Bayesian Nonresponse Models for the Analysis of Data from Small Areas: An Application to BMD and Age in NHANES III

by

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Abstract

We analyze data on bone mineral density (BMD) and age for white females age 20+ in the third National Health and Nutrition Examination Survey. For the sample the age of each individual is known, but some individuals did not have their BMD measured, mainly because they did not show up in the mobile examination centers. We have data from 35 counties, the small areas.

We use two types of models to analyze the data. In the ignorable nonresponse model, BMD does not depend on whether an individual responds or not. In the nonignorable nonresponse model, BMD is related to whether he/she responds. We incorporate this relationship in our model by using a Bayesian approach. We further divide these two types of models into continuous and categorical data models. Our nonignorable nonresponse models have one important feature: They are "close" to the ignorable nonresponse model thereby reducing the effects of the untestable assumptions so common in nonresponse models. In the continuous data models, because the age of all nonrespondents are known and there is a relation between BMD and age, age is used as a covariate. In the categorical data models BMD has three levels (normal, osteopenia, osteoporosis) and age has two levels (younger than 50 years, at least 50 years). Thus, age is a supplemental margin for the 2×3 categorical table. Our research on the categorical models is much deeper than on the continuous models.

Our models are hierarchical, a feature that allows a "borrowing of strength" across the counties. Individual inference for most of the counties is unreliable because there is large variation. This "borrowing of strength" is therefore necessary because it permits a substantial reduction in variation.

The joint posterior density of the parameters for each model is complex. Thus, we fit each model using Markov chain Monte Carlo methods to obtain samples from the posterior density. These samples are used to make inference about BMD and age, and the relation between BMD and age.

For the continuous data models, we show that there is an important relation between BMD and age by using a deviance measure, and we show that the nonignorable nonresponse models are to be preferred. For the categorical data models, we are able to estimate the proportion of individuals in each BMD and age cell of the categorical table, and we can assess the relation between BMD and age using the Bayes factor. A sensitivity analysis shows that there are differences, typically small, in inference that permits different levels of association between BMD and age. A simulation study shows that there is not much difference in inference between the ignorable nonresponse models and the nonignorable nonresponse models.

As expected, BMD depends on age and this inference can be obtained for some small counties. For the data we use, there are virtually no young individuals with osteoporosis. The nonignorable nonresponse models generalize the ignorable nonresponse models, and therefore, allow broader inference.

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Chapter 1

Introduction

The problem of missing data appearing as nonresponse is common in many surveys. Generally, there are two types of nonresponse: unit nonresponse and item nonresponse. Unit nonresponse means all variables are missing in some samples and item nonresponse means that a subset of variables is missing. Both unit nonresponse and item nonresponse can create bias in the sample estimates. If the nonrespondents significantly differ from respondents, the sample estimates may not represent the actual population value accurately and any inference made about the population is misleading. Even if the nonrespondents are the same as respondents, we still need to explore useful information about the population from missing data if there is item nonresponse. As a result, missing data requires serious attention in the analysis of survey data. Generally, high nonresponse rate can reflect serious bias.

We study bone mineral density (BMD) in the third National Health and Nutrition Examination Survey (NHANES III). About 31 percent of data on bone mineral density (BMD) is missing, and in some small counties, over 50 percent data are missing. Adjustments need to be made for missing data in the study of BMD.

In this thesis, we focus our study on the association between BMD and age accounting for missing data.

In this chapter, we provide a background on missing data for NHANES III.

1.1 Background

1.1.1 Definition and types of missing data

Missing data can be described as the incomplete data items in a dataset. Little and Rubin (1987) defined different types of missing data. A simple description in a summary form is given by Chantala et al. (*http://www.cpc.unc.edu/services/computer/presentations/mi_presentation2.pdf*) as follows:

- Missing Completely at Random (MCAR): the probability of missing data on a variable is unrelated to the variable itself or the other variables in the data set.

- Missing at Random (MAR): the nonresponse is related to the observed variable, but not to the missing variable.

- Missing not at Random (MNAR): the nonresponse is related to the observed variable, also to the missing variable.

1.1.2 General methods to deal with missing data

There are many methods to handle missing data problem:

- Data Deletion: Deleting the missing data directly.

- Mean imputation: Replacing the missing data by the mean value computed from the observed data.

- Regression imputation (Little and Rubin, 1987): Replacing the missing values by the predicted values from a regression of the missing item at times observed for the unit.

- Hot deck imputation: Substituting the missing values with the corresponding values drawn from similar observed units in the data set. For example, there are some nonresponse items in a survey, by hot deck imputation, the nonresponses are replaced by the responses from other individuals with similar demographic and economic backgrounds.

- Expectation Maximization (EM) approach (Dempster, Laird and Rubin, 1977): It is based on the idea of replacing a likelihood function which is difficult to maximize by a likelihood function with a sequence of easier maximizations whose limit is the answer to the original problem.

- Stochastic Regression Imputation (Little and Rubin, 1987): Replacing the missing values by a value predicted by the regression imputation plus a residual, drawn to reflect the uncertainty in the predicted value.

- Multiple Imputation (Lynch, 2002): Creating the multiple imputed datasets in which each dataset has different imputed values for the missing data. Then analyze each of the datasets and examine the parameter distributions in a bootstrap method.

It is important to note that all methods mentioned above generally assume data are MCAR or MAR.

1.1.3 Bayes method

The Bayesian method is a powerful tool to handle MCAR, MAR and MNAR data because it can build a sensible relationship between observed data and missing data.

Little and Rubin (1987) describes two types of Bayesian models:

- Ignorable nonresponse model: the observed data and missing data are assumed to have the same distributions.
- Nonignorable nonresponse model: the observed data and missing data are assumed to have different distributions.

Generally, MCAR and MAR missing data are treated by ignorable nonresponse model and MNAR data is treated by nonignorable nonresponse model. Cohen and Duffy (2002) point out that missing data in health surveys should be considered using nonignorable nonresponse models.

There are two methods to set up nonignorable nonresponse model: the selection approach and the pattern mixture approach. In the selection approach, the hypothetical complete data are modeled and a model for the nonresponse mechanism is added conditional on the hypothetical data. In the pattern mixture approach, the population is stratified into two patterns, respondents and nonrespondents, each being modeled separately and the final answer is obtained by a probabilistic mixture of these two.

For model selection, Bayes factor is a very useful tool (Kass and Raftery 1995). Suppose we fit two models, M_0 and M_1, to data $\underset{\sim}{y}$ under H_0 and H_1 hypothesis, the Bayes factor for comparing models M_1 and M_0 is defined as the ratio of the marginal densities of the data $\underset{\sim}{y}$ under the two hypothesis:

$$B_{10} = \frac{p(\underset{\sim}{y}|M_1)}{p(\underset{\sim}{y}|M_0)}$$

$p(\underset{\sim}{y}|\mathrm{M}_k)$ (k=0,1) are obtained by

$$p(\underset{\sim}{y}|\mathrm{M}_k) = \int p(\underset{\sim}{y}|\underset{\sim}{\theta}_k, \mathrm{M}_k) p(\underset{\sim}{\theta}_k|\mathrm{M}_k) d\underset{\sim}{\theta}_k$$

where $\underset{\sim}{\theta}_k$ is the parameter vector under H_k, $p(\underset{\sim}{y}|\underset{\sim}{\theta}_k, \mathrm{M}_k)$ is the likelihood function and $p(\underset{\sim}{\theta}_k|\mathrm{M}_k)$ is the prior density.

The Bayes factor summarizes the evidence provided by the data in favor of one scientific hypothesis. Kass and Raftery (1995) gave a comprehensive description of Bayes factors including their interpretation (Table 1.1).

We use the Bayes factor to test for association between BMD and age, and test whether a nonignorable nonresponse model holds with or without association.

$\log_e(B_{10})$	B_{10}	Evidence against H_0
0 to 1	1 to 3	Not worth more than a bare mention
1 to 3	3 to 20	Positive
3 to 5	20 to 150	Strong
>5	>150	Very strong

Table 1.1: Strength of evidence against H_0 using B_{10} = posterior odds/prior odds where odds = $P(H_1)/(1-P(H_1))$ (Kass and Raftery, 1995)

1.2 Description of Problem

Nandram, Han and Choi (2002) present a Bayesian nonignorable nonresponse model for small areas by studying body mass index (BMI) in the NHANES III survey. In their study, they propose a model for each of the eight age-race-sex cells by counties. More nonresponse arise from the young people for BMI variable. For the younger group, a nonignorable nonresponse model is used and for the older group, an ignorable nonresponse model is used, and then two groups are combined by using logistic regression. In our study we model the two groups simultaneously. Let x_{ijk} be the characteristic variable and y_{ijk} be the response variable where:

$$x_{ik} = \begin{cases} 1, & \text{if } k^{th} \text{ individual in } i^{th} \text{ county belongs to one of } j^{th} \text{ BMI level} \\ 0, & \text{if } k^{th} \text{ individual in } i^{th} \text{ county does not belong to one of } j^{th} \text{ BMI level} \end{cases}$$

and

$$y_{ijk} = \begin{cases} 1, & \text{if } k^{th} \text{ individual belonging to } j^{th} \text{ BMI in } i^{th} \text{ county responds} \\ 0, & \text{if } k^{th} \text{ individual belonging to } j^{th} \text{ BMI in } i^{th} \text{ county does not respond} \end{cases}$$

$$i = 1, \ldots, N, j = 1, \ldots, c, k = 1, \ldots, n_i.$$

Note N=34 counties and c=3 in their study.

For the igorable model, let p_{ij} be the probability that an individual in the i^{th} county belongs the j^{th} BMI level and π_i be the probability that an individual in the i^{th} county responds, $\underset{\sim}{\mu}_1 = (\mu_{11}, \mu_{12}, \ldots, \mu_{1c})'$, then,

$$\begin{aligned} \underset{\sim}{x}_{ik}|\underset{\sim}{p}_i &\overset{iid}{\sim} \text{Multinomial}(1, \underset{\sim}{p}_i), & y_{ijk}|\pi_i &\overset{iid}{\sim} \text{Bernoulli}(\pi_i), \\ \underset{\sim}{p}_i|\underset{\sim}{\mu}_1, \tau_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}_1\tau_1), & \pi_i|\mu_{21}, \tau_{21} &\overset{iid}{\sim} \text{Beta}(\mu_{21}\tau_{21}, (1-\mu_{21})\tau_{21}), \\ \underset{\sim}{\mu}_1 &\sim \text{Dirichlet}(\underset{\sim}{1}), & \mu_{21} &\sim \text{Uniform}(0, 1), \\ \tau_1 &\sim \text{Gamma}(\eta_1^{(0)}, \nu_1^{(0)}), & \tau_{21} &\sim \text{Gamma}(\eta_{21}^{(0)}, \nu_{21}^{(0)}). \end{aligned}$$

where

$$p(\underset{\sim}{p}_i|\underset{\sim}{\mu}_1, \tau_1) = \prod_{j=1}^{c} p_{ij}^{\mu_{1j}\tau_1 - 1}/D(\underset{\sim}{\mu}_1\tau_1),\ 0 < p_{ij} < 1,\ \sum_{j=1}^{c} p_{ij} = 1,$$
$$D(\underset{\sim}{\mu}_1\tau_1) = \prod_{j=1}^{c} \Gamma(\mu_{1j}\tau_1)/\Gamma(\tau_1),\ 0 < \mu_{1j} < 1,\ \sum_{j=1}^{c} \mu_{1j} = 1.$$

For the nonigorable model, let p_{ij} be the probability that an individual in the i^{th} county belonging to the j^{th} BMI level and π_{ij} be the probability that an individual at the j^{th} BMI level in the i^{th} county responds, then,

$$\underset{\sim}{x}_{ik}|\underset{\sim}{p}_i \overset{iid}{\sim} \text{Multinomial}(1, \underset{\sim}{p}_i),\ y_{ijk}|\{x_{ik} = (x_{i1l}, \ldots, x_{ick}),\ \pi_{ij}\} \overset{iid}{\sim} \text{Bernoulli}(\pi_{ij}),$$
$$x_{ijk} = 1, x_{ij'k} = 0, j \neq j' \text{ for } j, j' = 1, \ldots, c,$$
$$\underset{\sim}{p}_i|\underset{\sim}{\mu}_3, \tau_3 \overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}_3\tau_3),\ \underset{\sim}{\mu}_3 = (\mu_{31}, \mu_{32}, \ldots, \mu_{3c})',$$
$$\pi_{ij}|\mu_{4j}, \tau_{4j} \overset{iid}{\sim} \text{Beta}(\mu_{4j}\tau_{4j},\ (1 - \mu_{4j})\tau_{4j}),\ j = 1, \ldots, c,$$
$$\underset{\sim}{\mu}_3 \sim \text{Dirichlet}(\underset{\sim}{1}),\ \mu_{4j} \sim \text{Uniform}(0, 1),$$
$$\tau_3 \sim \text{Gamma}(\eta_3^{(0)}, \nu_3^{(0)}),\ \tau_{4j} \sim \text{Gamma}(\eta_{4j}^{(0)},\ \nu_{4j}^{(0)}),\ j = 1, \ldots, c.$$

In their work, they assumed the hyper-parameters $\nu_3^{(0)}$, τ_{4j}, $\eta_{4j}^{(0)}$ and $\nu_{4j}^{(0)}$ are known and then considered multinomial data obtained independently from several geographical areas.

In our thesis, we extend their work into two directions. We consider a general r × c table, but Nandram, Han and Choi (2002) has r=1. We also attack the problem of association for a general r × c table with missing data, and the issue of whether there is ignorability or not. We use the Bayes factor to access the association between BMD and age and to check for the ignorability.

1.3 Description of the NHANES III Data

NHANES III (1988-1994) is one of the surveys conducted by the National Center for Health Statistics to estimate a health status of the U.S. population.

NHANES III is based on a complete multistage probability sample design (Vital and Health Statistics, Series 2, Number 113, 1992). Data are collected through three stages: The first stage is the sample selection, in which samples were selected from households in 81 counties, but the final study comes from only the 35 largest counties; the second stage is the interview of sampled individuals for their medical history and socio-demographic information; the third stage is the medical examination of those sampled. Sampled individuals were asked to come to a mobile examination center (MEC) or visited by the examiner and data on the individual physical measurements, physical tests and biochemical measurements from blood and urine specimens were collected.

NHANES III generates the missing data by experiencing unit and item nonresponse (National Center for Health Statistics, 1996). The unit nonresponse generally occurs due to the sampled individuals screened, refused or absent from the interview. The item nonresponse occurs due to refusal or unwillingness to respond to specific questions or items. Therefore, such missing data might be nonignorable and requires special attention.

In our thesis, we study the association between BMD and age for white females, at least 20 years old. For obvious reasons, bone mineral density is not measured for persons younger than 20 years.

1.3.1 Description of BMD

BMD helps to estimate the risk of bone fracture. Generally, low BMD leads to osteoporosis which more frequently happens to old people who loose the bone mass progressively (Figure 1.1) and women who pass menopause (Figure 1.2).

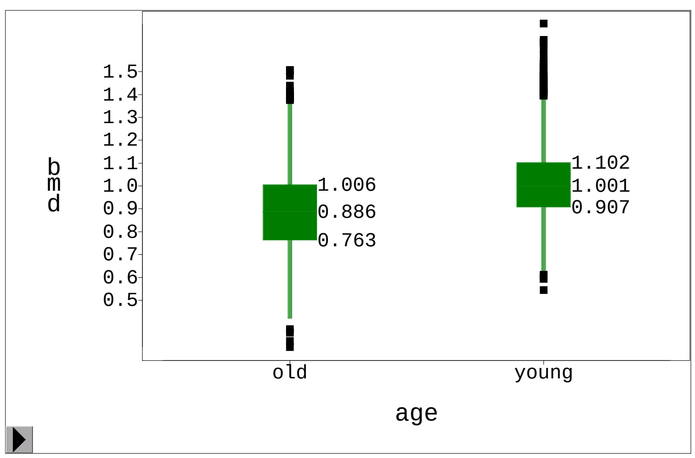


Figure 1.1: Box plot for observed BMD vs. age

BMD is tested by dual energy X-ray absorptiometry method (DXA) and the DXA systems employ two X-ray beams with different levels of energy. After subtraction of soft-tissue absorption, the absorption of each beam by the bone is used to calculate the BMD value. Because of its ability to accurately eliminate the soft-tissue absorption factor, DXA can measure BMD at central skeletal sites: the lumbar spine and the

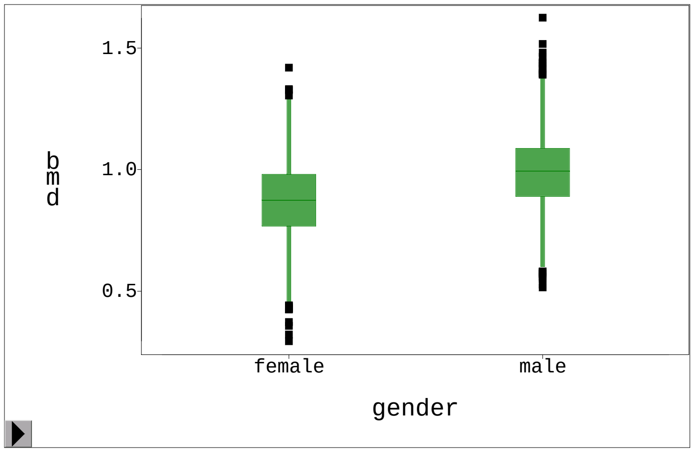


Figure 1.2: Box plot for observed BMD vs. gender

hip (including the femoral neck, Ward's triangle, and greater trochanter). DXA measurements at peripheral body sites (pDXA) can be performed using small, portable devices, in the physicians' offices or in mobile centers, with little or no discomfort or inconvenience to the patient. DXA is the most widely used BMD measurement technology with high accuracy (Merck).

The World Health Organization (WHO) has defined three levels for BMD:

a. Normal: BMD less than 1 standard deviation (SD) below the young Non-Hispanic white (NHW) adult mean.

b. Osteopenia: BMD from 1 to 2.5 SD below the young NHW adult mean.

c. Osteoporosis: BMD more than 2.5 SD below the young NHW adult mean.

Looker et al. (1997) defined the cutoff values of BMD in NHANES III based on the WHO criteria.

Level 1: Normal (BMD greater than 0.82);

Level 2: Osteopenia (BMD between 0.64 and 0.82);

Level 3: Osteoporosis (BMD less than 0.64).

Figure 1.3 shows BMD values in NHANES III are roughly normally distributed.

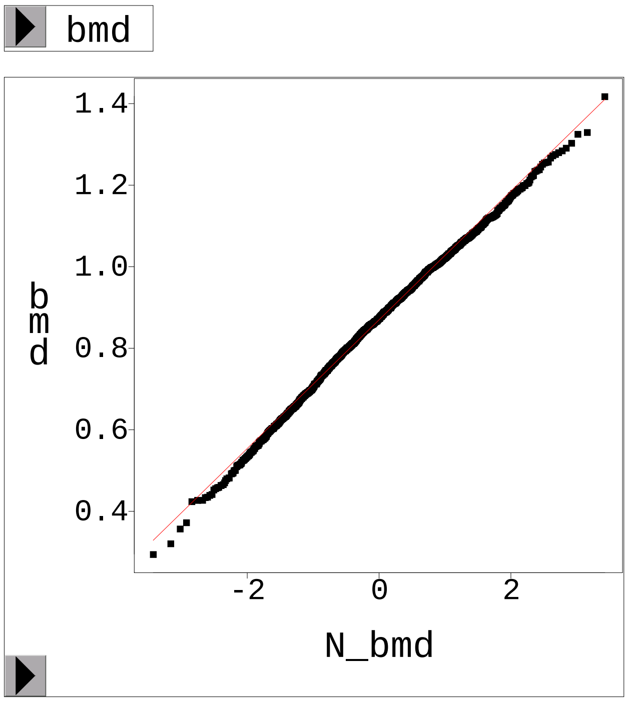


Figure 1.3: Normal quantile plot for the observed BMD data

1.3.2 Description of Age

We classify white females into two age groups (NHANES flyer, 2002):

a. Younger than 50 years (50-);

b. At least 50 years old (50+).

Age is not normally distributed and the Box-Cox transformation does not help. In fact age is really a discrete variable ranging from 21 to 90 in the data set.

1.3.3 Initial Exploration of the Relation Between BMD and Age

For each county, there is a downward trend of BMD versus age with some noise (see Figure 1.4). Table 1.2 shows the results obtained by fitting BMD on age by county for observed data. Low R^2 indicates that regression methods may not be suitable. Figure 1.5 shows the corresponding residual plot and Q-Q plot of BMD vs. age which are normally distributed.

To explore their association further, we use the Pearson Chi-square test to check their dependence for each county. Basically, the Pearson Chi-square test for two-way tables involves the difference between the observed and expected frequencies, where the expected frequencies are computed under the null hypothesis of independence. For a r $\times$ c table, the Chi-square statistic is

$$X^2 = \sum_j \sum_k \frac{(n_{jk} - e_{jk})^2}{e_{jk}}, \qquad e_{jk} = \frac{n_{j.} n_{.k}}{n},$$

where

$$n_{j.} = \sum_{k=1}^{c} n_{jk}, n_{.k} = \sum_{j=1}^{r} n_{jk} \quad \text{and} \quad n = \sum_{j=1}^{r} \sum_{k=1}^{c} n_{jk}.$$

Under the null hypothesis that the row and column variables are independent, X^2 has an asymptotic (as n $\rightarrow \infty$) Chi-square distribution with (r-1)(c-1) degrees of freedom. For large values of X^2, this test rejects the null hypothesis in favor of the alternative hypothesis of an association.

In our preliminary study, we use the Pearson Chi-square test in two different ways: only using observed data, and using both observed and missing data. When both the observed data and the missing data are used, we compute the missing cell counts by conditional probability (Wang, 2001 PhD thesis).

The conditional probability method estimates the missing cell counts by

$$\hat{n}_{ijk}^{(m)} = \frac{n_{ijk}^{(c)}}{n_{ij.}^{(c)}} \times n_{ij.}^{(m)}, \tag{1.1}$$

where, $n_{ijk}^{(c)}$, $n_{ij.}^{c}$ and $n_{i..}^{(c)}$ refer to the counts in the complete data and $n_{ijk}^{(m)}$, $n_{ij.}^{(m)}$ and $n_{i..}^{(m)}$ refer to the counts in the missing data, i=1, 2,. . ., N, j=1,. . .,r and k=1,2,. . ., c. N is the number of counties (N=35), r is the number of age levels (r=2) and c is the number of BMD levels (c=3). Here,

$$n_{ij.}^{(c)} = \sum_{k=1}^{c} n_{ijk}^{(c)}, n_{ij.}^{(m)} = \sum_{k=1}^{c} n_{ijk}^{(m)}, \quad n_{i.k}^{(c)} = \textstyle\sum_{j=1}^{r} n_{ijk}^{(c)} \quad \text{and } n_{i..}^{(c)} = \sum_{j=1}^{r} \sum_{k=1}^{c} n_{ijk}^{(c)}.$$

Table 1.3 shows BMD vs. age cell counts in the observed data and age cell counts in the missing data. We notice that the counts from some counties are very small, even in county 6037, there are small counts for osteoporosis, thus, small area estimation techniques are required.

Table 1.3 also shows the p_{values} of the Chi-square test. We get consistent results from no imputation method and conditional probability methods.

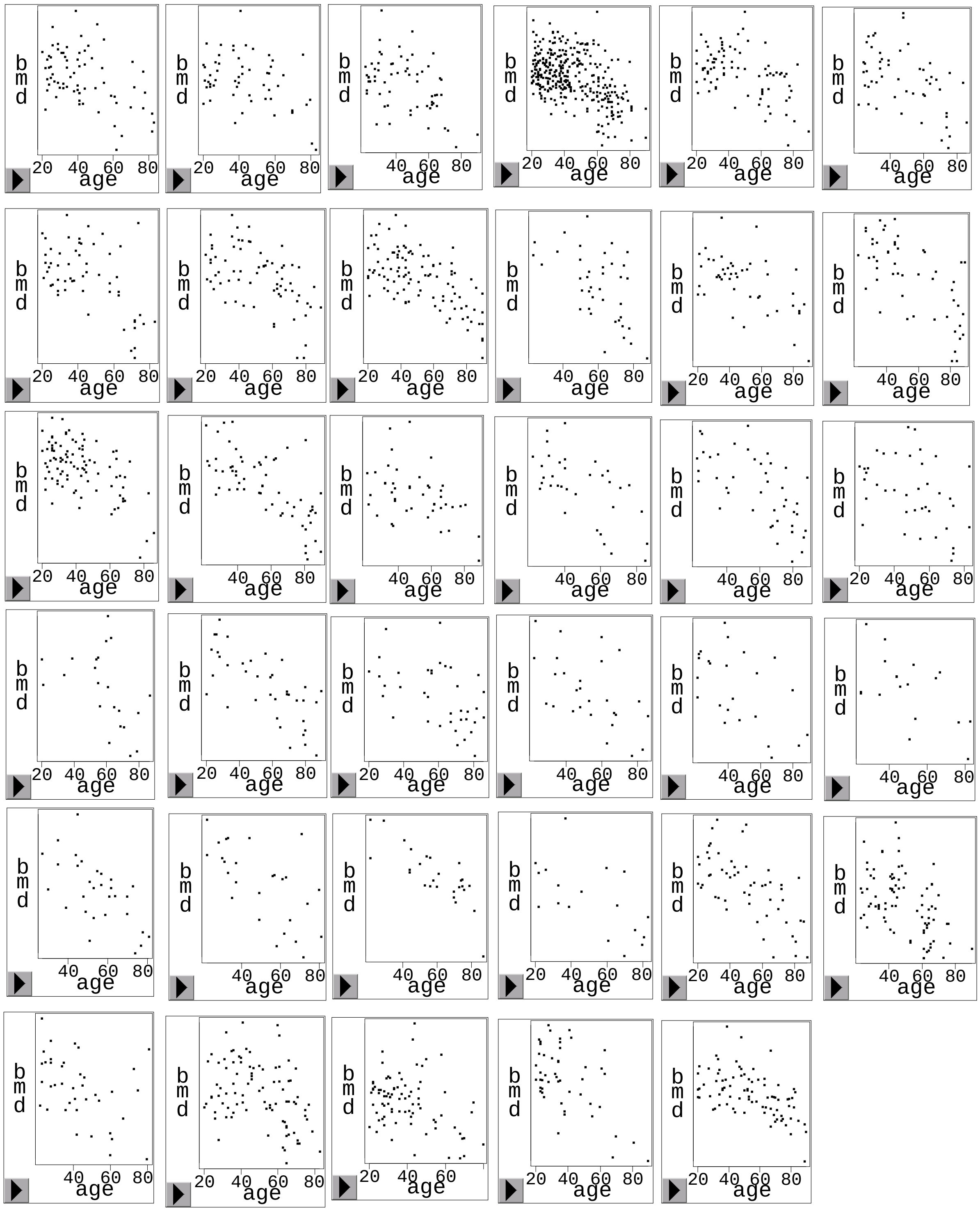


Figure 1.4: Scatter plots for BMD vs. age by county (from left to right and top to bottom is county 1, 2,...,35)

County	β_0	95% CI	$\beta_1 \times 10^{-2}$	95% CI (10^{-2})	R^2
4013	1.06	(0.98,1.15)	-0.42	(-0.61,-0.23)	0.22
6001	1.03	(0.93,1.13)	-0.33	(-0.54,-0.13)	0.16
6019	1.07	(0.98,1.16)	-0.37	(-0.56,-0.18)	0.20
6037	1.09	(1.05,1.13)	-0.42	(-0.51,-0.34)	0.23
6059	1.04	(0.96,1.11)	-0.37	(-0.51,-0.22)	0.24
6071	1.04	(0.94,1.14)	-0.38	(-0.57,-0.19)	0.24
6073	1.04	(0.96,1.12)	-0.39	(-0.56,-0.22)	0.28
6085	1.07	(0.98,1.15)	-0.42	(-0.58,-0.26)	0.29
6111	1.16	(1.08,1.22)	-0.55	(-0.68,-0.42)	0.41
12025	1.15	(0.97,1.33)	-0.58	(-0.87,-0.28)	0.30
12031	1.08	(0.96,1.19)	-0.49	(-0.71,-0.26)	0.31
12099	1.11	(1.02,1.21)	-0.54	(-0.70,-0.37)	0.47
17031	1.12	(1.05,1.19)	-0.48	(-0.62,-0.33)	0.32
25017	1.13	(1.04,1.21)	-0.57	(-0.72,-0.42)	0.49
26125	1.03	(0.92,1.14)	-0.35	(-0.56,-0.14)	0.21
26163	1.23	(1.09,1.36)	-0.59	(-0.84,-0.33)	0.40
29189	1.14	(1.02,1.26)	-0.54	(-0.73,-0.35)	0.43
36029	1.02	(0.91,1.13)	-0.29	(-0.50,-0.07)	0.17
36047	1.10	(0.86,1.34)	-0.45	(-0.85,-0.06)	0.22
36059	1.04	(0.95,1.13)	-0.47	(-0.63,-0.32)	0.51
36061	1.97	(0.85,1.09)	-0.32	(-0.52,-0.13)	0.25
36081	1.05	(0.90,1.19)	-0.41	(-0.66,-0.16)	0.31
36119	1.06	(0.92,1.20)	-0.41	(-0.69,-0.13)	0.30
39035	1.02	(0.86,1.18)	-0.40	(-0.70,-0.10)	0.34
39061	1.24	(1.07,1.42)	-0.65	(-0.96,-0.34)	0.40
42003	1.20	(1.03,1.36)	-0.63	(-0.94,-0.32)	0.41
42045	1.22	(1.08,1.36)	-0.74	(-0.96,-0.21)	0.64
42101	1.01	(0.85,1.17)	-0.41	(-0.70,-0.12)	0.36
44007	1.09	(1.00,1.17)	-0.48	(-0.63,-0.32)	0.42
48029	1.09	(1.00,1.88)	-0.44	(-0.62,-0.27)	0.24
48113	1.06	(0.91,1.21)	-0.47	(-0.79,-0.14)	0.19
48141	1.04	(0.95,1.12)	-0.32	(-0.48,-0.15)	0.14
48201	1.10	(1.01,1.19)	-0.37	(-0.59,-0.16)	0.13
48439	1.12	(1.03,1.22)	-0.50	(-0.72,-0.28)	0.31
53033	1.05	(0.97,1.13)	-0.40	(-0.54,-0.26)	0.28
Overall	1.10	(1.08,1.11)	-0.46	(-0.49,-0.43)	0.31

Table 1.2: Estimates with 95 % confidence interval for β_0 and β_1 in regression model: BMD = β_0 + β_1 age by county

	young			old			Missing		Chi−Square Tests	
County	1	2	3	1	2	3	young	old	I	II
4013	40	10	0	8	8	5	15	8	0.0001	<.0001
6001	29	6	1	10	8	2	10	8	0.0557	0.0189
6019	32	6	0	13	10	2	24	5	0.0128	0.0032
6037	189	22	0	47	54	13	100	51	<.0001	<.0001
6059	40	7	0	16	12	6	15	11	0.0003	<.0001
6071	24	5	0	10	9	4	31	19	0.0056	<.0001
6073	32	5	0	7	10	3	8	3	0.0002	<.0001
6085	28	6	0	15	16	6	9	11	0.0008	0.0001
6111	44	9	0	19	18	11	12	17	<.0001	<.0001
12025	5	0	0	15	12	7	2	11	0.0656	0.0236
12031	21	4	2	6	8	4	4	12	0.0117	0.0024
12099	23	3	0	7	9	9	8	9	<.0001	<.0001
17031	59	7	0	11	11	3	28	20	<.0001	<.0001
25017	20	5	0	9	19	8	11	17	<.0001	<.0001
26125	17	6	0	8	11	2	9	7	0.0363	0.0073
26163	18	1	0	8	5	2	1	10	0.0169	0.0096
29189	12	2	0	11	9	10	3	18	0.0067	0.0011
36029	15	3	0	10	10	1	7	14	0.0617	0.0125
36047	4	0	0	8	7	3	0	10	0.1304	0.1017
36059	12	2	0	5	12	7	6	14	0.0004	<.0001
36061	6	3	0	8	15	5	2	5	0.0910	0.0625
36081	9	4	0	3	8	3	8	26	0.0259	0.0004
36119	11	4	0	4	2	3	7	8	0.0551	0.0126
39035	6	3	0	4	3	2	3	8	0.3012	0.1889
39061	9	2	0	11	6	2	2	8	0.3288	0.1883
42003	12	2	0	6	3	4	11	24	0.0457	0.0033
42045	6	1	0	4	13	3	2	27	0.0080	0.0002
42101	6	3	0	2	3	4	1	9	0.0498	0.0166
44007	23	4	0	11	10	6	9	27	0.0017	<.0001
48029	41	6	0	12	21	3	26	38	<.0001	<.0001
48113	19	7	1	5	2	5	4	6	0.0010	0.0024
48141	35	9	0	24	18	4	15	4	0.0110	0.0034
48201	59	5	0	8	7	3	24	4	<.0001	<.0001
48439	34	4	0	4	4	3	8	8	0.0002	<.0001
53033	26	10	0	15	28	6	9	12	0.000	0.000
Total	966	176	4	364	401	164	434	489	<.0001	<.0001

Table 1.3: Counts for BMD by age for each of 35 counties (observed and missing) I and II are the p_{value} for the Chi-square tests for no imputation and imputation by conditional probability method respectively

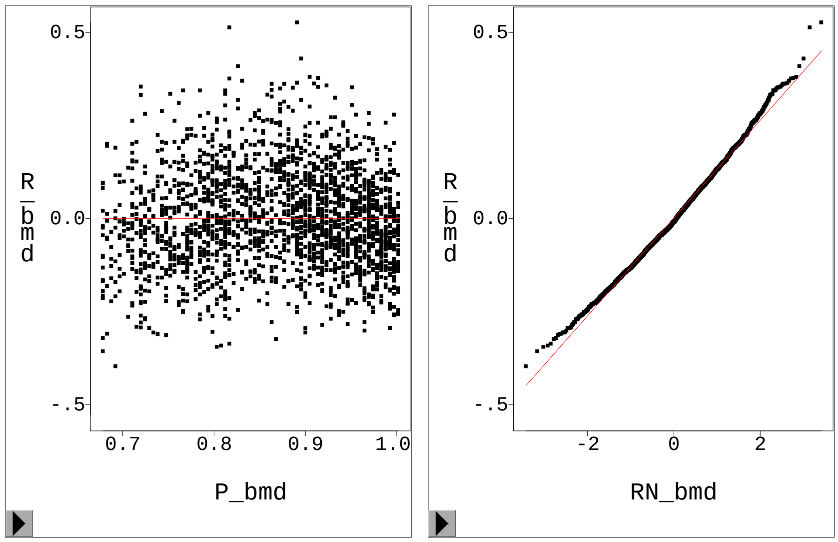


Figure 1.5: Residual plot and normal quantile plot for a simple regression of observed BMD on age

1.4 Thesis Overview

In this thesis, we introduce a Bayesian method to analyze data from small area when there are both unit nonresponse and item nonresponse. We illustrate our method by studying the data of BMD and age from NHANES III.

In Chapter 2, we treat BMD as a continuous variable and age as a covariate. We use the nested error regression model which is extended to accommodate nonresponse, and then we use Markov Chain Monte Carlo methods to fit an ignorable and a nonignorable nonresponse model.

However, because we would like to find out the percent of individuals in different BMD by age cells, we focus more on categorical models.

In Chapter 3, we construct categorical models for BMD and age accounting for missing data, and then we use Markov Chain Monte Carlo methods to fit the models. Then, we assess the association between BMD and age and ignorability of models by using Bayes factor.

In Chapter 4, we perform a sensitivity study to see how different are of our models by nonignorability and different degrees of association. We also conduct a simulation study to access how different are of the ignorable and the nonignorable nonresponse models. Finally, we make conclusions in this chapter.

Chapter 2

Continuous Models

In this chapter, we treat BMD as a continuous variable and age as a covariate variable. We use the nested error regression model (Prasad and Rao 1990, Battese, Harter and Fuller, 1988) which is extended to accommodate nonresponse.

The purpose of using the nested error regression model is to avoid the large standard errors produced by the estimators from small areas, and the model can give us alternative estimators where we allow a "borrowing of strength" from other related areas. The model is given by

$$\begin{aligned} y_{ij} &= \underset{\sim}{\mathrm{x}}'_{ij}\underset{\sim}{\beta} + \nu_i + e_{ij} \\ & \quad i = 1, \ldots, N, j = 1, \ldots, n_i, \end{aligned}$$

where y_{ij} is the character of interest for the j^{th} sample unit in the i^{th} area, $\underset{\sim}{x}_{ij} = (x_{ij1} \ldots x_{ijk})'$ is a k vector of corresponding auxiliary values, $\underset{\sim}{\beta} = (\beta_1 \ldots \beta_k)$ is a k vector of unknown parameters, and n_i is the number of sampling units observed in the i^{th} small area ($\sum_{i=1}^{N} n_i = n$). The random effects ν_i are assumed to be independent N(0, σ_ν^2) and independent of the e_{ij}, which are assumed to be independent N(0, σ_e^2).

2.1 The Nested Error Nonresponse Regression Models

2.1.1 Ignorable Nonresponse Models

Let y_{ij} be the BMD of j^{th} individual in the i^{th} county, a_{ij} is the corresponding age, i=1,...,N (N=35), j=1,...,n_i, n_i is the number of individuals in the sample in the i^{th} county.

We define the ignorable nonresponse model with covariate as

$$\begin{aligned} y_{ij} &= \beta_0 + \beta_1 a_{ij} + \nu_i + e_{ij},\ e_{ij} \overset{iid}{\sim} N(0,\sigma^2), \quad i=1,...,N, j=1,...,n_i, && (2.1)\\ &\nu_i \overset{iid}{\sim} N(0,\delta^2), \quad i=1,...,N, j=1,...,n_i, && (2.2)\\ &\delta^{-2} \sim \text{Gamma}(\frac{a}{2},\frac{b}{2}),\ \sigma^{-2} \sim \text{Gamma}(\frac{a}{2},\frac{b}{2}), && (2.3)\\ &p(\beta_0,\beta_1)=1,\ a,b=0.002. && (2.4) \end{aligned}$$

To check whether the covariate have substantial information, we define the ignorable nonresponse model without covariate as

$$\begin{aligned} y_{ij} = \beta_0 + \nu_i + e_{ij},\ e_{ij} &\overset{iid}{\sim} N(0,\sigma^2), \quad i=1,...,N, j=1,...,n_i, && (2.5)\\ \nu_i &\overset{iid}{\sim} N(0,\delta^2), && (2.6)\\ \delta^{-2} &\sim \text{Gamma}(\frac{a}{2},\frac{b}{2}),\ \sigma^{-2} \sim \text{Gamma}(\frac{a}{2},\frac{b}{2}), && (2.7)\\ p(\beta_0) &= 1,\ a,b=0.002. && (2.8) \end{aligned}$$

2.1.2 Nonignorable Nonresponse Model

We use the pattern mixture approach to construct the nonignorable nonresponse model.

Let r_{ij} (i=1, 2...,N and j=1, 2...,n_i) be the response indicator where

$$r_{ij} = \begin{cases} 1, & j^{th} \text{ individual in } i^{th} \text{ county responds;} \\ 0, & j^{th} \text{ individual in } i^{th} \text{ county does not respond.} \end{cases} \tag{2.9}$$

It is standard to assume that

$$r_{ij}|p_{ij} \overset{iid}{\sim} \text{Bernoulli}(p_{ij}) \tag{2.10}$$

and to link the response mechanism to age, we assume that

$$\log(\frac{p_{ij}}{1-p_{ij}}) = \alpha_0 + \alpha_1 a_{ij} + \eta_i \tag{2.11}$$

where a_{ij} is age of the j^{th} individual in the i^{th} county. Thus, we take

$$\begin{aligned} r_{ij}|\alpha_0, \alpha_1, \eta_i &\overset{ind}{\sim} \text{Bernoulli}\left\{\frac{e^{\alpha_0+\alpha_1 a_{ij}+\eta_i}}{1+e^{\alpha_0+\alpha_1 a_{ij}+\eta_i}}\right\}, & (2.12) \\ \eta_i &\overset{iid}{\sim} N(0, \sigma_\eta^2),\ \sigma_\eta^{-2} \sim \text{Gamma}(\frac{a}{2}, \frac{b}{2}),\ p(\underset{\sim}{\alpha}) = 1. & (2.13) \end{aligned}$$

For the BMD y_{ij}, we take

$$y_{ij} = \beta_0 + \nu_{0i} + (\beta_1 + \nu_{1i} r_{ij}) a_{ij} + e_{ij},\ e_{ij}|\sigma_e^2 \overset{iid}{\sim} N(0, \sigma_e^2), \qquad (2.14)$$

where

$$\begin{aligned} \nu_{0i}|\sigma_0^2 &\overset{iid}{\sim} N(0, \sigma_0^2),\ \ \nu_{1i}|\sigma_1^2 \overset{iid}{\sim} N(0, \sigma_1^2), & (2.15) \\ {\sigma_0}^{-2}, {\sigma_1}^{-2}, {\sigma_e}^{-2} &\sim \text{Gamma}(\frac{a}{2}, \frac{b}{2}), & (2.16) \\ p(\underset{\sim}{\beta}) &= 1,\ a, b = 0.002. & (2.17) \end{aligned}$$

Note that if $\nu_{1i} \equiv 0$, we get the ignorable nonresponse model and in this way, we have "centered" the nonignorable nonresponse model on the ignorable nonresponse one. This is a useful modeling strategy because it ties down the nonignorable nonresponse model close to the ignorable nonresponse model. This makes it possible to reduce the effect of untestable assumption associated with the nonrespondents.

2.2 Model Fitting

The joint posterior density of the parameters is complex, thus to make inference, we use the Gibbs sampler, an iterative simulation scheme for generating samples that converge to draws from a target distribution.

2.2.1 Ignorable Nonresponse Models

For the ignorable nonresponse model with covariate, by using Bayes' theorem, the joint posterior density of all the parameters is:

$$\begin{aligned} p(\underset{\sim}{\beta}, \underset{\sim}{\nu}, \underset{\sim}{\sigma}^2, \underset{\sim}{\delta}^2|\underset{\sim}{y}) \propto & \prod_{i=1}^{N}\prod_{j=1}^{n_i} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2\sigma^2}(y_{ij} - \beta_0 - \beta_1 a_{ij} - \nu_i)^2\right\} \\ & \times \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\delta^2}} exp\left\{-\frac{{\nu_i}^2}{2\delta^2}\right\}(\frac{1}{\sigma^2})^{\frac{a}{2}+1} exp\left\{-\frac{b}{2\sigma^2}\right\} \\ & \times (\frac{1}{\delta^2})^{\frac{a}{2}+1} exp\left\{-\frac{b}{2\delta^2}\right\}. \qquad (2.18) \end{aligned}$$

To use the Gibbs, we need the conditional posterior density of each parameter given the others.

Letting $\underset{\sim}{\beta} = (\beta_0, \beta_1)'$, $\underset{\sim}{d}_{ij} = (1, a_{ij})'$, $i = 1, ..., N$, $j = 1, ..., n_i$

$$\begin{aligned}
\underset{\sim}{\beta} | \underset{\sim}{\nu}, \underset{\sim}{\sigma}^2, \underset{\sim}{\delta}^2, \underset{\sim}{y} &\overset{ind}{\sim} \text{Normal}\Big\{ (\sum_{i=1}^{N}\sum_{j=1}^{n_i} \underset{\sim}{d}_{ij}\underset{\sim}{d}'_{ij})^{-1} \sum_{i=1}^{N}\sum_{j=1}^{n_i} \underset{\sim}{d}_{ij}(y_{ij} - \nu_i), \\
&\qquad \sigma^2 (\sum_{i=1}^{N}\sum_{j=1}^{n_i} \underset{\sim}{d}_{ij}\underset{\sim}{d}'_{ij})^{-1} \Big\}, \qquad (2.19)
\end{aligned}$$

$$\nu_i | \underset{\sim}{\beta}, \underset{\sim}{\sigma}^2, \underset{\sim}{\delta}^2, \underset{\sim}{y} \overset{ind}{\sim} \text{Normal}\Big\{ \frac{\sum_{j=1}^{n_i}(y_{ij} - \beta_0 - \beta_1 a_{ij})/\sigma^2}{1/\delta^2 + n_i/\sigma^2}, \frac{1}{1/\delta^2 + n_i/\sigma^2} \Big\},$$

$$\begin{aligned}
\sigma^{-2} | \underset{\sim}{\beta}, \underset{\sim}{\nu}, \delta^2, \underset{\sim}{y} &\sim \text{Gamma}\Big\{ \frac{a + \sum_{i=1}^{N} n_i}{2}, \\
&\qquad \frac{b + \sum_{i=1}^{N}\sum_{j=1}^{n_i}[y_{ij} - (\beta_0 + \beta_1 a_{ij} + \nu_i)]^2}{2} \Big\}, \qquad (2.20)
\end{aligned}$$

$$\delta^{-2} | \underset{\sim}{\beta}, \underset{\sim}{\nu}, \sigma^2, \underset{\sim}{y} \sim \text{Gamma}(\frac{a+N}{2}, \frac{b + \sum_{i=1}^{N} \nu_i^2}{2}). \qquad (2.21)$$

We start up the Gibbs sampler by taking

$$\nu_i = \hat{\nu}_i,\ \sigma^{-2} = \sum_{i=1}^{N}\sum_{j=1}^{n_i}(\hat{e_{ij}} - \overline{\hat{e_{ij}}})^2 / (\sum_{i=1}^{N} n_i - 1)\ , \delta^{-2} = \sum_{i=1}^{N}(\hat{\nu}_i - \overline{\hat{\nu}_i})^2/(N-1), \qquad (2.22)$$

where

$$\begin{aligned}
\hat{e_{ij}} &= y_{ij} - \hat{\beta}_0 - \hat{\beta}_1 a_{ij} - \hat{\nu}_i, \\
\hat{\underset{\sim}{\beta}} &= (\sum_{i=1}^{N}\sum_{j=1}^{n_i} \underset{\sim}{d}_{ij}\underset{\sim}{d}'_{ij})^{-1} (\sum_{i=1}^{N}\sum_{j=1}^{n_i} \underset{\sim}{d}_{ij} y_{ij}), \\
\hat{\nu}_i &= n_i^{-1} \sum_{j=1}^{n_i} \Big\{ (y_{ij} - \hat{\beta}_0 - \hat{\beta}_1 a_{ij}) \Big\}.
\end{aligned}$$

We draw 11000 iterates, throw out the first 1000, take every tenth and then make inferences. Table 2.1 shows that the autocorrelations (ACF) are small within ± 2 standard error; parameter estimates and their 95% credible intervals are shown in Table 2.3. $\nu_i + \beta_0$ are very close by counties. For the ignorable nonresponse model without covariate, we follow a similar process to get autocorrelations (ACF) and parameter estimates.

	Lag	ACF	Strr	Lag	ACF	Strr	Lag	ACF	Strr
$\nu_1 + \beta_0$	1	-0.06	0.032	11	0.06	0.031	20	0.02	0.031
$\nu_2 + \beta_0$	1	0.02	0.032	11	0.06	0.031	20	-0.02	0.031
$\nu_3 + \beta_0$	1	0.00	0.032	11	0.00	0.031	20	0.03	0.031
$\nu_4 + \beta_0$	1	0.03	0.032	11	0.02	0.031	20	-0.01	0.031
$\nu_5 + \beta_0$	1	-0.02	0.032	11	0.04	0.031	20	0.05	0.031
$\nu_6 + \beta_0$	1	0.08	0.032	11	-0.03	0.031	20	0.01	0.031
$\nu_7 + \beta_0$	1	0.06	0.032	11	0.00	0.031	20	0.01	0.031
$\nu_8 + \beta_0$	1	0.01	0.032	11	0.05	0.031	20	0.00	0.031
$\nu_9 + \beta_0$	1	-0.01	0.032	11	0.02	0.031	20	-0.02	0.031
$\nu_{10} + \beta_0$	1	-0.06	0.032	11	0.02	0.031	20	0.05	0.031
$\nu_{11} + \beta_0$	1	-0.03	0.032	11	0.00	0.031	20	0.01	0.031
$\nu_{12} + \beta_0$	1	0.02	0.032	11	0.00	0.031	20	-0.02	0.031
$\nu_{13} + \beta_0$	1	-0.01	0.032	11	0.02	0.031	20	-0.04	0.031
$\nu_{14} + \beta_0$	1	0.06	0.032	11	0.00	0.031	20	-0.01	0.031
$\nu_{15} + \beta_0$	1	-0.02	0.032	11	-0.03	0.031	20	-0.04	0.031
$\nu_{16} + \beta_0$	1	0.03	0.032	11	0.02	0.031	20	-0.05	0.031
$\nu_{17} + \beta_0$	1	-0.03	0.032	11	-0.01	0.031	20	0.04	0.031
$\nu_{18} + \beta_0$	1	0.01	0.032	11	0.00	0.031	20	-0.02	0.031
$\nu_{19} + \beta_0$	1	0.06	0.032	11	-0.02	0.031	20	-0.02	0.031
$\nu_{20} + \beta_0$	1	-0.03	0.032	11	-0.04	0.031	20	0.02	0.031
$\nu_{21} + \beta_0$	1	0.01	0.032	11	0.01	0.031	20	-0.03	0.031
$\nu_{22} + \beta_0$	1	0.05	0.032	11	0.04	0.031	20	0.00	0.031
$\nu_{23} + \beta_0$	1	0.02	0.032	11	-0.01	0.031	20	-0.04	0.031
$\nu_{24} + \beta_0$	1	-0.02	0.032	11	0.00	0.031	20	0.02	0.031
$\nu_{25} + \beta_0$	1	-0.03	0.032	11	0.06	0.031	20	-0.02	0.031
$\nu_{26} + \beta_0$	1	-0.03	0.032	11	0.03	0.031	20	-0.01	0.031
$\nu_{27} + \beta_0$	1	-0.03	0.032	11	0.01	0.031	20	-0.05	0.031
$\nu_{28} + \beta_0$	1	-0.04	0.032	11	-0.03	0.031	20	-0.03	0.031
$\nu_{29} + \beta_0$	1	0.02	0.032	11	0.00	0.031	20	-0.07	0.031
$\nu_{30} + \beta_0$	1	0.00	0.032	11	0.01	0.031	20	-0.02	0.031
$\nu_{31} + \beta_0$	1	0.00	0.032	11	0.03	0.031	20	0.03	0.031
$\nu_{32} + \beta_0$	1	0.00	0.032	11	-0.03	0.031	20	0.00	0.031
$\nu_{33} + \beta_0$	1	0.00	0.032	11	0.01	0.031	20	-0.03	0.031
$\nu_{34} + \beta_0$	1	-0.10	0.032	11	-0.03	0.031	20	0.03	0.031
$\nu_{35} + \beta_0$	1	0.03	0.032	11	-0.04	0.031	20	-0.08	0.031
β_1	1	-0.02	0.036	11	0.03	0.034	20	-0.00	0.031
σ	1	0.01	0.032	11	-0.01	0.031	20	-0.01	0.031
δ	1	-0.01	0.032	11	0.02	0.031	20	0.02	0.031

Table 2.1: Autocorrelation for ignorable nonresponse model with covariate

2.2.2 Nonignorable Nonresponse Models

The joint posterior density function for nonignorable nonresponse model is:

$$
\begin{aligned}
p(\underset{\sim}{y}, \underset{\sim}{r} | \underset{\sim}{\nu}, \underset{\sim}{\sigma}^2, \sigma_e^2, \underset{\sim}{\beta}, \underset{\sim}{\alpha}, \eta_i, \sigma_\eta^2) \quad \propto \quad & \prod_{i=1}^{N} \prod_{j=1}^{n_i} \Bigg\{ \frac{e^{(\alpha_0+\alpha_1 a_{ij}+\eta_i) r_{ij}}}{1+e^{\alpha_0+\alpha_1 a_{ij}+\eta_i}} \\
\times \quad & \frac{1}{(2\pi\sigma_e^2)^{1/2}} exp\Big\{ -\frac{1}{2\sigma_e^2}(y_{ij} - (\beta_0 + \nu_{0i} + (\beta_1 + \nu_{1i} r_{ij}) a_{ij}))^2 \Big\} \Bigg\} \\
\times \quad & \prod_{i=1}^{N} \Big\{ \frac{1}{(2\pi\sigma_0^2)^{1/2}} exp\{-\frac{\nu_{0i}^2}{2\sigma_0^2}\} \frac{1}{(2\pi\sigma_1^2)^{1/2}} exp\{-\frac{\nu_{1i}^2}{2\sigma_1^2}\} \Big\} \\
\times \quad & \prod_{i=1}^{N} \Big\{ \frac{1}{(2\pi\sigma_\eta^2)^{1/2}} exp\{-\frac{\eta_i^2}{2\sigma_\eta^2}\} \Big\} p(\underset{\sim}{\alpha}) \\
\times \quad & (\frac{1}{\sigma_0^2})^{\frac{a}{2}+1} exp\{-\frac{b}{2\sigma_0^2}\} (\frac{1}{\sigma_1^2})^{\frac{a}{2}+1} exp\{-\frac{b}{2\sigma_1^2}\} \\
\times \quad & (\frac{1}{\sigma_e^2})^{\frac{a}{2}+1} exp\{-\frac{b}{2\sigma_e^2}\} (\frac{1}{\sigma_\eta^2})^{\frac{a}{2}+1} exp\{-\frac{b}{2\sigma_\eta^2}\}. \qquad (2.23)
\end{aligned}
$$

Letting

$$
\begin{aligned}
&\underset{\sim}{\nu}_i = (\nu_{0i}, \nu_{1i})', \underset{\sim}{a}_{ij} = (1, a_{ij})', i = 1, ..., N, j = 1, ..., n_i, z_{ij} = y_{ij} - \nu_{0i} - r_{ij} a_{ij} \\
&\underset{\sim}{\mathrm{b}}_{ij} = (1, r_{ij} a_{ij})'
\end{aligned}
$$

$$
D = \begin{pmatrix} 0 & \frac{1}{\sigma_1^2} \\ \frac{1}{\sigma_2^2} & 0 \end{pmatrix}
$$

The conditional posterior densities are:

$$\underset{\sim}{\beta}|\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\nu},\underset{\sim}{\sigma},\sigma_e^2,\underset{\sim}{\alpha},\eta_i,\sigma_\eta^2 \overset{ind}{\sim} \text{Normal}\Big\{(\sum_{i=1}^{N}\sum_{j=1}^{n_i}\underset{\sim}{a}_{ij}\underset{\sim}{a'}_{ij})^{-1}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\underset{\sim}{a}_{ij}z_{ij}, \sigma_e^2(\sum_{i=1}^{N}\sum_{j=1}^{n_i}\underset{\sim}{a}_{ij}\underset{\sim}{a'}_{ij})^{-1}\Big\}, \tag{2.24}$$

$$\underset{\sim}{\nu}|\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\underset{\sim}{\sigma},\sigma_e^2,\underset{\sim}{\alpha},\eta_i,\sigma_\eta^2 \overset{ind}{\sim} \text{Normal}\Big\{\{\frac{\sum_{j=1}^{n_i} b_{ij}b'_{ij}}{{\sigma_e}^2}+D\}^{-1}\frac{\sum_{j=1}^{n_i} z_{ij}b'_{ij}}{{\sigma_e}^2}, \{\frac{\sum_{j=1}^{n_i} b_{ij}b'_{ij}}{{\sigma_e}^2}+D\}^{-1}\Big\}, \tag{2.25}$$

$$p(\eta_i|\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\underset{\sim}{\sigma},\sigma_e^2,\underset{\sim}{\alpha},\underset{\sim}{\nu},\sigma_\eta^2) \propto \prod_{j=1}^{n_i}\frac{e^{(\alpha_0+\alpha_1 a_{ij}+\eta_i)r_{ij}}}{1+e^{(\alpha_0+\alpha_1 a_{ij}+\eta_i)}} \times\frac{1}{(2\pi\sigma_\eta^2)^{1/2}}exp\Big\{-\frac{\eta_i^2}{2\sigma_\eta^2}\Big\}, \tag{2.26}$$

$$p(\underset{\sim}{\alpha}|\eta_i,\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\underset{\sim}{\sigma},\sigma_e^2,\underset{\sim}{\nu},\sigma_\eta^2) \propto \prod_{i=1}^{N}\prod_{j=1}^{n_i}\Big\{\frac{e^{(\alpha_0+\alpha_1 a_{ij}+\eta_i)r_{ij}}}{1+e^{\alpha_0+\alpha_1 a_{ij}+\eta_i}}\Big\}p(\underset{\sim}{\alpha}), \tag{2.27}$$

$$\sigma_e^{-2}|\underset{\sim}{\alpha},\eta_i,\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\underset{\sim}{\sigma},\underset{\sim}{\nu},\sigma_\eta^2 \sim \text{Gamma}\Big\{\frac{a+\sum_{i=1}^{N} n_i}{2}, \frac{b+\sum_{i=1}^{N}\sum_{j=1}^{n_i}[y_{ij}-(\nu_{0i}+\beta_0+(\beta_1+\nu_{ij})a_{ij})]^2}{2}\Big\}, \tag{2.28}$$

$$\sigma_0^{-2}|\sigma_e^2,\underset{\sim}{\alpha},\eta_i,\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\sigma_1,\underset{\sim}{\nu},\sigma_\eta^2) \sim \text{Gamma}(\frac{a+N}{2},\frac{b+\sum_{i=1}^{N}\nu_{0i}^2}{2}), \tag{2.29}$$

$$\sigma_1^{-2}|\sigma_e^2,\underset{\sim}{\alpha},\eta_i,\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\sigma_0,\underset{\sim}{\nu},\sigma_\eta^2 \sim \text{Gamma}(\frac{a+N}{2},\frac{b+\sum_{i=1}^{N}\nu_{1i}^2}{2}), \tag{2.30}$$

$$\sigma_\eta^{-2}|\sigma_e^2,\underset{\sim}{\alpha},\eta_i,\underset{\sim}{y},\underset{\sim}{r},\underset{\sim}{\beta},\sigma_0,\underset{\sim}{\nu},\sigma_1^2) \sim \text{Gamma}(\frac{a+N}{2},\frac{b+\sum_{i=1}^{N}\eta_i^2}{2}). \tag{2.31}$$

Since the conditional posterior densities of η_i and $\underset{\sim}{\alpha}$ are very complicated, we use Metropolis-Hastings sampler. We draw 11000 iterates, throw out the first 1000, take every tenth and then make inferences. The autocorrelations in Table 2.2 within ± 2 are small except for σ_2, β_0, σ_η. In Table 2.4, we compute the estimates of $\beta_0+\nu_i$ from the ignorable and the nonignorable nonresponse models and in Table 2.3 we compute $\beta_1+\nu_{1i}$ for the nonigorable model with β_1 for the ignorable nonresponse model.

Parameters	Lag	ACF	Strr	Lag	ACF	Strr	Lag	ACF	Strr
α_0	1	0.12	0.0316	11	0.05	0.0314	20	0.04	0.0313
α_1	1	-0.05	0.0316	11	-0.03	0.0314	20	0.01	0.0313
σ_η	1	0.38	0.0316	11	0.05	0.0314	20	-0.06	0.0313
β_0	1	0.43	0.0316	11	-0.04	0.0314	20	0.04	0.0313
β_1	1	-0.09	0.0316	11	-0.02	0.0314	20	-0.05	0.0313
σ_e	1	0.01	0.0316	11	-0.03	0.0314	20	0.01	0.0313
σ_1	1	0.00	0.0316	11	0.06	0.0314	20	-0.01	0.0313
σ_2	1	1.00	0.0316	11	0.99	0.0314	20	0.98	0.0313

Table 2.2: Autocorrelation for nonignorable nonresponse model

2.3 Residuals

We use a cross-validation analysis to evaluate the goodness of the fitted ignorable nonresponse model with covariate (Nandram and Choi 2002). We do so by using deleted residuals on the respondents' BMD values.

Let $(\underset{\sim}{x}_{(ij)}, \underset{\sim}{r}_{(ij)})$ denote the vector of all observations excluding the $(ij)^{th}$ observation (a_{ij}, r_{ij}). Then the $(ij)^{th}$ deleted residual is given by

$$\text{DRES}_{ij} = \{a_{ij} - E(a_{ij}|\underset{\sim}{x}_{(ij)}, \underset{\sim}{r}_{(ij)})\}/STD(a_{ij}|\underset{\sim}{x}_{(ij)}, \underset{\sim}{r}_{(ij)})$$

These values are obtained by performing a weighted importance sampling on the Gibbs sampler output. The posterior moments are obtained from

$$f(a_{ij}|\underset{\sim}{x}_{(ij)}, \underset{\sim}{r}_{(ij)}) = \int f(a_{ij}|\Omega)\pi(\Omega|\underset{\sim}{x}_{(ij)}, \underset{\sim}{r}_{(ij)})d\Omega$$

where

$$f(a_{ij}|\Omega) = \sum_{r_{ij}=0} f(a_{ij}|r_{ij}, \Omega)p(r_{ij}|\Omega)$$

We use the plots of DRES versus predicted BMD (PRED) for each county to examine the model. Figure 2.1 and Figure 2.2 show randomly scattered pattern, which indicates reasonable fit.

ignorable nonresponse model				nonignorable nonresponse model			
	PM	PSD	CI		PM	PSD	CI
$\nu_1 + \beta_0$	1.079	0.014	(1.052 , 1.107)	$\nu_1 + \beta_0$	0.883	0.015	(0.851 , 0.913)
$\nu_2 + \beta_0$	1.082	0.016	(1.052 , 1.113)	$\nu_2 + \beta_0$	0.869	0.016	(0.839 , 0.902)
$\nu_3 + \beta_0$	1.101	0.015	(1.070 , 1.132)	$\nu_3 + \beta_0$	0.914	0.016	(0.884 , 0.946)
$\nu_4 + \beta_0$	1.103	0.010	(1.085 , 1.123)	$\nu_4 + \beta_0$	0.904	0.007	(0.891 , 0.918)
$\nu_5 + \beta_0$	1.080	0.014	(1.052 , 1.108)	$\nu_5 + \beta_0$	0.861	0.014	(0.833 , 0.888)
$\nu_6 + \beta_0$	1.080	0.016	(1.046 , 1.109)	$\nu_6 + \beta_0$	0.862	0.018	(0.824 , 0.894)
$\nu_7 + \beta_0$	1.076	0.016	(1.045 , 1.105)	$\nu_7 + \beta_0$	0.863	0.017	(0.830 , 0.895)
$\nu_8 + \beta_0$	1.084	0.015	(1.055 , 1.114)	$\nu_8 + \beta_0$	0.848	0.015	(0.819 , 0.876)
$\nu_9 + \beta_0$	1.100	0.014	(1.072 , 1.127)	$\nu_9 + \beta_0$	0.862	0.013	(0.838 , 0.886)
$\nu_{10} + \beta_0$	1.079	0.018	(1.042 , 1.113)	$\nu_{10} + \beta_0$	0.800	0.019	(0.760 , 0.837)
$\nu_{11} + \beta_0$	1.070	0.017	(1.036 , 1.101)	$\nu_{11} + \beta_0$	0.823	0.018	(0.788 , 0.858)
$\nu_{12} + \beta_0$	1.075	0.016	(1.042 , 1.107)	$\nu_{12} + \beta_0$	0.826	0.017	(0.792 , 0.859)
$\nu_{13} + \beta_0$	1.102	0.014	(1.074 , 1.129)	$\nu_{13} + \beta_0$	0.906	0.013	(0.880 , 0.931)
$\nu_{14} + \beta_0$	1.071	0.016	(1.038 , 1.102)	$\nu_{14} + \beta_0$	0.812	0.015	(0.785 , 0.841)
$\nu_{15} + \beta_0$	1.084	0.017	(1.050 , 1.117)	$\nu_{15} + \beta_0$	0.851	0.018	(0.814 , 0.887)
$\nu_{16} + \beta_0$	1.123	0.018	(1.090 , 1.161)	$\nu_{16} + \beta_0$	0.901	0.021	(0.860 , 0.942)
$\nu_{17} + \beta_0$	1.085	0.018	(1.048 , 1.117)	$\nu_{17} + \beta_0$	0.814	0.018	(0.778 , 0.849)
$\nu_{18} + \beta_0$	1.095	0.018	(1.059 , 1.130)	$\nu_{18} + \beta_0$	0.861	0.019	(0.824 , 0.899)
$\nu_{19} + \beta_0$	1.092	0.020	(1.052 , 1.130)	$\nu_{19} + \beta_0$	0.821	0.024	(0.775 , 0.868)
$\nu_{20} + \beta_0$	1.056	0.019	(1.017 , 1.092)	$\nu_{20} + \beta_0$	0.783	0.020	(0.744 , 0.821)
$\nu_{21} + \beta_0$	1.066	0.019	(1.027 , 1.101)	$\nu_{21} + \beta_0$	0.792	0.020	(0.754 , 0.830)
$\nu_{22} + \beta_0$	1.077	0.019	(1.039 , 1.113)	$\nu_{22} + \beta_0$	0.810	0.022	(0.764 , 0.854)
$\nu_{23} + \beta_0$	1.084	0.019	(1.047 , 1.119)	$\nu_{23} + \beta_0$	0.861	0.023	(0.816 , 0.904)
$\nu_{24} + \beta_0$	1.073	0.021	(1.029 , 1.110)	$\nu_{24} + \beta_0$	0.814	0.026	(0.763 , 0.866)
$\nu_{25} + \beta_0$	1.108	0.018	(1.073 , 1.144)	$\nu_{25} + \beta_0$	0.870	0.021	(0.830 , 0.911)
$\nu_{26} + \beta_0$	1.093	0.019	(1.055 , 1.131)	$\nu_{26} + \beta_0$	0.853	0.022	(0.810 , 0.898)
$\nu_{27} + \beta_0$	1.069	0.019	(1.028 , 1.106)	$\nu_{27} + \beta_0$	0.777	0.023	(0.730 , 0.822)
$\nu_{28} + \beta_0$	1.066	0.021	(1.023 , 1.104)	$\nu_{28} + \beta_0$	0.795	0.026	(0.742 , 0.847)
$\nu_{29} + \beta_0$	1.079	0.016	(1.046 , 1.109)	$\nu_{29} + \beta_0$	0.823	0.017	(0.789 , 0.858)
$\nu_{30} + \beta_0$	1.093	0.014	(1.066 , 1.120)	$\nu_{30} + \beta_0$	0.859	0.014	(0.830 , 0.889)
$\nu_{31} + \beta_0$	1.067	0.018	(1.032 , 1.100)	$\nu_{31} + \beta_0$	0.848	0.019	(0.813 , 0.885)
$\nu_{32} + \beta_0$	1.097	0.014	(1.070 , 1.125)	$\nu_{32} + \beta_0$	0.881	0.013	(0.854 , 0.906)
$\nu_{33} + \beta_0$	1.120	0.014	(1.093 , 1.149)	$\nu_{33} + \beta_0$	0.950	0.014	(0.922 , 0.976)
$\nu_{34} + \beta_0$	1.097	0.015	(1.068 , 1.129)	$\nu_{34} + \beta_0$	0.909	0.017	(0.874 , 0.942)
$\nu_{35} + \beta_0$	1.080	0.015	(1.051 , 1.109)	$\nu_{35} + \beta_0$	0.833	0.014	(0.806 , 0.860)
β_1	-0.005	0.000	(-0.005 , -0.004)	α_0	0.835	0.044	(0.749 , 0.922)
σ	0.017	0.001	(0.016 , 0.018)	α_1	-0.212	0.039	(-0.290 , -0.139)
δ	0.001	0.000	(0.000 , 0.001)	σ_η	0.013	0.007	(0.005 , 0.029)
				σ_e	0.017	0.001	(0.016 , 0.018)
				σ_1	0.002	0.001	(0.001 , 0.004)
				σ_2	0.000	0.000	(0.000 , 0.000)

Table 2.3: Estimates for ignorable nonresponse model and nonignorable nonresponse model (NSE < 0.01): posterior mean (PM), posterior standard deviation (PSD) and 95 % credible interval (CI).

	$\beta_1+\nu_{1i}$	PM	PSD	NSE	CI
Nonignorable	1	-0.085	0.012	0.002	(-0.109, -0.061)
	2	-0.085	0.013	0.003	(-0.109, -0.060)
	3	-0.085	0.013	0.003	(-0.110, -0.058)
	4	-0.086	0.013	0.002	(-0.110, -0.060)
	5	-0.085	0.013	0.003	(-0.111, -0.060)
	6	-0.086	0.013	0.002	(-0.112, -0.061)
	7	-0.086	0.013	0.003	(-0.110, -0.062)
	8	-0.085	0.013	0.002	(-0.109, -0.059)
	9	-0.086	0.013	0.002	(-0.111, -0.062)
	10	-0.085	0.013	0.002	(-0.110, -0.061)
	11	-0.085	0.013	0.003	(-0.110, -0.060)
	12	-0.086	0.013	0.003	(-0.112, -0.060)
	13	-0.086	0.013	0.002	(-0.112, -0.061)
	14	-0.086	0.013	0.002	(-0.112, -0.061)
	15	-0.085	0.013	0.002	(-0.109, -0.061)
	16	-0.086	0.013	0.003	(-0.111, -0.061)
	17	-0.086	0.013	0.003	(-0.112, -0.062)
	18	-0.086	0.013	0.003	(-0.112, -0.061)
	19	-0.085	0.013	0.003	(-0.111, -0.059)
	20	-0.085	0.013	0.002	(-0.109, -0.061)
	21	-0.085	0.013	0.002	(-0.110, -0.058)
	22	-0.086	0.013	0.003	(-0.111, -0.060)
	23	-0.086	0.013	0.003	(-0.113, -0.060)
	24	-0.085	0.013	0.003	(-0.111, -0.058)
	25	-0.086	0.013	0.003	(-0.110, -0.061)
	26	-0.085	0.013	0.002	(-0.109, -0.060)
	27	-0.086	0.013	0.002	(-0.112, -0.060)
	28	-0.086	0.013	0.003	(-0.112, -0.059)
	29	-0.086	0.013	0.002	(-0.112, -0.061)
	30	-0.085	0.013	0.002	(-0.111, -0.060)
	31	-0.086	0.013	0.003	(-0.111, -0.061)
	32	-0.086	0.013	0.003	(-0.111, -0.060)
	33	-0.085	0.013	0.003	(-0.110, -0.060)
	34	-0.086	0.013	0.003	(-0.110, -0.059)
	35	-0.086	0.013	0.003	(-0.113, -0.060)
Ignorable	β_1	-0.005	0.00	0.003	(-0.005, -0.004)

Table 2.4: Estimates of $\beta_1+\nu_{1i}$ for nonignorable nonresponse model and β_1 for ignorable nonresponse model: posterior mean (PM), posterior standard deviation (PSD) and 95 % credible interval (CI).

	penalty	goodness	deviance
ignorable nonresponse model without covariate	685.00	48.32	733.31
ignorable nonresponse model with covariate	410.28	34.73	445.01

Table 2.5: Comparisons of models

2.4 Checking covariate

We use the deviance measures (Gelfand and Ghosh, 1998) to check the effect of age on BMD.

In this method from the ignorable nonresponse model,

$$\begin{aligned}
\mathrm{D} &\doteq \mathrm{P}+\mathrm{G} \\
\mathrm{D} &= \sum_{i=1}^{N}\mathrm{D}(i),\ D(i)=P(i)+G(i) \\
\mathrm{P} &= \sum_{i=1}^{N}\mathrm{P}(i),\ \mathrm{P}(i)=\sum_{j=1}^{r_i}\left\{\frac{1}{M}\sum_{h=1}^{M}\sigma^{2(h)}+\left\{\beta_0^{(h)}+\beta_1^{(h)}a_{ij}+\nu_i^{(h)}-\hat{y}_{ij}\right\}^2\right\} \\
\mathrm{G} &= \sum_{i=1}^{N}\mathrm{G}(i),\ \mathrm{G}(i)=\sum_{j=1}^{r_i}(\hat{y}_{ij}-y_{ij})^2 \\
& \quad i=1,\dots,N
\end{aligned}$$

where D is the deviance, P is the penalty and G is an error sum of squares $\hat{y}_{ij}$ is the predicted y_{ij} and $\hat{y}_{ij}=\frac{1}{M}\sum_{h=1}^{M}\{\beta_0^{(h)}+\beta_1^{(h)}a_{ij}+\nu_i^{(h)}\}$, r_i is the count of observed individuals in i^{th} county, $(\beta_0^{(h)},\beta_1^{(h)},\nu_i^{(h)},\sigma^{2(h)})$, h=1, ... M and M is iterate obtained from the Gibbs sampler.

We conclude that the ignorable nonresponse model with covariate is better than the ignorable nonresponse model without covariate; (see Table 2.5).

2.5 Conclusions

We compare the estimates of $\beta_1+\nu_i$ of nonignorable nonresponse model and β_1 of ignorable nonresponse model (Table 2.4), we found that the estimate of the ignorable nonresponse model is outside the 95 % credible interval and the nonignorable nonresponse model shows the negative relation with BMD.

We would like to find out the percent of individuals in different cells (e.g, young osteoporosis individuals). One can do so by using bivariate model on BMD and age, but since age is discrete, we use categorical methods.

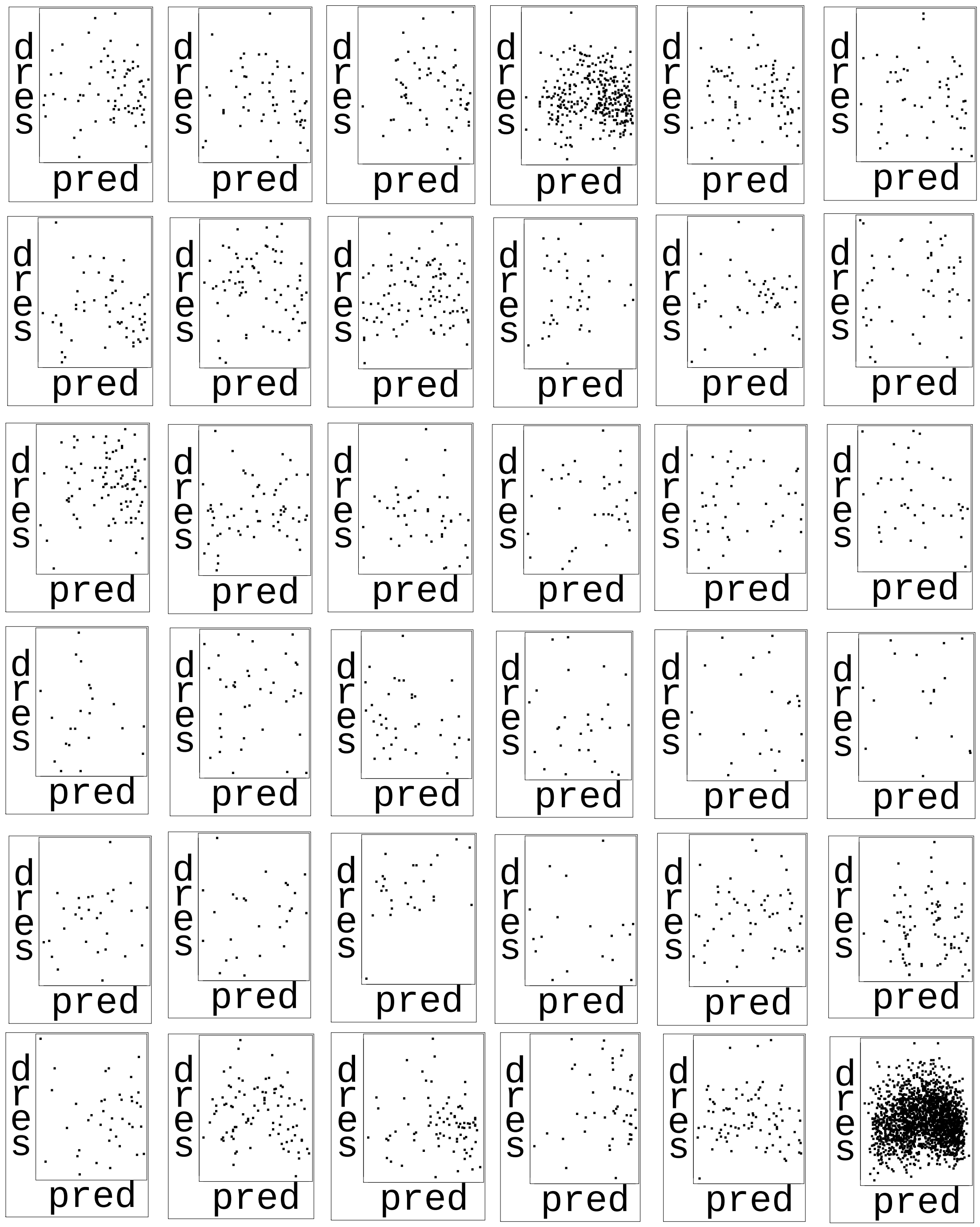


Figure 2.1: Cross-validation plots for dres vs. pred by counties (From top to bottom, left to right is for county 1,...,N and the last one is for the overall counties)

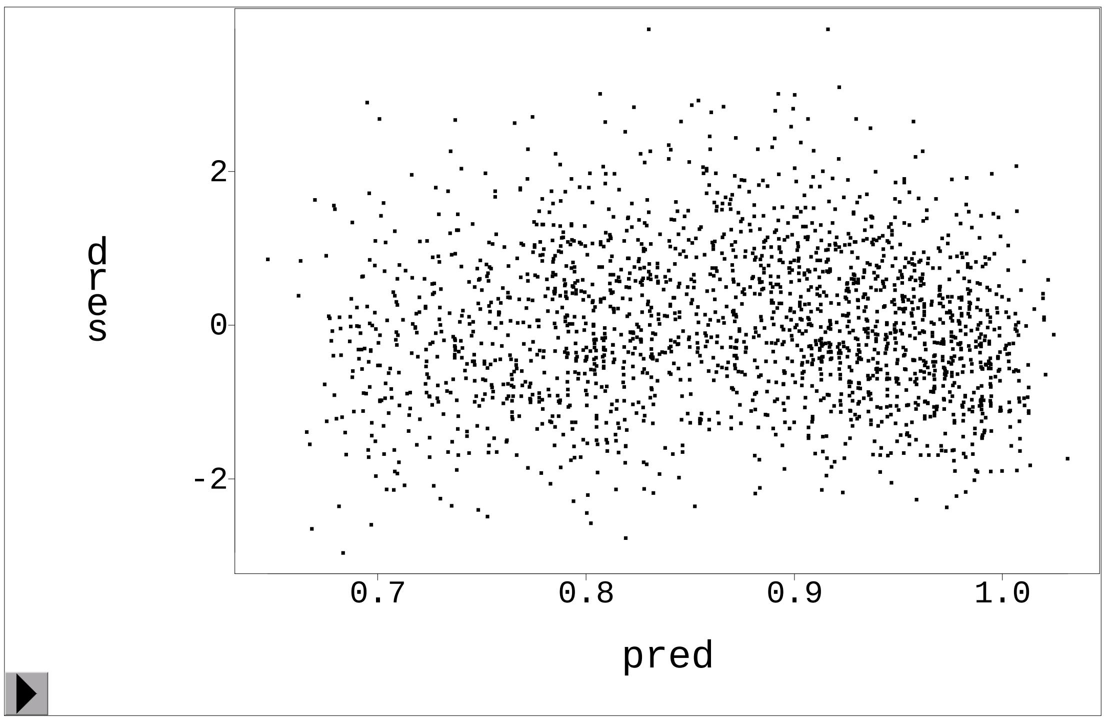


Figure 2.2: Cross-validation plots for dres vs. pred

Chapter 3

Categorical Models

In this chapter, we first construct categorical models for BMD and age accounting for missing data, and then we assess the association between BMD and age and the ignorability of models by using Bayes factor.

For the NHANES III data, there are three levels of BMD (normal, osteopenia and osteoporosis) and two levels of age (50- and 50+). We estimate the proportion of individuals in each of the 6 cells in the r $\times$ c table, and we assess the association of BMD and age.

3.1 Baseline Models

Let

$$\mathrm{I}_{ijk\ell} = \begin{cases} 1, & \text{if } \ell^{th} \text{ individual in } i^{th} \text{ county belongs to } j^{th} \text{ age and } k^{th} \text{ BMD level} \\ 0, & \text{if } \ell^{th} \text{ individual in } i^{th} \text{ county does not belong to } j^{th} \text{ age and } k^{th} \text{ BMD level} \end{cases}$$

and

$$r_{i\ell} = \begin{cases} 1, & \text{if } \ell^{th} \text{ individual in } i^{th} \text{ county responds} \\ 0, & \text{if } \ell^{th} \text{ individual in } i^{th} \text{ county does not respond} \end{cases}$$

where i=1,. . .,N, j=1,. . .,r (r=2), k=1,. . .,c (c=3), ℓ=1,. . .,n_i. Here N is the number of counties and N=35, n_i is the number of individuals in the i^{th} county.

Thus, $\sum_{\ell=1}^{n_i} \mathrm{I}_{ijk\ell} r_{i\ell} = y_{ijk}$ are the cell counts for the observed data belonging to j^{th} age and k^{th} BMD level within the i^{th} county, $\sum_{\ell=1}^{n_i} \mathrm{I}_{ijk\ell}(1-r_{i\ell}) = z_{ijk}$ are the cell counts for the missing data belonging to j^{th} age and k^{th} BMD level within the i^{th} county and z_{ijk} are latent variables (Figure 3.1). We construct a probabilistic structure to model $\mathrm{I}_{ijk\ell}$ and $r_{i\ell}$ using the assumptions of ignorability and nonignorability. The difference between the assumptions of ignorability and nonignorability depends on whether an individual responds independently of his/her status in r $\times$ c table.

3.1.1 Baseline Ignorable Nonresponse Model

Let π_i be the probability that an individual in the i^{th} county responds and it is independent of his/her status in r $\times$ c table; $\underset{\sim}{p}_i = \{p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c\}$, and p_{ijk} be the probability that an individual in the i^{th} county belongs to j^{th} age and k^{th} BMD level, i= 1, 2,. . .,N, j= 1,. . .,r, k=1,. . .,c. Thus, we assume

$r_{i\ell}|\underset{\sim}{\mathrm{I}}_{i\ell}, \pi_i \overset{iid}{\sim} \text{Bernoulli}(\pi_i)$ and $\underset{\sim}{I}_{i\ell}|\underset{\sim}{p}_i \overset{iid}{\sim} \text{Multinomial}(1, \underset{\sim}{p}_i)$ giving

$$\underset{\sim}{y}_i, \underset{\sim}{z}_i|\underset{\sim}{p}_i, \pi_i \overset{ind}{\sim} \text{Multinomial}\Big\{n_i, \pi_i p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, \\ (1 - \pi_i)p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c\Big\}. \quad (3.1)$$

Letting $\underset{\sim}{\mu} = \big\{\mu_{jk}, j = 1, \ldots, r, k = 1, \ldots, c\big\}$ and $\sum_{j=1}^{r}\sum_{k=1}^{c}\mu_{jk} = 1$, the remaining assumptions in the model are

$$\begin{aligned} \underset{\sim}{p}_i|\underset{\sim}{\mu}, \tau_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}\tau_1), i = 1, \ldots, N, \\ \pi_i|\nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1-\nu)\tau_2), i = 1, \ldots, N, \\ \nu &\sim \text{Uniform}(0,1),\ \underset{\sim}{\mu} \sim \text{Dirichlet}(\underset{\sim}{1}), \\ p(\tau_1) &= \frac{1}{(1+\tau_1)^2}, \tau_1 > 0,\ p(\tau_2) = \frac{1}{(1+\tau_2)^2}, \tau_2 > 0. \end{aligned} \quad (3.2)$$

3.1.2 Baseline Nonignorable Nonresponse Model

We assume that $\underset{\sim}{\pi}_i = \{\pi_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, \}$, π_{ijk} is the probability that an individual, in the i^{th} county belonging to the j^{th} age and k^{th} BMD level, responds and let $\underset{\sim}{\mu} = \big\{\mu_{11}, \mu_{12}, \mu_{1c}, \mu_{r1}, \mu_{r2}, \mu_{rc}\big\}$ and $\sum_{j=1}^{r}\sum_{k=1}^{c}\mu_{jk} = 1$. Thus, we assume $r_{i\ell}|\underset{\sim}{\mathrm{I}}_{i\ell}, \underset{\sim}{\pi}_i \overset{iid}{\sim} \text{Bernoulli}(\underset{\sim}{\pi}_i)$ and $\underset{\sim}{I}_{i\ell}|\underset{\sim}{p}_i \overset{iid}{\sim} \text{Multinomial}(1, \underset{\sim}{p}_i)$ giving

$$\underset{\sim}{y}_i, \underset{\sim}{z}_i|\underset{\sim}{p}_i, \underset{\sim}{\pi}_i \overset{ind}{\sim} \text{Multinomial}\Big\{n_i, \pi_{ijk} p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, \\ (1 - \pi_{ijk})p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c\Big\}. \quad (3.3)$$

Letting $\underset{\sim}{\mu} = \big\{\mu_{jk}, j = 1, \ldots, r, k = 1, \ldots, c\big\}$ and $\sum_{j=1}^{r}\sum_{k=1}^{c}\mu_{jk} = 1$, the remaining assumptions in the model are

$$\begin{aligned} \underset{\sim}{p}_i|\underset{\sim}{\mu}, \tau_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}\tau_1), i = 1, \ldots, N, \\ \pi_{ijk}|\nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1-\nu)\tau_2), i = 1, \ldots, N, \\ \nu &\sim \text{Uniform}(0,1),\ \underset{\sim}{\mu} \sim \text{Dirichlet}(\underset{\sim}{1}), \\ p(\tau_1) &= \frac{1}{(1+\tau_1)^2}, \tau_1 > 0,\ p(\tau_2) = \frac{1}{(1+\tau_2)^2}, \tau_2 > 0. \end{aligned}$$

3.2 Model Fitting

We use the Griddy Gibbs sampler to draw samples and make inference for the parameters in the model. The samples are drawn from the conditional posterior density and after a large number of iterations, they will converge to the joint posterior density.

3.2.1 Baseline Ignorable Nonresponse Model

First, by using Bayes' theorem, we obtain the joint posterior density of all the parameters.

$$\begin{aligned} p(\underset{\sim}{p}, \pi_i, \underset{\sim}{z}, \nu, \tau_2, \underset{\sim}{\mu}, \tau_1 | \underset{\sim}{y}) &\propto p(\underset{\sim}{y}, \underset{\sim}{z} | \pi_i, \underset{\sim}{p}) p(\pi_i | \nu, \tau_2) p(\underset{\sim}{p} | \underset{\sim}{\mu}, \tau_1) \\ &\quad \times p(\nu) p(\tau_2) p(\underset{\sim}{\mu}) p(\tau_1) \\ &= \prod_{i=1}^{N} \left\{ \prod_{j=1}^{r} \prod_{k=1}^{c} \left\{ \frac{(\mathrm{p}_{ijk}\pi_i)^{y_{ijk}}}{y_{ijk}!} \frac{(\mathrm{p}_{ijk}(1-\pi_i))^{z_{ijk}}}{z_{ijk}!} \right. \right. \\ &\quad \left. \times \frac{\mathrm{p}_{ijk}{}^{\mu_{jk}\tau_1 - 1}}{D(\underset{\sim}{\mu}\tau_1)} \right\} \frac{\pi_i{}^{\nu\tau_2 - 1}(1-\pi_i)^{(1-\nu)\tau_2 - 1}}{\mathrm{B}(\nu\tau_2, (1-\nu)\tau_2)} \\ &\quad \left. \times \frac{\nu^{\alpha_0 - 1}(1-\nu)^{\beta_0 - 1}}{\mathrm{B}(\alpha_0, \beta_0)} \frac{1}{(1+\tau_1)^2} \frac{1}{(1+\tau_2)^2} \right\}. \end{aligned} \quad (3.4)$$

Then, from the joint posterior density, we find the conditional posterior densities are:

$$\begin{aligned}
\underset{\sim}{p}_i|\underset{\sim}{y}_i,\underset{\sim}{z}_i,\pi_i,\nu,\tau_2,\underset{\sim}{\mu},\tau_1 &\overset{ind}{\sim} \text{Dirichlet}(y_{ijk}+z_{ijk}+\mu_{jk}\tau_1,\\
&\quad j=1,\dots,r,k=1,\dots,c), \qquad (3.5)
\end{aligned}$$

$$\begin{aligned}
\pi_i|\underset{\sim}{y}_i,\underset{\sim}{z}_i,\underset{\sim}{p}_i,\nu,\tau_2,\underset{\sim}{\mu},\tau_1 &\overset{ind}{\sim} \text{Beta}\Big(\sum_{j=1}^{r}\sum_{k=1}^{c} y_{ijk}+\nu\tau_2,\\
&\quad \sum_{j=1}^{r}\sum_{k=1}^{c} z_{ijk}+(1-\nu)\tau_2\Big),\\
&\quad j=1,\dots,r,k=1,\dots,c, \qquad (3.6)
\end{aligned}$$

$$\begin{aligned}
\underset{\sim}{z}_i|\underset{\sim}{y}_i,\underset{\sim}{p}_i,\pi_i,\nu,\tau_2,\underset{\sim}{\mu},\tau_1 &\overset{ind}{\sim} \text{Multinomial}\Big(n_i-r_i,\underset{\sim}{q}_i\Big),\\
&\quad r_i=\sum_{\ell=1}^{n_i} r_{i\ell},\, q_{ijk}=\frac{p_{ijk}(1-\pi_i)}{\sum_{j=1}^{r}\sum_{k=1}^{c} p_{ijk}(1-\pi_i)},\\
&\quad j=1,\dots,r,k=1,\dots,c, \qquad (3.7)
\end{aligned}$$

$$\begin{aligned}
p(\underset{\sim}{\mu},\tau_1|\underset{\sim}{y}_i,\underset{\sim}{z}_i,\underset{\sim}{p}_i,\pi_i,\nu,\tau_2) &\propto \prod_{i=1}^{N}\prod_{j=1}^{r}\prod_{k=1}^{c}\left\{\frac{p_{ijk}^{\mu_{jk}\tau_1-1}}{D(\underset{\sim}{\mu}\tau_1)}\right\}\frac{1}{(1+\tau_2)^2},\\
&\quad j=1,\dots,r,k=1,\dots,c, \qquad (3.8)
\end{aligned}$$

$$\begin{aligned}
p(\nu,\tau_2|\underset{\sim}{y}_i,\underset{\sim}{z}_i,\pi_i,\underset{\sim}{p}_i,\underset{\sim}{\mu},\tau_1) &\propto \prod_{i=1}^{N}\left\{\frac{\prod_{j=1}^{r}\prod_{k=1}^{c}\pi_i^{\nu\tau_2-1}(1-\pi_i)^{(1-\nu)\tau_2-1}}{\text{B}(\nu\tau_2,(1-\nu)\tau_2)}\right\}\\
&\quad \times\frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{(1+\tau_2)^2},\\
&\quad 0<\nu<1,j=1,\dots,r,k=1,\dots,c. \qquad (3.9)
\end{aligned}$$

The densities of μ, τ_1 in (3.8) and ν, τ_2 in (3.9) are complex so that we use grids to draw samples from them.

For example, we draw $\underset{\sim}{\mu}$ from its conditional probability density function.

a. Let $a_{jk}=\prod_{i=1}^{N} p_{ijk}$;

b. Let μ_{11}, μ_{12}, $\dots$ μ_{rc} be μ_1, μ_2, $\dots$ μ_m and a_{11}, a_{12}, $\dots$ a_{rc} be a_1, a_2, $\dots$ a_m, m = rc.

	lags	ACF	Str		lags	ACF	Str		lags	ACF	Str
μ_1	1	-0.23	0.0316	μ_4	1	-0.10	0.0316	τ_1	1	-0.21	0.0316
	5	0.01	0.0315		5	0.00	0.0315		5	0.04	0.0315
	10	-0.02	0.0314		10	-0.01	0.0314		10	0.01	0.0314
	15	0.07	0.0314		15	0.03	0.0314		15	0.05	0.0314
	20	0.02	0.0313		20	0.03	0.0313		20	0.04	0.0313
μ_2	1	-0.17	0.0316	μ_5	1	-0.08	0.0316	τ_2	1	0.02	0.0316
	5	0.04	0.0315		5	0.06	0.0315		5	-0.02	0.0315
	10	-0.03	0.0314		10	-0.03	0.0314		10	0.04	0.0314
	15	0.01	0.0314		15	0.04	0.0314		15	-0.01	0.0314
	20	0.04	0.0313		20	0.01	0.0313		20	0.01	0.0313
μ_3	1	-0.18	0.0316	μ_6	1	-0.14	0.0316	ν	1	0.02	0.0316
	5	0.02	0.0315		5	-0.04	0.0315		5	0.00	0.0315
	10	-0.01	0.0314		10	-0.02	0.0314		10	0.00	0.0314
	15	0.06	0.0314		15	0.04	0.0314		15	0.02	0.0314
	20	0.04	0.0313		20	0.00	0.0313		20	0.05	0.0313

Table 3.1: Sample autocorrelation for ignorable model (ACF is the autocorrelation and Str is the standard error)

c.

$$p(\underset{\sim}{\mu}|\underset{\sim}{p}, \pi_i, \underset{\sim}{y}, \underset{\sim}{z}, \nu, \tau_1, \tau_2) \propto \frac{\prod_{k=1}^{m} a_k^{\mu_k\tau_2-1}}{g(\underset{\sim}{\mu}\tau_2)}, g(\underset{\sim}{\mu}\tau_2) = \{D(\mu_1\tau_2, \ldots \mu_m\tau_2)\}^N,$$

$$\sum_{k=1}^{m} \mu_k = 1, \mu_k > 0, \mu_m = 1 - \sum_{k=1}^{m-1} \mu_k.$$

It follow that

$$p(\mu_{(k)}, |\underset{\sim}{\mu}_{(k)}, \underset{\sim}{p}, \pi, \underset{\sim}{z}, \nu, \tau_1, \tau_2, \underset{\sim}{y}) \propto \frac{a^{\mu_k\tau_2-1}{}_k a_m^{\mu_m\tau_2-1}}{g(\mu_k, \underset{\sim}{\mu}_{(k)}, \tau_2)},$$

$$0 < \mu_k < 1 - \sum_{k'=1, k'\neq k}^{m-1} \mu_{k'}, \; k = 1, \ldots, m-1.$$

We draw 11000 iterates, "burn in" 1000 iterates and pick every tenth iterate thereafter.

The samples are basically independent for the autocorrelations are within ± 2 (Table 3.1). Finally, we run our algorithm to obtain samples. Table 3.3 and Table 3.4 show our estimates of $\underset{\sim}{p}$ and π_i.

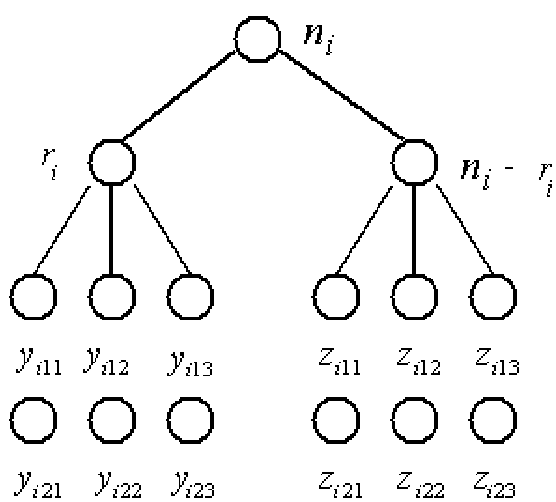


Figure 3.1: Nonresponse tree diagram for the r × c categorical table (y_{ijk} are the observed cell counts and z_{ijk} (latent variables) are the missing cell counts, i=1,...,N, j=1,...,r (r=2), k=1,...,c (c=3)) for the i^{th} county (Note that $\sum_{k=1}^{c} z_{ijk}$ is observed).

3.2.2 Baseline Nonignorable Nonresponse Model

The only difference from ignorable nonresponse model is π_{ijk} in stead of π_i, the joint posterior density for nonignorable nonresponse model is

$$
\begin{aligned}
p(\underset{\sim}{p}, \underset{\sim}{\pi}_i, \underset{\sim}{z}, \nu, \tau_2, \underset{\sim}{\mu}, \tau_1 | \underset{\sim}{y}) \quad \propto \quad & p(\underset{\sim}{y}, \underset{\sim}{z} | \underset{\sim}{\pi}_i, \underset{\sim}{p}) p(\underset{\sim}{\pi}_i | \nu, \tau_2) p(\underset{\sim}{p} | \underset{\sim}{\mu}, \tau_1) \\
& \times p(\nu) p(\tau_2) p(\underset{\sim}{\mu}) p(\tau_1) \\
= \quad & \prod_{i=1}^{N} \left\{ \prod_{j=1}^{r} \prod_{k=1}^{c} \left\{ \frac{(\mathrm{p}_{ijk} \pi_{ijk})^{y_{ijk}}}{y_{ijk}!} \frac{(\mathrm{p}_{ijk}(1-\pi_{ijk}))^{z_{ijk}}}{z_{ijk}!} \right. \right. \\
& \left. \times \frac{\mathrm{p}_{ijk}{}^{\mu_{jk}\tau_1 - 1}}{D(\underset{\sim}{\mu}\tau_1)} \right\} \frac{\pi_{ijk}{}^{\nu\tau_2 - 1} (1-\pi_{ijk})^{(1-\nu)\tau_2 - 1}}{\mathrm{B}(\nu\tau_2, (1-\nu)\tau_2)} \\
& \left. \times \frac{\nu^{\alpha_0 - 1}(1-\nu)^{\beta_0 - 1}}{\mathrm{B}(\alpha_0, \beta_0)} \frac{1}{(1+\tau_1)^2} \frac{1}{(1+\tau_2)^2} \right\}
\end{aligned}
\tag{3.10}
$$

and the conditional posterior densities of nonignorable nonresponse model are

$$
\begin{aligned}
\underset{\sim}{p}_i | \underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{\pi}_i, \nu, \tau_2, \underset{\sim}{\mu}, \tau_1 \overset{ind}{\sim} \; & \text{Dirichlet}(y_{ijk} + z_{ijk} + \mu_{jk}\tau_1, \\
& j = 1, \ldots, r, k = 1, \ldots, c),
\end{aligned}
\tag{3.11}
$$

$$
\begin{aligned}
\underset{\sim}{\pi}_i | \underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{p}_i, \nu, \tau_2, \underset{\sim}{\mu}, \tau_1 \overset{ind}{\sim} \; & \text{Beta}\Big(y_{ijk} + \nu\tau_2, \\
& z_{ijk} + (1-\nu)\tau_2\Big), \\
& j = 1, \ldots, r, k = 1, \ldots, c,
\end{aligned}
\tag{3.12}
$$

$$
\begin{aligned}
\underset{\sim}{z}_i | \underset{\sim}{y}_i, \underset{\sim}{p}_i, \underset{\sim}{\pi}_i, \nu, \tau_2, \underset{\sim}{\mu}, \tau_1 \overset{ind}{\sim} \; & \text{Multinomial}\Big(n_i - r_i, \underset{\sim}{q}_i\Big), \\
& r_i = \sum_{\ell=1}^{n_i} r_{i\ell}, q_{ijk} = \frac{p_{ijk}(1-\pi_{ijk})}{\sum_{j=1}^{r} \sum_{k=1}^{c} p_{ijk}(1-\pi_{ijk})}, \\
& j = 1, \ldots, r, k = 1, \ldots, c,
\end{aligned}
\tag{3.13}
$$

		lags	ACF	Str		lags	ACF	Str		lags	ACF	Str
		1	-0.22	0.0316		1	-0.07	0.0316		1	-0.27	0.0316
		5	0.01	0.0315		5	-0.04	0.0315		5	0.01	0.0315
	μ_1	10	-0.02	0.0314	μ_4	10	-0.08	0.0314	τ_1	10	0.01	0.0314
		15	-0.03	0.0314		15	0.00	0.0314		15	0.02	0.0314
		20	0.00	0.0313		20	-0.02	0.0313		20	0.01	0.0313
		1	-0.17	0.0316		1	-0.09	0.0316		1	-0.03	0.0316
		5	0.00	0.0315		5	0.01	0.0315		5	0.02	0.0315
h	μ_2	10	-0.02	0.0314	μ_5	10	0.01	0.0314	τ_2	10	-0.06	0.0314
		15	-0.02	0.0314		15	0.01	0.0314		15	0.05	0.0314
		20	-0.02	0.0313		20	-0.06	0.0313		20	0.01	0.0313
		1	-0.20	0.0316		1	-0.13	0.0316		1	0.34	0.0316
		5	-0.01	0.0315		5	0.01	0.0315		5	-0.01	0.0315
	μ_3	10	0.00	0.0314	μ_6	10	0.02	0.0314	ν	10	0.01	0.0314
		15	0.04	0.0314		15	-0.03	0.0314		15	0.02	0.0314
		20	-0.02	0.0313		20	-0.01	0.0313		20	0.02	0.0313

Table 3.2: Sample autocorrelation for nonignorable nonresponse model (ACF is the autocorrelation and Str is the standard error)

$$
\begin{aligned}
p(\underset{\sim}{\mu}, \tau_1 | \underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{p}_i, \underset{\sim}{\pi}_i, \nu, \tau_2) \quad \propto \quad & \prod_{i=1}^{N} \prod_{j=1}^{r} \prod_{k=1}^{c} \left\{ \frac{p_{ijk}^{\mu_{jk}\tau_1 - 1}}{D(\underset{\sim}{\mu}\tau_1)} \right\} \frac{1}{(1+\tau_2)^2}, \\
& j = 1, \ldots, r, k = 1, \ldots, c, \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
p(\nu, \tau_2 | \underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{\pi}_i, \underset{\sim}{p}_i, \underset{\sim}{\mu}, \tau_1) \quad \propto \quad & \prod_{i=1}^{N} \left\{ \frac{\prod_{j=1}^{r} \prod_{k=1}^{c} \pi_{ijk}^{\nu\tau_2 - 1} (1 - \pi_{ijk})^{(1-\nu)\tau_2 - 1}}{\mathrm{B}(\nu\tau_2, (1-\nu)\tau_2)} \right\} \\
& \times \frac{\nu^{\alpha_0 - 1}(1-\nu)^{\beta_0 - 1}}{(1+\tau_2)^2}, \\
& 0 < \nu < 1, j = 1, \ldots, r, k = 1, \ldots, c. \qquad (3.15)
\end{aligned}
$$

We also use grids to draw samples from μ, τ_1 and ν, τ_2, we draw, "burn in" and pick the same iterates as we do for ignorable nonresponse model. Table 3.2 shows that the autocorrelations are small and within ± 2. Table 3.3 and Table 3.4 give the estimates of $\underset{\sim}{p}$ and $\underset{\sim}{\pi}_i$.

Cell	Ig	Cou	PM	PSD	CI	Cou	PM	PSD	CI
	ig		0.529	0.055	(0.419, 0.634)		0.536	0.058	(0.425, 0.654)
11	nig		0.523	0.055	(0.415, 0.636)		0.531	0.059	(0.416, 0.644)
	ig		0.133	0.036	(0.072, 0.209)		0.091	0.035	(0.035, 0.165)
12	nig		0.140	0.039	(0.070, 0.221)		0.093	0.035	(0.036, 0.173)
	ig		0.002	0.004	(0.000, 0.015)		0.002	0.005	(0.000, 0.017)
13	nig		0.002	0.005	(0.000, 0.017)		0.002	0.007	(0.000, 0.022)
	ig	1	0.127	0.036	(0.066, 0.202)	7	0.135	0.040	(0.059, 0.221)
21	nig		0.129	0.037	(0.065, 0.210)		0.134	0.040	(0.068, 0.215)
	ig		0.130	0.039	(0.059, 0.211)		0.176	0.044	(0.097, 0.275)
22	nig		0.128	0.037	(0.063, 0.208)		0.176	0.043	(0.093, 0.264)
	ig		0.079	0.029	(0.031, 0.141)		0.061	0.029	(0.016, 0.124)
23	nig		0.078	0.027	(0.034, 0.139)		0.064	0.029	(0.019, 0.133)
	ig		0.481	0.057	(0.366, 0.586)		0.381	0.050	(0.286, 0.479)
11	nig		0.473	0.058	(0.364, 0.586)		0.378	0.049	(0.283, 0.474)
	ig		0.104	0.037	(0.044, 0.189)		0.087	0.031	(0.036, 0.157)
12	nig		0.107	0.038	(0.049, 0.191)		0.088	0.030	(0.037, 0.151)
	ig		0.016	0.015	(0.001, 0.054)		0.002	0.005	(0.000, 0.014)
13	nig	2	0.016	0.014	(0.001, 0.052)	8	0.002	0.004	(0.000, 0.013)
	ig		0.191	0.048	(0.108, 0.295)		0.211	0.043	(0.132, 0.300)
21	nig		0.190	0.048	(0.104, 0.285)		0.214	0.045	(0.134, 0.306)
	ig		0.156	0.044	(0.075, 0.254)		0.228	0.044	(0.146, 0.320)
22	nig		0.161	0.045	(0.085, 0.253)		0.227	0.045	(0.143, 0.319)
	ig		0.052	0.029	(0.009, 0.121)		0.091	0.032	(0.038, 0.157)
23	nig		0.053	0.029	(0.008, 0.120)		0.093	0.033	(0.039, 0.165)
	ig		0.537	0.053	(0.431, 0.640)		0.408	0.042	(0.326, 0.493)
11	nig		0.537	0.056	(0.427, 0.646)		0.404	0.043	(0.318, 0.487)
	ig		0.108	0.037	(0.045, 0.188)		0.087	0.026	(0.043, 0.144)
12	nig		0.109	0.037	(0.047, 0.189)		0.089	0.026	(0.046, 0.145)
	ig		0.002	0.006	(0.000, 0.017)		0.001	0.003	(0.000, 0.011)
13	nig	3	0.002	0.006	(0.000, 0.022)	9	0.001	0.003	(0.000, 0.010)
	ig		0.173	0.039	(0.102, 0.251)		0.198	0.035	(0.134, 0.270)
21	nig		0.172	0.041	(0.100, 0.263)		0.201	0.037	(0.132, 0.274)
	ig		0.140	0.036	(0.076, 0.214)		0.189	0.039	(0.119, 0.279)
22	nig		0.140	0.038	(0.078, 0.227)		0.190	0.037	(0.125, 0.269)
	ig		0.039	0.022	(0.007, 0.091)		0.116	0.031	(0.063, 0.178)
23	nig		0.040	0.022	(0.007, 0.091)		0.114	0.030	(0.061, 0.178)
	ig		0.578	0.023	(0.533, 0.625)		0.170	0.056	(0.073, 0.282)
11	nig		0.573	0.024	(0.526, 0.620)		0.167	0.053	(0.072, 0.276)
	ig		0.069	0.014	(0.044, 0.100)		0.022	0.021	(0.000, 0.074)
12	nig		0.074	0.016	(0.046, 0.110)		0.022	0.023	(0.000, 0.085)
	ig		0.000	0.001	(0.000, 0.003)		0.003	0.008	(0.000, 0.025)
13	nig	4	0.000	0.001	(0.000, 0.004)	10	0.003	0.007	(0.000, 0.023)
	ig		0.143	0.018	(0.107, 0.180)		0.350	0.072	(0.221, 0.508)
21	nig		0.144	0.018	(0.109, 0.183)		0.344	0.070	(0.218, 0.484)
	ig		0.167	0.019	(0.131, 0.205)		0.290	0.064	(0.176, 0.420)
22	nig		0.163	0.020	(0.125, 0.201)		0.293	0.064	(0.178, 0.421)
	ig		0.042	0.011	(0.024, 0.066)		0.165	0.051	(0.073, 0.284)
23	nig		0.044	0.011	(0.025, 0.067)		0.171	0.054	(0.075, 0.286)
	ig		0.473	0.048	(0.377, 0.564)		0.384	0.058	(0.274, 0.497)
11	nig		0.473	0.049	(0.381, 0.571)		0.384	0.058	(0.276, 0.501)
	ig		0.090	0.029	(0.042, 0.150)		0.080	0.032	(0.028, 0.151)
12	nig		0.091	0.030	(0.040, 0.154)		0.081	0.035	(0.027, 0.164)
	ig		0.002	0.004	(0.000, 0.016)		0.033	0.022	(0.004, 0.082)
13	nig	5	0.002	0.005	(0.000, 0.015)	11	0.033	0.021	(0.004, 0.091)
	ig		0.198	0.040	(0.124, 0.280)		0.173	0.053	(0.076, 0.283)
21	nig		0.198	0.038	(0.128, 0.279)		0.173	0.053	(0.076, 0.287)
	ig		0.157	0.037	(0.091, 0.232)		0.219	0.055	(0.115, 0.328)
22	nig		0.157	0.036	(0.088, 0.232)		0.216	0.056	(0.114, 0.335)
	ig		0.080	0.028	(0.034, 0.145)		0.112	0.043	(0.039, 0.207)
23	nig		0.080	0.029	(0.032, 0.147)		0.113	0.045	(0.039, 0.209)
	ig		0.467	0.052	(0.362, 0.574)		0.425	0.059	(0.314, 0.539)
11	nig		0.473	0.054	(0.370, 0.584)		0.423	0.059	(0.312, 0.543)
	ig		0.103	0.038	(0.040, 0.189)		0.067	0.030	(0.019, 0.136)
12	nig		0.099	0.038	(0.038, 0.181)		0.067	0.031	(0.019, 0.139)
	ig		0.002	0.006	(0.000, 0.019)		0.002	0.006	(0.000, 0.021)
13	nig	6	0.002	0.006	(0.000, 0.020)	12	0.002	0.006	(0.000, 0.019)
	ig		0.180	0.045	(0.102, 0.283)		0.153	0.045	(0.071, 0.250)
21	nig		0.182	0.044	(0.101, 0.270)		0.155	0.044	(0.079, 0.249)
	ig		0.168	0.042	(0.089, 0.254)		0.188	0.050	(0.101, 0.295)
22	nig		0.167	0.042	(0.091, 0.257)		0.187	0.049	(0.101, 0.285)
	ig		0.079	0.030	(0.029, 0.146)		0.165	0.046	(0.083, 0.262)
23	nig		0.077	0.030	(0.028, 0.143)		0.165	0.048	(0.082, 0.275)

Table 3.3: The posterior mean (PM), posterior standard error (PSD) and 95% confidence interval (CI) for the p by county (NSE < 0.015)

Cell	Ig	Cou	PM	PSD	CI	Cou	PM	PSD	CI
	ig		0.581	0.046	(0.494, 0.668)		0.182	0.070	(0.054, 0.321)
11	nig		0.574	0.044	(0.487, 0.660)		0.178	0.068	(0.052, 0.316)
	ig		0.075	0.025	(0.034, 0.129)		0.027	0.028	(0.000, 0.102)
12	nig		0.081	0.027	(0.038, 0.138)		0.025	0.027	(0.000, 0.089)
	ig		0.001	0.004	(0.000, 0.012)		0.003	0.010	(0.000, 0.030)
13	nig	13	0.001	0.004	(0.000, 0.010)	19	0.003	0.010	(0.000, 0.026)
	ig		0.146	0.034	(0.082, 0.218)		0.336	0.088	(0.183, 0.526)
21	nig		0.146	0.033	(0.087, 0.210)		0.340	0.092	(0.182, 0.539)
	ig		0.149	0.034	(0.085, 0.221)		0.309	0.086	(0.164, 0.494)
22	nig		0.149	0.034	(0.088, 0.218)		0.309	0.083	(0.163, 0.478)
	ig		0.048	0.022	(0.013, 0.098)		0.143	0.061	(0.045, 0.283)
23	nig		0.049	0.022	(0.012, 0.100)		0.145	0.063	(0.048, 0.299)
	ig		0.325	0.048	(0.231, 0.421)		0.300	0.059	(0.187, 0.409)
11	nig		0.320	0.049	(0.225, 0.419)		0.301	0.059	(0.191, 0.426)
	ig		0.082	0.031	(0.032, 0.152)		0.058	0.031	(0.012, 0.127)
12	nig		0.087	0.032	(0.035, 0.159)		0.062	0.033	(0.012, 0.137)
	ig		0.001	0.004	(0.000, 0.014)		0.003	0.007	(0.000, 0.025)
13	nig	14	0.002	0.005	(0.000, 0.014)	20	0.002	0.006	(0.000, 0.023)
	ig		0.163	0.042	(0.086, 0.247)		0.157	0.052	(0.067, 0.271)
21	nig		0.163	0.043	(0.086, 0.254)		0.156	0.054	(0.064, 0.273)
	ig		0.297	0.052	(0.199, 0.406)		0.306	0.065	(0.186, 0.441)
22	nig		0.297	0.055	(0.201, 0.421)		0.300	0.066	(0.186, 0.441)
	ig		0.132	0.039	(0.063, 0.214)		0.178	0.056	(0.085, 0.296)
23	nig		0.131	0.040	(0.064, 0.217)		0.179	0.058	(0.084, 0.312)
	ig		0.382	0.061	(0.272, 0.506)		0.205	0.061	(0.094, 0.331)
11	nig		0.382	0.061	(0.264, 0.504)		0.199	0.060	(0.090, 0.316)
	ig		0.128	0.044	(0.055, 0.226)		0.085	0.038	(0.026, 0.171)
12	nig		0.133	0.045	(0.058, 0.232)		0.084	0.039	(0.025, 0.172)
	ig		0.003	0.006	(0.000, 0.024)		0.003	0.007	(0.000, 0.025)
13	nig	15	0.003	0.008	(0.000, 0.023)	21	0.003	0.007	(0.000, 0.023)
	ig		0.185	0.050	(0.098, 0.302)		0.213	0.056	(0.111, 0.326)
21	nig		0.185	0.048	(0.095, 0.280)		0.217	0.059	(0.115, 0.341)
	ig		0.241	0.057	(0.139, 0.359)		0.363	0.069	(0.233, 0.504)
22	nig		0.237	0.055	(0.136, 0.353)		0.362	0.072	(0.230, 0.524)
	ig		0.061	0.032	(0.014, 0.136)		0.131	0.048	(0.055, 0.243)
23	nig		0.060	0.032	(0.010, 0.130)		0.136	0.049	(0.056, 0.249)
	ig		0.403	0.065	(0.278, 0.532)		0.255	0.060	(0.147, 0.372)
11	nig		0.400	0.068	(0.274, 0.531)		0.256	0.058	(0.149, 0.368)
	ig		0.040	0.027	(0.003, 0.108)		0.100	0.040	(0.036, 0.191)
12	nig		0.039	0.027	(0.003, 0.106)		0.102	0.043	(0.038, 0.201)
	ig		0.002	0.007	(0.000, 0.021)		0.003	0.007	(0.000, 0.021)
13	nig	16	0.003	0.007	(0.000, 0.021)	22	0.002	0.007	(0.000, 0.022)
	ig		0.271	0.069	(0.150, 0.414)		0.171	0.063	(0.064, 0.312)
21	nig		0.270	0.069	(0.149, 0.419)		0.165	0.066	(0.053, 0.311)
	ig		0.193	0.062	(0.088, 0.325)		0.330	0.080	(0.178, 0.494)
22	nig		0.199	0.066	(0.082, 0.333)		0.340	0.082	(0.189, 0.506)
	ig		0.090	0.046	(0.018, 0.193)		0.141	0.059	(0.041, 0.269)
23	nig		0.090	0.045	(0.023, 0.195)		0.135	0.061	(0.043, 0.285)
	ig		0.242	0.053	(0.139, 0.352)		0.395	0.073	(0.259, 0.548)
11	nig		0.240	0.051	(0.146, 0.346)		0.395	0.076	(0.256, 0.551)
	ig		0.046	0.025	(0.010, 0.103)		0.136	0.055	(0.045, 0.252)
12	nig		0.049	0.026	(0.009, 0.108)		0.139	0.057	(0.047, 0.268)
	ig		0.002	0.005	(0.000, 0.017)		0.004	0.012	(0.000, 0.038)
13	nig	17	0.002	0.006	(0.000, 0.021)	23	0.003	0.010	(0.000, 0.028)
	ig		0.264	0.061	(0.155, 0.391)		0.195	0.064	(0.085, 0.333)
21	nig		0.263	0.059	(0.157, 0.383)		0.196	0.066	(0.077, 0.333)
	ig		0.223	0.056	(0.125, 0.343)		0.138	0.061	(0.032, 0.275)
22	nig		0.227	0.058	(0.122, 0.354)		0.133	0.059	(0.031, 0.262)
	ig		0.224	0.058	(0.122, 0.347)		0.133	0.057	(0.042, 0.270)
23	nig		0.219	0.059	(0.121, 0.350)		0.133	0.057	(0.041, 0.264)
	ig		0.339	0.059	(0.232, 0.452)		0.297	0.075	(0.153, 0.452)
11	nig		0.344	0.059	(0.231, 0.459)		0.292	0.076	(0.148, 0.445)
	ig		0.077	0.035	(0.023, 0.155)		0.124	0.057	(0.035, 0.252)
12	nig		0.075	0.036	(0.019, 0.161)		0.128	0.058	(0.040, 0.262)
	ig		0.003	0.008	(0.000, 0.029)		0.004	0.010	(0.000, 0.032)
13	nig	18	0.003	0.007	(0.000, 0.023)	24	0.004	0.011	(0.000, 0.034)
	ig		0.263	0.061	(0.151, 0.389)		0.242	0.079	(0.101, 0.419)
21	nig		0.260	0.062	(0.147, 0.394)		0.243	0.081	(0.108, 0.431)
	ig		0.268	0.063	(0.156, 0.407)		0.206	0.079	(0.066, 0.376)
22	nig		0.265	0.064	(0.154, 0.398)		0.207	0.079	(0.068, 0.377)
	ig		0.051	0.034	(0.004, 0.126)		0.128	0.063	(0.031, 0.279)
23	nig		0.053	0.034	(0.004, 0.130)		0.126	0.059	(0.038, 0.258)

Cell	Ig	Cou	PM	PSD	CI	Cou	PM	PSD	CI
	ig		0.280	0.064	(0.150, 0.400)		0.423	0.065	(0.299, 0.553)
11	nig		0.280	0.066	(0.154, 0.414)		0.423	0.066	(0.299, 0.558)
	ig		0.066	0.037	(0.014, 0.153)		0.153	0.048	(0.073, 0.259)
12	nig		0.069	0.037	(0.014, 0.153)		0.153	0.048	(0.073, 0.263)
	ig		0.003	0.008	(0.000, 0.029)		0.021	0.021	(0.001, 0.075)
13	nig	25	0.003	0.008	(0.000, 0.025)	31	0.021	0.021	(0.001, 0.077)
	ig		0.348	0.079	(0.205, 0.507)		0.163	0.052	(0.071, 0.268)
21	nig		0.341	0.076	(0.208, 0.505)		0.163	0.052	(0.068, 0.273)
	ig		0.215	0.064	(0.104, 0.353)		0.099	0.047	(0.021, 0.206)
22	nig		0.220	0.065	(0.103, 0.363)		0.098	0.048	(0.017, 0.211)
	ig		0.088	0.047	(0.019, 0.189)		0.140	0.051	(0.058, 0.258)
23	nig		0.087	0.045	(0.018, 0.186)		0.142	0.049	(0.058, 0.246)
	ig		0.343	0.061	(0.228, 0.467)		0.420	0.047	(0.333, 0.518)
11	nig		0.342	0.057	(0.230, 0.461)		0.420	0.049	(0.327, 0.513)
	ig		0.068	0.037	(0.013, 0.151)		0.109	0.031	(0.054, 0.175)
12	nig		0.067	0.036	(0.013, 0.148)		0.112	0.034	(0.055, 0.182)
	ig		0.003	0.007	(0.000, 0.025)		0.001	0.004	(0.000, 0.012)
13	nig	26	0.002	0.007	(0.000, 0.022)	32	0.001	0.004	(0.000, 0.013)
	ig		0.252	0.069	(0.130, 0.398)		0.237	0.039	(0.164, 0.320)
21	nig		0.260	0.072	(0.128, 0.409)		0.235	0.039	(0.163, 0.318)
	ig		0.169	0.065	(0.054, 0.308)		0.183	0.036	(0.121, 0.259)
22	nig		0.165	0.065	(0.052, 0.304)		0.182	0.035	(0.117, 0.254)
	ig		0.166	0.063	(0.060, 0.303)		0.050	0.020	(0.019, 0.097)
23	nig		0.163	0.063	(0.061, 0.303)		0.050	0.021	(0.016, 0.099)
	ig		0.170	0.053	(0.076, 0.282)		0.693	0.050	(0.591, 0.790)
11	nig		0.171	0.051	(0.081, 0.278)		0.689	0.052	(0.587, 0.789)
	ig		0.037	0.027	(0.003, 0.106)		0.068	0.027	(0.025, 0.127)
12	nig		0.038	0.026	(0.004, 0.104)		0.072	0.028	(0.025, 0.134)
	ig		0.002	0.006	(0.000, 0.019)		0.001	0.003	(0.000, 0.010)
13	nig	27	0.002	0.006	(0.000, 0.021)	33	0.002	0.004	(0.000, 0.014)
	ig		0.193	0.068	(0.074, 0.341)		0.101	0.029	(0.051, 0.168)
21	nig		0.190	0.072	(0.067, 0.340)		0.102	0.030	(0.052, 0.166)
	ig		0.466	0.091	(0.301, 0.662)		0.093	0.029	(0.041, 0.150)
22	nig		0.471	0.092	(0.300, 0.661)		0.093	0.030	(0.041, 0.154)
	ig		0.131	0.058	(0.044, 0.265)		0.044	0.020	(0.012, 0.087)
23	nig		0.127	0.056	(0.035, 0.254)		0.043	0.019	(0.012, 0.085)
	ig		0.265	0.074	(0.130, 0.418)		0.585	0.064	(0.467, 0.705)
11	nig		0.264	0.075	(0.128, 0.417)		0.580	0.065	(0.456, 0.709)
	ig		0.112	0.052	(0.035, 0.230)		0.081	0.032	(0.029, 0.153)
12	nig		0.112	0.052	(0.034, 0.237)		0.084	0.034	(0.029, 0.158)
	ig		0.003	0.010	(0.000, 0.031)		0.002	0.006	(0.000, 0.021)
13	nig	28	0.004	0.010	(0.000, 0.031)	34	0.002	0.006	(0.000, 0.018)
	ig		0.178	0.076	(0.049, 0.334)		0.124	0.045	(0.045, 0.222)
21	nig		0.177	0.077	(0.051, 0.340)		0.124	0.046	(0.043, 0.220)
	ig		0.221	0.080	(0.082, 0.391)		0.125	0.042	(0.048, 0.212)
22	nig		0.225	0.083	(0.081, 0.415)		0.124	0.045	(0.047, 0.217)
	ig		0.221	0.089	(0.084, 0.422)		0.083	0.036	(0.028, 0.166)
23	nig		0.218	0.088	(0.075, 0.424)		0.086	0.038	(0.025, 0.174)
	ig		0.339	0.047	(0.253, 0.435)		0.310	0.045	(0.227, 0.401)
11	nig		0.337	0.050	(0.241, 0.432)		0.310	0.045	(0.223, 0.401)
	ig		0.065	0.027	(0.021, 0.125)		0.116	0.031	(0.062, 0.182)
12	nig		0.067	0.029	(0.020, 0.130)		0.116	0.033	(0.061, 0.186)
	ig		0.002	0.005	(0.000, 0.013)		0.001	0.004	(0.000, 0.014)
13	nig	29	0.002	0.004	(0.000, 0.016)	35	0.001	0.003	(0.000, 0.011)
	ig		0.239	0.057	(0.137, 0.353)		0.183	0.038	(0.114, 0.262)
21	nig		0.238	0.056	(0.136, 0.365)		0.183	0.039	(0.110, 0.264)
	ig		0.222	0.052	(0.128, 0.334)		0.312	0.045	(0.227, 0.404)
22	nig		0.226	0.055	(0.129, 0.345)		0.311	0.046	(0.225, 0.406)
	ig		0.133	0.046	(0.055, 0.233)		0.078	0.028	(0.030, 0.137)
23	nig		0.131	0.044	(0.061, 0.224)		0.078	0.028	(0.032, 0.141)
	ig		0.424	0.043	(0.348, 0.512)				
11	nig		0.422	0.041	(0.343, 0.504)				
	ig		0.068	0.025	(0.027, 0.126)				
12	nig		0.070	0.025	(0.031, 0.125)				
	ig		0.001	0.003	(0.000, 0.011)				
13	nig	30	0.001	0.004	(0.000, 0.013)				
	ig		0.174	0.040	(0.099, 0.260)				
21	nig		0.176	0.043	(0.102, 0.267)				
	ig		0.278	0.045	(0.195, 0.370)				
22	nig		0.278	0.046	(0.189, 0.369)				
	ig		0.054	0.025	(0.016, 0.114)				
23	nig		0.052	0.025	(0.013, 0.109)				

Cell	Ig	Cou	PM	PSD	CI	Cou	PM	PSD	CI
	ig		0.742	0.042	(0.655, 0.814)		0.801	0.043	(0.710, 0.878)
11	nig		0.658	0.043	(0.574, 0.743)		0.663	0.045	(0.578, 0.750)
	ig		0.742	0.042	(0.655, 0.814)		0.801	0.043	(0.710, 0.878)
12	nig		0.616	0.049	(0.520, 0.707)		0.606	0.051	(0.501, 0.709)
	ig		0.742	0.042	(0.655, 0.814)		0.801	0.043	(0.710, 0.878)
13	nig	1	0.597	0.054	(0.489, 0.701)	7	0.600	0.053	(0.497, 0.702)
	ig		0.742	0.042	(0.655, 0.814)		0.801	0.043	(0.710, 0.878)
21	nig		0.611	0.048	(0.512, 0.706)		0.621	0.048	(0.522, 0.717)
	ig		0.742	0.042	(0.655, 0.814)		0.801	0.043	(0.710, 0.878)
22	nig		0.613	0.049	(0.513, 0.707)		0.624	0.049	(0.531, 0.719)
	ig		0.742	0.042	(0.655, 0.814)		0.801	0.043	(0.710, 0.878)
23	nig		0.605	0.048	(0.508, 0.697)		0.605	0.051	(0.503, 0.702)
	ig		0.742	0.044	(0.647, 0.827)		0.762	0.040	(0.677, 0.837)
11	nig		0.648	0.045	(0.563, 0.738)		0.651	0.046	(0.560, 0.740)
	ig		0.742	0.044	(0.647, 0.827)		0.762	0.040	(0.677, 0.837)
12	nig		0.611	0.052	(0.506, 0.712)		0.609	0.049	(0.513, 0.705)
	ig		0.742	0.044	(0.647, 0.827)		0.762	0.040	(0.677, 0.837)
13	nig	2	0.599	0.052	(0.497, 0.700)	8	0.601	0.052	(0.495, 0.702)
	ig		0.742	0.044	(0.647, 0.827)		0.762	0.040	(0.677, 0.837)
21	nig		0.614	0.047	(0.523, 0.705)		0.627	0.046	(0.536, 0.715)
	ig		0.742	0.044	(0.647, 0.827)		0.762	0.040	(0.677, 0.837)
22	nig		0.612	0.050	(0.514, 0.711)		0.630	0.049	(0.536, 0.725)
	ig		0.742	0.044	(0.647, 0.827)		0.762	0.040	(0.677, 0.837)
23	nig		0.599	0.052	(0.499, 0.699)		0.609	0.052	(0.504, 0.712)
	ig		0.684	0.043	(0.598, 0.767)		0.766	0.034	(0.694, 0.830)
11	nig		0.603	0.043	(0.521, 0.687)		0.674	0.043	(0.592, 0.758)
	ig		0.684	0.043	(0.598, 0.767)		0.766	0.034	(0.694, 0.830)
12	nig		0.599	0.051	(0.492, 0.696)		0.618	0.048	(0.526, 0.711)
	ig		0.684	0.043	(0.598, 0.767)		0.766	0.034	(0.694, 0.830)
13	nig	3	0.596	0.052	(0.493, 0.696)	9	0.600	0.052	(0.495, 0.700)
	ig		0.684	0.043	(0.598, 0.767)		0.766	0.034	(0.694, 0.830)
21	nig		0.630	0.049	(0.534, 0.724)		0.628	0.046	(0.539, 0.717)
	ig		0.684	0.043	(0.598, 0.767)		0.766	0.034	(0.694, 0.830)
22	nig		0.621	0.050	(0.520, 0.720)		0.628	0.048	(0.530, 0.720)
	ig		0.684	0.043	(0.598, 0.767)		0.766	0.034	(0.694, 0.830)
23	nig		0.604	0.050	(0.504, 0.703)		0.617	0.051	(0.517, 0.714)
	ig		0.683	0.021	(0.642, 0.724)		0.727	0.054	(0.614, 0.828)
11	nig		0.662	0.026	(0.611, 0.712)		0.607	0.050	(0.507, 0.701)
	ig		0.683	0.021	(0.642, 0.724)		0.727	0.054	(0.614, 0.828)
12	nig		0.608	0.048	(0.514, 0.701)		0.597	0.048	(0.497, 0.682)
	ig		0.683	0.021	(0.642, 0.724)		0.727	0.054	(0.614, 0.828)
13	nig	4	0.598	0.052	(0.491, 0.696)	10	0.601	0.052	(0.496, 0.703)
	ig		0.683	0.021	(0.642, 0.724)		0.727	0.054	(0.614, 0.828)
21	nig		0.635	0.043	(0.546, 0.717)		0.628	0.049	(0.527, 0.714)
	ig		0.683	0.021	(0.642, 0.724)		0.727	0.054	(0.614, 0.828)
22	nig		0.640	0.043	(0.556, 0.725)		0.621	0.048	(0.524, 0.710)
	ig		0.683	0.021	(0.642, 0.724)		0.727	0.054	(0.614, 0.828)
23	nig		0.609	0.048	(0.509, 0.708)		0.611	0.050	(0.508, 0.705)
	ig		0.746	0.040	(0.662, 0.822)		0.724	0.051	(0.617, 0.812)
11	nig		0.656	0.043	(0.572, 0.741)		0.652	0.046	(0.569, 0.746)
	ig		0.746	0.040	(0.662, 0.822)		0.724	0.051	(0.617, 0.812)
12	nig		0.609	0.051	(0.513, 0.710)		0.611	0.050	(0.513, 0.712)
	ig		0.746	0.040	(0.662, 0.822)		0.724	0.051	(0.617, 0.812)
13	nig	5	0.597	0.052	(0.494, 0.697)	11	0.605	0.054	(0.494, 0.711)
	ig		0.746	0.040	(0.662, 0.822)		0.724	0.051	(0.617, 0.812)
21	nig		0.628	0.048	(0.528, 0.721)		0.601	0.052	(0.496, 0.697)
	ig		0.746	0.040	(0.662, 0.822)		0.724	0.051	(0.617, 0.812)
22	nig		0.618	0.050	(0.521, 0.710)		0.599	0.050	(0.500, 0.699)
	ig		0.746	0.040	(0.662, 0.822)		0.724	0.051	(0.617, 0.812)
23	nig		0.612	0.049	(0.508, 0.707)		0.599	0.051	(0.501, 0.702)
	ig		0.538	0.046	(0.447, 0.628)		0.736	0.046	(0.634, 0.820)
11	nig		0.562	0.044	(0.471, 0.646)		0.637	0.046	(0.547, 0.734)
	ig		0.538	0.046	(0.447, 0.628)		0.736	0.046	(0.634, 0.820)
12	nig		0.588	0.055	(0.476, 0.703)		0.602	0.050	(0.504, 0.698)
	ig		0.538	0.046	(0.447, 0.628)		0.736	0.046	(0.634, 0.820)
13	nig	6	0.597	0.052	(0.489, 0.690)	12	0.602	0.052	(0.494, 0.706)
	ig		0.538	0.046	(0.447, 0.628)		0.736	0.046	(0.634, 0.820)
21	nig		0.594	0.048	(0.501, 0.687)		0.606	0.048	(0.509, 0.703)
	ig		0.538	0.046	(0.447, 0.628)		0.736	0.046	(0.634, 0.820)
22	nig		0.593	0.051	(0.486, 0.693)		0.613	0.050	(0.517, 0.709)
	ig		0.538	0.046	(0.447, 0.628)		0.736	0.046	(0.634, 0.820)
23	nig		0.594	0.052	(0.492, 0.703)		0.616	0.051	(0.519, 0.716)

Table 3.4: The posterior mean (PM), posterior standard error (PSD) and 95% confidence interval (CI) for the π by county (NSE < 0.017)

Cell	Ig	Cou	PM	PSD	CI	Cou	PM	PSD	CI
	ig		0.658	0.038	(0.579, 0.732)		0.685	0.066	(0.545, 0.800)
11	nig		0.648	0.038	(0.573, 0.720)		0.614	0.050	(0.511, 0.706)
	ig		0.658	0.038	(0.579, 0.732)		0.685	0.066	(0.545, 0.800)
12	nig		0.601	0.050	(0.501, 0.693)		0.598	0.051	(0.496, 0.700)
	ig		0.658	0.038	(0.579, 0.732)		0.685	0.066	(0.545, 0.800)
13	nig	13	0.598	0.053	(0.491, 0.698)	19	0.598	0.053	(0.487, 0.699)
	ig		0.658	0.038	(0.579, 0.732)		0.685	0.066	(0.545, 0.800)
21	nig		0.592	0.049	(0.499, 0.688)		0.604	0.049	(0.499, 0.701)
	ig		0.658	0.038	(0.579, 0.732)		0.685	0.066	(0.545, 0.800)
22	nig		0.589	0.050	(0.484, 0.684)		0.601	0.050	(0.506, 0.696)
	ig		0.658	0.038	(0.579, 0.732)		0.685	0.066	(0.545, 0.800)
23	nig		0.596	0.051	(0.497, 0.696)		0.601	0.050	(0.504, 0.691)
	ig		0.684	0.044	(0.592, 0.769)		0.662	0.052	(0.559, 0.764)
11	nig		0.619	0.046	(0.530, 0.706)		0.617	0.049	(0.519, 0.714)
	ig		0.684	0.044	(0.592, 0.769)		0.662	0.052	(0.559, 0.764)
12	nig		0.602	0.049	(0.508, 0.700)		0.603	0.050	(0.500, 0.697)
	ig		0.684	0.044	(0.592, 0.769)		0.662	0.052	(0.559, 0.764)
13	nig	14	0.598	0.051	(0.501, 0.696)	20	0.600	0.053	(0.492, 0.705)
	ig		0.684	0.044	(0.592, 0.769)		0.662	0.052	(0.559, 0.764)
21	nig		0.606	0.049	(0.505, 0.698)		0.599	0.053	(0.493, 0.703)
	ig		0.684	0.044	(0.592, 0.769)		0.662	0.052	(0.559, 0.764)
22	nig		0.619	0.047	(0.523, 0.712)		0.607	0.048	(0.509, 0.697)
	ig		0.684	0.044	(0.592, 0.769)		0.662	0.052	(0.559, 0.764)
23	nig		0.607	0.052	(0.495, 0.706)		0.602	0.049	(0.502, 0.698)
	ig		0.719	0.050	(0.622, 0.814)		0.789	0.053	(0.677, 0.887)
11	nig		0.623	0.044	(0.530, 0.709)		0.614	0.049	(0.520, 0.710)
	ig		0.719	0.050	(0.622, 0.814)		0.789	0.053	(0.677, 0.887)
12	nig		0.607	0.050	(0.500, 0.700)		0.608	0.052	(0.503, 0.707)
	ig		0.719	0.050	(0.622, 0.814)		0.789	0.053	(0.677, 0.887)
13	nig	15	0.600	0.052	(0.485, 0.703)	21	0.601	0.051	(0.495, 0.699)
	ig		0.719	0.050	(0.622, 0.814)		0.789	0.053	(0.677, 0.887)
21	nig		0.612	0.049	(0.514, 0.706)		0.620	0.049	(0.522, 0.714)
	ig		0.719	0.050	(0.622, 0.814)		0.789	0.053	(0.677, 0.887)
22	nig		0.619	0.050	(0.517, 0.722)		0.636	0.048	(0.542, 0.732)
	ig		0.719	0.050	(0.622, 0.814)		0.789	0.053	(0.677, 0.887)
23	nig		0.602	0.052	(0.498, 0.702)		0.614	0.049	(0.514, 0.700)
	ig		0.733	0.056	(0.620, 0.833)		0.501	0.059	(0.386, 0.616)
11	nig		0.653	0.046	(0.561, 0.742)		0.603	0.048	(0.509, 0.698)
	ig		0.733	0.056	(0.620, 0.833)		0.501	0.059	(0.386, 0.616)
12	nig		0.602	0.052	(0.502, 0.705)		0.602	0.051	(0.500, 0.699)
	ig		0.733	0.056	(0.620, 0.833)		0.501	0.059	(0.386, 0.616)
13	nig	16	0.600	0.051	(0.495, 0.696)	22	0.599	0.051	(0.495, 0.698)
	ig		0.733	0.056	(0.620, 0.833)		0.501	0.059	(0.386, 0.616)
21	nig		0.602	0.049	(0.503, 0.699)		0.575	0.055	(0.461, 0.679)
	ig		0.733	0.056	(0.620, 0.833)		0.501	0.059	(0.386, 0.616)
22	nig		0.599	0.052	(0.501, 0.702)		0.559	0.050	(0.454, 0.662)
	ig		0.733	0.056	(0.620, 0.833)		0.501	0.059	(0.386, 0.616)
23	nig		0.598	0.050	(0.496, 0.697)		0.581	0.054	(0.466, 0.690)
	ig		0.677	0.052	(0.574, 0.773)		0.636	0.064	(0.510, 0.760)
11	nig		0.628	0.046	(0.538, 0.716)		0.611	0.048	(0.518, 0.701)
	ig		0.677	0.052	(0.574, 0.773)		0.636	0.064	(0.510, 0.760)
12	nig		0.605	0.049	(0.508, 0.706)		0.606	0.050	(0.505, 0.701)
	ig		0.677	0.052	(0.574, 0.773)		0.636	0.064	(0.510, 0.760)
13	nig	17	0.598	0.050	(0.494, 0.695)	23	0.598	0.050	(0.493, 0.691)
	ig		0.677	0.052	(0.574, 0.773)		0.636	0.064	(0.510, 0.760)
21	nig		0.601	0.048	(0.511, 0.691)		0.595	0.051	(0.493, 0.686)
	ig		0.677	0.052	(0.574, 0.773)		0.636	0.064	(0.510, 0.760)
22	nig		0.600	0.049	(0.504, 0.697)		0.594	0.051	(0.498, 0.698)
	ig		0.677	0.052	(0.574, 0.773)		0.636	0.064	(0.510, 0.760)
23	nig		0.604	0.050	(0.507, 0.701)		0.598	0.052	(0.495, 0.694)
	ig		0.656	0.053	(0.556, 0.753)		0.641	0.070	(0.490, 0.764)
11	nig		0.619	0.046	(0.534, 0.714)		0.611	0.049	(0.513, 0.709)
	ig		0.656	0.053	(0.556, 0.753)		0.641	0.070	(0.490, 0.764)
12	nig		0.605	0.052	(0.500, 0.703)		0.603	0.051	(0.506, 0.702)
	ig		0.656	0.053	(0.556, 0.753)		0.641	0.070	(0.490, 0.764)
13	nig	18	0.599	0.051	(0.498, 0.697)	24	0.597	0.053	(0.493, 0.693)
	ig		0.656	0.053	(0.556, 0.753)		0.641	0.070	(0.490, 0.764)
21	nig		0.599	0.050	(0.503, 0.698)		0.595	0.052	(0.492, 0.695)
	ig		0.656	0.053	(0.556, 0.753)		0.641	0.070	(0.490, 0.764)
22	nig		0.600	0.050	(0.505, 0.697)		0.595	0.050	(0.497, 0.692)
	ig		0.656	0.053	(0.556, 0.753)		0.641	0.070	(0.490, 0.764)
23	nig		0.593	0.051	(0.489, 0.686)		0.598	0.052	(0.492, 0.695)

Cell	Ig	Cou	PM	PSD	CI	Cou	PM	PSD	CI
	ig		0.728	0.058	(0.606, 0.832)		0.763	0.052	(0.660, 0.854)
11	nig		0.624	0.050	(0.526, 0.717)		0.650	0.046	(0.560, 0.741)
	ig		0.728	0.058	(0.606, 0.832)		0.763	0.052	(0.660, 0.854)
12	nig		0.606	0.053	(0.505, 0.710)		0.622	0.050	(0.523, 0.722)
	ig		0.728	0.058	(0.606, 0.832)		0.763	0.052	(0.660, 0.854)
13	nig	25	0.599	0.051	(0.498, 0.704)	31	0.599	0.052	(0.489, 0.695)
	ig		0.728	0.058	(0.606, 0.832)		0.763	0.052	(0.660, 0.854)
21	nig		0.617	0.047	(0.527, 0.710)		0.603	0.050	(0.507, 0.710)
	ig		0.728	0.058	(0.606, 0.832)		0.763	0.052	(0.660, 0.854)
22	nig		0.605	0.050	(0.507, 0.703)		0.596	0.054	(0.480, 0.695)
	ig		0.728	0.058	(0.606, 0.832)		0.763	0.052	(0.660, 0.854)
23	nig		0.597	0.051	(0.497, 0.699)		0.607	0.051	(0.495, 0.705)
	ig		0.494	0.055	(0.380, 0.598)		0.803	0.036	(0.729, 0.866)
11	nig		0.594	0.047	(0.492, 0.683)		0.647	0.042	(0.565, 0.733)
	ig		0.494	0.055	(0.380, 0.598)		0.803	0.036	(0.729, 0.866)
12	nig		0.593	0.050	(0.487, 0.687)		0.612	0.048	(0.516, 0.704)
	ig		0.494	0.055	(0.380, 0.598)		0.803	0.036	(0.729, 0.866)
13	nig	26	0.600	0.055	(0.488, 0.706)	32	0.599	0.052	(0.493, 0.699)
	ig		0.494	0.055	(0.380, 0.598)		0.803	0.036	(0.729, 0.866)
21	nig		0.567	0.052	(0.456, 0.662)		0.665	0.046	(0.575, 0.756)
	ig		0.494	0.055	(0.380, 0.598)		0.803	0.036	(0.729, 0.866)
22	nig		0.577	0.053	(0.464, 0.676)		0.654	0.047	(0.563, 0.748)
	ig		0.494	0.055	(0.380, 0.598)		0.803	0.036	(0.729, 0.866)
23	nig		0.577	0.054	(0.472, 0.675)		0.610	0.051	(0.507, 0.708)
	ig		0.530	0.059	(0.410, 0.641)		0.735	0.038	(0.654, 0.810)
11	nig		0.614	0.051	(0.514, 0.712)		0.658	0.037	(0.587, 0.735)
	ig		0.530	0.059	(0.410, 0.641)		0.735	0.038	(0.654, 0.810)
12	nig		0.601	0.050	(0.498, 0.695)		0.604	0.050	(0.506, 0.697)
	ig		0.530	0.059	(0.410, 0.641)		0.735	0.038	(0.654, 0.810)
13	nig	27	0.597	0.054	(0.485, 0.699)	33	0.598	0.052	(0.496, 0.702)
	ig		0.530	0.059	(0.410, 0.641)		0.735	0.038	(0.654, 0.810)
21	nig		0.580	0.054	(0.464, 0.679)		0.617	0.050	(0.515, 0.715)
	ig		0.530	0.059	(0.410, 0.641)		0.735	0.038	(0.654, 0.810)
22	nig		0.565	0.049	(0.459, 0.657)		0.613	0.050	(0.512, 0.705)
	ig		0.530	0.059	(0.410, 0.641)		0.735	0.038	(0.654, 0.810)
23	nig		0.585	0.051	(0.481, 0.679)		0.608	0.052	(0.496, 0.706)
	ig		0.659	0.071	(0.513, 0.789)		0.738	0.049	(0.639, 0.824)
11	nig		0.617	0.048	(0.518, 0.719)		0.664	0.044	(0.580, 0.754)
	ig		0.659	0.071	(0.513, 0.789)		0.738	0.049	(0.639, 0.824)
12	nig		0.611	0.052	(0.506, 0.718)		0.606	0.049	(0.509, 0.705)
	ig		0.659	0.071	(0.513, 0.789)		0.738	0.049	(0.639, 0.824)
13	nig	28	0.598	0.052	(0.496, 0.695)	34	0.598	0.050	(0.496, 0.696)
	ig		0.659	0.071	(0.513, 0.789)		0.738	0.049	(0.639, 0.824)
21	nig		0.593	0.051	(0.496, 0.686)		0.594	0.053	(0.483, 0.694)
	ig		0.659	0.071	(0.513, 0.789)		0.738	0.049	(0.639, 0.824)
22	nig		0.592	0.050	(0.494, 0.690)		0.594	0.053	(0.485, 0.694)
	ig		0.659	0.071	(0.513, 0.789)		0.738	0.049	(0.639, 0.824)
23	nig		0.595	0.053	(0.490, 0.690)		0.598	0.051	(0.494, 0.697)
	ig		0.615	0.047	(0.519, 0.709)		0.785	0.038	(0.708, 0.854)
11	nig		0.634	0.045	(0.546, 0.725)		0.647	0.046	(0.558, 0.739)
	ig		0.615	0.047	(0.519, 0.709)		0.785	0.038	(0.708, 0.854)
12	nig		0.605	0.051	(0.497, 0.701)		0.619	0.049	(0.522, 0.719)
	ig		0.615	0.047	(0.519, 0.709)		0.785	0.038	(0.708, 0.854)
13	nig	29	0.599	0.053	(0.492, 0.696)	35	0.600	0.051	(0.507, 0.697)
	ig		0.615	0.047	(0.519, 0.709)		0.785	0.038	(0.708, 0.854)
21	nig		0.586	0.051	(0.487, 0.687)		0.627	0.048	(0.533, 0.722)
	ig		0.615	0.047	(0.519, 0.709)		0.785	0.038	(0.708, 0.854)
22	nig		0.585	0.050	(0.487, 0.682)		0.654	0.047	(0.565, 0.756)
	ig		0.615	0.047	(0.519, 0.709)		0.785	0.038	(0.708, 0.854)
23	nig		0.589	0.052	(0.485, 0.687)		0.612	0.051	(0.511, 0.713)
	ig		0.578	0.037	(0.500, 0.652)				
11	nig		0.617	0.040	(0.538, 0.698)				
	ig		0.578	0.037	(0.500, 0.652)				
12	nig		0.600	0.051	(0.494, 0.695)				
	ig		0.578	0.037	(0.500, 0.652)				
13	nig	30	0.599	0.053	(0.489, 0.695)				
	ig		0.578	0.037	(0.500, 0.652)				
21	nig		0.575	0.050	(0.477, 0.668)				
	ig		0.578	0.037	(0.500, 0.652)				
22	nig		0.571	0.047	(0.480, 0.662)				
	ig		0.578	0.037	(0.500, 0.652)				
23	nig		0.590	0.052	(0.480, 0.693)				

	association	ONA	INA
Ignorable	Baseline	$\mu_{jk}=u_j v_k$	$p_{ijk}=t_{ij} s_{ik}$
Nonignorable	Baseline	$\mu_{jk}=u_j v_k$	$p_{ijk}=t_{ij} s_{ik}$

Table 3.5: The relationship between association hypothesis and ignorability hypothesis (ONA: overall no association, INA: individual no association)

3.3 Tests for association and ignorability

In this section, we use the Bayes factor to test the association between BMD and age and the ignorability. We test the association between BMD and age by checking the overall association and the individual association within each county. Under each association hypothesis, we also construct both the ignorable nonresponse model and the nonignorable nonresponse model for no association assumptions (See Table 3.5).

3.3.1 Hypothesis

Association Hypothesis

- The hypothesis of overall association:

$$\begin{cases} H_0 & : \text{BMD and age are not associated overall} \\ H_1 & : \text{BMD and age are associated overall} \end{cases}$$

- The hypothesis of individual association:

$$\begin{cases} H_0 & : \text{BMD and age are not associated in } i^{th} \text{ county} \\ H_1 & : \text{BMD and age are associated in } i^{th} \text{ county} \end{cases}$$

Ignorability Hypothesis

$$\begin{cases} H_0 & : \text{Ignorable nonresponse model is better than nonignorable nonresponse model} \\ H_1 & : \text{Ignorable nonresponse model is not better than nonignorable nonresponse model} \end{cases}$$

3.3.2 Models

Baseline models

Letting $\underset{\sim}{\mu} = \{\mu_{11}, \mu_{12}, \mu_{1c}, \mu_{r1}, \mu_{r2}, \mu_{rc}\}$ and $\sum_{j=1}^{r}\sum_{k=1}^{c}\mu_{jk} = 1$, the ignorable nonresponse model is

$$
\begin{aligned}
\underset{\sim}{y}_i, \underset{\sim}{z}_i | \underset{\sim}{p}_i, \pi_i &\overset{ind}{\sim} \text{Multinomial}\Big\{n_i, \pi_i p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c \\
&\qquad (1 - \pi_i) p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c\Big\} && (3.16) \\
\underset{\sim}{p}_i | \underset{\sim}{\mu}, \tau_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}\tau_1), i = 1, \ldots, N && (3.17) \\
\pi_i | \nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1 - \nu)\tau_2), i = 1, \ldots, N && (3.18) \\
\nu &\sim \text{Uniform}(0, 1) && (3.19) \\
\underset{\sim}{\mu} &\sim \text{Dirichlet}(\underset{\sim}{1}) && (3.20) \\
p(\tau_1) &= \frac{1}{(1 + \tau_1)^2}, \tau_1 > 0 && (3.21) \\
p(\tau_2) &= \frac{1}{(1 + \tau_2)^2}, \tau_2 > 0. && (3.22)
\end{aligned}
$$

The nonignorable nonresponse model is

$$
\begin{aligned}
\underset{\sim}{y}_i, \underset{\sim}{z}_i | \underset{\sim}{p}_i, \underset{\sim}{\pi}_i &\overset{ind}{\sim} \text{Multinomial}\Big\{n_i, \pi_{ijk} p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, \\
&\qquad (1 - \pi_{ijk}) p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c\Big\} && (3.23) \\
\underset{\sim}{p}_i | \underset{\sim}{\mu}, \tau_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}\tau_1), i = 1, \ldots, N && (3.24) \\
\pi_{ijk} | \nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1 - \nu)\tau_2), i = 1, \ldots, N && (3.25) \\
\nu &\sim \text{Uniform}(0, 1) && (3.26) \\
\underset{\sim}{\mu} &\sim \text{Dirichlet}(\underset{\sim}{1}) && (3.27) \\
p(\tau_1) &= \frac{1}{(1 + \tau_1)^2}, \tau_1 > 0 && (3.28) \\
p(\tau_2) &= \frac{1}{(1 + \tau_2)^2}, \tau_2 > 0. && (3.29)
\end{aligned}
$$

Overall no association models

In this case, we assume that $\underset{\sim}{p}_i$ depend on $u_j v_k$ in stead of μ_{jk} , where $u_j = \sum_{k=1}^{c} \mu_{jk}$, $v_k = \sum_{j=1}^{r} \mu_{jk}$, j=1,. . .,r and k=1,. . .,c.

Thus, the model with ignorability of no association is

$$\underset{\sim}{p}_i \overset{iid}{\sim} \text{Dirichlet}(u_1 v_1 \tau_1, u_1 v_2 \tau_1, u_1 v_c \tau_1, u_r v_1 \tau_1, u_r v_2 \tau_1, u_r v_c \tau_1) \quad (3.30)$$

$$\underset{\sim}{u} \sim \text{Dirichlet}(1, \ldots, 1) \quad (3.31)$$

$$\underset{\sim}{v} \sim \text{Dirichlet}(1, \ldots, 1) \quad (3.32)$$

$$\underset{\sim}{y}_i, \underset{\sim}{z}_i | \underset{\sim}{p}_i, \pi_i \overset{ind}{\sim} \text{Multinomial}(n_i, \pi_i p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, (1 - \pi_i) p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c) \quad (3.33)$$

$$\pi_i | \nu, \tau_2 \overset{iid}{\sim} \text{Beta}(\nu \tau_2, (1 - \nu) \tau_2), i = 1, \ldots, N \quad (3.34)$$

$$\nu \sim \text{Beta}(\alpha_0, \beta_0) \quad (3.35)$$

$$\underset{\sim}{\mu} \sim \text{Dirichlet}(1, \ldots, 1) \quad (3.36)$$

$$\underset{\sim}{\mu} = \{\mu_{11}, \mu_{12}, \mu_{1c}, \mu_{r1}, \mu_{r2}, \mu_{rc}\} \quad \text{and} \quad \sum_{j=1}^{r} \sum_{k=1}^{c} \mu_{jk} = 1$$

$$p(\tau_1) = \frac{1}{(1 + \tau_1)^2}, \tau_1 > 0 \quad (3.37)$$

$$p(\tau_2) = \frac{1}{(1 + \tau_2)^2}, \tau_2 > 0. \quad (3.38)$$

The model with nonignorability of no association is

$$\underset{\sim}{p}_i \overset{iid}{\sim} \text{Dirichlet}(u_1 v_1 \tau_1, u_1 v_2 \tau_1, u_1 v_c \tau_1, u_r v_1 \tau_1, u_r v_2 \tau_1, u_r v_c \tau_1) \quad (3.39)$$

$$\underset{\sim}{u} \sim \text{Dirichlet}(1, \ldots, 1) \quad (3.40)$$

$$\underset{\sim}{v} \sim \text{Dirichlet}(1, \ldots, 1) \quad (3.41)$$

$$\underset{\sim}{y}_i, \underset{\sim}{z}_i | \underset{\sim}{p}_i, \underset{\sim}{\pi}_i \overset{ind}{\sim} \text{Multinomial}(n_i, \pi_{ijk} p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, (1 - \pi_{ijk}) p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c) \quad (3.42)$$

$$\pi_{ijk} | \nu, \tau_2 \overset{iid}{\sim} \text{Beta}(\nu \tau_2, (1 - \nu) \tau_2), i = 1, \ldots, N \quad (3.43)$$

$$\nu \sim \text{Beta}(\alpha_0, \beta_0) \quad (3.44)$$

$$\underset{\sim}{\mu} \sim \text{Dirichlet}(1, \ldots, 1) \quad (3.45)$$

$$\underset{\sim}{\mu} = \{\mu_{11}, \mu_{12}, \mu_{1c}, \mu_{r1}, \mu_{r2}, \mu_{rc}\} \text{ and } \sum_{j=1}^{r} \sum_{k=1}^{c} \mu_{jk} = 1$$

$$p(\tau_1) = \frac{1}{(1 + \tau_1)^2}, \tau_1 > 0 \quad (3.46)$$

$$p(\tau_2) = \frac{1}{(1 + \tau_2)^2}, \tau_2 > 0. \quad (3.47)$$

Individual no association models

In this case, we assume that BMD and age for each county are not associated. That is, $p_{ijk} = t_{ij}s_{ik}$ for all j, k and each county, $\sum_{j=1}^{r} t_{ij} = 1$, $\sum_{k=1}^{c} s_{ik} = 1$, i=1,. . ., N.

Thus, the model with ignorability of no association is

$$\begin{aligned}
\underset{\sim}{y}_i, \underset{\sim}{z}_i | \pi_i, t_{ij}, s_{ik} &\overset{iid}{\sim} \text{Multinomial}(n_i, \pi_i t_{ij} s_{ik}, j = 1, \ldots, r, k = 1, \ldots, c, \\
&\quad (1 - \pi_i) t_{ij} s_{ik}, j = 1, \ldots, r, k = 1, \ldots, c) &(3.48)\\
\underset{\sim}{t}_i | \underset{\sim}{u}, \phi_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{u}\phi_1), i = 1, \ldots, N &(3.49)\\
\underset{\sim}{s}_i | \underset{\sim}{v}, \phi_2 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{v}\phi_2), i = 1, \ldots, N &(3.50)\\
\pi_i | \nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1 - \nu)\tau_2), i = 1, \ldots, N &(3.51)\\
\underset{\sim}{u} &\sim \text{Dirichlet}(1, 1) &(3.52)\\
\underset{\sim}{v} &\sim \text{Dirichlet}(1, 1, 1) &(3.53)\\
\nu &\sim \text{Beta}(\alpha_0, \beta_0) &(3.54)\\
p(\tau_2) &= \frac{1}{(1 + \tau_2)^2}, \tau_2 > 0 &(3.55)\\
p(\phi_1) &= \frac{1}{(1 + \phi_1)^2}, \phi_1 > 0 &(3.56)\\
p(\phi_2) &= \frac{1}{(1 + \phi_2)^2}, \phi_2 > 0. &(3.57)
\end{aligned}$$

The model with nonignorability of no association is

$$\begin{aligned}
\underset{\sim}{y}_i, \underset{\sim}{z}_i | \underset{\sim}{\pi}_i, t_{ij}, s_{ik} &\overset{iid}{\sim} \text{Multinomial}(n_i, \pi_{ijk} t_{ij} s_{ik}, j = 1, \ldots, r, k = 1, \ldots, c, \\
&\quad (1 - \pi_{ijk}) t_{ij} s_{ik}, j = 1, \ldots, r, k = 1, \ldots, c) &(3.58)\\
\underset{\sim}{t}_i | \underset{\sim}{u}, \phi_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{u}\phi_1), i = 1, \ldots, N &(3.59)\\
\underset{\sim}{s}_i | \underset{\sim}{v}, \phi_2 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{v}\phi_2), i = 1, \ldots, N &(3.60)\\
\pi_{ijk} | \nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1 - \nu)\tau_2), i = 1, \ldots, N &(3.61)\\
\underset{\sim}{u} &\sim \text{Dirichlet}(1, 1) &(3.62)\\
\underset{\sim}{v} &\sim \text{Dirichlet}(1, 1, 1) &(3.63)\\
\nu &\sim \text{Beta}(\alpha_0, \beta_0) &(3.64)\\
p(\tau_2) &= \frac{1}{(1 + \tau_2)^2}, \tau_2 > 0 &(3.65)\\
p(\phi_1) &= \frac{1}{(1 + \phi_1)^2}, \phi_1 > 0 &(3.66)\\
p(\phi_2) &= \frac{1}{(1 + \phi_2)^2}, \phi_2 > 0. &(3.67)
\end{aligned}$$

The models are fit in a manner similar to baseline ignorable and nonignorable nonresponse models. We use the Griddy Gibbs sampler to draw samples from these models (Appendix: A). The estimates of $\underset{\sim}{t}_i$, $\underset{\sim}{s}_i$ are shown in Table 3.3 and π_i is shown in Table 3.4.

3.3.3 Bayes Factor

Because the Bayes factor is the ratio of the marginal likelihoods, we show how to compute the marginal likelihoods (e.g.: See Nandram and Kim, 2002).

Suppose Ω is the set of parameters in the model. Then the marginal likelihood is

$$p(y) = \int p(y|\Omega)\pi(\Omega)d\Omega$$

It is possible to integrate out some of the parameters, so we can write

$$p(y) = \int g(\Omega_1|y)d\Omega_1$$

To compute $p(y)$, we use importance sampling. Here we construct an importance $f_a(\Omega_1|y)$, so that

$$p(y) = \int \frac{g(\Omega_1|y)}{f_a(\Omega_1|y)} f_a(\Omega_1|y)d\Omega_1$$

Suppose we draw a sample $\Omega_1^{(1)}, \ldots, \Omega_1^{(M)}$, from $f_a(\Omega_1|y)$. Then a Monte Carlo consistent estimator of $p(y)$ is $\widehat{p(y)}$, where

$$\begin{aligned} \widehat{p(y)} &= \frac{1}{M}\sum_{h=1}^{M} \exp\left\{ \log\{g(\Omega_1^{(h)}|y)\} - \log\{f_a(\Omega_1^{(h)}|y)\}\right\} \\ &= \frac{1}{M}\sum_{h=1}^{M} \exp\{k_h\} \end{aligned}$$

where $k_h = \log\{g(\Omega_1^{(h)}|y)\} - \log\{f_a(\Omega_1^{(h)}|y)\}$ and M is the Monte Carlo sample size. Thus, we state $g(\Omega_1|y)$ and $f_a(\Omega_1|y)$ for the various hypothesis tests. We calculate numerical standard error by $NSE \approx S/\sqrt{M}$ where $S^2 = \frac{1}{M}\sum_{h=1}^{M}(k_h - \bar{k})^2$.

Overall association hypothesis

For the ignorable baseline model, which is based on $\mu_{jk}=u_j v_k$, $g(\Omega_1|\underset{\sim}{y})$ and $f_a(\Omega_1|\underset{\sim}{y})$ are:

$$\begin{aligned} g(\Omega_1|\underset{\sim}{y}) &= \int\int\int\int\int\int\int \prod_{i=1}^{N}\left\{ n_i! \prod_{j=1}^{r}\prod_{k=1}^{c}\left\{ \frac{(p_{ijk}\pi_i)^{y_{ijk}}}{y_{ijk}!}\frac{(p_{ijk}(1-\pi_i))^{z_{ijk}}}{z_{ijk}!} \right.\right. \\ &\quad \times \left.\frac{{p_{ijk}}^{\mu_{jk}\tau_1-1}}{D(\underset{\sim}{\mu}\tau_1)}\right\} \frac{{\pi_i}^{\nu\tau_2-1}(1-\pi_i)^{(1-\nu)\tau_2-1}}{\mathrm{B}(\nu\tau_2,(1-\nu)\tau_2)} \\ &\quad \times \left.\frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{\mathrm{B}(\alpha_0,\beta_0)}\frac{1}{(1+\tau_1)^2}\frac{1}{(1+\tau_2)^2}\frac{1}{\mathrm{D}(1,1,1,1,1,1)}\right\} d\underset{\sim}{\mu}d\underset{\sim}{\nu}d\tau_1 d\tau_2 d\underset{\sim}{p}d\underset{\sim}{\pi}d\underset{\sim}{z} \\ &= \int\int\int\int\int \prod_{i=1}^{N}\left\{ \frac{n_i!(rc-1)!}{\prod_{j=1}^{r}\prod_{k=1}^{c} y_{ijk}!z_{ijk}!}\prod_{j=1}^{r}\prod_{k=1}^{c}\left\{\frac{\mathrm{B}(y_{ijk}+\nu\tau_2, z_{ijk}+(1-\nu)\tau_2)}{\mathrm{B}(\nu\tau_2,(1-\nu)\tau_2}\right.\right. \\ &\quad \times \left.\left.\frac{\mathrm{D}(y_{ijk}+z_{ijk}+\mu_{jk}\tau_1)}{\mathrm{D}(\underset{\sim}{\mu}\tau_1)}\right\}\frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{\mathrm{B}(\alpha_0,\beta_0)}\frac{1}{(1+\tau_1)^2}\frac{1}{(1+\tau_2)^2}\right\} \\ &\quad d\underset{\sim}{\mu}d\nu d\tau_1 d\tau_2 d\underset{\sim}{z} \end{aligned} \tag{3.68}$$

where rc=6, and

$$\begin{aligned} f_a(\Omega_1|\underset{\sim}{y}) &= \frac{b_1^{a_1}{\tau_1}^{a_1-1}e^{-b_1\tau_1}}{\Gamma(a_1)}\frac{b_2^{a_2}\tau_2^{a_2-1}e^{-b_2\tau_2}}{\Gamma(a_2)}\frac{\prod_{j=1}^{r}\prod_{k=1}^{c}\mu_{jk}^{\alpha_{jk}-1}}{\mathrm{D}(\alpha_{11},\ldots,\alpha_{rc})} \\ &\quad \times \frac{\nu^{a-1}(1-\nu)^{b-1}}{\mathrm{B}(a,b)}\prod_{i=1}^{N}\left\{\frac{n_{misi}!}{\prod_{j=1}^{r}\prod_{k=1}^{c} z_{ijk}!}\prod_{j=1}^{r}\prod_{k=1}^{c}\Big((1-\pi_i)p_{ijk})\Big)^{z_{ijk}}\right\} \end{aligned} \tag{3.69}$$

where π, p_{ijk}, a, b α_{11}, ..., α_{rc} are estimated from the Gibbs sampler. It is easy to draw samples from (3.69).

For the ignorable overall no association model, $g(\Omega_1|\underset{\sim}{y})$ and $f_a(\Omega_1|\underset{\sim}{y})$ are

$$
\begin{aligned}
g(\Omega_1|\underset{\sim}{y}) &= \int\int\int\int\int\int\int\int \prod_{i=1}^{N}\left\{ n_i! \prod_{j=1}^{r}\prod_{k=1}^{c}\left\{ \frac{(\mathrm{p}_{ijk}\pi_i)^{y_{ijk}}}{y_{ijk}!}\frac{(\mathrm{p}_{ijk}(1-\pi_i))^{z_{ijk}}}{z_{ijk}!} \right.\right.\\
&\quad \left. \times\frac{\pi_i{}^{\nu\tau_2-1}(1-\pi_i)^{(1-\nu)\tau_2-1}}{\mathrm{B}(\nu\tau_2,(1-\nu)\tau_2)}\frac{\mathrm{p}_{ijk}{}^{u_jv_k\tau_1-1}}{D(u_1v_1\tau_1,u_1v_2\tau_1,u_1v_c\tau_1,u_rv_1\tau_1,u_rv_2\tau_1,u_rv_c\tau_1)}\right\}\\
&\quad \left.\times\frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{\mathrm{B}(\alpha_0,\beta_0)}\frac{1}{(1+\tau_1)^2}\frac{1}{(1+\tau_2)^2}\frac{1}{\mathrm{D}(1,1)}\frac{1}{\mathrm{D}(1,1,1)}\right\}\\
&\quad d\underset{\sim}{\mu}d\nu d\tau_1 d\tau_2 d\underset{\sim}{p}d\underset{\sim}{\pi}d\underset{\sim}{z}d\underset{\sim}{v}\\
&= \int\int\int\int\int\int \prod_{i=1}^{N}\left\{\frac{n_i!(r-1)!(c-1)!}{\prod_{j=1}^{r}\prod_{k=1}^{c}y_{ijk}!z_{ijk}!}\right.\\
&\quad \times\prod_{j=1}^{r}\prod_{k=1}^{c}\left\{\frac{\mathrm{B}(y_{ijk}+\nu\tau_2,z_{ijk}+(1-\nu)\tau_2)}{\mathrm{B}(\nu\tau_2,(1-\nu\tau_2)}\right.\\
&\quad \left.\times\frac{\mathrm{D}(y_{ijk}+z_{ijk}+u_jv_k\tau_1,j=1,r,k=1,2,c)}{\mathrm{D}(u_1v_1\tau_1,u_1v_2\tau_1,u_1v_c\tau_1,u_rv_1\tau_1,u_rv_2\tau_1,u_rv_c\tau_1)}\right\}\\
&\quad \times\frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{\mathrm{B}(\alpha_0,\beta_0)}\frac{1}{(1+\tau_1)^2}\\
&\quad \left.\times\frac{1}{(1+\tau_2)^2}\right\}d\underset{\sim}{u}d\underset{\sim}{v}d\nu d\tau_1 d\tau_2 d\underset{\sim}{z} \qquad (3.70)
\end{aligned}
$$

and

$$
\begin{aligned}
f_a(\Omega_1|\underset{\sim}{y}) &= \frac{b_1^{a_1}\tau_1{}^{a_1-1}e^{-b_1\tau_1}}{\Gamma(a_1)}\frac{b_2^{a_2}\tau_2^{a_2-1}e^{-b_2\tau_2}}{\Gamma(a_2)}\frac{\prod_{j=1}^{r}u_j^{d_j-1}}{\mathrm{D}(d_1,d_2)}\frac{\prod_{k=1}^{c}v_k^{e_k-1}}{\mathrm{D}(e_1,e_2,e_3)}\\
&\quad \times\frac{\nu^{a-1}(1-\nu)^{b-1}}{\mathrm{B}(a,b)}\prod_{i=1}^{N}\left\{\frac{n_{misi}!}{\prod_{j=1}^{r}\prod_{k=1}^{c}z_{ijk}!}\prod_{j=1}^{r}\prod_{k=1}^{c}\Big((1-\pi_i)p_{ijk})\Big)^{z_{ijk}}\right\}
\end{aligned}
\qquad (3.71)
$$

Table 3.6 shows the results of log marginal likelihood and their standard errors. The log marginal likelihoods converge and the standard errors are small. Table 3.7 shows the logarithm Bayes factor for both ignorable and nonignorable nonresponse models. a,b, d_j, e_k, $\underset{\sim}{\pi}_i$ and $\underset{\sim}{p}$ are estimated from the Gibbs sampler.

	Ignorable nonresponse model				Nonignorable nonresponse model			
Simulation	$g(y)$	Std.	$g_0(y)$	Std.	$g(y)$	Std.	$g_0(y)$	Std.
1000	-790.56	2.09	-811.85	1.81	-590.76	1.29	-641.71	1.20
4000	-788.36	1.04	-802.95	0.87	-577.51	0.65	-639.81	0.59
7000	-787.88	0.79	-803.10	0.66	-577.67	0.49	-639.69	0.45
10000	-787.93	0.65	-802.95	0.55	-577.73	0.41	-639.36	0.38
13000	-787.81	0.58	-803.03	0.49	-577.59	0.36	-639.23	0.33
16000	-787.86	0.52	-802.96	0.44	-577.65	0.32	-639.28	0.30
19000	-787.90	0.47	-803.01	0.40	-577.68	0.30	-639.20	0.27
20000	-787.84	0.47	-803.06	0.39	-577.61	0.29	-639.26	0.27

Table 3.6: Log marginal likelihoods under overall association hypothesis (g(y) is the log marginal likelihood for baseline model, $g_0(y)$ is the log marginal likelihood for overall no association model and std. is the standard error)

	Overall Association		Ignorability	
Simulation	Ignorable	Nonignorable	Association	No association
1000	21.29	50.95	199.80	170.14
4000	14.59	62.30	210.85	163.14
7000	15.22	62.02	210.21	163.41
10000	15.02	61.63	210.20	163.59
13000	15.22	61.64	210.22	163.80
16000	15.10	61.63	210.21	163.68
19000	15.11	61.52	210.22	163.81
20000	15.22	61.65	210.23	163.80

Table 3.7: Logarithm of Bayes factors of overall association hypothesis under ignorability and nonignorability assumptions and Bayes factors of ignorability hypothesis under overall association and overall no association assumptions (i.e. the Bayes factor of overall association under ignorable nonresponse model is $g(y) - g_0(y)$)

Individual association hypothesis

For the ignorable individual no association model, which is based on $p_{ijk} = t_{ij}s_{ik}$, $g(\Omega_1|\underset{\sim}{y})$ and $f_a(\Omega_1|\underset{\sim}{y})$ are:

$$
\begin{aligned}
g(\Omega_1|\underset{\sim}{y}) = &\int\int\int\int\int\int\int\int\int\int \Bigg\{ n_i! \prod_{j=1}^{r}\prod_{k=1}^{c} \Bigg\{ \frac{(\mathrm{t}_{ij}\mathrm{s}_{ik}\pi_i)^{y_{ijk}}}{y_{ijk}!} \frac{\Big(\mathrm{t}_{ij}\mathrm{s}_{ik}(1-\pi_i)\Big)^{z_{ijk}}}{z_{ijk}!} \\
&\times \frac{\pi_i{}^{\nu\tau_2-1}(1-\pi_i)^{(1-\nu)\tau_2-1}}{\mathrm{B}(\nu\tau_2,(1-\nu)\tau_2)} \Bigg\} \prod_{j=1}^{r} \frac{\mathrm{t}_{ij}{}^{u_j\phi_1-1}}{D(u_1\phi_1,u_j\phi_1)} \prod_{k=1}^{c} \frac{\mathrm{s}_{ik}{}^{v_k\phi_2-1}}{D(v_1\phi_2,v_2\phi_2,v_c\phi_2)} \\
&\times \frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{\mathrm{B}(\alpha_0,\beta_0)} \frac{1}{(1+\phi_1)^2} \frac{1}{(1+\phi_2)^2} \frac{1}{(1+\tau_2)^2} \frac{1}{\mathrm{D}(1,1,1)} \frac{1}{\mathrm{D}(1,1)} \Bigg\} \\
&d\underset{\sim}{u}d\underset{\sim}{v}d\nu d\phi_1 d\phi_2 d\tau_2 d\underset{\sim}{t}d\underset{\sim}{s}d\underset{\sim}{\pi}d\underset{\sim}{z} \\
= &\int\int\int\int\int\int\int \Bigg\{ \frac{n_i!(r-1)!(c-1)!}{\prod_{j=1}^{r}\prod_{k=1}^{c} y_{ijk}!z_{ijk}!} \prod_{j=1}^{r}\prod_{k=1}^{c} \frac{\mathrm{B}(y_{ijk}+\nu\tau_2, z_{ijk}+(1-\nu)\tau_2)}{\mathrm{B}(\nu\tau_2,(1-\nu\tau_2)} \\
&\times \prod_{j=1}^{r} \frac{\mathrm{D}(\sum_{k=1}^{c}(y_{ijk}+z_{ijk})+u_j\phi_1)}{\mathrm{D}(u_1\phi_1,u_2\phi_1)} \prod_{k=1}^{c} \frac{\mathrm{D}(\sum_{j=1}^{r}(y_{ijk}+z_{ijk})+v_k\phi_2)}{\mathrm{D}(v_1\phi_2,v_2\phi_2,v_c\phi_2)} \\
&\times \frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{\mathrm{B}(\alpha_0,\beta_0)} \frac{1}{(1+\phi_1)^2} \frac{1}{(1+\phi_2)^2} \frac{1}{(1+\tau_2)^2} \Bigg\} \\
&d\underset{\sim}{u}d\underset{\sim}{v}d\nu d\phi_1 d\phi_2 d\tau_2 d\underset{\sim}{z}
\end{aligned} \tag{3.72}
$$

and

$$
\begin{aligned}
f_a(\Omega_1|\underset{\sim}{y}) = &\frac{b_0^{a_0}\phi_1{}^{a_0-1}e^{-b_0\phi_1}}{\Gamma(a_0)} \frac{b_1^{a_1}\phi_2{}^{a_1-1}e^{-b_1\phi_2}}{\Gamma(a_1)} \frac{b_2^{a_2}\tau_2^{a_2-1}e^{-b_2\tau_2}}{\Gamma(a_2)} \\
&\times \frac{\prod_{j=1}^{r} u_j^{\alpha_{1j}-1}}{\mathrm{D}(\alpha_{11},\alpha_{12})} \frac{\prod_{k=1}^{c} v_k^{\alpha_{2k}-1}}{\mathrm{D}(\alpha_{21},\alpha_{2r},\alpha_{2c})} \frac{\nu^{a-1}(1-\nu)^{b-1}}{\mathrm{B}(a,b)} \\
&\times \Bigg\{ \frac{n_{misi}!}{\prod_{j=1}^{r}\prod_{k=1}^{c} z_{ijk}!} \prod_{j=1}^{r}\prod_{k=1}^{c} \Big((1-\pi_i)t_{ij}s_{ik}\Big)^{z_{ijk}} \Bigg\}.
\end{aligned} \tag{3.73}
$$

The log marginal likelihoods of individual association and no association simulated 20000 times and their errors are shown in Table 3.8 and we get the logarithm Bayes factors in Table 3.9. Again, π_i, t_{ij}, s_{ik}, α_0, α_1, α_{1j} and α_{2k} are estimated from Monte Carlo methods.

	Ignorable nonresponse model				Nonignorable nonresponse model			
County	$g(y)$	Std.	$g_0(y)$	Std.	$g(y)$	Std.	$g_0(y)$	Std.
1	-41.75	2.78	-49.96	8.27	-38.72	5.80	-47.92	6.75
2	-41.48	1.16	-42.33	4.10	-37.82	4.91	-43.98	7.52
3	-42.56	3.40	-42.26	3.93	-35.67	1.10	-43.98	7.56
4	-84.02	6.09	-90.92	11.7	-47.01	5.73	-82.17	10.00
5	-46.43	5.51	-50.32	5.95	-39.66	0.58	-50.99	3.51
6	-44.33	6.76	-42.82	1.62	-33.62	4.01	-41.23	4.87
7	-37.99	5.05	-53.03	10.5	-42.45	5.31	-50.60	7.74
8	-41.21	3.31	-49.52	4.67	-39.18	2.15	-50.02	4.68
9	-51.21	7.83	-58.25	2.57	-42.54	3.94	-58.29	3.22
10	-32.54	0.19	-43.49	4.54	-34.70	6.22	-43.30	3.34
11	-39.35	3.24	-42.77	1.72	-35.74	4.97	-44.13	3.95
12	-41.22	5.41	-49.30	6.12	-35.91	2.29	-50.90	8.88
13	-50.75	2.51	-51.37	3.64	-38.44	0.67	-53.62	1.11
14	-37.35	2.45	-48.47	9.01	-34.51	5.85	-48.88	2.33
15	-36.15	3.76	-41.18	2.08	-33.57	3.46	-41.78	7.61
16	-32.82	4.39	-40.87	0.83	-34.59	3.39	-37.73	0.51
17	-33.34	4.68	-43.48	2.85	-33.40	3.69	-44.40	4.57
18	-36.81	1.26	-38.30	6.05	-31.77	0.56	-39.03	6.90
19	-31.07	2.17	-36.60	4.13	-30.10	0.27	-35.15	3.52
20	-35.64	0.68	-44.00	2.79	-31.85	2.69	-45.22	6.85
21	-33.30	5.18	-46.31	0.35	-36.05	2.95	-43.74	4.16
22	-34.58	5.29	-40.83	3.25	-30.28	0.99	-41.02	7.16
23	-30.28	1.29	-35.14	5.10	-30.48	1.16	-34.37	3.36
24	-26.33	2.21	-33.70	6.52	-27.41	2.08	-33.52	0.45
25	-31.49	4.06	-36.50	1.86	-31.91	3.39	-36.89	5.64
26	-30.07	3.84	-41.24	4.45	-32.95	3.17	-39.98	3.72
27	-33.71	2.12	-44.18	5.53	-32.96	0.51	-41.53	6.45
28	-23.89	4.70	-37.20	0.17	-29.35	1.61	-36.89	1.47
29	-33.66	4.27	-43.99	4.41	-35.66	2.36	-42.15	3.54
30	-48.98	4.21	-51.17	2.90	-34.25	0.45	-52.53	10.4
31	-32.43	3.41	-44.69	1.67	-37.34	5.97	-43.64	1.59
32	-46.75	1.04	-52.15	8.83	-47.87	3.82	-53.38	3.95
33	-53.05	5.31	-54.31	0.06	-41.44	3.36	-56.13	4.51
34	-42.85	4.14	-48.07	5.75	-38.72	6.04	-47.36	6.06
35	-42.51	3.97	-54.48	6.94	-42.75	2.75	-53.01	3.99

Table 3.8: Log marginal likelihoods under individual association hypothesis (g(y) is the log marginal likelihood for baseline model, $g_0(y)$ is the log marginal likelihood for individual no association model and std. is the standard error $\times 10^{-5}$)

	Individual Association		Ignorability	
County	Ignorable	Nonignorable	Association	No association
1	8.21	9.20	3.03	2.04
2	0.85	6.16	3.66	-1.65
3	-0.30	8.31	6.89	-1.72
4	6.90	35.16	37.01	8.75
5	3.89	11.33	6.77	-0.67
6	-1.51	7.61	10.71	1.59
7	15.04	8.15	-4.46	2.43
8	8.31	10.84	2.03	-0.50
9	7.04	15.75	8.67	-0.04
10	10.95	8.60	-2.16	0.19
11	3.42	8.39	3.61	-1.36
12	8.08	14.99	5.31	-1.60
13	0.62	15.18	12.31	-2.25
14	11.12	14.37	2.84	-0.41
15	5.03	8.21	2.58	-0.60
16	8.05	3.14	-1.77	3.14
17	10.14	11.00	-0.06	-0.92
18	1.49	7.26	5.04	-0.73
19	5.53	5.05	0.97	1.45
20	8.36	13.37	3.79	-1.22
21	13.01	7.69	-2.75	2.57
22	6.25	10.74	4.30	-0.19
23	4.86	3.89	-0.20	0.77
24	7.37	6.11	-1.08	0.18
25	5.01	4.98	-0.42	-0.39
26	11.17	7.03	-2.88	1.26
27	10.47	8.57	0.75	2.65
28	13.31	7.54	-5.46	0.31
29	10.33	6.49	-2.00	1.84
30	2.19	18.28	14.73	-1.36
31	12.26	6.30	-4.91	1.05
32	5.40	5.51	-1.12	-1.23
33	1.26	14.69	11.61	-1.82
34	5.22	8.64	4.13	0.71
35	11.97	10.26	-0.24	1.47

Table 3.9: Logarithm of Bayes factors of individual association hypothesis under ignorability and nonignorability assumptions and Bayes factors of ignorability hypothesis under individual association and individual no association assumptions (i.e. the Bayes factor of overall association under ignorable nonresponse model is $g(y) - g_0(y)$)

	lags	ACF	Str		lags	ACF	Str		lags	ACF	Str
μ_1	1	0.07	0.0316	μ_4	1	0.07	0.0316	τ_1	1	0.07	0.0316
	5	0.04	0.0315		5	0.02	0.0315		5	0.04	0.0315
	10	0.04	0.0314		10	0.04	0.0314		10	0.03	0.0314
	15	-0.01	0.0314		15	-0.02	0.0314		15	-0.04	0.0314
	20	-0.02	0.0313		20	-0.03	0.0313		20	-0.02	0.0313
μ_2	1	0.09	0.0316	μ_5	1	0.08	0.0316	τ_2	1	-0.04	0.0316
	5	0.01	0.0315		5	0.02	0.0315		5	-0.03	0.0315
	10	0.04	0.0314		10	0.04	0.0314		10	-0.04	0.0314
	15	-0.02	0.0314		15	-0.01	0.0314		15	-0.02	0.0314
	20	-0.03	0.0313		20	-0.02	0.0313		20	0.02	0.0313
μ_3	1	0.07	0.0316	μ_6	1	0.05	0.0316	ν	1	-0.07	0.0316
	5	0.08	0.0315		5	0.05	0.0315		5	0.00	0.0315
	10	0.03	0.0314		10	0.03	0.0314		10	-0.04	0.0314
	15	-0.04	0.0314		15	-0.02	0.0314		15	0.07	0.0314
	20	-0.01	0.0313		20	-0.01	0.0313		20	0.00	0.0313

Table 3.10: Sample autocorrelation for ignorable overall no association model (ACF is the autocorrelation and Str is the standard error)

3.3.4 Conclusions

We have conducted the test of association between BMD and age. We have used the Bayes factor to measure the strength of evidence. Table 3.7 and Table 3.9 show the logarithm Bayes factors and we use the table suggested by Kass and Raffery (1995) to make our conclusions.

First, we consider the test of association for the overall counties. For the ignorable model, the logarithm Bayes factors are about 15 showing very strong evidence that BMD and age are associated. For the nonignorable nonresponse model, the logarithm Bayes factors are much larger.

Next, we consider the test of association for the individual counties. A few counties show the evidence for an association is "not worth more than a bare mention" (e.g. counties 2, 3, 6, 13).

We also have conducted the test of ignorability. For the overall counties, there is very strong evidence for the nonignorable nonresponse model. For the individual counties, the ignorable nonresponse model is sometimes preferred (e.g. counties 7, 10, 16, 21, 24, 26, 28, 29, 31, 32).

	lags	ACF	Str		lags	ACF	Str		lags	ACF	Str
μ_1	1	0.40	0.0316	μ_4	1	0.41	0.0316	τ_1	1	0.49	0.0316
	5	0.07	0.0315		5	0.04	0.0315		5	0.04	0.0315
	10	0.02	0.0314		10	0.00	0.0314		10	0.01	0.0314
	15	-0.02	0.0314		15	0.00	0.0314		15	0.02	0.0314
	20	0.00	0.0313		20	0.02	0.0313		20	-0.01	0.0313
μ_2	1	0.40	0.0316	μ_5	1	0.39	0.0316	τ_2	1	-0.04	0.0316
	5	0.04	0.0315		5	0.04	0.0315		5	0.01	0.0315
	10	-0.01	0.0314		10	0.00	0.0314		10	0.02	0.0314
	15	0.01	0.0314		15	-0.01	0.0314		15	-0.02	0.0314
	20	0.04	0.0313		20	0.02	0.0313		20	-0.03	0.0313
μ_3	1	0.14	0.0316	μ_6	1	0.01	0.0316	ν	1	0.36	0.0316
	5	0.00	0.0315		5	-0.01	0.0315		5	0.00	0.0315
	10	-0.01	0.0314		10	-0.02	0.0314		10	0.00	0.0314
	15	-0.02	0.0314		15	-0.03	0.0314		15	0.00	0.0314
	20	0.03	0.0313		20	0.02	0.0313		20	-0.02	0.0313

Table 3.11: Sample autocorrelation for nonignorable overall no association model (ACF is the autocorrelation and Str is the standard error)

	lags	ACF	Str		lags	ACF	Str		lags	ACF	Str
μ_1	1	0.06	0.0316	μ_4	1	0.08	0.0316	τ_1	1	0.00	0.0316
	5	-0.03	0.0315		5	0.00	0.0315		5	0.01	0.0315
	10	0.04	0.0314		10	0.00	0.0314		10	0.01	0.0314
	15	-0.02	0.0314		15	0.03	0.0314		15	-0.02	0.0314
	20	-0.04	0.0313		20	-0.02	0.0313		20	-0.03	0.0313
μ_2	1	0.06	0.0316	μ_5	1	-0.03	0.0316	τ_2	1	-0.04	0.0316
	5	-0.03	0.0315		5	-0.04	0.0315		5	-0.03	0.0315
	10	0.04	0.0314		10	-0.04	0.0314		10	-0.01	0.0314
	15	-0.02	0.0314		15	-0.08	0.0314		15	-0.03	0.0314
	20	-0.04	0.0313		20	-0.02	0.0313		20	0.03	0.0313
μ_3	1	0.11	0.0316	μ_6	1	-0.01	0.0316	ν	1	0.05	0.0316
	5	-0.03	0.0315		5	0.03	0.0315		5	0.01	0.0315
	10	-0.03	0.0314		10	-0.03	0.0314		10	0.01	0.0314
	15	0.01	0.0314		15	-0.03	0.0314		15	0.01	0.0314
	20	0.00	0.0313		20	0.02	0.0313		20	0.01	0.0313

Table 3.12: Sample autocorrelation for ignorable individual no association model (ACF is the autocorrelation and Str is the standard error)

	lags	ACF	Str		lags	ACF	Str		lags	ACF	Str
v_{i1}	1	0.0049	0.0316	u_{i2}	1	0.0858	0.0316	ϕ_2	1	-0.0030	0.0316
	5	-0.0062	0.0315		5	-0.0033	0.0315		5	0.0217	0.0315
	10	0.0321	0.0314		10	0.0506	0.0314		10	-0.0005	0.0314
	15	-0.0014	0.0314		15	0.0131	0.0314		15	0.0586	0.0314
	20	-0.0837	0.0313		20	0.0147	0.0313		20	-0.0381	0.0313
v_{ir}	1	0.0049	0.0316	u_{ic}	1	-0.0274	0.0316	ν	1	0.3257	0.0316
	5	-0.0062	0.0315		5	0.0156	0.0315		5	-0.0267	0.0315
	10	0.0321	0.0314		10	0.0544	0.0314		10	-0.0676	0.0314
	15	-0.0014	0.0314		15	0.0010	0.0314		15	-0.0228	0.0314
	20	-0.0837	0.0313		20	0.0162	0.0313		20	0.0130	0.0314
u_{i1}	1	0.0859	0.0316	ϕ_1	1	-0.0009	0.0316	τ_2	1	-0.0379	0.0316
	5	-0.0210	0.0315		5	0.0226	0.0315		5	0.0059	0.0315
	10	0.0516	0.0314		10	-0.0411	0.0314		10	0.0230	0.0314
	15	0.0154	0.0314		15	-0.0117	0.0314		15	0.0450	0.0314
	20	-0.0093	0.0313		20	0.0056	0.0313		20	0.0263	0.0313

Table 3.13: Sample autocorrelation for nonignorable individual no association model (ACF is the autocorrelation and Str is the standard error)

Chapter 4

Sensitivity Study, Simulation Study and Conclusion

In this chapter, we perform a sensitivity study and a simulation study. The sensitivity study tests how sensitive our models are with respect to different levels of association and with respect to ignorability. The simulation study assesses how different the ignorable and the nonignorable nonresponse models are. Finally, we present our conclusions.

4.1 Sensitivity Study

The most important assumptions in the model are about p, π. Therefore, we compare them among the models of baseline, overall no association and individual no association under ignorable and nonignorable assumptions. Table 4.3 and Table 4.4 show the estimates of $\underset{\sim}{p}$ and $\underset{\sim}{\pi}$.

For $\underset{\sim}{p}$, the difference between ignorable and nonignorable nonresponse model is generally small. For the baseline and overall no association models, the difference is also small. But there is a huge difference between the individual and overall no association models. For example, for cell 11 in county 1, the PM of overall no association ignorable nonresponse model is 0.544 and that of individual no association ignorable nonresponse model is 0.400. This result varies with respect to the cells. For those cells in which p is small, difference is small and p is large, difference is large.

For $\underset{\sim}{\pi}$, ignorable nonresponse model has the same π for all cells. Nonignorable nonresponse model has different π's and the difference is small. Again the difference between the baseline and overall no association are small, but the difference between the baseline and individual no association model is not so small.

We also compare the $\hat{p}$ fitted from baseline, overall no association and individual no association under ignorable and nonignorable assumptions with p from only one county. Figure 4.1 and Figure 4.2 show the comparisons of posterior mean and posterior standard deviation for p within those different models. We find that there is no big difference among baseline ignorable nonresponse model, baseline nonignorable nonresponse model and overall no association ignorable nonresponse model and they show low variability compared with model from only one county. And individual no association models are very similar. Overall no association nonignorable nonresponse model and individual no association models show larger variability than model from only one county.

4.2 Simulation Study

We perform a simulation study to further compare the ignorable and the nonignorable nonresponse models.

First, we fit the nonignorable baseline model to the BMD/age data. We keep r=2, c=3, N=35 and the county size n_i in the original data, then we estimate $\underset{\sim}{\mu}$, τ_1, ν and τ_2 using the posterior means.

Second, we generate

$$\begin{aligned} \underset{\sim}{p}_i | \underset{\sim}{\mu}, \tau_1 &\overset{iid}{\sim} \text{Dirichlet}(\underset{\sim}{\mu}\tau_1), i = 1, \ldots, N \\ \pi_{ijk} | \nu, \tau_2 &\overset{iid}{\sim} \text{Beta}(\nu\tau_2, (1-\nu)\tau_2), i = 1, \ldots, N \end{aligned}$$

and with these values we generate the cell counts using

$$\begin{aligned} \underset{\sim}{y}_i, \underset{\sim}{z}_i | \underset{\sim}{p}_i, \underset{\sim}{\pi}_i \overset{ind}{\sim} \text{Multinomial}\Big\{ & n_i, \pi_{ijk} p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c, \\ & (1-\pi_{ijk}) p_{ijk}, j = 1, \ldots, r, k = 1, \ldots, c \Big\}. \end{aligned} \tag{4.1}$$

For the ignorable nonresponse model, we refer π_{ijk} to π. Let $p_{ijk}^{(s)}$ and $\pi^{(s)}$ (or $\pi_{ijk}^{(s)}$) denote the simulated values.

Third, we perform this procedure to get 1000 data sets from the ignorable nonresponse model and another 1000 data sets from the nonignorable nonresponse model. We fit each of these data sets using both the ignorable nonresponse model and the nonignorable nonresponse model. We compute the posterior means $p_{ijk}^{(e)}$, $\pi_{ijk}^{(e)}$ and their variance.

Finally, we compute the average (AVG) the ratio of $(p_{ijk}^{(e)} - p_{ijk}^{(s)})/p_{ijk}^{(s)}$ and $(\pi_{ijk}^{(e)} - \pi_{ijk}^{(s)})/\pi_{ijk}^{(s)}$ for each baseline model, both models are fitted to both data sets. Then we compute two average (AVG) and standard deviation (STD) over the 1000 data sets and the 35 counties. We also compute the ratio of the posterior standard deviation for ignorable nonresponse model to that for nonignorable nonresponse model (shown by Table 4.1 and Table 4.2).

We find that there exist slight difference among our results. First consider the results for the case in which the ignorable and nonignorable nonresponse models are fitted to data generated by ignorable nonresponse model. For p, the averages (see Figure 4.3 and standard deviations show slightly difference, but ignorable nonresponse model has slightly larger bias. For π, the standard deviation of ignorable nonresponse model is smaller than that of nonignorable nonresponse model.

Second, we consider the results for the case in which the ignorable and nonignorable nonresponse models are fitted to data generated by nonignorable nonresponse model. For p, the ignorable nonresponse model has larger average (see Figure 4.4 and standard deviation. For π, the nonignorable nonresponse model is very close to 0.

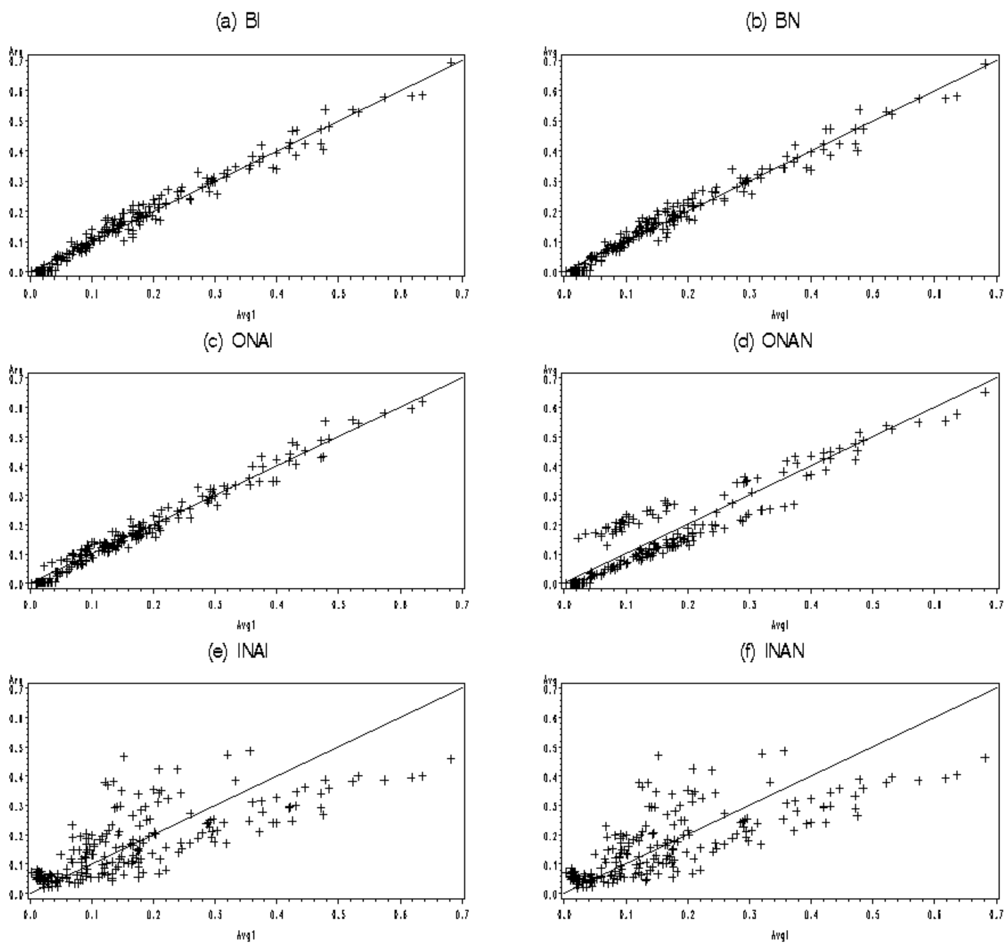


Figure 4.1: Scatter plots for average p comparisons (BI: baseline ignorable nonresponse model, BN: baseline nonignorable nonresponse model, ONAI: overall no association ignorable nonresponse model, ONAN: overall no association nonignorable nonresponse model, INAI: individual no association ignorable nonresponse model, INAN: individual no association nonignorable nonresponse model)

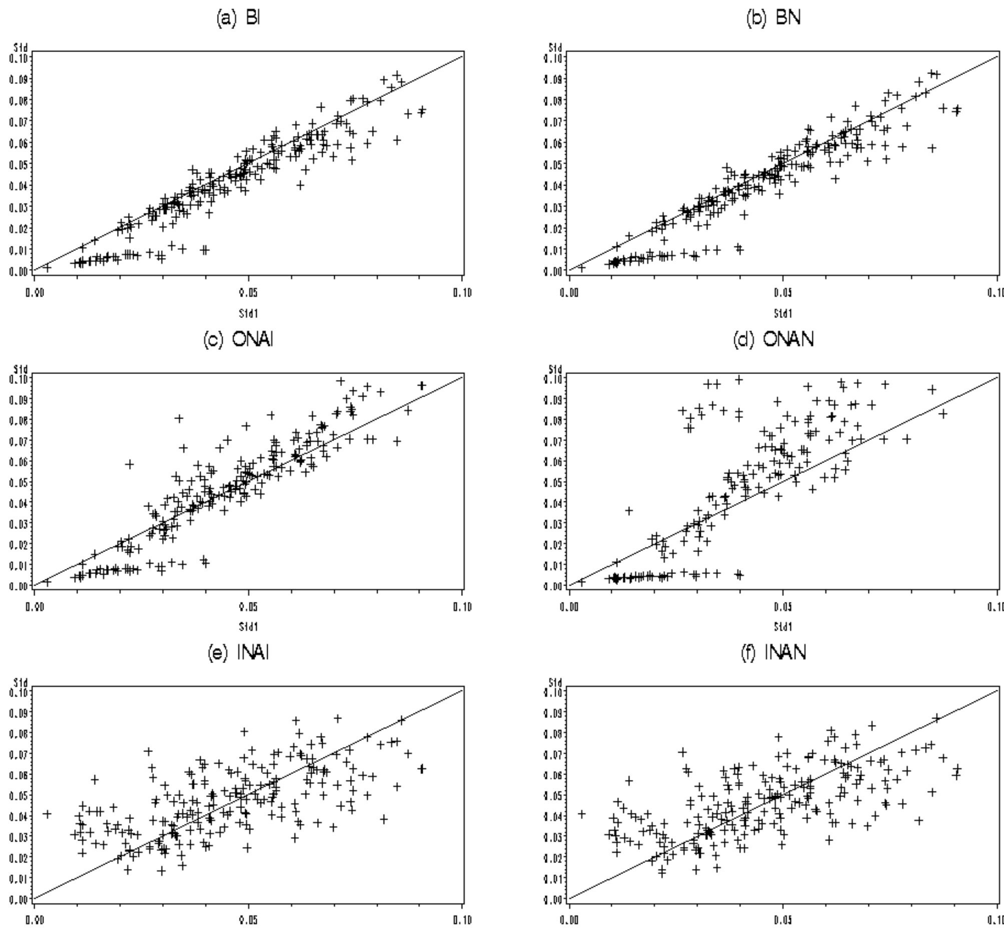


Figure 4.2: Scatter plots for standard deviation of p comparisons (BI: baseline ignorable nonresponse model, BN: baseline nonignorable nonresponse model, ONAI: overall no association ignorable nonresponse model, ONAN: overall no association nonignorable nonresponse model, INAI: individual no association ignorable nonresponse model, INAN: individual no association nonignorable nonresponse model)

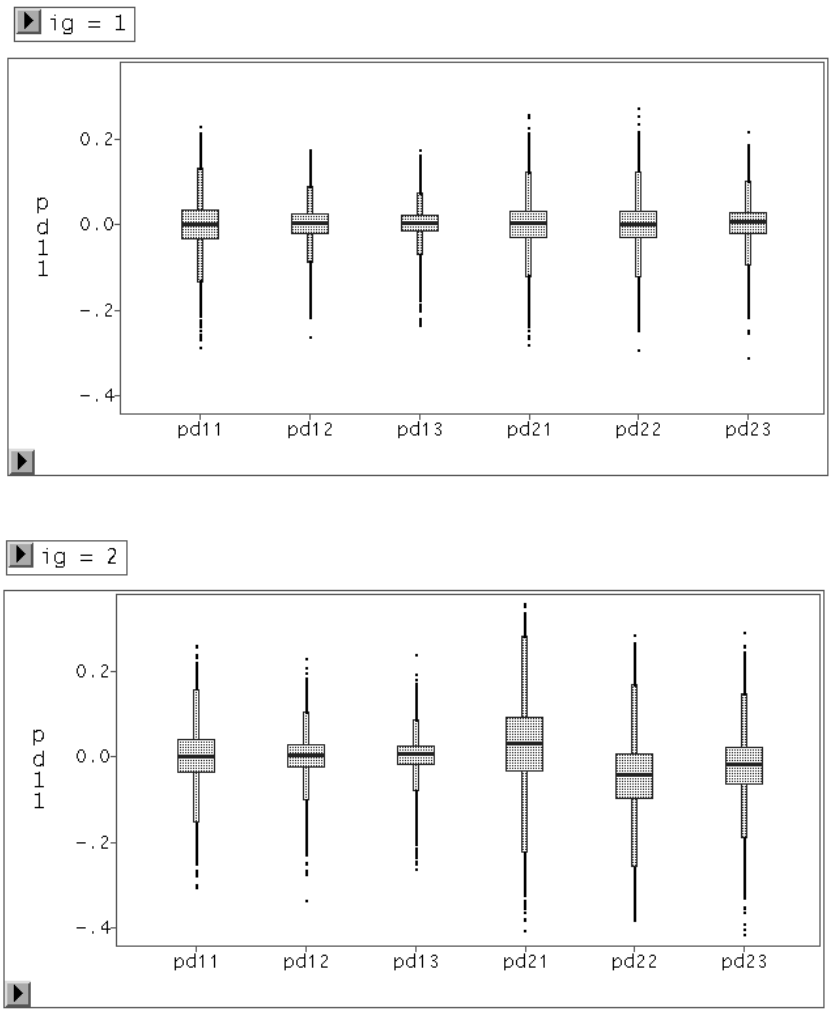
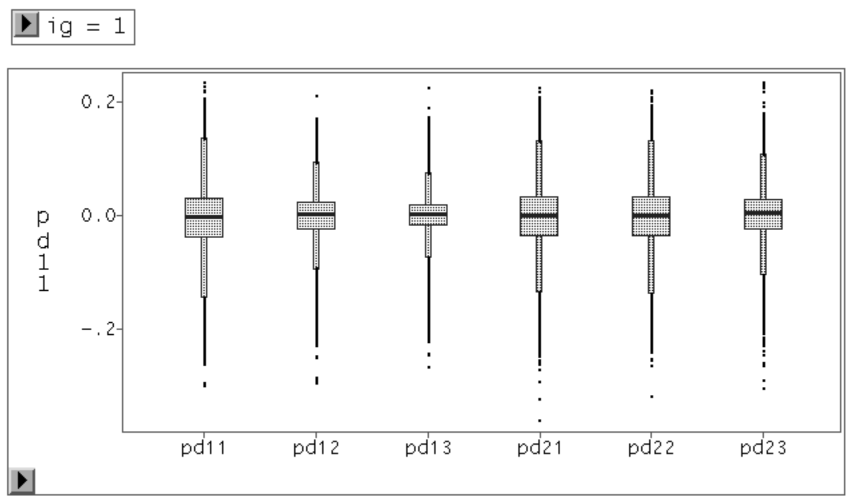


Figure 4.3: Box plots of average p for ignorable and nonignorable nonresponse model from data fitted from the ignorable nonresponse model (pd_{jk},j=1,...,2, k=1, ...,3 is the average p for j^{th} age and k^{th} BMD cell)



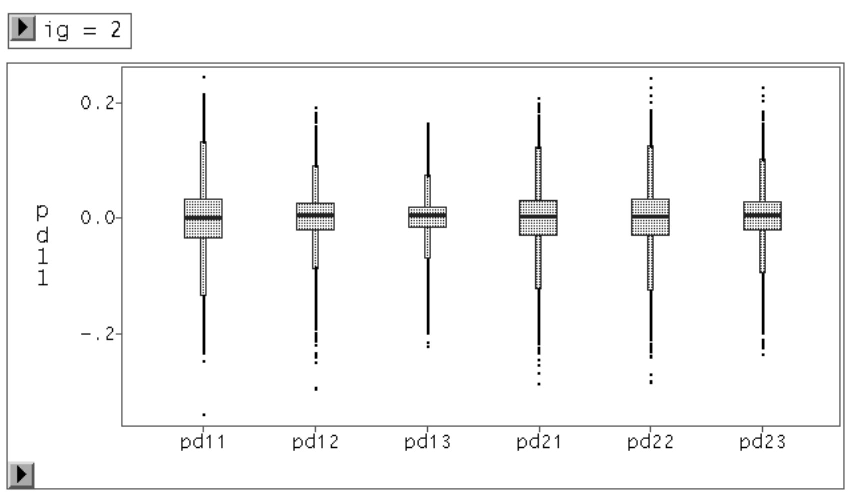


Figure 4.4: Box plots of average p for ignorable and nonignorable nonresponse model from data fitted from the nonignorable nonresponse model (pd_{jk},j=1,. . .,2, k=1, . . .,3 is the average p for j^{th} age and k^{th} BMD cell)

Data	Ignorable		Nonignorable	
Cell	AVG	STD	AVG	STD
11	1.018	0.235	1.015	0.228
12	1.092	0.527	1.093	0.527
13	1.148	0.667	1.147	0.683
21	1.028	0.312	1.030	0.312
22	1.024	0.300	1.025	0.300
23	1.069	0.452	1.070	0.456

Table 4.1: The standard deviation ratio of ignorable and nonignorable nonresponse model fitted by ignorable and nonignorable data for p

Data	Ignorable		Nonignorable	
Cell	AVG	STD	AVG	STD
11	1.022	0.202	0.943	0.148
12	0.976	0.208	0.899	0.155
13	0.967	0.209	0.891	0.157
21	0.998	0.205	0.921	0.152
22	1.001	0.205	0.924	0.151
23	0.979	0.207	0.902	0.155

Table 4.2: The standard deviation ratio of ignorable and nonignorable nonresponse model fitted by ignorable and nonignorable data for π

4.3 Conclusions

In our thesis, we present Bayesian nonignorable nonresponse models for small areas by studying BMD in NHANES III. We also study the problem of association and ignorability for a general r $\times$ c table with missing data.

First, we treated both BMD and age as continuous variables. We have constructed nested error regression models under ignorability and nonignorability assumptions. Because of the complexity of the joint posterior density, we fitted the model using Markov chain Monte Carlo methods. We confirmed age as the covariate and compared the estimates given by ignorable and nonignorable nonresponse models. The result shows that the nonignorable nonresponse model shows a negative relation with BMD.

Second, we treated both BMD and age as discrete variables. We analyzed multinomial data from r $\times$ c categorical tables for both ignorability and nonignorability. We constructed the table by setting up Bayesian ignorable and nonignorable nonresponse models. We also used Markov chain Monte Carlo methods to fit models. We tested association and ignorability by Bayes Factors. We set up overall no association and individual no association for both ignorable and nonignorable nonresponse models. Our Bayes factor shows "strong" evidence for overall association under the nonignorable nonresponse model.

Finally, we also have shown that by the sensitivity study and simulation study, nonignorable is "centered" to ignorable nonresponse model successfully, there is not much difference between them.

We have fitted the model $\underset{\sim}{n}_i \sim$ Multinomial(n_i, p_i) and $\underset{\sim}{\pi}_i \sim$ Dirichlet$(\underset{\sim}{1})$ for the i^{th} county, i=1, ..., 35. These models reflect what happens when only the data in the i^{th} county are used for inference. Compared the average, standard deviation, top 95 % confidence interval and bottom 95 % confidence interval (Figure 4.5) for the p estimated from the above model and nonignorable nonresponse model. We find those nonignorable nonresponse models have smaller values which indicates nonignorable nonresponse model is better.

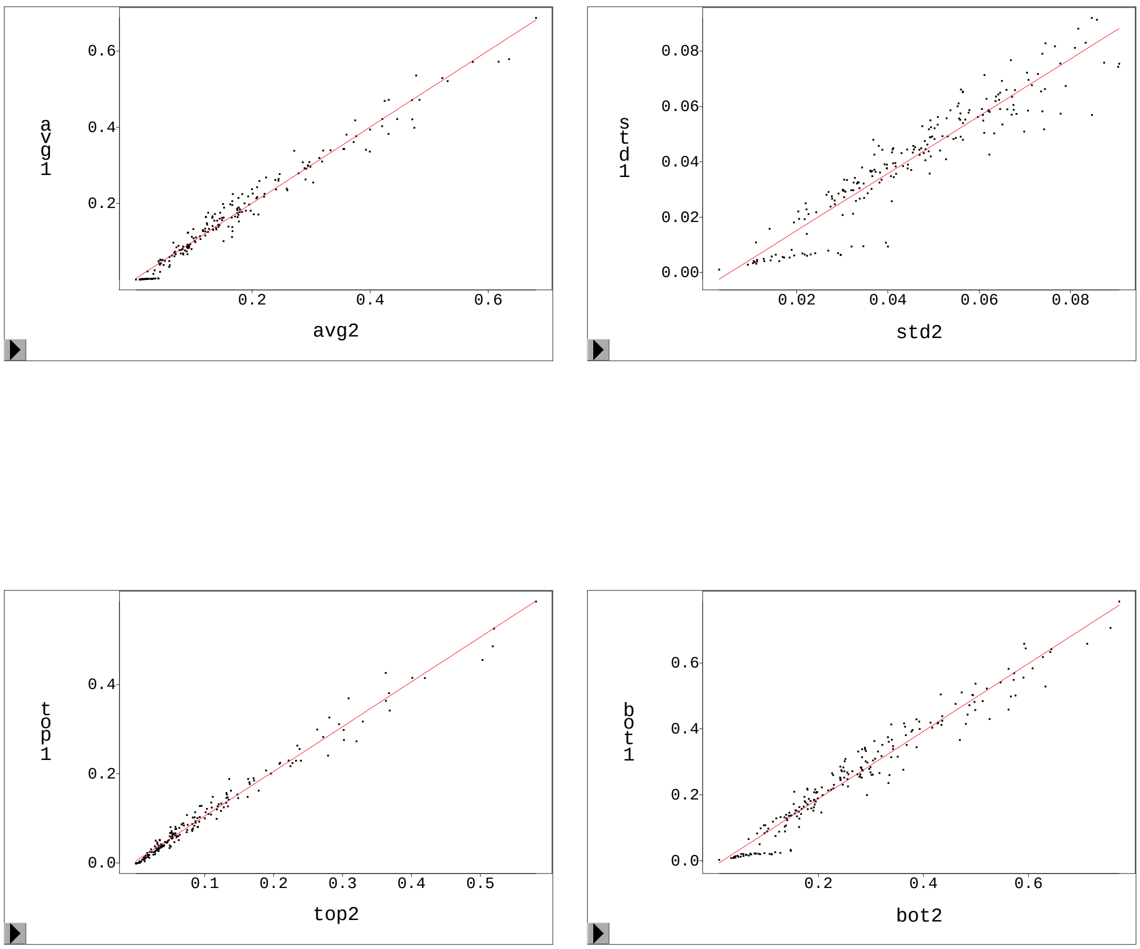


Figure 4.5: Scatter plots of model from one county versus nonignorable nonresponse model for average, standard deviation, lower end 95 % confidence interval and upper end of 95 % confidence interval

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.529	0.055	(0.419, 0.634)	0.544	0.052	(0.442, 0.643)	0.400	0.071	(0.260, 0.537)
	11	nig	0.523	0.055	(0.415, 0.636)	0.525	0.046	(0.425, 0.610)	0.396	0.070	(0.251, 0.531)
		ig	0.133	0.036	(0.072, 0.209)	0.163	0.045	(0.082, 0.259)	0.200	0.065	(0.091, 0.347)
	12	nig	0.140	0.039	(0.070, 0.221)	0.236	0.081	(0.102, 0.378)	0.201	0.063	(0.093, 0.343)
		ig	0.002	0.004	(0.000, 0.015)	0.002	0.006	(0.000, 0.019)	0.074	0.042	(0.014, 0.174)
	13	nig	0.002	0.005	(0.000, 0.017)	0.001	0.004	(0.000, 0.011)	0.076	0.044	(0.018, 0.183)
1		ig	0.127	0.036	(0.066, 0.202)	0.112	0.036	(0.053, 0.186)	0.193	0.042	(0.114, 0.277)
	21	nig	0.129	0.037	(0.065, 0.210)	0.093	0.043	(0.031, 0.186)	0.192	0.042	(0.118, 0.276)
		ig	0.130	0.039	(0.059, 0.211)	0.112	0.036	(0.051, 0.191)	0.096	0.033	(0.043, 0.169)
	22	nig	0.128	0.037	(0.063, 0.208)	0.089	0.042	(0.030, 0.187)	0.097	0.033	(0.043, 0.169)
		ig	0.079	0.029	(0.031, 0.141)	0.067	0.029	(0.023, 0.134)	0.036	0.022	(0.007, 0.094)
	23	nig	0.078	0.027	(0.034, 0.139)	0.056	0.031	(0.016, 0.135)	0.037	0.022	(0.008, 0.091)
		ig	0.481	0.057	(0.366, 0.586)	0.493	0.060	(0.377, 0.608)	0.357	0.069	(0.224, 0.490)
	11	nig	0.473	0.058	(0.364, 0.586)	0.486	0.056	(0.366, 0.582)	0.358	0.066	(0.238, 0.490)
		ig	0.104	0.037	(0.044, 0.189)	0.138	0.051	(0.049, 0.241)	0.182	0.057	(0.083, 0.306)
	12	nig	0.107	0.038	(0.049, 0.191)	0.220	0.099	(0.066, 0.391)	0.182	0.056	(0.085, 0.304)
		ig	0.016	0.015	(0.001, 0.054)	0.018	0.016	(0.001, 0.059)	0.067	0.038	(0.015, 0.159)
	13	nig	0.016	0.014	(0.001, 0.052)	0.013	0.013	(0.000, 0.048)	0.065	0.036	(0.013, 0.151)
2		ig	0.191	0.048	(0.108, 0.295)	0.176	0.052	(0.089, 0.287)	0.232	0.049	(0.143, 0.330)
	21	nig	0.190	0.048	(0.104, 0.285)	0.139	0.065	(0.046, 0.279)	0.233	0.048	(0.142, 0.331)
		ig	0.156	0.044	(0.075, 0.254)	0.142	0.047	(0.064, 0.250)	0.119	0.041	(0.053, 0.215)
	22	nig	0.161	0.045	(0.085, 0.253)	0.114	0.058	(0.033, 0.235)	0.119	0.038	(0.054, 0.202)
		ig	0.052	0.029	(0.009, 0.121)	0.034	0.024	(0.004, 0.095)	0.043	0.025	(0.009, 0.107)
	23	nig	0.053	0.029	(0.008, 0.120)	0.027	0.021	(0.003, 0.087)	0.043	0.025	(0.008, 0.101)
		ig	0.537	0.053	(0.431, 0.640)	0.552	0.055	(0.443, 0.652)	0.387	0.069	(0.254, 0.527)
	11	nig	0.537	0.056	(0.427, 0.646)	0.515	0.052	(0.409, 0.615)	0.389	0.067	(0.253, 0.527)
		ig	0.108	0.037	(0.045, 0.188)	0.141	0.046	(0.057, 0.237)	0.197	0.061	(0.087, 0.323)
	12	nig	0.109	0.037	(0.047, 0.189)	0.234	0.084	(0.093, 0.383)	0.197	0.062	(0.082, 0.324)
		ig	0.002	0.006	(0.000, 0.017)	0.003	0.007	(0.000, 0.023)	0.072	0.042	(0.013, 0.166)
	13	nig	0.002	0.006	(0.000, 0.022)	0.002	0.004	(0.000, 0.012)	0.074	0.041	(0.016, 0.176)
3		ig	0.173	0.039	(0.102, 0.251)	0.158	0.040	(0.087, 0.242)	0.202	0.044	(0.124, 0.296)
	21	nig	0.172	0.041	(0.100, 0.263)	0.129	0.054	(0.046, 0.239)	0.201	0.044	(0.121, 0.293)
		ig	0.140	0.036	(0.076, 0.214)	0.121	0.036	(0.060, 0.205)	0.103	0.034	(0.044, 0.179)
	22	nig	0.140	0.038	(0.078, 0.227)	0.101	0.047	(0.036, 0.213)	0.102	0.034	(0.041, 0.173)
		ig	0.039	0.022	(0.007, 0.091)	0.025	0.018	(0.003, 0.068)	0.038	0.022	(0.007, 0.092)
	23	nig	0.040	0.022	(0.007, 0.091)	0.019	0.015	(0.002, 0.060)	0.038	0.021	(0.008, 0.089)
		ig	0.578	0.023	(0.533, 0.625)	0.582	0.025	(0.534, 0.628)	0.387	0.063	(0.258, 0.510)
	11	nig	0.573	0.024	(0.526, 0.620)	0.550	0.026	(0.496, 0.598)	0.386	0.063	(0.258, 0.504)
		ig	0.069	0.014	(0.044, 0.100)	0.076	0.015	(0.050, 0.107)	0.190	0.057	(0.092, 0.319)
	12	nig	0.074	0.016	(0.046, 0.110)	0.130	0.036	(0.071, 0.204)	0.193	0.057	(0.090, 0.316)
		ig	0.000	0.001	(0.000, 0.003)	0.001	0.001	(0.000, 0.005)	0.072	0.041	(0.013, 0.163)
	13	nig	0.000	0.001	(0.000, 0.004)	0.001	0.001	(0.000, 0.005)	0.072	0.041	(0.014, 0.171)
4		ig	0.143	0.018	(0.107, 0.180)	0.141	0.019	(0.108, 0.179)	0.209	0.036	(0.144, 0.279)
	21	nig	0.144	0.018	(0.109, 0.183)	0.133	0.022	(0.091, 0.177)	0.207	0.035	(0.135, 0.273)
		ig	0.167	0.019	(0.131, 0.205)	0.161	0.019	(0.126, 0.197)	0.103	0.032	(0.047, 0.172)
	22	nig	0.163	0.020	(0.125, 0.201)	0.149	0.024	(0.105, 0.200)	0.104	0.031	(0.049, 0.173)
		ig	0.042	0.011	(0.024, 0.066)	0.039	0.010	(0.022, 0.060)	0.039	0.022	(0.007, 0.091)
	23	nig	0.044	0.011	(0.025, 0.067)	0.038	0.011	(0.019, 0.061)	0.039	0.022	(0.008, 0.093)
		ig	0.473	0.048	(0.377, 0.564)	0.486	0.049	(0.393, 0.578)	0.340	0.063	(0.215, 0.463)
	11	nig	0.473	0.049	(0.381, 0.571)	0.477	0.050	(0.369, 0.568)	0.333	0.062	(0.208, 0.456)
		ig	0.090	0.029	(0.042, 0.150)	0.115	0.039	(0.051, 0.198)	0.169	0.054	(0.075, 0.289)
	12	nig	0.091	0.030	(0.040, 0.154)	0.193	0.086	(0.073, 0.354)	0.172	0.054	(0.081, 0.282)
		ig	0.002	0.004	(0.000, 0.016)	0.002	0.005	(0.000, 0.017)	0.063	0.035	(0.012, 0.146)
	13	nig	0.002	0.005	(0.000, 0.015)	0.002	0.004	(0.000, 0.012)	0.066	0.036	(0.015, 0.156)
5		ig	0.198	0.040	(0.124, 0.280)	0.188	0.042	(0.111, 0.272)	0.254	0.050	(0.158, 0.349)
	21	nig	0.198	0.038	(0.128, 0.279)	0.153	0.058	(0.063, 0.273)	0.250	0.049	(0.155, 0.349)
		ig	0.157	0.037	(0.091, 0.232)	0.141	0.037	(0.080, 0.224)	0.126	0.042	(0.056, 0.215)
	22	nig	0.157	0.036	(0.088, 0.232)	0.117	0.046	(0.047, 0.216)	0.129	0.042	(0.058, 0.222)
		ig	0.080	0.028	(0.034, 0.145)	0.069	0.027	(0.024, 0.132)	0.047	0.027	(0.009, 0.116)
	23	nig	0.080	0.029	(0.032, 0.147)	0.058	0.030	(0.018, 0.127)	0.050	0.028	(0.011, 0.118)
		ig	0.467	0.052	(0.362, 0.574)	0.472	0.057	(0.357, 0.578)	0.346	0.063	(0.222, 0.474)
	11	nig	0.473	0.054	(0.370, 0.584)	0.450	0.060	(0.322, 0.553)	0.344	0.059	(0.226, 0.460)
		ig	0.103	0.038	(0.040, 0.189)	0.141	0.048	(0.057, 0.238)	0.170	0.055	(0.075, 0.281)
	12	nig	0.099	0.038	(0.038, 0.181)	0.227	0.084	(0.086, 0.376)	0.173	0.054	(0.077, 0.298)
		ig	0.002	0.006	(0.000, 0.019)	0.003	0.008	(0.000, 0.027)	0.064	0.036	(0.013, 0.154)
	13	nig	0.002	0.006	(0.000, 0.020)	0.002	0.004	(0.000, 0.014)	0.065	0.038	(0.013, 0.158)
6		ig	0.180	0.045	(0.102, 0.283)	0.169	0.046	(0.090, 0.270)	0.251	0.051	(0.159, 0.351)
	21	nig	0.182	0.044	(0.101, 0.270)	0.141	0.059	(0.053, 0.264)	0.248	0.050	(0.151, 0.351)
		ig	0.168	0.042	(0.089, 0.254)	0.149	0.043	(0.078, 0.246)	0.123	0.041	(0.051, 0.211)
	22	nig	0.167	0.042	(0.091, 0.257)	0.124	0.053	(0.043, 0.239)	0.124	0.039	(0.054, 0.207)
		ig	0.079	0.030	(0.029, 0.146)	0.065	0.031	(0.019, 0.138)	0.047	0.026	(0.009, 0.109)
	23	nig	0.077	0.030	(0.028, 0.143)	0.056	0.034	(0.011, 0.144)	0.046	0.027	(0.010, 0.111)
		ig	0.536	0.058	(0.425, 0.654)	0.558	0.059	(0.432, 0.667)	0.381	0.070	(0.236, 0.514)
	11	nig	0.531	0.059	(0.416, 0.644)	0.539	0.052	(0.432, 0.638)	0.378	0.067	(0.249, 0.512)
		ig	0.091	0.035	(0.035, 0.165)	0.127	0.050	(0.041, 0.228)	0.187	0.060	(0.083, 0.312)
	12	nig	0.093	0.035	(0.036, 0.173)	0.213	0.105	(0.060, 0.391)	0.193	0.059	(0.084, 0.320)
		ig	0.002	0.005	(0.000, 0.017)	0.003	0.008	(0.000, 0.024)	0.071	0.042	(0.012, 0.169)
	13	nig	0.002	0.007	(0.000, 0.022)	0.002	0.004	(0.000, 0.013)	0.072	0.039	(0.017, 0.162)
7		ig	0.135	0.040	(0.059, 0.221)	0.111	0.039	(0.045, 0.197)	0.216	0.051	(0.129, 0.328)
	21	nig	0.134	0.040	(0.068, 0.215)	0.087	0.046	(0.025, 0.191)	0.209	0.046	(0.126, 0.305)
		ig	0.176	0.044	(0.097, 0.275)	0.155	0.047	(0.075, 0.252)	0.106	0.037	(0.046, 0.188)
	22	nig	0.176	0.043	(0.093, 0.264)	0.123	0.060	(0.037, 0.247)	0.107	0.036	(0.044, 0.186)
		ig	0.061	0.029	(0.016, 0.124)	0.046	0.027	(0.011, 0.111)	0.040	0.024	(0.007, 0.098)
	23	nig	0.064	0.029	(0.019, 0.133)	0.037	0.026	(0.006, 0.102)	0.040	0.022	(0.009, 0.091)

Table 4.3: The posterior mean (PM), posterior standard error (PSD) and 95% confidence interval (CI) for p by county (NSE < 0.020)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.381	0.050	(0.286, 0.479)	0.397	0.053	(0.296, 0.500)	0.278	0.055	(0.176, 0.383)
	11	nig	0.378	0.049	(0.283, 0.474)	0.409	0.065	(0.279, 0.520)	0.280	0.053	(0.184, 0.390)
		ig	0.087	0.031	(0.036, 0.157)	0.113	0.042	(0.043, 0.209)	0.142	0.047	(0.063, 0.246)
	12	nig	0.088	0.030	(0.037, 0.151)	0.191	0.097	(0.053, 0.359)	0.139	0.043	(0.062, 0.228)
		ig	0.002	0.005	(0.000, 0.014)	0.002	0.006	(0.000, 0.018)	0.053	0.032	(0.009, 0.133)
	13	nig	0.002	0.004	(0.000, 0.013)	0.001	0.004	(0.000, 0.011)	0.053	0.029	(0.011, 0.126)
8		ig	0.211	0.043	(0.132, 0.300)	0.196	0.045	(0.114, 0.291)	0.311	0.060	(0.190, 0.428)
	21	nig	0.214	0.045	(0.134, 0.306)	0.162	0.067	(0.061, 0.288)	0.313	0.056	(0.209, 0.427)
		ig	0.228	0.044	(0.146, 0.320)	0.213	0.048	(0.123, 0.309)	0.158	0.051	(0.071, 0.267)
	22	nig	0.227	0.045	(0.143, 0.319)	0.174	0.072	(0.063, 0.311)	0.155	0.048	(0.073, 0.257)
		ig	0.091	0.032	(0.038, 0.157)	0.079	0.032	(0.029, 0.150)	0.058	0.035	(0.010, 0.143)
	23	nig	0.093	0.033	(0.039, 0.165)	0.064	0.034	(0.016, 0.140)	0.059	0.033	(0.013, 0.132)
		ig	0.408	0.042	(0.326, 0.493)	0.417	0.046	(0.330, 0.506)	0.293	0.053	(0.189, 0.397)
	11	nig	0.404	0.043	(0.318, 0.487)	0.424	0.052	(0.320, 0.517)	0.294	0.053	(0.186, 0.397)
		ig	0.087	0.026	(0.043, 0.144)	0.106	0.035	(0.048, 0.180)	0.149	0.046	(0.065, 0.244)
	12	nig	0.089	0.026	(0.046, 0.145)	0.177	0.081	(0.063, 0.320)	0.148	0.045	(0.069, 0.250)
		ig	0.001	0.003	(0.000, 0.011)	0.002	0.003	(0.000, 0.012)	0.055	0.031	(0.009, 0.124)
	13	nig	0.001	0.003	(0.000, 0.010)	0.001	0.003	(0.000, 0.009)	0.054	0.031	(0.011, 0.130)
9		ig	0.198	0.035	(0.134, 0.270)	0.188	0.038	(0.123, 0.268)	0.297	0.055	(0.196, 0.410)
	21	nig	0.201	0.037	(0.132, 0.274)	0.157	0.054	(0.068, 0.262)	0.299	0.053	(0.188, 0.402)
		ig	0.189	0.039	(0.119, 0.279)	0.178	0.039	(0.107, 0.258)	0.150	0.047	(0.069, 0.254)
	22	nig	0.190	0.037	(0.125, 0.269)	0.151	0.052	(0.068, 0.253)	0.151	0.046	(0.074, 0.254)
		ig	0.116	0.031	(0.063, 0.178)	0.109	0.031	(0.058, 0.180)	0.056	0.031	(0.011, 0.126)
	23	nig	0.114	0.030	(0.061, 0.178)	0.090	0.036	(0.035, 0.164)	0.055	0.031	(0.011, 0.132)
		ig	0.170	0.056	(0.073, 0.282)	0.175	0.077	(0.057, 0.328)	0.108	0.036	(0.049, 0.184)
	11	nig	0.167	0.053	(0.072, 0.276)	0.247	0.140	(0.058, 0.465)	0.105	0.033	(0.053, 0.174)
		ig	0.022	0.021	(0.000, 0.074)	0.058	0.058	(0.000, 0.190)	0.054	0.023	(0.019, 0.107)
	12	nig	0.022	0.023	(0.000, 0.085)	0.154	0.145	(0.000, 0.388)	0.053	0.022	(0.019, 0.103)
		ig	0.003	0.008	(0.000, 0.025)	0.003	0.008	(0.000, 0.026)	0.020	0.014	(0.003, 0.055)
	13	nig	0.003	0.007	(0.000, 0.023)	0.002	0.004	(0.000, 0.011)	0.020	0.012	(0.004, 0.052)
10		ig	0.350	0.072	(0.221, 0.508)	0.336	0.084	(0.189, 0.508)	0.486	0.087	(0.310, 0.650)
	21	nig	0.344	0.070	(0.218, 0.484)	0.261	0.134	(0.075, 0.510)	0.486	0.083	(0.321, 0.653)
		ig	0.290	0.064	(0.176, 0.420)	0.275	0.077	(0.136, 0.435)	0.242	0.075	(0.110, 0.400)
	22	nig	0.293	0.064	(0.178, 0.421)	0.212	0.112	(0.059, 0.417)	0.245	0.074	(0.113, 0.397)
		ig	0.165	0.051	(0.073, 0.284)	0.153	0.057	(0.060, 0.287)	0.091	0.052	(0.018, 0.210)
	23	nig	0.171	0.054	(0.075, 0.286)	0.124	0.073	(0.028, 0.280)	0.091	0.051	(0.019, 0.215)
		ig	0.384	0.058	(0.274, 0.497)	0.407	0.062	(0.279, 0.525)	0.301	0.061	(0.185, 0.427)
	11	nig	0.384	0.058	(0.276, 0.501)	0.425	0.071	(0.286, 0.540)	0.298	0.060	(0.182, 0.428)
		ig	0.080	0.032	(0.028, 0.151)	0.116	0.054	(0.029, 0.229)	0.150	0.049	(0.067, 0.261)
	12	nig	0.081	0.035	(0.027, 0.164)	0.198	0.117	(0.038, 0.395)	0.148	0.047	(0.068, 0.256)
		ig	0.033	0.022	(0.004, 0.082)	0.035	0.023	(0.006, 0.091)	0.056	0.032	(0.011, 0.132)
	13	nig	0.033	0.021	(0.004, 0.091)	0.027	0.021	(0.003, 0.084)	0.054	0.031	(0.010, 0.132)
11		ig	0.173	0.053	(0.076, 0.283)	0.149	0.056	(0.056, 0.271)	0.293	0.061	(0.175, 0.422)
	21	nig	0.173	0.053	(0.076, 0.287)	0.119	0.067	(0.027, 0.262)	0.297	0.058	(0.186, 0.410)
		ig	0.219	0.055	(0.115, 0.328)	0.198	0.061	(0.094, 0.328)	0.145	0.046	(0.065, 0.246)
	22	nig	0.216	0.056	(0.114, 0.335)	0.154	0.079	(0.044, 0.309)	0.148	0.047	(0.068, 0.256)
		ig	0.112	0.043	(0.039, 0.207)	0.096	0.047	(0.027, 0.212)	0.055	0.031	(0.011, 0.130)
	23	nig	0.113	0.045	(0.039, 0.209)	0.077	0.051	(0.012, 0.197)	0.054	0.031	(0.010, 0.123)
		ig	0.425	0.059	(0.314, 0.539)	0.440	0.061	(0.313, 0.552)	0.295	0.061	(0.178, 0.410)
	11	nig	0.423	0.059	(0.312, 0.543)	0.446	0.064	(0.316, 0.559)	0.296	0.058	(0.190, 0.419)
		ig	0.067	0.030	(0.019, 0.136)	0.099	0.050	(0.021, 0.210)	0.149	0.050	(0.065, 0.267)
	12	nig	0.067	0.031	(0.019, 0.139)	0.192	0.112	(0.036, 0.384)	0.148	0.049	(0.065, 0.261)
		ig	0.002	0.006	(0.000, 0.021)	0.003	0.007	(0.000, 0.025)	0.055	0.032	(0.010, 0.137)
	13	nig	0.002	0.006	(0.000, 0.019)	0.002	0.004	(0.000, 0.015)	0.056	0.032	(0.010, 0.133)
12		ig	0.153	0.045	(0.071, 0.250)	0.131	0.047	(0.058, 0.243)	0.296	0.062	(0.180, 0.414)
	21	nig	0.155	0.044	(0.079, 0.249)	0.104	0.057	(0.028, 0.231)	0.296	0.059	(0.183, 0.414)
		ig	0.188	0.050	(0.101, 0.295)	0.163	0.050	(0.076, 0.274)	0.149	0.048	(0.068, 0.259)
	22	nig	0.187	0.049	(0.101, 0.285)	0.131	0.066	(0.037, 0.277)	0.148	0.049	(0.064, 0.257)
		ig	0.165	0.046	(0.083, 0.262)	0.163	0.053	(0.076, 0.275)	0.056	0.034	(0.010, 0.135)
	23	nig	0.165	0.048	(0.082, 0.275)	0.126	0.064	(0.036, 0.264)	0.055	0.031	(0.011, 0.127)
		ig	0.581	0.046	(0.494, 0.668)	0.595	0.044	(0.509, 0.682)	0.395	0.068	(0.261, 0.532)
	11	nig	0.574	0.044	(0.487, 0.660)	0.554	0.043	(0.465, 0.638)	0.393	0.068	(0.252, 0.529)
		ig	0.075	0.025	(0.034, 0.129)	0.096	0.034	(0.040, 0.169)	0.196	0.058	(0.095, 0.323)
	12	nig	0.081	0.027	(0.038, 0.138)	0.183	0.076	(0.069, 0.325)	0.199	0.061	(0.088, 0.329)
		ig	0.001	0.004	(0.000, 0.012)	0.002	0.005	(0.000, 0.013)	0.071	0.040	(0.013, 0.168)
	13	nig	0.001	0.004	(0.000, 0.010)	0.001	0.004	(0.000, 0.011)	0.072	0.039	(0.015, 0.165)
13		ig	0.146	0.034	(0.082, 0.218)	0.134	0.035	(0.071, 0.210)	0.202	0.039	(0.125, 0.279)
	21	nig	0.146	0.033	(0.087, 0.210)	0.114	0.043	(0.045, 0.203)	0.199	0.039	(0.127, 0.280)
		ig	0.149	0.034	(0.085, 0.221)	0.135	0.035	(0.071, 0.208)	0.100	0.031	(0.046, 0.165)
	22	nig	0.149	0.034	(0.088, 0.218)	0.116	0.043	(0.046, 0.203)	0.100	0.032	(0.042, 0.172)
		ig	0.048	0.022	(0.013, 0.098)	0.038	0.021	(0.008, 0.090)	0.036	0.021	(0.007, 0.089)
	23	nig	0.049	0.022	(0.012, 0.100)	0.031	0.020	(0.005, 0.076)	0.037	0.020	(0.007, 0.087)
		ig	0.325	0.048	(0.231, 0.421)	0.331	0.057	(0.222, 0.447)	0.241	0.050	(0.142, 0.343)
	11	nig	0.320	0.049	(0.225, 0.419)	0.358	0.078	(0.219, 0.492)	0.240	0.048	(0.142, 0.337)
		ig	0.082	0.031	(0.032, 0.152)	0.113	0.044	(0.040, 0.207)	0.121	0.041	(0.051, 0.213)
	12	nig	0.087	0.032	(0.035, 0.159)	0.192	0.097	(0.057, 0.356)	0.123	0.041	(0.057, 0.216)
		ig	0.001	0.004	(0.000, 0.014)	0.002	0.006	(0.000, 0.020)	0.047	0.026	(0.008, 0.106)
	13	nig	0.002	0.005	(0.000, 0.014)	0.001	0.003	(0.000, 0.011)	0.046	0.028	(0.009, 0.119)
14		ig	0.163	0.042	(0.086, 0.247)	0.138	0.043	(0.068, 0.231)	0.349	0.065	(0.225, 0.475)
	21	nig	0.163	0.043	(0.086, 0.254)	0.117	0.054	(0.040, 0.234)	0.347	0.064	(0.218, 0.477)
		ig	0.297	0.052	(0.199, 0.406)	0.295	0.060	(0.189, 0.423)	0.174	0.056	(0.078, 0.297)
	22	nig	0.297	0.055	(0.201, 0.421)	0.236	0.096	(0.091, 0.411)	0.178	0.056	(0.083, 0.307)
		ig	0.132	0.039	(0.063, 0.214)	0.120	0.040	(0.054, 0.210)	0.068	0.039	(0.013, 0.166)
	23	nig	0.131	0.040	(0.064, 0.217)	0.097	0.047	(0.031, 0.202)	0.066	0.039	(0.013, 0.167)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.382	0.061	(0.272, 0.506)	0.399	0.065	(0.273, 0.527)	0.311	0.061	(0.188, 0.435)
	11	nig	0.382	0.061	(0.264, 0.504)	0.418	0.077	(0.257, 0.544)	0.309	0.063	(0.189, 0.435)
		ig	0.128	0.044	(0.055, 0.226)	0.175	0.060	(0.071, 0.299)	0.155	0.050	(0.068, 0.261)
	12	nig	0.133	0.045	(0.058, 0.232)	0.249	0.102	(0.090, 0.413)	0.155	0.049	(0.074, 0.268)
		ig	0.003	0.006	(0.000, 0.024)	0.003	0.008	(0.000, 0.027)	0.058	0.033	(0.012, 0.134)
	13	nig	0.003	0.008	(0.000, 0.023)	0.002	0.005	(0.000, 0.014)	0.059	0.033	(0.013, 0.143)
15		ig	0.185	0.050	(0.098, 0.302)	0.160	0.053	(0.071, 0.280)	0.283	0.060	(0.167, 0.411)
	21	nig	0.185	0.048	(0.095, 0.280)	0.124	0.065	(0.033, 0.260)	0.281	0.056	(0.171, 0.392)
		ig	0.241	0.057	(0.139, 0.359)	0.223	0.062	(0.116, 0.352)	0.140	0.046	(0.062, 0.242)
	22	nig	0.237	0.055	(0.136, 0.353)	0.174	0.089	(0.052, 0.342)	0.142	0.046	(0.064, 0.242)
		ig	0.061	0.032	(0.014, 0.136)	0.040	0.028	(0.005, 0.113)	0.053	0.030	(0.010, 0.126)
	23	nig	0.060	0.032	(0.010, 0.130)	0.032	0.028	(0.003, 0.106)	0.054	0.031	(0.012, 0.133)
		ig	0.403	0.065	(0.278, 0.532)	0.433	0.070	(0.294, 0.568)	0.268	0.059	(0.158, 0.384)
	11	nig	0.400	0.068	(0.274, 0.531)	0.451	0.071	(0.296, 0.564)	0.265	0.062	(0.153, 0.395)
		ig	0.040	0.027	(0.003, 0.108)	0.077	0.066	(0.002, 0.223)	0.133	0.046	(0.055, 0.237)
	12	nig	0.039	0.027	(0.003, 0.106)	0.173	0.147	(0.005, 0.401)	0.135	0.045	(0.060, 0.232)
		ig	0.002	0.007	(0.000, 0.021)	0.003	0.007	(0.000, 0.024)	0.052	0.031	(0.009, 0.125)
	13	nig	0.003	0.007	(0.000, 0.021)	0.002	0.006	(0.000, 0.015)	0.049	0.027	(0.010, 0.109)
16		ig	0.271	0.069	(0.150, 0.414)	0.259	0.082	(0.123, 0.431)	0.323	0.068	(0.199, 0.457)
	21	nig	0.270	0.069	(0.149, 0.419)	0.199	0.112	(0.047, 0.416)	0.325	0.065	(0.207, 0.457)
		ig	0.193	0.062	(0.088, 0.325)	0.163	0.069	(0.052, 0.320)	0.161	0.054	(0.072, 0.285)
	22	nig	0.199	0.066	(0.082, 0.333)	0.126	0.082	(0.024, 0.305)	0.166	0.053	(0.073, 0.277)
		ig	0.090	0.046	(0.018, 0.193)	0.064	0.044	(0.008, 0.175)	0.063	0.038	(0.011, 0.157)
	23	nig	0.090	0.045	(0.023, 0.195)	0.049	0.043	(0.004, 0.158)	0.061	0.035	(0.012, 0.144)
		ig	0.242	0.053	(0.139, 0.352)	0.252	0.063	(0.138, 0.374)	0.172	0.042	(0.094, 0.262)
	11	nig	0.240	0.051	(0.146, 0.346)	0.301	0.107	(0.132, 0.479)	0.169	0.041	(0.095, 0.257)
		ig	0.046	0.025	(0.010, 0.103)	0.082	0.053	(0.009, 0.200)	0.084	0.030	(0.033, 0.154)
	12	nig	0.049	0.026	(0.009, 0.108)	0.167	0.124	(0.017, 0.366)	0.085	0.032	(0.034, 0.157)
		ig	0.002	0.005	(0.000, 0.017)	0.003	0.007	(0.000, 0.023)	0.032	0.019	(0.007, 0.076)
	13	nig	0.002	0.006	(0.000, 0.021)	0.002	0.004	(0.000, 0.012)	0.032	0.018	(0.007, 0.075)
17		ig	0.264	0.061	(0.155, 0.391)	0.246	0.070	(0.124, 0.396)	0.425	0.073	(0.283, 0.569)
	21	nig	0.263	0.059	(0.157, 0.383)	0.198	0.100	(0.052, 0.391)	0.421	0.076	(0.272, 0.568)
		ig	0.223	0.056	(0.125, 0.343)	0.200	0.062	(0.091, 0.334)	0.208	0.065	(0.094, 0.349)
	22	nig	0.227	0.058	(0.122, 0.354)	0.159	0.080	(0.049, 0.324)	0.213	0.068	(0.093, 0.366)
		ig	0.224	0.058	(0.122, 0.347)	0.217	0.064	(0.111, 0.356)	0.080	0.044	(0.017, 0.181)
	23	nig	0.219	0.059	(0.121, 0.350)	0.173	0.089	(0.047, 0.354)	0.080	0.042	(0.016, 0.180)
		ig	0.339	0.059	(0.232, 0.452)	0.357	0.067	(0.225, 0.494)	0.248	0.055	(0.148, 0.355)
	11	nig	0.344	0.059	(0.231, 0.459)	0.380	0.087	(0.221, 0.520)	0.252	0.053	(0.155, 0.363)
		ig	0.077	0.035	(0.023, 0.155)	0.117	0.055	(0.030, 0.234)	0.126	0.045	(0.051, 0.233)
	12	nig	0.075	0.036	(0.019, 0.161)	0.203	0.115	(0.044, 0.392)	0.126	0.041	(0.055, 0.220)
		ig	0.003	0.008	(0.000, 0.029)	0.003	0.008	(0.000, 0.029)	0.047	0.027	(0.010, 0.112)
	13	nig	0.003	0.007	(0.000, 0.023)	0.002	0.004	(0.000, 0.013)	0.046	0.027	(0.010, 0.113)
18		ig	0.263	0.061	(0.151, 0.389)	0.250	0.067	(0.129, 0.388)	0.342	0.067	(0.212, 0.471)
	21	nig	0.260	0.062	(0.147, 0.394)	0.197	0.101	(0.055, 0.385)	0.342	0.065	(0.218, 0.471)
		ig	0.268	0.063	(0.156, 0.407)	0.247	0.069	(0.129, 0.387)	0.173	0.057	(0.075, 0.302)
	22	nig	0.265	0.064	(0.154, 0.398)	0.199	0.098	(0.059, 0.381)	0.171	0.053	(0.077, 0.282)
		ig	0.051	0.034	(0.004, 0.126)	0.024	0.024	(0.001, 0.088)	0.065	0.037	(0.014, 0.156)
	23	nig	0.053	0.034	(0.004, 0.130)	0.020	0.023	(0.000, 0.084)	0.063	0.036	(0.013, 0.151)
		ig	0.182	0.070	(0.054, 0.321)	0.193	0.098	(0.043, 0.400)	0.116	0.045	(0.046, 0.216)
	11	nig	0.178	0.068	(0.052, 0.316)	0.268	0.153	(0.056, 0.501)	0.113	0.040	(0.046, 0.203)
		ig	0.027	0.028	(0.000, 0.102)	0.070	0.080	(0.000, 0.241)	0.058	0.026	(0.017, 0.118)
	12	nig	0.025	0.027	(0.000, 0.089)	0.170	0.170	(0.000, 0.429)	0.058	0.025	(0.019, 0.112)
		ig	0.003	0.010	(0.000, 0.030)	0.004	0.010	(0.000, 0.032)	0.022	0.016	(0.004, 0.063)
	13	nig	0.003	0.010	(0.000, 0.026)	0.002	0.006	(0.000, 0.015)	0.021	0.015	(0.003, 0.059)
19		ig	0.336	0.088	(0.183, 0.526)	0.325	0.111	(0.138, 0.570)	0.473	0.086	(0.295, 0.637)
	21	nig	0.340	0.092	(0.182, 0.539)	0.251	0.159	(0.043, 0.537)	0.477	0.087	(0.316, 0.650)
		ig	0.309	0.086	(0.164, 0.494)	0.283	0.103	(0.112, 0.500)	0.238	0.075	(0.104, 0.395)
	22	nig	0.309	0.083	(0.163, 0.478)	0.216	0.141	(0.039, 0.497)	0.242	0.073	(0.110, 0.393)
		ig	0.143	0.061	(0.045, 0.283)	0.125	0.069	(0.026, 0.286)	0.092	0.055	(0.017, 0.229)
	23	nig	0.145	0.063	(0.048, 0.299)	0.093	0.079	(0.010, 0.285)	0.089	0.051	(0.016, 0.211)
		ig	0.300	0.059	(0.187, 0.409)	0.310	0.068	(0.181, 0.439)	0.212	0.050	(0.123, 0.314)
	11	nig	0.301	0.059	(0.191, 0.426)	0.348	0.098	(0.171, 0.505)	0.210	0.047	(0.116, 0.306)
		ig	0.058	0.031	(0.012, 0.127)	0.101	0.058	(0.013, 0.233)	0.106	0.037	(0.047, 0.185)
	12	nig	0.062	0.033	(0.012, 0.137)	0.191	0.124	(0.028, 0.392)	0.106	0.035	(0.049, 0.180)
		ig	0.003	0.007	(0.000, 0.025)	0.003	0.007	(0.000, 0.025)	0.040	0.024	(0.007, 0.099)
	13	nig	0.002	0.006	(0.000, 0.023)	0.002	0.005	(0.000, 0.014)	0.041	0.025	(0.008, 0.105)
20		ig	0.157	0.052	(0.067, 0.271)	0.127	0.053	(0.047, 0.259)	0.381	0.072	(0.237, 0.517)
	21	nig	0.156	0.054	(0.064, 0.273)	0.101	0.062	(0.023, 0.249)	0.377	0.070	(0.246, 0.514)
		ig	0.306	0.065	(0.186, 0.441)	0.289	0.077	(0.144, 0.438)	0.190	0.062	(0.083, 0.325)
	22	nig	0.300	0.066	(0.186, 0.441)	0.228	0.113	(0.067, 0.438)	0.191	0.058	(0.096, 0.311)
		ig	0.178	0.056	(0.085, 0.296)	0.170	0.057	(0.070, 0.285)	0.071	0.039	(0.013, 0.165)
	23	nig	0.179	0.058	(0.084, 0.312)	0.131	0.074	(0.029, 0.293)	0.074	0.043	(0.013, 0.179)
		ig	0.205	0.061	(0.094, 0.331)	0.217	0.082	(0.080, 0.388)	0.166	0.047	(0.090, 0.274)
	11	nig	0.199	0.060	(0.090, 0.316)	0.280	0.136	(0.081, 0.491)	0.166	0.045	(0.092, 0.267)
		ig	0.085	0.038	(0.026, 0.171)	0.131	0.066	(0.029, 0.268)	0.083	0.032	(0.034, 0.156)
	12	nig	0.084	0.039	(0.025, 0.172)	0.214	0.131	(0.033, 0.422)	0.084	0.033	(0.032, 0.158)
		ig	0.003	0.007	(0.000, 0.025)	0.003	0.008	(0.000, 0.026)	0.033	0.020	(0.007, 0.080)
	13	nig	0.003	0.007	(0.000, 0.023)	0.002	0.004	(0.000, 0.014)	0.031	0.019	(0.006, 0.078)
21		ig	0.213	0.056	(0.111, 0.326)	0.189	0.066	(0.085, 0.334)	0.423	0.080	(0.270, 0.576)
	21	nig	0.217	0.059	(0.115, 0.341)	0.145	0.087	(0.033, 0.324)	0.426	0.078	(0.267, 0.585)
		ig	0.363	0.069	(0.233, 0.504)	0.346	0.090	(0.187, 0.523)	0.211	0.066	(0.094, 0.348)
	22	nig	0.362	0.072	(0.230, 0.524)	0.269	0.147	(0.068, 0.529)	0.214	0.066	(0.098, 0.353)
		ig	0.131	0.048	(0.055, 0.243)	0.114	0.053	(0.037, 0.245)	0.084	0.048	(0.019, 0.199)
	23	nig	0.136	0.049	(0.056, 0.249)	0.090	0.062	(0.014, 0.235)	0.079	0.044	(0.015, 0.184)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.255	0.060	(0.147, 0.372)	0.265	0.071	(0.134, 0.404)	0.214	0.050	(0.126, 0.316)
	11	nig	0.256	0.058	(0.149, 0.368)	0.310	0.107	(0.134, 0.485)	0.214	0.046	(0.130, 0.302)
		ig	0.100	0.040	(0.036, 0.191)	0.144	0.060	(0.047, 0.266)	0.106	0.036	(0.046, 0.184)
	12	nig	0.102	0.043	(0.038, 0.201)	0.223	0.110	(0.057, 0.408)	0.107	0.036	(0.047, 0.192)
		ig	0.003	0.007	(0.000, 0.021)	0.004	0.009	(0.000, 0.029)	0.040	0.024	(0.008, 0.099)
	13	nig	0.002	0.007	(0.000, 0.022)	0.002	0.005	(0.000, 0.015)	0.041	0.024	(0.008, 0.099)
22		ig	0.171	0.063	(0.064, 0.312)	0.136	0.070	(0.036, 0.304)	0.380	0.070	(0.247, 0.522)
	21	nig	0.165	0.066	(0.053, 0.311)	0.102	0.073	(0.015, 0.279)	0.379	0.068	(0.238, 0.509)
		ig	0.330	0.080	(0.178, 0.494)	0.328	0.091	(0.166, 0.509)	0.189	0.059	(0.084, 0.306)
	22	nig	0.340	0.082	(0.189, 0.506)	0.273	0.137	(0.076, 0.528)	0.189	0.058	(0.087, 0.315)
		ig	0.141	0.059	(0.041, 0.269)	0.124	0.065	(0.023, 0.276)	0.071	0.040	(0.016, 0.173)
	23	nig	0.135	0.061	(0.043, 0.285)	0.090	0.064	(0.014, 0.249)	0.071	0.039	(0.015, 0.161)
		ig	0.395	0.073	(0.259, 0.548)	0.423	0.084	(0.257, 0.587)	0.327	0.070	(0.194, 0.479)
	11	nig	0.395	0.076	(0.256, 0.551)	0.434	0.083	(0.251, 0.566)	0.323	0.068	(0.205, 0.460)
		ig	0.136	0.055	(0.045, 0.252)	0.197	0.076	(0.068, 0.356)	0.162	0.056	(0.068, 0.287)
	12	nig	0.139	0.057	(0.047, 0.268)	0.275	0.116	(0.084, 0.453)	0.165	0.055	(0.075, 0.286)
		ig	0.004	0.012	(0.000, 0.038)	0.005	0.011	(0.000, 0.043)	0.062	0.035	(0.009, 0.148)
	13	nig	0.003	0.010	(0.000, 0.028)	0.002	0.006	(0.000, 0.017)	0.060	0.034	(0.012, 0.144)
23		ig	0.195	0.064	(0.085, 0.333)	0.168	0.073	(0.056, 0.342)	0.266	0.062	(0.153, 0.397)
	21	nig	0.196	0.066	(0.077, 0.333)	0.130	0.087	(0.021, 0.323)	0.267	0.063	(0.150, 0.391)
		ig	0.138	0.061	(0.032, 0.275)	0.088	0.054	(0.012, 0.221)	0.132	0.048	(0.053, 0.241)
	22	nig	0.133	0.059	(0.031, 0.262)	0.068	0.057	(0.007, 0.214)	0.136	0.046	(0.059, 0.236)
		ig	0.133	0.057	(0.042, 0.270)	0.119	0.063	(0.024, 0.263)	0.050	0.029	(0.008, 0.119)
	23	nig	0.133	0.057	(0.041, 0.264)	0.092	0.070	(0.010, 0.260)	0.049	0.028	(0.010, 0.117)
		ig	0.297	0.075	(0.153, 0.452)	0.321	0.096	(0.145, 0.500)	0.250	0.063	(0.139, 0.386)
	11	nig	0.292	0.076	(0.148, 0.445)	0.362	0.118	(0.141, 0.541)	0.251	0.063	(0.139, 0.390)
		ig	0.124	0.057	(0.035, 0.252)	0.191	0.084	(0.051, 0.359)	0.127	0.047	(0.051, 0.231)
	12	nig	0.128	0.058	(0.040, 0.262)	0.267	0.130	(0.058, 0.467)	0.128	0.046	(0.054, 0.228)
		ig	0.004	0.010	(0.000, 0.032)	0.005	0.012	(0.000, 0.041)	0.045	0.027	(0.009, 0.113)
	13	nig	0.004	0.011	(0.000, 0.034)	0.002	0.005	(0.000, 0.016)	0.047	0.028	(0.007, 0.114)
24		ig	0.242	0.079	(0.101, 0.419)	0.212	0.093	(0.058, 0.423)	0.342	0.074	(0.203, 0.487)
	21	nig	0.243	0.081	(0.108, 0.431)	0.171	0.122	(0.022, 0.435)	0.338	0.072	(0.205, 0.486)
		ig	0.206	0.079	(0.066, 0.376)	0.165	0.086	(0.039, 0.366)	0.173	0.057	(0.077, 0.300)
	22	nig	0.207	0.079	(0.068, 0.377)	0.121	0.097	(0.014, 0.354)	0.172	0.057	(0.076, 0.293)
		ig	0.128	0.063	(0.031, 0.279)	0.106	0.069	(0.013, 0.276)	0.062	0.036	(0.012, 0.149)
	23	nig	0.126	0.059	(0.038, 0.258)	0.077	0.070	(0.004, 0.267)	0.064	0.036	(0.012, 0.148)
		ig	0.280	0.064	(0.150, 0.400)	0.297	0.082	(0.146, 0.453)	0.205	0.052	(0.112, 0.311)
	11	nig	0.280	0.066	(0.154, 0.414)	0.343	0.113	(0.142, 0.525)	0.203	0.053	(0.113, 0.318)
		ig	0.066	0.037	(0.014, 0.153)	0.115	0.070	(0.012, 0.262)	0.104	0.040	(0.043, 0.197)
	12	nig	0.069	0.037	(0.014, 0.153)	0.206	0.140	(0.024, 0.420)	0.105	0.036	(0.047, 0.186)
		ig	0.003	0.008	(0.000, 0.029)	0.004	0.010	(0.000, 0.037)	0.039	0.024	(0.007, 0.100)
	13	nig	0.003	0.008	(0.000, 0.025)	0.002	0.006	(0.000, 0.013)	0.040	0.023	(0.007, 0.097)
25		ig	0.348	0.079	(0.205, 0.507)	0.336	0.096	(0.172, 0.534)	0.385	0.078	(0.228, 0.532)
	21	nig	0.341	0.076	(0.208, 0.505)	0.253	0.143	(0.064, 0.498)	0.380	0.071	(0.243, 0.515)
		ig	0.215	0.064	(0.104, 0.353)	0.187	0.073	(0.072, 0.351)	0.195	0.064	(0.084, 0.336)
	22	nig	0.220	0.065	(0.103, 0.363)	0.150	0.095	(0.032, 0.358)	0.197	0.061	(0.090, 0.318)
		ig	0.088	0.047	(0.019, 0.189)	0.061	0.042	(0.008, 0.171)	0.072	0.042	(0.014, 0.176)
	23	nig	0.087	0.045	(0.018, 0.186)	0.047	0.044	(0.004, 0.168)	0.075	0.042	(0.014, 0.173)
		ig	0.343	0.061	(0.228, 0.467)	0.348	0.070	(0.212, 0.482)	0.242	0.054	(0.144, 0.352)
	11	nig	0.342	0.057	(0.230, 0.461)	0.367	0.094	(0.200, 0.524)	0.238	0.051	(0.143, 0.353)
		ig	0.068	0.037	(0.013, 0.151)	0.110	0.054	(0.023, 0.231)	0.122	0.041	(0.055, 0.214)
	12	nig	0.067	0.036	(0.013, 0.148)	0.207	0.111	(0.046, 0.389)	0.123	0.041	(0.053, 0.210)
		ig	0.003	0.007	(0.000, 0.025)	0.004	0.009	(0.000, 0.029)	0.045	0.027	(0.008, 0.111)
	13	nig	0.002	0.007	(0.000, 0.022)	0.002	0.005	(0.000, 0.015)	0.047	0.026	(0.008, 0.107)
26		ig	0.252	0.069	(0.130, 0.398)	0.249	0.083	(0.105, 0.425)	0.351	0.069	(0.225, 0.491)
	21	nig	0.260	0.072	(0.128, 0.409)	0.198	0.109	(0.048, 0.414)	0.347	0.067	(0.211, 0.480)
		ig	0.169	0.065	(0.054, 0.308)	0.129	0.067	(0.031, 0.288)	0.176	0.058	(0.079, 0.296)
	22	nig	0.165	0.065	(0.052, 0.304)	0.102	0.072	(0.018, 0.285)	0.179	0.057	(0.079, 0.305)
		ig	0.166	0.063	(0.060, 0.303)	0.159	0.071	(0.050, 0.317)	0.064	0.037	(0.013, 0.150)
	23	nig	0.163	0.063	(0.061, 0.303)	0.123	0.082	(0.022, 0.312)	0.067	0.036	(0.012, 0.152)
		ig	0.170	0.053	(0.076, 0.282)	0.179	0.072	(0.060, 0.332)	0.116	0.034	(0.056, 0.192)
	11	nig	0.171	0.051	(0.081, 0.278)	0.250	0.135	(0.066, 0.460)	0.119	0.035	(0.058, 0.193)
		ig	0.037	0.027	(0.003, 0.106)	0.073	0.057	(0.002, 0.194)	0.062	0.025	(0.021, 0.120)
	12	nig	0.038	0.026	(0.004, 0.104)	0.161	0.137	(0.005, 0.378)	0.061	0.024	(0.024, 0.113)
		ig	0.002	0.006	(0.000, 0.019)	0.003	0.007	(0.000, 0.022)	0.021	0.013	(0.003, 0.052)
	13	nig	0.002	0.006	(0.000, 0.021)	0.002	0.004	(0.000, 0.012)	0.022	0.014	(0.003, 0.056)
27		ig	0.193	0.068	(0.074, 0.341)	0.156	0.074	(0.046, 0.322)	0.468	0.086	(0.301, 0.634)
	21	nig	0.190	0.072	(0.067, 0.340)	0.121	0.081	(0.021, 0.324)	0.472	0.081	(0.310, 0.627)
		ig	0.466	0.091	(0.301, 0.662)	0.481	0.108	(0.277, 0.688)	0.247	0.076	(0.110, 0.414)
	22	nig	0.471	0.092	(0.300, 0.661)	0.386	0.186	(0.117, 0.694)	0.241	0.074	(0.112, 0.410)
		ig	0.131	0.058	(0.044, 0.265)	0.108	0.061	(0.022, 0.249)	0.086	0.049	(0.016, 0.201)
	23	nig	0.127	0.056	(0.035, 0.254)	0.081	0.059	(0.011, 0.231)	0.086	0.049	(0.017, 0.214)
		ig	0.265	0.074	(0.130, 0.418)	0.292	0.096	(0.116, 0.472)	0.226	0.063	(0.117, 0.354)
	11	nig	0.264	0.075	(0.128, 0.417)	0.343	0.130	(0.116, 0.536)	0.225	0.060	(0.119, 0.351)
		ig	0.112	0.052	(0.035, 0.230)	0.173	0.085	(0.030, 0.341)	0.111	0.043	(0.045, 0.217)
	12	nig	0.112	0.052	(0.034, 0.237)	0.248	0.134	(0.050, 0.456)	0.116	0.045	(0.042, 0.218)
		ig	0.003	0.010	(0.000, 0.031)	0.004	0.011	(0.000, 0.032)	0.042	0.025	(0.006, 0.099)
	13	nig	0.004	0.010	(0.000, 0.031)	0.002	0.005	(0.000, 0.014)	0.042	0.024	(0.009, 0.101)
28		ig	0.178	0.076	(0.049, 0.334)	0.125	0.077	(0.023, 0.328)	0.370	0.078	(0.224, 0.517)
	21	nig	0.177	0.077	(0.051, 0.340)	0.101	0.087	(0.009, 0.331)	0.364	0.079	(0.221, 0.530)
		ig	0.221	0.080	(0.082, 0.391)	0.180	0.094	(0.046, 0.397)	0.183	0.063	(0.076, 0.327)
	22	nig	0.225	0.083	(0.081, 0.415)	0.137	0.106	(0.015, 0.380)	0.185	0.062	(0.081, 0.320)
		ig	0.221	0.089	(0.084, 0.422)	0.226	0.109	(0.058, 0.485)	0.068	0.038	(0.010, 0.155)
	23	nig	0.218	0.088	(0.075, 0.424)	0.170	0.127	(0.021, 0.441)	0.068	0.037	(0.015, 0.158)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.339	0.047	(0.253, 0.435)	0.349	0.054	(0.243, 0.454)	0.241	0.050	(0.147, 0.344)
	11	nig	0.337	0.050	(0.241, 0.432)	0.370	0.074	(0.231, 0.493)	0.240	0.049	(0.153, 0.339)
		ig	0.065	0.027	(0.021, 0.125)	0.093	0.043	(0.026, 0.185)	0.118	0.039	(0.053, 0.206)
	12	nig	0.067	0.029	(0.020, 0.130)	0.175	0.103	(0.035, 0.352)	0.121	0.038	(0.051, 0.199)
		ig	0.002	0.005	(0.000, 0.013)	0.002	0.005	(0.000, 0.016)	0.046	0.026	(0.008, 0.109)
	13	nig	0.002	0.004	(0.000, 0.016)	0.002	0.004	(0.000, 0.011)	0.045	0.026	(0.009, 0.102)
29		ig	0.239	0.057	(0.137, 0.353)	0.229	0.059	(0.124, 0.350)	0.355	0.065	(0.229, 0.480)
	21	nig	0.238	0.056	(0.136, 0.365)	0.188	0.082	(0.065, 0.350)	0.352	0.063	(0.232, 0.472)
		ig	0.222	0.052	(0.128, 0.334)	0.206	0.057	(0.103, 0.325)	0.174	0.057	(0.081, 0.301)
	22	nig	0.226	0.055	(0.129, 0.345)	0.168	0.076	(0.057, 0.325)	0.177	0.056	(0.077, 0.296)
		ig	0.133	0.046	(0.055, 0.233)	0.121	0.045	(0.044, 0.217)	0.068	0.039	(0.012, 0.160)
	23	nig	0.131	0.044	(0.061, 0.224)	0.096	0.050	(0.028, 0.213)	0.065	0.038	(0.012, 0.154)
		ig	0.424	0.043	(0.348, 0.512)	0.430	0.044	(0.342, 0.520)	0.291	0.054	(0.188, 0.396)
	11	nig	0.422	0.041	(0.343, 0.504)	0.421	0.053	(0.315, 0.512)	0.290	0.055	(0.182, 0.395)
		ig	0.068	0.025	(0.027, 0.126)	0.090	0.033	(0.033, 0.165)	0.149	0.048	(0.066, 0.254)
	12	nig	0.070	0.025	(0.031, 0.125)	0.168	0.076	(0.060, 0.312)	0.152	0.047	(0.071, 0.257)
		ig	0.001	0.003	(0.000, 0.011)	0.002	0.004	(0.000, 0.011)	0.054	0.032	(0.011, 0.128)
	13	nig	0.001	0.004	(0.000, 0.013)	0.001	0.003	(0.000, 0.010)	0.054	0.030	(0.012, 0.127)
30		ig	0.174	0.040	(0.099, 0.260)	0.161	0.038	(0.092, 0.244)	0.298	0.055	(0.194, 0.409)
	21	nig	0.176	0.043	(0.102, 0.267)	0.136	0.051	(0.056, 0.252)	0.294	0.055	(0.182, 0.396)
		ig	0.278	0.045	(0.195, 0.370)	0.278	0.048	(0.188, 0.378)	0.152	0.049	(0.067, 0.262)
	22	nig	0.278	0.046	(0.189, 0.369)	0.241	0.074	(0.118, 0.385)	0.154	0.048	(0.072, 0.257)
		ig	0.054	0.025	(0.016, 0.114)	0.039	0.022	(0.008, 0.090)	0.056	0.033	(0.011, 0.130)
	23	nig	0.052	0.025	(0.013, 0.109)	0.032	0.021	(0.005, 0.084)	0.055	0.031	(0.012, 0.128)
		ig	0.423	0.065	(0.299, 0.553)	0.453	0.071	(0.314, 0.595)	0.361	0.071	(0.230, 0.506)
	11	nig	0.423	0.066	(0.299, 0.558)	0.460	0.070	(0.306, 0.576)	0.358	0.067	(0.228, 0.485)
		ig	0.153	0.048	(0.073, 0.259)	0.199	0.064	(0.088, 0.334)	0.181	0.058	(0.083, 0.312)
	12	nig	0.153	0.048	(0.073, 0.263)	0.269	0.103	(0.109, 0.439)	0.181	0.054	(0.092, 0.299)
		ig	0.021	0.021	(0.001, 0.075)	0.024	0.022	(0.001, 0.075)	0.065	0.037	(0.012, 0.152)
	13	nig	0.021	0.021	(0.001, 0.077)	0.017	0.016	(0.001, 0.063)	0.069	0.039	(0.015, 0.167)
31		ig	0.163	0.052	(0.071, 0.268)	0.138	0.056	(0.049, 0.271)	0.233	0.055	(0.133, 0.347)
	21	nig	0.163	0.052	(0.068, 0.273)	0.108	0.069	(0.023, 0.262)	0.231	0.053	(0.132, 0.336)
		ig	0.099	0.047	(0.021, 0.206)	0.056	0.036	(0.008, 0.149)	0.117	0.040	(0.050, 0.204)
	22	nig	0.098	0.048	(0.017, 0.211)	0.047	0.039	(0.005, 0.150)	0.117	0.038	(0.056, 0.205)
		ig	0.140	0.051	(0.058, 0.258)	0.131	0.054	(0.048, 0.249)	0.042	0.025	(0.008, 0.098)
	23	nig	0.142	0.049	(0.058, 0.246)	0.099	0.065	(0.018, 0.251)	0.044	0.026	(0.010, 0.111)
		ig	0.420	0.047	(0.333, 0.518)	0.434	0.049	(0.340, 0.528)	0.316	0.058	(0.203, 0.425)
	11	nig	0.420	0.049	(0.327, 0.513)	0.431	0.059	(0.309, 0.532)	0.315	0.058	(0.207, 0.433)
		ig	0.109	0.031	(0.054, 0.175)	0.135	0.042	(0.060, 0.223)	0.159	0.050	(0.068, 0.265)
	12	nig	0.112	0.034	(0.055, 0.182)	0.208	0.082	(0.084, 0.354)	0.160	0.049	(0.071, 0.266)
		ig	0.001	0.004	(0.000, 0.012)	0.002	0.005	(0.000, 0.015)	0.062	0.036	(0.012, 0.146)
	13	nig	0.001	0.004	(0.000, 0.013)	0.002	0.004	(0.000, 0.013)	0.060	0.033	(0.012, 0.140)
32		ig	0.237	0.039	(0.164, 0.320)	0.222	0.043	(0.141, 0.317)	0.272	0.052	(0.177, 0.378)
	21	nig	0.235	0.039	(0.163, 0.318)	0.187	0.066	(0.085, 0.311)	0.274	0.051	(0.173, 0.379)
		ig	0.183	0.036	(0.121, 0.259)	0.170	0.038	(0.103, 0.257)	0.137	0.044	(0.061, 0.229)
	22	nig	0.182	0.035	(0.117, 0.254)	0.141	0.053	(0.060, 0.244)	0.139	0.043	(0.059, 0.232)
		ig	0.050	0.020	(0.019, 0.097)	0.037	0.019	(0.009, 0.080)	0.053	0.031	(0.011, 0.129)
	23	nig	0.050	0.021	(0.016, 0.099)	0.031	0.018	(0.007, 0.075)	0.052	0.029	(0.011, 0.124)
		ig	0.693	0.050	(0.591, 0.790)	0.713	0.044	(0.622, 0.793)	0.460	0.080	(0.308, 0.618)
	11	nig	0.689	0.052	(0.587, 0.789)	0.652	0.053	(0.555, 0.765)	0.462	0.078	(0.303, 0.605)
		ig	0.068	0.027	(0.025, 0.127)	0.095	0.038	(0.030, 0.175)	0.234	0.071	(0.105, 0.383)
	12	nig	0.072	0.028	(0.025, 0.134)	0.188	0.084	(0.057, 0.335)	0.229	0.071	(0.108, 0.376)
		ig	0.001	0.003	(0.000, 0.010)	0.002	0.005	(0.000, 0.017)	0.083	0.045	(0.016, 0.191)
	13	nig	0.002	0.004	(0.000, 0.014)	0.001	0.003	(0.000, 0.011)	0.086	0.047	(0.018, 0.197)
33		ig	0.101	0.029	(0.051, 0.168)	0.085	0.028	(0.040, 0.147)	0.132	0.031	(0.079, 0.199)
	21	nig	0.102	0.030	(0.052, 0.166)	0.071	0.032	(0.025, 0.147)	0.133	0.031	(0.078, 0.196)
		ig	0.093	0.029	(0.041, 0.150)	0.074	0.026	(0.032, 0.134)	0.067	0.024	(0.028, 0.123)
	22	nig	0.093	0.030	(0.041, 0.154)	0.062	0.029	(0.018, 0.125)	0.066	0.024	(0.029, 0.120)
		ig	0.044	0.020	(0.012, 0.087)	0.031	0.017	(0.007, 0.071)	0.024	0.014	(0.005, 0.058)
	23	nig	0.043	0.019	(0.012, 0.085)	0.025	0.016	(0.005, 0.067)	0.025	0.014	(0.005, 0.058)
		ig	0.585	0.064	(0.467, 0.705)	0.618	0.058	(0.502, 0.730)	0.401	0.074	(0.258, 0.560)
	11	nig	0.580	0.065	(0.456, 0.709)	0.578	0.055	(0.477, 0.697)	0.404	0.073	(0.265, 0.543)
		ig	0.081	0.032	(0.029, 0.153)	0.115	0.053	(0.029, 0.229)	0.204	0.067	(0.090, 0.351)
	12	nig	0.084	0.034	(0.029, 0.158)	0.206	0.110	(0.052, 0.391)	0.204	0.063	(0.091, 0.332)
		ig	0.002	0.006	(0.000, 0.021)	0.003	0.007	(0.000, 0.023)	0.075	0.044	(0.013, 0.188)
	13	nig	0.002	0.006	(0.000, 0.018)	0.002	0.004	(0.000, 0.013)	0.074	0.041	(0.015, 0.170)
34		ig	0.124	0.045	(0.045, 0.222)	0.097	0.043	(0.032, 0.199)	0.189	0.046	(0.112, 0.293)
	21	nig	0.124	0.046	(0.043, 0.220)	0.081	0.050	(0.016, 0.193)	0.189	0.045	(0.110, 0.281)
		ig	0.125	0.042	(0.048, 0.212)	0.098	0.040	(0.030, 0.190)	0.095	0.033	(0.039, 0.170)
	22	nig	0.124	0.045	(0.047, 0.217)	0.077	0.048	(0.017, 0.191)	0.095	0.033	(0.040, 0.164)
		ig	0.083	0.036	(0.028, 0.166)	0.069	0.037	(0.013, 0.153)	0.035	0.021	(0.006, 0.089)
	23	nig	0.086	0.038	(0.025, 0.174)	0.056	0.039	(0.008, 0.153)	0.035	0.020	(0.007, 0.087)
		ig	0.310	0.045	(0.227, 0.401)	0.321	0.050	(0.220, 0.421)	0.253	0.051	(0.153, 0.355)
	11	nig	0.310	0.045	(0.223, 0.401)	0.350	0.072	(0.219, 0.472)	0.255	0.049	(0.160, 0.355)
		ig	0.116	0.031	(0.062, 0.182)	0.140	0.040	(0.067, 0.228)	0.127	0.041	(0.056, 0.210)
	12	nig	0.116	0.033	(0.061, 0.186)	0.204	0.087	(0.076, 0.361)	0.127	0.039	(0.060, 0.214)
		ig	0.001	0.004	(0.000, 0.014)	0.002	0.005	(0.000, 0.016)	0.047	0.028	(0.009, 0.114)
	13	nig	0.001	0.003	(0.000, 0.011)	0.001	0.003	(0.000, 0.009)	0.048	0.027	(0.008, 0.110)
35		ig	0.183	0.038	(0.114, 0.262)	0.165	0.040	(0.096, 0.250)	0.340	0.063	(0.213, 0.463)
	21	nig	0.183	0.039	(0.110, 0.264)	0.139	0.054	(0.056, 0.247)	0.338	0.060	(0.219, 0.459)
		ig	0.312	0.045	(0.227, 0.404)	0.305	0.050	(0.217, 0.410)	0.171	0.053	(0.078, 0.285)
	22	nig	0.311	0.046	(0.225, 0.406)	0.251	0.089	(0.109, 0.407)	0.168	0.050	(0.079, 0.275)
		ig	0.078	0.028	(0.030, 0.137)	0.067	0.027	(0.024, 0.131)	0.063	0.038	(0.013, 0.157)
	23	nig	0.078	0.028	(0.032, 0.141)	0.056	0.028	(0.015, 0.122)	0.064	0.037	(0.012, 0.152)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.742	0.042	(0.655,0.814)	0.741	0.040	(0.658,0.814)	0.621	0.040	(0.544,0.695)
	11	nig	0.658	0.043	(0.574,0.743)	0.664	0.045	(0.581,0.751)	0.666	0.044	(0.251,0.531)
		ig	0.742	0.042	(0.655,0.814)	0.741	0.040	(0.658,0.814)	0.621	0.040	(0.544,0.695)
	12	nig	0.616	0.049	(0.520,0.707)	0.601	0.049	(0.506,0.694)	0.606	0.049	(0.093,0.343)
		ig	0.742	0.042	(0.655,0.814)	0.741	0.040	(0.658,0.814)	0.621	0.040	(0.544,0.695)
	13	nig	0.597	0.054	(0.489,0.701)	0.593	0.053	(0.485,0.692)	0.589	0.053	(0.018,0.183)
1		ig	0.742	0.042	(0.655,0.814)	0.741	0.040	(0.658,0.814)	0.621	0.040	(0.544,0.695)
	21	nig	0.611	0.048	(0.512,0.706)	0.609	0.049	(0.514,0.703)	0.597	0.050	(0.118,0.276)
		ig	0.742	0.042	(0.655,0.814)	0.741	0.040	(0.658,0.814)	0.621	0.040	(0.544,0.695)
	22	nig	0.613	0.049	(0.513,0.707)	0.605	0.051	(0.498,0.701)	0.615	0.050	(0.043,0.169)
		ig	0.742	0.042	(0.655,0.814)	0.741	0.040	(0.658,0.814)	0.621	0.040	(0.544,0.695)
	23	nig	0.605	0.048	(0.508,0.697)	0.601	0.054	(0.496,0.706)	0.611	0.050	(0.008,0.091)
		ig	0.742	0.044	(0.647,0.827)	0.741	0.045	(0.647,0.821)	0.608	0.042	(0.527,0.690)
	11	nig	0.648	0.045	(0.563,0.738)	0.654	0.046	(0.565,0.748)	0.654	0.046	(0.238,0.490)
		ig	0.742	0.044	(0.647,0.827)	0.741	0.045	(0.647,0.821)	0.608	0.042	(0.527,0.690)
	12	nig	0.611	0.052	(0.506,0.712)	0.599	0.052	(0.491,0.702)	0.602	0.050	(0.085,0.304)
		ig	0.742	0.044	(0.647,0.827)	0.741	0.045	(0.647,0.821)	0.608	0.042	(0.527,0.690)
	13	nig	0.599	0.052	(0.497,0.700)	0.596	0.052	(0.492,0.697)	0.596	0.052	(0.013,0.151)
2		ig	0.742	0.044	(0.647,0.827)	0.741	0.045	(0.647,0.821)	0.608	0.042	(0.527,0.690)
	21	nig	0.614	0.047	(0.523,0.705)	0.611	0.048	(0.518,0.706)	0.603	0.049	(0.142,0.331)
		ig	0.742	0.044	(0.647,0.827)	0.741	0.045	(0.647,0.821)	0.608	0.042	(0.527,0.690)
	22	nig	0.612	0.050	(0.514,0.711)	0.605	0.051	(0.506,0.702)	0.614	0.051	(0.054,0.202)
		ig	0.742	0.044	(0.647,0.827)	0.741	0.045	(0.647,0.821)	0.608	0.042	(0.527,0.690)
	23	nig	0.599	0.052	(0.499,0.699)	0.599	0.054	(0.491,0.699)	0.598	0.052	(0.008,0.101)
		ig	0.684	0.043	(0.598,0.767)	0.683	0.044	(0.593,0.763)	0.587	0.038	(0.516,0.666)
	11	nig	0.603	0.043	(0.521,0.687)	0.619	0.045	(0.527,0.708)	0.619	0.044	(0.253,0.527)
		ig	0.684	0.043	(0.598,0.767)	0.683	0.044	(0.593,0.763)	0.587	0.038	(0.516,0.666)
	12	nig	0.599	0.051	(0.492,0.696)	0.576	0.051	(0.477,0.673)	0.580	0.051	(0.082,0.324)
		ig	0.684	0.043	(0.598,0.767)	0.683	0.044	(0.593,0.763)	0.587	0.038	(0.516,0.666)
	13	nig	0.596	0.052	(0.493,0.696)	0.595	0.051	(0.490,0.695)	0.588	0.053	(0.016,0.176)
3		ig	0.684	0.043	(0.598,0.767)	0.683	0.044	(0.593,0.763)	0.587	0.038	(0.516,0.666)
	21	nig	0.630	0.049	(0.534,0.724)	0.629	0.049	(0.531,0.726)	0.625	0.049	(0.121,0.293)
		ig	0.684	0.043	(0.598,0.767)	0.683	0.044	(0.593,0.763)	0.587	0.038	(0.516,0.666)
	22	nig	0.621	0.050	(0.520,0.720)	0.620	0.049	(0.522,0.722)	0.623	0.051	(0.041,0.173)
		ig	0.684	0.043	(0.598,0.767)	0.683	0.044	(0.593,0.763)	0.587	0.038	(0.516,0.666)
	23	nig	0.604	0.050	(0.504,0.703)	0.601	0.052	(0.500,0.702)	0.602	0.050	(0.008,0.089)
		ig	0.683	0.021	(0.642,0.724)	0.683	0.021	(0.644,0.726)	0.650	0.022	(0.609,0.691)
	11	nig	0.662	0.026	(0.611,0.712)	0.683	0.030	(0.628,0.742)	0.719	0.033	(0.258,0.504)
		ig	0.683	0.021	(0.642,0.724)	0.683	0.021	(0.644,0.726)	0.650	0.022	(0.609,0.691)
	12	nig	0.608	0.048	(0.514,0.701)	0.555	0.052	(0.447,0.650)	0.513	0.050	(0.090,0.316)
		ig	0.683	0.021	(0.642,0.724)	0.683	0.021	(0.644,0.726)	0.650	0.022	(0.609,0.691)
	13	nig	0.598	0.052	(0.491,0.696)	0.594	0.053	(0.488,0.703)	0.558	0.055	(0.014,0.171)
4		ig	0.683	0.021	(0.642,0.724)	0.683	0.021	(0.644,0.726)	0.650	0.022	(0.609,0.691)
	21	nig	0.635	0.043	(0.546,0.717)	0.635	0.044	(0.547,0.720)	0.576	0.038	(0.135,0.273)
		ig	0.683	0.021	(0.642,0.724)	0.683	0.021	(0.644,0.726)	0.650	0.022	(0.609,0.691)
	22	nig	0.640	0.043	(0.556,0.725)	0.640	0.042	(0.562,0.723)	0.688	0.043	(0.049,0.173)
		ig	0.683	0.021	(0.642,0.724)	0.683	0.021	(0.644,0.726)	0.650	0.022	(0.609,0.691)
	23	nig	0.609	0.048	(0.509,0.708)	0.607	0.050	(0.503,0.706)	0.631	0.047	(0.008,0.093)
		ig	0.746	0.040	(0.662,0.822)	0.745	0.038	(0.668,0.819)	0.630	0.039	(0.553,0.703)
	11	nig	0.656	0.043	(0.572,0.741)	0.663	0.043	(0.581,0.743)	0.665	0.042	(0.208,0.456)
		ig	0.746	0.040	(0.662,0.822)	0.745	0.038	(0.668,0.819)	0.630	0.039	(0.553,0.703)
	12	nig	0.609	0.051	(0.513,0.710)	0.595	0.050	(0.487,0.686)	0.599	0.052	(0.081,0.282)
		ig	0.746	0.040	(0.662,0.822)	0.745	0.038	(0.668,0.819)	0.630	0.039	(0.553,0.703)
	13	nig	0.597	0.052	(0.494,0.697)	0.593	0.052	(0.492,0.694)	0.585	0.054	(0.015,0.156)
5		ig	0.746	0.040	(0.662,0.822)	0.745	0.038	(0.668,0.819)	0.630	0.039	(0.553,0.703)
	21	nig	0.628	0.048	(0.528,0.721)	0.625	0.046	(0.537,0.715)	0.613	0.048	(0.155,0.349)
		ig	0.746	0.040	(0.662,0.822)	0.745	0.038	(0.668,0.819)	0.630	0.039	(0.553,0.703)
	22	nig	0.618	0.050	(0.521,0.710)	0.618	0.048	(0.526,0.714)	0.623	0.049	(0.058,0.222)
		ig	0.746	0.040	(0.662,0.822)	0.745	0.038	(0.668,0.819)	0.630	0.039	(0.553,0.703)
	23	nig	0.612	0.049	(0.508,0.707)	0.607	0.051	(0.508,0.711)	0.615	0.051	(0.011,0.118)
		ig	0.538	0.046	(0.447,0.628)	0.538	0.047	(0.442,0.626)	0.506	0.035	(0.434,0.574)
	11	nig	0.562	0.044	(0.471,0.646)	0.578	0.044	(0.485,0.659)	0.584	0.046	(0.226,0.460)
		ig	0.538	0.046	(0.447,0.628)	0.538	0.047	(0.442,0.626)	0.506	0.035	(0.434,0.574)
	12	nig	0.588	0.055	(0.476,0.703)	0.560	0.054	(0.446,0.668)	0.564	0.053	(0.077,0.298)
		ig	0.538	0.046	(0.447,0.628)	0.538	0.047	(0.442,0.626)	0.506	0.035	(0.434,0.574)
	13	nig	0.597	0.052	(0.489,0.690)	0.596	0.054	(0.488,0.703)	0.579	0.053	(0.013,0.158)
6		ig	0.538	0.046	(0.447,0.628)	0.538	0.047	(0.442,0.626)	0.506	0.035	(0.434,0.574)
	21	nig	0.594	0.048	(0.501,0.687)	0.586	0.049	(0.485,0.677)	0.569	0.047	(0.151,0.351)
		ig	0.538	0.046	(0.447,0.628)	0.538	0.047	(0.442,0.626)	0.506	0.035	(0.434,0.574)
	22	nig	0.593	0.051	(0.486,0.693)	0.586	0.051	(0.488,0.688)	0.600	0.049	(0.054,0.207)
		ig	0.538	0.046	(0.447,0.628)	0.538	0.047	(0.442,0.626)	0.506	0.035	(0.434,0.574)
	23	nig	0.594	0.052	(0.492,0.703)	0.592	0.052	(0.485,0.688)	0.604	0.051	(0.010,0.111)
		ig	0.801	0.043	(0.710,0.878)	0.803	0.042	(0.715,0.883)	0.636	0.043	(0.556,0.720)
	11	nig	0.663	0.045	(0.578,0.750)	0.665	0.046	(0.579,0.760)	0.666	0.047	(0.249,0.512)
		ig	0.801	0.043	(0.710,0.878)	0.803	0.042	(0.715,0.883)	0.636	0.043	(0.556,0.720)
	12	nig	0.606	0.051	(0.501,0.709)	0.602	0.052	(0.497,0.704)	0.600	0.053	(0.084,0.320)
		ig	0.801	0.043	(0.710,0.878)	0.803	0.042	(0.715,0.883)	0.636	0.043	(0.556,0.720)
	13	nig	0.600	0.053	(0.497,0.702)	0.594	0.053	(0.488,0.696)	0.594	0.053	(0.017,0.162)
7		ig	0.801	0.043	(0.710,0.878)	0.803	0.042	(0.715,0.883)	0.636	0.043	(0.556,0.720)
	21	nig	0.621	0.048	(0.522,0.717)	0.617	0.049	(0.523,0.713)	0.610	0.051	(0.126,0.305)
		ig	0.801	0.043	(0.710,0.878)	0.803	0.042	(0.715,0.883)	0.636	0.043	(0.556,0.720)
	22	nig	0.624	0.049	(0.531,0.719)	0.624	0.049	(0.525,0.721)	0.627	0.049	(0.044,0.186)
		ig	0.801	0.043	(0.710,0.878)	0.803	0.042	(0.715,0.883)	0.636	0.043	(0.556,0.720)
	23	nig	0.605	0.051	(0.503,0.702)	0.604	0.051	(0.503,0.704)	0.605	0.050	(0.009,0.091)

Table 4.4: The posterior mean(PM),posterior standard error(PSD) and 95% confidence interval(CI) for π by county (NSE < 0.017)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.762	0.040	(0.677,0.837)	0.762	0.040	(0.676,0.835)	0.631	0.040	(0.550,0.709)
	11	nig	0.651	0.046	(0.560,0.740)	0.654	0.045	(0.567,0.744)	0.660	0.047	(0.184,0.390)
		ig	0.762	0.040	(0.677,0.837)	0.762	0.040	(0.676,0.835)	0.631	0.040	(0.550,0.709)
	12	nig	0.609	0.049	(0.513,0.705)	0.603	0.051	(0.497,0.701)	0.603	0.051	(0.062,0.228)
		ig	0.762	0.040	(0.677,0.837)	0.762	0.040	(0.676,0.835)	0.631	0.040	(0.550,0.709)
	13	nig	0.601	0.052	(0.495,0.702)	0.595	0.052	(0.489,0.703)	0.591	0.053	(0.011,0.126)
8		ig	0.762	0.040	(0.677,0.837)	0.762	0.040	(0.676,0.835)	0.631	0.040	(0.550,0.709)
	21	nig	0.627	0.046	(0.536,0.715)	0.625	0.049	(0.524,0.718)	0.615	0.046	(0.209,0.427)
		ig	0.762	0.040	(0.677,0.837)	0.762	0.040	(0.676,0.835)	0.631	0.040	(0.550,0.709)
	22	nig	0.630	0.049	(0.536,0.725)	0.627	0.049	(0.536,0.727)	0.634	0.049	(0.073,0.257)
		ig	0.762	0.040	(0.677,0.837)	0.762	0.040	(0.676,0.835)	0.631	0.040	(0.550,0.709)
	23	nig	0.609	0.052	(0.504,0.712)	0.608	0.053	(0.504,0.706)	0.614	0.052	(0.013,0.132)
		ig	0.766	0.034	(0.694,0.830)	0.763	0.035	(0.689,0.826)	0.654	0.038	(0.581,0.725)
	11	nig	0.674	0.043	(0.592,0.758)	0.682	0.044	(0.597,0.771)	0.686	0.045	(0.186,0.397)
		ig	0.766	0.034	(0.694,0.830)	0.763	0.035	(0.689,0.826)	0.654	0.038	(0.581,0.725)
	12	nig	0.618	0.048	(0.526,0.711)	0.608	0.049	(0.512,0.697)	0.610	0.049	(0.069,0.250)
		ig	0.766	0.034	(0.694,0.830)	0.763	0.035	(0.689,0.826)	0.654	0.038	(0.581,0.725)
	13	nig	0.600	0.052	(0.495,0.700)	0.597	0.051	(0.494,0.692)	0.587	0.052	(0.011,0.130)
9		ig	0.766	0.034	(0.694,0.830)	0.763	0.035	(0.689,0.826)	0.654	0.038	(0.581,0.725)
	21	nig	0.628	0.046	(0.539,0.717)	0.625	0.049	(0.523,0.721)	0.606	0.045	(0.188,0.402)
		ig	0.766	0.034	(0.694,0.830)	0.763	0.035	(0.689,0.826)	0.654	0.038	(0.581,0.725)
	22	nig	0.628	0.048	(0.530,0.720)	0.625	0.049	(0.520,0.723)	0.630	0.049	(0.074,0.254)
		ig	0.766	0.034	(0.694,0.830)	0.763	0.035	(0.689,0.826)	0.654	0.038	(0.581,0.725)
	23	nig	0.617	0.051	(0.517,0.714)	0.614	0.048	(0.519,0.710)	0.625	0.049	(0.011,0.132)
		ig	0.727	0.054	(0.614,0.828)	0.730	0.054	(0.620,0.825)	0.586	0.043	(0.508,0.673)
	11	nig	0.607	0.050	(0.507,0.701)	0.608	0.052	(0.507,0.706)	0.609	0.050	(0.053,0.174)
		ig	0.727	0.054	(0.614,0.828)	0.730	0.054	(0.620,0.825)	0.586	0.043	(0.508,0.673)
	12	nig	0.597	0.048	(0.497,0.682)	0.593	0.053	(0.484,0.690)	0.594	0.052	(0.019,0.103)
		ig	0.727	0.054	(0.614,0.828)	0.730	0.054	(0.620,0.825)	0.586	0.043	(0.508,0.673)
	13	nig	0.601	0.052	(0.496,0.703)	0.595	0.052	(0.488,0.697)	0.592	0.052	(0.004,0.052)
10		ig	0.727	0.054	(0.614,0.828)	0.730	0.054	(0.620,0.825)	0.586	0.043	(0.508,0.673)
	21	nig	0.628	0.049	(0.527,0.714)	0.622	0.048	(0.521,0.715)	0.616	0.047	(0.321,0.653)
		ig	0.727	0.054	(0.614,0.828)	0.730	0.054	(0.620,0.825)	0.586	0.043	(0.508,0.673)
	22	nig	0.621	0.048	(0.524,0.710)	0.619	0.049	(0.524,0.721)	0.622	0.049	(0.113,0.397)
		ig	0.727	0.054	(0.614,0.828)	0.730	0.054	(0.620,0.825)	0.586	0.043	(0.508,0.673)
	23	nig	0.611	0.050	(0.508,0.705)	0.609	0.051	(0.515,0.713)	0.612	0.051	(0.019,0.215)
		ig	0.724	0.051	(0.617,0.812)	0.721	0.051	(0.614,0.812)	0.589	0.040	(0.510,0.664)
	11	nig	0.652	0.046	(0.569,0.746)	0.653	0.046	(0.564,0.750)	0.654	0.048	(0.182,0.428)
		ig	0.724	0.051	(0.617,0.812)	0.721	0.051	(0.614,0.812)	0.589	0.040	(0.510,0.664)
	12	nig	0.611	0.050	(0.513,0.712)	0.604	0.049	(0.507,0.702)	0.603	0.050	(0.068,0.256)
		ig	0.724	0.051	(0.617,0.812)	0.721	0.051	(0.614,0.812)	0.589	0.040	(0.510,0.664)
	13	nig	0.605	0.054	(0.494,0.711)	0.603	0.051	(0.500,0.709)	0.600	0.053	(0.010,0.132)
11		ig	0.724	0.051	(0.617,0.812)	0.721	0.051	(0.614,0.812)	0.589	0.040	(0.510,0.664)
	21	nig	0.601	0.052	(0.496,0.697)	0.595	0.053	(0.483,0.695)	0.578	0.048	(0.186,0.410)
		ig	0.724	0.051	(0.617,0.812)	0.721	0.051	(0.614,0.812)	0.589	0.040	(0.510,0.664)
	22	nig	0.599	0.050	(0.500,0.699)	0.597	0.050	(0.492,0.698)	0.608	0.049	(0.068,0.256)
		ig	0.724	0.051	(0.617,0.812)	0.721	0.051	(0.614,0.812)	0.589	0.040	(0.510,0.664)
	23	nig	0.599	0.051	(0.501,0.702)	0.597	0.052	(0.494,0.695)	0.603	0.052	(0.010,0.123)
		ig	0.736	0.046	(0.634,0.820)	0.734	0.047	(0.632,0.815)	0.600	0.043	(0.523,0.689)
	11	nig	0.637	0.046	(0.547,0.734)	0.644	0.047	(0.551,0.735)	0.648	0.047	(0.190,0.419)
		ig	0.736	0.046	(0.634,0.820)	0.734	0.047	(0.632,0.815)	0.600	0.043	(0.523,0.689)
	12	nig	0.602	0.050	(0.504,0.698)	0.593	0.051	(0.486,0.686)	0.593	0.050	(0.065,0.261)
		ig	0.736	0.046	(0.634,0.820)	0.734	0.047	(0.632,0.815)	0.600	0.043	(0.523,0.689)
	13	nig	0.602	0.052	(0.494,0.706)	0.594	0.054	(0.488,0.698)	0.590	0.053	(0.010,0.133)
12		ig	0.736	0.046	(0.634,0.820)	0.734	0.047	(0.632,0.815)	0.600	0.043	(0.523,0.689)
	21	nig	0.606	0.048	(0.509,0.703)	0.609	0.050	(0.510,0.708)	0.590	0.050	(0.183,0.414)
		ig	0.736	0.046	(0.634,0.820)	0.734	0.047	(0.632,0.815)	0.600	0.043	(0.523,0.689)
	22	nig	0.613	0.050	(0.517,0.709)	0.613	0.050	(0.514,0.715)	0.616	0.050	(0.064,0.257)
		ig	0.736	0.046	(0.634,0.820)	0.734	0.047	(0.632,0.815)	0.600	0.043	(0.523,0.689)
	23	nig	0.616	0.051	(0.519,0.716)	0.610	0.050	(0.504,0.706)	0.624	0.054	(0.011,0.127)
		ig	0.658	0.038	(0.579,0.732)	0.658	0.037	(0.584,0.727)	0.588	0.034	(0.524,0.652)
	11	nig	0.648	0.038	(0.573,0.720)	0.664	0.041	(0.583,0.744)	0.666	0.040	(0.252,0.529)
		ig	0.658	0.038	(0.579,0.732)	0.658	0.037	(0.584,0.727)	0.588	0.034	(0.524,0.652)
	12	nig	0.601	0.050	(0.501,0.693)	0.578	0.051	(0.477,0.673)	0.580	0.052	(0.088,0.329)
		ig	0.658	0.038	(0.579,0.732)	0.658	0.037	(0.584,0.727)	0.588	0.034	(0.524,0.652)
	13	nig	0.598	0.053	(0.491,0.698)	0.595	0.053	(0.490,0.696)	0.585	0.054	(0.015,0.165)
13		ig	0.658	0.038	(0.579,0.732)	0.658	0.037	(0.584,0.727)	0.588	0.034	(0.524,0.652)
	21	nig	0.592	0.049	(0.499,0.688)	0.588	0.047	(0.492,0.678)	0.562	0.048	(0.127,0.280)
		ig	0.658	0.038	(0.579,0.732)	0.658	0.037	(0.584,0.727)	0.588	0.034	(0.524,0.652)
	22	nig	0.589	0.050	(0.484,0.684)	0.589	0.050	(0.488,0.682)	0.612	0.050	(0.042,0.172)
		ig	0.658	0.038	(0.579,0.732)	0.658	0.037	(0.584,0.727)	0.588	0.034	(0.524,0.652)
	23	nig	0.596	0.051	(0.497,0.696)	0.595	0.054	(0.481,0.699)	0.601	0.052	(0.007,0.087)
		ig	0.684	0.044	(0.592,0.769)	0.684	0.045	(0.593,0.772)	0.585	0.038	(0.514,0.661)
	11	nig	0.619	0.046	(0.530,0.706)	0.624	0.046	(0.536,0.721)	0.635	0.048	(0.142,0.337)
		ig	0.684	0.044	(0.592,0.769)	0.684	0.045	(0.593,0.772)	0.585	0.038	(0.514,0.661)
	12	nig	0.602	0.049	(0.508,0.700)	0.593	0.050	(0.494,0.691)	0.590	0.050	(0.057,0.216)
		ig	0.684	0.044	(0.592,0.769)	0.684	0.045	(0.593,0.772)	0.585	0.038	(0.514,0.661)
	13	nig	0.598	0.051	(0.501,0.696)	0.593	0.052	(0.488,0.696)	0.588	0.051	(0.009,0.119)
14		ig	0.684	0.044	(0.592,0.769)	0.684	0.045	(0.593,0.772)	0.585	0.038	(0.514,0.661)
	21	nig	0.606	0.049	(0.505,0.698)	0.603	0.053	(0.488,0.705)	0.578	0.049	(0.218,0.477)
		ig	0.684	0.044	(0.592,0.769)	0.684	0.045	(0.593,0.772)	0.585	0.038	(0.514,0.661)
	22	nig	0.619	0.047	(0.523,0.712)	0.618	0.046	(0.528,0.710)	0.627	0.050	(0.083,0.307)
		ig	0.684	0.044	(0.592,0.769)	0.684	0.045	(0.593,0.772)	0.585	0.038	(0.514,0.661)
	23	nig	0.607	0.052	(0.495,0.706)	0.606	0.049	(0.506,0.701)	0.615	0.050	(0.013,0.167)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.719	0.050	(0.622,0.814)	0.719	0.050	(0.618,0.815)	0.588	0.043	(0.503,0.673)
	11	nig	0.623	0.044	(0.530,0.709)	0.623	0.046	(0.532,0.717)	0.627	0.049	(0.189,0.435)
		ig	0.719	0.050	(0.622,0.814)	0.719	0.050	(0.618,0.815)	0.588	0.043	(0.503,0.673)
	12	nig	0.607	0.050	(0.500,0.700)	0.598	0.051	(0.493,0.696)	0.597	0.052	(0.074,0.268)
		ig	0.719	0.050	(0.622,0.814)	0.719	0.050	(0.618,0.815)	0.588	0.043	(0.503,0.673)
	13	nig	0.600	0.052	(0.485,0.703)	0.596	0.050	(0.498,0.694)	0.591	0.054	(0.013,0.143)
15		ig	0.719	0.050	(0.622,0.814)	0.719	0.050	(0.618,0.815)	0.588	0.043	(0.503,0.673)
	21	nig	0.612	0.049	(0.514,0.706)	0.611	0.055	(0.498,0.711)	0.606	0.052	(0.171,0.392)
		ig	0.719	0.050	(0.622,0.814)	0.719	0.050	(0.618,0.815)	0.588	0.043	(0.503,0.673)
	22	nig	0.619	0.050	(0.517,0.722)	0.616	0.048	(0.521,0.709)	0.621	0.049	(0.064,0.242)
		ig	0.719	0.050	(0.622,0.814)	0.719	0.050	(0.618,0.815)	0.588	0.043	(0.503,0.673)
	23	nig	0.602	0.052	(0.498,0.702)	0.600	0.052	(0.497,0.702)	0.599	0.051	(0.012,0.133)
		ig	0.733	0.056	(0.620,0.833)	0.733	0.057	(0.620,0.839)	0.581	0.044	(0.496,0.673)
	11	nig	0.653	0.046	(0.561,0.742)	0.652	0.049	(0.559,0.750)	0.654	0.048	(0.153,0.395)
		ig	0.733	0.056	(0.620,0.833)	0.733	0.057	(0.620,0.839)	0.581	0.044	(0.496,0.673)
	12	nig	0.602	0.052	(0.502,0.705)	0.596	0.054	(0.489,0.702)	0.598	0.052	(0.060,0.232)
		ig	0.733	0.056	(0.620,0.833)	0.733	0.057	(0.620,0.839)	0.581	0.044	(0.496,0.673)
	13	nig	0.600	0.051	(0.495,0.696)	0.600	0.053	(0.494,0.700)	0.592	0.052	(0.010,0.109)
16		ig	0.733	0.056	(0.620,0.833)	0.733	0.057	(0.620,0.839)	0.581	0.044	(0.496,0.673)
	21	nig	0.602	0.049	(0.503,0.699)	0.596	0.049	(0.500,0.690)	0.588	0.051	(0.207,0.457)
		ig	0.733	0.056	(0.620,0.833)	0.733	0.057	(0.620,0.839)	0.581	0.044	(0.496,0.673)
	22	nig	0.599	0.052	(0.501,0.702)	0.596	0.052	(0.492,0.696)	0.602	0.049	(0.073,0.277)
		ig	0.733	0.056	(0.620,0.833)	0.733	0.057	(0.620,0.839)	0.581	0.044	(0.496,0.673)
	23	nig	0.598	0.050	(0.496,0.697)	0.593	0.052	(0.487,0.698)	0.600	0.051	(0.012,0.144)
		ig	0.677	0.052	(0.574,0.773)	0.677	0.053	(0.566,0.774)	0.568	0.041	(0.486,0.654)
	11	nig	0.628	0.046	(0.538,0.716)	0.627	0.049	(0.531,0.724)	0.626	0.052	(0.095,0.257)
		ig	0.677	0.052	(0.574,0.773)	0.677	0.053	(0.566,0.774)	0.568	0.041	(0.486,0.654)
	12	nig	0.605	0.049	(0.508,0.706)	0.596	0.050	(0.498,0.688)	0.602	0.053	(0.034,0.157)
		ig	0.677	0.052	(0.574,0.773)	0.677	0.053	(0.566,0.774)	0.568	0.041	(0.486,0.654)
	13	nig	0.598	0.050	(0.494,0.695)	0.600	0.051	(0.498,0.701)	0.593	0.053	(0.007,0.075)
17		ig	0.677	0.052	(0.574,0.773)	0.677	0.053	(0.566,0.774)	0.568	0.041	(0.486,0.654)
	21	nig	0.601	0.048	(0.511,0.691)	0.599	0.050	(0.504,0.694)	0.585	0.049	(0.272,0.568)
		ig	0.677	0.052	(0.574,0.773)	0.677	0.053	(0.566,0.774)	0.568	0.041	(0.486,0.654)
	22	nig	0.600	0.049	(0.504,0.697)	0.602	0.051	(0.501,0.701)	0.605	0.049	(0.093,0.366)
		ig	0.677	0.052	(0.574,0.773)	0.677	0.053	(0.566,0.774)	0.568	0.041	(0.486,0.654)
	23	nig	0.604	0.050	(0.507,0.701)	0.604	0.049	(0.505,0.700)	0.616	0.051	(0.016,0.180)
		ig	0.656	0.053	(0.556,0.753)	0.656	0.053	(0.547,0.754)	0.556	0.039	(0.484,0.634)
	11	nig	0.619	0.046	(0.534,0.714)	0.624	0.049	(0.526,0.720)	0.625	0.049	(0.155,0.363)
		ig	0.656	0.053	(0.556,0.753)	0.656	0.053	(0.547,0.754)	0.556	0.039	(0.484,0.634)
	12	nig	0.605	0.052	(0.500,0.703)	0.592	0.053	(0.488,0.689)	0.593	0.050	(0.055,0.220)
		ig	0.656	0.053	(0.556,0.753)	0.656	0.053	(0.547,0.754)	0.556	0.039	(0.484,0.634)
	13	nig	0.599	0.051	(0.498,0.697)	0.594	0.054	(0.488,0.694)	0.593	0.054	(0.010,0.113)
18		ig	0.656	0.053	(0.556,0.753)	0.656	0.053	(0.547,0.754)	0.556	0.039	(0.484,0.634)
	21	nig	0.599	0.050	(0.503,0.698)	0.598	0.051	(0.494,0.694)	0.584	0.050	(0.218,0.471)
		ig	0.656	0.053	(0.556,0.753)	0.656	0.053	(0.547,0.754)	0.556	0.039	(0.484,0.634)
	22	nig	0.600	0.050	(0.505,0.697)	0.596	0.048	(0.499,0.688)	0.607	0.048	(0.077,0.282)
		ig	0.656	0.053	(0.556,0.753)	0.656	0.053	(0.547,0.754)	0.556	0.039	(0.484,0.634)
	23	nig	0.593	0.051	(0.489,0.686)	0.597	0.051	(0.496,0.695)	0.593	0.054	(0.013,0.151)
		ig	0.685	0.066	(0.545,0.800)	0.685	0.063	(0.553,0.795)	0.546	0.045	(0.460,0.636)
	11	nig	0.614	0.050	(0.511,0.706)	0.612	0.052	(0.502,0.711)	0.611	0.053	(0.046,0.203)
		ig	0.685	0.066	(0.545,0.800)	0.685	0.063	(0.553,0.795)	0.546	0.045	(0.460,0.636)
	12	nig	0.598	0.051	(0.496,0.700)	0.595	0.052	(0.488,0.690)	0.596	0.056	(0.019,0.112)
		ig	0.685	0.066	(0.545,0.800)	0.685	0.063	(0.553,0.795)	0.546	0.045	(0.460,0.636)
	13	nig	0.598	0.053	(0.487,0.699)	0.595	0.053	(0.485,0.692)	0.595	0.054	(0.003,0.059)
19		ig	0.685	0.066	(0.545,0.800)	0.685	0.063	(0.553,0.795)	0.546	0.045	(0.460,0.636)
	21	nig	0.604	0.049	(0.499,0.701)	0.599	0.051	(0.499,0.697)	0.592	0.054	(0.316,0.650)
		ig	0.685	0.066	(0.545,0.800)	0.685	0.063	(0.553,0.795)	0.546	0.045	(0.460,0.636)
	22	nig	0.601	0.050	(0.506,0.696)	0.602	0.049	(0.505,0.695)	0.606	0.051	(0.110,0.393)
		ig	0.685	0.066	(0.545,0.800)	0.685	0.063	(0.553,0.795)	0.546	0.045	(0.460,0.636)
	23	nig	0.601	0.050	(0.504,0.691)	0.597	0.052	(0.496,0.697)	0.598	0.051	(0.016,0.211)
		ig	0.662	0.052	(0.559,0.764)	0.661	0.054	(0.554,0.759)	0.554	0.042	(0.470,0.636)
	11	nig	0.617	0.049	(0.519,0.714)	0.617	0.048	(0.521,0.717)	0.621	0.048	(0.116,0.306)
		ig	0.662	0.052	(0.559,0.764)	0.661	0.054	(0.554,0.759)	0.554	0.042	(0.470,0.636)
	12	nig	0.603	0.050	(0.500,0.697)	0.592	0.052	(0.487,0.686)	0.590	0.051	(0.049,0.180)
		ig	0.662	0.052	(0.559,0.764)	0.661	0.054	(0.554,0.759)	0.554	0.042	(0.470,0.636)
	13	nig	0.600	0.053	(0.492,0.705)	0.597	0.052	(0.486,0.695)	0.590	0.053	(0.008,0.105)
20		ig	0.662	0.052	(0.559,0.764)	0.661	0.054	(0.554,0.759)	0.554	0.042	(0.470,0.636)
	21	nig	0.599	0.053	(0.493,0.703)	0.595	0.051	(0.491,0.694)	0.571	0.051	(0.246,0.514)
		ig	0.662	0.052	(0.559,0.764)	0.661	0.054	(0.554,0.759)	0.554	0.042	(0.470,0.636)
	22	nig	0.607	0.048	(0.509,0.697)	0.600	0.048	(0.507,0.699)	0.614	0.051	(0.096,0.311)
		ig	0.662	0.052	(0.559,0.764)	0.661	0.054	(0.554,0.759)	0.554	0.042	(0.470,0.636)
	23	nig	0.602	0.049	(0.502,0.698)	0.600	0.051	(0.498,0.699)	0.614	0.051	(0.013,0.179)
		ig	0.789	0.053	(0.677,0.887)	0.791	0.052	(0.679,0.889)	0.604	0.046	(0.515,0.699)
	11	nig	0.614	0.049	(0.520,0.710)	0.612	0.052	(0.505,0.716)	0.612	0.050	(0.092,0.267)
		ig	0.789	0.053	(0.677,0.887)	0.791	0.052	(0.679,0.889)	0.604	0.046	(0.515,0.699)
	12	nig	0.608	0.052	(0.503,0.707)	0.602	0.051	(0.496,0.698)	0.601	0.053	(0.032,0.158)
		ig	0.789	0.053	(0.677,0.887)	0.791	0.052	(0.679,0.889)	0.604	0.046	(0.515,0.699)
	13	nig	0.601	0.051	(0.495,0.699)	0.594	0.050	(0.496,0.689)	0.593	0.052	(0.006,0.078)
21		ig	0.789	0.053	(0.677,0.887)	0.791	0.052	(0.679,0.889)	0.604	0.046	(0.515,0.699)
	21	nig	0.620	0.049	(0.522,0.714)	0.619	0.049	(0.518,0.712)	0.610	0.050	(0.267,0.585)
		ig	0.789	0.053	(0.677,0.887)	0.791	0.052	(0.679,0.889)	0.604	0.046	(0.515,0.699)
	22	nig	0.636	0.048	(0.542,0.732)	0.634	0.048	(0.543,0.727)	0.639	0.048	(0.098,0.353)
		ig	0.789	0.053	(0.677,0.887)	0.791	0.052	(0.679,0.889)	0.604	0.046	(0.515,0.699)
	23	nig	0.614	0.049	(0.514,0.700)	0.611	0.049	(0.518,0.710)	0.612	0.051	(0.015,0.184)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.501	0.059	(0.386,0.616)	0.499	0.059	(0.375,0.611)	0.477	0.038	(0.396,0.552)
	11	nig	0.603	0.048	(0.509,0.698)	0.605	0.050	(0.503,0.704)	0.607	0.051	(0.130,0.302)
		ig	0.501	0.059	(0.386,0.616)	0.499	0.059	(0.375,0.611)	0.477	0.038	(0.396,0.552)
	12	nig	0.602	0.051	(0.500,0.699)	0.592	0.051	(0.489,0.692)	0.595	0.050	(0.047,0.192)
		ig	0.501	0.059	(0.386,0.616)	0.499	0.059	(0.375,0.611)	0.477	0.038	(0.396,0.552)
	13	nig	0.599	0.051	(0.495,0.698)	0.594	0.051	(0.486,0.692)	0.590	0.053	(0.008,0.099)
22		ig	0.501	0.059	(0.386,0.616)	0.499	0.059	(0.375,0.611)	0.477	0.038	(0.396,0.552)
	21	nig	0.575	0.055	(0.461,0.679)	0.576	0.054	(0.463,0.679)	0.531	0.054	(0.238,0.509)
		ig	0.501	0.059	(0.386,0.616)	0.499	0.059	(0.375,0.611)	0.477	0.038	(0.396,0.552)
	22	nig	0.559	0.050	(0.454,0.662)	0.547	0.051	(0.441,0.644)	0.580	0.049	(0.087,0.315)
		ig	0.501	0.059	(0.386,0.616)	0.499	0.059	(0.375,0.611)	0.477	0.038	(0.396,0.552)
	23	nig	0.581	0.054	(0.466,0.690)	0.578	0.054	(0.468,0.673)	0.591	0.053	(0.015,0.161)
		ig	0.636	0.064	(0.510,0.760)	0.633	0.063	(0.510,0.751)	0.531	0.043	(0.447,0.619)
	11	nig	0.611	0.048	(0.518,0.701)	0.613	0.049	(0.509,0.705)	0.615	0.047	(0.205,0.460)
		ig	0.636	0.064	(0.510,0.760)	0.633	0.063	(0.510,0.751)	0.531	0.043	(0.447,0.619)
	12	nig	0.606	0.050	(0.505,0.701)	0.597	0.050	(0.499,0.696)	0.602	0.053	(0.075,0.286)
		ig	0.636	0.064	(0.510,0.760)	0.633	0.063	(0.510,0.751)	0.531	0.043	(0.447,0.619)
	13	nig	0.598	0.050	(0.493,0.691)	0.597	0.052	(0.489,0.691)	0.589	0.053	(0.012,0.144)
23		ig	0.636	0.064	(0.510,0.760)	0.633	0.063	(0.510,0.751)	0.531	0.043	(0.447,0.619)
	21	nig	0.595	0.051	(0.493,0.686)	0.593	0.051	(0.492,0.697)	0.584	0.051	(0.150,0.391)
		ig	0.636	0.064	(0.510,0.760)	0.633	0.063	(0.510,0.751)	0.531	0.043	(0.447,0.619)
	22	nig	0.594	0.051	(0.498,0.698)	0.591	0.052	(0.482,0.689)	0.591	0.054	(0.059,0.236)
		ig	0.636	0.064	(0.510,0.760)	0.633	0.063	(0.510,0.751)	0.531	0.043	(0.447,0.619)
	23	nig	0.598	0.052	(0.495,0.694)	0.593	0.051	(0.493,0.688)	0.604	0.052	(0.010,0.117)
		ig	0.641	0.070	(0.490,0.764)	0.647	0.070	(0.508,0.779)	0.525	0.043	(0.444,0.612)
	11	nig	0.611	0.049	(0.513,0.709)	0.610	0.052	(0.508,0.710)	0.609	0.049	(0.139,0.390)
		ig	0.641	0.070	(0.490,0.764)	0.647	0.070	(0.508,0.779)	0.525	0.043	(0.444,0.612)
	12	nig	0.603	0.051	(0.506,0.702)	0.600	0.051	(0.496,0.701)	0.602	0.052	(0.054,0.228)
		ig	0.641	0.070	(0.490,0.764)	0.647	0.070	(0.508,0.779)	0.525	0.043	(0.444,0.612)
	13	nig	0.597	0.053	(0.493,0.693)	0.598	0.051	(0.493,0.700)	0.592	0.052	(0.007,0.114)
24		ig	0.641	0.070	(0.490,0.764)	0.647	0.070	(0.508,0.779)	0.525	0.043	(0.444,0.612)
	21	nig	0.595	0.052	(0.492,0.695)	0.591	0.052	(0.486,0.688)	0.584	0.051	(0.205,0.486)
		ig	0.641	0.070	(0.490,0.764)	0.647	0.070	(0.508,0.779)	0.525	0.043	(0.444,0.612)
	22	nig	0.595	0.050	(0.497,0.692)	0.594	0.050	(0.492,0.697)	0.593	0.053	(0.076,0.293)
		ig	0.641	0.070	(0.490,0.764)	0.647	0.070	(0.508,0.779)	0.525	0.043	(0.444,0.612)
	23	nig	0.598	0.052	(0.492,0.695)	0.593	0.052	(0.490,0.698)	0.599	0.051	(0.012,0.148)
		ig	0.728	0.058	(0.606,0.832)	0.726	0.059	(0.595,0.833)	0.570	0.046	(0.486,0.659)
	11	nig	0.624	0.050	(0.526,0.717)	0.623	0.050	(0.526,0.720)	0.624	0.050	(0.113,0.318)
		ig	0.728	0.058	(0.606,0.832)	0.726	0.059	(0.595,0.833)	0.570	0.046	(0.486,0.659)
	12	nig	0.606	0.053	(0.505,0.710)	0.598	0.050	(0.497,0.691)	0.598	0.053	(0.047,0.186)
		ig	0.728	0.058	(0.606,0.832)	0.726	0.059	(0.595,0.833)	0.570	0.046	(0.486,0.659)
	13	nig	0.599	0.051	(0.498,0.704)	0.594	0.049	(0.500,0.694)	0.594	0.053	(0.007,0.097)
25		ig	0.728	0.058	(0.606,0.832)	0.726	0.059	(0.595,0.833)	0.570	0.046	(0.486,0.659)
	21	nig	0.617	0.047	(0.527,0.710)	0.613	0.050	(0.511,0.708)	0.608	0.046	(0.243,0.515)
		ig	0.728	0.058	(0.606,0.832)	0.726	0.059	(0.595,0.833)	0.570	0.046	(0.486,0.659)
	22	nig	0.605	0.050	(0.507,0.703)	0.600	0.051	(0.490,0.697)	0.609	0.051	(0.090,0.318)
		ig	0.728	0.058	(0.606,0.832)	0.726	0.059	(0.595,0.833)	0.570	0.046	(0.486,0.659)
	23	nig	0.597	0.051	(0.497,0.699)	0.600	0.052	(0.500,0.701)	0.598	0.051	(0.014,0.173)
		ig	0.494	0.055	(0.380,0.598)	0.490	0.057	(0.375,0.601)	0.475	0.040	(0.392,0.552)
	11	nig	0.594	0.047	(0.492,0.683)	0.601	0.047	(0.506,0.688)	0.601	0.047	(0.143,0.353)
		ig	0.494	0.055	(0.380,0.598)	0.490	0.057	(0.375,0.601)	0.475	0.040	(0.392,0.552)
	12	nig	0.593	0.050	(0.487,0.687)	0.582	0.053	(0.474,0.686)	0.589	0.052	(0.053,0.210)
		ig	0.494	0.055	(0.380,0.598)	0.490	0.057	(0.375,0.601)	0.475	0.040	(0.392,0.552)
	13	nig	0.600	0.055	(0.488,0.706)	0.594	0.053	(0.484,0.700)	0.588	0.054	(0.008,0.107)
26		ig	0.494	0.055	(0.380,0.598)	0.490	0.057	(0.375,0.601)	0.475	0.040	(0.392,0.552)
	21	nig	0.567	0.052	(0.456,0.662)	0.558	0.054	(0.450,0.659)	0.535	0.053	(0.211,0.480)
		ig	0.494	0.055	(0.380,0.598)	0.490	0.057	(0.375,0.601)	0.475	0.040	(0.392,0.552)
	22	nig	0.577	0.053	(0.464,0.676)	0.580	0.054	(0.460,0.680)	0.578	0.054	(0.079,0.305)
		ig	0.494	0.055	(0.380,0.598)	0.490	0.057	(0.375,0.601)	0.475	0.040	(0.392,0.552)
	23	nig	0.577	0.054	(0.472,0.675)	0.573	0.054	(0.465,0.676)	0.592	0.051	(0.012,0.152)
		ig	0.530	0.059	(0.410,0.641)	0.531	0.058	(0.420,0.640)	0.496	0.041	(0.414,0.576)
	11	nig	0.614	0.051	(0.514,0.712)	0.611	0.053	(0.507,0.713)	0.612	0.051	(0.058,0.193)
		ig	0.530	0.059	(0.410,0.641)	0.531	0.058	(0.420,0.640)	0.496	0.041	(0.414,0.576)
	12	nig	0.601	0.050	(0.498,0.695)	0.597	0.050	(0.491,0.689)	0.594	0.050	(0.024,0.113)
		ig	0.530	0.059	(0.410,0.641)	0.531	0.058	(0.420,0.640)	0.496	0.041	(0.414,0.576)
	13	nig	0.597	0.054	(0.485,0.699)	0.596	0.052	(0.490,0.696)	0.593	0.055	(0.003,0.056)
27		ig	0.530	0.059	(0.410,0.641)	0.531	0.058	(0.420,0.640)	0.496	0.041	(0.414,0.576)
	21	nig	0.580	0.054	(0.464,0.679)	0.583	0.054	(0.477,0.692)	0.536	0.053	(0.310,0.627)
		ig	0.530	0.059	(0.410,0.641)	0.531	0.058	(0.420,0.640)	0.496	0.041	(0.414,0.576)
	22	nig	0.565	0.049	(0.459,0.657)	0.552	0.049	(0.449,0.644)	0.591	0.049	(0.112,0.410)
		ig	0.530	0.059	(0.410,0.641)	0.531	0.058	(0.420,0.640)	0.496	0.041	(0.414,0.576)
	23	nig	0.585	0.051	(0.481,0.679)	0.586	0.052	(0.481,0.684)	0.591	0.051	(0.017,0.214)
		ig	0.659	0.071	(0.513,0.789)	0.657	0.070	(0.509,0.784)	0.531	0.044	(0.444,0.615)
	11	nig	0.617	0.048	(0.518,0.719)	0.616	0.051	(0.511,0.712)	0.617	0.053	(0.119,0.351)
		ig	0.659	0.071	(0.513,0.789)	0.657	0.070	(0.509,0.784)	0.531	0.044	(0.444,0.615)
	12	nig	0.611	0.052	(0.506,0.718)	0.606	0.051	(0.500,0.704)	0.607	0.052	(0.042,0.218)
		ig	0.659	0.071	(0.513,0.789)	0.657	0.070	(0.509,0.784)	0.531	0.044	(0.444,0.615)
	13	nig	0.598	0.052	(0.496,0.695)	0.596	0.052	(0.493,0.694)	0.596	0.051	(0.009,0.101)
28		ig	0.659	0.071	(0.513,0.789)	0.657	0.070	(0.509,0.784)	0.531	0.044	(0.444,0.615)
	21	nig	0.593	0.051	(0.496,0.686)	0.591	0.053	(0.480,0.697)	0.575	0.052	(0.221,0.530)
		ig	0.659	0.071	(0.513,0.789)	0.657	0.070	(0.509,0.784)	0.531	0.044	(0.444,0.615)
	22	nig	0.592	0.050	(0.494,0.690)	0.593	0.051	(0.487,0.690)	0.595	0.051	(0.081,0.320)
		ig	0.659	0.071	(0.513,0.789)	0.657	0.070	(0.509,0.784)	0.531	0.044	(0.444,0.615)
	23	nig	0.595	0.053	(0.490,0.690)	0.593	0.051	(0.488,0.689)	0.603	0.050	(0.015,0.158)

Cou	Cell	Miss	Baseline			Overall			Individual		
			PM	PSD	CI	PM	PSD	CI	PM	PSD	CI
		ig	0.615	0.047	(0.519,0.709)	0.613	0.046	(0.519,0.694)	0.548	0.035	(0.479,0.616)
	11	nig	0.634	0.045	(0.546,0.725)	0.640	0.048	(0.550,0.740)	0.642	0.048	(0.153,0.339)
		ig	0.615	0.047	(0.519,0.709)	0.613	0.046	(0.519,0.694)	0.548	0.035	(0.479,0.616)
	12	nig	0.605	0.051	(0.497,0.701)	0.594	0.049	(0.494,0.694)	0.595	0.050	(0.051,0.199)
		ig	0.615	0.047	(0.519,0.709)	0.613	0.046	(0.519,0.694)	0.548	0.035	(0.479,0.616)
	13	nig	0.599	0.053	(0.492,0.696)	0.598	0.050	(0.496,0.700)	0.586	0.053	(0.009,0.102)
29		ig	0.615	0.047	(0.519,0.709)	0.613	0.046	(0.519,0.694)	0.548	0.035	(0.479,0.616)
	21	nig	0.586	0.051	(0.487,0.687)	0.577	0.052	(0.469,0.674)	0.549	0.051	(0.232,0.472)
		ig	0.615	0.047	(0.519,0.709)	0.613	0.046	(0.519,0.694)	0.548	0.035	(0.479,0.616)
	22	nig	0.585	0.050	(0.487,0.682)	0.580	0.053	(0.478,0.686)	0.594	0.047	(0.077,0.296)
		ig	0.615	0.047	(0.519,0.709)	0.613	0.046	(0.519,0.694)	0.548	0.035	(0.479,0.616)
	23	nig	0.589	0.052	(0.485,0.687)	0.587	0.052	(0.483,0.685)	0.606	0.053	(0.012,0.154)
		ig	0.578	0.037	(0.500,0.652)	0.578	0.038	(0.499,0.652)	0.536	0.031	(0.477,0.598)
	11	nig	0.617	0.040	(0.538,0.698)	0.633	0.043	(0.548,0.720)	0.642	0.043	(0.182,0.395)
		ig	0.578	0.037	(0.500,0.652)	0.578	0.038	(0.499,0.652)	0.536	0.031	(0.477,0.598)
	12	nig	0.600	0.051	(0.494,0.695)	0.573	0.051	(0.470,0.672)	0.568	0.051	(0.071,0.257)
		ig	0.578	0.037	(0.500,0.652)	0.578	0.038	(0.499,0.652)	0.536	0.031	(0.477,0.598)
	13	nig	0.599	0.053	(0.489,0.695)	0.595	0.052	(0.492,0.694)	0.584	0.051	(0.012,0.127)
30		ig	0.578	0.037	(0.500,0.652)	0.578	0.038	(0.499,0.652)	0.536	0.031	(0.477,0.598)
	21	nig	0.575	0.050	(0.477,0.668)	0.575	0.052	(0.477,0.680)	0.522	0.050	(0.182,0.396)
		ig	0.578	0.037	(0.500,0.652)	0.578	0.038	(0.499,0.652)	0.536	0.031	(0.477,0.598)
	22	nig	0.571	0.047	(0.480,0.662)	0.562	0.049	(0.463,0.660)	0.614	0.049	(0.072,0.257)
		ig	0.578	0.037	(0.500,0.652)	0.578	0.038	(0.499,0.652)	0.536	0.031	(0.477,0.598)
	23	nig	0.590	0.052	(0.480,0.693)	0.591	0.051	(0.487,0.687)	0.593	0.052	(0.012,0.128)
		ig	0.763	0.052	(0.660,0.854)	0.764	0.053	(0.650,0.853)	0.597	0.046	(0.516,0.695)
	11	nig	0.650	0.046	(0.560,0.741)	0.646	0.048	(0.554,0.740)	0.646	0.048	(0.228,0.485)
		ig	0.763	0.052	(0.660,0.854)	0.764	0.053	(0.650,0.853)	0.597	0.046	(0.516,0.695)
	12	nig	0.622	0.050	(0.523,0.722)	0.613	0.050	(0.507,0.713)	0.613	0.050	(0.092,0.299)
		ig	0.763	0.052	(0.660,0.854)	0.764	0.053	(0.650,0.853)	0.597	0.046	(0.516,0.695)
	13	nig	0.599	0.052	(0.489,0.695)	0.598	0.052	(0.497,0.699)	0.596	0.052	(0.015,0.167)
31		ig	0.763	0.052	(0.660,0.854)	0.764	0.053	(0.650,0.853)	0.597	0.046	(0.516,0.695)
	21	nig	0.603	0.050	(0.507,0.710)	0.603	0.053	(0.499,0.707)	0.595	0.052	(0.132,0.336)
		ig	0.763	0.052	(0.660,0.854)	0.764	0.053	(0.650,0.853)	0.597	0.046	(0.516,0.695)
	22	nig	0.596	0.054	(0.480,0.695)	0.593	0.052	(0.486,0.692)	0.594	0.052	(0.056,0.205)
		ig	0.763	0.052	(0.660,0.854)	0.764	0.053	(0.650,0.853)	0.597	0.046	(0.516,0.695)
	23	nig	0.607	0.051	(0.495,0.705)	0.600	0.051	(0.498,0.701)	0.612	0.048	(0.010,0.111)
		ig	0.803	0.036	(0.729,0.866)	0.801	0.036	(0.718,0.869)	0.667	0.041	(0.583,0.745)
	11	nig	0.647	0.042	(0.565,0.733)	0.652	0.044	(0.565,0.736)	0.657	0.044	(0.207,0.433)
		ig	0.803	0.036	(0.729,0.866)	0.801	0.036	(0.718,0.869)	0.667	0.041	(0.583,0.745)
	12	nig	0.612	0.048	(0.516,0.704)	0.600	0.050	(0.503,0.694)	0.603	0.048	(0.071,0.266)
		ig	0.803	0.036	(0.729,0.866)	0.801	0.036	(0.718,0.869)	0.667	0.041	(0.583,0.745)
	13	nig	0.599	0.052	(0.493,0.699)	0.595	0.054	(0.487,0.698)	0.592	0.052	(0.012,0.140)
32		ig	0.803	0.036	(0.729,0.866)	0.801	0.036	(0.718,0.869)	0.667	0.041	(0.583,0.745)
	21	nig	0.665	0.046	(0.575,0.756)	0.662	0.046	(0.569,0.753)	0.661	0.046	(0.173,0.379)
		ig	0.803	0.036	(0.729,0.866)	0.801	0.036	(0.718,0.869)	0.667	0.041	(0.583,0.745)
	22	nig	0.654	0.047	(0.563,0.748)	0.649	0.050	(0.555,0.756)	0.651	0.049	(0.059,0.232)
		ig	0.803	0.036	(0.729,0.866)	0.801	0.036	(0.718,0.869)	0.667	0.041	(0.583,0.745)
	23	nig	0.610	0.051	(0.507,0.708)	0.610	0.050	(0.512,0.708)	0.609	0.051	(0.011,0.124)
		ig	0.735	0.038	(0.654,0.810)	0.735	0.039	(0.656,0.805)	0.627	0.038	(0.555,0.701)
	11	nig	0.658	0.037	(0.587,0.735)	0.670	0.039	(0.597,0.745)	0.674	0.041	(0.303,0.605)
		ig	0.735	0.038	(0.654,0.810)	0.735	0.039	(0.656,0.805)	0.627	0.038	(0.555,0.701)
	12	nig	0.604	0.050	(0.506,0.697)	0.582	0.053	(0.466,0.693)	0.582	0.050	(0.108,0.376)
		ig	0.735	0.038	(0.654,0.810)	0.735	0.039	(0.656,0.805)	0.627	0.038	(0.555,0.701)
	13	nig	0.598	0.052	(0.496,0.702)	0.591	0.055	(0.477,0.691)	0.586	0.053	(0.018,0.197)
33		ig	0.735	0.038	(0.654,0.810)	0.735	0.039	(0.656,0.805)	0.627	0.038	(0.555,0.701)
	21	nig	0.617	0.050	(0.515,0.715)	0.614	0.050	(0.515,0.712)	0.611	0.048	(0.078,0.196)
		ig	0.735	0.038	(0.654,0.810)	0.735	0.039	(0.656,0.805)	0.627	0.038	(0.555,0.701)
	22	nig	0.613	0.050	(0.512,0.705)	0.612	0.049	(0.511,0.708)	0.618	0.052	(0.029,0.120)
		ig	0.735	0.038	(0.654,0.810)	0.735	0.039	(0.656,0.805)	0.627	0.038	(0.555,0.701)
	23	nig	0.608	0.052	(0.496,0.706)	0.603	0.052	(0.495,0.699)	0.607	0.053	(0.005,0.058)
		ig	0.738	0.049	(0.639,0.824)	0.737	0.048	(0.638,0.824)	0.599	0.043	(0.520,0.684)
	11	nig	0.664	0.044	(0.580,0.754)	0.670	0.046	(0.584,0.756)	0.670	0.045	(0.265,0.543)
		ig	0.738	0.049	(0.639,0.824)	0.737	0.048	(0.638,0.824)	0.599	0.043	(0.520,0.684)
	12	nig	0.606	0.049	(0.509,0.705)	0.601	0.051	(0.498,0.701)	0.599	0.050	(0.091,0.332)
		ig	0.738	0.049	(0.639,0.824)	0.737	0.048	(0.638,0.824)	0.599	0.043	(0.520,0.684)
	13	nig	0.598	0.050	(0.496,0.696)	0.594	0.052	(0.488,0.693)	0.595	0.053	(0.015,0.170)
34		ig	0.738	0.049	(0.639,0.824)	0.737	0.048	(0.638,0.824)	0.599	0.043	(0.520,0.684)
	21	nig	0.594	0.053	(0.483,0.694)	0.591	0.053	(0.486,0.689)	0.580	0.051	(0.110,0.281)
		ig	0.738	0.049	(0.639,0.824)	0.737	0.048	(0.638,0.824)	0.599	0.043	(0.520,0.684)
	22	nig	0.594	0.053	(0.485,0.694)	0.600	0.050	(0.499,0.695)	0.600	0.051	(0.040,0.164)
		ig	0.738	0.049	(0.639,0.824)	0.737	0.048	(0.638,0.824)	0.599	0.043	(0.520,0.684)
	23	nig	0.598	0.051	(0.494,0.697)	0.595	0.051	(0.491,0.685)	0.603	0.050	(0.007,0.087)
		ig	0.785	0.038	(0.708,0.854)	0.782	0.038	(0.701,0.851)	0.651	0.040	(0.573,0.731)
	11	nig	0.647	0.046	(0.558,0.739)	0.651	0.045	(0.565,0.743)	0.659	0.046	(0.160,0.355)
		ig	0.785	0.038	(0.708,0.854)	0.782	0.038	(0.701,0.851)	0.651	0.040	(0.573,0.731)
	12	nig	0.619	0.049	(0.522,0.719)	0.614	0.051	(0.511,0.715)	0.609	0.049	(0.060,0.214)
		ig	0.785	0.038	(0.708,0.854)	0.782	0.038	(0.701,0.851)	0.651	0.040	(0.573,0.731)
	13	nig	0.600	0.051	(0.507,0.697)	0.595	0.052	(0.485,0.690)	0.591	0.052	(0.008,0.110)
35		ig	0.785	0.038	(0.708,0.854)	0.782	0.038	(0.701,0.851)	0.651	0.040	(0.573,0.731)
	21	nig	0.627	0.048	(0.533,0.722)	0.628	0.048	(0.532,0.723)	0.615	0.046	(0.219,0.459)
		ig	0.785	0.038	(0.708,0.854)	0.782	0.038	(0.701,0.851)	0.651	0.040	(0.573,0.731)
	22	nig	0.654	0.047	(0.565,0.756)	0.651	0.046	(0.564,0.747)	0.661	0.048	(0.079,0.275)
		ig	0.785	0.038	(0.708,0.854)	0.782	0.038	(0.701,0.851)	0.651	0.040	(0.573,0.731)
	23	nig	0.612	0.051	(0.511,0.713)	0.611	0.051	(0.507,0.713)	0.612	0.051	(0.012,0.152)

Appendix:

Appendix A: Gibbs sampler for individual no association model with ignorability

First the joint posterior density of all parameters is:

$$
\begin{aligned}
p(\underset{\sim}{t}, \underset{\sim}{s}, \pi_i, \underset{\sim}{z}, \nu, \tau_2, \underset{\sim}{u}, \underset{\sim}{v}, \phi_1, \phi_2 | \underset{\sim}{y}) \;\propto\; & \prod_{i=1}^{N} \left\{ n_i! \prod_{j=1}^{r} \prod_{k=1}^{c} \left\{ \frac{(\mathrm{t}_{ij}\mathrm{s}_{ik}\pi_i)^{y_{ijk}}}{y_{ijk}!} \times \frac{(\mathrm{t}_{ij}\mathrm{s}_{ik}(1-\pi_i))^{z_{ijk}}}{z_{ijk}!} \right\} \right. \qquad (2) \\
\times\; & \frac{{\pi_i}^{\nu\tau_2 - 1}(1-\pi_i)^{(1-\nu)\tau_2 - 1}}{\mathrm{B}(\nu\tau_2, (1-\nu)\tau_2)} \times \prod_{j=1}^{r} \frac{{\mathrm{t}_{ij}}^{u_r\phi_1 - 1}}{D(u_1\phi_1, u_r\phi_1)} \\
\times\; & \left. \prod_{k=1}^{c} \frac{{\mathrm{s}_{ik}}^{v_k\phi_2 - 1}}{D(v_1\phi_2, v_2\phi_2, v_k\phi_2)} \right\} \times \frac{\nu^{\alpha_0 - 1}(1-\nu)^{\beta_0 - 1}}{\mathrm{B}(\alpha_0, \beta_0)} \\
\times\; & \frac{1}{(1+\phi_1)^2} \times \frac{1}{(1+\phi_2)^2} \times \frac{1}{(1+\tau_2)^2} \times \frac{1}{\mathrm{D}(1,1)} \times \frac{1}{\mathrm{D}(1,1,1)}
\end{aligned}
$$

And then, the conditional posterior probabilities are:

$$\underset{\sim}{t}_i|\underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{s}_i, \pi_i, \nu, \tau_2, \underset{\sim}{u}, \underset{\sim}{v}, \phi_1, \phi_2 \overset{ind}{\sim} \text{Dirichlet}(y_{ij.} + z_{ij.} + u_j\phi_1, \\ j = 1, \ldots, r) \quad (3)$$

$$\underset{\sim}{s}_i|\underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{t}_i, \pi_i, \nu, \tau_2, \underset{\sim}{u}, \underset{\sim}{v}, \phi_1, \phi_2 \overset{ind}{\sim} \text{Dirichlet}(y_{i.k} + z_{i.k} + v_k\phi_2, \\ k = 1, \ldots, c) \quad (4)$$

$$\pi_i|\underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{t}_i, \underset{\sim}{s}_i, \nu, \tau_2, \underset{\sim}{u}, \underset{\sim}{v}, \phi_1, \phi_2 \overset{ind}{\sim} \text{Beta}(\nu\tau_2 + \alpha_0, (1-\nu)\tau_2 + \beta_0) \quad (5)$$

$$\underset{\sim}{z}_i|\underset{\sim}{y}_i, \underset{\sim}{t}_i, \underset{\sim}{s}_i, \pi_i, \nu, \tau_2, \underset{\sim}{u}, \underset{\sim}{v}, \phi_1\phi_2 \overset{ind}{\sim} \text{Multinomial}\Big(n_i - r_i, \underset{\sim}{q}_i\Big), \\ q_{ijk} = \frac{t_{ij}s_{ik}(1-\pi_i)}{\sum_{j=1}^{r}\sum_{k=1}^{c} t_{ij}s_{ik}(1-\pi_i)} \\ j = 1, \ldots, r, k = 1, \ldots, c \quad (6)$$

$$p(\underset{\sim}{u}, \phi_1|\underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{t}_i, \underset{\sim}{s}_i, \pi_i, \nu, \tau_2, \underset{\sim}{v}, \phi_2) \propto \prod_{i=1}^{N}\prod_{j=1}^{r}\left\{\frac{t_{ij}^{u_j\phi_1-1}}{D(\underset{\sim}{u}\phi_1)}\right\} \times \frac{1}{(1+\phi_1)^2} \\ 0 < u_j < 1, \phi_1 > 0, j = 1, \ldots, r \quad (7)$$

$$p(\underset{\sim}{v}, \phi_2|\underset{\sim}{y}_i, \underset{\sim}{z}_i, \underset{\sim}{t}_i, \underset{\sim}{s}_i, \pi_i, \nu, \tau_2, \underset{\sim}{u}, \phi_1) \propto \prod_{i=1}^{N}\prod_{k=1}^{c}\left\{\frac{s_{ik}^{v_k\phi_2-1}}{D(\underset{\sim}{v}\phi_2)}\right\} \times \frac{1}{(1+\phi_2)^2} \\ 0 < v_k < 1, \phi_2 > 0, k = 1, \ldots, c \quad (8)$$

$$p(\nu, \tau_2|\underset{\sim}{y}_i, \underset{\sim}{z}_i, \pi_i, \underset{\sim}{t}_i, \underset{\sim}{s}_i, \underset{\sim}{u}, \underset{\sim}{v}, \phi_1, \phi_2) \propto \prod_{i=1}^{N}\left\{\frac{\prod_{j=1}^{r}\prod_{k=1}^{c}\pi_i^{\nu\tau_2-1}(1-\pi_i)^{(1-\nu)\tau_2-1}}{\text{B}(\nu\tau_2, (1-\nu)\tau_2)} \quad (9) \\ \times \frac{\nu^{\alpha_0-1}(1-\nu)^{\beta_0-1}}{(1+\tau_2)^2}\right\} \\ 0 < \nu < 1, \tau_2 > 0$$

We use the grids to draw samples from u, ϕ_1 in (8), v, ϕ_2 in (9) and ν, τ_2 in (3.9).

We also draw 11000 iterates, "burn in" 1000 iterates and pick every tenth iterate thereafter and get Table 3.12. The samples are independent, standard errors are small and steady. We also obtain estimates of $\underset{\sim}{t}_i$, $\underset{\sim}{s}_i$ (Table 3.3) and π_i (Table 3.4).

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