$$
\begin{aligned}
& \mathrm{m}_{\mathrm{g}}:=.367 \mathrm{~kg} \\
& \mathrm{l}_{\mathrm{wr}}:=\frac{23}{24} \cdot 14 \mathrm{in}=0.341 \mathrm{~m} \\
& \mathrm{l}_{\mathrm{wt}}:=\frac{23}{24} \cdot 4 \mathrm{in}=0.097 \mathrm{~m} \\
& \mathrm{l}_{\mathrm{wlr}}:=\mathrm{l}_{\mathrm{wt}}=0.097 \mathrm{~m} \\
& 1_{\text {wlt }}:=1.167 \mathrm{in}=0.03 \mathrm{~m} \\
& \mathrm{~b}_{\mathrm{w}}:=22.016 \mathrm{in}=0.559 \mathrm{~m} \\
& \mathrm{~b}_{\mathrm{wl}}:=\frac{23}{24} 4 \mathrm{in}=0.097 \mathrm{~m} \\
& \delta:=5 \mathrm{~mm}=5 \times 10^{-3} \mathrm{~m} \\
& \mathrm{~A}_{\mathrm{W}}:=2 \cdot 191.576 \mathrm{in}^{2}=0.247 \mathrm{~m}^{2} \\
& \mathrm{~A}_{\text {ftop }}:=16.357 \mathrm{in} \cdot 75 \mathrm{~mm}=0.031 \mathrm{~m}^{2} \\
& \mathrm{~A}_{\text {ffront }}:=75 \mathrm{~mm} \cdot 2.629 \mathrm{in}=50.082 \cdot \mathrm{~cm}^{2} \\
& \mathrm{~A}_{\mathrm{c} 1}:=\mathrm{A}_{\text {ffront }}+2 \delta \cdot\left(\mathrm{~b}_{\mathrm{w}}+\mathrm{b}_{\mathrm{wl}}\right)=115.74 \cdot \mathrm{~cm}^{2} \\
& \mathrm{~A}_{\mathrm{c} 2}:=\mathrm{A}_{\mathrm{ftop}}+\mathrm{A}_{\mathrm{w}}=2.784 \times 10^{3} \cdot \mathrm{~cm}^{2} \\
& \mathrm{~A}_{\text {wfront }}:=2 \cdot \delta \cdot\left(\mathrm{~b}_{\mathrm{w}}+\mathrm{b}_{\mathrm{wl}}\right)=65.657 \cdot \mathrm{~cm}^{2} \\
& \rho_{\text {all }}:=.0167 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}, .0168 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} . .1 .225 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \\
& \mathrm{AR}:=\frac{\mathrm{b}_{\mathrm{w}}^{2}}{\mathrm{~A}_{\mathrm{w}}}=1.265 \\
& \lambda:=\frac{1_{\mathrm{wt}}}{1_{\mathrm{wr}}}=0.286 \\
& 1_{\mu}:=\frac{2}{3} \cdot \frac{1+\lambda+\lambda^{2}}{1+\lambda} \cdot 1_{\mathrm{wr}}=0.242 \mathrm{~m} \\
& \rho_{\text {air }}:=1.225 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \\
& \alpha_{0}:=0{ }^{\circ}=0 \\
& \alpha_{5}:=5^{\circ}=0.087
\end{aligned}
$$

Mass of glider
Length of wing chord at root

Length of wing chord at tip Length of winglet chord at root Length of winglet chord a tip Span of wing from root to tip

Span of winglet
thickness of wings

Area of both wings
Cross sectional area of the fuselage from the top

Coss sectional are of the fuselage from the front
Cross sectional area of glider from the front

Cross sectional area of glider from top or bottom

Cross sectional area of wings and wingelts from the front
Range variable of air density from 100,000ft to sea level

Aspect Ratio of the wings

Taper ratio of the wings
mean chord length of swept wings
Density of air at sea level

Angle of attack $=0$
Angle of attack $=5$

$$
\begin{aligned}
& \alpha:=-.175,-.174 \ldots .175 \\
& \mathrm{~g}=9.807 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \\
& \mu_{\mathrm{air}}:=1.983 \cdot 10^{-5} \mathrm{Pas} \\
& \mathrm{~V}_{\mathrm{d}}:=20 \mathrm{mph}=8.941 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

Range variable of angle of attack in radians $\left(-10^{\circ}\right.$ to $10^{\circ}$
Acceleration of gravity

Viscocity of air
Desired airspeed of glider determined in simulation

Reynalds Number

$$
\mathrm{Re}:=\frac{\rho_{\mathrm{air}} \cdot \mathrm{~V}_{\mathrm{d}} \cdot 1 \mu}{\mu_{\mathrm{air}}}=1.334 \times 10^{5} \quad \text { Turbulant flow }
$$

boundry layer thickness

$$
\delta_{\mathrm{b}}:=\frac{.3821_{\mu}}{\operatorname{Re}^{\frac{1}{5}}}=8.712 \cdot \mathrm{~mm}
$$

WingLoading $:=\frac{\mathrm{m}_{\mathrm{g}}}{\mathrm{A}_{\mathrm{w}}}=1.485 \frac{\mathrm{~kg}}{\mathrm{~m}^{2}}$

Induced Drag

$$
\begin{array}{ll}
\mathrm{C}_{1}=2 \pi \alpha & \text { Equation for coefficient of lift of a flat plate } \\
c_{10}:=2 \pi \alpha_{0}=0 & \begin{array}{l}
\text { Coefficient of lift at } 0 \\
c_{15}:=2 \pi \alpha_{5}=0.548
\end{array} \\
c_{1}(\alpha):=2 \pi \alpha & \text { Coefficient of lift at } 5
\end{array}
$$

$$
\mathrm{C}_{\mathrm{di}}=\frac{\mathrm{C}_{1}^{2}}{\pi \mathrm{AR}} \quad \quad \text { Equation for coefficent of induced drag }
$$

$$
\mathrm{c}_{\mathrm{di} 0}:=\frac{\mathrm{c}_{10}^{2}}{\pi \mathrm{AR}}=0 \quad \text { Coefficent of induced drag at } 0
$$

$$
\mathrm{c}_{\mathrm{di5}}:=\frac{\mathrm{c}_{15}{ }^{2}}{\pi \mathrm{AR}}=0.076 \quad \underbrace{\text { Coefficent of induced drag at } 5}_{\circ}
$$

$$
\mathrm{c}_{\mathrm{di}}(\alpha):=\frac{(2 \pi \alpha)^{2}}{\pi \mathrm{AR}} \quad \text { Coefficent of induced drag at any given angle of attack }
$$



Does not account for flow seperation at angles of attack above 10*


## Form Drag [1]

$$
\begin{array}{ll}
\mathrm{c}_{\mathrm{dff}}:=.1 & \text { drag coefficient of long, streamlined body at } \mathrm{a}=0^{\circ} \\
\mathrm{c}_{\mathrm{dfw}}:=.005 & \text { drag coefficient of flat plate wings at } \mathrm{a}=0^{\circ}
\end{array}
$$

Friction (skin) Drag

$$
\tau=\mu_{\mathrm{air}} \cdot \frac{\mathrm{dV}}{\mathrm{dy}}=\mu_{\mathrm{air}} \cdot \frac{\mathrm{v}}{\frac{.3821_{\mu}}{\operatorname{Re}^{\frac{1}{5}}}} \quad \text { Equation for shear force of air on glider skin }
$$

$$
\mathrm{c}_{\mathrm{dfr}}=\frac{\tau}{\frac{1}{2} \rho_{\mathrm{air}} \cdot \mathrm{~V}^{2}} \quad \quad \text { Equation for coefficent of friction drag }
$$

$$
\mathrm{A}_{\mathrm{s}}:=(2 \cdot 22.118+2+46.926+3.058+8.3187+23.191+4.430+14.467+4 \cdot 9.879) \mathrm{in}^{2}+4 \cdot \mathrm{~A}_{\mathrm{w}}
$$

$$
\mathrm{A}_{\mathrm{s}}=1.109 \mathrm{~m}^{2} \quad \text { Surface area of entire glider }
$$

$$
\begin{array}{ll}
\mathrm{F}_{\mathrm{g}}:=\mathrm{m}_{\mathrm{g}} \cdot \mathrm{~g}=3.599 \cdot \mathrm{~N} & \text { Force of gravity on glider } \\
\mathrm{F}_{\mathrm{d}}=\frac{1}{2} \rho_{\mathrm{air}} \cdot \mathrm{v}^{2} \cdot \mathrm{c}_{\mathrm{d}} \cdot \mathrm{~A}_{\mathrm{c}} & \text { Equation for drag force } \\
\mathrm{F}_{\mathrm{g}}=\mathrm{F}_{\mathrm{d}} & \text { Equation to find terminal velocity }
\end{array}
$$

Given
$\mathrm{v}:=150 \mathrm{mph} \quad$ guess for the program
coefficnets of drag added together in proportion to the areas they affect


Terminal Velocity as a Function of Angle of Attack


Angle of Attack


Density of Atmosphere, $\left[\mathrm{kg} / \mathrm{m}^{\wedge} 3\right]$

Drag force at ideal terminal velocity

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{dt}}\left(5 \cdot^{\circ}, \rho_{\text {air }}\right)=134.43 \mathrm{~N} \\
& \text { Inital drag force on airframe } \\
& \text { turning 5* }
\end{aligned}
$$

$$
\mathrm{a}_{\mathrm{dt}}(\alpha):=\frac{\mathrm{F}_{\mathrm{dt}}\left(\alpha, \rho_{\mathrm{air}}\right)-\mathrm{F}_{\mathrm{g}}}{\mathrm{~m}_{\mathrm{g}} \cdot \mathrm{~g}}
$$

Initial decceleration on airframe at terminal velocity (in G's)



Drag force at any angle of attack, air density, and velocity:
$\mathrm{F}_{\mathrm{d} 1}\left(\alpha, \rho_{\mathrm{all}}, \mathrm{v}\right):=\frac{1}{2} \rho_{\mathrm{all}} \cdot(\mathrm{v})^{2}\left[\begin{array}{l}\mathrm{c}_{\mathrm{di}}(\alpha) \cdot\left(\sin \left(\frac{\pi}{2}-\alpha\right) \cdot \mathrm{A}_{\mathrm{c} 1}+\cos (\alpha) \mathrm{A}_{\mathrm{c} 2}\right)+\mathrm{c}_{\mathrm{dff}} \cdot \mathrm{A}_{\mathrm{ffront}}+\mathrm{c}_{\mathrm{dfw}} \cdot \mathrm{A}_{\mathrm{wfront}} \cdots \\ \\ +\frac{\mu_{\mathrm{air}} \cdot \frac{\mathrm{v}}{5.823 \mathrm{~mm}}}{\frac{1}{2} \rho_{\mathrm{air}} \cdot(\mathrm{v})^{2}} \cdot \mathrm{~A}_{\mathrm{s}}\end{array}\right]$
Initial Force on Airframe at Desired Velocity After Changing Angle of Attack


Bamboo Properties [2]

$$
\begin{array}{ll}
\sigma_{\mathrm{bb}}:=20.27 \frac{\mathrm{~N}}{\mathrm{~mm}^{2}}=20.27 \cdot \mathrm{MPa} & \text { Ultimate bending stress of bamboo } \\
\mathrm{r}_{\mathrm{b}}:=.125 \mathrm{in}=3.175 \mathrm{~mm} & \text { Radius of bamboo spars }
\end{array}
$$

Carbon fiber Properties [3]

$$
\begin{array}{ll}
\sigma_{\mathrm{bc}}:=89000 \mathrm{psi}=613.633 \cdot \mathrm{MPa} & \text { Ultimate bending stress of carbon fiber } \\
\mathrm{r}_{\mathrm{c}}:=\frac{3}{32} \mathrm{in}=2.381 \mathrm{~mm} & \text { Radius of carbon fiber spars }
\end{array}
$$

$$
\sigma=\frac{\mathrm{FL}}{\pi \mathrm{r}^{3}} \quad \mathrm{~F}_{\mathrm{b}}=\frac{\sigma \cdot \pi \cdot \mathrm{r}^{3}}{\mathrm{~L}} \quad \quad \text { Equation for maximum bending force on wings }
$$

Forces on differnt bamboo spar lengths
$\mathrm{F}_{\mathrm{bb} 1}:=\frac{\sigma_{\mathrm{bb}} \cdot \pi \cdot \mathrm{r}_{\mathrm{b}}{ }^{3}}{9.5 \mathrm{in}}=8.447 \mathrm{~N} \quad \mathrm{~F}_{\mathrm{bb} 2}:=\frac{\sigma_{\mathrm{bb}} \cdot \pi \cdot \mathrm{r}_{\mathrm{b}}{ }^{3}}{12 \mathrm{in}}=6.687 \mathrm{~N}$
Forces on different carbon fiber spar lengths
$\mathrm{F}_{\mathrm{bc} 1}:=\frac{\sigma_{\mathrm{bc} \cdot} \cdot \pi \cdot \mathrm{r}_{\mathrm{c}}{ }^{3}}{9.5 \mathrm{in}}=107.874 \mathrm{~N} \quad \mathrm{~F}_{\mathrm{bc} 2}:=\frac{\sigma_{\mathrm{bc} \cdot} \cdot \pi \cdot \mathrm{r}_{\mathrm{c}}{ }^{3}}{12 \mathrm{in}}=85.4 \mathrm{~N} \quad \mathrm{~F}_{\mathrm{bc} 3}:=\frac{\sigma_{\mathrm{bc} \cdot} \cdot \pi \cdot \mathrm{r}_{\mathrm{c}}{ }^{3}}{24 \mathrm{in}}=42.7 \mathrm{~N}$
$\begin{array}{ll}\mathrm{F}_{\mathrm{bbmax}}:=\mathrm{F}_{\mathrm{bb} 1}+3 \mathrm{~F}_{\mathrm{bb} 2}=28.507 \mathrm{~N} & \text { Force to break bamboo spars (4 spars) } \\ \mathrm{F}_{\mathrm{bcmax}}:=\mathrm{F}_{\mathrm{bc} 1}+\mathrm{F}_{\mathrm{bc} 2}+\mathrm{F}_{\mathrm{bc} 3}=235.974 \mathrm{~N} & \text { Force to break carbon fiber spars (3 spars) }\end{array}$
$\mathrm{W}_{\mathrm{bb}}:=\frac{\mathrm{F}_{\mathrm{bbmax}}}{\mathrm{g}}=2.907 \mathrm{~kg} \quad$ Equivalent mass on airframe to break bamboo
$\mathrm{W}_{\mathrm{bc}}:=\frac{\mathrm{F}_{\mathrm{bcmax}}}{\mathrm{g}}=24.063 \mathrm{~kg} \quad$ Equivalent mass on airframe to break carbon fiber
$\mathrm{V}_{\mathrm{cs}}:=0, .1 \frac{\mathrm{~m}}{\mathrm{~s}} . . \mathrm{V}_{\mathrm{t}}\left(0, \rho_{\mathrm{air}}\right)$
Range variable of velocities from 0 to ideal terminal velocity
$\sigma_{\mathrm{bps}}=\frac{3 \mathrm{~F} \cdot \mathrm{~L}}{2 \cdot \mathrm{~b} \cdot \mathrm{~d}^{2}}$
Equation to find bending stress of polystyrene

Experimental results:
$\mathrm{b}_{\mathrm{bps}}:=1 \mathrm{~cm}$
Width of polystyrene foam board test articles
$\mathrm{L}_{\mathrm{bps}}:=5.3 \mathrm{~cm}$
Length of polystyrene foam board test articles
$\mathrm{F}_{\mathrm{bps}}:=4.2 \mathrm{~N}$
Force applied when polystryene foam board failed
$\sigma_{\mathrm{bps}}:=\frac{3 \mathrm{~F}_{\mathrm{bps}} \cdot \mathrm{L}_{\mathrm{bps}}}{2 \cdot \mathrm{~b}_{\mathrm{bps}} \cdot \delta^{2}}=1.336 \times 10^{3} \cdot \mathrm{kPa}$
Ultimate bending stress of polystyrne foam baord.
$\mathrm{F}_{\text {bwing }}:=\int_{\mathrm{l}_{\mathrm{wt}}}^{\mathrm{l}_{\mathrm{wr}}} \frac{2 \sigma_{\mathrm{bps}} \cdot \delta^{2}}{3 \cdot \mathrm{~b}_{\mathrm{w}}} \mathrm{dl}=9.69 \cdot \mathrm{~N}$
Bending force to break wing at center of span

The wing technically will break before the bamboo or carbon fiber spars, however since the bending moment of both wings is centered on midpoint of the spars (inside the bulkhead), the wings experience much less bending force.

| $1 \times 10^{3}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \frac{\mathrm{F}_{\mathrm{d} 1}\left(0, \rho_{\text {air }}, V_{\mathrm{cs}}\right)}{\mathrm{F}_{\mathrm{d} 1}\left(1 \mathrm{deg}, \rho_{\mathrm{a}}, \mathrm{~V}_{\mathrm{oc}}\right)} \end{aligned}$ |  |  |  |  |
|  |  |  |  |  |
| $\mathrm{v}_{\mathrm{cs}}$ |  |  |  |  |
| Velocity, [m/s] |  |  |  |  |
| $\sigma_{\mathrm{Tp}}:=43.6 \frac{\mathrm{~N}}{\mathrm{~mm}^{2}} \quad$ Tensile strength of craft paper [4] |  |  |  |  |
| $\delta_{\text {paper }}:=.12 \mathrm{~mm}$ |  | Thickness of paper |  |  |
| $1_{\text {paper }}:=19.349 \mathrm{in}=0.491 \mathrm{~m}$ |  | Length of control surface |  |  |
| $\mathrm{A}_{\text {cpaper }}:=\delta_{\text {paper }}{ }^{-1} \text { paper }=0.59 \cdot \mathrm{~cm}^{2}$ |  | Area of control surface connection |  |  |
| $\mathrm{F}_{\text {tpaper }}:=\sigma_{\mathrm{Tp}} \cdot \mathrm{~A}_{\text {cpaper }}=2.571 \times 10^{3} \cdot \mathrm{~N}$ |  | Force required to tear off control surface |  |  |

Asuming force on control surfaces is akin to a fluid jet striking an angled, flat plate
$\mathrm{A}_{\mathrm{CS}}:=19.215 \mathrm{in} \cdot 1.772 \mathrm{in}=0.022 \mathrm{~m}^{2}$
$\theta_{\text {max }}:=60^{\circ}=1.047$
$\phi:=90^{\circ}-83.2^{\circ}=0.119$

Area of 1 control surface
Max deflection of control surface
Sweep angle of control surfaces
$\mathrm{F}_{\mathrm{csmax}}\left(\mathrm{V}_{\mathrm{cs}}\right):=\rho_{\mathrm{air}} \cdot \mathrm{A}_{\mathrm{cs}} \mathrm{V}_{\mathrm{cs}}{ }^{2} \cdot \sin \left(\theta_{\mathrm{max}}\right) \cdot \sin (\phi)$
$\mathrm{F}_{\text {csmax }}(20 \mathrm{mph})=0.221 \mathrm{~N}$
$\theta:=0 . .1 .047$
$\mathrm{F}_{\mathrm{cs}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right):=\rho_{\mathrm{air}} \cdot \mathrm{A}_{\mathrm{cc}} \cdot \mathrm{V}_{\mathrm{cs}}{ }^{2} \sin (\theta) \sin (\phi)$

Force on 1 control surface at maximum deflection

Force on 1 control surface at 20 mph

Range variable of control surface deflection, from $0^{\circ}$ to $60^{\circ}$
Force on 1 control surface at any velocity and delfection

Force on Control Surface at a Given Deflection


Deflection
Force on Control Surface at Max Deflection vs Force to Tear it off.



Velocity, [m/s]

$$
\begin{aligned}
& \mathrm{c}_{\mathrm{d}}:=1.2 \\
& \mathrm{~d}_{\mathrm{com}}:=7.14 \mathrm{in} \\
& \mathrm{~d}_{\text {center }}:=10.604 \mathrm{in}+\frac{75 \mathrm{~mm}}{2}=0.307 \mathrm{~m}
\end{aligned}
$$

Coefficient of drag of a perpendicular flat plate

Distance from COM

Distance from center of fuselage

$$
\mathrm{M}_{\mathrm{com}}\left(\theta, \mathrm{~V}_{\mathrm{cs}}\right):=\mathrm{d}_{\mathrm{com}} \cdot 2 \mathrm{~F}_{\mathrm{cs}}\left(\theta, \mathrm{~V}_{\mathrm{cs}}\right)
$$

$$
\mathrm{M}_{\text {center }}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right):=\mathrm{d}_{\text {center }} \cdot 2 \mathrm{~F}_{\mathrm{cs}}\left(\theta, \mathrm{~V}_{\mathrm{cs}}\right)
$$

$$
\mathrm{M}_{\text {commax }}\left(\mathrm{V}_{\mathrm{cs}}\right):=\mathrm{d}_{\mathrm{com}} \cdot 2 \mathrm{~F}_{\mathrm{csmax}}\left(\mathrm{~V}_{\mathrm{cs}}\right)
$$

$$
\mathrm{M}_{\mathrm{cmax}}\left(\mathrm{~V}_{\mathrm{cs}}\right):=\mathrm{d}_{\text {center }} \cdot 2 \mathrm{~F}_{\mathrm{csmax}}\left(\mathrm{~V}_{\mathrm{cs}}\right)
$$

Pitch moment

Roll moment

Max pitch moment
Max roll moment


Airspeed, [m/s]

| $\mathrm{T}_{\text {ServoStal11 }}:=1.8 \mathrm{kgf} \cdot \mathrm{cm}=0.177 \cdot \mathrm{~N} \cdot \mathrm{~m}$ | Max torque of SG90 Servos [5] |
| :--- | :--- |
| $\mathrm{T}_{\text {ServoStal12 }}:=22 \mathrm{ozf} \cdot \mathrm{in}=0.155 \cdot \mathrm{~N} \cdot \mathrm{~m}$ | Max torque of SM22 Servos [6] |
| $\mathrm{I}_{\mathrm{ch}}:=13.5 \mathrm{~mm}$ | Length of control horn |
| $\mathrm{I}_{\mathrm{sa}}:=14 \mathrm{~mm}$ | Length of Servo arm |
| $\mathrm{d}_{\mathrm{cs}}:=1.633 \mathrm{in}=41.478 \mathrm{~mm}$ | Distance between control surface center of <br> force and control horn |
| $\mathrm{M}_{\mathrm{CSonCH}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right):=\mathrm{d}_{\mathrm{cs}} \cdot \mathrm{F}_{\mathrm{cs}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right)$ | Moment of control surface on control horn |
| $\mathrm{M}_{\mathrm{CHonSA}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right):=\mathrm{M}_{\mathrm{CSonCH}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right) \cdot \frac{1_{\mathrm{ch}}}{1_{\mathrm{sa}}}$ | Moment of control horn on servo arm |

Torque Applied to Servos at Different Deflections


Velocity, [m/s]

Given
$\mathrm{V}_{\mathrm{cs}}:=100 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\mathrm{T}_{\text {ServoStall1 }}=\mathrm{d}_{\mathrm{cs}} \cdot \mathrm{F}_{\mathrm{cs}}\left(60^{\circ}, \mathrm{V}_{\mathrm{cs}}\right) \cdot \frac{1_{\mathrm{ch}}}{1_{\mathrm{sa}}}$
$\mathrm{V}_{\text {servostall60 }}:=\operatorname{Find}\left(\mathrm{V}_{\mathrm{cs}}\right)=39.993 \frac{\mathrm{~m}}{\mathrm{~s}} \quad(89.461 \mathrm{mph})$
Given
$\mathrm{V}_{\text {mas }}=100 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\mathrm{T}_{\text {ServoStall1 }}=\mathrm{d}_{\mathrm{cs}} \cdot \mathrm{F}_{\mathrm{cs}}\left(45^{\circ}, \mathrm{V}_{\mathrm{cs}}\right) \cdot \frac{1_{\mathrm{ch}}}{1_{\mathrm{sa}}}$
$\mathrm{V}_{\text {servostall45 }}:=\operatorname{Find}\left(\mathrm{V}_{\mathrm{cs}}\right)=44.259 \frac{\mathrm{~m}}{\mathrm{~s}} \quad(99.005 \mathrm{mph})$
Given

$$
\begin{aligned}
& \mathrm{V}_{\text {Masiv }}=100 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \mathrm{~T}_{\text {ServoStall1 }}=\mathrm{d}_{\mathrm{cs}} \cdot \mathrm{~F}_{\mathrm{cs}}\left(30^{\circ}, \mathrm{V}_{\mathrm{cs}}\right) \cdot \frac{1_{\mathrm{ch}}}{1_{\mathrm{sa}}} \\
& \mathrm{~V}_{\text {servostall30 }}:=\operatorname{Find}\left(\mathrm{V}_{\mathrm{cs}}\right)=52.633 \frac{\mathrm{~m}}{\mathrm{~s}} \quad(117.738 \mathrm{mph})
\end{aligned}
$$

Speeds at which the servos will stall at a given deflection

Given

$$
\begin{aligned}
& \mathrm{V}_{\text {Mest }}:=100 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \mathrm{~T}_{\text {ServoStall1 }}=\mathrm{d}_{\mathrm{cs}} \cdot \mathrm{~F}_{\mathrm{cs}}\left(15^{\circ}, \mathrm{V}_{\mathrm{cs}}\right) \cdot \frac{\mathrm{l}_{\mathrm{ch}}}{1_{\mathrm{sa}}} \\
& \mathrm{~V}_{\text {servostall15 }}:=\operatorname{Find}\left(\mathrm{V}_{\mathrm{cs}}\right)=73.156 \frac{\mathrm{~m}}{\mathrm{~s}} \quad(163.645 \mathrm{mph})
\end{aligned}
$$

Given

$$
\theta:=5^{\circ}
$$

$\mathrm{T}_{\text {ServoStall1 }}=\mathrm{d}_{\mathrm{cs}} \cdot \mathrm{F}_{\mathrm{cs}}\left(\theta, \mathrm{V}_{\mathrm{t}}\left(0, \rho_{\text {air }}\right)\right) \cdot \frac{1_{\mathrm{ch}}}{1_{\mathrm{sa}}}$

$$
\theta_{\operatorname{maxt}}:=\operatorname{Find}(\theta)=8.145 \cdot \operatorname{deg}
$$

Max deflection at ideal terminal velocity
$\mathrm{M}_{\text {Mainua }}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right):=\mathrm{d}_{\mathrm{com}} \cdot 2 \mathrm{~F}_{\mathrm{cs}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right)$
$\mathrm{M}_{\text {com }}\left(\theta_{\text {maxt }}, \mathrm{V}_{\mathrm{t}}\left(0, \rho_{\text {air }}\right)\right)=1.601 \cdot \mathrm{~N} \cdot \mathrm{~m}$

$$
\mathrm{V}_{\text {Masiv }}=0, .1 \frac{\mathrm{~m}}{\mathrm{~s}} . . \mathrm{V}_{\mathrm{t}}\left(0, \rho_{\text {air }}\right) \quad \text { reset } \mathrm{V}_{\mathrm{cs}}, \mathrm{M}_{\text {com }} \text { from calculations }
$$

Moments of Inertia


Division of wing for moment of inertia calculations. I, II, and III are the main sections while 1, 2, and 3 are subsections of I

$$
\begin{aligned}
& \mathrm{b}_{\mathrm{f}}:=16.357 \mathrm{in}=0.415 \mathrm{~m} \\
& \mathrm{~h}_{\mathrm{f}}:=75 \mathrm{~mm} \\
& \mathrm{~b}_{\mathrm{w} 1}:=12.01 \mathrm{in}=0.305 \mathrm{~m} \\
& \mathrm{~b}_{\mathrm{w} 2}:=1.406 \mathrm{in}=0.036 \mathrm{~m} \\
& \mathrm{~b}_{\mathrm{w} 3}:=2.427 \mathrm{in}=0.062 \mathrm{~m} \\
& \mathrm{~h}_{\mathrm{w} 1}:=22.213 \mathrm{in}=0.564 \mathrm{~m} \\
& \mathrm{~h}_{\mathrm{w} 2}:=22.213 \mathrm{in}=0.564 \mathrm{~m} \\
& \mathrm{~h}_{\mathrm{w} 3}:=22.213 \mathrm{in}=0.564 \mathrm{~m}
\end{aligned}
$$

Areas of each section of wing
$\mathrm{A}_{\mathrm{w} 1}:=\frac{1}{2} \mathrm{~b}_{\mathrm{w} 1} \cdot \mathrm{~h}_{\mathrm{w} 1}=0.086 \mathrm{~m}^{2}$
$\mathrm{A}_{\mathrm{w} 2}:=\mathrm{b}_{\mathrm{w} 2} \cdot \mathrm{~h}_{\mathrm{w} 2}=0.02 \mathrm{~m}^{2}$
$\mathrm{A}_{\mathrm{w} 3}:=\frac{1}{2} \mathrm{~b}_{\mathrm{w} 3} \cdot \mathrm{~h}_{\mathrm{w} 3}=8.695 \times 10^{-3} \mathrm{~m}^{2} \cdot 2$

Wing section 1 cut into more sections around COM

$$
\begin{array}{ll}
\mathrm{b}_{11}:=6.5 \mathrm{in}=0.165 \mathrm{~m} & \text { Base of each subsection of wing } \\
\mathrm{b}_{12}:=5.51 \mathrm{in}=0.14 \mathrm{~m} & \\
\mathrm{~b}_{13}:=5.51 \mathrm{in}=0.14 \mathrm{~m} &
\end{array}
$$

$$
\mathrm{h}_{11}:=12.021 \mathrm{in}=0.305 \mathrm{~m}
$$

$$
\mathrm{h}_{12}:=12.021 \mathrm{in}=0.305 \mathrm{~m}
$$

$$
\mathrm{h}_{13}:=10.191 \mathrm{in}=0.259 \mathrm{~m}
$$

Areas of each subsection of wing

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{w} 11}:=.5 \cdot \mathrm{~b}_{11} \cdot \mathrm{~h}_{11}=0.025 \mathrm{~m}^{2} \\
& \mathrm{~A}_{\mathrm{w} 12}:=\mathrm{h}_{12} \cdot \mathrm{~b}_{12}=0.043 \mathrm{~m}^{2} \\
& \mathrm{~A}_{\mathrm{w} 13}:=.5 \cdot \mathrm{~b}_{13} \cdot \mathrm{~h}_{13}=0.018 \mathrm{~m}^{2}
\end{aligned}
$$

$$
\mathrm{I}_{\mathrm{wx}}:=\int_{-\mathrm{A}_{\mathrm{w}}}^{\mathrm{A}_{\mathrm{w}}} 3\left(\mathrm{~h}_{\mathrm{w} 1}\right)^{2} \mathrm{dA}_{\mathrm{w}}+\int_{-\mathrm{A}_{\text {ftop }}}^{\mathrm{A}_{\text {ftop }}}\left(\frac{\mathrm{h}_{\mathrm{f}}}{2}\right)^{2} \mathrm{dA}_{\mathrm{ftop}}=0.472 \mathrm{~m}^{4}
$$

Second moment of area of wing around x axis (roll)

$$
\begin{aligned}
I_{w y}:= & 2 \cdot\left(\int_{0}^{A_{w 11}} h_{11}^{2} d A_{w 11}\right)-2 \cdot\left(\int_{0}^{A_{w 12}} h_{12}^{2} d A_{w 12}\right)-2 \cdot\left(\int_{0}^{A_{w 13}} h_{13}^{2} d A_{w 13}\right) \ldots=-5.879 \times 10^{-3} m^{4} \\
& +-2 \cdot\left(\int_{0}^{A_{w} 2} b_{w 2}^{2} d A_{w 2}\right)-2 \cdot\left(\int_{0}^{A_{w 3}} b_{w 3}^{2} d A_{w 3}\right)
\end{aligned}
$$

$$
\begin{array}{ll}
\mathrm{R}_{\mathrm{gx}}:=\sqrt{\frac{\mathrm{I}_{\mathrm{Wx}}}{\mathrm{~A}_{\mathrm{c} 1}}}=6.388 \mathrm{~m} & \text { Radius of gyration about the } \mathrm{x} \text { axis } \\
\mathrm{R}_{\mathrm{gy}}:=\sqrt{\frac{\left|\mathrm{I}_{\mathrm{wy}}\right|}{\mathrm{A}_{\mathrm{c} 1}}=0.713 \mathrm{~m}} & \text { Radius of gyration about the } \mathrm{y} \text { axis } \\
\mathrm{I}_{\mathrm{xx}}:=\mathrm{m}_{\mathrm{g}} \cdot \mathrm{R}_{\mathrm{gx}}^{2}=14.974 \mathrm{~m}^{2} \cdot \mathrm{~kg} & \text { Moment of inertia about } \mathrm{x} \text { axis (roll) } \\
\mathrm{I}_{\mathrm{yy}}:=\mathrm{m}_{\mathrm{g}} \cdot \mathrm{R}_{\mathrm{gy}}^{2}=0.186 \mathrm{~m}^{2} \cdot \mathrm{~kg} & \text { Moment of inertia about } \mathrm{y} \text { axis (pitch) }
\end{array}
$$

Roll rate [7]

$$
\begin{aligned}
& \frac{\mathrm{p} \cdot \mathrm{~b}_{\mathrm{w}}}{2 \mathrm{~V}_{\mathrm{cs}}}=\mathrm{constant} \\
& \mathrm{p}=-\frac{\mathrm{C}_{1 \delta \mathrm{a}}}{\mathrm{C}_{\mathrm{lp}}} \delta_{\mathrm{a}}\left(\frac{2 \mathrm{~V}_{\mathrm{cs}}}{\mathrm{~b}}\right) \\
& \frac{\mathrm{p} \cdot \mathrm{~b}_{\mathrm{w}}}{2 \mathrm{~V}_{\mathrm{cs}}}=-\frac{\mathrm{C}_{1 \delta \mathrm{a}}}{\mathrm{C}_{\mathrm{lp}}} \cdot \delta_{\mathrm{a}} \\
& \mathrm{~b}_{1}:=\frac{75 \mathrm{~mm}}{2}+1.006 \mathrm{in}=0.063 \mathrm{~m} \\
& \mathrm{~b}_{2}:=\frac{75 \mathrm{~mm}}{2}+20.213 \mathrm{in}=0.551 \mathrm{~m}
\end{aligned}
$$

$$
\mathrm{b}_{\mathrm{cs}}:=\frac{75 \mathrm{~mm}}{2}+10.611 \mathrm{in}=0.307 \mathrm{~m} \quad \text { Spanwise location of midpoint of control surface }
$$

$$
\mathrm{c}_{\mathrm{la}}:=\frac{\pi \cdot \mathrm{AR} \cdot 5\left[1+\cos \left[2 \cdot\left(0^{\circ}\right)\right]\right]}{1+\sqrt{1+\frac{\mathrm{AR}^{2}}{4} \cdot\left(1-.0390997^{2}\right)\left[\frac{1-\cos \left[2 \cdot\left(28.4^{\circ}\right)\right]}{1+\cos \left[2 \cdot\left(28.4^{\circ}\right)\right]}+1\right]}}=1.781 \quad \text { Coeffcient of lift of aileron [8] }
$$

$$
\mathrm{c}_{1 \delta \mathrm{a}}:=\mathrm{c}_{\mathrm{la}} \cdot \sqrt{\lambda} \cdot \frac{\mathrm{~A}_{\mathrm{cs}}}{\mathrm{~A}_{\mathrm{w}}} \cdot \frac{\mathrm{~b}_{\mathrm{cs}}}{\mathrm{~b}_{\mathrm{w}}}=0.046
$$

$$
\mathrm{C}_{1 \delta \mathrm{a}}:=\frac{\mathrm{c}_{1 \delta \mathrm{a}^{-1}} \mathrm{l}_{\mathrm{wr}}}{\mathrm{~A}_{\mathrm{w}} \cdot \mathrm{~b}_{\mathrm{w}}} \cdot\left[\left(\mathrm{~b}_{2}^{2}-\mathrm{b}_{1}^{2}\right)+\frac{4(\lambda-1)}{3 \cdot \mathrm{~b}_{\mathrm{w}}}\left(\mathrm{~b}_{2}^{3}-\mathrm{b}_{1}^{3}\right)\right]=1.74 \times 10^{-3}
$$

$\mathrm{C}_{\mathrm{lp}}:=-\frac{\left(\mathrm{c}_{\mathrm{la}}+\mathrm{c}_{\mathrm{dfw}}\right) \cdot \mathrm{I}_{\mathrm{wr}} \cdot \mathrm{b}_{\mathrm{w}}}{24 \cdot \mathrm{~A}_{\mathrm{w}}}(1+3 \cdot \lambda)=-0.107$

童: $=0 . .1 .047$
$\mathrm{p}_{\mathrm{roll}}\left(\theta, \mathrm{V}_{\mathrm{cs}}\right):=-\frac{\mathrm{C}_{\mathrm{lda}}}{\mathrm{C}_{\mathrm{lp}}} \theta \cdot \frac{2 \cdot \mathrm{v}_{\mathrm{cs}}}{\mathrm{b}_{\mathrm{w}}}$
$\mathrm{p}_{\text {roll }}\left(10^{\circ}, \mathrm{V}_{\mathrm{d}}\right)=5.221 \cdot \frac{\mathrm{deg}}{\mathrm{s}}$

Roll authority

Roll damping

Redefining $\theta$ for program

Roll Rate

Roll rate with $5^{\circ}$ deflection at 20 mph

Estimated Roll Rate as a Function of Aileron Delfection


Aileron Deflection
$\mathrm{p}_{\mathrm{x}}:=-\frac{\mathrm{C}_{\mathrm{l} \delta \mathrm{a}}}{\mathrm{C}_{\mathrm{lp}}} \cdot \frac{2}{\mathrm{~b}_{\mathrm{w}}}=0.05839 \frac{1}{\mathrm{~m}}$
Roll rate constant, multiply by control surface deflection (in degrees) and velocity (in $\mathrm{m} / \mathrm{s}$ ) to get roll rate in deg/s

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