Inquiry in Developing Trading Systems in FOREX Trading
An Interactive Qualifying Project
Submitted to the Faculty of Worcester Polytechnic Institute
In partial fulfillment of the requirements for the Degree of Bachelor of Science

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October 25, 2016 to October 26, 2017
Report submitted to esteemed faculty members
Acknowledgements

Our Interactive Qualifying Project team would like to thank all the key people for their contributions to the project.

Special thanks to Professor Frederick L. Huston for carving time out of his retirement to help us give finishing touches to the organization, grammar and structure of our paper.

Thank you also to our advisors, Professors Michael J. Radzicki and Hakim Hossein, for providing us with a strong foundational knowledge to effectively meet the challenges to complete this project.
Abstract

Maximizing profitability and minimizing risk in financial assets portfolios has been commonly solved with Mean-Variance Analysis (MVA). Trading systems, unlike investments, cannot be organized into portfolios through MVA due to discontinuous returns. Through a novel method to discretize trade system data into time series, MVA was then applied to custom forex trading systems and the optimized portfolio reduced risk by approximately 10% in comparison to several baseline portfolio configurations. The method provides a crude but general solution to optimizing trading system portfolios.
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Chapter 1: Background

1.1 Introduction

Both traders and investors have the same objective, they are looking to “beat the market” or to make as much profit as possible. The difference however, is in their approach. A trader is usually looking for an opportunity to make quick, short-term trades enter. Their trades might last anywhere from mere seconds to as long as several days, usually not more. On the contrary, investors enter the market and hold their position for much longer time. The longevity of their holding periods usually lasts anywhere from a year to more than a decade.

Traders are not focused so much on a particular company or a government currency, but rather on the news or other factors that may create predictable, short-term fluctuations of an asset that they can profit on. On the other hand, investors analyze more the company missions, government economies (in relation to currency) and invest in assets they believe will appreciate sometime in the long-term future.

That is why, investors may be less sensitive to the short-term fluctuations of the asset price. For example, the stock price may surge or decline to some extents due to recent news on media, but as long as it keeps its general direction favorably upward, the investors are more willing to buy and hold even due to unwelcome fluctuations. On the other hand, a trader in the same position would likely consider this fluctuation as a sign of a losing trade and quickly get out or let the trading system get them out of the trade automatically.

An investment can also be looked at as a gradual building of wealth that requires more long-term thinking. It may not offer short-term gratification, yet through compounding, it is possible to become wealthy if one holds the position in the financial markets for a long enough time.
Where an investor is patient, a trader may be less so in their trading style. Trading offers more short-term rewards, which will most likely require a higher engagement from traders’ side. Hence traders usually have a tendency to make quicker decisions because they most likely activate much more transactions during the day than typical investors do (by typical investors, we mean investors who invest in stocks, bonds and similar assets which generally require longer holding times before significant profit has been made).

1.2 Macroeconomics and Business Cycles

Macroeconomics is the study of how the aggregate economy, national or regional, behaves. It looks at the economy as a whole and focuses less on the individual decisions of companies or people. It deals with the structure of the economy and how its different components perform and interact with each other. Some of these components consumption, investments, savings and inflation. This is just to name a few. But the economy is never stable. In fact, it is always changing and it tends to move in cycles. Some call it business cycle, which are basically different stages of the economy such as expansion, peak, contraction and trough. The economy goes through this cycle over and over again and there are many factors playing in the background which determine or reflect which part of the cycle an economy should find itself at.

Besides these cycles, Macroeconomics also studies the relationships among different tradeable assets. With economy approaching a different stage in its business cycle, the relationships among assets is also changing. The relationship between stocks and bonds during contraction period of an economy may be different from how these two assets interact with each other during expansion.

Other economic terms which are pertinent here are inflation and deflation. Inflation means an increase in prices of commodities, goods and services; it happens at the same time when the economy is expanding, in another words when the output of all goods and services is going up. Similarly, deflation describes drop in prices and belongs to the contraction period of the economy. Economists use GDP to measure the
total value of goods and services provided in the economy and is a useful tool for observing business cycle of an economy.

1.3 Efficient Market Hypothesis vs. Behavioral Finance

Before we answer the question of what assets our system will trade, let’s review important economic theories which are pertinent to our project.

In finance there is a theory which states that it is impossible to beat the market; the theory supplies the argument which argues that since all pertinent information is automatically incorporated in the asset prices, there is no opportunity to make profitable traders or investments because assets are always trading at the fair price. For example, no stock trader or investor can make profit through purchase of an undervalued stock because there be no undervalued stocks. All stocks are continuously adjusting and reaching their fair prices based on all the relevant information presented around them. This theory is called Efficient Market Hypothesis (EMH) and it pushes the idea that studying the past in order to predict the future, in relation to the markets, is futile. Since asset prices can only reflect information that is available today, no one can know what new information might bubble up tomorrow and hence no one can predict what new prices are going to be (Investopedia, 2017).

There also is an opposing school of thought, which says that while EMH does make sense, it fails to explain the reasons for big downfalls in the markets such as the crash of 2008; it also fails to explain excess volatilities in the market and other speculative bubbles. This school of thought is called Behavioral Finance which attempts to explain these unexpected crashes and fluctuations in the markets. It says that there is a group of investors who are acting in very irrational ways and hence contribute to the market irregularities we observe from time to time. Some investors believe too much in their ability to pick winning trades, or incorrectly assume that there is some truth to the so called Gambler’s Fallacy, which wrongfully states the more frequent the happenings of the event, there is going to be a less chance that the same event happens next time; in stocks for instance, a Tesla stock may be going down for too long and
because of that an investor may incorrectly assume that Tesla stock has a higher chances of soaring up due to past performance. According to Behavioral Finance, these type of irrationalities cause some investors to make irrational decisions, which in turn cause irregularities in the markets that cannot be explained by EMH. Hence, one can argue that markets do not necessarily price their assets as fairly as expected and there may be a chance to “beat the market”.

One should also keep in mind that sticking strictly to one school of thought over the other may not be the wisest choice. It is more effective to build a trading system which attempts at incorporating reasonable elements from both theories. For example, we can build a trading system such that it trades low-risk assets to protect itself from excess volatilities; and alongside also trade high-risk assets in order to maximize one’s profits. At the end of the day, none of the two theories have so far been proved to be absolutely practical for real-life trading. However, if we had to choose, our trading system would lean more towards the ideas proposed by Behavioral Finance because it gives, from our perspective, more solid explanation to market riskiness and irregularities, which in turn may promise higher returns assuming the trading tools are used wisely. (Investopedia, Staff).

In the following paragraphs, we are going to touch on what is Macroeconomics and then quickly get into the 5 tradeable assets and which one of these we chose to trade in our trading system.

1.4 Why Choose Trading

After careful reflection, our team came to a decision to focus on trading rather than investing. We made this choice because trading fits well with our team’s ambition to devise a trading system that can immediately be tested for its worth in the real markets. Once the system is complete, anyone can use it and see almost immediately if it makes any profit. To be able to test the investment system is not possible because by definition, investing is a long-term game and hence users will have to wait for years before the system shows results. By the way, when we say “results”, we are not talking about backtesting the system, in case a reader wonders; rather, we talk about actually getting the feet wet in the real market and
producing measurable outcomes. To restate our choice, we choose trading because it is a short-term game which happens at much shorter timeframe and thus gives us an opportunity to immediately put the system to use, control it and potentially improve it later on.

1.5 The 5 Tradeable Asset Classes

As a trader or an investor, one’s aim is to build a profitable portfolio by buying/selling from five different asset classes. These asset classes are Equities, Bonds, Commodities, Futures and Currencies. There are several analysis tools that can help an investor or a trader to decide in what proportion to invest in two or more of the listed asset classes for the purpose of diversification (Investopedia, 2017).

1.5.1 Equities

Equity or stock represent a share of ownership in a publicly traded company. Companies offer a share of the ownership through IPO (Initial Public Offering) and sell it on equity markets around the world. Selling stock helps increase the company’s investment capital without having to take loans out from banks or use other sources of finance.

The company however, will have to give a certain amount of ownership and in some cases dividends based on to how much the trader has invested. Because the equities are bought and sold in markets, the investors are able to buy and sell shares relatively easily. Since stock exchanges are heavily regulated, access to shares is again relatively straightforward and considered a safe approach.

This market is the most common market for both traders and investors for all of the asset class markets. Companies that are well known such as Google, Amazon, Coca-Cola and Berkshire Hathaway have their shares available in these types of markets. Namely, NASDAQ or NYSE (New York Stock exchange).

As a trader or investor, the routes used to trade equities are different because of how the equities market are heavily regulated and taxed. A buyer can only buy stocks through a licensed brokerage firm. In order
to use these firms, there will be a premium they will make the buyer pay to use their services. These brokers will have different premiums for the classification of investors and traders.

In order to be classified as an investor, the buyer will need to buy and sell stocks on longer time frames. In order to be classified as a trader however, one will have to trade for more short-term gains. The high frequency of the trades used will classify a buyer as a trader. Since brokerage firms are mainly focused on making profits, they will charge premiums based on the frequency of their client’s trades. Determining how much tax and premiums an investor will have to pay will ultimately come from this classification. Therefore, the frequency of the trades will determine how much premium that will be paid and also the tax (based on short-term or long-term gains).

1.5.2 Bonds

Bonds are another major asset class. They are essentially loans an investor gives to a corporation or government. The issuer promises to pay an agreed upon interest rate during the life of the bond and also to repay the principal when it "matures," or comes due after a set period of time. At this date the bond will become active and will make the issuer pay the interest and the principal all together.

The main categories of bonds are municipal bonds, corporate bonds, U.S. Treasury bonds, notes and bills. Bonds can only be purchased by special investors in credit markets who are allowed (licensed) to buy and/or sell bonds.

The safest type of bonds are those issued by governments. They are safe because they are guaranteed by a government. Being able to print its currency (having a fiat currency) will make its ability to repay these debts helps in having this high credit. A country’s ability to pay back loans is a good indicator of their economic strength. These bonds have high credit quality that makes investors eager to buy more. These bonds are the safest type of bonds but don’t produce the highest returns.

Bonds issued by municipalities are relatively riskier than those issued by governments. These are usually used to fund infrastructure projects and other capital-intensive projects. These are riskier because they are
not able to print their own currency. Municipal Bonds attract investors as the interest paid for the bonds are usually tax-free.

Bonds issued by corporations are the riskiest but usually have higher interest rates to attract potential investors. These have a variety of interest rates and maturity dates that are categorized into three main lengths. Maturity of corporate bonds with less than five years are called short term bonds, five to twelve years are intermediate bonds, and more than twelve years are called long term bonds.

Bonds can also be bought and sold before their maturity date. This helps provide liquidity in a business and if the bond has a higher interest, it will become more of an interest to buyers as it will yield more as it goes. However, one should be wary of bonds with higher interest rates as there are many corporations who try to lure investors with these valuations and do not have stability to repay the bond.

1.5.3 Commodities

Commodities are an asset class that refers to physical goods. These goods can be traded electronically in a market environment. There are four sectors in commodities, the agricultural sector, the energy sector, the metal sector, and finally the livestock sector. These account for about 100 primary commodities and can be used as a source of asset for different traders.

Commodities can also be classified into three types, raw commodities, hard commodities, soft commodities. Raw commodities are the raw materials that are bought and sold in a commodity market. Hard commodities are commodities that are extracted through mining and extracted such as oil and gold. Soft commodities are the agricultural products such as wheat and livestock. This asset class is international and is the key reason behind the price changes in the raw materials we have today.

Asset classes like this are really made for larger companies that use the products themselves, but it is also open to traders who want to buy and sell commodities for profit. This asset uses one of the most basic economic principles (demand and supply). Unlike the other asset types, it is actual goods that are being circulated and traded on. It is especially interesting to trader due to the option of using futures contracts.
1.5.4 Futures

Future contracts are contracts between two parties where they agree to buy and sell a particular asset of specific quantity at a predetermined price at a specified date in the future.

For example, if an oil seeds producer wants to sell something he hasn’t produced yet but is afraid the prices will vary, he could just enter into the future contract with another buyer to sell his produce at a specified price and will make a set amount of profit even if the market has the price of that produce higher or lower. The commodities do not actually get delivered unless the contract is never resold or covered.

Many companies have taken advantage of these future contracts in their respective fields especially when prices are fluctuating for a given commodity. Examples include southwest airlines where they cut costs significantly by buying futures contracts for oil and successfully “beat the market”.

1.5.5 Currencies

Another major asset class is currency. This is a global market in which currencies are traded. It has the highest liquidity of all of the asset classes and also the highest valued market at approximately $5 trillion in movement each day. The market consists of central banks, commercial companies, investment management firms, hedge funds, retail forex brokers, and investors. Although the market is huge, the amount of money allotted to private citizens or non-government institutions is actually quite small.

The forex market is considered OTC (Over-the-counter) because it is decentralized and it is electronically run continuously 24 hours a day. It is also not as regulated like the other asset classes. Relatively speaking it is easier to get into the forex market compared to the challenges of entering the other asset classes because of the barriers to entry such as initial capital.

This market deals with currency strength relative to other currencies which lends itself to a number of different economic indicators. By looking at understood indicators such as the country’s political stability,
economic policies and bank’s interest rates, traders and investors can make informed decisions based on fundamental analysis.

High interest rates promote a strong currency. Because investors are looking for high yielding investments around the world that means if a currency has a higher interest rate given to it by its central bank, it will likely become strong. For example, if the U.S. treasury decides to increase the rates then the value of the dollar increases because foreign investors will invest in the dollar. This creates an increase in demand hence an increase in price. The opposite is true if the interest rates decrease.

The political climate of a country also affects the strength of a currency. A well-established rule of law and a history of constructive economic policies in a government make it an attractive investment. It is not that surprising that investors are willing to invest in currencies used by countries with stable governments. For example, currencies which are national currency of only one country are far riskier investments due to the risk of political turmoil in that region. This creates an unstable and volatile environment where investments are more likely to sink rather than bring dividends.

Finally, the economic policies given by a currency’s government can affect the attraction of investors. Policies are the implementation of general goals that a government has determined are necessary for it to survive and thrive in its environment. Examples of this include the balance between imports and exports in trade and how inflation rates are controlled.

Because of the many currencies that are used around the world, these currencies are traded in pairs. The eight most traded currency types are shown in the table below. One should also know that ‘the dollar is king’. Trading with the USD accounts for about 85% of all transactions with EUR/USD and JPY/USD making up 40% and 19%, respectively.
<table>
<thead>
<tr>
<th>Currency symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>American Dollar</td>
</tr>
<tr>
<td>EUR</td>
<td>Euro</td>
</tr>
<tr>
<td>GBP</td>
<td>British Pound</td>
</tr>
<tr>
<td>JPY</td>
<td>Japanese Yen</td>
</tr>
<tr>
<td>CHF</td>
<td>Swiss franc</td>
</tr>
<tr>
<td>NZD</td>
<td>New Zealand Dollar</td>
</tr>
<tr>
<td>AUD</td>
<td>Australian Dollar</td>
</tr>
<tr>
<td>CAD</td>
<td>Canadian Dollar</td>
</tr>
</tbody>
</table>

Table 1. List of FOREX currencies (Investopedia, 2017)
1.6 Why Forex

There was no single reason for why we chose to trade currencies over other assets. We definitely did not want to take on a labor of analyzing more than one asset for a trading system because it would be too complex and not effective. By focusing on only a single asset, we can direct all of our resources in figuring out all the different systems for trading Forex effectively. Compared to other tradeable assets, here are the reasons why we prefer to trade currencies:

1) Forex market is very volatile, giving traders opportunities to make significant returns off of high price changes.

2) Forex is the most liquid market, leaving traders with a huge room for experimentation. We have the ability to make lots of short-term trades on a daily basis and hence continuously improve our trading system via testing.

3) Leverage. Forex market offers a substantial leverage of up to 50:1, which is significantly higher than the general leverage availabilities for stocks (only 2:1).

Based on aforementioned reasons, our team decided to trade Forex. Before creating a system however, we have to make just one more decision with regards to whether we are going to stick to only one method of analyzing the markets or employ multiple at the same time. The following paragraphs talk about that.
1.7 Market Analysis

There are two main types of analysis used by investors and traders to predict the future price of a specific asset. One of them is fundamental analysis, which considers financial statements of the company, news and events and uses obtained knowledge to predict whether the price of a tradeable asset will rise or fall.

The second method for analyzing the markets is called technical analysis. The main reason why technical analysts believe their methodology is effective is that they assume that the history tends to repeat itself. There are certain patterns that the changing price has been making throughout the history of the market. If one can detect the pattern before it completes itself, there is an opportunity to make a very good guess on where the price might proceed to be – whether it goes up or down. In short, prices tend to move in trends. This can be very profitable for the trader. For example, if the trader catches the price surge at an early enough stage, he/she can trade an asset immediately and gain significant profit going forwards.

Seeing trends is one of the most useful ways of making profitable trades. A trend is a general direction to where the market is heading. The markets rarely move in straight lines. They are messy. In order to locate a trend, there is a general rule of thumb that technical analysts usually follow: if one observes a series of higher highs and higher lows in the price movement, then it is an uptrend. If there is a series of lower highs and lower lows, it is considered to be a downtrend. Detecting a trend line can also help traders identify resistance and support areas. These are areas where the market rebounds lower and higher respectively. If the market is observed to rebound lower more than twice at approximately the same area, then it is a resistance area. Conversely, if the market rebounds higher multiple times, then it is a sure sign that the market is touching the support area. Multiple rebounds validate the existence support/resistance area.

Detecting a rebound serve as an indicator for where the market may go. If one identifies the support or resistance area, he/she can predict that the next time the price approaches the same area, the market is
more likely to rebound and go in the opposite direction. The trader can choose to activate the trade at the precise moment when the rebound happens and then hold the trade as it goes in its newly established trend until gaining significant profit.

![Support and Resistance](image)

*Figure 1. Support & Resistance. This figure gives an example of support and resistance (Trader’s Day Trading, 2017)*

Another tool for technical analysis is an identification of breakouts. Breakouts are moments in time when the market, instead of rebounding off the resistance or support level, breaks through and continues on its trend. After breakouts, the price tends to move in the same direction for a long enough time allowing the trader to make significant profit.

While an investor may be confident in his/her ability to predict the market, there is always a chance of the market moving in the ‘wrong’ direction. Rumors around central bank’s decisions, political and other unexpected events can contribute to this. Investors cannot control the market, but what they can control is how much risk they take per trade. A stop-loss order is a very powerful tool for risk-minimization. The order is placed with a broker allowing the traders to get out of the trade if the market hits a percentage drop of the original price. For example, if an investor is betting that the EUR/USD currency will go up in value and instead, it goes down, the investor can order a stop-loss in advance and get the trader out of the
trade once the market hits the stop-loss. The order is placed close enough to the entry price to minimize losses. Yet, investors want to make sure the stop-loss is also far enough from the entry price in order to allow the market to go through minor fluctuations. This helps investors avoid the possibility of automatically canceling a winning trade (J. Wiley & Sons, 2012).

1.8 The Intermarket

Inter-Market Analysis is one type of technical analysis where one looks at the relationships between different asset classes, such as securities and bonds. These relationships vary along with cycles in the aggregate economy (Investopedia, 2017). For example, if the economy exhibits inflationary characteristics, then the stocks and bonds are moving in the same direction, most likely upwards. If the economy is deflationary, the stocks or securities and bonds have an inverse relationship. For example, if the interest rates fall, then more people will borrow more and spend more money. In turn, consumer spending increases. Company revenues will increase and consequently the stock prices of these companies may increase as well (Amadeo, K, 2017). Furthermore, because the interest rates have fallen, the demand for some bonds will be higher. This higher demand will drive up the prices on these bonds. And inversely, with a fall in interest rates, an increase in the value of stocks and bonds will be observed.

Out of these five asset classes, long-term investing in stocks has proven to be one of the most profitable endeavors. In the short-term, currencies may be better for a skilled trader. The disadvantage of choosing to invest in stocks is that it is relatively riskier and without a careful research, an investor may invest in the ‘wrong’ stocks and lose significant portions of their capital.

Bonds can be an option for lower-risk investment. Another attractive feature of bonds is that it pays on a regular basis just like fixed income. The disadvantage of bonds however, is that its returns are lower compared to stock returns. Another advantage of currency trading is that it requires minimal capital to get started, compared to the amount of capital one has to invest in stocks and bonds in order to be allowed to
participate in those markets. Also, the liquidity in Forex markets is the highest of the markets and that means it is possible to make a very large order of currency without any large price deviations. The disadvantage in currency investment is that the market is very volatile and consequently, there is a high risk of huge loss.

For commodities, the upside is that there are many opportunities for a particular commodity to go up in value and that increases the chances for some investors to see if their investments make profit. This is driven by the basic economic principle of supply and demand and serves another angle investors can use to perform fundamental analysis. Unlike stocks and bonds that sometimes go down in value due to inflation, the commodities go the other way. This is because during inflation, the prices of goods increase and so do the prices of commodities. The downside of investing in commodities is that even if some investor bought a commodity that is going up in value, unlike bonds and stocks, it does not provide dividends. The investor will have to sell the commodity to gain any potential profit from it.

### 1.8.1 Margins

For margins, currency investments and some very short-term bonds offer very low initial margin requirements; usually 1 or 2%. This means that a trader or an investor will only have to pay for the 1% of the whole investment risked; the other 99% is covered by the broker. The maintenance margin requirement is similar. For trading stocks however, the margin offers are less attractive. For example, in some cases, an investor has to pay for 50% of the whole investment to trade the 100% of that investment and is also required to maintain 30% of the whole investment in a margin account in order for the buying-on-margin process to work.

In the US, the minimum amount of deposit every day-trader should have is $25,000. If the balance falls below this amount, day-traders will not be allowed to invest in stocks. Usually, investors will need about $30,000 at least in order to protect themselves from going below $25,000 in case they experience loss. Also legally, day-traders are not able to risk more than 1% of their whole account balance in one day of
trading. Therefore, in this particular example, the investor may not put a stop-loss that allows for more than $300 loss out of the whole account balance.

1.8.2 Indicators

Indicators are statistical metrics computed from the price level and other quantifiable properties of the market of the asset being traded. They often allow a number of parameters to be specified. Technical indicators are indicators that exclusively rely on information about the price level and volume of the asset being traded and convey information regarding tendencies for the price levels movement. Some indicators simply predict that a reversal in the current movement may occur soon, while others may predict the range in which the price level may move when it does move. Indicators are often used in tandem with one another to better assign a degree of confidence to the predicted price movement. Even seemingly conflicting information from separate indicators may have its own interpretation. Technical indicators will be simply referred to as indicators.
1.9 Trading Systems and Automation

A trading system is a system capable of determining when and what interactions are to be made with a market based upon well-defined criteria, or simply put a set of rules. Such systems allow for an objective approach to trading and reduce human error that is often the result of emotion, stress, and lack of discipline or commitment to preplanned criteria. They also reduce costs and time associated with trading as their algorithmic nature makes their decision processes quick and efficient. A well-defined system allows for the remote execution of orders independent from the twitchy fingers of the trader; this often takes place in the form of a service selling the privilege to its users to have their capital managed by their proprietary trading system.

1.10 Electronic Systems

Electronic trading systems make up the vast majority of systems in use today, although technically speaking any system could be performed by any actor. Electronic trading systems programmatically define rules, parameters, and in many cases initiate a transaction with the market over the internet (with brokers acting as middlemen). The internal calculations of most live commercial trading systems can be performed nearly instantaneously with modern computers. This means real-time reaction to the market and other related financial data is at the disposal of almost anyone with a computer and internet connection. An exception to this includes ‘scalper’ trading systems that require calculations and transaction decisions to be made within microseconds and are essentially exclusive to specialized professionals. For our purposes, the terms ‘trading system’ and ‘system’ will refer to electronic non-scalping trading systems (Investopedia, 2004).

Like with most applications of a piece of technology, patterns have emerged among trading systems and they have been standardized in a number of ways. Firstly, their need to simultaneously support
complexity, customizability, and understandability to the user often require that they be implemented with higher level programming languages. Most commercial grade trading systems are implemented in languages created for that express usage. Secondly, because access to financial data and the management of one’s personal assets can become a cumbersome task even with the proper programing support, there is also software which builds on top of that functionality. An example of this are trading platforms which are larger pieces of software that come with an arsenal of common financial computations, can keep record of relevant account information, and can communicate transactions with the market or with a broker over the internet all out of the box.

1.11 Trading Criteria

The trading criteria of a system are up for the developer and/or user of the trading system to specify and, theoretically, can be arbitrarily complex. In practice however, the criteria are limited by the programmer’s ability to capture the complexity of the criteria and system’s abilities to perform its calculations quickly. For most well defined criteria based on relevant data, this is rarely a problem. A trading system’s criteria for performing various market interactions are what makes the system unique, while the underlying program that accomplishes this can be written in a number of languages and architectures (Investopedia, 2004).
Chapter 2: Methodology

When choosing indicators for trading system, we wanted to make sure that our chosen indicators complemented each other. We have developed three systems; out of three, two contain only one indicator and the third system is a combination of two, complementary indicators. The final product of our system is the aggregation of all three systems. Traders are welcome to use only one system; however, based on trial and error and the results of measurements, we have found that one indicator is never enough to make a solid trading decision. Using one system from our system is not going to provide the trader with much chance to beat the markets. However, with the right combination of system, where one indicator confirms the other, it is possible to make a trading decision with much more confidence.

After some of research, we have found that there is a large arsenal of indicators used for technical analysis and they can be divided into three categories – trend, volatility and momentum. Some of these indicators, such as Moving Averages, MACD and Bollinger Bands, may fall under two categories such as volatility and trend. Yet, some indicators like RSI and Stochastic belong strictly to only one category – momentum (A complementary approach to trading technical indicators, 2012).

After learning about this, we quickly realized that the best way to approach our trading system is to find indicators that are complementary. That is, to combine a momentum indicator with either a trend or a volatility indicator. In other words, our trading system contains three systems and each system will contain one or, at the most, two indicators each from a different category. The aim is to avoid signal redundancy, which is caused when a trader is analyzing the charts using the indicators of the same type (Investopedia, 2017). For example, a trader should not be using RSI and Stochastic indicators together. Both are momentum indicators and essentially represent the same thing – the strength of the trend. It makes more sense to use together different category of indicators because it will help the trader gather more diverse information about the particular, traded asset and avoid collecting repetitive, redundant data.
Our advice is to complement a momentum indicator such as RSI with a volatility indicator such as ATR indicator or Bollinger Bands. A volatility indicator, at the core, measures how much price fluctuation there is about a mean. High volatility means that the prices are going to be moving up or down frequently. High volatility is also associated with high risk. It is almost the opposite of momentum and that is what makes it such a strong complement. A trader can use a momentum indicator to understand if there is a strong trend in the markets and then confirm by looking at the volatility indicator and see if the volatility is low. This way, a trader has a method to reconfirm his/her observations using a complementary indicator.

Our first system is called a momentum system. It is using RSI as an indicator. The concept of how RSI works is very simple to understand. It is measured on 100-point scale. If RSI moves beyond the 70-point mark, the asset that the trader is looking at is overbought. If RSI moves below 30-point mark, then the asset is oversold. In other words, it indicates that the trend is getting stronger and moving in the same direction with a possibility of a future reversal. RSI is looking at the size of the opposing candlesticks in order to produce its value (See Figure 2).

The second system in this system employs complimentary indicators: one momentum and the other volatility indicator - RSI and Bollinger Bands, respectively. Bollinger Bands indicator is calculated using the standard deviation of a changing asset price. Wider bands in the indicator mean that the volatility in the market is high. If the bands are contracting, however, it means the volatility is decreasing. From this information, one can assume that RSI indicator will behave opposite to Bollinger Bands. If the market has a strong trend, the momentum need to be high because the market is moving in one direction. Consequently, if the market is strongly moving in one direction, then the volatility of this market will be relatively lower. If a trader, using both indicators, suspects a strong trend confirmed by RSI indicator, and a low volatility confirmed by Bollinger Bands indicator, then one’s chances of being correct in the prediction of the price movement increases and that is what we are looking for. (If a candlestick has a long body along with moving averages, this is a good indicator of high volatility.)
The third and the last system is to use Moving Averages. This indicator belongs to the category of trend indicators. The benefit of using a trend indicator is to predict the big picture – where is the market heading in general. While momentum indicators may provide a more direct insight to how the market is doing, Moving Averages is not as easy to understand and derive conclusions from. Yet it is still very useful. Trend indicators such as Moving Average incorporate the previous changes in prices in its value outputs and that helps a trader to understand the general direction of the market in the future.

The most optimal way to trade is to use three indicators of different categories together. Each indicator will show the unique characteristic of the market which may serve as a separate or a re-confirming signal of where the market is going.
2.1 Overview of Modern Portfolio Theory

Harry Markowitz created the Modern Portfolio Theory in an effort to help investors choose stocks and refine portfolios by maximizing profit and minimizing risk. It is a theory that relies on statistical representations of the performance and riskiness of the asset classes in the portfolio.

A basic concept for any trader/investor is to diversify or spread out their portfolio to get better returns at an acceptable risk. Modern Portfolio Theory (MPT) is a way of managing a portfolio with a variety of assets ranging from stocks to currency and optimize the return while also to decreasing the amount of risk taken as a system. The theory behind this methodology is to reduce the riskiness of the portfolio by combining many assets together. In addition to optimizing the portfolio for a certain risk level the theory also allows for the benefits of diversification to be quantified (Investopedia, 2017).

2.1.1 Mean Variance Analysis

Now that the trading system is set, the next step is to maximize the returns of the trading system for a given amount of risk. Firstly, the decision has been made to construct a portfolio, much like stocks portfolio; but instead of having different stocks, this portfolio contains different systems – namely the three systems that have already been discussed in the paper. Since the portfolio in this project only contains one asset, currencies, there had to be an alternative way to diversify a trader’s investments. For example, in portfolios that contain multiple asset classes, the most reasonable way to diversify this portfolio is to put a percentage of one’s investments into gold, a percentage into stocks and the rest into mutual funds (for example). In the case of this project, where there is only one asset class and one currency pair that is traded, what diversifies the portfolio is the three different systems for our system. If one looks at the asset prices of this portfolio, then one can realize immediately that since all systems deal with the same currency pair, the price movements of the given assets across all three systems will be identical.
However, since the trading systems are fundamentally different, the resulting returns are also different. The varying returns is already enough basis for this project to be able to calculate the risk associated with each trading system. Therefore, the fact that we are looking to construct a portfolio only around one currency pair is negligible for the purpose of this project.

Secondly, the decision was made on how to calculate risk for this portfolio. In technical analysis, there are multiple ways to calculate a risk associated with a particular investment. There are tools such as alpha, beta, standard deviation, R-squared and Sharpe ratio. Our team chose standard deviation as the basis for calculating all risks for the portfolio (Greenblatt, J, 1999).

The standard deviation has been calculated for each system using the rates of return over a 60-month period. Each system has been backtested over this period, where a monthly rate of return has been calculated; and based on that data, through mathematical calculations presented near the conclusion of this paper, we were able to quantify the standard deviation for the given rates of return that can be represented as a risk. (Bodie, Z., Kane, A., & Marcus, 2017).

If one looks at results in Table 3, a system that complements two indicators from different categories has a better risk to reward ratio. This provides a solid evidence that our assumptions have been correct and the results do increase when indicators of different types are complemented rather than trading with only one indicator or with more than one of the same category.

There is a need for determining the proper allocation of capital while minimizing riskiness across multiple trading systems for the currency markets (FOREX). This need fits the general motivation for Mean Variance Analysis (MVA) because of the tendency for trading systems to correlate just like stocks and other investments, yet it is not applicable directly. In this project, trade data was manipulated into a form where MVA can model the profitability and riskiness of trading systems.

A fundamental problem in diversified asset investing is how one can allocate capital in order to maximize profit and reduce risk associated with correlated securities. Harry Markowitz founded what is now known
as Modern Portfolio Theory (MPT) by initially exploring MVA, a part of MPT that is still fundamental to
its modern usages. Trading systems, as opposed to investments, cannot directly benefit from this form of
investment centric modeling. Without proper allocation methods, naive approaches to asset allocation
such as equal weighting, weighting based on overall profitability, and weighting based on overall variance
may be used and cause a reduction in the portfolio's profits and/or allowing for dangerously large
drawdowns.

The application of MPT to the three trading systems we developed has produced a portfolio which
performs significantly better than baseline portfolios. One of the baselines for testing, a portfolio which
assigned weights to trading systems in proportion to their overall profitability, had a roughly 10% larger
variance or risk for the same profitability compared an optimized portfolio through MVA. Another
baseline, a portfolio with equal weights, had 10% smaller profitability for the same risk compared to an
MPT portfolio. It should be noted these metrics are based on the metric used for modeling, the gain in
price level, and would manifest themselves even more dramatically if profitability and risk were
measured for the trade returns of the systems with traditional margin-based position sizing.
2.2 Trading Systems

2.2.1 Trade system #1 Swift RSI

In the first system, we use one indicator called RSI (Relative Strength Index) and make a trading decision based on its behavior as the market progresses. After experimenting with the RSI indicator, we started to see the value in making a trading system very simple. That is, after observing how helpful one indicator can be for making a trading decision, we intuitively came to a decision to make all three systems simple, using only one or maximum two indicators per one trading system.

As mentioned above, our first system uses only RSI. The graph of RSI oscillates from values 0 to roughly 78.0 units. Between values 30.0 and 70.0, RSI graph is oscillating the most frequently. If RSI goes over the upper limit value of 70.0, this means the currency pair that we are looking at is being overbought.

When a specific currency is overbought, this is an indication that the traders will start selling the currency and that of course will lead the currency pair to go down.

The picture below explains it graphically:
Analogously, if the RSI goes under the lower limit value of 30.0, this means the currency is being oversold and the given currency pair will start going up. The RSI indicator used a 14-bar period as this is the default and commonly utilized period. The market entry conditions were that if the RSI indicator crossed the entire threshold 30-40 (crossed above RSI=30 and above RSI=40) in the current bar, then a long trade was opened and if it crossed below the entire 60-70 threshold (crossed below RSI=70 and below RSI=60) then a short trade was opened. The exit conditions were that if a long trade was open and the RSI went below 28 or above 50 it would close that trade while if a short trade was open and it went above 72 or below 50 it would close. The 28/72 exit condition reflects that the RSI indicator has reversed its momentum and a fake signal of trend most caused by extreme random fluctuation in price level. The 50/50 exit condition reflects that momentum may be spent and the price level will not confidently continue its trend. The stop-loss was set to 30 pips. The stop-loss, inner threshold for RSI crossing, and levels in RSI for momentum being spent where optimized for on intervals of size 10.
2.2.2 Trade System #2: Swift RSI + Bollinger Bands

The indicators used in this system were RSI and Bollinger bands. The parameters for the Bollinger bands were a 20-bar period for its moving average and 2x multiplier for the deviation to be used in the outer bands as is default and popularly used. This an adaptation of the Trading System #1, Swift RSI, which further constrains entry conditions and widens exit conditions. If the RSI crosses the previously described thresholds and additionally the price-level goes outside of the Bollinger bands, then that is an entry signal. For example, if the during the current bar the RSI indicator crossed above 30 and crossed above 40 and the price level is below the lower Bollinger band, then a long trade is opened. In the first system, we use one indicator called RSI and make a trading decision based on its behavior as the market progresses. After experimenting with the RSI indicator, we started to see the value in making a trading system very simple. That is, after observing how helpful one indicator can be for making a trading decision, we intuitively came to a decision to make all three systems simple, using only one or maximum two indicators per one trading system.

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Analogously, if the RSI goes under the lower limit value of 30.0, this means the currency is being oversold and the given currency pair will start going up. The RSI indicator used a 14-bar period as this is the default and commonly utilized period. The market entry conditions were that if the RSI indicator crossed the entire threshold 30-40 (crossed above RSI=30 and above RSI=40) in the current bar, then a long trade was opened and if it crossed below the entire 60-70 threshold (crossed below RSI=70 and below RSI=60) then a short trade was opened. The exit conditions were that if a long trade was open and the RSI went below 28 or above 50 it would close that trade while if a short trade was open and it went above 72 or below 50 it would close. The 28/72 exit condition reflects that the RSI indicator has reversed its momentum and a fake signal of trend most caused by extreme random fluctuation in price level. The 50/50 exit condition reflects that momentum may be spent and the price level will not confidently continue its trend. The stop-loss was set to 30 pips. The stop-loss, inner threshold for RSI crossing, and levels in RSI for momentum being spent were optimized for on intervals of size 10.
Gross out of the Bollinger bands reflect also somewhat reflect overbought or oversold conditions and refine the condition for expecting that the price-level will follow a strong moderating trend. The exit conditions are that of the RSI swift or if the price-level crosses the mid-band. The crossing of the mid-band reflects that the momentum of a moderating trend may be over shortly, and locks in profits before reversals. The stop-loss was set to 50 pips. The stop-loss was optimized on intervals of size 10.

2.2.3 Trade System #3: MA Crossover

The only indicator used by this system is the simple moving average (SMA). There are 3 SMAs, one slow, one medium, and one fast with period parameters of 20, 10, and 5 respectively. This system relies on the classic crossover system where faster MAs crossing slower MAs all moving the same direction signals a trend in that direction. If the medium MA currently is above the long MA, the short MA has just crossed the medium MA, and the long MA has increased since bars 10 ago, a long trade is opened. The crossing and trend for the long MA are reversed to produce the rules for opening a short trade. The exit conditions were simply that if a faster moving MA went against the direction that the MAs were traveling at time of trade, that trade would be closed. The stop-loss was set to 20 pips. The stop-loss was tested on intervals of 10, and the number of bars from which the long MA must have moved in a particular direction was tested on intervals of 5.
Figure 4: Graphical explanation of Moving Average.
2.3 Optimizing and Analyzing Trading Systems

Back testing is the process of applying a trading system to market data sets and interpreting the results. The most important parameters which essentially constitute the frame of the backtest are the asset being examined, the date range, and the granularity of time with which price bars are formed. Interpreting backtests often involve looking at the general shape of the account balance curve and looking for things like how large and frequent gains and losses tend to be, noticeable drawdowns, and overall metrics such as profit. If a certain pattern manifests itself across multiple time frames and assets, the patterns may be extrapolated to future conditions and help inform an opinion on investing in a trading system. Interpretation may also overlap with debugging the programmed system by examining the price-level and indicators and seeing if the programmed trading system behaved as it was intended and if certain condition produce unforeseen results.

Optimization can be used to refine trading system’s behavior by reviewing its performance when back tested. Significant failures for a system to meet its objectives may call for more severe modification or rejection of the system, rather than optimization. Reasonably performing systems should be optimized by exploring a number of parameters used in the system, usually in the form of parameters for indicators and stop-losses and position sizes. One must however avoid the danger of overfitting their parameters which occurs when they are chosen to be optimal for the particular frame of the backtest, but perform poorly if the frame were to vary lightly. This can be avoided by backtesting over multiple frames and comparing optimal parameters and by checking if values surrounding optimal parameters produce similar results and form a plateau of sorts, rather than dropping off severely and forming peaks around the optimal parameters.

2.3.1 Review of Efficient MVA Derivation

Assuming the proper conditions are met for such optimization, one technique for optimization that falls under the umbrella of MPT is mean-variance analysis (MVA). MVA specifically seeks to maximize the
profitability and decrease the variance of a portfolio requires a time series of profits on fixed intervals for each asset in the portfolio. It does this by describing the weighting and expected values returned by assets and other conditions through the method of Lagrangian multipliers.

To summarize Lagrangian multipliers, if a function \( f(x_1, \cdots, x_n) \) is to be minimized and is also subject to \( M \) constraints of the form \( g_M(x_1, \cdots, x_n) = 0 \), then the solutions to the system of \( n + M \) equations described by the gradients \( \nabla_{x_1, \cdots, x_n, \lambda_1, \cdots, \lambda_M} \mathcal{L}(x_1, \cdots, x_n, \lambda_1, \cdots, \lambda_M) = 0 \) are the solutions for minimization where the Lagrangian expression is

\[
\mathcal{L}(x_1, \cdots, x_n, \lambda_1, \cdots, \lambda_M) = f(x_1, x_2, \cdots, x_n) - \sum_{k=1}^{M} \lambda_k g_k(x_1, \cdots, x_n)
\]

and \( \lambda_1, \cdots, \lambda_M \) are the actual Lagrangian multipliers.

A portfolio with \( n \) assets and profit time series \( x_1, \cdots, x_n \) and capital allocations or weights \( W_1, \cdots, W_n \) and average returns or expected returns \( E(x_i), \cdots, E(x_n) \) can be modeled in the following way. The net profit of the portfolio can be defined as \( \sum_1^n W_i E(x_i) \).

The variance of the portfolio can be defined by

\[
f(W_1, \cdots, W_n) = \text{Var} \left( \sum_{i=1}^{n} W_i x_i \right) = \\
\begin{bmatrix} W_1 & \cdots & W_n \end{bmatrix} \begin{bmatrix} \text{(Co)Var}(x_1, x_1) & \cdots & \text{Covar}(x_n, x_1) \\
\vdots & \ddots & \vdots \\
\text{Covar}(x_n, x_1) & \cdots & \text{(Co)Var}(x_n, x_n) \
\end{bmatrix} \begin{bmatrix} W_1 \\
\vdots \\
W_n \end{bmatrix}
\]

where \( \text{Var} \) is the variance of a data set and \( \text{Covar} \) is the covariance of two data sets.
The constraints of the portfolio can be defined by the sum all weights being 1 or \( W_1 + \cdots + W_n = 1 \) or 
\[
g_1(x_1, \cdots, x_n) = 1 - \sum_{i=1}^{n} W_i = 0
\]
and the net profit of the portfolio being at least \( k \) or 
\[
g_2(x_1, \cdots, x_n) = k - \sum_{i=1}^{n} W_i E(x_i) = 0.
\]

The Lagrangian expression then becomes 
\[
\mathcal{L}(W_1, \cdots, W_n, \lambda_1, \lambda_2) = \text{Var} \left( \sum_{i=1}^{n} W_i x_i \right) - \lambda_1 \left( 1 - \sum_{i=1}^{n} W_i \right) - \lambda_2 \left( k - \sum_{i=1}^{n} W_i E(x_i) \right).
\]

The system of equations to solve then becomes the \( n + 2 \) gradients 
\[
\nabla_{W_1, \cdots, W_n, \lambda_1, \lambda_2} \left[ \text{Var} \left( \sum_{i=1}^{n} W_i x_i \right) - \lambda_1 \left( 1 - \sum_{i=1}^{n} W_i \right) - \lambda_2 \left( k - \sum_{i=1}^{n} W_i E(x_i) \right) \right] = 0
\]

Which simplifies to the following matrix equation after much algebra 
\[
\begin{bmatrix}
(Co)\text{Var}(x_1, x_1) & \cdots & \text{Covar}(x_n, x_1) & 1 & E(x_1) \\
\vdots & \ddots & \vdots & \vdots & \vdots \\
\text{Covar}(x_n, x_1) & \cdots & (Co)\text{Var}(x_n, x_n) & 1 & E(x_n) \\
1 & \cdots & 1 & 0 & 0 \\
E(x_1) & \cdots & E(x_n) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
W_1 \\
\vdots \\
W_n \\
\lambda_1 \\
\lambda_2
\end{bmatrix} =
\begin{bmatrix}
0 \\
\vdots \\
\lambda_1 \\
0 \\
\lambda_2 \\
k
\end{bmatrix}
\]

Or as \( \text{EFFVCOV} \times \text{EFFW} = K \) and thus \( \text{EFFW} = \text{EFFVCOV}^{-1} \times K \), where EFFVOC is the efficient variance-covariance matrix and EFFW are the efficient weights. To summarize, this generates a matrix containing the optimal (for minimum portfolio variance) weighting for assets provided some minimum average profit. This known as an efficient variation of MVA (Iyengar, Efficient Frontier).

Note that computing the actual value of EFFW and the matrix computations involved were done through Microsoft Excel with standard matrix multiplication and inversion operations, and without the use of any solver functionality.
### 2.3.2 Efficient Frontier

The efficient frontier (EFF) in MVA is a curve that reflects risk-reward by plotting the minimum average profit on one axis, and the measure of risk (the variance or alternatively the standard deviation) on the other axis. The efficient frontier is also dubbed the Markowitz bullet due to the shape of the curve being like the tip of a bullet where the very tip is an extreme that reflects the overall least risky possible portfolio (a higher or counterintuitively a lower average profitability than this extreme will yield higher risk) (Investopedia, 2017).

### 2.4 A Method of Transforming Trading System Backtest

**Data for use in MVA**

To apply MVA to a set of time series data sets, they must all use the same fixed time intervals and each entry in the time series of an asset must theoretically represent the profitability of that system for that period of time. Applying MVA to an investment is simple given a time-series of the account balance that invests in that asset or alternatively the raw value of the asset. It can be done by generating a new time series of either absolute or relative changes from entry to entry in the original time series. In contrast, finding a way to apply MVA to the trading systems is more challenging due to the nature of trade data. We discuss and justify our approach to this problem.

More formally, the reason that MVA is able to model the profit and variance of investments through change in price levels is because the price level is a reflection of the investments return, discounting the negligible cost of a broker’s spread free (an assumption we make because we are using scalping trading systems). The profitability and variance of a trading systems is harder to model due to the often sparse collection of discrete periods of time in which trades have been opened and closed.
The profits of a trading system to be used in the time-series should be a reflection of system’s ability to generate signals for opening and closing trades that create the price level differential (in the direction of the trade long or short). Thus the change in value of the price level from open to close of trades should be observed, and not the change in account balance or free margin. This is because the account balance does necessarily not represent the same change in price level across time. This is also because trading systems which employ position sizing that is based on the account balance or free margin obscure the system’s ability to generate good signals with how much profit the system has made so far.

There is still a need to model the profit on a system on regular intervals. For this, we assume that taking the sum total of profitable changes in price level on regular intervals is a valid approach. It should be noted that the changes in price level are corrected to account for the direction of the trade (long or short) so that the profitable change in the price level is represented.

The last consideration we make is that intervals which are small enough to cause the majority of periods of time to contain no trades are undesirable. This is because systems which react similarly to certain market conditions will fail to show this correlation in the produced time series. To visualize this, imagine two systems which open and close trades around the same time, but not exactly at the same time. If the interval is too small and these trades are grouped in difference periods of time, the correlation is not observed. We experimentally determined that a monthly intervals of time allow all trading systems to have trades on a majority of periods.

Thus, our solution to this problem is the following: The sum total of changes in price level in the direction of and over the duration of trades along monthly intervals are used as the final time series to be fed into the MVA.
2.4.1 Trading System Development Process and Tools

The trading platform used for the creation of trading systems and their backtesting was Meta Trader 4 (MT4). MT4 is a trading platform for Forex and has its own language for creating trading systems which are known as Expert Advisors on this platform. MT4 like most trading platforms enables one to backtest trading systems on all currency pairs over as much data as is publically available.

To expedite the development of systems, the third-party software fxDreema was used as a tool to generate the MT4 expert advisor code. The logic of systems was defined through an acyclic flowchart created by the user that is composed interchangeable nodes that represent indicator conditions, logic gates, time filters, position opens and closes, and a wide variety of other common elements to trading systems.

2.4.2 Common System Properties

The three trading systems developed share the following commonalities:

· Position size of trades was set to a fixed value in pips or currency units. Although a live system should use a position size proportional to the account balance or free margin, a fixed value was chosen so that changes in price level over trades could be represented visually through the account balance curve and not be obscured by the size of the account balance didn’t we deem observing changes in price levels more important than changes in the account balance. This visual is important for eyeballing variance in price level for trades over time during the process of backtesting and optimization.

· Hourly bars are used because they were the largest bar-size for which the system would produce sufficiently many trades.

· Only one trade is open at any given moment for simplicity’s sake.

· Only one trade may be made per bar to avoid the eventuality that price level and indicators oscillate around levels that constantly open and close trades.
· Backtested on and optimized for the currency pair EURUSD because it is the largest currency pair market and thus tends to have the most complete price level data set for backtesting.

· Backtested on and optimized for the date range of 1/1/2012-1/1/2017 because this a round range over the last 5 years. Additionally, the year 2011 for the EURUSD market had seen some dramatic events with strong market momentums and resulted in disproportionately profitable trades in all systems, overshadowing typical variance in price levels profits.

· There is no take-profit, a condition which automatically close trades that have attained a certain amount of profit.

· There is a stop-loss, a condition which automatically closes trades that have attained a certain amount of loss, in the form of a fixed pips.

· Trades were not made from 12:00 AM to 6:00 AM on Mondays because over the weekend retail traders do not trade and historical trading data is not available, thus causing a jump in price-level following Monday 12:00 AM which may cause misleading information from indicators for several hour bars and produce bad trade signals.

· The entry conditions along with the exit conditions use symmetrical criteria for oscillator indicator’s readings (i.e. RSI cross above 30 is long signal and cross below 70 is short signal).
Chapter 3: Results & Analysis

Outcomes from the backtesting of the individual systems, the resulting efficient frontier curve (EFF), and comparisons between baseline portfolios with naive approaches to weighting and portfolios with weighting informed by EFF are covered in this section.

For brevity sake, the three systems we developed will be referred to with more concise names in this section:

- Swift RSI (RSI)
- Swift RSI + Bollinger Bands (RSI Bands)
- MA Crossover (MA3)

3.1 System Backtesting

Recall that backtesting data for each trading system was transformed into a time series of total pip gains (rates of return, see ‘MPT’ section) over monthly periods. Although the time period of month was chosen to avoid this, there are instances where trade systems did make trades during some months and in this case the total pip gain was assumed 0. This is despite the difference between a net gain of 0 and not making any trades. A table summarizing total trades and months without trades is as follows.

<table>
<thead>
<tr>
<th></th>
<th>RSI</th>
<th>RSI Bands</th>
<th>MA3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Trades</td>
<td>459</td>
<td>48</td>
<td>1203</td>
</tr>
<tr>
<td>Zero Count</td>
<td>2</td>
<td>29</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Total trades and months without trades for each Trading System
To characterize the profitability and variance (or risk) of each portfolio, we have the following table which provides the average pip return (on a monthly basis) and variance for each system.

<table>
<thead>
<tr>
<th></th>
<th>RSI</th>
<th>RSI Bands</th>
<th>MA 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Monthly Profit (pips)</td>
<td>11</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>Variance</td>
<td>5163</td>
<td>877</td>
<td>12986</td>
</tr>
</tbody>
</table>

Table 3: A comparison of returns vs. variance reveals the anticipated trend of higher returns for higher risk.
3.2 Resulting Efficient Frontier

Using the described time series and applying the previously derived equations, the EFF can be produced. The following are a data table and graph that represent the return vs variance (or, quantified return vs quantified risk) relationship that is the EFF curve. Note that for the sake of simplicity, the data table displayed does not display all data points, rather it only shows the points on intervals of 0.5 for the target profit.
<table>
<thead>
<tr>
<th>Profit</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>814</td>
</tr>
<tr>
<td>8.5</td>
<td>790</td>
</tr>
<tr>
<td>9.0</td>
<td>816</td>
</tr>
<tr>
<td>9.5</td>
<td>891</td>
</tr>
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<tr>
<td>16.0</td>
<td>6383</td>
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</table>

Table 4: A table showing the profit and variance of portfolios whose weighting was determined by EFF.

Note that yellow highlighted rows begin when the derived EFF weights contain one or more negative weights for the trading systems and thus are technically not possible. There is always a certain target profit for which this phenomenon will occur. In our portfolio, this limit was at a target profit of approximately 13.8 pips.

One may discard the portion of the EFF to the left of the Markowitz bullet as we are not concerned with such suboptimal weighting. The data corresponding to the portion of the curve that is relevant (to the right of the bullet) starts past the row demarcated by highlighting. The blue highlighted row represents where the EFF yields absolute minimal variance. One can clearly see the trade-off between return and variance.
(rewards vs risk). Recall that for any point on this curve, what is being represented is the optimal variance that be achieved through EFF weighting as a function of a target return. The system weightings associated with each point on the EFF curve can be seen in the following data table. Note that the range of target profits covered in this table are restricted for viewing convenience.

![Variance vs. Profit of Portfolio with EFF Weighting](image)

*Figure 5: Variance vs Profit of EFF weighted Portfolios*
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<th>Profit</th>
<th>Variance</th>
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<td>11.9</td>
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Table 5: The return and variance of EFF weighted portfolios (over a smaller range), and the weights derived
Notice that the higher the desired profit, the larger the weights that are assigned to the RSI and MA3 systems relative to the RSI Bands system. This is expected given the higher returns of RSI and MA3 and the lower risk of RSI Bands as shown in Table 5, further demonstrating that increased returns comes at the cost of increased risk (given less weight to the less profitable but less risky system).

### 3.3 Portfolio Comparisons

In order to evaluate the benefits of a portfolio that uses EFF weighting versus more naive weighting strategies, we construct a number of intuitive baseline weighting strategies and simulate those portfolios.

One weighting strategy dubbed ‘Equal Weighting’ assigns equals weights to each of the three systems. Another strategy dubbed ‘Return Based Weighting’ weights each system in proportion to their average returns. A final strategy dubbed ‘Return over STD’ weights each system in proportion to the ratio between their average return and their standard deviation.

For each portfolio based on one these baseline weighting strategies, we construct two additional portfolios which use EFF informed weighting that match 1) The return and 2) The variance. This allows us to make comparisons between EFF weighted portfolios and naively weighted portfolios in the following ways: 1) How much less risky is the portfolio for the same target profit; 2) How much more profitable is the portfolio for the same variance. We make these comparisons in the following tables:

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<tr>
<th></th>
<th>Equal Weight</th>
<th>Efficient Matched Return</th>
<th>Efficient Matched Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>12.9</td>
<td>12.9</td>
<td>13.3</td>
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<tr>
<td>Variance</td>
<td>3104.144</td>
<td>2718.456</td>
<td>3084.143</td>
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</table>

Table 6: A comparison of profit and variance of portfolios. The baseline portfolio (left column), the EFF weighted portfolio that matched the baseline portfolio’s profit (middle column), and the EFF weighted portfolio matching the variance (right column).
Decreases variance by 12% for EFF system with matched profit and increases profit by 3% for EFF system with matched variance

<table>
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<th>Return Weight</th>
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<th>Efficient Matched Risk**</th>
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</thead>
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<tr>
<td>Variance</td>
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<td>4613.968</td>
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</table>

*Table 7: A comparison of profit and variance of portfolios.*

Decreases variance by 9% for EFF system with matched profit and increases profit by 2% for EFF system with matched variance

** These two EFF based weighting strategy are technically not possible due to them having negative weights for one or more trading systems in the portfolio

<table>
<thead>
<tr>
<th></th>
<th>Return over STD</th>
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<th>Efficient Matched Risk</th>
</tr>
</thead>
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<tr>
<td>Variance</td>
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<td>2305.977</td>
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</table>

*Table 8: A comparison of profit and variance of portfolios.*

Decreases variance by 10% for EFF system with matched profit and increases profit by 2% for EFF system with matched variance

For comparisons between EFF informed portfolios and baseline portfolios, we observed differences in favor of the EFF portfolios for their risk by a magnitude of approximately 10% and for their profit a magnitude of approximately 2%.

It should be noted that in Table 7 the weighting strategies which are derived by the EFF in order to match the profit and variance of the baseline weighting strategy (Return Weight based) are technically not possible due to the limitations discussed in the previous section.
Figure 6: A graph of the variance versus profit of EFF weighted portfolios, along with the portfolios with baseline weighting strategies. The baseline weighting strategies are 'Return over STD' (left), 'Equal Weight' (middle), and 'Return Weight' (right).

The above graph compares the performance of the EFF weighted strategies with that of the baseline weight strategies. Each baseline strategy used for comparison is represented by a group of three points on this graph. The point not lying on the EFF curve represents the performance of the actual baseline strategy. The point lying on the curve directly below the baseline strategy point is the performance of the ‘Matched Return’ EFF portfolio, and their difference in variance is significant of the EFF’s ability to reduce risk given a target profit. The point lying on the curve directly to the right of the baseline strategy point is the performance of the ‘Matched Risk’ EFF portfolio, and their difference in profit is significant of the EFF’s ability to increase profit given an allowable risk.
Chapter 4: Conclusion and Limitations

4.1 Conclusion

The goal of diversifying investments can be accomplished through a number of techniques. Modern Portfolio Theory offers a number of techniques that help quantify the performance and risk of portfolios, with Mean Variance Analysis being our technique of choice. The data we have used to showcase MVA’s value was based on a portfolio of three FOREX trading systems that we engineered. Mean Variance Analysis (MVA) is rarely applied to trading systems due to the nature of the data produced, and so we found a method of translating that data into a more useable format.

Our process going from an idea to a viable portfolio can be broken down into four stages:

In the first stage, we have done an in-depth analysis to find the right indicators based on the parameters which correspond to our chosen methodology of complementary indicators; that is to use and complement different types of indicator while trading. These types include momentum, trend and volatility.

In the second stage, we have put together our chosen indicators into three trading systems. The first trading system uses RSI indicator; the second system includes a combination of RSI and Bollinger Bands; the third system includes Moving Average. In order to stay true to the aforementioned methodology, we recommend the reader to trade using all three strategies together in order to maximize their chances of choosing profitable trades, minimizing the redundancy of information and potentially lowering the risk to reward ratio.

In the third stage, we put our three trading systems into a portfolio in order to find out how one can use our system and allocate capital resulting in the most efficient trading - trading for maximum returns for a given risk. We have applied a Mean Variance Analysis (MVA) of a Modern Portfolio Theory (MPT) to
our trading system in order to determine various options for allocating that capital and quantifying the risk and returns for each.

The fourth stage, which is also the final stage, is to find the best option for allocating capital across our trading system portfolio, so that for a chosen level of reduced risk, a trader can find which option returns a maximum amount of profits.

According to our research results, using MVA produced better portfolios compared to the baseline. We found that the weighting strategies of an Efficient Frontier Curve (EFF) produced by applying MVA to the portfolio can marginally reduce risks and/or increase profits. The EFF weighted portfolios performed better in the followings ways. For a given target profit, they reduced risk by roughly 10%. For a given allowable risk, they increased profit by roughly 2%.

Our results demonstrate that applying MVA is a viable approach for balancing portfolios of trading systems because it can optimize profit for a given risk.
4.2 Limitations

There are limitations in the methodology of our project and the methods we have used for analysis. The following is a collection of these limitations:

- Our baseline weighting strategies of choice were simple, intuitive, and not exhaustive. They may fail to include other common weighting strategies which perform considerably better than those we have chosen. This means it is possible that the degree to which MVA weighted portfolios outperform baseline strategies has been over stated.

- The time series data used for MVA was collected from backtesting each trading system over a five year period on one FOREX currency pair, as mentioned previously. It is possible that the weights derived by MVA are overfit for this data set. This means that simulating a portfolio of these systems with a set of weights chosen along the EFF may produce riskier or less profitable systems over a different currency pair of time period.

- When finding portfolio weightings along the EFF that will match the return or variance of a baseline weighted portfolio, we did not find exact matches. We simply searched among precomputed instances along the EFF and selected the instance that matched closest. Note that these precomputed instances were in intervals of 0.1 pips of profit.

- We attempt to showcase the utility of MVA as it applies to a portfolio of trading systems, but we only consider portfolios using the three trading systems we have developed. Using a different set of trading systems will yield different results with regards our metrics comparing MVA weighted portfolios to baseline weight strategies. We do however believe that regardless of trading systems used, an MVA optimized portfolio would never perform worse than any intuitive weighting strategy.

- There has not been enough research done to give a convincing proof that the technique of complementing various indicators is significantly better than trading with only one indicator, even though it may sound very reasonable to some group for traders.
• The risk associated with balancing portfolios has been represented by variance, which may not be the best risk indicator because high variance of an asset price, while denoting riskiness, may not always be unfavorable. For example, a stock price may deviate highly from its mean in a favorable direction and hence be a good investment for anyone who wants to ride the occasional positive wave. If the associated risk is only represented by variance, then it may mislead many investors into thinking that high variance equals high risk and have them wrongfully conclude in the favorability of an investment.

• The time series we have used for our trading systems’ profits was formed by aggregating trade profits over monthly intervals. While we were crudely determined that monthly intervals worked well, we cannot conclude that this choice of interval is optimal.
### Appendix

#### Variance-Covariance Matrix

<table>
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<th>RSI Bands</th>
<th>RSI</th>
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</tr>
</thead>
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</table>

This matrix displays the covariance of profitability (pips) of all pairs of systems among our three trading systems.

#### Simulation of Portfolios: Baselines and EFF Matched

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<th>Date</th>
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<th>Efficient Matched Return</th>
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Professor Radzicki’s Lecture Notes
Link: https://drive.google.com/file/d/0B-c5TIM-JhHbZERlN2tobndRaUU/view

Project Results in Excel Sheet
Link: https://drive.google.com/file/d/0B-c5TIM-JhHbaklFbU5vYjhlZzQ/view
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