Statistical Analysis of the Variance of Luck in Poker

A Major Qualifying Project, submitted to the faculty of Worcester Polytechnic Institute in partial fulfillment of the requirements for the

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Introduction

Poker is an interesting and unique game to play. There are very few other examples of games where players are governed by a similar balance of skill and luck. Poker is a game where the fate of the individual player is not entirely under his or her control. On any given session, in any given hand, any player has a chance to win. Regardless of how long you've been playing, how well you understand the numbers of the game, or how well you can read your opponents. In any given hand you can lose. The best hand doesn't always win, the best player doesn't always win, and because of this fact people forget about the skill aspect involved in playing cards and simply write it off as a game of chance.

While it is true that anyone can win on any given hand or during any given session of poker, that does not mean everyone has the same likelihood of success at the tables. For example, the best starting hand in Texas Holdem poker is AA regardless of the suits of the cards and the worst possible starting hand is 72 off suit or unsuited (shorthand 72o). If two players get all their money in the middle right at the start of the hand, the player with AA has a 90% chance to win while the player that has the 72 has only a 10% chance of victory. Both players have a possibility of coming out in the end with a win on this hand, but one of these two players is in far better shape than the other. The player with the AA has a definite advantage in this matchup and if you were to run this hand out hundreds of thousands of times the player with AA is tremendously more likely to be a winning hand than the 72 and therefore the AA is going to be incredibly more profitable in the long run.

Playing just the numbers alone, it is easy to see how poker is a game where the player with the better hand is more likely to win money, however poker doesn't happen in a vacuum. There is more to the game than just numbers. If every player at the table played the exact same way with the exact same skill level, no one would be able to make money. The money would just go back and forth across the table to the player who happened to have the best hand in that particular moment. The other aspect of poker is that there is a very human element to the game. You are testing your wits and skills against another human being rather than betting against a computer that will always make the same plays all the time. Because you are playing against other people, and people have tendencies that they like to follow, you can learn these tendencies and play off of them rather than just playing a purely statistical game.

When you factor in what your opponent is doing, the math behind each poker hand you play changes. This is called putting an opponent on a *range of hands*. Let's take two players in particular at your table. Say Player 1 is playing around 50% of the hands he or she is dealt at the table and Player 2 is playing only 6% of the hands he or she is dealt. The way you play a hand against Player 1 will need to be different than the way you play a hand against Player 2. Let's say you have a strong starting hand like AQ suited (shorthand AQs). Using a poker odds calculator such as Poker Stove[1] we can calculate our likelihood of winning against Player 1's range of hands. Using the calculator we see that our AQs has a 64.682% chance to have the best hand while Player 1 has only a 35.318% chance. Those numbers also include the probability that the hand will result in a tie and both players will have the same hand. The

[1] Professionals, By. *PokerStove Home*. Web. 01 Oct. 2010. <http://www.pokerstove.com/>.

math is very different against Player 2 however. When playing AQs against Player 2's range of hands we see that AQs has only a 43.227% likelihood of being the best hand when action is completed while Player 2's range holds 56.773% likelihood. The numbers vary greatly depending on the opponent you are playing against. Appendix A Example 1 shows the detailed output from Poker Stove on these hands.

Poker is a game with incredible detail and many variables involved in every decision. The more you play and study the game, the more accuracy you will have with putting an opponent on a range of hands, you will develop a better grasp of the probabilities in the game, and your skill advantage in the game will grow. The better you becomes as a player, the more likely you are to make money and the more likely you are to earn enough money to make a living playing the game. Simply put, the better poker players will win money over time, and those players with the biggest advantages will be able to earn a very comfortable living. Poker is a game where skill is the dominating factor in the overall outcome.

Structure of the Game

When talking about poker, it is very easy to get lost in the possibilities. There are many different versions of poker to play, different formats, different betting structures, each of which has its own calculations required to make the correct decisions. For the purposes of this paper the focus will be placed on one single game-type of poker known as Texas Holdem, one specific type of betting structure known as No Limit, and two formats to play the game in known as

Tournaments and Cash Games. Each of these attributes adds structure and specific rules to the game.

Texas Holdem poker is a blind based game. This means that before any cards are dealt two of the players at the table must put in bets. These are called *blind bets* and are commonly referred to as *blinds*. The first player to the left of the dealer puts in one of these two bets known as the *small blind* and the player to his left doubles that bet which is called the *big blind.* These blinds are predetermined values and will remain constant unless the *table stakes* are changed. For example, if you are playing at a table where the table stakes are $1/$2, that means the SB is $1 forever and the BB is $2 forever unless otherwise noted. Each player is dealt two cards face down to begin with. There is then a round of betting from each player based on the two cards he or she was dealt. Each player has three actions that can be taken with the hand; you are allowed to *call* the largest bet made to that point in the hand, you can *raise* the *bet*, or you can *fold* your hand and simply stop playing until a new hand is dealt to you. All of the bets made on the hand go into the middle in a pile known as the *pot* which will be awarded to the eventual winner of the hand.

When the first round of betting is completed the players remaining in the hand move to the next phase which is known as the *flop*. The flop is three cards placed up in the middle of the table by the dealer. These cards are community cards that any of the players can use combined with hole cards they were each given to make the best possible poker hand. Another round of betting follows where players have the same three options, call, raise, or fold. Upon completion of this second round of betting the dealer deals another community card called the

turn. Another round of betting follows same as the first two and then the dealer deals one final community cards called the *river*. There is one final round of betting after the river is dealt. Once all the betting is completed all the remaining players in the hand turn their two hole cards face up. The player with the best 5-card poker hand using any combination of their two cards and the 5 cards on the board wins the pot.

The betting structure that will be the focus of this paper is No Limit. This refers to the amount of money you can wager during any of the individual betting rounds. In a No Limit Texas Holdem game, each player has the ability to put all of their money into the pot at any time during the hand. This is known as going *all in*. If any of your opponents want to play the hand after you go all in, they will need to match the amount of money you have put into the pot just as they would if you put in a smaller bet size. If one of your opponents wants to play the hand and does not have enough to match the bet that has been put out there, they must put all their remaining chips into the middle and go all in for less than the bet size. The most money that player can win is what he or she put into the middle from the other player. For example let's say you go all in for $50 and your opponent wants to call you. If your opponent only has $30 in front of him or her, then that $30 goes into the middle from each player and the pot will be $60. The player with only $30 cannot win the additional $20 you bet so that money is returned to you. If there was a third player in the hand that wanted to play and that person had $50 to play with, then you and player 2 you put $30 in the *main pot* along with player 1 and put the other $20 in a *side pot* that only you and player 2 can win. At the end of the hand the

player with the better hand between you and player 2 wins the side pot which will be $40 and that player tries to beat player 1's hand for the $90 main pot.

Cash Game Poker

In general, cash game poker is defined by the following attributes. Anytime a cash game is mentioned, it is assumed to have these parameters attached to it.

- The players at the table all buy into the game with a certain amount of cash. This number is predetermined to coincide with the table stakes. For No Limit games, there will be a preset minimum buy in and maximum *buy in*. In most cash games, players are given chips to play with rather than loose cash. These chips have a one-to-one correspondence between their face value and their monetary value (Ex: A $5 poker chip in a cash game is worth $5 cash).
- The table stakes of a cash game will remain constant over time and the game being played will not change. If the game began as a $1/$2 No Limit Holdem game, it will remain to be that as long as the game is running. In some cases, players are allowed to change the game or the stakes if all players agree to the proposed changes.
- If a player loses the amount of money they put up for their initial buy in then that player is allowed to put more of his or her money at the table, so long as it meets with the minimum and maximum requirements of the table. A player may also do this without going completely broke. This is called a *reload*.

- A player may choose to leave at any time he or she is not in a hand. If a player leaves while still in a hand, the hand is automatically folded. Once a player leaves a new player can take the empty seat and join the game. Unless a player leaves the game entirely, he or she is not allowed to remove any money from the table. Also, if a player leaves and returns to the same game within a given timeframe (say 2 hours for example) the player must buy in for the full amount that he or she left with.

Major Factors

There are many variables that change the calculations for each hand. It is impossible to label every individual variable that comes up, however there are a universal few that will constantly come into play during your calculations.

Position

Position is one of the most overlooked aspects of poker when players just start learning the dynamics of the game. Once learned, position becomes one of the most vital pieces of information at the table. Position is determined by which player is the dealer that round. The two players to the dealers left are in the blinds. To their left are those players in early position, then middle position, and finally late position which is capped by the player who is acting as the dealer in this hand. Your position in a hand dictates what kind of hands you should be playing, how you should play them, how your opponents will react to your plays, and so on. Position also dictates the flow of money around the table. Assuming skill level is equal in all the opponents playing in a particular game, the money would move in a definite pattern around the table where the players who are in certain positions will win money and

those who are in other positions will be destined to lose money. Position at the table is broken up into 4 categories, early position, middle position, late position, and the blinds.

The least profitable place to be in any hand of poker is in one of the blinds. The blinds are designed as bets to start the action in Texas Holdem. The major disadvantage to these seats is that you do not get to see your whole cards before you put money into the pot. Unlike all the other players who are at the table, if you fold pre-flop you will show a net loss for that hand while all the other players lose nothing by folding. In addition the player in the blinds will be the first players to act for the rest of the hand so all of your opponents get to see you play your hand and are afforded that much more information as a result. Those disadvantages are usually too much to overcome and as a result the players in the blinds typically lose money on every hand they play. The first player to put up a blind is known as the small blind or SB for short and the player to his left is the big blind or BB for short.

Being in early position means you are one of the first players to act on any of the betting rounds. This is a bad position to be in at the table for a few reasons. For starters you have all of the other players yet to act. You have no information on any of their hands and have no idea how each player will proceed when it's his or her turn to act. In addition to this, each player acting after you gets to see what you do before he or she needs to even look at the cards. Your opponents will have information about your hand and can base their own decisions on this additional information. All of this is compiled by the fact the position does not change during the hand in a game of Texas Holdem. Therefore, any players that have a positional advantage on you in the beginning of the hand will have that advantage on you at the end of the hand. It is typical because of all of the pitfalls of early position that players sitting in these seats tend to win less money than those players that sit in later positions and to counteract this fact it is common practice to only play the strongest of hands from early position. The person who is first to act

pre-flop is known as *Under the Gun* or UTG for short. The player to his left is UTG+1 and then UTG+2 to the left one more seat. Those three players are in early position for the hand.

After early position comes middle position. Middle position is where players start to have significantly more information at their disposal than their counterparts in the blinds and early position. In middle position you get to see at least 3 players act before you need to make your own decision about your hand. You get to see how many players have put money into the pot to this point, how big the pot size already is, if the players in front of you raised it can tell you they have a strong hand while just calling the BB typically means something weaker. Middle position players still have other players left to act behind them, but that number is far less than for those in early position. Middle position players are just denoted as MP and there is only one MP player in a 9 handed game of poker. If you add another player you add a MP+1 slot and if you remove a player from a game and play with 8 the MP slot is effectively removed from the game.

Finally we come to late position, the strongest place to be in a poker hand and where the money tends to congregate overtime. When in late position a player only has a couple of players left to act behind if there are any at all. The first player in late position is called the *Hijack* written HJ for short. This player only has to deal with two players left to act behind not including the blinds pre-flop. Just after the HJ is the *Cutoff* or CO for short and following that is the best position in the game, the dealer which can be written either as D or B for short (the "B" referring to the plastic disk moved around the table denoting who the dealer is this hand, which is known as "the button"). The players acting in later position get to see everyone else act before they make and decisions about their own hands. This wealth of information makes a huge difference in terms of the amount of money that can be won from these positions compared to the earlier positions at the table.

Stack Sizes

The amount of money that you can win on any given hand is directly related to the amount of money each player participating in the hand has. You can't win more money than what you have in front of you however, so if you are playing against a player who has $300 and you have $100, the most money you can win this hand is $100 profit from the other player.

Players in the Hand

The number of players in the hand changes a few things. As the number of players in the hand increases, the amount of money you can win increases. Going with the example given in stack sizes, let's say you have $100, your first opponent has $300, and another player with $155 also comes into the hand. The potential amount of money you can win this hand is $200 in profit rather than $100 when you only play one of these opponents. With this increase in players however, you notice that the relative likelihood of you having the best hand decreases. Let's say that we are playing just the one player with $300 in front of him. Let's say we have the best hand *pre-flop* with AA and our opponent has KK. Using Poker Stove we can calculate that we have an 82.637% to have the best hand verses our opponent who only has a 17.363% chance. By putting a third player into the mix and giving him a hand of QQ to start we see that this player only has a 15.098% chance to have the best hand by the end. The problem here though is that the chance of the KK has not significantly been affected by the introduction of this new player. In this spot KK has a 17.232% chance of winning the hand. This means that the probability of AA winning has significantly decreased to 67.670%. While you can win 50% more money with the introduction of this third player, you are 18.11% less likely to win the hand (1 – (67.67%/82.637%)).

Pot Odds

Using pot odds properly in a hand of poker contains several of the components that have previously been mentioned. The pot odds of a particular decision consist of two components, the amount of money currently in the pot and the amount you need to call to continue playing the hand. For example, say there is $10 in the pot and your only opponent in the hand bets $2 into you. Your pot odds on this particular hand are 10/2 (read as "ten to two") which reduces to 5/1.

This information is used often as the starting point to making a decision. This can be best illustrated by a hand. Looking at the previous example your opponent has given you 5/1 odds to make the call. Knowing this information, how does it apply to the decision making process. Let's say my opponent has a better hand than I do at the moment and that my hand only has a 25% probability to win when all the cards are dealt. That means my odds of winning the hand are 75/25 or 3/1. Looking at the odds a different way, for me to break even on calling my opponent with only a 25% chance to win I have to win $3 for every $1 that I put into the pot. Now the pot is laying me 5/1 odds, which means I will get paid $5 for the $1 I put into the pot. Since the odds the pot is laying me are larger than the odds of me winning the hand, the call becomes profitable to make.

Range of Hands of Your Opponent(s)

In a few examples so far the likelihood of you winning a hand has been discussed. This was calculated by taking the hand your opponent has and calculating the percentage of the time that hand beats your hand. It is extremely rare in poker to have that kind of information at your disposal. You will almost never know exactly what two cards your opponent has until he turns the cards face up to you. What you have to do is base your decision not just on one single hand, but on all the likely hands your

opponent could have in the position he or she is in. This process is known as putting your opponent on a range of hands.

This is one of the major points of skill for a poker player. Accurately assessing what your opponent has is the basis for all of the calculations that follow. The range of hands your opponent is likely to have is based on all of the factors we have discussed so far. The position of the player, the stack size, the number of other players in the hand when that opponent has already acted, and the pot odds all play a role in assessing what an opponent might have.

Let's take the following example. Say you have AA at a $50 No Limit Texas Holdem ($50 NLHE shorthand) and you are in the CO in a 9-handed game. You make a bet to $1.50 and your opponent on the button calls. Right here is where you have to start thinking about what hands are in this player's range. You know a few things about this player right away. The player is in late position, called your bet, has a certain amount of chips in front of himself or herself, is facing no other players in the hand other than you, and the player called $1.50 into what was already a $2.25 pot (which includes the SB of $0.25 and the BB of $0.50) meaning that player had 1.5/1 odds to make the call. All of this information needs to be taken into account when assessing what hands a player might have.

There is one additional factor you need to know when it comes to the call your opponent just made in the previous example, what are the tendencies of this player? A player's tendencies are how a player plays a given hand in a given spot. In the example we provided some players will tend to play 87s while others will be playing sneaky with a big hand like KK and still others will play a smaller pair like 22 hoping to hit a third 2 on the flop. These tendencies occur overtime and all players have them. There is some data that can help determine what a particular player tends to do.

After playing for a while with a particular player for a certain number of hands, you will notice that the player bets some of the time, raises some of the time, folds some of the time, and calls some of the time. How often a player conducts each of these 4 actions is what helps to define that player's tendencies. Each of those actions can be represented by a numerical value.

- VP$IP (or VPIP) – Stands for "Voluntarily Put Money Into the Pot", this statistic indicates the number of times a player has put money into the pot in anyway. A VPIP of 25 means that 25% of the time the player is dealt a hand that player puts money into the pot.
- PFR – Stands for "Pre-flop Raise" and is the percentage of the time the player bets when dealt a hand at random. The maximum value this can be is equal to a player's VPIP number. A player who has a PFR of 19 bets 19% of the time. Coupled with the VPIP you can calculate the percentage of the time an opponent just calls when putting money into the middle. In our two examples the player has numbers of 25/19 which means 6% of the time the player puts money into the pot he or she is calling either the BB or someone else's bet.
- PF3B – Stands for "Pre-flop Three Bet." A three bet (3B) is when a player raises after someone else has already bet, and a pre-flop 3B is only including the times this happens in the pre-flop round. A PF3B of 15 means that 15% of the time an opponent bets out before the player gets a chance to act, that player raises rather than just calling or folding the hand.
- FPF3B – Stands for "Fold to Pre-flop Three Bet." This is the number of times when this player bets and an opponent raises that bet that the player folds the hand. A FPF3B of 90 means that 90% of the time that player is raised him or her folds the hand.

Examples of each of these stats exist on later streets as well. There is a "Turn Three Bet" (T3B) as well as a "Fold to River Three Bet" (FR3B). Depending on where you are in the betting rounds determines what statistics you should focus on. If your opponent had numbers that read 41/5/0/85 (VPIP/PFR/PF3B/FPF3B) you would play differently than if that player had numbers more like 10/3/2/65. Also these tendencies each player shows changes what the range of hands would likely be.

Going back to our example we can start to put together a better idea of what our opponent can have in this spot. The villain is in late position and has position on us so his hand doesn't need to be as strong. The stats are 15/3/0/85 which means he is putting money into the pot 15% of the time and is calling about 12% of the time. Our villain only has us to beat to win the hand and has $45 in front of him or her and we have $65 in front of us, making our effective stacks $45 for the hand. The villain also put in $1.50 hoping to win $2.25 which means the pot odds of 1.5/1 are not significantly high. Now we can start to build our range of hands.

With our opponent playing 15% of the hands he or she is dealt we can type that into PokerStove and come up with a range of 77+, A7s+, K9s+, QTs+, JTs, ATo+, KTo+, QJo which represents a typical 15% range of starting hands in Texas Holdem. Our villain is raising the top 3% of hands that he or she is dealt which is represented by 99+, AKs. Since our villain called rather than putting in a raise it is less likely that he or she has one of these 3% hands, but since it is not totally out of the realm of possibility the hands are not removed from consideration. Using our opponent's stack size and the pot odds as well as our experience with the player at the table we can further cut down or widen the range of hands accordingly. For example, just because a player is calling 15% of the time doesn't mean that player is calling only the top 15% of hands in the deck. Using all this information to assign a range to your opponent makes that range more accurate and therefore makes the calculations based off of this range of hands more reliable.

Online Poker v. Live Poker

These are the two mediums in which the game of poker exists and can be played. Each has its own attributes that make it unique from the other. The biggest difference between the two is that when playing online poker each player only has the mathematical information and patterns of his or her opponent to go one when making decisions. There are some tells that players with have the lend information about the true strength of their hand but for the most part the information you can rely on is all from the betting patterns each opponent you face has. In a live poker game you are provided much more qualitative information in a given hand. You have the ability to see your opponents and how uncomfortable they are with their hands. You can talk to them to try and get more information, or they can start talking to you to try and throw you off their trail.

When playing online, that data for the hands can be more easily collected and analyzed. All of the calculations can be done in both mediums so long as you just keep a log of all the information we have discussed to this point (stack sizes, hands an opponent plays, etc.) however the information is more readily available to you online than live because since you are on a computer, you can run simple programs to track the players at the table you are playing on. There are many types of tools you can use, but for the purposes of this paper two specific programs were used.

PokerStove

Poker Stove is calculation software that is used by players to help calculate the profitability of certain hands. Poker stove calculates the all-in equity of certain hands. This is slightly different than the

odds of a particular hand winning. The software runs millions of simulations of two or more hands and comes out with the overall equity that each hand has.[2] For example, take the following output.

339,036,192 games 0.285 secs 1,189,600,673 games/sec

Board:
Dead:

	equity	win	tie	pots won	pots tied		
Hand 0:	**80.704%**	79.03%	01.67%	267941940	5672994.00	{ AA }	
Hand 1:	**19.296%**	17.62%	01.67%	59748264	5672994.00	{ 99+, AKs }	

In this example Hand 0 has 80.704% equity in the hand. This means that Hand 0 can expect to see a return of roughly 81% of whatever money was invested into the pot by both players. So if each player invests $1 into the pot, Hand 0 can expect to get $1.61 back while Hand 1 can expect only $0.39. Poker Stove is a very important tool for poker players. It is particularly useful after playing a long session and looking back at some of the hands that were played. Using this software a player can determine if a move was mathematically correct or not based on the assumptions the player made about his or her opponent(s).

PokerTracker 3

PokerTracker 3 is a collection of many tools that allow the user to track and analyze data about the hands the user plays and witnesses at the table. You have the ability to see hands you've played from many angles. You can track how much money you've won with each hand, from what position at the table are you most profitable, how you have done against a particular opponent in the past, all of this information at your disposal with relative ease. The software also allows the user to display key statistics about an opponent while playing. The stats that have been brought up such as VPIP and 3B

[2] Replacing, By. "PokerStove: Poker Software and Analysis." *PokerStove Home*. Web. 25 Oct. 2010. <http://www.pokerstove.com/pokerstove/faq.php>.

will be displayed in the software HUD (Head-Up-Display) and can be customized to fit the user's specific needs.[3]

Statistical Breakdown

Poker is a game that happens in the long run. The results on a per hand basis are too affected by variance to rely on them as accurate predictors for the level of skill a particular player has. These results are also too short term to identify trends a particular opponent has. It is only after tens of thousands of hands when you see the short term variance become less significant than a player's skill.

Let's take an example. Say a player is playing a game of poker and their win rate is 2 big blinds per 100 hands (2B/100). This means that on every 100 hands dealt on average this player can expect to win 2 big blinds, but that same game has a standard deviation of 18BB/100. This means that in the short term view of 100 hands the luck factor is the far greater factor in who wins and who loses. The variance you experience in a session is directly proportional to the number of hands you play and the stakes you play at. Using the definition of variance as $E[x^2] – E[x]^2$ we note that the variance doubles every time you double the number of hands you play and it quadruples when you double the stake you are playing at. Looking at the square root of the variance, known as the standard deviation, we notice an interesting trend. We see that if the variance is X and the standard deviation is sqrt(x) per every 100 hands, then the variance for 400 hands would be 4X and the standard deviation is sqrt(4x), or 2 * sqrt(x) which is 200. When we quadruple our sample size we only double or standard deviation.[4]

[3] *PokerTracker - Online Poker Software, Player Stats Tracking & HUD*. Web. 01 Oct. 2010. <http://www.pokertracker.com/products/PT3/>.
[4] "Poker Variance - How to Calculate Variance in Poker." *Play Online Poker - Online Poker Site - Online Poker Strategy and News*. Web. 01 Oct. 2010. <http://www.aintluck.com/strategy/basic/poker-variance/>.

Going back to our same example we notice that after 10,000 hands we see that we earn 2BB * 100 (remember the 2BB is already per 100 hands) for 200BB and the standard deviation is 18BB * sqrt(100) which is 180BB. At 10,000 hands the money won from the standard deviation become less significant than the money you earned by your skill advantage. After 1 million hands the difference becomes even greater, 2000BB earned via skill factor verses 1800BB earned due to standard deviation. As we increase the number of hands played the luck factor will become less effective in the overall results.

Using software like PokerTracker 3[5] we can gather all this data and graph the trends that occur.

Green Line: Profits

When breaking down a particular player's statistics there are a few factors involved. Let's look at the following graph:

[5] *PokerTracker - Online Poker Software, Player Stats Tracking & HUD*. Web. 01 Oct. 2010. <http://www.pokertracker.com/>.

This graph shows the profits a player has earned as a green line with profits on the Y-axis and the number of hands played on the X-axis. As you can see the numbers fluctuate violently in any short term example but as your focus widens you can see trends going in either the positive or negative direction. The green line is obviously the most important statistic a poker player can worry about in the long run, once variance has been accounted for. The results on any given hand are irrelevant. What the player needs to look for is the equity of a particular hand and whether or not that play will be profitable in the long run.

To calculate the equity of a hand, a player takes into account the money in the pot, the money he or she invests to win the hand, and how often he or she will win.[6] For example let's say you have AA pre-flop, a $200 stack in a NLHE game, and you are in the BB. A player with $200 UTG goes all in and

[6] "Poker Math - How to Calculate Pot Odds and Equity: Equity - PokerListings.com." *Online Poker Sites Toplists & Bonus Offers, Poker Reviews and News*. Web. 03 Oct. 2010. <http://www.pokerlistings.com/strategy/poker-math/how-to-calculate-pot-odds-and-equity-equity>.

everyone folds around to you. Now you must assign a range of hands to your opponent using the statistics you have acquired on this player. His numbers through the 100 hands you've played with him are 10/3/0/98.3, so he is raising 3% of his hands pre-flop. Using that number we can calculate his range is 99+, AKs. We now have enough information to calculate our equity in this hand using PokerStove. Plugging in our information we see the following results:

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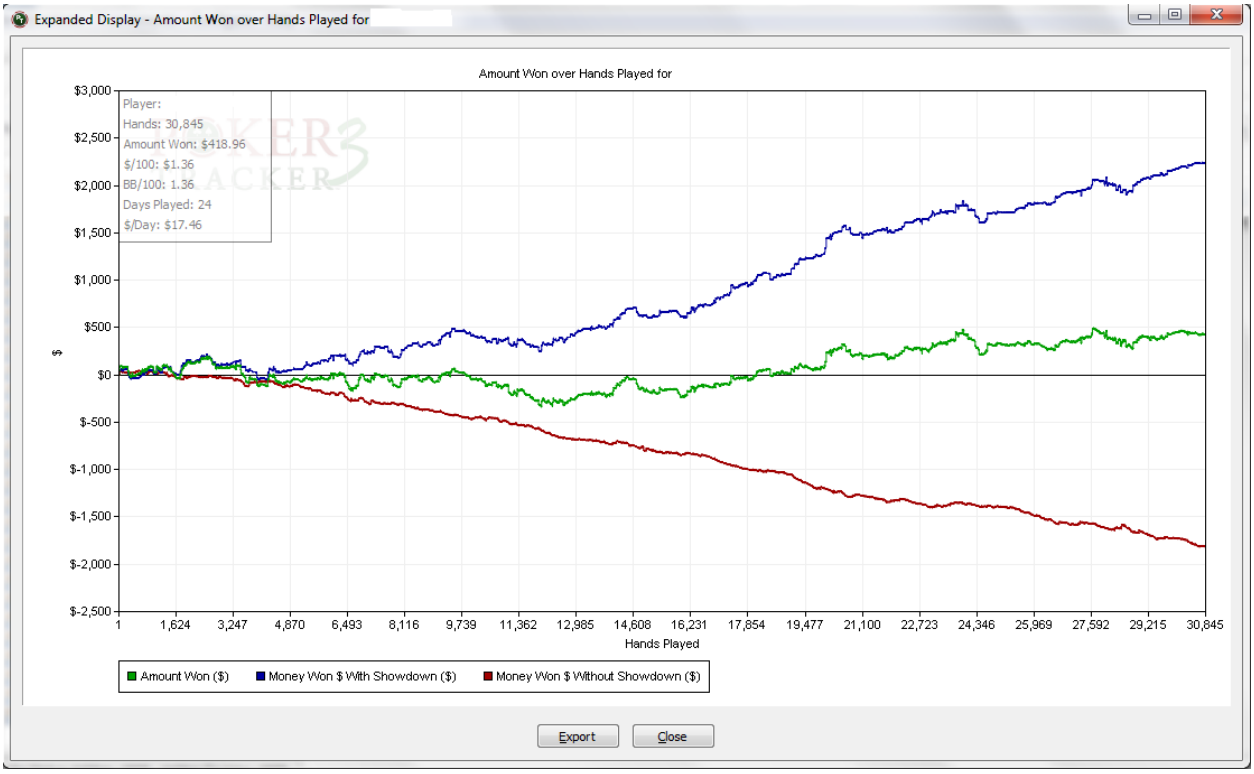
Board:
Dead:

	equity	win	tie	pots won	pots tied		
Hand 0:	**80.704%**		79.03%	01.67%	267941940	5672994.00	{ AA }
Hand 1:	**19.296%**		17.62%	01.67%	59748264	5672994.00	{ 99+, AKs }

We see that in this hand against this range of hands for our opponent we have an 80.704% chance to win the hand. If we call the pot will be $401 dollars (also adding in the SB who folded to the raise) so our long term equity on this hand is (.80704)*($401) which is $323.63. We only need to invest $200 to win this money. Running this hand an infinite number of times we will expect to gain $123.63 making this play very profitable. Even if on any one hand we lose to our opponent, the play is still correct because we will earn money making this play. Conversely our opponent's equity will be (.19296)*(401) which is $77.37. Since he invested $200 into this pot his equity is -$123.63 meaning he will lose money making this play long term.

Red Line and Blue Line

The profits a poker player earns can be broken down further than just the money a player has earned. We can further dissect the data so see when and where an opponent wins money.



The above graph has the profit lines broken down into two separate pieces, the red line which indicates the money won without a showdown and the blue line which indicates the number of money won at showdown.

Each of these new lines requires a little explanation. Let's take the following 4 hands into consideration:

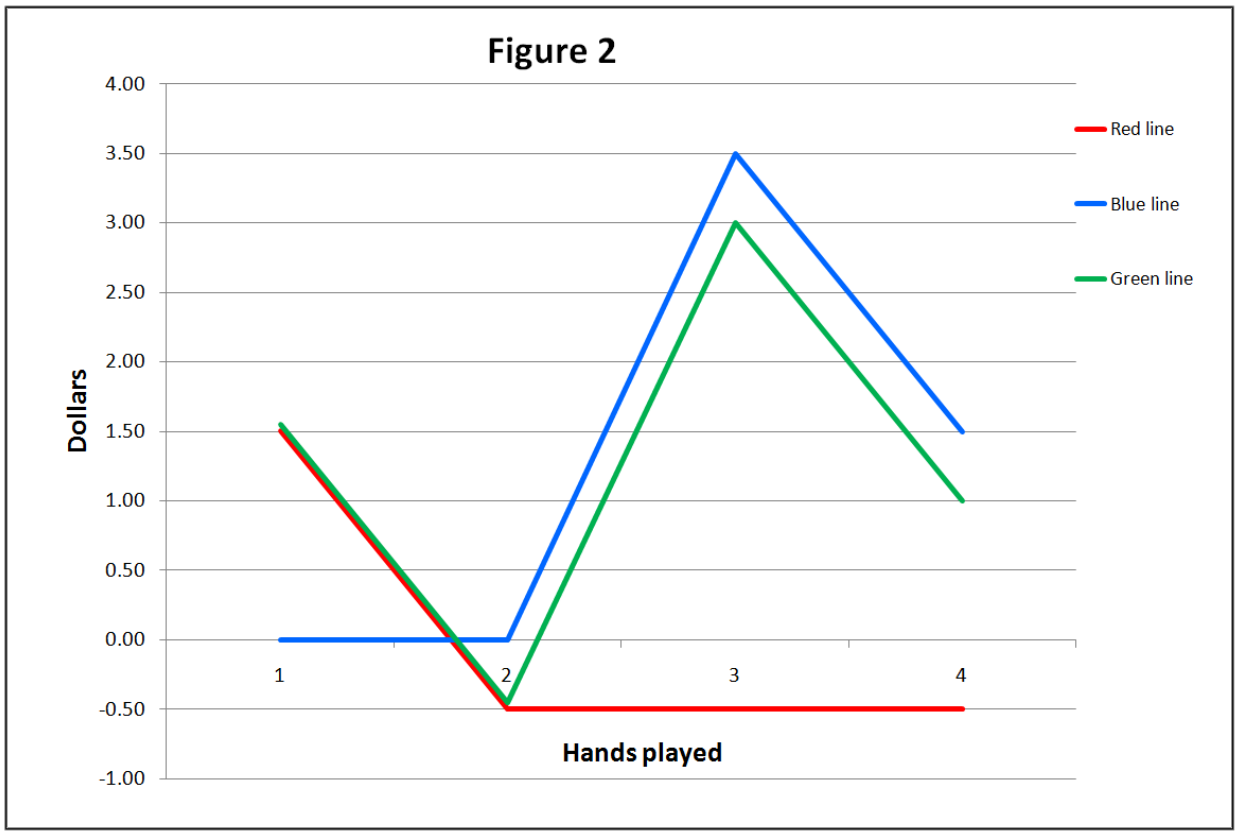
- Hand 1: Player 1 bets $2 pre-flop and all other players fold. Player 1 wins the blinds amounting to $1.50.
- Hand 2: Player 1 bets $2 pre-flop and is raised by another player behind him. Player 1 folds without seeing any more cards.

- Hand 3: Player 1 bets $2 pre-flop and is called by a player behind him. Both players check on the flop, turn, and river and turn their hands over. Player 1 has the best hand and wins the pot.
- Hand 4: Player 1 bets $2 pre-flop and is called by a player behind him. Both players check on the flop, turn, and river and turn their hands over. Player 1 has the worst hand and loses the pot.

We can take these 4 hands and graph the green line profits of Player 1:

We see that over these 4 hands Player 1 has won a net of $3, but where and when this money was won is also of importance. Breaking down the hands further we see that Player 1 earned -$0.50 without getting to a showdown and earned $3.50 when there was a showdown and the best hand won.

We now add this data to the graph and now have a more in depth idea of how Player 1 is earning money playing poker:



Let's assume Player 1 plays 10,000 hands and has results that exactly follow this pattern. We see that he has earned -$1,250 when hands do not get to showdown and $8,750 when hands do get to showdown. This tells us that Player 1 plays more quality hands and tries to get the money into the pot when he has the best hand. If he doesn't have the best hand in his mind, he folds. Now let's contrast this with Player 2 who has earned $8,750 when hands do not get to showdown and -$1,250 when hands do get to showdown. Player 2 is earning her money by forcing her opponents to fold their hands before she has to show what she has. Her cards are irrelevant in her overall profits. Both of these are viable options for earning profits in poker but the question is when each method is more appropriate.

Player 1 is looking around for a table to join. He has two options, he can either sit at a table with 8 other players that player exactly like Player 2 or he can sit at a table with players that play exactly like him. In this case he would want to sit at the table with all the players like Player 2 because Player 1's profit is coming from having the best hand and the Player 2 type players have the best hand less often than he does. Conversely, Player 2 when provided the same two options should sit in the game full of Player 1 type players because her value comes from making other players fold their hands which Player 1 type players are more likely to do.

These two players both have their own style of play and both can be equally effective at the right table. Both players are eventually going to run into the same problem. If Player 1's only option is to sit at the table where he is playing with only Player 1 type players, he will be unable to make any money unless he just gets more good hands than his opponents. Player 1 must rely on luck to beat the table full of players like him to win. If however Player 1 plays like Player 2 at this very same table, he will be able to earn plenty of money in the game without needing to rely on just luck. By switching his play he has turned an unprofitable situation into one that he can exploit for value. Player 2 can do the same thing as well. This illustrates a crucial part of the skill of poker. Making adjustments at the table based on your opponents can make or break a particular session. There is no one style of player that is perfect in every situation. You need to be able to mix up your play to be a long term winner in poker.

Also, the breakdown of each line can be related back to the overall equity discussion. A blue line equity calculation would be the same as what we developed earlier. Now let's change the example just a bit. Let's say that instead of going all in our opponent only bets $100, half the money in front of him. All the other players fold around to us and we have AA in the BB. We now have the option of going all in ourselves, which based on our previous calculations we should take because we know that we have over an 80% chance to win the hand and tremendous equity in this pot. But now we have two

ways of winning the hand, we can also force our opponent to fold in which case we will win the pot 100% of the time he does this. That 100% goes directly into our red line profits.

With our opponents range being 99+, AKs we can calculate how many hand-combinations he really has. Each pair has 6 combinations of happening with exception of AA which he can only have 1 way since we hold two of the 4 aces. He can also make AKs 2 ways with the two remaining aces and the two kings in the deck that match their suit. So we see that our villain has 6 * 5 + 1 + 2 hand-combinations for a total of 33 hands. Looking at our villain's stats we see that he is folding to a 3bet 98.3% of the time which means he will fold 99, TT, and JJ in this spot. That means 18 times our of 33 we win the pot 100% of the time and 15 times out of 33 we win the pot 80.704% of the time. When our opponent folds we earn $301 dollars for a $200 investment netting us $101 and as calculated earlier we earn $323.63 for our $200 investment earning us $123.63.

We can now calculate our equity in the hand with the information we have. We see that our equity in this case is $101(18/33) + $121.63(15/33) which is $111.29. The second half of the equation is the money that goes into our blue line and the first half of the equation is money that will go into our red line. The red line portion is also known as your fold equity in the hand. Fold equity is just the likelihood of your opponent folding multiplied by the money you win when he or she does fold. This number is what your red line is based on.

Conclusion

Every decision in poker can be boiled down into a mathematical problem. How likely a player is to win a hand, how much money he or she can win, how likely and opponent of yours is to fold, the list goes on and on. Everything about poker can be turned into a simple calculation and evaluated as a good

or bad play based on the long term equity of a hand. We can also see that mathematically the luck factor of a game may be greater in the short term, but a player's long term win rate will have a significantly greatly affect the more hands that are played.

All of these factors can be put together in a real world example. The following is a real hand history from October 16th of 2010. This hand can be broken down just like the examples provided to this point.

Cereus Network - $0.50 NL - Holdem - 6 players
Hand converted by PokerTracker 3

BTN: $4.85
SB: $81.40
BB: $40.45
UTG: $48.75
Hero (MP): $91.35
CO: $49.90

SB posts SB $0.25, BB posts BB $0.50

Pre Flop: *(pot: $0.75)* **Hero** has


UTG calls $0.50,

The first decision of the hand is often the most important. The hand itself isn't really all that important, but your position and plan for this hand are what really matters here. In this particular case we are 2nd to act pre-flop in a 6-handed game and we have some history with the players at the table. We can look at each player's stats to give us a better idea of how to play and what our opponents' tendencies are.

Hero calls $0.50, *fold*, BTN calls $0.50, *fold*, BB checks

Flop: *($2.25, 4 players)*

BB checks, **UTG** checks,

Now that we have hit a big hand, what is the best way to proceed? That depends on what we know about our opponents and how we have played the hand until this point. Do our opponents like to call on the flop with a wide range of hands? If we bet are we likely to get a call or would it be a better idea to simply play the hand slowly and let our opponents do the betting for us?

Hero bets $1.50, *fold*, *fold*, **UTG** calls $1.50

Turn: *($5.25, 2 players)*

UTG checks, **Hero** bets $4.00, **UTG** raises to $17.25,

Most importantly, how do we get the most value from the hands we have? Depending on what position we are in, how much money we effectively have left, and the strength of our hand as well as everything else to this point, does it make sense to go for maximum value or should we slow down?

Hero raises to $57.00, **UTG** calls $29.50

River: *($98.75, 2 players)*

Even with all this concrete, mathematical evidence, the real skill in poker is in knowing when to apply each calculation. As illustrated by the real world hand history example, it is easy to get bogged

down in questions. There are dozens of factors that play a part in each decision at the tables. The true skill comes in finding what information is relevant to the situation you find yourself in. Knowing when a player is likely to fold depends not just on the numbers but on how that player plays a particular hand. Understanding an opponent's range of hands comes from careful observation and practice. Simply knowing the numbers isn't enough, you need to be able to really understand each piece of data in depth and be able to analyze how relevant that data is to your purpose. The entirety of the math in the game will only get you so far, being able to read the trends as they develop can only come with practice and repetition.

In short, poker boils down to a simple fact that the results you experience are not reliant on the luck of the draw. It takes more than just a seat at the table to make money and you have more control over your fate than you realize. Poker is a game of skill where luck plays a small role rather than just another casino game like craps or roulette. A little luck in your favor can certainly help, but over a longer period of time if you are losing money, you simply have yourself to blame.

Glossary

All in -> To put all of your money into the pot.

Bet -> Term used to describe the action of putting money into the pot before anyone else has.

Blinds -> Blind bets of a specified amount put up in a Texas Holdem poker game to get the action started.

> **Big Blind ->** The second blind of the hand which is put up by the player to the left of the small blind. It's value is twice that of the small blind.
>
> **Small Blind ->** The first blind which is put up by the first player to the left of the dealer.

Buy In -> The amount of money a player starts with at the table.

Flop -> The first three community cards in a game of Texas Holdem put out after the first round of betting.

> **Pre-flop ->** Term used to describe all action before the flop.

Fold -> The act of surrendering one's cards.

Implied Odds -> Odds that factor in the odds the pot is affording plus the odds that the opponent's stack is providing. Implied odds assumes you will get your opponent to go all in when you hit your hand.

Pot -> Amount of money that has been wagered in the hand to this point.

> **Main Pot ->** Money that all the players in the hand are playing for.

Side Pot -> In a multi-way pot, when one player is all in and the other two participants are still betting, the money is put into the side pot which only those two players can win.

Raise -> The act of increasing the amount wagered after someone else has put in a bet.

Range of hands -> All of the potential card combinations a player is going to have during a given hand of poker.

Suit -> Spades, hearts, clubs, and diamonds.

Suited -> Two cards that are of the same suit

Table Stakes -> Refers to the blinds and antes that a particular game is being played at.

Appendix A: Poker Stove Calculations

Example 1:

Text results appended to pokerstove.txt

410,952,960 games 0.423 secs 971,520,000 games/sec

Board:
Dead:

	equity	win	tie	pots won	pots tied	
Hand 0:	56.773%	53.73%	03.04%	220823484	12485934.00	{ 88+, ATs+, KQs, AKo }
Hand 1:	43.227%	40.19%	03.04%	165157608	12485934.00	{ AQs }

3,931,449,984 games 4.155 secs 946,197,348 games/sec

Board:
Dead:

	equity	win	tie	pots won	pots tied	
Hand 0:	35.318%	33.74%	01.58%	1326370496	62125052.00	{ 33+, A2s+, K2s+, Q2s+, J4s+, T6s+, 96s+, 86s+, 76s, 65s, A2o+, K5o+, Q7o+, J7o+, T8o+, 98o }
Hand 1:	64.682%	63.10%	01.58%	2480829384	62125052.00	{ AQs }

Example 2:

Text results appended to pokerstove.txt

1,712,304 games 0.001 secs 1,712,304,000 games/sec

Board:
Dead:

	equity	win	tie	pots won	pots tied	
Hand 0:	82.637%	82.36%	00.27%	1410336	4654.00	{ AcAd }
Hand 1:	17.363%	17.09%	00.27%	292660	4654.00	{ KcKd }

1,370,754 games 0.026 secs 52,721,307 games/sec

Board:
Dead:

	equity	win	tie	pots won	pots tied	
Hand 0:	67.670%	67.47%	00.20%	924864	2728.67	{ AcAd }
Hand 1:	17.232%	17.03%	00.20%	233476	2728.67	{ KcKd }
Hand 2:	15.098%	14.90%	00.20%	204228	2728.67	{ QcQd }

Appendix B: Rank of Hands

Royal Flush:

AKQJT all of the same suit.

Straight Flush:

5 connecting cards that make a straight all of the same suit (Ex: 5s6s7s8s9s)

Four of a Kind:

4 cards of the same rank in the hand (Ex: JJJJ5)

Full House:

3 cards of the same rank combined with 2 cards of a different rank. The higher 3 card rank is the bigger hand (Ex: QQQ55 beats JJJAA).

Flush:

5 cards together of any rank all of same suit. The highest combination of ranked and suited cards wins the pot (Ex: AhKh7h4h3h beats QhJhTh5h2h)

Straight:

5 cards ranked in consecutive order regardless of suit (Ex: T9876)

Three of a Kind:

3 cards of the same rank coupled with two other cards of different rank from both the matched 3 and the other unranked card (Ex: 555JQ)

Two Pair:

2 cards of the same rank coupled with 2 cards of a different rank that are the same as one another with one card of unique rank. The better hand is the hand with the highest pair, in the event the biggest pair each player has is the same the winner is the hand with the better second pair. If there is a tie still at this point the higher unpaired card wins (Ex: AA225 beats KKQQ7, AA55T beats AA225, and AA22K beats AA225)

One Pair:

2 cards of the same rank with 3 cards of unique rank in the same hand. If the ranked cards are the same, the pot goes to the player with the highest unique card, then the second highest if the highest cards are the same, and the third highest if the second highest card is also the same (Ex: AA543 beats KKJT9, AAT43 beats AA543, AAT53 beats AAT43, and AAT53 beats AAT32)

High Card:

5 cards that are unconnected, do not all share a common suit, and are all unique in rank. The value of the hand is based on the highest ranked card in a players hand and goes in descending order from there (Ex: AT874 beats A9874)