Project Number: MB1 MQP 0001

Developing an Excel VBA Application to Set a Generic SD Model in Equilibrium

&

Modeling the US Domestic Economy using an Accounting Framework in SD

A Major Qualifying Project Report:

submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

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Date: April 26th, 2012

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Acknowledgements

I would like to extend many thanks to Prof. Blais and Prof. Radzicki for their unyielding support, and wise insights. Having worked with them has been a tremendous opportunity, one that I am quite grateful for.

Executive Summary

The project consists of two independent endeavors. First, an application that automates the process of setting a System Dynamics (SD) model in equilibrium is developed. Second, based on a previously conceived model of an economy, a SD model of the domestic U.S. Economy is developed and calibrated to fit historical data.

System Dynamics is a modeling discipline whose fundamental building blocks are sets of non-linear differential equations. We characterize a system of Ordinary Differential Equations (ODE) as being in equilibrium when the level variables are invariable with respect to the independent variable. The application is therefore concerned with solving for the exogenous variables such that, for each level variable, the rate of change (in SD terminology: "Net Flow") is equal to zero. A prevalent need arising in SD modeling practices motivated the project. Specifically, in order to test the effects of policies on the behavior of a system, SD modelers often first set the model in equilibrium. Doing so allows modelers to isolate the effect of policies on the system, and consequently derive conclusions about how the tested decisions impact the systemic behavior.

The second endeavor is a generic model of the U.S. Domestic Economy whose structure is primarily based on work done by Professor Kaoru Yamagushi at Doshisha Business School in Japan. After surveying models developed by a variety of scholars, Yamagushi's work was chosen because it relied on the double-entry accounting system. Such an approach provided a powerful, yet flexible and generic, framework for analyzing the relationship between the key players of an economy. When testing for the veracity of the model, the endogenous behavior of some economic variables is compared against historical data. The trend of a number of variables is successfully replicated. Further work needs to be conducted in order better calibrate the behavior of other variables, as well as to extend the boundaries of the system by, for instance, including an international sector.

Part I

Developing an Excel VBA Application to Set a Generic System Dynamics Model in Equilibrium

Introduction and Motivation

This section presents the development of an application that numerically computes the conditions under which a System Dynamics model can be set in equilibrium. System Dynamics is a modeling discipline whose fundamental building blocks are sets of non-linear differential equations. A prevalent need arising in SD modeling practices motivated the project. Specifically, in order to test the effects of policies on the behavior of a system, SD modelers often first set the model in equilibrium. Doing so allows modelers to isolate the effect of policies on the system; doing so helps modelers understand how the system reacts to policy changes.

While some analytic heuristics were devised for finding the equilibrium points of a subset of non-linear dynamic model (Barlas & Yaşarcan), there currently are no applications available for explicitly finding equilibrium points of SD models. For example, Barlas and Yaşarcan provide an analytical approach for reducing a system of polynomials in order to, thereafter, solve it using known numerical methods aimed at one-variable nonlinear equations. The current work, however, attempts to numerically estimate the equilibrium values of a *generic* system of equation.

This section of the report begins with an explanation of the reasons for choosing Microsoft Excel Visual Basic for Application as the environment in which to program the application. Thereafter, the report proceeds to elaborate on what the mathematical definition of equilibrium in System Dynamics entails and how to analytically find the solution. Next, two numerical methods are presented, namely "The Tracing Method" and "The Simultaneous Method." In these sections, the fundamental logic of the algorithm is discussed rather than the details of the code. Then, an overview of the user's interaction with the Graphical User Interface is presented[1]. Finally, the report delves into the limitations of the current application, and proposes suggestions for future work.

Why Microsoft Excel

The application was developed in Microsoft Excel's Visual Basic for Application (VBA.) Excel provided both the intuitive interface and computational capabilities needed to design the envisioned application. When compared with other coding environments, namely Matlab and Microsoft Visual Studio, Excel clearly emerged as more suitable for the task at hand.

At first, Matlab appears to be an appropriate choice, for it possesses a rich library of numerical algorithms necessary for creating the application, as well as a set of tools for designing a Graphical User Interface (GUI.) However, Matlab is inefficient for controlling symbolic variables. Because SD modelers tend to refer to variables by name (translated into strings when coding,) it was unnecessarily complicated, although possible, to utilize Matlab.

[1] The user interested in solely learning how to use the application may directly proceed to "The User Experience" section.

Microsoft Visual Studio is equipped with powerful features to design GUI, and is arguably the friendliest programming environment. However, Visual Studio lacked the algorithms for solving a system of simultaneous equations, and these would have been manually programmed if the software were chosen. Doing so was a possibility, albeit a redundant task given their inherent availability in Microsoft Excel.

Microsoft Excel is equipped with both the GUI tools and computational algorithms sufficient to effectively devise the application. Moreover, Excel tends to be more commonly found in personal computers than Matlab or Visual Studio, which reinforced Excel's appropriateness.

<u>Setting a SD Model in Equilibrium: An Analytical Solution</u>

Prior to delving the developed numerical algorithm, it is necessary to discuss what setting a SD model in equilibrium entails. Recall that when a system of differential equations is in equilibrium, all of the level variables, referred to as stocks in SD, remain unchanged over time. In other words, for each of the stocks in the system, the net flow (sum of inflows minus sum of outflows) must equal zero. Mathematically speaking, the following condition must be satisfied in order for the system to be in equilibrium:

$$\sum inflows - \sum outflows = 0 \ \forall \ stocks$$

To illustrate, consider the following example:

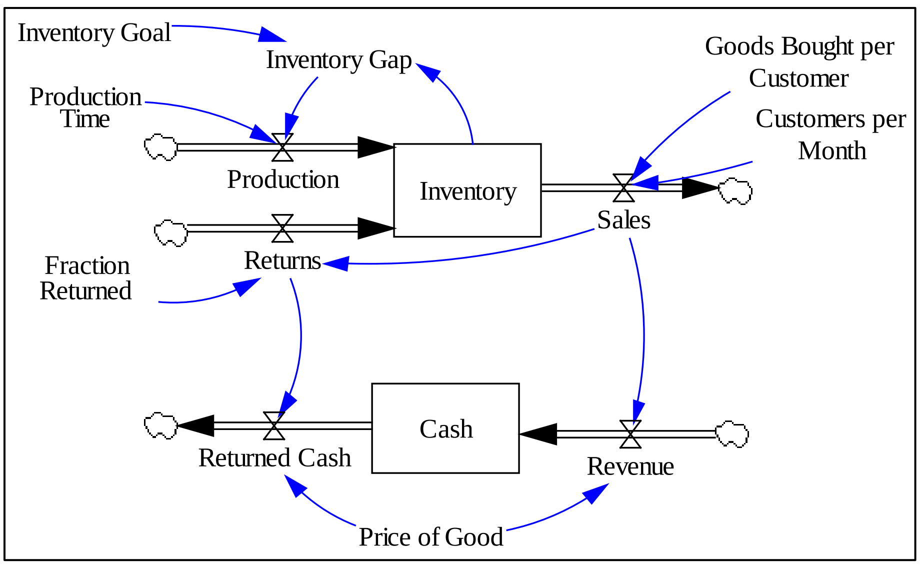

Figure 1.1: Inventory-Cash SD Model

Each of the equations for each of the variables can be found in the table below.

Table 1.1: Stock-Flow-Parameters Equations Table

Stocks	**Equation**
Inventory	INTEGRAL(Production + Returns – Sales) ; Initial=1000
Cash	INTEGRAL(Revenue – Returned Cash); Initial=10,000
Flows	
Production	Inventory Gap / Production Time
Returns	Sales * Fraction Returned
Sales	Customers per Month * Goods Bought per Customer
Revenue	Sales * Price of Good
Returned Cash	Returns * Price of Good
Parameters	
Inventory Gap	Inventory Goal – Inventory Gap
Inventory Goal	2500
Production Time	1
Fraction Returned	0.1
Customers per Month (CM)	1000
Goods Bought per Customer (GBC)	2
Price of Good	10

Given such a system, let us analytically find a solution that will set the system in equilibrium. In order to so, it seems appropriate to begin by invoking the equilibrium condition that each stock's net flow must be equal to zero. Doing so yields the following set of equations:

$$\int Production + \int Returns - \int Sales = 0 \; for\ the\ Stock\ of\ "Inventory"$$

$$\int Revenue - \int Returned\ Cash = 0 \; for\ the\ Stock\ of\ "Cash"$$

However, it is worthwhile to note that, because the model should be set in equilibrium from the outset of the simulation, the integrals are unnecessary. In other words, assuming there are no shocks in the system, such as PULSE functions, as long as the system is initially in equilibrium, it will remain so throughout the simulation.

Hence, removing the integral signs and replacing each of the flows by their associated equation, the following system of equations is attained:

$$\frac{Inventory\ Goal - Inventory}{Production\ Time} + (GBC * CM) * Fraction\ Returned - (GBC * CM) = 0 \; for\ "Inventory"$$

$$(GBC * CM) * Price\ of\ Good - (GBC * CM) * Fraction\ Returned * Price\ of\ Good = 0\ for\ "Cash"$$

Clearly, in order for the equality to hold true, “Fraction Returned” must equal 1. Plugging-in this newfound value into the above equation yields the following relation:

$$\frac{Inventory\ Goal - Inventory}{Production\ Time} = 0$$

And thus,

$$Inventory\ Goal = Inventory$$

In other words, as long as the “Fraction Returned” is equal to one, and the “Inventory Goal”- an exogenous variable- is equal to the initial value of “Inventory”- a stock- the system will be in equilibrium. Clearly, because there are two equations to be satisfied and three variables, the system of equations is *underdetermined*. Note that setting “Fraction Returned” equal to one, and “Inventory Goal” equal to “Inventory” is not the only feasible solution. Indeed, it is clear that are *infinitely* many solutions, for the system is *underdetermined*[2]. In analytically solving the system, a solution, perhaps the most elegant one, was found; the numerical algorithm presented later in the report may output a different set of values that will set the model in equilibrium.

To recapitulate, the methodology behind finding equilibrium solutions of a SD involves first setting the sum of the inflows equal to the sum of the outflows for each stock, and then identifying the variables that are causally related to each of the flow. The process is iteratively repeated until either an exogenous variable or a stock is met, at which point the value of such a variable must be modified such that the net flow of each stock is equal to zero. In the previous example, “Fraction Returned” was the exogenous variable found when traveling through the causal chain of variables affecting the flow “Returns”; similarly “Inventory Goal” and “Inventory” were found for the inflow “Production.” Moreover, the “Sales” inflow is causally linked to “Goods Bought per Customer” (GBC) and “Customers per Month” (CM,) and “Revenue” and “Returned Cash” are linked to “Price of Good.” Some of these variables, as discerned in the calculations above, were cancelled out in the simplification process, which allowed for simplified analysis.

However, systems of equations with an easily computable analytical solution are only a subset of the much wider set of nonlinear systems. Specifically, much of the systems encountered “outside of textbooks” are so intricate that an analytical solution may be infeasible to find. In other words, an *exact* equilibrium (such as solutions of analytical methods) solution may be difficult to find given the nonlinearities inherent in the equations, and it may be necessary to *approximate* the

[2] In Algebra, a system of equations is underdetermined when the number of variables exceeds the number of equations.

values of the exogenous variables that render the system stable, that is static over time. In order to do so, numerical methods can be utilized.

Numerical Algorithm: The Tracing Method

Searching for Variables in Causal Link

Inspired from the methodology for analytically setting a model in equilibrium, the Tracing Method needs to search for the exogenous variables linked to each stock, as depicted in the figure below.

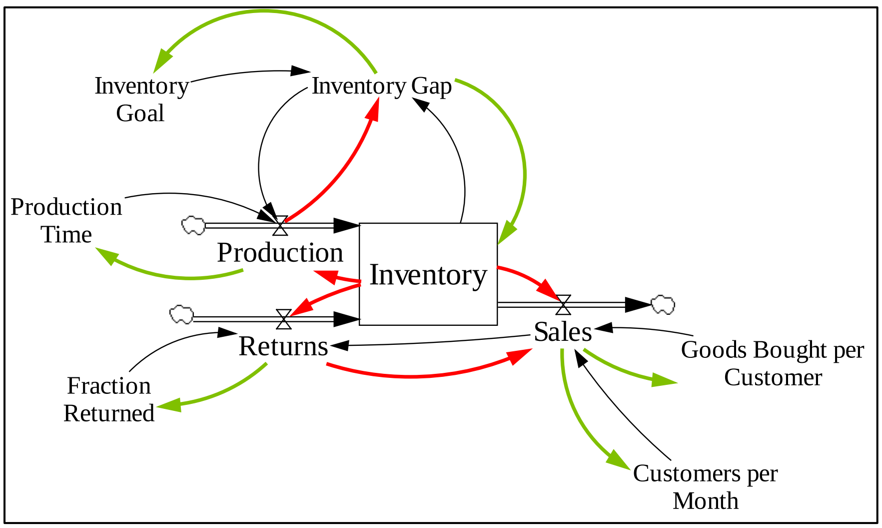

Figure 1.2: Illustration of Numerical Algorithm

In the diagram above, the black arrows represent the relationships needed to write the equations of the system. On the other hand, the red and green arrows represent the direction in which the algorithm traces the variable. The red arrow denotes that the variable that the algorithm reached is endogenous, and consequently, the algorithm needs to continue the search. The green arrow denotes that the variable reached is either exogenous (e.g. from "Sales" to "Customers per Month") or that it is a Stock (e.g. from "Inventory Gap" to "Inventory,") at which point the algorithm halts the search. For example, if the algorithm reached the variable "Returns" then it will gather the following variables[3]:

- "Fraction Returned"→ Stop search because variable is exogenous
- "Sales" → Continue search because variable is endogenous

[3] The method for gathering variables can be found under the "SeparateVariables" function in the VBA code.

When “Sales” is considered, the algorithm will gather the following variables

- Good Bought per Customer → Stop because variable is exogenous
- Customers per Month → Stop because variable is exogenous

Note, moreover, that if the algorithm will halt if it encounters a Stock variable. For example, if the algorithm is considering the variable “Inventory Gap”, it will gather the following variables:

- Inventory Goal → Stop because variable is exogenous
- Inventory → Stop because variable is a Stock

In this algorithm, a Stock’s *initial value* can be freely modified, if need be. Once the algorithm reaches an exogenous variable (or a Stock,) it will be in a position to change its value such that the associated Stock’s net flow is equal to zero.

The next section further elaborates on how the algorithm changes the variables in order to set the system in equilibrium.

Controlling Exogenous Variables

Once exogenous variables are met, the algorithm needs to modify their values while maintaining the condition that the Stock’s net flow is equal to zero. For example, suppose that the algorithm computed that in order for the net flow of “Inventory” to equal to zero, “Sales” must equal to 2. Since,

$$Sales = Goods\ Bought\ per\ Customer\ (GBC) * Customers\ per\ Month(CM) = 2$$

then the equation is *underdetermined.* In other words, the algorithm may choose to modify the value of both variables- “GBC” and “CM”- to satisfy $Sales = 2$, however only modifying one is needed. The other variable will remain fixed.

Initially, the algorithm was developed in such a way that all exogenous variables would be controlled. While such a methodology did work, it rendered the computations slow. Moreover, under such an approach, the solution was numerically convoluted, which was undesirable when and a number of other, significantly more simple, solutions existed. For example, in equation

$$Goods\ Bought\ per\ Customer\ (GBC) * Customers\ per\ Month(CM) = 2$$

There are infinitely many solutions; if the algorithm is allowed to modify both variables-“GBC” and “CM”- simultaneously, then the solver may arbitrarily choose unnecessarily complicated values. For example, the solver may yield “GBC”=2/213 and “CM”= 213; such numerical complications will slow down the algorithm. However, if one variable is fixed, say “CM”=2, then the algorithm will solely examine how changing “GBC” will yield “Sales”=2; in this case, there is a unique solution, “GBC”=1. Not only will the solution be more elegant and precise, but it will also be significantly faster.

To illustrate how the algorithm will search for variables, the following diagrams portray potential pathways:

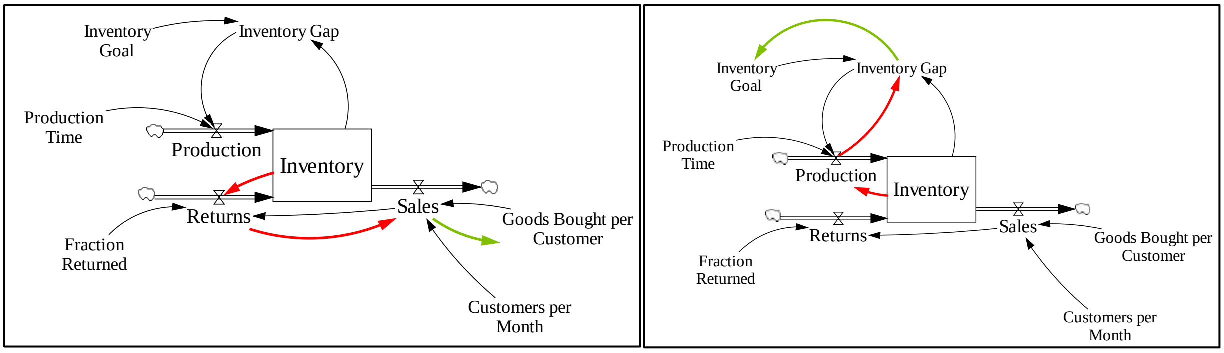

Figure 1.3 & 1.4: Possible Variable Search Routes

Evidently, there are a number of other potential combinations for considering the variables[4]. However, it is critical to mention that at every iteration and for each stock, the algorithm undergoes only *one* route. Indeed, each stock has only one equation (disregarding of the number of flows and parameters associated with it,) and consequently, changing only one variable is theoretically sufficient in order to set the net flow of that stock equal to zero.

Therefore, a pseudocode for the variable search method can be conceived:

1. *FOR Each Stock*
   a. *Find the Flows that change the Stock's Level*
   b. *Choose one Flow*
   c. *Set Flow=Dummy Variable*
   d. *Set Stopping Condition = 0*
   e. *WHILE Stopping Condition= 0*
      i. *Find Variables that Change the Dummy Variable's Value*
      ii. *Choose one Variable*
      iii. *Set Chosen Variable=Dummy Variable*
      iv. *If Dummy Variable ≠ Stock THEN*
         1. *IF Dummy Variable=Endogenous Variable THEN*
            a. *Stopping Condition=0 (continue the search)*
         2. *Else (i.e. Dummy Variable= Exogenous Variable)*
            a. *Stopping Condition=1*
         3. *End If*
      v. *Else (i.e. Dummy Variable=Stock)*
         1. *Stopping Condition =1 (stop the search)*
      vi. *End IF*
   f. *End WHILE*
2. *Move to Next Stock*

[4] The method for choosing which variables the algorithm considers can be found in the "SetRange" function in the VBA code.

Implementing Excel's Solver during Variable Search

One of the primary reasons for choosing Excel VBA to design the application is the inherent non-linear equation solver found in Microsoft Excel. From the outset of the variable search algorithm, Excel's Solver can be utilized to find what the flow's value needs to be in order to set the Stock in equilibrium. Thereafter, as the casual link between variables is traced until an exogenous variable or stock is met, Excel's Solver will be used to determine what the value of each related variable should be. The table below is an example of how the algorithm controls the value of variables by following the route illustrated in Figure 1.4:

Table 1.2: List of Variables Traced by Search Variable Algorithm

Current Variable Considered	Current Variable Formula	Current Variable Objective Value	Next Variable Considered	Next Variable Objective Value
Inventory (Stock)	$Production + Returns\ (1000) - \ Sales\ (3000)$	0	Production (Flow)	2000
Production (Flow)	$\frac{Inventory\ Goal - Inventory\ (1000)}{Production\ Time\ (1)}$	2000	Inventory Goal (Parameter)	3000
Inventory Goal (Paramter)	*None (Variable is Exogenous)*	**3000**		

Note: The number in parenthesis in the variable's formula refers to the value that associated variable held at the time that the algorithm reached the considered variable.

From the table above, in the second row, it is clear that in order for the Inventory to equal to zero, Production must equal to 2000, since Returns and Sales are equal to 1000 and 3000, respectively. After choosing to consider the variable "Production", the algorithm examines its formula and choses to vary "Inventory Goal". In order for "Production" to equal to 2000, "Inventory Goal" must equal to 3000, given that "Inventory" is equal to 1000. Finally, since "Inventory Goal" is exogenous, the algorithm halts, sets "Inventory Goal" equal to 3000 (which renders "Inventory" equal to zero,) and can continue to the next stock, if any. During this process, Excel's solver is used to determine the variables' objective value, that is, the value in the fifth column.

Consequently, one can modify the pseudocode generated above such that it includes the solver's contributions. The additions are in blue:

1. *FOR Each Stock*
    a. *Find the Flows that change the Stock's Level*
    b. *Choose one Flow*
    c. *Set DummyVariable =Chosen Flow*
    d. *Solve Stock Value = 0 by changing the value of DummyVariable*
    e. *Save the value of the DummyVariable that sets Stock Value = 0 in a variable named DummyObjectiveValue*
    f. *Set Stopping Condition = 0*
    g. *WHILE Stopping Condition= 0*
        i. *Find Variables that Change the Dummy Variable's Value*
        ii. *Save these Variables in an Array named VariableRange*
        iii. *Choose one Variable in VariableRange (denote it by ChosenVariable)*
        iv. *Solve DummyVariable = DummyObjectiveValue by changing the value of ChosenVariable*
        v. *Save the value of ChosenVariable that sets Dummy Variable = DummyObjectiveValue.*
        vi. *Set DummyVariable= ChosenVariable*
        vii. *If DummyVariable ≠ Stock THEN*
            1. *IF Dummy Variable=Endogenous Variable THEN*

a. *Stopping Condition=0 (continue the search)*
2. *Else (i.e. Dummy Variable= Exogenous Variable)*
   a. *Set VariableSolution=DummyObjectiveValue*
   b. *Output VariableSolution to User*
   c. *Stopping Condition=1*
3. *End If*

viii. *Else (i.e. Dummy Variable=Stock)*
1. *Set VariableSolution=DummyObjectiveValue*
2. *Output VariableSolution to User*
3. *Stopping Condition =1 (stop the search)*

ix. *End IF*

h. *End WHILE*

2. *Move to Next Stock*

The algorithm above formalizes the process described in Table 1.2.

In addition to using Excel's Solver capability to find the objective value of each of the variables in the causal chain, the algorithm creates an overall objective function that will be used to check whether a solution has been found. This function will be the sum of the absolute values of the Stocks' net flows. Mathematically speaking, the overall objective function, referred to "OverallObjective" in this report will be:

$$OverallObjective = \sum_{i=1}^{n} |StockFormula(i)|$$

Where "n" refers to the total number of stocks in the system, and "StockFormula(i)" refers to Stock i's net flow (net flow=sum of inflows - sum of outflows.) The reason for considering the absolute value of each Stock's formula is clear. Examine the flaw that may mislead the solution if either one of the following changes was made:

1. $OverallObjective = \sum_{i=1}^{n} StockFormula(i)$
2. $OverallObjective = |\sum_{i=1}^{n} StockFormula(i)|$

In either one of the cases, if there were two stocks, where StockFormula(1) = 1 and StockFormula(2) = -1, then OverallObjective = 0. The application would then, wrongfully so, inform the user that a solution has been found. Consider the absolute value of each Stock's formula, and then adding them up, remedies such an issue.

As the algorithm seeks to find how to set "StockFormula" equal to 0 for each Stock, "OverallObjective" will be automatically updated. Evidently, a value of zero for "OverallObjective" is sought, however, in some systems, particularly in non-linear ones, exact solutions may be unfeasible, and numerical approximations are given instead. The algorithm accounts for such situations by prompting the user, in the main form, to input an acceptable tolerance level, and will check whether "OverallObjective" < Error Tolerance. If so, then the application informs the user that a solution has been found. Otherwise, the application will inform the user that the algorithm has failed to converge to a solution, and subsequently, the user will have the option of running the solution method again.

Finally, it is worthwhile noting that in order to conduct the steps outlined in the pseudocode, the algorithm saves information pertaining to the variables in the system in arrays. Primarily, the algorithm saves the following information:

1. The variable's address in excel (e.g. the cell in the first column and the first row would be referred to A1-the dollar sign is utilized to "fix" the cell, however whether it is included is irrelevant for our purposes.)
2. The variable's formula (e.g. if the formula is CM*GBC, and the address of CM and GBC are "A1" and "A2" respectively, then the formula saved will be "=A1*A2"; if the specified cell is numeric, then the value will be saved.)
3. The variable's objective value, which at first is initialized to zero. This information, updated iteratively, refers to the value that each value needs to assume in order for the system to be in equilibrium.
4. Each Stock's initial value.

The remaining details underlying the Tracing Method can be found commented in the code.

Numerical Algorithms: The Simultaneous Method

An alternate approach for setting a model in equilibrium is the "Simultaneous Method." The following method attempts to change every exogenous variable such that the following objective function is satisfied:

$$\sum_{i=1}^{n} |StockFormula(i)| = 0$$

Where StockFormula(i) refers to stock's i net flow, where the net flow is the difference between the sum of the inflows and the sum of the outflows.

The algorithm thus begins by searching for the exogenous variables. For the example presented previously, the following diagram depicts in red which variables are suitable to be changed in order to satisfy the objective function:

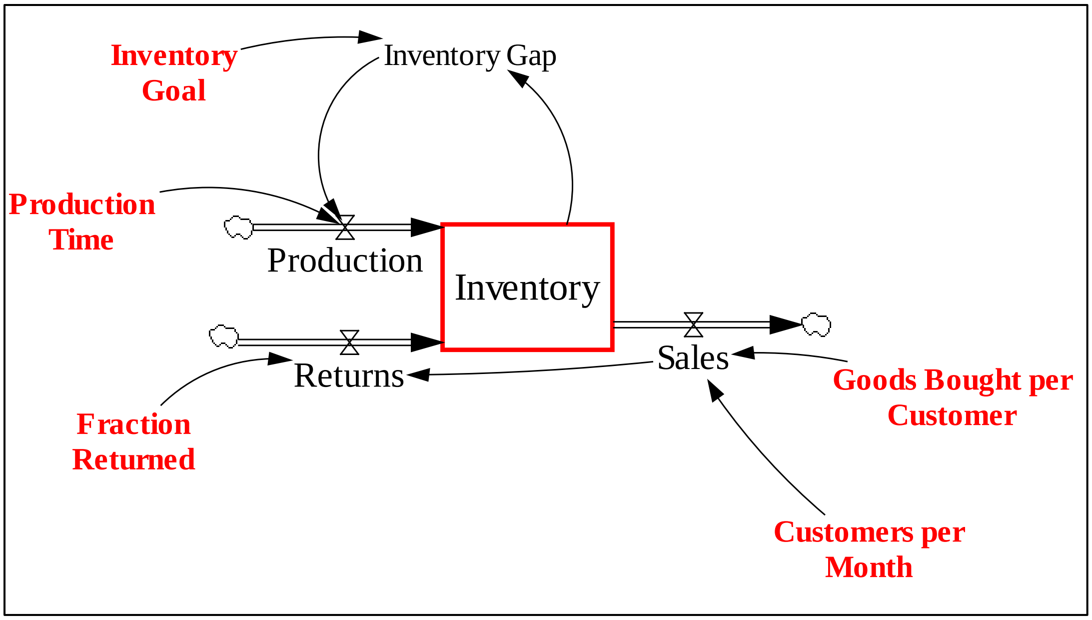

Figure 1.5: Identifying Variables to Change in the Simultaneous Method

Clearly, as discussed previously, solely one variable is needed to set the net flow of inventory equal to zero. However, the Simultaneous method will attempt to find a combination of values for the exogenous variables that renders the stock of inventory in equilibrium.

The User Experience with the Graphical User Interface

This section introduces the user interface of the application and illustrates the major steps a use needs to follow in order to attain a solution. User-friendliness was paramount in the development of the application (at least we hope the user finds it to be easy to use!) A form was created in order to avoid potential errors the user may cause.

Consider the Romeo & Juliette model described by the following set of differential equations.

$$\frac{dR}{dt} = a * J$$

$$\frac{dJ}{dt} = b * R$$

Where J=Juliette's Love, R=Romeo's Love (functions with respect to time, t,) and "a" and "b" are constant parameters, with a=-3 and b=2 (the magnitude of "a" and "b" are arbitrarily chosen.)

The solution is clear; this example is used to illustrate the process of setting up a model using the application. Here are the typical steps:

1. Open the Microsoft Excel file that contains the application. When Excel loads, the following spreadsheet should appear:

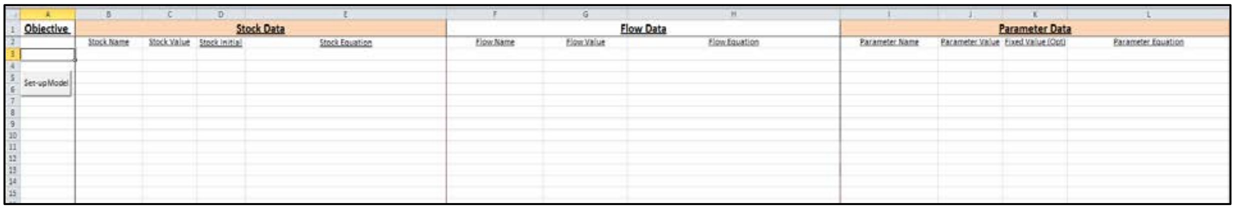

Figure 1.6: Main Spreadsheet

The user will note three sections pertaining the "Stock Data", "Flow Data", and "Parameter Data." Within each section lie a number of columns that will hold necessary information about each variable.

2. Note the "Set-up Model" button on the left-hand side of the spreadsheet. Clicking it will invoke the following form.

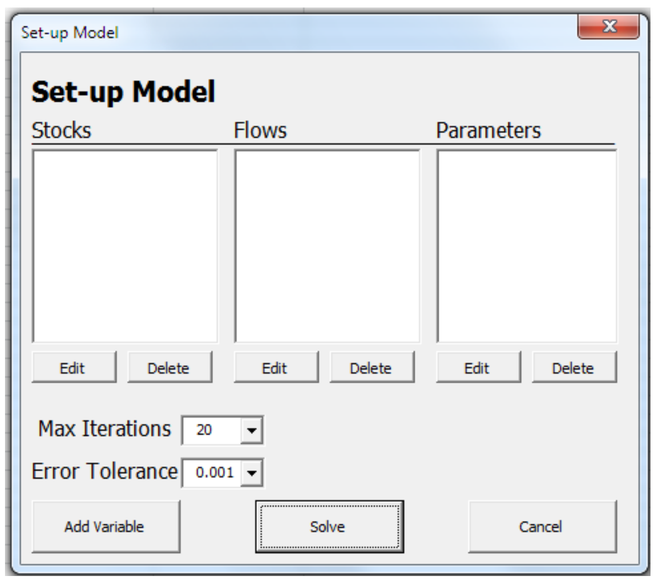

Figure 1.7: Main Form

3. Input variables' information. In order to do so, first click on the "Add Variable" button, which invokes the following form.

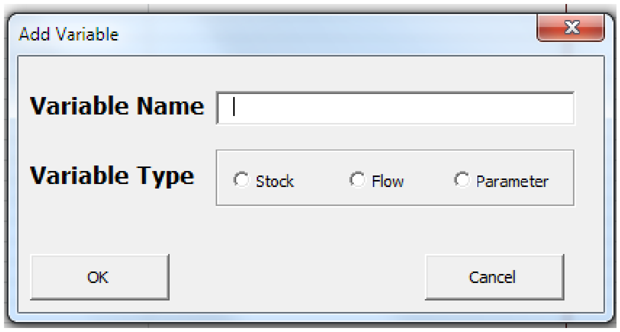

Figure 1.8: "Add Variable" Form

The user is thereafter required to input the variable's name and specify its type[5]. Doing so will only add the variable name in the appropriate section, without information about the formula. Every time a new variable is added, the user will note that in the list form, as well as in the spreadsheet, the variable's name will appear. For the Romeo & Juliette model, the form should look similar to the following, after all the variables are introduced:

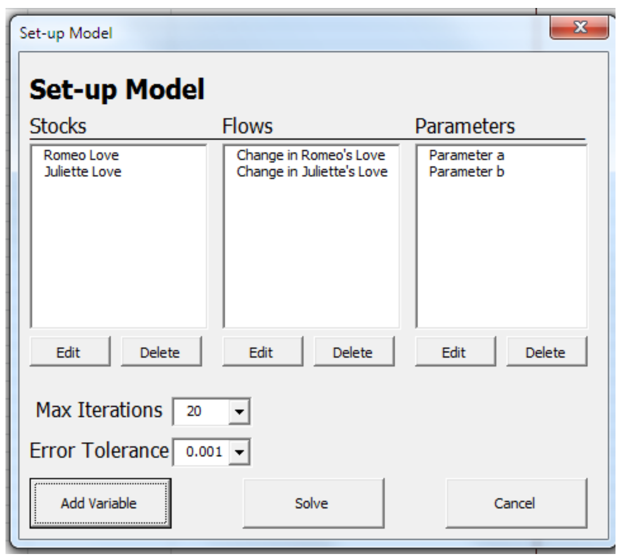

Figure 1.9: Main Form with Variables Added

4. Edit the formula for each variable. When editing the stocks, the following form will appear:

[5] **Attention:** When writing the variable's name *do not include any of symbols representing major operations.* Specifically, the following operators are not allowed in a variable's name: "+", "-", "*", "/","^". Parentheses are also not allowed. For example, naming a variable as "Population (age 16 and under)" is not allowed. Spaces *are* allowed.

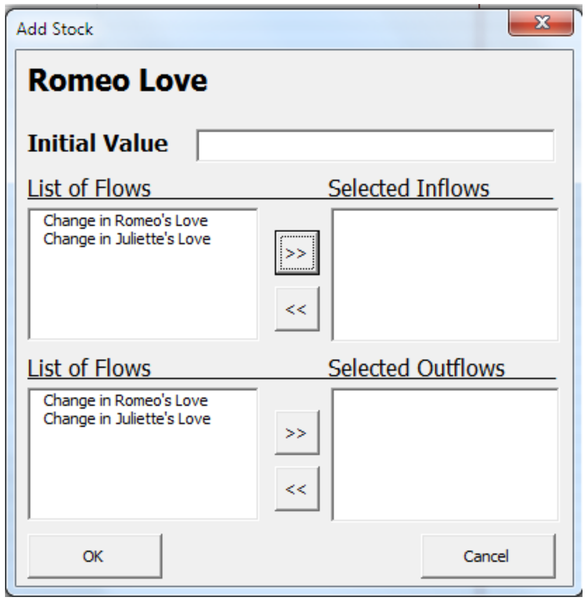

Figure 1.10: Add Stock Form

Note that the chosen stock's name will appear at the top right corner of the form. In this form the user is first prompted to enter the stock's initial value. Thereafter, the user is promoted to choose which inflows and/or outflows are causally linked to the stock. To choose, the user selects the inflow/outflow from the list, then can either *double-click* on the entry or *click on the button* with the ">>" symbol. Similarly for removing inflows/outflows, but the "<<" button is used instead. To submit the choice, the user clicks on the "OK" button. The application will generate a message box if there is an error in the input, such as for example, if the user inputs a non-numeric initial value. The information entered by the user will then appear in the spreadsheet. By clicking on the "Cancel" button, any choices made by the user will be revoked and no changes will be done to the current spreadsheets.

5. When editing the flows or parameters the following form appears:

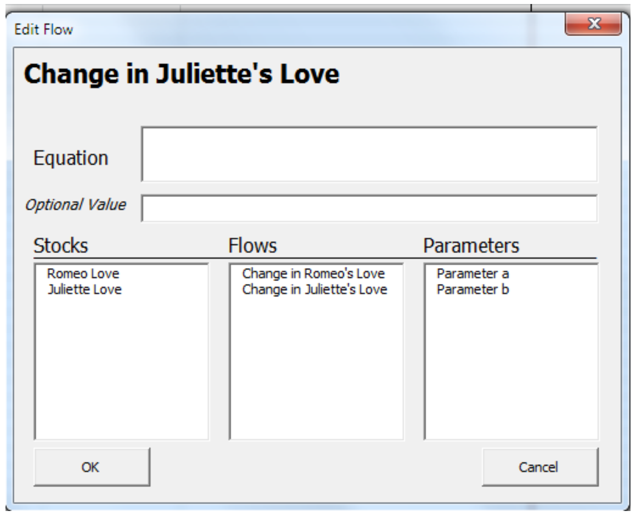

Figure 1.11: Add Flow/Parameter Form

The user is then prompted to enter the equation of the variable in question. In order to do so, the user must click on the location within textbox where he or she wishes to add the variable, and then double-click on the entry from the listboxes. Upon double-clicking, the variable's name will appear in the textbox.
The equation supports the main operators supported by Excel, namely addition ("+",) subtraction ("-",) multiplication ("*",) division ("/",) and exponent ("^".) If need be, the user can also add parenthesis ("(" , ")".)[6]

Under the equation textbox can be found another textbox labeled "Optional Value." This textbox allows fixing some variable's value[7][8]. For example, when solving, if the algorithm seeks to set find a solution for equation $\frac{Inventory}{Delay} = 0$ by changing "Delay" then it will increase the denominator to a very large value, until the error tolerance level is met. However, there are more sensible ways for setting the equation equal to zero, namely by rendering "Inventory" null. The user is allowed is fix "Delay" equal to some value, say one, at which point the algorithm will seek other variables to change. It will then attempt to set solve the equation by setting "Inventory" equal to zero. In case all the variables in an equation are fixed, the algorithm will halt and display a message prompting the user to free at least one of the variables.

[6] **Attention:** When writing the equation *do not include spaces between the variable names or the operators.* If a variable's name has spaces (e.g. "Cash in Vault") is acceptable, whereas spaces between different variables and operators is not (e.g. "Population * Birth".)

[7] Note that the "Optional Value" feature is only available for parameters.

[8] Fixing variables' values is only allowed for exogenous parameters. The reason is that endogenous variables can be also implicitly fixed by finding and modifying the exogenous variables to which the former are related.

When the equation is complete, pressing the "OK" button will edit the selected variable's formula. Pressing the "Cancel" button will exit the form without making changes. If the algorithm detects an error it will output a message box prompting the user to change the input. There are two possible causes for errors: exogenous

- *The formula written is not computable* (e.g. "Population *+ Birth".) To trap for such errors, the code is written such that if Excel cannot evaluate the formula, then the algorithm will generate an error message.
- *A variable is not recognized* (e.g. in the Romeo & Juliette model, writing "Romeo Loves" as opposed to "Romeo Love" which is how the stock has been defined will trigger an error.)

6. Once the model is set up and the all the equations are entered, the user is ready to solve the system for the equilibrium values. In order to do so, the user must click on the button "Solve" in the main form.

7. Once one the algorithm is completed, a form will appear to inform the user of the results. Specifically, if

$$OverallObjective = \sum_{i=1}^{n} |StockFormula(i)| \leq \mathcal{E}$$

Where Ɛ is some error tolerance inputted by the user, then the message box will display that "a solution was found." Otherwise, the message box will display that "No solution was found." In either case, the user has the option to "try again" or "keep results."[9][10]

Limitations & Future Work

The application has a number of limitations that are noted in the footnotes. However, for clarity, these will be reiterated in this section.

[9] **Attention:** The user must keep in mind that the algorithm determines whether a solution was found based on the error tolerance. However, solely examining the message box is not sufficient to assess the validity of the results. For example, the OverallObjective variable may be equal to 0.004. However, with an error tolerance of 0.001, the algorithm will state that no solution was reached. However, it is possible that 0.004 is the best value that Excel's Solver was able to reach, given the equations and the values of the other variables in the system. If the user is unsatisfied with an OverallObjective of 0.004, he or she may attempt to change the value of the variables in the system, and run the algorithm again. Excel's Solver, with a different set of values, may reach a better solution.

[10] **Attention:** Although the value of OverallObjective may be small, say 0.001, when the values found by the algorithm are entered into the System Dynamics model, the small initial discrepancy-0.001- may amplify over time, and push the system away from equilibrium.

1. When writing the variable's name *do not include any of symbols representing major operations.* Specifically, the following operators are not allowed in a variable's name: "+", "-", "*", "/","^". Parentheses are also not allowed. For example, naming a variable as "Population (age 16 and under)" is not allowed. Spaces *are* allowed.
   When considering a variable's equation entered by the user, the application splits the string based on the major operation symbols[11]. For example, if the user enters "Historical Population*Birth Rate" into the equation textbox, the application will search for "*" and consider everything on its right to be one variable, and everything on its left to be another variable. Thus, entering "Population (Historical)*Birth Rate" will force the algorithm to split the string at "(", which will improperly split the string into "Population" and "Historical)*Birth rate". Future work can attempt to remedy this issue.

2. When writing the equation in the edit flow/parameter window *do not include spaces between the variable names or the operators.* If a variable's name has spaces (e.g. "Cash in Vault") is acceptable, whereas spaces between different variables and operators is not (e.g. "Population * Birth".) The reason, similarly to the first limitation, is because spaces should be allowed to be part of a variable's name. By implementing the splitting method to find the variables that are part of an equation, spaces will interfere with the string division if allowed. Thus, they are omitted. Future work can be done to alter the code in such a way that spaces can be placed between variables in the textbox.

3. In the Tracing Method, the user does not currently have the choice to direct the algorithm through a certain chosen causal link of variables. Instead, the algorithm picks the first variable it finds in the equation that is suited to be modified. However, the user may prefer to change a certain variable to set some stock in equilibrium. Indeed, changing some variables may help the algorithm find a more accurate solution faster. It seems appropriate therefore, to design a Graphical User Interface that directs the algorithm to the user's chosen exogenous variables when it attempts to solve for the equilibrium values of a system.

Conclusion

The application presents two numerical algorithms for find the equilibrium solutions of a System Dynamics model. Naturally, in order to improve the application (algorithm and user experience,) sharing it with the System Dynamics community seems appropriate. Thus, in the future, the application should be released, and feedback should be gathered to understand what the strengths of the algorithm are, how to improve the algorithm to be efficient with some subsets of nonlinear systems, and how to improve the user-friendliness of the application.

[11] The splitting method is used to separate parts of a string once a user defined symbol is encountered. For example "1+2", given the symbol "+" will produce two strings "1" and "2", and will thereafter place each string in an entry of an array.

Part II

Modeling the US Domestic Economy using an Accounting Framework in System Dynamics

Introduction

The overarching purpose of this research project is to build a generic model of the US Economy that modelers can utilize as a launching point for testing the effect of policies on the Economy. How do changes in Fiscal or Monetary policies affect employment levels in the US? Does the Buffet Rule improve the US Government's deficit? Would the robustness of the banking system improve if lending institutions adopted a full-reserve system? Is the Central Bank necessary in the US Economy? These are only some of the questions that the model should be able to provide answers for.

Based on Professor Kaoru Yamagushi's contributions in Economic modeling using System Dynamics, the work presented in this report is concerned with calibrating Yamagushi's Accounting-based model of an economy to fit US data. Several models were considered, as described in the literature review, each having a different orientation. Ultimately, however, Yamagushi's model was chosen for two primary reasons. First, being based on the double-entry accounting system, the model clearly illustrated the interactions that occurred between the various players of the US Economy. Second, the model is clearly built and supported with significant literature, which helped replicate the model under the time constraint and within the scope of this undergraduate project. Indeed, clearly the endeavor described in the opening paragraph of the introduction is vast; it also requires continuous and iterative follow-up. This project is an initial launch to the grander endeavor.

The report begins with an examination of the three models considered in the literature, namely those devised by Forrester, Mashayekhi, and Yamagushi. Thereafter, the report delves into, using supporting illustrations, how Yamagushi's model was calibrated, along with some preliminary simulation results. Finally, limitations and challenges faced during the model calibration process are discussed, and avenues for future work are suggested.

Literature Review

Forrester's National Model

The overarching purpose of Forrester's System Dynamics National Model is twofold (Forrester, 1984). First, the model should endogenously explain how the interaction of the economy's microstructures generates the aggregate behavior of the Macroeconomy in the United States. Specifically, Forrester is interested in certain key economic phenomena that have dominated economists' attention since the field has begun, namely, business cycles, inflation, stagflation, the economic long wave and growth. Second, the National Model can be utilized as a suitable interface to test the effect of various economic policies on variables of interest.

The economics discipline is currently divided in two primary and complementary subfields: macroeconomics and microeconomics. But these fields should not be treated as separate entities; rather, they are heavily interrelated, and "microeconomics [should] provided an adequate basis for explaining much of the behavior seen in national economies." (Forrester) And therefore, System Dynamics, due to its emphasis on dynamic relationships among variables, appears to be a suitable tool for modeling the Economy.

Forrester pursues his modeling by examining and observing the economy, and is not relying on economic theory that is already well established.

Forrester's model consists of six main sectors that represent the domestic U.S. Economy:

1. Consumer Producing Sector
2. Capital Producing Sector
3. Household Sector
4. Government Sector
5. Labor Sector
6. Financial Sector

The following diagram portrays the major physical flows between the national players in the National Model (Graham, 1984):

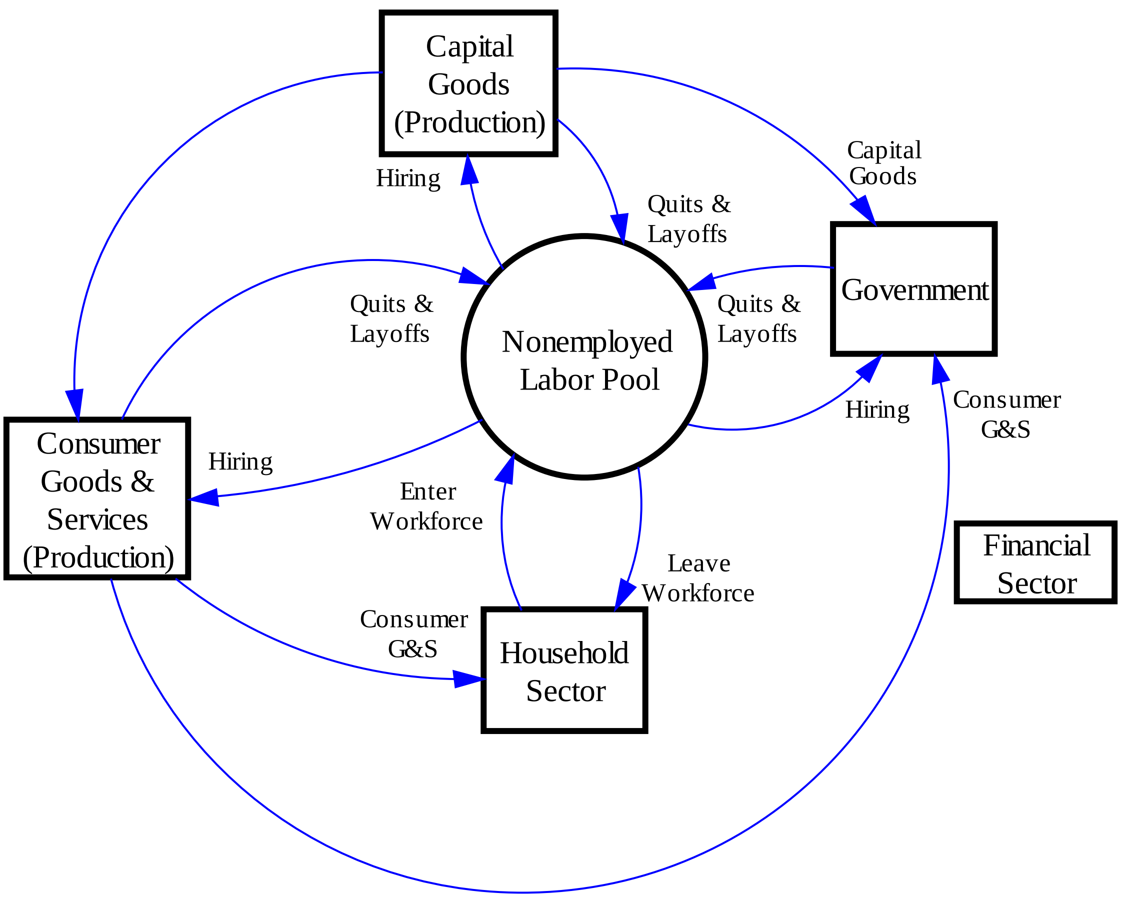

Figure 2.1: Physical Flow in Forrester's National Model

The production as a whole represents an aggregation of the economy's firms and corporations. Forrester divides the production sector into two distinct entities, each of which has a different output type. On the one hand, the "Consumer Goods & Services" provides the household and government sector with goods and services that become obsolete once consumed. On the other hand, the "Capital Goods" (hereafter referred to as Capital Sector) produces capital goods, such as plant and equipment that are used in the production of other capital items or consumer goods and services. Moreover, the Capital Sector distributes its output to the Government Sector, the Consumer Sector, and back to itself. It is noteworthy that the Production Sector borrows money from financial institutions, pays tax to the Government, distributes dividends to the shareholders, and maintains retained earnings.

The Household Sector is home to the population that is not in the workforce and provides the economy with a potential labor force. Consistent with the basic economic reality, the household sector is paid wages and interest based on the work that it provides and the savings it commits to financial institutions. Moreover, the Household sector pays taxes to the Government and spends its disposable income in the Consumer Sector.

The Labor Sector is the platform on which hirings and layoffs are made. Based on the availability of labor and inflation, wage is also determined. The Government, the Capital, and the Consumer Sectors refer to the labor pool in order to hire workers. Moreover, workers return to the labor pool (for potential employment) if one of the three hiring sectors chooses to terminate the employment of some workers.

The Financial Sector aggregates banks and financial institutions. It allows the household sector to gain interest on its savings and hold demand deposit accounts; it also allows the production sector to borrow money. As of its interaction with the Government, the Financial Sector holds Government bonds and must maintain a reserve level. Therefore, it is clear that the Financial Sector is paid by the Production and Government Sector, and uses the funds to pay the Household sector the interest. In case the Production sector defaults on its debt, the financial sector can rely on Federal deposit insurance.

Evidently, information flow (including transactions) needs to be accounted for in the model, and Forrester does so. It is clear, based on the papers describing the National Model, that Forrester models transactions using the Accounting System as the underlying framework. The following diagram, for instance, portrays how Forrester modeled the interaction between the production and the financial sector.

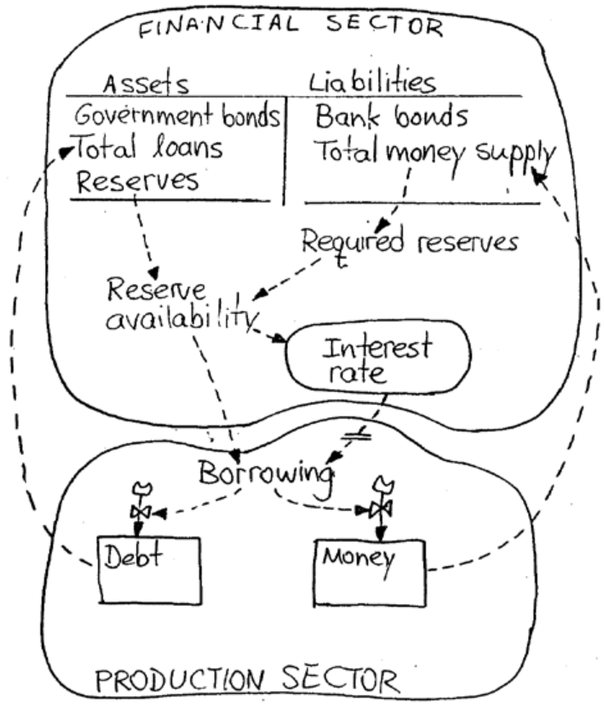

Figure 2.2:

There are similarities in how these transactions are modeled in Forrester and Yamagushi's models. The details will be explored later in the report.

Mashayekhi's Economy Model

Ali Mashayekhi, a MIT Sloan doctorate graduate, developed a System Dynamics model involving the Iranian economy as part of his Ph.D dissertation (Mashayeki, 1978). Specifically, the model was targeted at examining the transition of the Iranian economy into one that is oil-independent. One of his primary arguments is that the Iranian economy would face a crisis if it did not reduce its dependency on oil, a non-renewable resource. The System Dynamics model mirrors segments of the Iranian economy, and shows how, if the oil-related policies were kept unchanged, the economy would hurt as a result. While the purpose of the model is significantly different than ours, the structure of the Iranian economy can provide a feasible starting point for modeling the U.S. economy. Having said so, examining the sectors of the Mashayekhi's model, hereafter referred to as the M-model, will help develop important insights about the workings of the U.S. economy.

The primary sectors of his model are:

1) Allocation of Production Factors between Sectors
2) Agricultural Sector
3) Production Capacity in the Industrial Sector
4) Allocation of Income and Capital Accumulation

5) Foreign Trade: Import and Export Mechanisms
6) Oil Exportation
7) Population Structure and Change
8) Education Sector and Growth of Education Level
9) Technology Transfer

The M-model considers three factors of production: capital, labor, and educational level. While the former two are conventionally included in economic models, the third, educational level, is a more subtle one. It nevertheless is quite important in the ultimate determination of total output. Mashayekhi argues, based on the work of academic economists, that improving educational level can have a greater impact on economic growth than the accumulation of capital and labor.

In the M-model, the population sector provides labor. Capital is endogenously determined, through the accumulation of domestic and imported investment. The educational level is a function of the number of graduates. The three production factors are then allocated to the agricultural and non-agricultural sectors in order to produce output. The way that the M-model determines how to allocate the production factors is by comparing the availability of the outputs in the agricultural versus the non-agricultural sectors. If a certain sector, say the agricultural one, is low on output, then the number of production factors of high productivity allocated there will be increased.

The agricultural sector is concerned with the production of food. The reason that Mayashekhi emphasizes the agricultural sector and its output is because the economy will face food shortage when-as the M-model suggests-the availability of oil will be decay. The total food supply in the economy is a function of the domestic production and imports. The domestic production depends on the amount of production factors allocated to the agricultural sectors, as well as the level of technology and the land space.

The production factors that are not allocated to the agricultural sector are directed to the industrial, non-agricultural, sector. In turn, this sector represents the aggregate production capacity of the economy outside of the agricultural sector. With the power to produce, the economy must then decide how to distribute this capability. The M-model allows for the production capacity to be transferred to four sectors:

1) Capital goods sector
2) Intermediate goods sector
3) Education sector
4) Consumption goods sector’

The M-model determines how to allocate production capacity in each of these four sectors by comparing the desired production capacity against the total actual total output.

The capital goods sector produces the equipment required for production. In Iran, capital goods are mostly imported. The performance of this sector directly relates to the economic growth or decay. Clearly, a lack of capital equipment will slow down production, and as a consequence, economic growth.

Intermediate goods are semi-finished goods that are used as input in the production of other final goods. These include consumer goods or services or capital. Similarly to the capital goods market, the performance of this sector affects economic growth.

Next, a substantial model of an economy should account for the education level in the population. With education, labor productivity increases. Therefore, education influences the adoption of new technology, and ultimately productivity of the labor force. The development of a nation, economically, scientifically and socially, is a function of the education level.

Demand for education is a function of the income per capita in the country, among other variables. Clearly, an increase in the demand for education, which is endogenous in the M-model, will lead to a higher education output if the capacity of the sector is able to accommodate the demand. The Education sector produces graduates who are ready to produce goods and services. It is noteworthy that the level of education is measured in man-years of schooling.

In the M-Model, the Consumer Goods sector produces the non-agricultural goods and services to be consumed by household. The amount of available goods is the summation of the domestically produced goods and the imported goods. Similarly to the Capital and Intermediary goods sectors, the output changes proportionally with the capacity utilization.

In order to take into account the effect of technological progress on the economy, Mayashekhi includes a technology sector. Clearly, the performance of this sector affects economic growth, for better technology implies higher productivity, better output quality, more resourceful managers, and more efficient social and political institutions. In Iran, technological progress is achieved by importing technology from industrialized nations.

The transfer of technology is a function of the amount of available technology in industrial nations (that is not already utilized by Iran), and Iran’s ability to transfer the technology. In turn, transferring technology depends on the average education level of the workforce as well as the Iran’s ability to conduct foreign trade.

The manner in which the nation’s disposable income is distributed to account for the various demand types is important. In the M-model, the nation can use total income, a summation of income from agricultural, capital, intermediate, consumption goods and oil, to demand food, investment products, and consumer goods and services. Changes in per-capita income does not proportionally change the amount of goods of a certain type demanded. For instance, if the per-capita income increased, the amount of food demanded does not necessarily increase. In fact, food, a percentage of total demanded goods, may decrease, despite potentially having the total expenditure on food increase. An aggregate demand function governs how the income is spent. Either domestic production or importation satisfies demand for a final good.

Trade is an important determinant of economic growth, and discrepancies between supply and demand in the Iranian economy can be adjusted using trading policies. Indeed, the amount imported and exported is determined in this sector, based on domestic production, domestic demand, foreign-exchange availability, and governmental policies concerning trade.

The demographic character of a nation plays a strong role in shaping the economy. Therefore, considering population change and its effect on the economy is important. Appropriately so, the population size is endogenously determined, and is the accumulation (i.e. integration) of birth rate minus death rate. Birth rate depends on the population size at a given point in time, food per capital, and the educational level of the population. Food per capital changes population in the same direction, and educational level in the opposite direction. Death rate depends on industrialization and food per capita, and changes in both of these affect population in the opposite direction.

Finally, because the M-model examines the effect of Iran's heavy reliance on oil on the performance of the economy, examining oil trading policies is essential. Because the oil sector does not employ a large percentage of the labor force and investment in relationship to Gross Domestic Capital is low, the M-model ignores the production factors in the oil sector.

<u>Yamagushi's Model</u>

Yamagushi's System Dynamics model came about being somewhat as a reaction to the 2008 financial crisis that the U.S. Economy experienced (Yamagushi, On the Liquidation of Government Debt under A Debt-Free Money System-Modeling the American Monetary Act, 2010). The author first claims that the crisis was engendered due to a systematic failure of the debt money system. Therefore, replicating the structure of the system through which money and loaned out and borrowed, using System Dynamics, can lead to insights about the forces leading to the crisis. Moreover, such a SD model would allow the user to develop policies to cope with the liquidation of government debt, which would improve unemployment and foreign economies' recessions. Furthermore, the model is a powerful interface for examining how the economy would react to the *public money system* as proposed by the American Monetary Act. Yamagushi finds that reliance on public money would help control inflation and recession in the economy.

It is thus clear that the model's primary purpose is to analyze the monetary system currently instituted in the U.S. Economy, the debt-money system, and compare it to an alternative, the public-money system (Yamagushi, Macroeconomic Dynamics- Accounting System Dynamics Approach, 2011)

The ultimate objective of Yamagushi's pursuit is different from ours, but can nevertheless provide us with a promising starting point.

The Macroeconomic system comprises of six main sectors:

1. Central Bank
2. Commercial Banks
3. Consumers (households)
4. Producers (Firms)
5. Government
6. Foreign Sector

He identifies his system as being a Capitalist Market Economy. Such a system implies that private ownership is allowed; landowners own land and homes, shareholders own corporate shares, corporations own capital. Moreover, there is a platform- the market- that allows sellers and consumers to conduct transactions.

Next, a short description of each of the sectors is presented[12].

In order to operate, the Government in Yamagushi's model gains revenue by taxing the consumer (via an income tax) and the producer (via a corporate tax.) As for its expenditures, the Government dedicates a portion of its spending to the public, and another to paying back, with interest, the consumers for lending money to the Government in the form of securities. In this model, the spendings are endogenously determined, and are a function of either the growth-dependent expenditures or tax revenue-dependent expenditures. It is clear that if spending is greater than the tax revenue, then the Government must borrow cash from consumers and banks, again the form of securities. The foreign sector of the model also includes a Government whose role and activities are similar to that of the domestic sector.

The producers represent the players in the economy that provide goods and services to the economy and seek to generate a profit. Clearly, in order to operate, they must pay wages to the labors (who in turn are also the consumers), interest to the banks for the loans, and must take into account the depreciating value of the capital. These are the expenses considered in the model; subtracting the expense sum from the company's revenue, and afterwards considering Government taxes, is considered the profit. Some of the profit is paid out to the owners (i.e. the consumers in this model) as dividends and the remainder is considered retaining earning. Following such a model, producers are continuously in a state of cash flow deficit and must

[12] Note that there are many similarities between Yamagushi's orginal model and the modified version. In the next section-"Calibrating Yamagushi's Model to Fit the US Economy's Structure and Data"- the model is explored using supporting illustrations, and some of the content will overlap with the description provided in the literature review. However, for the purpose of the literature review, a verbal account of Yamagushi's model is maintained.

therefore borrow money from banks (and pay interest) in order to finance the investments. Moreover, this model considers a foreign sector, which mirrors the domestic sector in its structure. Trade occurs between the foreign and domestic sector in order to promote the production process, and producers are allowed to invest abroad.

Because they provide a source of labor supply in the economic system, consumers receive a wage which in turn is either spent consuming the producers' goods and services, or saved in financial institutions or the Government. For the savings, consumers naturally receive interest. Consumers also own shares of firms in the producers sector, and receive dividends as a result. Therefore there are three main sources of revenue for the consumers: wages, interest, and dividends. In addition, consumers are paid cash whenever securities they previously held are demanded back from the Government. Consumers naturally owe the Government tax fees; the difference between gross revenue and income tax is considered the consumers' disposable income, which can then be either spent on consumption and government securities or saved in banks. Furthermore, given the existence of a foreign sector, consumers are allowed to invest abroad. The amount invested is considered a fraction of the consumers' deposits, and returns from investment returns are calculated using deposit returns. In that same fashion, the Government and the producers pay foreign investors interest when they inject into the domestic economy.

The role of banks is to use the deposits they receive from consumers and make loans to producers. Clearly, banks must make an interest payment to consumers and collect one from the producers at a prime rate. Moreover, as required by the Federal Reserve., it is mandatory for banks to maintain a reserve level with the central bank. In case banks need additional funds for loans, they are able to borrow money from the Federal Reserves at a discount rate. The retained earnings, consisting of the amount that remains after paying the consumer, the central bank, and receiving interest from the producers, if positive, are used to pay bank employees who in turn are consumers. Furthermore, commercial banks interact with the foreign sector when there is demand from producers, consumers and the central bank. Foreign exchange is maintained in the domestic banks' reserves, and its value is determined according to the exchange rate. Gains or losses that are a result of foreign exchanges are accounted for in the retained earnings. The dollar is considered to be the vehicle currency, that is, the currency that is used to denominate the value of assets or contracts. Therefore, foreign banks must set-up a foreign exchange account. That is one instance in the model where the foreign sector does not mirror the domestic one.

Finally, the Central Bank is the one entity in the system that has to authority to control the currency level in the economy. It can do so through a few main transactions:

1) Issuing money against gold deposited by the public
2) Issuing money against government securities bought by consumers and banks in open market operations

3) Issuing currency against credit loan made out to commercial banks (Yamagushi refers to this activity as *money out of nothing)*
4) Withdrawing money by selling government securities to the public and banks,
5) Withdrawing money by collecting interest payments from banks
6) Controlling the required reserve ratio
7) Implementing monetary policies in open market operations and modifying discount rate
8) Controlling the amount of credit loans to banks, known as *window guidance* in Japan
9) Buying/Selling in the foreign exchange market

The next section will discuss how Yamagushi's model was modified in an attempt to calibrate it to US data.

Calibrating Yamagushi's Model to Fit the US Economy's Structure and Data

Model Sectors

The major sectors that Yamagushi introduced in his model are maintained. The US Domestic Economy consists of the following sectors:

1) Household Sector
2) Government
3) Banks
4) Central Bank
5) Consumers
6) Producers
7) Aggregate Production

These are all gathered from Yamagushi's work (some sectors, such as "Household Sector," are not discussed in the literature review; they do, however, appear in other versions of his work, other than the one surveyed in the literature review.) The "Foreign Sector", however, is omitted, and variables that pertain to the foreign sector, such as "Net Export," are kept exogenous.

Next, each of the sectors in the model is illustrated. Note that the following diagrams do not represent snapshots of the actual SD model, but rather simplified versions with some links removed. The purpose of these illustrations is to convey how the interactions between the sectors translate into a SD model without necessarily focusing on the mathematical equations.

Meet the Players

In the following sections, the dynamics of each of the identified sectors will be described; however, prior to doing so, it is useful to first meet the players, and introduce the logo that will represent each of the players.

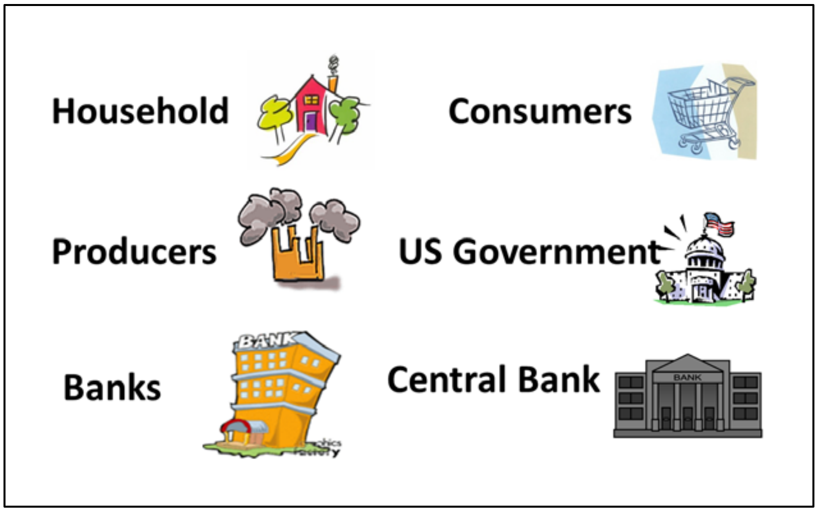

Figure 2.3: The Players of the US Domestic Economy

The Household Sector

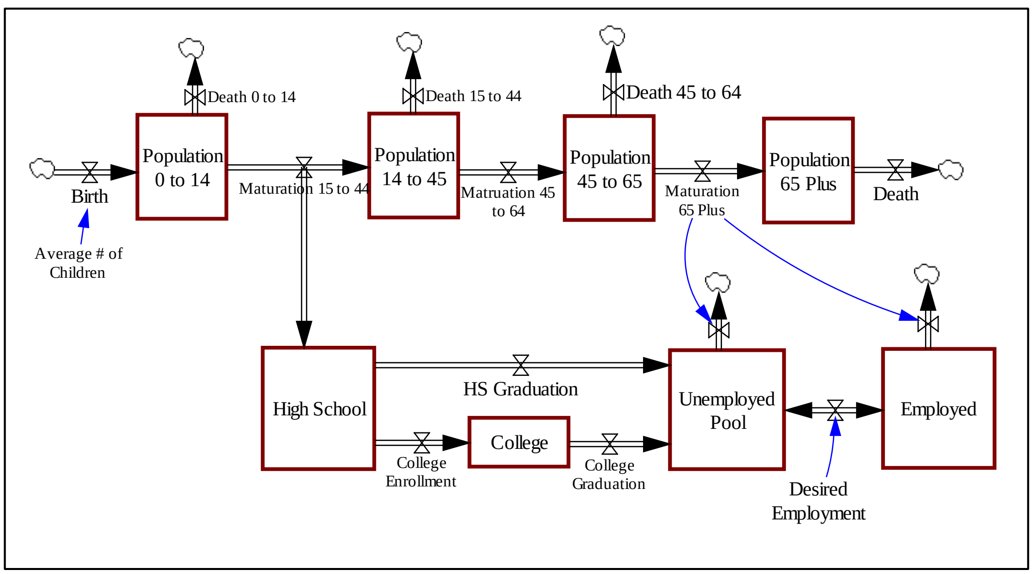

Figure 2.4: Illustration of the Household Sector

Reflecting the conventional dynamics of birth, education, employment, ageing, and death, the household sector computes population growth over time, and by doing so, derives the dynamics of the labor pool. Specifically, given an average number of children born every year, people mature and join their respective group age, as reflected by the four stocks (“Population 0 to 14”- “Population 65 Plus”.) Upon reaching age 15, people typically enroll in high schools. Four years later, a certain proportion of high school students enroll in colleges; the remainder directly join the labor market. Thus, the pool of unemployed people thus changes based on the number of high school and college graduates, as well as based on changes in desired employment in the economy. When individuals retire, which is assumed to occur at age 65, they leave the labor pool.

Endogenously calculating "Desired Employment" is arguably the most difficult piece of the "Household Sector." However, it also is quite essential, for an accurate formula for desired employment will help link the household sector to other sectors, such as through government spending, change in excise tax, or change in interest rate. Further research needs to be conducted in order to determine how desired employment changes over time; however, for the purpose of the current model, historical employment rate, an exogenous variable, is utilized instead.

When calibrated for mortality rates and average number of children, the model endogenously generates the total population. Indeed, in the graph below, the simulated total population, from year 1980 (year 0) to year 2010 (year 30,) is in red, whereas the historical total population is in blue.

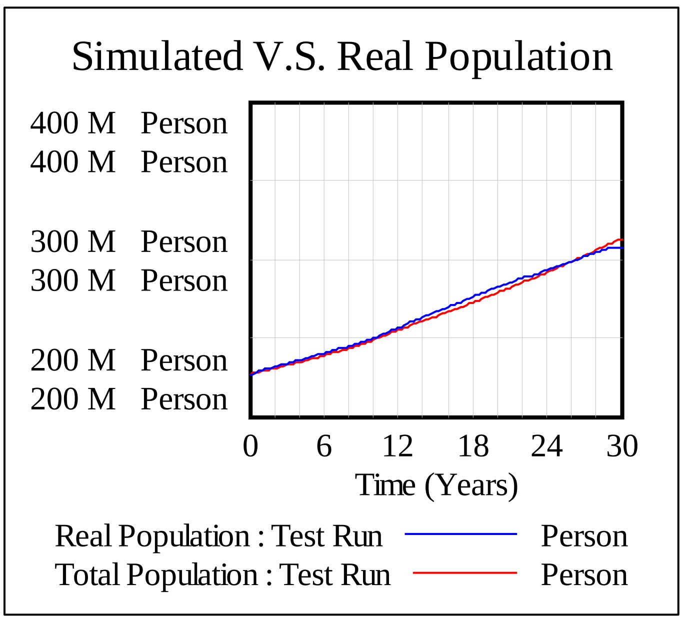

Figure 2.5: Simulated versus Historical Total Population

Moreover, when the historical employment rates are used to determine desired employment, the model generates an employment level (blue line,) which is compared against historical employment level (red line.)

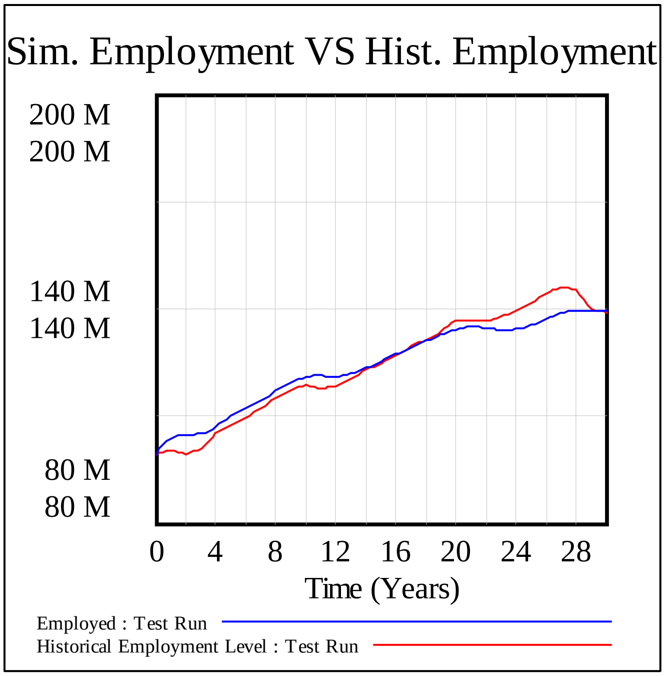

Figure 2.6: Simulated versus Historical Employment Levels

The Consumer Sector

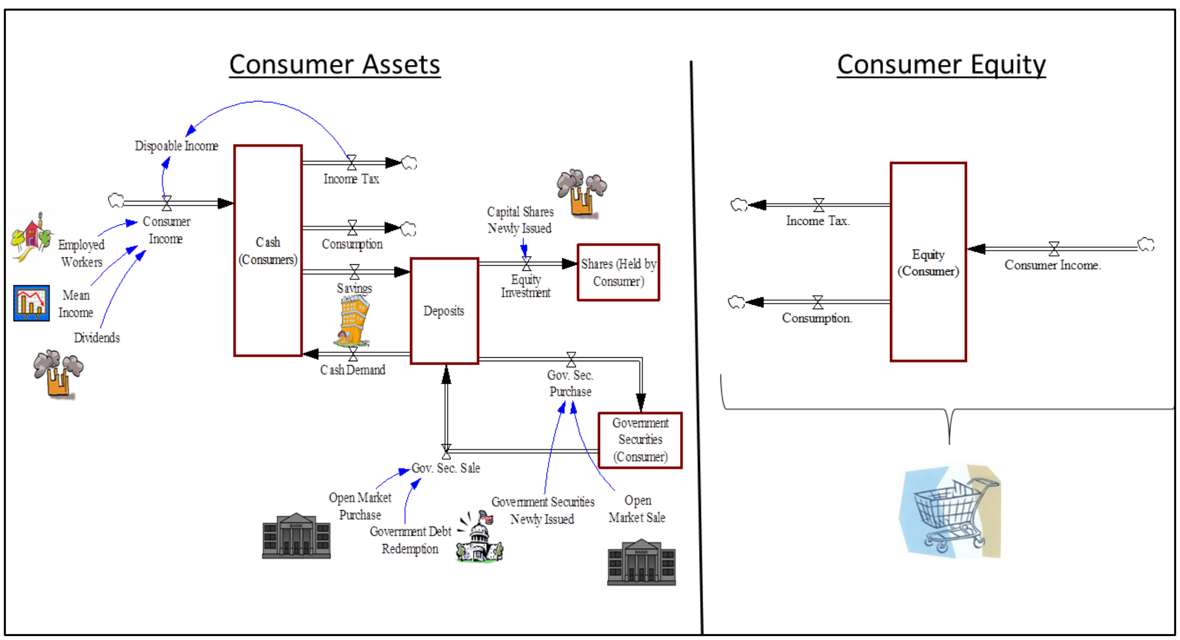

Figure 2.7: Illustration of the Consumer Sector

The consumer sector can be examined using the double-entry accounting framework. Consumers have assets, which, given that consumers do not hold debt in this model, are translated into equity. While the assumption that consumers do not hold debt may appear to be unreasonable, it does maintain a simple and generic model. Debts are assumed to be undertaken by producers when these seek to pursue new ventures or invest. Realistically, however, consumers may hold debt, such as mortgages, which, as the 2008 Financial Crisis demonstrated, are important to model. However, such analysis on the housing sector is an example of how this generic version of the US Domestic Economy can be enhanced to test for policies to solve and understand the dynamics of the housing crisis.

Within the consumer's assets, income-the addition of aggregate mean income (employed workers multiplied by historical mean income) and dividends from the producers sector-increases the consumer's level of cash. Income tax, a possible fiscal policy used by Government, decreases consumers' cash level. In addition, consumption is based on Price, Price Elasticity, and Marginal Propensity to Consume, and also reduces the consumers' cash level. After paying income tax, consumers' disposable income is computed.

Once the cash level is determined, the consumer sector interacts with the bank sector, by either saving or retrieving cash from deposits. While the decision to save is relatively easy to model and can be assumed to be the fraction of disposable income not consumed, the decision to demand cash is more difficult to model. Yamagushi suggests that the flow "cash demand" is based on "currency ratio" and "cashing time." If this model is to be expanded, further research can be done to derive a formula for capturing cash demand trends in the US.

Finally, consumers have the option to invest in either private equity or in fixed income. By undertaking the former, consumers interact with producers and buy shares in the primary market. Indeed, solely the primary market is necessary to be modeled in this system, for transactions that occur in the secondary market happen between consumers. Because consumers are modeled from an aggregate perspective, the model is not concerned with which consumer is holding a producer's share, but rather that the stock is in some consumer's hand. As for investing in fixed income, consumers can only buy or sell Government securities (Treasuries) in this model. The supply of Government securities is affected by the Government's release of new securities when deficit needs to be financed, and by the Central Bank's open market operations.

The Producer Sector

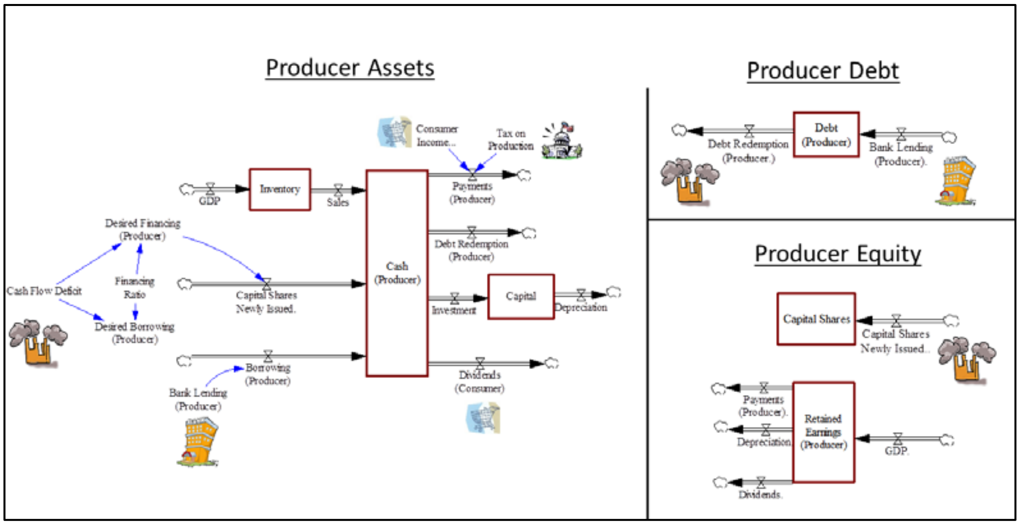

Figure 2.8: Illustration of the Producer Sector

Producers have assets, debt and equity. Evidently, the producers' cash increases when sales are made, which in turn decreases aggregate inventory. However, what determines inventory in an economy? Gross Domestic Product (GDP) is the aggregation of an economy's production, and accumulates into a stock called "Inventory." Furthermore, producers have operations that need to be financed in order to produce their goods and services. When operating at a cash flow deficit, producers must find the funds necessary to conduct their activities. In order to do so, producers can either issue new capital shares (equity bought by the consumer) or borrow from the banks (which depends on the bank's lending capacity.) Whether the funds originate from debt or equity is determined by an exogenous variable called "Financing Ratio."

Factors that reduce the producers' cash are tax on production paid to the Government and income paid to the consumer. In addition, the producers' cash will diminish when debts are repaid after a fixed time period has elapsed. Additionally, producers may choose to invest, which accumulates into a stock of Capital, and depreciates over time. Finally, producers may opt to pay their shareholders (the consumers in this model) dividend.

Evidently, debt accumulates when banks loans are made to the producer, and diminishes when debt is repaid. Accounting for these dynamics represents the debt portion of the producers' balance sheet. The remaining inflows and outflows are found in the equity sector. Note how the sum of the inflows/outflows in the assets portion of the balance sheet is equivalent to the sum of the inflows/outflows in the debt and equity sectors.

The Government Sector

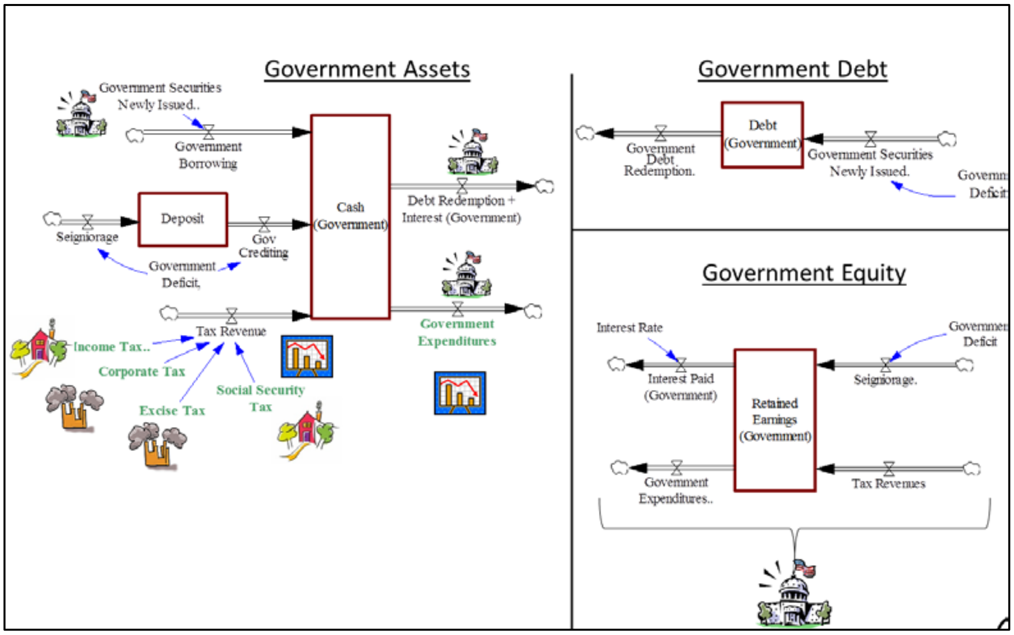

Figure 2.9: Illustration of Government Sector

Similarly to producers and consumers, the US Government also holds cash in this model. Cash is primarily increased when Treasuries are newly issued, which represents the Government's way of borrowing funds. In addition, tax revenues evidently increase the Government's cash level, and, in the US, can be divided into four main kinds: "Income Tax," "Corporate Tax," "Excise Tax," and "Social Security Tax." In the model, historical tax rates are used to compute the amount raised in each form of tax. Moreover, the tax related variables are in green because they are intended to remain exogenous to test the effect of fiscal policies on the economy. Currently, effective rates are utilized for simplicity; however, detailed analysis of the effect of tax changes will require further division of the tax rates.

An additional source of revenue for the Government is seigniorage. When the Government issues new securities, there exists an opportunity for monetary gain by selling the bonds at a slightly higher price than their face value[13]. The difference is seigniorage and adds on to a deposit stock that can be used as credit for additional cash if need be. Government crediting is nullified in the model for reasons of simplicity. The factors decreasing the Government's cash reserve are expenditures, seen in green because it is a fiscal policy, as well as debt redemption. Government Expenditures are kept exogenous, whereas debt redemption depends on the amount of debt accumulated. As seen under in the debt portion of the Government's balance sheet, debt accumulates when positive deficit occurs.

[13] Another example of how seigniorage can occur involves Gold transactions. For example, when one buys an ounce of gold, the item may only hold 93% of gold, with 7% made of another metal. However, the buyer will nevertheless paid the value of an entire ounce. The difference is revenue for the seller.

The following graph illustrates a comparison between the simulated and historical Government tax revenues, based on exogenous rates entered in the system:

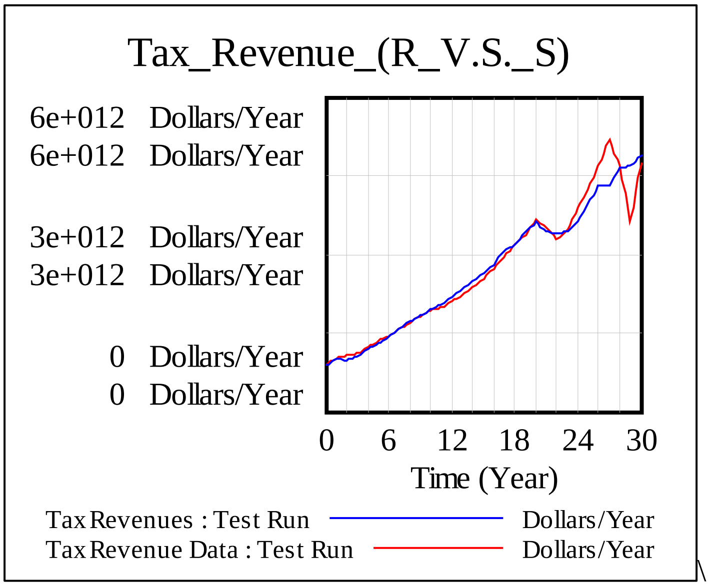

Figure 2.10: Simulated versus Historical Government Tax Revenue

Moreover, the following graph depicts simulated versus historical Government debt:

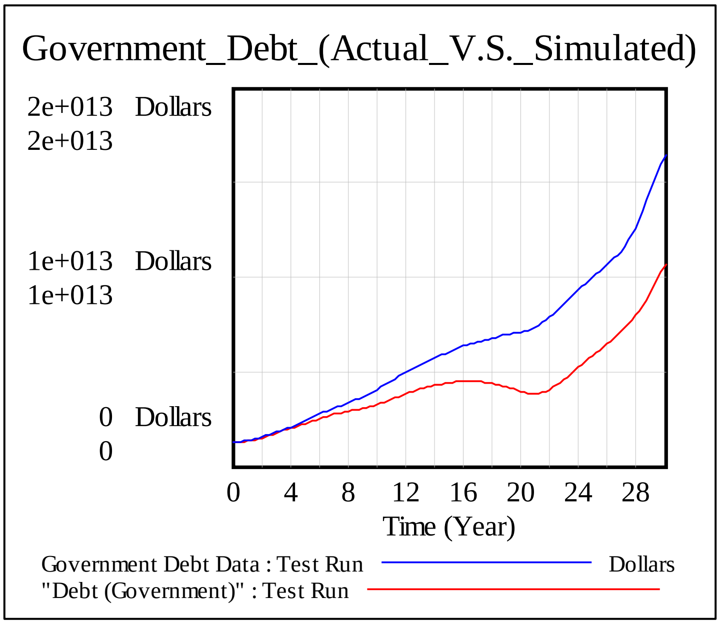

Figure 2.11: Simulated versus Historical Government Debt

While the values are not exactly replicated, the trends of the simulated and historical values are similar. This similarity motivates further research into the reasons behind the difference. Perhaps the calculation of Government deficit needs to be calibrated or the formula accounting for the Government's debt redemption needs to be reassessed.

The Bank Sector

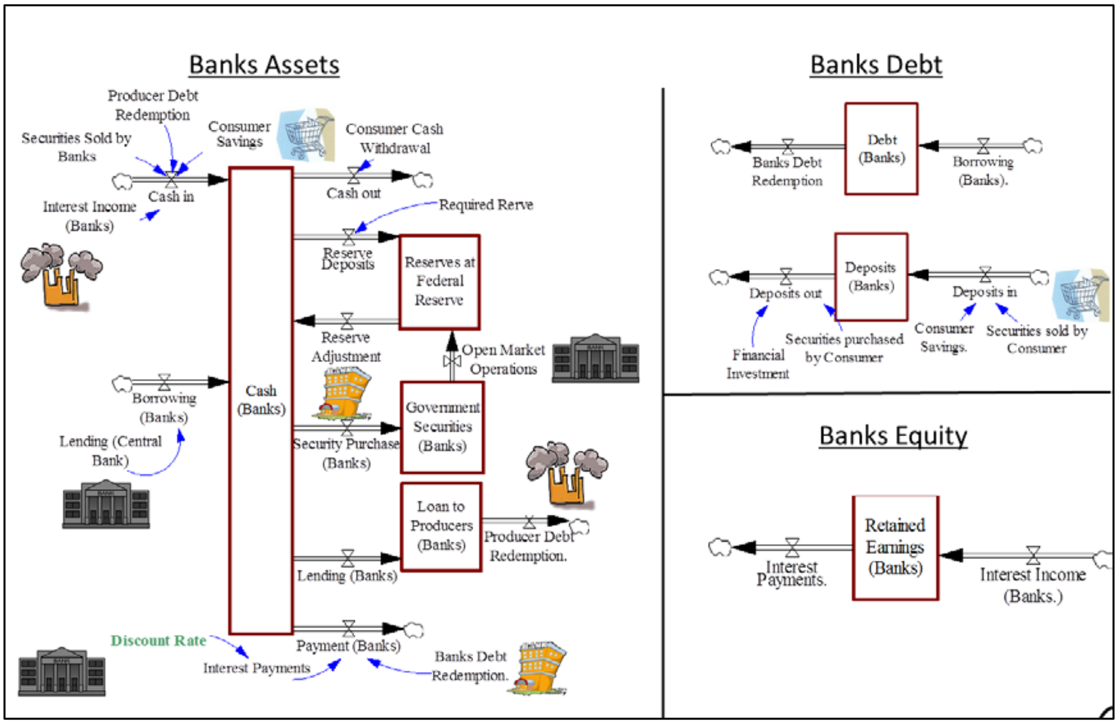

Figure 2.12: Illustration of Bank Sector

The banks hold cash their vaults. Consumer Savings, Producer Debt Redemption, Securities Sold by Banks, and Interest Earned increase the Banks' cash level. Moreover, Banks may choose to borrow funds from the Central Bank at a Discount Rate in order to increase their cash level and meet required reserves. While discount rates is modeled in this system, and represents one of the primary tools utilized by the Federal Reserve for conducting monetary policy, Federal Funds Rate are not considered. Federal Funds Rate represents the overnight rate at which banks would lend to each other, typically in order to satisfy the reserve requirements set by the Central Bank. However, because this model examines the aggregation of banks in one balance sheet, the Federal Funds Rate is irrelevant. This model is not concerned with banks' individual balance sheet.

The cash level in banks decreases when consumers choose to withdraw their savings. In addition, the Federal Reserve sets a Required Reserve Ratio, which may either decrease or increase the cash level, based on whether the current reserve greater or less than the required reserve level. Moreover, banks have the option of buying Government Securities, which accumulate into a stock of Government Securities, and decreases through open market operations (Central Bank purchases.) Further, it is evident, that banks' cash reserve decreases through loans made to the producers, which accumulate into a stock named "Loan to Producers." This stock in turn decreases when the producers pay their debt. Finally, banks need to pay interest when they choose to buy debt, which will decrease the banks' cash level.

The Central Bank

The Central Bank is concerned primarily with monetary policy. In doing so, it can utilize three main tools: the Federal Funds Rate, the Discount Rate, and Open Market Operations (Akhtar, 1997). The first two were previously discussed. The Central Bank sector should ideally be able to endogenously determine how Government Securities purchase/sale decisions by the Federal Reserve affect the money supply, and consequently the interest rates at which banks lend to one another (the Federal Funds Rate) and to producers. Due to time constraints and the relative complexity of the task, these dynamics were not explored in this project. However, modeling open market operations using an Accounting-Based System Dynamics approach is essential to completing the model. Some preliminary work has been conducted for systemically modeling open market operations (Radzicki), and refining it to fit in the grander picture that Yamagushi's model presents are important future steps in this research project.

Aggregate Production

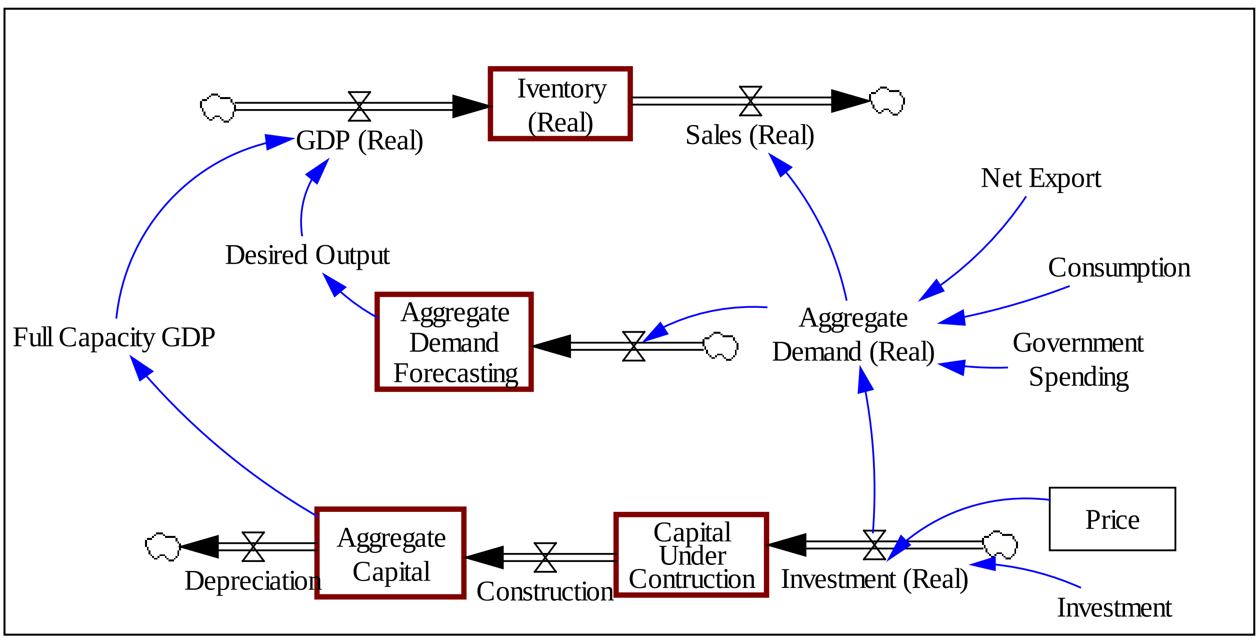

Figure 2.13: Illustration of Aggregate Production

The above diagram depicts a snapshot of how aggregate production is computed in the Yamagushi model. Based on price, and investment decisions made by the production sector, real investment is undertaken.[14] Investment decisions lead to capital under construction, which

[14] Real Investment refers to the actual goods and services, rather than their nominal value.

eventually accumulates into an aggregate capital level. Evidently, coupled with other variables such as technological change, aggregate capital contributes to determining the economy's production capacity- the "Full Capacity GDP." This capacity for aggregate production is accompanied with the economy's Aggregate Demand, which is equivalent to the addition of Net Export, Consumption, Government Spending and Real Investment. Using information about aggregate demand, desired output can, with some time delay, be computed. Amalgamating information about Full Capacity GDP and Desired Output helps determine the economy's GDP in a given year. Indeed, an economy produces the minimum of its full capacity and its desired output. Naturally, GDP accumulates into a stock of aggregate inventory, which decreases due to aggregate demand.

Quite interestingly, the behavior of real investment follows the overarching trend that describes the growth pattern of the US Economy, namely "growth and oscillation." The following graph shows the investment behavioral mode:

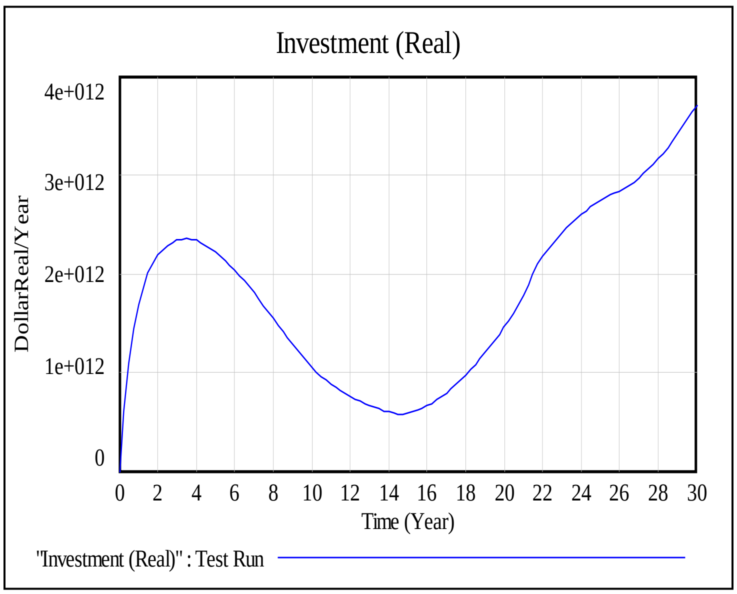

Figure 2.14: Simulated Investment

Finally, entering information about Net Export (exogenous,) Government Spending (exogenous,) Real Investment (endogenous) and Consumption (endogenous,) the model yields a GDP trend similar to historical behavior. In the following graph, the blue line depicts the simulated GDP behavior, whereas the red line illustrates the historical GDP of the US.

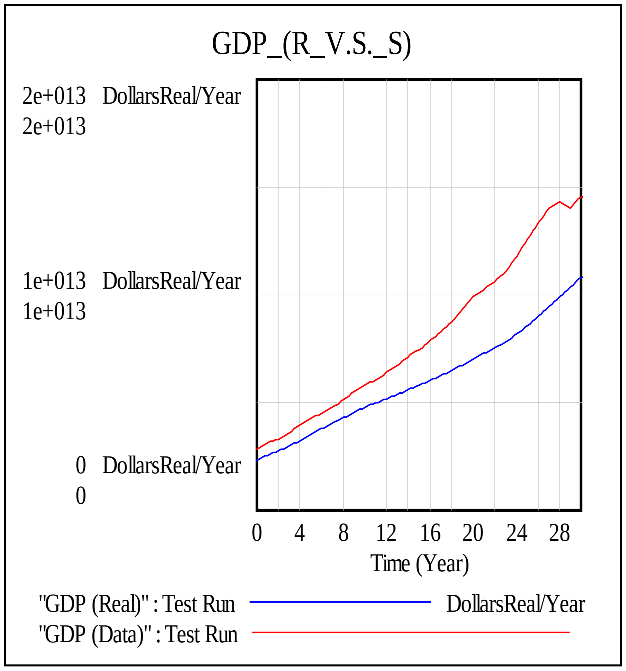

Figure 2.15: Simulated versus Historical US GDP

Model Limitations & Future Work

Several limitations and encountered challenges encountered during the modeling process were presented in the previous section. However, to reiterate and to elaborate, these will be highlighted again in this section.

First, endogenously modeling "desired employment" in an economy, that is the rules that dictate the rate at which workers move from the unemployed to the employed pool and vice versa, is necessary to link the household sector to the remainder of the model. The challenge in this pursuit is to first determine what variables determine how employment levels change, and then to attempt to devise a formula that seems to be generating a trend similar to historical values. Moreover, studying the economic literature on how employment changes over time is a useful foundation that may be quite applicable to this model.

Second, a model of open market operations is still quite primitive and needs to be further examined. The link between money supply in bank vaults (a direct effect of open market operations) and interest rates is unknown and not deterministic. One approach for modeling open market operations is to assume that the Federal Reserve always achieves the interest rates it sets out to reach during the open market operations. Another possible approach is to assume a worst and best case scenario, and examine how the system would react under each of these scenarios. For example, if the open market operations did not decrease the interest rates to the extent that the Fed hoped for, then this situation may be thought as a "worst case." Similarly, the situation in which Fed did efficiently reach its interest rate targets in the time frame it had expected may be thought of as the “best case.” In sum, however, modeling open market operations may be a complex task to be methodologically approached with caution.

Third, a sector that endogenously calculates money supply, as well as the resulting inflation and interest rates, is necessary to fully grasp the effects of financial policies on the system. Yamagushi proposes a structure in his model that is worth examining. Finally, further work should include a foreign sector once the domestic sector is fully complete and calibrated. In order to do so, creating a model that mirrors the structure of the US Domestic Economy but has different parameter values is one such feasible approach, as suggested by Yamagushi. Other approaches also exist. For example, the foreign sector may modeled as a sector that aggregates all the trades conducted with other economies, or a sector that generates the price dynamics of natural resources such as oil. Evidently then, in the future, this model has the potential to grow in a number of directions.

Conclusion

In sum, this project is concerned with calibrating a generic model of the domestic US economy. After reviewing a number of models, Yamagushi’s work seemed most appropriate, given the amount of supporting literature and the flexibility of the Accounting-based modeling paradigm. The behavioral mode of some variables, such as GDP and Total Population, was successfully generated by the model. Further work needs to be conducted to calibrate other key economic indicators and to add other sectors into the model, as discussed in the “Model Limitation & Future Work” section. The preliminary results that the model generated are promising, motivating further work on the model.

Bibliography

Akhtar, M. (1997). *Understanding Open Market Operations.* New York, NY: Federal Reserve Bank of New York.

Barlas, Y., & Yaşarcan, H. (n.d.). *A Method For Finding Equilibrium Points of a Non-Linear Dynamic Model.* İstanbul, Turkey.

Forrester, J. W. (1984). The System Dynamics National Model-Objectives, Philosophy, and Status. *International System Dynamics Conference.* Oslo, Norway.

Graham, A. K. (1984). *Introduction to the System Dynamics National Model Structure.*

Mashayeki, A. N. (1978). *Strategy of Economic Development in Iran: A Case of Development Based on Exhaustible Resources.* Cambridge, MA: Massachusetts Institute of Technology.

Yamagushi, K. (2010). *On the Liquidation of Government Debt under A Debt-Free Money System-Modeling the American Monetary Act.* Seoul, Korea.

Yamagushi, K. (2011). *Macroeconomic Dynamics- Accounting System Dynamics Approach.* Kyoto, Japan.