

Predicting Stock Prices

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Abstract

Using the stochastic processes called Markov Chains, we sought out to predict the immediate future stock prices for a given company. We found the moving averages for the data and the grouped them into four different states of results. We then applied Markov Chain calculations to the data to create a 4x4 transitional probability matrix. Using this transition matrix we solved a system of equations and found four steady states that were variables that represented the probability that a stock price for a given day would fall into one of the four states. When we use this information we can apply our actual data to these equations and predict the next stock prices for the near future. We were able to successfully predict the next few days of stock prices using this method.

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1. Introduction

In our project, we were asked to analyze a year's worth of stock portfolio for a company and apply moving averages and Markov Chains to the data in hopes to predict the stock prices for the near future. We chose Google, as it is a company that everyone knows and the stock price data was well diverse and had enough information for many applications of moving averages.

The first thing we did was to apply moving averages to create an approximate evaluation of the data. In order to find moving averages, we first had to apply a moving average with an increment of five. This involves taking the sum of five days of stock and then dividing it by five. However, one can only do this starting at day five, because there was enough data to actually create a moving average. We eventually had a data set that included a moving average price and a closing price. We then needed to find a difference data set to apply Markov Chains too. We took the difference of the closing price and the moving average price. These differences were going to be what we applied Markov Chains too.

However, we first had to group the differences into four blocks. The reason why we did this was to create more accurate observations that it makes it more exact when we analyze the data. We then create a transition matrix. The entries in the matrix represented how many times the data points go from one block to another. This leads to 16 observations of data. For example, the first row of entries the matrix represents the number of times the data goes from the first block and stays in the first block, the number of times the data goes from the first block to the second block, the number of times the data goes from the first block to the third block, and the last entry of the row represents the number of times the data goes from the first block to the fourth block. All entries needed to be in decimal form, so the total number of observation points divided each entry.

With the matrix, we could now apply Markovian properties to our data. In other words, using Markovian properties we created a system of equations with the unknown variables being our steady states that we are aiming to obtain. These equations are sums of probabilities multiplied by our

unknown variables. We then aimed to solve the system of equations to find our steady state probabilities.

With our steady state probabilities we were now able to predict where each immediate stock price can fall into an interval. These probabilities are now good indicators of where the stock prices will fall. We then observed the new data and made some observations. We noticed that a moving average with an interval of 5, was too small and it's resulting steady states were too close together to see an actual difference in intervals. We then increased it to an interval of 10 and found that there was an improved steady state differences, but still a little close. We were able to predict what the possible price range for a given day could be, but we also came to the conclusion that only four interval blocks may have not been a large enough selection. However, due to time constraints, if we did increase the number of blocks, there would be issues in completing the project on time. In conclusion, applying Markov Chains is an effective way to predict stock prices, but one needs to create a large enough intervals to get better results.

2. Background

2.1 Markov Chains

A Markov Chain is a *stochastic process* that has the *Markovian property*.

Definition 1.1: A *stochastic process* is defined to be an indexed collection of random variables $\{X_t\}$, where the index t runs through a given set T , generally labeled with the set of nonnegative integers. The variable X_t is meant to represent a measurable characteristic, or point of interest. For example, if one were to look at a collection of muffins, some with blueberries and some without blueberries, the variable X_t can be used to label muffins with blueberries. Supposedly, if there were four blueberry muffins within a given collection, the set X_t could be designated as the set of blueberry muffins, with each muffin labeled as $X_1, X_2, X_3,$ or X_4 . Thus, it is evident from this example that stochastic processes are discrete collections of random variables.

A *stochastic process* often has the following structure:

The current status of the system can fall into any one of a set of $(M+1)$ mutually exclusive categories called *states*. For convenience, these states are labeled with integers from 0 to M . The random variable X_t represents the *state of the system* at time t , so its only possible values are 0 to M . The system is observed at particular points of time, labeled $t= 0$ to M . Thus, the *stochastic process* $\{X_t\} = \{X_0, X_1, X_2, \dots\}$ provides a mathematical representation of how the status of the physical system evolves over time.¹ Using the previous example of a collection of muffins, the variable X_2 here would represent the number of blueberry muffins at time, $t=2$.

Definition 1.2: A stochastic process $\{X_t\}$ is said to have the *Markovian property* if $P\{X_{t+1}=j | X_0 = k_0, X_1 = k_1, \dots, X_{t-1} = k_{t-1}, X_t = i\} = P\{X_{t+1} = j | X_t = i\}$, for $t = 0, 1, 2, \dots$ and every sequence $i, j, k_0, k_1, \dots, k_{t-1}$.² This is saying that the probability of X_{t+1} being equal to j is solely dependent upon the preceding event of what X_t equals.

2.2 Transition Probabilities

Conditional probabilities for Markov Chains are called *transition probabilities*.

Definition 1.3: If Conditional probabilities are defined as $P\{X_{t+1} = j | X_t = i\}$ then, for each i and j , *stationary one-step transition probabilities* for a Markov Chain are defined as,

$$P\{X_{t+1} = j | X_t = i\} = P\{X_1 = j | X_0 = i\} \text{ for all } t = 1, 2, \dots$$

Stationary transition probabilities indicate that transition probabilities do not change over time.

Aside from one-step transition probabilities, Markov Chains can also have *n-step transition probabilities*, which is the conditional probability that the process will be in state j after n -steps provided that it starts in state i at time t .

¹ Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research, Eighth Edition* (New York: McGraw Hill, 2005), 732.

² Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research, Eighth Edition* (New York: McGraw Hill, 2005), 734.

Definition 1.4: *n-step transition probabilities* are defined as the conditional probability $P\{X_{t+n} = j | X_t = i\} = P\{X_n = j | X_0 = i\}$ for all $t = 0, 1, \dots$

Therefore, a Markov Chain is a stochastic process that states that the conditional probability of a future event relies on the present state of the process, rather than any past states, or events.

A conventional way to note stationary transition probabilities that will be seen later in this paper is,

$$P_{ij} = P\{X_{t+1} = j | X_t = i\},$$

$$P_{ij}^{(n)} = P\{X_{t+n} = j | X_t = i\}.^3$$

2.2.1 Chapman-Kolmogorov Equations

We use Chapman-Kolmogorov Equations to provide a method to compute all of the *n-step transition probabilities*:

$$p_{ij}^{(n)} = \sum_{k=0}^M p_{ik}^{(m)} p_{kj}^{(n-m)} \text{ For all } i = 0, 1, \dots, M,$$

$$j = 0, 1, \dots, M,$$

$$\text{And any } m = 1, 2, \dots, n-1,$$

$$n = m+1, m+2, \dots.^4$$

These equations are used to point out that when we go from one steady state to another in n steps, the process will be in some other state after exactly m (m is less than n) states. Thus the summation is just the conditional probability that, given a starting point in one state, the process goes to the other state after m steps and then to the next state in $n-m$ steps.

Therefore, by summing up these conditional probabilities over all the possible steady states must yield

³ Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research, Eighth Edition* (New York: McGraw Hill, 2005), 734.

⁴ Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research, Eighth Edition* (New York: McGraw Hill, 2005), 739

$$p_{ij}^{(n)} = \sum_{k=0}^M p_{ik} p_{kj}^{(n-1)}$$

And

$$p_{ij}^{(n)} = \sum_{k=0}^M p_{ik}^{(n-1)} p_{kj}$$

This means that these expressions allow us to obtain the *n-step probabilities* from the one-step transition probabilities recursively.

2.2.2 Transition Matrix

The conditional probabilities for a stochastic process can be organized into an *n-step transition matrix*. Such a matrix is of the form

$$\mathbf{P}^{(n)} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ P_{31} & P_{32} & \ddots & & \\ \vdots & \vdots & & & \\ P_{n1} & P_{n2} & & & P_{nn} \end{bmatrix}$$

A transition matrix shows the transition probability in a particular column and row as the transition from the row state to the column state. Since transition matrices are comprised of conditional probabilities, each entry of a transition matrix is nonnegative and less than 1. Each row of a transition matrix must also sum to the value 1 since each row signifies a state of the overall stochastic process, and each entry within each row is a conditional probability for the process to be in that state.

2.2.3 State Transition Diagrams

A convenient and useful method to visualize the state of Markov Chains when they have stationary transition probabilities and a finite number of states is through the use of a *state transition diagram*. In such diagram, each state of a Markov chain is drawn as a numbered node,

and the conditional probability of moving from one state to another is drawn by connecting the nodes with an edge and labeling the edge with the numbered probability. ⁵For example, if the weather in a certain town is given by the following stochastic process $\{X_t\}$ for $t = 0, 1, 2, \dots$

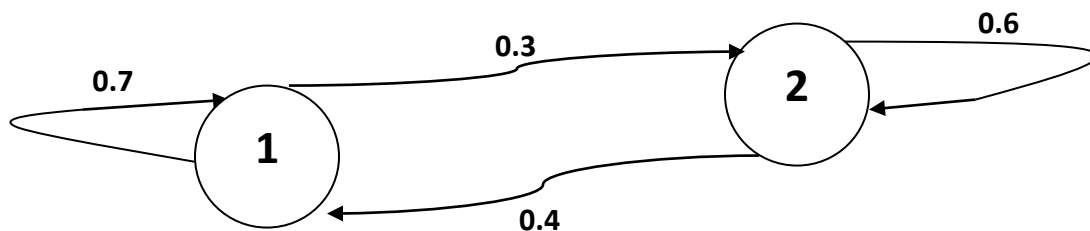
where:

$$X_t = \begin{cases} 1 & \text{if day } t \text{ is sunny} \\ 2 & \text{if day } t \text{ is cloudy} \end{cases}$$

And

$$P\{X_{t+1} = 1 | X_t = 1\} = 0.7$$

$$P\{X_{t+1} = 2 | X_t = 1\} = 0.4 \text{ Then the state transition diagram for this example is:}$$



2.3 Categorizing States of Markov Chains

Since Markov chains are long run stochastic processes that include transitional probabilities which indicate the likelihood the process will move from one state to another, it is often necessary to categorize, or classify, the varying types of states.

Definition 1.5: A state k is said to be *accessible* from a state j if $P_{jk}^{(n)} > 0$ for some $n \geq 0$, or simply stated, the system can eventually move from state j to state k .

Definition 1.6: If a state j is accessible from a state k and k is accessible from state j then states j and k are said to *communicate* with one another.

⁵ Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research, Eighth Edition* (New York: McGraw Hill, 2005), 735-737.

In a Markov chain, every state communicates with itself, since $P_{jj}^{(0)} = P\{X_0 = j | X_0 = j\} = 1$, and if a state j is said to communicate with another state k then k communicates with j . If a state j communicates with k and k communicates with l then state j communicates with state l .⁶

Since different states can communicate with one another within the same system, Markov Chains can be placed into *classes*, which are groupings of states that only communicate with one another. If every state of a Markov chain communicates with every other state within the chain, that is if the entire Markov chain is in itself one class, then the chain is said to be *irreducible*.

2.3.1 Transient States

When studying a Markov chain, certain states may only be accessible from others and often times whether or not the system will move from one state to another is convenient to know.

Definition 1.6: If some state j is accessible from state k , but k is not accessible from j (provided that $j \neq k$), then j is considered a *transient state*. Therefore a transient state is one where once the process enters the state, the process can never return to the state, and once the process enters the state, there exists a positive probability that the process will move to another state and will never return to the original state. Thus, the chain will only enter transient states a finite number of times.⁷

2.3.2 Recurrent States and Absorbing States

Aside from transient states where once the process enters, it can leave, but will never return, other states have the characteristic that the process will most certainly return to it after

⁶ Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research, Eighth Edition* (New York: McGraw Hill, 2005), 742-743.

⁷ Hillier and Lieberman, *Introduction to Operations Research*, 743.

it has entered it once, and certain others have the characteristic that once the process enters the state, it will never leave the state.

Definition 1.7: If a stochastic process, such as a Markov chain, enters a state, and will definitely return to it, the state is said to be *recurrent*. Hence, recurrent states cannot be transient; however, they can be *absorbing*.

Definition 1.8: A state is considered to be *absorbent*, if after entering the state, the process will never leave the state. If for example, the state j is an absorbing state, then $P_{jj} = 1$.

From the above definitions, it is apparent that when grouping the states of a Markov chain into classes, each state belonging to a class is either transient or recurrent. For an irreducible finite-state Markov chain, every state is recurrent, and for any finite-state Markov chain, all the states cannot be transient.

2.3.3 Periodicity and Ergodicity

Periodicity is defined as the following:

Definition 1.9: For a state j in a Markov chain, the *period* is the largest integer t (where $t > 0$) such that $P_{jj}^{(n)} = 0$ for all values of n other than $t, 2t, 3t, \dots$

If a process can be in a state j at times m and $m+1$, the state of the period is 1, and is called *aperiodic*. Every state within a class of a Markov chain shares the same period. For finite-state Markov chains, aperiodicity can lead to ergodicity.

Definition 1.10: In a finite-state Markov chain, recurrent states that are aperiodic are called *ergodic* states, and a Markov chain is called ergodic if all of its states are ergodic (or, aperiodic) states.⁸

⁸ Hillier and Lieberman, *Introduction to Operations Research*, 743-744

2.4 Properties of Markov Chains in the Long Run

2.4.1 Steady State Probabilities

After the n -step transition probabilities for a Markov chain have been calculated, the Markov chain will display the characteristic of a steady state. Meaning, that if the value of n is large enough, every row of the matrix will be the same, and such, the probability that the process is in each state does not depend on the initial state of the process. Therefore, the probability that the process will be in each state k after a certain number of transitions is a limiting probability that exists independently of the initial state. This can be defined as:

For any irreducible ergodic Markov chain, $\lim_{n \rightarrow \infty} P_{ij}^{(n)}$ exists and is independent of i .

Furthermore,

$$\lim_{n \rightarrow \infty} P_{ij}^{(n)} = \pi_j > 0$$

where the π_j uniquely satisfy the following steady-state equations

$$\pi_j = \sum_{i=0}^M \pi_i p_{ij}, \text{ for } j = 0, 1, \dots, M,$$

$$\sum_{j=0}^M \pi_j = 1.$$

The steady state probabilities of the Markov chain are π_j . These values indicate that after a large number of transitions the probability of finding the process in a particular state such as j tends to the value of π_j which is independent of the initial state. The π_j are also known as stationary probabilities, when if the initial probability of being in state j is given by π_j for all j , then the probability of finding the process in state j at time $n=1,2,\dots$ is also given by π_j , or

$$P\{X_n = j\} = \pi_j.^9$$

⁹ Hillier and Lieberman, *Introduction to Operations Research*, 743-745.

2.4.2 Applying Steady State Probabilities to the Transition Matrix

In order to solve for the steady state probabilities discussed above, the aforementioned formulas must be applied to the transition matrix, and the linear system needs to be solved.

Suppose, for a given Markov chain, the one-step transition matrix is the following:

$$\mathbf{P} = \begin{bmatrix} p_{01} & p_{02} & p_{03} \\ p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.1 & 0.5 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

Then the corresponding π_j 's would be

$$\sum_{j=0}^3 \pi_j = 1.$$

If we solve the following:

$$\begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.1 & 0.5 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \end{bmatrix}, \text{ which leads to the corresponding linear system:}$$

$$\begin{aligned} \pi_0 p_{01} + \pi_1 p_{02} + \pi_2 p_{03} &= \pi_0 = \pi_0 0.5 + \pi_1 0.4 + \pi_2 0.1 \\ \pi_0 p_{11} + \pi_1 p_{12} + \pi_2 p_{13} &= \pi_1 = \pi_0 0.3 + \pi_1 0.1 + \pi_2 0.5 \\ \pi_0 p_{21} + \pi_1 p_{22} + \pi_2 p_{23} &= \pi_2 = \pi_0 0.2 + \pi_1 0.4 + \pi_2 0.4 \\ \pi_0 + \pi_1 + \pi_2 &= 1. \end{aligned}$$

Therefore, the steady state probabilities for this example are:

2.5 First Passage Times

The term *first passage time* refers to the amount of time it takes for the transitions to go to a state i to a state j when i is not equal to j . When $i = j$ this is just the number of transitions it takes to go from one state and back to itself. In this case, the *first passage time* is known as the *recurrence time* for state i .

In general, the *first passage times* are random variables. These probability associated with them depend upon with the transition probabilities of the process. We can denote $f_{ij}^{(n)}$ denote the probability that the *first passage time* from state i to j is equal to n . For $n > 1$, this *first passage time* is n if the first transition is from state i to some state k , k is not equal to j , and

then the *first passage time* from state k to state j is $n - 1$. Therefore, these probabilities satisfy the following recursive relationships:

$$\begin{aligned} f_{ij}^{(1)} &= p_{ij}^{(1)} = p_{ij}, \\ f_{ij}^{(2)} &= \sum_{k \neq j} p_{ik} f_{kj}^{(1)}, \\ f_{ij}^{(n)} &= \sum_{k \neq j} p_{ik} f_{kj}^{(n-1)}. \end{aligned}$$

The problem with these relationships is that the summations of all these relationships can be strictly less than 1, which implies that a process initially in state i may never actually reach state j . This means that these relationships aren't the best measure of *first passage times*. The good thing is that there is another way to calculate *first passage times*.

We will denote the expectation of the *first passage time* as μ_{ij} , which is defined by

$$\mu_{ij} = \begin{cases} \infty & \text{if } \sum_{n=1}^{\infty} f_{ij}^{(n)} < 1 \\ \sum_{n=1}^{\infty} n f_{ij}^{(n)} & \text{if } \sum_{n=1}^{\infty} f_{ij}^{(n)} = 1 \end{cases}$$

Whenever

$$\sum_{n=1}^{\infty} f_{ij}^{(n)} = 1,$$

$$\mu_{ij} = 1 + \sum_{k \neq j} p_{ik} \mu_{kj}.$$

This new equation states that the first transition from state i can be to either state j or to some other state k . If it does go to state j , then the *first passage time* is 1. This means that the conditional expected *first passage time* from state i to state j is $1 + \mu_{kj}$. When we combine all of these new facts, and we sum over all the possibilities for the first transition, leads us to this equation.¹⁰

¹⁰ Hillier and Lieberman, *Introduction to Operations Research*, pages 750 - 751

2.6 Stock Terminology

In order to proceed with the application of Markov chains to the prediction of stock prices, a set of terminology regarding stocks is also useful to have. This project in particular uses opening and closing prices.

Definition 2.5.1: A *stock market* is an exchange where security trading is conducted by professional stockbrokers. It is a public market in which shares of different companies are bought and sold. The technical analysis for it is anticipating future price movements using historical prices, trading volume, open interest, and other trading data to study price patterns.^{11,12}

Definition 2.5.2: The *opening price* refers to the price of each individual share on the beginning of that trading day.¹³

Definition 2.5.3: The *closing price* refers to the price of each individual share at the end of that trading day¹⁴.

Once a list of stock prices has been found, calculating a moving average for the prices provides a method for forecasting the stock prices. Moving averages show the general tendency of the stock prices over the long run, and therefore provide a simple and useful way to predict the future of the prices.

Definition 2.5.4: Given a sequence, $\{a_i\}_{i=1}^N$, an *n-moving average* is a new sequence $\{s_i\}_{i=1}^{N-n+1}$ defined from the a_i by taking the average of the subsequence of n terms¹⁵,

$$s_i = \frac{1}{n} \sum_{j=1}^{i+n-1} a_j .$$

¹¹ "Aridni Glossary," Aridni, accessed January 26, 2011, <http://aridni.com/glossary>

¹² "Wordnet, Princeton," Princeton Wordnet, accessed January 26, 2011, <http://wordnetweb.princeton.edu/perl/webwn?s=stock%20market>

¹³ Little, Jeffrey B. & Lucien Rhodes, *Understanding Wall Street*, Fourth Edition, (New York: McGraw Hill, 2004), 58

¹⁴ Little, Jeffrey B. & Lucien Rhodes, *Understanding Wall Street*, Fourth Edition, (New York, McGraw Hill, 2004), 58

¹⁵ Weisstein, Eric W. "Moving Average." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/MovingAverage.html>

3. Methodology

As mentioned earlier, the final objective of this project is to examine a set of stock prices and use the probability method of Markov chains to predict the values of the stock prices in their immediate future. Thus to conduct this work, a data set of such prices was first collected, examined, and then the probability method was applied.

3.1 Procedural Overview

The data set of stock prices chosen for this project is one year's worth of prices for the company Google, Inc., which was available on the company's website.¹⁶ An important note is Google's stock was chosen at random during our collection time period. The company's stock holds no specific personal interest to us other than for its academic use in this project. The set of both opening and closing prices used begins on September 2, 2009 and ends on September 3, 2010, which brings a total of 263 days of prices that were used for this project. This entire table of stock prices can be found in the appendices of this paper.

Once the stock prices were found, the first step towards applying Markov chains to the data set began with the calculation of moving averages. Moving averages provide a forecast for future prices and therefore are crucial to our work here. Using the difference between the forecasted and actual prices enables us to make our predictions for the possibility of where future prices may lie. These moving averages were calculated for both opening and closing prices for intervals (called *i*) of 3, 5, and 10 days. This was done so that observations regarding any similarities or differences on the prices, and their predictions could be made at a later date and in a clear manner. It is important to know that the moving averages of the closing prices were the primary focus in the beginning because the closing prices of one day provided for the opening prices for the day after. Regardless, both sets of moving averages were calculated. Each of the moving averages calculated with the data set can be found in the appendices.

¹⁶ "Google, Inc. NASDAQ:GOOG" First accessed September 2, 2010, ©2010 Google, <http://www.google.com/finance/historical?cid=694653&startdate=Sep+2%2C+2009&enddate=Sep+3%2C+2010&num=30>

After the three intervals of moving averages were calculated for the set of stock prices, the difference between each actual price and the moving average of each individual day was calculated. This information is what we would use to predict future stock prices. Once the difference between each day's price was calculated, we then focused on binning each of the difference prices into four intervals set within the larger interval from the lowest difference price to the highest difference price. The formation of the four intervals was done so through the use of histograms. A histogram of the difference prices showed where the prices fell. This helped to establish the four bin ranges, or intervals.

When the prices were sorted, the lowest difference prices that fell as outliers compared to the rest of the difference prices were grouped in one interval, and the same was done to the higher end of the difference prices. The middle two intervals were then established at random, but so as to keep the remaining set of difference prices relatively equivalent. Each of the intervals were labeled P1, P2, P3, P4. After the intervals were established for each data sheet of difference prices, each individual difference price was labeled as to which interval it fell in.

Once each difference price was labeled with its corresponding interval, the number of transitions for each individual difference price interval to the next difference price interval was counted. For example, if Day 115's difference price belonged to interval P2, and Day 116's difference price belonged to interval P3, then a one count was added to the transition from P2 to P3 (labeled for convenience as P_{23}). Every such transition from each interval, or state, was counted and recorded. The number of points belonging to each interval was also recorded.

Once all the above information was recorded, a one-step transition matrix was ready to be prepared. Each entry of the matrix is supposed to be the probability of the data points moving from, or transitioning from, one state to another, with the states corresponding to the appropriate rows and columns. In order to calculate each entry of the matrix, the values of p_{ij} were divided by the total number of difference prices in the interval p_i , which corresponds to the aforementioned P_i .

After the one-step transition matrices were built for each interval on which the moving averages were created, the steady state probabilities can be found. The steady state probabilities are found solving the linear systems with the transition matrix multiplied with the vector π_j . The steady states indicate the probability that the difference of the prices will be within the aforementioned intervals. This provides a percentage of where future difference prices may fall, and thus provides for a prediction of what the future holds for these stocks.

3.2 Procedure with Data

In order to better understand the process of using Markov chains and how we reached the data that we have, this section explains the procedure with a corresponding data set so as to better explain our work.

Once we had our data set, we calculated the corresponding moving average. In this case the moving average is over an interval of 3 days. For the other data sets that were used for this project, please see the appendices.

Figure 3.2.1

Date	Closing Prices	Moving Average, interval of 3	Difference of Prices (Actual-Mov.Avg)	Opening Prices	Moving Average of Opening Prices(Interval =3)	Difference of Prices (Actual-Mov. Avg)
1-Sep-09	455.76	#N/A		459.68	#N/A	
2-Sep-09	453.01	#N/A		455.82	#N/A	
3-Sep-09	457.52	455.43	2.09	455.82	457.1066667	-1.286666667
4-Sep-09	461.3	457.2766667	4.023333333	457.57	456.4033333	1.166666667
8-Sep-09	458.62	459.1466667	-0.526666667	464.29	459.2266667	5.063333333
9-Sep-09	463.97	461.2966667	2.673333333	459.06	460.3066667	-1.246666667
10-Sep-09	470.94	464.51	6.43	466.65	463.3333333	3.316666667
11-Sep-09	472.14	469.0166667	3.123333333	470.4	465.37	5.03
14-Sep-09	475.12	472.7333333	2.386666667	470.51	469.1866667	1.323333333
15-Sep-09	477.54	474.9333333	2.606666667	475.08	471.9966667	3.083333333
16-Sep-09	488.29	480.3166667	7.973333333	479.8	475.13	4.67
17-Sep-09	491.72	485.85	5.87	490.57	481.8166667	8.753333333
18-Sep-09	491.46	490.49	0.97	496.77	489.0466667	7.723333333

21-Sep-09	497	493.3933333	3.606666667	487.74	491.6933333	-3.953333333
22-Sep-09	499.06	495.84	3.22	500.92	495.1433333	5.776666667
23-Sep-09	498.81	498.29	0.52	500.78	496.48	4.3
24-Sep-09	496.77	498.2133333	-1.443333333	500.47	500.7233333	-0.253333333
25-Sep-09	492.48	496.02	-3.54	494.29	498.5133333	-4.223333333
28-Sep-09	498.53	495.9266667	2.603333333	494.84	496.5333333	-1.693333333
29-Sep-09	498.53	496.5133333	2.016666667	499.53	496.22	3.31
30-Sep-09	495.85	497.6366667	-1.786666667	500	498.1233333	1.876666667
1-Oct-09	487.2	493.86	-6.66	493	497.51	-4.51
2-Oct-09	484.58	489.21	-4.63	483.74	492.2466667	-8.506666667
5-Oct-09	488.52	486.7666667	1.753333333	487.65	488.13	-0.48
6-Oct-09	498.74	490.6133333	8.126666667	491.7	487.6966667	4.003333333
7-Oct-09	517.54	501.6	15.94	499	492.7833333	6.216666667
8-Oct-09	514.18	510.1533333	4.026666667	519.57	503.4233333	16.146666667
9-Oct-09	516.25	515.99	0.26	516.65	511.74	4.91
12-Oct-09	524.04	518.1566667	5.883333333	523.42	519.88	3.54
13-Oct-09	526.11	522.1333333	3.976666667	524.39	521.4866667	2.903333333
14-Oct-09	535.32	528.49	6.83	532.46	526.7566667	5.703333333
15-Oct-09	529.91	530.4466667	-0.536666667	533.75	530.2	3.55
16-Oct-09	549.85	538.36	11.49	547.33	537.8466667	9.483333333
19-Oct-09	552.09	543.95	8.14	552.69	544.59	8.1
20-Oct-09	551.72	551.22	0.5	551.64	550.5533333	1.086666667
21-Oct-09	551.1	551.6366667	-0.536666667	549.91	551.4133333	-1.503333333
22-Oct-09	554.09	552.3033333	1.786666667	550	550.5166667	-0.516666667
23-Oct-09	553.69	552.96	0.73	555.25	551.72	3.53
26-Oct-09	554.21	553.9966667	0.213333333	555.75	553.6666667	2.083333333
27-Oct-09	548.29	552.0633333	-3.773333333	550.97	553.99	-3.02
28-Oct-09	540.3	547.6	-7.3	547.87	551.53	-3.66
29-Oct-09	551.05	546.5466667	4.503333333	543.01	547.2833333	-4.273333333
30-Oct-09	536.12	542.49	-6.37	550	546.96	3.04
2-Nov-09	533.99	540.3866667	-6.396666667	537.08	543.3633333	-6.283333333
3-Nov-09	537.29	535.8	1.49	530.01	539.03	-9.02
4-Nov-09	540.33	537.2033333	3.126666667	540.8	535.9633333	4.836666667
5-Nov-09	548.65	542.09	6.56	543.49	538.1	5.39
6-Nov-09	551.1	546.6933333	4.406666667	547.72	544.0033333	3.716666667
9-Nov-09	562.51	554.0866667	8.423333333	555.45	548.8866667	6.563333333
10-Nov-09	566.76	560.1233333	6.636666667	562.73	555.3	7.43
11-Nov-09	570.56	566.61	3.95	570.48	562.8866667	7.593333333
12-Nov-09	567.85	568.39	-0.54	569.56	567.59	1.97
13-Nov-09	572.05	570.1533333	1.896666667	569.29	569.7766667	-0.486666667
16-Nov-09	576.28	572.06	4.22	575	571.2833333	3.716666667
17-Nov-09	577.49	575.2733333	2.216666667	574.87	573.0533333	1.816666667

18-Nov-09	576.65	576.8066667	-0.156666667	576.65	575.5066667	1.143333333
19-Nov-09	572.99	575.71	-2.72	573.77	575.0966667	-1.326666667
20-Nov-09	569.96	573.2	-3.24	569.5	573.3066667	-3.806666667
23-Nov-09	582.35	575.1	7.25	576.49	573.2533333	3.236666667
24-Nov-09	583.09	578.4666667	4.623333333	582.5	576.1633333	6.336666667
25-Nov-09	585.74	583.7266667	2.013333333	586.41	581.8	4.61
26-Nov-09	585.74	584.8566667	0.883333333	585.74	584.8833333	0.856666667
27-Nov-09	579.76	583.7466667	-3.986666667	572	581.3833333	-9.383333333
30-Nov-09	583	582.8333333	0.166666667	580.63	579.4566667	1.173333333
1-Dec-09	589.87	584.21	5.66	588.13	580.2533333	7.876666667
2-Dec-09	587.51	586.7933333	0.716666667	590.98	586.58	4.4
3-Dec-09	585.74	587.7066667	-1.966666667	589.04	589.3833333	-0.343333333
4-Dec-09	585.01	586.0866667	-1.076666667	593.02	591.0133333	2.006666667
7-Dec-09	586.25	585.6666667	0.583333333	584.23	588.7633333	-4.533333333
8-Dec-09	587.05	586.1033333	0.946666667	583.5	586.9166667	-3.416666667
9-Dec-09	589.02	587.44	1.58	587.5	585.0766667	2.423333333
10-Dec-09	591.5	589.19	2.31	590.44	587.1466667	3.293333333
11-Dec-09	590.51	590.3433333	0.166666667	594.68	590.8733333	3.806666667
14-Dec-09	595.73	592.58	3.15	595.24	593.4533333	1.786666667
15-Dec-09	593.14	593.1266667	0.013333333	593.3	594.4066667	-1.106666667
16-Dec-09	597.76	595.5433333	2.216666667	598.91	595.8166667	3.093333333
17-Dec-09	593.94	594.9466667	-1.006666667	596.44	596.2166667	0.223333333
18-Dec-09	596.42	596.04	0.38	596.03	597.1266667	-1.096666667
21-Dec-09	598.68	596.3466667	2.333333333	597.95	596.8066667	1.143333333
22-Dec-09	601.12	598.74	2.38	601.34	598.44	2.9
23-Dec-09	611.68	603.8266667	7.853333333	603.5	600.93	2.57
24-Dec-09	618.48	610.4266667	8.053333333	612.93	605.9233333	7.006666667
25-Dec-09	618.48	616.2133333	2.266666667	618.48	611.6366667	6.843333333
28-Dec-09	622.87	619.9433333	2.926666667	621.66	617.69	3.97
29-Dec-09	619.4	620.25	-0.85	624.74	621.6266667	3.113333333
30-Dec-09	622.73	621.6666667	1.063333333	618.5	621.6333333	-3.133333333
31-Dec-09	619.98	620.7033333	-0.723333333	624.75	622.6633333	2.086666667
1-Jan-10	619.98	620.8966667	-0.916666667	619.98	621.0766667	-1.096666667
4-Jan-10	626.75	622.2366667	4.513333333	626.95	623.8933333	3.056666667
5-Jan-10	623.99	623.5733333	0.416666667	627.18	624.7033333	2.476666667
6-Jan-10	608.26	619.6666667	-11.40666667	625.86	626.6633333	-0.803333333
7-Jan-10	594.1	608.7833333	-14.68333333	609.4	620.8133333	-11.41333333
8-Jan-10	602.02	601.46	0.56	592	609.0866667	-17.08666667
11-Jan-10	601.11	599.0766667	2.033333333	604.46	601.9533333	2.506666667
12-Jan-10	590.48	597.87	-7.39	598.08	598.18	-0.1
13-Jan-10	587.09	592.8933333	-5.803333333	576.49	593.01	-16.52
14-Jan-10	589.85	589.14	0.71	583.9	586.1566667	-2.256666667

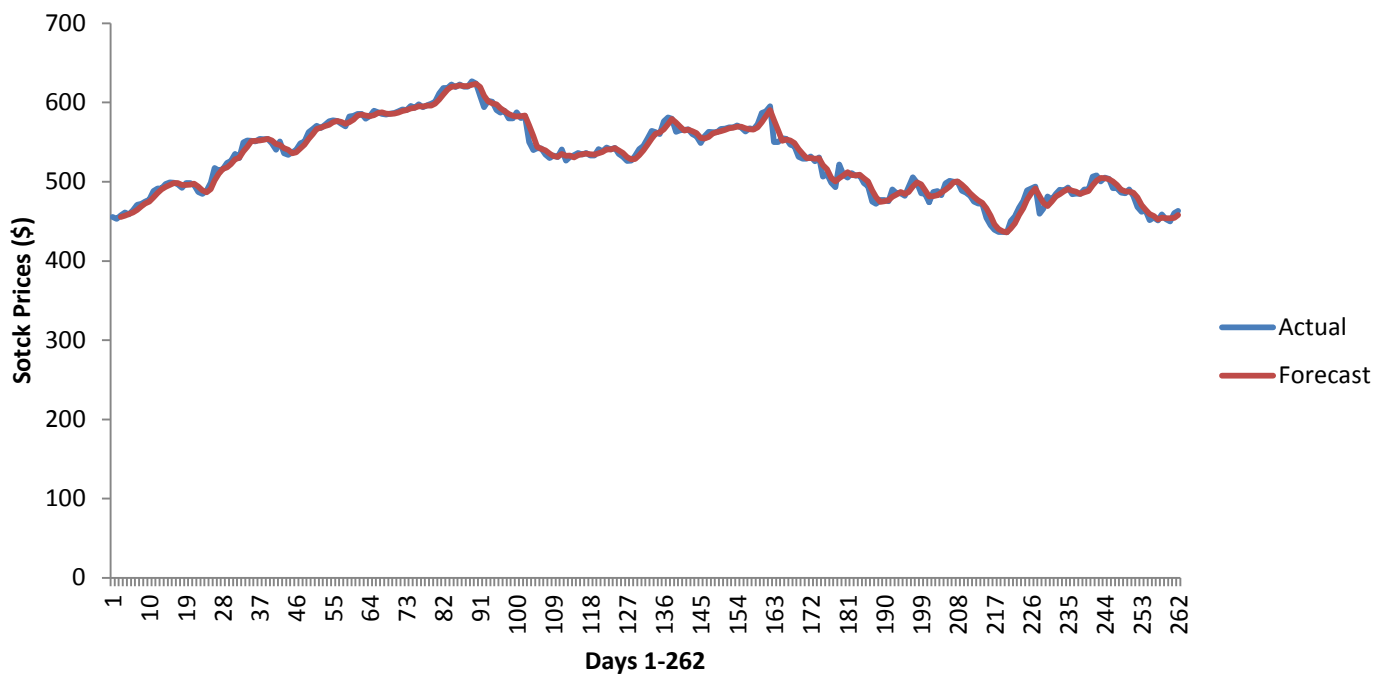
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18-Jan-10	580	583.2833333	-3.283333333	580	585.1633333	-5.163333333
19-Jan-10	587.62	582.54	5.08	581.49	584.36	-2.87
20-Jan-10	580.41	582.6766667	-2.266666667	585.98	582.49	3.49
21-Jan-10	582.98	583.67	-0.69	583.44	583.6366667	-0.196666667
22-Jan-10	550.01	571.1333333	-21.12333333	564.5	577.9733333	-13.47333333
25-Jan-10	540	557.6633333	-17.66333333	546.59	564.8433333	-18.25333333
26-Jan-10	542.42	544.1433333	-1.723333333	537.97	549.6866667	-11.71666667
27-Jan-10	542.1	541.5066667	0.593333333	541.27	541.9433333	-0.673333333
28-Jan-10	534.29	539.6033333	-5.313333333	544.49	541.2433333	3.246666667
29-Jan-10	529.94	535.4433333	-5.503333333	538.49	541.4166667	-2.926666667
1-Feb-10	533.02	532.4166667	0.603333333	534.6	539.1933333	-4.593333333
2-Feb-10	531.12	531.36	-0.24	534.96	536.0166667	-1.056666667
3-Feb-10	540.82	534.9866667	5.833333333	528.67	532.7433333	-4.073333333
4-Feb-10	526.78	532.9066667	-6.126666667	537	533.5433333	3.456666667
5-Feb-10	531.29	532.9633333	-1.673333333	528.4	531.3566667	-2.956666667
8-Feb-10	533.47	530.5133333	2.956666667	532.5	532.6333333	-0.133333333
9-Feb-10	536.44	533.7333333	2.706666667	539.54	533.48	6.06
10-Feb-10	534.44	534.7833333	-0.343333333	534.07	535.37	-1.3
11-Feb-10	536.4	535.76	0.64	532.25	535.2866667	-3.036666667
12-Feb-10	533.12	534.6533333	-1.533333333	532.97	533.0966667	-0.126666667
15-Feb-10	533.12	534.2133333	-1.093333333	533.12	532.78	0.34
16-Feb-10	541.3	535.8466667	5.453333333	537.14	534.41	2.73
17-Feb-10	538.21	537.5433333	0.666666667	542	537.42	4.58
18-Feb-10	543.22	540.91	2.31	537.54	538.8933333	-1.353333333
19-Feb-10	540.76	540.73	0.03	541	540.18	0.82
22-Feb-10	542.8	542.26	0.54	547.35	541.9633333	5.386666667
23-Feb-10	535.07	539.5433333	-4.473333333	543	543.7833333	-0.783333333
24-Feb-10	531.47	536.4466667	-4.976666667	534.39	541.58	-7.19
25-Feb-10	526.43	530.99	-4.56	527.12	534.8366667	-7.716666667
26-Feb-10	526.8	528.2333333	-1.433333333	527.42	529.6433333	-2.223333333
1-Mar-10	532.69	528.64	4.05	529.2	527.9133333	1.286666667
2-Mar-10	541.06	533.5166667	7.543333333	535.48	530.7	4.78
3-Mar-10	545.32	539.69	5.63	542.36	535.68	6.68
4-Mar-10	554.59	546.99	7.6	546.5	541.4466667	5.053333333
5-Mar-10	564.21	554.7066667	9.503333333	561.35	550.07	11.28
8-Mar-10	562.48	560.4266667	2.053333333	564.78	557.5433333	7.236666667
9-Mar-10	560.19	562.2933333	-2.103333333	559.85	561.9933333	-2.143333333
10-Mar-10	576.45	566.3733333	10.07666667	563.76	562.7966667	0.963333333
11-Mar-10	581.14	572.5933333	8.546666667	574.26	565.9566667	8.303333333
12-Mar-10	579.54	579.0433333	0.496666667	588.14	575.3866667	12.753333333
15-Mar-10	563.18	574.62	-11.44	566.68	576.36	-9.68

16-Mar-10	565.2	569.3066667	-4.106666667	561.83	572.2166667	-10.386666667
17-Mar-10	565.56	564.6466667	0.913333333	568.3	565.6033333	2.696666667
18-Mar-10	566.4	565.72	0.68	564.72	564.95	-0.23
19-Mar-10	560	563.9866667	-3.986666667	566.23	566.4166667	-0.186666667
22-Mar-10	557.5	561.3	-3.8	556.11	562.3533333	-6.243333333
23-Mar-10	549	555.5	-6.5	557.04	559.7933333	-2.753333333
24-Mar-10	557.33	554.61	2.72	545.51	552.8866667	-7.376666667
25-Mar-10	562.88	556.4033333	6.476666667	559.02	553.8566667	5.163333333
26-Mar-10	562.69	560.9666667	1.723333333	565.27	556.6	8.67
29-Mar-10	562.45	562.6733333	-0.223333333	563	562.43	0.57
30-Mar-10	566.71	563.95	2.76	562.83	563.7	-0.87
31-Mar-10	567.12	565.4266667	1.693333333	565.05	563.6266667	1.423333333
1-Apr-10	568.8	567.5433333	1.256666667	571.35	566.41	4.94
2-Apr-10	568.8	568.24	0.56	568.8	568.4	0.4
5-Apr-10	571.01	569.5366667	1.473333333	570.9	570.35	0.55
6-Apr-10	568.22	569.3433333	-1.123333333	569.46	569.72	-0.26
7-Apr-10	563.54	567.59	-4.05	567.3	569.22	-1.92
8-Apr-10	567.49	566.4166667	1.073333333	563.32	566.6933333	-3.373333333
9-Apr-10	566.22	565.75	0.47	568	566.2066667	1.793333333
12-Apr-10	572.73	568.8133333	3.916666667	567.35	566.2233333	1.126666667
13-Apr-10	586.77	575.24	11.53	572.53	569.2933333	3.236666667
14-Apr-10	589	582.8333333	6.166666667	590.06	576.6466667	13.413333333
15-Apr-10	595.3	590.3566667	4.943333333	592.17	584.92	7.25
16-Apr-10	550.14	578.1466667	-28.006666667	563	581.7433333	-18.743333333
19-Apr-10	550.1	565.18	-15.08	548.75	567.9733333	-19.223333333
20-Apr-10	555.04	551.76	3.28	554.17	555.3066667	-1.136666667
21-Apr-10	554.3	553.1466667	1.153333333	556.46	553.1266667	3.333333333
22-Apr-10	547.06	552.1333333	-5.073333333	552	554.21	-2.21
23-Apr-10	544.99	548.7833333	-3.793333333	547.25	551.9033333	-4.653333333
26-Apr-10	531.64	541.23	-9.59	544.97	548.0733333	-3.103333333
27-Apr-10	529.06	535.23	-6.17	528.94	540.3866667	-11.446666667
28-Apr-10	529.19	529.9633333	-0.773333333	532.1	535.3366667	-3.236666667
29-Apr-10	532	530.0833333	1.916666667	533.37	531.47	1.9
30-Apr-10	525.7	528.9633333	-3.263333333	531.13	532.2	-1.07
3-May-10	530.6	529.4333333	1.166666667	526.5	530.3333333	-3.833333333
4-May-10	506.37	520.89	-14.52	526.52	528.05	-1.53
5-May-10	509.76	515.5766667	-5.816666667	500.98	518	-17.02
6-May-10	498.67	504.9333333	-6.263333333	508.75	512.0833333	-3.333333333
7-May-10	493.14	500.5233333	-7.383333333	499.97	503.2333333	-3.263333333
10-May-10	521.65	504.4866667	17.163333333	513.97	507.5633333	6.406666667
11-May-10	509.05	507.9466667	1.103333333	515.67	509.87	5.8
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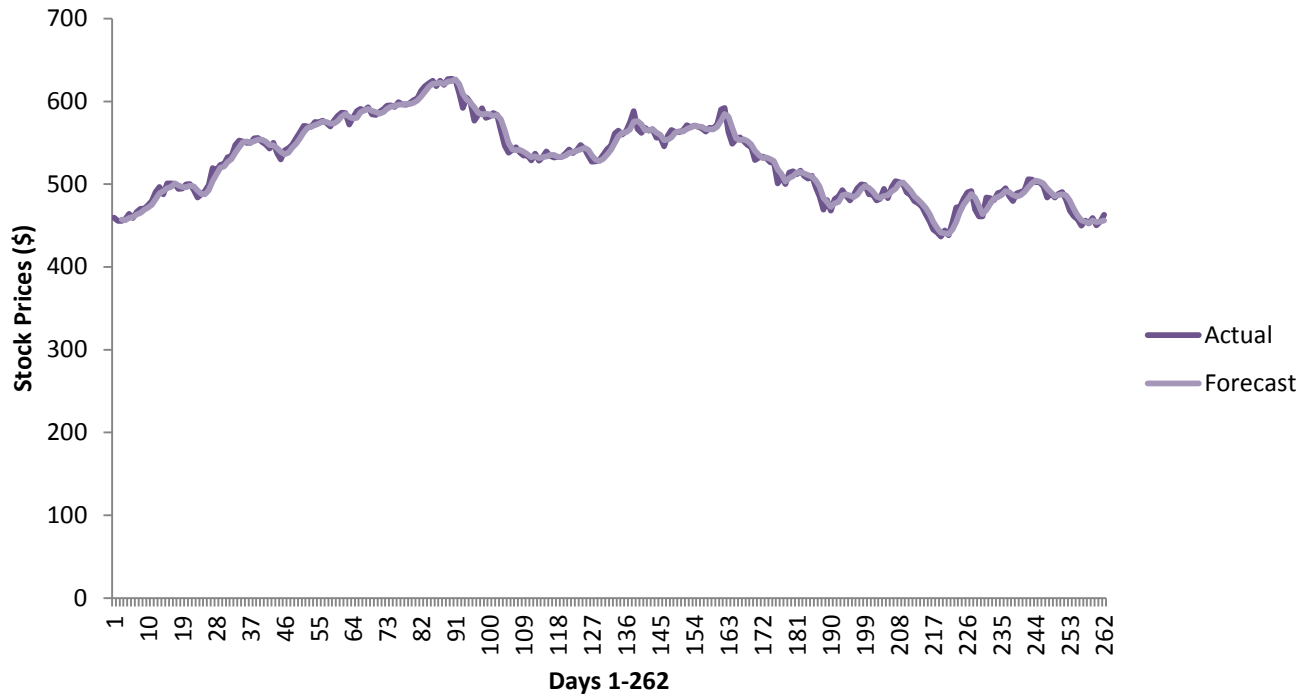
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17-May-10	507.97	508.7933333	-0.823333333	506.78	511.0166667	-4.236666667
18-May-10	498.37	504.6233333	-6.253333333	510	508.85	1.15
19-May-10	494.43	500.2566667	-5.826666667	496.26	504.3466667	-8.086666667
20-May-10	475.01	489.27	-14.26	485.07	497.11	-12.04
21-May-10	472.05	480.4966667	-8.446666667	469.06	483.4633333	-14.403333333
24-May-10	477.16	474.74	2.42	480.73	478.2866667	2.443333333
25-May-10	477.07	475.4266667	1.643333333	468.16	472.65	-4.49
26-May-10	475.47	476.5666667	-1.096666667	482.07	476.9866667	5.083333333
27-May-10	490.46	481	9.46	484.86	478.3633333	6.496666667
28-May-10	485.63	483.8533333	1.776666667	492.74	486.5566667	6.183333333
31-May-10	485.63	487.24	-1.61	485.63	487.7433333	-2.113333333
1-Jun-10	482.37	484.5433333	-2.173333333	480.43	486.2666667	-5.836666667
2-Jun-10	493.37	487.1233333	6.246666667	486.68	484.2466667	2.433333333
3-Jun-10	505.6	493.78	11.82	495.11	487.4066667	7.703333333
4-Jun-10	498.72	499.23	-0.51	499.72	493.8366667	5.883333333
7-Jun-10	485.52	496.6133333	-11.09333333	499.06	497.9633333	1.096666667
8-Jun-10	484.78	489.6733333	-4.893333333	487.85	495.5433333	-7.693333333
9-Jun-10	474.02	481.44	-7.42	487.22	491.3766667	-4.156666667
10-Jun-10	487.01	481.9366667	5.073333333	480.37	485.1466667	-4.776666667
11-Jun-10	488.5	483.1766667	5.323333333	482.5	483.3633333	-0.863333333
14-Jun-10	483.19	486.2333333	-3.043333333	494.48	485.7833333	8.696666667
15-Jun-10	497.99	489.8933333	8.096666667	483.08	486.6866667	-3.606666667
16-Jun-10	501.27	494.15	7.12	496.17	491.2433333	4.926666667
17-Jun-10	500.08	499.78	0.3	503.45	494.2333333	9.216666667
18-Jun-10	500.03	500.46	-0.43	502.51	500.71	1.8
21-Jun-10	488.56	496.2233333	-7.663333333	499.9	501.9533333	-2.053333333
22-Jun-10	486.25	491.6133333	-5.363333333	489.9	497.4366667	-7.536666667
23-Jun-10	482.05	485.62	-3.57	486.89	492.23	-5.34
24-Jun-10	475.1	481.1333333	-6.033333333	479.66	485.4833333	-5.823333333
25-Jun-10	472.68	476.61	-3.93	477.06	481.2033333	-4.143333333
28-Jun-10	472.08	473.2866667	-1.206666667	472.59	476.4366667	-3.846666667
29-Jun-10	454.26	466.34	-12.08	463.44	471.03	-7.59
30-Jun-10	444.95	457.0966667	-12.14666667	454.96	463.6633333	-8.703333333
1-Jul-10	439.49	446.2333333	-6.743333333	445.29	454.5633333	-9.273333333
2-Jul-10	436.55	440.33	-3.78	441.62	447.29	-5.67
5-Jul-10	436.55	437.53	-0.98	436.55	441.1533333	-4.603333333
6-Jul-10	436.07	436.39	-0.32	444	440.7233333	3.276666667
7-Jul-10	450.2	440.94	9.26	438.31	439.62	-1.31
8-Jul-10	456.56	447.61	8.95	453.55	445.2866667	8.263333333
9-Jul-10	467.49	458.0833333	9.406666667	471.96	454.6066667	17.353333333

12-Jul-10	475.83	466.6266667	9.203333333	472.37	465.96	6.41
13-Jul-10	489.2	477.5066667	11.69333333	482.25	475.5266667	6.723333333
14-Jul-10	491.34	485.4566667	5.883333333	489.88	481.5	8.38
15-Jul-10	494.02	491.52	2.5	491.73	487.9533333	3.776666667
16-Jul-10	459.6	481.6533333	-22.05333333	469.12	483.5766667	-14.45666667
19-Jul-10	466.18	473.2666667	-7.086666667	461.01	473.9533333	-12.94333333
20-Jul-10	481.59	469.1233333	12.46666667	461.03	463.72	-2.69
21-Jul-10	477.5	475.09	2.41	484	468.68	15.32
22-Jul-10	484.81	481.3	3.51	483.23	476.0866667	7.143333333
23-Jul-10	490.06	484.1233333	5.936666667	480.77	482.6666667	-1.896666667
26-Jul-10	488.97	487.9466667	1.023333333	489.09	484.3633333	4.726666667
27-Jul-10	492.63	490.5533333	2.076666667	490.58	486.8133333	3.766666667
28-Jul-10	484.35	488.65	-4.3	494.94	491.5366667	3.403333333
29-Jul-10	484.99	487.3233333	-2.333333333	485.95	490.49	-4.54
30-Jul-10	484.85	484.73	0.12	479.65	486.8466667	-7.196666667
2-Aug-10	490.41	486.75	3.66	488.99	484.8633333	4.126666667
3-Aug-10	489.83	488.3633333	1.466666667	490.5	486.38	4.12
4-Aug-10	506.32	495.52	10.8	492.18	490.5566667	1.623333333
5-Aug-10	508.1	501.4166667	6.683333333	505.89	496.19	9.7
6-Aug-10	500.22	504.88	-4.66	505.4	501.1566667	4.243333333
9-Aug-10	505.35	504.5566667	0.793333333	502.25	504.5133333	-2.263333333
10-Aug-10	503.71	503.0933333	0.616666667	502.35	503.3333333	-0.983333333
11-Aug-10	491.74	500.2666667	-8.526666667	497.73	500.7766667	-3.046666667
12-Aug-10	492.01	495.82	-3.81	483.94	494.6733333	-10.73333333
13-Aug-10	486.35	490.0333333	-3.683333333	489	490.2233333	-1.223333333
16-Aug-10	485.59	487.9833333	-2.393333333	483.68	485.54	-1.86
17-Aug-10	490.52	487.4866667	3.033333333	488.53	487.07	1.46
18-Aug-10	482.15	486.0866667	-3.936666667	490.44	487.55	2.89
19-Aug-10	467.97	480.2133333	-12.24333333	481.01	486.66	-5.65
20-Aug-10	462.02	470.7133333	-8.693333333	467.97	479.8066667	-11.83666667
23-Aug-10	464.07	464.6866667	-0.616666667	461.5	470.16	-8.66
24-Aug-10	451.39	459.16	-7.77	457.7	462.39	-4.69
25-Aug-10	454.62	456.6933333	-2.073333333	450	456.4	-6.4
26-Aug-10	450.98	452.33	-1.35	456.06	454.5866667	1.473333333
27-Aug-10	458.83	454.81	4.02	452.56	452.8733333	-0.313333333
30-Aug-10	452.69	454.1666667	-1.476666667	459.15	455.9233333	3.226666667
31-Aug-10	450.02	453.8466667	-3.826666667	450.11	453.94	-3.83
1-Sep-10	460.34	454.35	5.99	454.98	454.7466667	0.233333333
2-Sep-10	463.18	457.8466667	5.333333333	462.84	455.9766667	6.863333333
		Max Difference	17.16333333		Max Difference	17.35333333
		Min Difference	-28.00666667		Min Difference	-19.22333333

Moving Average of Closing Prices with Interval 3



Moving Average of Opening Prices with Interval of 3



After the moving averages were calculated, we created a histogram to help us establish the four P intervals that would be used to build the transition matrix.

Bin	Frequency
-28.0067	1
-25.1835	0
-22.3604	0
-19.5373	2
-16.7142	1
-13.891	4
-11.0679	6
-8.24479	4
-5.42167	23
-2.59854	30
0.224583	48
3.047708	71
5.870833	30
8.693958	24
11.51708	9
14.34021	4
More	2

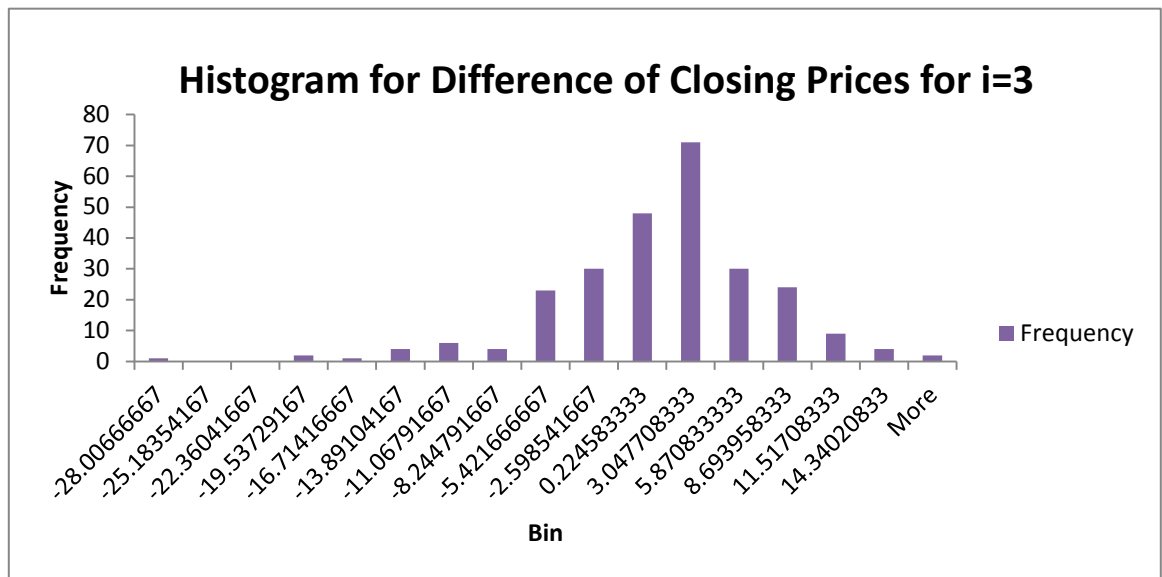
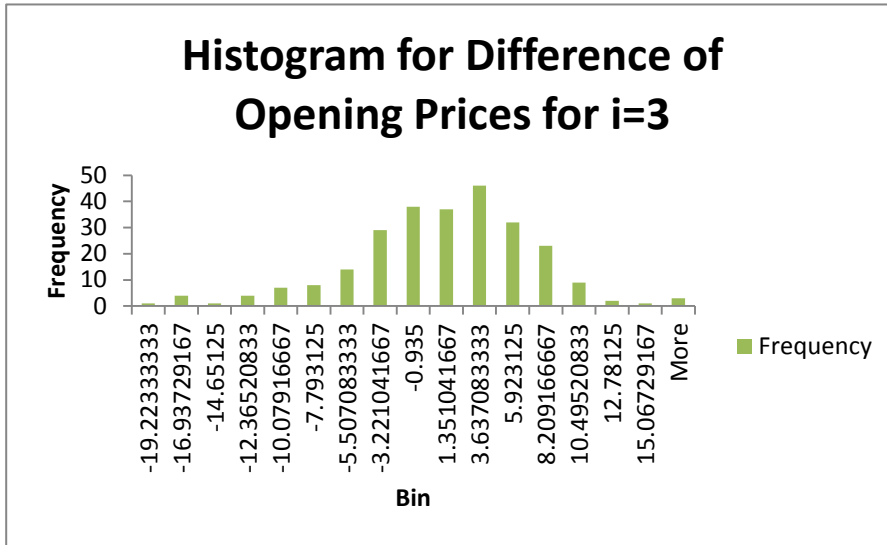


Figure 3.2.2, Histogram and bin table for i=3

Figure 3.2.2 was used as a reference for creating four intervals (labeled I_1, I_2, I_3, I_4). The following are the intervals created from the above figure.



Bin	Frequency
-19.2233	1
-16.9373	4
-14.6513	1
-12.3652	4
-10.0792	7
-7.79313	8
-5.50708	14
-3.22104	29
-0.935	38
1.351042	37
3.637083	46
5.923125	32
8.20916667	23
10.49520833	9
12.78125	2
15.06729167	1
More	3

Figure 3.2.3

Histogram and bin table for i=3 Opening Prices

Lowest Point	-28.00666667
Highest Point	17.16333333
Total Interval	[-28.0066667, 17.1633333]
Interval I1	[-28.0066667, -5.421666666666668]
Interval I2	(-5.421666666666668, 0.224583333]
Interval I3	(0.2245833333333314, 3.04770833333331]
Interval I4	(3.047708333, 17.16333333]

Figure 3.2.4, Table of Intervals for i=3 Closing Prices

Lowest Point	-19.2233
Highest Point	17.35333
Interval 1	[-19.2233, -5.50708]
Interval 2	(-5.50708, 3.637083]
Interval 3	(3.637083, 10.49521]
Interval 4	(10.49521, 17.35333]

Figure 3.2.5, Table of Intervals for i=3 Opening Prices

Once the intervals were created, each of the data points were labeled with the name of the interval to which they corresponded. After the intervals were labeled, the transition from each data point to the next, specifically, the transition of the interval from one data point to the next was calculated. The following table demonstrates this step.

Figure 3.2.6, Interval i=3 Closing Prices, labeled and counted (note, Count of "P1" column refers to P1 as I1, they are interchangeable here)

Difference of Closing Prices (Actual-Mov.Avg)	P State Count	Transition Count
2.09	P3	
4.023333333	P4	P_34
-0.526666667	P2	P_42
2.673333333	P3	P_23
6.43	P4	P_34
3.123333333	P4	P_44
2.386666667	P3	P_43
2.606666667	P3	P_33
7.973333333	P4	P_34
5.87	P4	P_44
0.97	P3	P_43
3.606666667	P4	P_34
3.22	P4	P_44
0.52	P3	P_43
-1.443333333	P2	P_32
-3.54	P2	P_22
2.603333333	P3	P_23
2.016666667	P3	P_33
-1.786666667	P2	P_32
-6.66	P1	P_21
-4.63	P2	P_12
1.753333333	P3	P_23
8.126666667	P4	P_34
15.94	P4	P_44
4.026666667	P4	P_44
0.26	P3	P_43
5.883333333	P4	P_34

Count of P1	41
Count of P2	78
Count of P3	72
Count of P4	69
Count of P_11	16
Count of P_12	13
Count of P_13	7
Count of P_14	5
Count of P_21	14
Count of P_22	27
Count of P_23	20
Count of P_24	17
Count of P_31	8
Count of P_32	29
Count of P_33	22
Count of P_34	13
Count of P_41	3
Count of P_42	9
Count of P_43	22
Count of P_44	34

3.976666667	P4	P_44
6.83	P4	P_44
-0.536666667	P2	P_42
11.49	P4	P_24
8.14	P4	P_44
0.5	P3	P_43
-0.536666667	P2	P_32
1.786666667	P3	P_23
0.73	P3	P_33
0.213333333	P2	P_32
-3.773333333	P2	P_22
-7.3	P1	P_21
4.503333333	P4	P_14
-6.37	P1	P_41
-6.396666667	P1	P_11
1.49	P3	P_13
3.126666667	P4	P_34
6.56	P4	P_44
4.406666667	P4	P_44
8.423333333	P4	P_44
6.636666667	P4	P_44
3.95	P4	P_44
-0.54	P2	P_42
1.896666667	P3	P_23
4.22	P4	P_34
2.216666667	P3	P_43
-0.156666667	P2	P_32
-2.72	P2	P_22
-3.24	P2	P_22
7.25	P4	P_24
4.623333333	P4	P_44
2.013333333	P3	P_43
0.883333333	P3	P_33
-3.986666667	P2	P_32
0.166666667	P2	P_22
5.66	P4	P_24
0.716666667	P3	P_43
-1.966666667	P2	P_32
-1.076666667	P2	P_22
0.583333333	P3	P_23
0.946666667	P3	P_33
1.58	P3	P_33

2.31	P3	P_33
0.166666667	P2	P_32
3.15	P4	P_24
0.013333333	P2	P_42
2.216666667	P3	P_23
-1.006666667	P2	P_32
0.38	P3	P_23
2.333333333	P3	P_33
2.38	P3	P_33
7.853333333	P4	P_34
8.053333333	P4	P_44
2.266666667	P3	P_43
2.926666667	P3	P_33
-0.85	P2	P_32
1.063333333	P3	P_23
-0.723333333	P2	P_32
-0.916666667	P2	P_22
4.513333333	P4	P_24
0.416666667	P3	P_43
-11.406666667	P1	P_31
-14.683333333	P1	P_11
0.56	P3	P_13
2.033333333	P3	P_33
-7.39	P1	P_31
-5.803333333	P1	P_11
0.71	P3	P_13
-5.646666667	P1	P_31
-3.283333333	P2	P_12
5.08	P4	P_24
-2.266666667	P2	P_42
-0.69	P2	P_22
-21.123333333	P1	P_21
-17.663333333	P1	P_11
-1.723333333	P2	P_12
0.593333333	P3	P_23
-5.313333333	P2	P_32
-5.503333333	P1	P_21
0.603333333	P3	P_13
-0.24	P2	P_32
5.833333333	P4	P_24
-6.126666667	P1	P_41
-1.673333333	P2	P_12

2.956666667	P3	P_23
2.706666667	P3	P_33
-0.343333333	P2	P_32
0.64	P3	P_23
-1.533333333	P2	P_32
-1.093333333	P2	P_22
5.453333333	P4	P_24
0.666666667	P3	P_43
2.31	P3	P_33
0.03	P2	P_32
0.54	P3	P_23
-4.473333333	P2	P_32
-4.976666667	P2	P_22
-4.56	P2	P_22
-1.433333333	P2	P_22
4.05	P4	P_24
7.543333333	P4	P_44
5.63	P4	P_44
7.6	P4	P_44
9.503333333	P4	P_44
2.053333333	P3	P_43
-2.103333333	P2	P_32
10.07666667	P4	P_24
8.546666667	P4	P_44
0.496666667	P3	P_43
-11.44	P1	P_31
-4.106666667	P2	P_12
0.913333333	P3	P_23
0.68	P3	P_33
-3.986666667	P2	P_32
-3.8	P2	P_22
-6.5	P1	P_21
2.72	P3	P_13
6.476666667	P4	P_34
1.723333333	P3	P_43
-0.223333333	P2	P_32
2.76	P3	P_23
1.693333333	P3	P_33
1.256666667	P3	P_33
0.56	P3	P_33
1.473333333	P3	P_33
-1.123333333	P2	P_32

-4.05	P2	P_22
1.073333333	P3	P_23
0.47	P3	P_33
3.916666667	P4	P_34
11.53	P4	P_44
6.166666667	P4	P_44
4.943333333	P4	P_44
-28.00666667	P1	P_41
-15.08	P1	P_11
3.28	P4	P_14
1.153333333	P3	P_43
-5.073333333	P2	P_32
-3.793333333	P2	P_22
-9.59	P1	P_21
-6.17	P1	P_11
-0.773333333	P2	P_12
1.916666667	P3	P_23
-3.263333333	P2	P_32
1.166666667	P3	P_23
-14.52	P1	P_31
-5.816666667	P1	P_11
-6.263333333	P1	P_11
-7.383333333	P1	P_11
17.16333333	P4	P_14
1.103333333	P3	P_43
-6.64	P1	P_31
2.44	P3	P_13
-0.403333333	P2	P_32
-0.823333333	P2	P_22
-6.253333333	P1	P_21
-5.826666667	P1	P_11
-14.26	P1	P_11
-8.446666667	P1	P_11
2.42	P3	P_13
1.643333333	P3	P_33
-1.096666667	P2	P_32
9.46	P4	P_24
1.776666667	P3	P_43
-1.61	P2	P_32
-2.173333333	P2	P_22
6.246666667	P4	P_24
11.82	P4	P_44

-0.51	P2	P_42
-11.09333333	P1	P_21
-4.893333333	P2	P_12
-7.42	P1	P_21
5.073333333	P4	P_14
5.323333333	P4	P_44
-3.043333333	P2	P_42
8.096666667	P4	P_24
7.12	P4	P_44
0.3	P3	P_43
-0.43	P2	P_32
-7.663333333	P1	P_21
-5.363333333	P2	P_12
-3.57	P2	P_22
-6.033333333	P1	P_21
-3.93	P2	P_12
-1.206666667	P2	P_22
-12.08	P1	P_21
-12.14666667	P1	P_11
-6.743333333	P1	P_11
-3.78	P2	P_12
-0.98	P2	P_22
-0.32	P2	P_22
9.26	P4	P_24
8.95	P4	P_44
9.406666667	P4	P_44
9.203333333	P4	P_44
11.69333333	P4	P_44
5.883333333	P4	P_44
2.5	P3	P_43
-22.05333333	P1	P_31
-7.086666667	P1	P_11
12.46666667	P4	P_14
2.41	P3	P_43
3.51	P4	P_34
5.936666667	P4	P_44
1.023333333	P3	P_43
2.076666667	P3	P_33
-4.3	P2	P_32
-2.333333333	P2	P_22
0.12	P2	P_22
3.66	P4	P_24

1.466666667	P3	P_43
10.8	P4	P_34
6.683333333	P4	P_44
-4.66	P2	P_42
0.793333333	P3	P_23
0.616666667	P3	P_33
-8.526666667	P1	P_31
-3.81	P2	P_12
-3.683333333	P2	P_22
-2.393333333	P2	P_22
3.033333333	P3	P_23
-3.936666667	P2	P_32
-12.24333333	P1	P_21
-8.693333333	P1	P_11
-0.616666667	P2	P_12
-7.77	P1	P_21
-2.073333333	P2	P_12
-1.35	P2	P_22
4.02	P4	P_24
-1.476666667	P2	P_42
-3.826666667	P2	P_22
5.99	P4	P_24
5.333333333	P4	P_44

Figure 3.2.7, Interval i=3 Opening Prices, labeled and counted

Difference of OpeningPrices (Actual-Mov. Avg)	States	Transition Counts
-1.286666667	P2	
1.166666667	P2	P_22
5.063333333	P3	P_23
-1.246666667	P2	P_32
3.316666667	P2	P_22
5.03	P3	P_23
1.323333333	P2	P_32
3.083333333	P2	P_22
4.67	P3	P_23
8.753333333	P3	P_33
7.723333333	P3	P_33
-3.953333333	P2	P_32
5.776666667	P3	P_23
4.3	P3	P_33
-0.253333333	P2	P_32
-4.223333333	P2	P_22
-1.693333333	P2	P_22
3.31	P2	P_22
1.876666667	P2	P_22
-4.51	P2	P_22
-8.506666667	P1	P_21
-0.48	P2	P_12
4.003333333	P3	P_23
6.216666667	P3	P_33
16.146666667	P4	P_34
4.91	P3	P_43
3.54	P2	P_32
2.903333333	P2	P_22
5.703333333	P3	P_23
3.55	P2	P_32
9.483333333	P3	P_23
8.1	P3	P_33
1.086666667	P2	P_32

Number of points in P1	39
Number of Points in P2	151
Number of points in P3	64
Number of points in P4	6
Counting of P_11	15
Counting of P_12	21
Counting of P_13	3
Counting of P_14	0
Counting of P_21	21
Counting of P_22	100
Counting of P_23	28
Counting of P_24	2
Counting of P_31	2
Counting of P_32	29
Counting of P_33	28
Counting of P_34	4
Counting of P_41	1
Counting of P_42	0
Counting of P_43	5
Counting of P_44	0

-1.503333333	P2	P_22
-0.516666667	P2	P_22
3.53	P2	P_22
2.083333333	P2	P_22
-3.02	P2	P_22
-3.66	P2	P_22
-4.273333333	P2	P_22
3.04	P2	P_22
-6.283333333	P1	P_21
-9.02	P1	P_11
4.836666667	P3	P_13
5.39	P3	P_33
3.716666667	P3	P_33
6.563333333	P3	P_33
7.43	P3	P_33
7.593333333	P3	P_33
1.97	P2	P_32
-0.486666667	P2	P_22
3.716666667	P3	P_23
1.816666667	P2	P_32
1.143333333	P2	P_22
-1.326666667	P2	P_22
-3.806666667	P2	P_22
3.236666667	P2	P_22
6.336666667	P3	P_23
4.61	P3	P_33
0.856666667	P2	P_32
-9.383333333	P1	P_21
1.173333333	P2	P_12
7.876666667	P3	P_23
4.4	P3	P_33
-0.343333333	P2	P_32
2.006666667	P2	P_22
-4.533333333	P2	P_22
-3.416666667	P2	P_22
2.423333333	P2	P_22
3.293333333	P2	P_22
3.806666667	P3	P_23
1.786666667	P2	P_32
-1.106666667	P2	P_22
3.093333333	P2	P_22
0.223333333	P2	P_22

-1.096666667	P2	P_22
1.143333333	P2	P_22
2.9	P2	P_22
2.57	P2	P_22
7.006666667	P3	P_23
6.843333333	P3	P_33
3.97	P3	P_33
3.113333333	P2	P_32
-3.133333333	P2	P_22
2.086666667	P2	P_22
-1.096666667	P2	P_22
3.056666667	P2	P_22
2.476666667	P2	P_22
-0.803333333	P2	P_22
-11.41333333	P1	P_21
-17.08666667	P1	P_11
2.506666667	P2	P_12
-0.1	P2	P_22
-16.52	P1	P_21
-2.256666667	P2	P_12
7.596666667	P3	P_23
-5.163333333	P2	P_32
-2.87	P2	P_22
3.49	P2	P_22
-0.196666667	P2	P_22
-13.47333333	P1	P_21
-18.25333333	P1	P_11
-11.71666667	P1	P_11
-0.673333333	P2	P_12
3.246666667	P2	P_22
-2.926666667	P2	P_22
-4.593333333	P2	P_22
-1.056666667	P2	P_22
-4.073333333	P2	P_22
3.456666667	P2	P_22
-2.956666667	P2	P_22
-0.133333333	P2	P_22
6.06	P3	P_23
-1.3	P2	P_32
-3.036666667	P2	P_22
-0.126666667	P2	P_22
0.34	P2	P_22

2.73	P2	P_22
4.58	P3	P_23
-1.353333333	P2	P_32
0.82	P2	P_22
5.386666667	P3	P_23
-0.783333333	P2	P_32
-7.19	P1	P_21
-7.716666667	P1	P_11
-2.223333333	P2	P_12
1.286666667	P2	P_22
4.78	P3	P_23
6.68	P3	P_33
5.053333333	P3	P_33
11.28	P4	P_34
7.236666667	P3	P_43
-2.143333333	P2	P_32
0.963333333	P2	P_22
8.303333333	P3	P_23
12.75333333	P4	P_34
-9.68	P1	P_41
-10.38666667	P1	P_11
2.696666667	P2	P_12
-0.23	P2	P_22
-0.186666667	P2	P_22
-6.243333333	P1	P_21
-2.753333333	P2	P_12
-7.376666667	P1	P_21
5.163333333	P3	P_13
8.67	P3	P_33
0.57	P2	P_32
-0.87	P2	P_22
1.423333333	P2	P_22
4.94	P3	P_23
0.4	P2	P_32
0.55	P2	P_22
-0.26	P2	P_22
-1.92	P2	P_22
-3.373333333	P2	P_22
1.793333333	P2	P_22
1.126666667	P2	P_22
3.236666667	P2	P_22
13.41333333	P4	P_24

7.25	P3	P_43
-18.74333333	P1	P_31
-19.22333333	P1	P_11
-1.136666667	P2	P_12
3.333333333	P2	P_22
-2.21	P2	P_22
-4.653333333	P2	P_22
-3.103333333	P2	P_22
-11.44666667	P1	P_21
-3.236666667	P2	P_12
1.9	P2	P_22
-1.07	P2	P_22
-3.833333333	P2	P_22
-1.53	P2	P_22
-17.02	P1	P_21
-3.333333333	P2	P_12
-3.263333333	P2	P_22
6.406666667	P3	P_23
5.8	P3	P_33
-1.853333333	P2	P_32
1.763333333	P2	P_22
-3	P2	P_22
-4.236666667	P2	P_22
1.15	P2	P_22
-8.086666667	P1	P_21
-12.04	P1	P_11
-14.40333333	P1	P_11
2.443333333	P2	P_12
-4.49	P2	P_22
5.083333333	P3	P_23
6.496666667	P3	P_33
6.183333333	P3	P_33
-2.113333333	P2	P_32
-5.836666667	P1	P_21
2.433333333	P2	P_12
7.703333333	P3	P_23
5.883333333	P3	P_33
1.096666667	P2	P_32
-7.693333333	P1	P_21
-4.156666667	P2	P_12
-4.776666667	P2	P_22
-0.863333333	P2	P_22

8.696666667	P3	P_23
-3.606666667	P2	P_32
4.926666667	P3	P_23
9.216666667	P3	P_33
1.8	P2	P_32
-2.053333333	P2	P_22
-7.536666667	P1	P_21
-5.34	P2	P_12
-5.823333333	P1	P_21
-4.143333333	P2	P_12
-3.846666667	P2	P_22
-7.59	P1	P_21
-8.703333333	P1	P_11
-9.273333333	P1	P_11
-5.67	P1	P_11
-4.603333333	P2	P_12
3.276666667	P2	P_22
-1.31	P2	P_22
8.263333333	P3	P_23
17.353333333	P4	P_34
6.41	P3	P_43
6.723333333	P3	P_33
8.38	P3	P_33
3.776666667	P3	P_33
-14.456666667	P1	P_31
-12.943333333	P1	P_11
-2.69	P2	P_12
15.32	P4	P_24
7.143333333	P3	P_43
-1.896666667	P2	P_32
4.726666667	P3	P_23
3.766666667	P3	P_33
3.403333333	P2	P_32
-4.54	P2	P_22
-7.196666667	P1	P_21
4.126666667	P3	P_13
4.12	P3	P_33
1.623333333	P2	P_32
9.7	P3	P_23
4.243333333	P3	P_33
-2.263333333	P2	P_32
-0.983333333	P2	P_22

-3.046666667	P2	P_22
-10.733333333	P1	P_21
-1.223333333	P2	P_12
-1.86	P2	P_22
1.46	P2	P_22
2.89	P2	P_22
-5.65	P1	P_21
-11.836666667	P1	P_11
-8.66	P1	P_11
-4.69	P2	P_12
-6.4	P1	P_21
1.473333333	P2	P_12
-0.313333333	P2	P_22
3.226666667	P2	P_22
-3.83	P2	P_22
0.233333333	P2	P_22
6.863333333	P3	P_23

After the difference prices and the transitions were labeled with the appropriate intervals, the transition matrix was built and the ensuing linear system of equations was solved to find the steady states.

The transition matrix for $i=3$, Closing Prices

$$\begin{bmatrix} \frac{16}{41} & \frac{13}{41} & \frac{7}{41} & \frac{5}{41} \\ \frac{7}{39} & \frac{9}{26} & \frac{10}{39} & \frac{17}{78} \\ \frac{1}{9} & \frac{29}{72} & \frac{11}{36} & \frac{13}{72} \\ \frac{1}{23} & \frac{3}{23} & \frac{22}{69} & \frac{34}{69} \end{bmatrix}$$

$$\begin{cases} \pi_1=1/4 \\ \pi_2=1/4 \\ \pi_3=1/4 \\ \pi_4=1/4 \end{cases}$$

The transition matrix for $i=3$, Opening Prices

$$\begin{bmatrix} \frac{15}{39} & \frac{21}{39} & \frac{3}{39} & \frac{0}{39} \\ \frac{21}{151} & \frac{100}{151} & \frac{28}{151} & \frac{2}{151} \\ \frac{2}{64} & \frac{29}{64} & \frac{28}{64} & \frac{4}{64} \\ \frac{1}{6} & \frac{0}{6} & \frac{5}{6} & \frac{0}{6} \end{bmatrix}$$

Π_1	0.24087024
Π_2	0.24118104
Π_3	0.23869464
Π_4	0.27925408

The steady states provide for us the probability that the difference price will be in any of the four previously established intervals. According to our work, for the closing prices with the moving average interval of 3, the probability that the difference prices for the day after our data ends is 25% for each interval. For the opening prices for the moving average interval of 3, the probability that the difference price will be in interval I1 (also known as P_1) is $\Pi_1=24.09\%$. The probability that the difference price will be in interval I2 (also known as P_2) is $\Pi_2=24.12\%$, while the probability that the difference price will be in $\Pi_3=23.87\%$, and the probability that the difference price will be $\Pi_4=27.93\%$ (provided we round to 4 significant figures).

Using these percentages the following table summarizes the probabilities of the next day's stock prices:

Summary of Closing Prices*		Summary of Opening Prices	
25% chance that next day's price will be in this interval	[435.173, 457.758]	24.09% chance that next day's price will be in this interval	[443.617, 457.333]
25% chance that next day's price will be in this interval	(457.758, 463.405]	24.12% chance that next day's price will be in this interval	(457.333, 466.477]
25% chance that next day's price will be in this interval	(463.405, 466.228]	23.87% chance that next day's price will be in this interval	(466.477, 473.335]
25% chance that next day's price will be in this interval	(466.228, 480.343]	27.93% chance that next day's price will be in this interval	(473.335, 480.193]

*Note: All figures in this table are rounded to six significant digits.

4. Observations

As we were carrying out our calculations for our data sets, we came across several interesting results. With the moving averages, we noticed that for intervals 3 and 5, the difference between the actual prices and forecasted prices were not as great as the difference between the actual and forecasted prices for the interval of 10. For intervals 3 and 5, the difference ranged from [-28.0066667, 17.1633333] and [-43.04, 20.878] respectively, whereas the difference for the interval of 10 was [-36.844, 623.194]. This indicates that the moving average forecast for intervals 3 and 5 are closer to the actual prices than for the interval 10. This is understandable since the increments of 3 and 5 are significantly smaller than the increment of 10, so that when the moving average is calculated the variation between the actual and a small increment is far less than the variation between the actual price and a large increment.

Aside from the variations in moving averages, another interesting point of note is for our steady state calculations, almost every steady state, regardless of moving average increment size, revolved around an approximate of 25%. This indicates that as a general baseline, using four intervals to calculate the steady states may not be an accurate enough calculation to understand the probabilities of the difference prices for the future.

5. Conclusions

Our work thus far has implemented the use of Markov chains to predicting stock prices. Using the difference between forecast prices and actual prices, we have calculated the possible steady state, or probability of the future of the difference price. While much of our work was streamlined with the use of MAPLE, and Microsoft Excel 2010, there are some anomalies to note. For example, the dividing of the difference prices into four intervals, or binning, was completed in a rather random manner. While both low and high outliers were grouped within their own intervals, the actual barriers of the intervals were decided upon randomly. This was done to ensure that the results would not be compromised due any personal preference we may have had.

Our final results are summarized in the following tables:

- For the moving average interval of 3:

Summary of Closing Prices		Summary of Opening Prices	
25% chance that next day's price will be in this interval	[435.173, 457.758]	24.09% chance that next day's price will be in this interval	[443.617, 457.333]
25% chance that next day's price will be in this interval	(457.758, 463.405]	24.12% chance that next day's price will be in this interval	(457.333, 466.477]
25% chance that next day's price will be in this interval	(463.405, 466.228]	23.87% chance that next day's price will be in this interval	(466.477, 473.335]
25% chance that next day's price will be in this interval	(466.228, 480.343]	27.93% chance that next day's price will be in this interval	(473.335, 480.193]

- For the moving average interval of 5:

Summary of Closing Prices		Summary of Opening Prices	
24.80% chance that next day's price will be in this interval	[417.270, 449.229]	25% chance that next day's price will be in this closed interval	[437.030, 458.489]
24.72% chance that next day's price will be in this interval	(449.229, 461.214]	25% chance that next day's price will be in this interval	(458.489, 464.620]
24.45% chance that next day's price will be in this interval	(461.214, 469.203]	25% chance that next day's price will be in this interval	(464.620, 473.816]
25.98% chance that next day's price will be in this interval	(469.203, 481.188]	25% chance that next day's price will be in this interval	[473.816, 486.078]

- For the moving average interval of 10

Summary of Closing Prices	
24.58% chance that next day's price will be in this closed interval	[426.336, 470.339]
24.87% chance that next day's price will be in this interval	(470.339, 910.364]
24.63% chance that next day's price will be in this interval	(910.364, 998.369]
25.91% chance that next day's price will be in this interval	(998.369, 1086.37]

As mentioned earlier in the Observations, the steady states for all our moving average intervals approximate to a value of 25%, indicating that the interval of moving averages may not affect steady state values, as much as the number of intervals in which differences prices are grouped. Had we used a different number of steady states we may have gotten more varied results. Perhaps if we were able to weigh our steady states, or have more than four steady states, the results could be more conclusive. More exploration is needed in this area.

Regardless, the use of Markov Chains for predicting stock prices is an area of financial mathematics that needs to be explored more fully. While in our work here we have found results that do not prove to be worthwhile for a stock broker on Wall Street, it is no indication that Markov Chains cannot be used for such predictions.

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APPENDIX A

Moving Average Calculations and Plots
Date: September 1, 2009 -September 2, 2010
Moving Average for interval of 5

Date	Closing Prices	Moving Average (Interval = 5)	Difference of Prices (Actual-Mov.Avg)	Opening Prices	Moving Average of Opening Prices(Interval =5)	Difference of Prices (Actual-Mov. Avg)
1-Sep-09	454.42	#N/A		459.68	#N/A	
2-Sep-09	452.59	#N/A		455.82	#N/A	
3-Sep-09	455	#N/A		455.82	#N/A	
4-Sep-09	455.78	#N/A		457.57	#N/A	
8-Sep-09	455.84	454.726	1.114	464.29	458.636	5.654
9-Sep-09	458.8	455.602	3.198	459.06	458.512	0.548
10-Sep-09	462	457.484	4.516	466.65	460.678	5.972
11-Sep-09	467.63	460.01	7.62	470.4	463.594	6.806
14-Sep-09	470.05	462.864	7.186	470.51	466.182	4.328
15-Sep-09	472.71	466.238	6.472	475.08	468.34	6.74
16-Sep-09	478.48	470.174	8.306	479.8	472.488	7.312
17-Sep-09	487.15	475.204	11.946	490.57	477.272	13.298
18-Sep-09	491.23	479.924	11.306	496.77	482.546	14.224
21-Sep-09	486.22	483.158	3.062	487.74	485.992	1.748
22-Sep-09	497.81	488.178	9.632	500.92	491.16	9.76
23-Sep-09	497.71	492.024	5.686	500.78	495.356	5.424
24-Sep-09	493	493.194	-0.194	500.47	497.336	3.134
25-Sep-09	492	493.348	-1.348	494.29	496.84	-2.55
28-Sep-09	493.3	494.764	-1.464	494.84	498.26	-3.42
29-Sep-09	493.01	493.804	-0.794	499.53	497.982	1.548
30-Sep-09	487.24	491.71	-4.47	500	497.826	2.174
1-Oct-09	487	490.51	-3.51	493	496.332	-3.332
2-Oct-09	482.6	488.63	-6.03	483.74	494.222	-10.482
5-Oct-09	483.34	486.638	-3.298	487.65	492.784	-5.134
6-Oct-09	491.7	486.376	5.324	491.7	491.218	0.482
7-Oct-09	497.81	488.49	9.32	499	491.018	7.982
8-Oct-09	513.34	493.758	19.582	519.57	496.332	23.238
9-Oct-09	514.5	500.138	14.362	516.65	502.914	13.736
12-Oct-09	519.32	507.334	11.986	523.42	510.068	13.352
13-Oct-09	521.38	513.27	8.11	524.39	516.606	7.784
14-Oct-09	530	519.708	10.292	532.46	523.298	9.162
15-Oct-09	527.27	522.494	4.776	533.75	526.134	7.616
16-Oct-09	544.53	528.5	16.03	547.33	532.27	15.06
19-Oct-09	548.73	534.382	14.348	552.69	538.124	14.566

20-Oct-09	540.7	538.246	2.454	551.64	543.574	8.066
21-Oct-09	549	542.046	6.954	549.91	547.064	2.846
22-Oct-09	548	546.192	1.808	550	550.314	-0.314
23-Oct-09	551.2	547.526	3.674	555.25	551.898	3.352
26-Oct-09	550.89	547.958	2.932	555.75	552.51	3.24
27-Oct-09	544.16	548.65	-4.49	550.97	552.376	-1.406
28-Oct-09	538.25	546.5	-8.25	547.87	551.968	-4.098
29-Oct-09	541	545.1	-4.1	543.01	550.57	-7.56
30-Oct-09	534.24	541.708	-7.468	550	549.52	0.48
2-Nov-09	528.24	537.178	-8.938	537.08	545.786	-8.706
3-Nov-09	528.3	534.006	-5.706	530.01	541.594	-11.584
4-Nov-09	536.42	533.64	2.78	540.8	540.18	0.62
5-Nov-09	542.66	533.972	8.688	543.49	540.276	3.214
6-Nov-09	545.5	536.224	9.276	547.72	539.82	7.9
9-Nov-09	554.23	541.422	12.808	555.45	543.494	11.956
10-Nov-09	562	548.162	13.838	562.73	550.038	12.692
11-Nov-09	565.86	554.05	11.81	570.48	555.974	14.506
12-Nov-09	565.5	558.618	6.882	569.56	561.188	8.372
13-Nov-09	566.61	562.84	3.77	569.29	565.502	3.788
16-Nov-09	572.78	566.55	6.23	575	569.412	5.588
17-Nov-09	573.72	568.894	4.826	574.87	571.84	3.03
18-Nov-09	572.07	570.136	1.934	576.65	573.074	3.576
19-Nov-09	570	571.036	-1.036	573.77	573.916	-0.146
20-Nov-09	569.4	571.594	-2.194	569.5	573.958	-4.458
23-Nov-09	575.86	572.21	3.65	576.49	574.256	2.234
24-Nov-09	576.54	572.774	3.766	582.5	575.782	6.718
25-Nov-09	582.69	574.898	7.792	586.41	577.734	8.676
26-Nov-09	585.74	578.046	7.694	585.74	580.128	5.612
27-Nov-09	570.97	578.36	-7.39	572	580.628	-8.628
30-Nov-09	577.11	578.61	-1.5	580.63	581.456	-0.826
1-Dec-09	583	579.902	3.098	588.13	582.582	5.548
2-Dec-09	586.22	580.608	5.612	590.98	583.496	7.484
3-Dec-09	585	580.46	4.54	589.04	584.156	4.884
4-Dec-09	579.18	582.102	-2.922	593.02	588.36	4.66
7-Dec-09	581	582.88	-1.88	584.23	589.08	-4.85
8-Dec-09	582	582.68	-0.68	583.5	588.154	-4.654
9-Dec-09	583.58	582.152	1.428	587.5	587.458	0.042
10-Dec-09	590.41	583.234	7.176	590.44	587.738	2.702
11-Dec-09	587.73	584.944	2.786	594.68	588.07	6.61
14-Dec-09	592.61	587.266	5.344	595.24	590.272	4.968
15-Dec-09	590.99	589.064	1.926	593.3	592.232	1.068
16-Dec-09	596.64	591.676	4.964	598.91	594.514	4.396
17-Dec-09	593.76	592.346	1.414	596.44	595.714	0.726
18-Dec-09	595	593.8	1.2	596.03	595.984	0.046

21-Dec-09	595.67	594.412	1.258	597.95	596.526	1.424
22-Dec-09	598.85	595.984	2.866	601.34	598.134	3.206
23-Dec-09	602.85	597.226	5.624	603.5	599.052	4.448
24-Dec-09	612.27	600.928	11.342	612.93	602.35	10.58
25-Dec-09	618.48	605.624	12.856	618.48	606.84	11.64
28-Dec-09	618.48	610.186	8.294	621.66	611.582	10.078
29-Dec-09	618.29	614.074	4.216	624.74	616.262	8.478
30-Dec-09	618.01	617.106	0.904	618.5	619.262	-0.762
31-Dec-09	619.98	618.648	1.332	624.75	621.626	3.124
1-Jan-10	619.98	618.948	1.032	619.98	621.926	-1.946
4-Jan-10	624.24	620.1	4.14	626.95	622.984	3.966
5-Jan-10	621.54	620.75	0.79	627.18	623.472	3.708
6-Jan-10	606.36	618.42	-12.06	625.86	624.944	0.916
7-Jan-10	592.65	612.954	-20.304	609.4	621.874	-12.474
8-Jan-10	589.11	606.78	-17.67	592	616.278	-24.278
11-Jan-10	594.04	600.74	-6.7	604.46	611.78	-7.32
12-Jan-10	588	594.032	-6.032	598.08	605.96	-7.88
13-Jan-10	573.9	587.54	-13.64	576.49	596.086	-19.596
14-Jan-10	582.81	585.572	-2.762	583.9	590.986	-7.086
15-Jan-10	578.04	583.358	-5.318	591.59	590.904	0.686
18-Jan-10	580	580.55	-0.55	580	586.012	-6.012
19-Jan-10	576.29	578.208	-1.918	581.49	582.694	-1.204
20-Jan-10	575.29	578.486	-3.196	585.98	584.592	1.388
21-Jan-10	572.25	576.374	-4.124	583.44	584.5	-1.06
22-Jan-10	534.86	567.738	-32.878	564.5	579.082	-14.582
25-Jan-10	535.51	558.84	-23.33	546.59	572.4	-25.81
26-Jan-10	536.29	550.84	-14.55	537.97	563.696	-25.726
27-Jan-10	535.31	542.844	-7.534	541.27	554.754	-13.484
28-Jan-10	530.6	534.514	-3.914	544.49	546.964	-2.474
29-Jan-10	525.61	532.664	-7.054	538.49	541.762	-3.272
1-Feb-10	530.3	531.622	-1.322	534.6	539.364	-4.764
2-Feb-10	527.61	529.886	-2.276	534.96	538.762	-3.802
3-Feb-10	528.23	528.47	-0.24	528.67	536.242	-7.572
4-Feb-10	525.56	527.462	-1.902	537	534.744	2.256
5-Feb-10	522.46	526.832	-4.372	528.4	532.726	-4.326
8-Feb-10	531.53	527.078	4.452	532.5	532.306	0.194
9-Feb-10	535.07	528.57	6.5	539.54	533.222	6.318
10-Feb-10	527.69	528.462	-0.772	534.07	534.302	-0.232
11-Feb-10	529.5	529.25	0.25	532.25	533.352	-1.102
12-Feb-10	530.5	530.858	-0.358	532.97	534.266	-1.296
15-Feb-10	533.12	531.176	1.944	533.12	534.39	-1.27
16-Feb-10	534.3	531.022	3.278	537.14	533.91	3.23
17-Feb-10	537.61	533.006	4.604	542	535.496	6.504
18-Feb-10	536.14	534.334	1.806	537.54	536.554	0.986

19-Feb-10	539.7	536.174	3.526	541	538.16	2.84
22-Feb-10	541	537.75	3.25	547.35	541.006	6.344
23-Feb-10	532.29	537.348	-5.058	543	542.178	0.822
24-Feb-10	530.51	535.928	-5.418	534.39	540.656	-6.266
25-Feb-10	520	532.7	-12.7	527.12	538.572	-11.452
26-Feb-10	523.48	529.456	-5.976	527.42	535.856	-8.436
1-Mar-10	527.74	526.804	0.936	529.2	532.226	-3.026
2-Mar-10	535.01	527.348	7.662	535.48	530.722	4.758
3-Mar-10	539.25	529.096	10.154	542.36	532.316	10.044
4-Mar-10	546.2	534.336	11.864	546.5	536.192	10.308
5-Mar-10	559.9	541.62	18.28	561.35	542.978	18.372
8-Mar-10	561.01	548.274	12.736	564.78	550.094	14.686
9-Mar-10	556.5	552.572	3.928	559.85	554.968	4.882
10-Mar-10	562.21	557.164	5.046	563.76	559.248	4.512
11-Mar-10	574.2	562.764	11.436	574.26	564.8	9.46
12-Mar-10	579.16	566.616	12.544	588.14	570.158	17.982
15-Mar-10	556	565.614	-9.614	566.68	570.538	-3.858
16-Mar-10	560.76	566.466	-5.706	561.83	570.934	-9.104
17-Mar-10	564.25	566.874	-2.624	568.3	571.842	-3.542
18-Mar-10	562.96	564.626	-1.666	564.72	569.934	-5.214
19-Mar-10	557.28	560.25	-2.97	566.23	565.552	0.678
22-Mar-10	554.28	559.906	-5.626	556.11	563.438	-7.328
23-Mar-10	542	556.154	-14.154	557.04	562.48	-5.44
24-Mar-10	539.7	551.244	-11.544	545.51	557.922	-12.412
25-Mar-10	558.66	550.384	8.276	559.02	556.782	2.238
26-Mar-10	560.02	550.932	9.088	565.27	556.59	8.68
29-Mar-10	560.57	552.19	8.38	563	557.968	5.032
30-Mar-10	560.28	555.846	4.434	562.83	559.126	3.704
31-Mar-10	562.81	560.468	2.342	565.05	563.034	2.016
1-Apr-10	565.55	561.846	3.704	571.35	565.5	5.85
2-Apr-10	568.8	563.602	5.198	568.8	566.206	2.594
5-Apr-10	569	565.288	3.712	570.9	567.786	3.114
6-Apr-10	565.4	566.312	-0.912	569.46	569.112	0.348
7-Apr-10	561.86	566.122	-4.262	567.3	569.562	-2.262
8-Apr-10	560.05	565.022	-4.972	563.32	567.956	-4.636
9-Apr-10	564	564.062	-0.062	568	567.796	0.204
12-Apr-10	566.22	563.506	2.714	567.35	567.086	0.264
13-Apr-10	571.13	564.652	6.478	572.53	567.7	4.83
14-Apr-10	584.01	569.082	14.928	590.06	572.252	17.808
15-Apr-10	588.29	574.73	13.56	592.17	578.022	14.148
16-Apr-10	549.63	571.856	-22.226	563	577.022	-14.022
19-Apr-10	545	567.612	-22.612	548.75	573.302	-24.552
20-Apr-10	551.06	563.598	-12.538	554.17	569.63	-15.46
21-Apr-10	552.16	557.228	-5.068	556.46	562.91	-6.45

22-Apr-10	543.35	548.24	-4.89	552	554.876	-2.876
23-Apr-10	542.27	546.768	-4.498	547.25	551.726	-4.476
26-Apr-10	529.21	543.61	-14.4	544.97	550.97	-6
27-Apr-10	527.24	538.846	-11.606	528.94	545.924	-16.984
28-Apr-10	521.03	532.62	-11.59	532.1	541.052	-8.952
29-Apr-10	526.67	529.284	-2.614	533.37	537.326	-3.956
30-Apr-10	525.44	525.918	-0.478	531.13	534.102	-2.972
3-May-10	525.08	525.092	-0.012	526.5	530.408	-3.908
4-May-10	504.21	520.486	-16.276	526.52	529.924	-3.404
5-May-10	500.47	516.374	-15.904	500.98	523.7	-22.72
6-May-10	460	503.04	-43.04	508.75	518.776	-10.026
7-May-10	481.33	494.218	-12.888	499.97	512.544	-12.574
10-May-10	512.6	491.722	20.878	513.97	510.038	3.932
11-May-10	508.22	492.524	15.696	515.67	507.868	7.802
12-May-10	502	492.83	9.17	512.04	510.08	1.96
13-May-10	510.37	502.904	7.466	516.5	511.63	4.87
14-May-10	496.25	505.888	-9.638	509.77	513.59	-3.82
17-May-10	498.35	503.038	-4.688	506.78	512.152	-5.372
18-May-10	497.07	500.808	-3.738	510	511.018	-1.018
19-May-10	487.74	497.956	-10.216	496.26	507.862	-11.602
20-May-10	473.8	490.642	-16.842	485.07	501.576	-16.506
21-May-10	464.4	484.272	-19.872	469.06	493.434	-24.374
24-May-10	476.8	479.962	-3.162	480.73	488.224	-7.494
25-May-10	464.01	473.35	-9.34	468.16	479.856	-11.696
26-May-10	475	470.802	4.198	482.07	477.018	5.052
27-May-10	481.05	472.252	8.798	484.86	476.976	7.884
28-May-10	483	475.972	7.028	492.74	481.712	11.028
31-May-10	485.63	477.738	7.892	485.63	482.692	2.938
1-Jun-10	480.12	480.96	-0.84	480.43	485.146	-4.716
2-Jun-10	481.46	482.252	-0.792	486.68	486.068	0.612
3-Jun-10	494.7	484.982	9.718	495.11	488.118	6.992
4-Jun-10	496.7	487.722	8.978	499.72	489.514	10.206
7-Jun-10	483.15	487.226	-4.076	499.06	492.2	6.86
8-Jun-10	477.54	486.71	-9.17	487.85	493.684	-5.834
9-Jun-10	472	484.818	-12.818	487.22	493.792	-6.572
10-Jun-10	475.84	481.046	-5.206	480.37	490.844	-10.474
11-Jun-10	481.62	478.03	3.59	482.5	487.4	-4.9
14-Jun-10	483.19	478.038	5.152	494.48	486.484	7.996
15-Jun-10	482.18	478.966	3.214	483.08	485.53	-2.45
16-Jun-10	496.11	483.788	12.322	496.17	487.32	8.85
17-Jun-10	496.69	487.958	8.732	503.45	491.936	11.514
18-Jun-10	498.13	491.26	6.87	502.51	495.938	6.572
21-Jun-10	484.89	491.6	-6.71	499.9	497.022	2.878
22-Jun-10	485.73	492.31	-6.58	489.9	498.386	-8.486

23-Jun-10	478.16	488.72	-10.56	486.89	496.53	-9.64
24-Jun-10	473.26	484.034	-10.774	479.66	491.772	-12.112
25-Jun-10	470.56	478.52	-7.96	477.06	486.682	-9.622
28-Jun-10	469.01	475.344	-6.334	472.59	481.22	-8.63
29-Jun-10	451.12	468.422	-17.302	463.44	475.928	-12.488
30-Jun-10	444.72	461.734	-17.014	454.96	469.542	-14.582
1-Jul-10	433.63	453.808	-20.178	445.29	462.668	-17.378
2-Jul-10	436	446.896	-10.896	441.62	455.58	-13.96
5-Jul-10	436.55	440.404	-3.854	436.55	448.372	-11.822
6-Jul-10	433.63	436.906	-3.276	444	444.484	-0.484
7-Jul-10	435.38	435.038	0.342	438.31	441.154	-2.844
8-Jul-10	449.66	438.244	11.416	453.55	442.806	10.744
9-Jul-10	462.78	443.6	19.18	471.96	448.874	23.086
12-Jul-10	471.08	450.506	20.574	472.37	456.038	16.332
13-Jul-10	480.28	459.836	20.444	482.25	463.688	18.562
14-Jul-10	486.46	470.052	16.408	489.88	474.002	15.878
15-Jul-10	482.68	476.656	6.024	491.73	481.638	10.092
16-Jul-10	459.52	476.004	-16.484	469.12	481.07	-11.95
19-Jul-10	457.52	473.292	-15.772	461.01	478.798	-17.788
20-Jul-10	460.6	469.356	-8.756	461.03	474.554	-13.524
21-Jul-10	475.43	467.15	8.28	484	473.378	10.622
22-Jul-10	482.48	467.11	15.37	483.23	471.678	11.552
23-Jul-10	480.01	471.208	8.802	480.77	474.008	6.762
26-Jul-10	484.88	476.68	8.2	489.09	479.624	9.466
27-Jul-10	490.17	482.594	7.576	490.58	485.534	5.046
28-Jul-10	482.67	484.042	-1.372	494.94	487.722	7.218
29-Jul-10	479.33	483.412	-4.082	485.95	488.266	-2.316
30-Jul-10	479.14	483.238	-4.098	479.65	488.042	-8.392
2-Aug-10	486.94	483.65	3.29	488.99	488.022	0.968
3-Aug-10	486.76	482.968	3.792	490.5	488.006	2.494
4-Aug-10	491.05	484.644	6.406	492.18	487.454	4.726
5-Aug-10	503.56	489.49	14.07	505.89	491.442	14.448
6-Aug-10	496.05	492.872	3.178	505.4	496.592	8.808
9-Aug-10	501.36	495.756	5.604	502.25	499.244	3.006
10-Aug-10	498.57	498.118	0.452	502.35	501.614	0.736
11-Aug-10	491.5	498.208	-6.708	497.73	502.724	-4.994
12-Aug-10	483.94	494.284	-10.344	483.94	498.334	-14.394
13-Aug-10	486.01	492.276	-6.266	489	495.054	-6.054
16-Aug-10	483.68	488.74	-5.06	483.68	491.34	-7.66
17-Aug-10	486.1	486.246	-0.146	488.53	488.576	-0.046
18-Aug-10	481.55	484.256	-2.706	490.44	487.118	3.322
19-Aug-10	467.25	480.918	-13.668	481.01	486.532	-5.522
20-Aug-10	462.18	476.152	-13.972	467.97	482.326	-14.356
23-Aug-10	457.73	470.962	-13.232	461.5	477.89	-16.39

24-Aug-10	450.92	463.926	-13.006	457.7	471.724	-14.024
25-Aug-10	450	457.616	-7.616	450	463.636	-13.636
26-Aug-10	450.44	454.254	-3.814	456.06	458.646	-2.586
27-Aug-10	448.31	451.48	-3.17	452.56	455.564	-3.004
30-Aug-10	452.42	450.418	2.002	459.15	455.094	4.056
31-Aug-10	448	449.834	-1.834	450.11	453.576	-3.466
1-Sep-10	452.5	450.334	2.166	454.98	454.572	0.408
2-Sep-10	460.31	452.308	8.002	462.84	455.928	6.912
	Max Difference -->	20.878		Max Difference - ->	23.238	
	Min Difference -->	-43.04		Min Difference - ->	-25.81	

Moving Average Calculations for interval of 10

Date	Closing Prices	Moving Average, Interval = 10	Difference of Prices (Actual-Mov.Avg)	Opening Prices	Moving Average of Opening Prices(Interval =5)	Difference of Prices (Actual-Mov. Avg)
1-Sep-09	455.76	#N/A		459.68	#N/A	
2-Sep-09	453.01	#N/A		455.82	#N/A	
3-Sep-09	457.52	#N/A		455.82	#N/A	
4-Sep-09	461.3	#N/A		457.57	#N/A	
8-Sep-09	458.62	#N/A		464.29	#N/A	
9-Sep-09	463.97	#N/A		459.06	#N/A	
10-Sep-09	470.94	#N/A		466.65	#N/A	
11-Sep-09	472.14	#N/A		470.4	#N/A	
14-Sep-09	475.12	#N/A		470.51	#N/A	
15-Sep-09	477.54	464.592	12.948	475.08	463.488	11.592
16-Sep-09	488.29	467.845	468.286	479.8	465.5	14.3
17-Sep-09	491.72	471.716	20.004	490.57	468.975	21.595
18-Sep-09	491.46	475.11	-5.31	496.77	473.07	496.77
21-Sep-09	497	478.68	18.32	487.74	476.087	11.653
22-Sep-09	499.06	482.724	16.336	500.92	479.75	21.17
23-Sep-09	498.81	486.208	12.602	500.78	483.922	16.858
24-Sep-09	496.77	488.791	495.115	500.47	487.304	13.166
25-Sep-09	492.48	490.825	1.655	494.29	489.693	494.29
28-Sep-09	498.53	493.166	3.69	494.84	492.126	2.714
29-Sep-09	498.53	495.265	3.265	499.53	494.571	4.959
30-Sep-09	495.85	496.021	-0.171	500	496.591	3.409
1-Oct-09	487.2	495.569	-8.369	493	496.834	-3.834
2-Oct-09	484.58	494.881	490.093	483.74	495.531	483.74
5-Oct-09	488.52	494.033	-5.513	487.65	495.522	-7.872
6-Oct-09	498.74	494.001	7.04	491.7	494.6	-2.9
7-Oct-09	517.54	495.874	21.666	499	494.422	4.578
8-Oct-09	514.18	497.615	16.565	519.57	496.332	23.238
9-Oct-09	516.25	499.992	16.258	516.65	498.568	516.65
12-Oct-09	524.04	502.543	503.231	523.42	501.426	21.994
13-Oct-09	526.11	505.301	20.809	524.39	503.912	20.478
14-Oct-09	535.32	509.248	2.86	532.46	507.158	25.302
15-Oct-09	529.91	513.519	16.391	533.75	511.233	22.517
16-Oct-09	549.85	520.046	29.804	547.33	517.592	547.33
19-Oct-09	552.09	526.403	25.687	552.69	524.096	28.594
20-Oct-09	551.72	531.701	535.677	551.64	530.09	21.55
21-Oct-09	551.1	535.057	16.043	549.91	535.181	14.729
22-Oct-09	554.09	539.048	4.09	550	538.224	11.776
23-Oct-09	553.69	542.792	10.898	555.25	542.084	555.25
26-Oct-09	554.21	545.809	8.401	555.75	545.317	10.433

27-Oct-09	548.29	548.027	0.263	550.97	547.975	2.995
28-Oct-09	540.3	548.525	539.889	547.87	549.516	-1.646
29-Oct-09	551.05	550.639	0.411	543.01	550.442	-7.432
30-Oct-09	536.12	549.266	-13.88	550	550.709	550
2-Nov-09	533.99	547.456	-13.466	537.08	549.148	-12.068
3-Nov-09	537.29	546.013	-8.723	530.01	546.985	-16.975
4-Nov-09	540.33	544.936	-4.606	540.8	546.074	-5.274
5-Nov-09	548.65	544.392	541.683	543.49	545.423	-1.933
6-Nov-09	551.1	544.133	6.967	547.72	544.67	547.72
9-Nov-09	562.51	544.963	7.06	555.45	544.64	10.81
10-Nov-09	566.76	546.81	19.95	562.73	545.816	16.914
11-Nov-09	570.56	549.836	20.724	570.48	548.077	22.403
12-Nov-09	567.85	551.516	16.334	569.56	550.732	18.828
13-Nov-09	572.05	555.109	555.108	569.29	552.661	569.29
16-Nov-09	576.28	559.338	16.942	575	556.453	18.547
17-Nov-09	577.49	563.358	2.62	574.87	560.939	13.931
18-Nov-09	576.65	566.99	9.66	576.65	564.524	12.126
19-Nov-09	572.99	569.424	3.566	573.77	567.552	6.218
20-Nov-09	569.96	571.31	-1.35	569.5	569.73	569.5
23-Nov-09	582.35	573.294	574.187	576.49	571.834	4.656
24-Nov-09	583.09	574.927	8.163	582.5	573.811	8.689
25-Nov-09	585.74	576.445	-0.67	586.41	575.404	11.006
26-Nov-09	585.74	578.234	7.506	585.74	577.022	8.718
27-Nov-09	579.76	579.005	0.755	572	577.293	572
30-Nov-09	583	579.677	3.323	580.63	577.856	2.774
1-Dec-09	589.87	580.915	584.361	588.13	579.182	8.948
2-Dec-09	587.51	582.001	5.509	590.98	580.615	10.365
3-Dec-09	585.74	583.276	-3.3	589.04	582.142	6.898
4-Dec-09	585.01	584.781	0.229	593.02	584.494	593.02
7-Dec-09	586.25	585.171	1.079	584.23	585.268	-1.038
8-Dec-09	587.05	585.567	1.483	583.5	585.368	-1.868
9-Dec-09	589.02	585.895	583.991	587.5	585.477	2.023
10-Dec-09	591.5	586.471	5.029	590.44	585.947	4.493
11-Dec-09	590.51	587.546	-4.17	594.68	588.215	594.68
14-Dec-09	595.73	588.819	6.911	595.24	589.676	5.564
15-Dec-09	593.14	589.146	3.994	593.3	590.193	3.107
16-Dec-09	597.76	590.171	7.589	598.91	590.986	7.924
17-Dec-09	593.94	590.991	589.652	596.44	591.726	4.714
18-Dec-09	596.42	592.132	4.288	596.03	592.027	596.03
21-Dec-09	598.68	593.375	0.73	597.95	593.399	4.551
22-Dec-09	601.12	594.782	6.338	601.34	595.183	6.157
23-Dec-09	611.68	597.048	14.632	603.5	596.783	6.717
24-Dec-09	618.48	599.746	18.734	612.93	599.032	13.898
25-Dec-09	618.48	602.543	600.867	618.48	601.412	618.48

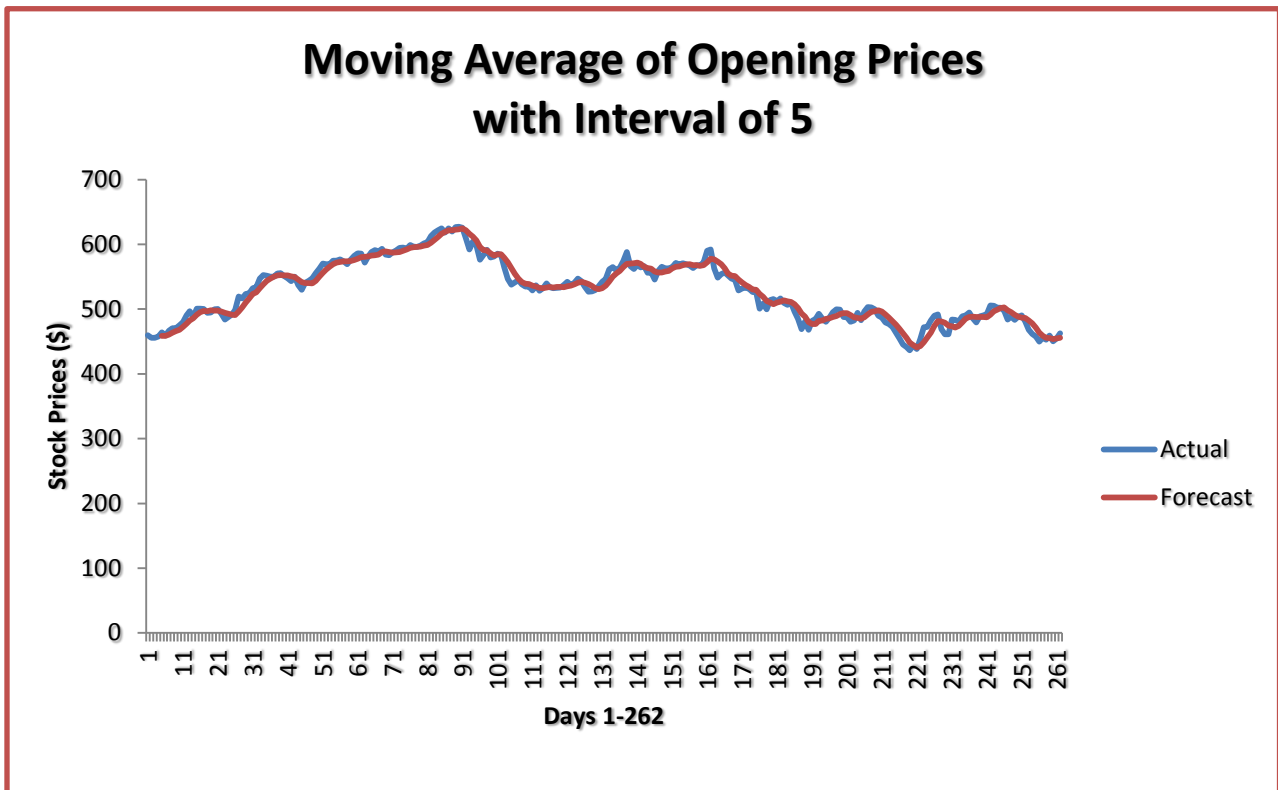
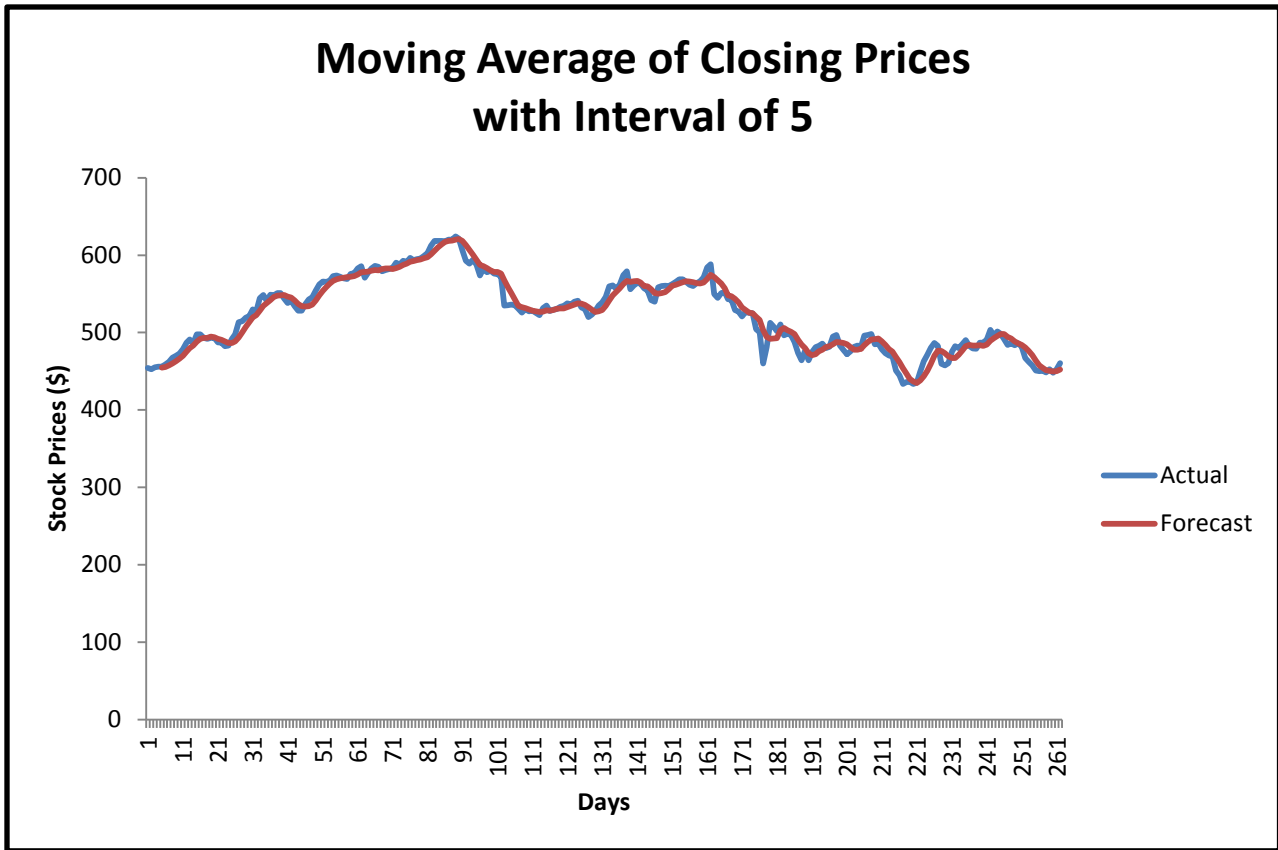
28-Dec-09	622.87	605.257	17.613	621.66	604.054	17.606
29-Dec-09	619.4	607.883	-5.34	624.74	607.198	17.542
30-Dec-09	622.73	610.38	12.35	618.5	609.157	9.343
31-Dec-09	619.98	612.984	6.996	624.75	611.988	12.762
1-Jan-10	619.98	615.34	4.64	619.98	614.383	619.98
4-Jan-10	626.75	618.147	623.194	626.95	617.283	9.667
5-Jan-10	623.99	620.434	3.556	627.18	619.867	7.313
6-Jan-10	608.26	620.092	-17.6	625.86	622.103	3.757
7-Jan-10	594.1	617.654	-23.554	609.4	621.75	-12.35
8-Jan-10	602.02	616.008	-13.988	592	619.102	592
11-Jan-10	601.11	613.832	-12.722	604.46	617.382	-12.922
12-Jan-10	590.48	610.94	610.766	598.08	614.716	-16.636
13-Jan-10	587.09	607.376	-20.286	576.49	610.515	-34.025
14-Jan-10	589.85	604.363	5.95	583.9	606.43	-22.53
15-Jan-10	580	600.365	-20.365	591.59	603.591	591.59
18-Jan-10	580	595.69	-15.69	580	598.896	-18.896
19-Jan-10	587.62	592.053	-4.433	581.49	594.327	-12.837
20-Jan-10	580.41	589.268	585.586	585.98	590.339	-4.359
21-Jan-10	582.98	588.156	-5.176	583.44	587.743	-4.303
22-Jan-10	550.01	582.955	-14.49	564.5	584.993	564.5
25-Jan-10	540	576.844	-36.844	546.59	579.206	-32.616
26-Jan-10	542.42	572.038	-29.618	537.97	573.195	-35.225
27-Jan-10	542.1	567.539	-25.439	541.27	569.673	-28.403
28-Jan-10	534.29	561.983	561.327	544.49	565.732	-21.242
29-Jan-10	529.94	556.977	-27.037	538.49	560.422	538.49
1-Feb-10	533.02	552.279	-1.58	534.6	555.882	-21.282
2-Feb-10	531.12	546.629	-15.509	534.96	551.229	-16.269
3-Feb-10	540.82	542.67	-1.85	528.67	545.498	-16.828
4-Feb-10	526.78	537.05	-10.27	537	540.854	-3.854
5-Feb-10	531.29	535.178	532.345	528.4	537.244	528.4
8-Feb-10	533.47	534.525	-1.055	532.5	535.835	-3.335
9-Feb-10	536.44	533.927	-3.1	539.54	535.992	3.548
10-Feb-10	534.44	533.161	1.279	534.07	535.272	-1.202
11-Feb-10	536.4	533.372	3.028	532.25	534.048	-1.798
12-Feb-10	533.12	533.69	-0.57	532.97	533.496	532.97
15-Feb-10	533.12	533.7	526.538	533.12	533.348	-0.228
16-Feb-10	541.3	534.718	6.582	537.14	533.566	3.574
17-Feb-10	538.21	534.457	-3.79	542	534.899	7.101
18-Feb-10	543.22	536.101	7.119	537.54	534.953	2.587
19-Feb-10	540.76	537.048	3.712	541	536.213	541
22-Feb-10	542.8	537.981	4.819	547.35	537.698	9.652
23-Feb-10	535.07	537.844	541.147	543	538.044	4.956
24-Feb-10	531.47	537.547	-6.077	534.39	538.076	-3.686
25-Feb-10	526.43	536.55	-0.69	527.12	537.563	-10.443

26-Feb-10	526.8	535.918	-9.118	527.42	537.008	527.42
1-Mar-10	532.69	535.875	-3.185	529.2	536.616	-7.416
2-Mar-10	541.06	535.851	5.209	535.48	536.45	-0.97
3-Mar-10	545.32	536.562	528.429	542.36	536.486	5.874
4-Mar-10	554.59	537.699	16.891	546.5	537.382	9.118
5-Mar-10	564.21	540.044	2.86	561.35	539.417	561.35
8-Mar-10	562.48	542.012	20.468	564.78	541.16	23.62
9-Mar-10	560.19	544.524	15.666	559.85	542.845	17.005
10-Mar-10	576.45	549.022	27.428	563.76	545.782	17.978
11-Mar-10	581.14	554.493	561.367	574.26	550.496	23.764
12-Mar-10	579.54	559.767	19.773	588.14	556.568	588.14
15-Mar-10	563.18	562.816	-3.5	566.68	560.316	6.364
16-Mar-10	565.2	565.23	-0.03	561.83	562.951	-1.121
17-Mar-10	565.56	567.254	-1.694	568.3	565.545	2.755
18-Mar-10	566.4	568.435	-2.035	564.72	567.367	-2.647
19-Mar-10	560	568.014	570.016	566.23	567.855	566.23
22-Mar-10	557.5	567.516	-10.016	556.11	566.988	-10.878
23-Mar-10	549	566.397	-8.04	557.04	566.707	-9.667
24-Mar-10	557.33	564.485	-7.155	545.51	564.882	-19.372
25-Mar-10	562.88	562.659	0.221	559.02	563.358	-4.338
26-Mar-10	562.69	560.974	1.716	565.27	561.071	565.27
29-Mar-10	562.45	560.901	556.792	563	560.703	2.297
30-Mar-10	566.71	561.052	5.658	562.83	560.803	2.027
31-Mar-10	567.12	561.208	2.07	565.05	560.478	4.572
1-Apr-10	568.8	561.448	7.352	571.35	561.141	10.209
2-Apr-10	568.8	562.328	6.472	568.8	561.398	568.8
5-Apr-10	571.01	563.679	7.331	570.9	562.877	8.023
6-Apr-10	568.22	565.601	570.902	569.46	564.119	5.341
7-Apr-10	563.54	566.222	-2.682	567.3	566.298	1.002
8-Apr-10	567.49	566.683	4.17	563.32	566.728	-3.408
9-Apr-10	566.22	567.036	-0.816	568	567.001	568
12-Apr-10	572.73	568.064	4.666	567.35	567.436	-0.086
13-Apr-10	586.77	570.07	16.7	572.53	568.406	4.124
14-Apr-10	589	572.258	568.608	590.06	570.907	19.153
15-Apr-10	595.3	574.908	20.392	592.17	572.989	19.181
16-Apr-10	550.14	573.042	-12.86	563	572.409	563
19-Apr-10	550.1	570.951	-20.851	548.75	570.194	-21.444
20-Apr-10	555.04	569.633	-14.593	554.17	568.665	-14.495
21-Apr-10	554.3	568.709	-14.409	556.46	567.581	-11.121
22-Apr-10	547.06	566.666	566.613	552	566.449	-14.449
23-Apr-10	544.99	564.543	-19.553	547.25	564.374	547.25
26-Apr-10	531.64	560.434	-13.33	544.97	562.136	-17.166
27-Apr-10	529.06	554.663	-25.603	528.94	557.777	-28.837
28-Apr-10	529.19	548.682	-19.492	532.1	551.981	-19.881

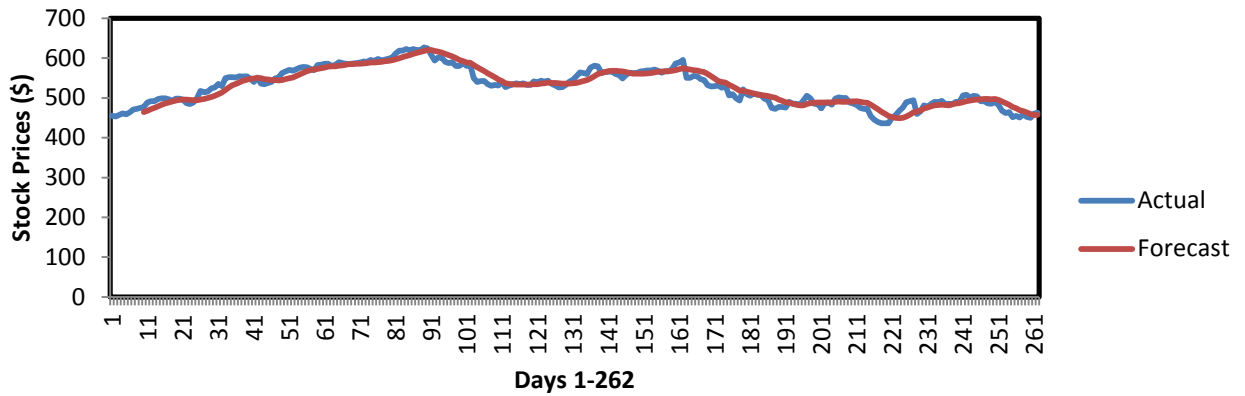
29-Apr-10	532	542.352	-10.352	533.37	546.101	-12.731
30-Apr-10	525.7	539.908	533.058	531.13	542.914	531.13
3-May-10	530.6	537.958	-7.358	526.5	540.689	-14.189
4-May-10	506.37	533.091	-20.15	526.52	537.924	-11.404
5-May-10	509.76	528.637	-18.877	500.98	532.376	-31.396
6-May-10	498.67	523.798	-25.128	508.75	528.051	-19.301
7-May-10	493.14	518.613	-25.473	499.97	523.323	499.97
10-May-10	521.65	517.614	528.213	513.97	520.223	-6.253
11-May-10	509.05	515.613	-6.563	515.67	518.896	-3.226
12-May-10	505.39	513.233	-6.65	512.04	516.89	-4.85
13-May-10	510.88	511.121	-0.241	516.5	515.203	1.297
14-May-10	507.53	509.304	-1.774	509.77	513.067	509.77
17-May-10	507.97	507.041	0.929	506.78	511.095	-4.315
18-May-10	498.37	506.241	508.648	510	509.443	0.557
19-May-10	494.43	504.708	-10.278	496.26	508.971	-12.711
20-May-10	475.01	502.342	-10.06	485.07	506.603	-21.533
21-May-10	472.05	500.233	-28.183	469.06	503.512	469.06
24-May-10	477.16	495.784	-18.624	480.73	500.188	-19.458
25-May-10	477.07	492.586	-15.516	468.16	495.437	-27.277
26-May-10	475.47	489.594	472.562	482.07	492.44	-10.37
27-May-10	490.46	487.552	2.908	484.86	489.276	-4.416
28-May-10	485.63	485.362	-7.11	492.74	487.573	492.74
31-May-10	485.63	483.128	2.502	485.63	485.458	0.172
1-Jun-10	482.37	481.528	0.842	480.43	482.501	-2.071
2-Jun-10	493.37	481.422	11.948	486.68	481.543	5.137
3-Jun-10	505.6	484.481	494.028	495.11	482.547	12.563
4-Jun-10	498.72	487.148	11.572	499.72	485.613	499.72
7-Jun-10	485.52	487.984	-13.54	499.06	487.446	11.614
8-Jun-10	484.78	488.755	-3.975	487.85	489.415	-1.565
9-Jun-10	474.02	488.61	-14.59	487.22	489.93	-2.71
10-Jun-10	487.01	488.265	-1.255	480.37	489.481	-9.111
11-Jun-10	488.5	488.552	493.618	482.5	488.457	482.5
14-Jun-10	483.19	488.308	-5.118	494.48	489.342	5.138
15-Jun-10	497.99	489.87	14.91	483.08	489.607	-6.527
16-Jun-10	501.27	490.66	10.61	496.17	490.556	5.614
17-Jun-10	500.08	490.108	9.972	503.45	491.39	12.06
18-Jun-10	500.03	490.239	9.791	502.51	491.669	502.51
21-Jun-10	488.56	490.543	493	499.9	491.753	8.147
22-Jun-10	486.25	490.69	-4.44	489.9	491.958	-2.058
23-Jun-10	482.05	491.493	-4.84	486.89	491.925	-5.035
24-Jun-10	475.1	490.302	-15.202	479.66	491.854	-12.194
25-Jun-10	472.68	488.72	-16.04	477.06	491.31	477.06
28-Jun-10	472.08	487.609	-15.529	472.59	489.121	-16.531
29-Jun-10	454.26	483.236	486.914	463.44	487.157	-23.717

30-Jun-10	444.95	477.604	-32.654	454.96	483.036	-28.076
1-Jul-10	439.49	471.545	-5.8	445.29	477.22	-31.93
2-Jul-10	436.55	465.197	-28.647	441.62	471.131	441.62
5-Jul-10	436.55	459.996	-23.446	436.55	464.796	-28.246
6-Jul-10	436.07	454.978	-18.908	444	460.206	-16.206
7-Jul-10	450.2	451.793	443.579	438.31	455.348	-17.038
8-Jul-10	456.56	449.939	6.621	453.55	452.737	0.813
9-Jul-10	467.49	449.42	-4.47	471.96	452.227	471.96
12-Jul-10	475.83	449.795	26.035	472.37	452.205	20.165
13-Jul-10	489.2	453.289	35.911	482.25	454.086	28.164
14-Jul-10	491.34	457.928	33.412	489.88	457.578	32.302
15-Jul-10	494.02	463.381	500.106	491.73	462.222	29.508
16-Jul-10	459.6	465.686	-6.086	469.12	464.972	469.12
19-Jul-10	466.18	468.649	5.17	461.01	467.418	-6.408
20-Jul-10	481.59	473.201	8.389	461.03	469.121	-8.091
21-Jul-10	477.5	475.931	1.569	484	473.69	10.31
22-Jul-10	484.81	478.756	6.054	483.23	476.658	6.572
23-Jul-10	490.06	481.013	483.417	480.77	477.539	480.77
26-Jul-10	488.97	482.327	6.643	489.09	479.211	9.879
27-Jul-10	492.63	482.67	2.05	490.58	480.044	10.536
28-Jul-10	484.35	481.971	2.379	494.94	480.55	14.39
29-Jul-10	484.99	481.068	3.922	485.95	479.972	5.978
30-Jul-10	484.85	483.593	1.257	479.65	481.025	479.65
2-Aug-10	490.41	486.016	487.42	488.99	483.823	5.167
3-Aug-10	489.83	486.84	2.99	490.5	486.77	3.73
4-Aug-10	506.32	489.722	14.14	492.18	487.588	4.592
5-Aug-10	508.1	492.051	16.049	505.89	489.854	16.036
6-Aug-10	500.22	493.067	7.153	505.4	492.317	505.4
9-Aug-10	505.35	494.705	10.645	502.25	493.633	8.617
10-Aug-10	503.71	495.813	508.522	502.35	494.81	7.54
11-Aug-10	491.74	496.552	-4.812	497.73	495.089	2.641
12-Aug-10	492.01	497.254	8.07	483.94	494.888	-10.948
13-Aug-10	486.35	497.404	-11.054	489	495.823	489
16-Aug-10	485.59	496.922	-11.332	483.68	495.292	-11.612
17-Aug-10	490.52	496.991	-6.471	488.53	495.095	-6.565
18-Aug-10	482.15	494.574	504.741	490.44	494.921	-4.481
19-Aug-10	467.97	490.561	-22.591	481.01	492.433	-11.423
20-Aug-10	462.02	486.741	-5.95	467.97	488.69	467.97
23-Aug-10	464.07	482.613	-18.543	461.5	484.615	-23.115
24-Aug-10	451.39	477.381	-25.991	457.7	480.15	-22.45
25-Aug-10	454.62	473.669	-19.049	450	475.377	-25.377
26-Aug-10	450.98	469.566	458.964	456.06	472.589	-16.529
27-Aug-10	458.83	466.814	-7.984	452.56	468.945	452.56
30-Aug-10	452.69	463.524	-6.46	459.15	466.492	-7.342

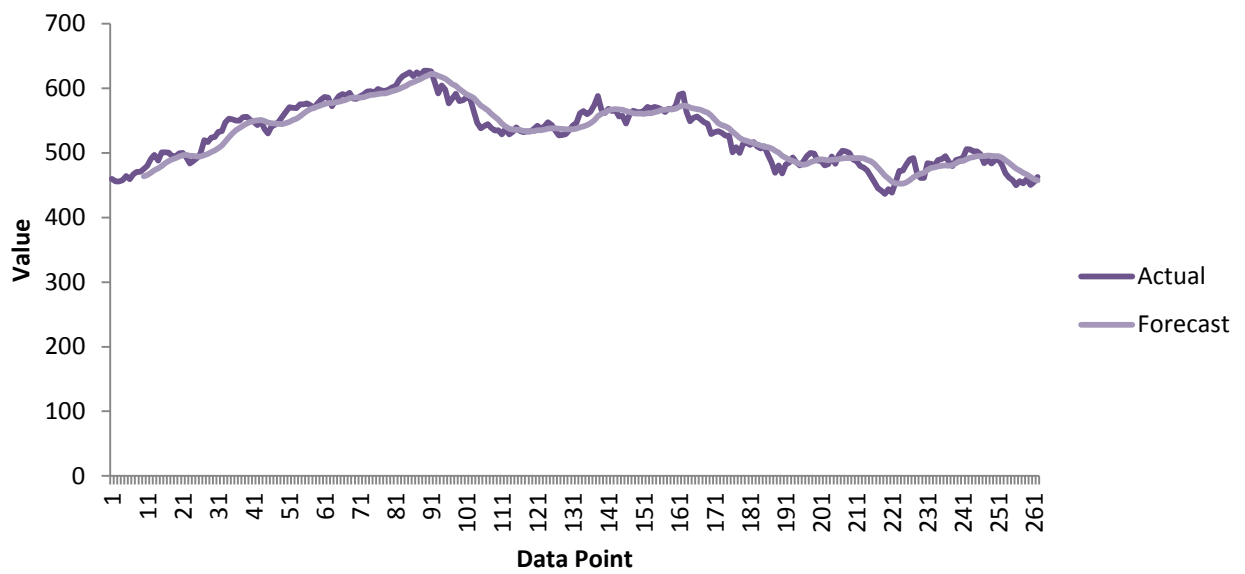
31-Aug-10	450.02	459.474	-9.454	450.11	462.65	-12.54
1-Sep-10	460.34	457.293	3.047	454.98	459.104	-4.124
2-Sep-10	463.18	456.814	6.366	462.84	457.287	5.553
	Max Difference	623.194		Max Difference	619.98	
	Min Difference	-36.844		Min Difference	619.98	



Moving Average of Closing Prices with Interval of 10



Moving Average of Opening Prices with Interval of 10



APPENDIX B

For i=5, Closing Prices

Difference of Closing Prices (Actual-Mov.Avg) For i=5	P State Automated Count	Automated Transition Labeling
1.114	P3	
3.198	P3	P_33
4.516	P3	P_33
7.62	P3	P_33
7.186	P3	P_33
6.472	P3	P_33
8.306	P3	P_33
11.946	P4	P_34
11.306	P4	P_44
3.062	P3	P_43
9.632	P4	P_34
5.686	P3	P_43
-0.194	P2	P_32
-1.348	P2	P_22
-1.464	P2	P_22
-0.794	P2	P_22
-4.47	P2	P_22
-3.51	P2	P_22
-6.03	P2	P_22
-3.298	P2	P_22
5.324	P3	P_23
9.32	P4	P_34
19.582	P4	P_44
14.362	P4	P_44
11.986	P4	P_44
8.11	P3	P_43
10.292	P4	P_34
4.776	P3	P_43
16.03	P4	P_34
14.348	P4	P_44
2.454	P3	P_43
6.954	P3	P_33

1.808	P3	P_33
3.674	P3	P_33
2.932	P3	P_33
-4.49	P2	P_32
-8.25	P2	P_22
-4.1	P2	P_22
-7.468	P2	P_22
-8.938	P2	P_22
-5.706	P2	P_22
2.78	P3	P_23
8.688	P3	P_33
9.276	P4	P_34
12.808	P4	P_44
13.838	P4	P_44
11.81	P4	P_44
6.882	P3	P_43
3.77	P3	P_33
6.23	P3	P_33
4.826	P3	P_33
1.934	P3	P_33
-1.036	P2	P_32
-2.194	P2	P_22
3.65	P3	P_23
3.766	P3	P_33
7.792	P3	P_33
7.694	P3	P_33
-7.39	P2	P_32
-1.5	P2	P_22
3.098	P3	P_23
5.612	P3	P_33
4.54	P3	P_33
-2.922	P2	P_32
-1.88	P2	P_22
-0.68	P2	P_22
1.428	P3	P_23
7.176	P3	P_33
2.786	P3	P_33
5.344	P3	P_33
1.926	P3	P_33
4.964	P3	P_33
1.414	P3	P_33
1.2	P3	P_33
1.258	P3	P_33
2.866	P3	P_33

5.624	P3	P_33
11.342	P4	P_34
12.856	P4	P_44
8.294	P3	P_43
4.216	P3	P_33
0.904	P3	P_33
1.332	P3	P_33
1.032	P3	P_33
4.14	P3	P_33
0.79	P2	P_32
-12.06	P1	P_21
-20.304	P1	P_11
-17.67	P1	P_11
-6.7	P2	P_12
-6.032	P2	P_22
-13.64	P1	P_21
-2.762	P2	P_12
-5.318	P2	P_22
-0.55	P2	P_22
-1.918	P2	P_22
-3.196	P2	P_22
-4.124	P2	P_22
-32.878	P1	P_21
-23.33	P1	P_11
-14.55	P1	P_11
-7.534	P2	P_12
-3.914	P2	P_22
-7.054	P2	P_22
-1.322	P2	P_22
-2.276	P2	P_22
-0.24	P2	P_22
-1.902	P2	P_22
-4.372	P2	P_22
4.452	P3	P_23
6.5	P3	P_33
-0.772	P2	P_32
0.25	P2	P_22
-0.358	P2	P_22
1.944	P3	P_23
3.278	P3	P_33
4.604	P3	P_33
1.806	P3	P_33
3.526	P3	P_33
3.25	P3	P_33

-5.058	P2	P_32
-5.418	P2	P_22
-12.7	P1	P_21
-5.976	P2	P_12
0.936	P3	P_23
7.662	P3	P_33
10.154	P4	P_34
11.864	P4	P_44
18.28	P4	P_44
12.736	P4	P_44
3.928	P3	P_43
5.046	P3	P_33
11.436	P4	P_34
12.544	P4	P_44
-9.614	P2	P_42
-5.706	P2	P_22
-2.624	P2	P_22
-1.666	P2	P_22
-2.97	P2	P_22
-5.626	P2	P_22
-14.154	P1	P_21
-11.544	P1	P_11
8.276	P3	P_13
9.088	P4	P_34
8.38	P3	P_43
4.434	P3	P_33
2.342	P3	P_33
3.704	P3	P_33
5.198	P3	P_33
3.712	P3	P_33
-0.912	P2	P_32
-4.262	P2	P_22
-4.972	P2	P_22
-0.062	P2	P_22
2.714	P3	P_23
6.478	P3	P_33
14.928	P4	P_34
13.56	P4	P_44
-22.226	P1	P_41
-22.612	P1	P_11
-12.538	P1	P_11
-5.068	P2	P_12
-4.89	P2	P_22
-4.498	P2	P_22

-14.4	P1	P_21
-11.606	P1	P_11
-11.59	P1	P_11
-2.614	P2	P_12
-0.478	P2	P_22
-0.012	P2	P_22
-16.276	P1	P_21
-15.904	P1	P_11
-43.04	P1	P_11
-12.888	P1	P_11
20.878	P4	P_14
15.696	P4	P_44
9.17	P4	P_44
7.466	P3	P_43
-9.638	P2	P_32
-4.688	P2	P_22
-3.738	P2	P_22
-10.216	P2	P_22
-16.842	P1	P_21
-19.872	P1	P_11
-3.162	P2	P_12
-9.34	P2	P_22
4.198	P3	P_23
8.798	P3	P_33
7.028	P3	P_33
7.892	P3	P_33
-0.84	P2	P_32
-0.792	P2	P_22
9.718	P4	P_24
8.978	P4	P_44
-4.076	P2	P_42
-9.17	P2	P_22
-12.818	P1	P_21
-5.206	P2	P_12
3.59	P3	P_23
5.152	P3	P_33
3.214	P3	P_33
12.322	P4	P_34
8.732	P3	P_43
6.87	P3	P_33
-6.71	P2	P_32
-6.58	P2	P_22
-10.56	P2	P_22
-10.774	P2	P_22

-7.96	P2	P_22
-6.334	P2	P_22
-17.302	P1	P_21
-17.014	P1	P_11
-20.178	P1	P_11
-10.896	P2	P_12
-3.854	P2	P_22
-3.276	P2	P_22
0.342	P2	P_22
11.416	P4	P_24
19.18	P4	P_44
20.574	P4	P_44
20.444	P4	P_44
16.408	P4	P_44
6.024	P3	P_43
-16.484	P1	P_31
-15.772	P1	P_11
-8.756	P2	P_12
8.28	P3	P_23
15.37	P4	P_34
8.802	P3	P_43
8.2	P3	P_33
7.576	P3	P_33
-1.372	P2	P_32
-4.082	P2	P_22
-4.098	P2	P_22
3.29	P3	P_23
3.792	P3	P_33
6.406	P3	P_33
14.07	P4	P_34
3.178	P3	P_43
5.604	P3	P_33
0.452	P2	P_32
-6.708	P2	P_22
-10.344	P2	P_22
-6.266	P2	P_22
-5.06	P2	P_22
-0.146	P2	P_22
-2.706	P2	P_22
-13.668	P1	P_21
-13.972	P1	P_11
-13.232	P1	P_11
-13.006	P1	P_11
-7.616	P2	P_12

-3.814	P2	P_22
-3.17	P2	P_22
2.002	P3	P_23
-1.834	P2	P_32
2.166	P3	P_23
8.002	P3	P_33

Interval grouping and count for i=5, Closing Prices

Lowest point:	-43.04
Highest point:	20.878
Interval	[-43.04, 20.878]
Interval 1:	[-43.04, -11.081]
Interval 2:	(-11.081, 0.903625]
Interval 3:	(0.903625, 8.893375]
Interval 4:	(8.893375, 20.878]
32.000000	COUNTIF for I1
96.000000	COUNTIF for I2
92.000000	COUNTIF for I3
38.000000	COUNTIF for I4
258.000000	Total Rows of Data
COUNTIFS Total P_11	19
COUNTIFS Total P_12	11
COUNTIFS Total P_13	1
COUNTIFS Total P_14	1
COUNTIFS Total P_21	11
COUNTIFS Total P_22	68
COUNTIFS Total P_23	15
COUNTIFS Total P_24	2
COUNTIFS Total P_31	1
COUNTIFS Total P_32	15
COUNTIFS Total P_33	61
COUNTIFS Total P_34	14
COUNTIFS Total P_41	1
COUNTIFS Total P_42	2
COUNTIFS Total P_43	14
COUNTIFS Total P_44	21

Steady-State	
Π_1	0.248013205
Π_2	0.247218486
Π_3	0.244956596
Π_4	0.259811713

For i=5, Opening Prices

Difference of Opening Prices (Actual-Mov. Avg)	Automated State Count	Transition Labeling	Diff. Prices Sorted (least to greatest)
5.6540	P3		-25.8100
0.5480	P2	P_32	-25.7260
5.9720	P3	P_23	-24.5520
6.8060	P3	P_33	-24.3740
4.3280	P3	P_33	-24.2780
6.7400	P3	P_33	-22.7200
7.3120	P3	P_33	-19.5960
13.2980	P4	P_34	-17.7880
14.2240	P4	P_44	-17.3780
1.7480	P2	P_42	-16.9840
9.7600	P3	P_23	-16.5060
5.4240	P3	P_33	-16.3900
3.1340	P3	P_33	-15.4600
-2.5500	P2	P_32	-14.5820
-3.4200	P2	P_22	-14.5820
1.5480	P2	P_22	-14.3940
2.1740	P3	P_23	-14.3560
-3.3320	P2	P_32	-14.0240
-10.4820	P1	P_21	-14.0220
-5.1340	P1	P_11	-13.9600
0.4820	P2	P_12	-13.6360
7.9820	P3	P_23	-13.5240
23.2380	P4	P_34	-13.4840
13.7360	P4	P_44	-12.5740
13.3520	P4	P_44	-12.4880
7.7840	P3	P_43	-12.4740
9.1620	P3	P_33	-12.4120
7.6160	P3	P_33	-12.1120
15.0600	P4	P_34	-11.9500
14.5660	P4	P_44	-11.8220
8.0660	P3	P_43	-11.6960
2.8460	P3	P_33	-11.6020
-0.3140	P2	P_32	-11.5840
3.3520	P3	P_23	-11.4520
3.2400	P3	P_33	-10.4820
-1.4060	P2	P_32	-10.4740
-4.0980	P2	P_22	-10.0260
-7.5600	P1	P_21	-9.6400

0.4800	P2	P_12	-9.6220
-8.7060	P1	P_21	-9.1040
-11.5840	P1	P_11	-8.9520
0.6200	P2	P_12	-8.7060
3.2140	P3	P_23	-8.6300
7.9000	P3	P_33	-8.6280
11.9560	P4	P_34	-8.4860
12.6920	P4	P_44	-8.4360
14.5060	P4	P_44	-8.3920
8.3720	P3	P_43	-7.8800
3.7880	P3	P_33	-7.6600
5.5880	P3	P_33	-7.5720
3.0300	P3	P_33	-7.5600
3.5760	P3	P_33	-7.4940
-0.1460	P2	P_32	-7.3280
-4.4580	P1	P_21	-7.3200
2.2340	P3	P_13	-7.0860
6.7180	P3	P_33	-6.5720
8.6760	P3	P_33	-6.4500
5.6120	P3	P_33	-6.2660
-8.6280	P1	P_31	-6.0540
-0.8260	P2	P_12	-6.0120
5.5480	P3	P_23	-6.0000
7.4840	P3	P_33	-5.8340
4.8840	P3	P_33	-5.5220
4.6600	P3	P_33	-5.4400
-4.8500	P1	P_31	-5.3720
-4.6540	P1	P_11	-5.2140
0.0420	P2	P_12	-5.1340
2.7020	P3	P_23	-4.9940
6.6100	P3	P_33	-4.9000
4.9680	P3	P_33	-4.8500
1.0680	P2	P_32	-4.7640
4.3960	P3	P_23	-4.7160
0.7260	P2	P_32	-4.6540
0.0460	P2	P_22	-4.6360
1.4240	P2	P_22	-4.4760
3.2060	P3	P_23	-4.4580
4.4480	P3	P_33	-4.3260
10.5800	P3	P_33	-4.0980
11.6400	P4	P_34	-3.9560
10.0780	P3	P_43	-3.9080
8.4780	P3	P_33	-3.8580
-0.7620	P2	P_32	-3.8200

3.1240	P3	P_23	-3.8020
-1.9460	P2	P_32	-3.5420
3.9660	P3	P_23	-3.4660
3.7080	P3	P_33	-3.4200
0.9160	P2	P_32	-3.4040
-12.4740	P1	P_21	-3.3320
-24.2780	P1	P_11	-3.2720
-7.3200	P1	P_11	-3.0260
-7.8800	P1	P_11	-3.0040
-19.5960	P1	P_11	-2.9720
-7.0860	P1	P_11	-2.8760
0.6860	P2	P_12	-2.8440
-6.0120	P1	P_21	-2.5860
-1.2040	P2	P_12	-2.5500
1.3880	P2	P_22	-2.4740
-1.0600	P2	P_22	-2.4500
-14.5820	P1	P_21	-2.3160
-25.8100	P1	P_11	-2.2620
-25.7260	P1	P_11	-1.9460
-13.4840	P1	P_11	-1.4060
-2.4740	P2	P_12	-1.2960
-3.2720	P2	P_22	-1.2700
-4.7640	P1	P_21	-1.2040
-3.8020	P2	P_12	-1.1020
-7.5720	P1	P_21	-1.0600
2.2560	P3	P_13	-1.0180
-4.3260	P2	P_32	-0.8260
0.1940	P2	P_22	-0.7620
6.3180	P3	P_23	-0.4840
-0.2320	P2	P_32	-0.3140
-1.1020	P2	P_22	-0.2320
-1.2960	P2	P_22	-0.1460
-1.2700	P2	P_22	-0.0460
3.2300	P3	P_23	0.0420
6.5040	P3	P_33	0.0460
0.9860	P2	P_32	0.1940
2.8400	P3	P_23	0.2040
6.3440	P3	P_33	0.2640
0.8220	P2	P_32	0.3480
-6.2660	P1	P_21	0.4080
-11.4520	P1	P_11	0.4800
-8.4360	P1	P_11	0.4820
-3.0260	P2	P_12	0.5480
4.7580	P3	P_23	0.6120

10.0440	P3	P_33	0.6200
10.3080	P3	P_33	0.6780
18.3720	P4	P_34	0.6860
14.6860	P4	P_44	0.7260
4.8820	P3	P_43	0.7360
4.5120	P3	P_33	0.8220
9.4600	P3	P_33	0.9160
17.9820	P4	P_34	0.9680
-3.8580	P2	P_42	0.9860
-9.1040	P1	P_21	1.0680
-3.5420	P2	P_12	1.3880
-5.2140	P1	P_21	1.4240
0.6780	P2	P_12	1.5480
-7.3280	P1	P_21	1.7480
-5.4400	P1	P_11	1.9600
-12.4120	P1	P_11	2.0160
2.2380	P3	P_13	2.1740
8.6800	P3	P_33	2.2340
5.0320	P3	P_33	2.2380
3.7040	P3	P_33	2.2560
2.0160	P3	P_33	2.4940
5.8500	P3	P_33	2.5940
2.5940	P3	P_33	2.7020
3.1140	P3	P_33	2.8400
0.3480	P2	P_32	2.8460
-2.2620	P2	P_22	2.8780
-4.6360	P1	P_21	2.9380
0.2040	P2	P_12	3.0060
0.2640	P2	P_22	3.0300
4.8300	P3	P_23	3.1140
17.8080	P4	P_34	3.1240
14.1480	P4	P_44	3.1340
-14.0220	P1	P_41	3.2060
-24.5520	P1	P_11	3.2140
-15.4600	P1	P_11	3.2300
-6.4500	P1	P_11	3.2400
-2.8760	P2	P_12	3.3220
-4.4760	P1	P_21	3.3520
-6.0000	P1	P_11	3.5760
-16.9840	P1	P_11	3.7040
-8.9520	P1	P_11	3.7080
-3.9560	P2	P_12	3.7880
-2.9720	P2	P_22	3.9320
-3.9080	P2	P_22	3.9660

-3.4040	P2	P_22	4.0560
-22.7200	P1	P_21	4.3280
-10.0260	P1	P_11	4.3960
-12.5740	P1	P_11	4.4480
3.9320	P3	P_13	4.5120
7.8020	P3	P_33	4.6600
1.9600	P3	P_33	4.7260
4.8700	P3	P_33	4.7580
-3.8200	P2	P_32	4.8300
-5.3720	P1	P_21	4.8700
-1.0180	P2	P_12	4.8820
-11.6020	P1	P_21	4.8840
-16.5060	P1	P_11	4.9680
-24.3740	P1	P_11	5.0320
-7.4940	P1	P_11	5.0460
-11.6960	P1	P_11	5.0520
5.0520	P3	P_13	5.4240
7.8840	P3	P_33	5.5480
11.0280	P4	P_34	5.5880
2.9380	P3	P_43	5.6120
-4.7160	P1	P_31	5.6540
0.6120	P2	P_12	5.8500
6.9920	P3	P_23	5.9720
10.2060	P3	P_33	6.3180
6.8600	P3	P_33	6.3440
-5.8340	P1	P_31	6.5040
-6.5720	P1	P_11	6.5720
-10.4740	P1	P_11	6.6100
-4.9000	P1	P_11	6.7180
7.9960	P3	P_13	6.7400
-2.4500	P2	P_32	6.7620
8.8500	P3	P_23	6.8060
11.5140	P4	P_34	6.8600
6.5720	P3	P_43	6.9120
2.8780	P3	P_33	6.9920
-8.4860	P1	P_31	7.2180
-9.6400	P1	P_11	7.3120
-12.1120	P1	P_11	7.4840
-9.6220	P1	P_11	7.6160
-8.6300	P1	P_11	7.7840
-12.4880	P1	P_11	7.8020
-14.5820	P1	P_11	7.8840
-17.3780	P1	P_11	7.9000
-13.9600	P1	P_11	7.9820

-11.8220	P1	P_11	7.9960
-0.4840	P2	P_12	8.0660
-2.8440	P2	P_22	8.3720
10.7440	P3	P_23	8.4780
23.0860	P4	P_34	8.6760
16.3320	P4	P_44	8.6800
18.5620	P4	P_44	8.8080
15.8780	P4	P_44	8.8500
10.0920	P3	P_43	9.1620
-11.9500	P1	P_31	9.4600
-17.7880	P1	P_11	9.4660
-13.5240	P1	P_11	9.7600
10.6220	P3	P_13	10.0440
11.5520	P4	P_34	10.0780
6.7620	P3	P_43	10.0920
9.4660	P3	P_33	10.2060
5.0460	P3	P_33	10.3080
7.2180	P3	P_33	10.5800
-2.3160	P2	P_32	10.6220
-8.3920	P1	P_21	10.7440
0.9680	P2	P_12	11.0280
2.4940	P3	P_23	11.5140
4.7260	P3	P_33	11.5520
14.4480	P4	P_34	11.6400
8.8080	P3	P_43	11.9560
3.0060	P3	P_33	12.6920
0.7360	P2	P_32	13.2980
-4.9940	P1	P_21	13.3520
-14.3940	P1	P_11	13.7360
-6.0540	P1	P_11	14.1480
-7.6600	P1	P_11	14.2240
-0.0460	P2	P_12	14.4480
3.3220	P3	P_23	14.5060
-5.5220	P1	P_31	14.5660
-14.3560	P1	P_11	14.6860
-16.3900	P1	P_11	15.0600
-14.0240	P1	P_11	15.8780
-13.6360	P1	P_11	16.3320
-2.5860	P2	P_12	17.8080
-3.0040	P2	P_22	17.9820
4.0560	P3	P_23	18.3720
-3.4660	P2	P_32	18.5620
0.4080	P2	P_22	23.0860
6.9120	P3	P_23	23.2380

Interval grouping and count for $i=5$, Opening Prices

Lowest point	-25.8100
Highest Point:	23.2380
Interval:	[-25.81, 23.238]
Interval 1:	[-25.81,-4.3515]
Interval 2:	(-4.3515, 1.7795]
Interval 3:	(1.7795, 10.976]
Interval 4:	(10.976, 23.2380]
Number of points in I1:	76
Number of points in I2:	64
Number of points in I3:	94
Number of points in I4:	24
Number of transitions P_11	48
Number of transitions P_12	21
Number of transitions P_13	7
Number of transitions P_14	0
Number of transitions P_21	20
Number of transitions P_22	20
Number of transitions P_23	24
Number of transitions P_24	0
Number of transitions P_31	7
Number of transitions P_32	21
Number of transitions P_33	52
Number of transitions P_34	13
Number of transitions P_41	1
Number of transitions P_42	2
Number of transitions P_43	10
Number of transitions P_44	11

Steady States

$\pi_1=1/4$
$\pi_2=1/4$
$\pi_3=1/4$
$\pi_4=1/4$

For i=10, Closing Prices

Difference of Closing Prices (Actual-Mov.Avg)	Labeling of P-State	Transitions
12.948	P2	
468.286	P3	P_23
20.004	P2	P_32
-5.31	P1	P_21
18.32	P2	P_12
16.336	P2	P_22
12.602	P2	P_22
495.115	P3	P_23
1.655	P1	P_31
3.69	P1	P_11
3.265	P1	P_11
-0.171	P1	P_11
-8.369	P1	P_11
490.093	P3	P_13
-5.513	P1	P_31
7.04	P1	P_11
21.666	P2	P_12
16.565	P2	P_22
16.258	P2	P_22
503.231	P3	P_23
20.809	P2	P_32
2.86	P1	P_21
16.391	P2	P_12
29.804	P2	P_22
25.687	P2	P_22
535.677	P4	P_24
16.043	P2	P_42
4.09	P1	P_21
10.898	P2	P_12
8.401	P2	P_22
0.263	P1	P_21
539.889	P4	P_14
0.411	P1	P_41
-13.88	P1	P_11
-13.466	P1	P_11
-8.723	P1	P_11
-4.606	P1	P_11

541.683	P4	P_14
6.967	P1	P_41
7.06	P1	P_11
19.95	P2	P_12
20.724	P2	P_22
16.334	P2	P_22
555.108	P4	P_24
16.942	P2	P_42
2.62	P1	P_21
9.66	P2	P_12
3.566	P1	P_21
-1.35	P1	P_11
574.187	P4	P_14
8.163	P2	P_42
-0.67	P1	P_21
7.506	P2	P_12
0.755	P1	P_21
3.323	P1	P_11
584.361	P4	P_14
5.509	P1	P_41
-3.3	P1	P_11
0.229	P1	P_11
1.079	P1	P_11
1.483	P1	P_11
583.991	P4	P_14
5.029	P1	P_41
-4.17	P1	P_11
6.911	P1	P_11
3.994	P1	P_11
7.589	P2	P_12
589.652	P4	P_24
4.288	P1	P_41
0.73	P1	P_11
6.338	P1	P_11
14.632	P2	P_12
18.734	P2	P_22
600.867	P4	P_24
17.613	P2	P_42
-5.34	P1	P_21
12.35	P2	P_12
6.996	P1	P_21
4.64	P1	P_11
623.194	P4	P_14

3.556	P1	P_41
-17.6	P1	P_11
-23.554	P1	P_11
-13.988	P1	P_11
-12.722	P1	P_11
610.766	P4	P_14
-20.286	P1	P_41
5.95	P1	P_11
-20.365	P1	P_11
-15.69	P1	P_11
-4.433	P1	P_11
585.586	P4	P_14
-5.176	P1	P_41
-14.49	P1	P_11
-36.844	P1	P_11
-29.618	P1	P_11
-25.439	P1	P_11
561.327	P4	P_14
-27.037	P1	P_41
-1.58	P1	P_11
-15.509	P1	P_11
-1.85	P1	P_11
-10.27	P1	P_11
532.345	P3	P_13
-1.055	P1	P_31
-3.1	P1	P_11
1.279	P1	P_11
3.028	P1	P_11
-0.57	P1	P_11
526.538	P3	P_13
6.582	P1	P_31
-3.79	P1	P_11
7.119	P1	P_11
3.712	P1	P_11
4.819	P1	P_11
541.147	P4	P_14
-6.077	P1	P_41
-0.69	P1	P_11
-9.118	P1	P_11
-3.185	P1	P_11
5.209	P1	P_11
528.429	P3	P_13
16.891	P2	P_32
2.86	P1	P_21

20.468	P2	P_12
15.666	P2	P_22
27.428	P2	P_22
561.367	P4	P_24
19.773	P2	P_42
-3.5	P1	P_21
-0.03	P1	P_11
-1.694	P1	P_11
-2.035	P1	P_11
570.016	P4	P_14
-10.016	P1	P_41
-8.04	P1	P_11
-7.155	P1	P_11
0.221	P1	P_11
1.716	P1	P_11
556.792	P4	P_14
5.658	P1	P_41
2.07	P1	P_11
7.352	P2	P_12
6.472	P1	P_21
7.331	P2	P_12
570.902	P4	P_24
-2.682	P1	P_41
4.17	P1	P_11
-0.816	P1	P_11
4.666	P1	P_11
16.7	P2	P_12
568.608	P4	P_24
20.392	P2	P_42
-12.86	P1	P_21
-20.851	P1	P_11
-14.593	P1	P_11
-14.409	P1	P_11
566.613	P4	P_14
-19.553	P1	P_41
-13.33	P1	P_11
-25.603	P1	P_11
-19.492	P1	P_11
-10.352	P1	P_11
533.058	P3	P_13
-7.358	P1	P_31
-20.15	P1	P_11
-18.877	P1	P_11
-25.128	P1	P_11

-25.473	P1	P_11
528.213	P3	P_13
-6.563	P1	P_31
-6.65	P1	P_11
-0.241	P1	P_11
-1.774	P1	P_11
0.929	P1	P_11
508.648	P3	P_13
-10.278	P1	P_31
-10.06	P1	P_11
-28.183	P1	P_11
-18.624	P1	P_11
-15.516	P1	P_11
472.562	P3	P_13
2.908	P1	P_31
-7.11	P1	P_11
2.502	P1	P_11
0.842	P1	P_11
11.948	P2	P_12
494.028	P3	P_23
11.572	P2	P_32
-13.54	P1	P_21
-3.975	P1	P_11
-14.59	P1	P_11
-1.255	P1	P_11
493.618	P3	P_13
-5.118	P1	P_31
14.91	P2	P_12
10.61	P2	P_22
9.972	P2	P_22
9.791	P2	P_22
493	P3	P_23
-4.44	P1	P_31
-4.84	P1	P_11
-15.202	P1	P_11
-16.04	P1	P_11
-15.529	P1	P_11
486.914	P3	P_13
-32.654	P1	P_31
-5.8	P1	P_11
-28.647	P1	P_11
-23.446	P1	P_11
-18.908	P1	P_11
443.579	P2	P_12

6.621	P1	P_21
-4.47	P1	P_11
26.035	P2	P_12
35.911	P2	P_22
33.412	P2	P_22
500.106	P3	P_23
-6.086	P1	P_31
5.17	P1	P_11
8.389	P2	P_12
1.569	P1	P_21
6.054	P1	P_11
483.417	P3	P_13
6.643	P1	P_31
2.05	P1	P_11
2.379	P1	P_11
3.922	P1	P_11
1.257	P1	P_11
487.42	P3	P_13
2.99	P1	P_31
14.14	P2	P_12
16.049	P2	P_22
7.153	P1	P_21
10.645	P2	P_12
508.522	P3	P_23
-4.812	P1	P_31
8.07	P2	P_12
-11.054	P1	P_21
-11.332	P1	P_11
-6.471	P1	P_11
504.741	P3	P_13
-22.591	P1	P_31
-5.95	P1	P_11
-18.543	P1	P_11
-25.991	P1	P_11
-19.049	P1	P_11
458.964	P3	P_13
-7.984	P1	P_31
-6.46	P1	P_11
-9.454	P1	P_11
3.047	P1	P_11
6.366	P1	P_11

Interval grouping and counting for $i=10$, closing prices

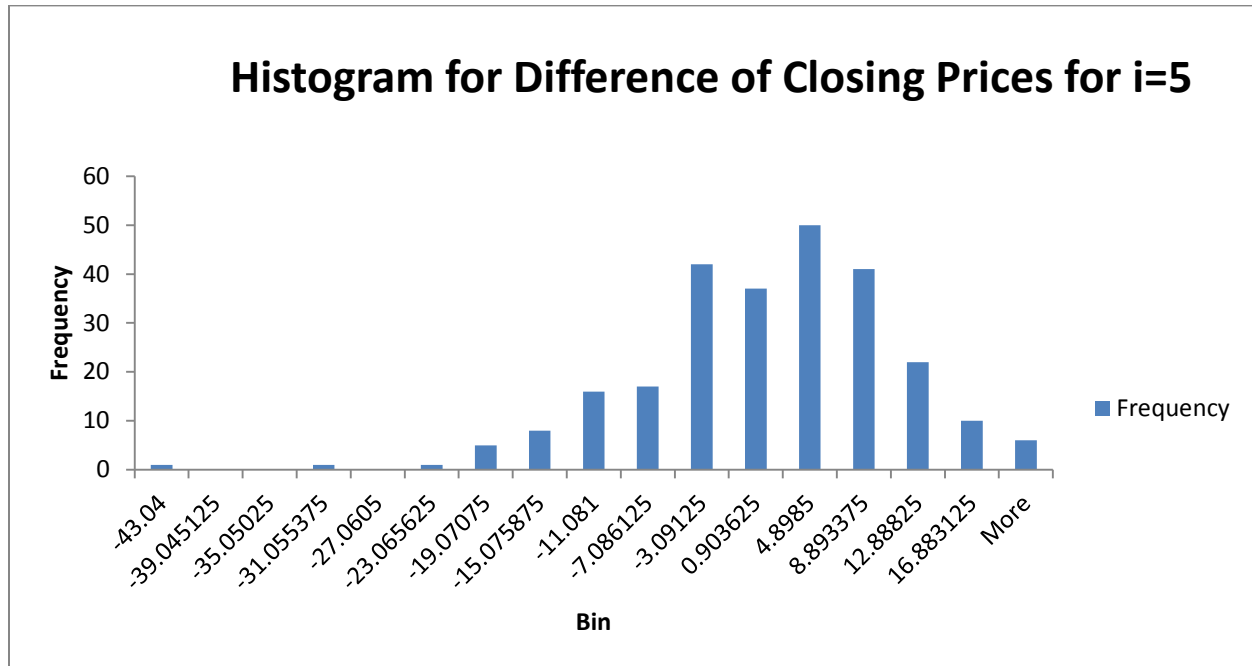
Interval 1	[36.844, 7.158533]
Interval 2	(7.15833, 447.1839]
Interval 3	(447.1839, 535.1889]
Interval 4	(535.1889, 623.194]
Points in P1	161
Points in P2	51
Points in P3	21
Points in P4	20
Transitions for P_11	111
Transitions for P_12	22
Transitions for P_13	14
Transitions for P_14	13
Transitions for P_21	19
Transitions for P_22	18
Transitions for P_23	7
Transitions for P_24	7
Transitions for P_31	17
Transitions for P_32	4
Transitions for P_33	0
Transitions for P_34	0
Transitions for P_41	14
Transitions for P_42	6
Transitions for P_43	0
Transitions for P_44	0

Steady States

Π_1	0.245794
Π_2	0.248738
Π_3	0.246354
Π_4	0.259114

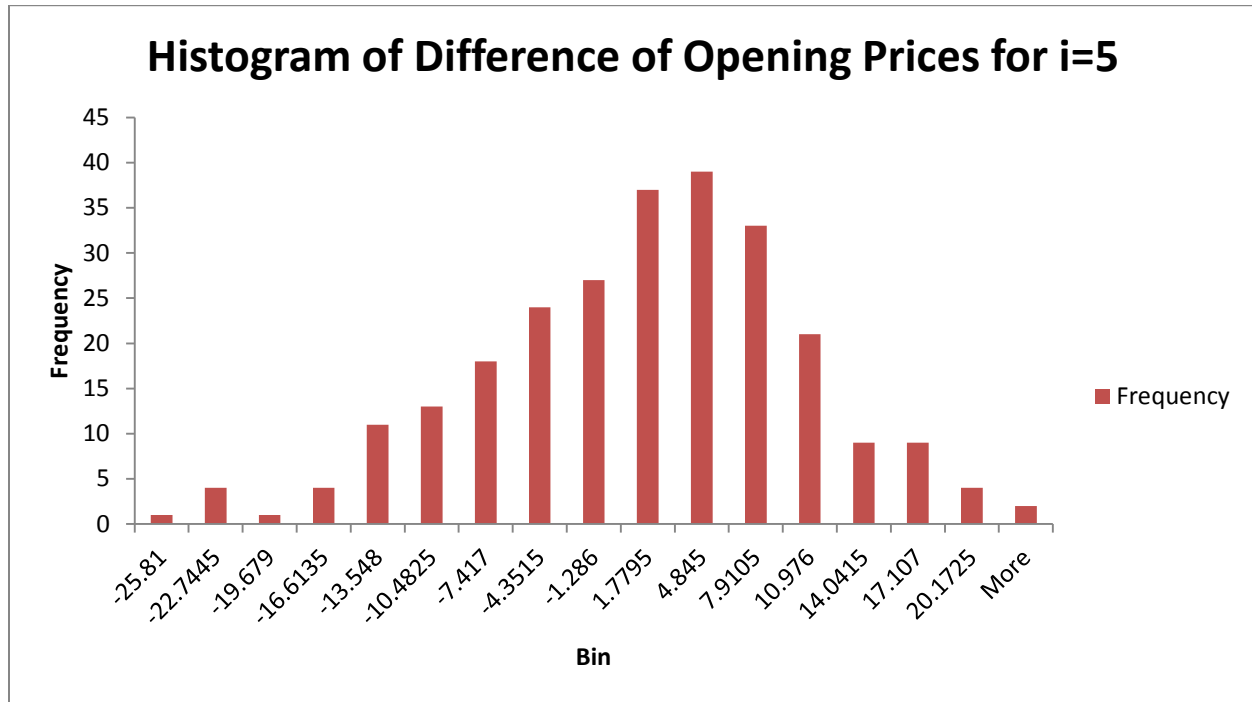
APPENDIX C

Histogram and bins for $i=5$, Closing Prices



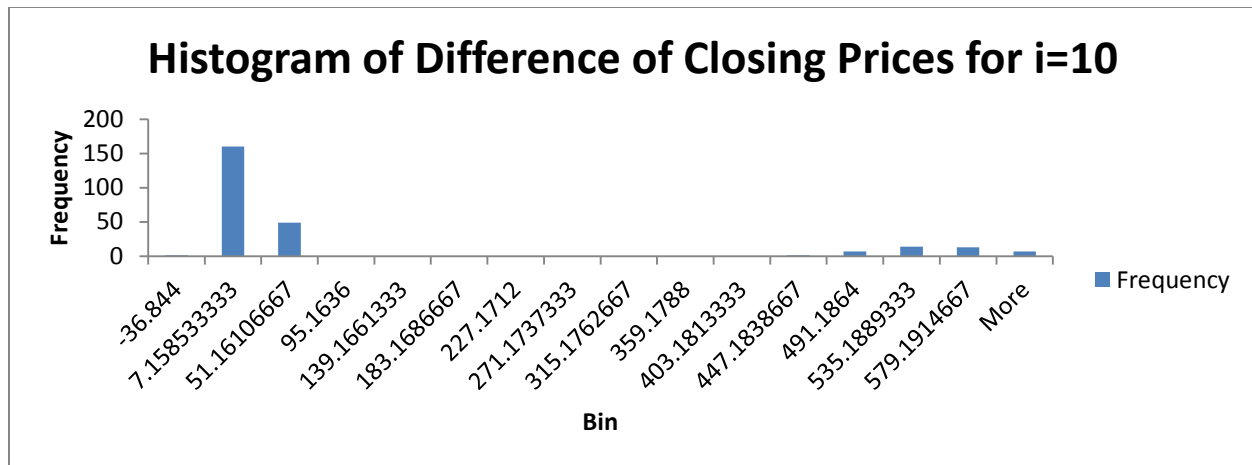
<i>Bin</i>	<i>Frequency</i>
-43.04	1
-39.0451	0
-35.0503	0
-31.0554	1
-27.0605	0
-23.0656	1
-19.0708	5
-15.0759	8
-11.081	16
-7.08613	17
-3.09125	42
0.903625	37
4.8985	50
8.893375	41
12.88825	22
16.88313	10
More	6

Histogram and bins for i=5, Opening Prices

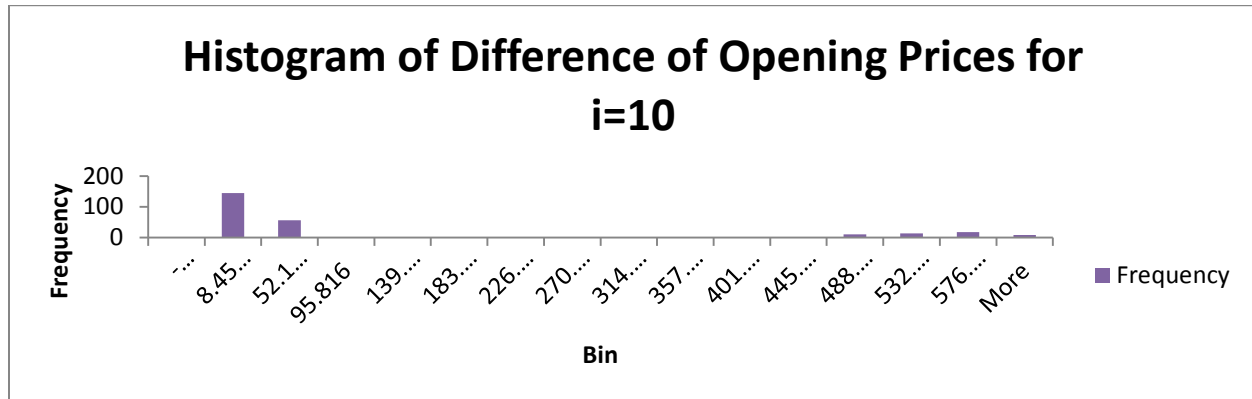


<i>Bin</i>	<i>Frequency</i>
-25.81	1
-	-
22.7445	4
-19.679	1
-	-
16.6135	4
-13.548	11
-	-
10.4825	13
-7.417	18
-4.3515	24
-1.286	27
1.7795	37
4.845	39
7.9105	33
10.976	21
14.0415	9
17.107	9
20.1725	4
More	2

Histograms and bins for i=10, Closing Prices



<i>Bin</i>	<i>Frequency</i>
-36.844	1
7.158533	160
51.16107	49
95.1636	0
139.1661	0
183.1687	0
227.1712	0
271.1737	0
315.1763	0
359.1788	0
403.1813	0
447.1839	1
491.1864	7
535.1889	14
579.1915	13
More	7

Histogram and bins for $i=10$, Opening Prices

<i>Bin</i>	<i>Frequency</i>
-35.225	1
8.45333	145
52.13567	56
95.816	0
139.4963	0
183.1767	0
226.857	0
270.5373	0
314.2177	0
357.898	0
401.5783	0
445.2587	1
488.939	10
532.6193	13
576.2997	18
More	8

APPENDIX D

Transition Matrix for i=5, Closing Prices

0.59375	0.34375	0.03125	0.03125
0.114583	0.708333333	0.15625	0.020833333
0.010869565	0.163043478	0.663043478	0.152173913
0.026315789	0.052631579	0.368421053	0.552631579

Transition Matrix for i=5, Opening Prices

0.63157895	0.2763158	0.0921053	0
0.3125	0.3125	0.375	0
0.07446809	0.2234043	0.5531915	0.1382979
0.04166667	0.0833333	0.4166667	0.4583333

Transition Matrix for i=10, Closing Prices

0.689441	0.136646	0.086957	0.080745
0.372549	0.352941	0.137255	0.137255
0.809524	0.190476	0	0
0.7	0.3	0	0