

# Materials Fire and Thermal Performance: Low-E Windows

This Major Qualifying Project report is submitted in partial fulfillment of the degree requirements of Worcester Polytechnic Institute. The views and opinions expressed herein are those of the authors and do not necessarily reflect the positions or opinions Worcester Polytechnic Institute.

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## **Abstract**

The use of Low-Emissivity windows has increased over the past 20 years to account for over 80% of residential windows currently in use. In specific cases these windows have caused vinyl siding deformation, or melting, on neighboring houses due to the focusing of sun energy from curved window surfaces. To better understand this problem, our goal was to create a tool to predict the possibility of siding damage in a given house and neighboring house set up in any location in the United States. Important factors include the prediction of sun angles and intensity, model of the sun intensity focusing from the window surface, application of a transient temperature analysis of a multilayered surface, and analysis of the damage due to time exposed to increased temperatures. From the use of this tool, we were able to identify the key factors that lead to the deformation of siding.

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## Table of Nomenclature

<u>Abbreviation/Symbol</u>	<u>Name/Description</u>
A	Area of the Wall Section
a	Semi-major Axis of Earth's Orbit Around the Sun
$a_0$	Constant used to Calculate Atmospheric Transmittance
$a_1$	Constant used to Calculate Atmospheric Transmittance
$A_i$	Area of the Reflected Image
$\alpha$	Thermal Diffusivity of a Material
alt	Altitude Angle
$A_{tude}$	Altitude of House Location in the United States
Az	Azimuth Angle
$Az_{surf}$	Azimuth Angle of the Surface
$\gamma$	Constant used to Calculate Atmospheric Transmittance
CF	Concentration Factor
$c_p$	Specific Heat of a Material (Material Property)
D	Daylight Savings Value (0 or 1)
d	Declination Angle
Day	Day
Defl	Window Deflection Measurement
Dnh	Distance to Neighboring House
$\Delta t$	Transient Heat Analysis Time Step
$\Delta x$	Finite Difference Method Node Size
$d_i$	Distance Between the Window and the Reflected Image
$d_s$	Distance of Between the Sun and Earth
$\dot{e}$	Energy Generating Element
EOT	Equation of Time
$\varepsilon$	Eccentricity of the Earth's Orbit around the Sun
f	Focal Point Distance
FDM	Finite Difference Method

$G_B$	Direct Solar Radiation on Ground
$G_{B,90}$	Direct Solar Radiation to Window Surface, Perpendicular to $G_B$ ,
$G_D$	Diffuse Solar Radiation on Ground $G_D$
$G_{D, Tilted}$	Diffuse Solar Radiation on Siding, Perpendicular to $G_D$
$G_0$	Solar Irradiance
$G_{0h}$	Solar Irradiance at a Given Location on Earth
$G_\chi$	Solar Irradiance Reflected by Surroundings
$G_{R,90}$	Reflected Solar Irradiance on Siding, Perpendicular to $G_\chi$
$G_{sc}$	Solar Constant
$G_T$	Total Solar Irradiance
H	Hour Angle
$h_{in}$	Inside Convective Heat Transfer Coefficient
$h_{out}$	Outside Convective Heat Transfer Coefficient
Ht	Height of the Window
i	Time Step Number
IGU	Insulating Glass Unit
Inc	Incidence Angle
k	Thermal Conductivity of a Material (Material Property)
$\kappa$	Absorptivity of a Material (Material Property)
Lat	Latitude of House Location
LC	Longitudinal Correction
LCT	Local Clock Time
Lon	Longitude
Low-E	Low-Emissivity
M	Magnification Ratio
m	Finite Difference Method Node Number
Mon	Month
N	Day of the Year (Out of 365)
Per	Period of the Earth's Orbit Around the Sun

Q	Heat Power
$q''$	Heat flux
R	Radius of Curvature the Glass
Refl	Reflectance Property of the Window (Value between 0 and 1)
$\rho$	density of a material (material property)
$r_0$	Variable used to Calculate Atmospheric Transmittance
$r_1$	Variable used to Calculate Atmospheric Transmittance
$r_\gamma$	Variable used to Calculate Atmospheric Transmittance
$r_i$	Radius of the reflected image of the sun (light spot)
$r_s$	Radius of the Sun
$\sigma$	Stefan Boltzmann Constant, ( $5.67 \cdot 10^8 \text{ W/m}^2$ )
SHGC	Solar Heat Gain Coefficient
T	Temperature
$T_s$	Temperature at the Surface of the Sun, 5785 Kelvin
$\Delta T$	Change in Temperature
Tilt	Angle w.r.t the Earth's Surface of the Window
$\tau$	Mesh Fourier Number
$\tau_B$	Atmospheric Transmittance of Beam Radiation
$\tau_D$	Atmospheric Transmittance of Diffuse Radiation
$T_\infty$ & $T_{\text{amb}}$	Ambient Inside or Outside Air
ts	Solar Time
$\nu$	Angle of Earth with respect to the Sun
$\forall$	Volume
W	Width of the Window
x	angle related to Hour Angle
$\chi$	Reflected Radiation Subscript
Year	Current Calendar Year
Z	Zenith Angle

## Chapter 1 – Introduction

Significant strides have been made to increase the energy efficiency of buildings over the past 40 years. Consumption of energy per unit of floor space has been reduced by 11% since 1980. Considering that buildings account for over 40% of the total energy consumption of the United States, the improvement of energy usage in this area has a huge impact.<sup>1</sup> This increased energy efficiency is attributed almost entirely to appliance efficiency, population migration (to warmer areas that require less heating), changes to more environmentally responsible usage habits, and building structural changes.

Structurally, energy efficiency can be improved by sealing the building envelope. The building envelope consists of the components that surround the heated or cooled sections of a building, like walls, floors, roofs, windows, and doors.<sup>1</sup> All these components lose energy at different rates. Energy losses from windows alone account for around 40 billion dollars of lost energy every year.<sup>2</sup> In order to reduce these losses, government funded organizations that promote the use of energy efficient products began to investigate and advocate new window designs in the 1980s.<sup>3</sup> From these investigations on window technology, Low-Emissivity (Low-E) coatings were identified for implementation in common window design. In 1985, only 5% of residential housing utilized Low-E windows. Today, Low-E windows account for over 80% of residential windows.<sup>2</sup>

While the implementation of Low-E windows has contributed to lowering energy usage, one significant problem has been identified. Low-E windows are so effective at reflecting solar energy that reflection in a window that has a slight curvature can result in energy focusing. This focusing can result in the energy focusing on nearby homes or other objects that can reach temperatures of over 120 degrees Celsius.<sup>2</sup> This high temperature has caused documented damage to vinyl siding in over 16 states as of 2010.<sup>2</sup> It has been found that this energy focusing is caused



by a combination of factors including window deflection, sun orientation, and sun intensity. In order to further understand the occurrence of this issue, we have compiled information to calculate sun angles, sun intensity, window energy focusing properties, and lastly siding temperature for any location in the United States. These calculations are compiled in a Siding Temperature Calculation Tool that can be used to determine if a given house and neighboring house setup will likely result in the deformation of siding materials. This paper outlines the background information necessary to understand the problem, how it was applied to predict the occurrence of siding deformation using the Siding Temperature Calculation Tool, and the process used to confirm the accuracy of the tool.

## **Chapter 2 – Background**

### **2.1 Siding**

Siding is the external layer of material that protects a home or building from weathering, provides insulation to the house, and determines the appearance or style of a building. Siding can be made from a variety of materials including wood, stone, brick, metal, plastic/vinyl, or composite materials, which all have different properties.

#### ***2.1.1 Siding Properties***

The important properties of siding related to our project are absorptivity and heat distortion temperature. Absorptivity is the amount of radiation that is not reflected but instead held inside of a material. This is important for the heat analysis that will be discussed in Chapter 3.5. The heat distortion temperature is the temperature at which a material starts to. Depending on the material properties, the heat distortion temperature can vary widely. Materials like vinyl have a lower heat distortion temperatures, 61-74°C.<sup>4</sup> This means it is easier to deform them but unlikely to result in

fire ignition. Other materials like stone, wood, and brick are unlikely to deform at temperatures commonly observed at the siding of houses. These properties along with ranges of absorptivity and emissivity of the common types of house siding can be seen in Appendix A.1.

### ***2.1.2 Vinyl Siding Deformation***

As shown below in Figure 1, the siding produces buckling and wrinkles. According to the report by Cardinal Glass Industries, it is because the vinyl sidings tend to shrink as it reaches the glass transition temperature. This can occur from heating from Low-E window sunlight focusing that will be discussed more in Section 3.4 Focal Point.<sup>5</sup> Mechanically induced thermal degradation takes place during melt processing of polymers at high temperature and, modifies mechanical properties and weather resistance of the material.<sup>6</sup>



**Figure 1. Vinyl Siding Distortion<sup>5</sup>**

The distortion temperature does not specifically indicate the temperature that vinyl siding will permanently distort. The distortion temperature is a measure of a polymer's resistance to distortion under a given heat-load at elevated temperatures.<sup>6</sup> The distortion is due to chemical degradation in polymer structure and occurs with a required activation energy. Degraded chemical bonds cannot be recovered, and thus the exposure to high temperature over time is cumulative.<sup>5</sup> The distortion of the siding depends on both temperature and time.<sup>5</sup> The following figure shows the vinyl siding samples tested under different under different temperatures but same duration.

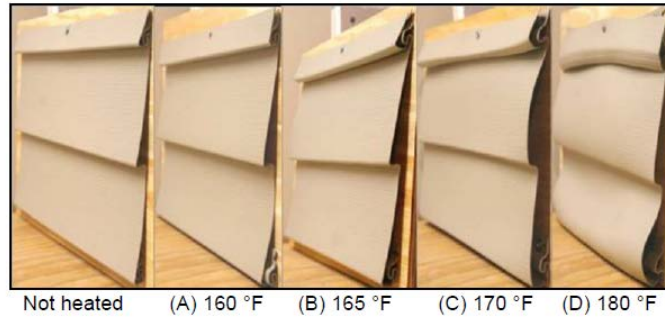


Figure 2. Level of Distortions of Heated Vinyl Sidings for One Hour Under Respective Temperatures<sup>7</sup>

Permanent distortion of vinyl siding can be influenced by orientation of the siding surface, architectural design and proximity of the heat sources also govern the distortion of the vinyl siding.<sup>8</sup> The detailed explanation of these factors is in Appendix A.2 Vinyl Siding Properties and Heat-Buildup and Appendix A.3 Environmental Factors Affecting Siding.

## 2.2 Windows

### 2.2.1 Window Properties

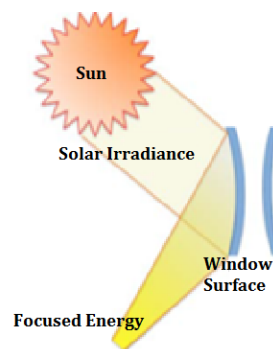
There are a variety of properties that need to be taken into account when studying windows. Windows are rated by their thermal transmittance, which is known as its U-factor. The thermal transmittance through the window assembly includes the heat transfer by conduction, convection, and radiation. A lower U-factor, means less heat transfer through the window, and therefore, a more energy efficient window. Further discussion on thermal transmittance of windows can be found in Appendix A.4 R-Value Nomenclature and Measurement Methods. There are three methods window manufactures use to lower thermal transmittance. These include are using multiple panes, gas fillings, and special coatings.<sup>10</sup>

Coatings are used to decrease the radiant heat flow. These coatings are applied to one surface of only one pane of glass in an insulated glass unit. There are two types of Low-E coatings: a warm climate variety that is intended to minimize the Solar Heat Gain Coefficient or the cold

climate variety that is intended to minimize the U-factor. These are achieved by the coating being applied to the inner pane or the outer pane, respectively. Low-Emissivity means that the amount of heat reradiated by the window is low. Standard clear glass emits 84% of the infrared radiant energy it absorbs.<sup>9</sup> Low-E windows are classified by their Solar Heat Gain Coefficient, a dimensionless number between 0 and 1. A window's SHGC is found by dividing the amount of solar heat gain through the glass by the glass' incident solar heat.<sup>10</sup>

### ***2.2.2 Window Deflections***

Insulating glass units are constructed with two or more panes of glass with air or gas sealed between them. If the pressure inside the window pane changes or the window glass was not manufactured completely flat, this can result in deflection.<sup>8</sup> Many factors, identified by Lawrence Berkeley Lab, can play a role in the variations in pressure, temperature, or mechanical properties of the window. These factors are outlined in Appendix A.5 Deflection Causing Factors of IGUs and Appendix A.6 Factors Influencing Extent and Shape of IGUs.



**Figure 3. Glass deflection and illustration of solar irradiance concentration<sup>8</sup>**

As shown above, when a window becomes concave due to deflection, it can result in concentration of the rays. This concentration of energy will be discussed further in Section 3.4 Focal Point.

## 2.3 Sun Energy

### 2.3.1 Predicting Solar Patterns

The earth revolves around the sun on its orbit throughout the year. As shown in Figure 4, the earth's orbit has an ellipse shape. Therefore, the distance from the Earth to the Sun is different according to the day of the year. Calculations regarding the method used to determine this distance can be found in Appendix A.9.

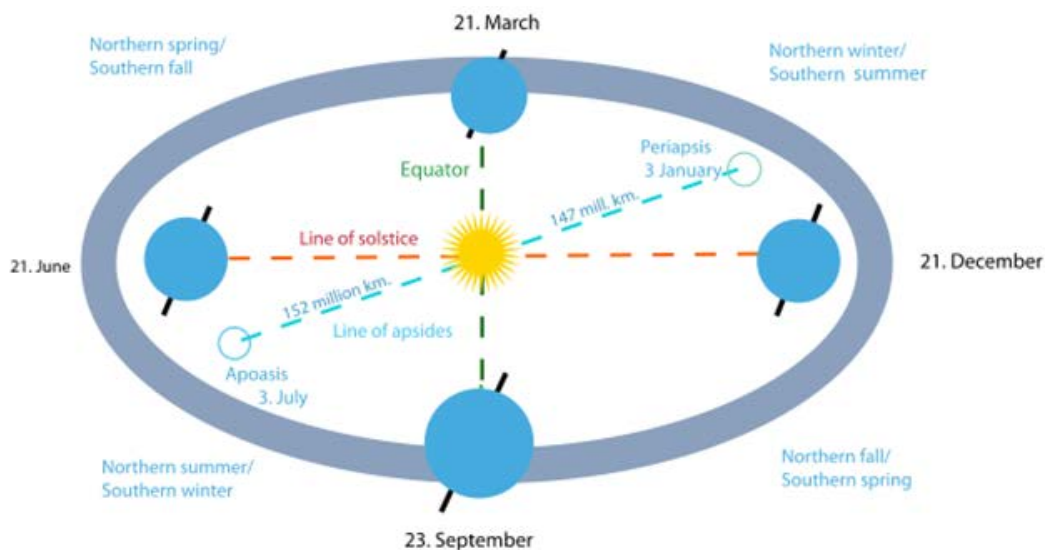


Figure 4. The Solar Orbit of the Earth<sup>11</sup>

In addition to revolving around the sun on its orbit, the earth also spins on its axis that is titled  $23.25^\circ$ .<sup>9</sup> This tilt causes the location on Earth that receives the most direct sun rays to change throughout the year. In the summer months of the northern hemisphere, the sun rays are most direct between tropic of cancer and equator. In the winter months of the northern hemisphere, the sun rays direct between the tropic of Capricorn and equator. The position of the Earth with respect to the Sun is the cause of solar angles that occur between the sun in relation to a specific location on the Earth. There are six different angles that together account for the positioning of the Earth and the Sun that are discussed in detail in Section 3.2 and Appendix A.10. With these angles, it is possible to determine how much solar radiation is reaching the Earth's surface.

### 2.3.2 Solar Radiation

Since the radiation from sun travels in parallel rays to the earth, the amount of radiation that the earth receives depends on the distance that the Sun's rays have to travel. The radiation will vary depending on the distance between the Earth and the Sun and the angles that result from the orientation between the two. The Sun's intensity also varies according to latitude on the earth. The latitude is the angle of north-to-south distance from the equator with the equator being  $0^\circ$ .<sup>12</sup> Considering the sun rays are parallel, the radiation intensity has cosine effect.<sup>13</sup>

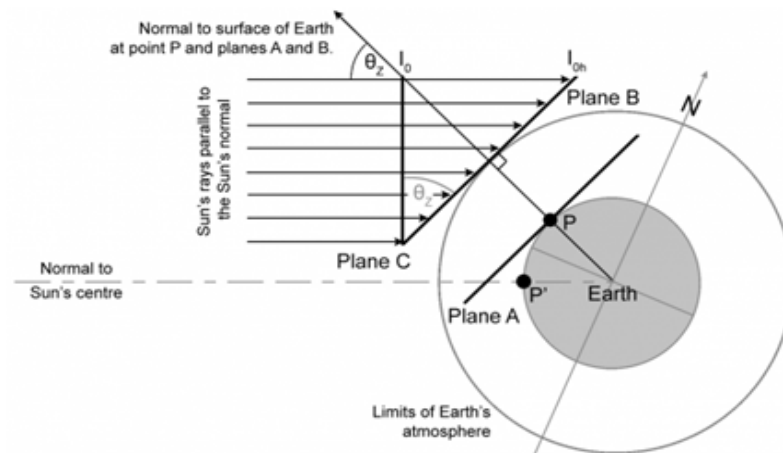


Figure 5. The cosine effect on the solar radiation<sup>13</sup>

As shown in Figure 5, solar rays that reach the northern and southern parts of the Earth have to travel more distance and will not have the same intensity as sun rays that are normal to Earth.

When the solar radiation reaches the Earth, it has to pass through the atmosphere. The Earth's atmosphere scatters and absorbs some of the radiation that is incident on the Earth's surface.<sup>13</sup> Approximately 1.6% to 11% of solar radiation is reflected back into space, 11% to 30% is absorbed by the atmosphere, 5% to 26% is scattered to the Earth as diffused radiation and 83% to 33% reaches to Earth as direct beam. Below, Figure 6 shows solar irradiance reaches the Earth's surface in three ways: beam, diffuse, and reflected irradiance.

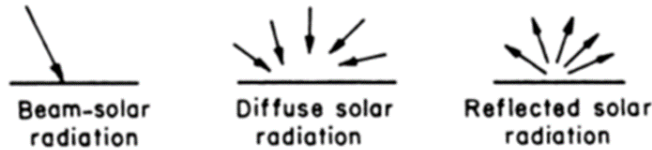


Figure 6. Different Types of Solar Radiation<sup>14</sup>

## Chapter 3 – Siding Temperature Calculation Tool

The Siding Temperature Calculation Tool allows users to determine if the configuration of a house and neighboring house in the United States could result in siding deformation on the neighboring house. A process to calculate sun intensity and angles, energy focusing, and heat transfer to the siding has been organized based on characteristics of a situation input by the user.

### 3.1 Layout

The tool is organized by Microsoft Excel worksheet and a MATLAB file. In the Excel file there are three sheets: user inputs, siding intensity calculations, and damage analysis. The first section is user inputs. The input sheet has a list of inputs, a detailed description of how to obtain each one, and a diagram indicating the orientation of important reference angles. A summary of all of the inputs and how they can be determined is located in Appendix A.14.

Input	Input Value	Unit	Description
State	Massachusetts		
Latitude	42.26	Degrees	Latitude and longitude can be found using google maps. Type in the address of the home being evaluated and click the location on the map closest to the window that is considered to be reflecting sunlight. The latitude and longitude can be found in the bottom right hand corner of the page in this format: Latitude, longitude. Enter those values here. <a href="https://www.google.com/maps/">https://www.google.com/maps/</a>
Longitude	71.78	Degrees	
Time Zone	Eastern		Atlantic, Eastern, Central, Mountain, Pacific, Alaska, Hawaii-Aleutian (locations city name) in kilometers" and click search. Enter the value at the top of the results list into the box to the left. For example if you were searching for the city of Worcester MA, you would enter "Altitude of Worcester MA in kilometers" the result is 0.146km. You would then type 0.146 in the box to the left.
Altitude	0.146	km	
Window Width	0.9144	Meters	Measure or identify, from the specifications of the product, the dimensions of your window.
Window Height	0.6096	Meters	
Tilt of Window (Tilt)	90	Degrees	Indicate the tilt of the window. If the window is in an exterior wall that is the most common perpendicular orientation, enter "90" in the box to the left. If the window is tilted, estimate the tilt of the window by determining the angle from directly perpendicular to the ground to a line normal to the surface of the window. A diagram of this angle can be seen to the right.
Azimuth Angle of Surface (Azsurf)	175	Degrees	Determine the angle of the perpendicular to the window surface measured from north. A diagram of this angle can be seen to the right.
Absorptivity of Siding	0.5		
Type of Low-e Coating (Reflectance)	Low Solar Gain		Indicate the type of low-e coating by examining the specifications for the window being considered. Select the type from the drop down list to the left.
Deflection	Low (0.5mm)	mm	

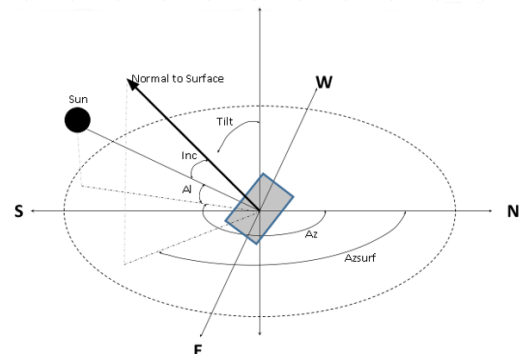


Figure 7. Image of the Input Page for the Siding Temperature Calculation Tool

Using these inputs, calculations are done to determine the intensity of the sun on the siding. The results are exported and used in the MATLAB file to perform a transient heat analysis on the siding. With these results, a damage analysis is done in in Excel. The following list shows the calculation processes that are utilized in the tool:

1. Sun Angle Calculations
2. Sun Intensity Calculations
3. Focal Point and Concentration Factor Calculation
4. Siding Temperature Calculations
5. Damage Analysis

### 3.2 Sun Angles

Sun angles that describe the relative location of the Sun and the Earth have been well investigated. The direction of the energy emitted by the sun is important to the development of the Siding Temperature Calculation Tool, to calculate the sun intensity at a given location on the Earth's surface. The sun angles can be calculated using the latitude and longitude of the house containing the window, the date and time, the tilt of the window, and the orientation of the window. These calculations have been integrated into the tool utilizing two main sources: Power from the Sun by Stine and Geyer <sup>15</sup> and ITACA's compilation The Sun as a Source of Energy.<sup>16</sup> Figure 9 shows five of the size angles that must be calculated to determine the irradiance of the sun.

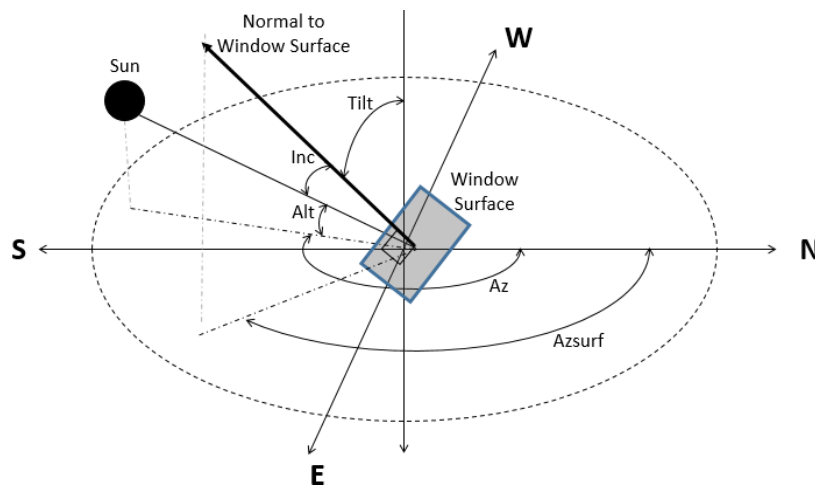


Figure 8. Altitude, Inclination, Azimuth, Azimuth of Surface, and Tilt Angle Diagram



Two of these angles, the incidence angle and the hour angle are used directly in the calculations of the solar intensity. A full detailed outline of each angle calculation is located in Appendix A.10 and a sample calculation of sun angles for a test case in Worcester, MA is in Appendix A.14.

### **3.3 Solar Intensity**

Using the sun angle calculation results, solar intensity on the siding is determined by combining direct focused beam irradiance, diffused irradiance and reflected irradiance.

#### ***3.3.1 Direct Irradiance***

As described in Section 2.3, solar radiation reaches the surface of the window as a direct source from the sun. This is the greatest source of irradiance because it is reflected by the window and focused into a smaller spot. The intensity of this radiation is integrated in the calculation of the beam irradiance of the reflected ray from the window in the focal point section.

#### ***3.3.2 Diffused Irradiance***

Some of the solar radiation absorbed by the atmosphere reaches the ground from all directions. As it is not directional and does not reflect back as a concentrated beam from the window. Therefore, diffused irradiance is not included in the calculation of the intensity of the reflected beam. However, it is utilized in the calculation of the siding temperature. To determine this diffusion in the atmosphere, the location on earth, climate and cloudiness are necessary.

#### ***3.3.3 Reflected Irradiance***

The reflected radiation from the surrounding ground cannot be neglected when the temperature of the siding is calculated. This varies with the ground surrounding the house location can be described as dry grass, sand, vegetation, dark soil, pale soil, or dry bare ground. In this tool,

the user has a variety of choices for his or her house's surrounding materials and from that the tool calculates reflected irradiation. The steps of all irradiance calculations is found in Appendix 9.

### 3.4 Focal Point

Once the intensity of the direct irradiance at the window's surface is known, it can be used to calculate the beam irradiance. For the purpose of this tool, we modeled the sunlight interacts with a Low-E window as a concave mirror. This concept is further explained in Appendix A.8. Two important values are the focal point distance and the concentration factor. The focal point is shown in Figure 9.

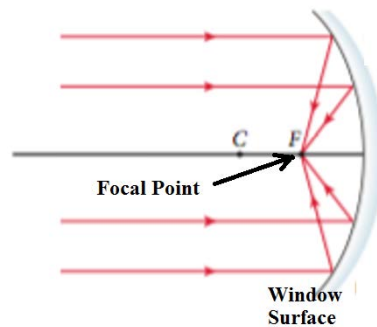


Figure 9. Focal Point Diagram

Conceptually, all rays reflecting are perpendicularly traveling to the concave pane of glass, where they reflect towards the focal point. The equations for finding the location of the focal point are explained in Appendix A.12. With the location of the focal point, the concentration factor can be found for a given distance from the surface of the window. To find the concentration factor, two like triangles are considered; one for if the neighboring house is before the focal point, and one for if the neighboring house is after the focal point. Using these like triangles, the area of the reflected image is proportionately scaled based on the ratio of the window to the area of the spot at the focal point. The concentration factor,  $CF$ , is then determined by Equation 1, where  $A_i$  is the area of the reflected image, or spot, found from the similar triangles.

$$CF = \frac{Ht * W}{A_i} = \frac{Ht * W}{r_i^2 \pi}$$

Equation 1

The concentration factor is then used to find the irradiance reflected to this spot using the reflectance value of the window. A detailed explanation of the focal point calculations can be found in Appendix A.8. The beam irradiance is therefore found by the following equation:

$$Beam\ Irradiance = Refl * G_{B, 90} * CF.$$

Equation 2

### 3.5 Siding Temperature Calculations

The ultimate goal of the tool is to evaluate damage based on the temperature of siding over a given period of time. This will allow the tool to indicate whether or not a Low-E window will cause a high temperature, capable of damaging surrounding siding. To represent this temperature the implicit finite difference method, FDM was used.

#### 3.5.1 Steady State Initial Conditions

The first step in the heat calculations was to set up a one-dimensional heat conduction representation of a wall section. For the sample calculation, the wall section was assumed to have four layers: a sheet of gypsum board, a fiberglass batt blanket, a sheet of plywood, and a layer of vinyl siding. This wall section is depicted by Figure 10

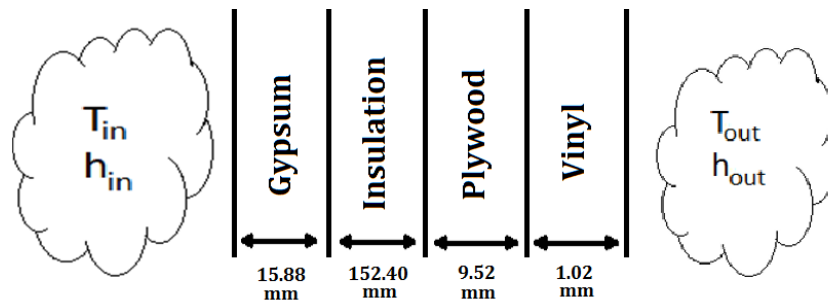


Figure 10. Representation of Wall Section Layers

To solve for the temperature at different points in the wall section, an energy balance is performed on small differential volume elements, along the length of section. Using the finite difference method, accurate results were obtained by replacing differential temperature quantities by small temperature differences per discrete position points, known as nodes.<sup>17</sup> For this tool, a 20 node finite difference representation was set up. A system is considered at steady-state when its variables are constant over time.<sup>17</sup> For the wall section, it was determined that once the sun went down, the system would approach and reach steady state because it was no longer subject to the solar heat flux. All of the state variables start each morning at steady state.

In efforts to simplify the linear temperature analysis of the wall section, each of the four layers was analyzed separately. Using a popular textbook—Cengel’s *Heat Transfer: A practical approach*—as a reference, it was found that the 20 node representation used three variations of the one-dimensional heat conduction equation, Equation 3. The temperatures were assumed to not be changing with time, and therefore the left side of Equation 3 was set to equal zero. The full derivation of the equations used for the steady state analysis can be found in Appendix A.13.

$$\sum_{All\ sides} \dot{Q} + \dot{E}_{gen,element} = \rho V_{element} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t} = \frac{\Delta E_{element}}{\Delta t}$$

Equation 3

Due to the complexity of having 21 equations and 21 unknown temperatures, MATLAB software was used to write a module to construct a 21x21 matrix of the unknown temperatures and then used to solve the system of equations. The benefit of using MATLAB for this steady state heat conduction problem was the code could be written using variables, and the code could be run for varying input conditions.

### **3.5.2 Transient Analysis**

After establishing steady state conditions for the wall section, a set of linear equations was written for the transient heat analysis. For this set of calculations, a heat flux was introduced. This heat flux accounted for all solar irradiance: diffused, reflected from the surroundings, and reflected from the Low-E window. In transient analysis, the temperature changes with position and time. To calculate the siding temperature at a given point in the day, the temperature at each position point is calculated at small time steps to determine the temperature after the solar irradiance has been introduced to the wall section at sun rise.

The transient analysis uses the steady state conditions found in Section 3.5.1 in a new energy balance at each node in the wall section. The same energy balance, from Equation 3, was used, however, the right side of the equation changed with time and therefore was not equal to zero. Four variations of the energy balance were found that could be applied to the 20 node wall section. These four equations and their derivations are found in Appendix A.13. This set of equations was made into a 21x1 matrix in a third MATLAB module that imported the initial steady state conditions module and the mesh Fourier module. The transient analysis module, found in Appendix A.16, is run in a loop and calculates the temperature gradient of the wall after a period of time--given by the number of loops times the time step of two minutes--of exposure to the solar irradiance. The program can be run over the period of a day to analyze the siding temperature with changing irradiance based on time of day, changing ambient air temperatures and heat transfer coefficients. The resulting temperatures are then used for a damage analysis.

### **3.6 Damage Analysis**

Using the temperature results from the process described in Section 3.5, the lifetime of the siding and the level of distortion can be predicted using the Arrhenius Relationship. This

relationship has been used by numerous other studies to predict the lifetime of vinyl siding.<sup>18</sup> The degradation of vinyl siding due to heat is based on the destruction of chlorine bonds in the material. The chart below shows the rate of dehydrochlorination of PVC. The detailed calculation and explanation of dehydrochlorination is described in Appendix A.17.

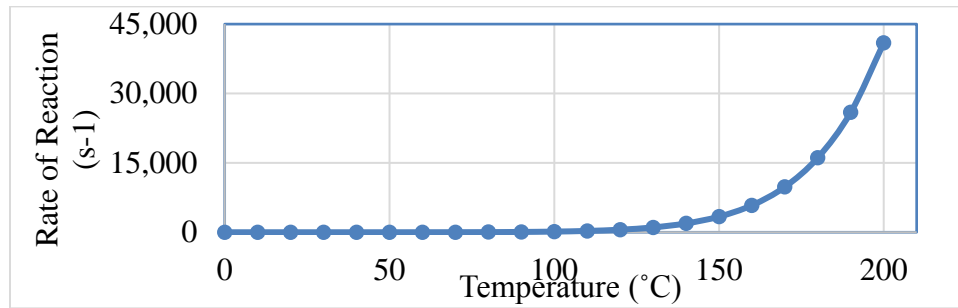


Figure 11. Rate of Reactions at Different Temperatures

The Arrhenius relationship assumes the degradation process is controlled by Equation 4.

$$\text{Rate of Reaction [s}^{-1}\text{]} = Ce^{-E/RT}$$

Equation 4

The exponent variables represent activation energy,  $E$  with units in  $\text{Jmol}^{-1}$ , the gas constant  $R$ , which is equal to  $8.314 \text{ Jmol}^{-1}\text{K}^{-1}$ , and absolute temperature  $T$ , with units in Kelvins. The variable  $C$  represents the pre-exponential factor that is adjusted to a scale consisting of four damage levels: one being almost unnoticeable damage and four being extreme siding damage.<sup>18</sup>

## Chapter 4 – Analysis

After creating the tool and checking the calculation processes, the accuracy of it and the impact of inputs were tested. A sensitivity analysis was performed and a damage analysis was done for a known test case to check the results of the Siding Temperature Calculation Tool.

### 4.1 Sensitivity Analysis

A one at a time sensitivity analysis was performed on each of the seven identified inputs to the tool. For each one, a single parameter is varied at a time to identify how that variable impacts

the results. A full outline of the data and the analysis methods used is in Appendix 17. For each of the parameters, a range of realistic values was identified for a specific house and neighboring house configuration the tool has evaluated. With that, comparisons between the change in siding temperature and change in each variable were made. In these comparisons, the siding temperature range and variable ranges were normalized to be able to compare them more directly. The following equation was used for each of the variables:

$$\frac{(\Delta T / T_{ref})}{(\Delta \text{Variable Value} / \text{Variable Ref})} = \text{Sensitivity Value}$$

Equation 5

**Table 1. Table of Calculated Sensitivity Values**

<b>Variable [unit]</b>	<b>Sensitivity Value</b>
Deflection [mm]	0.228
Heat Transfer Coefficient [W/m <sup>2</sup> K]	0.059
Surrounding Ground Reflectivity	0.04
Ambient Temperature [K]	0.801
Neighboring House Distance [m]	0.005
Reflectance of Window	0.073
Absorptivity of Siding	0.164

The results from these calculations are summarized in Table 1. From these results and further details that can be found in Appendix A.17, some conclusions can be made. The variables that have the greatest influence to cause siding deformation are deflection, ambient temperature, and the absorptivity of the siding. The deflection impacts the temperature of the siding four times as much as the surrounding ground reflectivity and three times as much as the reflectance of the window. The reflectance has a lower impact likely because standards require a certain range of reflectance values for windows which lessens the variation that can occur from this factor.

According to our results, the ambient temperature has a much greater impact than the deflection. We believe this is not the case because there is a relationship between the ambient temperature and the deflection of the window that is not accounted for. Studies have shown that deflection of windows often occur in the winter months when temperatures are lower. If this deflection goes away or is reduced in the summer when the ambient temperature increases, then the intensity and therefor the temperature of the siding would be reduced. In this analysis we did a one a time analysis assuming a constant deflection of the window throughout the year. While this may be accurate if rare cases where the deflection has been caused by manufacturing and or installation, it does not account for cases where the deflection is seasonal. Refer the Chapter 5 for suggestions for future improvements and how this issue could be addressed.

#### 4.2 Damage Results from Worcester Case Study

To understand the damage of the vinyl siding, an analysis was done for the Worcester case study house using the input parameters outlined in Appendix A.14. More information about the location and images of the damage that occurred is found in Appendix A.20. Analysis shows that the temperature peaks as the solar beam crosses a spot on the siding (shown in Figure 12). That is also when the rate of chemical reaction in the siding increases as shown in Figure 13.

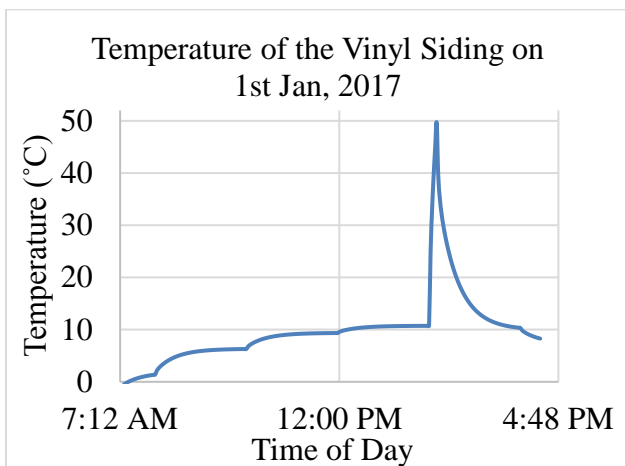


Figure 12: Temperature of Siding on Jan 1st, 2017

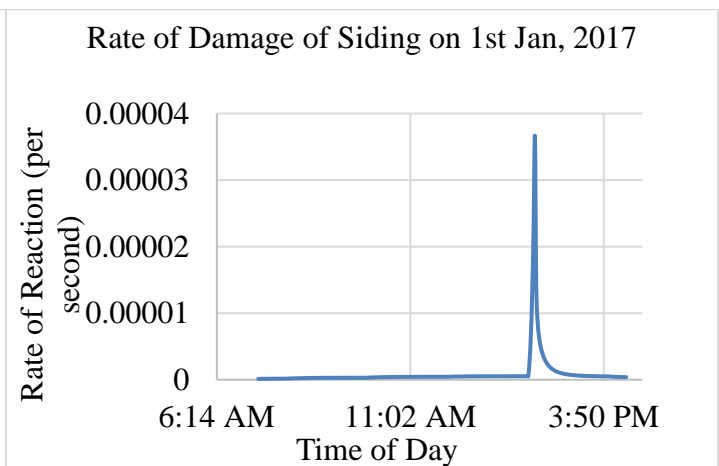


Figure 13: Rate of Damage of Siding on 1st Jan, 2017



Although the damage acquired during a day is very low, the damage accumulated throughout the year reaches damage level four, corresponding to extreme damage to the siding.

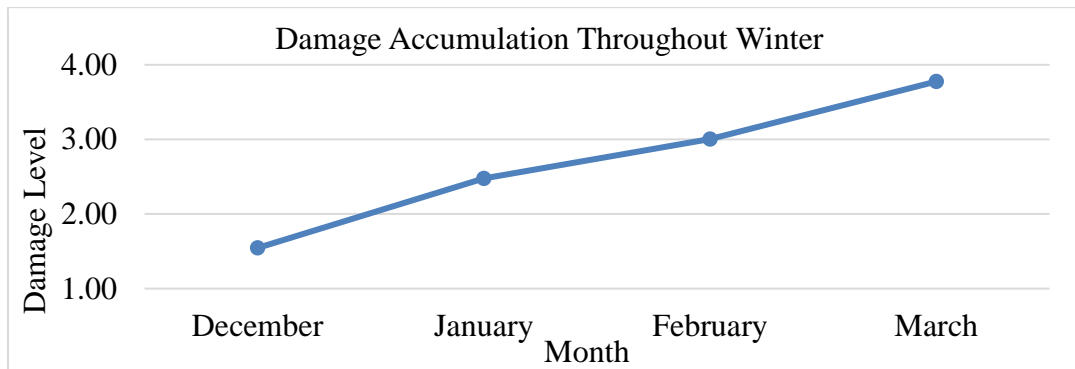


Figure 14: Vinyl Siding Damage Accumulation Throughout Winter Months

After accumulating damage throughout the year, it was determined that the results were too high. In its current configuration, the tool does not automatically predict how the deflection of the window will respond to the ambient temperature change. Because we know that in this case the deflection is caused by the temperature change throughout the year, we predict that the deflection occurs only in the winter months. In Figure 14 the damage predicted over four months of moderate deflection of the window is shown. The damage shown aligns with the information provided about the case study, suggesting that over four months of full exposure significant damage will occur to the siding. To verify that when the deflection decreases as the ambient temperature increases in the summer months, the tool was used to predict the temperature of the siding at the same location with deflection of the window decreased almost to zero.

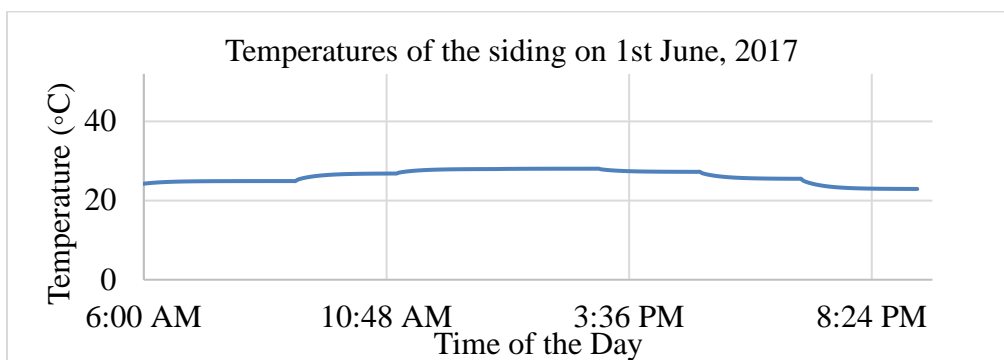


Figure 15: Temperature of the Siding on 1st June, 2017

The results, in Figure 16, demonstrate that the temperature is steady and low around 20 degree Celsius (shown in Figure 16). Therefore, it is predicted that very low deformation is taking place during summer months and damage acquired can be neglected as shown in Figure 17.

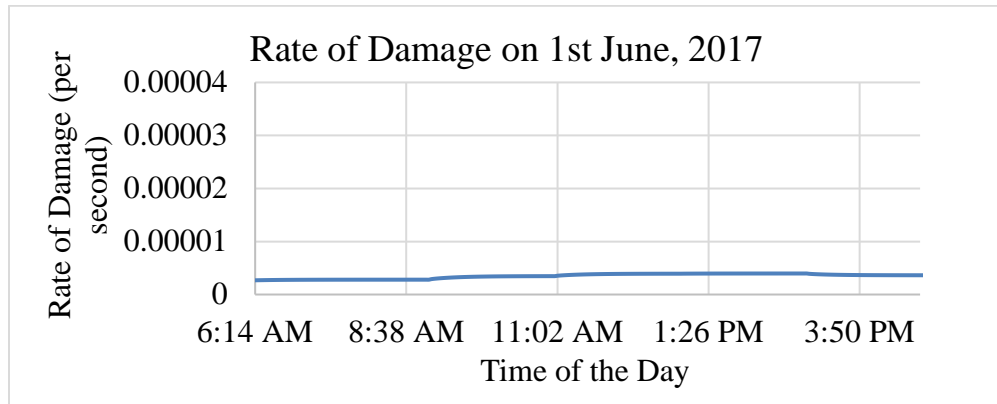


Figure 16: Rate of Damage of Siding on 1st June, 2017

### 4.3 Damage Results from Pennsylvania Case Study

To understand the capabilities of the tool, we tested on a known case in Pennsylvania. In reality, the vinyl sidings of this house were also severely distorted. From the output of the tool, we had a high peak of temperature (shown in Figure 18) similar to the case in Worcester.

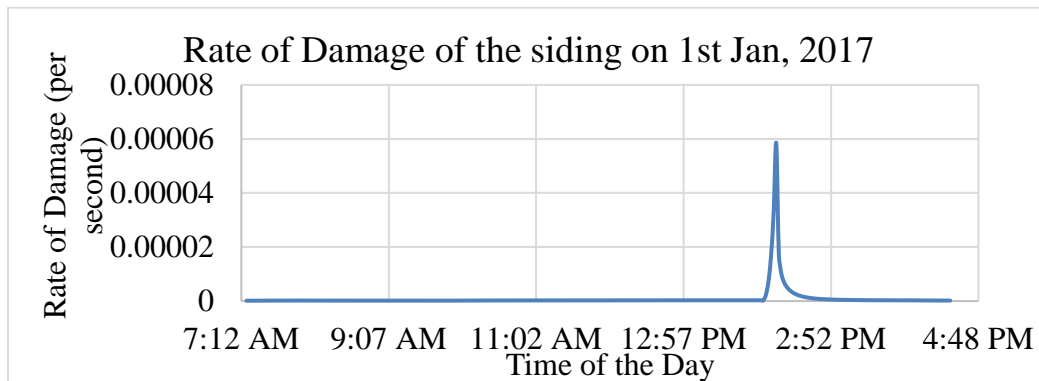


Figure 17: Rate of Damage of Vinyl Sidings of PA Case

With that peak, the rate of damage was also high at that point (Shown in Figure 17). As we calculated, we can see the level 1 damage just in one month of January.

## **Chapter 5 – Future Work**

The tool developed for this project was developed understand the processes and the factors that contribute to causing siding deformation. In order to further develop the tool, the next steps would be to investigate the occurrence of window deflection and quantify the factors that cause it. The ability to predict deflection would provide more accuracy for the overall ability of the tool to predict siding deformation. It would make it possible to automate the tool more for ease of use. In addition, to make the tool more useable it would be necessary to automate more processes and inputs to make it more user friendly.

## **Chapter 6 – Conclusion**

From the development and analysis of the tool we have confirmed the applicability of this method to predicting and understanding vinyl siding deformation. The assumptions made and calculation processes developed have been validated through testing of the tool in multiple conditions. By observing the reactions of the output of the tool to varied inputs we have been able to identify how different factors combine and accumulate to cause cases of siding deformation. Future work to be able to predict the amount of deflection based on environmental conditions is recommended to continue to develop the Siding Temperature Calculation Tool and further quantify this issue.

## Chapter 7 – References

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## Appendix A.1 Siding information

### 1) Vinyl Siding

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Most popular siding in US.

Durable/Low cost/Fire retardant.

No insulation value/ Will be damaged from heat.

No warranty coverage for damage from sun reflections.

ASTM standards require 0.035" minimum thickness for vinyl siding. But a preferred thickness is 0.42-0.45.

Material:

PVC vinyl releases toxic dioxin when burned.

**VINYL:**

- Synthetic material
- Type of plastic made from combination of ethylene and chlorine which form polyvinyl Chloride otherwise known as PVC resin.
- Resistant to moisture and humidity, strong and durable, can produce a variety of colors and other visible properties like transparency.
- Low cost
- Environmentally friendly because it can be easily recycled.
- Melting point 100-260°C depending on specific composition.  
Many coatings available that can impact the heat/light absorption properties of the siding.<sup>1</sup>

### 2) Wood Siding

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Popular in western US.

No insulation.

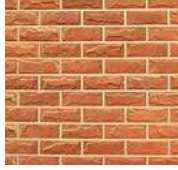
Environmentally friendly

Requires painting/ staining

Can cause growth of algae, mildew or moss causing rot and deterioration if not cared for well.

### 3) Brick Siding

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Long lasting, low maintenance.  
High heat resistance but expensive.

4) Fiber-cement siding



More expensive than vinyl.  
Resistant to fire, wind and rain damage.  
Primed and painted. Has to be repainted.  
Not flammable.

5) Stucco Siding



Popular in warm/dry climates.  
Expensive installation because it takes a skilled person to install.  
Made from mix of cement, sand and water.  
Durable and long lasting.

6) Stone/ stone-veneer



Thickness 1"  
15lbs/ft<sup>2</sup>  
Made from real stone or artificial alternative  
Alternative materials are usually a thin layer of stone (slate, sandstone, or mica schist) on top of a composite material  
High melting temperatures does not experience the deformation from window reflection/focusing.<sup>2</sup>



**Table 2. Table of Siding Properties**

Siding Type	Heat Distortion Temperature (°F)	Ignition Temperature (°F)	Absorptivity	Emissivity
Vinyl	160-165	730	0.7-0.9	0.84-0.95
Wood	248	190-260		0.86-0.9
Brick			0.5-0.8	0.7-0.9
Stone	3000		0.3-0.5	0.7-0.8
Metal	842		0.4-0.65	0.9-0.95
Stucco			0.3-0.5	0.9

## **A.1 References**

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## Appendix A.2 Vinyl Siding Properties and Heat-Buildup

Table 3. Properties of vinyl siding that may affect the heat-build up and permanent deformation <sup>1</sup>

Factors	Description
Total solar absorptance	Generally, darker colors will absorb more energy and have greater heat build-up. However, even for two materials with the same apparent color, the heat build-up may vary because of the specific pigment system used and its absorptance in the IR range.
Thickness and profile shape	Thicker siding, as well as certain shapes (deeper profiles with more bends), may have increased resistance to distortion.
Rigid insulating backing	When insulation is formed and bonded to the siding profile it may increase the siding's rigidity and distortion resistance. Insulation behind the siding may also increase the siding temperature when exposed to radiation, which may counteract the increased rigidity.
Heat distortion and glass transition temperatures	The HDT and glass transition temperatures may be used as reference points for the temperature range in which vinyl siding may permanently distort. Time of exposure, temperature, method of loading, and fiber stress all contribute to how and when distortion will actually occur.
Durability	Physical aging, such as from weathering, typically stiffens vinyl siding and may slightly increase its HDT. The complete effects of aging on distortion resistance are unknown (Rabinovitch 1992).
Extrusion process	Improperly processed vinyl siding may be more susceptible to distortions such as oil-canning at elevated temperatures.
Installation	ASTM D4756 and individual manufacturer installation procedures are designed to allow for thermal expansion of vinyl siding. Improper installation, or expansion greater than is allowed for in installation, may leave the siding more susceptible to, or result in, permanent distortion.

### A.2 References

<sup>1</sup> Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

## Appendix A.3 Environmental Factors Affecting Siding

Table 4. Environmental factors that affect the temperature of vinyl siding<sup>1</sup>

Factors	Description
Orientation of the siding surface	The sidings view factor (fraction of radiation that leaves one surface and is intercepted by another surface) of energy sources and nearby surfaces, and its direction relative to local winds helps determine the equilibrium temperature reached by the siding surface.
Direct and diffuse solar radiation	The energy from direct solar and diffuse sky radiation typically has a large impact on siding surface temperature. The combination of solar azimuth, solar altitude, and orientation of the siding surface determine the extent of direct and diffuse solar exposure.
Reflected solar radiation	Direct reflected solar energy from smooth surfaces such as window glass and swimming pools and diffuse reflections from rough surfaces such as painted walls, patios, sidewalks, and driveways may heat the siding surface.
Long-wave radiated energy from nearby surfaces	Re-radiated heat from nearby surfaces, such as asphalt, may increase the temperature of the siding surface. The heat island effect is an example of this (LBNL Heat Island Group 2010).
Physical obstructions	Objects such as foliage, fences, or adjacent houses may block direct or reflected solar radiation from reaching the siding surface.
Local climate	Local wind speed and air temperature affect the heat-transfer rate of the siding surface to the ambient air.
Architectural design	Alcoves, overhangs, and other architectural features may block wind or trap heat near the siding surface.
Proximity of heat sources	The distance of the siding from the source of direct or reflected radiation will help determine the quantity of radiation that reaches the surface. Air conditioner compressors, barbeque grills, and other common residential heat sources may heat siding if located close to the siding surface.

### A.3 References

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<sup>1</sup> Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

## **Appendix A.4 R-Value Nomenclature and Measurement Methods**

There are three properties with similar relationships that are important to understand when examining the energy efficiency of building materials: U-factor, k-factor, and R-value. The U-factor is simply the reciprocal of the R-value, where the R-value is the measure of thermal resistance of a material. It is the ratio of a temperature unit per heat flux. The k-factor, the thermal conductivity for a unit of thickness of a material, is also the reciprocal of the R-value. The difference between the U-factor and the k-factor are that the k-factor is for a single material while the U-factor is for a cross-section of layered materials. A window is therefore rated with its U-factor because its thermal transmittance is dependent on the multiple components it contains, such as the sash, each pane of glass and the spacers between panes.<sup>1</sup> The American Society for Testing and Materials (ASTM) has established three test methods used for measuring the R-value of materials: ASTM C-177, ASTM C-518, and ASTM C-1363.

### **A.4.1 ASTM C-177: Standard Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties of the Guarded Hot Plate Apparatus.**

In this test method, the thermal conductivity of a test specimen is measured then calculated using Fourier's Law. A diagram of the test conditions can be found below. To conduct this test, two homogenous test specimen of the same thickness are needed. One specimen is placed on each side of a hot plate. The other side of each specimen is touching a "cold plate," which is chilled by a liquid cooling system. The hot plate is providing the test system with a known heat flux. The system is insulated on its edges to prevent lateral heat flow, which would corrupt the results.<sup>2</sup> The effective thermal conductivity,  $k$ , is then calculated using the following equation, where  $\Delta x$ =specimen thickness,  $\Delta T=T_{\text{hot}}-T_{\text{cold}}$ ,  $A$  is the area of "Metered Area",  $Q$  is the heat power supplied to the metered area (this power is at an assigned rate.)

$$k_{eff} = Q/2 * \Delta x / (\Delta T * A)$$

Equation 6

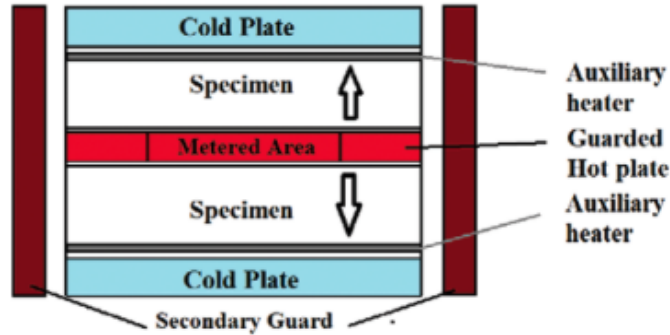


Figure 18. ASTM C 177 Test Apparatus<sup>2</sup>

#### A.4.2 ASTM C-518: Standard Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus

This test method is similar to the ASTM C 177, however, the testing apparatus uses a heat flux transducer placed between the specimen and the hot plate. Rather than using the power supplied to the hot plate to apply Fourier’s law, the transducer is measuring the heat flux applied to it by measuring the voltage drop across it. This change in thermovoltage is proportional to the drop in temperature occurring throughout the plate.<sup>3</sup> The thermal conductivity,  $k$ , of the specimen is then found by the following equation where  $q''$  is the heat flux measured by the transducer,

$$k_{eff} = -q'' * \Delta x / \Delta T$$

Equation 7

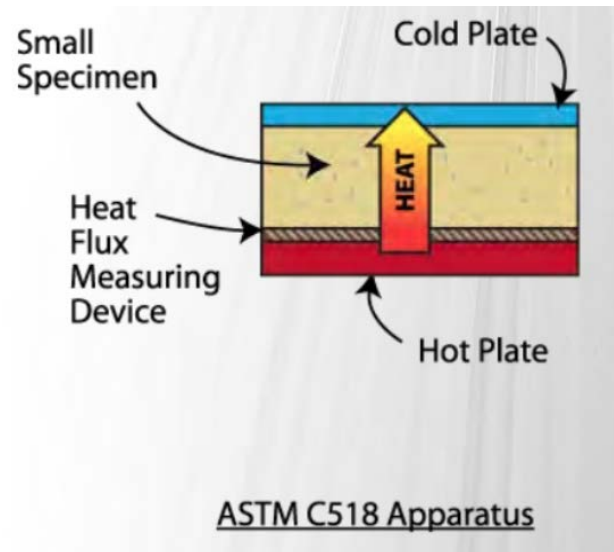


Figure 19. ASTM C 518 Test Apparatus<sup>3</sup>

#### A.4.3 ASTM C-1363: Standard Test Method for Thermal Performance of Building Assemblies by Means of a Calibrated Hot Box

ASTM C 1363 is the method commonly used to test the R-value of large specimens, often times including windows. The apparatus sandwiches a specimen between two chambers. One chamber is kept at a steady state cool temperature. The apparatus measures the power required to heat the opposite, warmer chamber at a constant temperature. Fourier's law is then used to calculate the thermal conductivity,  $k$ , of the specimen under the steady state conditions, where  $Q$  is the power needed to maintain the warm chamber's temperature,  $\Delta x$  is the thickness of the sample,  $\Delta T$  is the difference between warm chamber and the cold chamber, and  $A$  is the area of the specimen being measured.<sup>4</sup>

$$k_{eff} = Q * \Delta x / (\Delta T * A)$$

Equation 8

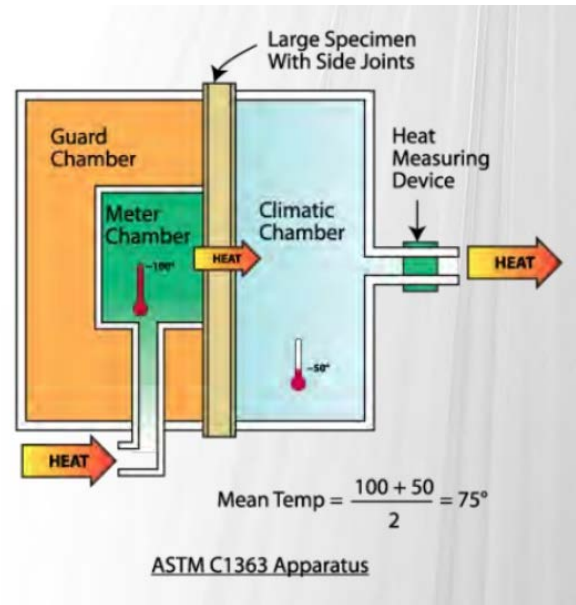


Figure 20. ASTM C-1363 Apparatus<sup>4</sup>

#### A.4 References

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## Appendix A.5 Deflection Causing Factors of IGUs

Table 5. Factors that may cause deflections on the glass surface of IGUs<sup>1</sup>

Factors	Description
Pressure differences	When the external (atmospheric) pressure increases relative to the interior (sealed) pressure of the unit, e.g., when interior temperature or altitude decreases or barometric pressure increases, the pressure differential may cause the glass to deflect inward (become concave). A 1/2 psi (3.4 kilopascal [kPa]) pressure differential is approximately equal to a change of 20°F (11°C), 1,000 ft. (305 m), or 1 in. mercury (Hg) (3.4 kPa).
Manufacturing procedure	Improper manufacturing by sealing an IGU at an elevated internal gas temperature when using a heat activated edge sealant may reduce the amount of gas sealed inside the unit. This will bias the unit toward a lower internal pressure, which may result in concave glass once the unit is cooled.
Gas diffusion	In argon-filled units the driving force for out-diffusion of argon and in-diffusion of air is the concentration gradient resulting from using pure argon in the IGUs sealed volume. Argon diffusion is faster through most sealant materials than nitrogen and oxygen diffusion is, so the result with some sealants is a net loss of molecules in the sealed volume causing a lower internal pressure in the IGU and inward (concave) glass deflection (Czanderna 2000).
External pressure	Wind forces on the exterior pane of glass and internal building pressures (from air conditioners or ventilation systems) on the interior pane of glass may both contribute to glass deflection (Cardinal IG 2008).
Installation	Installing windows out of square, plumb, or plane to the wall may cause deflections in the window frame and on the IGU surface. Frame joint misalignments or glazing sealant thickness variations may also cause deflection at the glass edge.
Glass treatment	The tempering, or heat treating, of glass may introduce some bowing that typically appear as waves on the glass surface.

### A.5 References

<sup>1</sup> Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

## Appendix A.6 Factors Influencing Extent and Shape of IGUs

Table 6. Factors that may influence the extent and shape of IGU deflection<sup>1</sup>

Factors	Description
Glass thickness	Thinner glass deflects more in response to the same force than thicker glass and can be more difficult to keep flat during manufacture. Single-strength 2.5-mm (3/32-in.) glass is typically the thinnest used; double-strength 3-mm (1/8-in.) is also common in residential windows.
IGU size and aspect ratio ASTM	ASTM E1300 is typically used by manufacturers to determine the appropriate glass thickness to use based on area, aspect ratio, edge support, and load conditions. Higher loads and larger sizes require thicker or treated glass to meet the same strength and load criteria.
Glass support	The extent of unsupported glass edges and the stiffness of the frame on the supported glass edges both affect deflection.
Edge configuration	Spacer and sealant stiffness affect edge deflection when under load.
Internal gap width	Larger internal gap widths contain more gas fill, which results in greater internal pressure differentials from changes in temperature and altitude. Larger internal gaps also place the glass plates farther apart, which increases the stiffness of the system.

### A.6 References

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<sup>1</sup> Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

## Appendix A.7 IGUs Used in LBL Experiment

Table 7. Types of IGUs and buildings used for experiment<sup>1</sup>

Site A	Windows	<ul style="list-style-type: none"> <li>· Group A-1: 93 single sliders, 2-pane, 90% Argon gas fill; Typical Size: 845 x 1290 mm</li> <li>· Group A-2: 92 fixed, 2-pane, 90% Argon gas fill; Typical Size: 830 x 380 mm</li> <li>· Vinyl window frames</li> </ul>
	Building Type	Hotel, conditioned but unoccupied for winter shutdown
	Measurement	Winter only because no equivalent warm-weather shutdown available
Site B	Windows	<ul style="list-style-type: none"> <li>· Group B-1: 70 casements, 2-pane, 90% Argon gas fill; Typical size: 1495 x 760 mm</li> <li>· Group B-2: 32 fixed, 2-pane, 90% Argon gas fill; Typical size: 760 x 355 mm</li> <li>· Group B-3: 30 casements, 3-pane, 90% Krypton gas fill; Typical size: 760 x 1495 mm</li> <li>· Group B-4: 30 casements, 3-pane, 90% Argon gas fill; Typical size: 760 x 1495 mm</li> <li>· Vinyl and aluminum-clad wood window frames</li> </ul>
	Building Type	Single family homes, conditioned but unoccupied
	Measurement	Winter and summer
Site C	Windows	<ul style="list-style-type: none"> <li>· Group C-1: 48 double hung, 2-pane, 90% Argon gas fill; Typical size: 775 x 1320 mm</li> <li>· Group C-2: 82 double hung, 3-pane, 90% Krypton gas fill; Typical size: 775 x 1320 mm</li> <li>· Vinyl window frames</li> </ul>
	Building Type	Duplexes, conditioned but unoccupied
	Measurement	Winter and summer
Site D	Windows	<ul style="list-style-type: none"> <li>· Group D-1: 424 double hung, 2-pane, 90% Argon gas fill; Typical size: 850 x 1770 mm</li> <li>· Vinyl window frames</li> </ul>
	Building Type	Single family homes, conditioned but unoccupied
	Measurement	Winter and summer

### A.7 References

<sup>1</sup> Hart, R., Goudey, H., Arasteh, D., & Curcija, D. C. (2012). Thermal performance impacts of center-of-glass deflections in installed insulating glazing units. *Energy and Buildings*, 54, 453-460.

## Appendix A.8 Window Deflection: Windows Acting as Spherical Mirrors

### A.8.1 Reflectance Value for Window

The reflectance value for the window is a ratio of the amount of energy reflected by the window to the amount of energy hitting the window. This numerical value is not one of the properties listed on the label from the factory. However, a user of this application has the option to choose one of four options that will account for the reflectance value of the window in question. The ultimate goal of this application is to help identify if a Low-E window will cause siding deformation problems. Therefore, the highest reflectance value for each of the four option's ranges was assumed. This should be made known to the user so he or she knows that the final suggestion of the application is for the worst-case scenario. Table 5 gives the range of solar energy reflected by each window type as a percentage (i.e. the reflectance value multiplied by 100.) So for example, the user in our example that has a window with a Solar Heat Gain Coefficient of 0.24 would choose the "2 Pane, Low Solar gain Low-E option." The spreadsheet would then assume a reflectance value of 44%, despite the fact that the window's actual reflectance value may be lower than that.

Table 8. Table Identifying SHGC Ranges for Reflectance Value Used by Program<sup>1,2</sup>

IGU Type	SHGC	IGU Total Solar Reflectance Out
2 Pane (P), clear	0.80	13%
2P, High Solar gain, Low-E	> 0.4	16-25%
2P, Moderate Solar gain Low-E	0.25 < SHGC < 0.4	35-30%
2P, Low Solar gain Low-E	< 0.25	42-44%

### A.8.2 Deformation of Window

The concentrating effect produced by the window of the reflective solar energy is caused by the outer pane of glass of an IGU to go from a flat surface to a concave surface, as seen below in Figure 21.



Figure 21. (Left) Two Pane IGU with No Deformation. (Right) Two Pane IGU with Deformation Causing Concave Shape<sup>3</sup>

When the two panes of glass of an IGU cause a concave shape, the outer pane becomes a circular segment with properties that can be derived with geometric and trigonometric relations. Figure 22 has been constructed to illustrate those relationships with reference to the original shape. Assume that this diagram is a side view of the window. Below, the dashed line indicates the window's original shape when it experiences no deformation. If looking at the image on the left, the curved segment of to the right of the dashed line is representative of the deformed window shape. The measurement denoted  $x$  was the measured distance between the centers of glass of the panes at their starting positions. Variable  $R$  is representative of the radius of curvature of the pane. The height of the window and deflection are denoted as  $H_t$  and  $Defl$ , respectively. The radius of curvature is equated using Equation 3.4.3 found immediately after the diagram.<sup>4</sup>

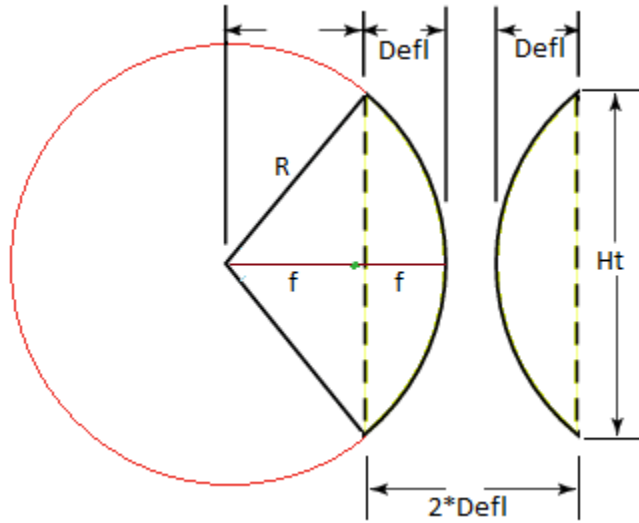


Figure 22. Imaginary circular relationships of concave glass

$$R = defl + \frac{0.125 Ht^2}{2 * defl}$$

Equation 9

Spherical mirrors have two main equations that describe their reflective properties: the mirror equation and the equation of magnification. There are two variations of the mirror equation which can be seen in Equation 10 and Equation 11.<sup>5</sup>

$$\frac{1}{d_s} + \frac{1}{d_i} = \frac{2}{R}$$

Equation 10

$$\frac{1}{d_s} + \frac{1}{d_i} = \frac{1}{f}$$

Equation 11

In these equations, R and f still represent the radius of curvature and focal points, respectively. The variable  $d_s$  is the distance between the sun and the earth from Section 3.3.2, which again is a time

dependent variable on the day of year. The variable  $d_i$  is the distance between the window and the reflected image of the sun, i.e. the light “spot” on the adjacent home. By manipulating the two equations above, the following relationship can be found in Equation 12.

$$f = \frac{R}{2}$$

Equation 12

### A.8.3 Concave Mirrors

Light can be observed reflecting off of a concave mirror in one of six ways, which can be seen below in Figure 23.

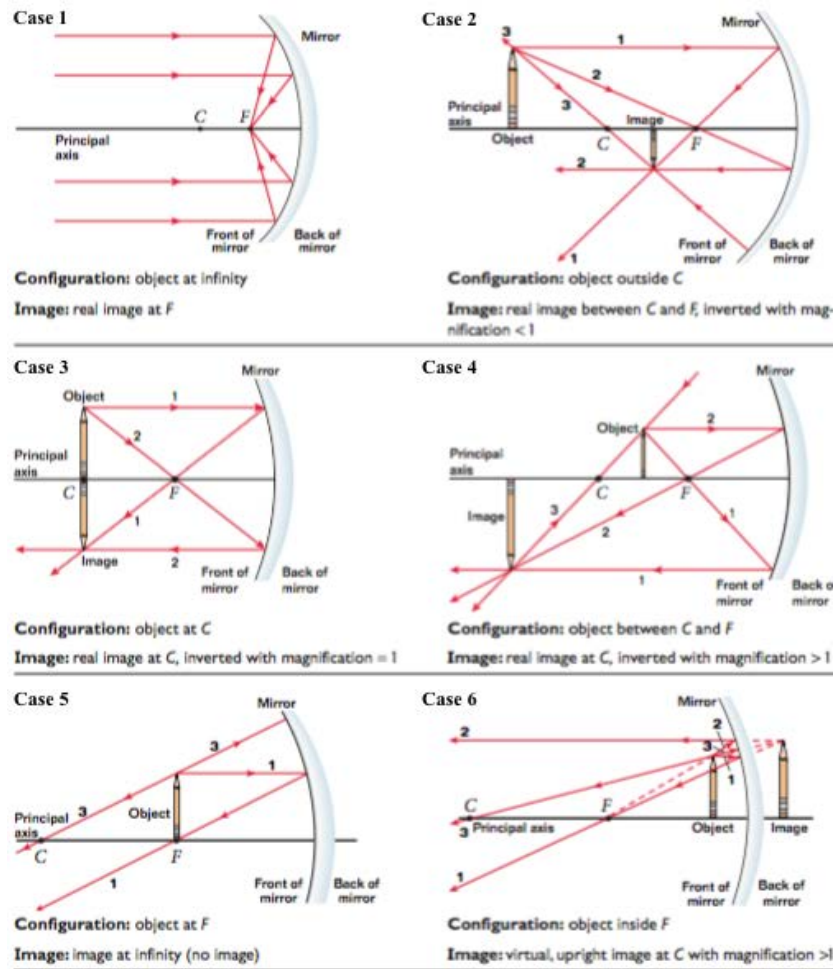


Figure 23. Six Observed Behaviors of Light Reflected Off of a Concave Mirror<sup>5</sup>



For objects at an infinite distance from the mirror, the light hits the mirror in parallel rays and the reflected rays all pass through a focal point found on the mirror's principal axis. The focal point,  $f$ , is at a distance equal to one half of the radius of curvature,  $R$ , of the curved mirrored surface. This case is illustrated in Case 1 from the figure above. The rays of light from the sun behave according to those in Case 1. The solar energy reflected by the mirror is concentrated at any point located near the focal point. Of course, the greatest concentration of the energy is found directly at the focal point. The concentration is a ratio of the size of the object being reflect--the sun in this case to the size of the reflected image. Here, the reflected image is the light "spot" which appears on the neighbor's siding, which is the cause of siding deformation in this study.

#### A.8.4 References

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<sup>1</sup> Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

<sup>2</sup> "Measuring Performance: ENERGY STAR Windows." *Efficient Windows Collaboration*.  
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<sup>3</sup> Hart, R., Goudey, H., Arasteh, D., & Curcija, D. C. (2012). Thermal performance impacts of center-of-glass deflections in installed insulating glazing units. *Energy and Buildings*, 54, 453-460.

<sup>4</sup> "Circular Segment." Wolfram Mathworld.

<http://mathworld.wolfram.com/CircularSegment.html>

<sup>5</sup> Serway, R. & J. Faugh. (2006) *Physics*. Holt, Rinehart, and Winston. 455-60.

## Appendix A.9 Distance from the Sun to the Earth

The distance from the sun to the earth can be found using the polar equation of a conic section in Equation 13. In this equation,  $d_s$  is representative of the radius of the ellipse,  $\epsilon$  is the eccentricity of the trajectory and  $a$  is the semi-major axis. The angle  $\nu$  is dependent on the location of the earth relative to its period.

$$d_s = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos \nu}$$

Equation 13

For the purpose of this tool, the angle  $\nu$  is approximated, using Equation 14. It is an approximation because the formula for angle  $\nu$  is assuming the Earth rotates at a constant angular velocity. However, any body rotates faster when it is closer to its center of gravitation, i.e. the sun in this instance.<sup>1</sup> It is important to note that the circumference of the unit circle is computed in radians. The term *Per* in this equation is the period required to make one full rotation around the ellipse. The values used for calculating the distance of the sun as they relate to conic sections can be found in Table 9.

$$\nu = \frac{2\pi * Day}{Per}$$

Equation 14

Table 9. Numerical Values for Constants Used in Conical Sections Computations<sup>1</sup>

Parameter	Constant Value
Semi-major axis, $a$	$1.496 * 10^{11}$ m
Eccentricity, $\epsilon$	0.0167
Period, <i>Per</i>	365.2
Radius of Sun, $R_s$	$696 * 10^6$ m

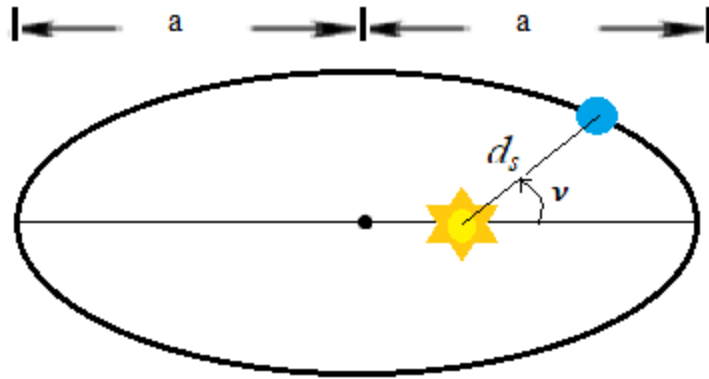


Figure 24. Earth's elliptical orbit around the sun

## A.9 References

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<sup>1</sup> Hale, F. (1994). Introduction to Space Flight. Prentice Hall. P. 11;343.

## Appendix A.10 Sun Angles Calculations Outline

### A.10.1 Hour Angle

The hour angle is the angular measurement of the earth's rotation around the polar axis. When the earth is at the highest point in the sky, the hour angle is zero. The equation for the hour angle is given by Equation 15.

$$H = 15 * (ts - 12)$$

Equation 15<sup>1</sup>

In this equation,  $ts$  is the solar time and  $H$  is the hour angle. Solar time is based on local clock time, the equation of time, and daylight savings time. The equation of time is the difference between the true solar time and mean solar time. Local clock time, or  $LCT$ , is the time based on the 24 hour clock. The equation of time,  $EOT$ , is given by Equation 16

$$EOT = 0.258 * \cos(x) - 7.416 * \sin(x) - 3.648 * \cos(2x) - 9.228 * \sin(2x)$$

Equation 16<sup>1</sup>

In Equation 17,  $x$  is an angle that is dependent on the day of the year.

$$x = 360 * (N - 1)/365.242$$

Equation 17<sup>1</sup>

Daylight savings time is another variable that can affect the solar time. If daylight savings time is in effect, a value of one, represented below in the equation as  $D$ , is added to the solar time. If daylight savings time is not currently in effect,  $D$  is equal to zero.

The last variable that affects the solar time is the longitude correction. This is defined by Equation 18. This corrects for the time variation within the time zone. The longitude of standard time zone meridian is a value that depends on the location of the house. Table 10 was constructed to provide these values for different United States time zones<sup>2</sup> and is found following Equation 18.

$$LC = (\textit{longitude} - \textit{longitude of standard time zone meridian})/15$$

Equation 18<sup>1</sup>

Table 10. North-American Zone Meridians

<b>Time Zone</b>	<b>Zone Meridian (West Longitude)</b>
Atlantic	60°
Eastern	75°
Central	90°
Mountain	105°
Pacific	120°
Alaska	135°
Hawaii-Aleutian	150°

Equation 19, on the following page, provides the equation for solar time,  $ts$ . This takes into account the local clock time, the equation of time, the longitude correction and daylight savings time. Once  $ts$  is calculated, that value can be used to find the hour angle. The hour angle is used with the declination angle and the location latitude and longitude to calculate the altitude, azimuth, zenith, and incidence angle.

$$ts = LCT + EOT/60 + LC + D$$

Equation 19<sup>1</sup>

### A.10.2 Declination Angle

The declination angle is shown in the figure below. It is an angle that is dependent only on day of the year. The declination angle varies throughout the year as can be seen below.

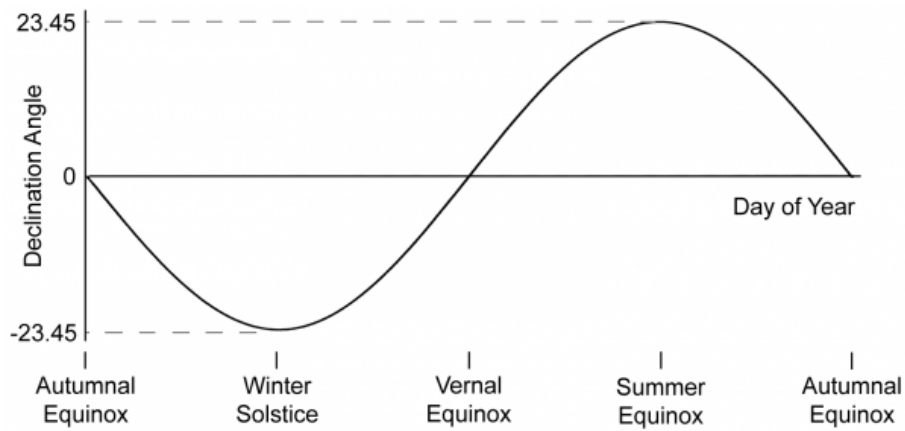


Figure 25. Declination Variation Over a Year<sup>3</sup>

This angle is consistently the same everywhere on the Earth on any specific day.

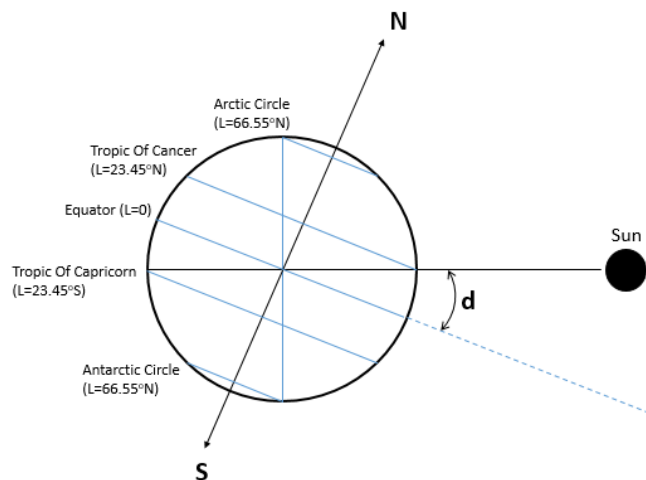


Figure 26. Declination Angle Diagram

The equation for the declination angle can be found below in Equation 20.

$$d = 23.45 * \sin\left(\frac{360}{365}(284 + N)\right)$$

Equation 20<sup>3</sup>

The declination angle can vary between +23.45 degrees and -23.45 degrees throughout the year.<sup>3</sup> The accuracy of these equations is based off of the accuracy of the calendar in predicting the motion of the Earth around the sun. These equations are not exact because the calendar year is not an exact representation of the Earth's path around the sun. With the declination angle and the hour angle, it is possible to calculate the altitude angle.

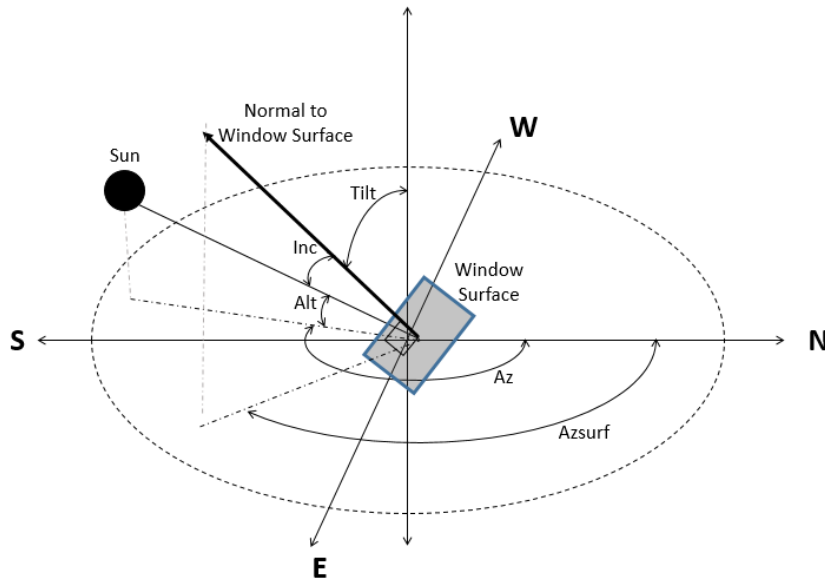


Figure 27. Altitude, Inclination, Azimuth, Azimuth of Surface, and Tilt Angle Diagram

The altitude angle, *alt*, as shown in the figure above, is the angle between the sun and a horizontal surface considered the observation point. This angle is calculated based on the declination angle, an observation latitude, and the hour angle using Equation 21.

$$alt = \sin(d) * \sin(Lat) + \cos(d) * \cos(H) * \cos(Lat)$$

Equation 21<sup>3</sup>

The zenith angle,  $Z$ , is the complementary angle of the altitude angle. To find the zenith angle, the equation is simply 90 degrees minus the altitude angle in degrees, as seen below in Equation 22.

After calculating the zenith angle, the azimuth angle can be calculated.

$$z = 90^\circ - alt$$

Equation 22<sup>3</sup>

### A.10.3 Azimuth Angle

The azimuth angle is also shown in Figure 36, above. It can have values ranging from +180° to -180°. It depends on the time of day, location and date. Its equation, Equation 23, is based off of the hour angle, declination angle, and zenith angle as shown below. This equation is for the angle of the horizontal measured clockwise from north and is specific to the location of the house based on the latitude that the house is located at.

$$Az = asin(-sin(H) * cos(d)/sin(Z))$$

Equation 23<sup>3</sup>

### A.10.4 Incidence Angle

The angle of incidence is the last angle needed to calculate for the sun intensity. The equation for the incident angle allows us to consider windows that are oriented at angles other than just horizontally. The equation for the incidence angle is as follows:

$Inc$

$$\begin{aligned} = & \text{acos}([sin(d) * sin(Lat) * cos(Tilt)] + [sin(d) * cos(Lat) * sin(Tilt) * cos(Azsurf)]) \\ & + [cos(d) * cos(Lat) * cos(Tilt) * cos(H)] - [cos(d) * sin(Lat) * sin(Tilt) * cos(Azsurf) \\ & * cos(H)] - [cos(d) * sin(Tilt) * sin(Azsurf) * sin(H)] \end{aligned}$$

Equation 24<sup>3</sup>



When the tilt angle is 90°, this equation simplifies to become Equation 25 below.

*Inc*

$$= \text{acos}([\sin(d) * \cos(Lat) * \sin(Tilt) * \cos(Azsurf)] + \cos(d) * \sin(Lat) * \cos(Azsurf) * \cos(H)) - [\cos(d) * \sin(Azsurf) * \sin(H)]$$

Equation 25<sup>3</sup>

For application in the tool, the first equation, Equation 24, for the incidence angle will be used so tilted windows can be considered. A sample calculation for a specific location is done with these equations in Appendix A.14 Sample Calculation Process. Once the Incidence angle has been calculated, the Solar Intensity can be evaluated.

## A.10 Reference

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<sup>1</sup> Stine, W. and M. Geyer. (1986) Power From The Sun: Chapter 3. John Wiley and Sons, Inc.

Web. Retrieved 12 December 2016, from

<http://www.powerfromthesun.net/Book/chapter03/chapter03.html>

<sup>2</sup> "Time-Zone and Meridian Offset Reference Information." (1998) Vixen North America: Web.

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[http://www.company7.com/library/vixen/VixenNA\\_TimeZoneOffset.pdf](http://www.company7.com/library/vixen/VixenNA_TimeZoneOffset.pdf)

<sup>3</sup> "Part 3: Calculating Solar Angles." The sun as source of energy. ITCA. Retrieved 12 December 2016, from

<http://www.itacanet.org/the-sun-as-a-source-of-energy/part-3-calculating-solar-angles/>

## **Appendix A.11 Solar Intensity Calculations**

In order to calculate the temperature of the sidings, the solar intensity reaching to the glass surface of the windows was needed in the calculation.

As mentioned in Section 2.3, the sun intensity that the earth receive varies with the time of the year and location on Earth. Therefore, to include the sun intensity in the tool, we cannot use the same value of sun intensity at different places on Earth. To include the sun intensity in the “Siding Temperature Calculation Tool”, we calculated the sun intensity at specific location of the earth.

The solar intensity at specific location depends on the latitude and altitude of the location and different sun angles that are mentioned in Section 2.2. From those inputs, solar intensity is calculated through the following steps.

1. Calculating solar constant
2. Calculating solar irradiance
3. Calculating solar intensity of beam radiation
4. Calculating solar intensity of diffuse radiation
5. Calculating solar reflected intensity

### **A.11.1 Calculating Solar Constant**

In this calculation, the sun considered to be producing the constant amount of energy. At the surface of the sun, the radiation is about  $6.33 \times 10^7$  watts per square meter.<sup>1</sup> However, the radiation spreads out and has to travel the distance between the earth and sun. Therefore, when the solar radiation reaches the earth, it has reduced a lot.<sup>1</sup>

Solar constant is the radiation intensity that falls on the imaginary surface at the edge of the atmosphere of the earth. In the calculation, the radiation from the sun is considered to travel in parallel rays and the imaginary surface is normal to those rays as shown in the Figure 28.



Figure 28. The Sun's rays incident on earth<sup>1</sup>

The solar constant,  $G_{sc}$ , is calculated by following Equation 26. In the calculation, temperature of the sun's surface, the radius of the sun and average distance from the sun to the earth are taken into account. The Stefan-Boltzmann Constant,  $\sigma$  equals  $5.67 \times 10^{-8} \text{ W/m}^2$ . The temperature of the sun's surface is known to be 5785 Kelvin, and is denoted using the variable  $T$  in the equation. The radius of the sun,  $R_s$  is  $6.96 \times 10^6$  meters, and the average distance between the sun and the earth is  $1.5 \times 10^{12}$  meters. From the calculation, the solar constant is found to be  $1367 \text{ W/m}^2$ . An exact method for calculating the distance between the sun and the earth can be found in Appendix A.9 Distance from the Sun to the Earth.

$$G_{sc} = \sigma T^4 \left( \frac{4\pi r_s^2}{4\pi d_s} \right) = 1367 \text{ W/m}^2$$

Equation 26

### A.11.2 Calculating Solar Irradiance

As mentioned in the Section 2.1, the amount of solar radiation the earth receive varies throughout the year. It happens because of the change of declination angle caused by the tilt of

earth by  $23.25^\circ$  on its axis and the earth revolving around the sun on its orbit., found in Section 3.2, shows the declination angle and the line joining the center of the sun (the line parallel to the rays from the sun).

The declination depends on the time of the year on earth. Therefore, the date of the year is considered to calculate the solar irradiance. Solar irradiance is the intensity of solar radiation that falls on the imaginary plane that is normal to the sun rays at the edge of the atmosphere as shown in Figure 11, but at the specific day of the year.

The solar irradiance is calculated by Equation 27 below.<sup>2</sup> Here,  $G_{sc}$  represents the solar constant and  $N$  is the day number—where  $N$  is equal to one when evaluating for January 1<sup>st</sup>. A sample calculation can be found in Appendix A.14 Sample Calculation Process.

$$G_0 = G_{sc} \left( 1 + 0.033 \cos \left( \frac{360 * N}{365} \right) \right)$$

Equation 27

Considering the sun's rays come parallel to the earth, all the paths of the earth do not receive the solar radiation equally. As shown in the Figure 29, some rays reach the earth perpendicularly and some rays reaches the earth tangentially. The tangential rays produce an angle called zenith angle. The calculation of zenith angle is mentioned in Section 3.2.

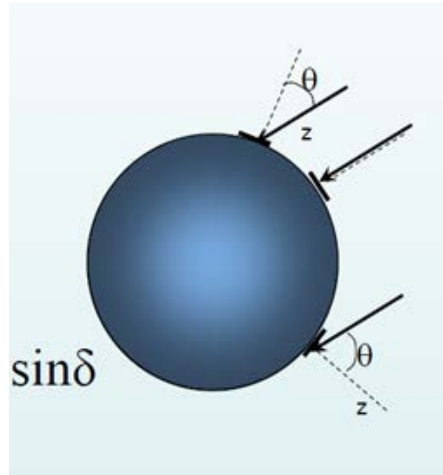


Figure 29. Radiation normal and tangent on the outer surface of the atmosphere<sup>2</sup>

The radiation intensity at the specific location,  $G_{0h}$ , is calculated by the following equation.<sup>2</sup> In this equation,  $G_0$  represents the solar irradiance and  $Z$  represents the zenith angle. The sample calculation using inputs for a specific house configuration in Worcester, MA can be found in Appendix A.14 Sample Calculation Process.

$$G_{0h} = G_0 \cos(Z)$$

Equation 28

### A.11.3 Calculating Solar Intensity of Beam Radiation

As explained in the Section 2.3, the solar radiation divides into beam radiation and diffused radiation as it enters the atmosphere. The intensity of beam radiation depends on the air mass, which relates to altitude of the location, and different sun angles at the specific time of the day and year. To calculate the solar intensity of beam radiation, the atmospheric transmittance of the beam radiation is needed. It is calculated by the following equations:

$$\tau_B = a_0 + a_1 * e^{-\gamma/\cos Z}$$

Equation 29<sup>2</sup>

In Equation 29,  $\tau_B$  represents the atmospheric transmittance of beam radiation. The variables  $a_0$ ,  $a_1$ ,  $\gamma$  are all constants that depend on the climate type and altitude of the location. They are calculated by the following equations.

$$a_0 = r_0(0.4234 - 0.00821(6 - A_{tude})^2)$$

Equation 30<sup>2</sup>

$$a_1 = r_1(0.5055 - 0.00595(6.5 - A_{tude})^2)$$

Equation 31<sup>2</sup>

$$\gamma = r_\gamma(0.2711 - 0.01825(2.5 - A_{tude})^2)$$

Equation 32<sup>2</sup>

In the previously stated three equations,  $A_{tude}$  is the observer's altitude in kilometers, and the values for the variables  $r_0$ ,  $r_1$  and  $r_\gamma$  can be found below in Table 11.

Climate Type	$r_0$	$r_1$	$r_\gamma$
Tropical	0.95	0.98	1.02
Midlatitude Summer	0.97	0.99	1.02
Subarctic Summer	0.99	0.99	1.01
Midlatitude Winter	1.03	1.01	1.00

Table 11.  $r_0$ ,  $r_1$  and  $r_\gamma$  at Different Climate Types<sup>2</sup>

After calculating the atmospheric transmittance of beam radiation, the solar intensity of beam radiation on the horizontal surface on earth can be calculated by the following equation.<sup>2</sup> Equation 33 relates the product of the atmospheric transmittance for beam radiation,  $\tau_B$ , and the solar irradiance at specific location,  $G_{0h}$  to the solar intensity of beam radiation on the horizontal surface on the earth.

$$G_B = G_{0h} * \tau_B$$

Equation 33

In addition, the face of the window is not horizontal on the surface of the earth. It is tilted 90° from the surface of the Earth. Therefore, to consider the solar beam radiation on the window we cannot use the above value of the beam radiation intensity.

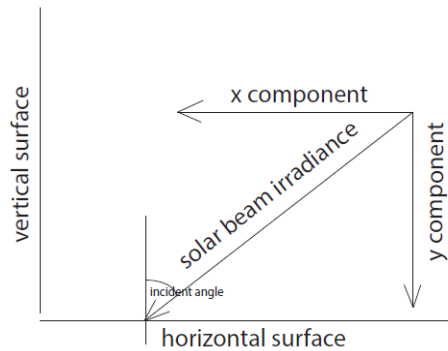


Figure 30. Reference Angles to the Origin of Position of the Surface of the Earth

As the  $G_B$  is measured on horizontal surface, it is the y component of the directional beam irradiance. To calculate the beam radiation at the vertical surface, we need to calculate the x component the beam irradiance as shown above in Figure 30. Reference Angles to the Origin of Position of the Surface of the Earth. Using the law of sine, solar beam irradiance on the vertical surface can be calculated by Equation 34. In this equation, solar intensity of beam radiation on the vertical surface is represented by  $G_{B,90}$ ,  $G_B$  is the solar intensity of beam radiation on the horizontal surface, and  $Inc$  is the inclination angle.

$$G_{B,90} = G_B * \frac{\sin(Inc)}{\sin(90 - Inc)}$$

Equation 34

#### A.11.4 Calculating Solar Intensity of Diffused Radiation

The account for all the sources of irradiation, we need to consider the solar radiation that reaches the surface of the Earth as diffuse radiation. To calculate the intensity of diffused radiation, the atmospheric transmittance for diffuse radiation is needed. It is calculated below by Equation 35, where  $\tau_D$  is the atmospheric transmittance for diffuse radiation and  $\tau_B$  is the atmospheric transmittance for beam radiation.

$$\tau_D = 0.271 - 0.294 * \tau_B$$

Equation 35

After calculating the atmospheric transmittance for diffuse radiation, the intensity of diffuse radiation on the horizontal surface can be calculated by the following equation.<sup>2</sup> In this equation,  $G_D$  is the intensity of diffused radiation on horizontal surface,  $G_{oh}$  is the solar irradiance at the specific location and  $\tau_D$  is the atmospheric transmittance for diffuse radiation

$$G_D = G_{oh} * \tau_D$$

Equation 36

In this case, the window is tilted  $90^\circ$  to the surface of the earth. Therefore, the same as the beam radiation, the intensity at the tilted surface is also needed to calculate for the diffused radiation. The calculation is performed using Equation 37. Here, the intensity of diffused radiation on tilted surface is given by  $G_{D, Tilted}$ , the variable Tilt is the angle tilted of the surface, and  $G_D$  is the intensity of diffused radiation on horizontal surface. A full sample calculation can be found in Appendix A.10.

$$G_{D, Tilted} = G_D * \frac{1 + \cos(Tilt)}{2}$$

Equation 37



### A.11.5 Calculating the Solar Intensity of Reflected Radiation

The reflected radiation is explained in section 2.3. The intensity of reflected radiation takes into account of the reflectivity of the surrounding surface. The calculation goes according Equation 38.<sup>3</sup> Here,  $G_{R,90}$  is the intensity of reflected radiation on the tilted surface at 90 degrees. Additionally,  $G$  is the intensity of solar radiation reaching to earth ( $G_B + G_D$ ) and  $\chi$  is the reflectivity of surrounding.

$$G_{R,90} = G\chi \frac{1 - \cos(\text{Tilt})}{2}$$

Equation 38

Table 12. Reflectivity,  $\chi$ , of Different Surroundings<sup>4</sup>

dry bare ground	0.2
dry grassland	0.3
desert sand	0.4
snow	0.5-0.8
pale soil	0.3
dark soil	0.1
water	0.1
vegetation	0.2

### A.11.6 Uncertainties

All the above calculations are made by the assumption of the weather of the day is nice and clear. However, solar intensity can vary by many different considerations such as cloudiness of the day, fog and obstacles such as shadows of the buildings or tree. Therefore, there are uncertainties for the above solar intensity calculations. Although they are calculated as accurately as possible on paper, the actual intensity of solar radiation can be a little different from the calculated intensity.

### A.11 References

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<sup>1</sup> “Part 2: Solar Energy Reaching the Earth Surface” | The sun as source of energy | ITCA

<http://www.itacanet.org/the-sun-as-a-source-of-energy/part-2-solar-energy-reaching-the-earths-surface/>

<sup>2</sup> Kharseh, M. “Solar Radiation Calculation” | Quinnipiac University

<https://www.researchgate.net/file.PostFileLoader.html?id=553e4871d685ccd10e8b4618&assetKey=AS%3A273765705945088%401442282238044>

<sup>3</sup> Perino M. “Available Solar Radiation” | IDES-edu

<https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=5&cad=rja&uact=8&sqi=2&ved=0ahUKewibINbkqvHQAhVI5SYKHZRqD68QFghEMAQ&url=http%3A%2F%2Fwww.ides-edu.eu%2Fwp-content%2Fuploads%2F2013%2F04%2F4-Available-Solar-Radiation.pdf&usg=AFQjCNEog3HI08Zz2rA7VQhIsWTr6LV6Lw&sig2=RVFSJAqXfoHlnuBJE0JgOw&bvm=bv.141320020,d.cG>

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<sup>4</sup> “Part 4: Irradiation Calculation” | The sun as source of energy | ITCA [“Part 4: Irradiation Calculation” | The sun as source of energy | ITCA](http://www.itacanet.org/the-sun-as-a-source-of-energy/part-4-irradiation-calculations/) <http://www.itacanet.org/the-sun-as-a-source-of-energy/part-4-irradiation-calculations/>

## Appendix A.12 Focal Point Calculations

Once the intensity of the radiation at the window's surface is known, how the window reflects the energy and how that affects the intensity of the reflection is determined. The distance between the sun and the earth is an important variable in determining the ultimate concentration factor. As previously mentioned, the equations used to determine this distance can be found in Appendix A.9 Distance from the Sun to the Earth. This distance, which is denoted  $d_s$ , is used to find the distance between the center of glass deflection and the focal point. However, it is important to note that the distance between the sun and the earth is a function of time. It is therefore important for the user to specify the day of year which is the same value used in the sun angle calculations. The equations used to find distance  $d_s$  are Equation 12 and Equation 13, which are again found in Appendix A.9 Distance from the Sun to the Earth.

The next step in the focal point and concentration factor calculations is to analyze the window's shape due to deformation caused by pressure differences. As previously mentioned in this report, siding deformation results from solar energy concentrated from concave shaped Low-E windows. The outer pane of an IGU experiencing pressure differences becomes a circular segment with properties that can be derived with geometric and trigonometric relations. An in-depth derivation of the window's geometric and trigonometric relations can be found in Appendix A.8 Window Deflection: Windows Acting as Spherical Mirrors. The focal point distance is the most important value needed in this stage of the tool's calculations. The focal point is found by Equation 39 on the following page where  $R$  is the radius of the window's curvature.

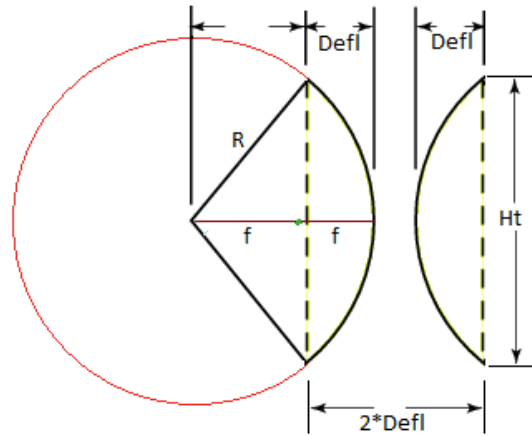


Figure 31. Imaginary circular relationships of concave glass

$$f = \frac{R}{2}$$

Equation 39

This distance,  $f$ , is the distance between the center of glass deflection and the focal point found on the mirror's principal axis. This concept is introduced in Appendix A.8 Window Deflection: Windows Acting as Spherical Mirrors of this report. This is the distance from the window glass at which the greatest concentration of intensity occurs. Light interacts with concave mirrors in six different ways, which can be seen in Figure 23, found in Appendix A.8 Window Deflection: Windows Acting as Spherical Mirrors. For the purpose of this study, sunlight interacts with a Low-E window acting as a concave mirror as an object at an infinite distance from the mirror.

Conceptually, all rays reflecting are perpendicularly traveling to the concave pane of glass, where they reflect towards the focal point. The size of the reflected light,  $r_i$  can then be calculated using the equation for magnification, given by Equation 40.

$$M = \frac{r_i}{r_s} = -\frac{d_i}{d_s}$$

Equation 40

The negative sign implies that the image is inverted, however, this is irrelevant because the sun is completely symmetrical. The reflected image of the sun will appear to be a “spot” wherever it is focused. This spot is the reflected solar intensity with a radius of size  $r_i$ . The intensity is concentrated in this spot on whatever surface it is hitting. The concentration factor is the ratio of the area of the window, off which the intensity is reflecting to the area of the spot. Up until this point, the reflected image has been considered as a one-dimensional quantity. From this point forward the reflected image shall be considered as a two-dimensional image, and shall be introduced into the set of equations in the concentration factor—a ratio of two-dimensional quantities, i.e. the area of the window to the area of the reflected image.

To find the area of the reflected image, two like triangles are considered; one for an example of if the neighboring house is before the focal point, and one for if the neighboring house is after the focal point. Using these like triangles, the area of the reflected image is proportionately scaled based on the ratio of the window to the area of the spot at the focal point. The Excel spreadsheet tool then uses an if statement. It runs two simultaneous calculations, and it uses the proper like triangle for the neighboring house distance being either before or after the focal point. The concentration factor is then determined by Equation 1, where  $A_i$  is the area of the reflected image, or spot, found from the similar triangles.

$$CF = \frac{Ht * W}{A_i} = \frac{Ht * W}{r_i^2 \pi}$$

Equation 41

Last, the focal point section of the tool calculates the amount of irradiance reflected to this spot. This value is dependent on the reflectance value that is input into the program from the reflectance range option. A detailed explanation of this option of reflectance ranges can be found

in Appendix A.8 Window Deflection: Windows Acting as Spherical Mirrors. The reflected irradiance is therefore found by the following equation

$$\text{Reflected Irradiance} = \text{Refl} * \text{Direct Irradiance} * \text{CF}$$

Equation 42

## Appendix A.13 Siding Temperature Calculations

This section follows the process of using the energy balance equation to derive the equations used for both the steady state and transient heat transfer analysis through a wall component. The following diagram seen in Figure 32 illustrates the wall component determined to be the most common wall section for a house utilizing vinyl siding. For transient heat transfer analysis, boundary conditions at a steady state were first established. Then, a transient heat transfer analysis was carried out using the implicit Finite Difference Method. The equations found in this section were derived using Cengel's *Heat Transfer: A practical approach* as a reference.<sup>1</sup> The FDM analysis applied a 20 node representation to calculate the temperature at each node at each time step. This ultimately allowed us to determine the temperature of the siding at two minute intervals over the course of a day. Following the diagram, Equation 43 is the energy balance equation each derivation uses to start.

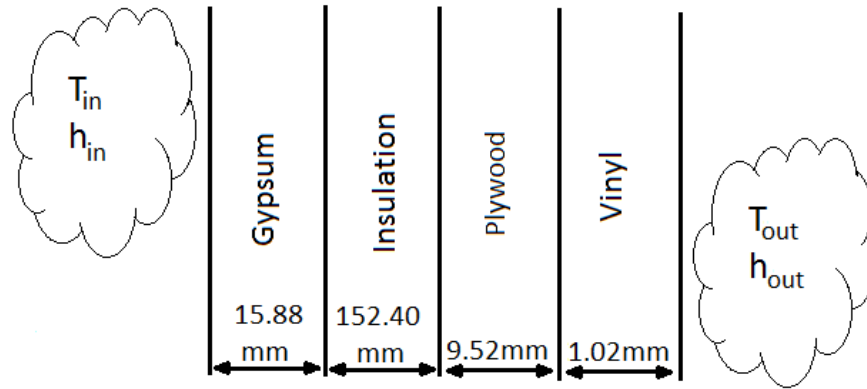


Figure 32. Steady State Heat Transfer Diagram for Common Wall Section

$$\sum_{All\ sides} \dot{Q} + \dot{E}_{gen,element} = \rho \forall_{element} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t} = \frac{\Delta E_{element}}{\Delta t}$$

Equation 43

### A.13.1 Steady State Heat Transfer Analysis

The initial conditions of our transient heat transfer analysis are established for the wall component when the sun first rises. At this point, the temperature of the ambient air is stable and there is no heat flux into the system. Therefore, the initial conditions are determined using a steady state analysis. For the steady state analysis, the right side of the energy balance equation is set to be equal to zero because the temperature is not changing with respect to time. For each of the nodes at the time equal to zero, the energy balance will start with the following equation, Equation 44.

$$\sum_{All\ sides} \dot{Q} + \dot{E}_{gen,element} = 0$$

Equation 44



### ***A.13.1.1 Interior Nodes***

For the purpose of this subsection, regular nodes shall be defined as interior nodes that are subject to only one material's mechanical properties subject to only conductive heat transfer. Starting with Equation 44, the equation for these regular nodes can be written as Equation 45.

$$kA \frac{T_{m-1} - T_m}{\Delta x} + kA \frac{T_{m+1} - T_m}{\Delta x} + \dot{e}_m A \Delta x = 0$$

Equation 45

Additionally, the wall is not generating any energy so the third component of this equation can be set to zero. Both sides of the equation can then be multiplied by  $\Delta x$ . The last step is to divide both sides of the equation by  $kA$ , thus yielding the final equation used for these interior nodes which can be seen in Equation 46.

$$T_{m+1} - 2T_m + T_{m-1} = 0$$

Equation 46

### ***A.13.1.2 Boundary Nodes Subject to Natural Convection***

For the purpose of this subsection, boundary nodes shall be defined as Node 0 and Node 20. Each of these nodes is subject to a convective heat transfer coefficient and an air temperature in its respective environment—inside and outside, respectively. Starting with Equation 44, the equation for these regular nodes can be written as Equation 47.

$$hA(T_\infty - T_m) + kA \frac{T_{m+1} - T_m}{\Delta x} + \dot{e}_m \left( \frac{A \Delta x}{2} \right) = 0$$

Equation 47

Again, the wall is not generating any energy so the third component of this equation can be set to zero. The area, A is the same for all each component so A can be canceled from the equation. This simplifies the energy balance equation to Equation 48.

$$h(T_\infty - T_m) + k \frac{T_{m+1} - T_m}{\Delta x} = 0$$

Equation 48

Finally, each temperature variable is separated. The ambient air temperature is a known constant, so it is moved to the right side of the equation. The entire equation is multiplied by negative one, resulting with Equation 49 being the final equation at the boundary nodes subject to natural convection.

$$\left(h + \frac{k}{\Delta x}\right) T_m - \frac{k}{\Delta x} T_{m+1} = hT_\infty$$

Equation 49

### A.13.1.3 Interior Nodes at a Material Interface Boundary

For the purpose of this subsection, interface boundary nodes shall be defined as Nodes 5, 10 and 15. Each of these nodes is subject to conductive heat transfer of two materials. Starting with Equation 44, the equation for these interior interface nodes can be written as Equation 50.

$$k_A A \frac{T_{m-1} - T_m}{\Delta x} + k_B A \frac{T_{m+1} - T_m}{\Delta x} + \dot{e}_{A,m} \left(\frac{A\Delta x}{2}\right) + \dot{e}_{B,m} \left(\frac{A\Delta x}{2}\right) = 0$$

Equation 50

Again, the wall is not generating any energy so both the third and fourth components of this equation can be set to zero. The area, A is the same for all each component so A can be canceled from the equation. This simplifies the energy balance equation to Equation 51.

$$k_A \frac{T_{m-1} - T_m}{\Delta x} + k_B \frac{T_{m+1} - T_m}{\Delta x} = 0$$

Equation 51

Finally, each temperature variable is separated. The resulting equation is given as Equation 52.

$$\frac{k_A}{\Delta x} T_{m-1} - \left(\frac{k_A}{\Delta x} + \frac{k_B}{\Delta x}\right) T_m + \frac{k_B}{\Delta x} T_{m+1} = 0$$

Equation 52

### A.13.2 Transient Heat Transfer Analysis

Once we had established steady state conditions for the wall section, a set of linear equations was written for the transient heat analysis. For this set of calculations, a heat flux was introduced. This heat flux accounted for all solar irradiance: diffused, reflected from the surroundings, and reflected from the Low-E window. In transient analysis, the temperature not only changes with position, it also changes with time. Therefore, to calculate the siding temperature at a given point in the day, the temperature at each position point needs to be calculated at small time steps to determine the temperature after the solar irradiance has been introduced to the wall section after the sun rises. A diagram of the transient heat transfer analysis can be seen on the following page in Figure 33.

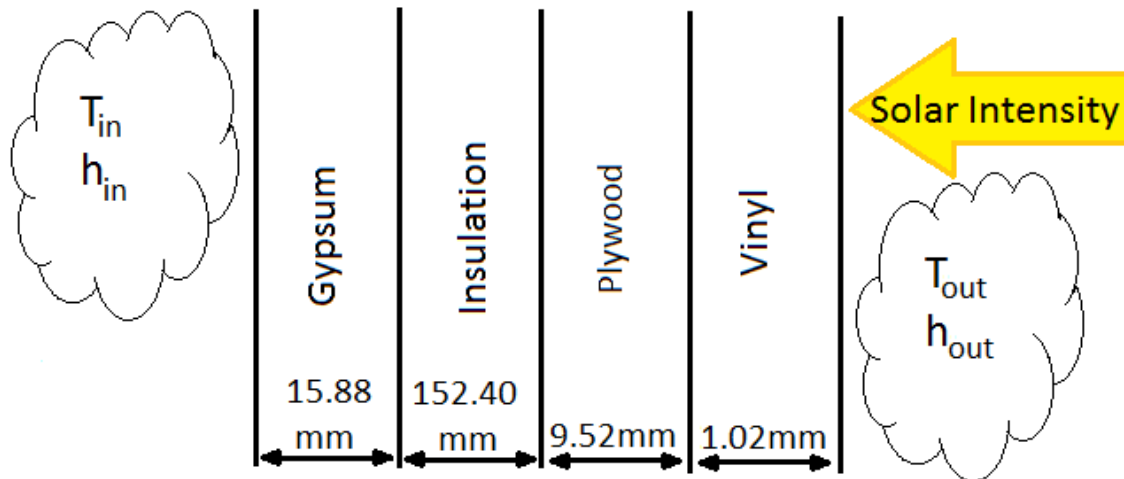


Figure 33. Transient Heat Transfer Diagram

The transient analysis uses the steady state conditions found in A.13.1 Steady State Heat Transfer Analysis in a new energy balance at each node in the wall section. Starting with Equation 43, an energy balance of each node was derived. It was found that there were four variations of the energy balance that could be applied to the 20 node wall section. To simplify the equations, the

mesh Fourier number could be used to replace a number of variables in the equations that were only dependent on one material, therefore all of those that were not at an interface boundary node, such as those at Nodes 5, 10, and 15. The mesh Fourier number is given by Equation 53, where  $\alpha$  is the thermal diffusivity of a material which is equal to the thermal conductivity of a material,  $k$ , divided by the product of the material's specific heat,  $c_p$ , and its density,  $\rho$ .

$$\tau = \frac{\alpha \Delta t}{\Delta x^2}$$

Equation 53

Node 0 was again a boundary condition, which was subjected to the natural convection inside of the house, and was represented by **Error! Reference source not found.** Nodes 5, 10, and 15 were represented by an interface boundary equation, represented by **Error! Reference source not found.** Interior nodes were represented by **Error! Reference source not found.** However, Node 20 was introduced to the solar irradiance during the transient analysis, thus introducing a heat flux component. **Error! Reference source not found.** was used to find the energy balance at Node 20. The superscripts found in **Error! Reference source not found.** through **Error! Reference source not found.** represent time steps. Therefore solving for  $T^{i+1}$  allowed us to explicitly find the time at the preceding time using the previously calculated temperature.

#### ***A.13.2.1 Regular Nodes***

For the regular nodes, which are defined as the same regular interior nodes used for the steady state analysis, we started with Equation 43. These nodes are again only subjected to heat transfer through conduction, only this time the right side of the equation is not equal to zero. After the performing the energy balance equation, the equation becomes Equation 54.

$$kA \frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x} + kA \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x} + e_m A \Delta x = \rho A \Delta x c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 54

The area, A is the same for all each component so A can be canceled from the equation. The wall is still not generating any electricity so the third component can also be set to zero. This simplifies the energy balance equation to Equation 55, found on the following page.

$$k \frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x} + k \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x} = \rho \Delta x c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 55

The thermal conductivity, k, is then divided by both sides of the equation yielding Equation 56.

$$\frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x} + \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x} = \frac{\rho \Delta x c_p}{k} \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 56

The product of density specific heat divided by thermal conductivity can be replaced such that using Equation 57 results in Equation 58.

$$\frac{1}{\alpha} = \frac{\rho c_p}{k}$$

Equation 57

$$\frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x} + \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x} = \frac{\Delta x}{\alpha} \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 58

The node size,  $\Delta x$ , is the multiplied by both sides makes the equation become that which is seen in Equation 59.

$$T_{m-1}^{i+1} - T_m^{i+1} + T_{m+1}^{i+1} - T_m^{i+1} = \frac{\Delta x^2}{\alpha} \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 59

Next, using the mesh Fourier number in Equation 53, our equation becomes Equation 60.

$$T_{m-1}^{i+1} - T_m^{i+1} + T_{m+1}^{i+1} - T_m^{i+1} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

Equation 60

Multiplying both sides of the equation by the mesh Fourier number, the equation becomes Equation 61.

$$\tau(T_{m-1}^{i+1} - T_m^{i+1} + T_{m+1}^{i+1} - T_m^{i+1}) = T_m^{i+1} - T_m^i$$

Equation 61

Last, the mesh Fourier number is distributed to each temperature variable, and the temperature at the “i+1” time step is subtracted from both sides of the equation. Finally, after combining like terms, the final equation for regular interior nodes becomes Equation 62.

$$\tau T_{m-1}^{i+1} - (2\tau - 1)T_m^{i+1} + \tau T_{m+1}^{i+1} = -T_m^i$$

Equation 62

### ***A.13.2.2 Exterior Boundary Condition Subject to Interior Natural Convection***

During the transient analysis, the boundary node subject to the inside air no longer uses the same equation as the boundary node subject to the outside air because the outside node is now subject to a solar heat flux. An energy balance at Node 0 therefore yields the same equation as the steady state equation only, the right side of the equation accounts for the temperature change over the time step. This energy balance yields Equation 63.

$$h_{\infty,in}A(T_{\infty,in} - T_0^{i+1}) + kA \frac{T_1^{i+1} - T_0^{i+1}}{\Delta x} + \frac{\dot{e}_0 A \Delta x}{2} = \rho A \frac{\Delta x}{2} c_p \frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Equation 63

Similar to all of the previous derivations, the area, A, can again be canceled. There is still no energy generation in the wall, so the third component of the equation can also be canceled. The results of those manipulations can be seen in Equation 64 on the following page.

$$h_{\infty,in}(T_{\infty,in} - T_0^{i+1}) + k \frac{T_1^{i+1} - T_0^{i+1}}{\Delta x} = \rho \frac{\Delta x}{2} c_p \frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Equation 64

Next, the thermal conductivity, k, is divided from both sides of the equation, which produces Equation 65.

$$\frac{h_{\infty,in}(T_{\infty,in} - T_0^{i+1})}{k} + \frac{T_1^{i+1} - T_0^{i+1}}{\Delta x} = \frac{\rho \Delta x c_p}{2k} \frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Equation 65

Thermal diffusivity,  $\alpha$ , can then replace the corresponding variables on the right side of the equation to produce Equation 66.

$$\frac{h_{\infty,in}(T_{\infty,in} - T_0^{i+1})}{k} + \frac{T_1^{i+1} - T_0^{i+1}}{\Delta x} = \frac{\Delta x}{2\alpha} \frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Equation 66

Next, the node size,  $\Delta x$ , and the number are the multiplied through the equation, yielding Equation 67.

$$\frac{2h_{\infty,in}\Delta x(T_{\infty,in} - T_0^{i+1})}{k} + 2(T_1^{i+1} - T_0^{i+1}) = \frac{\Delta x^2}{\alpha} \frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Equation 67

The mesh Fourier number can then be used to replace the corresponding variables on the right side of the equation. This results in Equation 68.

$$\frac{2h_{\infty,in}\Delta x(T_{\infty,in} - T_0^{i+1})}{k} + 2(T_1^{i+1} - T_0^{i+1}) = \frac{T_0^{i+1} - T_0^i}{\tau}$$

Equation 68

Finally, the mesh Fourier number can be multiplied through the equation to result in Equation 69.

$$\frac{2\tau h_{\infty,in}\Delta x(T_{\infty,in} - T_0^{i+1})}{k} + 2\tau(T_1^{i+1} - T_0^{i+1}) = T_0^{i+1} - T_0^i$$

Equation 69

The last manipulation is to distribute the coefficients to each of the temperature variables, combine like terms, and bring known constants to the right side of the equation. The final linear equation for the interior boundary node subject to natural convection is Equation 70.

$$\left(\frac{2\tau h_{\infty,in}\Delta x}{k} + 2\tau + 1\right) T_0^{i+1} - 2\tau T_1^{i+1} = \frac{2\tau h_{\infty,in}\Delta x}{k} T_{\infty,in} + T_0^i$$

Equation 70

### ***A.13.2.3 Exterior Boundary Condition Subject to Outside Natural Convection and Solar Radiation***

The transient heat analysis equation for Node 20 is similar to that of Node 0 except that it also incorporates the total solar irradiance calculated from the Focal Point calculation,  $\dot{Q}$ , multiplied by the absorbance value of the vinyl siding,  $\kappa$ . The energy balance for this node is given by Equation 71.

$$h_{\infty,out}A(T_{\infty,out} - T_{20}^{i+1}) + kA\frac{T_{19}^{i+1} - T_{20}^{i+1}}{\Delta x} + \frac{e_{20}A\Delta x}{2} + \dot{Q}_{total}\kappa A = \rho A\frac{\Delta x}{2}c_p\frac{T_{20}^{i+1} - T_{20}^i}{\Delta t}$$

Equation 71



Similar to all of the other equations, the total wall area,  $A$ , is the same for each component, so the  $A$  can be canceled from the equation. Again, the wall is not generating any energy so the third component can also be canceled from the equation. This produces Equation 72.

$$h_{\infty,out}(T_{\infty,out} - T_{20}^{i+1}) + k \frac{T_{19}^{i+1} - T_{20}^{i+1}}{\Delta x} + \dot{q}''_{total} \kappa = \rho \frac{\Delta x}{2} c_p \frac{T_{20}^{i+1} - T_{20}^i}{\Delta t}$$

Equation 72

Next, the equation was multiplied by node size,  $\Delta x$ , divided by thermal conductivity,  $k$ . The result is Equation 73.

$$\frac{h_{\infty,out} \Delta x}{k} (T_{\infty,out} - T_{20}^{i+1}) + (T_{19}^{i+1} - T_{20}^{i+1}) + \frac{\dot{q}'' \kappa \Delta x}{k} = \frac{\rho \Delta x^2 c_p}{2k} \frac{T_{20}^{i+1} - T_{20}^i}{\Delta t}$$

Equation 73

Thermal diffusivity is then used to replace three variables found on the right side of the equation to produce Equation 74.

$$\frac{h_{\infty,out} \Delta x}{k} (T_{\infty,out} - T_{20}^{i+1}) + (T_{19}^{i+1} - T_{20}^{i+1}) + \frac{\dot{q}''_{total} \kappa \Delta x}{k} = \frac{\Delta x^2}{2\alpha} \frac{T_{20}^{i+1} - T_{20}^i}{\Delta t}$$

Equation 74

Below, Equation 75 uses the mesh Fourier number in place of the corresponding variables from Equation 74.

$$\frac{h_{\infty,out} \Delta x}{k} (T_{\infty,out} - T_{20}^{i+1}) + (T_{19}^{i+1} - T_{20}^{i+1}) + \frac{\dot{q}''_{total} \kappa \Delta x}{k} = \frac{T_{20}^{i+1} - T_{20}^i}{2\tau}$$

Equation 75

Next, both sides of the equation are multiplied by  $2\tau$ . The coefficients are then distributed to the temperature variables. The temperature variable at the “i+1” time step is then subtracted from both sides leaving us with Equation 76.

$$\frac{2\tau h_{\infty,out}\Delta x}{k}T_{\infty,out} - \frac{2\tau h_{\infty,out}\Delta x}{k}T_{20}^{i+1} + 2\tau T_{19}^{i+1} - 2\tau T_{20}^{i+1} - T_{20}^{i+1} + \frac{2\tau \dot{Q}_{total}\kappa\Delta x}{k} = -T_{20}^i$$

Equation 76

All known variables with known temperatures are then moved to the right side of the equation. This is done by subtracting the  $T_{\infty,out}$  variable from both sides. There are also no unknown temperatures associated with the solar heat flux, so that component is also subtracted from both sides. The resulting equation can be seen in Equation 77.

$$-\frac{2\tau h_{\infty,in}\Delta x}{k}T_{20}^{i+1} + 2\tau T_{19}^{i+1} - 2\tau T_{20}^{i+1} - T_{20}^{i+1} = -T_{20}^i - \frac{2\tau h_{\infty,in}\Delta x}{k}T_{\infty,in} - \frac{2\tau \dot{Q}_{total}\kappa\Delta x}{k}$$

Equation 77

The whole equation is then multiplied by negative one. The last step is to pair like terms, and the final equation used for the transient analysis of Node 20 was derived to be Equation 78.

$$-2\tau T_{19}^{i+1} + \left(1 + 2\tau + \frac{2\tau h_{\infty,in}\Delta x}{k}\right)T_{20}^{i+1} = T_{20}^i + \frac{2\tau h_{\infty,in}\Delta x}{k}T_{\infty,in} + \frac{2\tau \dot{Q}_{total}\kappa\Delta x}{k}$$

Equation 78

#### A.13.2.4 Interior Interface Nodes

The fourth energy balance equation needed for the transient analysis of the wall component was required for the interior interface nodes, which occur at Node 5, 10, and 15. Starting with the energy balance equation, Equation 79 can be written for the conductive heat transfer these three nodes exhibit. Note that the area, A is equal for each component of this equation and has already been canceled to keep the equation as one line of text.

$$k_a \frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x_a} + k_b \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x_b} + e_a \Delta x_a + e_b \Delta x_b = (\rho_a \Delta x_a c_{p,a} + \rho_b \Delta x_b c_{p,b}) \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 79

The wall has no energy generation and there for the third and fourth components in this equation are equal to zero and are canceled. The result can be seen in Equation 80.

$$k_a \frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x_a} + k_b \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x_b} = (\rho_a \Delta x_a c_{p,a} + \rho_b \Delta x_b c_{p,b}) \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 80

In efforts to prevent losing any coefficients in this long equation, we defined the following parameter which is equivalent to six known constants. We defined N as  $N = (\rho_a \Delta x_a c_{p,a} + \rho_b \Delta x_b c_{p,b})$  and replace it in our equation to produce Equation 81.

$$k_a \frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta x_a} + k_b G_T \frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta x_b} = N \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Equation 81

Both sides of the equation are then multiplied by the time step,  $\Delta t$  divided by N. This manipulation yields Equation 82.

$$\frac{k_a \Delta t}{N \Delta x_a} (T_{m-1}^{i+1} - T_m^{i+1}) + \frac{k_b \Delta t}{N \Delta x_b} (T_{m+1}^{i+1} - T_m^{i+1}) = T_m^{i+1} - T_m^i$$

Equation 82

The coefficients are then distributed to the unknown temperature variables, and the unknown time variable at time step “i+1” is subtracted from both sides. The result is Equation 83, below.

$$\frac{k_a \Delta t}{N \Delta x_a} T_{m-1}^{i+1} - \frac{k_a \Delta t}{N \Delta x_a} T_m^{i+1} + \frac{k_b \Delta t}{N \Delta x_b} T_{m+1}^{i+1} - \frac{k_b \Delta t}{N \Delta x_b} T_m^{i+1} - T_m^{i+1} = -T_m^i$$

Equation 83

Finally, like terms are combine to leave us with a final transient analysis derivation of Equation 84..

$$\frac{k_a \Delta t}{N \Delta x_a} T_m^{i+1} - \left( \frac{k_a \Delta t}{N \Delta x_a} + \frac{k_b \Delta t}{N \Delta x_b} + 1 \right) T_m^{i+1} + \frac{k_b \Delta t}{N \Delta x_b} T_{m+1}^{i+1} = -T_m^i$$

Equation 84

The equations derived in this appendix were used in the MATLAB code, which can be found in Appendix A.16 Siding Temperature Calculations: MATLAB Code. The MATLAB modules referenced are the method used to calculate the temperature of the siding over the hours of daylight, which were used to perform a damage analysis.

### A.13 References

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<sup>1</sup> Cengel, Y. (1998) Heat Transfer: A practical approach.

## Appendix A.14 Sample Calculation Process

Table 13. Inputs for Sample Calculation:

Input	Name	Value	Unit
Absorptivity	Abs	0.5	
Altitude	Atude	0.143	kilometers
Azimuth Angle of Surface	Azsurf	175 3.05433	Degrees rad
Day	Day	2	
Day of Year (out of 365)	N	2	
Daylight Savings Time (1 for yes, 0 for no)	dst	0	
Deflection	Defl	2	meters
Emissivity	Emiss	0.92	
Focal Point Distance	f		meters
Heat Transfer Coefficient	h	5	Watts/[(meters <sup>2</sup> ) (Kelvins)]
Latitude	Lat	45.26 0.7899	Degrees Rad
Longitude	Lon	71.78 1.2528	Degrees Rad
Local Clock Time	Time	15:00	
Month (1-12)	Mon	1	
Neighboring House Information	Dnh	15	meters
Radius of Window's Curvature	R		meters

Reflectance	Refl	0.44	
Temperature of Environment Surroundings	Tamb	283	K
Tilt of Window	Tilt	90	degrees
Window Height	Ht	0.6096	meters
Window Width	W	0.9144	meters
Year	Year	2016	

### **A.14.1 Sun Angles**

#### ***A.14.1.1 Hour Angle***

In this appendix the whole calculation process will be evaluated for an example location found in Worcester, Massachusetts. The equations that follow are discussed thoroughly in the preceding Appendices, and therefore, they will not be discussed in great detail again. Rather, they will simply be evaluated using the input values found immediately above this paragraph in Table 13. The first value we need to determine is the angle, which is dependent of on the day of the year. For this example, we are considering January 2, which is the second day of the year. Specifically that means N equals 2 and Equation 17 from Appendix A.10.1 Hour Angle. Below, the angle x is calculated and determined to be 0.986 degrees.

$$x = 360 * \frac{N - 1}{365.242} = 360 * \frac{2 - 1}{365.242} = 0.986^\circ$$

Equation 17<sup>1</sup> (Being Evaluated)

Next, Equation 16, on the following page, is evaluated to find the equation of time, EOT. The equation of time is calculated to be -3.833 minutes. The reason this value is negative is because on this day of the year the sun seems comparatively slower than a clock at the given time.

$$\begin{aligned}
 EOT &= 0.258 * \cos(x) - 7.416 * \sin(x) - 3.648 * \cos(2x) - 9.228 * \sin(2x) \\
 &= 0.258 * \cos(0.986) - 7.416 * \sin(0.986) - 3.648 * \cos(2 * 0.986) - 9.228 * \sin(2 * 0.986) \\
 EOT &= -3.833 \text{ minutes}
 \end{aligned}$$

Equation 16<sup>1</sup> (Being Evaluated)

After the equation of time is calculated, the longitude correction, *LC*, is calculated using Equation 18 from Appendix A.10.1 Hour Angle. This corrects for the time variation within the time zone. The longitude of standard time zone meridian is a value that depends on the location of the house. Table 10 is copied from Appendix A.10 below. The Eastern value was used in the evaluation of Equation 18. Longitudinal correction is found following Table 10 to be -3.22/15 hours.

Table 10. North-American Zone Meridians<sup>2</sup>

<b>Time Zone</b>	<b>Zone Meridian (West Longitude)</b>
Atlantic	60°
Eastern	75°
Central	90°
Mountain	105°
Pacific	120°
Alaska	135°
Hawaii-Aleutian	150°

$$LC = (\textit{longitude} - \textit{longitude of standard time zone meridian})/15$$

$$LC = (71.78 - 75)/15$$

$$LC = -3.22/15 \text{ hours}$$

Equation 18<sup>1</sup> (Being Evaluated)

The equation for solar time  $ts$ , given by Equation 19 is then solved. In this equation, the variable  $LCT$  represents the local clock time which is an input in the Siding Temperature Calculation Tool. The variable  $D$  in this equation takes on one of two values: either one or zero.  $D$  is equal to 1 if daylight savings time is currently in effect, or it is equal to zero if it is not in effect.

$$ts = LCT + EOT/60 + LC + D$$

$$ts = 15.00 + (-3.833)/60 + (-3.22)/15 + 0$$

$$ts = 14.785 \text{ hours}$$

Equation 19<sup>1</sup> (Being EvaluateG\_Td)

Lastly, the hour angle,  $H$ , can then be evaluated using Equation 15. The hour angle at 3:00 PM on January 2 is determined to be  $47.2^\circ$ .

$$H = 15 * (ts - 12)$$

$$H = 15 * (14.785 - 12)$$

$$H = 47.2^\circ$$

Equation 15<sup>1</sup> (Being Evaluated)

#### ***A.14.1.2 Declination Angle***

As previously mentioned in Appendix A.10.2 Declination Angle, the declination angle is dependent only on the day of the year, and it has the same value for all locations on Earth on that particular day. It can be seen on the following page in Figure 34. The variation of the angle's value through the year can be seen in Figure 35 on the following page. When evaluated for the sample day, January 2, the declination angle is found to be  $-22.93^\circ$ . This calculation is carried through using Equation 20 on the succeeding page.



$$d = 23.45 * \sin(360/365(284 + N))$$

$$d = 23.45 * \sin(360/365(284 + 2))$$

$$d = -22.93^\circ$$

Equation 20<sup>3</sup> (Being Evaluated)

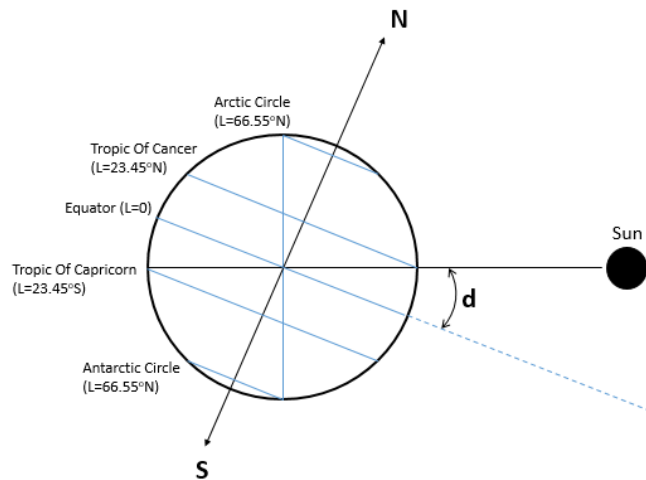


Figure 34. Declination Angle Diagram

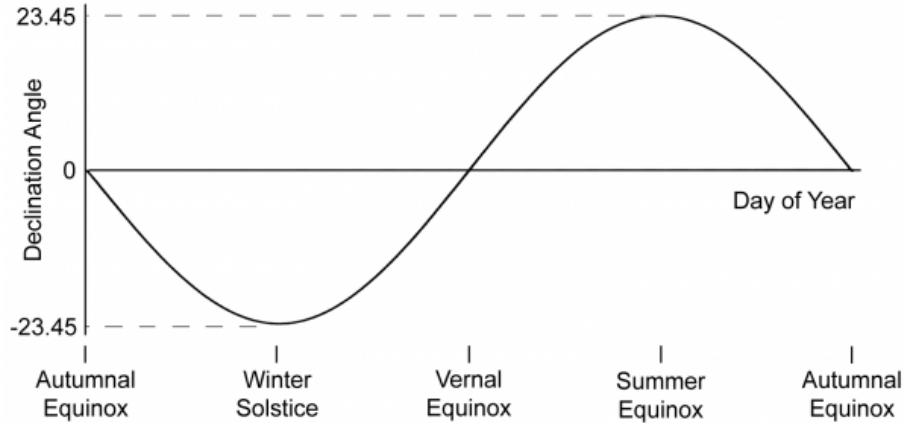


Figure 35. Declination Angle Variation During the Year<sup>3</sup>

### A.14.1.3 Altitude Angle

The angle between the sun and a horizontal surface, which is considered the observation point, is known as the altitude angle, *alt*. A graphical representation of the altitude angle can be seen on the following page in Figure 36. The equation used to solve for the altitude angle is found

in Equation 21. This equation requires the declination angle, hour angle, and the latitude in order for it to be solved.

$$\sin(\text{alt}) = \sin(d) * \sin(\text{Lat}) + \cos(d) * \cos(H) * \cos(\text{Lat})$$

$$\sin(\text{alt}) = \sin(-22.93^\circ) * \sin(45.26^\circ) + \cos(-22.93^\circ) * \cos(47.2^\circ) * \cos(45.26^\circ)$$

$$\sin(\text{alt}) = 0.164$$

$$\text{alt} = 9.42^\circ$$

Equation 21<sup>3</sup> (Being Evaluated)

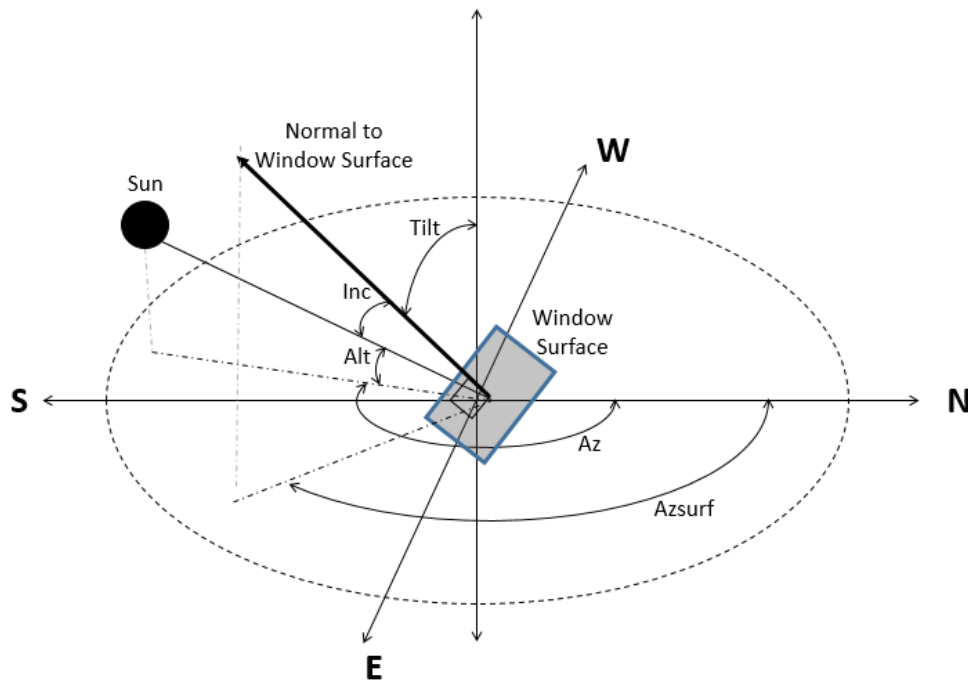


Figure 36. Altitude, Inclination, Azimuth, Azimuth of Surface, and Tilt Angle Diagram

#### A.14.1.4 Zenith Angle

The zenith angle,  $Z$ , is the complementary angle of the altitude angle. To find the zenith angle, the equation is simply  $90^\circ$  minus the altitude angle in degrees. Equation 22 on the following page shows calculation of the zenith angle for the sample calculation. The value was found to be  $80.6^\circ$

$$z = 90^\circ - alt$$

$$z = 90^\circ - 9.42^\circ$$

$$z = 80.6^\circ$$

Equation 22<sup>3</sup> (Being Evaluated)

#### ***A.14.1.5 Azimuth Angle***

The azimuth angle can also be seen in Figure 36, above. The azimuth angle, *Az*, depends on the time of day, location and date. The equation is based off of the hour angle, declination angle, and zenith angle as shown below in Equation 23. For the sample calculation the azimuth angle was found to be  $-43.2^\circ$ . This equation is for the angle of the horizontal measured clockwise from north and is specific to the location of the house based on the latitude that the house is located at.

$$\sin(Az) = -\sin(H) * \cos(d) / \sin(Z)$$

$$\sin(Az) = -\sin(47.2^\circ) * \cos(-22.93^\circ) / \sin(80.6^\circ)$$

$$\sin(Az) = -0.685$$

$$Az = -43.2^\circ$$

Equation 23<sup>3</sup> (Being Evaluated)

#### ***A.14.1.5 Incidence Angle***

Finally, the incidence angle, ***Inc***, is the last sun angle needed to find the solar intensity reflected off a Low-E window. The equation for the incident angle allows us to consider windows that are oriented at angles other than just horizontally. Equation 24, below, is the equation used to find the incidence angle, which is dependent on the tilt of the window, the declination angle, the latitude of the location, and hour angle. Due to the fact the window in the sample calculation is not

at an incline, the simplified equation for the angle of incidence can be used. This is Equation 24, below, which was also discussed in Appendix A.10 Sun Angles Calculations Outline.

$$\cos(Inc)$$

$$= \sin(-22.93^\circ) * \cos(45.26^\circ) * \sin(90^\circ) * \cos(175^\circ) + \cos(-22.93^\circ) * \sin(45.26^\circ) * \cos(175^\circ) * \cos(47.2^\circ) - \cos(-22.93^\circ) * \sin(175^\circ) * \sin(47.2^\circ)$$

$$\cos(Inc) = -0.228$$

$$Inc = 103.2^\circ$$

Equation 24<sup>3</sup> (Being Evaluated)

#### ***A.14.1.6 Sun Angle Outputs***

Hour Angle	<b><i>H</i></b>	47.2°
Declination angle	<b><i>d</i></b>	-22.9°
Altitude Angle	<b><i>Alt</i></b>	9.42°
Azimuth Angle	<b><i>Az</i></b>	-43.2°
Zenith Angle	<b><i>Z</i></b>	80.6°
Incidence angle	<b><i>Inc</i></b>	103.2°

#### **A.14.2 Solar Intensity**

##### ***A.14.2.1 The Solar Constant***

Equation 26 is used to calculate the solar constant,  $G_{sc}$ . Here, the radius of the sun,  $R_S$  is  $6.96 \times 10^6$  meters, and the average distance between the sun and the earth,  $D_s$ , is  $1.5 \times 10^{12}$  meters. The Stefan-Boltzmann Constant,  $\sigma$ , equals  $5.67 \times 10^{-8} \text{ W/m}^2$ . The temperature of the sun's surface,  $T_s$ , is known to be 5785 Kelvin, and is denoted using the variable. From the calculation, the solar

constant is found to be  $1367 \text{ W/m}^2$ . An exact method for calculating the distance between the sun and the earth can be found in Appendix A.9 Distance from the Sun to the Earth.

$$G_{sc} = \sigma T^4 \left( \frac{4\pi r_s^2}{4\pi d_s} \right) = 1,367 \text{ W/m}^2$$

Equation 26

#### A.14.2.2 Solar Irradiance

The solar irradiance,  $G_o$ , is calculated by Equation 27 below. Here,  $G_{sc}$  represents the solar constant and  $N$  is the day number—where  $N$  is still equal to two when evaluating for January 2nd. Figure 37 shows how  $G_o$  varies over the course of the calendar year.

$$G_o = G_{sc} \left( 1 + 0.033 \cos \left( \frac{360 * N}{365} \right) \right)$$

$$G_o = 1,367 \left( 1 + 0.033 \cos \left( \frac{360 * 2}{365} \right) \right) = 1,412 \text{ W/m}^2$$

Equation 27 (Being Evaluated)

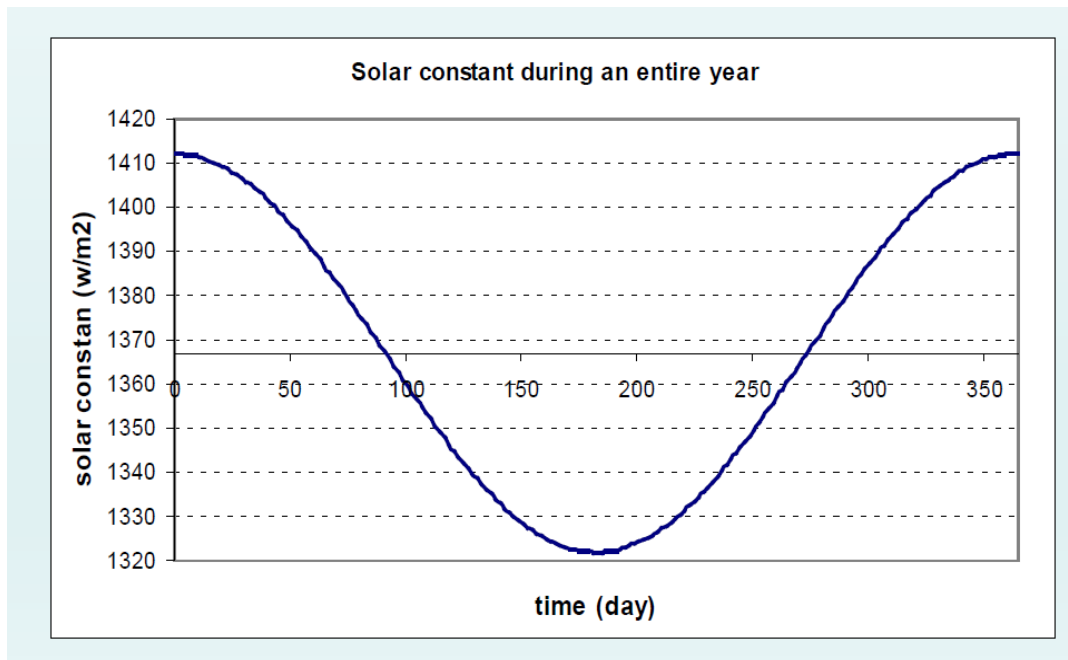


Figure 37. Solar Irradiance versus Day of Year

Extraterrestrial radiation  $G_{oh}$  is the radiation incident on the surface tangent on the outer surface of the atmosphere, which is illustrated below in Figure 38. Extraterrestrial radiation is calculated by Equation 28, which is found below Figure 38. In this equation,  $Z$  is the zenith angle found in Section A.14.1.4 Zenith Angle, which was calculated to be  $80.6^\circ$ .  $G_o$  is the value calculated using Equation 27, which was  $1,412 \text{ W/m}^2$ .

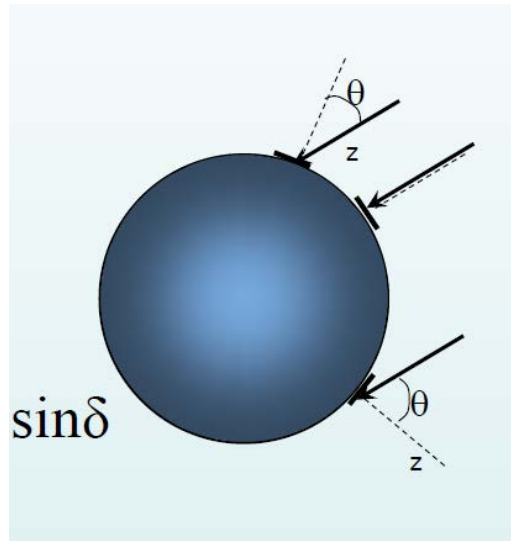


Figure 38. Radiation Normal and Tangent on the Outer Surface of the Atmosphere

$$G_{oh} = G_o \cos(Z)$$

$$G_{oh} = 1,412 * \cos(80.6^\circ) = 230.6 \text{ W/m}^2$$

Equation 28

#### A.14.2.3 Beam Radiation

Beam radiation was explained in Sections 2.3.2 Solar Radiation, and 3.3.1 Direct Irradiance, and A.11.3 Calculating Solar Intensity of Beam Radiation. Solar rays reaching the ground without changing direction account for beam radiation. Beam radiation received on the horizontal surface at ground surface is calculated by Equation 33 found on the following page. In

this equation,  $G_{oh}$  is the value found in the preceding section, which was calculated as 230.6 W/m<sup>2</sup>. The atmospheric transmittance of beam radiation,  $\tau_B$ , is a slightly harder variable to calculate. The calculation process was described with more detail in A.11.3 Calculating Solar Intensity of Beam Radiation. Equation 29 is used for solving for  $\tau_B$ , and this equation has three constants on which it is dependent:  $a_0$ ,  $a_1$ , and  $\gamma$ . These constants are dependent on the climate type and altitude of the location. They are calculated using Equation 30.

$$G_B = G_{oh} * \tau_B$$

Equation 33

$$\tau_B = a_0 + a_1 * e^{-\gamma/\cos Z}$$

Equation 29

$$a_0 = r_0(0.4234 - 0.00821(6 - A_{tude})^2)$$

Equation 302

$$a_1 = r_1(0.5055 - 0.00595(6.5 - A_{tude})^2)$$

Equation 312

$$\gamma = r_\gamma(0.2711 - 0.01825(2.5 - A_{tude})^2)$$

Equation 322

In the previously stated three equations,  $A_{tude}$  is the observer's altitude in kilometers, and the values for the variables  $r_0$ ,  $r_1$  and  $r_\gamma$  can be found in Table 11 on the following page.

Table 11.  $r_0$ ,  $r_1$  and  $r_\gamma$  at Different Climate Types2

Climate Type	$r_0$	$r_1$	$r_\gamma$
Tropical	0.95	0.98	1.02
Midlatitude Summer	0.97	0.99	1.02
Subarctic Summer	0.99	0.99	1.01
Midlatitude Winter	1.03	1.01	1.00

The altitude of Worcester, Massachusetts is 0.146 kilometers above sea level. On January 2<sup>nd</sup>, out of the four categories present, the area would be best classified as Midlatitude Winter, thus using the associated  $r_0$ ,  $r_1$  and  $r_\gamma$  values with that climate type for the calculations.

$$a_0 = r_0(0.4234 - 0.00821(6 - A_{tude})^2)$$

$$a_0 = 1.03 * (0.4234 - 0.00821(6 - 0.146)^2)$$

$$a_0 = 0.146$$

Equation 302

$$a_1 = r_1(0.5055 - 0.00595(6.5 - A_{tude})^2)$$

$$a_1 = 1.01(0.5055 - 0.00595(6.5 - 0.146)^2)$$

$$a_1 = 0.268$$

Equation 312

$$\gamma = r_\gamma(0.2711 - 0.01825(2.5 - A_{tude})^2)$$

$$\gamma = 1.00(0.2711 - 0.01825(2.5 - 0.146)^2)$$

$$\gamma = 0.170$$

Equation 322

Next, we are able to calculate the atmospheric transmittance,  $\tau_B$ .



$$\tau_B = a_0 + a_1 * e^{-\gamma/\cos(Z)}$$

$$\tau_B = 0.146 + 0.268 * e^{-0.170/\cos(80.6)}$$

$$\tau_B = 0.241$$

Equation 29

Finally, we are able to use Equation 33 to calculate the solar beam irradiance.

$$G_B = G_{0h} * \tau_B$$

$$G_B = 230.6 * 0.241$$

$$G_B = 55.5 \text{ W/m}^2$$

Equation 33

The solar beam irradiance is such that its quantity is measured on a horizontal surface. To evaluate the solar beam irradiance on the window, which is perpendicular to the calculated  $G_B$  from Equation 33. Equation 34 below is used to find the solar beam irradiance that the window experiences.

$$G_{B,90} = G_B * \frac{\sin(\text{Inc})}{\sin(90 - \text{Inc})}$$

$$G_{B,90} = 55.5 * \frac{\sin(103.2)}{\sin(90 - 103.2)}$$

$$G_{B,90} = 236.6 \text{ W/m}^2$$

Equation 34

#### ***A.14.2.4 Diffused Solar Radiation***

To account for all the sources of irradiation, we need to consider the solar radiation that reaches the surface of the Earth as diffuse radiation. To calculate the diffused irradiance of which

the siding is subjected diffused radiation, the solar irradiance at a specific location,  $G_{oh}$ , is multiplied by the atmospheric transmittance for diffuse radiation. The diffused atmospheric transmittance is calculated using **Error! Reference source not found.**. The quantity of diffuse irradiance on the horizontal ground can then be calculated using Equation 33.

$$\tau_D = 0.271 - 0.294 * \tau_B$$

$$\tau_D = 0.271 - 0.294 * 0.241$$

$$\tau_D = 0.200$$

Error! Reference source not found.

$$G_D = G_{oh} * \tau_D$$

$$G_D = 230.6 * 0.200$$

$$G_D = 46.1 \text{ W/m}^2$$

Equation 33

The irradiance from diffuse solar energy then has to be translated from the horizontal ground to the wall tilted  $90^\circ$  with respect to the ground. This calculation is performed using Equation 37.

$$G_{D,Tilted} = G_D * \frac{1 + \cos(Tilt)}{2}$$

$$G_{D,Tilted} = 46.1 * \frac{1 + \cos(90^\circ)}{2}$$

$$G_{D,Tilted} = 23.1 \text{ W/m}^2$$

Equation 34

#### ***A.14.2.4 Reflected Solar Radiation from the Surrounding***

The reflected radiation was briefly explained in Section 2.3. The irradiation of reflected radiation takes into account of the reflectivity of the surrounding surface. The calculation is

performed using Equation 38.<sup>4</sup> Here,  $G_{R,90}$  is the intensity of reflected radiation on the tilted surface at 90 degrees. Additionally,  $G$  is the intensity of solar radiation reaching to earth ( $G_B + G_D$ ) and  $\chi$  is the reflectivity of surrounding. The corresponding value of  $\chi$  can be looked up in. For January 2<sup>nd</sup> in Massachusetts, we will assume a snow cover with a  $\chi$  value of 0.5.

Table 14. Reflectivity,  $\chi$ , of Different Surroundings<sup>5</sup>

Ground Coverage	$\chi$ value
dry bare ground	0.2
dry grassland	0.3
desert sand	0.4
snow	0.5-0.8
pale soil	0.3
dark soil	0.1
water	0.1
vegetation	0.2

$$G_{R,90} = G\chi \frac{1 - \cos(\text{Tilt})}{2}$$

$$G_{R,90} = (G_B + G_D) \chi \frac{1 - \cos(\text{Tilt})}{2}$$

$$G_{R,90} = (55.5 + 46.1) * 0.5 * \frac{1 - \cos(90)}{2}$$

$$G_{R,90} = 25.4$$

Equation 35

### A 14.3 Focal Point Calculations

In the next step the distance of the sun from the earth is important, and it is a value that changes. The distance can be found using the polar equation of a conic section, where  $\epsilon$  is the eccentricity of the trajectory and  $a$  is the semi-major axis. The angle  $\nu$  is dependent on the location of the earth relative to its period. It can be approximated using Equation 85.

$$\nu = \frac{2\pi * N}{Per}$$

$$\nu = \frac{2\pi * 2}{365}$$

$$\nu = 0.034 \text{ radians}$$

Equation 85

The distance of the sun from the earth,  $d_s$ , can then be calculated. This distance is important to analysis the spherical mirror-like qualities of the deformed window. The values for the following equation, Equation 13, can be found in Table 9, both of which are found in Appendix A.9.

$$d_s = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos \nu}$$

$$d_s = \frac{1.496 * 10^{11}(1 - 0.0167^2)}{1 + 0.0167 * \cos(0.034)}$$

$$d_s = 1.471 * 10^{11}m$$

Equation 13

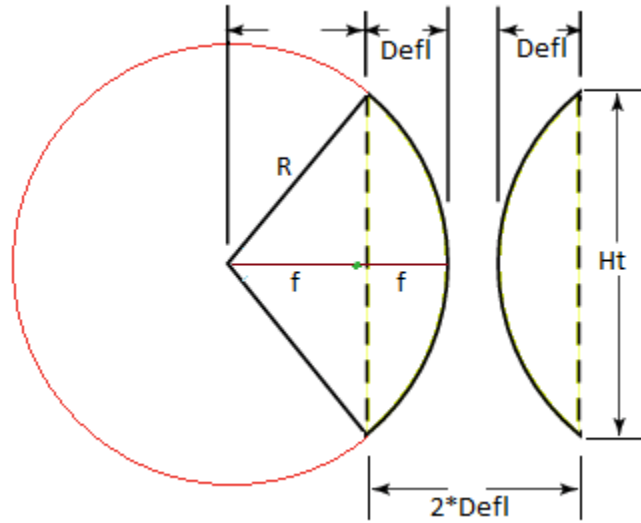


Figure 39. Imaginary Circular Relationships of Concave Glass

In the above image, the dashed lines is representative of the two panes of glass in an insulated glass unit, IGU, when it experiences no deformation. The left pane is exposed to the outside, and the right pane is the inside. Assume that we are looking at the window from the side. The standard gap size,  $x$ , is 15 mm.<sup>6</sup> For the sample equation, a deflection of 2 mm was assumed, which in this image is represented by the term  $Defl$ . The first value calculated is the radius of curvature,  $R$ , of the theoretical circle related to the concave shape. This is done using

$$R = Defl + \frac{0.125Ht^2}{Defl}$$

$$R = 2 + \frac{0.125(600)^2}{2}$$

$$R = 22502 \text{ mm} = 22.5 \text{ m}$$

Equation 86

Next, the mirror equation is manipulated using two variations to find the focal point with regards to previously calculated radius of curvature. Where  $d_s$  is the distance to the sun and  $d_i$  is the distance to the reflected image. In the following equation, the same two distances can be related to the focal point of reflection,  $f$ .

$$\frac{1}{d_s} + \frac{1}{d_i} = \frac{2}{R} = \frac{1}{f}$$

Equation 87

By rewriting Equation 87, the following derivation can be made to find the focal point will occur 11.5 meters away from the center of the deflected glass window unit.

$$f = \frac{R}{2} = \frac{22.5}{2} = 11.25 \text{ m}$$

Equation 88

Figure 40 on the following page represents rays of light coming from a distant object where  $d_s$  is  $\infty$ , or very large in comparison to  $d_i$ , therefore, all light converges at the focal point, with  $d_i = f$ .<sup>7</sup> The reflected image is inverted, however, this is irrelevant because the sun is completely symmetrical. Using to the relationship  $d_i = f$  we can rewrite the equation for magnification, taking the absolute value, and solving for  $r_i$  the following derivation can be found in Equation 44.

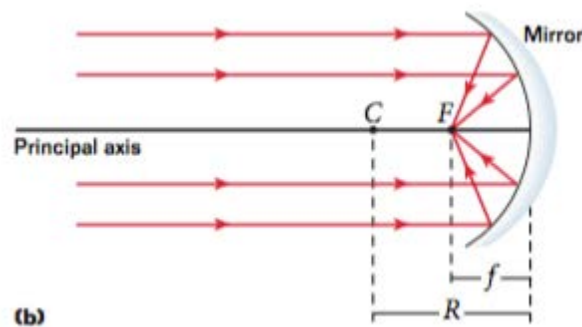


Figure 40. Light Rays at an Infinite Distance from a Convex Mirror All Converge at the Focal Point

The derivation shown in **Error! Reference source not found.** can be used to find the size of the sun’s reflected image on a neighboring home that located is exactly at the focal point distance away from a Low-E window experiencing deflection. The reflected image of the sun will appear to be a “spot” wherever it is focused. This spot is the reflected solar energy with a radius of size  $r_i$ .

$$M = \frac{r_i}{r_s} = -\frac{d_i}{d_s}$$

$$\left| \frac{r_i}{r_s} \right| = \left| -\frac{d_i}{d_s} \right|$$

$$r_i = \frac{d_i}{d_s} r_s$$

$$r_i = \frac{f}{d_s} r_s$$

$$r_i = \frac{11.25}{1.471 * 10^{11}} (696 * 10^6)$$

$$r_i = 0.054 \text{ m} = 54 \text{ mm}$$

However, it is unlikely that a house be located at the exact distance where the focal point will land. Therefore, the Excel Spreadsheet Tool needed to be able to determine how big the sun's reflected image would be if it were being reflected off a Low-E window experiencing deflection. A like triangle was considered to calculate the size of a concentrated solar light spot. Spot on whatever surface it is hitting. The concentration factor is the ratio of the area of the window, off which the intensity is reflecting to the area of the spot. This concentration factor can be found with the following ratio. The area of the window pane to the area of the reflected image produces the concentration factor.

$$CF = \frac{Ht * W}{A_i}$$

$$CF = \frac{0.6096 * 0.9144}{0.09}$$

$$CF = 5.94$$

**Equation 89**

The amount of energy reflected to this spot is dependent on the reflectance value that is input into the program and the concentration factor. This is found by simply multiplying the three values together. This can be seen below in Equation 90.

$$(G_{D,Tilted} + G_{R,90} + G_{B,90}) * Refl * CF = G_T$$

$$(236.6 + 25.4 + 23.1) * 0.44 * 5.92 = G_T$$

$$G_T = 742.6 \text{ W/m}^2$$

Equation 90

This is the amount of solar irradiance that is hitting the neighboring house at 3 PM on January 3, under the given input parameters of the sample calculation. The transient heat analysis that was previously described in this paper was too complex to do by hand. The equations were derived by hand and self-coded, but they were not executed by hand. Thus, this is the end of the sample calculation process. The rest of the results and analysis were generated using a computer.

## A.14 References

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<sup>1</sup> Stine, W. and M. Geyer. (1986) Power From The Sun: Chapter 3. John Wiley and Sons, Inc.  
Web. Retrieved 12 Decemeber 2016, from

<http://www.powerfromthesun.net/Book/chapter03/chapter03.html>

<sup>2</sup> "Time-Zone and Meridian Offset Reference Information."(1998) Vixen North America: Web.  
Retrieved 8 December 2016, from

[http://www.company7.com/library/vixen/VixenNA\\_TimeZoneOffset.pdf](http://www.company7.com/library/vixen/VixenNA_TimeZoneOffset.pdf)

<sup>3</sup> "Part 3: Calculating Solar Angles." The sun as source of energy. ITCA. Retrieved 12 December 2016, from

<http://www.itacanet.org/the-sun-as-a-source-of-energy/part-3-calculating-solar-angles/>



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<sup>4</sup> Perino M. “Available Solar Radiation” | IDES-edu

<https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=5&cad=rja&uact=8&sqi=2&ved=0ahUKEwibINbkqyHQAhVI5SYKHZRqD68QFghEMAQ&url=http%3A%2F%2Fwww.ides-edu.eu%2Fwp-content%2Fuploads%2F2013%2F04%2F4-Available-Solar-Radiation.pdf&usg=AFQjCNEog3HI08Zz2rA7VQhIsWTr6LV6Lw&sig2=RVFSJAqXfoHlnuBJE0JgOw&bvm=bv.141320020,d.cG>

<sup>5</sup> “Part 4: Irradiation Calculation” | The sun as source of energy | ITCA [“Part 4: Irradiation Calculation” | The sun as source of energy | ITCA http://www.itacanet.org/the-sun-as-a-source-of-energy/part-4-irradiation-calculations/](http://www.itacanet.org/the-sun-as-a-source-of-energy/part-4-irradiation-calculations/)

<sup>6</sup> Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

<sup>7</sup> Serway, R. & J. Faugh. (2006) *Physics*. Holt, Rinehart, and Winston. 455-60.

## Appendix A.15 Siding Temperature Calculation Tool

Table 15. Table of all of the Inputs for the Siding Temperature Calculation Tool

Input	Reference Name	Description
Latitude	Lat	The latitude of house A. The angle of the house's north-to-south distance from the equator with the equator being 0°. <sup>1</sup> This value can be easily and very accurately obtained from maps.google.com
Longitude	Lon	The longitude of house A. The angle of the house's east-to-west distance from the prime meridian. <sup>1</sup>
Altitude	Atude	The altitude of the house in kilometer
Day *	Day	The day of the month that is to be considered.
Month (1-12) *	Mon	The month of the year input as a number 1 through 12.
Year *	Year	The current calendar year.
Day of Year *	N	The day of the year is the number day out of 365. This is currently an input to simplify the process of the tool so we can make sure that it is working properly, but will eventually be automatic.
Local Clock Time *	Time	The time of day in military time.
Daylight Savings Time (1 for yes, 0 for no) *	D	This is the value that represents whether the location is currently practicing daylight savings time. This will affect the sun angle calculations. <sup>2</sup>
Window Width	W	The window width is the horizontal measurement of only the glass part of one pane of the window in meters.
Window Height	Ht	The height of the window is the horizontal measurement of the glass section of one pane of the window in meters.
Tilt of Window	Tilt	The tilt of the window is the angle of the normal to the front face of the window to the normal of the surface of the earth. A diagram including the tilt angle can be seen in Figure 8. Altitude, Inclination, Azimuth, Azimuth of Surface, and Tilt Angle Diagram
Azimuth Angle of Surface	Azsurf	The azimuth angle of the surface is the angle, measured clockwise from the north direction

		to the direction of the normal of the front face of the window. <sup>3</sup> This angle is shown more clearly in Figure 8.
Absorptivity of the Siding	$\kappa$	The absorptivity of siding was tested by Lawrence Berkeley Lab using a spectrophotometer. This value is assumed to be constant for all vinyl siding from LBL's data. <sup>4</sup>
Emissivity of the Siding	Emiss	The emissivity of the siding again was measured by LBL using a spectrophotometer and assumed to be consistent for this project. <sup>4</sup>
Reflectance of the Window	Refl	This value is selected from one of four ranges. It is not a readily available quantity so it is selected from a range approximated from known values. <sup>5</sup>
Deflection *	Defl	The deflection for our tool currently is an input value that we can be approximating based off of previous research from Lawrence Berkeley Lab. <sup>5</sup>
Average Temperature	Tamb	This is an input that the user will enter the average temperature range of the area to provide the information for the the heat transfer equations.
Heat Transfer Coefficient	h	Heat transfer coefficient for natural convection alone is 10W/m <sup>2</sup> s. For cases with wind speed, the heat transfer coefficient can be found for wind speeds ranging from 2-20m/s as shown in Appendix A.3.
Distance of Neighboring House	Dnh	The distance of house A from house B measured in meters.

## A.15 References

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<sup>1</sup> "Latitude." Latitude and Longitude Defined. MapTools, 2016. Web. Retrieved 16 Dec. 2016, from

[http://maptools.com/tutorials/lat\\_lon/definitions](http://maptools.com/tutorials/lat_lon/definitions)

<sup>2</sup> "Daylight Saving Time Around the World". 2016. Web. Retrieved 16 December 2016 from,

<https://www.timeanddate.com/time/dst/2016.html>

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<sup>3</sup> "Part 3: Calculating Solar Angles." ITACA RSS. ITACA, n.d. Web. 12 Dec. 2016.

<http://www.itacanet.org/the-sun-as-a-source-of-energy/part-3-calculating-solar-angles/>

<sup>4</sup> Hart, R. (2011). Research Needs: Glass Solar Reflectance and Vinyl Siding.

<sup>5</sup> Hart, R., Goudey, H., Arasteh, D., & Curcija, D. C. (2012). Thermal performance impacts of center-of-glass deflections in installed insulating glazing units. *Energy and Buildings*, 54, 453-460.

## Appendix A.16 Siding Temperature Calculations: MATLAB Code

### A.16.1 Initial\_Conditions.m

```
%%Global Variables
h_out = 6 %%Initial Boundary Conditions
absorb_vinyl = 0.92
h_in = 5
k_vinyl = 0.25 %%Thermal conductivity of wall components
k_ply = 0.12
k_ins = 0.04
k_gyp = 0.17
T_in = 294 %%Initial Boundary Temperatures
T_out = 271
thick_vinyl = 0.00102 %%Delta X values for wall components
thick_ply = 0.009525
thick_ins = 0.1524
thick_gyp = 0.015875
dx_gyp = thick_gyp/5
dx_ins = thick_ins/5
dx_ply = thick_ply/5
dx_vinyl = thick_vinyl/5
%%Gypsum matrix
Gyp_mat = [(h_in+k_gyp/dx_gyp), -k_gyp/dx_gyp, 0, 0, 0, 0, 0;
1, -2, 1, 0, 0, 0, 0;
0, 1, -2, 1, 0, 0, 0;
0, 0, 1, -2, 1, 0, 0;
0, 0, 0, 1, -2, 1, 0;
0, 0, 0, 0, k_gyp/dx_gyp, -(k_gyp/dx_gyp+k_ins/dx_ins), k_ins/dx_ins]
%% Insulation matrix
Ins_mat = [k_gyp/dx_gyp, -(k_gyp/dx_gyp+k_ins/dx_ins), k_ins/dx_ins, 0, 0, 0,
0, 0;
0, 1, -2, 1, 0, 0, 0, 0;
0, 0, 1, -2, 1, 0, 0, 0;
0, 0, 0, 1, -2, 1, 0, 0;
0, 0, 0, 0, 1, -2, 1, 0;
0, 0, 0, 0, 0, k_ins/dx_ins, -(k_ins/dx_ins+k_ply/dx_ply), k_ply/dx_ply]
%% Plywood matrix
Ply_mat = [k_ins/dx_ins, -(k_ins/dx_ins+k_ply/dx_ply), k_ply/dx_ply, 0, 0, 0,
0, 0;
0, 1, -2, 1, 0, 0, 0, 0;
0, 0, 1, -2, 1, 0, 0, 0;
0, 0, 0, 1, -2, 1, 0, 0;
0, 0, 0, 0, 1, -2, 1, 0;
0, 0, 0, 0, 0, k_ply/dx_ply, -(k_ply/dx_ply+k_vinyl/dx_vinyl),
k_vinyl/dx_vinyl]
%% Vinyl matrix
Vinyl_mat = [k_ply/dx_ply, -(k_ply/dx_ply+k_vinyl/dx_vinyl),
k_vinyl/dx_vinyl, 0, 0, 0, 0;
0, 1, -2, 1, 0, 0, 0;
0, 0, 1, -2, 1, 0, 0;
0, 0, 0, 1, -2, 1, 0;
0, 0, 0, 0, 1, -2, 1;
0, 0, 0, 0, 0, -k_vinyl/dx_vinyl, (h_out+k_vinyl/dx_vinyl)]
%%Combine matrix solution
```

```

X = [Gyp_mat(1:5,:), zeros(5,5);zeros(6,4),Ins_mat]
Y = [X(1:10,:),zeros(10,5);zeros(6,9),Ply_mat]
Z = [Y(1:15,:),zeros(15,4);zeros(6,14),Vinyl_mat]
B_mat = [h_in*T_in;
zeros(19,1);
h_out*T_out]
%%Solved Temperature Matrix
T = linsolve(Z,B_mat)

```

## A.16.2 Mesh\_Fourier.m

```

cp_gyp = 1090 %% Properties of gypsum
rho_gyp = 800
cp_ins = 700 %% Properties of insulation
rho_ins = 7
cp_ply = 1210 %% Properties of plywood
rho_ply = 540
cp_vinyl = 1250 %% Properties of vinyl
rho_vinyl = 1400
%%thermal diffusivity
alpha_g = k_gyp/(cp_gyp*rho_gyp)
alpha_i = k_ins/(cp_ins*rho_ins)
alpha_p = k_ply/(cp_ply*rho_ply)
alpha_v = k_vinyl/(cp_vinyl*rho_vinyl)
%%time step
delta_t = 120 %seconds
%%mesh fourier numbers
tau_g = alpha_g*delta_t/(dx_gyp*dx_gyp)
tau_i = alpha_i*delta_t/(dx_ins*dx_ins)
tau_p = alpha_p*delta_t/(dx_ply*dx_ply)
tau_v = alpha_v*delta_t/(dx_vinyl*dx_vinyl)

```

## A.16.3 Transient\_Analysis.m

```

clear all
Initial_Conditions; %%Calls initial condition to be run
Mesh_Fourier; %%Calls stability criterion to be run
%%Interface constants
gyp_ins = (rho_gyp*dx_gyp*cp_gyp/2)+(rho_ins*dx_ins*cp_ins/2)
ins_ply = (rho_ins*dx_ins*cp_ins/2)+(rho_ply*dx_ply*cp_ply/2)
ply_vin = (rho_ply*dx_ply*cp_ply/2)+(rho_vinyl*dx_vinyl*cp_vinyl/2)
T_start = [0;T(21,1)]
T_day = [T_start]
%7:16 Sunrise to 8am
for i = 1:22
q_prime = 37
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]

```

```

i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;
0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]
v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);
-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-
T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end
%8am to 10am
for i = 23:82
q_prime = 93
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]
i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;
0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]

```

```

v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);
-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-
T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end
%10am to Noon
for i = 83:142
q_prime = 131
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]
i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;
0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]
v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);
-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-

```



```

T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end
%Noon until 2
for i = 143:202
q_prime = 148
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]
i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;
0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]
v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);
-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-
T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end
%2:00 - Spot starts at 2:00, determines temp after 10 min spot passes
for i = 203:207
q_prime = 1344
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]

```

```

0, 0, 0, tau_g, -(1+2*tau_g), tau_g]
i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;
0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]
v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);
-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-
T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end
%2:10 until 4
for i = 208:262
q_prime = 138
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]
i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;

```

```

0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]
v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);
-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-
T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end
%4 until 4:26
for i = 263:275
q_prime = 105
g_mat = [2*h_in*tau_g*dx_gyp/k_gyp+2*tau_g+1, -2*tau_g, 0, 0, 0, 0;
tau_g, -(1+2*tau_g), tau_g, 0, 0, 0;
0, tau_g, -(1+2*tau_g), tau_g, 0, 0;
0, 0, tau_g, -(1+2*tau_g), tau_g, 0;
0, 0, 0, tau_g, -(1+2*tau_g), tau_g]
i_mat = [delta_t*k_gyp/(dx_gyp*gyp_ins), -
(delta_t*k_gyp/(gyp_ins*dx_gyp)+k_ins*delta_t/(gyp_ins*dx_ins)+1)
delta_t*k_ins/(dx_ins*gyp_ins), 0, 0, 0, 0;
0, tau_i, -(1+2*tau_i), tau_i, 0, 0, 0;
0, 0, tau_i, -(1+2*tau_i), tau_i, 0, 0;
0, 0, 0, tau_i, -(1+2*tau_i), tau_i, 0;
0, 0, 0, 0, tau_i, -(1+2*tau_i), tau_i]
p_mat = [delta_t*k_ins/(dx_ins*ins_ply), -
(delta_t*k_ins/(ins_ply*dx_ins)+k_ply*delta_t/(ins_ply*dx_ply)+1)
delta_t*k_ply/(dx_ply*ins_ply), 0, 0, 0, 0;
0, tau_p, -(1+2*tau_p), tau_p, 0, 0, 0;
0, 0, tau_p, -(1+2*tau_p), tau_p, 0, 0;
0, 0, 0, tau_p, -(1+2*tau_p), tau_p, 0;
0, 0, 0, 0, tau_p, -(1+2*tau_p), tau_p]
v_mat = [delta_t*k_ply/(dx_ply*ply_vin), -
(delta_t*k_ply/(ply_vin*dx_ply)+k_vinyl*delta_t/(ply_vin*dx_vinyl)+1)
delta_t*k_vinyl/(dx_vinyl*ply_vin), 0, 0, 0, 0;
0, tau_v, -(1+2*tau_v), tau_v, 0, 0, 0;
0, 0, tau_v, -(1+2*tau_v), tau_v, 0, 0;
0, 0, 0, tau_v, -(1+2*tau_v), tau_v, 0;
0, 0, 0, 0, tau_v, -(1+2*tau_v), tau_v;
0, 0, 0, 0, 0, -2*tau_v, 2*tau_v*h_out*dx_vinyl/k_vinyl+2*tau_v+1]
Trans_mat = [g_mat,zeros(5,15);zeros(5,4),i_mat,zeros(5,10);
zeros(5,9),p_mat,zeros(5,5);zeros(6,14),v_mat]
T_i = [T(1,1)+(2*h_in*dx_gyp*tau_g/k_gyp)*T_in;-T(2,1);-T(3,1);-T(4,1);-
T(5,1);-T(6,1);-T(7,1);-T(8,1);-T(9,1);-T(10,1);

```

```

-T(11,1);-T(12,1);-T(13,1);-T(14,1);-T(15,1);-T(16,1);-T(17,1);-T(18,1);-
T(19,1);-
T(20,1);T(21,1)+(2*tau_v*dx_vinyl/k_vinyl)*q_prime*absorb_vinyl+2*tau_v*h_out
*dx_vinyl/k_vinyl*T_out]
T_plus1 = linsolve(Trans_mat,T_i)
T_time = [i;T_plus1(21,1)]
T_day = [T_day(:,:);T_time(:,:)]
T = T_plus1
end

```

## Appendix A.17 Thermal Degradation of PVC

Introduced in Section 2.1.2, the distortion of the vinyl siding is mainly because of the accelerated thermal degradation of PVC. Dehydrochlorination is the first step of the process.<sup>1</sup> With the break of C-Cl bonds, the chlorine radicals are released and reacts with hydrogen to form HCl. This explains the weight loss during the degradation. During this process, both molecular reaction and radical propagations support the release of HCl.<sup>1</sup> However, as the purpose of this calculation in this tool is to predict the siding deformation, the breakage of C-Cl will be focused rather than the whole dehydrochlorination (release of HCl) process.

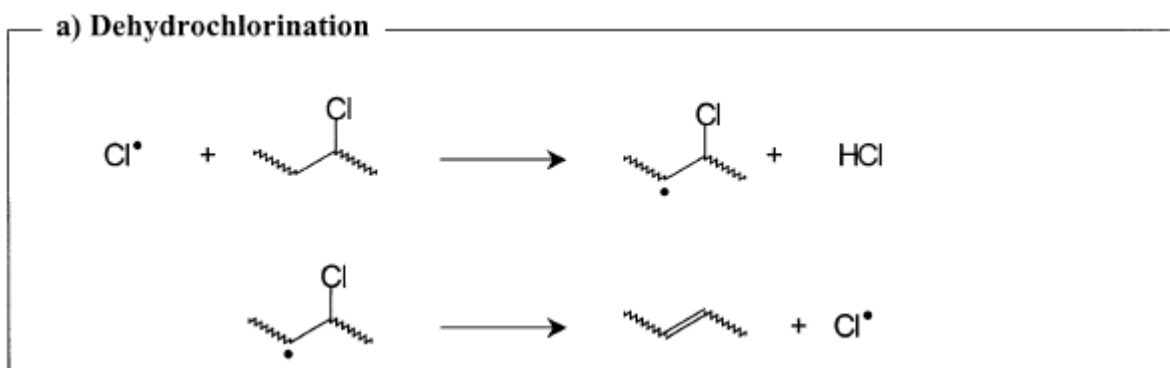


Figure 41. Chemical equation of dehydrochlorination<sup>1</sup>.

During this process, the breakage of C-Cl bond that results in decomposition. This reaction is responsible for the chlorine formation. The bond energy of C-Cl is lower than C-C and thus, the formation of chlorine radicals is easier than the primary radicals. According to the literature, the activation energy needed for this process is  $83 \text{ kJmol}^{-1}$  and the Arrhenius parameter  $A = 6 \times 10^{13} \text{ s}^{-1}$ .<sup>1</sup> Substituting this data into the Arrhenius relationship,

$$k = 6 \times 10^{13} \exp(-83000/8.314 \times T)$$

Equation 91

Then using the excel spreadsheet, the rate of reactions from  $0^\circ\text{C}$  to  $200^\circ\text{C}$  are plotted.

Table 16. Calculation of k and Ln(k)

T (°C)	T (K)	k	Ln(k)
0	273	0.007883452	-5.670612677
10	283	0.028701056	-4.349200015
20	293	0.095669481	-3.117986168
30	303	0.294534194	-1.968040563
40	313	0.843903459	-0.89157391
50	323	2.265389456	0.118238461
60	333	5.731076683	1.067401441
70	343	13.73490244	1.96121964
80	353	31.32625358	2.804396582
90	363	68.27508587	3.601117494
100	373	142.7163586	4.355118785
110	383	287.0542824	5.069746642
120	393	557.1979457	5.748006668
130	403	1046.549855	6.392606147
140	413	1906.573967	7.005990155
150	423	3376.221311	7.590372555
160	433	5822.976033	8.147762697
170	443	9798.785066	8.679988498
180	453	16114.62982	9.188716471
190	463	25937.96525	9.675469153
200	473	40917.66653	10.14164033

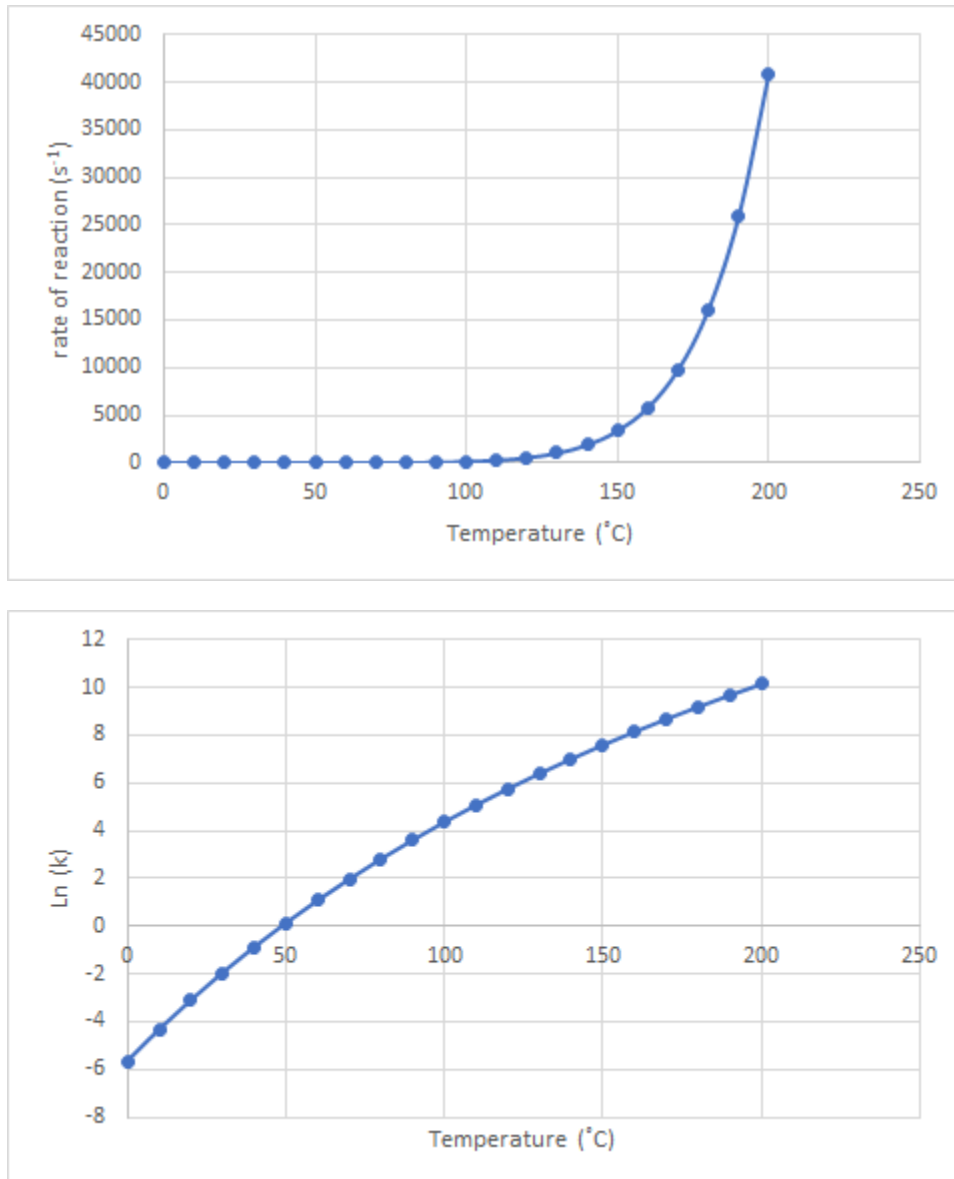


Figure 42. Plots showing the rate of reaction at different temperature

The damage of the vinyl siding is predicted by integrating the rates of the reactions at respective temperatures of the vinyl siding. The results of temperature analysis and reaction rates, explained in section 3.5 and 3.6, are used in the prediction. The integral equation of predicting damage is given by Equation 92. Here, the rate of reaction,  $k$ , is a function of temperature in

Kelvin. The other variables associated with this equation are time,  $t$ , the lower and upper time limits  $a$  and  $b$ , respectively, and  $C$  which is a constant of integration.

$$\int_b^a k(T) dx = 10^9 \exp\left(-\frac{83000}{8.314 * T}\right) t + C$$

Equation 92

For the convenience of the tool user, the damage is shown in the range from level 1-4, where level 1 corresponds to a visible damage and level 4 corresponds the very bad condition of the siding. The standards of those levels are set the damage of the vinyl siding resulted from heating 60 minutes at the following temperatures. The detailed calculation of these levels can be found in Appendix A.19 Damage Level Calculations.

**Level 1:** deformation of the siding from heating at **71 degree Celsius** for one hour.

**Level 2:** deformation of the siding from heating at **82 degree Celsius** for one hour.

**Level 3:** deformation of the siding from heating at **86 degree Celsius** for one hour.

**Level 4:** deformation of the siding from heating at **90 degree Celsius** for one hour.

## A.17 Reference

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<sup>1</sup> Braun, D. (1971). Thermal degradation of polyvinyl chloride. *Pure and Applied Chemistry*, 26(2), 173-192.



## Appendix A.18 Sensitivity Analysis

### A.18.1 Standard Reference Values and Processes for Sensitivity Analysis

$$T_{ref} = T_{distortion} = 347$$

The reference temperature,  $T_{ref}$ , was defined as the heat distortion temperature,  $T_{distortion}$ , for vinyl siding because all of the sensitivity tests are done for a situation involving vinyl siding on a home.

#### Calculation Process for Sensitivity Values:

For each of the following section of this Appendix (A.17) the sensitivity values were calculated using the following process:

$$\left(\frac{\Delta T}{T_{Ref}}\right) / \left(\frac{\Delta Var}{Var_{Ref}}\right) = \text{Sensitivity Value}$$

Equation 93

Where:

$$\Delta T = T_{High} - T_{Low} \quad \text{and} \quad \Delta Var = Var_{High} - Var_{Low}$$

$$T_{Ref} = 347K$$

$$Var_{Ref} = \text{an average value for the variable at hand}$$

### A.18.2 Deflection Analysis:

Reference Values:

- High: 5mm
- Base: 2.5
- Low: 0.01mm

Values based off of measurements from the following article: Hart, R. (2012). Research Needs: Glass Solar Reflectance and Vinyl Siding.

Table 17. (Left) Sensitivity Information When Varying the Deflection Input Value  
 Table 18. (Right) Table of Input Conditions

Deflection [mm]	Siding Temperature [K]
0.01	293.8
0.25	293.8
0.5	295.7
0.75	301.2
1	303.4
1.25	306.4
1.5	309.2
1.75	314.1
2	322.6
2.1	330.2
2.15	332.9
2.2	337.2
2.25	342.4
2.3	358.7
2.4	349.3
2.5	334.0
2.75	320.3
3	313.7
3.25	309.5
3.5	306.3
3.75	303.7
4	301.7
4.25	300.0
4.5	298.5
4.75	297.2
5	296.1

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Dalight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	0.5
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	Low Solar Gain
Deflection	Varied
Temperature of Environment Surroundings	271
Heat Transfer Coefficient	6
Neighboring House Distance	10
Climate Type	Midlatitude Winter
Tamb	271
Surrounding Ground	Vegetation

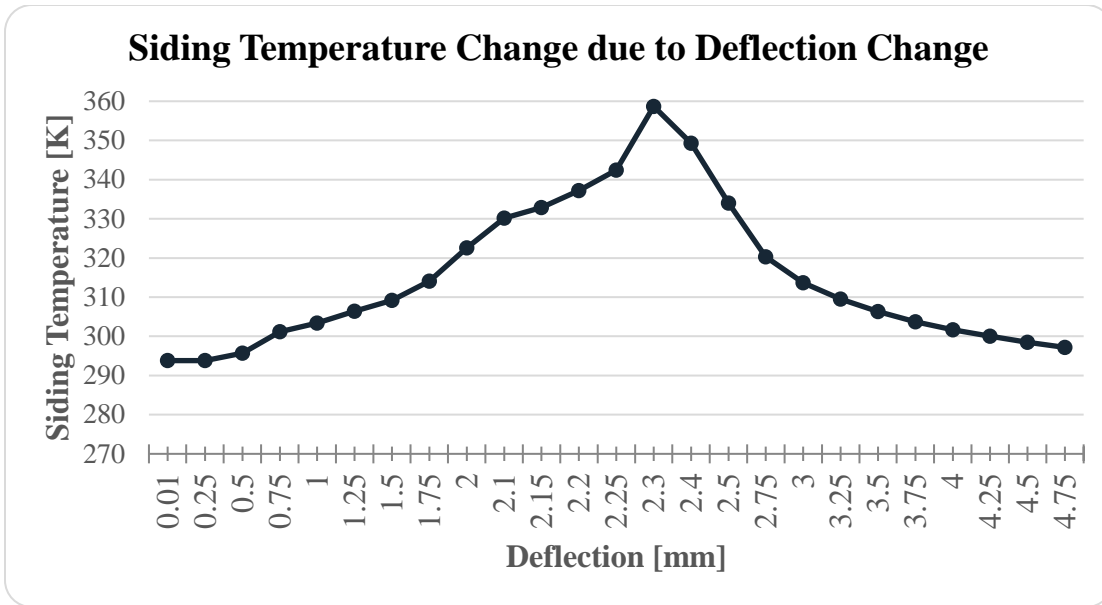


Figure 43. Graph of Tool Temperature Outputs When the Deflection Input Parameter is Varied

Table 19. Calculation of the Sensitivity Value for Deflection Following the Process Outlined in Appendix 17.1

Sensitivity Value:	
$\Delta T =$	64.9
$T_{ref} =$	347
$\Delta T / T_{ref} =$	0.1870317
$\Delta d =$	2.05
$d_{ref} =$	2.5
$\Delta d / \Delta d_{norm}$	0.82
$(\Delta T / T_{ref}) / (\Delta d / \Delta d_{norm})$	0.228087439

### A.18.3 Heat Transfer Coefficient Analysis:

Reference Values:

High: 100 W/m<sup>2</sup>K

Base: 55 W/m<sup>2</sup>K

Low: 10 W/m<sup>2</sup>K

Values based off of measurements from the following article: Whitelaw, J. H. (2011). Thermopedia. *Convective Heat Transfer*. doi:10.1615/AtoZ.c.convective\_heat\_transfer

Table 20. (Left) Sensitivity Information When Varying the Heat Transfer Coefficient Input Value  
Table 21. (Right) Table of Input Conditions

Heat Transfer Coefficient [W/m <sup>2</sup> K]	Siding Temperature [K]
10	310.9
15	302.4
20	296.8
30	290.0
40	286.0
50	283.4
60	281.5
70	280.1
80	279.1
90	278.2
100	277.5

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Dalight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	0.5
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	Low Solar Gain
Deflection	2
Temperature of Environment Surroundings	271
Heat Transfer Coefficient	<b>Varied</b>
Neighboring House Distance	10
Climate Type	Midlatitude Winter
Tamb	271
Surrounding Ground	Vegetation

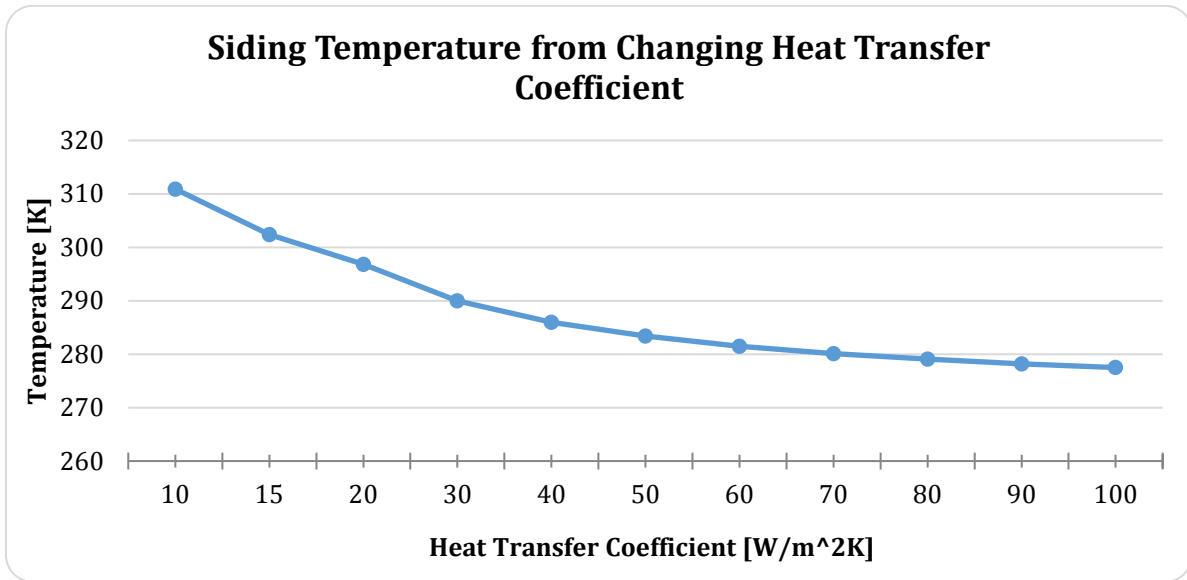


Figure 44. Chart of Tool Temperature Outputs When the Heat Transfer Coefficient Input Parameter is Varied

Table 22. Calculation of the Sensitivity Value for Heat Transfer Coefficient Following the Process Outlined in Appendix 17.1

Sensitivity Value:		
	$\Delta T =$	33.4
	$T_{ref} =$	347
$\Delta T/T_{ref} =$		0.096253602
	$\Delta h =$	90
	$h_{ref} =$	55
$\Delta h/h_{ref}$		1.636363636
$(\Delta T/T_{ref})/(\Delta h/h_{ref})$		
		0.05882165

### A.18.4 Reflectivity from Surrounding Ground Conditions Analysis:

Reference Values:

High: 0.65 (snow cover conditions)

Base: 0.375

Low: 0.1 (dark soil or water surroundings)

Values based off of information from the following article: Kharseh, M. "Solar Radiation Calculation" | Quinnipiac University

Table 23. (Left) Sensitivity Information When Varying the Surrounding Ground Reflectance Input Value  
Table 24. (Right) Table of Input Conditions

Reflectivity	Siding Temperature [K]
0.2	322.6
0.2	322.6
0.1	318.9
0.1	318.9
0.3	326.3
0.3	326.3
0.4	330.0
0.65	339.2

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Dalight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	0.5
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	Low Solar Gain
Deflection	2
Temperature of Environment Surroundings	271
Heat Transfer Coefficient	6
Neighboring House Distance	10
Climate Type	Midlatitude Winter
Tamb	271
Surrounding Ground	<b>Varied</b>

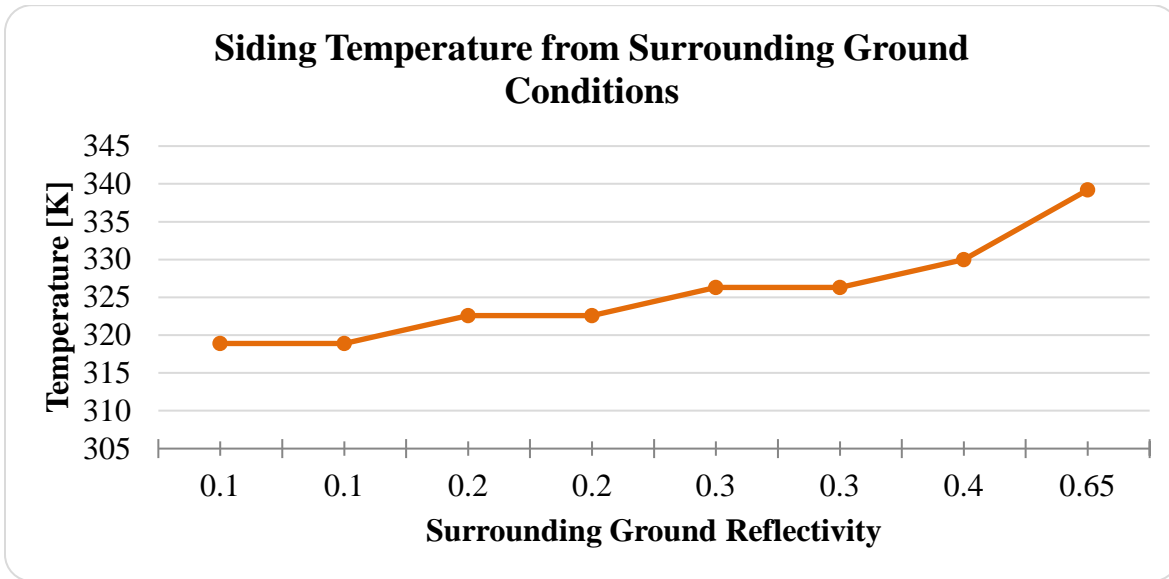


Figure 45. Chart of Tool Temperature Outputs When the Surrounding Ground Reflectance Input Parameter is Varied

Table 25. Calculation of the Sensitivity Value for Surrounding Ground Reflectivity Following the Process Outlined in Appendix 17.1

Sensitivity Value:		
	$\Delta T =$	16.6
	$T_{ref} =$	347
$\Delta T / T_{ref} =$		0.04783862
	$\Delta \text{reflectivity} =$	0.45
	$\text{reflectivity}_{ref} =$	0.375
$\Delta \text{reflectivity} / \text{reflectivity}_{ref}$		1.2
$(\Delta T / T_{ref}) / (\Delta \text{reflectivity} / \text{reflectivity}_{ref})$		
		0.039865514

### A.18.5 Ambient Temperature Analysis:

Reference Values:

High: 300.4 K

Base: 284 K

Low: 267.6 K

Values based off of data collected by US Climate Data website.

Table 26. (Left) Sensitivity Information When Varying the Ambient Temperature Input Value  
Table 27. (Right) Table of Input Conditions

Tamb [K]	Siding Temperature [K]
267.6	318.8
273	324.6
278.6	330.0
284	335.1
289.6	340.5
295	345.7
300.4	350.9

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Dalight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	0.5
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	Low Solar Gain
Deflection	2
Temperature of Environment Surroundings	<b>Varied</b>
Heat Transfer Coefficient	6
Neighboring House Distance	10
Climate Type	Midlatitude Winter
Surrounding Ground	Vegetation



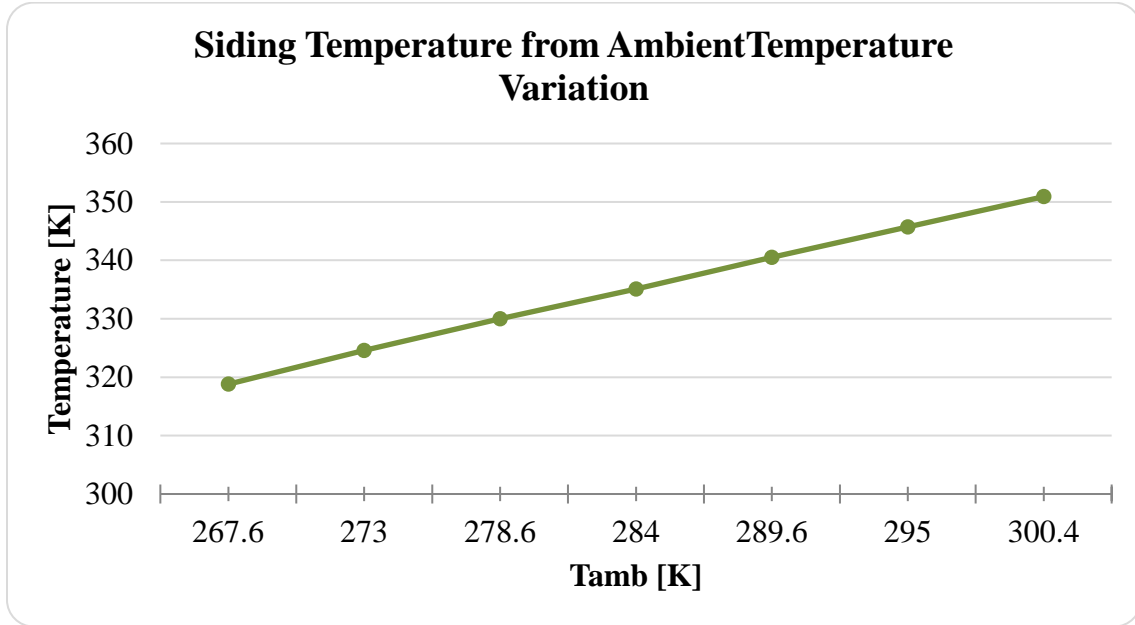


Figure 46. Chart of Tool Temperature Outputs When the Ambient Temperature Input Parameter is Varied

Table 28. Calculation of the Sensitivity Value for Ambient Temperature Following the Process Outlined in Appendix 17.1

Sensitivity Value:	
	$\Delta T = 32.1$
	$T_{ref} = 347$
$\Delta T / T_{ref} =$	0.0925
	$\Delta T_{amb} = 32.8$
	$T_{ambref} = 284$
$\Delta T_{amb} / T_{ambref}$	0.1155
$(\Delta T / T_{ref}) / (\Delta T_{amb} / T_{ambref})$	
	0.800977016

### A.18.6 Neighboring House Distance Analysis:

Reference Values:

High: 30m

Base: 16m

Low: 2m

Values determined assuming houses have a minimum distance between them of 2 meters and that a maximum likely to occur with no interference of objects or brush would be 30 meters.

Table 29. (Left) Sensitivity Information When Varying the Neighboring House Distance  
Table 30. (Right) Table of Input Conditions

Distance Of Neighboring House (m)	Siding Temperature [K]
2	291.3
6	296.6
10	322.6
14	311.8
18	295.1
22	290.8
26	288.8
30	387.6

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Daylight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	0.5
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	Low Solar Gain
Deflection	2
Temperature of Environment Surroundings	271
Heat Transfer Coefficient	6
Neighboring House Distance	<b>Varied</b>
Climate Type	Midlatitude Winter
Tamb	271
Surrounding Ground	Vegetation

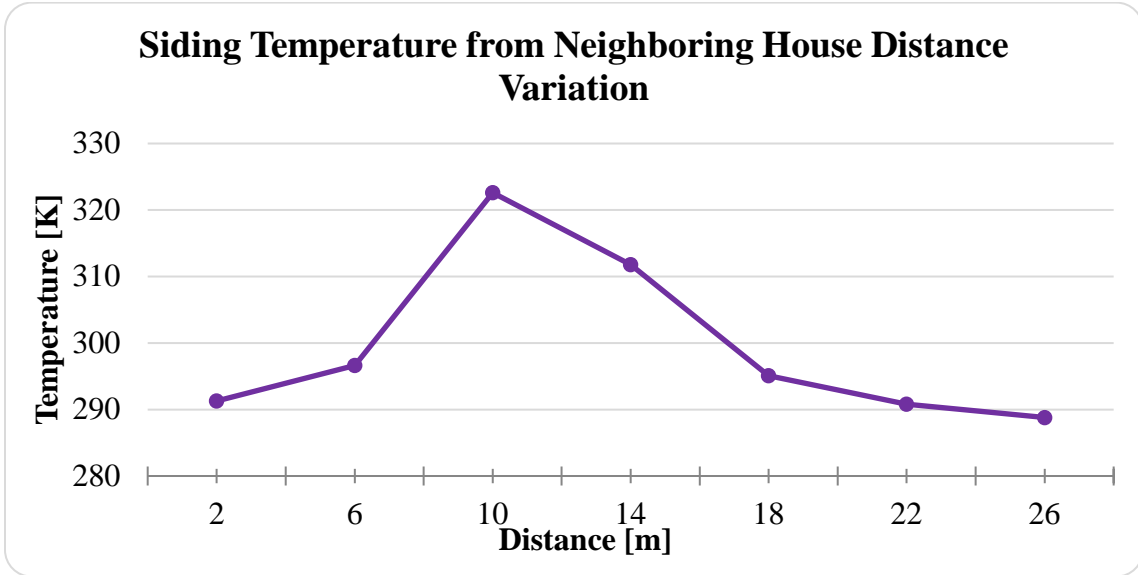


Figure 47. Chart of Tool Temperature Outputs When the Neighboring House Distance Input Parameter is Varied

Table 31. Calculation of the Sensitivity Value for Neighboring House Distance Following the Process Outlined in Appendix 17.1

Sensitivity Value:	
$\Delta T =$	2.5
$T_{ref} =$	347
$\Delta T/T_{ref} =$	0.007204611
$\Delta d_{nh} =$	24
$d_{nhref} =$	16
$\Delta d_{nh}/d_{nhref}$	1.5
$(\Delta T/T_{ref})/(\Delta d_{nh}/d_{nhref})$	
	0.004803074

### A.18.7 Window Reflectance Analysis

Reference Values:

High: 0.44

Base: 0.285

Low: 0.13

Values determined by observing the properties of Low-E windows and other common window types to develop a range of possible reflectance values with 0.44 being the highest possible value and 0.13 as the lowest possible reflectance. Based on “Measuring Performance: ENERGY STAR Windows.” *Efficient Windows Collaboration.*” (n.d.). Web. 18 Dec. 2016.

Table 32. (Left) Sensitivity Information When Varying the Window Reflectance Value  
Table 33. (Right) Table of Input Conditions

Reflectance Value	Siding Temperature [K]
0.13	295
0.25	305.7
0.35	314.6
0.44	322.6

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Dalight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	0.5
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	<b>Varied</b>
Deflection	2
Temperature of Environment Surroundings	271
Heat Transfer Coefficient	6
Neighboring House Distance	10
Climate Type	Midlatitude Winter
Tamb	271
Surrounding Ground	Vegetation

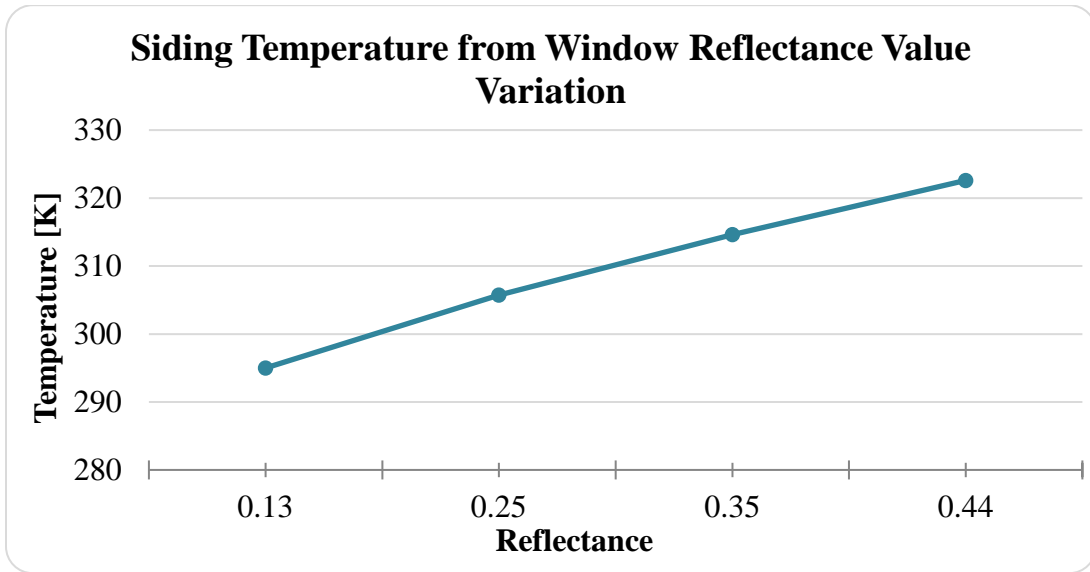


Figure 48. Chart of Tool Temperature Outputs When the Window Reflectance Input Parameter is Varied

Table 34. Calculation of the Sensitivity Value for Window Reflectance Following the Process Outlined in Appendix 17.1

Sensitivity Value:	
$\Delta T =$	27.6
$T_{ref} =$	347
$\Delta T / T_{ref} =$	0.079538905
$\Delta \text{Reflectance}_W =$	0.31
$\text{Reflectance}_{Wref} =$	0.285
$\Delta \text{Reflectance}_W / \text{Reflectance}_{Wref} =$	1.087719298
$(\Delta T / T_{ref}) / (\Delta \text{Reflectance}_W / \text{Reflectance}_{Wref})$	
	0.073124477

### A.18.7 Siding Absorptivity Analysis:

Reference Values:

High: 0.92

Base: 0.56

Low: 0.2

Values based off of ranges given in the following article: Cardinal IG. (n.d.). Vinyl Siding Distortion.

Table 35. (Left) Sensitivity Information When Varying the Siding Absorptivity Value

Table 36. (Right) Table of Input Conditions

Absorptivity Value	Siding Temperature [K]
0.2	292.2
0.43666	316.2
0.53333	326.0
0.62999	335.8
0.7267	345.7
0.8233	355.5
0.92	365.3

Input Values	
Input	Value
Latitude	42.26
Longitude	71.78
Time Zone	Eastern
Altitude	0.146
Date (MM/DD/YYYY)	1/1/17
Local Clock Time	12
Daylight Savings Time	0
Window Width	0.9144
Window Height	0.6096
Tilt of Window (Tilt)	90
Azimuth Angle of Surface (Azsurf)	175
Absorptivity of Siding	<b>Varied</b>
Emissivity of Siding	0.92
Type of Low-E Coating (Reflectance)	Low Solar Gain
Deflection	2
Temperature of Environment Surroundings	271
Heat Transfer Coefficient	6
Neighboring House Distance	10
Climate Type	Midlatitude Winter
Tamb	271
Surrounding Ground	Vegetation

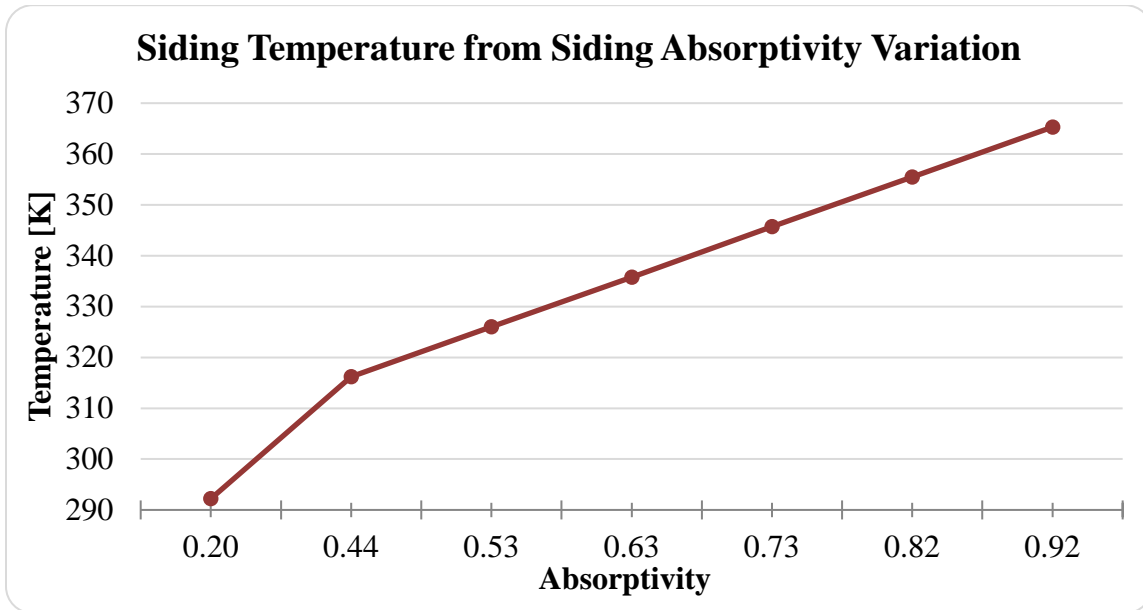


Figure 49. Chart of Tool Temperature Outputs When the Siding Absorptivity Input Parameter is Varied

Table 37. Calculation of the Sensitivity Value for Siding Absorptivity Following the Process Outlined in Appendix 17.1

Sensitivity Value:	
$\Delta T =$	73.1
$T_{ref} =$	347
$\Delta T / T_{ref} =$	0.210662824
$\Delta \text{ReflectanceW} =$	0.72
$\text{ReflectanceW}_{ref} =$	0.56
$\Delta \text{ReflectanceW} / \text{ReflectanceW}_{ref}$	1.285714286
$(\Delta T / T_{ref}) / (\Delta \text{dnh} / \text{dnh}_{ref})$	
	0.163848863

### A.18.8 Input Variables Sensitivity Compared

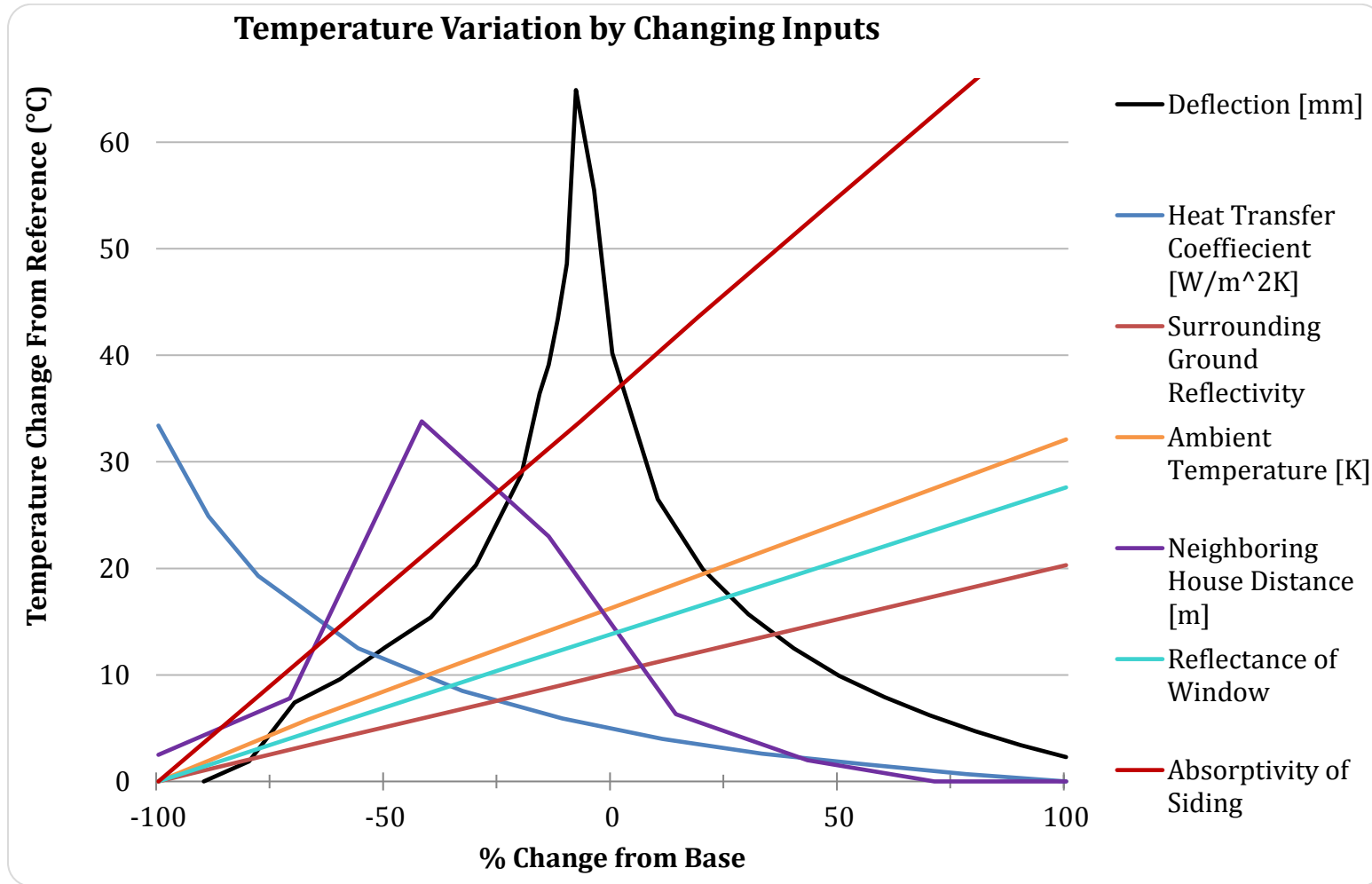


Figure 50. Temperature Variation by Changing Inputs Chart



**Table 38. Table of Calculated Sensitivity Values**

<b>Variable</b>	<b>Sensitivity Value</b>
Deflection [mm]	0.228
Heat Transfer Coefficient [W/m <sup>2</sup> K]	0.059
Surrounding Ground Reflectivity	0.04
Ambient Temperature [K]	0.801
Neighboring House Distance [m]	0.005
Reflectance of Window	0.073
Absorptivity of Siding	0.164

Sensitivity values were calculated in each separate appendix above. Reference values were determined by researching realistic ranges for each input parameter.

## Appendix A.19 Damage Level Calculations

The equation is adjusted to:

$$\int_b^a k(T) dx = 10^9 \exp\left(-\frac{83000}{8.314 * T}\right) t + C$$

T = temperature in Kelvin

t = time

Sample Calculation

Deformation after heating the siding at 71 degree Celsius for one hour

$$\int_0^{3600} k(344) dx = 10^9 \exp\left(-\frac{83000}{8.314*344}\right) 3600 = 0.9 \sim 1$$

Deformation after heating the siding at 82 degree Celsius for one hour

$$\int_0^{3600} k(355) dx = 10^9 \exp\left(-\frac{83000}{8.314*344}\right) 3600 = 2.2 \sim 2$$

Deformation after heating the siding at 86 degree Celsius for one hour

$$\int_0^{3600} k(359) dx = 10^9 \exp\left(-\frac{83000}{8.314*344}\right) 3600 = 3.0 \sim 3$$

Deformation after heating the siding at 86 degree Celsius for one hour

$$\int_0^{3600} k(363) dx = 10^9 \exp\left(-\frac{83000}{8.314*363}\right) 3600 = 4.1 \sim 4$$

Temperature throughout a day for the Pennsylvania test case discussed in Section 4.3 is below.

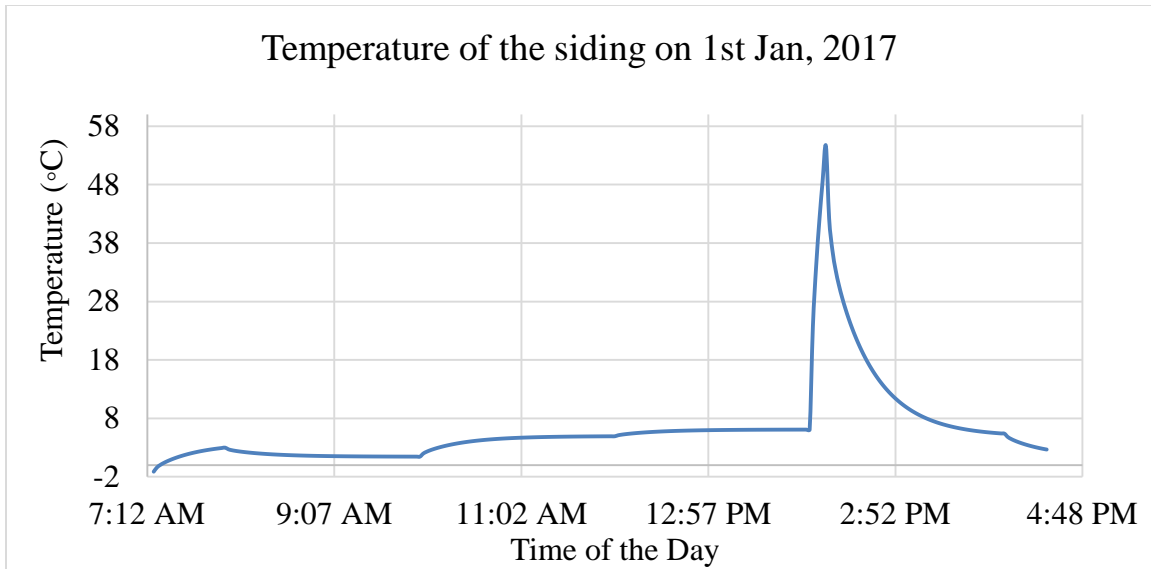


Figure 51: Temperature of Vinyl Sidings of PA Case

## Appendix A.20 Worcester Case Study House

A known case of siding damage was supplied to the team. The following information was supplied by Charles Wilson as a basis for the MQP.



The geographic location for this home was provided and siding and properties were determined by observation and educated assumptions knowing when the windows were installed and that they are Low-E windows.

The same case was investigated by WCVB anchor Susan Wornick. This can be viewed at the following source:

Wornick, S. (2012, July 6). Melting cars, homes tied to energy-efficient windows. *WCBB*. Retrieved April 26, 2017, from <http://www.wcvb.com/article/melting-cars-homes-tied-to-energy-efficient-windows/8171869>

## Appendix A.21 Images of the Tool Format

	A	B	C	G	H
1	INPUTS				
2	Enter values as accurately as possible in the "Input Value"				
3	<b>Input</b>	<b>Variable Name</b>	<b>Input Value</b>	<b>Unit</b>	<b>Description</b>
4	State		Massachusetts		
5	Latitude	Lat	42.26	Degrees	Latitude and longitude can be found using google maps. Type in the address of the home being evaluated and click the location on the map closest to the window that is considered to be reflecting sunlight. The latitude and longitude can be found in the bottom right hand corner of the page in this format: Latitude, longitude. Enter those values here.
6	Longitude	Lon	71.78	Degrees	
7	Time Zone	tz	Eastern		Atlantic, Eastern, Central, Mountain, Pacific, Alaska, Hawaii-Aleutian
8	Altitude	A	0.146	km	To find the altitude of a city, go to the google search engine. Type "altitude of (locations city name) in kilometers" and click search. Enter the value at the top of the results list into the box to the left. For example if you were searching for the city of Worcester MA, you would enter "Altitude of Worcester MA in kilometers" the result is 0.146km. You would then type 0.146 in the box to the left.
12	Window Width	W	0.9144	Meters	Measure or identify, from the specifications of the product, the dimensions of your window.
13	Window Height	Ht	0.6096	Meters	
14	Tilt of Window (Tilt)	Tilt	90	Degrees	is the most common perpendicular orientation, enter "90" in the box to the left. If the window is tilted, estimate the tilt of the window by determining the angle from directly perpendicular to the ground to a line normal to the surface of the window. A diagram of this angle can be seen to the right.
15	Azimuth Angle of Surface (Azsurf)	Azsurf	175	Degrees	Determine the angle of the perpendicular to the window surface measured from north. A diagram of this angle can be seen to the right.
16	Absorptivity of Siding	Abs	0.5		
17	Emissivity of Siding	Emiss	0.92		
18	Type of Low-e Coating (Reflectance)	Refl	Low Solar Gain		the window being considered. Select the type from the drop down list to the left.
19	Deflection	Defl	Low (0.5mm)	mm	
22	Neighboring House Distance	dnh	20	meters	distance to the best accuracy possible to the closest point of the neighboring house.
25	Surrounding Ground		dry bare ground		Select the surrounding ground conditions and that will determine the reflectivity value of the ground.

Figure 52. Screenshot of Tool Front Page/User Input Interface

	A	B	C	D	E	F	G	H	I	J	K
1	DROP DOWNS AND OTHER DATA										
2											
3	<b>General</b>										
4	<b>Time Zones</b>		<b>Daylight Savings Time</b>		<b>PI</b>		<b>Degrees to Rad</b>				
5					3.141592654		0.0174533				
6	Atlantic	60									
7	Eastern	75									
8	Central	90									
9	Mountain	105									
10	Pacific	120									
11	Alaska	135									
12	Hawaii-Aleutian	150									
13											
14											
15											
16	<b>Solar Intensity</b>										
17	<b>Stephen Boltzman's Constant [σ]</b>		<b>Radius of the Sun [R]</b>		<b>Average Distance Between Earth and Sun [D]</b>		<b>Solar Constant [Gsc]</b>				
18	5.67E-08		695700000		1.5147E+12		1367				
19											
20											
21			<b>Distance From The Sun, Ds</b>		<b>Sun Data (Rs/Ds)</b>						
22			1.4895E+11		0.004670689						
23											
24	<b>State Data</b>										
25	Average Temperature by Month										
26	<b>State</b>	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	
27	1 Alabama	46.5	50.5	57.5	64.5	72.5	79	81.5	81.5	76	
28	2 Alaska	17	20.5	26.5	36.5	48	55.5	58.5	57	48.5	
29	3 Arizona	56.5	60	65	72.5	82	91	94.5	93.5	88.5	
30	4 Arkansas	41.5	45	53.5	62	71	79	82.5	82.5	75.5	
31	5 California	46.5	50.5	54.5	58.5	65.5	71.5	75	74.5	71.5	
32	6 Colorado	31.5	32.5	40.5	47.5	57.5	67	74.5	72.5	72.5	
33	7 Connecticut	30	32.5	39	49.5	59.5	68.5	74	73.5	66	
34	8 District of Colum	34.5	36.5	44	54	64.5	73	77.5	75	68.5	

Figure 53. Image 1 of 2 of the Stored Data





	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		Varied Date & Time Inputs		Climate and Surroundings Varied Inputs								
3	Series	Date	Local Clock Time	Daylight Savings Time	Climate Type	r0	r1	rK	Surrounding Ground	Input Ground Reflectivity	Hour Angle [H]	
4	1	1	6	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-94.1	-1.6
5	2	1	8	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-64.1	-1.1
6	3	1	10	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-34.1	-0.6
7	4	1	12	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-4.1	-0.1
8	5	1	14	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	25.9	0.5
9	6	1	16	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	55.9	1.0
10	7	1	18	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	85.9	1.5
363	1032	358	10	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-34.2	-0.6
364	1033	358	12	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-4.2	-0.1
365	1034	358	14	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	25.8	0.5
366	1035	358	16	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	55.8	1.0
367	1036	358	18	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	85.8	1.5
368	1037	365	6	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-96.7	-1.7
369	1038	365	8	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-66.7	-1.2
370	1039	365	10	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-36.7	-0.6
371	1040	365	12	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	-6.7	-0.1
372	1041	365	14	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	23.3	0.4
373	1042	365	16	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	53.3	0.9
374	1043	365	18	0	Midlatitude Winter	1.03	1.01	1.00	dry bare ground	0.2	83.3	1.5

Figure 55. Image 1 of 3 of the Intensity on Siding Sheet

K		L		M		N		O		P		Q		R		S		T		U		V		W		X		Y	
Angles												Solar Intensity																	
Hour Angle		Declination		Zenith Angle		Incidence		G0h		τB		G B		Dirtr		G D		Difftr		Refr									
[H]	[d]	[Z]	[Inc]																										
-94.1	-1.6	-17.9	-0.3	104.9	1.8	104.9	1.8	-362.0	3.4	-1226.1	4623.0	262	131	-96															
-64.1	-1.1	-17.9	-0.3	84.2	1.5	84.2	1.5	143.5	0.2	23.8	232.7	32	16	6															
-34.1	-0.6	-17.9	-0.3	67.8	1.2	67.8	1.2	532.5	0.4	226.8	557.1	78	39	30															
-4.1	-0.1	-17.9	-0.3	60.2	1.1	60.2	1.1	700.7	0.5	351.1	614.3	87	43	44															
25.9	0.5	-17.9	-0.3	64.7	1.1	64.7	1.1	603.1	0.5	277.6	587.8	82	41	36															
55.9	1.0	-17.9	-0.3	79.2	1.4	79.2	1.4	265.7	0.2	66.4	346.5	52	26	12															
85.9	1.5	-17.9	-0.3	99.0	1.7	99.0	1.7	-220.9	8.4	-1850.3	11683.1	484	242	-137															
-34.2	-0.6	-22.7	-0.4	72.2	1.3	72.2	1.3	430.5	0.4	158.2	494.1	70	35	23															
-4.2	-0.1	-22.7	-0.4	65.1	1.1	65.1	1.1	594.4	0.5	271.3	584.4	81	41	35															
25.8	0.5	-22.7	-0.4	69.2	1.2	69.2	1.2	500.8	0.4	204.8	539.8	75	38	28															
55.8	1.0	-22.7	-0.4	82.9	1.4	82.9	1.4	174.7	0.2	32.0	256.7	38	19	7															
85.8	1.5	-22.7	-0.4	102.1	1.8	102.1	1.8	-296.5	4.6	-1369.2	6374.1	322	161	-105															
-96.7	-1.7	-13.4	-0.2	103.9	1.8	103.9	1.8	-338.9	3.7	-1262.6	5106.9	279	140	-98															
-66.7	-1.2	-13.4	-0.2	82.6	1.4	82.6	1.4	181.9	0.2	34.2	263.2	39	20	7															
-36.7	-0.6	-13.4	-0.2	65.1	1.1	65.1	1.1	595.2	0.5	271.8	584.8	81	41	35															
-6.7	-0.1	-13.4	-0.2	56.0	1.0	56.0	1.0	790.2	0.5	420.9	623.3	90	45	51															
23.3	0.4	-13.4	-0.2	59.6	1.0	59.6	1.0	714.6	0.5	361.8	616.6	87	44	45															
53.3	0.9	-13.4	-0.2	74.0	1.3	74.0	1.3	388.6	0.3	132.2	461.7	66	33	20															
83.3	1.5	-13.4	-0.2	94.1	1.6	94.1	1.6	-100.2	146.5	-14685.8	206364.3	4290	2145	-1040															

Figure 56. Image 2 of 3 of the Intensity on Siding Sheet

Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI
Focal Point Calculations										
Refr	Distance From The Sun, Ds	Sun Data (Rs/Ds)	Focal Point	d1 (Theoretical Distance of 0 Area for house Closer than FP)	d1 (Theoretical Distance of 0 Area for House father than FP)	Area of Suns Reflected Image at House Siding	Concentration Factor	Beam Intensit y on Siding	Total Intensit y	Tamb
-96	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	2780	2815	271
6	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	140	161	271
30	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	335	404	271
44	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	369	457	271
36	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	353	430	271
12	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	208	246	271
-137	147057163708	0.004730813396	11.6	12	3.2	0.41	1.37	7026	7132	271
23	147075895532	0.004730210872	11.6	12.0	3.2	0.41	1.37	297	355	275
35	147075895532	0.004730210872	11.6	12.0	3.2	0.41	1.37	351	427	275
28	147075895532	0.004730210872	11.6	12.0	3.2	0.41	1.37	325	390	275
7	147075895532	0.004730210872	11.6	12.0	3.2	0.41	1.37	154	180	275
-105	147075895532	0.004730210872	11.6	12.0	3.2	0.41	1.37	3834	3890	275
-98	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	3071	3113	275
7	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	158	185	275
35	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	352	428	275
51	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	375	471	275
45	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	371	459	275
20	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	278	331	275
-1040	147056822732	0.004730824365	11.6	12.0	3.2	0.41	1.37	124111	125217	275

Figure 57. Image 3 of 3 of the Intensity on Siding Sheet