An Analysis of University Endowment Spending

A Major Qualifying Project

Submitted to the Faculty of

Worcester Polytechnic Institute

in partial fulfillment of the requirements for the

Degree of Bachelor of Science

in

Actuarial Science

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Date: 4/27/2020

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Abstract

The goal of this project is to identify the best spending rule to maximize the purchasing power of a university's endowment, while also maintaining steady yearly spending each year. An endowment is an asset owned that should support an organization in perpetuity; in the case of most universities, it is a large sum of money that is invested in various assets, and managers decide how much to spend each year. Researchers analyze rules' performance in various financial metrics for 1,000 economic scenarios designed to replicate annual returns earned by endowments over a 50-year period. The results of this analysis suggest that universities should calculate yearly spending from their endowment using the recursive moving average spending rule, a blend of the hybrid and moving average rules.

Introduction

Endowments are vital assets to colleges and universities worldwide. While their components can vary, endowments are expected to support the university in perpetuity. The financial assets of the endowment are typically invested to retain their purchasing power, or inflation-adjusted value while still spending consistently every year. For example, $1 in 1920 could buy as much as $12.93 in 2020, so it is crucial that cash from donations be invested so that no purchasing power is lost over time. Funds can be invested in any type of financial instrument and returns from these investments are typically an average of 60% of the returns from the S&P 500 index.

Spending from the endowment must be carefully managed so that it accomplishes two goals: maintaining the purchasing power of the fund and spending a consistent amount of money each year. These two goals conflict with each other; the best way to maximize the purchasing power of the fund is to spend nothing, while spending consistently from year to year would involve spending an inflation-adjusted value each year. Spending the same amount each year would diminish purchasing power in years such as 2008 where there were negative returns, making the spending amount a greater percentage of the market value of the fund and decreasing the endowment's capacity for growth.

There are many ways an institution can decide how much money to spend out of its endowment each year in order to accomplish these goals. Most institutions currently use the **moving average rule**, where the amount spent each year is equal to a spending rate multiplied by the average market value of the endowment over the previous 3 years. Another rule which is gaining popularity is the **hybrid rule**, where a weighted average of the previous year's spending and a proportion of the market value of the portfolio is spent each year.

In January of 2019, James Yaworski developed two new rules for spending out of an endowment: the **purchasing power rule** and the **blended rule**. The purchasing power rule, unlike most other existing rules, correlates its spending with the market in an effort to spend more when the economy is good and spend less when the economy is bad, thus maintaining the overall purchasing power of the fund. The blended rule spends a weighted average of the purchasing power rule spending and the hybrid rule spending, hoping to combine the strengths of each rule (maintaining purchasing power and consistent spending amounts, respectively).

The goal of this project is to analyze the best spending rules for university endowments. Each rule was analyzed via handpicked economic scenarios such as a bull run, bear run, and historical returns from the S&P 500 using a beta of 0.6, as well as randomly generated economic scenarios. For each scenario, many metrics were calculated for each rule:

1. Retention of purchasing power
2. Total spent (with and without inflation)
3. Average spent per year (with and without inflation)
4. Standard deviation of yearly spending (with and without inflation)
5. Coefficient of variation of yearly spending (with and without inflation)

These criteria were summarized using TOPSIS analysis to rank all of the rules for each scenario. Using the results from the randomly generated returns, these ranks were aggregated for the rules and the process was repeated for the top four rules, giving a clear idea of which rules were best.

We will show that the **recursive moving average rule**, a new rule that we developed, is preferable to all existing rules because use of the rule results in the best overall performance of the above metrics while being a relatively easy rule to present.

Background

Introduction to Endowments

Universities rely on several sources of income to remain in business each year. In addition to income from tuition, commercial operations such as grants, and philanthropic giving, most universities are fortunate enough to have an endowment. A university's endowment consists of donations from donors in the form of money or property. The university invests donations, and utilizes the investment income (Smith, 2019). The goal of this project is to find the optimal method to determine how much money should be spent annually from the endowment in order to preserve the endowment in perpetuity while providing consistent assistance to the university.

Endowments provide an extra layer of financial stability for universities. The endowment manager’s goal is to spend as much as possible from the endowment while maintaining the inflation-adjusted market value of the fund. To preserve the endowment’s value, only a small percentage of the endowment is spent each year, and the remainder is invested to provide financial support for future expenses. The portion that is spent often funds larger campus initiatives, such as construction of new buildings or upgrading the departmental equipment. Universities also use their endowments to fund scholarships, fellowships, and financial aid to students (Understanding College and University Endowments, 2014).

The size of a university’s endowment is reliant on the university’s connection with their alumni and the community alike. U.S. News & World Report lists the universities with the largest endowments by the end of the fiscal year 2018 as follows:

1. Harvard University, $37.1B
2. Yale University, $27.2B
3. Stanford University, $24.8B
4. Princeton University, $23.4B
5. Massachusetts Institute of Technology (MIT), $14.8B

...

Worcester Polytechnic Institute (WPI), $519.6M

Schools with grand reputations are likely to attract large donations. However, endowments of this size are not representative of the majority of universities’ endowments. U.S. News & World Report claimed the median endowment value among universities to be $65.1 million by the end of fiscal year 2018. By this metric, Worcester Polytechnic Institute’s endowment is greater than 50% of all other university endowments as the its market value stood above $519.6 million at the end of Fiscal Year 2018 (Kerr, 2019).

Composition and Management of University Endowments

Donations are an important contribution to a university's endowment. With each donation, a university must be able to manage these funds effectively in order to maintain a relationship with donors as well as funding for future expenses. It is important to prioritize their fundraising strategy in order to rely on sources of income other than tuition, government support, and contracted research. In order to maximize both short-term and long-term contributions to a university, the university must prioritize three strategies:

1. Fundraising strategy
2. Investment strategy
3. Spending strategy

These elements are highly valued by universities such that they tend to be embedded into the governance structure of the endowment, which typically consists of the Board of Trustees of the endowment, the investment committee, and the endowment’s operating investment management staff (Franz and Kranner, 2019). The remainder of this section will discuss the various fundraising and investment strategies a university could implement when managing their endowment. Spending strategies will be touched upon in brief, but further analysis and detail will be provided in later sections.

<u>Fundraising Strategies of Endowments</u>

To stabilize and maximize the amount of donations a university receives, the Board of Trustees should be involved when considering a change in their endowment strategy or when launching a large fundraising campaign (Franz and Kranner, 2019). It is important to note that a donation of money tends to be given under specific conditions provided by the donor, so the university must abide by these conditions in how the donation is spent. For example, if a donation was given with the intent of being a long-term fund, the university must choose an appropriate investment strategy that earns enough return to cover both the distribution of this donation and any inflation, market shocks, and costs of managing these funds in the long term (Franz and Kranner, 2019). Alternatively, a university could use their inflow of donations towards current expenses.

The key to successful fundraising is a good and long-term relationship with donors, transparency and communication with stakeholders, and preservation and adaptation of the fundraising infrastructure (Franz and Kranner, 2019). To maintain a good relationship with donors, it is important for the university to ensure that donations are being used to satisfy the intent the donor had when contributing that sum of money. If a university is successful in supporting their donors in this way, it increases the chances that the same donor will make additional contributions to the university. In fact, a general rule of thumb is that it takes about five years from the first interaction with a potential donor until a large contribution is made to the university (Franz and

Kranner, 2019). Therefore, establishing a strong relationship with donors is crucial for future funding. Note that this analysis ignores the role of donations in performance of spending rules.

Investment Strategies of Endowments

To maximize total returns of an endowment, strategizing ways to invest the endowment is crucial for any university. These strategies are mainly influenced by the endowment's characteristics, which can be categorized into the following: the size of the endowment, the availability of "know-how", and the network of the institution (Franz and Kranner, 2019).

Typically, universities with smaller endowments can follow similar investment strategies as those with larger endowments to achieve high level, risk-adjusted returns for their own portfolio (Azlen and Zermati, 2017). The size of an endowment can limit a university's ability to invest in certain asset classes as well as the university's access to highly qualified endowment managers. In addition, the composition of expenses by a university will vary depending on the size of their endowment; for instance, a larger endowment will yield a higher proportion of management and incentive fees with lower operating costs than that of a smaller endowment (Franz and Kranner, 2019). However, an endowment's size does not typically influence the overall expense ratio.

The term "know-how" relates to having a knowledgeable understanding of investment strategies to generate returns to continue to fund future spending, cover inflation, and to buffer for drastic changes in the market. This understanding is valuable when hiring consultants and assigning investment mandates to a group of asset-class managers (Franz and Kranner, 2019). This strong network of individuals provides a solid pool of managers for the institution, resulting in access to knowledgeable management for the endowment. Understanding an institution's network also helps a university on how to maximize engagement with donors and stakeholders.

When it comes to investment styles historically, most endowments do not reveal or report their specific investment styles. The best depiction of the investment styles used by universities can be broken up into three approaches:

1. The passive market approach
2. Systematic strategies
3. A skillful selection of active managers (Franz and Kranner, 2019)

The passive market approach is generally used when resources are scarce, resulting in a focus on minimizing costs and maximizing diversification in asset-allocation. Systematic strategies are used typically by medium-sized and large endowments as a result of growth in endowment size; these strategies require greater "know-how" and skillful managers to implement these strategies. Having a skillful selection of active managers additionally allows for a university to utilize their network while having the potential to outperform the market in the long run. Only top managers tend to bring in excess returns after costs, so this investment style requires both "know-how" and a solid network in determining quality management.

When implementing these investment styles, the following approaches can be taken:

1. Internal management by university employees
2. A portfolio of external managers selected by the endowment staff
3. The appointment of an Outsourced Chief Investment Officer (OCIO) (Franz and Kranner, 2019)

The three characteristics of an endowment; size, "know-how", and network, help dictate the university's implementation style. Other factors like management cost and transparency should also be considered when determining implementation styles. In-house management tends to come with a discount on an asset manager's salary in comparison to salaries given by outside investment firms; this style ultimately makes it difficult to keep top asset managers in house when higher salaries are provided elsewhere (Franz and Kranner, 2019).

In turn, in-house management implies high levels of transparency as management is kept within the university, which also suggests full responsibility of the endowment by the university. External management agrees on costs when management is assigned. Since in-house management requires more extensive risk management, entry costs might end up being higher than originally estimated. Given these higher costs associated with in-house management, this reasoning suggests that most universities use an external manger approach (Franz and Kranner, 2019).

<u>How Endowments are Invested</u>

Top leaders in US endowment funds such as Harvard University and Yale University act as good indicators of what dictates strong asset allocation. Although the sizes of their endowments exceed that of the average university, universities with smaller endowments can obtain high levels of risk-adjusted returns for their own portfolios by adopting similar asset allocation principles (Azlen and Zermati, 2017). Typically, university endowments invest 70% of their endowment in traditional asset classes, which are composed of public and private equity, bonds, and cash, and the other 30% in alternative assets; these alternative assets are composed of investments in real estate, commodities, natural resources, and absolute return strategies. For a stronger asset allocation portfolio, following a 55%, 45% split between traditional asset classes and alternative assets respectively allows for additional diversification (Azlen and Zermati, 2017); this diversification is what allows universities like Harvard and Yale to perform successfully in investing long term.

Purchasing Power

While the value of an endowment allows comparisons between universities at one moment of time, purchasing power is a more accurate way to compare the value of a single fund over time. Purchasing power is defined as the amount of goods and services that can be purchased by the value of the fund. Even if the value of the fund increases, if the price of what it buys increases by the same amount, then the purchasing power of the fund will remain the same (Hayes, 2019).

For example, if you make $600 a week and apples cost $10 a bag, then you can buy 60 bags of apples. Now suppose you get a raise to $900 a week, but the price of a bag of apples also increases to $20 a bag. Now you are only able to buy 45 bags of apples, 25% less than you were previously able to buy, even though your salary increased by 50%. In this case, your purchasing power decreased by 15 bags of apples, or 25% of your original purchasing power.

Now suppose that the price of apples increases to $12. You are now able to buy 75 bags of apples, a 25% increase in purchasing power from when you were able to buy 60 bags of apples. However, your salary increased 50%, so even though purchasing power increased, inflation caused that increase in purchasing power to be 25% less than it could have been in an inflation-free world.

The Bureau of Labor Statistics has multiple price indices to measure the overall price level within the economy. The following are some of the many indices tailored to the specific buying habits of a person or business in order to estimate inflation for the consumer:

1. Consumer Price Index
2. Producer Price Index
3. Higher Education Price Index

The two main indices are the Consumer Price Index (CPI), which measures the price level for the average consumer, and the Producer Price Index (PPI), which measures the price level for those that produce goods. The value of these indices is compared to a base year, which is normalized to have a price level of 100. There are other indices tailored to specific industries, such as the Higher Education Price Index (HEPI), which measures the price level associated with the costs of colleges and universities. Furthermore, there are many variations to the CPI and the PPI in order to more accurately measure the price of buying or selling goods, respectively. The PPI has specific indices based on classifications such as commodity and industry level, and the CPI has indices based on whether a person is living on Social Security (Bureau of Labor Statistics, 2019). These price indices are calculated by aggregating the cost of a certain “basket” of goods purchased by the group and then normalizing it to a base year. The actual value of the CPI for a specific can be calculated as follows:

$$CPI(year\ t) = \frac{Market\ Basket\ Cost\ (year\ t)}{Market\ Basket\ Cost\ (Base\ Year)} * 100$$

The inflation rate is the yearly change in prices. In the apple example, when the price of apples increased from $10 to $12, and if those were the prices on two subsequent years, then the inflation rate for that period would be 20%, because the price increased 20% over the span of one year. Using the inflation rate, the purchasing power can be adjusted from year to year to account for changes in price levels. From any given Price index, we can calculate the inflation rate and purchasing power as follows:

$$Inflation\ Rate\ (year\ t) = \frac{PI\ (year\ t+1) - PI(year\ t)}{PI\ (year\ t)},$$

where PI is the price index used.

$$PP\ (year\ t\ +\ 1) = PP\ (year\ t) * \frac{(1-s)*(1+r)}{(1+inflation\ rate\ (year\ t)},$$

where *PP* is the purchasing power of the fund, *r* is the rate of growth of the fund in year x and *s* is the percentage of the fund spent in year x.

Generally, however, the goal is not just calculating changes in purchasing power from year to year, but rather calculating it for a wide range of years and comparing it to a base year. The non-recursive formula for purchasing power is as follows:

$$PP\ (year\ t) = Fund\ Value\ (year\ t) * \frac{PI(base\ year)}{PI(year\ t)}$$

This formula can be used to calculate the purchasing power for all years compared to a base year. Using this formula enables the tracking of the real value of a fund over a long period of time. Even if the fund increases significantly over time, if inflation increases at a faster rate, the real value of the fund will decrease. Furthermore, even though the spending rate is seen only in the recursive formula and not the general formula, the impact of spending is extremely significant over time. As universities spend more, they have less to invest and it is more difficult to retain purchasing power.

Endowment Spending

University endowments are not intended to be a nest egg to be spent all at once; they are meant to support the university in perpetuity. As such, endowment managers must be careful to allocate spending appropriately to preserve the endowment for the future. There are many ways that they can choose a spending rate for the upcoming year based on factors such as market value of the portfolio and inflation. The method which managers use to spend from the endowments are referred to as spending rules, and these rules have a dramatic effect on the purchasing power of the endowment over time (Yaworski, 2019).

There are two main goals when spending from an endowment: maintaining the real value of the portfolio and spending a consistent amount from year to year. Because the returns on an endowment are dependent on the market and therefore are not consistent, these two goals often contradict each other. For example, if the spending rule is spending 5% of the market value of the endowment, but returns are only 2% in the previous year and the inflation rate is also 2% then the endowment would retain only 95% of the purchasing power from the year before. Similarly, if a rule was chosen to preserve purchasing power and spend only the earnings above inflation, then the spending would be extremely inconsistent from year to year due to the inconsistent returns of the market. To ameliorate this conflict, in 1974 Litvack, Malkiel, and Quandt defined three criteria for a spending rule for endowments (Litvack et. al, 1974):

1. The spending rule should be independent of investment decisions.
2. The spending rule should protect the real value of the core of the endowment fund.
3. The spending rule should generate reasonably stable income for the university.

These criteria arose from problems with traditional methods of spending. Under traditional spending rules, the principal of the bonds and the number of securities must be preserved. With these antiquated guidelines, the manager is not allowed to spend anything but the coupons of the bond and the dividends of the stocks regardless of the returns of the portfolio (Litvack et. al, 1974).

If managers developed investment strategy based on traditional spending methods, then they could (and most likely would) invest mainly in coupon bonds in order to maximize the spendable income for the year. Since coupon bonds have a lower average return than other non-coupon financial instruments, the total return of the portfolio would be sub-optimal. All the spendable income would have to be reinvested without spending in order to maintain the real value of the portfolio. If universities invest in non-dividend paying stocks, then they will have no spendable income from that investment, so it is undesirable to do so under traditional rules. Rather than have managers base their investment strategy around spending, their goal should be to maximize the total return of the fund (Litvack et. al, 1974).

Furthermore, spending only the dividends and interest earned on the portfolio is not enough to meet universities' needs. Reinvesting capital gains of the portfolio allows these earnings to be saved for the future instead of spent on university needs now. Generally, this strategy leads to a low amount of spending overall that would not meet the needs of the university. While these traditional rules maintain and grow the real value of the fund, they do not give managers leeway in choosing how much they wish to spend from year to year (Litvack et. al, 1974).

These traditional rules for endowment spending have become obsolete in recent years. **Table 1** shows the usage of spending rules by institutions, filtered by endowments greater than $1B and less than $1B, according to fiscal year 2017 data from NACUBO. According to the data, only 3% of institutions with endowments still use the traditional method of spending. The vast majority, making up 73% of institutions, now use a moving average spending rule to calculate spending, and the remaining institutions mainly use inflation banded methods or a blend between the inflation banded and moving average approaches. However, the popularity of these methods change when looking at institutions with endowments over $1 billion; only 48% of these endowments follow the moving average approach, 12% use an inflation banded approach and 21% use a weighted average of the moving average and inflation banded methods (Franz and Kranner, 2019).

Spending Rules		Percentage of institutions following the rule		
Rule	Description	Average	>$1B	<$1B
Decide Each Year	Spending decided each year	9%	6$	9%
Spend all current income	Current Cash Flows (e.g. dividends and interest)	3%	2%	3%
Inflation Banded	Last year's spending rate plus inflation, with upper and lower bands depending on the endowment value	5%	12%	4%
Moving average	A pre-specified percentage of a moving average of the endowment's market value - usually based on the past three years	73%	48%	76%
Weighted average or hybrid	Combination of, for example, 20% of the inflation rule and 80% of the moving average rule	9%	21%	7%
Average spending rate		4.4%	4.8%	4.4%

Table 1: Types of Spending Rules and Frequency of Use (Franz and Kranner, 2019)

Even though the moving average rule is extremely popular among universities with average endowments, it lacks the ability to retain purchasing power. The moving average rule takes the average market value of the previous three years and multiplies it by a desired spending rate to calculate dollars spent this year. The problem comes when the market value of the portfolio decreases; the dollar amount spent is then a greater percentage of the current market value of the portfolio than anticipated. This phenomenon results in a true spending rate that is negatively correlated with the market. Assume that for four consecutive years the market value of a university's endowment is $100, $110, $120, and $100, respectively, and the spending rate chosen by the university is 5%. Then the moving average rule would suggest that in the fourth year I spend $5.5, which is 5.5% of the market value of the endowment in the fourth year. This level of spending is higher than their desired spending rate of 5%. This example showcases the negative relationship between spending and market returns when using the moving average rule, which takes a toll on purchasing power over time. A study by James Yaworski showed that a portfolio of which 60% were stocks and 40% were bonds lost 12% of its purchasing power between 1997 and 2017, using a desired spending rate of 4.80% (Yaworski, 2019).

The other rules have a similar problem. In the inflation banded rule, the previous year's spending is adjusted by increasing the previous year's spending by a set factor (usually inflation), and if the true spending rate is not within the preset "bands", then spending is simply the desired spending rate multiplied by the market value of the portfolio.

The **simple rule** is a less complicated version of the moving average rule. This rule simply multiplies the market value of the endowment at a given time by the desired spending rate determined by the institution. Although this rule would be easily explainable to trustees, it's lack of dynamic qualities causes it to rarely be used in the industry.

The hybrid rule (by Yaworski's definition) calculates spending as a weighted average of inflation-adjusted spending from the previous year and the target spending rate multiplied by the market value of the endowment. The components of this rule produce spending negatively correlated to the market value of the endowment, and as such they provide more consistent dollar amounts of spending but do not retain the real value of the portfolio. According to an analysis by Yaworski, the hybrid rule only retains 29% of purchasing power after 100 years of typical market returns (Yaworski, 2019).

Yaworski proposes a rule for spending called the purchasing power rule, which is designed so that the true spending rate is correlated with the market value of the portfolio. This results in lower spending in times of economic stress and higher spending in times of prosperity. The issue with this rule is that it leads to inconsistent payments. However, by performing a weighted average of this rule with the hybrid rule, a university can trade some of its purchasing power for less distribution decline. The calculation that takes the weighted average of the purchasing power rule and the hybrid rule is known as the blended rule. The flexibility of this rule allows the university to cater to its own specific needs, however it is unknown whether any universities use this policy to spend their endowments (Yaworski, 2019).

Table 2 shows the calculations of the spending rules mentioned above. The calculations are in terms of *M(t)*, *r*, *s*, *I(t)*, *S(t)*, and *w* that can be defined like so:

M(t)	Market value of endowment at time *t*
r	Desired spending rate as % of market value
s	Actual spending rate at time *t* as % of market value
I(t)	Inflation from year *t* to year *t+1*
S(t)	Spending at time *t*
w	Weights of rules

Spending Rule	Formula
Moving Average Rule	$S(t) = AVERAGE(M(t-1), M(t-2), M(t-3)) * r$
Simple Rule	$S(t) = M(t) * r$
Band Rule	If *r* is within a specified range: $S(t) = S(t-1) * I(t-1)$ Else: $S(t) = M(t) * r$
Hybrid Rule	$S(t) = w1 * S(t-1) + w1 * r * M(t)$
Purchasing Power Rule	$S(t) = M(t) * S * \frac{M(t)}{M(T-1) * I(t-1) + D}$
Blended Rule	$S(t) = w1 * S(purchasing\ power) + w2 * S(hybrid)$

Table 2: Spending Rules and their Formulas

These spending rules all have their advantages and disadvantages in meeting the criteria of a good spending rule, but universities still have yet to find a rule that maximizes the purchasing power of their endowments while maintaining consistent spending power (Yaworski, 2019).

The intent of this project is to identify the most effective spending rule for universities' endowments. By analyzing a rule's effect on purchasing power and spending consistency, this project seeks to find a universal rule that fulfills all the needs of a university without compromise.

Methodology

Understanding Existing Rules

This project focuses specifically on analyzing all existing spending rules to determine if any rules have inherent advantages or disadvantages. Using Microsoft Excel and VBA, we were able to design a spreadsheet that calculated key performance metrics for each rule under various economic scenarios and aggregate results.

<u>Parameters of Existing Rules</u>

Before analyzing how these existing rules performed, we had to initialize the parameters within certain rules' formulas. The most common of these parameters is the target spending rate, which is the percentage of the endowment that the manager wishes to spend each year. The national average for this parameter is 4.8%, but we used 4.7% because that is the target spending rate of Worcester Polytechnic Institute.

Certain spending methods have rule-specific parameters. For example, the formula for spending using the **band rule** is as follows:

If $lower\ bound < \frac{Spending_{t-1} * \frac{CPI_t}{CPI_{t-1}}}{Market\ Value} < upper\ bound$:

$$Spending_t = Spending_{t-1} * \frac{CPI_t}{CPI_{t-1}}$$

Otherwise:

$$Spending_t = Spending_{t-1} * \frac{CPI_t}{CPI_{t-1}}$$

Like the target spending rate, the lower and upper bounds for the spending rate have widely adopted values of 4% and 6% respectively, so those are the values we used in our analysis.

The hybrid and blended rules use a weighted average of different spending methods. The hybrid rule uses a weighted average of the previous year's spending and the simple rule's spending, where the first year's spending is set to the simple rule. To decide how much weight to put on the simple rule for subsequent years, we analyzed three metrics under each value of the weights (using an arbitrary endowment with a market value starting at $1,000):

1. Total dollar amount spent, with an inflation-adjustment
2. Retention of purchasing power
3. Standard deviation of yearly spending, with an inflation-adjustment

Assuming a starting market value of $1,000, **Figure 1** shows the total spent (adjusted for inflation) from a hypothetical endowment under each composition of the hybrid rule. When the weight of the simple rule is 0% (resulting in a weight of previous spending of 100%), the hybrid rule spends the most inflation-adjusted money, but this parametrization leads to a drastic decrease in retained purchasing power. Once the weight of the simple rule surpasses 30%, additional weight of the simple rule results in additional inflation-adjusted spending, which is preferred.

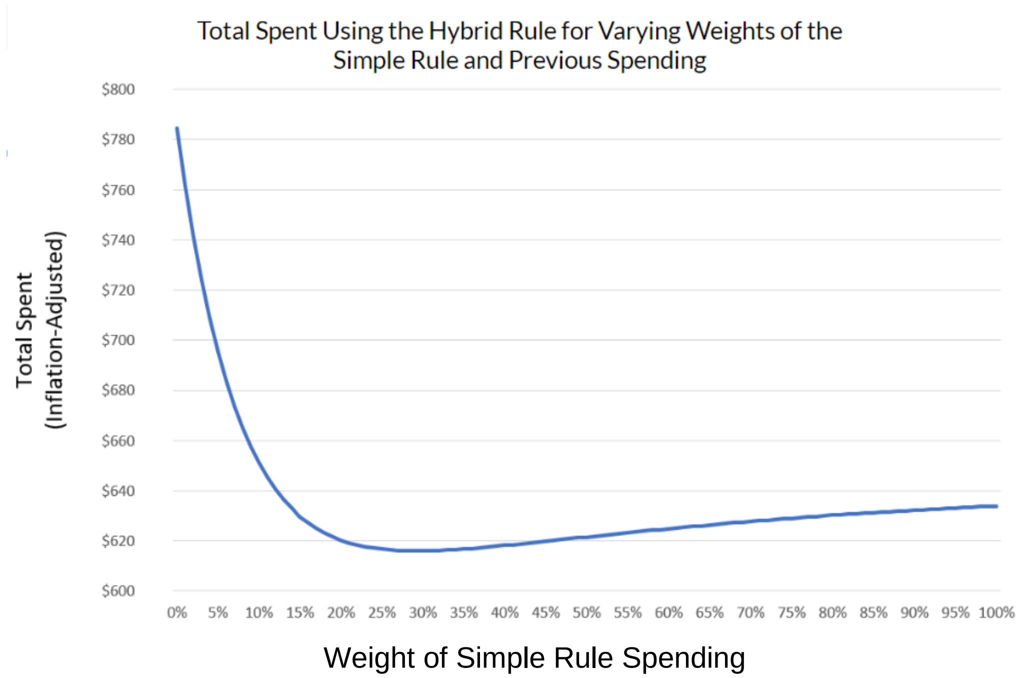

Figure 1

Figure 2 shows the percentage of purchasing power retained under the same economic scenario for every possible composition of the hybrid rule. Purchasing power is suboptimal when the weight of the simple rule is less than 30%. After the weight of the simple rule surpasses 30%, additional weight of the simple rule results in a slight increase in retained purchasing power; therefore, maximizing this parameter is ideal for this metric.

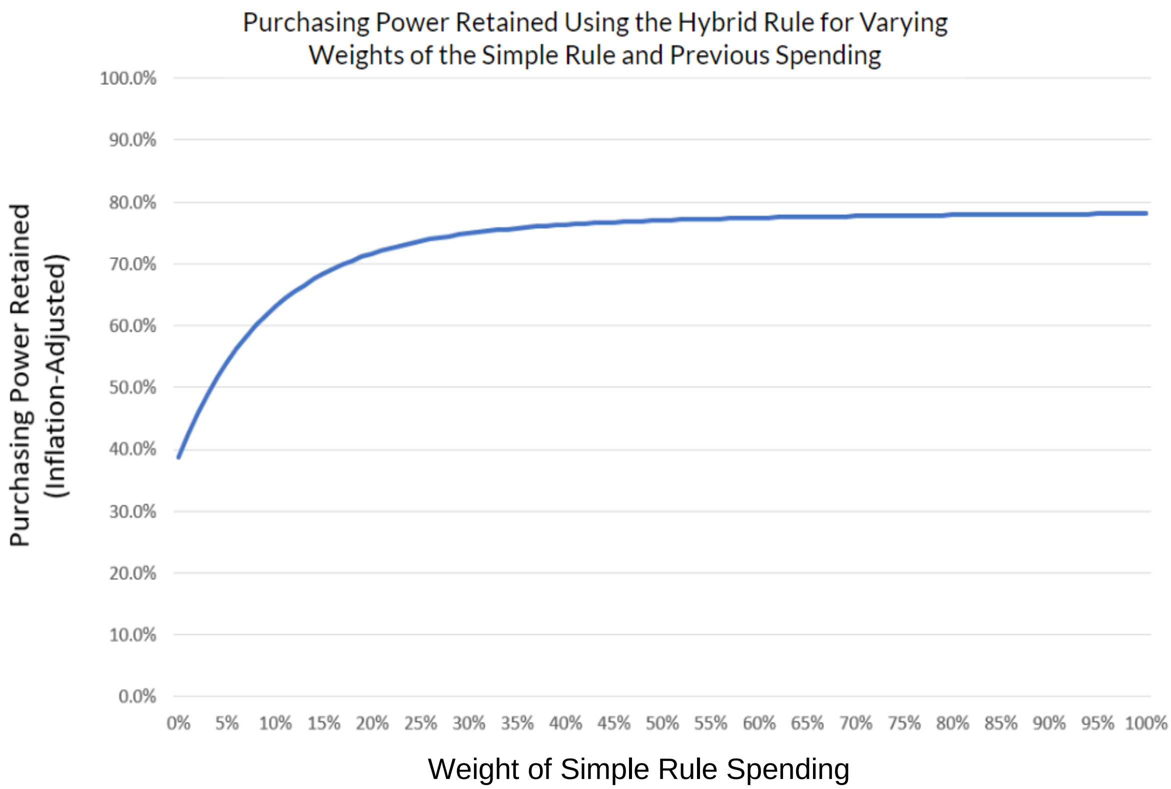

Figure 2

Under the same economic scenario with a starting market value of $1,000, **Figure 3** shows the standard deviation of yearly spending (adjusted for inflation) for every possible composition of the hybrid rule. While the volatility is minimized when the weight of the simple rule is 0%, it should be noted that the low volatility results from spending the same amount each year, an idea that defeats the purpose of creating a spending rule. Therefore, using a weight of the simple rule of approximately 60% minimizes the standard deviation of yearly inflation-adjusted spending.

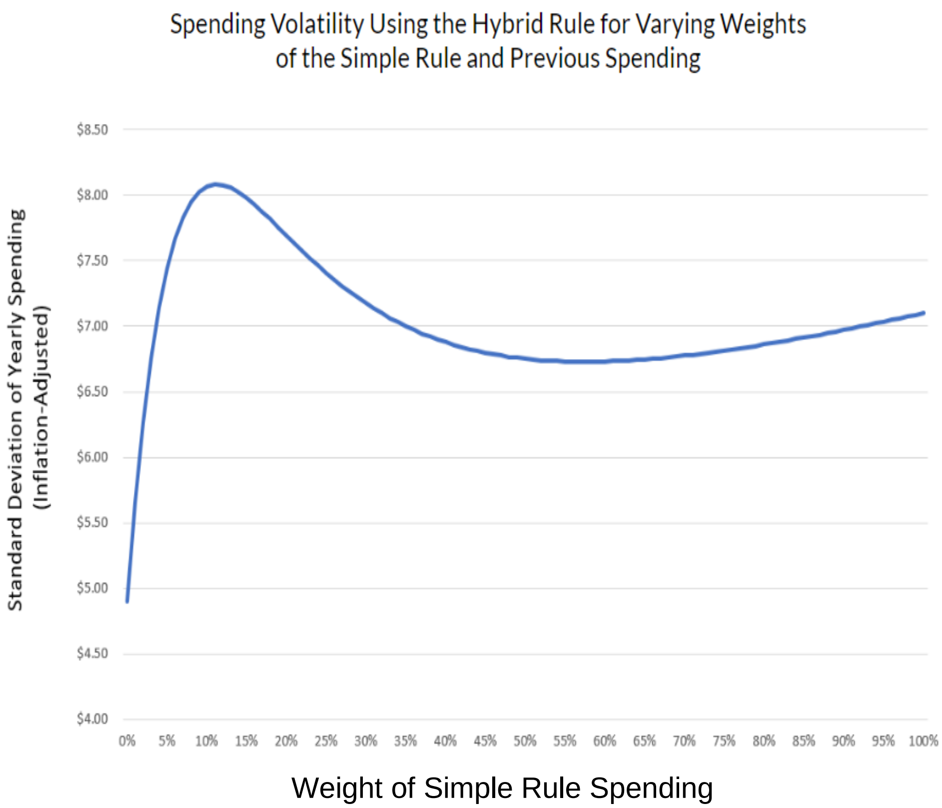

Figure 3

Since the optimal rule spends consistently and retains purchasing power, the optimal weight would maximize the total amount spent and purchasing power retained while minimizing the standard deviation of yearly spending. The data suggests that the standard deviation of spending is minimized at a weight of the simple rule of 63% and there are few additional benefits to increasing that amount within the hybrid rule. Increasing the weight of the simple rule would increase the standard deviation of yearly spending, which would offset those benefits. Therefore, we used a weight of 37% for the simple rule and 63% for the weight of previous spending to compose the hybrid rule for our simulations.

Similarly, the blended rule is a weighted average of the purchasing power rule and the hybrid rule. We used the same process for creating the weights of the blended rule while using the optimized values of the hybrid rule derived above. **Figure 4** shows total spent under the blended rule for each possible composition. As the weight of the purchasing power rule increases, the total inflation-adjusted spending increases as well. The increase in spending occurs because the purchasing power rule spends without consistency to enable more spending while maintaining purchasing power. This metric is therefore optimized for the maximum possible weight of the purchasing power rule.

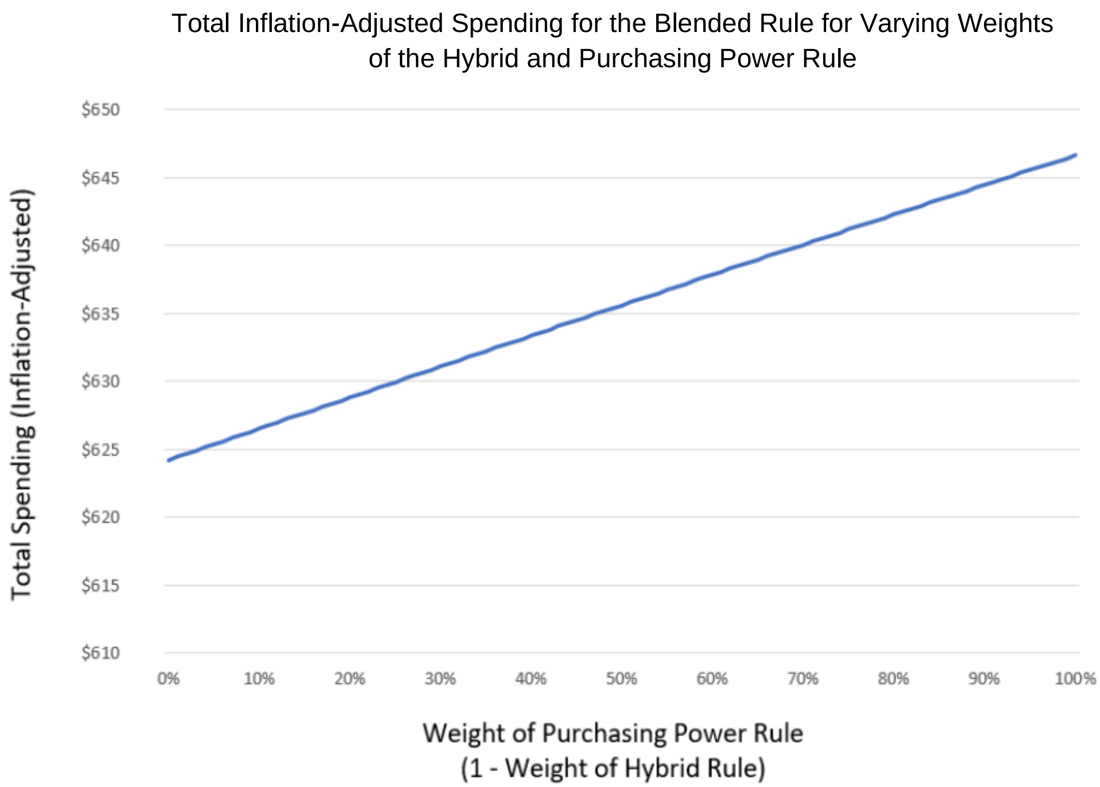

Figure 4

Figure 5 shows the retention of purchasing power using the blended rule. As anticipated, retained purchasing power for the blended rule increases with more weight of the purchasing power rule. Therefore, this metric is also optimized with more weight placed on the purchasing power rule.

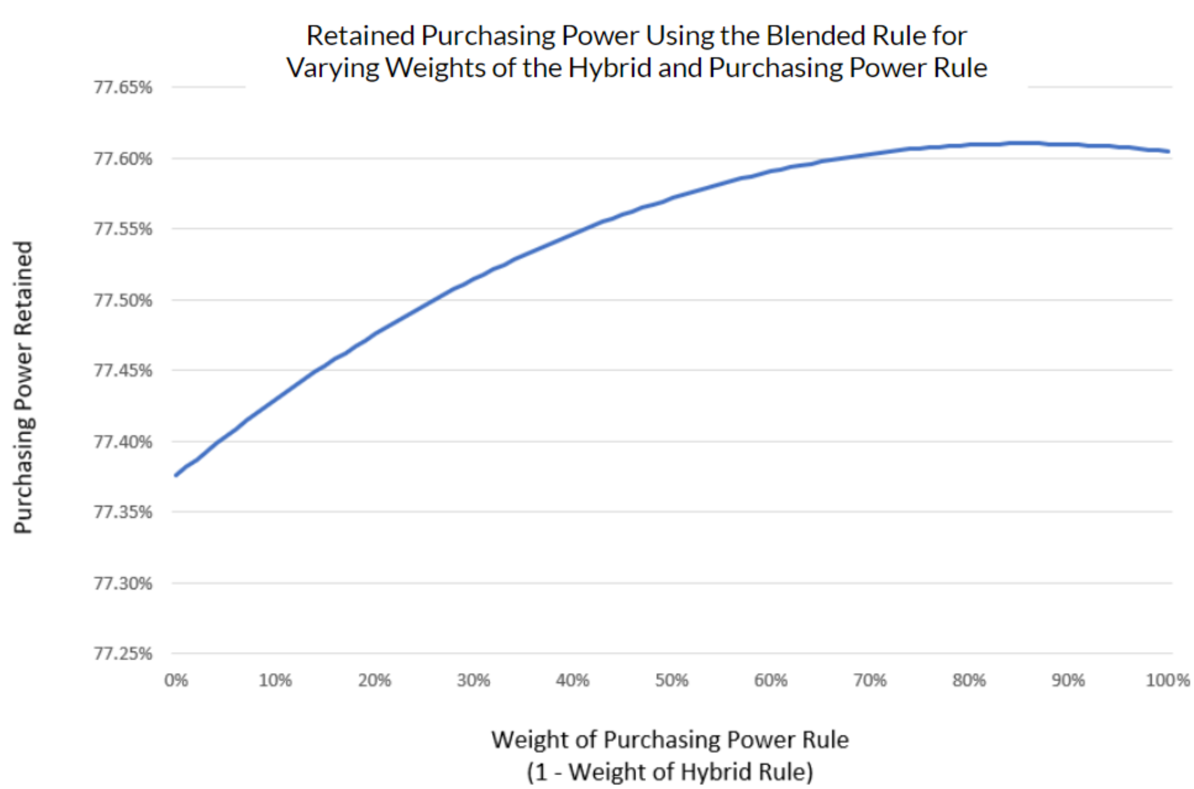

Figure 5

Figure 6 shows the standard deviation of yearly spending under the blended rule. As the weight of the hybrid rule increases, the overall volatility is reduced significantly until the weight of the hybrid rule is about 80%. Unlike the other two metrics, this metric is optimized with less weight of the purchasing power rule.

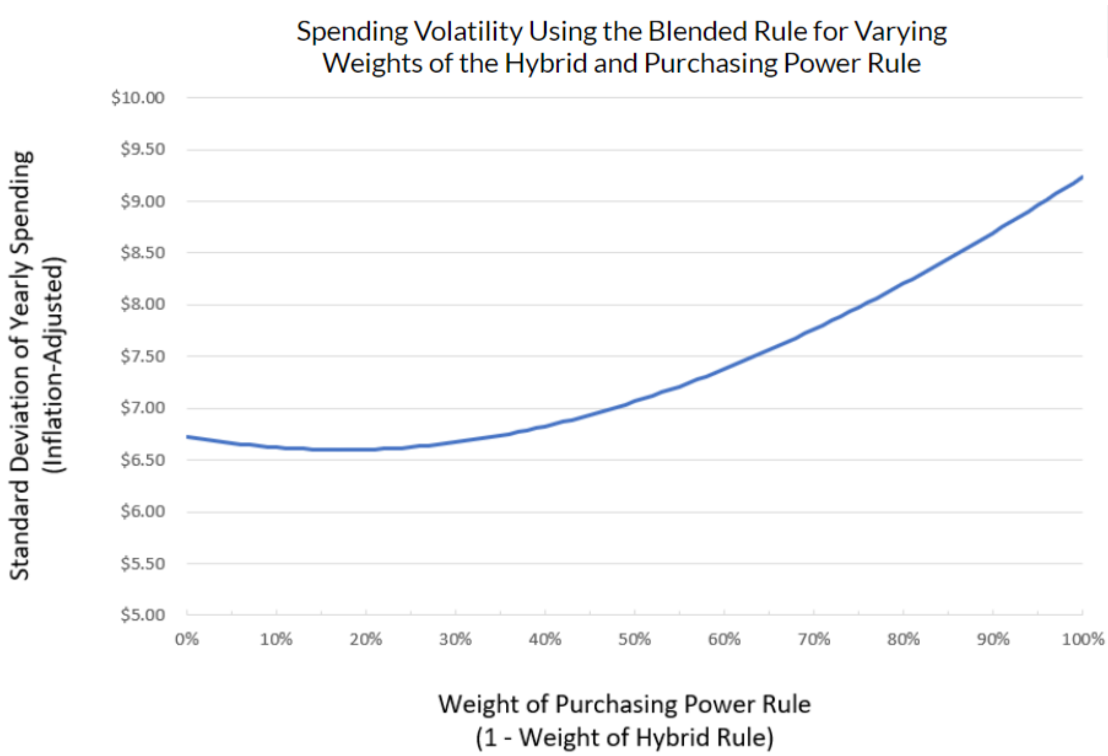

Figure 6

Since an increase in weight of the purchasing power rule resulted in more inflation-adjusted spending and retained purchasing power but also more volatility in spending, we opted to use 50% weight in the hybrid rule and 50% weight in the purchasing power rule to compose the blended rule.

<u>Metrics for Analysis of Spending Rules</u>

In order to compare and analyze the effects of these spending rules on a university's endowment, the following list of metrics were outlined to summarize the performance of these rules in different economic scenarios:

1. Percentage of retention of purchasing power
2. Average yearly change in purchasing power
3. Average yearly change in endowment value
4. Total dollar amount spent in the given time period
5. Average dollar amount spent per year
6. Standard deviation of yearly spending
7. Total dollar amount spent in the given time period with an inflation adjustment
8. Average dollar amount spent per year with an inflation adjustment
9. Standard deviation of yearly spending with an inflation adjustment
10. Coefficient of variation of endowment spending with an inflation adjustment
11. Simplicity of rule

These metrics will be used in the analysis, ranking, and comparison of the spending rules. However, it would be more beneficial to highlight the metrics that are important to endowment managers when choosing a spending rule. For more information on this matter, we met with the Chief Financial Officer (CFO) of WPI, Jeffrey S. Solomon, along with his financial consultant, Jeffrey R. Croteau, from Prime Buchholz, an investment consulting firm. We met with each separately and asked each of them about their priorities in a spending rule. They both relayed the following information:

1. The goal when managing an endowment is to not drastically change the dollar amount of spending from year to year. It is important to have as little volatility as possible.
2. It is important to have a rule that is simple to explain to those important to management decisions, such as the Board of Trustees, to help gain engagement, trust, and understanding.
3. When choosing a spending rule, it is ideal for that spending rule to be able to retain the purchasing power of the endowment.

With the responses from both Jeff Solomon and Jeff Croteau, we decided to highlight the following metrics:

1. the total dollar amount spent over time with an inflation adjustment
2. the percentage of retention of purchasing power
3. the coefficient of variation of endowment spending with an inflation adjustment
4. the simplicity of rule

Inflation adjusted metrics were highlighted in order to better compare the spending rules and their effects on endowment spending.

Metric 1: Total Dollar Amount Spent (with an Inflation Adjustment)

When deciding how much to spend out of the endowment, one important factor for the university and the endowment manager is being able to safely spend the maximum amount given any economic scenario. Most universities want to maximize the dollar amount spent out of the endowment, so this metric aims to highlight this focus. Spending rules will be ranked based on having a higher total dollar amount spent, which has been inflation-adjusted.

Metric 2: Retention of Purchasing Power

Purchasing power is an accurate way to measure the value of the endowment. Endowment managers want to assess how a spending rule is maintaining the endowment's original value throughout market performances and endowment spending over time. Therefore, it is important to highlight the retention of purchasing power as a metric for comparing rules. Endowment managers can observe which spending rule allows for a higher retention of the endowment's purchasing power.

Metric 3: Coefficient of Variation (with Inflation Adjustment)

The coefficient of variation, *CV*, is generally described as a ratio of the standard deviation and the average of a data set, as outlined by the following formula:

$$CV = \frac{\sigma}{\mu} * 100$$

In the context of endowment spending, σ denotes the standard deviation of yearly spending with an inflation adjustment, and μ denotes the average dollar amount spent per year also with an inflation adjustment. Since the coefficient of variation is a statistic used for comparing the degree of variation across a data set, highlighting this metric is useful for endowment managers; it coincides with the goal of minimizing volatility through striving to keep yearly spending consistent. This metric allows us to measure each rule's variation in spending to judge which rules have the lowest volatility in spending.

Metric 4: Simplicity of Rule

When discussing and making decisions on endowment spending, it is crucial for an endowment manager to be able to simply explain the rule behind yearly suggested spending amounts. By doing so, the endowment manager can gain trust from others by knowing how the numbers are being calculated. Understanding the rules allows for better discussion on the amount that should be spent out of the endowment that year. To help quantify the metric of simplicity, we counted the number of operations used in calculating spending for each rule. The spending rules are then ranked such that the spending rule with the fewest calculations is preferable, as fewer operations typically suggests a simpler algorithm and are easier to understand. However, since simplicity can lend itself to being subjective, the other three metrics are weighted more in the mathematical ranking of the rules.

TOPSIS

In order to summarize what spending rules performed the best, we used a statistical analysis method called "Technique for Order Preference by Similarity to Ideal Solution" (TOPSIS). TOPSIS assigns numerical ranks to different alternatives based each alternatives performance in some certain criterion. In our situation, the different "alternatives" were the six existing spending methods. These spending methods were assigned a numerical rank based on how well they retained purchasing power, the total amount spent from the endowment throughout the economic scenario, the minimization of the coefficient of variation, and the simplicity of the spending rule. TOPSIS analysis ranks the spending method with the shortest Euclidean distance from the ideal solution as first. The calculations for the ranks that TOPSIS analysis generates are as follows:

1. Determine a weight for each criterion such that the sum of the weights equal one. An arbitrary weight vector is shown below:

   $w = [w_1, w_2, \ldots, w_n]$such that $w_1 + w_2 + \cdots + w_n = 1$ and $w_n \in \Re$

   Criterion can be cost functions where less is ideal, like the minimization of the coefficient of variation or the simplicity of the rule. Criterion can also be benefit functions where more is ideal, like retention of purchasing power or the total amount spent from the endowment. Each weight is assigned to an entry in the decision matrix. An arbitrary decision matrix can be denoted as $X = (x_{ij})$ such that $x_{ij} \in \Re$.

2. Because every criterion has different properties, formats and units, the decision matrix must be normalized in order to accurately draw conclusions about the data. Three common practices of calculating the normalized value, n_{ij}, such that $i = 1, \ldots, m$ and $j = 1, \ldots,$ n, are shown below:

   a. $n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij}^2)}}$

   b. $n_{ij} = \frac{x_{ij}}{\max x_{ij}}$

   c. Given a benefit criterion, then $n_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}}$

      Given a cost criterion, then $n_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}$

3. The weighted normalized decision matrix is then calculated by multiplying each normalized value with the appropriate weight to produce values v_{ij} like so:

   $v_{ij} = w_j * n_{ij}$ for $i = 1, \ldots, \mathrm{m}$ and $j = 1, \ldots, \mathrm{n}$

   This holds such that w_j is the weight of the j-th criterion and the sum of the w_j vector is 1.

4. Every normalized value is then compared to an ideal solution. The performance of every alternative is compared to a positive and negative ideal solution. The positive ideal solution minimizes the cost criterion and maximizes the benefit criterion, while the negative ideal solution minimizes the benefit criterion and maximizes the cost criterion. The calculations for the positive and negative ideal solutions are shown below as A^+ and A^- respectively:

   $$A^+ = ({v_1}^+, {v_2}^+, \ldots, {v_n}^+) = \left[\left(\max v_{ij} \middle| \, j \in \mathrm{I}\right), \left(\min v_{ij} \middle| \, j \in \mathrm{J}\right)\right]$$

   $$A^- = ({v_1}^-, {v_2}^-, \ldots, {v_n}^-) = \left[\left(\min v_{ij} \middle| \, j \in \mathrm{I}\right), \left(\max v_{ij} \middle| \, j \in \mathrm{J}\right)\right]$$

   such that I and J represent benefit and cost criterion respectively, and $i = 1, \ldots, \mathrm{m}$; $j = 1, \ldots, \mathrm{n}$

5. The Euclidean distance is then calculated for each weighted normalized value, v_{ij}, from the ideal positive and negative solution as ${d_i}^+$ and ${d_i}^-$ respectively:

   $${d_i}^+ = \left(\sum_{j=1}^{n} \left(v_{ij} - {v_j}^+\right)^{\mathrm{p}}\right)^{\frac{1}{\mathrm{p}}}, \text{ such that } i = 1, 2, \ldots, \mathrm{m}$$

   $${d_i}^- = \left(\sum_{j=1}^{n} \left(v_{ij} - {v_j}^-\right)^{\mathrm{p}}\right)^{\frac{1}{\mathrm{p}}}, \text{ such that } i = 1, 2, \ldots, \mathrm{m}$$

6. These Euclidean distances are then compared by their relative closeness, R_i, to the positive ideal solution. R_i can be calculated as:

   $R_i = \frac{{d_i}^-}{{d_i}^- + {d_i}^+}$ such that $0 \leq R_i \leq 1$ and $i = 1, 2, \ldots, \mathrm{m}$

   The relative closeness values, R_i, can then be used to rank the alternatives in descending order.

Creating New Rules

Using techniques used by previously existing rules, we decided to create spending rules that capitalize on the strengths of the strongest rules. In doing so, we hope that our rules can outperform all the previously existing rules.

<u>The Recursive Moving Average Rule</u>

The two rules used most often are the moving average rule and the hybrid rule. In order to capitalize on the strengths of both, we created the recursive moving average rule. Its formula is as follows:

$$Spending_t = 0.37 * Spending_{t-1} + 0.63 * Moving\ Average\ Rule\ Spending$$

This rule is a modification of the hybrid rule in which the simple rule spending is replaced with moving average rule spending. This substitution is intended to reduce volatility in inflation-adjusted spending. Since the moving average rule is an average of the market value of the portfolio from the previous three years, and the spending of the first year is spent based on the moving average rule, the spending can be shown as a function of previous year's market value, causing the weight of each year to decrease as it moves further into the past. **Table 3** shows the decomposition of the recursive moving average rule as a function of the market value of the endowment. The market value of the portfolio in years *t-1* and *t-2* are weighted the most because they are included in both last year's spending and this year's spending. Less of an emphasis is placed on the market value of year *t* and years *t-3*, and market values of earlier years are weighted significantly less because they are not included in the original moving average formula.

Recursive Moving Average Rule Spending Formula
$= 0.63 * 0.05 * \left(\frac{MKT_t}{3} + \frac{MKT_{t-1}}{3} + \frac{MKT_{t-2}}{3}\right) + 0.37 * S_{t-1}$
$= 0.63 * 0.05 * \left(\frac{MKT_t}{3} + \frac{MKT_{t-1}}{3} + \frac{MKT_{t-2}}{3}\right) + 0.37 * \left(0.63 * 0.05 * \left(\frac{MKT_{t-1}}{3} + \frac{MKT_{t-2}}{3} + \frac{MKT_{t-3}}{3}\right) + 0.37 * S_{t-2}\right)$
$= 0.63 * 0.05 * \left(\frac{MKT_t}{3} + \frac{MKT_{t-1}}{3} + \frac{MKT_{t-2}}{3}\right) + 0.37 * \left(0.63 * 0.05 * \left(\frac{MKT_{t-1}}{3} + \frac{MKT_{t-2}}{3} + \frac{MKT_{t-3}}{3}\right) + 0.37 * 0.63 * 0.05 * \left(\frac{MKT_{t-2}}{3} + \frac{MKT_{t-3}}{3} + \frac{MKT_{t-4}}{3}\right) + 0.37 * S_{t-3}\right)$
$= 0.0105 * MKT_t + 0.0144 * MKT_{t-1} + 0.0158 * MKT_{t-2} + \cdots + 0.37^3 * S_{t-3}$

Table 3: The Formula for the Recursive Moving Average Rule Over Time
(using 0.05 as the desired spending rate, S_t as the spending amount at time t and MKT_t as the market value of the portfolio at time t)

Weighted Moving Average Rule

Since the moving average rule is one of the most popular rules used by universities, we wanted to explore whether it was the most rewarding for an endowment manager to equally weight the endowment portfolio values at times, $t, t-1$, and $t-2$. Therefore, we decided to create a version of the moving average rule where these weights can be adjusted. The formula for this rule, which we decided to call the **weighted moving average rule**, is as follows:

$$WMA = w_1 * M(t) + w_2 * M(t-1) + w_3 * M(t-2)$$

Here, w_1, w_2, and w_3 denote the weights chosen for each respective time period, and *M(t)* denotes the market value of the portfolio at the respective time, *t*. These weights were adjusted to maximize the performance of this rule while analyzing the "Endowment Scenario" outlined in the following section.

Analysis of Rules

Economic Scenario Generator (ESG)

In order to analyze the performance of the different spending rules, we needed data to test the rules on. We created a variety of different economic scenarios to simulate how the spending rules would perform in different economic markets. A strategy that we explored was mirroring the S&P 500 to generate endowment returns. After meeting with Jeff Solomon, it was brought to our attention that this scenario was not very representative of the returns that a standard endowment would generate; the S&P 500 is far too volatile. According to Jeff Croteau, an average university endowment's returns are correlated with the S&P 500 returns with $\alpha = 0$ and $\beta = 0.6$, so we modified the S&P 500 returns by multiplying each year's return by 0.6 to produce more representative data. The "Endowment Scenario" was created using these methods to replicate the returns of an endowment.

To explore various market trends, we created five additional economic scenarios that we felt represented various economic conditions:

1. Consistently increasing market
2. Market with no change
3. Bull market
4. Bear market
5. Uncertain market

The constantly increasing market scenario modeled a market with 6% returns, the average return for the WPI endowment according to Jeff Solomon. We also created a scenario called "market with no change" that exhibited no returns to model an endowment invested entirely in cash. The bull and bear market scenarios were created using uniformly generated returns between 8% to 13% and -1% to -6%, respectively, to simulate both desirable and catastrophic economic scenarios. Lastly, the uncertain market scenario uniformly generated returns from -2.5% to 2.5% to model an economic market with no growth trend.

In addition to these economic scenarios, we also tested the spending rules against one thousand economic scenarios that were randomly generated from an economic scenario generator. The economic scenario generator produced returns were normally distributed with a mean of 12% and standard deviation of 18%, designed to replicate the S&P 500. We then modified the data by multiplying each return by the advised 0.6 to reduce volatility and used these returns to test the different spending methods.

<u>Comparing the Rules</u>

Using these economic scenarios, metrics for rule performance, and TOPSIS analysis, we wrote a macro in VBA that aggregated all the data from using all rules for each of the 1,000 economic scenarios. The aggregated results of the scenarios allowed us to see which rules were stronger than others, so we took the top four performing rules and pitted them against each other, once again running the macro to compare these four rules in each of the economic scenarios to see which rule reigned supreme.

Results and Recommendations

Results

Comparison of Moving Average and Weighted Moving Average Rules

Before ranking all rules together, it is worth comparing the performances of the moving average rule and the weighted moving average rule to see if adjusting the weights is significant. The key difference between the weighted moving average rule and the more common moving average rule is the value of the weights placed for the endowment portfolio values. For the typical moving average rule, each of the portfolio values at *t*, *t-1*, and *t-2* is weighted equally at about 0.33. In order to optimize these weights, we decided to compare the TOPSIS rankings between these two rules while adjusting the weights of the weighted moving average rule. We focused on using scenarios where the moving average rule performed to its highest and lowest rankings, allowing us to test the consistency of specific weights in a more simplified manner. Through testing, we were able to compute these weights to optimize the performance of the weighted moving average rule. These weights can be found in **Table 4**.

Portfolio Values at Respective Years	Weights of Values
t	0.34
t-1	0.32
t-2	0.34

Table 4: Optimized Weights of the Weighted Moving Average Rule

To further test the weighted moving average rule against the moving average rule, we ran these rules through the thousand random scenarios. We analyzed their rankings by looking at their average ranking, standard deviation of their rankings, and their best and worst rankings. **Table 5** shows the data while considering simplicity:

	Moving Average	Weighted Moving Average
Average Rank	2.601	4.386
Standard Deviation of Rank	0.581	0.513
Best Ranking	1	3
Worst Ranking	4	5

Table 5: Ranking Results of Moving Average and Weighted Moving Average while Considering Simplicity

Considering simplicity, the weighted moving average rule adds another operation to the moving average rule through changing the values of the weights. Therefore, based on the number of operations alone, the extra calculation makes the weighted moving average rule slightly more complicated than the moving average rule. The extra complication shows through the fact that the moving average rule tends to perform better on average with a ranking of 2.601 in comparison to the weighted average rule's average ranking of 4.386 in the simplicity model. In addition, the weighted moving average rule only placed third as its best ranking in comparison to the moving average rule ranking first; similar results are seen with each rule's worst ranking as the weighted moving average rule has ranked lower than the moving average rule at some point in the simulation.

Given the subjective nature that comes with judging simplicity, we also collected the data without considering the simplicity of these rules. The results highlight each rule's true capability in these scenarios. **Table 6** shows the data without factoring in simplicity:

	Moving Average	Weighted Moving Average
Average Rank	4.377	3.656
Standard Deviation of Rank	0.912	1.025
Best Ranking	2	1
Worst Ranking	7	7

Table 6: Ranking Results of Moving Average and Weighted Moving Average without Considering in Simplicity

Based on the formulas of the rules alone, the weighted moving average rule tends to perform slightly better than the moving average rule, with average rankings of 3.656 and 4.377 respectively. Similar standard deviations of the rank, 0.912 and 1.025 for the moving average rule and weighted moving average rule respectively, indicate that each rule ranks on average at the same consistency. Therefore, it can be observed that the weighted moving average rule tends to perform better against the moving average rule in most cases.

However, considering the tradeoff of a slightly more complicated rule for insignificantly better results, the moving average rule is just as reliable as the weighted moving average. Therefore, the current weights of the moving average rule of 0.33 for each *t* value are reliable for the performance of the rule.

Rankings

Using the methods outlined in the methodology, we averaged the TOPSIS ranks for each of the 1,000 scenarios for all rules. The following are the rankings for the rules with equal weight placed on total inflation-adjusted spending, retention of purchasing power, and coefficient of variation of inflation-adjusted spending:

1. Recursive moving average rule
2. Blended rule
3. Weighted moving average rule
4. Hybrid rule
5. Moving average rule
6. Purchasing power rule
7. Simple rule
8. Band rule

While the hybrid rule and the moving average rule beat the simple rule and the band rule, two rules that are likely unknown to institutions, the recursive moving average rule and the blended rule, took the first two rankings. In these 50-year economic scenarios, the recursive moving average rule retained 48.5% of its purchasing power with our parameters, while also maintaining the lowest coefficient of variation of spending. The blended rule spent 0.5% more on average, with an adjustment for inflation, over the lifetime of the simulation, but it retained 5% less purchasing power over that same period. Similarly, the hybrid rule spent 0.03% more inflation-adjusted money than the recursive moving average rule and retained 3% less purchasing power on average. The super moving average rule can trade a minimal disadvantage for a large advantage.

When weighting the simplicity factor as 7% of the total TOPSIS score and the three metrics used above as 31%, the rankings were:

1. Simple rule
2. Hybrid rule
3. Moving average rule
4. Weighted moving average rule
5. Recursive moving average rule
6. Purchasing power rule
7. Band rule
8. Blended rule

With simplicity being weighted, the simple rule is ranked first by this model, but the simple rule's performance is not the strongest in any of the financial metrics. In the endowment scenario, using the simple rule would result in less overall spending and retained purchasing power than the blended rule, which placed last in the simplicity model's rankings. In addition, use of the simple rule would result in a greater coefficient of variation in spending than the blended rule, suggesting that the first-place rule was worse in all three financial metrics than the last-place rule. Most notably, the band rule is ranked more favorably than the blended rule even though the band rule has the worst performance in all financial metrics, and it is the second most complicated rule. Furthermore, the rankings generated from the model using simplicity are the same as if the model only used simplicity as a metric, with the only difference being that the moving average rule ranks higher than the weighted moving average rule. This model essentially ignores the spending rules' financial performance to a large degree. These rankings are biased because while there is variance in the financial performance of each of the rules, there is no variance in the simplicity of each rule; the formula is preset, so simplicity will always work in favor of the simpler rule, regardless of economic scenario. Since the intention of this model was to give more sophisticated rankings to rules with similar financial metrics and not to completely redo the rankings, these biased results suggest that we should use the model ignoring simplicity to compare the performance of the rules.

We selected recursive moving average rule, blended rule, hybrid rule, and moving average rule for the analysis of the top four rules. Using the number of first place rankings a rule achieved in the TOPSIS model without simplicity, we observed the following final rankings:

1. Recursive moving average rule (best rule in 62% of scenarios)
2. Blended rule (best rule in 34% of scenarios)
3. Hybrid rule (best rule in 3% of scenarios)
4. Moving average rule (best rule in 1% of scenarios)

Spending Rule Performance

We utilized the one thousand, 50-year economic scenarios produced from the economic scenario generator to test the success of each spending rule. The summary statistics chosen to quantify the performance of each spending rule and the results are shown below in **Table 7**.

	Moving Average Rule	Simple Rule	Band Rule	Purchasing Power Rule	Hybrid Rule	Blended Rule	Recursive Moving Average Rule
Retention of Purchasing Power	43%	41%	37%	41%	44%	43%	47%
Total Spent (Inflation-Adjusted)	$1,177	$1,178	$1,174	$1,185	$1,175	$1,181	$1,174
Coefficient of Variation of Yearly Spending (Inflation-Adjusted)	47%	49%	53%	49%	48%	47%	46%
Simplicity	4	1	6.5	6	4	13	6

Table 7: Spending Rule Performance

The summary statistics suggest that the recursive moving average rule retains purchasing power the most efficiently and has the lowest coefficient of variation. It is important to note that due to the omission of donations, no rule can effectively maintain purchasing power in scenarios where there is significant volatility. However, we are still able to compare the rules with each other, even though this great of a reduction of purchasing power would not happen under active management. The conjunction of these two metrics and the rule's satisfactory performance in other categories makes the recursive moving average rule a standout rule. The purchasing power rule and simple rule were top performers in terms of maximization of the total amount spent from the endowment and the simplicity of the spending rule respectively. Although these rules were top performers in their respective categories, their performance in other categories were only average. The moving average rule, hybrid rule, and blended rule could all be considered middle tier rules based on their overall average performance as well. Lastly, the band rule significantly underperformed as it retained the least purchasing power, spent the least amount of money, and had the largest coefficient of variation.

Recommendations

The spending rule that performed the best on average in the random scenarios is the recursive moving average rule, closely followed by the blended rule. The more commonly used rules such as the hybrid rule and the moving average rules rank as third and fourth most preferable, respectively.

While our study was able to closely replicate and expand upon the work of James Yaworski and others, there are numerous external factors that could affect which rule should be picked.

<u>Simplicity</u>

While we omitted simplicity in the mathematical analysis of the spending rules, simplicity should not be ignored entirely. We ranked each rule based on the average number of mathematical operations needed to calculate the spending for each year. **Table 8** shows each rule and the simplicity factor used for each rule:

Rule	**Average # of Operations**
Simple Rule	1
Moving Average Rule	4
Hybrid Rule	4
Weighted Moving Average Rule	5
Recursive Moving Average Rule	6
Purchasing Power Rule	6
Band Rule	6.5
Blended Rule	13

Table 8: Number of Operations Used in the Formulas of Each Spending Rule

The board of directors is more willing to selecting a spending rule if it is easy to understand how they are deciding to spend. Clearly, the blended rule is much more complex than any other rule, with at least six more operations than any other rule. While the benefits to using this rule over the moving average rule are great, explaining both the hybrid rule and purchasing power rule, then how they are mixed, may be too complex for many boards. While the recursive moving average rule performs extremely well, it may be difficult to convince a board to use this rule simply because it requires understanding of both the hybrid rule and the moving average rule. While it is not terribly complicated, it still takes some time to process why the rule has distinct advantages over the other rules, and if the rule they are currently using has performed well for them, it will be even more difficult to make the change. Even if a rule has a clear advantage over the rule currently in use, it may still be difficult to convince enough people to use it.

Performance Consistency

When looking at the rankings of each of these spending rules, it is important to also consider the consistency at which these rules perform at their average ranking. After running the economic scenarios both while considering and disregarding simplicity, we collected results on the rules' rankings. **Table 9** shows the results when disregarding the simplicity of the rules in the TOPSIS ranking:

	Moving Average Rule	Simple Rule	Band Rule	Purchasing Power Rule	Hybrid Rule	Blended Rule	Recursive Moving Average Rule	Weighted Moving Average Rule
Average Rank	4.377	5.675	7.720	4.504	3.826	3.380	2.862	3.656
Standard Deviation of Rank	0.912	1.551	0.911	2.729	1.999	1.516	2.505	1.025
Best Ranking	2	1	1	1	1	1	1	1
Worst Ranking	7	8	8	8	8	7	8	7

Table 9: Ranking Results of Spending Rules Disregarding Simplicity

On average, the top five performing rules while disregarding simplicity are, in order:

1. Recursive Moving Average Rule (2.862)
2. Blended Rule (3.380)
3. Weighted Moving Average Rule (3.656)
4. Hybrid Rule (3.826)
5. Moving Average Rule (4.377)

Although the average rankings of these rules are important to consider, it is also important to consider the consistency of these rules' performance. A rule's consistency of rank can be measured by the standard deviation of the rules' ranks, where a lower standard deviation indicates more consistent ranking within the 1,000 scenarios. The following rank from lowest to highest standard deviation of rank for the best performing rules:

1. Moving Average Rule (0.912)
2. Weighted Moving Average Rule (0.1.025)
3. Blended Rule (1.516)
4. Hybrid Rule (1.999)
5. Recursive Moving Average Rule (2.505)

Although the recursive moving average rule overall ranks the best on average, if an endowment manager is concerned more about which rule consistently can perform the best in a variety of economic scenarios, they may prefer using either the blended rule or the hybrid rule. However, considering the blended rule is significantly more complicated than many of the spending rules, they may decide that the hybrid rule or a version of the moving average rule is preferable.

Validity of Economic Scenarios

The economic scenario generator used in this study rendered returns from a normal distribution with a mean of 12% and a standard deviation of 18%. We then multiplied these returns by 0.6 to more closely model the returns from an endowment. Scaling the distribution of returns results in a new normal distribution with a mean of 7.2% and a standard deviation of 10.8%. While the returns are designed to replicate the returns from the S&P 500 index, they may not accurately replicate future returns. In some scenarios, there were long periods of no economic growth, lasting up to 10 years. Realistically, the government would likely take action to ensure that the economy continues to grow. While these scenarios are good for our purposes, we recognize that these scenarios are most likely not the most accurate way to simulate 50-year scenarios from the S&P 500.

Conclusion

Endowments are vital assets to universities and other institutional investors. Endowment managers are faced with the task of preserving the purchasing power of the endowment, while also spending consistent amounts from year to year. Existing spending methods have different strengths among the metrics we analyzed, and the "ideal" spending method for a given university is up to the discretion of the endowment manager. Currently the moving average rule is the most commonly accepted practice among universities, however our analysis suggests that the recursive moving average rule is the best rule for retaining purchasing power and minimizing spending variance, with very little impact to the total amount spent. Furthermore, it is far easier to understand than other rules with similar advantages. Even though implementing a new spending method is an involved process up to university discretion, we suggest the use of the recursive moving average rule to endowment managers.

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