This is the RBE 2001 lecture on operational amplifiers.

**INTRODUCTION**

**Section 1-1**

~~Slide 3~~

You were first introduced to operational amplifiers or op-amps in RBE 1001. They are used in many different kinds of devices such as:

* Comparators,
* Amplifiers,
* Battery monitors,
* D/A and A/D converters,
* Linear motor controls,
* Many other applications

~~Slide 4~~

When you buy an op-amp today it generally comes as an integrated circuit or IC. Here you can see some eight-pin DIPS or dual in-line packages. But we're getting ahead of ourselves. The first question you should be asking is "What exactly is an op-amp?"

Well the answer is that an op-amp is just a really good voltage amplifier!

~~Slide 5~~

The first thing we need to know in using op-amps is understanding how they work. And to understand how they work we're going to need to build a model. We'll begin with a visual model and then work up to a mathematical model.

**Section 1-2**

We'll start with the symbol for an op-amp - the triangle in the middle of the picture. Looking at the left hand side of the triangle we can see that there are two inputs: one with a positive sign and one with a negative sign. The input with the positive sign is called the non-inverting input and the input with the negative sign is called the inverting input. We can see that there may be different voltages - in this case v1 and v2 - connected to the non-inverting and inverting inputs respectively. We'll come back to the inputs momentarily.

Unlike some of the circuits we've looked at previously, an op-amp is an active circuit. It therefore needs to be powered. We're showing in this model that the power is supplied as positive 15 volts (shown here as VCC) and -15 volts or VEE. These values will of course vary depending on the particular op-amp you are using and the application.

Now let's look at the right hand side of the op-amp. There we have a single output. Note that all of the inputs and outputs are referenced to ground as shown in the diagram.

**Section 2-1**

The output of the op-amp is given by the following formula:

As you can see the output is given by the difference between the and multiplied by the constant . is the open-loop gain and for most op-amps is a very large value. How large? It often is as large as a million or more.

Let’s think about the implications of that statement for a second. Suppose the difference between and is just one volt and . That means the output voltage is going to be one million volts! Really???

We’ll, actually no. Let’s look at the next figure to see what really happens.

~~Slide 6~~

Let’s plot the difference between the two input voltages on the horizontal axis. On the vertical axis we’ll plot the output voltage . From the formula given previously we can see that the relationship between the difference in input voltages and the output voltage is going to be a straight line with a slope of . [ Note that the slope of the line shown in the figure is much, much smaller than the real slope would be. ]

So what keeps the output voltage from heading off to some huge value? Well, the op-amp circuitry cannot generate an output voltage any larger than VCC or VEE. Those voltages (VCC and VEE) are often called the “rail” voltages and most op-amps cannot generate an output voltage that is equal to a rail voltage. What happens is that the op-amp circuitry “saturates” a little shy of the rail voltage. The voltage difference between the saturation voltage and the rail voltage varies by op-amp but is typically a few tenths of a volt. [ Note that some op-amps actually can generate an output voltage equal to the rail voltages. These are called ‘rail-to-rail’ op-amps. ]

~~Slide 7~~

So far, we have generated a high level model of the op-amp. Let’s now drill down to a bit more detail. We can model the basic operation of an op-amp with the following circuit. On the left hand side of the circuit we have the non-inverting the input – given by – and the inverting input given by. The difference between those two inputs is given bywhich is shown as the voltage across the resistor where the subscript *i* indicates ‘input’. Now let’s look at the right hand side of the circuit. Let’s start with a red diamond. That’s a symbol you might not have seen before. It indicates a voltage-controlled voltage source. By ‘voltage-controlled’ it is meant that the output of the voltage source is not fixed that rather is specified by another voltage in the circuit. A formula is used to specify the voltage; in this case the formula is: .

The last circuit element is the series output resistor. **~~It should be obvious~~** that with the output of the op-amp open-circuited as is shown in the figure, the output voltage will be equal to the formula we saw earlier.

What is missing from this model?

<< Insert some type of hidden multiple choice assessment at this point. The answer I’m looking for is the fact that this model does not saturate and therefore clip the output voltage at the rail voltages. In fact the model makes no mention of the rail voltages at all.>>

~~Slide 8~~

Let’s carry on with the model we have started developing. We’ll separate it into two parts: the input and the output. Let’s look at the input (left-hand) part first. Assume some arbitrary circuitry connected to the input of the op-amp. We will model that circuitry as a voltage source in series with some resistance . You’ve seen this before; we’re modeling that source circuitry as a Thévenin equivalent circuit.

Now let’s look at the right hand side of the circuit. The voltage-controlled voltage source is in series with the output resistor and that is in turn in series with a load resistor. The output voltage is measured across the load resistor.

We will now extend this model to what is called an ‘ideal’ op-amp. In the ideal op-amp the input resistance approaches infinity. As the input resistance approaches infinity and the current flowing through the input resistor approaches zero. This means that the currents entering the inverting and non-inverting inputs also approach zero.

It was previously stated that the open-loop gain was very large; we will now let it approached infinity. The output, however, must remain a bounded number. Given that is heading off to infinity, the only way that can happen is if the difference in the input voltages approaches zero. This in turn means that. Finally, in order for , the output resistance must also approach zero.

So in summary, the ideal op-amp has the following characteristics:

* the input resistance ,
* the open-loop gain ,
* the output resistance,
* the input currents , and
* the input voltages .

~~Slide 9~~

Having an amplifier with enormous gain is useful, and we will certainly exploit that eventually, but let’s first look at an op-amp circuit that has a much lower gain. The circuit that is shown is called an ‘inverting’ amplifier. We’ll see why it’s called that later on.

In the circuit there is one input voltage source and one load resistor. In addition, it has two other resistors and . Notice that the resistor is connected between the output of the op-amp and the inverting input. Because this resistor is feeding some of the output of the op-amp back into an input it’s called a ‘feedback’ resistor. If

[ Need to add an explanation here of the role of the feedback resistor and how the feedback is actually negative. ]

Let’s now develop a mathematical model for this circuit. We’ll assume that were using an ideal op-amp.

The first thing we should notice is that the non-inverting input is grounded. What does that tell us about the voltage at the inverting input?

<< I’d like to see a ‘reveal’ at this point. The correct answer is that the non-inverting input must also be at zero volts. >>

With that in mind, let’s now figure out the current that’s flowing through the resistor. We know that the voltage at the left end of the resistor is given by and the voltage of the right end of the resistor is zero volts. We can therefore use Ohm’s law to calculate the current flowing through the resistor as .

The next step in our analysis is to do KCL at the node labeled ‘A’. The current is flowing into node A; assume a current is flowing out of the node through the resistor . What about the last branch connecting to node A – the inverting input to the op-amp? How much current is flowing in that branch?

<< Another ‘reveal’ here… The answer in this case is zero. Recall that the current flowing into either of the inputs of an ideal op-amp approaches zero. >>

So that means the current flowing out of node A must equal the current flowing into node A, or in other words .

Finally, Kirchoff’s voltage law (KVL) can be used to create one additional equation. The loop will start at node A and end at the load resistor which is specified to have the voltage across it.

<< Why can we start the KVL loop at node A? It’s because the non-inverting input to the op-amp is grounded – which in turn means that the inverting input of the op-amp (node A) is also effectively grounded. >>

This results in the equation. Plugging back in the previous result for we get.

The quantity that is of particular interest is the ratio of the output voltage to the input voltage or . This is formally called the closed-loop gain of the circuit (‘closed’ because of the presence of the feedback resistor), but we’ll call it just the gain of the circuit. Doing a little algebra, we get the following interesting result:.

Two things in particular should be noted. The first is that the gain is simply the ratio of the resistors to . Because resistors come in many different standard fixed as well as variable values it should be apparent that any desired gain including fractional values can be arranged. The other important thing to notice is that the gain is negative. This means that the output voltage will be of opposite polarity to the input voltage. The name of the circuit, i.e., an ‘inverting’ amplifier, comes from this characteristic.

~~Slide 10~~

Let’s now work through an example. Notice in this case we have two voltage sources andboth of which are connected to the inverting input through the resistors and respectively. This can be handled just as we did before. The difference in this case is that there will be two currents coming into node A – one for each voltage source.

~~Slide 11~~

We began by doing KVL around the loop indicated by the red arrow. This results in the following equation . This can be rearranged to give us the current . The other voltage source is handled in the same way resulting in. As was done previously KCL is done at node A resulting in the following equation which can be solved for the current through the feedback resistor giving us .

~~Slide 12~~

KVL can now be used to relate the output voltage to the current through the feedback resistor. The resulting equation is which leads us to. Plugging in the previous result for results in or  
  
. << Make this equation X >>

This is an interesting and powerful result. Note that the output voltage is a function of both of the input voltages and , but each of those input voltages is subject to a potentially different gain because the resistors and need not be the same.

Let’s take this example a little bit further. Suppose we needed to create the following functional relationship between the input voltages and the output voltage: . How might we do that? Assume at this point that you can pick pretty much any value you want for each of the resistors. Since the feedback resistor is common to both terms in equation X let’s start by picking a value for that. Let’s go with 10 K ohms. Having picked that value for the feedback resistor it should be obvious that the values for the resistors and need to be 1 K ohms and 2 K ohms respectively. We can see that this generates the desired result: .

That’s pretty cool! We can actually use op-amps to perform algebraic operations on voltages. Actually, that’s where the word ‘operational’ in the phrase operational amplifier comes from. As we continue, we’ll see that there are many other mathematical operations that op-amps can perform.

~~Slide 13~~

You have all of the basics of inverting op-amps at this point, so it’s now time to start putting them together to solve more complicated problems. In the following circuit you can see that there are not only two voltage sources but two op-amps as well. The goal of this exercise is to figure out the relationship between the output voltage and the two input voltages. That result is also going to be dependent on the values of the various resistors in the circuit.

So where do we begin? We can see that the right-hand side of the circuit looks very much like the problem we just solved. The difference is that there’s no voltage source directly connected to the resistor . Or is there?

Actually, there is a voltage source connected to the resistor . It is the output of the left-hand op-amp. If we can figure out what that voltage is, then we can use that to solve the right-hand part of the circuit just as we did before.

So let’s isolate the left-hand part of the circuit and call the output voltage of the left hand op-amp . We already know how to solve this circuit so we’ll skip over the details and go straight to the result: .

~~Slide 14~~

We can now proceed as we did in the previous problem. First, an equation can be developed that expresses the contribution to the output voltage from the voltage ‘source’ called : .

~~Slide 15~~

Then we can develop an equation for the contribution from the other voltage source. This gives us: . Combining these two contributions we get our final result:.

That wasn’t so bad – was it? The circuit looked pretty intimidating at first but by simply breaking it down in a logical fashion and solving each of the resulting parts piecemeal we solved it quite readily.

Now it’s time to learn about a different kind of op-amp circuit.

~~Slide 16~~

So far, we have been looking at inverting op-amp circuits. We’ll now turn our attention to the noninverting op-amp circuit. Consider the circuit shown in the following diagram. Notice that the input voltage is this time connected to the non-inverting input. We again have two additional resistors in the circuit. The feedback resistor again connects the output to the inverting input, but this time the other resistor acts to form a voltage divider with.

~~Slide 17~~

Before going on let’s examine whether the feedback is still negative. We’ll start by assuming the supply voltage is positive as shown and therefore the output voltage will be positive (and possibly large). Since the resistors and form a voltage divider, some portion of the output voltage will appear across the resistor; let’s call that voltage.

We know that the differential input voltage is given by the equation: . It should be apparent that as (and therefore ) get larger then will get smaller (given a constant ). Thus is driven towards zero and we indeed have negative feedback.

<< At this point in the lecture I had a clicker slide asking what would happen if there was positive feedback instead of negative. The answer, of course, is that with positive feedback an increase in the output would generate an even larger input which in turn would generate a larger output and so on… The output of the op-amp would therefore ‘run away’ so to speak. This is something we’ll find very useful in yet another op-amp based circuit. >>

~~Slide 19~~

Now that we know the feedback is negative we can use the analysis techniques we have seen previously. More specifically, we can use the summing point constraints which are that: and . If we apply KVL around the left most loop as shown we can see that and therefore . Since , and form a voltage divider which gives us: . Putting it all together we can finally see that the gain is given by: .

As you can see, the gain in this case is always going to be a positive value. It is again, however, influenced by the ratio of the resistors to .

~~Slide 20~~

There are a few interesting things to note about the op-amp circuits that we’ve been studying so far. The first is that the input resistance of the circuit is theoretically infinite (resulting in ). The implication of this is that the op-amp places no load on the circuits that are driving it. Also note that the load resistance has not shown up in either of the voltage gain equations developed so far. This means that the output voltage is independent of the load resistance. The implication of this fact is that the output resistance is zero.

**It is important to remember that we have been dealing with an ideal op-amp in each of these analyses and real op-amps made behave somewhat differently.** The good news, however, is that real op-amps don’t behave all that much differently from the ideal op-amp model so we are justified in using that as the basis for our analyses.

<< Insert an example problem here using a non-inverting op-amp. >>

~~Slide 21~~

We can build on our model of the non-inverting op-amp to produce another very useful circuit. Recall that the gain of the circuit is given by: . Suppose we want the gain to approach unity. How could we achieve that?

One way is to let the value of the resistor go to zero. Another way we could achieve the same result is to let the value of the resistor approach infinity. Actually, since both of these approaches achieve the result we desire, why not use both? If we left approach infinity and approach zero, we get the following circuit.

~~Slide 22~~

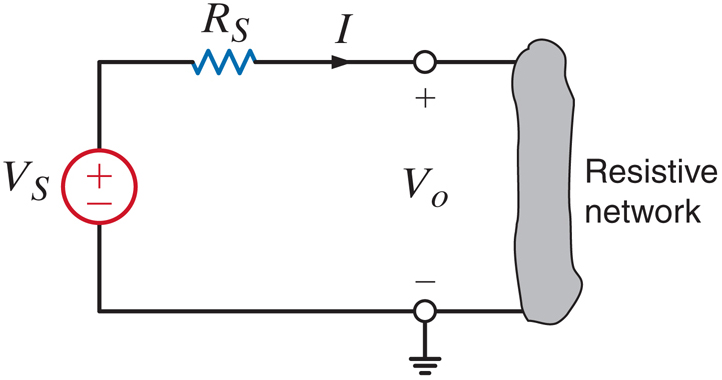
This circuit is called a unity gain buffer (because the voltage gain is exactly 1). It’s also often called a voltage follower as the output voltage exactly follows the input voltage.

OK, that’s cool – but what’s the point? Why go to all the work of creating a circuit that does nothing?

Well, the answer to that question is that the circuit actually does do something and it’s something that’s very useful. Let’s take a closer look…

~~Slide 23~~

Consider the network in the following figure:



Here we have a voltage source in series with a resistor . These components are driving some unknown resistive network at the right of the circuit (the gray blob). It should be clear that the voltage measured across the resistive network is not equal to but rather is given by . Because the current flowing through is non-zero there’s a voltage drop across which leads to being different from . This is because the resistive network ‘loads’ the voltage source .

~~Slide 24~~

Now suppose that we use a unity gain buffer or voltage follower to isolate the source circuitry from the resistive network as shown in the following figure. We know from what we’ve studied previously that the current flowing into the op-amp will be zero (or very nearly so for real op-amps). We also know that the output voltage of the op-amp will be equal to the source voltage as there will be no voltage drop across the resistor .

The question you may be asking yourself is: “Where does the energy that is dissipated in the resistive network come from?” The answer is that the energy supplied to the resistive network comes from the power supplies that are driving the op-amp.

Summary

In this chapter we’ve examined op-amps in a little more detail than you saw in RBE 1001. We developed a model for the behavior of an ideal op-amp and we also examined the behavior of real opamps. We have looked at the standard circuits for inverting as well as non-inverting op-amps. Finally, we also looked at the circuit for a voltage follower (a.k.a. a unity gain buffer) and saw how that may be used to isolate a voltage source from a load.

I THINK THAT WE SHOULD ADD REVIEW OF IMPORTANT EQUATIONS! - OH