li> <li><math>dy/dx=(r\cos\theta+r'\sin\theta)/(-r\sin\theta+r'\cos\theta)</math></li> <li><math>Area = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta</math></li> </ul>	<p><b>Essential Question(s):</b></p> <ul style="list-style-type: none"> <li>How do I find area inside a polar graph?</li> <li>How do I find area inside parts of a polar graph?</li> </ul>
<p><b>Content Objectives:</b>  <b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>Polar is an alternative to the Cartesian plane</li> <li><math>X=r\cos\theta</math> , <math>y=r\sin\theta</math></li> <li><math>R=\sqrt{(x^2+y^2)}</math></li> <li><math>\tan\theta=y/x</math></li> <li><math>dy/dx=(r\cos\theta+r'\sin\theta)/(-r\sin\theta+r'\cos\theta)</math></li> <li><math>Area = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta</math></li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>Convert between polar to cartesian</li> <li>Graph polar</li> <li>Find the area of a polar graph</li> </ul>	<p><b>Language Objectives:</b>                      ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Solve problems with mixed symbols and words.</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Rephrase symbolic language in a grammatically correct sentence or two.</li> <li>Solve problems with mixed symbols and words.</li> </ul>
<p><b>Key Vocabulary</b>  <math>\theta</math>, <math>r</math>, <math>y</math>, <math>x</math>, ordered pair, radius, radians, degrees, wedge, Area, integral, with respect to</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Defining key vocabulary in original language</li> <li>Completing packet problems, ultimately without assistance, and showing appropriate work.</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>Assisting others</li> <li>Asking insightful questions</li> <li>Connecting concepts with past experiences.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Can they explain the polar system in their own words?</li> <li>Is the packet's answers and work correct?</li> </ul>	

## Stage 3- Learning Plan

### Learning Activities:

Do Now/Bell Ringer/Opener: Graph polar equations by hand

#### Learning Activity 1: Area of a circle

Read Introduction. Then we will derive area of a circle, then area of a partial circle.

#### Learning Activity 2: Area of polar packet

We will solve for the area of various shapes, then parts of shapes. I will model examples of each type of problem.

#### Learning Activity 3: Overview of quiz

I will explain that we're going to have a quiz on what we learned. There will be series convergence, approximation, interpolation, related rates, polar to Cartesian conversion, Cartesian to polar conversion, graphing polar, and polar slope.

#### Closing: simple summary

We'll summarize what we've learned about polar so far, then I'll release them.

### Multiple Intelligences Addressed:

- |   |  |  |   |
|---|--|--|---|
| <input type="checkbox"/> Linguistic         | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial | <input type="checkbox"/> Interpersonal                   | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic       |

### Student Grouping

- Whole Class    
 Small Group    
 Pairs    
 Individual

The whole class is a small group of 3 (4 with me).

### Instructional Delivery Methods

- |  |                                  |   |
|--|----------------------------------|---|
| <input checked="" type="checkbox"/> Teacher Modeling/Demonstration | <input type="checkbox"/> Lecture | <input type="checkbox"/> Discussion                 |
| <input type="checkbox"/> Cooperative Learning                      | <input type="checkbox"/> Centers | <input checked="" type="checkbox"/> Problem Solving |
| <input type="checkbox"/> Independent Projects                      |                                  |   |

### Accommodations

All students are seated near the teacher.

### Modifications

Window is opened, even though it's January, because students are hot after gym.

### Homework/Extension Activities:

Because this is an optional math class, and they have completed their requirements, this class has no homework

### Materials and Equipment Needed:

- List of quiz topics and sample questions of each
- Polar slope packet
- Graphing calculators
- Polar graphing paper



## Lesson Plan Title: Basic Roller Coaster Continuity

Teacher's Name: Priscilla Cole      Subject/Course: Applications of Calculus  
 Unit: F(2 variables), tangent approx., and optimization      Grade Level: 12

### Overview of and Motivation for Lesson:

To review continuity and have some fun.

Stage 1-Desired Results	
<p><b>Standard(s):</b></p> <ul style="list-style-type: none"> <li>• These students have completed Mass. Standards.</li> </ul>	
<p><b>Understanding(s):</b>  <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>• Continuity requires both derivatives and y values to match</li> <li>• Arc Length is <math display="block">L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx</math></li> </ul>	<p><b>Essential Question(s):</b></p> <ul style="list-style-type: none"> <li>• How do I make continuous piecewise functions?</li> <li>• How do I find arc length of Cartesian coordinates?</li> </ul>
<p><b>Content Objectives:</b>  <b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>• Continuity requires both derivatives and y values to match</li> <li>• Arc Length is <math display="block">L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx</math></li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>• Design a continuous rollercoaster</li> </ul>	<p><b>Language Objectives:</b>                      ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Describe their roller coaster</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Describe their roller coaster</li> </ul>
<p><b>Key Vocabulary</b>                      Continuous, derivative, roller coaster</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Designing roller coaster</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>• Assisting others</li> <li>• Asking insightful questions</li> <li>• Connecting concepts with past experiences.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Can they explain the polar system in their own words?</li> <li>• Is the packet's answers and work correct?</li> </ul>	

## Stage 3- Learning Plan

### Learning Activities:

Do Now/Bell Ringer/Opener: Week off high-low

Learning Activity 1: Instructions for roller coaster project  
Read Instructions. Take questions.

Learning Activity 2: Roller Coaster Project  
Students will work on roller coaster project

\_\_\_\_\_ This will take multiple days \_\_\_\_\_

Closing: Clean up, Clean up, everybody, everywhere..  
We'll clean up, but we won't sing, although that would have invoked the ever-challenging musical intelligence.

### Multiple Intelligences Addressed:

- |   |  |  |  |
|---|--|--|--|
| <input type="checkbox"/> Linguistic         | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input checked="" type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial | <input type="checkbox"/> Interpersonal                   | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic                  |

### Student Grouping

- Whole Class       Small Group       Pairs       Individual

The whole class is a small group of 3 (4 with me).

### Instructional Delivery Methods

- |  |                                  |  |
|--|----------------------------------|--|
| <input checked="" type="checkbox"/> Teacher Modeling/Demonstration | <input type="checkbox"/> Lecture | <input type="checkbox"/> Discussion      |
| <input type="checkbox"/> Cooperative Learning                      | <input type="checkbox"/> Centers | <input type="checkbox"/> Problem Solving |
| <input checked="" type="checkbox"/> Independent Projects           |                                  |  |

### Accommodations

All students are seated near the teacher.

### Modifications

Window is opened, even though it's January, because students are hot after gym.

### Homework/Extension Activities:

Because this is an optional math class, and they have completed their requirements, this class has no homework

### Materials and Equipment Needed:

- Graphing Calculators
- Instructions sheet with Rubric
- Printer paper
- Graphing paper
- Colored pencils
- Stapler with staples

## Lesson Plan Title: 3-D

**Teacher's Name: Priscilla Cole      Subject/Course: Applications of Calculus**  
**Unit: F(2 variables), tangent approx., and optimization      Grade Level: 12**

### Overview of and Motivation for Lesson:

The students are interested in learning multivariable calculus. They think it will be cool, and frankly I agree with them. Multivariable calculus is used in fluid dynamics, thermodynamics, and high finance fields. It also gives us an opportunity to review 2-D calculus in an interesting way, and is used to derive many formulas.

Stage 1-Desired Results	
<b>Standard(s):</b> <ul style="list-style-type: none"> <li>These students have completed Mass. Standards.</li> </ul>	
<b>Understanding(s):</b> <i>Students will understand that . . .</i> <ul style="list-style-type: none"> <li>Contour plots show lines with equal z values</li> </ul>	<b>Essential Question(s):</b> <ul style="list-style-type: none"> <li>What is a contour line?</li> </ul>
<b>Content Objectives:</b> <b>Knowledge:</b> <i>Students will know . . .</i> <ul style="list-style-type: none"> <li>Contour plots show lines with equal z values</li> <li></li> </ul> <b>Skills/Performance:</b> <i>Students will be able to . . .</i> <ul style="list-style-type: none"> <li>Draw contour plots</li> </ul>	<b>Language Objectives:</b> ELD Level 3 <i>Students will be able to . . . in English</i> <ul style="list-style-type: none"> <li>Define key vocabulary</li> </ul> ELD Level 4 <i>Students will be able to . . . in English</i> <ul style="list-style-type: none"> <li>Define key vocabulary clearly</li> </ul>
<b>Key Vocabulary</b> Contour lines=level sets=isolines=lines of equal value=lines of equal potential	
Stage 2-Assessment Evidence	
<b>Performance Task(s) or Key Evidence</b> <ul style="list-style-type: none"> <li>Defining key vocabulary in original language</li> <li>Completing packet problems, ultimately without assistance, and showing appropriate work.</li> </ul>	<b>Other Evidence:</b> <ul style="list-style-type: none"> <li>Assisting others</li> <li>Asking insightful questions</li> <li>Connecting concepts with past experiences.</li> </ul>
<b>Key Criteria to measure Performance Task (s) or Key Evidence</b> <ul style="list-style-type: none"> <li>Can they explain the key vocabulary in their own words?</li> <li>Is the packet's answers and work correct?</li> </ul>	

## Stage 3- Learning Plan

### Learning Activities:

Do Now/Bell Ringer/Opener: None

Learning Activity 1: Overview instructions online

<https://www.khanacademy.org/math/multivariable-calculus/thinking-about-multivariable-function/ways-to-represent-multivariable-functions/a/contour-maps>

It's a great resource, and I'll probably put it on the projector, even for our little class of three.

Learning Activity 2: Reading notes

We will read the notes silently

Learning Activity 3: Independent problems

Students will solve problems independently, graph them, and compare.

Learning Activity 4: cardboard contour plots

Students make 3-d cardboard models of the graphs, stacking the contour plots. Wish me luck!

Closing: simple summary

We'll summarize what we've learned about derivatives, then I'll release them.

### Multiple Intelligences Addressed:

- |   |  |  |  |
|---|--|--|--|
| <input type="checkbox"/> Linguistic         | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input checked="" type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial | <input type="checkbox"/> Interpersonal                   | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic                  |

### Student Grouping

- Whole Class     Small Group     Pairs     Individual

The whole class is a small group of 3 (4 with me).

### Instructional Delivery Methods

- |  |                                  |   |
|--|----------------------------------|---|
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| <input type="checkbox"/> Cooperative Learning                      | <input type="checkbox"/> Centers | <input checked="" type="checkbox"/> Problem Solving |
| <input checked="" type="checkbox"/> Independent Projects           |                                  |   |

### Accommodations

All students are seated near the teacher.

### Modifications

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### Homework/Extension Activities:

Because this is an optional math class, and they have completed their requirements, this class has no homework

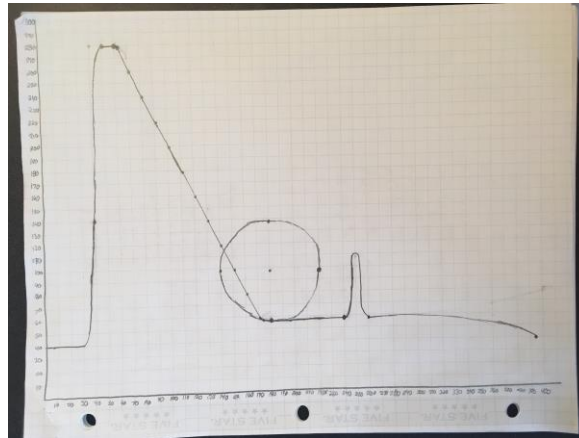
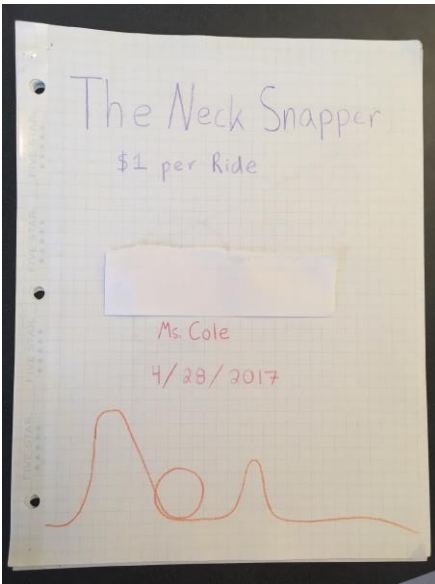
### Materials and Equipment Needed:

- Scissors
- Cardboard
- Projection and internet access
- Worksheet
- Graph paper and writing implements



3-D Cardboard Contour plots made by myself (two on the far right) and my applications of calculus students.





Ryan Grady

$$f(x) = \begin{cases} 40 & 0 \leq x \leq 30 \\ -(x-30)^2 + 40 & 30 \leq x \leq 40 \\ -(x-50)^2 + 280 & 40 \leq x \leq 50 \\ 280 & 50 \leq x \leq 60 \\ -(x-60)^2 + 280 & 60 \leq x \leq 70 \\ -2x + 401 & 70 \leq x \leq 171 \\ \frac{58}{x-170} & 170 \leq x \leq 171 \\ -\frac{1000(x-200)^2 + 98}{100} & 170 \leq x \leq 200 \\ \frac{1000(x-200)^2 + 98}{100} & 200 \leq x \leq 220 \\ -\frac{1000(x-200)^2 + 98}{100} & 200 \leq x \leq 220 \\ \frac{58}{x-200} & 200 \leq x \leq 220 \\ -\frac{20 \cos(\frac{\pi}{10}(x-200)) + 58}{100} & 200 \leq x \leq 250 \\ \frac{1000(x-200)^2 + 98}{100} & 200 \leq x \leq 250 \end{cases}$$

$(x-172)^2 + 57, 171 \leq x \leq 172$

$\lim_{x \rightarrow 0^+} f(x) = 40, \lim_{x \rightarrow 0^+} f'(x) = 0$   
 $\lim_{x \rightarrow 30^+} f(x) = 40, \lim_{x \rightarrow 30^+} f'(x) = 0$   
 $\lim_{x \rightarrow 40^+} f(x) = 40, \lim_{x \rightarrow 40^+} f'(x) = 0$   
 $\lim_{x \rightarrow 50^+} f(x) = 280, \lim_{x \rightarrow 50^+} f'(x) = 0$   
 $\lim_{x \rightarrow 60^+} f(x) = 280, \lim_{x \rightarrow 60^+} f'(x) = 0$   
 $\lim_{x \rightarrow 70^+} f(x) = 280, \lim_{x \rightarrow 70^+} f'(x) = 0$   
 $\lim_{x \rightarrow 171^+} f(x) = 280, \lim_{x \rightarrow 171^+} f'(x) = 0$   
 $\lim_{x \rightarrow 170^+} f(x) = -\infty, \lim_{x \rightarrow 170^+} f'(x) = 0$   
 $\lim_{x \rightarrow 171^-} f(x) = \infty, \lim_{x \rightarrow 171^-} f'(x) = 0$   
 $\lim_{x \rightarrow 200^+} f(x) = -\infty, \lim_{x \rightarrow 200^+} f'(x) = 0$   
 $\lim_{x \rightarrow 200^-} f(x) = \infty, \lim_{x \rightarrow 200^-} f'(x) = 0$   
 $\lim_{x \rightarrow 220^+} f(x) = 58, \lim_{x \rightarrow 220^+} f'(x) = 0$   
 $\lim_{x \rightarrow 220^-} f(x) = 58, \lim_{x \rightarrow 220^-} f'(x) = 0$   
 $\lim_{x \rightarrow 250^+} f(x) = 58, \lim_{x \rightarrow 250^+} f'(x) = 0$   
 $\lim_{x \rightarrow 250^-} f(x) = 58, \lim_{x \rightarrow 250^-} f'(x) = 0$

$\lim_{x \rightarrow 172^+} f(x) = (172-172)^2 + 57 = 57, \lim_{x \rightarrow 172^+} f'(x) = 0$   
 $\lim_{x \rightarrow 172^-} f(x) = 57, \lim_{x \rightarrow 172^-} f'(x) = 0$   
 $\lim_{x \rightarrow 171^+} f(x) = 57, \lim_{x \rightarrow 171^+} f'(x) = 0$   
 $\lim_{x \rightarrow 171^-} f(x) = 57, \lim_{x \rightarrow 171^-} f'(x) = 0$   
 $\lim_{x \rightarrow 200^+} f(x) = 98, \lim_{x \rightarrow 200^+} f'(x) = 0$   
 $\lim_{x \rightarrow 200^-} f(x) = 98, \lim_{x \rightarrow 200^-} f'(x) = 0$   
 $\lim_{x \rightarrow 220^+} f(x) = 58, \lim_{x \rightarrow 220^+} f'(x) = 0$   
 $\lim_{x \rightarrow 220^-} f(x) = 58, \lim_{x \rightarrow 220^-} f'(x) = 0$   
 $\lim_{x \rightarrow 250^+} f(x) = 58, \lim_{x \rightarrow 250^+} f'(x) = 0$   
 $\lim_{x \rightarrow 250^-} f(x) = 58, \lim_{x \rightarrow 250^-} f'(x) = 0$

Length Calculation

Segment 1 = 30 feet  
( $0 \leq x \leq 30$ )

Segment 2 =  $\int_{30}^{40} \sqrt{1 + (-2(x-30))^2} dx = 101.05$  feet  
( $30 \leq x \leq 40$ )

Segment 3 =  $\int_{40}^{50} \sqrt{1 + (-2(x-50))^2} dx = 101.05$  feet  
( $40 \leq x \leq 50$ )

Segment 4 = 10 feet  
( $50 \leq x \leq 60$ )

Segment 5 =  $\int_{60}^{70} \sqrt{1 + (-2(x-60))^2} dx = 1.48$  feet  
( $60 \leq x \leq 70$ )

Segment 6 =  $\int_{70}^{171} \sqrt{1 + (-2)^2} dx = 245.97$  feet  
( $70 \leq x \leq 171$ )

Segment 7 =  $\int_{171}^{172} \sqrt{1 + (2(x-172))^2} dx = 1.48$  feet  
( $171 \leq x \leq 172$ )

Segment 8 = 0 feet  
( $172 \leq x \leq 172$ )

Segment 9 =  $C = 2\pi r, C = 2\pi(40) = 251.33$  feet  
( $172 \leq x \leq 200$ )  
entire loop is circle w/ radius of 40 feet so...

Segment 10 = 60 feet  
( $200 \leq x \leq 200$ )

Segment 11 =  $\int_{200}^{220} \sqrt{1 + (30 \sin(\frac{\pi}{10}(x-200)))^2} dx = 48.62$  feet  
( $200 \leq x \leq 220$ )

Segment 12 =  $\int_{220}^{250} \sqrt{1 + (\frac{-2(x-200)}{100})^2} dx = 155.75$  feet  
( $220 \leq x \leq 250$ )

Total Length = 994.73 feet

**WARNING!**

Must be 4'5" to ride the Neck Snapper

Please take proper precautions before enjoying this ride.

Failing to take proper precautions will result in a tragic death.

G-Forces can surpass deadly levels at certain segments of this ride.

Proper Precautions include wearing belts, sunglasses, loose clothing, and not getting on the ride.

$-(x-50)^2 + 280$  @  $(50, 280)$

$w = \sqrt{\frac{r}{r_1}}$   $r = \frac{1}{2}$   $m = 60$  m/s

$v = \sqrt{\frac{2(4900)}{60}} = 7$  m/s at first peak  $\rightarrow 1,568$  mph

$(x-172)^2 + 57$  @  $(172, 57)$

$r = \frac{1}{2}$

$v = \sqrt{\frac{2(4900)}{50}} = 7$  m/s at first bottom

$-30 \cos(\frac{\pi}{10}(x-200)) + 58$  @  $x = 250$

$(1 + 0.029)^2$   $r = 344.83$  m

$v = \sqrt{\frac{100(344.83)}{50}} = 8.39$  m/s

$40^2 = r_1 y^2$  @  $x = 190, x = 220$

$v = \sqrt{\frac{40(4900)}{50}} = 6.26$  m/s  $v = \sqrt{\frac{40(180)}{50}} = 2.8$  m/s

$v = \sqrt{\frac{40(200)}{50}} = 6.85$  m/s

Roller Coaster Project: Students made a rollercoaster from by making a continuous and differentiable piecewise function, calculating its length, radius of curvature, and the g-forces on riders. As noted on the warning sign, this ride was survivable because the hills are too tight but the loop is too big.

# HONORS CALCULUS

## Lesson Plan Title: Integrals are Signed Area

**Teacher's Name: Priscilla Cole      Subject/Course: Honors Calculus**

**Unit: Understanding what integrals are**

**Grade Level: 10-12**

### Overview of and Motivation for Lesson:

We are beginning integration. There are several ways to understand integrals. My goal is for each student to understand integrals 1 of the 4 ways, and for all students to be able to evaluate integrals and apply them to numerical problems and word problems. Learning the properties will help them apply methods of integration.

Stage 1-Desired Results	
<p><b>Standard(s):</b></p> <ul style="list-style-type: none"> <li>These students have completed Mass. Standards.</li> </ul>	
<p><b>Understanding(s):</b> <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>Integrals are signed area under a curve</li> <li><math>\int_a^b f(x)dx</math></li> <li>Integrals have the following properties: Order of Integration, Zero, Constant Multiple, Sum and Difference, Additivity (with the bounds)</li> </ul>	<p><b>Essential Question(s):</b> What are the properties of integrals?</p>
<p><b>Content Objectives:</b> <b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>Integrals have the following properties: Order of Integration, Zero, Constant Multiple, Sum and Difference, Additivity (with the bounds), and they mean . . .</li> <li>What <math>\int_a^b f(x)dx</math> means</li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>Define Integrals</li> <li>Apply the Order of Integration, Zero, Constant Multiple, Sum and Difference and Additivity properties to definite integrals to which are similar to evaluated integrals</li> <li>Define the Order of Integration, Zero, Constant Multiple, Sum and Difference and Additivity properties</li> </ul>	<p><b>Language Objectives:</b> ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Apply the 5 properties</li> <li>Explain the 5 properties by demonstration</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Do the above</li> <li>Define the 5 properties</li> </ul>
<p><b>Key Vocabulary</b> Order of Integration, Zero, Constant Multiple, Sum and Difference and Additivity properties</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Completing similar problems, ultimately without assistance.</li> <li>Completing novel problems</li> <li>Explains ideas in their own words.</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>Assisting others</li> <li>Asking insightful questions</li> <li>Connecting concepts with life experiences and other classes.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Exit ticket is correct and complete</li> </ul>	



### Stage 3- Learning Plan

**Learning Activities:**

Do Now/Bell Ringer/Opener: Going over the homework  
I'll be teaching from a powerpoint, going over their homework.

**Learning Activity 1: Flip chart for 5 properties of integrals**

Students will make a flip chart with the 5 properties of integrals, using my guidance from a powerpoint presentation.

\_\_\_\_\_this may take more than one day\_\_\_\_\_

**Closing: Exit ticket**

Evaluate these  $g(x)$ 's and state the properties used.

**Learning Activity 2: Going over homework (which may be done in class)**

We will review (and/or reteach) the properties and signed area methods

**Learning Activity 3: Using the calculators**

In assigned pairs, students will learn to evaluate definite integrals on their calculators. I'll be using the TI-84 on the ENO board.

**Learning Activity 4: Proving the properties with calculators (finish for hw)**

In those pairs, using their calculators, students will prove each of these properties and add an original example to each page of their flip chart, including the original integral, it's evaluation, the modified integral, its evaluation, and how the fits into the properties discussed.

**Multiple Intelligences Addressed:**

- |   |  |  |  |
|---|--|--|--|
| <input type="checkbox"/> Linguistic         | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input checked="" type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial | <input checked="" type="checkbox"/> Interpersonal        | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic                  |

**Student Grouping**

- Whole Class     Small Group     Pairs     Individual

**Instructional Delivery Methods**

- Teacher Modeling/Demonstration     Lecture     Discussion  
 Cooperative Learning     Centers     Problem Solving  
 Independent Projects

**Accommodations**

Low threat environment will be maintained for students with anxiety.

**Modifications**

Franny will be in the back of the room.

**Homework/Extension Activities:**

worksheets

**Materials and Equipment Needed:**

- Projection
- Flip charts
- Calculators
- worksheets

Please take out your homework

## 5 Properties of Integrals

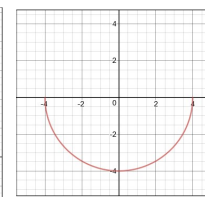
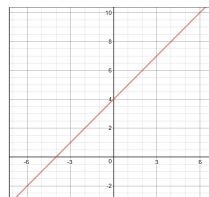
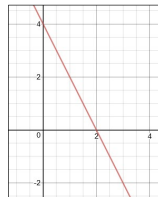
Order of Integration, Zero, Constant Multiple, Sum and Difference, and Additivity

Please take out your homework

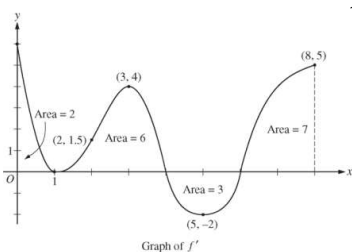
$$\int_{-2}^2 -2x + 4 = 16$$

$$\int_{-6}^4 x + 4 = 28$$

$$\int_{-4}^4 -\sqrt{16-x^2} = -8\pi$$



Please take out your homework



1) Using the graph of  $f'$  above, evaluate the following definite integrals.

a.  $\int_0^4 f'(x) dx = 8$

b.  $\int_4^6 f'(x) dx = -3$

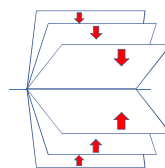
c.  $\int_6^8 f'(x) dx = 7$

d.  $\int_0^6 f'(x) dx = 5$

e.  $\int_0^8 f'(x) dx = 12$

f. What do you notice about these integrals?

Get 3 blank sheets of paper



**Title: 5 Properties of Integrals**

1. Order of Integration
2. Zero
3. Constant Multiple
4. Sum and Difference
5. Additivity (with the bounds)

Please take out your homework

$$\int_{-2}^2 -2x + 4 dx = A_{\text{triangle}} = \frac{1}{2} * 8 * 4 = 16$$

$$\int_{-6}^4 x + 4 dx = A_{\text{triangle}\#1} + A_{\text{triangle}\#2} = -4 + 32 = 28$$

$$\int_{-4}^4 -\sqrt{16-x^2} dx = -A_{\text{semicircle}} = -\frac{1}{2}\pi r^2 = -\frac{1}{2}\pi * 4^2 = -8\pi$$

### #1) Order of Integration

• If the upper bound and the lower bound are switched, then the integral is the negative of its original value.

$$\bullet \int_b^a f(x) dx = -\int_a^b f(x) dx$$

$$\bullet \int_7^0 2x + 9 dx = -\int_0^7 2x + 9 dx$$

$$\bullet \int_9^{-3} 4x dx =$$

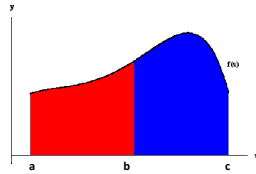
## #2) Zero

- If the upper bound and the lower bound are the same, then the integral is equal to zero.

$$\int_a^b f(x) dx = 0$$

$$\int_7^7 2x + 9 dx = 0$$

$$\int_{-3}^{-3} e^x dx =$$



## #4) Sum and difference (in the integrand)

- The integral of the difference of two terms is the integral of the first term minus the integral of the second term.

$$\int_a^b f(x) - g(x) dx = \int_a^b f(x) dx - \int_a^b g(x) dx$$

$$\int_a^b e^x - 2 dx = \int_a^b e^x dx - \int_a^b 2 dx$$

$$\int_{-5}^{-2} 4 + x dx =$$

## #3) Constant multiple

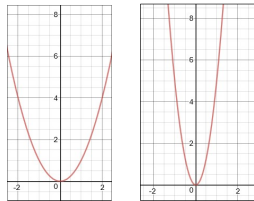
Constant: an number with a fixed value, such as 3, -9.12345, or  $\pi$ .

- If an integral is multiplied by a constant, the integral equals the original integrals value times that constant.

$$\int_a^b c * f(x) dx = c * \int_a^b f(x) dx$$

$$\int_0^5 7 * f(x) dx = 7 * \int_0^5 f(x) dx$$

$$\int_{-2}^2 5 * x^2 dx = 5 * \int_{-2}^2 x^2 dx$$

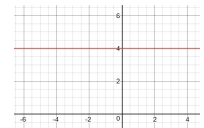
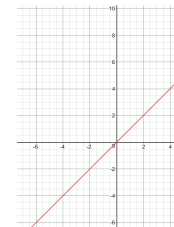
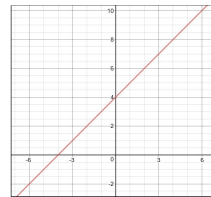


## Please take your homework back out

$$\int_{-6}^4 x + 4 = 28$$

$$\int_{-6}^4 x =$$

$$\int_{-6}^4 4 =$$



## #4) Sum and difference (in the integrand)

- The integral of the sum of two terms is the integral of the first term plus the integral of the second term.

$$\int_a^b f(x) + g(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$\int_a^b 7x + 2 dx = \int_a^b 7x dx + \int_a^b 2 dx$$

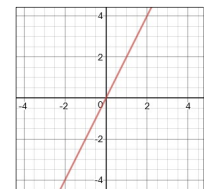
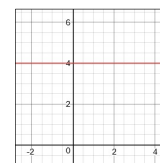
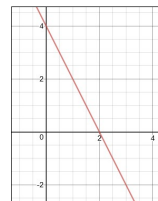
$$\int_0^5 10 + x dx =$$

## Please take your homework back out

$$\int_{-2}^2 4 - 2x = 16$$

$$\int_{-2}^2 4 =$$

$$\int_{-2}^2 2x =$$



### #5) Additivity (in the bounds)

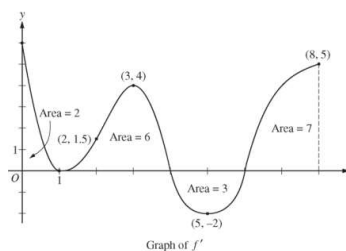
• The value of an integral from a to c is that integral from a to b plus that integral from b to c.

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

$$\int_0^{10} e^x dx = \int_0^3 e^x dx + \int_3^{10} e^x dx$$

$$\int_{-7}^{21} 4 + x dx =$$

### Please take your homework back out



$$\int_6^8 f'(x) dx = 7$$

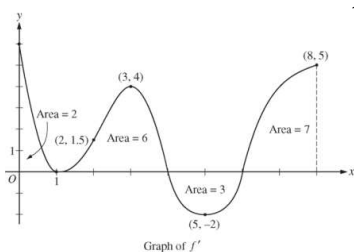
$$\int_8^6 f'(x) dx =$$

$$\int_8^8 f'(x) dx =$$

$$\int_6^8 3f'(x) dx =$$

$$\int_6^8 -f'(x) dx =$$

### Please take your homework back out



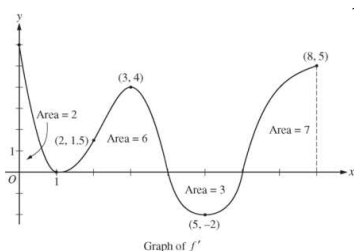
1) Using the graph of  $f'$  above, evaluate the following definite integrals.

$$a. \int_0^4 f'(x) dx = 8$$

$$b. \int_4^6 f'(x) dx = -3$$

$$c. \int_6^8 f'(x) dx = 7$$

### Please take your homework back out



1) Using the graph of  $f'$  above, evaluate the following definite integrals.

$$c. \int_6^8 f'(x) dx = 7$$

$$d. \int_0^6 f'(x) dx = 5$$

$$e. \int_0^8 f'(x) dx = 12$$

### Sources

- <https://www.desmos.com/calculator>
- <http://www.sosmath.com/calculus/integ/integ02/integ02.html>
- [http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13\\_frq\\_calculus\\_ab.pdf](http://media.collegeboard.com/digitalServices/pdf/ap/apcentral/ap13_frq_calculus_ab.pdf)
- Calculus Graphical, Numerical, Algebraic 3<sup>rd</sup> Edition

## Lesson Plan Title: Integrals on Calculators

**Teacher's Name:** Priscilla Cole  
**Calculus**

**Subject/Course:** Honors

**Unit:** Understanding what integrals are

**Grade Level:** 10-12

### Overview of and Motivation for Lesson:

We are beginning integration. There are several ways to understand integrals. My goal is for each student to understand integrals 1 of the 4 ways, and for all students to be able to evaluate integrals and apply them to numerical problems and word problems. We will learn to evaluate integrals with calculators.

Stage 1-Desired Results	
<p><b>Standard(s):</b></p> <ul style="list-style-type: none"> <li>These students have completed Mass. Standards.</li> </ul>	
<p><b>Understanding(s):</b> <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>Integrals are signed area under a curve</li> <li>Integrals are the limit of a Riemann sum</li> <li><math>\int_a^b f(x) dx</math></li> <li><math>h_{average} = \frac{1}{b-a} \int_a^b h(x) dx</math></li> </ul>	<p><b>Essential Question(s):</b></p> <p>How do I find average value of an integrable function?</p> <p>How do I evaluate an integral?</p>
<p><b>Content Objectives:</b> <b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>Integrals are signed area under a curve</li> <li>Integrals are the limit of a Riemann sum</li> <li>What <math>\int_a^b f(x) dx</math> means</li> <li><math>h_{average} = \frac{1}{b-a} \int_a^b h(x) dx</math></li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>Find average value using integrals evaluated on a calculator.</li> <li>Find average value using integrals evaluated by area under a graph.</li> <li>Find average value using integrals evaluated from a list of value.</li> </ul>	<p><b>Language Objectives:</b></p> <p>ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Read word problems with pictures</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>Read word problems and explain them</li> </ul>
<p><b>Key Vocabulary</b> Integral, Average, Integral, Integrable, mean</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Completing similar problems, ultimately without assistance.</li> <li>Completing novel problems</li> <li>Explains ideas in their own words.</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>Assisting others</li> <li>Asking insightful questions</li> <li>Connecting concepts with life experiences and other classes.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>Are their answers correct?</li> <li>Is the work shown correct?</li> </ul>	

### Stage 3- Learning Plan

**Learning Activities:**

Do Now/Bell Ringer/Opener: Checking Homework

I'll go over the homework, and have them pair share their definitions.

Learning Activity 1: Powerpoint on Average Value Formula

I'll give direct instruction and work through examples with them.

Closing: Begin Homework

3 question worksheet: 1 formula, 1 graph, 1 table

**Multiple Intelligences Addressed:**

- |  |  |  |  |
|--|--|--|--|
| <input checked="" type="checkbox"/> Linguistic | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input type="checkbox"/> Bodily-kinesthetic      |
| <input checked="" type="checkbox"/> Spatial    | <input checked="" type="checkbox"/> Interpersonal        | <input type="checkbox"/> Intrapersonal | <input checked="" type="checkbox"/> Naturalistic |

**Student Grouping**

- Whole Class     
  Small Group     
  Pairs     
  Individual

**Instructional Delivery Methods**

- |  |   |   |
|--|---|---|
| <input checked="" type="checkbox"/> Teacher Modeling/Demonstration | <input checked="" type="checkbox"/> Lecture | <input type="checkbox"/> Discussion                 |
| <input type="checkbox"/> Cooperative Learning                      | <input type="checkbox"/> Centers            | <input checked="" type="checkbox"/> Problem Solving |
| <input type="checkbox"/> Independent Projects                      |   |   |

**Accommodations**

Low threat environment will be maintained for students with anxiety.

**Modifications**

Franny will be in the back of the room.

**Homework/Extension Activities:**

Review derivatives

**Materials and Equipment Needed:**

- Projection
- Calculators
- Worksheets
- Yesterday's HW

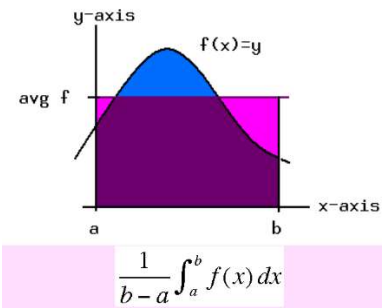
Please take out your homework

## Using integrals to find Average Value

$$h_{average} = \frac{1}{b-a} \int_a^b h(x) dx$$

Avg. Value

The average value can be found by rearranging area under the curve to form a rectangle



Old definition of Average (mean)

$$\bullet \text{Average} = \frac{a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n}{n}$$



• If there are 5 koala exhibits in New York, and they have 2, 3, 4, 5, and 6 koalas, what is the average number of koalas per exhibit?

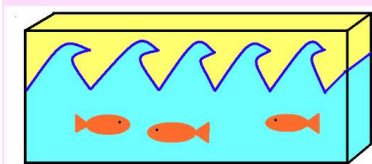
Avg. Value

The average value can be found by rearranging area under the curve to form a rectangle

Now we can answer our fish tank question!  
(That is, if the waves were described by an integrable function)

How high would the water level be if the waves all settled?

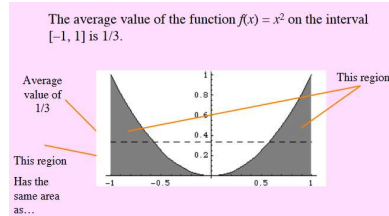
Consider the following picture:



How high would the water level be if the waves all settled?

Avg. Value

The average value can be found by rearranging area under the curve to form a rectangle



$$h_{average} = \frac{1}{b-a} \int_a^b h(x) dx$$

The average value of the function  $f(x) = x^2$  on the interval  $[-1, 1]$  is  $1/3$ .

Average value of  $1/3$

This region Has the same area as...

What is the average number of deaths under 5 yoa?

### Falling faster

Number of deaths of under-5-year-olds\*, m

- Actual
- At 1990-2000 trend
- At 1996-2001 trend

$$h_{average} = \frac{1}{b-a} \int_a^b h(x) dx$$

Find the average value of  $f(t) = 2^t$  on the interval  $[0, 2]$ .  
First, make a graphical estimate:

What is the average number of adult penguins from 1910 to 2010?

$y = 600e^{-.02t}$

$$h_{average} = \frac{1}{b-a} \int_a^b h(x) dx$$

Find the average value of  $f(t) = 2^t$  on the interval  $[0, 2]$ .  
First, make a graphical estimate:

$$\frac{1}{2-0} \int_0^2 2^t dt = \frac{1}{2} \int_0^2 2^t dt$$

So the average value is about  $(1/2)(4.328)$  or 2.164

### Sources

- <https://www.desmos.com/calculator>
- [http://www.math.vt.edu/people/qlfang/class\\_home/Lesson8.pdf](http://www.math.vt.edu/people/qlfang/class_home/Lesson8.pdf)
- Calculus Graphical, Numerical, Algebraic 3<sup>rd</sup> Edition
- <https://www.gatesnotes.com/2017-Annual-Letter>



**Lesson Plan Title: Evaluating Integrals Flip chart: exponents and review**

**Teacher’s Name: Priscilla Cole**

**Subject/Course: Honors Calculus**

**Unit: Evaluating Integrals at last!**

**Grade Level: 10-12**

**Overview of and Motivation for Lesson:**

Now that we understand what integrals are, we are learning how to evaluate them. We will study the methods for integrating basic functions in this unit. My goal is for all students to be able to evaluate integrals with basic rules and the 5 properties we studied last week. We will also review how to evaluate integrals with calculators.

Stage 1-Desired Results	
<p><b>Standard(s):</b></p> <ul style="list-style-type: none"> <li>• These students have completed Mass. Standards.</li> </ul>	
<p><b>Understanding(s):</b> <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>• Integration is the inverse of differentiation (FTC).</li> <li>• <math>\int x^n dx = \frac{x^{n+1}}{n+1} + C</math></li> <li>• <math>\int \frac{1}{x} dx = \ln x  + C</math></li> <li>• <math>\int e^x dx = e^x + C</math></li> <li>• <math>\int \cos x dx = \sin x + C</math></li> <li>• <math>\int \sin x dx = -\cos x + C</math></li> </ul>	<p><b>Essential Question(s):</b></p> <p>Why is integration sometimes called antidifferentiation?</p> <p>How do I take the derivative without a calculator?</p>
<p><b>Content Objectives:</b> <i>Knowledge: Students will know . . .</i></p> <ul style="list-style-type: none"> <li>• The above rules</li> <li>• Integrals are the inverse of differentiation</li> </ul> <p><b>Skills/Performance: Students will be able to . . .</b></p> <ul style="list-style-type: none"> <li>• Evaluate integrals using the above rules.</li> <li>• Apply the FTC to symbolic problems</li> </ul>	<p><b>Language Objectives:</b></p> <p>ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Read symbolic problems aloud</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Read symbolic problems aloud</li> </ul>
<p><b>Key Vocabulary</b> Integral, derivative, inverse, power rule, exponent rule, logarithm, natural logarithm,</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Completing similar problems, ultimately without assistance.</li> <li>• Defining terms in their own words.</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>• Assisting others</li> <li>• Asking insightful questions</li> <li>• Connecting concepts with life experiences and other classes.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Are their answers correct?</li> <li>• Is the work shown correct?</li> </ul>	

### Stage 3- Learning Plan

**Learning Activities:**

**Do Now/Bell Ringer/Opener:** Collecting Homework and taking out flip charts  
I'll collect the homework. We are continuing from yesterday's flip chart.

**Learning Activity 1:** Powerpoint lesson on Integration rule for  $e^x$   
I'll give direct instruction, alternating between direct instruction and practice.

**Learning Activity 2:** Make Review posters  
In groups, students will make posters for the rules we've learned, including the application of the FTC, the formula, and examples, AND THEIR NAMES.

**Learning Activity 3:** Look at Review posters  
In groups, students will rotate and see the other groups posters.

**Closing:** Reminder to review properties

**Multiple Intelligences Addressed:**

- |  |  |  |  |
|--|--|--|--|
| <input checked="" type="checkbox"/> Linguistic | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input checked="" type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial    | <input checked="" type="checkbox"/> Interpersonal        | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic                  |

**Student Grouping**

- Whole Class     
 Small Group     
 Pairs     
 Individual

**Instructional Delivery Methods**

- Teacher Modeling/Demonstration     
 Lecture     
 Discussion  
 Cooperative Learning     
 Centers     
 Problem Solving  
 Independent Projects

**Accommodations**

Low threat environment will be maintained for students with anxiety.

**Modifications**

Franny will be in the back of the room.

**Homework/Extension Activities:**

Review your properties!

**Materials and Equipment Needed:**

- Projection
- Mini-worksheet
- Internet access
- Large poster-sized paper
- Markers

Please take out your flip charts



## Evaluating Integrals: $e^x$



$$\int e^x dx = e^x + C$$

## The Fundamental Theorem of Calculus

Integration and derivation undo each other

$$f(x) = \frac{d}{dx} \int_0^x f(x) dx$$

$$\int e^x dx = e^x + C$$

$$\frac{d}{dx} e^x + C = e^x$$

$$\int e^x dx =$$

Please group up for poster making

Each group gets a topic



1. Power rule ( $x^3$ )
2. Exception to the power rule ( $1/x$ )
3. Trigonometric functions ( $\sin x$ )
4. Exponential functions ( $e^x$  or  $a^x$ )



Practice

$$\int e^x dx = e^x + C$$



$$\int e^x dx =$$

$$\int 5e^x dx =$$

Each review poster must have:

- A clear title and your **names**
- The icon/clipart attached
- The formula
- Examples of **each** application of the rule
- Application of the Fundamental Theorem of Calculus
- Don't forget about +C





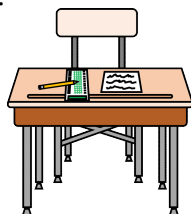
## Sources

- <https://openclipart.org/user-detail/barnheartow/>
- Calculus Graphical, Numerical, Algebraic 3<sup>rd</sup> Edition

Quiz coming! Quiz coming!

Quiz coming! Quiz coming!

- **Rules**
- You will need +C
- A Bonus Question



## Power Rule

ex)  $\int x^a dx = \frac{1}{a+1} x^{a+1} = \frac{1}{8} x^8 = \frac{x^8}{8}$

ex)  $\int x^3 dx = \frac{1}{3+1} x^{3+1} = \frac{1}{4} x^4 = \frac{x^4}{4}$

Application FTC:

$$\frac{d}{dx} x^5 = \int 5x^4 dx = \frac{5x^{4+1}}{5} = \frac{d}{dx} x^5 + C$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

## THE Power Rule

Formula:  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

examples:

$$\int x^1 dx = \frac{1}{1+1} x^{1+1} + C = \frac{1}{2} x^2 + C$$

$$\int x^6 dx = \frac{1}{6+1} x^{6+1} + C = \frac{1}{7} x^7 + C$$

Fundamental Theorem of Calculus:

$$\int x^{-2} dx = -\frac{1}{-2+1} x^{-2+1} + C = \frac{1}{-1} x^{-1} + C = -\frac{1}{x} + C$$

$$\frac{d}{dx} x^{-1} = -x^{-2} = -\frac{1}{x^2}$$

## Evaluating Integrals $e^x$


$$\int e^x dx = e^x + C$$

$$\frac{d}{dx} e^x + C = e^x$$

## Exception to the Power Rule



$$\int \frac{1}{x} dx = \int x^{-1} dx = \frac{1}{-1+1} (x)^{-1+1} + C = \frac{1}{0} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$



## EVALUATING INTEGRALS

### TRIG RULES

 Fundamental Theorem of Calculus: 

Integration and Derivation undo each other

Using this rule:

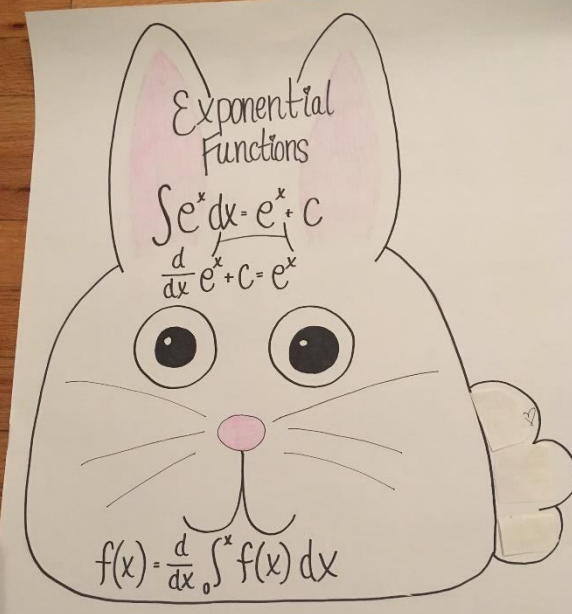
$\int \sin x dx = -\cos x + C$	$\frac{d}{dx} -\cos x + C = \sin x$
$\int \cos x dx = \sin x + C$	$\frac{d}{dx} \sin x + C = \cos x$
$\int \sec^2 x dx = \tan x + C$	$\frac{d}{dx} \tan x + C = \sec^2 x$
$\int -\csc^2 x dx = \cot x + C$	$\frac{d}{dx} \cot x + C = -\csc^2 x$
$\int \sec x \tan x dx = \sec x + C$	$\frac{d}{dx} \sec x + C = \sec x \tan x$
$\int -\csc x \cot x dx = \csc x + C$	$\frac{d}{dx} \csc x + C = -\csc x \cot x$

## Exponential Functions

$$\int e^x dx = e^x + C$$



$$\frac{d}{dx} e^x + C = e^x$$

$$f(x) = \frac{d}{dx} \int_0^x f(x) dx$$



## Trigonometric Functions

By \_\_\_\_\_

$$\int \sin x dx = -\cos x + C$$

$$\frac{d}{dx} -\cos x = \sin x$$

$$\int \cos x dx = \sin x + C$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\int \sec^2 x dx = \tan x + C$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int -\csc^2 x dx = \cot x + C$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\int -\csc x \cot x dx = \csc x + C$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

**SPECIAL NEEDS  
SMALL GROUP MATH**



**Teacher's Name: Priscilla Cole**  
**Unit: Functions**

**Subject/Course: Pullout math**  
**Grade Level: 8-10**

**Overview of and Motivation for Lesson:**

Solving equations of two variables is a learning goal for my students. It practices fractions (which they need) and understanding of variables and how to manipulate them. Plugging numbers into a formula someone else wrote is a useful skill. Calculating paychecks, calculating handyman supplies, evaluating prices, and understanding financial instruments all involve finding points in a function. Also this topic is on the PARC and the MCAS.

Stage 1-Desired Results	
<p><b>Standard(s): Algebra: Functions (pages 92-95)</b>  <b>Seeing Structure in Expressions</b></p> <ul style="list-style-type: none"> <li>• Interpret the structure of expressions.</li> <li>• Write expressions in equivalent forms to solve problems.</li> </ul> <p><b>Creating Equations</b></p> <ul style="list-style-type: none"> <li>• Create equations that describe numbers or relationships.</li> </ul> <p><b>Reasoning with Equations and Inequalities</b></p> <ul style="list-style-type: none"> <li>• <u>Understand solving equations as a process of reasoning and explain the reasoning.</u></li> <li>• Solve equations and inequalities in one variable.</li> <li>• <u>Solve systems of equations.</u></li> <li>• Represent and solve equations and inequalities graphically.</li> </ul>	
<p><b>Understanding(s):</b>  <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>• 2 lines intercept where <math>y_1=y_2</math></li> <li>• Algebra is about balancing both sides of an equal sign.</li> <li>• In algebra you can check your work by plugging the numbers back into the equations</li> </ul>	<p><b>Essential Question(s):</b></p> <p>How do I use a function?          How do I use the Cartesian plane?          What can I find using the equation of a line?          How do I find intersections?</p>
<p><b>Content Objectives:</b>  <b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>• 2 lines intercept where <math>y_1=y_2</math></li> <li>• Algebra is about balancing both sides of an equal sign.</li> <li>• In algebra you can check your work by plugging the numbers back into the equations</li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>• Find where to lines intersect</li> <li>• Check their work</li> </ul>	<p><b>Language Objectives:</b>          ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Perform the math problem requested with explanations, rephrasing, and some setup</li> <li>• Understand math as modeled</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Perform the math problem requested with explanations and rephrasing</li> <li>• Understand math as modeled</li> <li>• Explain reasoning</li> </ul>
<p><b>Key Vocabulary</b>          intercept, intersect, cross, solution, check, x-axis, y-axis, equality</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Answering questions and prompts</li> <li>• Completing worksheet problems</li> <li>• Decreasing levels of support required.</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>• Assisting others</li> <li>• Asking insightful questions</li> <li>• Connecting concepts.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Are the answers correct?</li> <li>• Is the work shown correct?</li> <li>• How much support was required?</li> </ul>	



### Stage 3- Learning Plan

**Learning Activities:**

Do Now/Bell Ringer/Opener: homework check  
I'll go over the homework.

Learning Activity 2: Flip chart lesson: substitution, welcome back  
I'll demonstrate on the board how to find the intersection of two lines using the substitution method, and check my work by plugging into both equations and students will record this process in their flip charts.

Learning Activity 3: Worksheets with steps, in pairs  
The students will work on worksheets for simultaneous equations, color-coding their work with colored pencils according to which equation they are working from, pairing the students carefully. There will be two sheets required, with a bonus for Easter candy.

Closing: Assigning Homework

**Multiple Intelligences Addressed:**

- |   |  |  |   |
|---|--|--|---|
| <input type="checkbox"/> Linguistic         | <input checked="" type="checkbox"/> Logical-Mathematical | <input type="checkbox"/> Musical       | <input type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial | <input type="checkbox"/> Interpersonal                   | <input type="checkbox"/> Intrapersonal | <input type="checkbox"/> Naturalistic       |

**Student Grouping**

- Whole Class     Small Group     Pairs     Individual

The whole class is 4 students

**Instructional Delivery Methods**

- |  |   |   |
|--|---|---|
| <input checked="" type="checkbox"/> Teacher Modeling/Demonstration | <input checked="" type="checkbox"/> Lecture | <input type="checkbox"/> Discussion                 |
| <input type="checkbox"/> Cooperative Learning                      | <input type="checkbox"/> Centers            | <input checked="" type="checkbox"/> Problem Solving |
| <input type="checkbox"/> Independent Projects                      |   |   |

**Accommodations**

See IEP (it's a book!)

**Modifications**

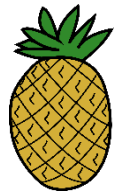
Basically, this class. See IEP (it's a book!)

**Homework/Extension Activities:**

Worksheet with equations of 2 variables to solve

**Materials and Equipment Needed:**

- Graphing paper
- Rulers
- worksheets
- Calculators
- Large white board with markers and erasers
- Colored pencils



Name: \_\_\_\_\_

1. Simplify the equations
2. Multiply equation(s) by a constant to get a pair of matching coefficients
3. Add or subtract the equations to cancel out a variable
4. Solve for the other variable in an original equation
5. Check your work

Find the solution to the following equations, checking off each step as you go:

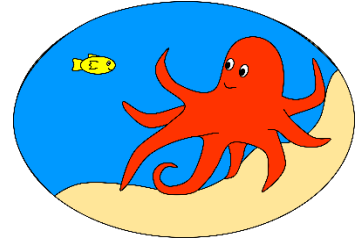
$$y = -3x - 3$$

$$y = x + 5$$



Name: \_\_\_\_\_

1. Write the system of equations
2. Isolate y in one equation
3. Plug that into the other equation
4. Solve for x
5. Solve for y
6. Check your work



Find the solution to the following equations, checking off each step as you go:

$$3x + 2y = 6$$

$$x - 2y = -2$$

**Teacher's Name: Priscilla Cole**  
**Unit: Functions**

**Subject/Course: Pullout math**  
**Grade Level: 8-10**

**Overview of and Motivation for Lesson:**

Solving equations of two variables is a learning goal for my students. It practices fractions (which they need) and understanding of variables and how to manipulate them. Plugging numbers into a formula someone else wrote is a useful skill. Calculating paychecks, calculating handyman supplies, evaluating prices, and understanding financial instruments all involve finding points in a function. Also this topic is on the PARC and the MCAS. Multiplying and dividing fractions is useful on its own too, both for those tests, and for life math.

Stage 1-Desired Results	
<p><b>Standard(s): Algebra: Functions (pages 92-95)</b>  <b>Seeing Structure in Expressions</b></p> <ul style="list-style-type: none"> <li>• Interpret the structure of expressions.</li> <li>• Write expressions in equivalent forms to solve problems.</li> </ul> <p><b>Creating Equations</b></p> <ul style="list-style-type: none"> <li>• Create equations that describe numbers or relationships.</li> </ul> <p><b>Reasoning with Equations and Inequalities</b></p> <ul style="list-style-type: none"> <li>• Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>• Solve equations and inequalities in one variable.</li> <li>• <u>Solve systems of equations.</u></li> <li>• Represent and solve equations and inequalities graphically.</li> </ul>	
<p><b>Understanding(s):</b>  <i>Students will understand that . . .</i></p> <ul style="list-style-type: none"> <li>• To multiply fractions, we multiply across</li> <li>• To divide fractions, we keep change flip</li> </ul>	<p><b>Essential Question(s):</b>            How do I perform basic functions with fractions?</p>
<p><b>Content Objectives:</b>  <b>Knowledge:</b> <i>Students will know . . .</i></p> <ul style="list-style-type: none"> <li>• To multiply fractions, we multiply across</li> <li>• To divide fractions, we keep change flip</li> </ul> <p><b>Skills/Performance:</b> <i>Students will be able to . . .</i></p> <ul style="list-style-type: none"> <li>• Multiply fractions by multiplying across</li> <li>• Divide fractions using keep change flip</li> <li>•</li> </ul>	<p><b>Language Objectives:</b>            ELD Level 3 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Perform the math problem requested with explanations, rephrasing, and some setup</li> </ul> <p>ELD Level 4 <i>Students will be able to . . . in English</i></p> <ul style="list-style-type: none"> <li>• Perform the math problem requested with explanations and rephrasing</li> <li>• Understand math as modeled</li> <li>• Explain reasoning</li> </ul>
<p><b>Key Vocabulary</b>            Fraction, denominator, numerator, Inverses, reciprocal</p>	
Stage 2-Assessment Evidence	
<p><b>Performance Task(s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Answering questions and prompts</li> <li>• Completing worksheet problems</li> <li>• Decreasing levels of support required.</li> </ul>	<p><b>Other Evidence:</b></p> <ul style="list-style-type: none"> <li>• Assisting others</li> <li>• Asking insightful questions</li> <li>• Connecting concepts.</li> </ul>
<p><b>Key Criteria to measure Performance Task (s) or Key Evidence</b></p> <ul style="list-style-type: none"> <li>• Are the answers correct?</li> <li>• Is the work shown correct?</li> <li>• How much support was required?</li> </ul>	

## Stage 3- Learning Plan

**Learning Activities:**

Do Now/Bell Ringer/Opener: Log into a computer

Learning Activity 1: Watching fraction function song for multiplication  
[https://www.youtube.com/playlist?list=PLWphMREEQDrg1tc8uNo\\_0TKv8KWCj-6-p](https://www.youtube.com/playlist?list=PLWphMREEQDrg1tc8uNo_0TKv8KWCj-6-p)  
 google “fractions song numberrock”

Learning Activity 2: Work sheets with demo  
 The students will work on the problem grid, following the color-coded demo problem.

Learning Activity 3: Watching fraction function song for division  
[https://www.youtube.com/playlist?list=PLWphMREEQDrg1tc8uNo\\_0TKv8KWCj-6-p](https://www.youtube.com/playlist?list=PLWphMREEQDrg1tc8uNo_0TKv8KWCj-6-p)  
 google “fractions song numberrock”

Learning Activity 4: Work sheets with demo  
 The students will work on the problem grid, following the color-coded demo problem.

**Closing:** numberrock songs and behavior address  
 The students can watch any numberrock songs they like with headphones on, or bring their homework (another problem grid) If the students focus well, I thank them for that. If not, I’ll speak accordingly.

**Multiple Intelligences Addressed:**

- |  |  |   |   |
|--|--|---|---|
| <input checked="" type="checkbox"/> Linguistic | <input checked="" type="checkbox"/> Logical-Mathematical | <input checked="" type="checkbox"/> Musical | <input type="checkbox"/> Bodily-kinesthetic |
| <input checked="" type="checkbox"/> Spatial    | <input type="checkbox"/> Interpersonal                   | <input type="checkbox"/> Intrapersonal      | <input type="checkbox"/> Naturalistic       |

**Student Grouping**

- Whole Class     Small Group     Pairs     Individual

The whole class is 4 students

**Instructional Delivery Methods**

- |  |   |   |
|--|---|---|
| <input checked="" type="checkbox"/> Teacher Modeling/Demonstration | <input checked="" type="checkbox"/> Lecture | <input type="checkbox"/> Discussion                 |
| <input type="checkbox"/> Cooperative Learning                      | <input type="checkbox"/> Centers            | <input checked="" type="checkbox"/> Problem Solving |
| <input type="checkbox"/> Independent Projects                      |   |   |

**Accommodations**

See IEP (it’s a book!)

**Modifications**

Basically, this class. See IEP (it’s a book!)

**Homework/Extension Activities:**

Problem grid like the one in class

**Materials and Equipment Needed:**

- Problem grid worksheets
- Calculators
- Large white board with markers and erasers
- Colored pencils
- Computers
- headphones

**Multiplication**

**Division**

$\frac{3}{2}$ $\frac{1}{9}$	$\frac{3}{2} * \frac{1}{9} = \frac{3 * 1}{2 * 9} = \frac{3}{18}$	$\frac{3}{2} \div \frac{1}{9} = \frac{3}{2} * \frac{9}{1} = \frac{3 * 9}{2 * 1} = \frac{27}{2} = 13.5$
$\frac{1}{3}$ $\frac{5}{6}$		
$\frac{7}{8}$ $\frac{1}{3}$		
$\frac{2}{4}$ $\frac{3}{1}$		

**Multiplication**

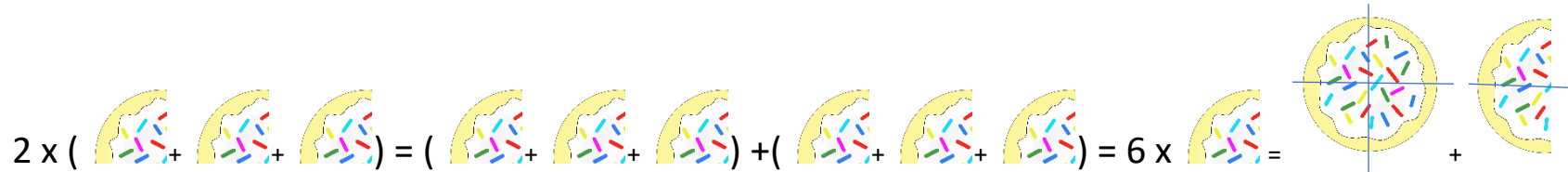
**Division**

$\frac{3}{2}$ $\frac{1}{9}$	$\frac{3}{2} * \frac{1}{9} = \frac{3 * 1}{2 * 9} = \frac{3}{18}$	$\frac{3}{2} \div \frac{1}{9} = \frac{3}{2} * \frac{9}{1} = \frac{3 * 9}{2 * 1} = \frac{27}{2} = 13.5$
$\frac{1}{5}$ $\frac{2}{3}$		
$\frac{4}{5}$ $\frac{6}{7}$		
$\frac{8}{9}$ $\frac{1}{2}$		



You will use paper circles instead of sugar cookies (You may eat **actual cookies** once you finish is activity). Solve the following problems as shown below:

$$2 * \frac{3}{4} = \frac{2}{1} * \frac{3}{4} = \frac{2*3}{1*4} = \frac{6}{4} = 1.5$$



Your turn:

$$3 * \frac{1}{4} =$$

$$4 * \frac{1}{2} =$$

$$2 * \frac{3}{4} =$$

$$3 * \frac{2}{3} =$$

$$4 * \frac{2}{4} =$$

**Point slope form:**  $y - y_1 = m(x - x_1)$   
Remember, slope is  $m$

Example: slope is 2, the line passes through (4, 5)  
 $y - 5 = 2(x - 4)$

Example: slope is 4, the line passes through (-1, -6)  
 $y - (-6) = (4)(x - (-1))$   
 $y + 6 = 4(x + 1)$

**Put the following lines into point slope form:**

slope is 1, the line passes through (2, 3)

$$y - 3 = 1(x - 2)$$

slope is -3, the line passes through (4, 15)

$$y - 15 = -3(x - 4)$$

slope is 2, the line passes through (-9, 3)

$$y - 3 = 2(x - (-9))$$

slope is 5, the line passes through (-1, -4)

$$y - (-4) = 5(x - (-1))$$

slope is  $\frac{1}{2}$ , the line passes through (7, 1)

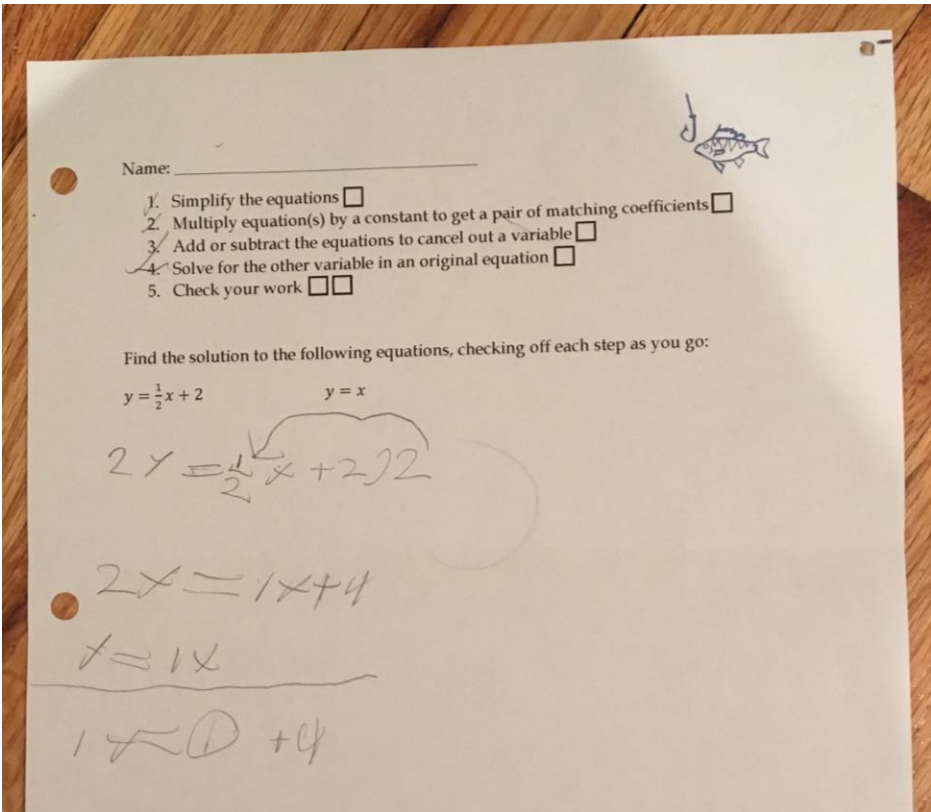
$$y - 1 = \frac{1}{2}(x - 7)$$

slope is -2, the line passes through  $(\frac{3}{4}, 1)$

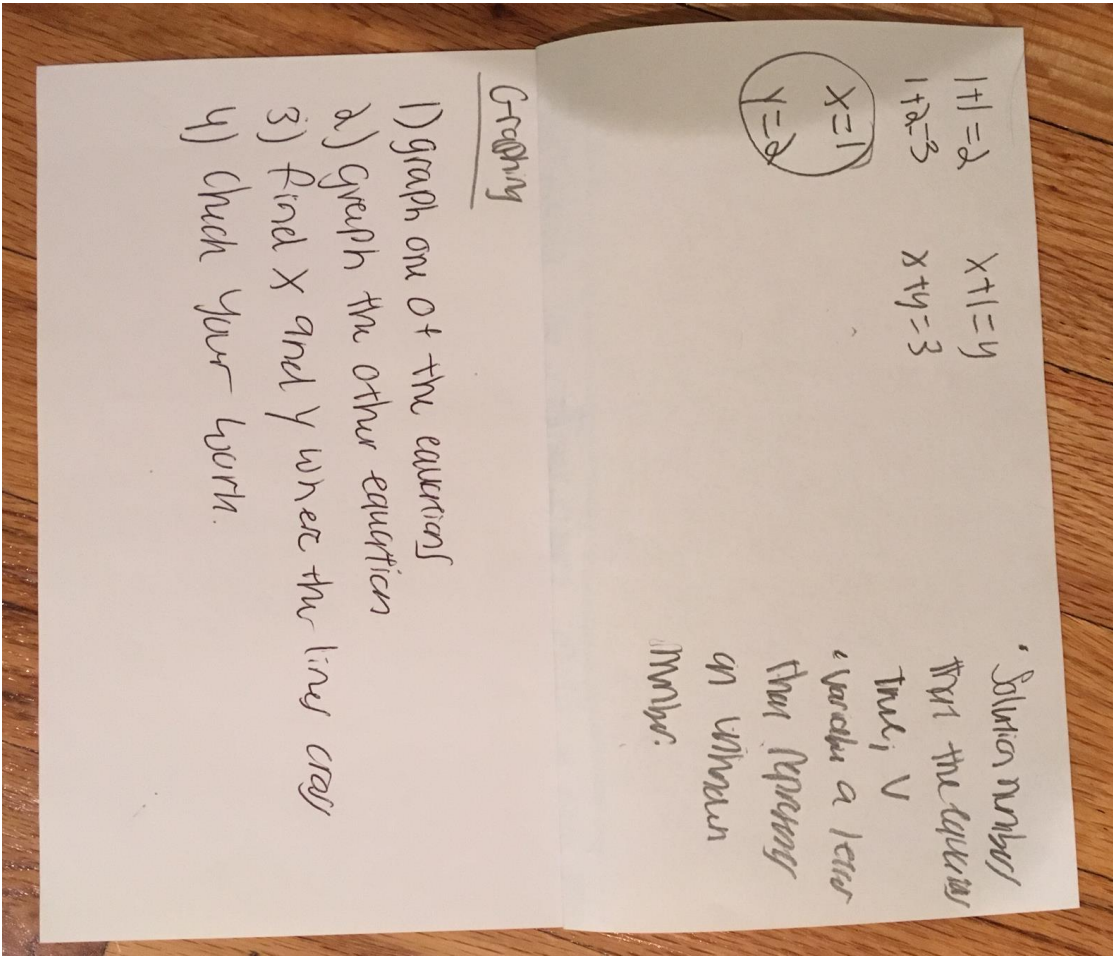
$$y - 1 = -2(x - \frac{3}{4})$$

$$y - 5 = 2(x - 4)$$

Student example of color-coding point-slope form



Student example of using the elimination method to solve equations of two variables with a fraction.



Student-made flip chart from SPED math