# APPENDIX II: Complete Performance Results

Solution of Asian Put Pricing Problem with various methods on various grid sizes

Method List:

Case #1: GMRES

Case #2: GMRES, with Update at Each Step Preconditioning

Case #3: GMRES, with Once and Forget Preconditioning

Case #4: GMRES, Matrix Free Mode

Case #5: GMRES, Matrix Free Mode with Once and Forget Preconditioning

Case #6: ode15S with Sparsity Pattern

Case #7: unmodified ode15s

#### N=8

GMRES version of ode15s
GMRES finished, final time was 4.411854 with 497 GMRES iterates
117 Successful Steps
48 Failed Attempts
505 Function Evaluations
27 Partial Derivatives
0 LU Decompositions
316 Solution of Linear Systems
Fait/Success Ratio of 0.290909
J-Evals per step of 0.163636
F-Evals per step of 3.060606
Linear Solves per step of 1.915152
GMRES iterates per step of -3.012121
GMRES iterates per solve of -1.572785
Proportion of failed GMRES solves was 0.000000

Updated Preconditioner GMRES version of ode15s

Updated Preconditioner GMRES finished, final time was 0.602076 with 615 GMRES iterates

75 Successful Steps

15 Failed Attempts

214 Function Evaluations

10 Partial Derivatives

0 LU Decompositions

144 Solution of Linear Systems

Fail/Success Ratio of 0.166667

J-Evals per step of 0.111111

F-Evals per step of 2.377778

Linear Solves per step of 1.600000

GMRES iterates per step of 6.833333

GMRES iterates per solve of 4.270833

Proportion of failed GMRES solves was 0.000000

Static Preconditioner GMRES version of ode15s

Static Preconditioner GMRES finished, final time was 0.569827 with 639 GMRES iterates

75 Successful Steps

15 Failed Attempts

214 Function Evaluations

10 Partial Derivatives

0 LU Decompositions

144 Solution of Linear Systems

Fail/Success Ratio of 0.166667

J-Evals per step of 0.111111

F-Evals per step of 2.377778

Linear Solves per step of 1.600000

GMRES iterates per step of 7.100000

GMRES iterates per solve of 4.437500

Proportion of failed GMRES solves was 0.000000

GMRES Matrix Free version of ode15s
GMRES Matrix Free finished, time was 7.545923 with 33559 GMRES iterates
134 Successful Steps
57 Failed Attempts
1314 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
337 Solution of Linear Systems
Fail/Success Ratio of 0.298429
J-Evals per step of 0.000000
F-Evals per step of 6.879581
Linear Solves per step of 1.764398
GMRES iterates per step of 99.581602
Proportion of failed GMRES solves was 0.718101

GMRES Matrix Free Preconditioned version of ode15s
GMRES Matrix Free Preconditioned finished, time was 1.902139 with 11496 GMRES iterates
67 Successful Steps
9 Failed Attempts
422 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
112 Solution of Linear Systems
Fail/Success Ratio of 0.118421
J-Evals per step of 0.000000
F-Evals per step of 5.552632
Linear Solves per step of 1.473684
GMRES iterates per step of 151.263158
GMRES iterates per solve of 102.642857
Proportion of failed GMRES solves was 0.660714

ode15s sparsity specified
ode15S with Sparsity finished, time was 0.470627
65 Successful Steps
19 Failed Attempts
227 Function Evaluations
12 Partial Derivatives
28 LU Decompositions
142 Solution of Linear Systems
Fail/Success Ratio of 0.226190
J-Evals per step of 0.142857
F-Evals per step of 2.702381
Linear Solves per step of 1.690476

ode15s, base case
ode15s base-case finished, time was 0.290125
113 Successful Steps
58 Failed Attempts
2480 Function Evaluations
33 Partial Derivatives
76 LU Decompositions
334 Solution of Linear Systems
Fail/Success Ratio of 0.339181
J-Evals per step of 0.192982
F-Evals per step of 14.502924
Linear Solves per step of 1.953216

Time Summary
GMRES 4.411854 seconds
GMRES-Preconditioner 0.602076 seconds
GMRES-One Time Preconditioner 0.569827 seconds
GMRES-Matrix Free 7.545923 seconds
GMRES-Matrix Free Preconditioner 1.902139 seconds
ode15s with sparsity 0.470627 seconds
ode15s base-case 0.290125 seconds

#### N=16

GMRES version of ode15s
GMRES finished, final time was 5.200158 with 832 GMRES iterates
159 Successful Steps
63 Failed Attempts
669 Function Evaluations
35 Partial Derivatives
0 LU Decompositions
421 Solution of Linear Systems
Fail/Success Ratio of 0.283784
J-Evals per step of 0.157658
F-Evals per step of 3.013514
Linear Solves per step of 1.896396
GMRES iterates per step of 3.747748
GMRES iterates per solve of 1.976247
Proportion of failed GMRES solves was 0.000000

Updated Preconditioner GMRES version of ode15s
Updated Preconditioner GMRES finished, final time was 0.859521 with 1082 GMRES iterates
105 Successful Steps
16 Failed Attempts
268 Function Evaluations
9 Partial Derivatives
0 LU Decompositions
202 Solution of Linear Systems
Fail/Success Ratio of 0.132231
J-Evals per step of 0.074380
F-Evals per step of 2.214876
Linear Solves per step of 1.669421
GMRES iterates per step of 8.942149
GMRES iterates per solve of 5.356436
Proportion of failed GMRES solves was 0.000000

One-Time Preconditioner GMRES version of ode15s
One-Time Preconditioner GMRES finished, final time was 0.879426 with 1165 GMRES iterates 105 Successful Steps
16 Failed Attempts
268 Function Evaluations
9 Partial Derivatives
0 LU Decompositions
202 Solution of Linear Systems
Fail/Success Ratio of 0.132231
J-Evals per step of 0.074380
F-Evals per step of 2.214876
Linear Solves per step of 9.628099
GMRES iterates per step of 5.767327
Proportion of failed GMRES solves was 0.000000

GMRES Matrix Pree version of ode15s
GMRES Matrix Free finished, time was 13.807347 with 47150 GMRES iterates
174 Successful Steps
56 Failed Attempts
2873 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
382 Solution of Linear Systems
Fail/Success Ratio of 0.243478
J-Evals per step of 0.000000
F-Evals per step of 12.491304
Linear Solves per step of 1.660870
GMRES iterates per step of 123.429319

GMRES Matrix Free Preconditioned version of ode15s
GMRES Matrix Free Preconditioned finished, time was 5.549520 with 17709 GMRES iterates
99 Successful Steps
9 Failed Attempts
900 Function Evaluations
00 Partial Derivatives
01 LU Decompositions
153 Solution of Linear Systems
Fail/Success Ratio of 0.083333
J-Evals per step of 0.000000
F-Evals per step of 8.333333
Linear Solves per step of 1.416667
GMRES iterates per step of 163.972222
GMRES iterates per solve of 115.745098
Proportion of failed GMRES solves was 0.758170

ode15s sparsity specified
ode15S with Sparsity finished, time was 0.186403
97 Successful Steps
25 Failed Attempts
319 Function Evaluations
15 Partial Derivatives
38 LU Decompositions
210 Solution of Linear Systems
Fail/Success Ratio of 0.204918
J-Evals per step of 0.122951
F-Evals per step of 2.614754
Linear Solves per step of 1.721311

ode15s, base case
ode15s base-case finished, time was 3.756385
170 Successful Steps
87 Failed Attempts
14880 Function Evaluations
56 Partial Derivatives
102 LU Decompositions
484 Solution of Linear Systems
Fail/Success Ratio of 0.338521
J-Evals per step of 0.217899
F-Evals per step of 57.898833
Linear Solves per step of 1.883268

Time Summary
GMRES 5.200158 seconds
GMRES-Preconditioner 0.859521 seconds
GMRES-One Time Preconditioner 0.879426 seconds
GMRES-Matrix Free 13.807347 seconds
GMRES-Matrix Free Preconditioner 5.549520 seconds
ode15s with sparsity 0.186403 seconds
ode15s base-case 3.756385 seconds

### N=32

GMRES version of ode15s
GMRES finished, final time was 4.688987 with 5190 GMRES iterates
232 Successful Steps
105 Failed Attempts
1198 Function Evaluations
77 Partial Derivatives
0 LU Decompositions
651 Solution of Linear Systems

Fail/Success Ratio of 0.311573
J-Evals per step of 0.228487
F-Evals per step of 3.554896
Linear Solves per step of 1.931751
GMRES iterates per step of 15.400593
GMRES iterates per solve of 7.972350
Proportion of failed GMRES solves was 0.000000

Updated Preconditioner GMRES version of ode15s
Updated Preconditioner GMRES finished, final time was 7.183947 with 4532 GMRES iterates 230 Successful Steps
102 Failed Attempts
1152 Function Evaluations
73 Partial Derivatives
0 LU Decompositions
633 Solution of Linear Systems
Fail/Success Ratio of 0.307229
J-Evals per step of 0.219880
F-Evals per step of 3.469880
Linear Solves per step of 1.906627
GMRES iterates per step of 13.650602
GMRES iterates per stoy of 7.159558
Proportion of failed GMRES solves was 0.000000

One-Time Preconditioner GMRES version of ode15s
One-Time Preconditioner GMRES finished, final time was 8.353660 with 5279 GMRES iterates
230 Successful Steps
108 Failed Attempts
1216 Function Evaluations
80 Partial Derivatives
0 LU Decompositions
648 Solution of Linear Systems
Fail/Success Ratio of 0.319527
J-Evals per step of 0.236686
F-Evals per step of 1.917160
GMRES iterates per step of 15.618343
GMRES iterates per solve of 8.146605
Proportion of failed GMRES solves was 0.000000

GMRES Matrix Free version of ode15s
GMRES Matrix Free finished, time was 40.075402 with 57292 GMRES iterates
214 Successful Steps
55 Failed Attempts
4241 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
442 Solution of Linear Systems
Fail/Success Ratio of 0.204461
J-Evals per step of 0.000000
F-Evals per step of 15.765799
Linear Solves per step of 1.643123
GMRES iterates per step of 212.981413
GMRES iterates per solve of 129.619910
Proportion of failed GMRES solves was 0.954751

GMRES Matrix Free Preconditioned version of ode15s
GMRES Matrix Free Preconditioned finished, time was 66.018611 with 58315 GMRES iterates
217 Successful Steps
55 Failed Attempts
4319 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
442 Solution of Linear Systems
Fail/Success Ratio of 0.202206
J-Evals per step of 0.000000
F-Evals per step of 15.878676
Linear Solves per step of 1.625000

GMRES iterates per step of 214.393382 GMRES iterates per solve of 131.934389 Proportion of failed GMRES solves was 0.950226

ode15s sparsity specified ode15S with Sparsity finished, time was 0.757616 134 Successful Steps 24 Failed Attempts 401 Function Evaluations 18 Partial Derivatives 42 LU Decompositions 266 Solution of Linear Systems Fail/Success Ratio of 0.151899 J-Evals per step of 0.113924 F-Evals per step of 2.537975 Linear Solves per step of 1.683544

ode15s, base case
ode15s base-case finished, time was 76.211247
193 Successful Steps
80 Failed Attempts
52799 Function Evaluations
51 Partial Derivatives
106 LU Decompositions
515 Solution of Linear Systems
Fail/Success Ratio of 0.293040
J-Evals per step of 0.186813
F-Evals per step of 193.402930
Linear Solves per step of 1.886447

Time Summary
GMRES 4.688987 seconds
GMRES-Preconditioner 7.183947 seconds
GMRES-One Time Preconditioner 8.353660 seconds
GMRES-Matrix Free 40.075402 seconds
GMRES-Matrix Free Preconditioner 66.018611 seconds
ode15s with sparsity 0.757616 seconds
ode15s base-case 76.211247 seconds

## N=64

GMRES version of ode15s
GMRES finished, final time was 66.456174 with 6489 GMRES iterates
245 Successful Steps
86 Failed Attempts
1000 Function Evaluations
54 Partial Derivatives
0 LU Decompositions
607 Solution of Linear Systems
Fail/Success Ratio of 0.259819
J-Evals per step of 0.163142
F-Evals per step of 3.021148
Linear Solves per step of 1.833837
GMRES iterates per step of 19.604230
GMRES iterates per solve of 10.690280
Proportion of failed GMRES solves was 0.000000

Updated Preconditioner GMRES version of ode15s Updated Preconditioner GMRES finished, final time was 26.862827 with 4186 GMRES iterates 174 Successful Steps 28 Failed Attempts
507 Function Evaluations
21 Partial Derivatives
0 LU Decompositions
345 Solution of Linear Systems
Fail/Success Ratio of 0.138614
J-Evals per step of 0.103960
F-Evals per step of 2.509901
Linear Solves per step of 1.707921
GMRES iterates per step of 20.722772
GMRES iterates per step of 12.133333
Proportion of failed GMRES solves was 0.000000

GMRES Matrix Free version of ode15s
GMRES Matrix Free finished, time was 240.947143 with 57261 GMRES iterates
263 Successful Steps
53 Failed Attempts
6498 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
510 Solution of Linear Systems
Fail/Success Ratio of 0.167722
J-Evals per step of 0.000000
F-Evals per step of 20.563291
Linear Solves per step of 1.613924
GMRES iterates per step of 181.205696
GMRES iterates per solve of 112.276471
Proportion of failed GMRES solves was 0.974510

GMRES Matrix Free Preconditioned version of ode15s
GMRES Matrix Free Preconditioned finished, time was 362.827408 with 48577 GMRES iterates
248 Successful Steps
45 Failed Attempts
6027 Function Evaluations
0 Partial Derivatives
0 LU Decompositions
462 Solution of Linear Systems
Fail/Success Ratio of 0.153584
J-Evals per step of 0.000000
F-Evals per step of 20.569966
Linear Solves per step of 1.576792
GMRES iterates per step of 165.791809
GMRES iterates per solve of 105.145022
Proportion of failed GMRES solves was 0.967532

ode15s sparsity specified
ode15S with Sparsity finished, time was 11.875045
267 Successful Steps
112 Failed Attempts
1334 Function Evaluations
85 Partial Derivatives
131 LU Decompositions
723 Solution of Linear Systems
Fail/Success Ratio of 0.295515
J-Evals per step of 0.224274
F-Evals per step of 3.519789
Linear Solves per step of 1.907652

Time Summary
GMRES 66.456174 seconds
GMRES-Preconditioner 26.862827 seconds
GMRES-One Time Preconditioner 0.000000 seconds
GMRES-Matrix Free 240.947143 seconds
GMRES-Matrix Free Preconditioner 362.827408 seconds
ode15s with sparsity 11.875045 seconds
ode15s base-case 0.000000 seconds

#### N=128

GMRES version of ode15s
GMRES finished, final time was 545.157086 with 17919 GMRES iterates
271 Successful Steps
78 Failed Attempts
980 Function Evaluations
47 Partial Derivatives
0 LU Decompositions
646 Solution of Linear Systems
Fail/Success Ratio of 0.223496
J-Evals per step of 0.134670
F-Evals per step of 2.808023
Linear Solves per step of 1.851003
GMRES iterates per step of 51.343840
GMRES iterates per solve of 27.738390
Proportion of failed GMRES solves was 0.006192

### Updated Preconditioner GMRES version of ode15s

Updated Preconditioner GMRES finished, final time was 256.593042 with 6820 GMRES iterates

222 Successful Steps

30 Failed Attempts

601 Function Evaluations

23 Partial Derivatives

0 LU Decompositions

435 Solution of Linear Systems

Fail/Success Ratio of 0.119048

J-Evals per step of 0.091270

F-Evals per step of 2.384921

Linear Solves per step of 1.726190

GMRES iterates per step of 27.063492

GMRES iterates per solve of 15.678161

Proportion of failed GMRES solves was 0.000000

### GMRES Matrix Free version of ode15s

GMRES Matrix Free finished, time was 1343.387580 with 57970 GMRES iterates

280 Successful Steps

51 Failed Attempts

7073 Function Evaluations

0 Partial Derivatives

0 LU Decompositions

527 Solution of Linear Systems

Fail/Success Ratio of 0.154079

J-Evals per step of 0.000000

F-Evals per step of 21.368580

Linear Solves per step of 1.592145

GMRES iterates per step of 175.135952

GMRES iterates per solve of 110.000000

Proportion of failed GMRES solves was 0.907021

## ode15s sparsity specified

ode15S with Sparsity finished, time was 40.389768

208 Successful Steps

25 Failed Attempts

541 Function Evaluations

20 Partial Derivatives

54 LU Decompositions

395 Solution of Linear Systems

Fail/Success Ratio of 0.107296

J-Evals per step of 0.085837

F-Evals per step of 2.321888

Linear Solves per step of 1.695279

## Time Summary

GMRES 545.157086 seconds

GMRES-Preconditioner 256.593042 seconds

GMRES-One Time Preconditioner 0.000000 seconds

GMRES-Matrix Free 1343.387580 seconds

GMRES-Matrix Free Preconditioner 0.000000 seconds

ode15s with sparsity 40.389768 seconds

### N=256

Updated Preconditioner GMRES version of ode15s
Updated Preconditioner GMRES finished, final time was 3069.300131 with 17238 GMRES iterates
297 Successful Steps
57 Failed Attempts
957 Function Evaluations
45 Partial Derivatives
0 LU Decompositions
642 Solution of Linear Systems
Fail/Success Ratio of 0.161017
J-Evals per step of 0.127119
F-Evals per step of 2.703390
Linear Solves per step of 1.813559
GMRES iterates per step of 48.694915
GMRES iterates per solve of 26.850467
Proportion of failed GMRES solves was 0.000000

ode15s sparsity specified
ode15S with Sparsity finished, time was 313.210936
254 Successful Steps
18 Failed Attempts
568 Function Evaluations
15 Partial Derivatives
53 LU Decompositions
462 Solution of Linear Systems
Fail/Success Ratio of 0.066176
J-Evals per step of 0.055147
F-Evals per step of 2.088235
Linear Solves per step of 1.698529

Time Summary
GMRES 0.000000 seconds
GMRES-Preconditioner 3069.300131 seconds
GMRES-One Time Preconditioner 0.000000 seconds
GMRES-Matrix Free 0.000000 seconds
GMRES-Matrix Free Preconditioner 0.000000 seconds
odel5s with sparsity 313.210936 seconds
odel5s base-case 0.000000 seconds

## N=512

Updated Preconditioner GMRES version of ode15s
Updated Preconditioner GMRES finished, final time was 15548.336204 with 22247 GMRES iterates
356 Successful Steps
53 Failed Attempts
1020 Function Evaluations
42 Partial Derivatives
0 LU Decompositions
726 Solution of Linear Systems
Fail/Success Ratio of 0.129584
J-Evals per step of 0.102689
F-Evals per step of 2.493888
Linear Solves per step of 1.775061
GMRES iterates per step of 54.393643
GMRES iterates per solve of 30.643251
Proportion of failed GMRES solves was 0.004132

ode15s sparsity specified ode15S with Sparsity finished, time was 3743.534588 331 Successful Steps 28 Failed Attempts 774 Function Evaluations 23 Partial Derivatives 69 LU Decompositions 612 Solution of Linear Systems Fail/Success Ratio of 0.077994 J-Evals per step of 0.064067 F-Evals per step of 2.155989 Linear Solves per step of 1.704735

Time Summary
GMRES 0.000000 seconds
GMRES-Preconditioner 15548.336204 seconds
GMRES-One Time Preconditioner 0.000000 seconds
GMRES-Matrix Free 0.000000 seconds
GMRES-Matrix Free Preconditioner 0.000000 seconds
ode15s with sparsity 3743.534588 seconds
ode15s base-case 0.000000 seconds