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# INTEGRATIVE SPINE DYNAMICS WITH RESPECT TO PUSHING, PULLING AND LIFTING

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## ABSTRACT

There is a growing need to understand spine dynamics with respect to safety in manufacturing environments. Lower back pain is becoming an increasing problem for manufacturing employees and expensive for manufacturing companies. Excitation loads and motion on the lumbar spine due to manufacturing activities such as pushing, pulling and lifting objects can cause spinal instability, and may even lead to lower back pain. Because of this, there is a need to create a safer working environment.

The goal of this project is to formulate a mathematical spine model that predicts forces and motion trajectories for safely and effectively pushing, pulling and lifting objects in manufacturing environments. An inverted pendulum sufficed as a single degree of freedom model, and the pivot of which represents the most problematic lumbar joint, L4-L5. The flexor and extensor trunk muscles allow the spine to bend forward and backward with respect to the L4-L5 joint. The flexor and extensor muscles are represented with a set of springs and dampers, while the muscle reflex delays provide modulations to feedback gains.

Dynamic equations, which determine forces, moments, velocities and accelerations of the spine are integrated with spinal stability indices. The stability indices determine how robust the lumbar spine is against perturbations while pushing, pulling and lifting objects. The proposed model provides a framework for new safety guidelines in manufacturing environments.

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## ACKNOWLEDGEMENTS

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# CHAPTER I: SPINE AND RELATED DYNAMICS

## 1. Introduction

There is a growing need to understand spine dynamics related to manufacturing activities. Manual labor in manufacturing environments often requires workers to perform repeated motions such as pushing, pulling and lifting objects. When performing these activities, a worker performs the task within a particular range of motion and is subjected to reaction forces from the object being moved. The forces acting on the individual's spine, if excessive, can cause instability. This instability can increase the risk of a back injury.

In the past, back injuries have been difficult not only to diagnose, but also to treat and cure. Back injuries caused by working conditions in manufacturing environments can become costly for companies. Whether workers are taking days off to rest sore back muscles, or receiving workman's compensation for a more serious back injury, it is always at the company's expense. It is much more efficient for a manufacturing company to create, or revamp a workspace. The parameters of the workspace should limit the range of motion and subjected forces to promote better safety standards and spinal health for the employees.

The first objective of this Major Qualifying Project is to research the spine dynamics required for the specific manufacturing activities of pushing, pulling and lifting objects. The dynamic equations are incorporated into the mathematical model and all variables are kept generic for experimental verification. The dynamic motion and forces experienced by the spine ultimately determine spine stability.

The next objective is to determine the spine stability caused by the dynamic motion and forces with respect to pushing, pulling and lifting. The stability indices for the spine relate forces and dynamic motion, and in turn, determine how robust the spinal system is against a debilitating perturbation. With the help of Biomechanics Professor, Jacek Cholewicki, data and stability calculations from EMG readings of volunteer spine experiments of Yale University are incorporated into the model.

Next, a simplified mathematical spine model is proposed to simplify and accommodate assumptions. For this project, an inverted pendulum representing a single degree of freedom spinal system is considered. Attached to the model are springs and dampers which represent the flexors and extensor muscles, accordingly. In addition to this model, reflex delay response from the muscles is represented as gain. The focus of the spine model is on the L4-L5 joint which is represented as the pendulum pivot point. The pendulum contains a center of mass at the top, which is located at the T9 vertebra. This center of mass represents the force from the upper body above the hips. All perturbations are acted on this mass.

Using a mathematical spine model can determine the parameters essential for a healthy individual to safely execute the dynamic activities of pushing, pulling and carrying weight. Also, various amounts of weight and spinal flexion, either one time or in repetition, may be accounted for. This model can therefore be used by various manufacturing companies to either revamp a workspace or promote customized safety guidelines to create healthier working environments for their employees.

The remaining portions of this paper are as follows: Chapter II contains extensive research on spine dynamics, including: lower back pain, spine physiology, biomechanics, dynamics, stability and model formation. Chapter III contains the results of spine dynamics with respect to manufacturing activities, including: the spine model, previous work done on similar models, governing equations and how they vary to different activities. Finally, Chapter IV contains a discussion and conclusion.

# CHAPTER II: SPINE DYNAMICS

## 2. Introduction

Spine dynamics play an important role in the stability of the spine and subsequently, good spinal health. By finding the causes of spinal instability, risk of lower back injury is reduced. Extensive spine research is conducted to obtain a better understanding of how to model the spine during the activities of pushing, pulling and lifting.

Position, velocity and acceleration determine the dynamics of the spine. Physiological limits to these factors are provided by research into spine physiology and biomechanics. Dynamic equations are derived for pushing, pulling and lifting. The equations uniquely describe potential forces and motions experienced during each of the three activities. Spine stability is determined by integrating reflex delays of the muscle and the dynamic equations.

## 2.1 Lower Back Pain

Manufacturing activities are a major factor causing lower back injuries. Table 1 depicts the significance of lower back pain. Statistics from occupational companies report that overexertion of the lower back makes up a quarter of all on-the-job injuries. The three activities of pushing, pulling and lifting are high risk activities for back injuries. Out of all reports of lower back injury, lifting accounts for 66%, while pushing and pulling account for about 20%. The results of these injuries are very costly to industries. Just in North America, the total industry cost for lower back injuries is in the billions of dollars per year [64].

**Table 1: Lower Back Pain Statistics [64]**

| Percentage of Reported Occupational Injuries | |
|---|---|
| Overexertion of Lower Back | 25% |
| Occupational Activities Causing Lower Back Pain | |
| Lifting | Pushing and Pulling |
| 66% | 17-20% |
| American Lower Back Pain Statistics | |
| Report Back Pain in Lifetime | 80% |
| Cases of Lower Back Pain per Year | 880,000 |
| Total Industry Expenses | $20-100 billion |

"Physical work that requires heavy lifting and frequent twisting is the most likely to cause back problems" [2]. Even jobs that require sitting or standing for long periods of time can cause lower back pain. Even though the causes for lower back pain can vary, "the majority of back injuries involve damage to muscles, tendons, and ligaments—your body's *soft tissues*" [3]. A back sprain or strain are caused during movements such as twisting, lifting and bending and may take several weeks or months to completely heal [4].

The activities of pushing, pulling and lifting can cause instability in the spine and put a worker at risk of back injury. According to *Occupational Biomechanics*, there are two different modes of back failure. The first mode is called a single overexertion event, known otherwise as a strain to the tissue surrounding the spine. The result of this type of failure is an inflammatory reaction accompanied with pain and temporary or permanent impairment acute failure [30].

The second mode of failure is due to repeated sub-maximal exertions. This is usually due to repeated motions where muscle fatigue is present and in which micro-strain injuries arise. The result of this injury is a decrease in tissue tolerance, and if prolonged, the “capacity of tissue drops below of induced tissue strains accompanying each repeated exertion” [30]. This is a serious injury that can result from the cumulative trauma [30].

Other risk factors include age, weight, gender and general health. Also, less common back injuries may include fractures to the vertebrae themselves, or slipped or herniated intervertebral discs [5]. The focus for this project is on the muscles; mainly the flexors and extensors for a model. The effects of prevailing health problems are complicated to decipher and will not be discussed in detail.

In order to decrease the risk of back injuries, manufacturing companies conventionally provide training aids for their workers. These training aids outline a preferred method of performing a given task. A common example of a training aid is the lifting guidelines which inform individuals to keep a straight back and lift only with the legs. Even with these guidelines, it is still hard to predict if a worker is at risk for injury. This is mainly because every individual is different; some people are taller than others, heavier, stronger etc. Therefore, each individual has different safety parameters.

As seen in figure 1, the parameters for the working environment are planned and calculated using various heights and positions [1]. For example, a shelf at height $V_D$ which has objects that need to be repeatedly placed at a table at height $V_o$ will provide a trajectory of motion. Dimensions such as these provide an idea of the motions and forces involved for the spine dynamics. The dynamics therefore predict how the spine will behave during a given activity.

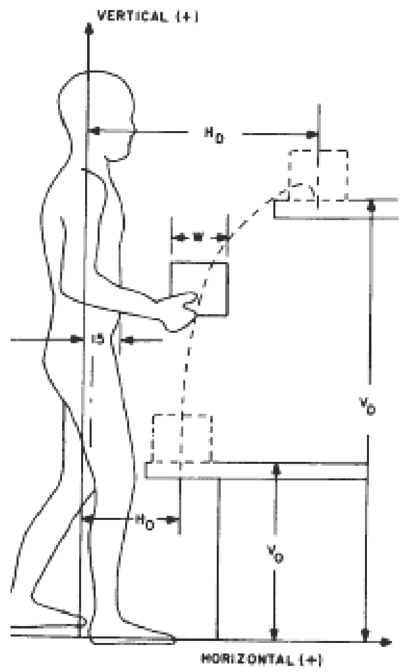



**Figure 1: Manufacturing Environment Parameters**

The common consequence of poorly planned working environments or insufficient training guides is lower back pain. Lower back pain may easily cause decreased efficiency of the worker to continue his/her job, days off from work, or at worst, an expensive medical condition, all of which are at the expense of the company.

By using a model which has variable physiological parameters, a quick and customized calculation can be made. Such parameters may include an individual's height, weight, etc, and parameters unique to a given activity, such as the range of motion and the weight of a load.  This can easily determine whether an individual is safely executing a task, or at risk for a back injury.  A model can therefore be used to create customized training aids or help to plan better workspace dimensions on the manufacturing line.

## 2.2 Spine Physiology and Biomechanics

The physiology and the relative biomechanics of the spine provide the basis for the limits of motion and forces the spine can endure. A healthy spine is able to transfer weight and bending moments of the head, trunk and pelvis safely. With the help of surrounding trunk muscles, the spine allows physiologic motions between the head, trunk and pelvis as well. Also, the spine and trunk muscles protect the spine cord from experiencing damaging forces [6].

The vertebrae provide support, while the intervertebral discs acts as a pivot point between vertebrae. Therefore, the vertebrae are considered levers, while the discs are considered as confined joints [6]. When the spine experiences forces greater than it can resist, the skeletal tissues will ultimately fail.

The muscles surrounding the spine allow the vertebral column to bend or twist and also provided stability and support. The muscles offer stiffness and are therefore thought of as the actuators of the system [6]. Spinal muscles can fail to properly support the spine if they experience too much force.

Finally, the nervous system provides reflex delay time from the muscles. The spinal cord is very delicate and requires protection from the skeletal and muscular system in order to avoid damage. In order to prevent injury, the muscle reflexes of the back must be strong and fast enough to compensate for a sudden load. Without the nervous system, the muscles would not fire in a timely fashion and therefore not stabilize the spine before injury ensues. In a mathematical spine model, the nervous system will be represented as feedback gains, where changes in milliseconds can make the difference between a healthy spine and a possible injury.

### 2.2.1 Spinal Skeletal System and Motion

The spine's structure and basic function is depicted by the skeletal system. The vertebral column allows the spine to bend forward, backward, side to side and turn and rotate on its central axis [7]. The vertebral column is made up of a series of vertebra and discs which allow for the numerous degrees of freedom.

The spinal column is broken into four curvatures, or sections, as seen in figure 2. The primary curves are the thoracic and pelvic curvatures which are both concave anteriorly, (towards the front of the body). The thoracic curvature is located between the ribs, and the pelvic curvature makes up the pelvic area which includes fused bones in the sacrum and coccyx.

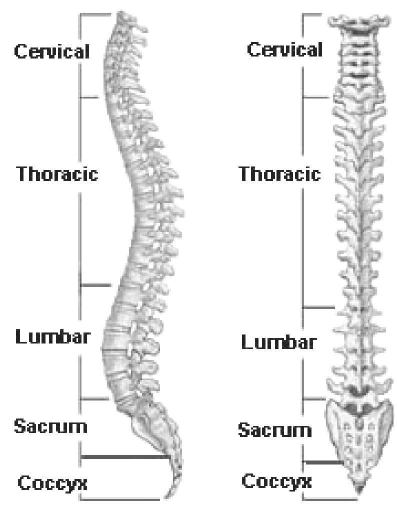



**Figure 2: Main Curvatures of the Spine**

The secondary curves are the cervical and lumbar curvatures which are both convex anteriorly [7]. The cervical curvature makes up the neck and supports the head. The lumbar section, on the other hand, supports the weight of the body above it. It also allows for maximum bending and twisting in the spine. [7]

Since the lumbar section of the spine allows for the most movement and supports the most weight, it is most vulnerable to injury when performing various physical activities [8]. The L4-L5 joint is notorious for being the most problematic joint of the lumbar section. Therefore, the focus of a model should be on this joint.

The lumbar section of the spine is made up of a series of vertebrae of different shapes and sizes. The vertebrae are separated by intervertebral discs which are connected together by ligaments [7]. An example of an intervertebral disc and a vertebra can be observed in figures 3(a) and 3(b). These discs act as joints from which each vertebra can pivot on [7].

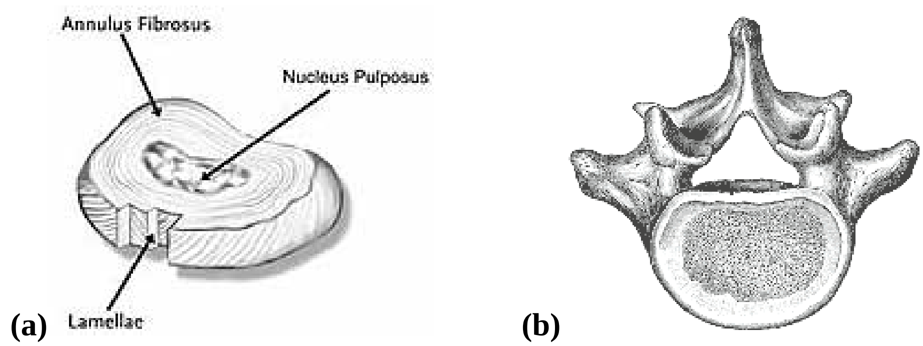



**Figure 3: Generic (a) Intervertebral Disc and (b) Vertebra**

The intervertebral disc between at the L4-L5 joint has key properties for understanding how the spine operates for an accurate model. The intervertebral discs are composed of a tough outer layer of fibrocartilage (annulus forbrosus) and an elastic central mass (nucleus polpusus) [7] as seen in figure 3(a).

The L4-L5 disc is subjected to various compressive forces. Not only is this disc subjected to the weight of the upper body, but also forces due to dynamic motions such as walking, jumping, etc. Loads can be subjected in short or long duration. Short duration loads can be high in amplitude and cause irreparable structural damage. Long duration loading applies to many manufacturing activities where the spine is subjected to a lighter load over a longer period, or repeated loading. Long duration loads are responsible for fatigue failure in the discs, which they are most prone to [9].

Interestingly, the discs behave differently depending on the magnitude of the load it is subjected to. The intervertebral discs provide little resistance at lower loads to allow for more flexibility and movement. For larger loads, on the other hand, the discs become stiffer to help increase stability for the spine.

When the spine is bending, one half of the disc is subject to compression forces, while the other half is subjected to forces in tension [10]. Generally the lumbar discs, such as the L4-L5 disc, exhibit larger torsional strength in the posterior and anterior sections. This is to help accommodate larger bending moments [10].

The intervertebral discs are composed of a viscoelastic structure, and therefore display viscoelastic behavior. Due to this behavior, a phenomenon called hysteresis is observed which helps the discs effectively absorb shock away from the brain [11]. The greatest amount of hysteresis is observed in the lower lumbar discs, including the L4-L5 joint [12]. It is also interesting to note that the hysteresis decreases significantly when the same disc is loaded a second time [11]. This may prove to be important when looking into repetitive loading in manufacturing activities.

Finally, the spinal ligaments add restraints to the system. These ligaments act like rubber bands which resist tensile forces, but buckle when subjected to compression [13]. An example of the variety of ligaments in the lumbar section can be seen in figure 4.

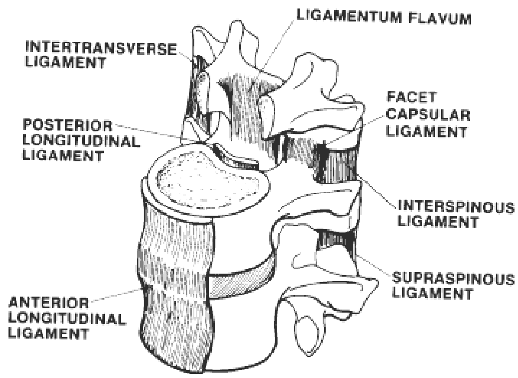



**Figure 4: Various Ligaments of the Lumbar Spine**

They restrict the motion of the vertebrae within well defined limits. The ligaments therefore help provide stability to the spine by reducing the amount of motion it can achieve [13].

### 2.2.2 Spinal Muscular System and Stiffness

The motions of the spine and trunk are controlled by muscles found in the back and abdominal sections of the human body. In order to formulate an accurate spine model, essential muscles and their functions are identified in this section. Key ideas and assumptions regarding muscle function are deliberated. The relationship between motion, muscle length and stiffness is also discussed in this section.

Muscles related to the movements and reflexes of the spine when bending forward and backward are separated into two main groups; the flexors and extensors. These two muscle groups work in conjunction to keep the spine upright, control motion and protect the spine and spinal column by sufficiently reacting to external forces. Large muscles are used to create larger trunk movements and provide stiffness. Small muscle groups provide precise control of the large movements [14]. A model is kept simple by collectively grouping the muscles.

Flexor muscles are found in the abdominal region of the human body. There are four abdominal muscles: the external oblique, internal oblique, transversus abdominis, rectus abdominis as seen in figure 5. They are also called the prevertebral muscles and, for the scope of the model, these muscles are treated as one unit as the flexor muscles [15]. The flexor muscles' primary function is motion control, but they also assist the body with expiration and inspiration [16].

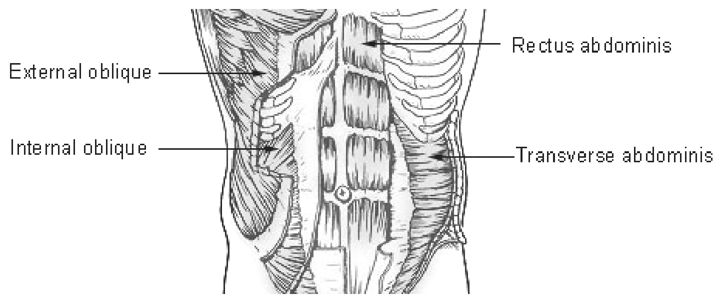



**Figure 5: Trunk Flexor Muscles**

The extensor muscles are found on the back side of the human body. The muscles of the back are separated into three groups, deep, intermediate and superficial. These three groups are collectively referred to as the postvertebral muscles [14]. For the scope of the model, these muscles are also treated as one unit, the extensor muscles. The extensor muscles have several functions. They keep the spine and head upright, assist in respiratory functions, control large and small movements of the back as well as provide dynamic stability to prevent injury [17]. A more comprehensive list of muscles related to the spine and its movement can be found under Appendix A.

The way muscles work at the molecular level relates to the different contractions muscles can produce. This also develops the relation between muscle length and stiffness, which will be discusses later. At the molecular level, muscles are made up of parallel filaments of proteins [18]. When the brain wants to contract a muscle, the larger of the two filaments creates crossbridges that link to the smaller of the filaments. The larger filaments are made from a protein called myosin and the smaller filaments are made from a protein called actin [19].

Once the myosin crossbridges are established, the myosin heads curve and create a pulling action on the actin filament. Figure 6 shows two myosin heads attached to the darker actin filaments, forming a crossbridge. The left half of the figure shows the myosin bending, showing a pulling action with respect to the actin filament.

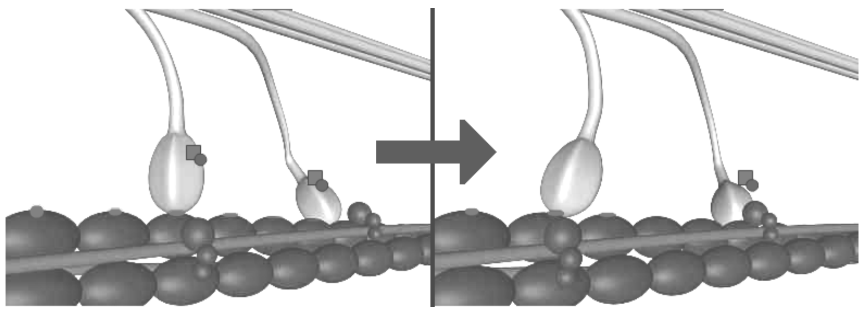


**Figure 6: Myosin Filaments Forming Crossbridge with Actin Filaments**

Next, the myosin head can release the actin filament or hold the onto the actin filament. If the myosin head releases the crossbridge, it can then establish another one and keep pulling the muscle. This action would be analogous to climbing a rope. There are thousands upon thousands of such crossbridges within each muscle that create the different kinds of contractions muscles can experience [20].

There are three different types of muscle contraction [21]. Static contractions occur while the muscle length remains constant. An example would be the bicep muscle when the forearm is flexed and held without any movement. This type of contraction relates primarily to holding the body still while counteracting external forces, such as gravity [21]. During static contraction, the myosin heads hold on to the actin filaments, maintaining the crossbridges and therefore providing stiffness [17].

Concentric contractions occur simultaneously as the length of the muscle decreases. An example would be the biceps during the flexion of the forearm. While the muscle length is shortening, the muscle can cause movement of the body. During concentric contractions, the myosin heads continually create crossbridges, pull on the actin filament, release the crossbridge, unbend and create another crossbridge further down the actin filament. This cycle is similar to a human climbing a rope. In the case of the myosin, more 'arms' are involved [21].

Eccentric contractions occur as the length of the muscle increases but tension is still present. An example of this would be the bicep as the forearm is being extended. The tension generated by the muscle during eccentric contraction is aimed at controlling the movement of the body, such as decelerating the arm as a ball is thrown. During eccentric contraction, myosin filaments form crossbridge with actin filaments for only short periods of time [21].

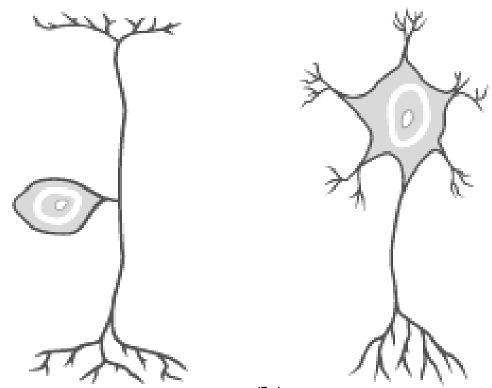
The definition of the stiffness coefficient within biomechanics is the ratio of resistance offered to the displacement imposed [22]. According to Cholewicki, the number of crossbridges present within a muscle determines its stiffness. From this, stiffness can be formulated based upon type of contraction. Static contractions cause the muscles to be the stiffest because the number of crossbridges at the molecular level is at its highest.

During concentric contraction, some myosin crossbridges have to be disconnected in order to create more pull on the muscle. Because of this, the stiffness of the muscle is not as high as in static contraction. The muscles experience the least amount of stiffness when in eccentric contraction because the number of crossbridges at any one point is the generally the lowest relative to the other contractions [23]. A better relationship between muscle length and stiffness can be made through a study of the motions as well as the contraction types. This will be done in a later section.

### 2.2.3 Spinal Nervous System and Reflex Delays

The nervous system relays nerve impulse signals between the brain and muscles, and is responsible for reaction time of the spinal system. The nervous system is a complex electro-chemical system that works as a nerve loop function. This nerve loop function takes time to execute, and this time delay is unique to every person [8]. Even though the delay takes place within fractions of a second, time differences from one person to the next can mean the difference between a healthy back, and one that is prone to lower back injury [8].

An example of a nerve loop function would be the ‘knee-jerk’ reaction where the patella tendon is tapped; this then causes a chain reaction. Since this tendon is temporarily pulled, the quadriceps muscle will be stretched. This change in length will then be sensed by sensory neurons which send impulse signals to the spinal cord. Motor neurons are located in the spinal cord, and they receive nerve impulse signals. Both the sensory and motor neurons can be seen in figure 7(a) and (b) respectively. From there, the motor neurons return the signal to the quadriceps muscle which, upon receiving the signal, will contract causing the leg to kick [24].



**(a)** **(b)**

**Figure 7: Example of a (a) Sensory Neuron and (b) Motor Neuron**

The knee-jerk loop is very similar system for the extensor and flexor muscles in the back. While in motion, the body responds to stimuli and the nervous system helps calculate how to compensate within a second's time. An example of the nerve loop for supporting spine flexor and extensor muscles can be seen in figure 8. Muscles are shown on the left, as well as the neurons connecting it to the spine, which can be seen on the right. The sensory neuron is the small feature seen above the spinal cord, while the motor neuron is located inside the spine.

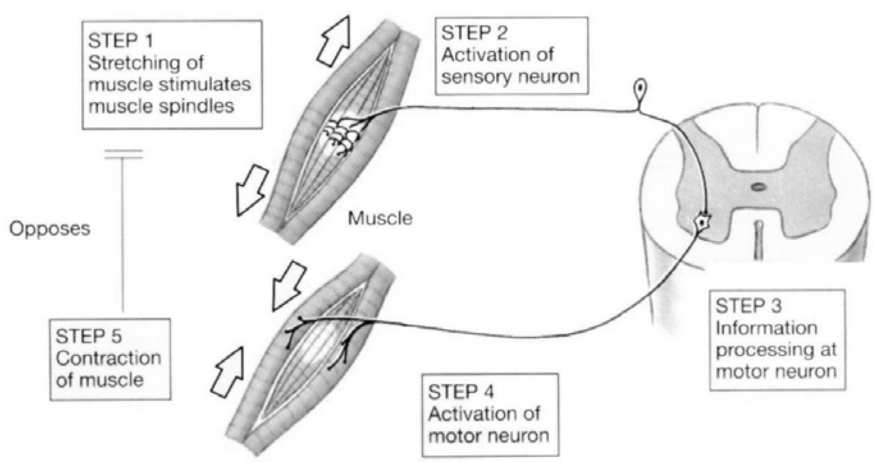



**Figure 8: Reaction Nerve Loop for Spine**

The axons are the hair like extension of a nerve cell that carries messages as seen in figure 9. They play an important role for the functioning of the action potential which is the electrical part of a neuron's two-part, electrical-chemical message. The action potential consists of a brief pulse of electrical current that travels along the axon. A neurotransmitter release is triggered when the action potential reaches the axon terminal, which can also be seen in figure 9 [25].

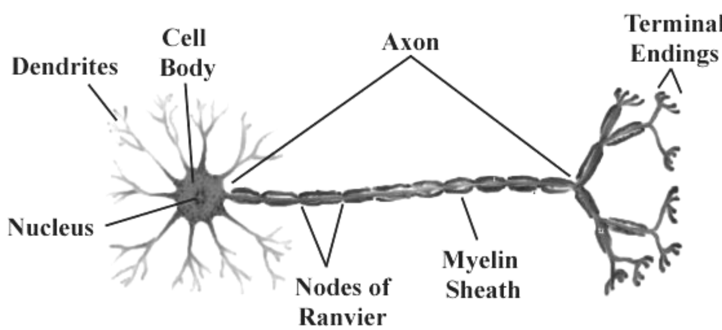



**Figure 9: Parts of a Neuron**

The action potential is the "long distance" signal that carries information in the nervous system with a strong enough stretch causing multiple action potentials. The membrane potential is created by the difference in electric voltage across the cell membrane. On the other hand, the resting potential is when there is no stretch detected in the muscle membrane [26].

The action potential is important when dealing with reflex delay. When there is a stretch in the muscle, there is a reflex delay depending on the distance of the recording site to the muscle. The action potential propagates without decrement through the neuron with a relatively low speed. The amplitude of the signal remains unchanged and the reflex delay of the signal is about 0.1 seconds from the muscle to the spinal cord. The average velocity is 15 meters per second, with the highest speed reaching 100 meters per second [27]. This delay is directly related to the stability of the spine, and therefore whether the spine is at risk for a back injury.

Interestingly, after a neuron fires an action potential there is a short period called the absolute refractory period as seen in figure 10. During this period it is impossible to trigger another action potential. The refractory period lasts about 1 millisecond this limits the firing rate of a neuron to about 1000 action potential per second [27]. The relative refractory period may allow a second action potential trigger, but the intensity is far less.

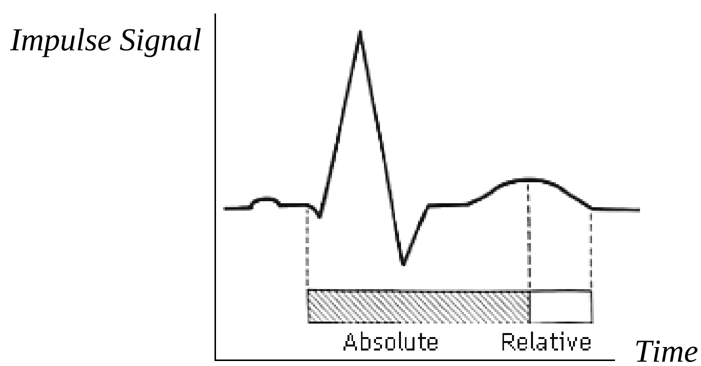



**Figure 10: Refractory Period**

In the case for flexors and extensors of the spine, the nervous system can determine the velocity of the spine bending forward as an example. In order to stop the spine from continuing at that speed and bending too far too fast, the nervous system relays information to stop the flexor from pulling and initiate the extensors to pull in the opposite direction. By doing this, the muscles can gain stability and protect the spine.

## 2.3 Spine Dynamics

The spine dynamics section describes the three activities of pushing, pulling and lifting in detail. Forces, dimensions and other parameters are kept in variable form to allow for later application of the model. This section also describes how to locate forces acting on the spine and how this relates to a single degree of freedom model.

First, relevant information about the kinematics of the spine is described for range of motion and stiffness. Second, a general description of the biomechanics of lifting, pushing and pulling is described with respect to maximum forces and effective handle heights. Instability of the spine is also discussed with respect to factors such as friction forces of the feet and exaggerated body positions while pushing and pulling.

Since the focus of the model is on the lumbar spine, the range of motion and following stiffness coefficients will be directed towards the lumbar section, and specifically the L4-L5 joint when possible. The limits of motion (in degrees) for the lumbar spine can be seen in table 2. The combined flexion/extension values in this table are represented as θ in later diagrams such as figures 11, 12 and 13. Not surprisingly, the greatest range of motion is observed in combined flexion and extension (bending forward and backwards respectively), while the most limited range of motion is for the one side axial rotation.

**Table 2: L4-L5 Range of Motion [degrees] [28]**

| | |
|---|---|
| Combined Flexion/Extension | 9-21 |
| One Side Lateral Bending | 3-9 |
| One Side Axial Rotation | 1-3 |

Average stiffness coefficients will be used for the spring coefficients in the spine model. As seen in table 3, the average stiffness of the lumbar region varies between different motions. The orientation of the spine in table 3 is seen in figure 15. The highest stiffness values are observed when the lumbar section is in compression and in an axial rotation moment. The lowest amount of stiffness can be observed during anterior shear motion and a flexion (bending forward) moment. Stiffness is a main factor in affecting the stability of the spine and will play an important role during creation of a spine model.

**Table 3: Average Lumbar Stiffness Coefficients [N/mm] and [Nm/deg] [29]**

| **Forces (N/mm)** | | | | |
|---|---|---|---|---|
| Tension | Compression | Ant. Shear | Post. Shear | Lat. Shear |
| (+FY) | (-FY) | (+FZ) | (-FZ) | (FX) |
| 770 | 2,000 | 121 | 170 | 145 |
| **Moments (Nm/degree)** | | | | |
| Flexion | Extension | Lat. Bending | Axial Rotation | |
| (+MX) | (-MX) | (MZ) | (MY) | |
| 1.36 | 2.08 | 1.75 | 5 | |

Depending on the direction a person is experiencing a load and its magnitude can affect the trunk stiffness as a whole. As seen in table 4, as the magnitude of the load increases, the effective trunk stiffness does as well. The loads are given in percent body weight. Therefore, the more weight an individual experiences, the stiffer the spinal system will be.

**Table 4: Effects of Load Direction and Magnitude on Trunk Stiffness [Nm/rad] [32]**

| Horizontal load (%BW) | 0 | 10 | 20 |
|---|---|---|---|
| Extension | 1237 (698) | 1839 (829) | 2004 (1042) |
| Flexion | 1253 (760) | 1707 (716) | 1872 (816) |
| Left lateral bending | 1180 (722) | 1512 (715) | 1828 (743) |
| Right lateral bending | 1191 (685) | 1816 (724) | 2120 (849) |
| **Vertical load (%BW)** | **0** | **10** | **20** |
| Extension | 1493 (616) | 1606 (1030) | 1980 (965) |
| Flexion | 1028 (688) | 1586 (800) | 2218 (865) |
| Left lateral bending | 1202 (662) | 1514 (624) | 1804 (891) |
| Right lateral bending | 1225 (603) | 1819 (746) | 2083 (764) |
| (standard deviations are in parenthesis) | | | |

The first publicized set of weight-lifting limits was created by the International Labor Organization in 1962. These limits were published to help reduce back injuries due to occupational biomechanics and were based on the opinions of medical experts. They specified "safe" weight limits for different ages and genders. The problem with these first limits was that the frequency and size of the object being lifted was not taking into consideration, and because of this, no decrease in back injuries resulted. This early implementation for occupational biomechanics provides information on how other factors, such as frequency and size, will drastically affect the performance of the spine during manufacturing activities [31].

A diagram of a worker lifting can be seen in figure 11. As seen in this figure, the center of mass for the worker is at the T9 level. The back is assumed to be rigid, like the inverted pendulum model. The arms holding the object are also assumed rigid. If the arms are to be bent, then the resulting force vector would be in the direction from the shoulders to the hands. The forces acting on the center of mass are the force of the box; $F_o$ and the body weight; $m \cdot g$. The reactant forces are shown as $F_p$ and the Normal force. All possible angles for position variations are also shown.

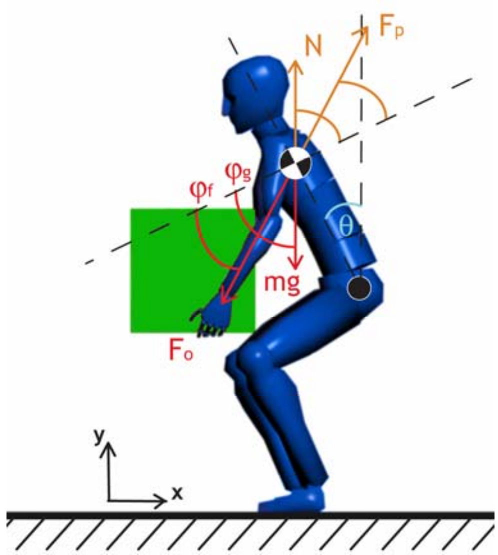



**Figure 11: Lifting Diagram**

On the other hand, about 17-20% of overexertion injuries are associated with the activities of pushing and pulling (not accounting for foot slippage) [33]. On average, the maximum pushing and pulling hand forces when moving masses up to 68 kg is a range between 40-120 N. For larger, stronger males moving masses up to 450 kg, the peak hand forces are as large as 500 N [33].

The diagrams for pushing and pulling can be seen in figures 12 and 13. The back is also assumed rigid as well as the arm position. In both diagrams the various forces acting on the center of mass can be seen. The force of gravity on the individual as well as the weight of the box, $F_o$, are shown at their respective angles, as well as the resultant forces; the normal force and the force the worker is asserting on the object; $F_p$. The model focuses on two main areas. First, the position of the spine is shown relative to the neutral position, represented by the value of θ. Second, the direction and magnitude of a perturbation force.

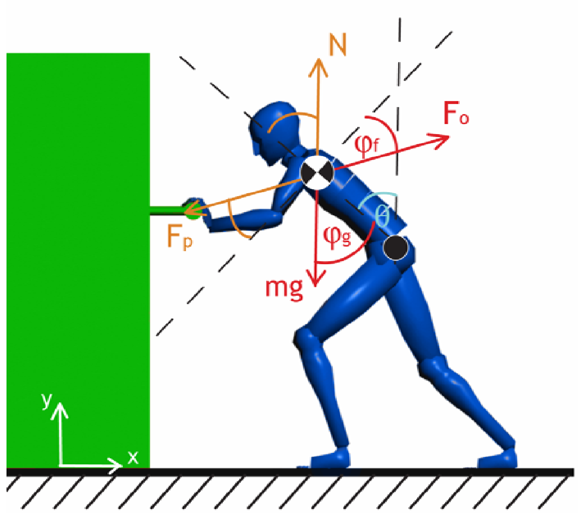



**Figure 12: Pushing Diagram**

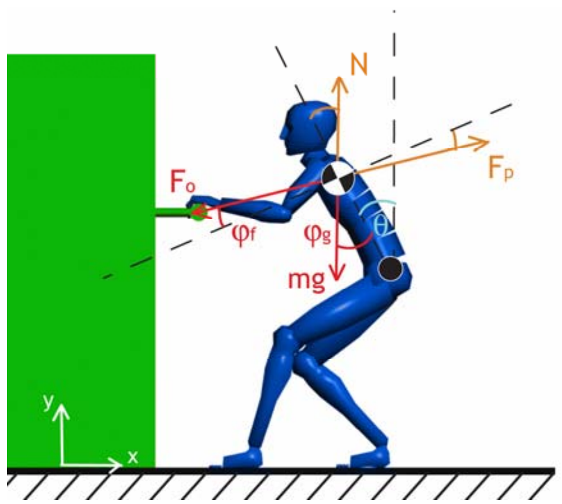



**Figure 13: Pulling Diagram**

A significant aspect to note is the hand position relative to the body. It has been found that "the vertical height of the handle against which one pushes and pulls on high-traction flooring is of critical importance" [33]. The optimal handle height when pushing or pulling is about 91 to 114 cm from the floor. This is about hip height for males, as seen in figures 12 and 13. This lower posture allows the worker to position his/her feet farther from the object (leaning farther further) when pushing and vice-versa when pulling.

This kind of positioning allows the person to use their own body weight to assist in the given activity, however, at the same time also creates a more unstable situation due to the extreme position. This is a type of situation where the floor conditions are vital for the proper foot friction to keep the person from falling and harming oneself.

The hand positions also play a significant role as far as vertical force components. If there is any vertical component to the hand forces, then depending on the position, it will either add to, or subtract from the body weight [34]. By adding to the body weight, the foot friction also increases, and vise-versa. There is a key relationship between the hand force components, floor friction and body posture which all interact in a complicated fashion [34]. This relationship ultimately determines the maximum output forces the body can produce and should be considered when describing forces on the spine. Tables for the recommended pushing and pulling forces can be seen in tables 5 and 6.

**Table 5: Recommended (90th Percentile) Male and Female Pushing Forces [N] [35]**

| | **Distance Pushed** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 m | | | 15 m | | | 45 m | | |
| **Repetition Rate** | 1 min | 5 min | 8 h | 1 min | 5 min | 8 h | 1 min | 5 min | 8 h |
| **Male** | | | | | | | | | |
| Initial (Peak) Force | 260 | 280 | 340 | 220 | 230 | 280 | 140 | 190 | 230 |
| Sustained (Average) Force | 160 | 190 | 230 | 110 | 130 | 160 | 70 | 90 | 130 |
| **Female** | | | | | | | | | |
| Initial (Peak) Force | 170 | 200 | 220 | 140 | 160 | 170 | 120 | 150 | 180 |
| Sustained (Average) Force | 90 | 100 | 130 | 60 | 70 | 100 | 50 | 60 | 80 |

From the recommended pushing forces in table 5, one can see how far the object was pushed, as well as how long the object was acted on, and the resulting force, either peak or average observed. The lowest amount of force can be observed for the longest distance and shortest time period. On the other hand, the largest forces can be observed for the shortest distance with the longest about of time, either initial or sustained force.

**Table 6: Recommended (90th Percentile) Male and Female Pulling Forces [N] [35]**

| | Distance Pulled | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 m | | | 15 m | | | 45 m | | |
| **Repetition Rate** | 1 min | 5 min | 8 h | 1 min | 5 min | 8 h | 1 min | 5 min | 8 h |
| **Male** | | | | | | | | | |
| Initial (Peak) Force | 250 | 270 | 320 | 210 | 230 | 280 | 140 | 180 | 230 |
| Sustained (Average) Force | 160 | 190 | 240 | 120 | 140 | 170 | 70 | 100 | 140 |
| **Female** | | | | | | | | | |
| Initial (Peak) Force | 180 | 210 | 230 | 140 | 160 | 180 | 130 | 150 | 180 |
| Sustained (Average) Force | 100 | 110 | 140 | 70 | 80 | 110 | 50 | 60 | 90 |

From the recommended pulling forces in table 6, the smallest required force can be observed for the longest distance and shortest duration for both initial and sustained forces. The largest forces can be observed for the shortest distance, longest duration for both initial and sustained forces.

It is important that forces are correlated with mass and acceleration for each of the activities. Position, velocity and acceleration equations are developed based on forces present during lifting, pushing and pulling. Figures 11 through 13 help illustrate how the equations are derived. The dynamic forces for lifting, pushing and pulling are evaluated as follows:

$$\sum M = m \cdot h^2 \cdot \alpha \,, \qquad \text{[E-1]}$$

$$F_p \cdot h \cdot \cos(\phi_f) - F_o \cdot h \cdot \cos(\phi_f) + N \cdot h \cdot \cos(\phi_g) - mg \cdot h \cdot \cos(\phi_g) = m \cdot h^2 \cdot \alpha \,, \qquad \text{[E-2]}$$

and $m \cdot h^2 \cdot \alpha = (F_p - F_o) \cdot h \cdot \cos(\phi_f) + (N - mg) \cdot h \cdot \cos(\phi_g)$. [E-3]

Angular acceleration, α, is integrated to acquire an equation for angular velocity. The angular velocity, ω, is also integrated to determine the position equation, which is represented by θ. The equations for the acceleration are solved, first by solving for the angular acceleration, namely

$$\alpha = \frac{\left(F_p - F_o\right)\cdot h \cdot \cos\left(\phi_f\right) + \left(N - mg\right)\cdot h \cdot \cos\left(\phi_g\right)}{m \cdot h^2}, \qquad \text{[E-4a]}$$

which simplifies to

$$\alpha = \frac{\left(F_p - F_o\right)\cdot \cos\left(\phi_f\right) + \left(N - mg\right)\cdot \cos\left(\phi_g\right)}{m \cdot h}. \qquad \text{[E-4b]}$$

Since the angular acceleration is the derivative of angular velocity, that is $\alpha = \frac{d\omega}{dt}$, we thus have

$$d\omega = \alpha \cdot dt = \left[\frac{\left(F_p - F_o\right)\cdot \cos\left(\phi_f\right) + \left(N - mg\right)\cdot \cos\left(\phi_g\right)}{m \cdot h}\right]\cdot dt\,. \qquad \text{[E-5]}$$

Integrating this yields

$$\omega = \left[\frac{\left(F_p - F_o\right)\cdot \cos\left(\phi_f\right) + \left(N - mg\right)\cdot \cos\left(\phi_g\right)}{m \cdot h}\right]\cdot t + c_1. \qquad \text{[E-6]}$$

Also, we know that $\omega = \frac{d\theta}{dt}$, and from this we can obtain

$$d\theta = \omega \cdot dt = \left[\frac{\left(F_p - F_o\right)\cdot \cos\left(\phi_f\right) + \left(N - mg\right)\cdot \cos\left(\phi_g\right)}{m \cdot h}\right]\cdot t \cdot dt + c_1 \cdot dt\,. \qquad \text{[E-7]}$$

A further integration of [E-7] gives us

$$\theta = \left[ \frac{\left(F_p - F_o\right) \cdot \cos\left(\phi_f\right) + \left(N - mg\right) \cdot \cos\left(\phi_g\right)}{2 \cdot m \cdot h} \right] \cdot t^2 + c_1 \cdot t + c_2 , \qquad \text{[E-8]}$$

whose constants, $c_1$ and $c_2$, are set by initial conditions.

Referring back to figures 11 through 13, the relationship between all the variables can be seen. The angle θ represents the orientation of the spine from its neutral position. The force $F_p$ represents the force exerted by the person. $F_o$ represents the force exerted by the box on the person. The angle $\varphi_f$ is the angle between a force and a perpendicular axis to the pendulum. N and mg represent the normal force and the force due to gravity. The angle $\varphi_g$ is the angle between forces relating to mg and normal forces and the perpendicular axis to the pendulum. The main reaction force of the box onto the person is based on the friction coefficient of between the box and moving surface.

The diagrams shown specify the three manufacturing activities of pushing, pulling and lifting. Correct posture while performing each activity is assumed. This requires the lifter to maintain a straight back and use with the legs instead of arms when lifting an object. Pushing and pulling an object while maintaining proper posture is also assumed. The handle height is at the hip level and body weight is used to help perform the activities of pushing and pulling. These assumptions will also carry over to the mathematical model. Tables 5 and 6 depict recommended pushing and pulling forces, and are used to give an idea of expected forces for the model. Stiffness of the spine in the mathematical model is taken from table 4.

Finally the dynamic equations are used in the mathematical model to determine spinal position, velocity and acceleration due to experienced forces. These equations help determine spine stability and are used in the model to predict a safe zone of motions and forces.

## 2.4 Spine Stability

The loads and motions exerted on the human body can cause instability in the lumbar spine. The state of spinal instability can lead to injury. In order to formulate an acceptable spine model, the state of instability needs to be more clearly defined. The American Heritage Dictionary defines stability as "the ability of an object to maintain equilibrium or resume its original upright position after displacement" [36]. This definition holds true for the human body and, more specifically, the spine as well.

The stability of the spine is determined by its ability to return to its original upright position after experiencing a perturbation causing unexpected motion or applied load. The spine becomes unstable when it reaches a point where it cannot return to its original position. For example, if the motion of the human body is fast and severe enough to cause spinal instability, then the spine will not be able to return to a normal position soon enough and regain stability before it goes beyond the limit of its healthy range of motion. At that point, the spine is at risk for injury.

There are several physiological circumstances that can affect when, how and why the spine becomes unstable. Fatigue determines how large of a cyclical applied load or motion the spine can resist over time. Reflex response loops tie into spine fatigue by controlling when the spine muscles attempt to return to their original state. Physical deterioration of elements within and around the spine determines the stability, response time, and the ability to cope with motions and external loads on the system [37].

Fatigue occurs when there is a reduction in the ability of muscle to exert a force in response to voluntary effort [38]. Static loads experienced in the body, such as those due to gravity, are compensated for by the use of voluntary muscle through an unconscious effort. If the body experiences such loads for extended periods of time, the brain will eventually become aware of the use of these muscles. Because the neuromuscular system has adapted to prevent damage to the muscles, a conscious effort will be required to keep the body at equilibrium once fatigue is realized. The muscles will no longer be able to exert their full potential tension [39]. If an external dynamic load is applied to the body with an existing static load, the muscles, in their weakened state, might not be able to provide enough tension to ensure stability.

Reflex loops also utilize voluntary muscle to prevent damage to the body as well as ensure stability. The process of constantly using the reflexes to make small balance adjustments can fatigue muscles greatly, and as a result can reduce the maximum activation of muscle motor units [39]. Therefore, fatigued muscles reduce the range of motion and loads through which the spine can maintain its stability.

A human mind that senses muscle fatigue not only experiences impairment in activating muscles, but in reflexes as well [39]. Reflex loops may cause the peripheral to fatigue, which then fails to propagate action potentials along motoneurons, impairs transmission across neuromuscular junction and declines the magnitude of the action potential. In other words, the response of the muscle to the reflex slows, allowing more time for an external load to cause displacement, and motion to propagate beyond the range of stability.

Physical factors can determine the response time and capability to handle loads. It is known that intervertebral discs are very important to the stability of the spine [40]. A previously injured or deteriorated disc decreases the range of motion of the spine, which becomes unstable if the range of motion is exceeded by as much as fifty percent [41]. This causes the range of stability to decrease significantly. Flexor and extensor muscles that are weak or out of shape cannot create the same amount of tension compared to stronger muscles. Therefore they cannot react to large loads or displacements.

An example of how reflex delay plays an important role can be seen in figure 14. Studies have been performed to determine the reflex response time to a perturbation in vivo. This graph was created by conducting an experiment where a volunteer experienced a load which is unexpectedly released. This experiment is further discussed in chapter three.

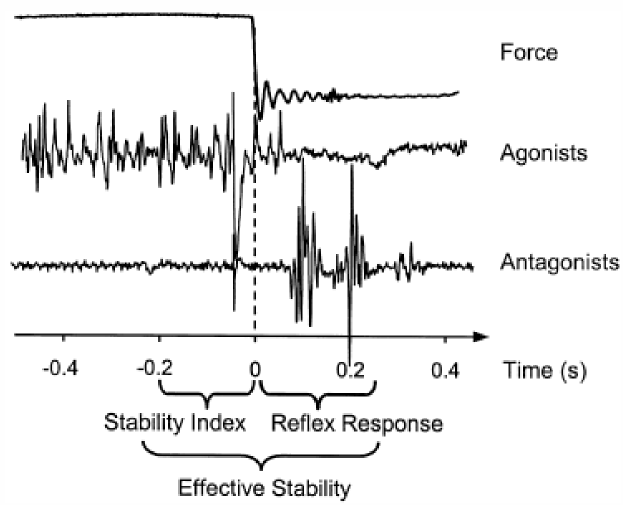



**Figure 14: Clinical Study of Reflex Response**

The effective stability region in figure 14 shows how there is a delay between shutting off the agonist muscles and effectively activating the antagonist muscles to regain stability [42]. This delay time may differ from person to person. For example, an athlete may have less delay time than someone with lower back pain. It is still not understood if subjects are predisposed with a longer delay time, which makes them more vulnerable to lower back pain, or if lower back pain causes a longer delay time because the muscles are already fatigued to compensate for the injury.

Any condition that causes the spine to be considered clinically unstable also reduces the spine's capability to remain biomechanically stable. Clinical instability is defined as "the loss of the ability of the spine under physiological loads to maintain its pattern of displacement so that there is no initial or additional neurological deficit, no major deformity, and no incapacitating pain" [43]. Other conditions such as osteoporosis, scoliosis, and other spinal diseases do impact spine stability. For simplicity, the team has chosen not to include these factors.

The understanding of fatigue, reflexes and physical factors is essential in determining the gains found within the model. Based on research, appropriate gains can be established for flexor and extensor muscles, which play an active role maintaining functional upright and sitting stability of the spine [44]. The quality and quantity of motion also help determine the gains [42]. A heavy load or large number of repetitions will cause fatigue, which in turn will deteriorate reflexes and will increase the chance of instability in the spine.

## 2.5 Model Formation

A mathematical model can be used to analyze potential risk involved with manufacturing activities. The spine is a complicated system, consisting of numerous degrees of freedom and complicated, non-linear viscoelastic dynamics. The spine also features changing stiffness created by the various trunk muscles. By using the physiological background, assumptions can be made to make a simplified, yet accurate representation of the spine.

The first assumption is that this model only has one degree of freedom. This assumption allows only for the greatest ranges of motion, bending forward and back. The team decided that this would suffice for the activities of pushing, pulling and lifting since these activities don't require lateral bending. This also simplifies the dynamics involved. The position, velocity and acceleration of the inverted pendulum are only in one plane.

The next assumption goes hand in hand with a one degree of freedom system. That is, the muscles will only be grouped into two sets; the flexors and extensors. The stiffness and damping coefficients may change during the dynamic activities, but their main function is to activate quickly enough to overcome a perturbation.

Finally, the model will only create linear stability indices. The nonlinear viscoelastic behavior of the tissues will be assumed linear since the nonlinear contribution of tissues to spine stability is negligible.

### 2.5.1 Single Degree of Freedom Mathematical Spine Model

The physiological model must first be established and described before formulating a mathematical representation. Figure 15 shows a functional spinal unit with all possible forces, moments, translations and rotations. In order to keep the system as straightforward as possible, the model is limited to rotation about the x-axis as shown in figure 15 [45]. The muscles involved with motion of the spine in the x-axis rotational directions are combined into two groups, the flexors and extensors. The spine is treated as a rigid body, with the joint representing the L4-L5 vertebrae.

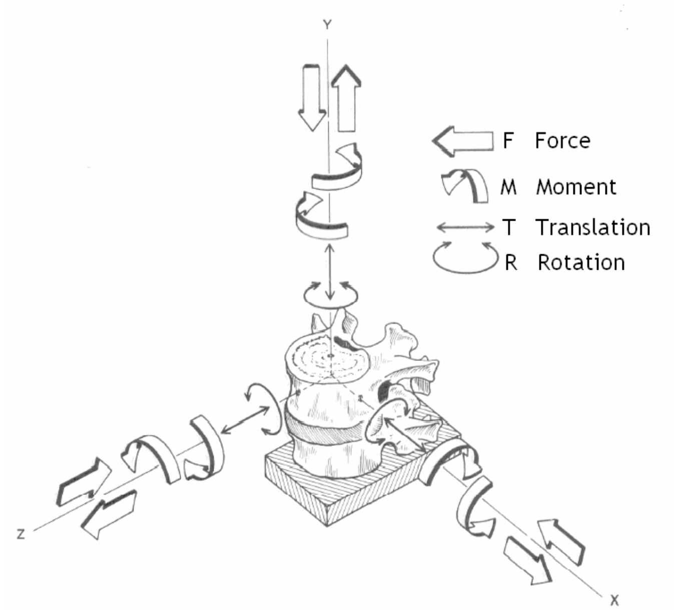



**Figure 15: Functional Spinal Unit**

The human body exhibits viscoelastic mechanical properties. Muscles, tendons, bones, and reflex response loops all contribute to the viscoelastic behavior found in the body. To make the model as straightforward as possible, muscles, and their tendons, are treated as two systems consisting of springs and dampers. One system represents the flexors and associated tendons. The other system represents the extensors and associated tendons. This system also takes into account the inability of muscle to exert a pushing force. The delay created by reflex response loops is taken into account in the mathematical model.

It is necessary to discuss the correlation between muscle forces and muscle length. Voluntary muscles create the greatest amount of tension, or force, at their resting length [46]. As the muscle lengthens, the number of possible crossbridges decreases and the amount of tension the muscle is able to create decreases. When the muscle shortens, the filaments overlap and the number of possible crossbridges decreases and the amount of tension the muscle can create also decreases. This relationship is best illustrated in Figure 16 [38].

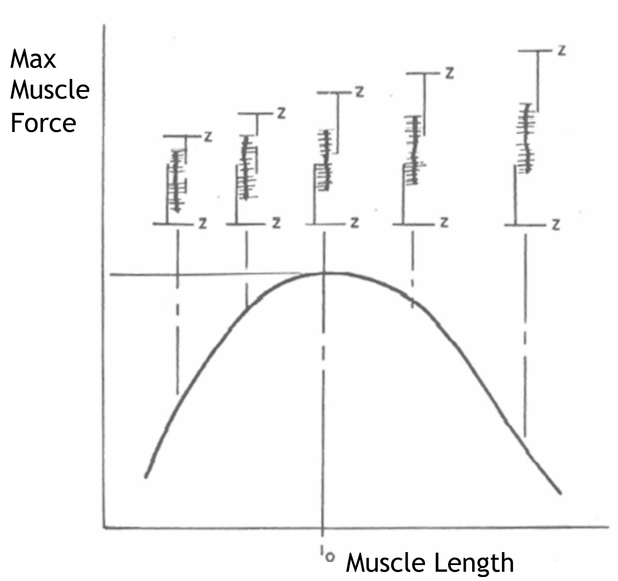


**Figure 16: Muscle Length versus Tension**

The length-tension relationship is not solely dependent on the filament and crossbridge relationship. The elastic fibrous tissue network also plays an important role [46]. When an external load is applied to a muscle, a preloading condition occurs. An example would be the forces of gravity acting on the muscles. The preloading condition changes the muscle length-tension relationship. The total amount of tension the muscle can exert decreases [46].

There also exists a relationship between muscle tension and the velocity of the muscle changing length. As the velocity of a shortening muscle increases, the muscle's tension production capability decreases [38]. Inefficient coupling at the crossbridges causes loss in tension as filaments slide quickly past each other. Also, fluid viscosity of muscle causes viscous friction to develop within the muscle. This friction must be overcome in order for the muscle to move. These factors limit the maximum tension production of the muscles.

Electromyogram (EMG) readings are used to record muscle activity. Electrical potentials within muscles show motor unit activation [47]. The primary use of EMG is to predict muscle tension. An increase in muscle tension causes an increase in the amplitude of the EMG signal. This signal has to be processed before useful data is extracted [48]. The active force producing capability of muscle is dependent on the relative size of the muscle, the length of the muscle and the speed at which the muscle changes length [49]. These vary for each individual and the EMG signal is processed to meet the changing demands.

### 2.5.2 Mathematical Model

The inverted pendulum is used to represent the physiological system and can be seen in figure 17.

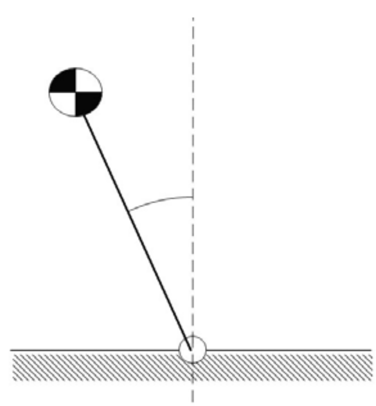


**Figure 17: 1 DOF Inverted Pendulum Model**

It is used because of its simple yet accurate representation of the human spine in flexion and extension. Figure 17 shows the model in its most simple state. A more illustrative version can be seen in figure 18.The spine is represented by the link between the mass and pivot joint. The flexors and extensors are represented by the spring and damper systems.

The model seen in figures 17 and 18can be represented by mathematical equations. A central equation is derived from a basic differential equation,

$$f(t) = m\ddot{\theta} + (c_e + c_f)\dot{\theta} + (k_e + k_f)\theta + \mu_e[\theta(t-\tau_e) + \dot{\theta}(t-\tau_e)] + \mu_f[\theta(t-\tau_f) + \dot{\theta}(t-\tau_f)] \qquad \text{[E-9]}$$

The coefficients $c_e$ and $c_f$ represent the damping forces of the extensors and flexors respectively. The coefficients $k_e$ and $k_f$ represent the spring forces of the extensors and flexors respectively. Reflex delays are represented by gains $\mu_f$ for the flexors and $\mu_e$ for the extensors. The time delay in both the flexors and extensors is represented by $\tau$. Mass of body above the L4-L5 joint is represented by m. The variables $\theta$, $\dot{\theta}$ and $\ddot{\theta}$ represent position, velocity and acceleration respectively. The equation is simplified by first replacing the damping coefficients with a resulting damping,

$$c_R = c_e + c_f\,. \qquad \text{[E-10]}$$

The same is done for the stiffness coefficients,

$$k_R = k_e + k_f\,. \qquad \text{[E-11]}$$

A resulting natural frequency is calculated from the natural frequencies of the flexors and extensors,

$$\omega_R = \omega_e + \omega_f\,. \qquad \text{[E-12]}$$

The relationship between the resulting natural frequency, resulting stiffness and mass is established,

$$\frac{k_R}{m} = \omega_R^2 .$$ [E-13]

A correlation between feedback loop gain factor, resulting natural frequency, stiffness, extensor feedback loop gain and mass is shown by,

$$\mu_1 = \frac{\omega_R^2}{k_R}\mu_e = \frac{\mu_e}{m} .$$ [E-14]

Reflex delay of the extensors corresponds to the generic reflex delay $\tau_1$,

$$\tau_e = \tau_1$$ [E-15]

A correlation between feedback loop gain factor, resulting natural frequency, stiffness, flexor feedback loop gain and mass is shown by,

$$\mu_2 = \frac{\omega_R^2}{k_R}\mu_f = \frac{\mu_f}{m} .$$ [E-16]

Reflex delay of the flexors is shown to corresponds to the generic reflex delay $\tau_2$,

$$\tau_f = \tau_2$$ [E-17]

The variable $\delta_R$, representing the damping factor with respect to both the flexor and extensor muscles, is defined as,

$$\delta_R = \frac{c_R}{2\sqrt{mk_R}} .$$ [E-18]

The relationship between the damping factor, resulting damping, mass and natural frequency is established as,

$$\frac{c_R}{m} = \frac{c_R}{m} \times \frac{2\omega_R}{2\omega_R} = 2\omega_R \frac{c_R}{2m}\sqrt{\frac{m}{k_R}} .$$ [E-19]

The equation E-19 is simplified to,

$$\frac{c_R}{m} = 2\omega_R \delta_R. \quad \text{[E-20]}$$

The equations are then substituted into the basic equation to give the central governing equation,

$$\ddot{\theta} + 2\omega_R \delta_R \dot{\theta} + \omega_R^2 \theta + \mu_1[\theta(t-\tau_1) + \dot{\theta}(t-\tau_1)] + \mu_2[\theta(t-\tau_2) + \dot{\theta}(t-\tau_2)] = f(t). \quad \text{[E-21]}$$

The neutral delay differential equation is then solved depending on initial conditions. The reflex delay gain parameters, $\mu_1$ and $\mu_2$, help establish the initial conditions. Since the body responds in a non-linear manner, the reflex delay, $\tau_1$ and $\tau_2$, are the main factor in determining the initial conditions. Based on the Eigen value solutions, stability is determined. If the solution crosses the imaginary plane, stability is compromised. This will be further developed in a later section.

# CHAPTER III: SPINE DYNAMICS WITH RESPECT TO MANUFACTURING ACTIVITIES

## 3. Introduction

Studies have shown that various dynamic activities impact spine stability. This chapter will discuss the major factors that determine a spine's stability. First, an overview of previous work done on spine stability models established by Jacek Cholewicki is provided. Next, the equations used to describe the motion and stability of the model are discussed, including the governing dynamic equations, used to determine forces and motions due to the three manufacturing activities, and the stability equations and indices which ultimately determine risk of injury to the lumbar spine.

The goal of this section is to integrate spine dynamics and spine stability to create a mathematical model. This model provides safety parameters for manufacturing workers while performing the activities of pushing, pulling and lifting without the risk of lower back injury. The model for this project is based off the work of Professors, Dr. Jacek Cholewicki from Biomechanics Department of Yale University. The team will be using an inverted pendulum model originally developed by Cholewicki. Also, the results of the stability indices are discussed in detail.

## 3.1 Spine Model Formulation

The spine model is formulated by integrating the dynamic equations and physiological parameters to determine forces and motions. From these forces and motions, stability indices can be formulated. The inverted pendulum which the mathematical equations are based off of can be seen in figure 18.

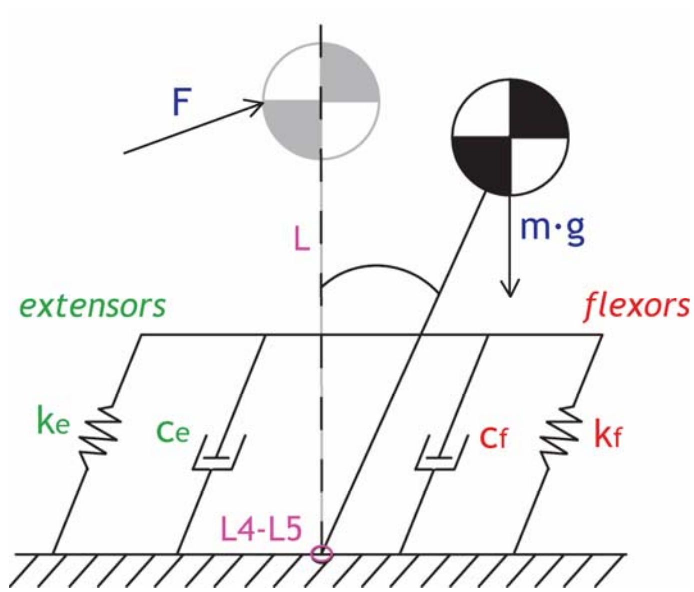



**Figure 18: Spine Model**

As seen in this figure, the extensors and the flexors are grouped together. The extensors are on the left and are represented with a spring and damping coefficient, $k_e$ and $c_e$, while the flexors on the right are represented with $k_f$ and $c_e$. The joint is represented as the L4-L5 pivot. The center of mass, located at the T9 vertebra, represents all the mass of the trunk, head and arms. The force acting on the center of mass is the perturbation force.

There are many assumptions for the mathematics that coincide with the model, which will be discussed later. The resulting equations utilize the motion and forces acting on the pendulum to determine a relative stability of the system. The basis for the mathematics and the subsequent research that went into formulating the original model is discussed in the next section to understand the assumptions of the model.

### 3.1.1 Previous Work by Prof. Jacek Cholewicki

The spine stability data for this project is based off of the work of Yale University's Biomechanics Professor, Dr. Jacek Cholewicki. The work of Prof. Cholewicki and colleges has focused on lumbar stability. They have created a mathematical spine model to determine why patients have lower back pain. The goal of this MQP is to further develop Prof. Cholewicki's model by incorporating dynamic motion and modifying the activities towards the manufacturing tasks of pushing, pulling and lifting. Prof. Cholewicki and his colleges have submitted numerous articles to medical journals including *The Journal of Biomechanics*. This section will describe previous work done by Cholewicki relating to the modified model created by this team.

One of the methods used by Cholewicki et al. to obtain accurate data for their spine model was to investigate the mechanical stability of *in vivo* lumbar spine. They have accomplished this by creating a test apparatus which holds the volunteer in a semi-seated position that allows the torso to move in all directions while restricting the motion of the hips as seen in figure 19 [50].

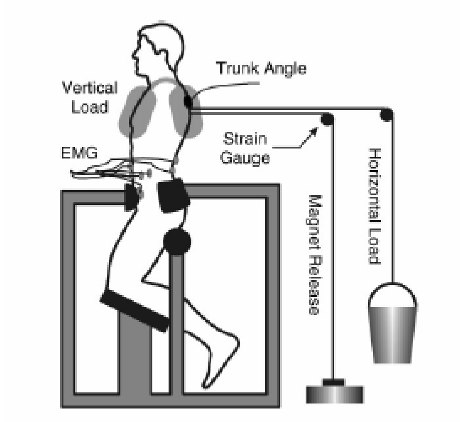



**Figure 19: Yale Test Apparatus**

The volunteer is then hooked up to an EMG machine to measure the activity level of the various muscles surrounding the spine. The individual is attached to a pulley system with cables at the T9 level. At the end of the pulley were a weight and an electromagnetic that could be released by the researchers [50]. The weight can vary as well as the direction it acts on the subject.

The electromagnet was released at random during the three trials when the volunteers reached and maintained 35% of their maximum force. This force averaged 172 (SD 54) for the 6 male and 6 female volunteers [50]. Similar to the inverted pendulum, the volunteers were asked to keep their upper bodies rigid by crossing their arms against their chest [51]. From these trials, it is assumed that the 200 ms of muscle activity prior to loading determines the spine stability [42]. After the perturbation was activated, Cholewicki et al. found that trunk muscle reaction time averaged between 40 and 80ms in this experiment [52].

The theory for these tests is that the active control of the spine is ultimately achieved by the force of the spine muscles. The force of the muscles is linearly proportional to the stiffness of the muscle. By cocontracting the surrounding muscles, the stiffness of the spine increases as well as the stability [53]. The results proved the hypothesis that added weight before a perturbation results in the increased muscles stiffness prior to perturbation. Therefore, there is more stability after the perturbation occurs. It is believed that a lack of preparation for a perturbation, i.e. no prior stiffness of the muscles can lead to an injury and lower back pain [52]. The results of the different weights and directions on the spine stability index from prior loading, or cocontraction of the spine muscles, can be seen in table 7 below [32].

**Table 7: Effects of Load Direction and Magnitude on SI [Nm/rad] [54]**

| **Horizontal load (%BW)** | **0** | **10** | **20** |
|---|---|---|---|
| Extension* | 423 (85) | 477 (94) | 532 (102) |
| Flexion* | 270 (46) | 309 (59) | 320 (52) |
| Left lateral bending* | 335 (58) | 380 (70) | 425 (82) |
| Right lateral bending* | 315 (57) | 371 (77) | 417 (84) |
| **Vertical load (%BW)** | **0** | **10** | **20** |
| Extension | 473 (92) | 474 (96) | 486 (97) |
| Flexion* | 322 (62) | 291 (48) | 285 (45) |
| Left lateral bending | 382 (73) | 376 (68) | 382 (69) |
| Right lateral bending | 374 (93) | 360 (64) | 369 (67) |
| (standard deviations are in parenthesis) | | | |

As seen in the table, the higher the SI, the more stable the system is. These values were calculated with a mathematical model that incorporated measured stiffness values (see table 4) from the EMG readings from the 12 volunteers.

The mathematical model that was created for similar tests carried out by Cholewicki et al. is much more complicated than the model proposed for this project. The Cholewicki model includes 3 axes of rotation for each vertebral joint, ending up with an 18 DOF system [55]. 90 separate muscle simulators were used along with the EMG data as previously discussed [57]. An example of this model can be seen in figure 20.

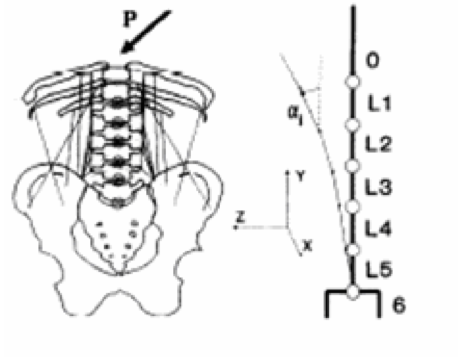



**Figure 20: 18 DOF Yale Spine Model**

A simpler version of this model, a similar to the inverted pendulum model the MQP group is using can be seen in figure 21. The length of the pendulum L, is the distance between the L4-L5 joint and the T9 vertebrae [57]. Also in this model, the stiffness and damping coefficients are assumed to be constant. However, this is not entirely accurate since the reflex response determines how quickly and how much stiffness is required constantly. Therefore the coefficients change throughout movement of the spine [52].

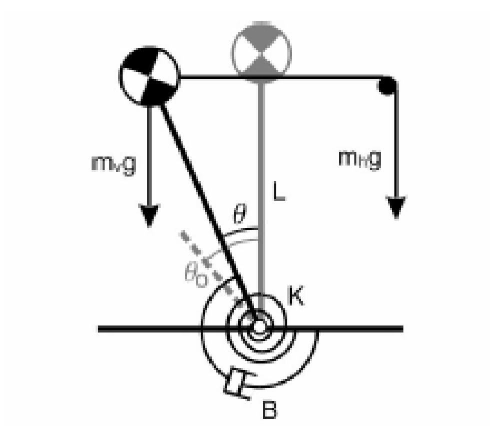



**Figure 21: Yale Inverted Pendulum Model**

The previous stability analysis conducted by Cholewicki was a static analysis. The stability indices were created by analyzing each pose during various activities. An example of the mathematical static analysis to create the stability indices of Cholewicki may be seen in Appendix B. A similar approach will be used by a mathematical approach except adapted to dynamic movement.

### 3.1.2 Governing Equation

In order to generate stability indices with respect to pushing, pulling and lifting, the governing equation has to be first solved to formulate the characteristic equation. The solutions to the characteristic equation yield the indices and are dependent on assumed initial conditions.

The self excited case is examined with the initial governing equation,

$$\ddot{\theta}+2\omega_R\delta_R\dot{\theta}+\omega_R^2\theta+\mu_1[\theta(t-\tau_1)+\dot{\theta}(t-\tau_1)]+\mu_2[\theta(t-\tau_2)+\dot{\theta}(t-\tau_2)]=f\left(t\right). \quad \text{[E-22]}$$

In this case, motion of the spine is generated by the muscles rather than by outside perturbation forces. Because of this assumption, the governing equation can be set equal to zero,

$$\ddot{\theta}+2\omega_R\delta_R\dot{\theta}+\omega_R^2\theta+\mu_1[\theta(t-\tau_1)+\dot{\theta}(t-\tau_1)]+\mu_2[\theta(t-\tau_2)+\dot{\theta}(t-\tau_2)]=0. \quad \text{[E-23]}$$

Based on the previous assumption, it can also be assumed that the action of any reflex delays with respect to perturbation forces will also be negligible. This results in $\dot{\theta}\left(t-\tau_1\right)$ and $\dot{\theta}\left(t-\tau_2\right)$ both equaling to zero. The outcome yielding a retarded differential equation with multiple delays, $\tau_1$ and $\tau_2$,

$$\ddot{\theta}+2\omega_R\delta_R\dot{\theta}+\omega_R^2\theta+\mu_1\left[\theta(t-\tau_1)\right]+\mu_2\left[\theta(t-\tau_2)\right]=0. \quad \text{[E-24]}$$

Several cases can be specified for gain factors and reflex delays. The first case assumes that $\tau_1 = \tau_2$. Physiologically this assumption means that the flexors and extensors both have the same reflex response time. The second case assumes that the reflex delays are not equal, $\tau_1 \neq \tau_2$. A reflex signal has to travel from sensory input, to the spine and then to the responding muscle. This causes a delay that is dependent on the location of the muscle. The second case represents the flexors and extensors most accurately because its shows a different reflex delay time for each muscle group. In the third case, the second reflex delay is the product of the first reflex delay and some coefficient, $\tau_2 = \alpha \cdot \tau_1$. This case is not as physiologically accurate as the second case, mostly because the reflex delays are dependent on many variables. These variables change often and do not necessarily cause the reflex delays to remain proportional.

The fourth case presumes that the first feedback loop gain is not equal to zero, $\mu_1 \neq 0$, while the second feedback loop gain is equal to zero, $\mu_2 = 0$. In general, when one set of muscles is activated to perform a contraction, such as the flexors, the antagonistic muscle group will be deactivated. The final case, assumes that the first feedback loop gain is equal to zero, $\mu_1 = 0$, while the second feedback loop gain is not equal to zero, $\mu_2 \neq 0$.

To develop a characteristic equation for self-excited flexion of the spine, the fourth case is assumed, with $\mu_1$ and $\mu_2$ representing the flexor and extensor muscles respectively. The resulting equation from this scenario is,

$$\ddot{\theta} + 2\omega_R \delta_R \dot{\theta} + \omega_R^2 \theta + \mu_1 \left[ \theta(t - \tau_1) \right] = 0 . \qquad \text{[E-25]}$$

The solution of $\theta = e^{\lambda \cdot t}$ [E-26] is used to solve equation E25. The result of solving for position with respect to the difference between time and reflex delay is,

$$\theta\left(t-\tau_1\right)=e^{\lambda\cdot t\left(t-\tau_1\right)}=e^{\lambda\cdot t}e^{-\lambda\cdot\tau_1}. \qquad \text{[E-27]}$$

By deriving the position equation, an equation for velocity is obtained,

$$\dot{\theta}=\lambda\cdot e^{\lambda\cdot t}. \qquad \text{[E-28]}$$

Taking the double derivate of the position equation 27, or a derivate of velocity equation 30, an equation for acceleration is obtained,

$$\ddot{\theta}=\lambda^2\cdot e^{\lambda\cdot t} \qquad \text{[E-29]}$$

The solutions for equations of position 27, velocity 28 and acceleration 29 are then substituted into equation 27,

$$e^{\lambda\cdot t}\cdot\left\{\lambda^2+2\cdot\omega_R\cdot\delta_R\cdot\lambda+\omega_R^2+\mu_1\cdot e^{-\lambda\cdot\tau_1}\right\}=0 \qquad \text{[E-30]}$$

Since $e^{\lambda\cdot t}=e^0=1$, the equation will only work if

$$\lambda^2+2\cdot\omega_R\cdot\delta_R\cdot\lambda+\omega_R^2+\mu_1\cdot e^{-\lambda\cdot\tau_1}=0. \qquad \text{[E-31]}$$

The equation 31 is also referred to as the transcendental characteristic equation. From this equation, a Lyanpanov stability index can be created based upon the location Eigen values, λ, in the complex plane. This will be discussed in a later portion of the report.

### 3.1.3 Varied to different activities

There are many factors contributing to spine stability during the dynamic tasks of pushing, pulling and lifting. These factors include cocontraction of the surrounding spine muscles prior to loading, muscle stiffness which can vary throughout the duration of a given activity [52], and the reflex delay of intrinsic spine muscles to adjust quickly to regain balance [58]. Posture also plays an important role in the stability of the spine [59]. Depending on the spine dynamics of the given manufacturing activity (such as the magnitude of subjected forces or range of motion), certain factors, or combination of factors may play a more significant role in the stability of the spine and therefore mathematically represented in the model.

It has been studied that the L4-L5 joint experiences it highest compression force during lifting and, in vivo, the loads can range from 6,000N for everyday activities, to 18,000N in activities such as power lifting [60]. During the activity of lifting, cocontraction prior to loading allows the spine to prepare for heavy loads by increasing stiffness [56] to resist excessive motion and therefore decrease the risk injury. Once the spine is loaded, stiffness must be maintained appropriately to execute the task. It seems that the stability of the lumbar spine actually increases during the most demanding tasks [59]. If on the other hand, the spine is subjected to unexpected loads, it is crucial for the reflex response delay of the muscles to be quick and strong enough to regain stability to prevent the spine from buckling [49].

In the case of primed heavy lifting the muscles are cocontracted and stiff, providing high spine stability. Therefore, injury is most likely to occur due to compressive loads large enough to exceed tissue tolerance which leads to failure [59]. In the case of unexpected heavy loading, the crucial factor is the reflex delay time. If the reflexes are not quick enough to recruit both intrinsic muscles to balance and large muscles to provide stiffness and stability, then the back will experience a muscle spasm or a tissue overload [61].

On the other hand, it has been studied that the spine is most vulnerable during flexion [60]. For the sake of conserving energy, the spine muscles are not as stiff when performing lighter tasks [56] and allow for more movement. In fact, the spine has very low stability when a person is standing upright [62]. Therefore spine stability is much more dependant on a quick and properly functioning reflex response [52], controlled by the central nervous system [53]. During the tasks of pushing and pulling, the spine is in a flexed posture. The team hypothesizes that even a slight unexpected change in force, (either due to a change in friction of the object being acted on, or if the person's feet slip) would be enough to put the spine at risk of instability and possible injury.

In the case of pushing and pulling, lower back pain can be caused due to a moment of instability which may cause a slip and fall (generating an unexpected loading or unloading) [53]. Since the subjected forces due to pushing and pulling are not as strenuous as heavy lifting, the stiffness of the spine is not as high. In this case, the team hypothesizes that it is a combination of some prior stiffness to the perturbation in combination with reflex time delay that will determine if the spine can regain stability. Also, the amount of prior stiffness will determine how significant the role of reflex time delay will be. For example, a stiffer spine prior to perturbation may compensate for a slower reflex delay time in order to regain stability.

The team conducted the activities of pushing, pulling and lifting to gather information from observation. From the activities of pushing and pulling, the team members noticed an initial force to overcome the friction between the object and the floor in order to get the object in motion. Once the object was in motion, the team also noticed a cyclical force, which is less than the initial due to the momentum of the object from which increased in force when the object slowed down. The team suspects this may be more relevant for an object on wheels, or if the friction is very low between the floor and the object. The team believes that a sudden increase or decrease in the force, as observed in these cases, may cause the highest instance of spinal instability.

As for lifting, the team also noticed that the initial force require to lift the object took a little time to stabilize. The team members also suspect a sudden drop in the box may cause instability in the spine. These unexpected forces are considered the perturbation forces in the model.

## 3.2 Linear Stability Indices

Based upon the characteristic equation 31, a stability index can be created. The stability indices determine if a neutral, stable position can be achieved given an external perturbation. The solution of the characteristic equation is given as,

$$\Delta(\lambda, \mu_1) = \lambda^2 + 2 \cdot \omega_R \cdot \delta_R \cdot \lambda + \omega_R^2 + \mu_1 \cdot e^{-\lambda \cdot \tau_1} = 0. \quad \text{[E-32]}$$

This equation can be simplified to its Eigen value solution,

$$\lambda_{1,2} = \pm j\omega. \quad \text{[E-33]}$$

The Eigen value solution is a complex conjugate pair and determines the stability of the spine.

The spine can be considered stable when the Eigen values of the characteristic equation are in the negative portion of the real plane, $\mathrm{Re}\,\lambda_{1,2} < 0$. Motion of the Eigen values from the negative real plane across the complex plane and into the positive real plane constitutes instability. The Eigen values always cross in pair as seen in figure 22. The proximity of the Eigen values to the positive real plane is determines how likely the Eigen values will cross over and become unstable.

Values directly on the boundary, or on the complex plane, show a spine that experiences no damping, $\delta_R = 0$, and are considered unstable. The further the Eigen values are from the positive portion of the real plane, the more stable the spine is considered; $\mathrm{Re}\,\lambda_{1,2} \ll 0$. In this state, the spine is said to be asymptotically stable. Any occurrence of Eigen values within the positive portion of the real plane, $\mathrm{Re}\,\lambda_{1,2} > 0$, constitutes an unstable state of the spine.

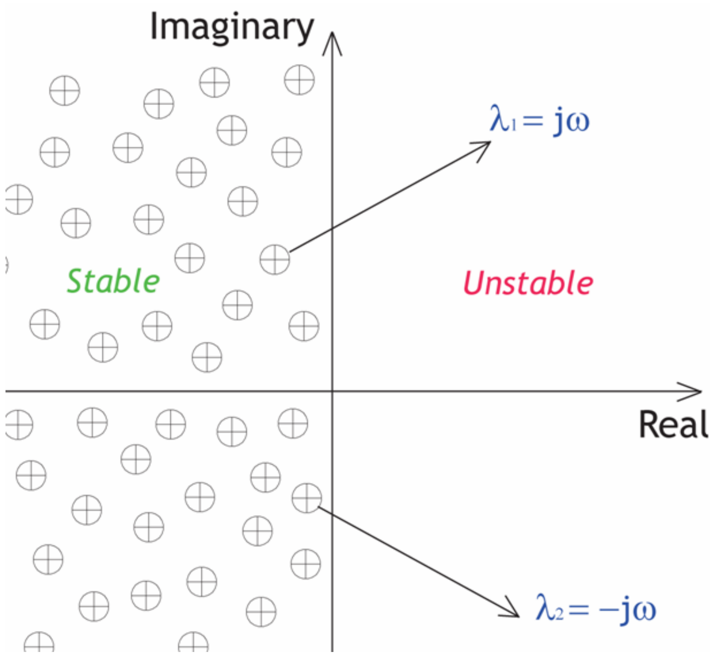



**Figure 22: Location of Eigen Values in Complex Plane**

When the characteristic equation is graphed, the resulting stability regions and hyperbola can be seen in figure 23. Depending on the conditions of the feedback loop gains, $\mu_1$ and $\mu_2$, each of the lettered regions can represent either a stable state or an unstable state. For example, if $\mu_1$ was greater than $\mu_2$, region *b* could be unstable and region *a* would be considered as stable.

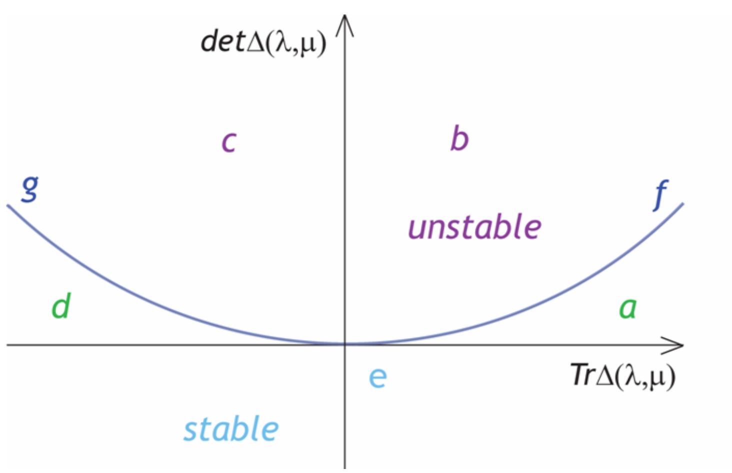



**Figure 23: Stability Regions**

The stability index can also be viewed as a three dimensional graph with respect to position and perturbation force. An example can be seen in figure 24. The most stable region is the peak. The slope of the curve leading up to the peak represents how quickly the spine can return to a neutral, stable position. If the location on the curve changes from the neutral position, and if the slope is too great, the spine will not be able to return to its stable state. Also, the further the spine wonders away from the neutral position, the less likely it will return to the stable state.

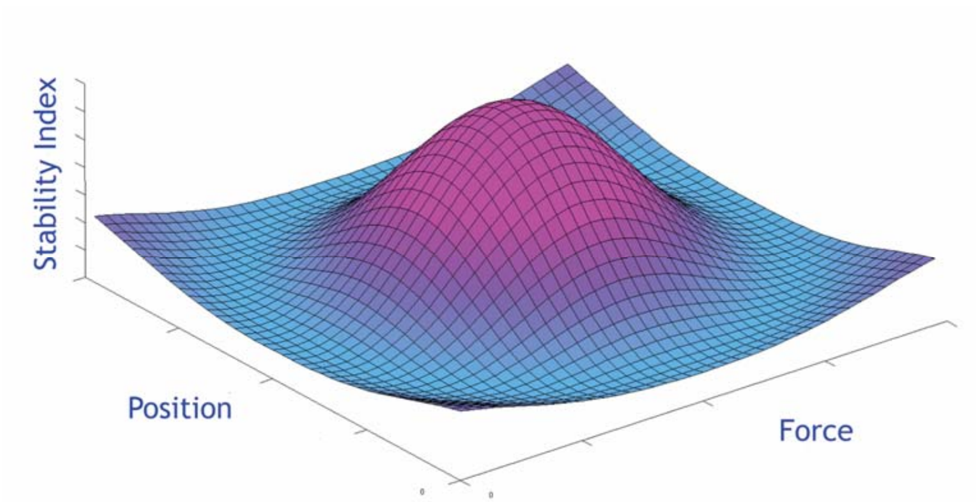



**Figure 24: Three Dimensional Stability Index Model**

The shape of this curve can change drastically depending on physiological parameters of each individual. For example, a person with stronger trunk muscles and faster reflexes will have a deeper curve with a flatter top, portraying a more robust system against perturbation, and a larger asymptotically stable area.

The stability indices can also be viewed as trajectories with respect to position and velocity. Figure 25 shows such a graph. If the trajectory of the spine remains close to the neutral position, represented by the origin, then it is more likely to return to its initial stable state. A trajectory that spends a lot of time wondering away from its neutral position will most likely become unstable. Any trajectory that leaves the stable region will become unstable and will not be able to return to the stable region. The further the trajectory wonders away from its neutral position, the more likely it will become unstable.

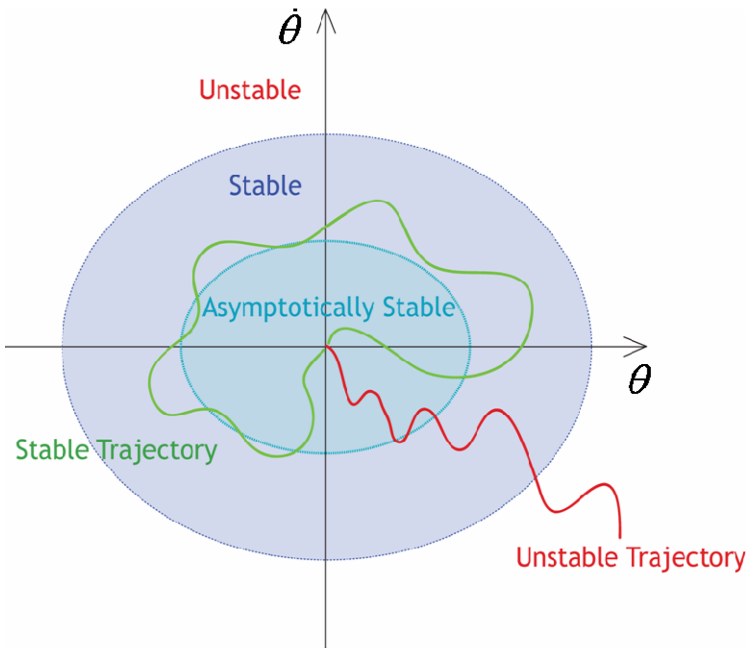



**Figure 25: Stable and Unstable Trajectories**

The asymptotically stable region is considered to be the most stable, meaning that the spine will most likely return to its neutral position at the end of its trajectory. The stable and asymptotically stable region changes in shape and size depending on many factors including physiological parameters of individuals.

## CHAPTER IV: CONCLUSION AND DISCUSSION

The results of this project correspond to the work of Jacek Cholewicki, with a similar approach to the stability indices. The main difference between this project and previous work done by Professor Cholewicki, is that this model incorporates dynamic movement. However, there are still many limitations for this model.

The next steps for this project include creating a user interface, further testing in a manufacturing environment, improving the accuracy of the model and finally, the applicability of this model for manufacturing companies. The team also has recommendations for students who may continue this project.

The first step the team recommends for the model is to create a user interface that will easily allow a user to type in various inputs and receive clear, comprehensible data. Known variables relating to a worker's physiology and activity will be prompted by the user interface with blank boxes, for example. Variables for a worker's physiology may include height, length from hips to shoulders, weight, etc. Variables for a given activity may include weight of the object, which activity being performed, how long the task takes, the distance the object is being moved vertically or horizontally, how many repetitions executed, etc.

Once the inputs are typed in by the user, a macro may run in the background calculating the stability indices. The display should outline ranges of motions and weights which may be executed safely, and also those which, when executed, will put the worker at risk for a back injury. Since it is abstract to tell a worker that they should only execute the activity within 20 degrees of bending, a visual of a person should be provided with the safe ranges of motions shown, or correct the positioning. On the other hand, a table listing safe weights and their repetitions will suffice since it is straight forward. The output must also be in a printer-friendly format to allow for easy dispersal.

The model must be tested in a working environment. Once a user interface has been created, numerical values can be entered to create specific guidelines for workers. The results of the model must be tested to determine if the guidelines help prevent injury. Adjustments to the model can therefore be made accordingly.

There are many adjustments that can be made to improve the accuracy of the model. Increasing the number of joints allow for an increase in degrees of freedom of the system. More springs and dampers can be added to represent a more complicated and realistic muscular system. Non-linear elements can be added to account for viscoelastic tissues.

Finally, the MQP team has some recommendations for those who may want to continue this project. One of the recommendations concerns the make-up of the team members. If possible, a mechanical engineer, biomechanical engineer, mathematician, and a computer science majors should all be represented to provide proficient understanding in the respective majors and can therefore effectively specialize in each aspect of this project.

Another recommendation would be to create a physical model. The vertebrae may be machined out of a comparable material to bone, as well as the intervertebral joint composed with a solid core surrounded by a supportive viscoelastic material. Actual springs and dampers may be used with a gain to activate the system. A perturbation force must somehow be represented for the three manufacturing activities.

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## APPENDIX A: MUSCLES OF BACK AND ABDOMEN

**Table 8: Muscles of the Back and Abdomen [65]**

| Muscles Related to the Spine | | |
|---|---|---|
| **Muscles of the Back** | | |
| | | |
| **First Layer** | **Fourth Layer** | **Fifth Layer** |
| Trapezius | ***Sacral and lumbar regions*** | Simispinalis Dorsi |
| Latissimus dorsi | Erector spinae | Simispinalis colli |
| | ***Dorsal region*** | Multifidus spinae |
| **Second Layer** | Ilio-costalis | Rotatores spinae |
| Levator anguli scapulae | Musculus accessorius ad ilio-costalem | Supraspinales |
| Rhomboideus minor | Longissimus dorsi | Interspinales |
| Rhomboideus major | Spinalis dorsi | Extensor coccygis |
| | ***Cervical Region*** | Intertransversales |
| **Third Layer** | Cervicalis ascendens | Rectus capitis posticus major |
| Serratus posticus superior | Transversalis cervicis | Rectus capitis posticus minor |
| Serratus posticus inferior | Trachelo-mastoid | Obliquus capitis inferior |
| Splenius capitis | Complexus | Obliquus capitis superior |
| Splenius colli | Biventer cervicis | |
| | Spinalis colli | |
| | | |
| **Muscles of the Abdomen** | | |
| | | |
| **Superficial** | **Deep** | |
| Obliquus Externus | Psoas magnus | |
| Obliquus Internus | Psoas parvus | |
| Transversalis | Iliacus | |
| Rectus | Quadratus lumborum | |
| Pyramidalis | | |

## APPENDIX B: STATIC STABILITY ANALYSIS

Static Stability Analysis [Jacek Cholewicki] [63]

At any given frame, the potential of the spine system (V) is expressed as the sum of the elastic energy stored in the linear springs ($U_L$) (muscles and tendons), elastic energy stored in the torsional springs ($U_T$) (lumped intervertebral joint discs, ligaments and other passive tissues) minus the work performed on the external load (W):

$$V = U_L + U_T - W \quad \text{(B1)}$$

Partial derivatives of the potential V were calculated separately for each component taking the Euler angles $\alpha_i$ (3 rotation angles x 6 joints = 18 df) as the generalized coordinates:

$$\frac{\partial V}{\partial \alpha_i} = \frac{\partial U_L}{\partial \alpha_i} + \frac{\partial U_T}{\partial \alpha_i} - \frac{\partial W}{\partial \alpha_i}$$
$$\frac{\partial^2 V}{\partial \alpha_i \partial \alpha_j} = \frac{\partial^2 U_L}{\partial \alpha_i \partial \alpha_j} + \frac{\partial^2 U_T}{\partial \alpha_i \partial \alpha_j} - \frac{\partial^2 W}{\partial \alpha_i \partial \alpha_j} \quad \text{(B2)}$$

The energy stored in linear springs ($U_L$) can be expressed as follows:

$$U_L = \sum_{m=1}^{90} F_m (l_{pm} - l_{om}) + \frac{1}{2} K_m (l_{pm} - l_{om})^2 \quad \text{(B3)}$$

where
$F_m$ = instantaneous muscle force (N)
$K_m$ = instantaneous muscle stiffness (N/m)
$l_{om}$, $l_{pm}$ = original ('frozen' in a given frame) and perturbed muscle lengths (m) and

$$\frac{\partial U_L}{\partial \alpha_i} = \sum_{m=1}^{90} [F_m + K_m (l_{pm} - l_{om})] \frac{\partial l_{pm}}{\partial \alpha_i}$$
$$\frac{\partial^2 U_L}{\partial \alpha_i \partial \alpha_j} = \sum_{m=1}^{90} K_m \frac{\partial l_{pm}}{\partial \alpha_j} \frac{\partial l_{pm}}{\partial \alpha_i} + [F_m + K_m (l_{pm} - l_{om})] \frac{\partial^2 l_{pm}}{\partial \alpha_j \partial \alpha_i} \quad \text{(B4)}$$

Since the partial derivatives are evaluated at the unperturbed point of equilibrium, $l_{pm} - l_{om} = 0$ and the Equations (B4) reduce to the following:

$$\frac{\partial U_L}{\partial \alpha_i} = \sum_{m=1}^{90} F_m \frac{\partial l_{pm}}{\partial \alpha_i}$$

$$\frac{\partial^2 U_L}{\partial\alpha_i\partial\alpha_j} = \sum_{m=1}^{90} K_m \frac{\partial l_{pm}}{\partial\alpha_j}\frac{\partial l_{pm}}{\partial\alpha_i} + F_m \frac{\partial^2 l_{pm}}{\partial\alpha_j\partial\alpha_i} \tag{B5}$$

If the muscle length is represented with a sum of n sections (when the muscle passes through the nodal point), its potential energy derivatives consist of a sum of its sections with some additional terms. Thus, if $l_{om} = l_{om1} + l_{om2} + \ldots + l_{omn}$ and $l_{pm} = l_{pm1} + l_{pm2} + \ldots + l_{pmn}$ then

$$\frac{\partial U_{Lm}}{\partial\alpha_i} = \sum_{n=1}^{nodes+1} \frac{\partial U_{Lm}}{\partial\alpha_i}$$

$$\frac{\partial^2 U_{Lm}}{\partial\alpha_i\partial\alpha_j} = \sum_{n=1}^{nodes+1} \frac{\partial^2 U_{Lm}}{\partial\alpha_i\partial\alpha_j} + K_m \sum_{r\neq s}^{nodes+1} \frac{\partial l_{pm}}{\partial\alpha_j}\frac{\partial l_{pm}}{\partial\alpha_i} \tag{B6}$$

Since the length of a given muscle $l_p$ (dropping the muscle subscript 'm' at this point) is given by the vector sum of the length components in the X, Y and Z axes direction,

$$l_p = (l_{px}^2 + l_{py}^2 + l_{pz}^2)^{1/2} \tag{B7}$$

Then

$$\frac{\partial l_p}{\partial\alpha_i} = (l_{px}^2 + l_{py}^2 + l_{pz}^2)^{-1/2}(l_{px}\frac{\partial l_{px}}{\partial\alpha_i} + l_{py}\frac{\partial l_{py}}{\partial\alpha_i} + l_{pz}\frac{\partial l_{pz}}{\partial\alpha_i}) \tag{B8}$$

And

$$\begin{aligned}\frac{\partial^2 l_p}{\partial\alpha_i\partial\alpha_j} &= -(l_{px}^2 + l_{py}^2 + l_{pz}^2)^{-3/2}(l_{px}\frac{\partial l_{px}}{\partial\alpha_i} + l_{py}\frac{\partial l_{py}}{\partial\alpha_i} + l_{pz}\frac{\partial l_{pz}}{\partial\alpha_i})\\ &\cdot(l_{px}\frac{\partial l_{px}}{\alpha_j} + l_{py}\frac{\partial l_{py}}{\alpha_j} + l_{pz}\frac{\partial l_{pz}}{\alpha_j}) + (l_{px}^2 + l_{py}^2 + l_{pz}^2)^{-1/2}(\frac{\partial l_{px}}{\partial\alpha_j}\frac{\partial l_{px}}{\partial\alpha_i}\\ &+ l_{px}\frac{\partial^2 l_{px}}{\partial\alpha_i\partial\alpha_j} + \frac{\partial l_{py}}{\partial\alpha_j}\frac{\partial l_{py}}{\partial\alpha_i} + l_{py}\frac{\partial^2 l_{py}}{\partial\alpha_i\partial\alpha_j} + \frac{\partial l_{pz}}{\partial\alpha_j}\frac{\partial l_{pz}}{\partial\alpha_i} + l_{pz}\frac{\partial^2 l_{pz}}{\partial\alpha_i\partial\alpha_j})\end{aligned} \tag{B9}$$

Substituting (B6), (B7) and (B8) into (B4) yields

$$\frac{\partial U_{Lm}}{\partial\alpha_i} = F_m(l_{px}^2 + l_{py}^2 + l_{pz}^2)^{1/2}(l_{px}\frac{\partial l_{px}}{\partial\alpha_i} + l_{py}\frac{\partial l_{py}}{\partial\alpha_i} + l_{pz}\frac{\partial l_{pz}}{\partial\alpha_i}) \tag{B10}$$

And

$$\frac{\partial^2 U_{Lm}}{\partial\alpha_i\partial\alpha_j} = (K_m l_p^{-2} - F_m l_p^{-3})[(l_{px}\frac{\partial l_{px}}{\partial\alpha_i} + l_{py}\frac{\partial l_{py}}{\partial\alpha_i} + l_{pz}\frac{\partial l_{pz}}{\partial\alpha_i})$$

$$\cdot (l_{px}\frac{\partial l_{px}}{\partial\alpha_j} + l_{py}\frac{\partial l_{py}}{\partial\alpha_j} + l_{pz}\frac{\partial l_{pz}}{\partial\alpha_j})] + F_m l_p^{-1}(\frac{\partial l_{px}}{\partial\alpha_j}\frac{\partial l_{px}}{\partial\alpha_i} + l_{px}\frac{\partial^2 l_{px}}{\partial\alpha_i\partial\alpha_j} \qquad \text{(B11)}$$

$$+\frac{\partial l_{py}}{\partial\alpha_j}\frac{\partial l_{py}}{\partial\alpha_i} + l_{py}\frac{\partial^2 l_{py}}{\partial\alpha_i\partial\alpha_j} + \frac{\partial l_{pz}}{\partial\alpha_j}\frac{\partial l_{pz}}{\partial\alpha_i} + l_{pz}\frac{\partial^2 l_{pz}}{\partial\alpha_i\partial\alpha_j})$$

It remains to evaluate partial derivatives of muscle length components, $l_{px}$, $l_{py}$, $l_{pz}$ in relation to all 18 rotation angles $\alpha_i$. If the muscle originates on a skeletal segment ‘w’ and inserts onto the segment ‘u’ (Figure 3), then its length vector

$$\begin{bmatrix} l_{px} \\ l_{py} \\ l_{pz} \end{bmatrix} = [\lambda_u]\begin{bmatrix} X_u - 0X_u \\ Y_u - 0Y_u \\ Z_u - 0Z_u \end{bmatrix} - [\lambda_w]\begin{bmatrix} X_w - 0X_w \\ Y_w - 0Y_w \\ Z_w - 0Z_w \end{bmatrix}$$

$$+[\lambda_{u+1}][L_{u+1}] + \ldots + [\lambda_w][L_w] \qquad \text{(B12)}$$

$$u, w = 0,\ldots 6, w\rangle u$$

Where

λ is a rotation matrix.

L is the vector of vertebral segment lengths taken between the adjacent joints,

X, Y, Z are coordinates of the muscle attachment points in the reference posture.

0X, 0Y, 0Z are coordinates of the rotation (a joint) of a given segment.

Partial derivatives of the elements of rotation matrices were easily programmed on a computer by inserting the appropriate derivatives of the trigonometric functions.

To obtain the elastic energy, which is stored in all of the torsional springs, we need to integrate the Equation (1) with respect to the relative joint angles and sum it over the 6 joints:

$$U_{Tx} = \sum_{j=0}^{5} \int M_{xj} d(\phi_j - \phi_{j+1}) = \sum_{j=0}^{5} \frac{a_{xj}}{b_{xj}} [e^{bxj(\phi j - \phi j+1)}$$
$$- b_{jx}(\phi_j - \phi_{j+1})] + K(\psi_j - \psi_{j+1})(\phi_j - \phi_{j+1})$$
$$U_{Ty} = \sum_{j=0}^{5} \int M_{yj} d(\psi_j - \psi_{j+1}) = \sum_{j=0}^{5} \frac{a_{yj}}{b_{yj}} [e^{byj(\psi j - \psi j+1)} \tag{B13}$$
$$- b_{jy}(\psi_j - \psi_{j+1})] + K(\phi_j - \phi_{j+1})(\psi_j - \psi_{j+1})$$
$$U_{Tz} = \sum_{j=0}^{5} \int M_{zj} d(\theta_j - \theta_{j+1}) = \sum_{j=0}^{5} \frac{a_{zj}}{b_{zj}} [e^{bzj(\theta j - \theta j+1)}$$
$$- b_{jz}(\theta_j - \theta_{j+1})]$$

The first partial derivatives of $U_T$ will have two terms belonging to the two adjacent intervertebral joints:

$$\frac{\partial U_T}{\partial \phi_j} = a_{xj}[e^{bxj(\phi j - \phi j+1)} - 1] + K(\psi_j - \psi_{j+1}) - a_{x(j-1)}$$
$$\cdot [e^{bx(j-1)(\phi j-1-\phi j)} - 1] - K(\psi_{j-1} - \psi_j)$$
$$\frac{\partial U_T}{\partial \psi_j} = a_{yj}[e^{byj(\psi j - \psi j+1)} - 1] + K(\phi_j - \phi_{j+1}) - a_{y(j-1)} \tag{B14}$$
$$\cdot [e^{by(j-1)(\psi j-1-\psi j)} - 1] - K(\phi_{j-1} - \phi_j)$$
$$\frac{\partial U_T}{\partial \theta_j} = a_{zj}[e^{bzj(\theta j - \theta j+1)} - 1] + a_{z(j-1)}$$
$$\cdot [e^{bzj(j-1)(\theta j-1-\theta j)} - 1]$$

For the negative angles, coefficients ‘a’ and ‘b’ will appear with a minus sign and the appropriate constant will be inserted in the case of flexion.  Now, there are six second partial derivatives of the $U_T$ possible for the general case:

$$\frac{\partial^2 U_T}{\partial\phi_j \partial\phi_{j-1}} = -a_{x(j-1)} b_{x(j-1)} e^{b x(j-1)(\phi j-1-\phi j)}$$

$$\frac{\partial^2 U_T}{\partial\phi_j^2} = a_{xj} b_{xj} e^{bxj(\phi j-1+\phi j)} + a_{x(j-1)} b_{x(j-1)} e^{b x(j-1)(\phi j-1-\phi j)}$$

$$\frac{\partial U_T}{\partial\phi_j \partial\phi_{j+1}} = -a_{xj} b_{xj} e^{bxj(\phi i-1+\phi j)} \tag{B15}$$

$$\frac{\partial^2 U_T}{\partial\phi_j \partial\psi_{j-1}} = \frac{\partial^2 U_T}{\partial\phi_j \partial\phi_{j+1}} = -K$$

$$\frac{\partial^2 U_T}{\partial\phi_j \partial\psi_j} = 2K$$

An identical equation format results if the $U_T$ formulation of twist is differentiated twice. Flexion/extension has the same general format as (B15), except K = 0 in this case.

The external work W performed by the load P is a dot product of the force and displacement vectors:

$$W = \vec{P} \bullet \Delta\vec{h} = P_x(h_{px} - h_{ox}) + P_y(h_{py} - h_{oy}) + P_z(h_{pz} - h_{oz}) \tag{B16}$$

where $h_p$ and $h_o$ are the perturbed and the original points of force application. Thus,

$$\frac{\partial W}{\partial\alpha_i} = F_x \frac{\partial h_{px}}{\partial\alpha_i} + F_y \frac{\partial h_{py}}{\partial\alpha_i} + F_z \frac{\partial h_{pz}}{\partial\alpha_i}$$

$$\frac{\partial^2 W}{\partial\alpha_i \partial\alpha_j} = F_x \frac{\partial^2 h_{px}}{\partial\alpha_i \partial\alpha_j} + F_y \frac{\partial^2 h_{py}}{\partial\alpha_i \partial\alpha_j} + F_z \frac{\partial^2 h_{pz}}{\partial\alpha_i \partial\alpha_j} \tag{B17}$$

Since the load P is always applied to the ribcage,

$$\begin{bmatrix} h_{px} \\ h_{py} \\ h_{pz} \end{bmatrix} = [\lambda_0] \begin{bmatrix} X_{h0} - 0X_0 \\ Y_{h0} - 0Y_0 \\ Z_{h0} - 0Z_0 \end{bmatrix} + [\lambda_1][L_1] + \ldots + [\lambda_6][L_6] \tag{B18}$$

The derivatives of the rotation matrix [λ] are the same in Equation (B12). Because the global axes system is imbedded into the pelvis, the last term in Equation (B18) vanishes upon the differentiation. Once calculated, all partial derivatives were inserted into the Hessian matrix in Equation (2).

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